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Sphaleron/Instanton Induced Processes

Kazuki Sakurai

(University of Warsaw)

17/10/2022, Grieg conference @ Warsaw

Q. What is sphaleron/instanton induced processes?

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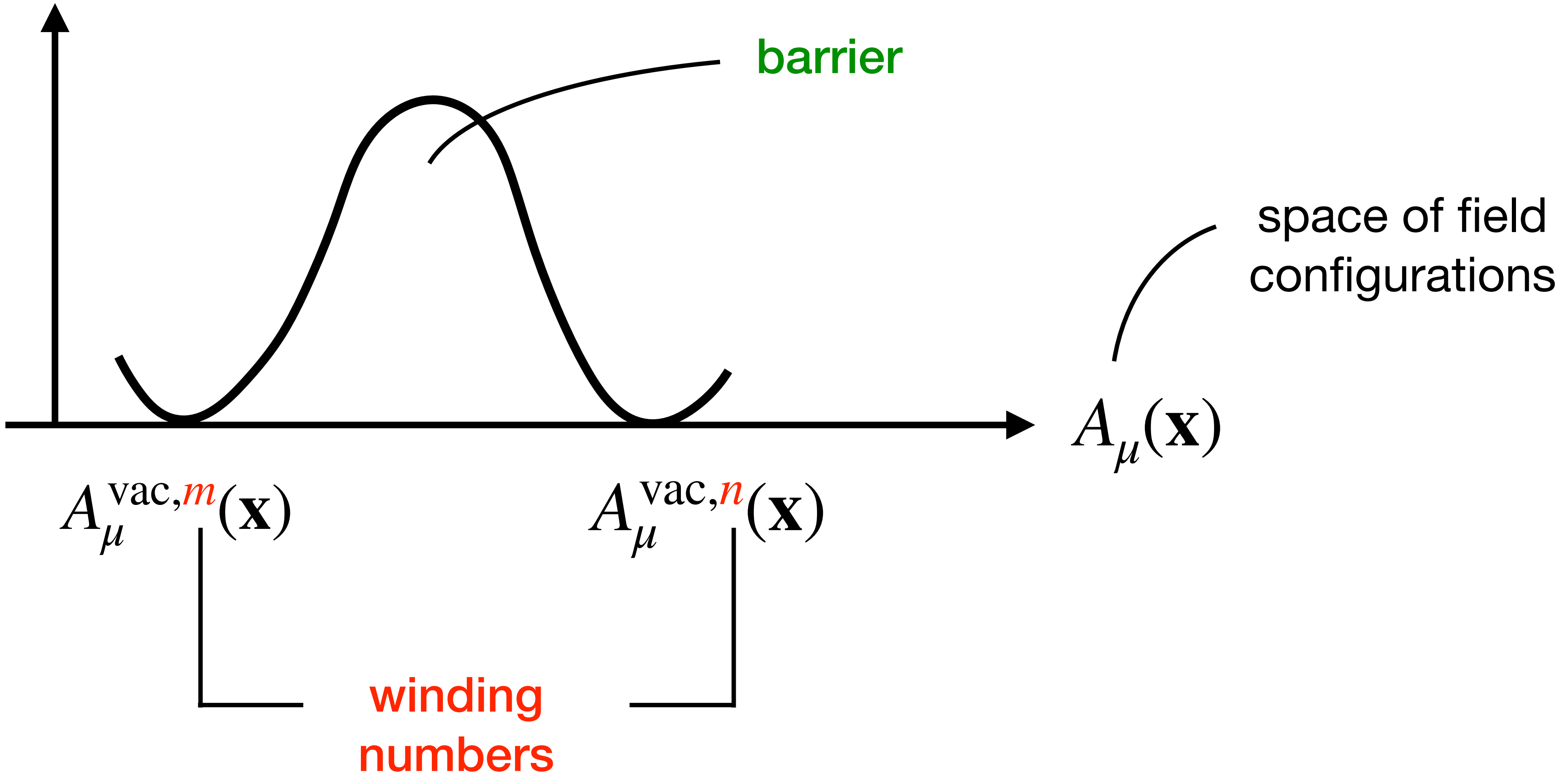
total energy of the configuration

energy density

$$E(\{A_\mu(\mathbf{x})\}) = \int d^3\mathbf{x}\, \mathscr{E}(A_\mu(\mathbf{x}))$$



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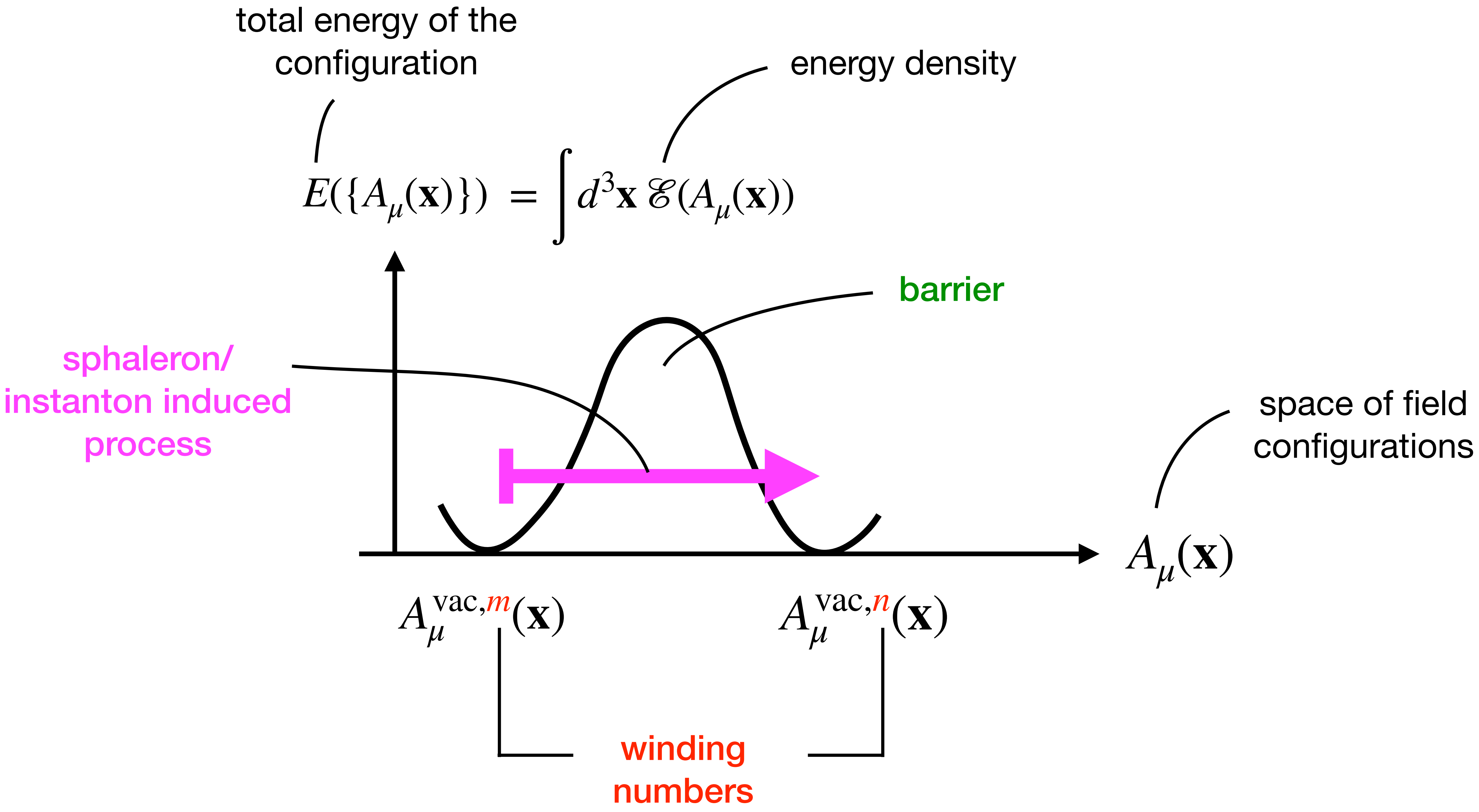

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$$E(\{A_\mu(\mathbf{x})\}) = \int d^3\mathbf{x}\, \mathscr{E}(A_\mu(\mathbf{x}))$$

barrier

sphaleron/ instanton induced process

space of field configurations

$A_\mu(\mathbf{x})$

$A_\mu^{\mathrm{vac},m}(\mathbf{x})$

$A_\mu^{\mathrm{vac},n}(\mathbf{x})$

winding numbers

- Yan-Mils Lagrangian (SU(2)):

energy density

$$\mathscr{L}_{\text{YM}} = -\frac{1}{2}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] \quad \Longrightarrow \quad \mathscr{E}(A_\mu(\mathbf{x})) = \frac{1}{2}\mathrm{Tr}\left[F_{ij}F_{ij}\right] \geq 0$$

$$\boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \qquad A_\mu = A_\mu^a T^a \qquad T^a : \text{generator of SU(2)}}$$

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- $\mathscr{L}_{\mathrm{YM}}$ and $\mathscr{E}(A_\mu(\mathbf{x}))$ are invariant under the gauge transformation:

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Q. What is the lowest energy (vacuum) field configuration?

A. Obviously $A_\mu(\mathbf{x}) = 0$ gives $\mathscr{E} = 0$ (vacuum).

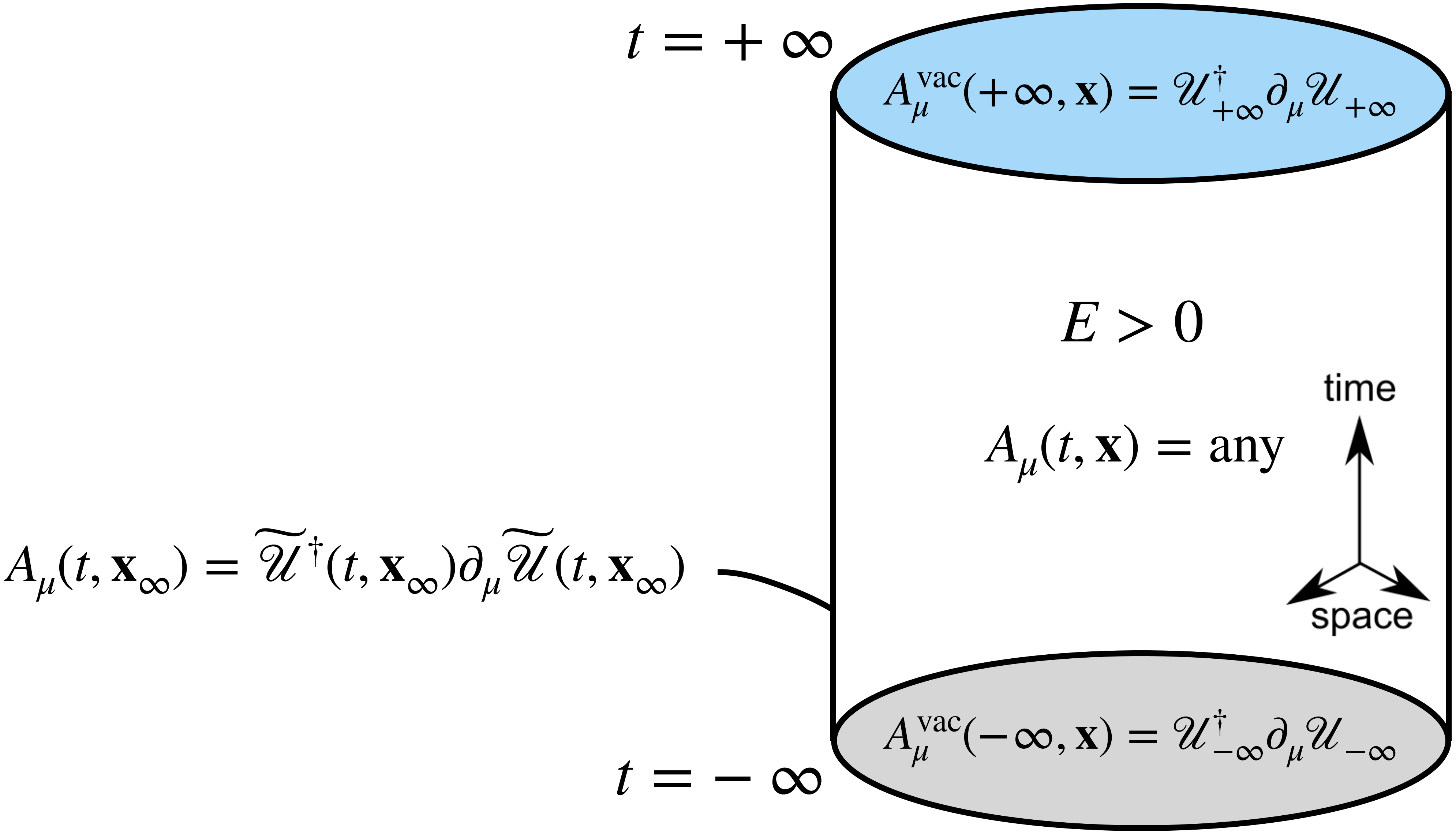
But due to the gauge symmetry, any configuration that is gauge transformed from $A_\mu(\mathbf{x}) = 0$ also gives $\mathscr{E} = 0$ (vacuum)

$$A_\mu^{\text{vac}}(\mathbf{x}) \equiv -\frac{i}{g}\mathscr{U}^\dagger(\mathbf{x})[\partial_\mu \mathscr{U}(\mathbf{x})]$$

$$\boxed{\mathscr{U}(\mathbf{x}) = e^{i\omega^a(\mathbf{x})T^a}}$$

(pure gauge configuration)

- Consider a transition between two vacua, which costs only a **finite energy**.
- During the transition, the energy density must vanish at $\mathbf{x} \to \mathbf{x}_\infty$



$t = +\infty$

$A_\mu^{\text{vac}}(+\infty, \mathbf{x}) = \mathcal{U}^\dagger_{+\infty}\partial_\mu \mathcal{U}_{+\infty}$

$E > 0$

$A_\mu(t, \mathbf{x}) = \text{any}$

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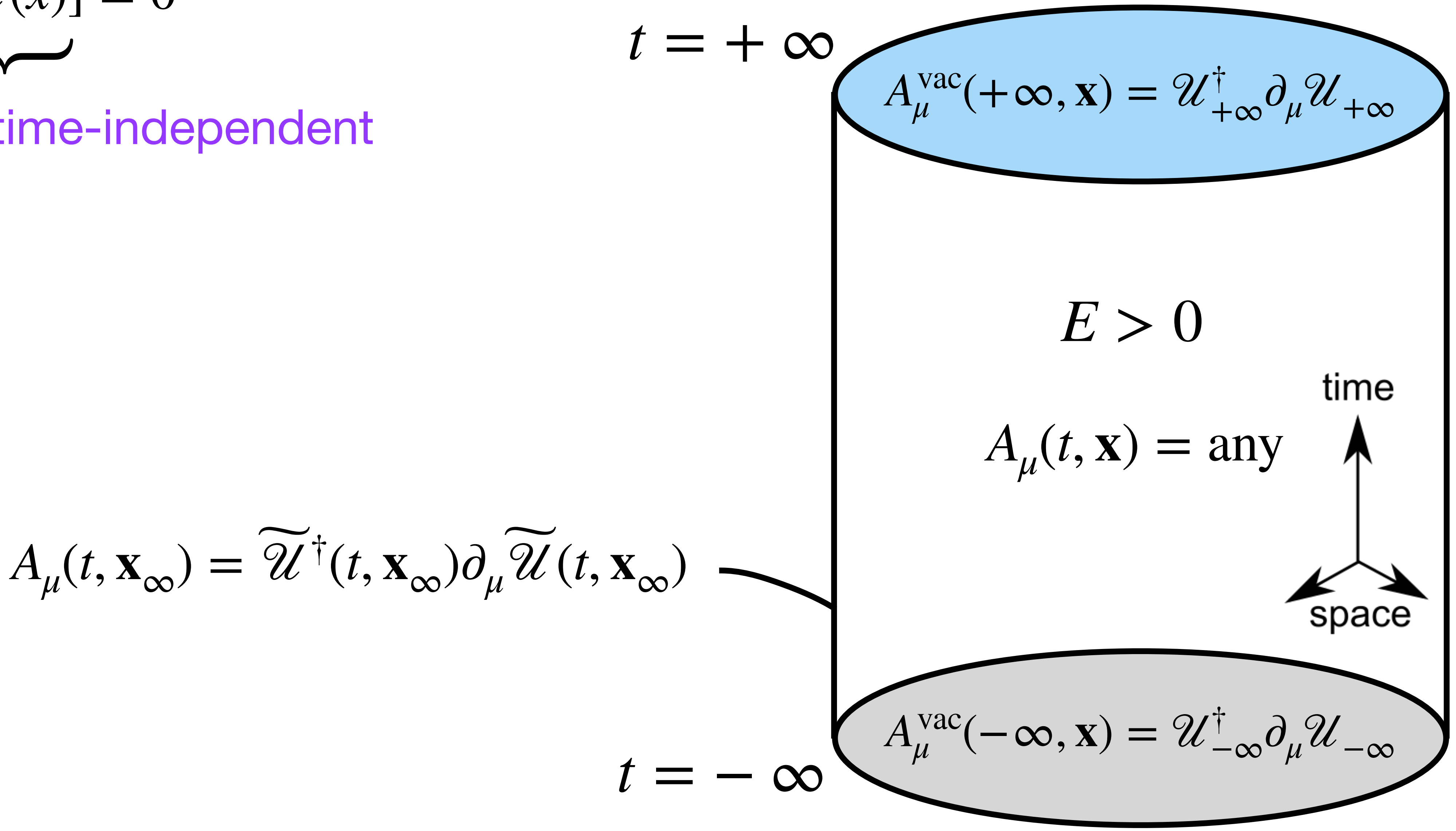
$$A_\mu(x) \to U^\dagger(x) A_\mu(x) U(x) - \frac{i}{g} U^\dagger(x)[\partial_\mu U(x)]$$

