Exploring new avenues to probe CP violation in τ Yukawa interaction

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(Based on ongoing work with Janusz Rosiek and Stefan Pokorski)

Conference of Norwegian Financial Mechanism
“Early Universe” project

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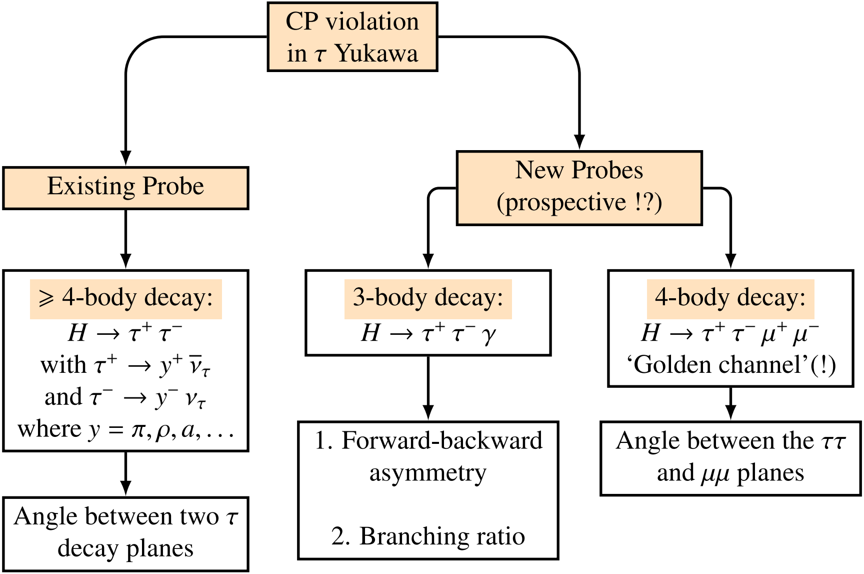
Overview



CP violation in τ Yukawa

Existing Probe

$\geqslant$ 4-body decay:
$H \to \tau^+ \tau^-$
with $\tau^+ \to y^+ \bar{\nu}_\tau$
and $\tau^- \to y^- \nu_\tau$
where $y = \pi, \rho, a, \ldots$

Angle between two τ decay planes

New Probes (prospective !?)

3-body decay:
$H \to \tau^+ \tau^- \gamma$

1. Forward-backward asymmetry
2. Branching ratio

4-body decay:
$H \to \tau^+ \tau^- \mu^+ \mu^-$
'Golden channel'(!)

Angle between the $\tau\tau$ and $\mu\mu$ planes

CP violating Lagrangian

- CP violating $H\tau\tau$ Yukawa interaction is written using various notations in the literature. For simplicity we shall use the following,

$$\mathscr{L}_{H\tau\tau} = -\frac{m_\tau}{v}\,\bar{\tau}\left(a_\tau + i\,\gamma^5\,b_\tau\right)\tau\,H\,,$$

where $v = \left(\sqrt{2}\,G_F\right)^{-1/2} \simeq 246$ GeV, and $a_\tau^{\text{SM}} = 1$, $b_\tau^{\text{SM}} = 0$ in the SM.

- $b_\tau \neq 0 \implies$ CP violation. Both a_τ and b_τ are real.

- Measurement of e^- EDM suggest[1]: $|b_\tau| \lesssim 0.29$ at 90% C.L.

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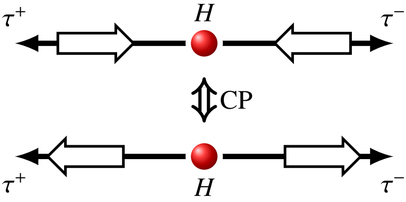
[1] J. Alonso-Gonzalez, A. de Giorgi, L. Merlo and S. Pokorski, JHEP **05**, 041 (2022).

The 2-body decay $H \to \tau^+ \tau^-$



- Branching ratio in SM: $\sim 6.15\%$
- Energies and momenta of $\tau^\pm$ fixed in H rest frame.
- Very highly boosted τs:

  $\beta_\tau = 0.99960\, c$.

- Only 2 helicity configurations allowed: $\tau^+_L \tau^-_L \xleftrightarrow{\text{CP}} \tau^+_R \tau^-_R$.
- Partial decay rate: $\Gamma_{\tau\tau} = \dfrac{m_H}{8\,\pi} \dfrac{m_\tau^2}{v^2} \left(a_\tau^2 \left(1 - \dfrac{4\, m_\tau^2}{m_H^2} \right) + b_\tau^2 \right) \sqrt{1 - \dfrac{4\, m_\tau^2}{m_H^2}}$.

  Constraint: $a_\tau^2 + b_\tau^2 \simeq 1$. Experimentally[2] $0.99 \lesssim a_\tau^2 + b_\tau^2 \lesssim 1.01$

- Both helicity configurations equally likely:

$$|\mathscr{M}_{++}|^2 = |\mathscr{M}_{--}|^2 = \left(\frac{m_\tau}{v} \right)^2 \left[\left(a_\tau^2 + b_\tau^2 \right) m_H^2 - 4\, a_\tau^2 m_\tau^2 \right].$$

$\therefore$ No way to measure CP violation, if we study this 2-body decay only.

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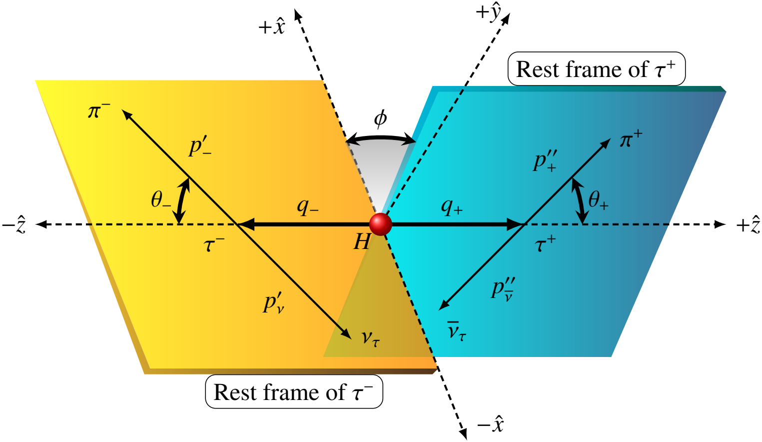
[2] J. Alonso-Gonzalez, A. de Giorgi, L. Merlo and S. Pokorski, JHEP **05**, 041 (2022).

