Phase transition in models with extra-dimensions

Norway grants

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Universe & Collider
Grieg

HR EXCELLENCE IN RESEARCH

Understanding the Early Universe:
interplay of theory and collider experiments

Joint research project between the University of Warsaw & University of Bergen

UNIVERSITY OF WARSAW

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CONTENTS

INTRO

- A concrete description for the Cosmological history of the Universe requires the understanding of undergoing Phase Transitions (PTs)

- During the Early Universe evolution, phase transitions can happen at various scales and take various forms: 1st or 2nd -order and crossover

- Matter - Anti matter asymmetry and Gravitational waves among others are indicators about the nature of the PT of the Early Universe

↓

It underwent a 1st order PT

INTRO

- Focus on 1st-order PTs: A metastable state is separated from an energetically favourable state by a potential barrier

$\downarrow$

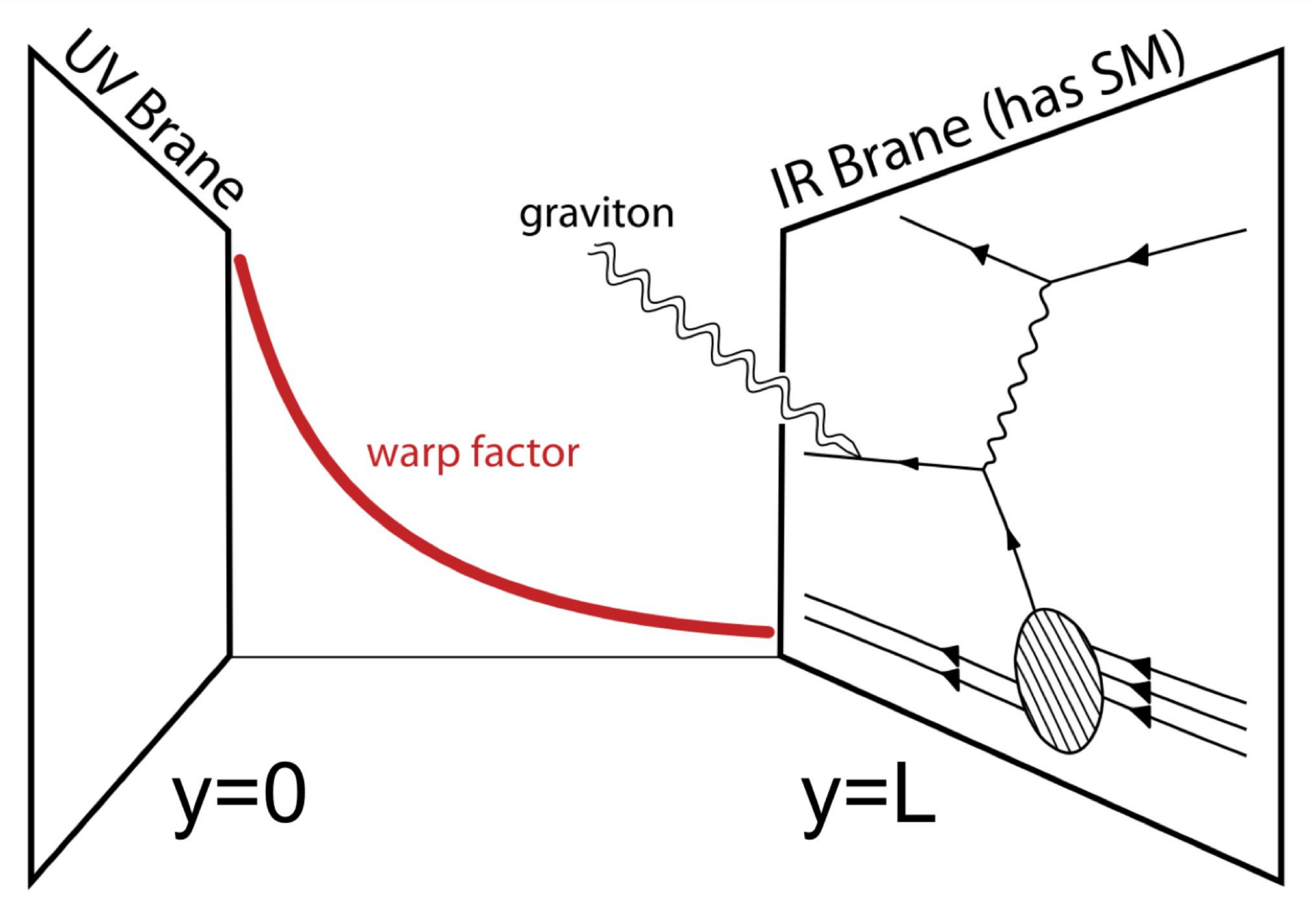
The transition proceeding through the nucleation of bubbles

- Such a 1st-order PT in the early universe naturally leads to the production of gravitational waves (bubble collisions)

- For temperature range $100\text{GeV} - 1\text{TeV}$ (ElectroWeak scale), the gravitational wave signal could lie in the frequency range of the upcoming space-based gravitational wave detector LISA

- How to study this 1st order PT? → The stabilized Randall-Sundrum (RS) model

L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370–3373
W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. 83 (1999) 4922–4925



1. *Single warped, spatial extra dimension*
2. *UV* (Planck) *and IR* (TeV) *brane*
3. *Standard Model is confined to IR brane*
4. *Why this model? Natural solution to the hierarchy problem*

Credit: J. P. Manuel et al, 2020

INTRO

- However RS by itself deviates from standard Cosmology already at order $\sim$ 1TeV

J. M. Cline et al, Phys. Rev. Lett. 83 (1999) 4245
P. Binetruy et al, Nucl. Phys. B565 (2000) 269–287
C. Csaki et al, Phys. Rev. D62 (2000) 045015
M. Peloso et al, Phys. Lett. B489 (2000) 411

- Describe RS under the framework of AdS/CFT correspondence

P. Creminelli, A. Nicolis and R. Rattazzi, JHEP 03 (2002) 051

- $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$: Relates a $(d+1)$-dimensional gravitational model described asymptotically by AdS space to a d-dimensional conformal field theory (strongly coupled)

J. M. Maldacena, Int. J. Theor. Phys. 38 (1999) 1113–1133

- Under the previous arguments the picture for the 1st-order PT is clear:

1. For $T > T_c$ the stable phase is described by an AdS_5 − Schwarzschild ($\mathrm{AdS}_{\mathrm{BH}}$) geometry

$$ds^2_{\mathrm{BH}} = e^{-2A(y)}(-h(y)dt^2 + \delta_{ij}dx^i dx^j) + h(y)^{-1}dy^2$$

The TeV brane is replaced by the horizon of a 5d Schwarzschild black hole (BH)

The Hawking temperature, T_H, of this black hole corresponds to the temperature of the 4d CFT.