1) $A_0(t, \mathbf{x}) = 0$ (temporal gauge)

$$\Downarrow$$

$$A_0^{\mathrm{pure}}(x) = \mathcal{U}^\dagger(x)[\underbrace{\partial_0 \mathcal{U}(x)]}_{\mathcal{U}(x)\text{ is time-independent}} = 0$$



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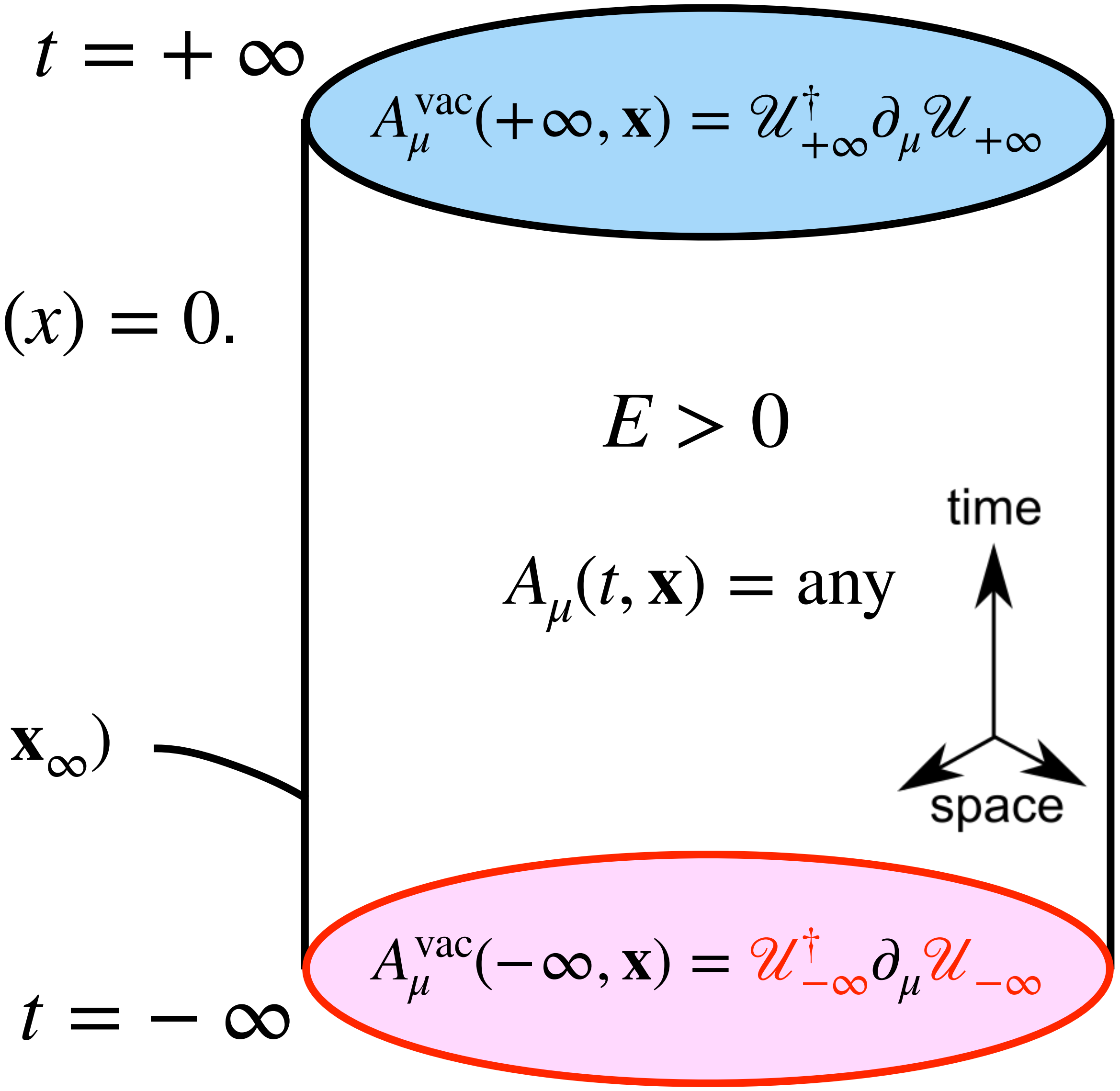
$\mathcal{U}(x)$ is time-independent

2) time-independent gauge trans doesn't spoil $A_0(x) = 0$.

Using this freedom we fix $\mathcal{U}_{-\infty}(\mathbf{x}) = \mathbf{1}$.



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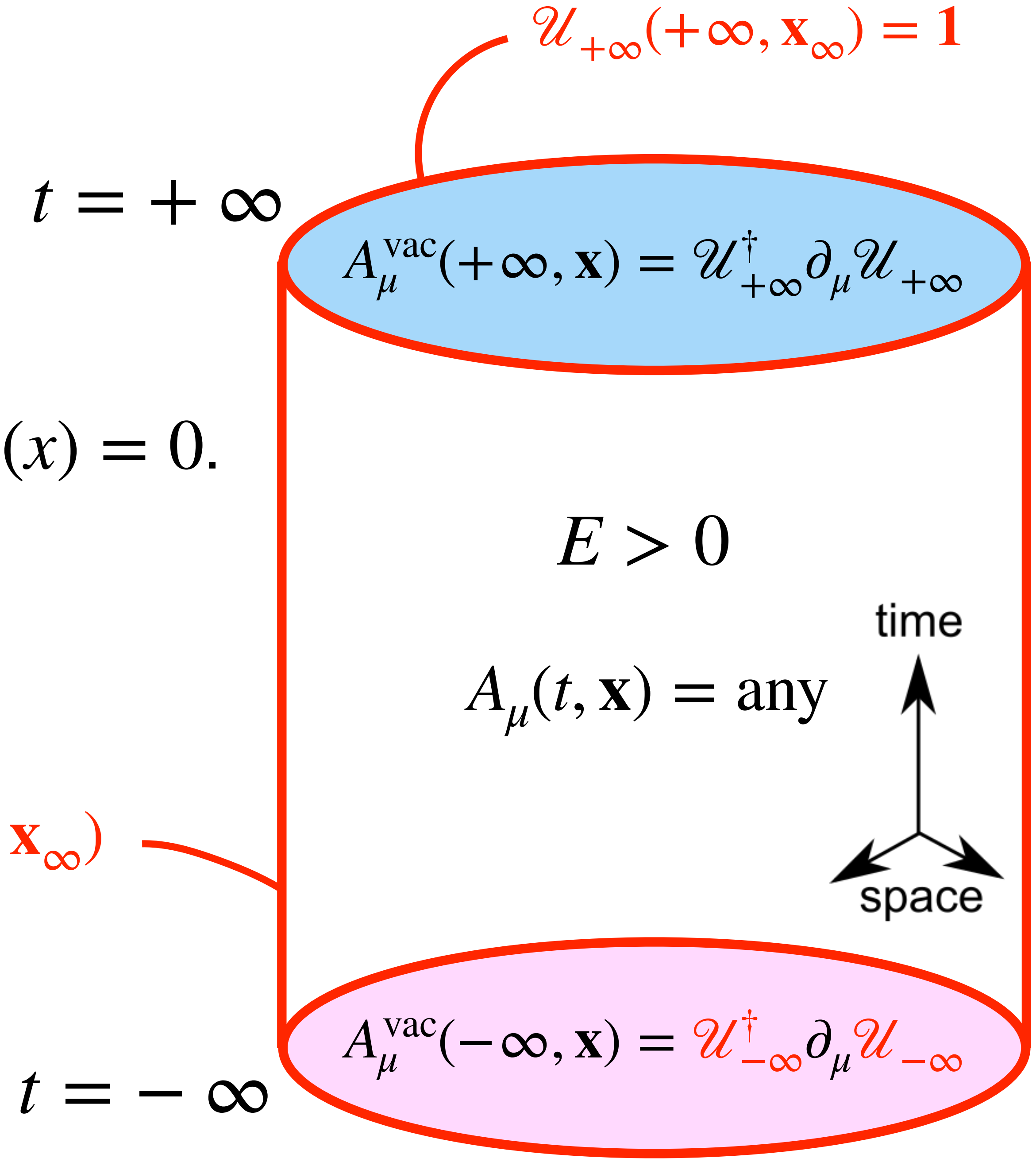

$\mathcal{U}_{+\infty}(+\infty, \mathbf{x}_\infty) = \mathbf{1}$

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What field configurations are allowed in the final state vacuum?

- final state vacuum is characterised by $\mathscr{U}_{+\infty}(\mathbf{x})$ with the condition $\mathscr{U}_{+\infty}(\mathbf{x}_\infty) = \mathbf{1}$
- $\mathscr{U}_{+\infty}(\mathbf{x})$ is a rule to assign a matrix $\mathscr{U} \in \mathrm{SU}(2)$ at every space point $\mathbf{x} \in \mathbb{R}^3$

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  $\Rightarrow$ all points at spacial infinity can be identified to be the same point.

base space: $\mathbb{R}^3 \to \mathbb{S}^3$

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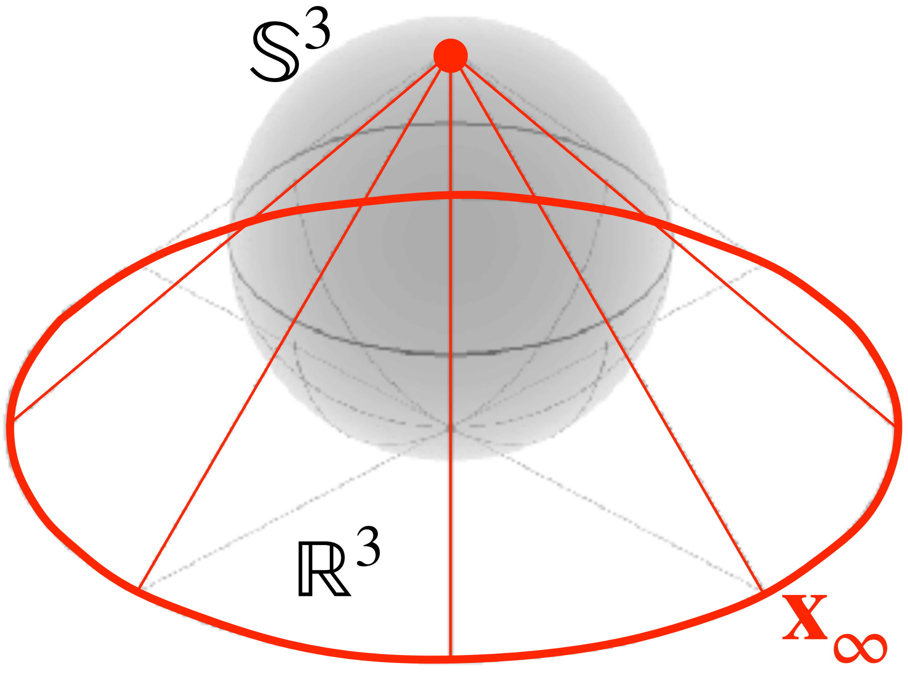

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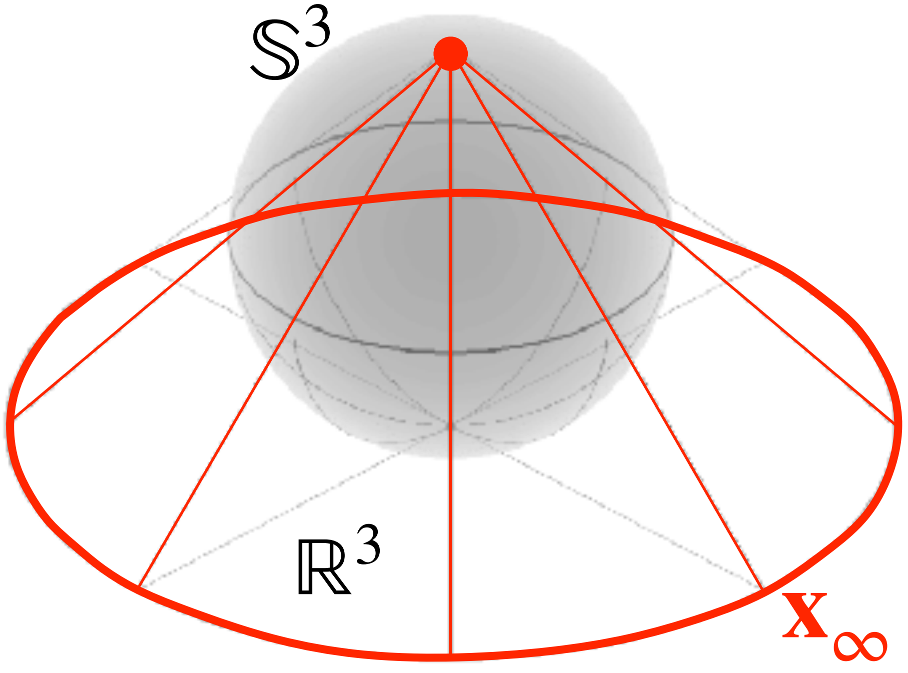
- SU(2) matrix can be written as:

$$\mathscr{U} = \begin{bmatrix} a & -b^* \\ b & a^* \end{bmatrix} \qquad a, b \in \mathbb{C} \qquad \underbrace{|a|^2 + |b|^2 = 1}_{\text{target space: } \mathrm{SU}(2) \cong \mathbb{S}^3}$$

- The vacuum configuration is equivalent to

$$\mathrm{map}: \underset{\underset{\text{space}}{\uparrow}}{\mathbb{S}^3} \mapsto \underset{\underset{\text{SU(2)}}{\uparrow}}{\mathbb{S}^3}$$



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target space: $\mathrm{SU}(2) \cong \mathbb{S}^3$

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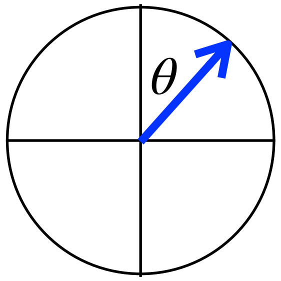
$$\left.\text{map}:\ \underset{\substack{\uparrow \\ \text{space}}}{\mathbb{S}^3} \mapsto \underset{\substack{\uparrow \\ \text{SU(2)}}}{\mathbb{S}^3}\ \right\}$$

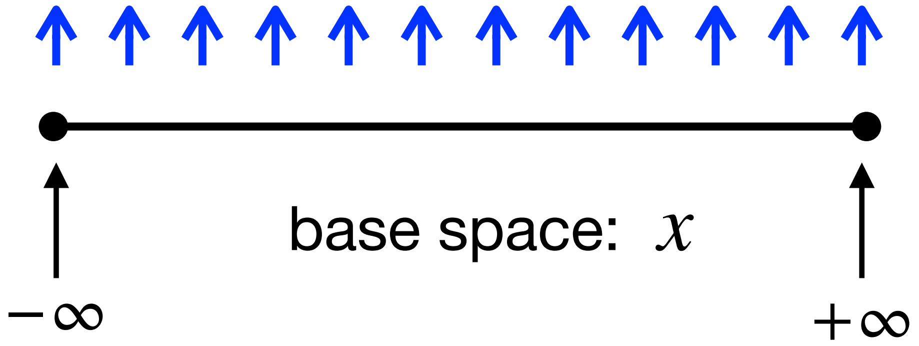
this map is classified by the integer winding number

$$\pi_3(\mathbb{S}^3) = \mathbb{Z}$$

map : $\mathbb{S}^1 \mapsto \mathrm{U}(1) \cong \mathbb{S}^1$

target space: $\mathscr{U} = e^{i\theta} \cong$

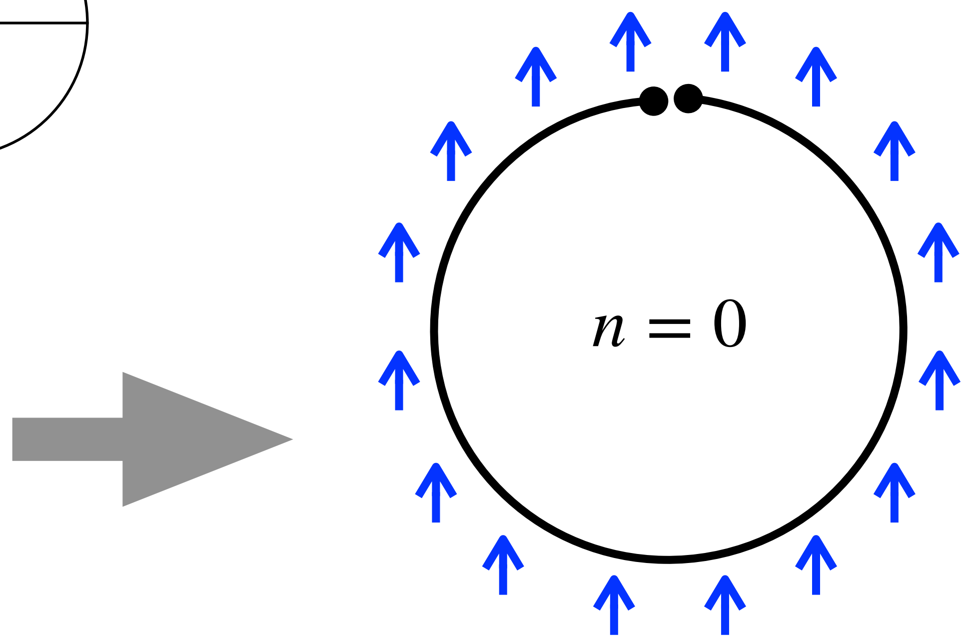




base space: x

$-\infty$

$+\infty$



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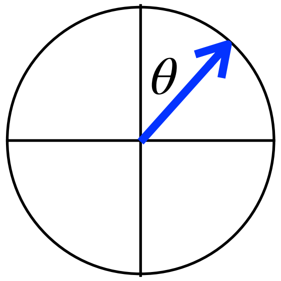