The 4-body decay $H \to \tau^+ \tau^- \to \pi^+ \pi^- \nu_\tau \bar{\nu}_\tau$

- Final state has two missing particles: τ reconstruction issues
- Much richer kinematics: 3 uni-angular distributions possible



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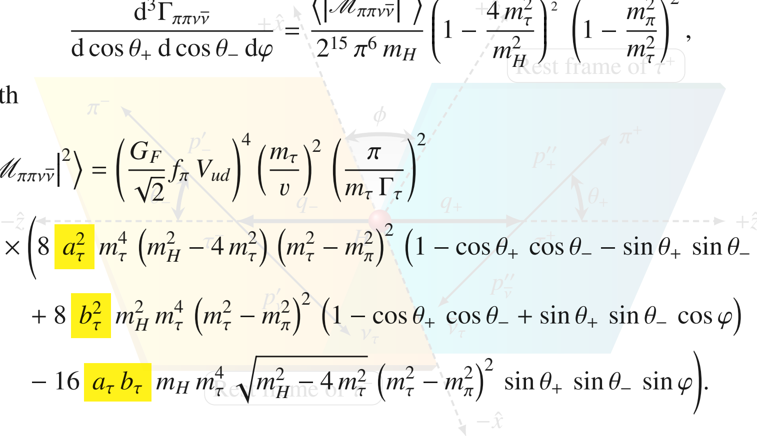


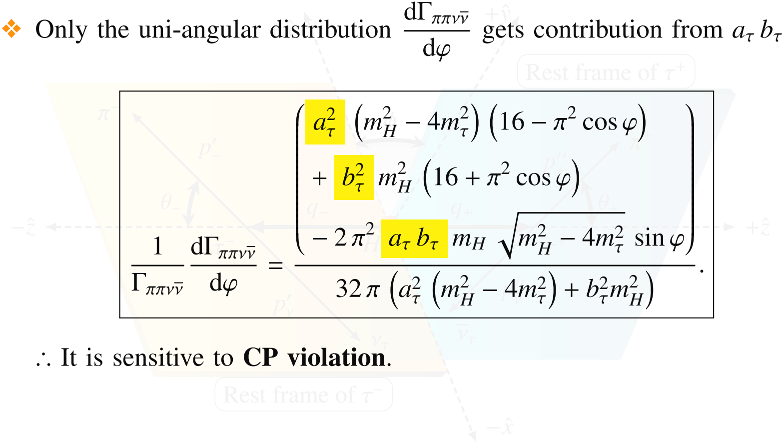
$$\frac{\mathrm{d}^3\Gamma_{\pi\pi\nu\bar{\nu}}}{\mathrm{d}\cos\theta_+\,\mathrm{d}\cos\theta_-\,\mathrm{d}\varphi} = \frac{\left\langle\left|\mathscr{M}_{\pi\pi\nu\bar{\nu}}\right|^2\right\rangle}{2^{15}\,\pi^6\,m_H}\left(1-\frac{4\,m_\tau^2}{m_H^2}\right)^{\frac{1}{2}}\left(1-\frac{m_\pi^2}{m_\tau^2}\right)^2,$$

with

$$\begin{aligned}\left\langle\left|\mathscr{M}_{\pi\pi\nu\bar{\nu}}\right|^2\right\rangle = {} & \left(\frac{G_F}{\sqrt{2}}f_\pi\,V_{ud}\right)^4\left(\frac{m_\tau}{v}\right)^2\left(\frac{\pi}{m_\tau\,\Gamma_\tau}\right)^2 \\ & \times\Big(8\,a_\tau^2\,m_\tau^4\left(m_H^2-4\,m_\tau^2\right)\left(m_\tau^2-m_\pi^2\right)^2\left(1-\cos\theta_+\,\cos\theta_--\sin\theta_+\,\sin\theta_-\,\cos\varphi\right) \\ & \quad +8\,b_\tau^2\,m_H^2\,m_\tau^4\left(m_\tau^2-m_\pi^2\right)^2\left(1-\cos\theta_+\,\cos\theta_-+\sin\theta_+\,\sin\theta_-\,\cos\varphi\right) \\ & \quad -16\,a_\tau\,b_\tau\,m_H\,m_\tau^4\sqrt{m_H^2-4\,m_\tau^2}\left(m_\tau^2-m_\pi^2\right)^2\,\sin\theta_+\,\sin\theta_-\,\sin\varphi\Big).\end{aligned}$$

The 4-body decay $H \to \tau^+ \tau^- \to \pi^+ \pi^- \nu_\tau \bar{\nu}_\tau$

- Final state has two missing particles: τ reconstruction issues
- Much richer kinematics: 3 uni-angular distributions possible

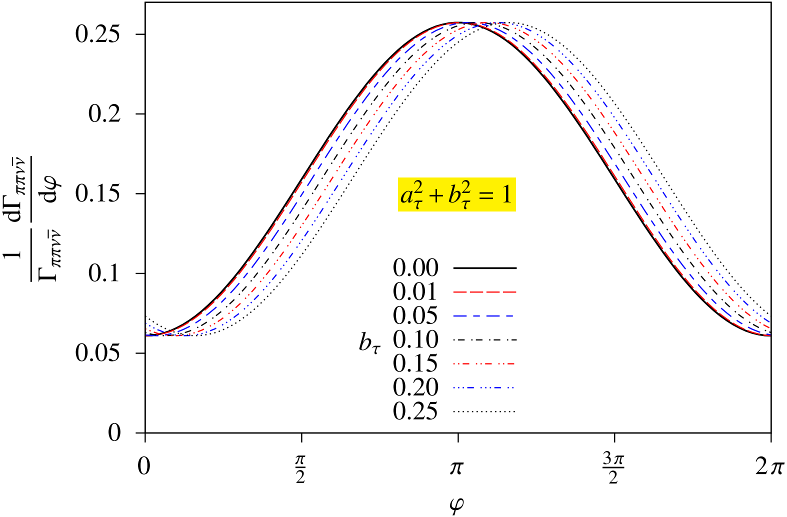


- Only the uni-angular distribution $\frac{\mathrm{d}\Gamma_{\pi\pi\nu\bar{\nu}}}{\mathrm{d}\varphi}$ gets contribution from $a_\tau\, b_\tau$.

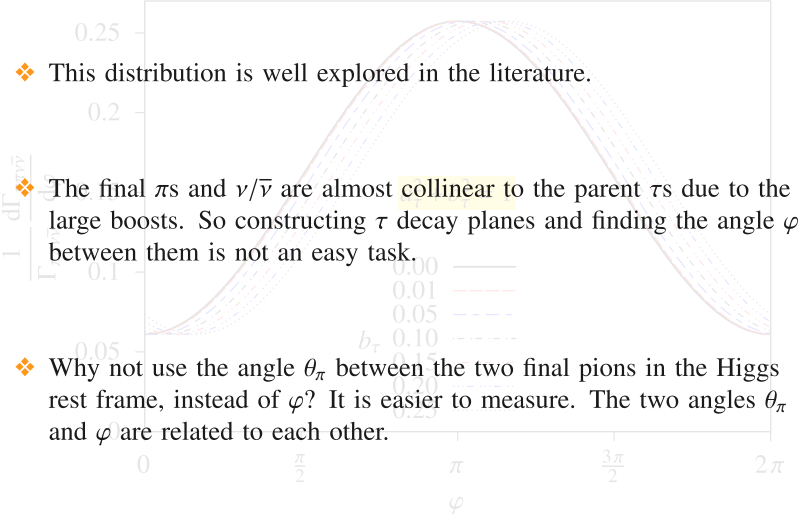
$$\boxed{\frac{1}{\Gamma_{\pi\pi\nu\bar{\nu}}}\frac{\mathrm{d}\Gamma_{\pi\pi\nu\bar{\nu}}}{\mathrm{d}\varphi} = \frac{\begin{pmatrix} a_\tau^2 \left(m_H^2 - 4m_\tau^2\right)\left(16 - \pi^2 \cos\varphi\right) \\ + b_\tau^2\, m_H^2 \left(16 + \pi^2 \cos\varphi\right) \\ -\, 2\,\pi^2\, a_\tau\, b_\tau\, m_H \sqrt{m_H^2 - 4m_\tau^2}\, \sin\varphi \end{pmatrix}}{32\,\pi \left(a_\tau^2 \left(m_H^2 - 4m_\tau^2\right) + b_\tau^2 m_H^2\right)}.}$$

$\therefore$ It is sensitive to **CP violation**.