2. For $T < T_c$ the RS model $\mathrm{AdS}_{\mathrm{RS}}$ becomes energetically favourable and a phase transition occurs

$$ds^2_{\mathrm{RS}} = e^{-2A(y)}(-dt^2 + \delta_{ij}dx^i dx^j) + dy^2$$

- The TeV brane replaces the black hole horizon

$<\chi> \neq 0$ → The distance between the two branes is stabilised

This phase transition is known as the holographic phase transition

- The dynamics of the holographic phase transition is well studied

G. Nardini, M. Quiros, and A. Wulzer, JHEP 09 (2007) 077
L. Randall and G. Servant, JHEP 05 (2007) 054
T. Konstandin, G. Nardini, and M. Quiros, Phys. Rev. D82 (2010) 083513
B. Bellazzini, C. Csaki, J. Hubisz, J. Serra and J. Terning, ArXiv:1305.3919
E. Megías, G. Nardini and M. Quirós, JHEP 95 (2018)
E. Megías, G. Nardini and M. Quirós, Phys. Rev. D 102 (5) (2020) 055004

- The plan:

*1. Stick to the **strong back-reaction**: $A(y)$ such that we cannot expand around AdS in the IR*

*2. Focus on the **pure soft-wall scenario**: Keep $|\Lambda_2| \equiv \Lambda$ high enough to push the IR brane towards the singularity*

*3. Consider 3 **benchmark potentials** compatible with 2.*

4. Match the results with the effective dilaton potential from the 4d point of view

5. Analyze the promptness of the 1st-order PT and its implications on the generated Stochastic Gravitational Wave Background (SGWB), in contact with the relevant parameters

A 4D EFFECTIVE THEORY

- Start analyzing the PT from the dilaton (radion's dual) point of view

- Confined phase: Assume an effective potential for the dilaton

$$\mathcal{L}_{\rm dil} = \frac{3N^2}{4\pi^2}\left(\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(\chi)\right) \qquad V(\chi) = \frac{\tilde{m}^2}{4}\chi^4\left(\log\frac{\chi}{\chi_0} - \frac{1}{4}\right)$$

P. Creminelli et al, '02
C. Csaki et al, '13

- $\chi_0 \equiv <\chi>$, $\tilde{m} = \frac{m_\chi}{\chi_0}$, $N >> 1$ the rank of $SU(N)$

- Deconfined phase: Harder to characterize. Assume negligible contribution to the bounce action

A 4D EFFECTIVE THEORY

- Only the potential in the minimum is needed

$$V_{BH} = -a\frac{\pi^2 N^2}{8}T^4$$

- a defined from AdS$_{\rm BH}$, $a \sim 1(10^{-2})$ for small (strong) back-reaction

M. Quirós et al, '18
A. Pomarol et al, '19

- $V(\chi_0) = V_{\rm BH}$ defines the critical temperature

$$T_c^4 = \frac{3\tilde{m}^2}{8\pi^4 a}\chi_0^4$$

- For T_n compute the $O(3)$ and $O(4)$ bounce actions $\frac{S_3}{T}$ and S_4

A 4D EFFECTIVE THEORY

- The bounce actions, the EOMs and BCs:

$$S_3 = 4\pi \frac{3N^2}{4\pi^2} \int dr r^2 \left(\frac{1}{2} \chi'(r)^2 + \bar{V}(\chi(r)) \right) \longrightarrow \frac{\mathrm{d}^2\chi}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\chi}{\mathrm{d}r} - V'(\chi) = 0$$

$$S_4 = 2\pi^2 \frac{3N^2}{4\pi^2} \int dr r^3 \left(\frac{1}{2} \chi'(r)^2 + \bar{V}(\chi(r)) \right) \longrightarrow \frac{\mathrm{d}^2\chi}{\mathrm{d}r^2} + \frac{3}{r}\frac{\mathrm{d}\chi}{\mathrm{d}r} - V'(\chi) = 0$$

$$\chi'|_{r=0} = 0 \qquad \chi'|^2_{\chi=0} = -\frac{8\pi^2}{3N^2} V_{BH} = \frac{a\pi^4}{3} T_n^4$$

- $\bar{V}(\chi) = V(\chi) + \frac{1}{2}\chi'^2|_{\chi=0}$, the independent parameters $m_\chi = \chi_0 \tilde{m}, N, a$

A 4D EFFECTIVE THEORY

- Calculating $\frac{T_n}{T_c} \equiv \frac{T_n}{T_c}(m_\chi, N, a)$ determines the promptness of the PT

- Defining $\hat{\chi} = \tilde{m}\chi$ and $V(\hat{\chi}) = \frac{1}{4}\hat{\chi}^4 \left(\log \frac{\hat{\chi}}{\hat{\chi}_0} - \frac{1}{4}\right) \longrightarrow$

$$\frac{d^2\hat{\chi}}{dr^2} + \frac{2}{r}\frac{d\hat{\chi}}{dr} + \hat{\chi}^3 \log \frac{\hat{\chi}}{\hat{\chi}_0} = 0 \qquad O(3)$$

$$\frac{d^2\hat{\chi}}{dr^2} + \frac{3}{r}\frac{d\hat{\chi}}{dr} + \hat{\chi}^3 \log \frac{\hat{\chi}}{\hat{\chi}_0} = 0 \qquad O(4)$$

- The solution is $\chi_{O(k)}(r) = \chi_0 f_{O(k)}\left(\tilde{\chi}_n; \tilde{m}\,\chi_0 r\right)$ with $\tilde{\chi} = \chi_n/\chi_0$, $k = 3,4$. The BCs give

$$T_n^4 = \frac{3}{a\pi^4}\tilde{m}^2\chi_0^4 f'^2_{O(k),0}(\tilde{\chi}_n)$$

$$\frac{T_n^4}{T_c^4} = 8 f'^2_{O(k),0}(\tilde{\chi}_n)$$

A 4D EFFECTIVE THEORY

- The total bounce actions

$$S_3 = \frac{2^{\frac{3}{4}} 3^{\frac{3}{4}} N^2 a^{\frac{1}{4}}}{\tilde{m}^{\frac{3}{2}}} T_c \tilde{S}_3(\tilde{\chi}_n) \qquad S_4 = \frac{3N^2}{2\tilde{m}^2} \tilde{S}_4(\tilde{\chi}_n)$$

$$\tilde{S}_k(\tilde{\chi}_n) = \int dx\, x^{k-1} \left(\frac{1}{2} \left(f'^2_{O(k)}(\tilde{\chi}_n; x) + f'^2_{O(k),0}(\tilde{\chi}_n) \right) + \frac{1}{4} f^4_{O(k)}(\tilde{\chi}_n; x) \left(\log f_{O(k)}(\tilde{\chi}_n; x) - \frac{1}{4} \right) \right)$$

