NATIONAL SCIENCE CENTRE
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Faculty of Physics

Quantum information and CP measurement in $H \to \tau^+\tau^-$ at future high energy lepton colliders

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Outline of talk

- The EPR argument
- CHSH inequality in Local hidden variable theory and in QM
- Bell inequality in $H \to \tau^+\tau^-$ at high energy lepton colliders
- Result
- Conclusion

The EPR argument

> *"If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exist an element of physical reality corresponding to this physical quantity"*
>
> → *Einstien, Podolski and Rosen, 1935*

❑ Reality

(On based of criteria of physical reality, they argued that QM was not a complete theory)

Ex: In QM, in the case of two physical quantities described by non-commutating operators, the knowledge of (say, spin component in x direction) makes impossible the knowledge of the other (y, z component of the spin).

EPR argue on that: In QM

Either the description of reality given by the wave function is *not complete.*

Or, these two quantities cannot have *simultaneous reality.*

❑ Locality

(The result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past)

QM violates both local and real requirements

The EPR argument

- As per EPR, the QM behavior could be explained by additional variables called Local Hidden variables (LHV). These would restore locality and causality to the theory (and they demonstrated it for the Stern Gerlach experimental observations).

- It seems difficult that time to experimentally discriminate QM and general hidden variable theories.

- In 1964, John Bell, made a fundamental contribution, showing that no deterministic hidden variable theory can reproduce al the statistical predictions of quantum mechanics(1964) derived simple inequalities that can discriminate QM from any local-real hidden variable theories: Bell inequalities

- He showed we can't explain all QM statistical predication by LHV, it can be easily show mathematically for maximally entangled states.

I will go through this step by step:

1. What is an entangled state
2. What is the Bell inequality

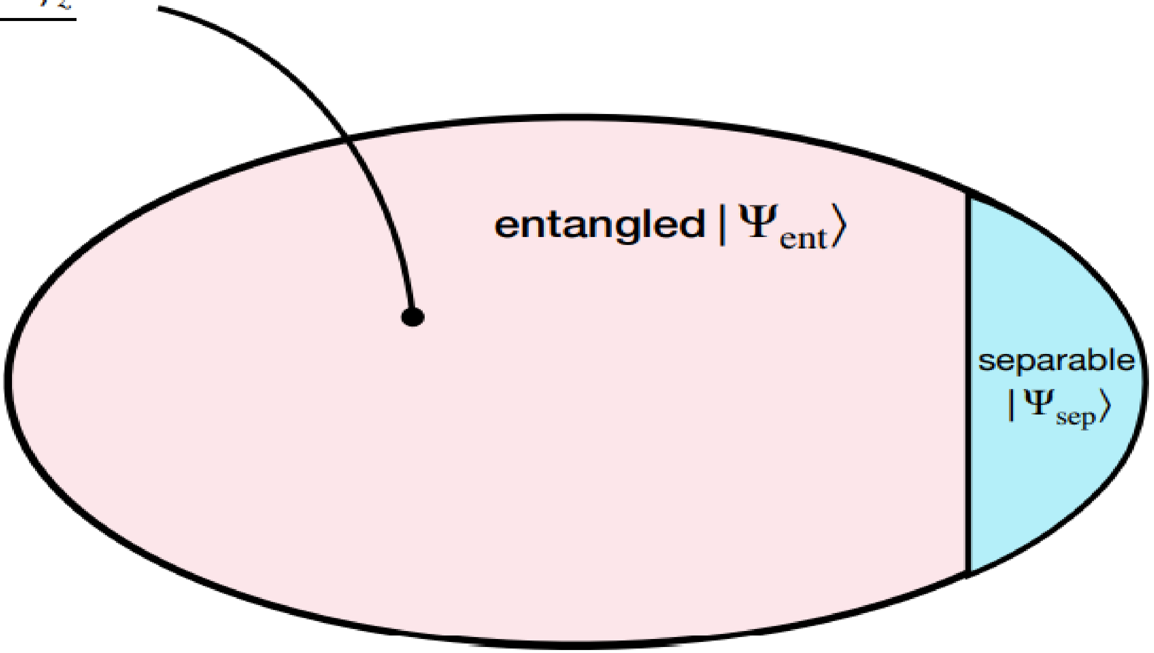
Entanglement

general: $|\Psi\rangle \doteq c_{11}|++\rangle_z + c_{12}|+-\rangle_z + c_{21}|-+\rangle_z + c_{22}|--\rangle_z$

separable: $|\Psi_{\text{sep}}\rangle \doteq \left[c_1^\alpha|+\rangle_z + c_2^\alpha|-\rangle_z\right] \otimes \left[c_1^\beta|+\rangle_z + c_2^\beta|-\rangle_z\right]$

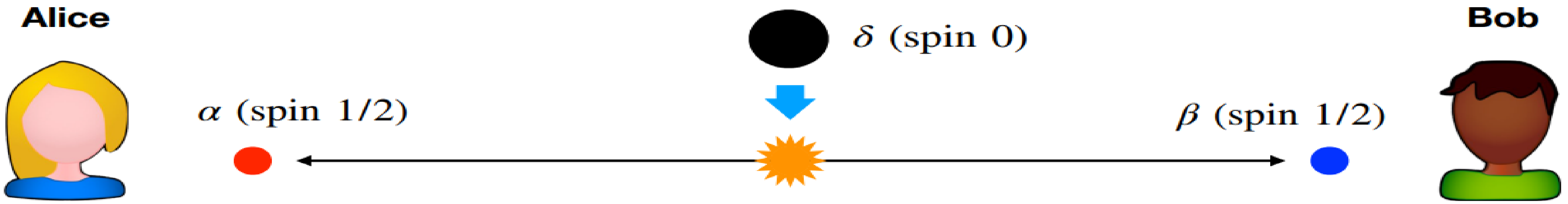
entangled: $|\Psi_{\text{ent}}\rangle \xcancel{\doteq} \left[c_1^\alpha|+\rangle_z + c_2^\alpha|-\rangle_z\right] \otimes \left[c_1^\beta|+\rangle_z + c_2^\beta|-\rangle_z\right]$

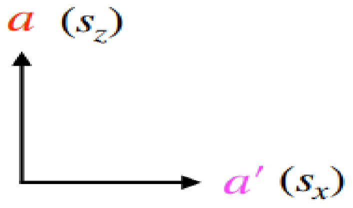
entangled: $|\Psi^{(0,0)}\rangle \doteq \frac{|+-\rangle_z - |-+\rangle_z}{\sqrt{2}}$

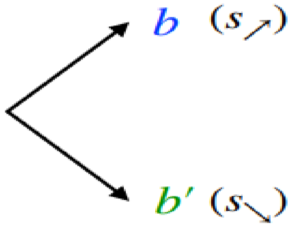


CHSH *inequality*

[Clauser, Horne, Shimony, Holt, 1969]







The experiment consists of 4 sessions:

1) Alice and Bob measure $s_a[\alpha]$ and $s_b[\beta]$, respectively. Repeat the measurement many times and calculate $\langle s_a \cdot s_b \rangle$.
2) Repeat (1) but for a and b'.
3) Repeat (1) but for a' and b.
4) Repeat (1) but for a' and b'.