The 4-body decay $H \to \tau^+ \tau^- \to \pi^+ \pi^- \nu_\tau \bar{\nu}_\tau$



The 4-body decay $H \to \tau^+ \tau^- \to \pi^+ \pi^- \nu_\tau \bar{\nu}_\tau$



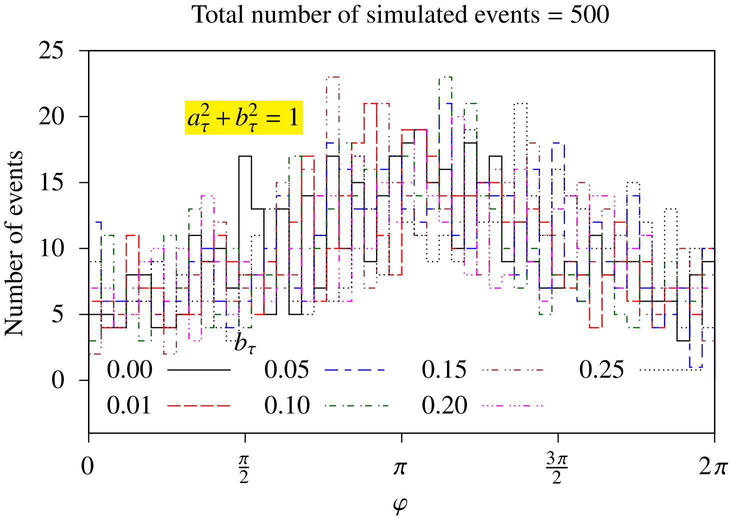
- This distribution is well explored in the literature.

- The final πs and $\nu/\bar{\nu}$ are almost collinear to the parent τs due to the large boosts. So constructing τ decay planes and finding the angle φ between them is not an easy task.

- Why not use the angle θ_π between the two final pions in the Higgs rest frame, instead of φ? It is easier to measure. The two angles θ_π and φ are related to each other.

The 4-body decay $H \to \tau^+ \tau^- \to \pi^+ \pi^- \nu_\tau \bar{\nu}_\tau$

Comparison of φ and $\cos\theta_\pi$ distributions



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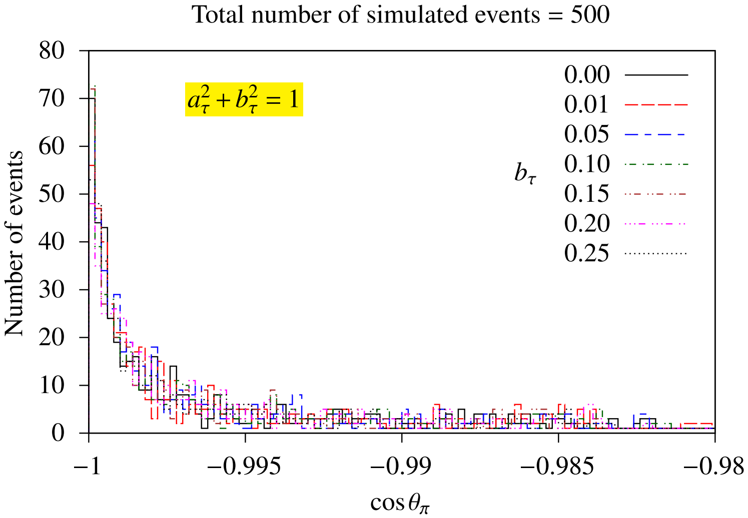


The 4-body decay $H \to \tau^+ \tau^- \to \pi^+ \pi^- \nu_\tau \bar{\nu}_\tau$

Comparison of φ and $\cos\theta_\pi$ distributions

- Numerical comparison of the two angular distributions vindicates the choice of φ. The $\cos\theta_\pi$ distributions for different a_τ and b_τ are extremely closely spaced with peaks close to $\theta_\pi = 180°$ and no significant differences can be noticed.

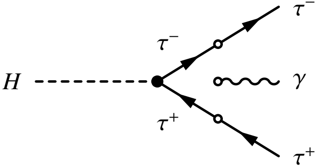
- The final state π's could be replaced by ρ, a mesons which decay to two or three pions. Such studies have already been considered in the literature.

- We do not have any new meaningful observable in this scenario. Only experimental studies with more statistics, better angular resolutions, seem to be the way forward.

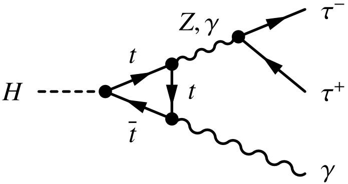
The 3-body decay $H \to \tau^+ \tau^- \gamma$

The idea

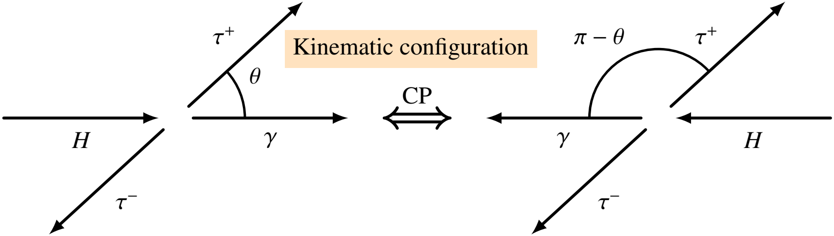
Tree level contribution



1-loop level contribution

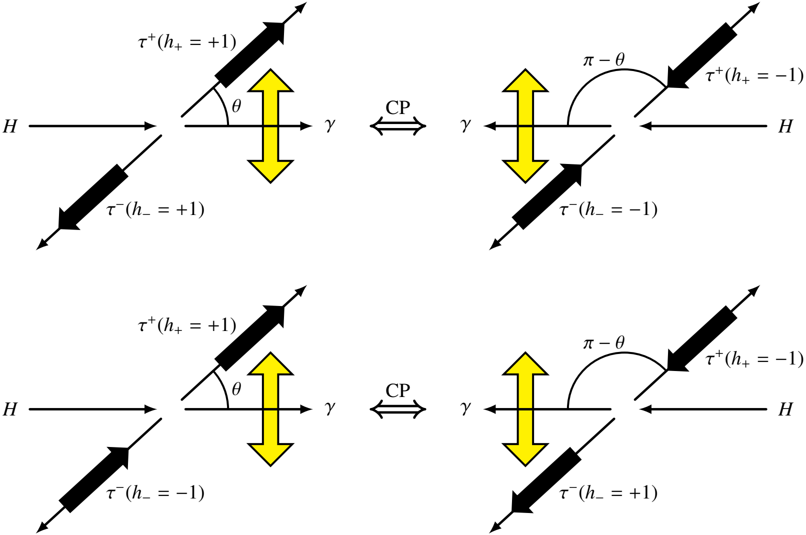


Kinematic configuration



The 3-body decay $H \to \tau^+ \tau^- \gamma$

All τ helicity configurations possible here unlike the case in $H \to \tau^+ \tau^-$



The 3-body decay $H \to \tau^+ \tau^- \gamma$

The difference between helicity amplitude squares

- Energies and momenta of $\tau^\pm$ in H rest frame are no longer fixed.
- Another uni-angular distribution ($\cos\theta$ distribution) is at our disposal.
- All helicities of τs are possible, unlike the 2-body decay $H \to \tau^+ \tau^-$.