- For $T_n \sim$ TeV, $\frac{S_3}{T_n}$ and $S_4 \sim 140$

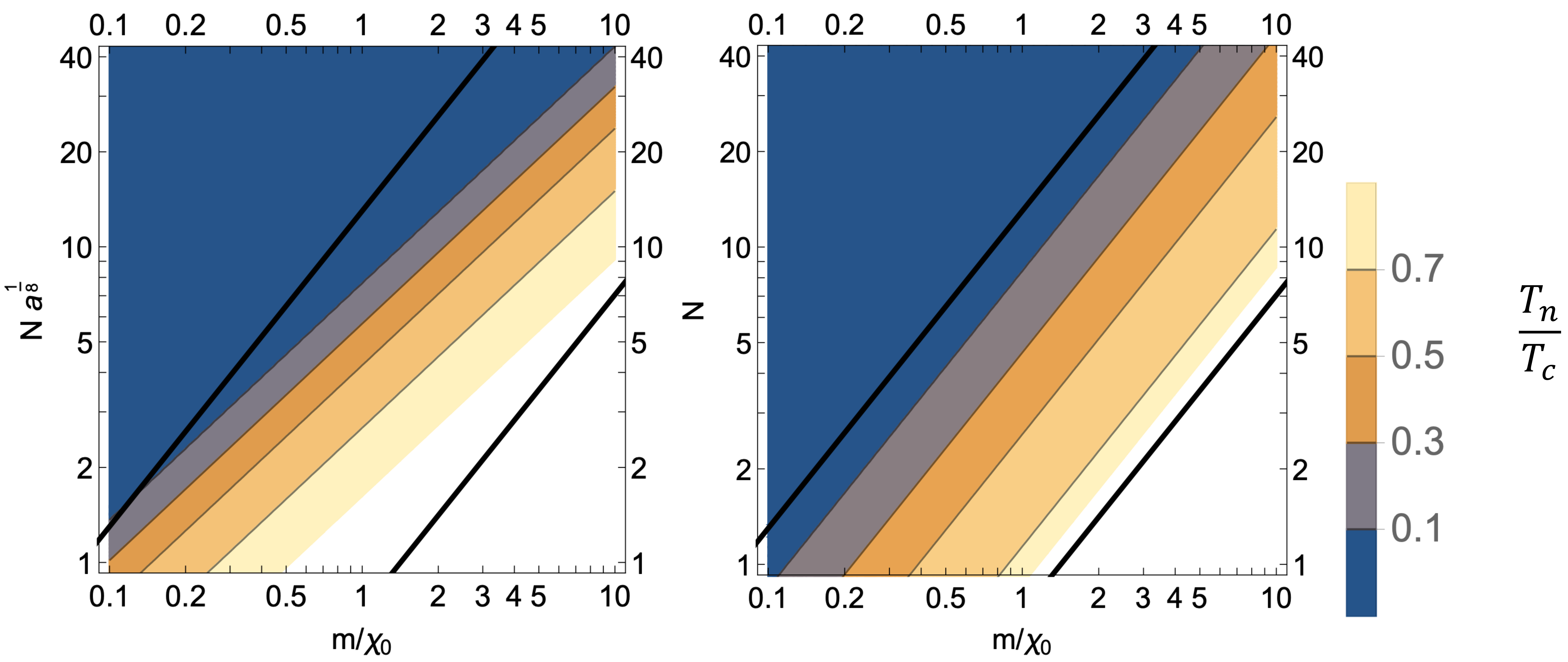
T. Konstandin, G. Nardini and M. Quiros, Phys. Rev. D82 (2010) 083513

D. Bunk, J. Hubisz and B. Jain Eur. Phys. J. C78 (2018) 78

- Given $\frac{T_n}{T_c}$ we get $\tilde{\chi}_n$ and

$$N^2 a^{\frac{1}{4}} = \frac{140}{2^{\frac{3}{4}} 3^{\frac{3}{4}}} \frac{T_n}{T_c} \frac{1}{\tilde{S}_3(\tilde{\chi}_n)} \tilde{m}^{\frac{3}{2}} \qquad N^2 = \frac{280}{3} \frac{1}{\tilde{S}_4(\tilde{\chi}_n)} \tilde{m}^2$$

A 4D EFFECTIVE THEORY



- For $0.65 \lesssim \frac{T_n}{T_c} \lesssim 0.8$ and $10 \leq N \leq 20$ ⟶ $\frac{m_\chi}{<\chi>} \gtrsim 4.3$

- What about the 5d picture and the realist scenarios that match the 4d picture?

- Confined phase: $ds^2_{\rm RS} = e^{-2A(y)}(-dt^2 + \delta_{ij}dx^i dx^j) + dy^2$

$$S_{\rm 5d} = S_{\rm GR} + S_\phi \longrightarrow$$

$$S_{\rm GR} = \frac{1}{2\kappa^2}\int d^5x\Big(\frac{\sqrt{-g}}{2}R_5 + \sum_{i=1,2}\sqrt{-\bar{g}}\big|_{y_i}\delta(y-y_i)K_i\Big)$$

$$S_\phi = \frac{1}{2}\int d^5x\Big[\sqrt{-g}\Big(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\Big) - \sum_{i=1,2}\sqrt{-\bar{g}}\big|_{y_i}\delta(y-y_i)U_i(\phi)\Big]$$

- $i = \{1,2\} \equiv \{\mathrm{UV}, \mathrm{IR}\}$, $K_{1,2}$ are the GHY boundary terms, $\kappa^2 = M_5^{-3} = \frac{4\pi^2}{N^2 k^3}$ from AdS/CFT, set $k = 1$

- $V(\phi)$ and $U_{1,2}(\phi)$ are the bulk and brane potentials of $\phi \equiv \phi(x, y)$. EOMs and BCs

$$
\begin{aligned}
A'' &= 2A'^2 + \frac{\kappa^2}{6}\phi'^2 + \frac{\kappa^2}{3}V \\
\phi'' &= 4A'\phi' + V' \\
A'^2 &= \frac{\kappa^2}{12}\phi'^2 - \frac{\kappa^2}{6}V
\end{aligned}
\qquad
\begin{aligned}
\lim_{y\to y_i^\pm} \phi' &= \pm\frac{1}{2}U_i'(\phi(y_i)) \\
\lim_{y\to y_i^\pm} A' &= \pm\frac{\kappa^2}{6}U_i(\phi(y_i))
\end{aligned}
$$

$$
A(y) \underset{y\to-\infty}{\sim} ky \qquad \lim_{y\to-\infty} V(\phi(y)) = -6M_5^3 k^2
$$

PHASE TRANSITIONS IN SOFT-WALL MODEL

- Solving the EOMs using the superpotential method

$$V(\phi) = \frac{1}{8}\left(\frac{\partial W}{\partial \phi}\right)^2 - \frac{1}{6M_5^3}W^2(\phi) \qquad \phi' = \frac{1}{2}\frac{\mathrm{d}W}{\mathrm{d}\phi}, \qquad A' = \frac{1}{6M_5^3}W$$