Finally, we construct

$$R_{\text{CHSH}} \equiv \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$

CHSH *inequality in* LHV Theories

$$R_{\text{CHSH}} \equiv \frac{1}{2}\left|\langle s_a s_b\rangle - \langle s_a s_{b'}\rangle + \langle s_{a'} s_b\rangle + \langle s_{a'} s_{b'}\rangle\right| \leq 1$$

$$|\langle ab\rangle - \langle ab'\rangle| = \left|\int d\lambda\,(ab - ab')\,P\right|$$

$$\pm\, aba'b'P - (\pm\, aba'b'P) = 0$$

$$= \int d\lambda\,|ab(1 \pm a'b')P - ab'(1 \pm a'b)P|$$

$$\leq \int d\lambda\Big(|ab||1 \pm a'b'|P + |ab'||1 \pm a'b|P\Big)$$

$$|ab| = |ab'| = 1$$

$$|1 \pm a'b'|, |1 \pm a'b| \geq 0$$

$$= \int d\lambda\Big[(1 \pm a'b')P + (1 \pm a'b)P\Big]$$

$$= 2 \pm (\langle a'b'\rangle + \langle a'b\rangle)$$

$$a = s_a$$
$$b = s_b$$
$$\vdots$$

$$\Rightarrow \tilde{R}_{\text{CHSH}} = \frac{1}{2}\left(|\langle ab\rangle - \langle ab'\rangle| + |\langle a'b\rangle + \langle a'b'\rangle|\right) \leq 1$$

$$\max_{(\vec{a},\vec{b},\vec{a}',\vec{b}')} (R_{\text{CHSH}}) = \max_{(\vec{a},\vec{b},\vec{a}',\vec{b}')} \left(\tilde{R}_{\text{CHSH}}\right)$$

$$\boxed{\langle ab\rangle = \int a(\lambda)b(\lambda)P(\lambda)d\lambda}$$

$$\boxed{\int P(\lambda)d\lambda = 1}$$

CHSH *inequality in* QM

- Lets consider an QM wavefunction of singlet state of two spin ½ particles

$$|\psi^{(0,0)}\rangle = \frac{|+-\rangle_z - |-+\rangle_z}{\sqrt{2}}$$

one can show

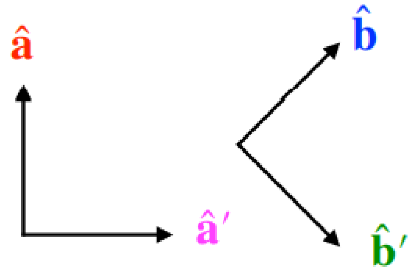
$$\langle s_a s_b \rangle = \langle \Psi^{(0,0)} | s_a s_b | \Psi^{(0,0)} \rangle = (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})$$

therefore

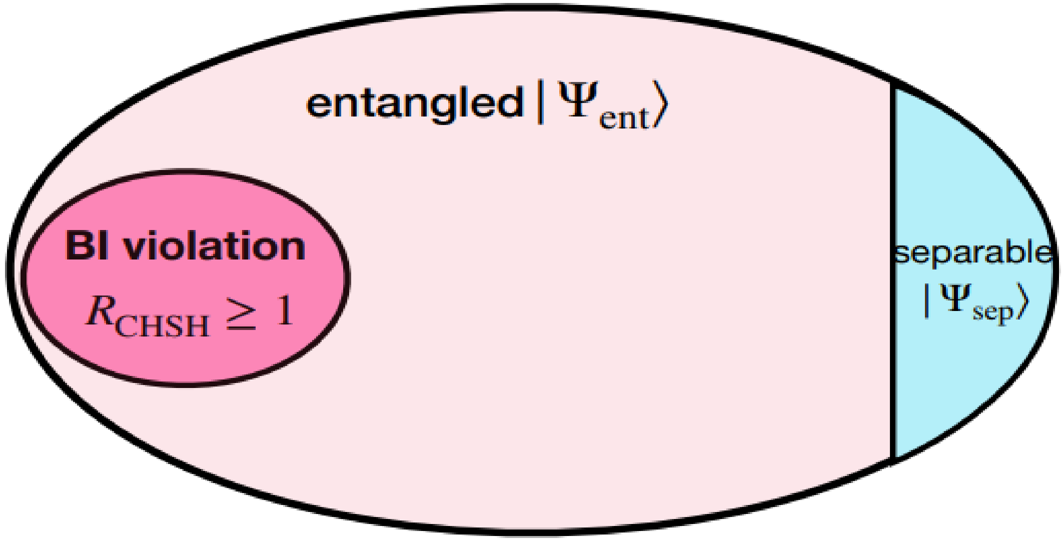
violates the upper bound of hidden variable theories!

$$R_{\text{CHSH}} = \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$

$$= \frac{1}{2} \left| \underbrace{(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})}_{\frac{1}{\sqrt{2}}} - \underbrace{(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}')}_{-\frac{1}{\sqrt{2}}} + \underbrace{(\hat{\mathbf{a}}' \cdot \hat{\mathbf{b}})}_{\frac{1}{\sqrt{2}}} + \underbrace{(\hat{\mathbf{a}}' \cdot \hat{\mathbf{b}}')}_{\frac{1}{\sqrt{2}}} \right| = \sqrt{2}$$



$$R_{\text{CHSH}} \le \begin{cases} 1 & \text{(HV theories)} \\ \sqrt{2} & \text{(QM)} \end{cases}$$



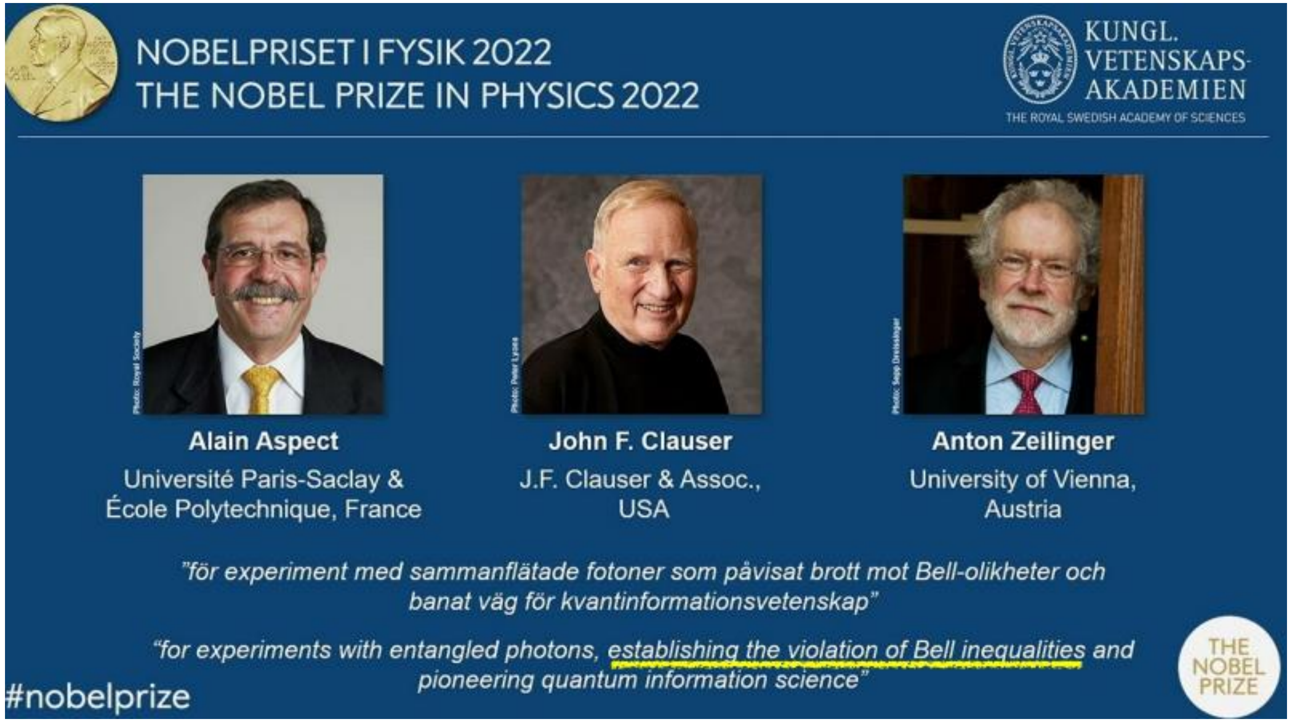
Q: Could we check this experimentally?