$$\left|\mathscr{M}_{\tau\tau\gamma}^{(+,+)}\right|^2 = \left|\mathscr{M}_{\tau\tau\gamma}^{(-,-)}\right|^2 = \frac{8\, e^2\, m_\tau^2\, m_{+-}^2\, \left(1 - \cos^2\theta\right)}{v^2\, \left(m_H^2 - m_{+-}^2\right)^2\, \left(m_{+-}^2 - \left(m_{+-}^2 - 4\, m_\tau^2\right)\cos^2\theta\right)^2}$$

$$\times\Bigg(\left(32\, m_{+-}^2\, m_\tau^4 - 10\, m_{+-}^4\, m_\tau^2 - 4\, m_H^2\, m_{+-}^2\, m_\tau^2 - 2\, m_H^4\, m_\tau^2 + m_{+-}^6 + m_H^4\, m_{+-}^2\right)\, a_\tau^2$$

$$- \left(2\, m_{+-}^4\, m_\tau^2 + 4\, m_H^2\, m_{+-}^2\, m_\tau^2 + 2\, m_H^4\, m_\tau^2 - m_{+-}^6 - m_H^4\, m_{+-}^2\right)\, b_\tau^2\Bigg),$$

$$\left|\mathscr{M}_{\tau\tau\gamma}^{(+,-)}\right|^2 = \left|\mathscr{M}_{\tau\tau\gamma}^{(-,+)}\right|^2 = \frac{16\, e^2\, m_\tau^4\, m_{+-}^2\, \left(1 + \cos^2\theta\right)\, \left(b_\tau^2 + a_\tau^2\right)}{v^2\, \left(m_{+-}^2 - \left(m_{+-}^2 - 4\, m_\tau^2\right)\cos^2\theta\right)^2}.$$

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$$\left|\mathcal{M}_{\tau\tau\gamma}^{(+,+)}\right| \qquad \frac{8\,e^2\,m_\tau^2\,m_{+-}^2\,\left(1-\cos^2\theta\right)}{)^2\,\left(m_{+-}^2-\left(m_{+-}^2-4\,m_\tau^2\right)\cos^2\theta\right)^2}$$

$$^2_{+-}\,m_\tau^2-2\,m_H^4\,m_\tau^2+m_{+-}^6+m_H^4\,m_{+-}^2\Big)\,a_\tau^2$$

$$^4_H\,m_\tau^2-m_{+-}^6-m_H^4\,m_{+-}^2\Big)\,b_\tau^2\Bigg),$$

$$\left|\mathcal{M}_{\tau\tau\gamma}^{(+,-)}\right|^2=\Big| \qquad \frac{_{+-}\,\left(1+\cos^2\theta\right)\,\left(b_\tau^2+a_\tau^2\right)}{v^2\,\left(m_{+-}^2-\left(m_{+-}^2-4\,m_\tau^2\right)\cos^2\theta\right)^2}.$$

No term linear in $\cos\theta$, when *only Yukawa* contribution is considered

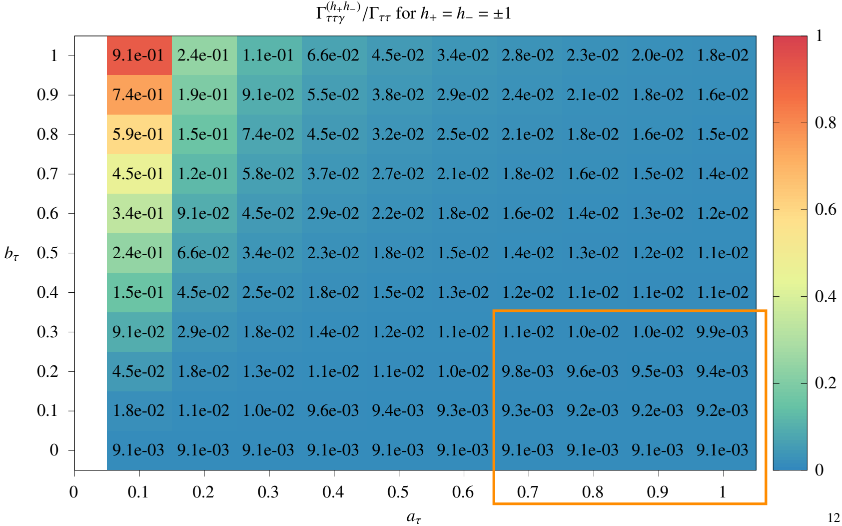
The 3-body decay $H \to \tau^+ \tau^- \gamma$

The differential decay rate for specific τ helicities

$$
\begin{aligned}
\frac{1}{\Gamma_{\tau\tau}} \left(\frac{\mathrm{d}\Gamma^{(h_+ h_-)}_{\tau\tau\gamma}}{\mathrm{d}m^2_{+-}\, \mathrm{d}\cos\theta} \right)_{\text{com}} = {} & \frac{\alpha}{2\pi \sqrt{m_H^2 - 4\, m_\tau^2}\, \left(a_\tau^2 \left(m_H^2 - 4\, m_\tau^2 \right) + b_\tau^2 \right)} \\
& \times \frac{m^2_{+-} \sqrt{m^2_{+-} - 4\, m_\tau^2}}{\left(m_H^4 - m_{+-}^4 \right) \left(m_{+-}^2 - \left(m_{+-}^2 - 4\, m_\tau^2 \right) \cos^2\theta \right)^2} \\
& \times \Bigg(-\, a_\tau^2 \Big(m_{+-}^2 \left(32\, m_\tau^4 - 8\, m_{+-}^2\, m_\tau^2 - 8\, m_H^2\, m_\tau^2 + m_{+-}^4 + m_H^4 \right) \left(h_-\, h_+ \cos^2\theta - 1 \right) \\
& \qquad - \left(4\, m_\tau^2 - m_{+-}^2 \right) \left(8\, m_{+-}^2\, m_\tau^2 - m_{+-}^4 - m_H^4 \right) \left(h_-\, h_+ - \cos^2\theta \right) \Big) \\
& \quad - b_\tau^2 \Big(m_{+-}^2 \left(8\, m_H^2\, m_\tau^2 - m_{+-}^4 - m_H^4 \right) \left(1 - h_-\, h_+ \cos^2\theta \right) \\
& \qquad + \left(m_{+-}^4 + m_H^4 \right) \left(4\, m_\tau^2 - m_{+-}^2 \right) \left(h_-\, h_+ - \cos^2\theta \right) \Big) \Bigg).
\end{aligned}
$$

The 3-body decay $H \to \tau^+ \tau^- \gamma$

Numerical estimates of $\Gamma^{(h_+ h_-)}_{\tau\tau\gamma}/\Gamma_{\tau\tau}$ for $h_+ = h_-$ with a cut on photon energy, $E_\gamma > 20$ GeV