- Focus on **stiff-wall** approximation: $\phi'(y_i) = v_i, A'(y_i) = \mp\frac{\kappa^2}{6}|\Lambda_i|$

- Focus on **pure soft-wall** models: $V(\phi \to \infty)$ strongly affects the solution

$$V(\phi) \underset{\phi\to\infty}{\sim} \begin{cases} \phi^2 \\ \phi^4 \\ e^{2\gamma\phi} \end{cases}$$

PHASE TRANSITIONS IN SOFT-WALL MODEL

- 2 choices for the superpotential

$$W_0 \underset{\phi\to\infty}{\sim} A\sqrt{-V}$$

$$W \underset{\phi\to\infty}{\sim} \exp\left[\frac{2}{\sqrt{3}} M_5^{-3/2}\phi\right]$$

$\longrightarrow\ T_c \neq 0$

$$\begin{aligned}
W^{(2)}(\phi,\gamma,N) &= \frac{6N^2}{4\pi^2}\sqrt{1+\gamma\phi^2(y)} \\
W^{(4)}(\phi,\gamma,N) &= \frac{6N^2}{4\pi^2}\left(1+\gamma\phi^2(y)\right) \\
W^{(e)}(\phi,\gamma,N) &= \frac{6N^2}{4\pi^2}\left(1+e^{\gamma\phi(y)}\right)
\end{aligned}$$

$\xrightarrow{|\Lambda_2| > W(v_2)}\ T_c \neq 0$

$\longrightarrow$

$$\begin{aligned}
V^{(2)}(\phi,\gamma,N) &= \frac{-6N^2}{4\pi^2}\left[1-\frac{3N^2}{16\pi^2}\frac{\gamma^4\phi^2(y)}{1+\gamma^2\phi^2(y)}+\gamma^2\phi^2(y)\right] \\
V^{(4)}(\phi,\gamma,N) &= \frac{-6N^2}{4\pi^2}\left[1+\left(2\gamma^2-\frac{3N^2}{4\pi^2}\gamma^4\right)\phi^2(y)+\gamma^4\phi^4(y)\right] \\
V^{(e)}(\phi,\gamma,N) &= \frac{-6N^2}{4\pi^2}\left[1+2e^{\gamma\phi(y)}+\left(1-\frac{3N^2}{16\pi^2}\gamma^2\right)e^{2\gamma\phi(y)}\right]
\end{aligned}$$

PHASE TRANSITIONS IN SOFT-WALL MODEL

- Asymptotic behaviour

$$V^{(2)}(\phi) \underset{\phi\to\infty}{\sim} -6k^2M_5^3\gamma^2\phi^2$$

$$V^{(4)}(\phi) \underset{\phi\to\infty}{\sim} -6k^2M_5^3\gamma^4\phi^4$$

$$V^{(e)}(\phi) \underset{\phi\to\infty}{\sim} -6k^2M_5^3\left(1-\frac{3}{4}M^3\gamma^2\right)e^{2\gamma\phi}$$

- The 4d effective Lagrangian: $\mathcal{L}_{\text{eff}} = -V_{\text{eff}}(\chi) - \frac{C(\chi)}{2}(\partial\chi)^2 + O(\partial^4)$

↓

J. M. Lizana, M. Olechowski and S. Pokorski JHEP 09 (2020) 092

- $\phi(y) \equiv \phi(y, N, \gamma, \Lambda, v_2)$ and $A(y) \equiv A(y, N, \gamma, \Lambda, v_2)$

$$\mathcal{L}_{\text{eff}} = -V_{\text{eff}}(\chi) - \frac{C(\chi)}{2}(\partial\chi)^2 + O(\partial^4)$$

↓

$$V_{\text{eff}} = \frac{1}{2}\left(-6M_5^3 A' + \Lambda_1\right)\Big|_{y_1^+} + \frac{1}{2}\, e^{-4(A(y_2)-A(y_1))}\left(6M_5^3 A' + \Lambda_2\right)\Big|_{y_2^-}$$

$$C = -\frac{18M_5^6}{k^2}\left(\int_{y_1}^{y_2} \mathrm{d}s\; e^{4A-4A(y_2)} \frac{V(\phi)}{\phi'^2}\Big|_s\right)^{-1} \int_{y_1}^{y_2} \mathrm{d}s\; e^{4A-4A(y_2)} \frac{A'P}{\phi'^2}\Big|_s$$

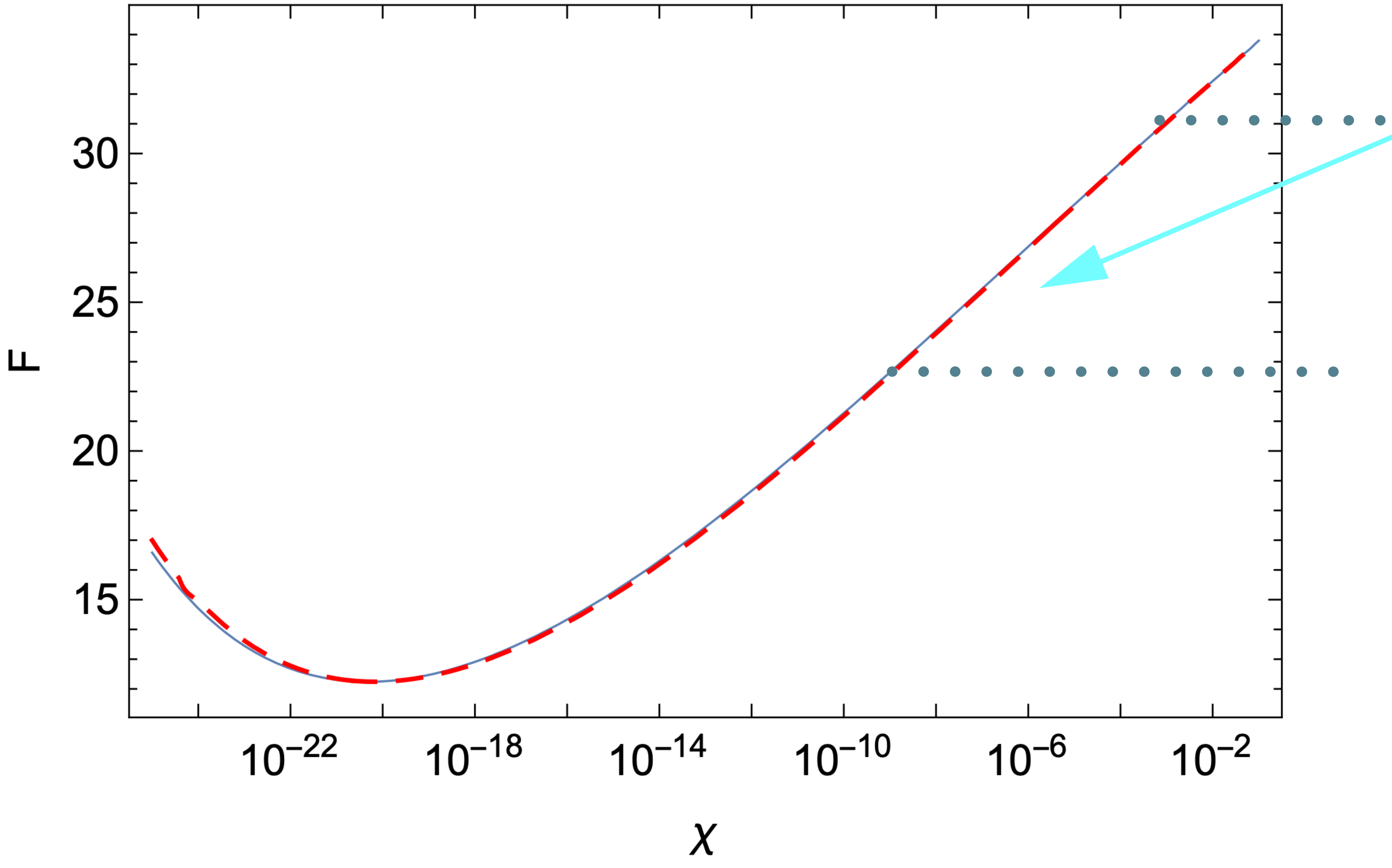
- The general form of the effective potential is

