INSTITUT POLYTECHNIQUE DE PARIS

QUANTUM GRAVITY AND THE SWAMPLAND (REVIEW)

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« Early Universe », Warsaw

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Outline

0) Fundamental interactions and quantum gravity*

1) Quantum Gravity and Swampland

- No exact global symmetries, completeness conjecture
- The weak gravity conjecture (WGC), naturalness
- UV/IR connections from black holes
- De Sitter conjecture
- Festina Lente

2) Causality and unitarity conditions*

0) Fundamental interactions and quantum gravity

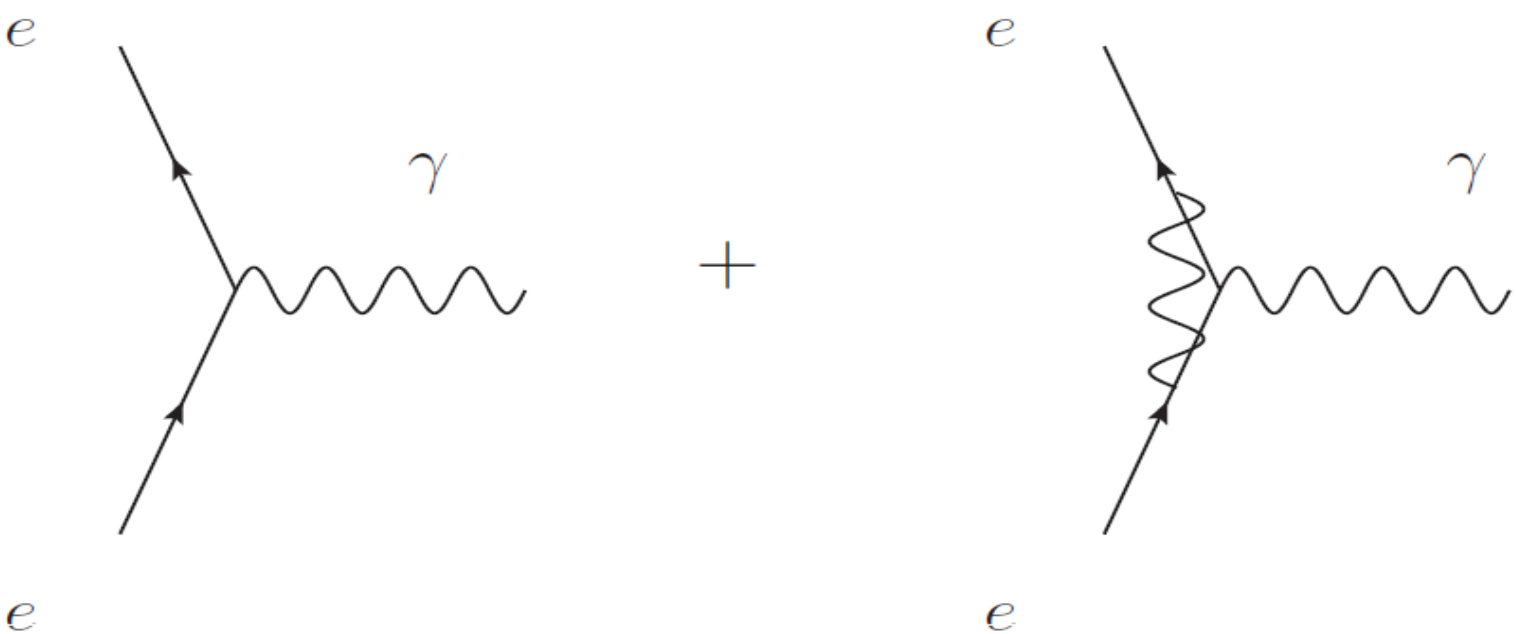
There are four fundamental interactions in nature :

Interaction	Description	distances
$Gravitation$	$Rel.\ gen.$	$Infinity$
$Electromagn.$	$Maxwell$	$Infinity$
$Strong$	$Yang-Mills\ (QCD)$	$10^{-15}m$
$Weak$	$Weinberg-Salam$	$10^{-17}m$