A: We already has been observed Bell inequality violation ($R_{CHSH} \geq 1$) in low energy experiments:

- Entangled photon pairs (from decays of Calcium atoms)
  Crauser, Horne, Shimony, Holt (1969), Freedman and Clauser (1972), A. Aspect et. al. (1981, 1982), Y. H. Shih, C. O. Alley (1988), L. K. Shalm et al. (2015) [5σ]

- Entangled proton pairs (from decays of 2_{He})
  M. M. Lamehi-Rachti, W. Mitting (1972), H. Sakai (2006)

- $K^0\overline{K}^0, B^0\overline{B}^0$ flavour oscillation
  CPLEAR (1999), Belle (2004, 2007)



NOBELPRISET I FYSIK 2022
THE NOBEL PRIZE IN PHYSICS 2022

KUNGL. VETENSKAPS-AKADEMIEN
THE ROYAL SWEDISH ACADEMY OF SCIENCES

Alain Aspect
Université Paris-Saclay &
École Polytechnique, France

John F. Clauser
J.F. Clauser & Assoc.,
USA

Anton Zeilinger
University of Vienna,
Austria

"för experiment med sammanflätade fotoner som påvisat brott mot Bell-olikheter och banat väg för kvantinformationsvetenskap"

"for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

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THE NOBEL PRIZE

Can we test Bell inequality violation ($R_{CHSH} \geq 1$) and entanglement at High energy colliders?

- Entanglement in pp → tt ¯ @ LHC – Y. Afik, J. R. M. de Nova (2020)
- Bell inequality test in pp → tt ¯ @ LHC M. Fabbrichesi, R. Floreanini, G. Panizzo (2021) C. Severi, C. D. Boschi, F. Maltoni, M. Sioli (2021) J. A. Aguilar-Saavedra, J. A. Casas (2022)
- Bell inequality test in H → WW* @ LHC – A.J. Barr(2021)

We are interested in study of Quantum property test in H → τ τ
@ high energy colliders e+ e−

Density matrix

probability of having $|\Psi_1\rangle$

- For a statistical ensemble $\{\{p_1 : |\Psi_1\rangle\}, \{p_2 : |\Psi_2\rangle\}, \{p_3 : |\Psi_3\rangle\}, \cdots\}$, we define the **density operator/matrix**

$$\hat{\rho} \equiv \sum_k p_k |\Psi_k\rangle\langle\Psi_k| \qquad \rho_{ab} \equiv \langle e_a|\hat{\rho}|e_b\rangle$$

$$0 \leq p_k \leq 1$$

$$\sum_k p_k = 1$$

$$\langle e_a|e_b\rangle = \delta_{ab}$$

- Density matrices satisfy the conditions:
  - $\hat{\rho}^\dagger = \hat{\rho}$
  - $\mathrm{Tr}\,\hat{\rho} = 1$
  - $\hat{\rho}$ is positive definite, that is ${}^\forall|\varphi\rangle;\ \langle\varphi|\hat{\rho}|\varphi\rangle \geq 0.$

- The expectation of an observable $\hat{O}$ is calculated by

$$\langle\hat{O}\rangle = \mathrm{Tr}\left[\hat{O}\hat{\rho}\right]$$

Density matrix

Spin 1/2 biparticle system

- The spin system of α and β particles has 4 independent bases:

$$\left(\, |e_1\rangle, |e_2\rangle, |e_3\rangle, |e_4\rangle \,\right) = \left(\, |++\rangle, |+-\rangle, |-+\rangle, |--\rangle \,\right)$$

- ==> ρ_{ab} is a 4 x 4 matrix (hermitian, Tr=1). It can be expanded as

$$\rho = \frac{1}{4}\left(\mathbf{1} \otimes \mathbf{1} + B_i \cdot \sigma_i \otimes \mathbf{1} + \overline{B}_i \cdot \mathbf{1} \otimes \sigma_i + C_{ij} \cdot \sigma_i \otimes \sigma_j\right) \qquad B_i, \overline{B}_i, \overset{\overset{\text{3x3 matrix}}{\downarrow}}{C_{ij}} \in \mathbb{R}$$

- For the spin operators $\hat{s}^\alpha$ and $\hat{s}^\beta$,

spin-spin correlation

$$\langle \hat{s}_i^\alpha \rangle = \mathrm{Tr}\left[\hat{s}_i^\alpha \hat{\rho}\right] = B_i \qquad \langle \hat{s}_i^\beta \rangle = \mathrm{Tr}\left[\hat{s}_i^\beta \hat{\rho}\right] = \overline{B}_i \qquad \boxed{\langle \hat{s}_i^\alpha \hat{s}_j^\beta \rangle = \mathrm{Tr}\left[\hat{s}_i^\alpha \hat{s}_j^\beta \hat{\rho}\right] = C_{ij}}$$

Once we know C matrix. We can compute different quantum information like Entanglement etc.
Q: How to compute C matrix in high Energy Physics?

Entanglement

- If the state is separable (not entangled),

$$\rho = \sum_k p_k \rho_k^\alpha \otimes \rho_k^\beta$$

$$0 \leq p_k \leq 1$$

$$\sum_k p_k = 1$$

then, a modified matrix by the partial transpose

$$\rho^{T_\beta} \equiv \sum_k p_k \rho_k^\alpha \otimes [\rho_k^\beta]^T$$

is also a physical density matrix, i.e. Tr=1 and non-negative.

- For biparticle systems, entanglement $\Longleftrightarrow$ ρ^{T_β} to be non-positive.

Peres-Horodecki
(1996, 1997)

- A simple sufficient condition for entanglement is:

$$E \equiv C_{11} + C_{22} - C_{33} > 1$$

Density matrix of $H \to \tau^+\tau^-$

$$\mathscr{L}_{\text{int}} = -\frac{m_\tau}{v_{\text{SM}}}\kappa H\bar{\psi}_\tau(\cos\delta + i\gamma_5\sin\delta)\psi_\tau$$

SM: $(\kappa, \delta) = (1{,}0)$

The spin density matrix for the two taus in $H \to \tau^+\tau^-$ is given by

$$\rho_{mn,\bar{m}\bar{n}} = \frac{\mathcal{M}^{*n\bar{n}}\mathcal{M}^{m\bar{m}}}{\sum_{m\bar{m}}|\mathcal{M}^{m\bar{m}}|^2} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i2\delta} & 0 \\ 0 & e^{i2\delta} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Using spin density matrix we can compute B and C matrix

where

$$\mathcal{M}^{m\bar{m}} = c\,\bar{u}^m(p)(\cos\delta + i\gamma_5\sin\delta)v^{\bar{m}}(\bar{p})$$

$$B_i = \overline{B}_i = 0$$

$$C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

SM (δ)=0

$$C_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Density matrix of $H \to \tau^+\tau^-$