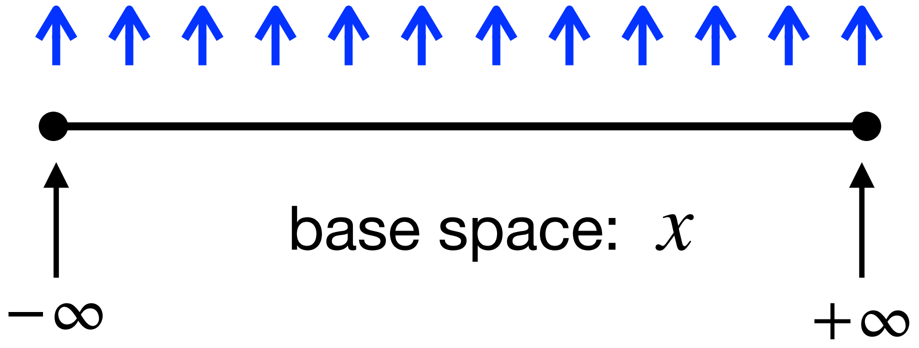

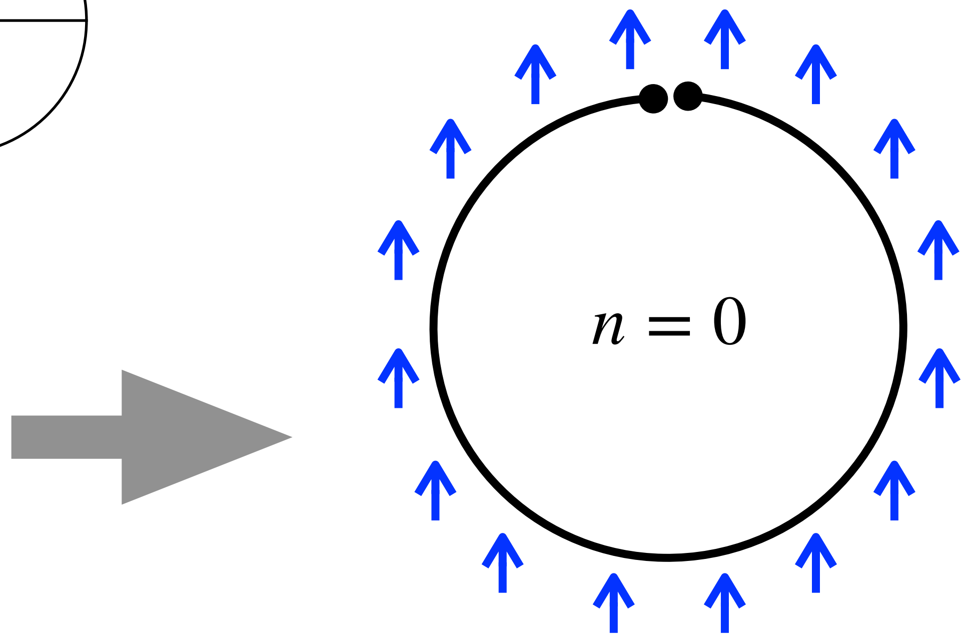

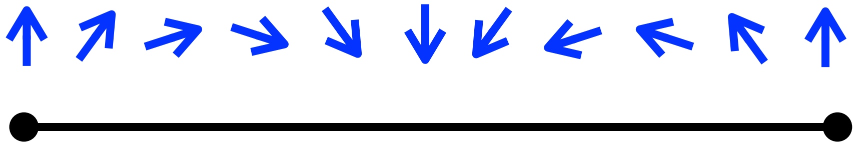

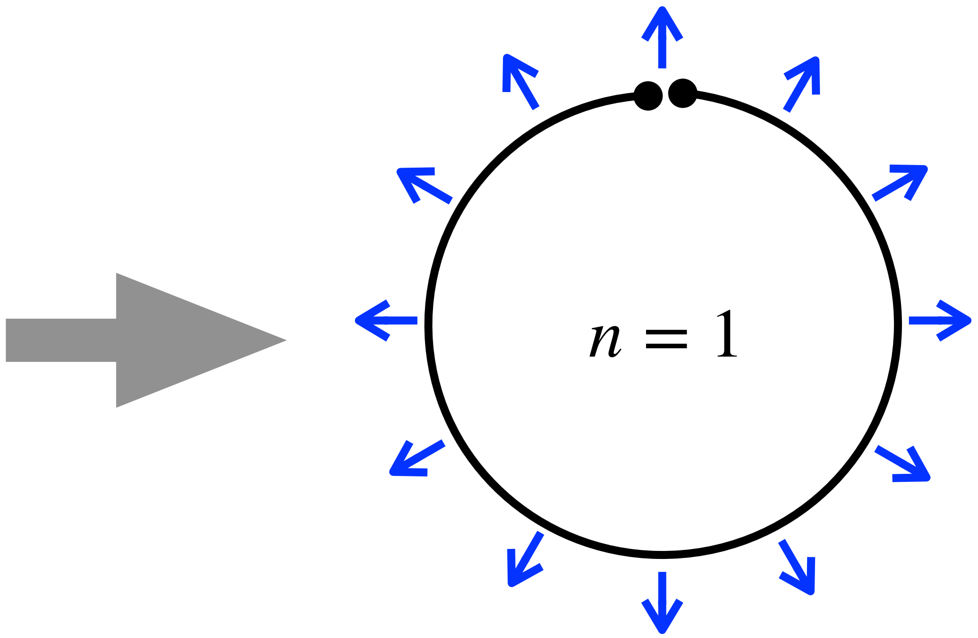

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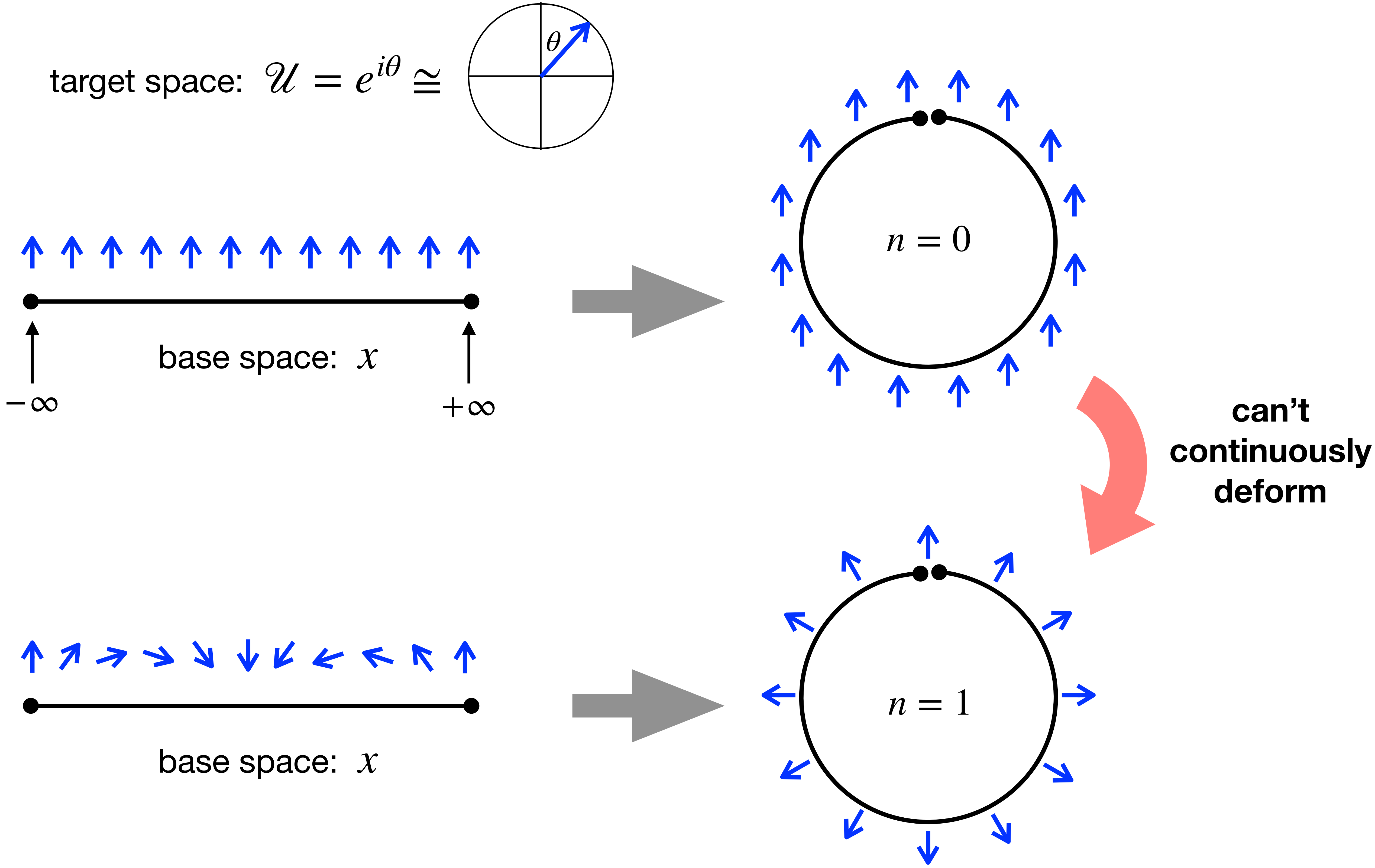