$$V_{\text{eff}}(\chi) = F(\chi)\,\chi^4$$

PHASE TRANSITIONS IN SOFT-WALL MODEL

- The function $F(\chi)$ shows when $V_{\text{eff}}(\chi)$ matches $V(\chi)$

$$|\Lambda_2| \equiv \Lambda >> W^{(2,4,e)}(v_2) \rightarrow F(\chi) \sim \ln\chi$$



$$V_{\text{eff}}(\chi) \rightarrow V(\chi)$$

$$W \underset{\phi\rightarrow\infty}{\sim} \exp\left[\frac{2}{\sqrt{3}} M_5^{-3/2}\phi\right]$$

IR brane shifts towards the singularity (soft-wall limit)

PHASE TRANSITIONS IN SOFT-WALL MODEL

- For which parameters is the PT prompt?

1. Fix $v_2 >> 1$

- The promptness depends on N, γ, Λ and v_2:

2. Vary $(W^{(2,4,e)}(v_2) <<) a_1 \leq \Lambda \leq a_2$

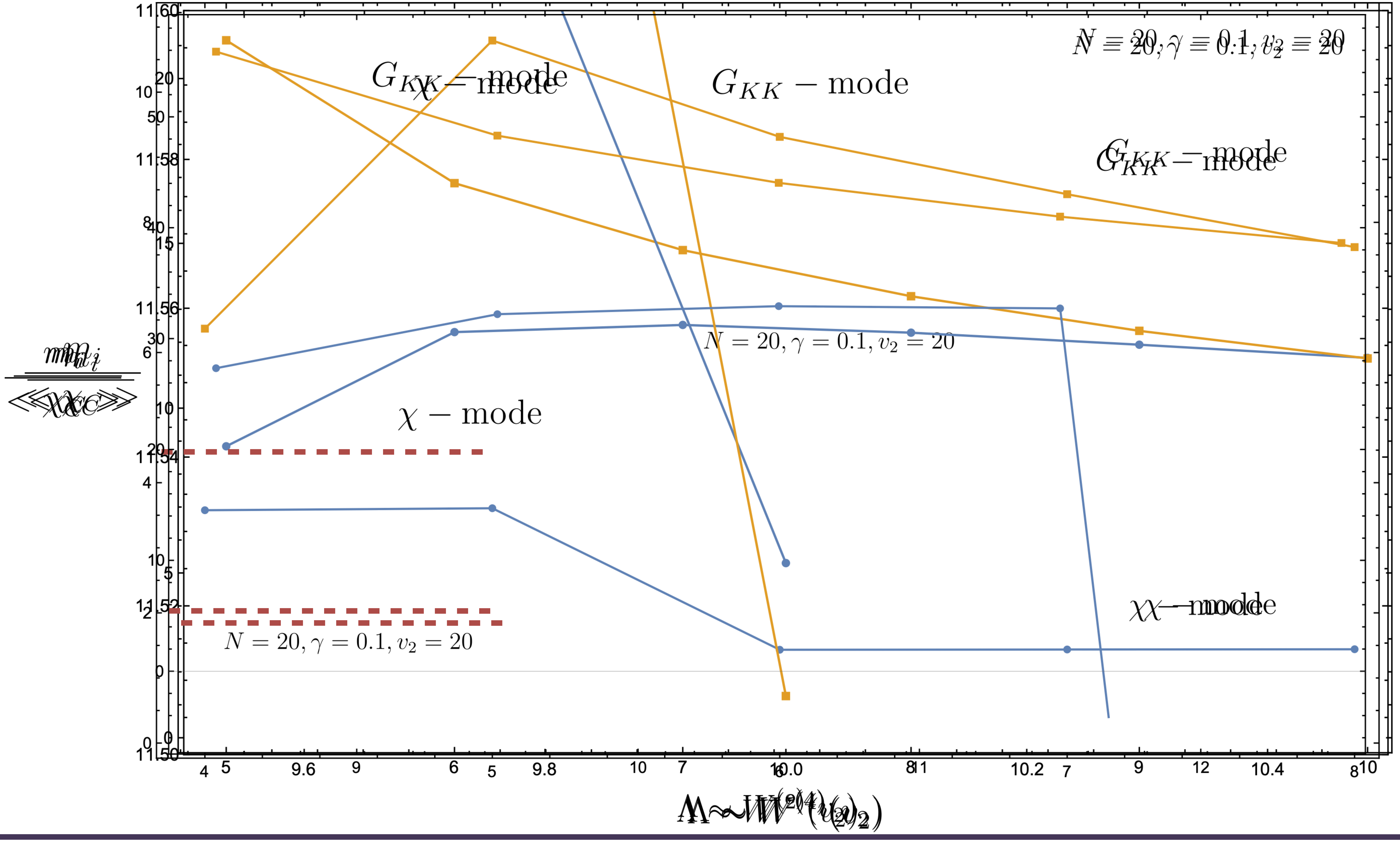
3. $N = \{10,20\}$ *and* $\gamma = \{0.01,0.03,0.1\}$

- Constraints:

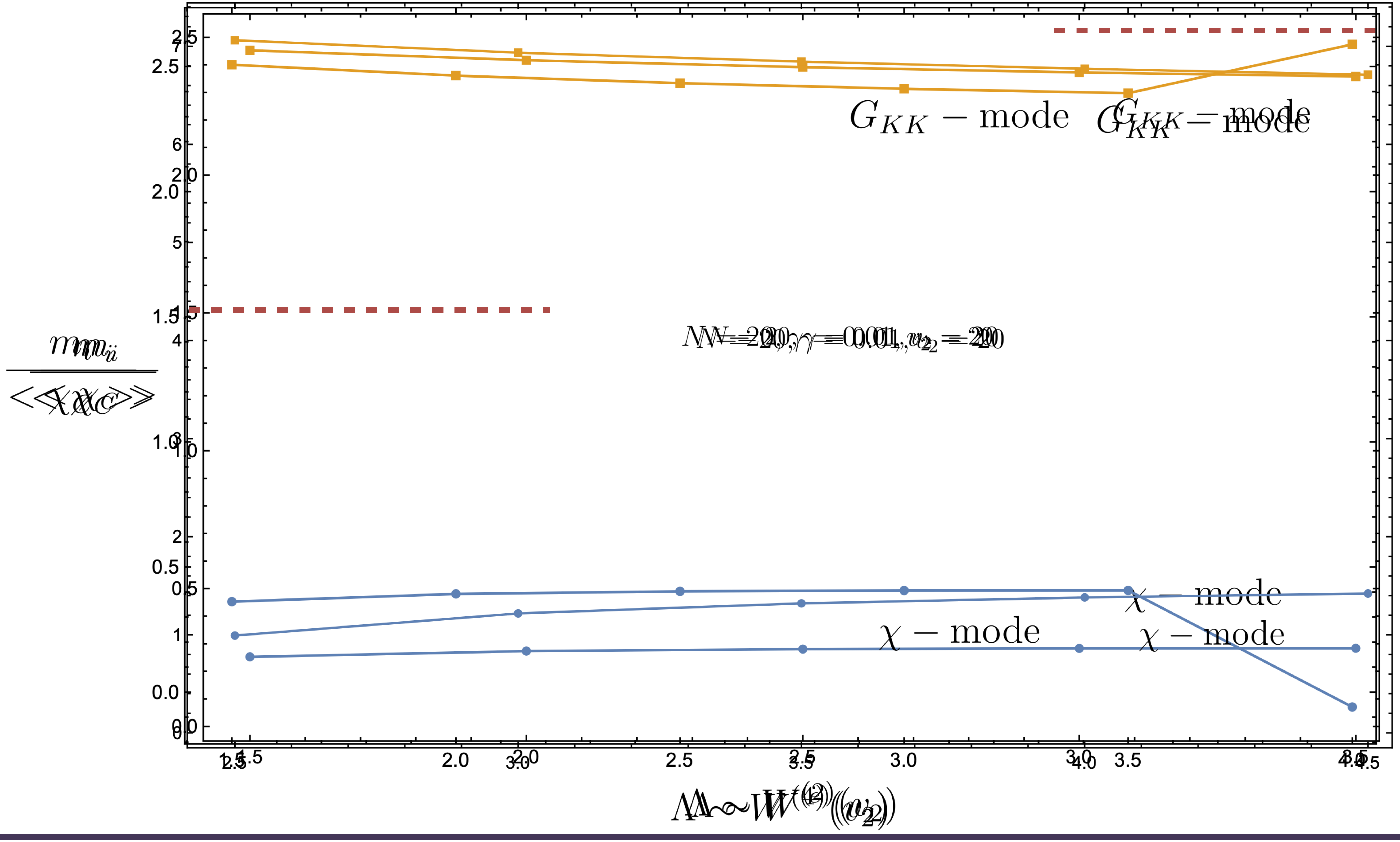
1. Hierarchy of mass scales: $m_\chi(N,\gamma,\Lambda,v_2) < m_{KK}^G(N,\gamma,\Lambda,v_2) < m_{KK}^h(N,\gamma,\Lambda,v_2)$