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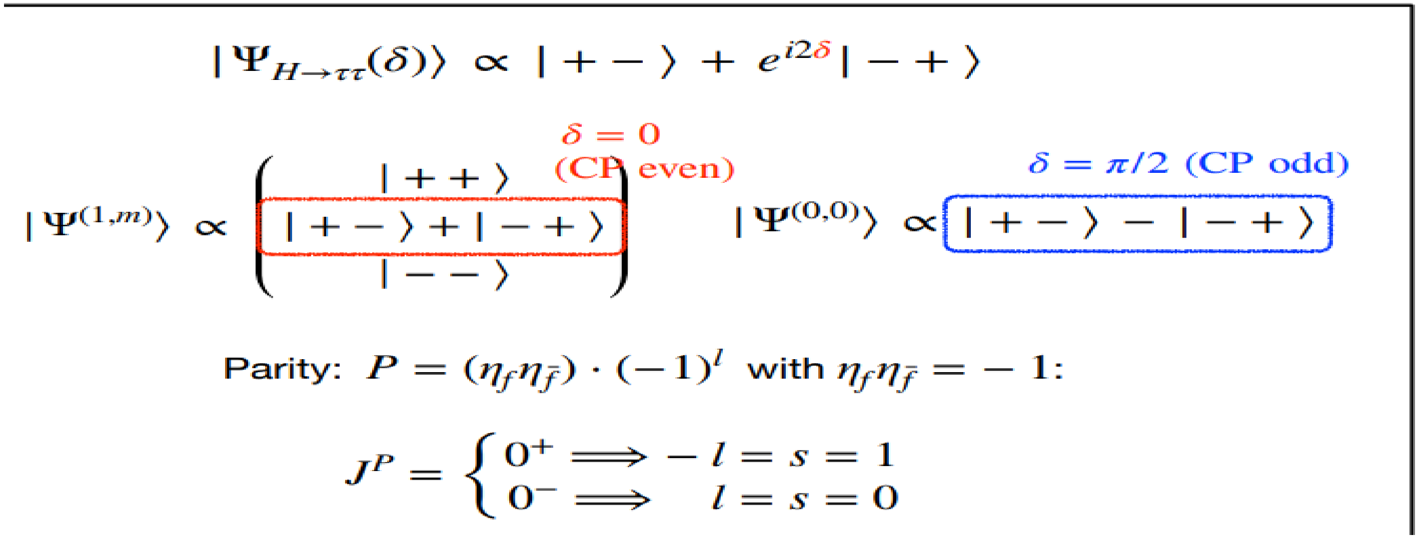


$$|\Psi_{H\to\tau\tau}(\delta)\rangle \propto |+-\rangle + e^{i2\delta}|-+\rangle$$

$\delta = 0$ (CP even)

$\delta = \pi/2$ (CP odd)

$$|\Psi^{(1,m)}\rangle \propto \begin{pmatrix} |++\rangle \\ \boxed{|+-\rangle + |-+\rangle} \\ |--\rangle \end{pmatrix} \qquad |\Psi^{(0,0)}\rangle \propto \boxed{|+-\rangle - |-+\rangle}$$

Parity: $P = (\eta_f\eta_{\bar{f}}) \cdot (-1)^l$ with $\eta_f\eta_{\bar{f}} = -1$:

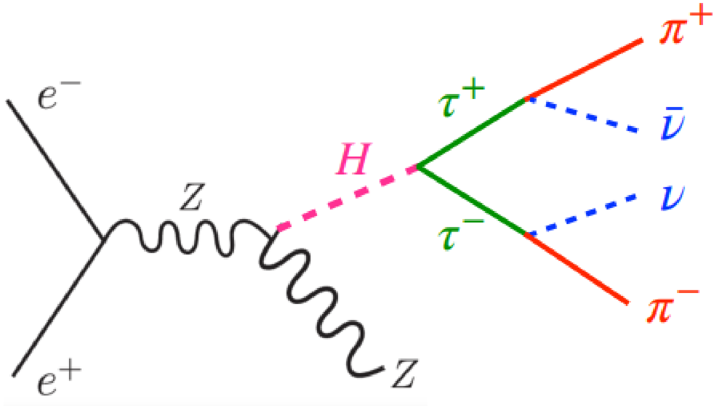
$$J^P = \begin{cases} 0^+ \Longrightarrow -l = s = 1 \\ 0^- \Longrightarrow \quad l = s = 0 \end{cases}$$

$$B_i = \overline{B}_i = 0$$

$$C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Spin-correlation matrix and CHSH *in lepton collider*

- Let’s suppose a spin 1/2 particle τ is at rest and spinning in the s direction



- τ- decays into a measurable particle $\pi_{\tau-}$ and neutrino.
- The decay distribution is generally given by

$$\frac{d\Gamma}{d\Omega} \propto 1 + x_\tau(\hat{I}_\tau . s)$$

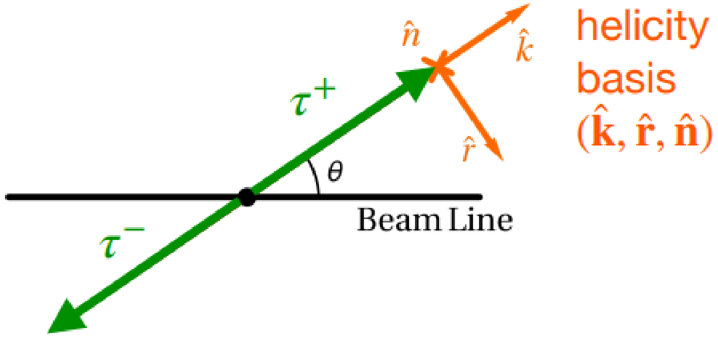
- $\hat{I}_\tau$ Is a unit direction vector of pion of tau+ measured at the rest frame of tau.
- x $\in [-1,1]$ *is called spin − analysing power*. And it depends on decay mode. For $\tau^- \rightarrow \pi^- + \vartheta_\tau$ $\Longrightarrow$ x=1
- We can show for $\tau^- + \tau^+ \rightarrow (\pi_{\tau-} + \vartheta_\tau) + (\pi_{\tau+} + \vartheta_\tau)$ and and $\xi_{ij} = (\hat{I}_{\tau-})_i(\hat{I}_{\tau+})_j$

$$\frac{d\sigma}{d\xi_{ij}} = (1 - C_{ij}) \cdot \ln\left(\frac{1}{\xi_{ij}}\right)$$

Spin-correlation matrix and CHSH *in lepton collider*

$$R_{\mathrm{CHSH}} \equiv \frac{1}{2}\left|\langle s_a s_b\rangle - \langle s_a s_{b'}\rangle + \langle s_{a'} s_b\rangle + \langle s_{a'} s_{b'}\rangle\right|$$

$$= \frac{9}{2|x_\alpha x_\beta|}\left|\left\langle(\hat{\mathbf{l}}_\alpha)_a(\hat{\mathbf{l}}_\beta)_b\right\rangle - \left\langle(\hat{\mathbf{l}}_a)(\hat{\mathbf{l}}_\beta)_{b'}\right\rangle + \left\langle(\hat{\mathbf{l}}_\alpha)_{a'}(\hat{\mathbf{l}}_\beta)_b\right\rangle + \left\langle(\hat{\mathbf{l}}_\alpha)_{a'}(\hat{\mathbf{l}}_\beta)_{b'}\right\rangle\right|$$



R_{CHSH} can be directly calculated

once the unit vectors $(\hat{\mathbf{a}}, \hat{\mathbf{a}}', \hat{\mathbf{b}}, \hat{\mathbf{b}}')$ are fixed.

- we define Helicity basis at the Higgs rest frame

r ≡ (h − k cos θ)/ sin θ

- In the $\tau^{+(-)}$ rest frame, we measure the direction of $\pi^{+(-)}$, $\hat{\mathbf{l}}^+$ and $\hat{\mathbf{l}}^-$, and calculate R_{CHSH} directly with

$$(\hat{\mathbf{a}}, \hat{\mathbf{a}}', \hat{\mathbf{b}}, \hat{\mathbf{b}}') = (\hat{\mathbf{k}}, \hat{\mathbf{r}}, \frac{1}{\sqrt{2}}(\hat{\mathbf{k}}+\hat{\mathbf{r}}), \frac{1}{\sqrt{2}}(\hat{\mathbf{k}}-\hat{\mathbf{r}}))$$

and measure C_{ij}

What do we want to study?