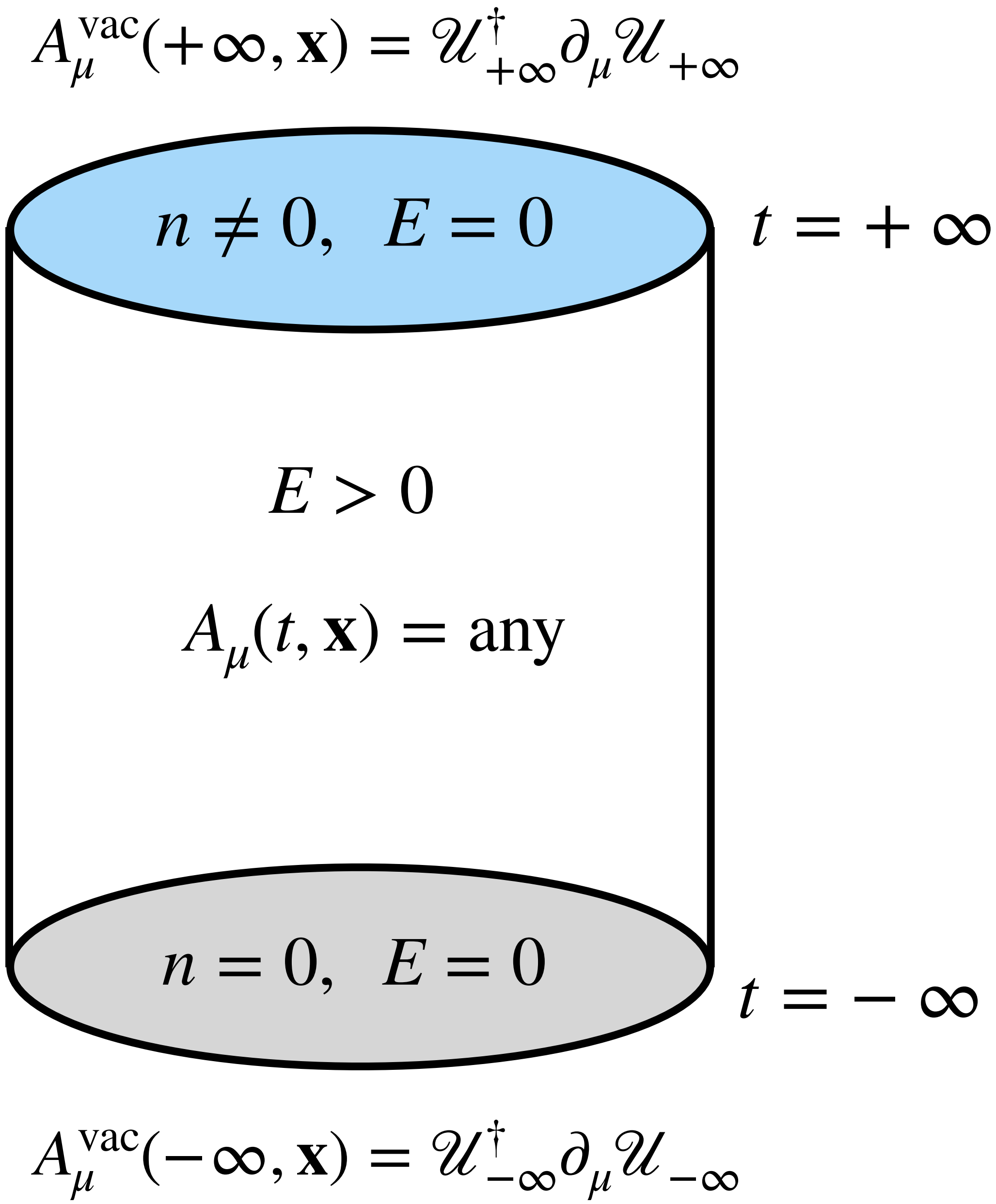
base space: x

$n = 0$

can’t continuously deform

base space: x

$n = 1$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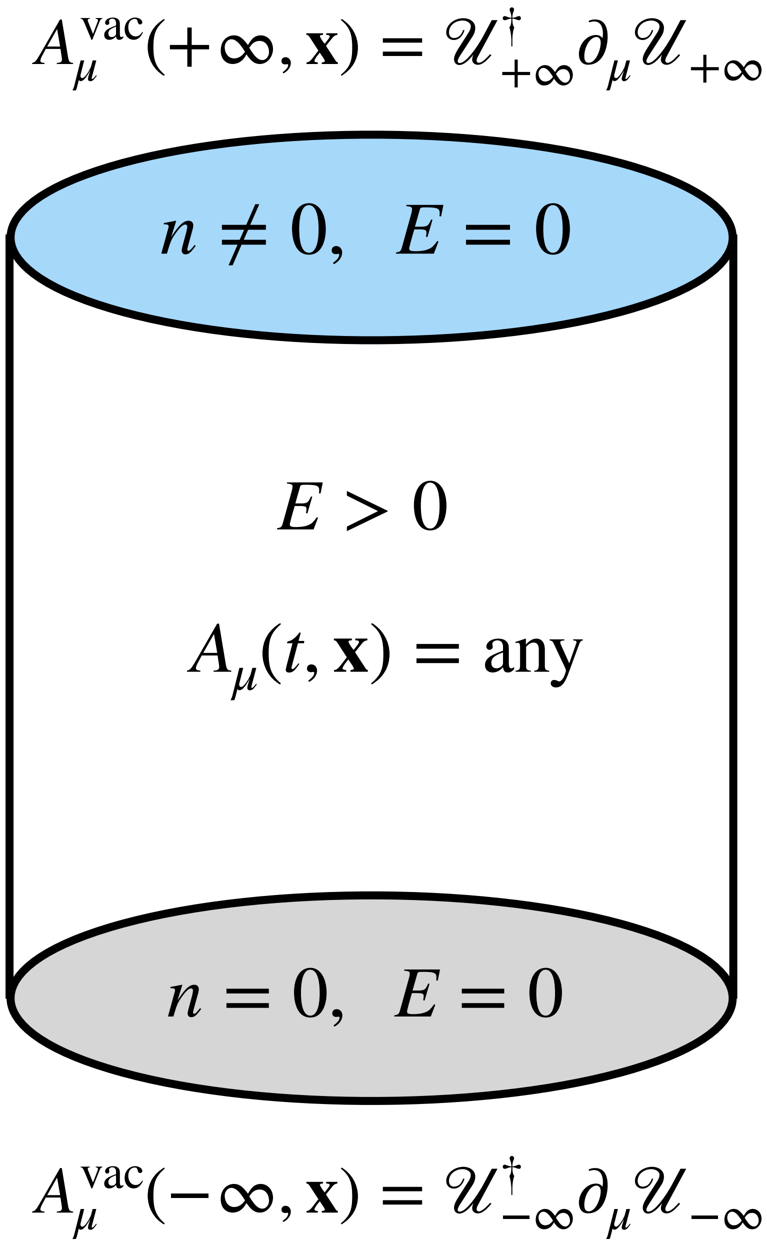
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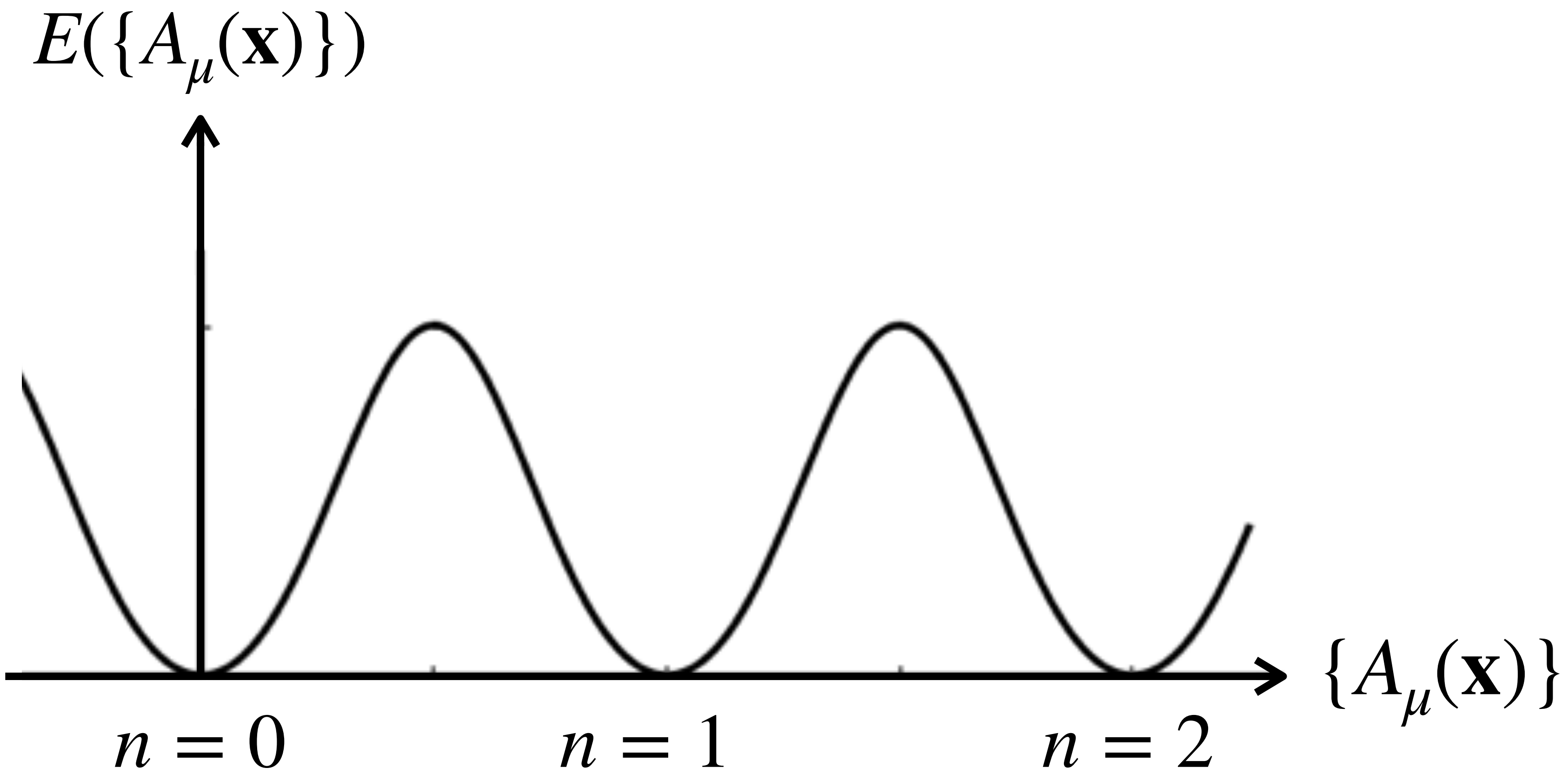
$E(\{A_\mu(\mathbf{x})\})$
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$n = 1$
$n = 2$

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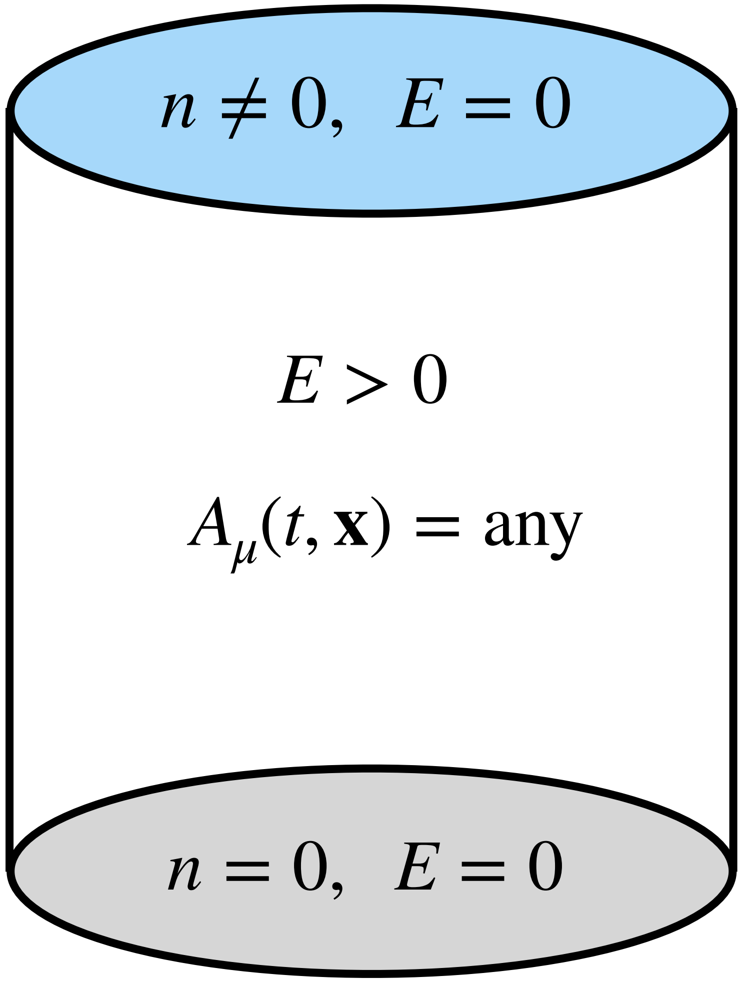

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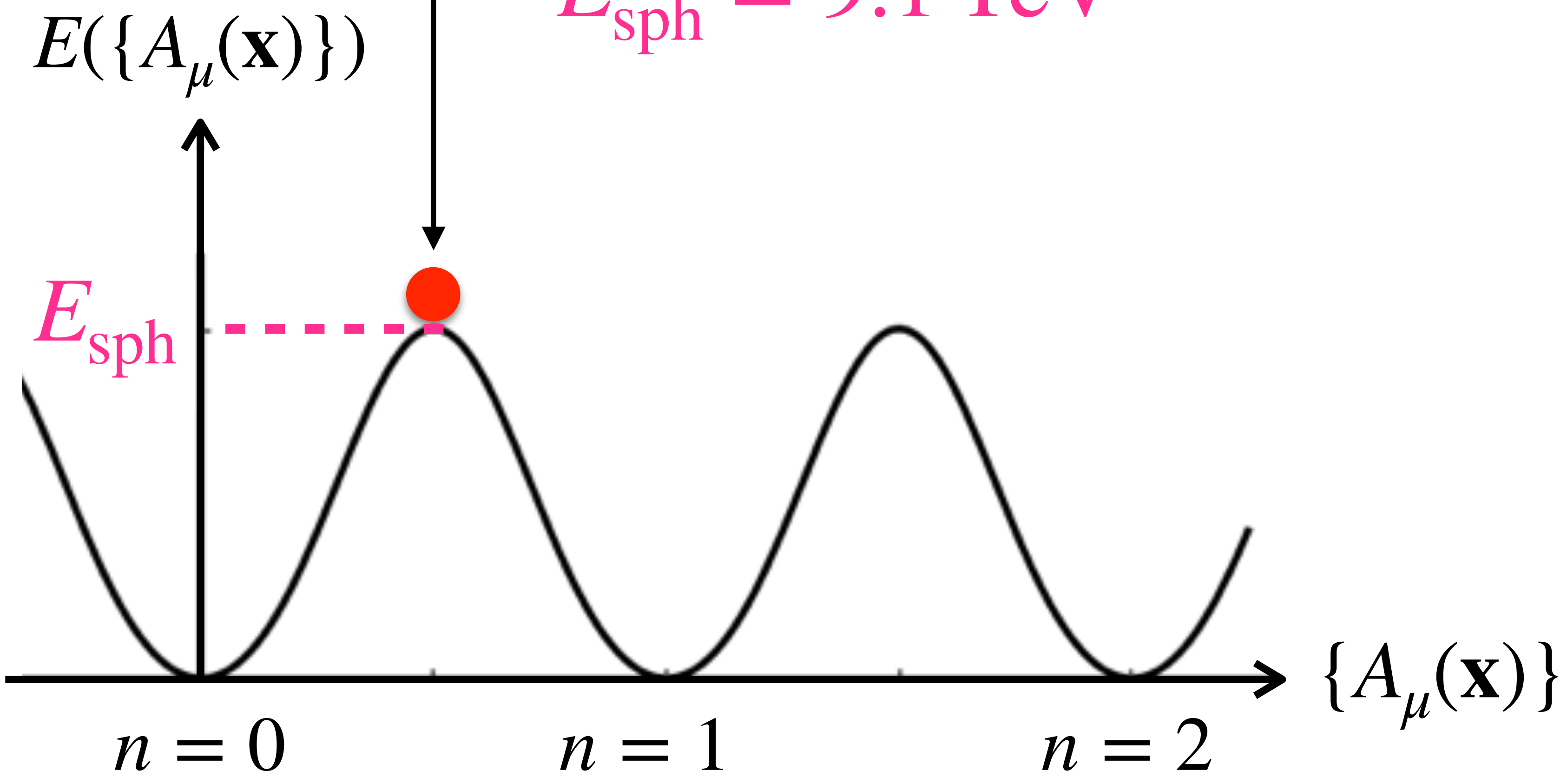
[Klinkhamer, Manton 1984]

sphaleron

unstable solution sitting at the top of barrier (saddle point)

$$E_{\mathrm{sph}} = 9.1\ \mathrm{TeV}$$



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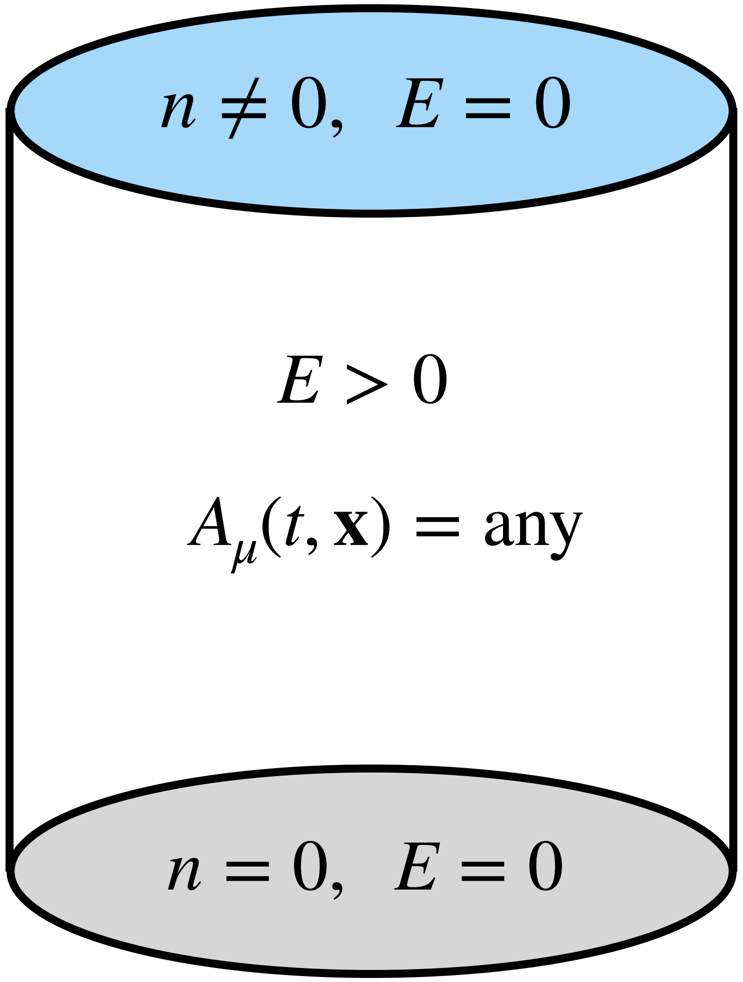

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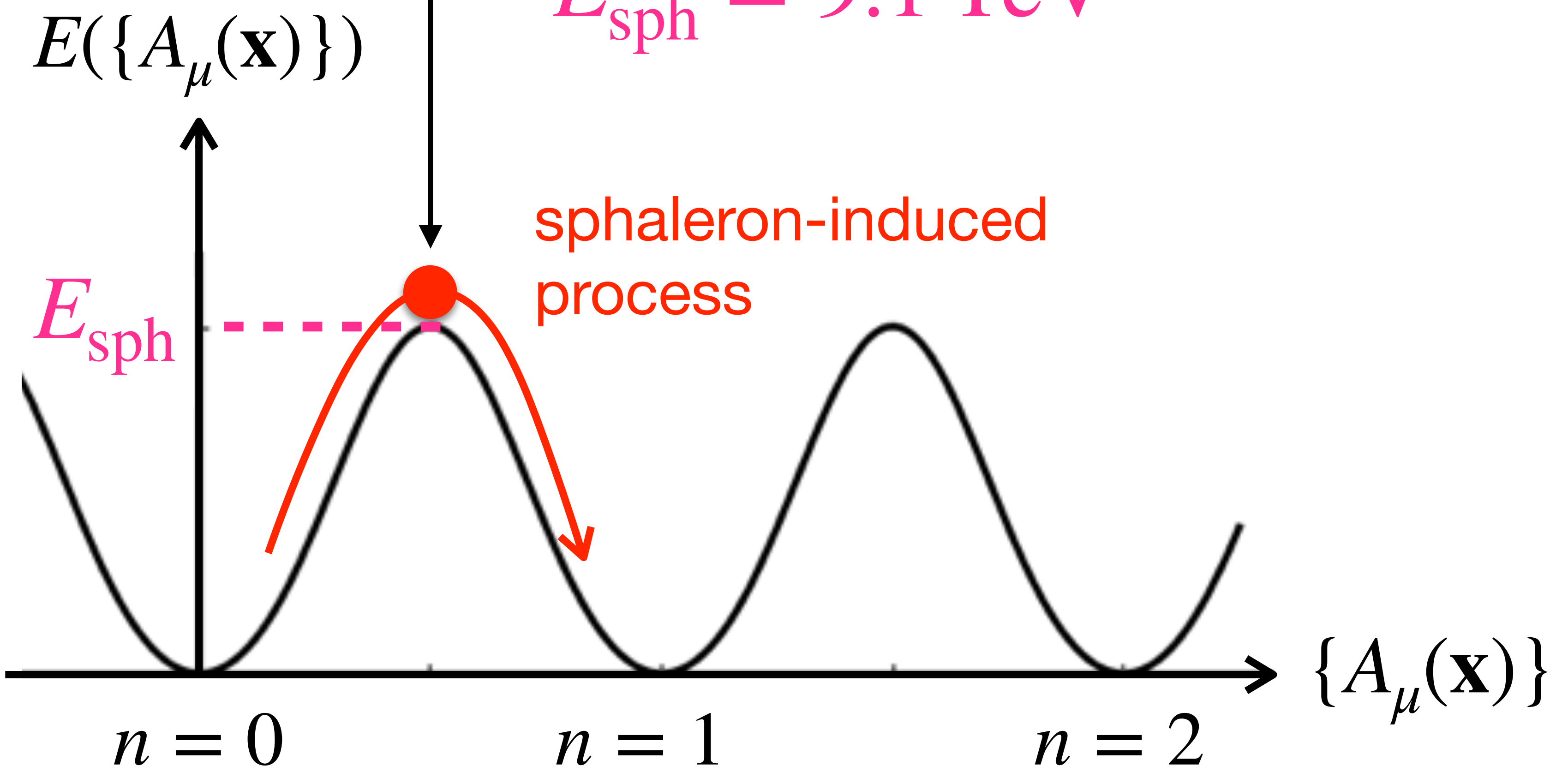