2. Positivity condition $\phi(y,N,\gamma,\Lambda,v_2) > 0$

- Find $\frac{m_\chi}{<\chi>}$ as a function of Λ



$G_{KK} - \text{mode}$
$\chi - \text{mode}$
$N = 20, \gamma = 0.1, v_2 = 20$


G_{KK} − mode
G_{KK} − mode
χ − mode
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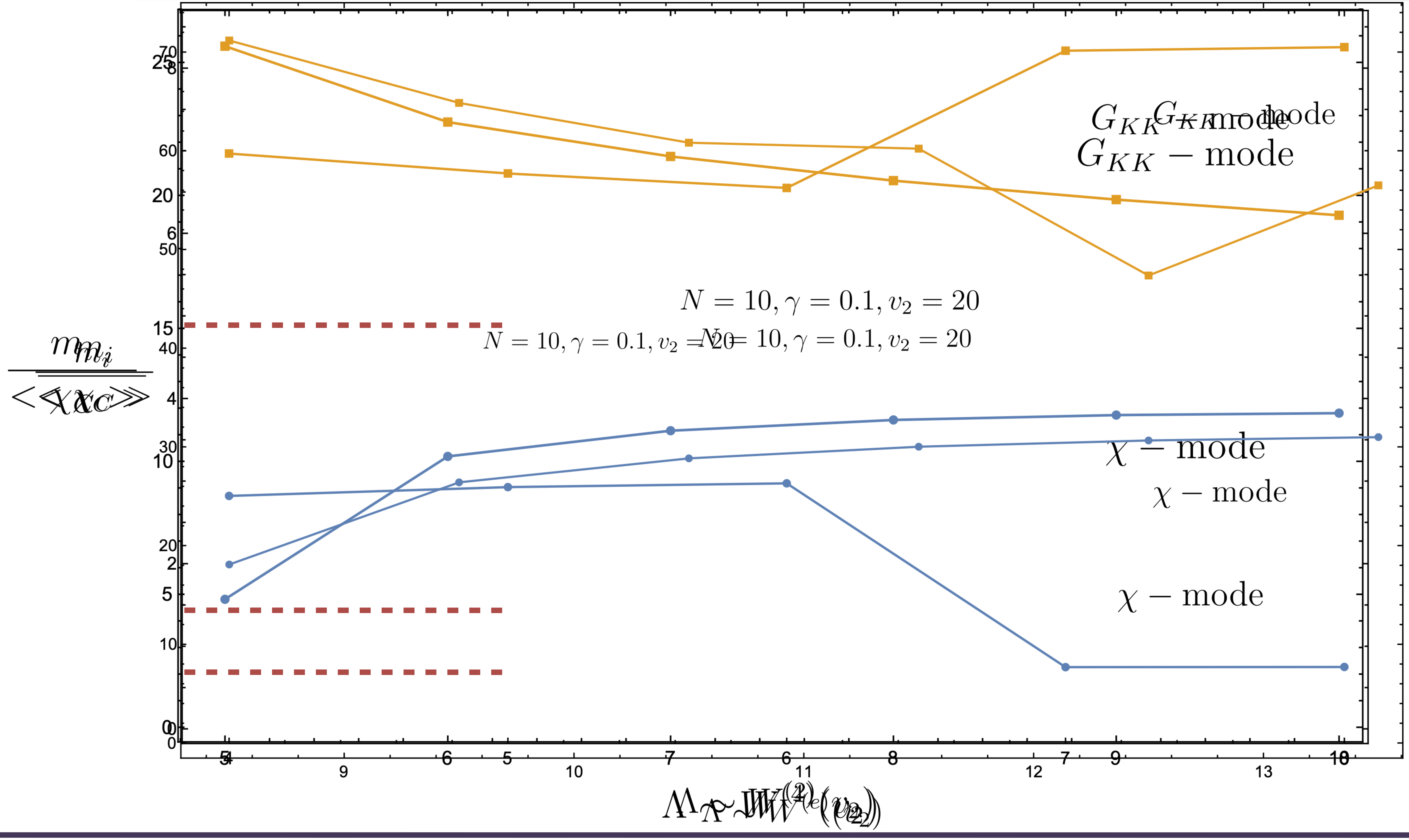
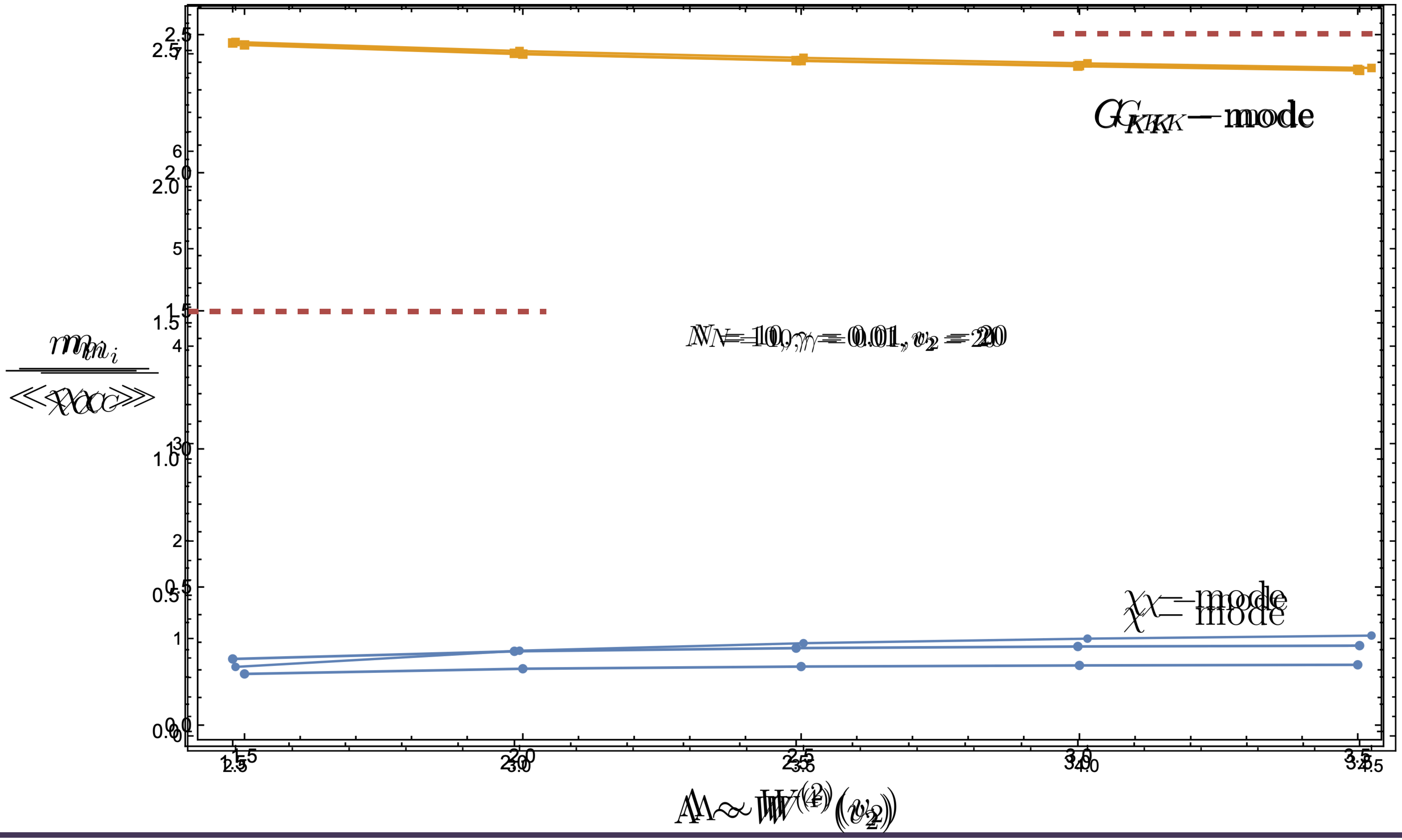
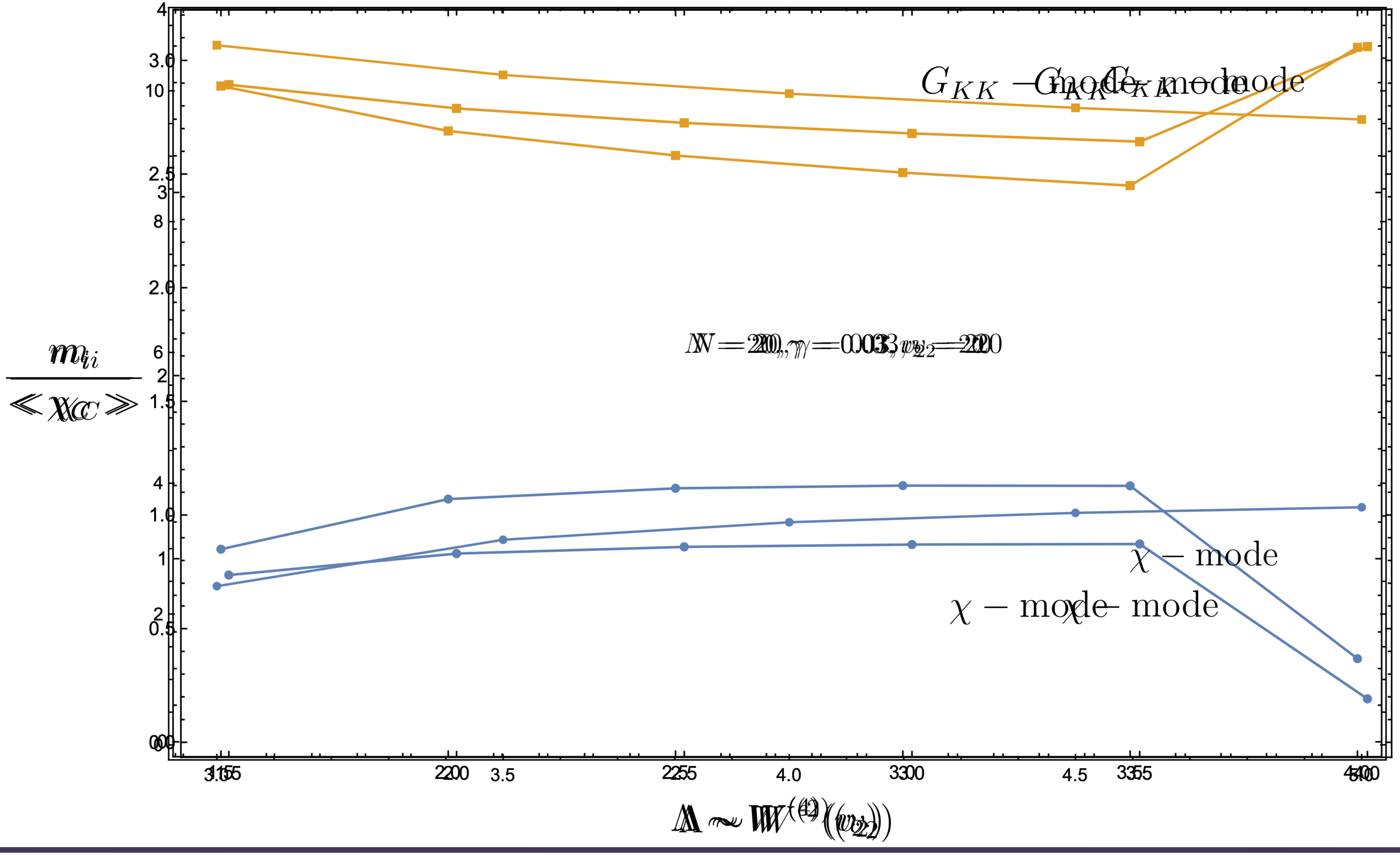