$\Gamma^{(h_+ h_-)}_{\tau\tau\gamma}/\Gamma_{\tau\tau}$ for $h_+ = h_- = \pm 1$

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	9.1e-01	2.4e-01	1.1e-01	6.6e-02	4.5e-02	3.4e-02	2.8e-02	2.3e-02	2.0e-02	1.8e-02
0.9	7.4e-01	1.9e-01	9.1e-02	5.5e-02	3.8e-02	2.9e-02	2.4e-02	2.1e-02	1.8e-02	1.6e-02
0.8	5.9e-01	1.5e-01	7.4e-02	4.5e-02	3.2e-02	2.5e-02	2.1e-02	1.8e-02	1.6e-02	1.5e-02
0.7	4.5e-01	1.2e-01	5.8e-02	3.7e-02	2.7e-02	2.1e-02	1.8e-02	1.6e-02	1.5e-02	1.4e-02
0.6	3.4e-01	9.1e-02	4.5e-02	2.9e-02	2.2e-02	1.8e-02	1.6e-02	1.4e-02	1.3e-02	1.2e-02
0.5	2.4e-01	6.6e-02	3.4e-02	2.3e-02	1.8e-02	1.5e-02	1.4e-02	1.3e-02	1.2e-02	1.1e-02
0.4	1.5e-01	4.5e-02	2.5e-02	1.8e-02	1.5e-02	1.3e-02	1.2e-02	1.1e-02	1.1e-02	1.1e-02
0.3	9.1e-02	2.9e-02	1.8e-02	1.4e-02	1.2e-02	1.1e-02	1.1e-02	1.0e-02	1.0e-02	9.9e-03
0.2	4.5e-02	1.8e-02	1.3e-02	1.1e-02	1.1e-02	1.0e-02	9.8e-03	9.6e-03	9.5e-03	9.4e-03
0.1	1.8e-02	1.1e-02	1.0e-02	9.6e-03	9.4e-03	9.3e-03	9.3e-03	9.2e-03	9.2e-03	9.2e-03
0	9.1e-03	9.1e-03	9.1e-03	9.1e-03	9.1e-03	9.1e-03	9.1e-03	9.1e-03	9.1e-03	9.1e-03

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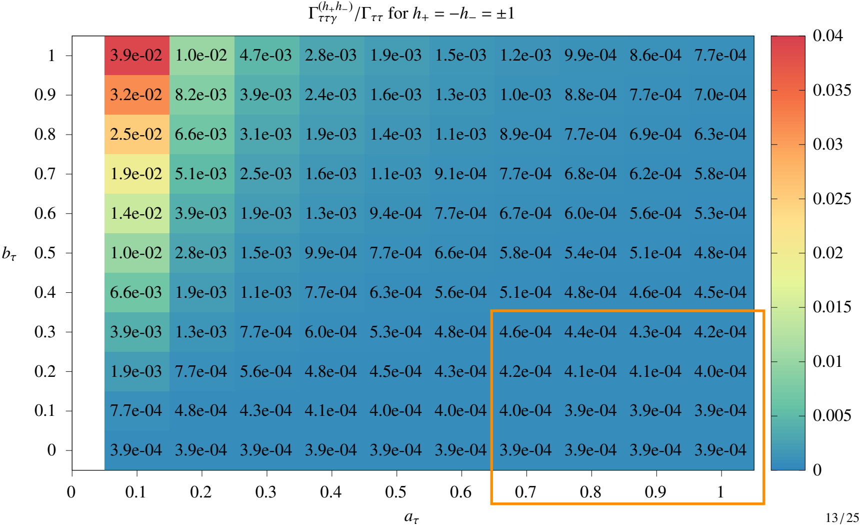
Numerical estimates of $\Gamma_{\tau\tau\gamma}^{(h_+h_-)}/\Gamma_{\tau\tau}$ for $h_+ = -h_-$ with a cut on photon energy, $E_\gamma > 20$ GeV

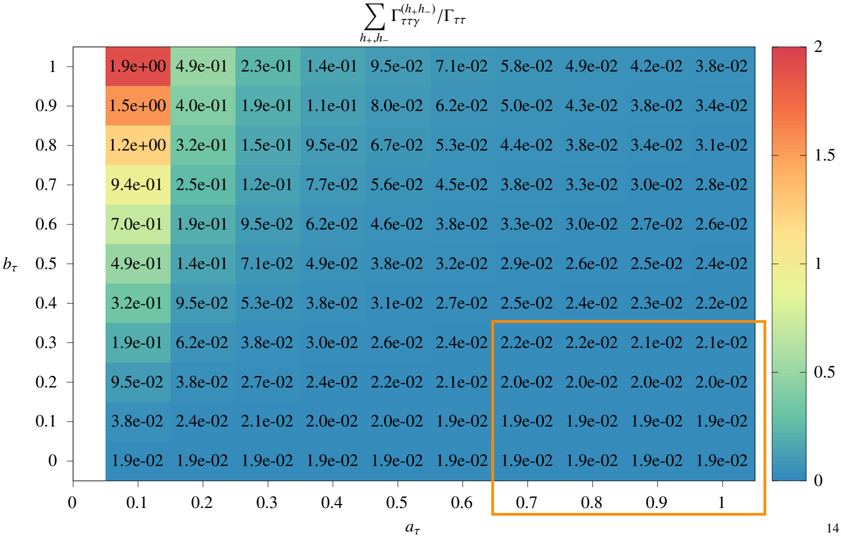


$\Gamma_{\tau\tau\gamma}^{(h_+h_-)}/\Gamma_{\tau\tau}$ for $h_+ = -h_- = \pm 1$

b_τ \ a_τ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	3.9e-02	1.0e-02	4.7e-03	2.8e-03	1.9e-03	1.5e-03	1.2e-03	9.9e-04	8.6e-04	7.7e-04
0.9	3.2e-02	8.2e-03	3.9e-03	2.4e-03	1.6e-03	1.3e-03	1.0e-03	8.8e-04	7.7e-04	7.0e-04
0.8	2.5e-02	6.6e-03	3.1e-03	1.9e-03	1.4e-03	1.1e-03	8.9e-04	7.7e-04	6.9e-04	6.3e-04
0.7	1.9e-02	5.1e-03	2.5e-03	1.6e-03	1.1e-03	9.1e-04	7.7e-04	6.8e-04	6.2e-04	5.8e-04
0.6	1.4e-02	3.9e-03	1.9e-03	1.3e-03	9.4e-04	7.7e-04	6.7e-04	6.0e-04	5.6e-04	5.3e-04
0.5	1.0e-02	2.8e-03	1.5e-03	9.9e-04	7.7e-04	6.6e-04	5.8e-04	5.4e-04	5.1e-04	4.8e-04
0.4	6.6e-03	1.9e-03	1.1e-03	7.7e-04	6.3e-04	5.6e-04	5.1e-04	4.8e-04	4.6e-04	4.5e-04
0.3	3.9e-03	1.3e-03	7.7e-04	6.0e-04	5.3e-04	4.8e-04	4.6e-04	4.4e-04	4.3e-04	4.2e-04
0.2	1.9e-03	7.7e-04	5.6e-04	4.8e-04	4.5e-04	4.3e-04	4.2e-04	4.1e-04	4.1e-04	4.0e-04
0.1	7.7e-04	4.8e-04	4.3e-04	4.1e-04	4.0e-04	4.0e-04	4.0e-04	3.9e-04	3.9e-04	3.9e-04
0	3.9e-04	3.9e-04	3.9e-04	3.9e-04	3.9e-04	3.9e-04	3.9e-04	3.9e-04	3.9e-04	3.9e-04

The 3-body decay $H \to \tau^+ \tau^- \gamma$

Numerical estimates of $\sum_{h_+,h_-} \Gamma_{\tau\tau\gamma}^{(h_+h_-)}/\Gamma_{\tau\tau}$ with a cut on photon energy, $E_\gamma > 20$ GeV