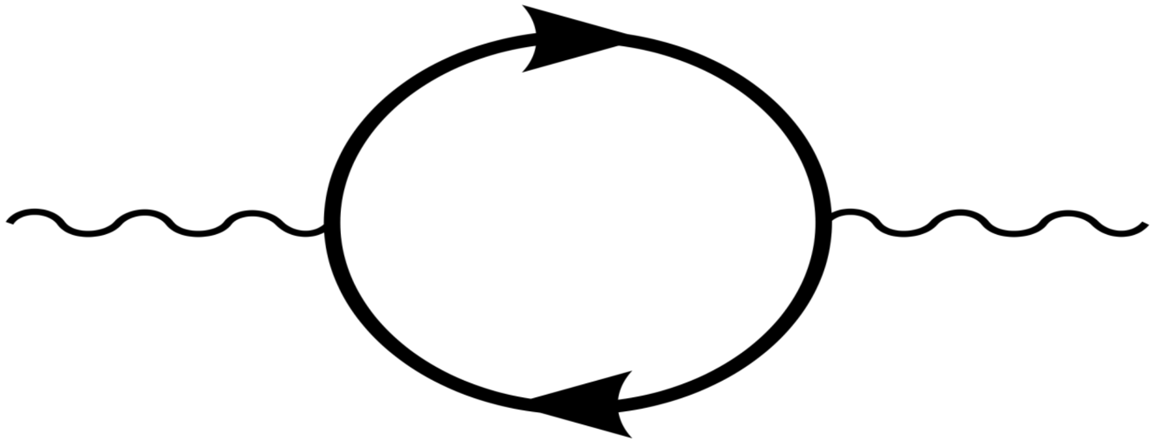
With the exception of gravity, all other interactions are described by renormalizable quantum field theories (QFT).

QFT = relativity + quantum mechanics

Computation of physical obervables is based on perturbation theory:



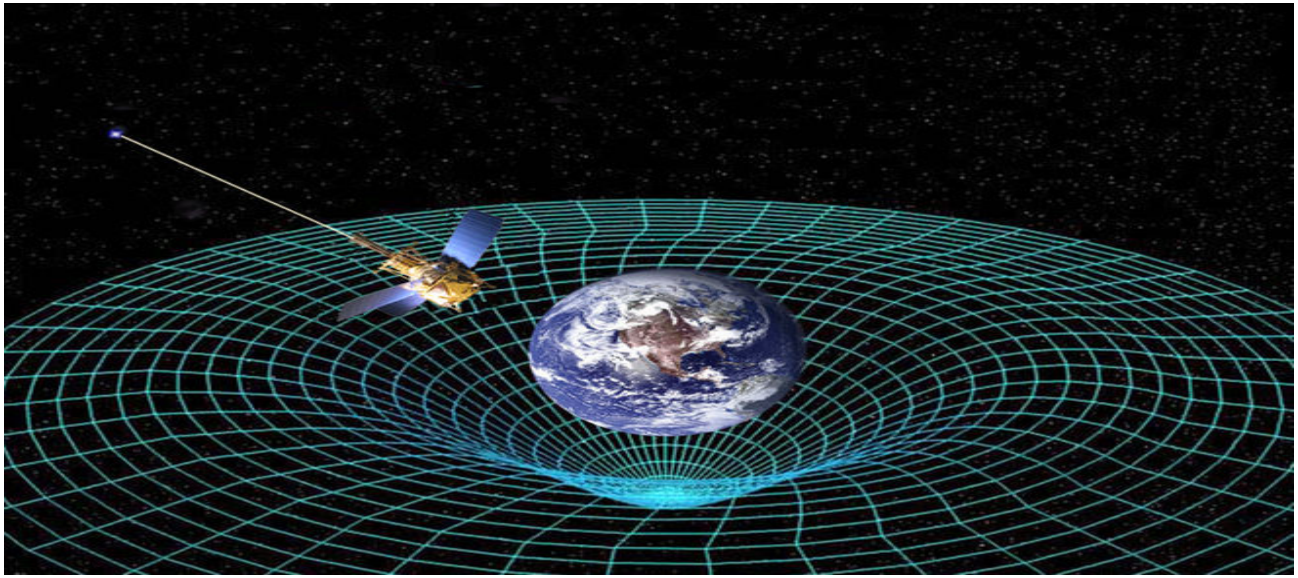
Point-like interactions in Feynman diagrams generate ultraviolet (UV) divergences



- Renormalizable theory: the UV divergences can be « hidden » into a finite number of parameters (charges,masses)
- Renormalization predicts the variation with energy of the fine structure constant, confirmed at CERN.

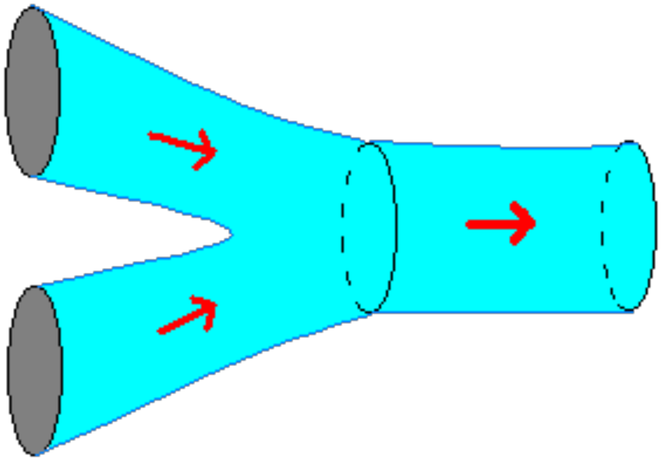
Einstein general relativity is a classical theory :

Mass/energy ⟹ spacetime geometry $g_{\mu\nu}$



Its quantization $g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_P} h_{\mu\nu}$ leads to UV divergences which cannot be reabsorbed in a finite number of parameters ⟹ non-renormalizable

- String theory has no point-like interactions (minimal length $l_s = M_s^{-1/2}$)



→ no UV divergences.