➢Entanglement

- A simple sufficient condition for entanglement is:

$$E \equiv C_{11} + C_{22} - C_{33} > 1$$

J. A. Aguilar-Saavedra and J. A. Casas 2022

➢Steerability

- For unpolarized cases,$< \hat{s}_i^A > = < \hat{s}_i^B \geq 0$, a necessary and sufficient condition for steerability is given by: [Jevtic, Hall, Anderson, Zwierz, Wiseman 2015]

$$\mathcal{S}[\rho] \equiv \frac{1}{2\pi} \int d\Omega_{\mathbf{n}} \sqrt{\mathbf{n}^T C^T C \mathbf{n}}\,, \qquad \mathcal{S}[\rho] > 1$$

➢Bell-inequality Violation

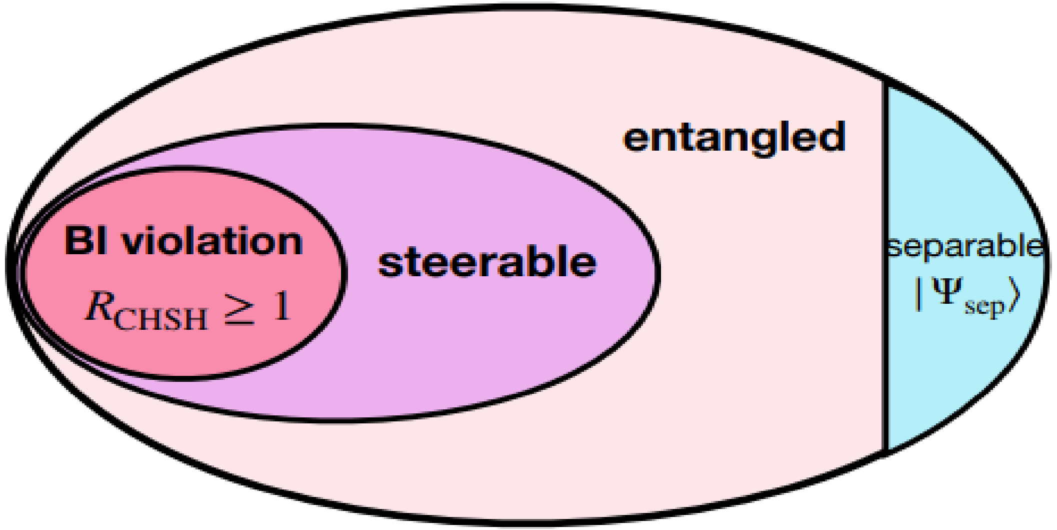
- It can be directly calculated using unit direction of pion measured at the rest frame of tau, once he unit vectors($\hat{a}, \hat{a}', \hat{b}, \hat{b}'$) are fixed.

Steering

[Schrödinger 1935]

- Steering for Alice is Alice's ability to "steer" Bob's local state by her measurement.
- Suppose Alice and Bob measure the observables $\mathscr{A}$ and $\mathscr{B}$, and obtained the outcomes a and b. The state is said to be *steerable* by Alice, if it is ***not*** possible to write this probability in a form: [Jones, Wiseman, Doherty 2007]

$$p(a,b) = \sum_{\lambda} P(\lambda)\, p(a|\lambda)\, p_Q(b|\lambda)\,, \qquad p_Q(b\,|\,\lambda) = \mathrm{Tr}\left[\overset{\substack{\text{Bob's local state}\\ \downarrow}}{\rho_B}(\lambda)\,|\,b\rangle\langle b\,|\right]$$



Quantum information of $H \to \tau^+\tau^-$ in Standard Model

SM values: $C_{ij} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$

$E = 3$ **Entanglement** $\Longrightarrow E > 1$

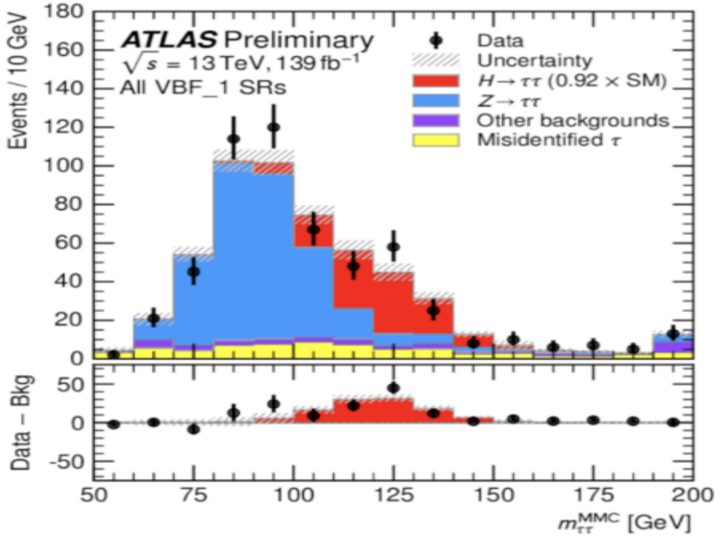
$\mathcal{S}[\rho] = 2$ **Steerablity** $\Longrightarrow \mathcal{S}[\rho] > 1$

$R_{\text{CHSH}} = \sqrt{2} \simeq 1.414$ **Bell-nonlocal** $\Longrightarrow R_{\text{CHSH}} > 1$

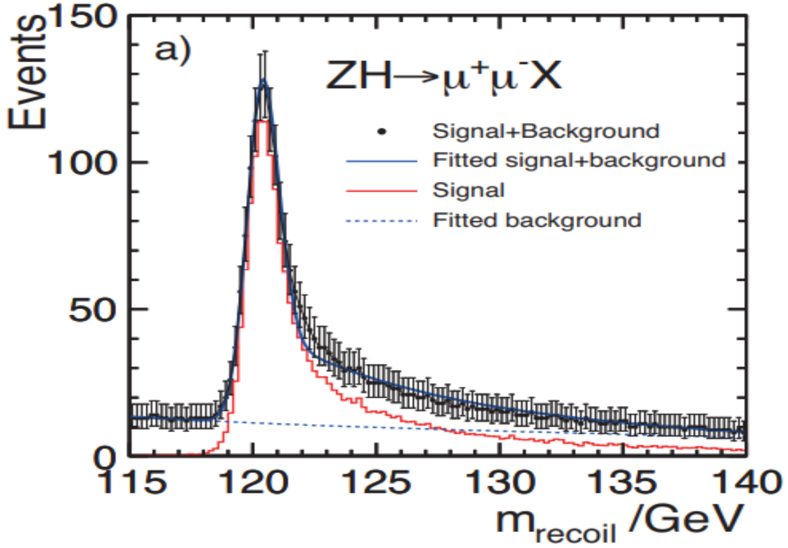
$H \rightarrow \tau^{+}\tau^{-}$ *at Lepton colliders*

- At LHC, main production mode is $g\ g \rightarrow H \rightarrow\ \tau^{-}\tau^{+}$, which is loop-induced.
- Final state $\tau^{-}\tau^{+}$ have large background due to tree-level $q\ \bar{q} \rightarrow$ Z* $\rightarrow \tau^{-}\tau^{+}$.
- The main handle for signal/background is the invariant mass of the visible decay products of two taus, due to neutrinos in tau decays, invariant mass have long tails and therefore signal and background overleap.
- At Lepton colliders, main production channel near threshold is $e^{-}e^{+}$ →ZH, and main background is $e^{-}e^{+}$ →Z $\tau^{-}\tau^{+}$, where pair of taus comes from an offshell photon.
- We know initial 4-momentum, can reconstruct Higgs momentum, independent from Higgs decay mode.