$\sum_{h_+,h_-} \Gamma_{\tau\tau\gamma}^{(h_+h_-)}/\Gamma_{\tau\tau}$

b_τ \ a_τ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
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0.9	1.5e+00	4.0e-01	1.9e-01	1.1e-01	8.0e-02	6.2e-02	5.0e-02	4.3e-02	3.8e-02	3.4e-02
0.8	1.2e+00	3.2e-01	1.5e-01	9.5e-02	6.7e-02	5.3e-02	4.4e-02	3.8e-02	3.4e-02	3.1e-02
0.7	9.4e-01	2.5e-01	1.2e-01	7.7e-02	5.6e-02	4.5e-02	3.8e-02	3.3e-02	3.0e-02	2.8e-02
0.6	7.0e-01	1.9e-01	9.5e-02	6.2e-02	4.6e-02	3.8e-02	3.3e-02	3.0e-02	2.7e-02	2.6e-02
0.5	4.9e-01	1.4e-01	7.1e-02	4.9e-02	3.8e-02	3.2e-02	2.9e-02	2.6e-02	2.5e-02	2.4e-02
0.4	3.2e-01	9.5e-02	5.3e-02	3.8e-02	3.1e-02	2.7e-02	2.5e-02	2.4e-02	2.3e-02	2.2e-02
0.3	1.9e-01	6.2e-02	3.8e-02	3.0e-02	2.6e-02	2.4e-02	2.2e-02	2.2e-02	2.1e-02	2.1e-02
0.2	9.5e-02	3.8e-02	2.7e-02	2.4e-02	2.2e-02	2.1e-02	2.0e-02	2.0e-02	2.0e-02	2.0e-02
0.1	3.8e-02	2.4e-02	2.1e-02	2.0e-02	2.0e-02	1.9e-02	1.9e-02	1.9e-02	1.9e-02	1.9e-02
0	1.9e-02	1.9e-02	1.9e-02	1.9e-02	1.9e-02	1.9e-02	1.9e-02	1.9e-02	1.9e-02	1.9e-02

The 3-body decay $H \to \tau^+ \tau^- \gamma$

Including SM 1-loop contribution to $H \to \mathscr{V} \gamma$ with $\mathscr{V} \to \tau^+ \tau^-$ affects the angular distribution

- The Lorentz invariant and gauge invariant, effective Lagrangian describing $H \to \mathscr{V} \gamma$ where $\mathscr{V} = Z, \gamma$ is given by[3]

$$\mathscr{L}_{H\mathscr{V}\gamma} = \frac{H}{4\,v}\left(2A_2^{Z\gamma} F^{\mu\nu} Z_{\mu\nu} + 2A_3^{Z\gamma} F^{\mu\nu} \widetilde{Z}_{\mu\nu} + A_2^{\gamma\gamma} F^{\mu\nu} F_{\mu\nu} + A_3^{\gamma\gamma} F^{\mu\nu} \widetilde{F}_{\mu\nu}\right),$$

  where $\mathscr{V}_{\mu\nu} = \partial_\mu \mathscr{V}_\nu - \partial_\nu \mathscr{V}_\mu$, $\widetilde{\mathscr{V}}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} \mathscr{V}^{\rho\sigma}$, and $A_2^{\mathscr{V}\gamma}$, $A_3^{\mathscr{V}\gamma}$ are two dimensionless form factors.

- The amplitude for $H \to \tau^+ \tau^- \gamma$ can be split into 3 components:

$$\mathscr{M}_{\tau\tau\gamma} = \mathscr{M}_{\tau\tau\gamma}^{(\text{Yukawa})} + \mathscr{M}_{\tau\tau\gamma}^{(Z\gamma)} + \mathscr{M}_{\tau\tau\gamma}^{(\gamma\gamma)}.$$

- The amplitude square is thus given by,

$$\begin{aligned} \left|\mathscr{M}_{\tau\tau\gamma}\right|^2 = &\left|\mathscr{M}_{\tau\tau\gamma}^{(\text{Yukawa})}\right|^2 + \left|\mathscr{M}_{\tau\tau\gamma}^{(Z\gamma)}\right|^2 + \left|\mathscr{M}_{\tau\tau\gamma}^{(\gamma\gamma)}\right|^2 + 2\,\text{Re}\left(\mathscr{M}_{\tau\tau\gamma}^{(\gamma\gamma)}\, \mathscr{M}_{\tau\tau\gamma}^{(Z\gamma)*}\right) \\ &+ 2\,\text{Re}\left(\mathscr{M}_{\tau\tau\gamma}^{(\text{Yukawa})}\, \mathscr{M}_{\tau\tau\gamma}^{(Z\gamma)*}\right) + 2\,\text{Re}\left(\mathscr{M}_{\tau\tau\gamma}^{(\text{Yukawa})}\, \mathscr{M}_{\tau\tau\gamma}^{(\gamma\gamma)*}\right). \end{aligned}$$

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[3] Y. Chen, A. Falkowski, I. Low and R. Vega-Morales, Phys.Rev.D **90**, no.11, 113006 (2014).

The 3-body decay $H \to \tau^+\tau^-\gamma$

Including SM 1-loop contribution to $H \to \mathscr{V}\gamma$ with $\mathscr{V} \to \tau^+\tau^-$ affects the angular distribution

$$\left|\mathcal{M}^{(\text{Yukawa})}_{\tau\tau\gamma}\right|^2 = \frac{16\,e^2\,m_{+-}^2\,m_\tau^2}{v^2\left(m_H^2-m_{+-}^2\right)^2\left(m_{+-}^2-\cos^2\theta\,\left(m_{+-}^2-4m_\tau^2\right)\right)^2}$$
$$\times\left(m_{+-}^2\left(\left(a_\tau^2+b_\tau^2\right)\left(m_H^4+m_{+-}^4\right)-8m_\tau^2\left(a_\tau^2\left(m_H^2+m_{+-}^2\right)+b_\tau^2m_H^2\right)+32a_\tau^2m_\tau^4\right)\right.$$
$$\left.-\cos^2\theta\,\left(m_{+-}^2-4m_\tau^2\right)\left(\left(a_\tau^2+b_\tau^2\right)\left(m_H^4+m_{+-}^4\right)-8a_\tau^2m_{+-}^2m_\tau^2\right)\right),$$

$$\left|\mathcal{M}^{(Z\gamma)}_{\tau\tau\gamma}\right|^2 = \frac{g_Z^2\left(\left(A_2^{Z\gamma}\right)^2+\left(A_3^{Z\gamma}\right)^2\right)\left(m_H^2-m_{+-}^2\right)^2}{16v^2\left(\left(m_{+-}^2-m_Z^2\right)^2+\Gamma_Z^2m_Z^2\right)}$$
$$\times\left(\left(\left(c_A^\tau\right)^2+\left(c_V^\tau\right)^2\right)\left[3m_{+-}^2+\cos 2\theta\,\left(m_{+-}^2-4m_\tau^2\right)\right]+4m_\tau^2\left(\left(c_V^\tau\right)^2-3\left(c_A^\tau\right)^2\right)\right),$$

$$\left|\mathcal{M}^{(\gamma\gamma)}_{\tau\tau\gamma}\right|^2 = \frac{e^2\left(\left(A_2^{\gamma\gamma}\right)^2+\left(A_3^{\gamma\gamma}\right)^2\right)\left(m_H^2-m_{+-}^2\right)^2\left(\cos 2\theta\,\left(m_{+-}^2-4m_\tau^2\right)+3m_{+-}^2+4m_\tau^2\right)}{4m_{+-}^4v^2},$$