G_KK − mode
N = 10, γ = 0.1, v_2 = 20
χ − mode
χ − mode
χ − mode


$G_{KK}-$mode
$\chi-$mode
$N=10, \gamma=0.01, v_2=20$



G_{KK} − mode
$N = 20, \gamma = 0.03, v_2 = 20$
χ − mode
χ − mode
$\frac{m_i}{\langle \chi_C \rangle}$
$\Lambda \sim W^{(4)}(v_2)$

FUTURE STEP

- What we learn from the deconfined phase?

$$ds^2_{\mathrm{BH}} = e^{-2A(y)}\left(-h(y)dt^2 + \delta_{ij}dx^i dx^j\right) + h(y)^{-1}dy^2$$

- EOMs again

$$\begin{aligned} A'' &= \frac{4\pi^2}{3N}\phi'^2 \\ A'^2 &= \frac{h'}{4h}A' - \frac{4\pi^2}{6N^2k^3}\frac{V}{h} + \frac{4\pi^2}{12N^2k^3}\phi'^2 \\ h'' &= 4A'h' \end{aligned}$$

$\rightarrow$ $h(y, N, \gamma, \Lambda, v_2)$

$$T_H = \frac{e^{-A(y_h)}}{4\pi}h'(y)\Big|_{y=y_h}$$

$$a(y_h) = \left|\frac{4k^3}{h'(y_h)}\right| = \left|\frac{4}{h'(y_h)}\right|^3$$

$$a(y_h, N, \gamma, \Lambda, v_2) \simeq 0.03$$

FUTURE STEP

- What we learn from the deconfined phase?

$$ds^2_{\rm BH} = e^{-2A(y)}(-h(y)dt^2 + \delta_{ij}dx^i dx^j) + h(y)^{-1}dy^2$$

- EOMs again

$$\begin{aligned} A'' &= \frac{4\pi^2}{3N}\phi'^2 \\ A'^2 &= \frac{h'}{4h}A' - \frac{4\pi^2}{6N^2k^3}\frac{V}{h} + \frac{4\pi^2}{12N^2k^3}\phi'^2 \\ h'' &= 4A'h' \end{aligned}$$

$\longrightarrow$ $h(y, N, \gamma, \Lambda, v_2)$

$$T_H = \frac{e^{-A(y_h)}}{4\pi}h'(y)\Big|_{y=y_h}$$

$$a(y_h) = \left|\frac{4k}{h'(y_h)}\right|^3 \equiv \left|\frac{4}{h'(y_h)}\right|^3$$

$a(y_h, N, \gamma, \Lambda, v_2) \approx 0.03$

FUTURE STEP

- Deconfined/Confined 1st order PT produces SGWB

- SGWB depends on the PT paramters

C. Caprini et al, JCAP 1604 (2016) 001

- 3 main contributions

1. *Collision of expanding bubble walls and shocks in the plasma*
2. *Sound waves left in the plasma after bubble collision*
3. *Magnetohydrodynamic (MHD) turbulence forming in the plasma after bubble collision*

$$h^2\Omega(f) \approx h^2\Omega_{\rm col}(f) + h^2\Omega_{\rm sw}(f) + h^2\Omega_{\rm turb}(f)$$

FUTURE STEP

- Consider the envelope approximation for the power spectrum

$$h^2\Omega_{\rm env}(f) = 1.67\times 10^{-5}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa_{\rm env}\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3}\left(\frac{0.11v_w^3}{0.42+v_w^2}\right)S_{\rm env}(f)$$

$$S_{\rm env}(f) = \frac{3.8(f/f_{\rm env})^{2.8}}{1+2.8(f/f_{\rm env})^{3.8}} \qquad f_{\rm env} = 16.5\,\mu{\rm Hz}\left(\frac{f_*}{\beta}\right)\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}$$

A. Kosowsky, '92
C. Caprini, R. Durrer, and G. Servant,'08
J. Huber and T. Konstandin, '08

- Bigger $\alpha \equiv \alpha(T_n)$ and smaller $\frac{\beta}{H_*}$, the stronger the SGWB signal

CONCLUSIONS

- Cosmological phase transitions are studied using the stabilized Randall-Sundrum model under the framework of AdS/CFT. This model exhibits a 1st-order phase transition between an AdS_5 −Schwarzschild geometry for $T > T_c$ to the usual Randall-Sundrum for $T < T_c$

- From the boundary point of view the construction of an effective 4d action for the dilaton (radion's dual) field gives us an estimate about the promptness of the phase transition through the m_χ as a function of $\frac{T_n}{T_c}, N, a$.

- On the 5d model we focus on the pure soft-wall limit and choosing 3 benchmark potentials we solve the EOMs for the gravity-scalar system.

- Then we obtain the effective action and estimate numerically the mass of the radion which tells us for which potentials what range of parameters the phase transition is prompt

- Future step is to incorporate the obtained technology to the analysis of the stochastic gravitational waves background derived from the $\mathrm{AdS}_{\mathrm{BH}} - \mathrm{AdS}_{\mathrm{RS}}$ 1st-order phase transition

THANK YOU