LHC



ILC



$H \rightarrow \tau^{+}\tau^{-}$ *at Lepton colliders*

- Second advantage, ability of reconstructing two tau momenta by solving kinematical constraints because we know initial state 4-momentum with good precision.
- This is important for the C-matrix measurement and bell inequality test based on angular distributions of $\pi^{+}$and π^{-} in tau rest frame.
- Since taus are heavily boosted a small error on the tau momentum leads to a large error on the angular distribution
- Precise reconstruction of the tau momenta is therefore crucial for the C-matrix measurement and Bell inequality test

$H \to \tau^+\tau^-$ at Lepton colliders

- To determine the tau momenta, we have to reconstruct the unobserved neutrino momenta $(p_x^{\nu}, p_y^{\nu}, p_z^{\nu})$, $(p_x^{\bar{\nu}}, p_y^{\bar{\nu}}, p_z^{\bar{\nu}})$.

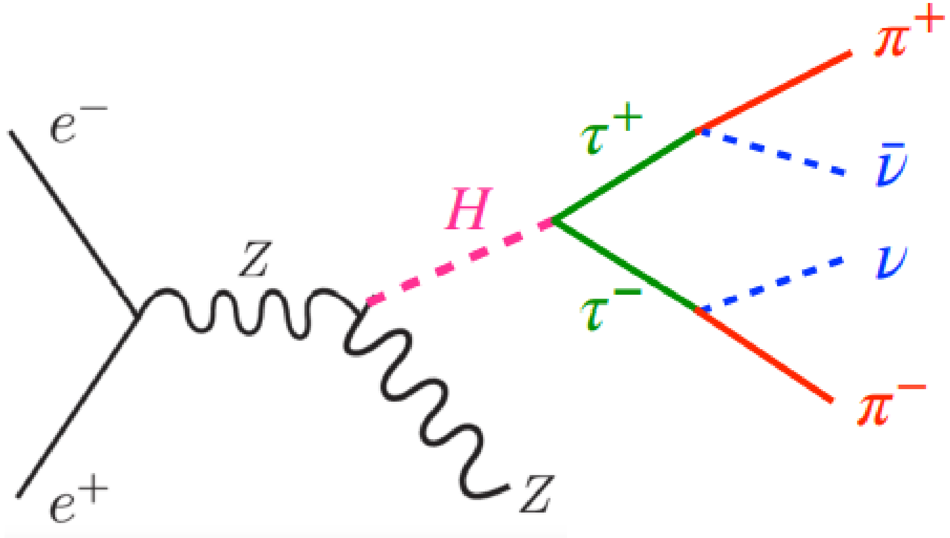
- 6 unknowns can be constrained by 2 mass-shell conditions and 4 energy-momentum conservation.

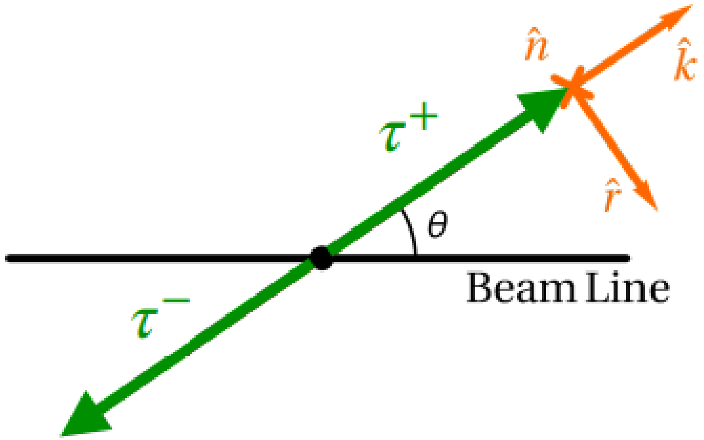
$$m_\tau^2 = (p_{\tau^+})^2 = (p_{\pi^+} + p_{\bar{\nu}})^2$$

$$m_\tau^2 = (p_{\tau^-})^2 = (p_{\pi^-} + p_{\nu})^2$$

$$(p_{ee} - p_Z)^\mu = p_H^\mu = \left[(p_{\pi^-} + p_\nu) + (p_{\pi^+} + p_{\bar{\nu}})\right]^\mu$$

- With the reconstructed momenta, we define $(\hat{\mathbf{k}}, \hat{\mathbf{r}}, \hat{\mathbf{n}})$ basis at the Higgs rest frame.





helicity basis $(\hat{\mathbf{k}}, \hat{\mathbf{r}}, \hat{\mathbf{n}})$

Simulation

	ILC	FCC-ee
energy (GeV)	250	240
luminosity (ab^{-1})	3	5
beam resolution e^+ (%)	0.18	$0.83 \cdot 10^{-4}$
beam resolution e^- (%)	0.27	$0.83 \cdot 10^{-4}$
$\sigma(e^+e^- \to HZ)$ (fb)	240.1	240.3
# of signal ($\sigma \cdot \mathrm{BR} \cdot L$)	414	691

- Generate the SM events $(\kappa, \delta) = (1{,}0)$ with **MadGraph5**.

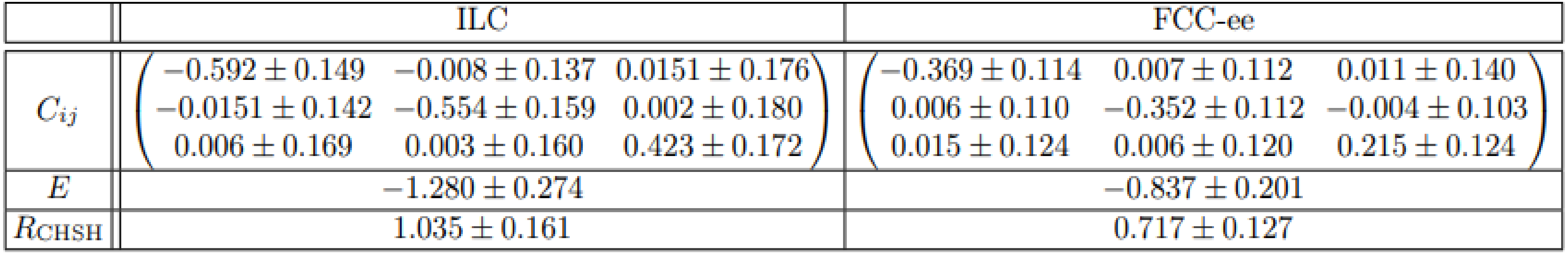
- We incorporate the detector effect by **smearing energies** of visible particles with

$$E^{\text{true}} \to E^{\text{obs}} = (1 + \sigma_E \cdot \underset{\uparrow}{\omega}) \cdot E^{\text{true}} \qquad \sigma_E = 0.03$$

random number from the normal distribution

- We perform **100 pseudo-experiments** to estimate the statistical uncertainties of the measurements.

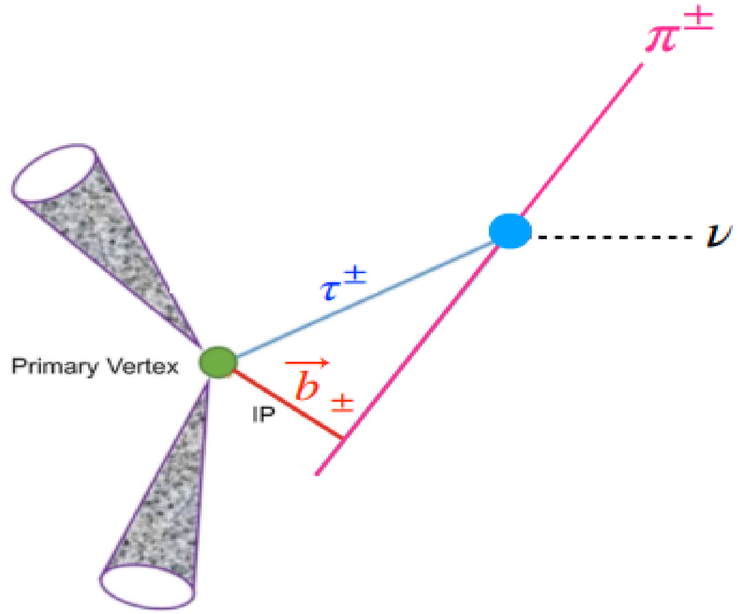
Results



	ILC	FCC-ee
C_{ij}	$\begin{pmatrix} -0.592 \pm 0.149 & -0.008 \pm 0.137 & 0.0151 \pm 0.176 \\ -0.0151 \pm 0.142 & -0.554 \pm 0.159 & 0.002 \pm 0.180 \\ 0.006 \pm 0.169 & 0.003 \pm 0.160 & 0.423 \pm 0.172 \end{pmatrix}$	$\begin{pmatrix} -0.369 \pm 0.114 & 0.007 \pm 0.112 & 0.011 \pm 0.140 \\ 0.006 \pm 0.110 & -0.352 \pm 0.112 & -0.004 \pm 0.103 \\ 0.015 \pm 0.124 & 0.006 \pm 0.120 & 0.215 \pm 0.124 \end{pmatrix}$
E	-1.280 ± 0.274	-0.837 ± 0.201
R_{CHSH}	1.035 ± 0.161	0.717 ± 0.127

- The result is catastrophic. It may be blamed to the detector effect, since the reconstruction of tau-rest frames is very sensitive to the energy resolution.