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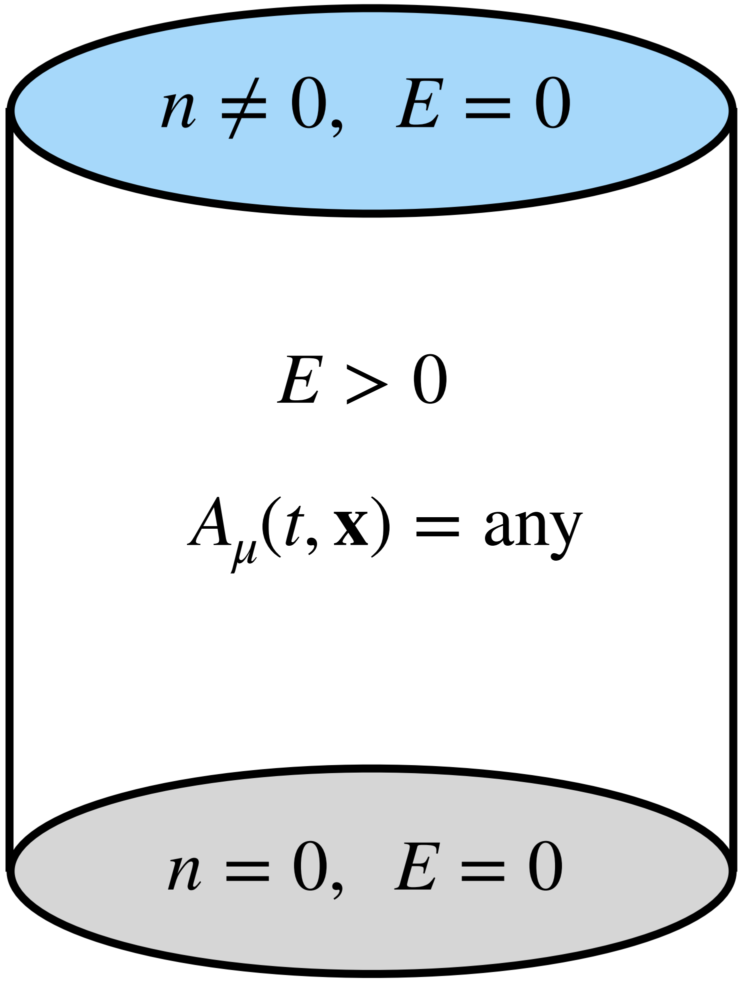

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instanton [t'Hooft 1976]

quantum tunnelling process

- tunnelling rate is small at zero energy

$$\Gamma = e^{-\frac{4\pi}{\alpha_W}} \sim 10^{-170}$$

[Klinkhamer, Manton 1984]

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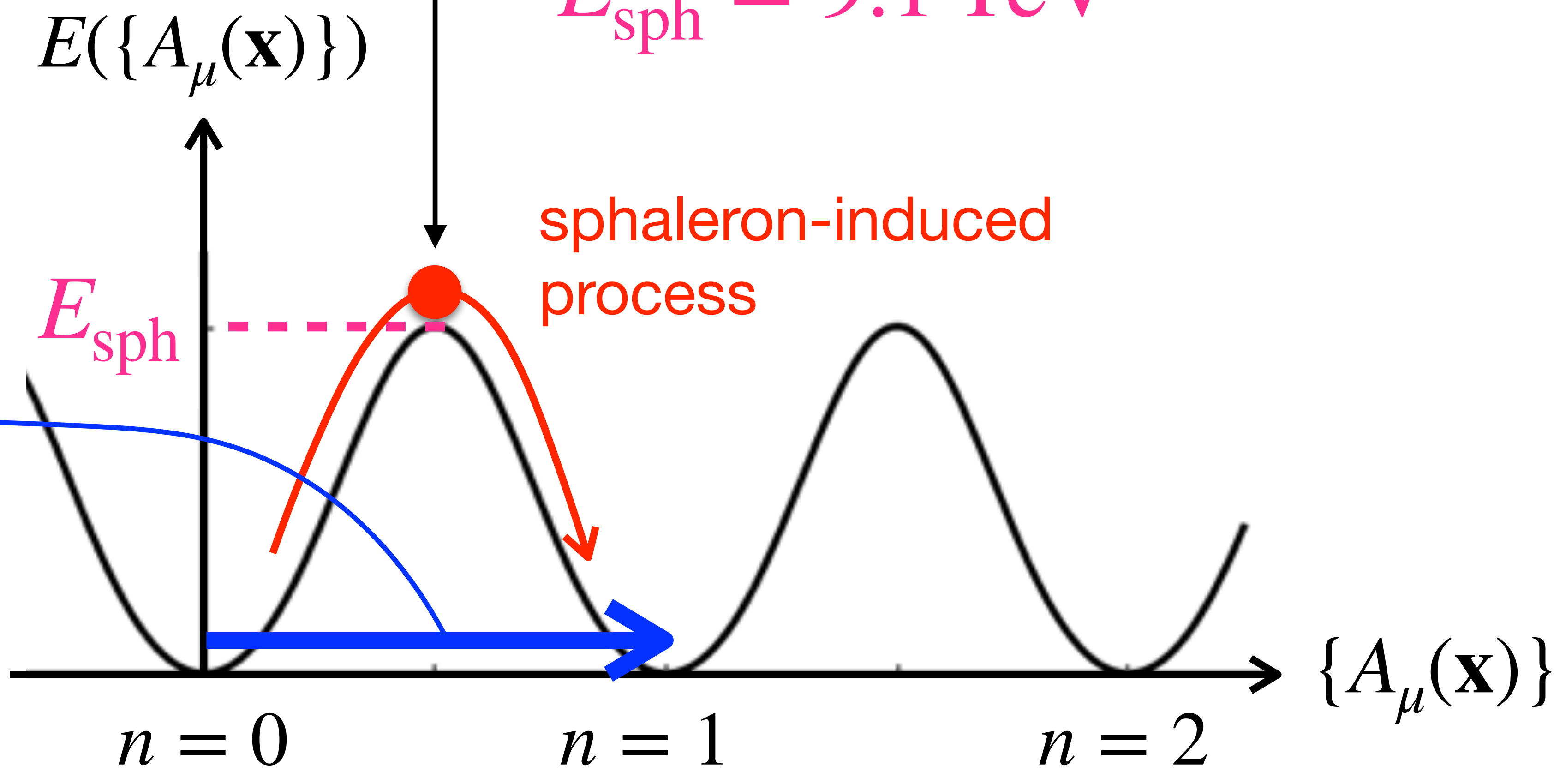

What does the sphaleron/instanton-induced process look like in a lab?

winding number changing process

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winding number changing process

- a “current” K_μ carrying the winding number must satisfy:

$$K_\mu(A_\mu(\mathbf{x})) \qquad Q = \int d^3\mathbf{x}\, K_0(A_\mu(\mathbf{x})) \qquad \int d^3\mathbf{x}\, K_0(\underbrace{A_\mu^{\mathrm{vac},n}(\mathbf{x})}_{\mathcal{U}_n^\dagger[\partial_\mu \mathcal{U}_n]}) = n$$

What does the sphaleron/instanton-induced process look like in a lab?

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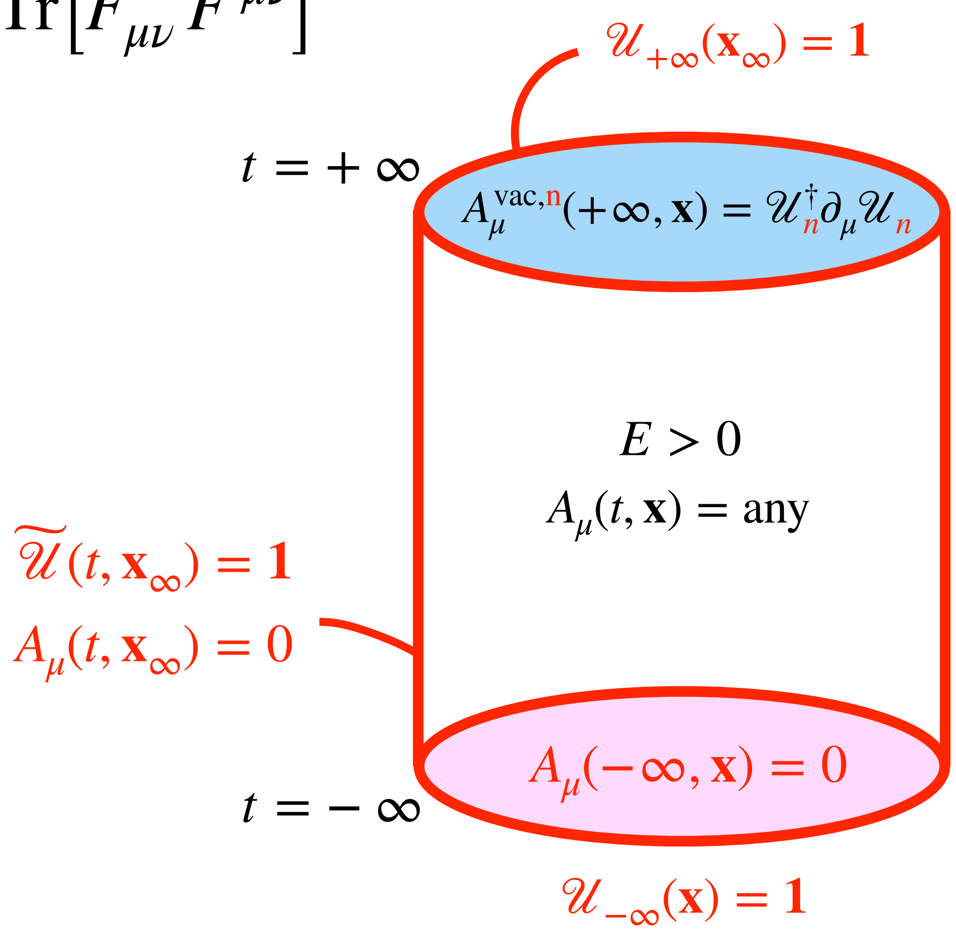
$$= \underbrace{\int d^3\mathbf{x}\, K^0}_{0}\Big|_{t=-\infty} + \underbrace{\int dt\, d\vec{\mathbf{x}}_\infty\cdot\vec{\mathbf{K}}}_{0}\Big|_{\mathbf{x}_\infty} + \underbrace{\int d^3\mathbf{x}\, K^0(A_\mu^{\mathrm{vac},n})}_{n}\Big|_{t=+\infty}$$



$\mathcal{U}_{+\infty}(\mathbf{x}_\infty) = \mathbf{1}$

$t = +\infty$

$A_\mu^{\mathrm{vac,n}}(+\infty, \mathbf{x}) = \mathcal{U}_n^\dagger \partial_\mu \mathcal{U}_n$

$E > 0$

$A_\mu(t, \mathbf{x}) =$ any

$\widetilde{\mathcal{U}}(t, \mathbf{x}_\infty) = \mathbf{1}$

$A_\mu(t, \mathbf{x}_\infty) = 0$

$t = -\infty$

$A_\mu(-\infty, \mathbf{x}) = 0$

$\mathcal{U}_{-\infty}(\mathbf{x}) = \mathbf{1}$

In the sphaleron/instanton-induced process: $|\Omega_0\rangle \rightarrow |\Omega_n\rangle$

$$n = \int d^4x \frac{1}{16\pi^2} \mathrm{Tr}\left[F_{\mu\nu} \widetilde{F}^{\mu\nu}\right]$$

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change of fermion number
that coupled to SU(2) gauge field

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$$n = \int d^4x \frac{1}{16\pi^2} \mathrm{Tr}\left[F_{\mu\nu}\widetilde{F}^{\mu\nu}\right] \underset{\text{anomaly}}{=} \left\{ \begin{array}{l} \Delta N_{q_1^r} \\ \Delta N_{q_1^g} \\ \Delta N_{q_1^b} \\ \Delta N_{\ell_1} \end{array} \right. \quad \times 3 \text{ flavour}$$

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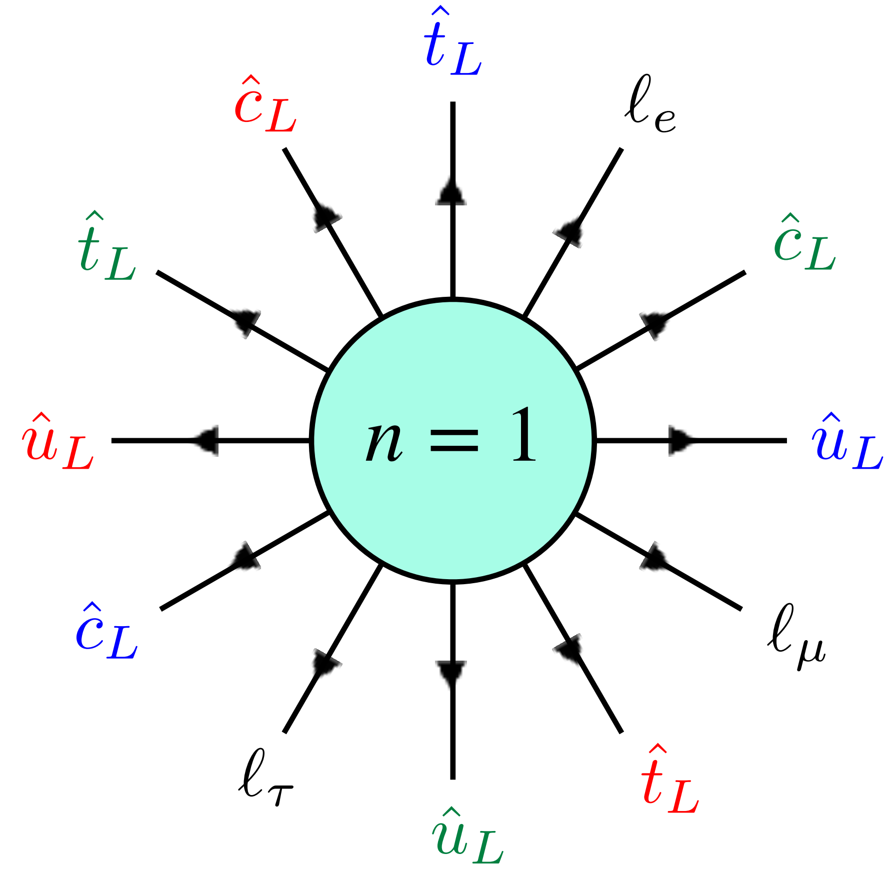
For $n = 1$, all 12 left-handed fermions are produced from the vacuum:

$\Delta B = \Delta L = 3n$

$\Delta(B + L) = 6n$

$\Delta(B - L) = 0$



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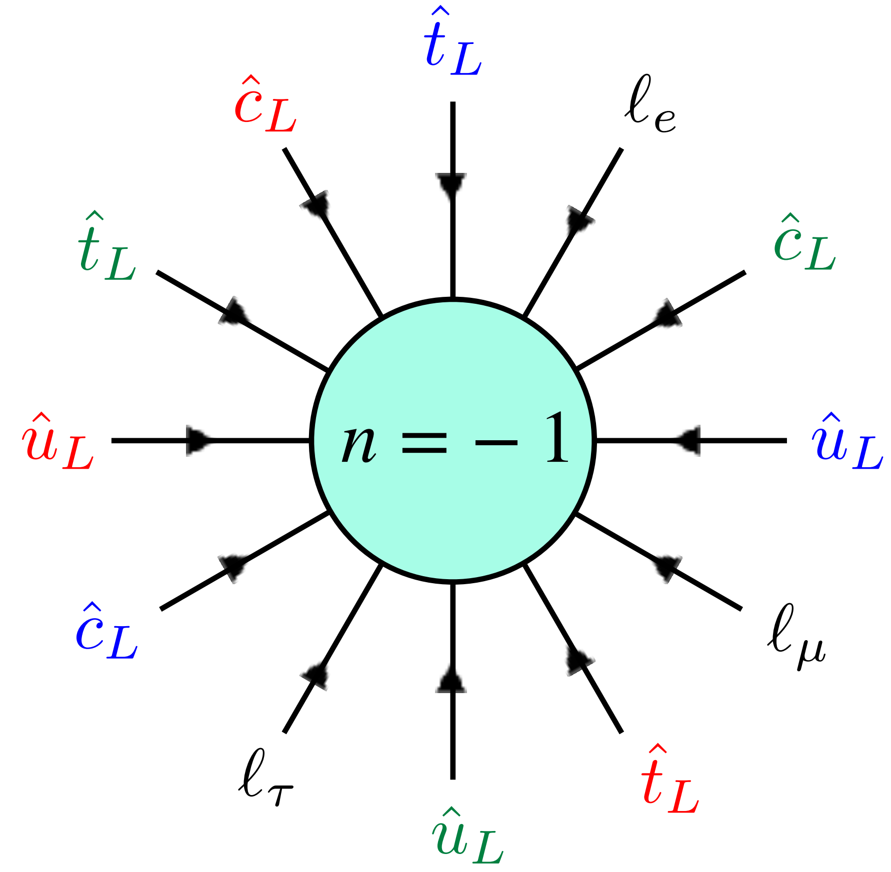
For $n = -1$,



The final state:

$\{e^+, \bar{\nu}_e\}$

$\{\mu^+, \bar{\nu}_\mu\}$

$\{\tau^+, \bar{\nu}_\tau\}$

$\{\bar{u}, \bar{d}\} \times 1$

$\{\bar{c}, \bar{s}\} \times 3$

$\{\bar{t}, \bar{b}\} \times 3$

+ some EW bosons

ex)

$$uu \to e^+ \bar{\nu}_\mu \bar{\nu}_\tau \bar{d}\, \bar{c}\, \bar{s}\, \bar{s}\, \bar{t}\, \bar{b}\, \bar{b} \quad \Rightarrow 1e^+ + 4j + 1\bar{t} + 2b + E^T_{\text{miss}}$$