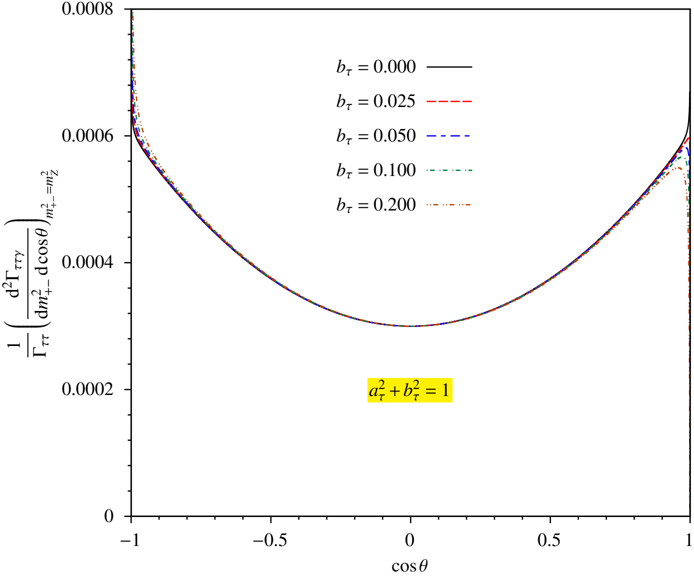
The 3-body decay $H \to \tau^+ \tau^- \gamma$

Including SM 1-loop contribution to $H \to \mathscr{V}\gamma$ with $\mathscr{V} \to \tau^+\tau^-$ affects the angular distribution

$$\begin{aligned}
\operatorname{Re}\left(\mathscr{M}^{(\gamma\gamma)}_{\tau\tau\gamma}\,\mathscr{M}^{(Z\gamma)*}_{\tau\tau\gamma}\right) = &-\frac{e\,g_Z\left(m_H^2 - m_{+-}^2\right)^2}{8m_{+-}^2 v^2\left(\left(m_{+-}^2 - m_Z^2\right)^2 + \Gamma_Z^2 m_Z^2\right)} \\
&\times\Bigg(4\Gamma_Z c_A^\tau m_{+-} m_Z \cos\theta\,\sqrt{m_{+-}^2 - 4m_\tau^2}\,\left(A_2^{\gamma\gamma}A_3^{Z\gamma} - A_2^{Z\gamma}A_3^{\gamma\gamma}\right) \\
&\quad + c_V^\tau\left(m_{+-}^2 - m_Z^2\right)\left(A_2^{\gamma\gamma}A_2^{Z\gamma} + A_3^{\gamma\gamma}A_3^{Z\gamma}\right)\left(\cos 2\theta\left(m_{+-}^2 - 4m_\tau^2\right) + 3m_{+-}^2 + 4m_\tau^2\right)\Bigg), \\
\operatorname{Re}\left(\mathscr{M}^{(\mathrm{Yukawa})}_{\tau\tau\gamma}\,\mathscr{M}^{(Z\gamma)*}_{\tau\tau\gamma}\right) = &-\frac{2\,e\,g_Z\,m_{+-}\,m_\tau^2}{v^2\left(\left(m_{+-}^2 - m_Z^2\right)^2 + \Gamma_Z^2 m_Z^2\right)\left(\cos^2\theta\left(m_{+-}^2 - 4m_\tau^2\right) - m_{+-}^2\right)} \\
&\times\Bigg(2\Gamma_Z c_A^\tau m_Z \cos\theta\,\left(m_H^2 - m_{+-}^2\right)\sqrt{m_{+-}^2 - 4m_\tau^2}\left(A_3^{Z\gamma}a_\tau - A_2^{Z\gamma}b_\tau\right) \\
&\quad + c_V^\tau m_{+-}\left(m_{+-}^2 - m_Z^2\right)\Big(A_2^{Z\gamma}a_\tau\left(2m_H^2 - m_{+-}^2 - 4m_\tau^2 - \cos 2\theta\left(m_{+-}^2 - 4m_\tau^2\right)\right) \\
&\qquad + 2A_3^{Z\gamma}b_\tau\left(m_H^2 - m_{+-}^2\right)\Big)\Bigg), \\
\operatorname{Re}\left(\mathscr{M}^{(\mathrm{Yukawa})}_{\tau\tau\gamma}\,\mathscr{M}^{(\gamma\gamma)*}_{\tau\tau\gamma}\right) = &\frac{8\,e^2\,m_\tau^2\left(A_2^{\gamma\gamma}a_\tau\left(m_H^2 - 4m_\tau^2 - \cos^2\theta\left(m_{+-}^2 - 4m_\tau^2\right)\right) + A_3^{\gamma\gamma}b_\tau\left(m_H^2 - m_{+-}^2\right)\right)}{v^2\left(\cos^2\theta\left(m_{+-}^2 - 4m_\tau^2\right) - m_{+-}^2\right)}.
\end{aligned}$$

The 3-body decay $H \to \tau^+ \tau^- \gamma$

Including SM 1-loop contribution to $H \to \mathscr{V} \gamma$ with $\mathscr{V} \to \tau^+ \tau^-$ affects the angular distribution



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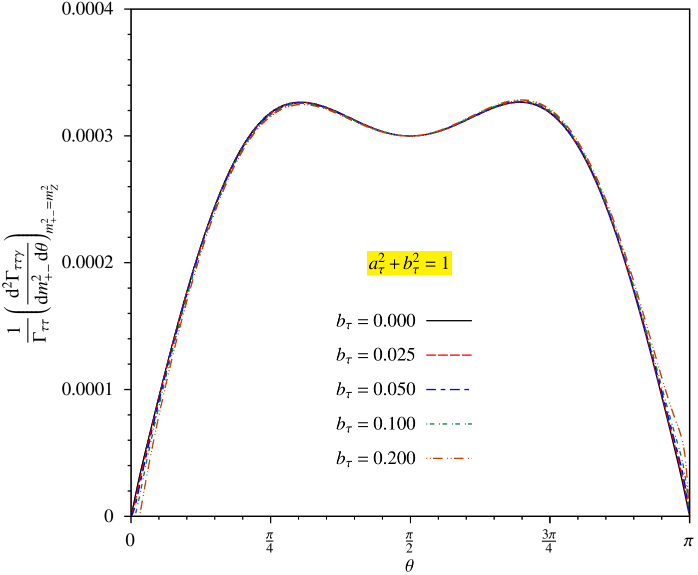
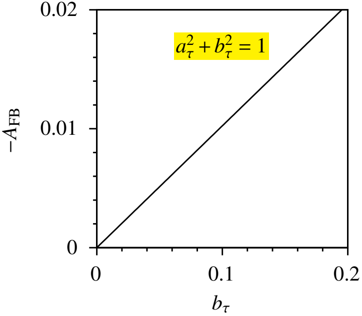


The 3-body decay $H \to \tau^+ \tau^- \gamma$

Including SM 1-loop contribution to $H \to \mathscr{V} \gamma$ with $\mathscr{V} \to \tau^+ \tau^-$ affects the angular distribution