Impact parameter effect



Use impact parameter information

- We use the information of impact parameter $\vec{b}_\pm$ measurement of $\pi^\pm$ to "correct" the observed energies of $\tau^\pm$ and Z decay products
- We check whether the reconstructed τ momenta are consistent with the measured impact parameters.
- We construct the likelihood function and search for the most likely τ momenta.

$$\boxed{E_\alpha(\delta_\alpha) = (1 + \sigma^E_\alpha \cdot \delta_\alpha) \cdot E^{\text{obs}}_\alpha}$$

$$\vec{b}_+ = |\vec{b}_+| \left(\sin^{-1}\Theta_+ \cdot \vec{e}_{\tau^+} - \tan^{-1}\Theta_+ \cdot \vec{e}_{\pi^+}\right)$$

$$\vec{\Delta}^i_{b_+}(\{\delta\}) \equiv \vec{b}_+ - |\vec{b}_+| \left(\sin^{-1}\Theta^i_+(\{\delta\}) \cdot \vec{e}^{\,i}_{\tau^+}(\{\delta\}) - \tan^{-1}\Theta^i_+(\{\delta\}) \cdot \vec{e}_{\pi^+}\right)$$

$$L^i_\pm(\{\delta\}) = \frac{[\Delta^i_{b_\pm}(\{\delta\})]^2_x + [\Delta^i_{b_\pm}(\{\delta\})]^2_y}{\sigma^2_{b_T}} + \frac{[\Delta^i_{b_\pm}(\{\delta\})]^2_z}{\sigma^2_{b_z}}$$

$$\boxed{L^i(\{\delta\}) = L^i_+(\{\delta\}) + L^i_-(\{\delta\})}$$

Results

	ILC	FCC-ee
C_{ij}	$\begin{pmatrix} 0.7803 \pm 0.195 & 0.019 \pm 0.162 & 0.046 \pm 0.180 \\ -0.001 \pm 0.171 & 0.858 \pm 0.165 & 0.000 \pm 0.178 \\ -0.024 \pm 0.188 & -0.010 \pm 0.162 & -0.678 \pm 0.184 \end{pmatrix}$	$\begin{pmatrix} 0.925 \pm 0.131 & -0.001 \pm 0.122 & 0.023 \pm 0.109 \\ 0.014 \pm 0.128 & 0.968 \pm 0.128 & -0.018 \pm 0.121 \\ -0.009 \pm 0.131 & -0.009 \pm 0.131 & -0.928 \pm 0.126 \end{pmatrix}$
E	2.182 ± 0.309 $\sim 4\sigma$	2.797 ± 0.191 $\gg 5\sigma$
$\mathcal{S}[\rho]$	1.626 ± 0.187 $\sim 3\sigma$	1.922 ± 0.155 $\sim 5\sigma$
R_{CHSH}	0.821 ± 0.167	1.273 ± 0.093 $\sim 3\sigma$

SM values:

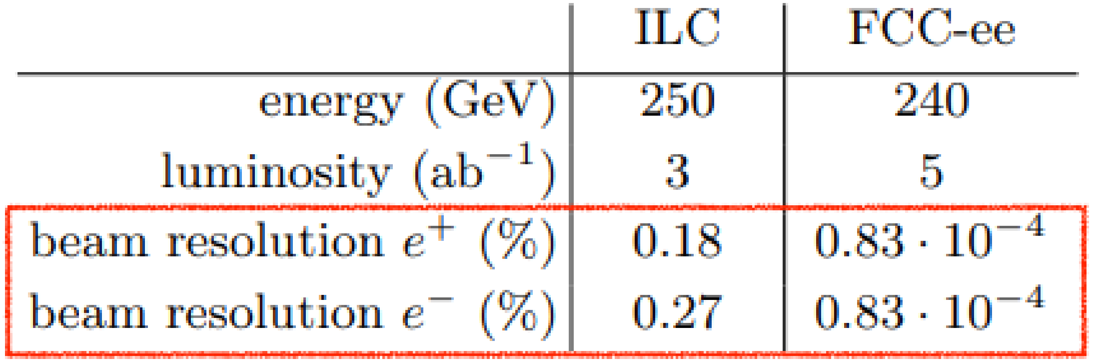
$$C_{ij} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$$

$E = 3$ **Entanglement** $\Longrightarrow E > 1$

$\mathcal{S}[\rho] = 2$ **Steerablity** $\Longrightarrow \mathcal{S}[\rho] > 1$

$R_{\text{CHSH}} = \sqrt{2} \simeq 1.414$ **Bell-nonlocal** $\Longrightarrow R_{\text{CHSH}} > 1$

Superiority of FCC-ee over ILC is due to a better beam resolution



	ILC	FCC-ee
energy (GeV)	250	240
luminosity (ab^{-1})	3	5
beam resolution e^+ (%)	0.18	$0.83 \cdot 10^{-4}$
beam resolution e^- (%)	0.27	$0.83 \cdot 10^{-4}$

CP measurement

- Under CP, the spin correlation matrix transforms: $C \overset{CP}{\longrightarrow} C^T$
- This can be used for a *model-independent* test of CP violation. We define:

$$A \equiv (C_{rn} - C_{nr})^2 + (C_{nk} - C_{kn})^2 + (C_{kr} - C_{rk})^2 \;\; \geq 0$$

- Observation of $A \neq 0$ immediately confirms CP violation.

- From our simulation, we observe

$$A = \begin{cases} 0.204 \pm 0.173 & \text{(ILC)} \\ 0.112 \pm 0.085 & \text{(FCC-ee)} \end{cases} \quad \longleftarrow$$

consistent with absence of CPV

- This model independent bounds can be translated to the constraint on the CP-phase δ

$$\mathscr{L}_{\text{int}} \propto H \bar{\psi}_\tau (\cos\delta + i\gamma_5 \sin\delta)\psi_\tau \;\Rightarrow\; C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix} \;\Rightarrow\; A(\delta) = 4\sin^2 2\delta$$

CP measurement

- Focusing on the region near $|\delta| = 0$, we find the 1-σ bounds:

$$|\delta| < \begin{cases} 8.9^o & \text{(ILC)} \\ 6.4^o & \text{(FCC-ee)} \end{cases}$$

- Other studies:

$\Delta\delta \sim 11.5^o$ (HL-LHC) [Hagiwara, Ma, Mori 2016]

$\Delta\delta \sim 4.3^o$ (ILC) [Jeans and G. W. Wilson 2018]

Summary

- High energy tests of entanglement and Bell inequality has recently attracted an attention.
- $\tau^+\tau^-$ pairs from $H \to \tau^+\tau^-$ form the EPR triplet state $|\Psi^{(1,0)}\rangle = \frac{|+,-\rangle + |-,+\rangle}{\sqrt{2}}$, and maximally entangled.
- We investigated feasibility of quantum property tests @ ILC and FCC-ee.
- Quantum test requires to a precise reconstruction of the tau rest frames and IP information is crucial to achieve this.
- Spin correlation is sensitive to CP-phase and we can measure the CP-phase as a byproduct of the quantum property measurement.