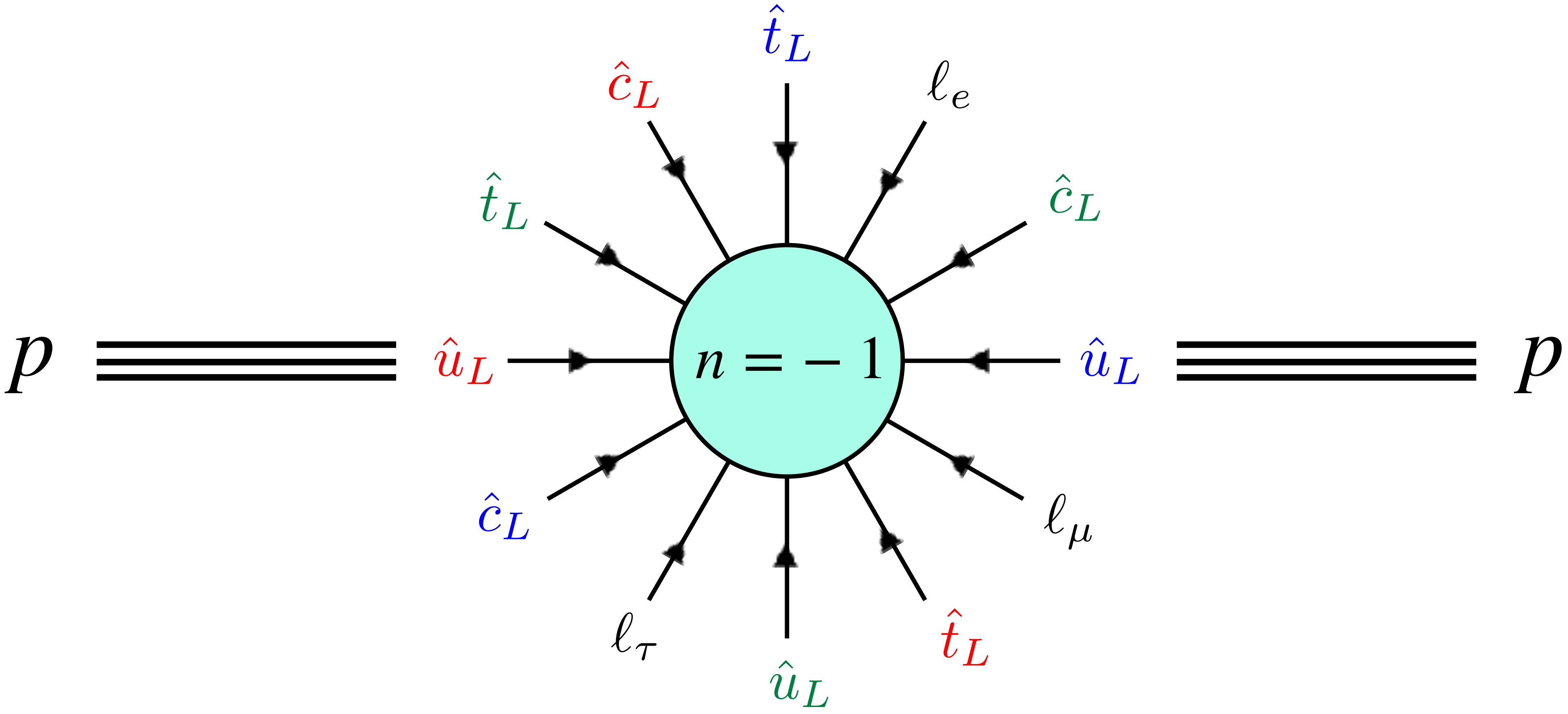
$$uu \to \bar{\nu}_e \mu^+ \tau^+ \bar{d}\, \bar{c}\, \bar{s}\, \bar{s}\, \bar{b}\, \bar{b}\, \bar{b} \quad \Rightarrow 1\mu^+ + 1\tau^+ + 4j + 3b + E^T_{\text{miss}}$$

$$ud \to \bar{\nu}_e \mu^+ \bar{\nu}_\tau \bar{d}\, \bar{c}\, \bar{s}\, \bar{s}\, \bar{t}\, \bar{t}\, \bar{b} \quad \Rightarrow 1\mu^+ + 4j + 2\bar{t} + 1b + E^T_{\text{miss}}$$

+ some EW bosons

Many particles are produced

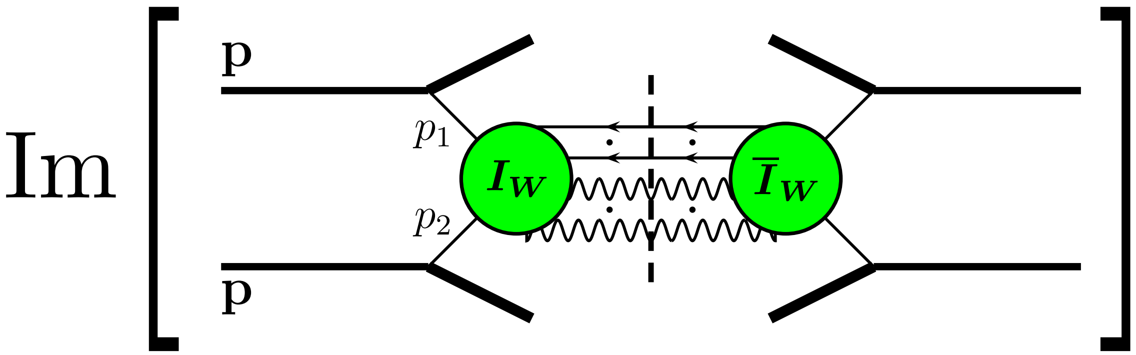
Confused with BH events?



Cross section (?)

- The inclusive cross-section can be estimated using the dispersion relations (optical theorem).

[Khoze, Ringwald ’91]



$$\hat{\sigma}^{\rm BV}_{\rm incl} \equiv \sum_{n_W,n_h} \hat{\sigma}^{\rm BV}_{n_W,n_h} = \frac{P(\epsilon)}{m_W^2} \exp\left[-\frac{4\pi}{\alpha_W} F\left(\epsilon\right)\right]$$

$$F(\epsilon) = 1 - \frac{9}{8}\epsilon^{4/3} + \frac{9}{16}\epsilon^2 - \frac{3}{32}\left(4 - 3\frac{m_h^2}{m_W^2}\right)\epsilon^{8/3}\ln\frac{1}{\epsilon} + \mathcal{O}(\epsilon^{8/3}\cdot \mathrm{const})$$

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$$\epsilon \equiv \frac{\sqrt{\hat{s}}}{M_0} \qquad M_0 \equiv \sqrt{6}\pi\frac{m_W}{\alpha_W} \simeq 18.3\ \mathrm{TeV}$$

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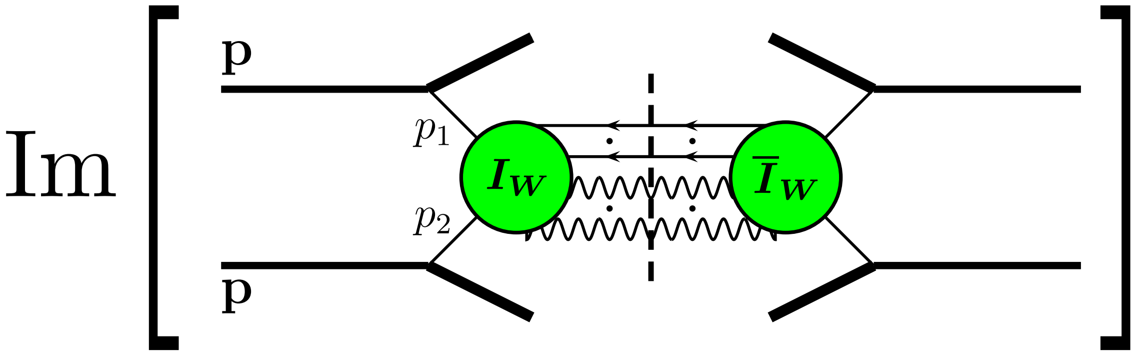

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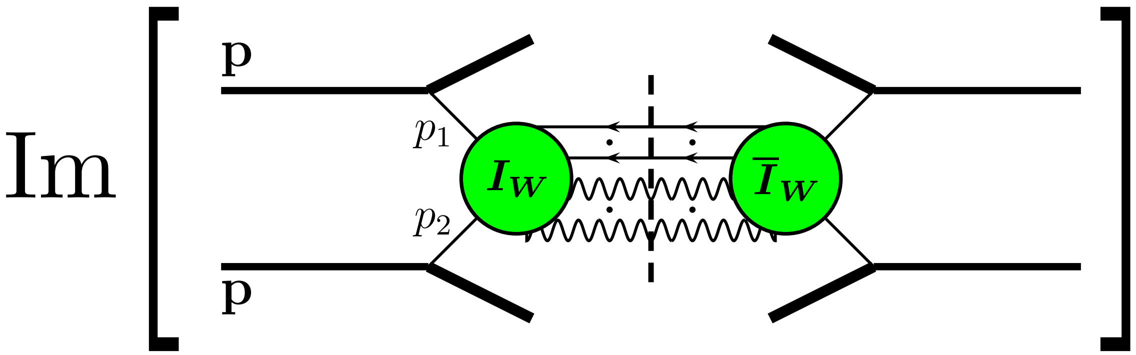

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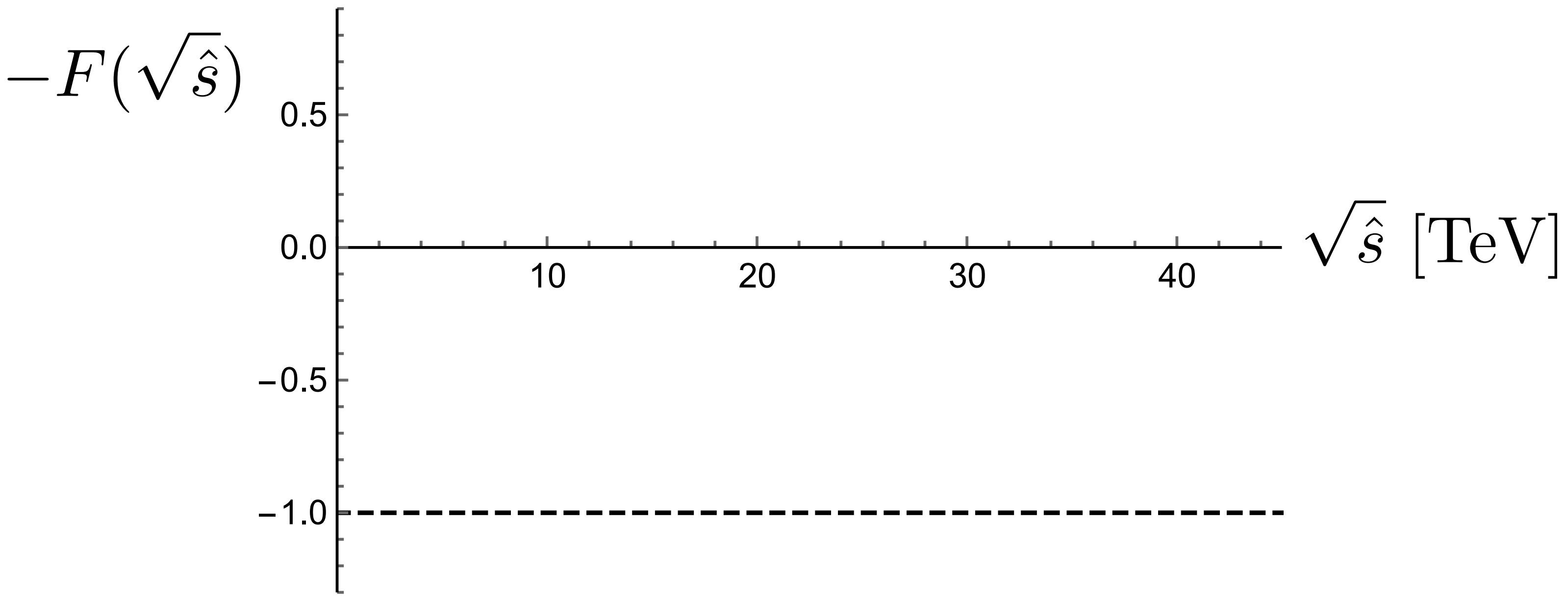

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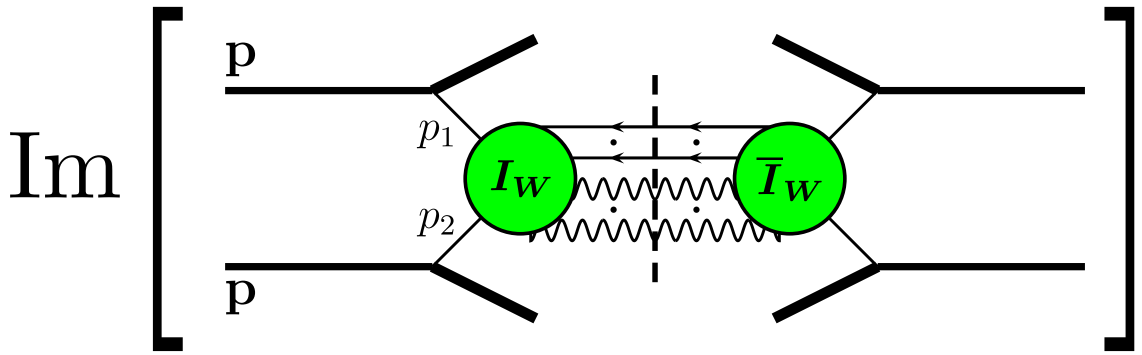

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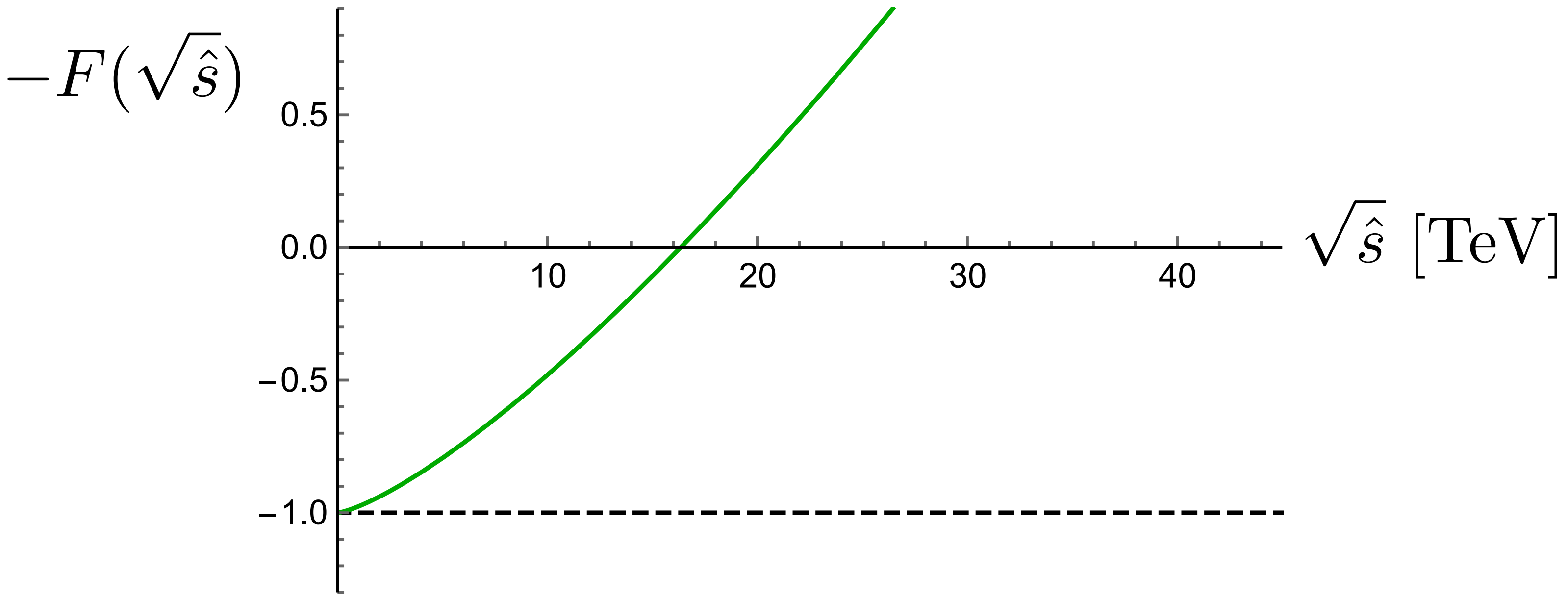

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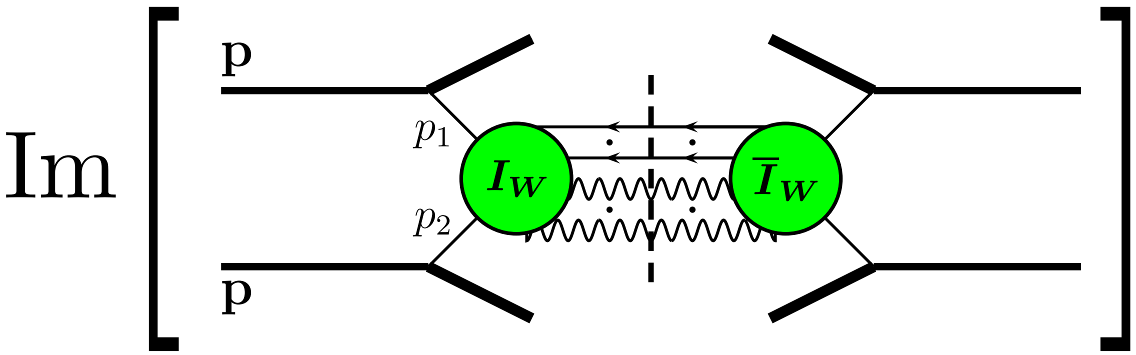