Forward-backward asymmetry:

$$\boxed{A_{\mathrm{FB}} = \frac{\displaystyle\int_0^1 \left(\frac{\mathrm{d}^2 \Gamma_{\tau\tau\gamma}}{\mathrm{d}m_{+-}^2 \, \mathrm{d}\cos\theta} \right)_{m_{+-}^2 = m_Z^2} \mathrm{d}\cos\theta - \int_{-1}^0 \left(\frac{\mathrm{d}^2 \Gamma_{\tau\tau\gamma}}{\mathrm{d}m_{+-}^2 \, \mathrm{d}\cos\theta} \right)_{m_{+-}^2 = m_Z^2} \mathrm{d}\cos\theta}{\displaystyle\int_{-1}^1 \left(\frac{\mathrm{d}^2 \Gamma_{\tau\tau\gamma}}{\mathrm{d}m_{+-}^2 \, \mathrm{d}\cos\theta} \right)_{m_{+-}^2 = m_Z^2} \mathrm{d}\cos\theta}.}$$



The 3-body decay $H \to \tau^+ \tau^- \gamma$

Summary

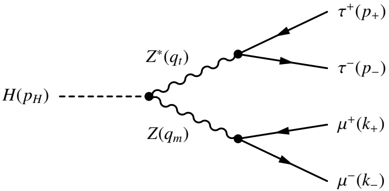
There are two clear experimental observables that can exploit $H \to \tau^+ \tau^- \gamma$:

- Branching ratio: it can be used to rule out large values of b_τ.
- Forward-backward asymmetry A_{FB}: can be used to probe $b_\tau \lesssim 0.2$, if A_{FB} can be probed at percent level accuracy.

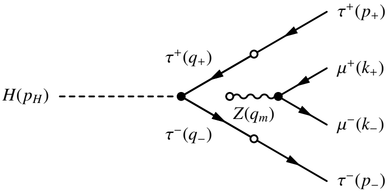
A more thorough numerical study is ongoing.

The 4-body decay $H \to \tau^+ \tau^- \mu^+ \mu^-$ (‘Golden channel’!?)

Another probe of CP violation



(a)

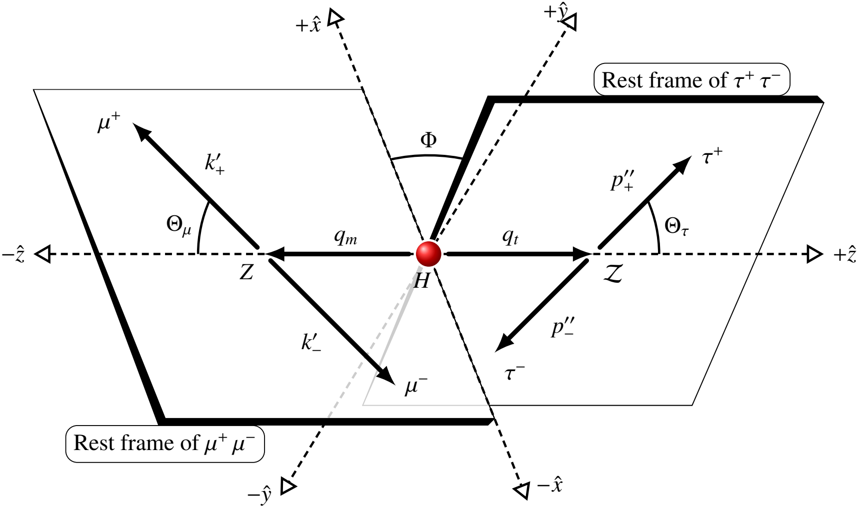


(b)

- Decay amplitude: $\mathscr{M}_{\tau\tau\mu\mu} = \mathscr{M}^{ZZ}_{\tau\tau\mu\mu} + \mathscr{M}^{\text{Yukawa}}_{\tau\tau\mu\mu}$.
- The interference term $\propto b_\tau$, so it is sensitive to the CP violation.
- Huge expression for amplitude square.

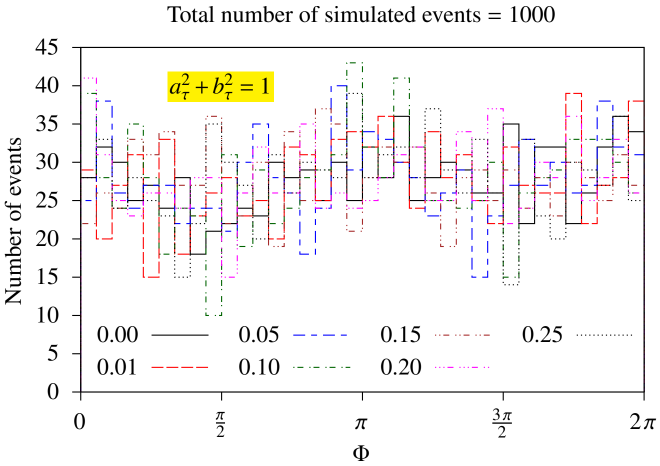
The 4-body decay $H \to \tau^+ \tau^- \mu^+ \mu^-$ (‘Golden channel’!?)

The CP violating contribution in the interference term is proportional to **$\sin\Phi$**



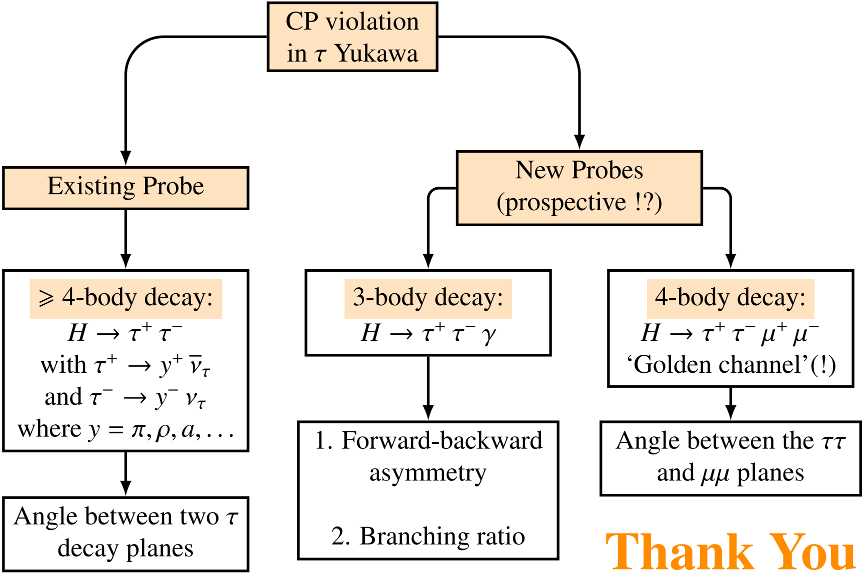
The 4-body decay $H \to \tau^+ \tau^- \mu^+ \mu^-$ ('Golden channel'!?)

The CP violating contribution in the interference term is proportional to sin Φ



- The angle Φ between the two decay planes is sensitive to CP violation.
- Numerical study is ongoing.

Summary



CP violation in τ Yukawa

Existing Probe

$\geqslant$ 4-body decay:
$H \to \tau^+ \tau^-$
with $\tau^+ \to y^+ \bar{\nu}_\tau$
and $\tau^- \to y^- \nu_\tau$
where $y = \pi, \rho, a, \ldots$

Angle between two τ decay planes

New Probes (prospective !?)

3-body decay:
$H \to \tau^+ \tau^- \gamma$

1. Forward-backward asymmetry
2. Branching ratio

4-body decay:
$H \to \tau^+ \tau^- \mu^+ \mu^-$
'Golden channel'(!)

Angle between the $\tau\tau$ and $\mu\mu$ planes

Thank You