	Entanglement	Steering	Bell-inquality	CP-phase
ILC	$\sim 4\sigma$	$\sim 3\sigma$		8.9^o
FCC-ee	$\gg 5\sigma$	$\sim 5\sigma$	$\sim 3\sigma$	6.4^o

Thank you for the attention!

The explanation in QM is very different.

Although their outcomes are different in each decay, QM says *the state of the particles are exactly the same for all decays*:

$$|\Psi^{(0,0)}\rangle \underset{\uparrow \atop \text{up to a phase } e^{i\theta}}{\doteq} \frac{|\overset{\alpha}{+}\overset{\beta}{-}\rangle_z - |-+\rangle_z}{\sqrt{2}}$$

- Before the measurements, particles have no definite spin. Outcomes are undetermined.

(no realism)

- At the moment when Alice makes her measurement, the state collapses into:

$$|\Psi\rangle \xrightarrow[\text{Alice's measurement}]{} \begin{cases} |+\textcircled{$-$}\rangle_z & \cdots \text{ Alice finds } S_z[\alpha] = +1 \\ |-\textcircled{$+$}\rangle_z & \cdots \text{ Alice finds } S_z[\alpha] = -1 \end{cases}$$

Bob's outcome is completely determined (before his measurement) and 100% anti-correlated with Alice's

(non-local)

Bell inequality

$$\left\langle s_a^\alpha \cdot s_b^\beta \right\rangle = \hat{a}_i \hat{b}_j \cdot \left\langle s_i^\alpha \cdot s_j^\beta \right\rangle = \hat{a}_i C_{ij} \hat{b}_i$$

unit vectors: $\hat{a}, \hat{a}', \hat{b}, \hat{b}'$

$$R_{\mathrm{CHSH}} \equiv \frac{1}{2} \left| \left\langle s_a^\alpha \cdot s_b^\beta \right\rangle - \left\langle s_a^\alpha \cdot s_{b'}^\beta \right\rangle + \left\langle s_{a'}^\alpha \cdot s_b^\beta \right\rangle + \left\langle s_{a'}^\alpha \cdot s_{b'}^\beta \right\rangle \right|$$

$$= \frac{1}{2} \left| \hat{a}_i C_{ij} (\hat{b} - \hat{b}')_j + \hat{a}'_i C_{ij} (\hat{b} + \hat{b}')_j \right|$$

$$\max_{\hat{a}, \hat{a}', \hat{b}, \hat{b}'} \left[R_{\mathrm{CHSH}} \right] = \sqrt{\lambda_1 + \lambda_2}$$

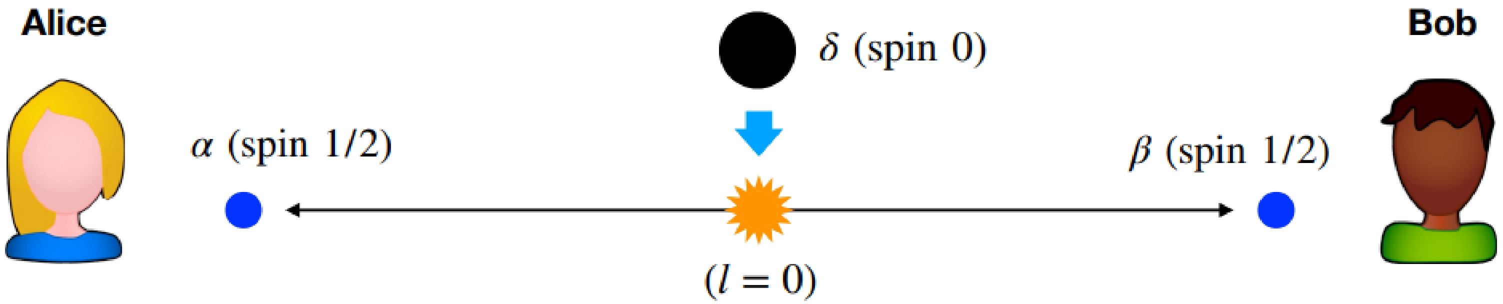
($\lambda_1 \geq \lambda_2 \geq \lambda_3$ are 3 eigenvalues of $C^T C$)

Violation of Bell inequality implies

$$\boxed{\sqrt{\lambda_1 + \lambda_2} > 1}$$

M. Fabbrichesi, R. Floreanini,
G. Panizzo (2021)

$$\begin{aligned}
\sigma(e^+e^- \to HZ)\big|_{\sqrt{s}=240\mathrm{GeV}} &= 240.3\,\mathrm{fb} \\
BR(H \to \tau^+\tau^-) &= 0.0632 \\
BR(\tau^- \to \pi^-\nu_\tau) &= 0.109 \\
BR(Z \to jj, \mu\mu, ee) &= 0.766 \\
\sigma(e^+e^- \to HZ)^{\mathrm{unpol}}_{240} \cdot BR_{H\to\tau\tau} \cdot [BR_{\tau\to\pi\nu}]^2 \cdot BR_{Z\to jj,\mu\mu,ee} &= 0.1382\,\mathrm{fb}
\end{aligned}$$



- Alice and Bob receive particles α and β, respectively, and measure the spin z-component of their particles. Repeat the process many times.
- Alice and Bob will find their results are completely random (+1 and -1 50-50%)
- Nevertheless, their result is 100% anti-correlated due to the angular momentum conservation. If Alice's result is +1, Bon's result is always -1 and vice versa.

Alice	+	+	-	+	-	-	+	+	+	-	+	-
Bob	-	-	+	-	+	+	-	-	-	+	-	+
$S_z^\alpha \cdot S_z^\beta$	-	-	-	-	-	-	-	-	-	-	-	-

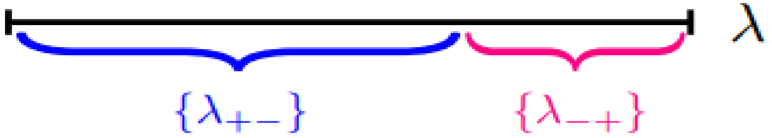
$$\langle S_z^\alpha \cdot S_z^\beta \rangle = -1$$

The most natural explanation would be as follows:

- Since their result is sometimes +1 and sometimes -1, it is natural to think that *the state of α and β are different in each decay.* The result look random, since we don't know in which sate the α and β particles are in each decay.

- This means we can parametrise the state of *α* and *β* by a set of unknown (hidden) variables, λ. For *i*-th decay, their states are:

$$\alpha(\lambda_i), \qquad \beta(\lambda_i)$$



$$\text{If } \lambda_i \in \{\lambda_{+-}\} \implies S_z[\alpha(\lambda_i)] = +1, \;\; S_z[\beta(\lambda_i)] = -1$$

$$\text{If } \lambda_i \in \{\lambda_{-+}\} \implies S_z[\alpha(\lambda_i)] = -1, \;\; S_z[\beta(\lambda_i)] = +1$$

$$P(\lambda \in \{\lambda_{+-}\}) = P(\lambda \in \{\lambda_{-+}\}) = \frac{1}{2}$$

In this explanation:

- Particles have definite properties regardless of the measurement (realism)

- Alice's measurement has no influence on Bob's particle (locality)

$$C_{ij} = 4 \cdot \frac{N(\xi_{ij} > 0) - N(\xi_{ij} < 0)}{N(\xi_{ij} > 0) + N(\xi_{ij} < 0)}$$