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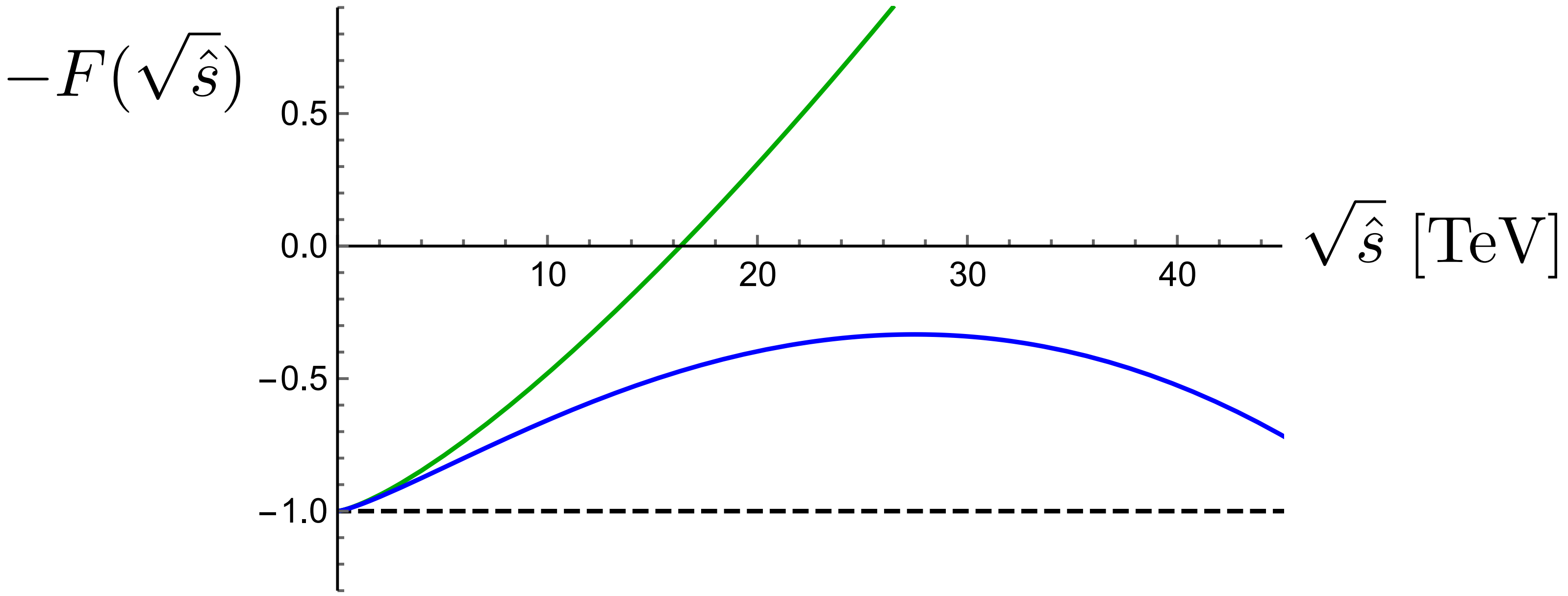

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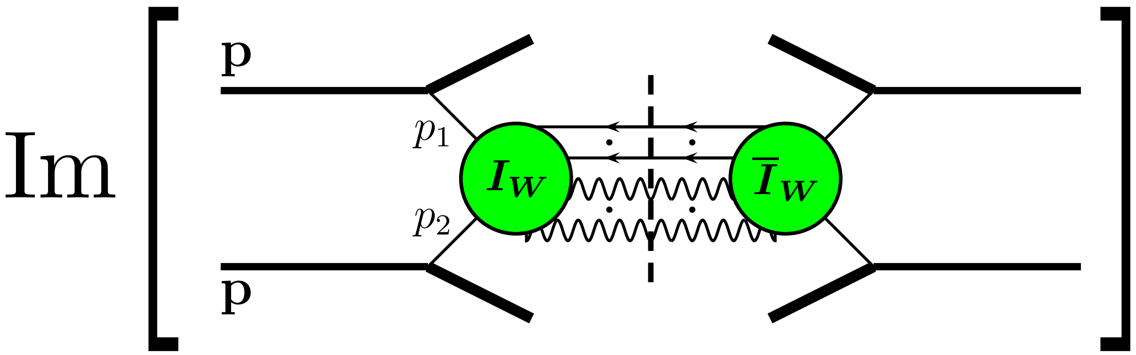

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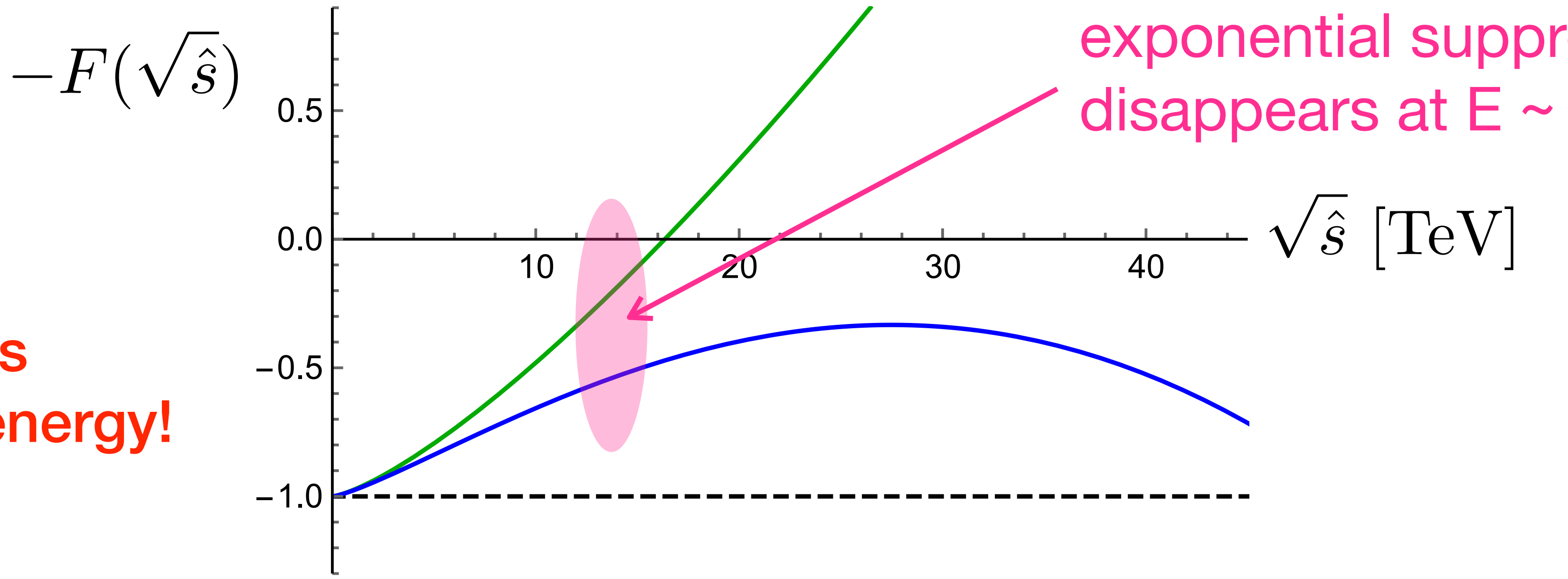


exponential suppression disappears at E ~ E_{sph}?

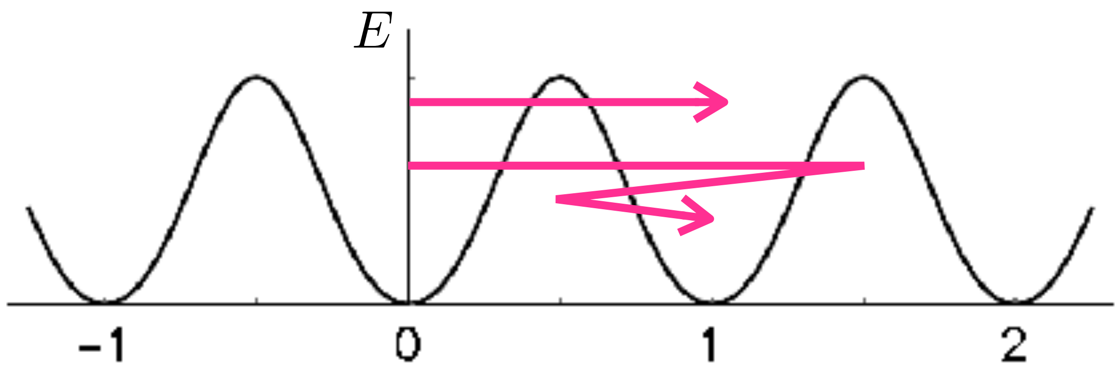
Cross-section grows exponentially with energy!

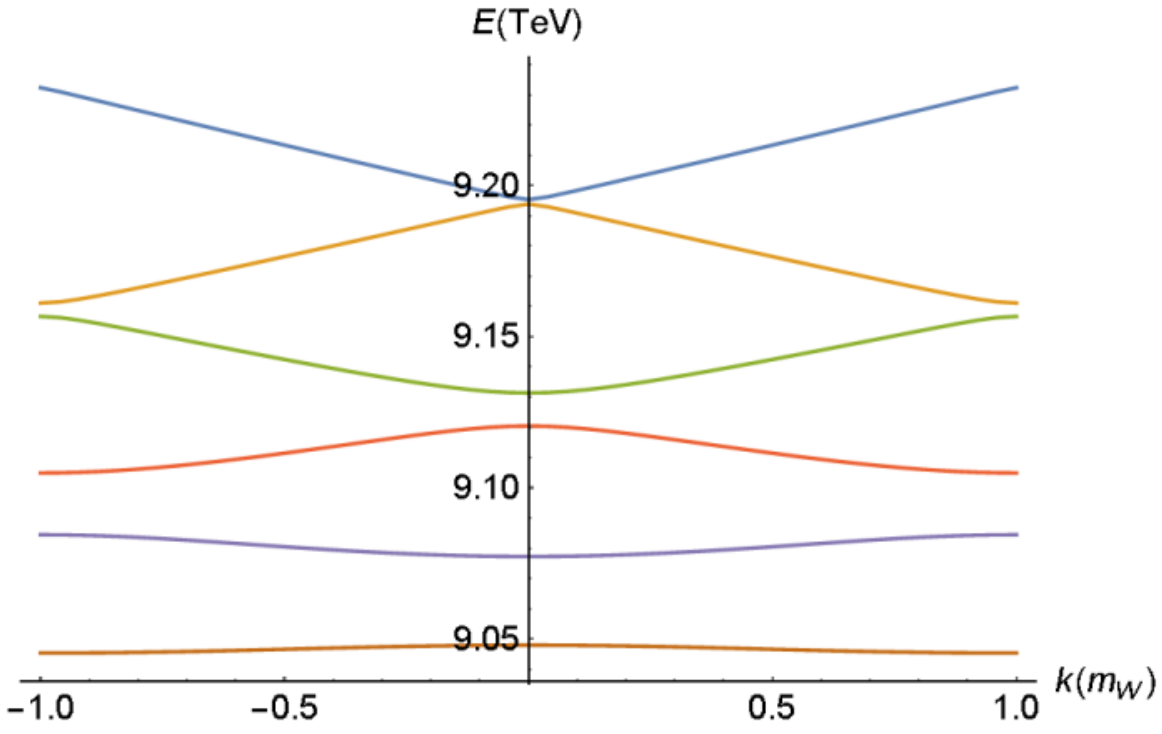
- More recently (2015), it has been pointed out that at zero temperature instanton rate may be able to overcome the exponential suppression for $E > E_{sph}$ ~ 9 TeV, if the periodicity of the EW potential is taken into account, due to *resonant tunnelling*.

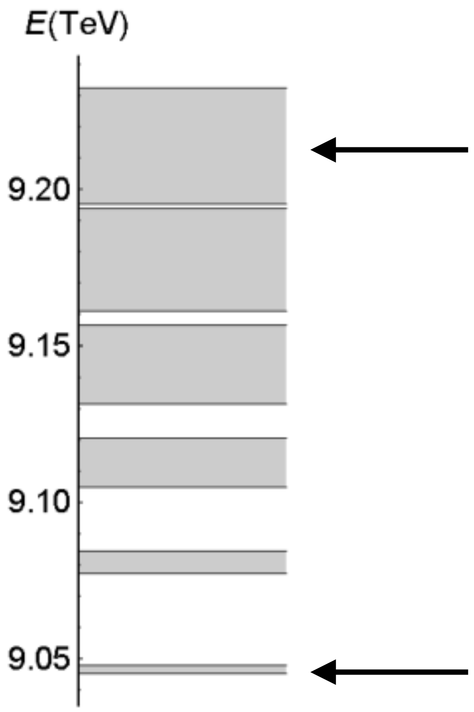
Tye, Wong [1505.03690, 1710.07223]
Qiu, Tye [1812.07181]

Resonant tunneling:

Different paths coherently interfere at particular energies, forming a conducing band structure







the band widths are unsuppressed for E > 9.1 TeV.

Exponentially small widths for E < 9.1 TeV corresponds to the exponential suppression in instantons

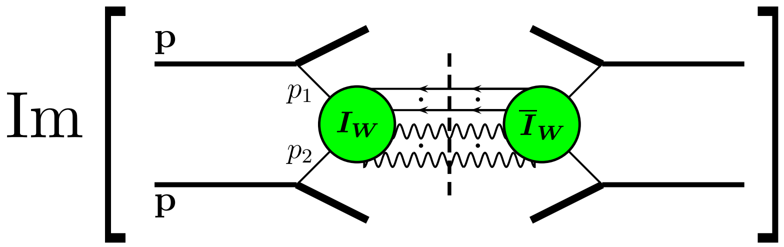
Cross-section Estimate

$$\boxed{\hat{\sigma}^{\mathrm{BV}}_{\mathrm{incl}} \equiv \sum_{n_W, n_h} \hat{\sigma}^{\mathrm{BV}}_{n_W, n_h} = \frac{P(\epsilon)}{m_W^2} \exp\left[-\frac{4\pi}{\alpha_W} \underline{F\left(\epsilon\right)}\right]}$$

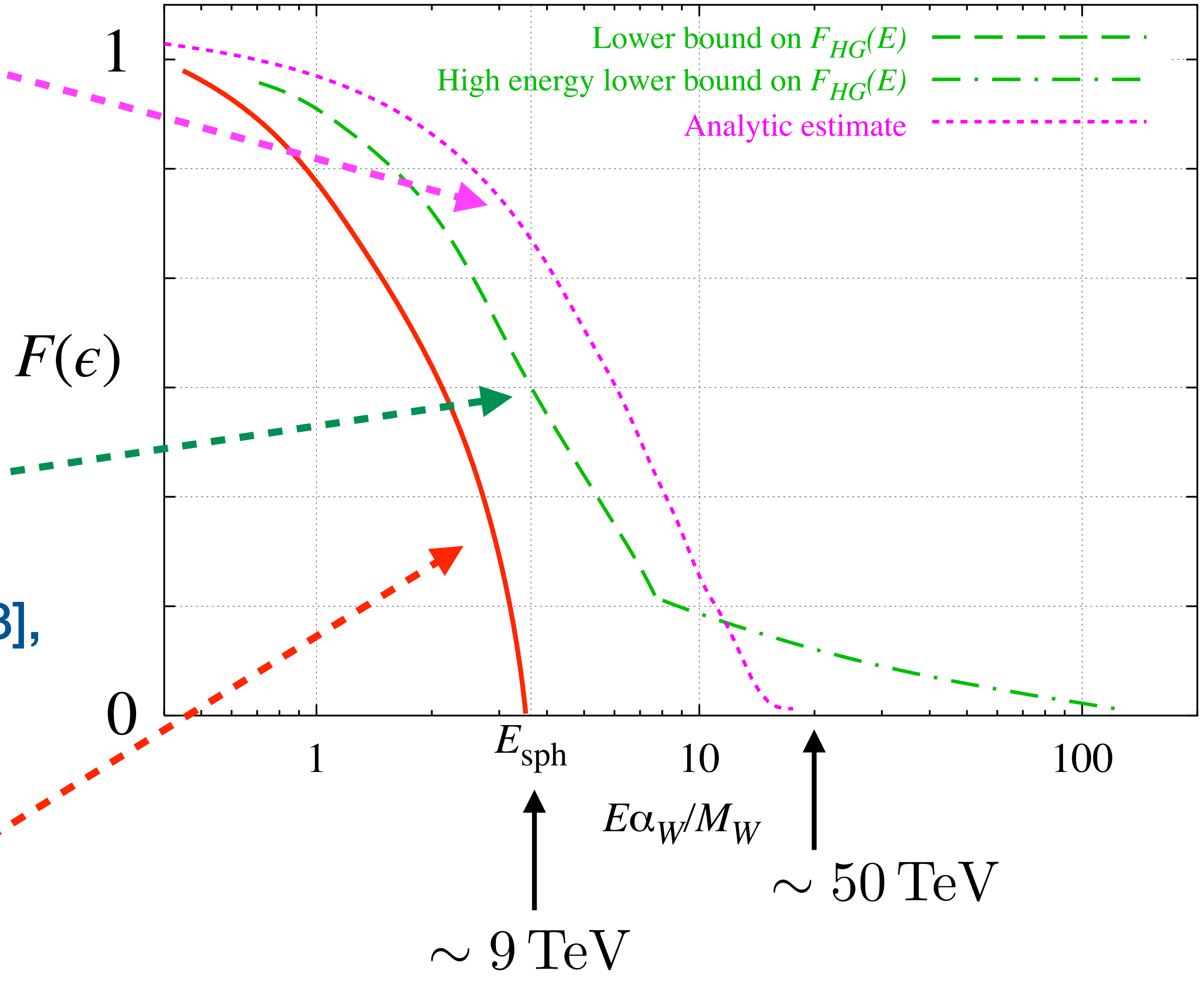
$$\epsilon \equiv E \cdot \frac{\alpha_W}{4\pi m_W}$$

- **Optical theorem**

  [Khoze, Ringwald '91], …



[Bezrukov, Levkov, Rebbi, Rubakov, Tinyakov '03]



- **Semi-Classical method**

  [Bezrukov, Levkov, Rebbi, Rubakov, Tinyakov '03],

  [Rubakov, Tinyakov '92], …

- **Treating $\mathrm{N_{CS}}$ as a dynamical variable**

  [Tye, Wong '15 '16]

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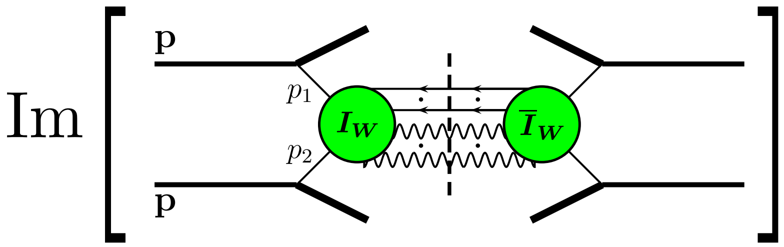

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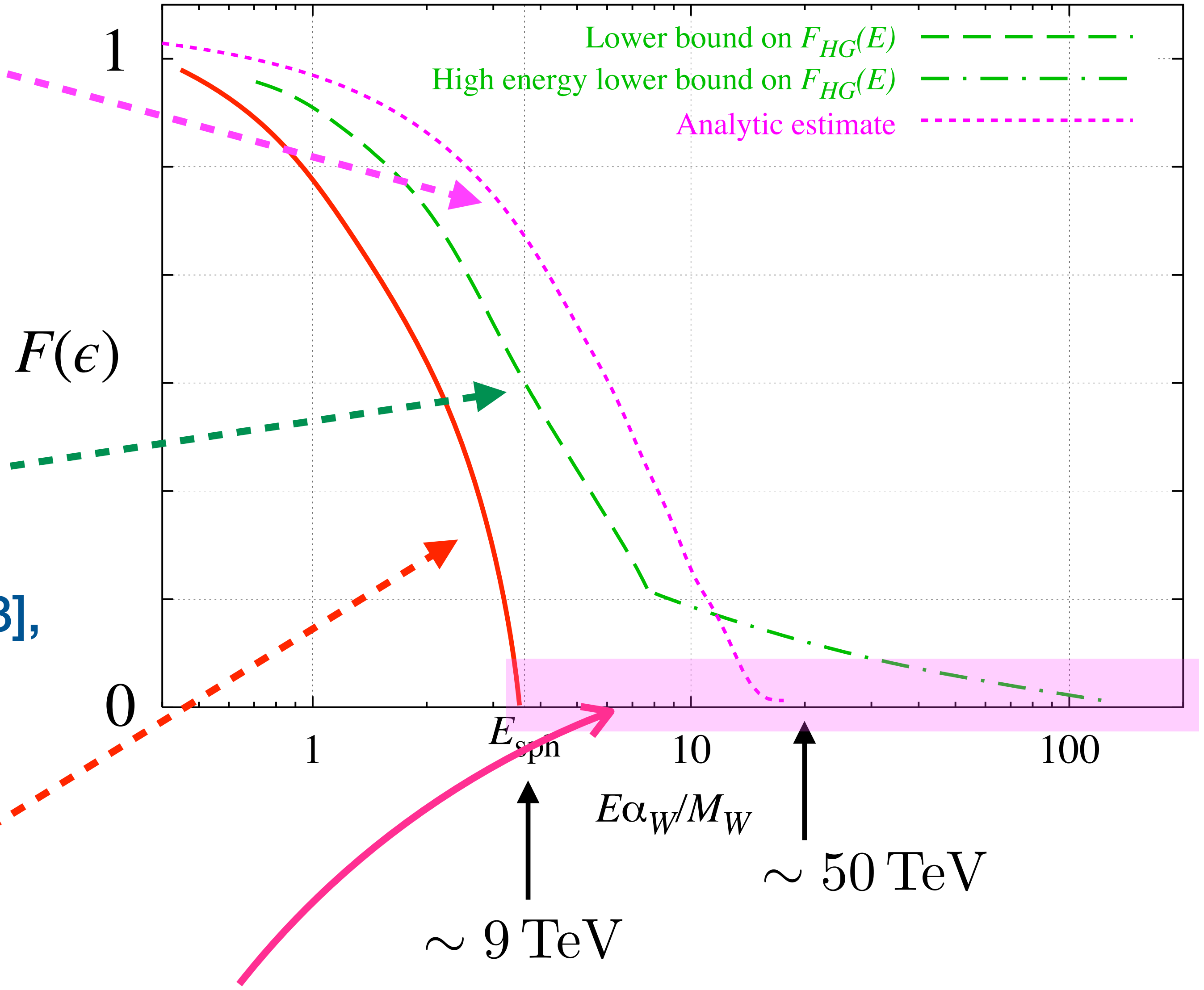

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- **Treating N_{CS} as a dynamical variable**

  [Tye, Wong '15 '16]

huge theo. unc. on the energy at which σ terns on

Phenomenological parametrization

partonic:

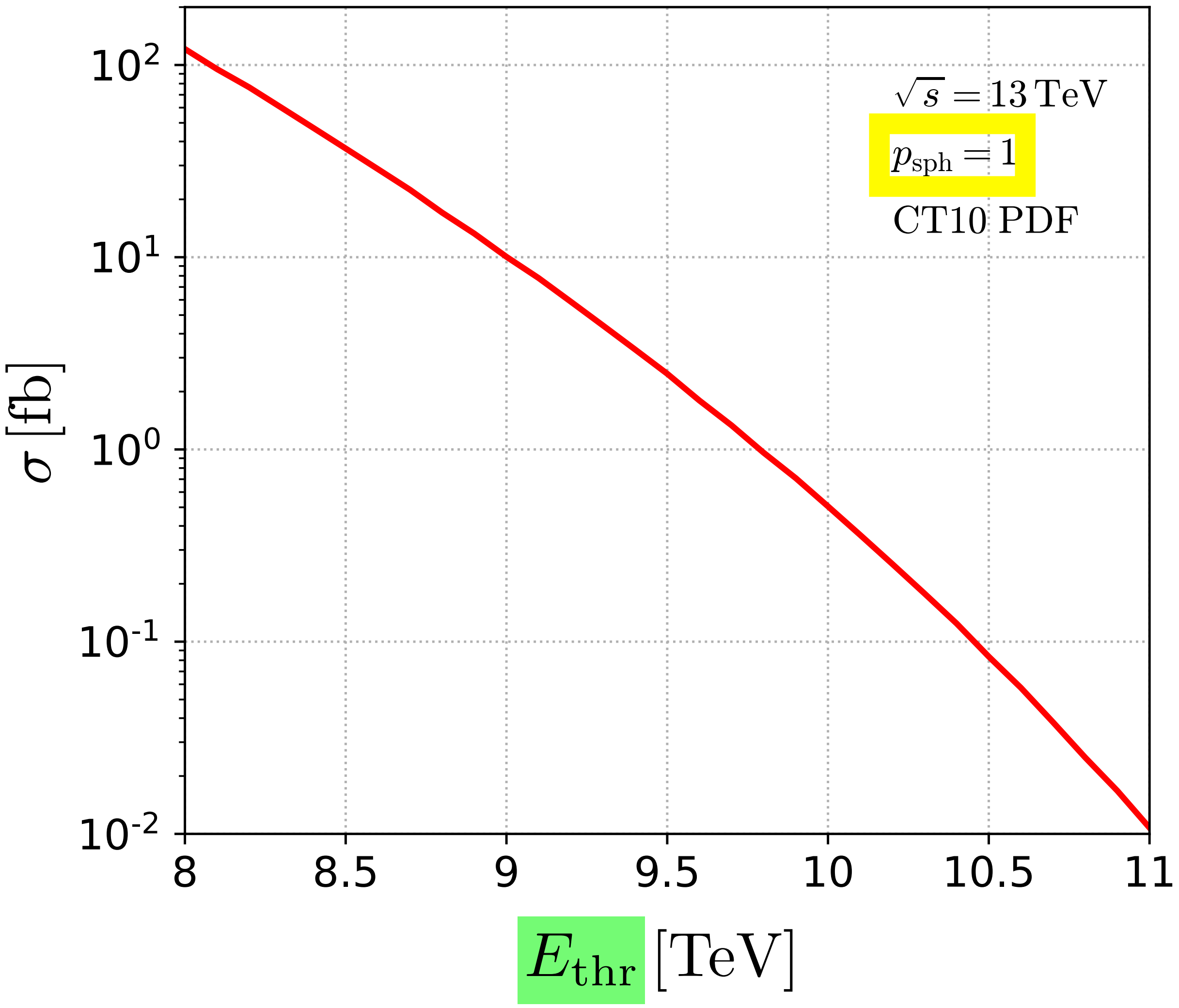
$$\boxed{\hat{\sigma}_0(\sqrt{\hat{s}}) = \frac{p_{\rm sph}}{m_W^2}\Theta(\sqrt{\hat{s}} - E_{\rm thr})}$$

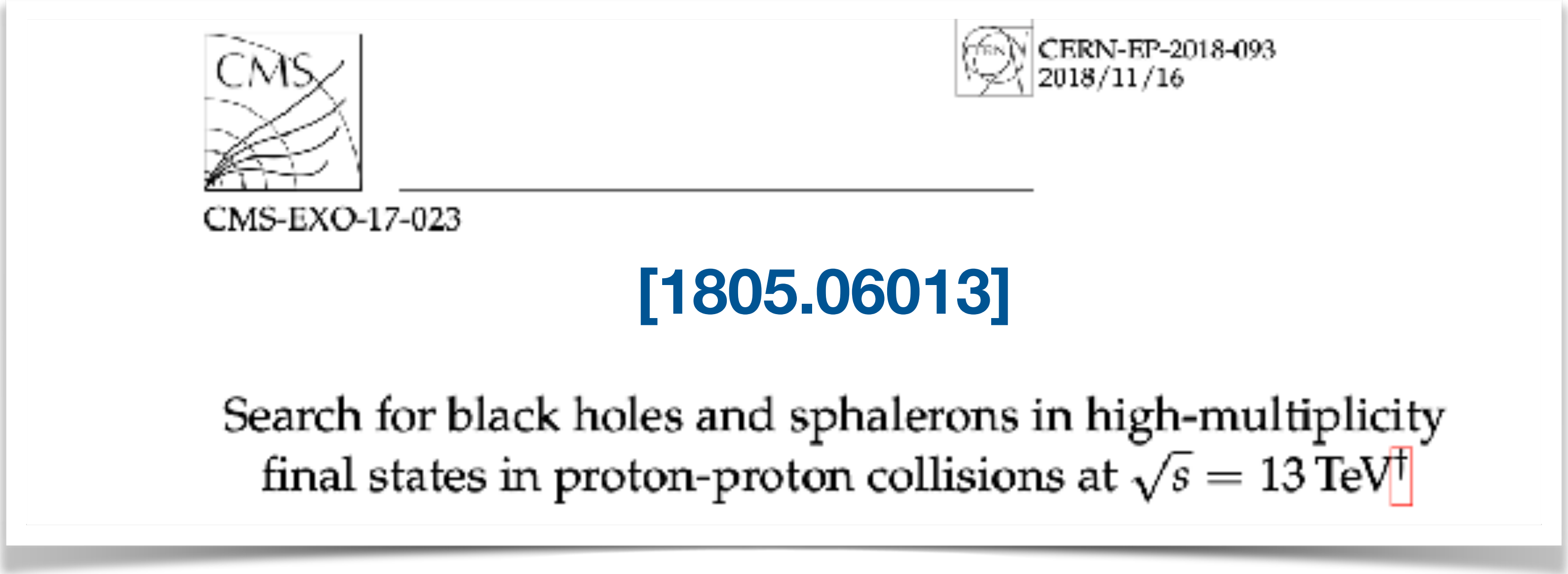
$$\hat{\sigma}^{\rm BV}_{\rm incl} \equiv \sum_{n_W,n_h} \hat{\sigma}^{\rm BV}_{n_W,n_h} = \frac{P(\epsilon)}{m_W^2}\exp\left[-\frac{4\pi}{\alpha_W}F(\epsilon)\right]$$

hadronic:

$$\sigma_{pp}(\sqrt{s}) \sim \sum_{ab}\left(\frac{1}{2}\right)^2 \int dx_1 dx_2 f_a(x_1) f_b(x_2) \hat{\sigma}_0(\sqrt{s x_1 x_2})$$

[Ellis, KS, 1601.03654]





CMS

CERN-EP-2018-093
2018/11/16

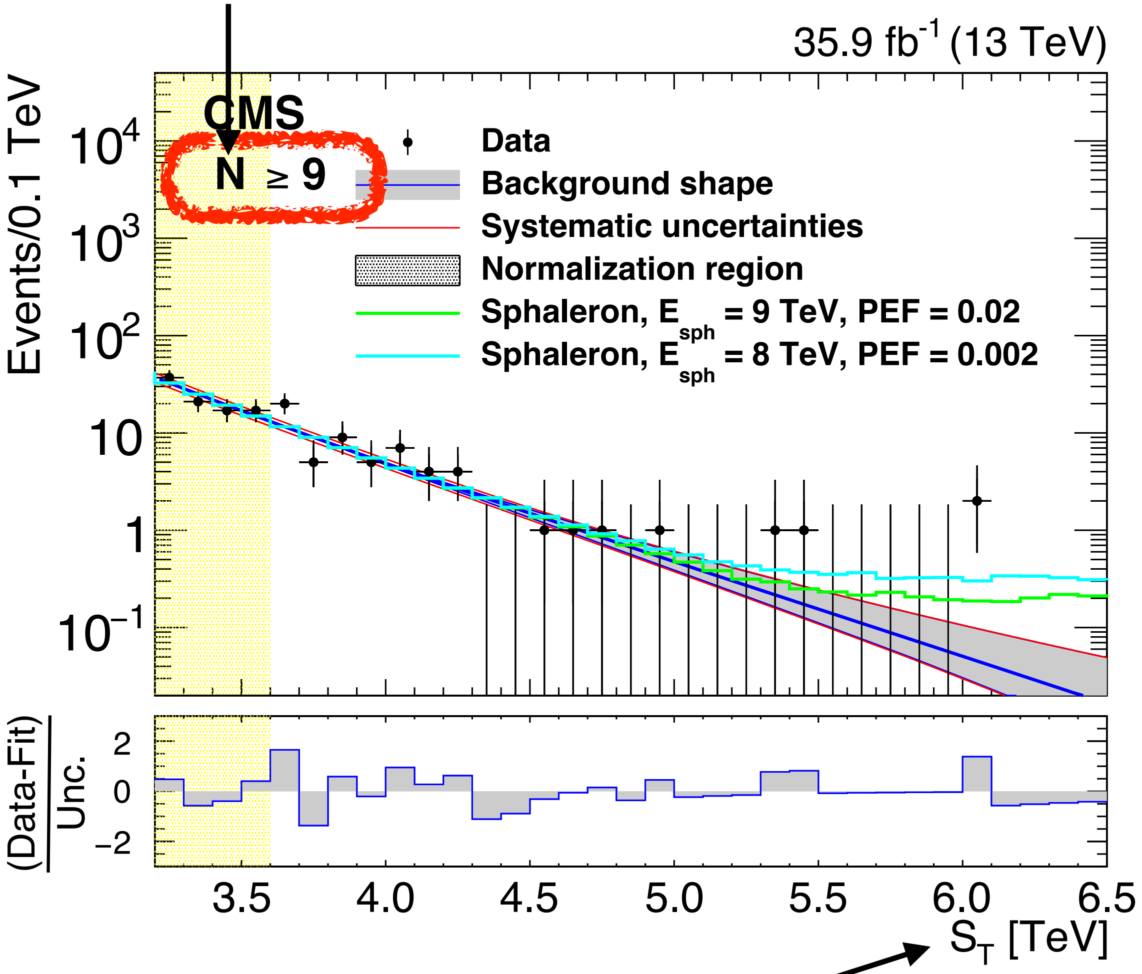
CMS-EXO-17-023

[1805.06013]

Search for black holes and sphalerons in high-multiplicity final states in proton-proton collisions at $\sqrt{s} = 13$ TeV[†]

$$\hat{\sigma}(\sqrt{\hat{s}}) = \frac{p_{\mathrm{sph}}}{m_W^2}\Theta(\sqrt{\hat{s}} - E_{\mathrm{thr}})$$

# of jets + leptons + photons



Sum of all pT in the final state