String theory contains all four fundamental interactions. The only complete theory of Quantum Gravity to date.

1) Swampland and Quantum Gravity

Are all consistent Quantum Field Theories obtainable from String Theory ?

Probably NO

Swampland = the set of consistent QFT not obtainable from String Theory

There are various swampland conjectures: (Vafa+Ooguri, review E.Palti)

- **No exact global symmetries**, completeness conjecture

- de Sitter: impossibility of constructing a vacuum with positive cosmological constant →

- quintessence-like dark energy models are then the only viable possibility ?

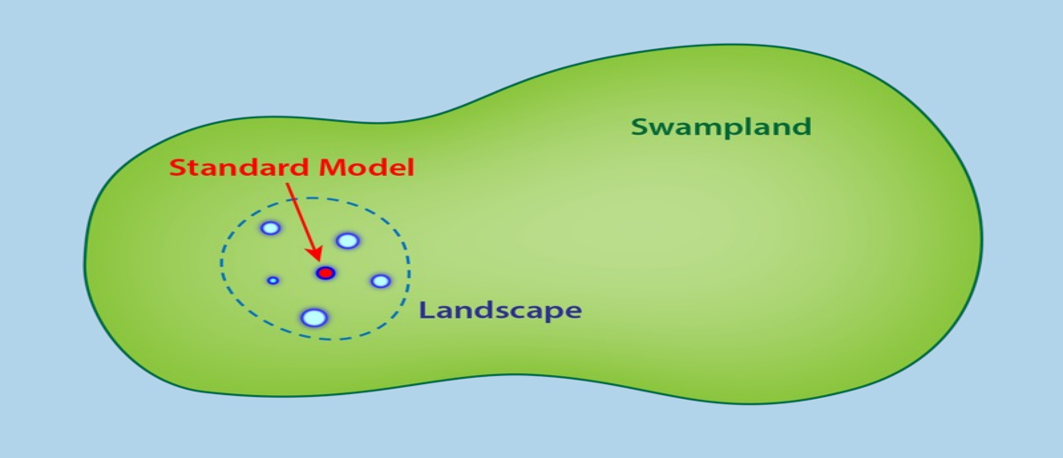
- Weak Gravity Conjecture (WGC) ↔ Gravity is the weakest force

- The distance conjecture: not possible to have super-Planckian field excursions

- Festina Lente, etc

Final goal swampland program ?

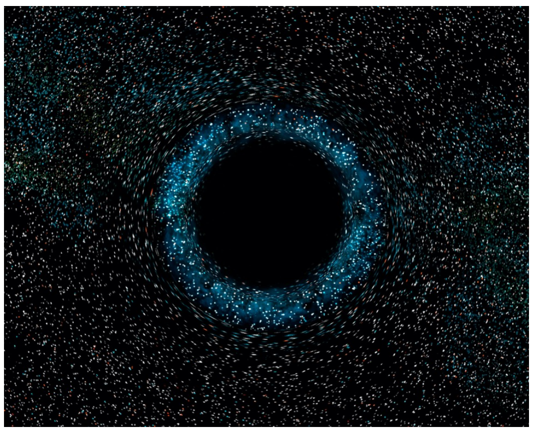
Supplement rules of effective QFT with additional constraints, which would guide Beyond the Standard Model and cosmology constructions.

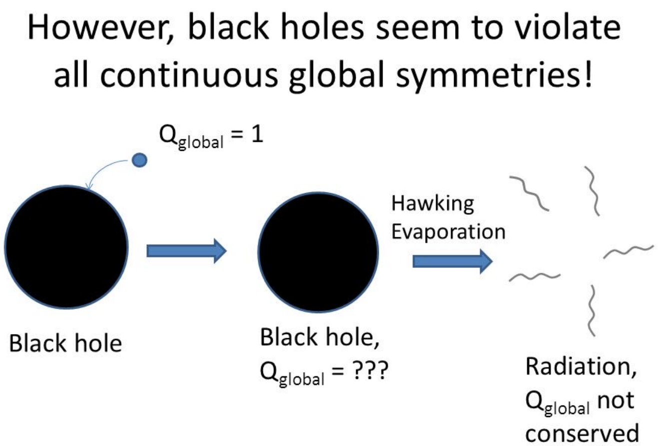


- No exact global symmetries ?

It is believed that a theory of quantum gravity cannot have exact global symmetries. True for String Theory.

Black hole argument:



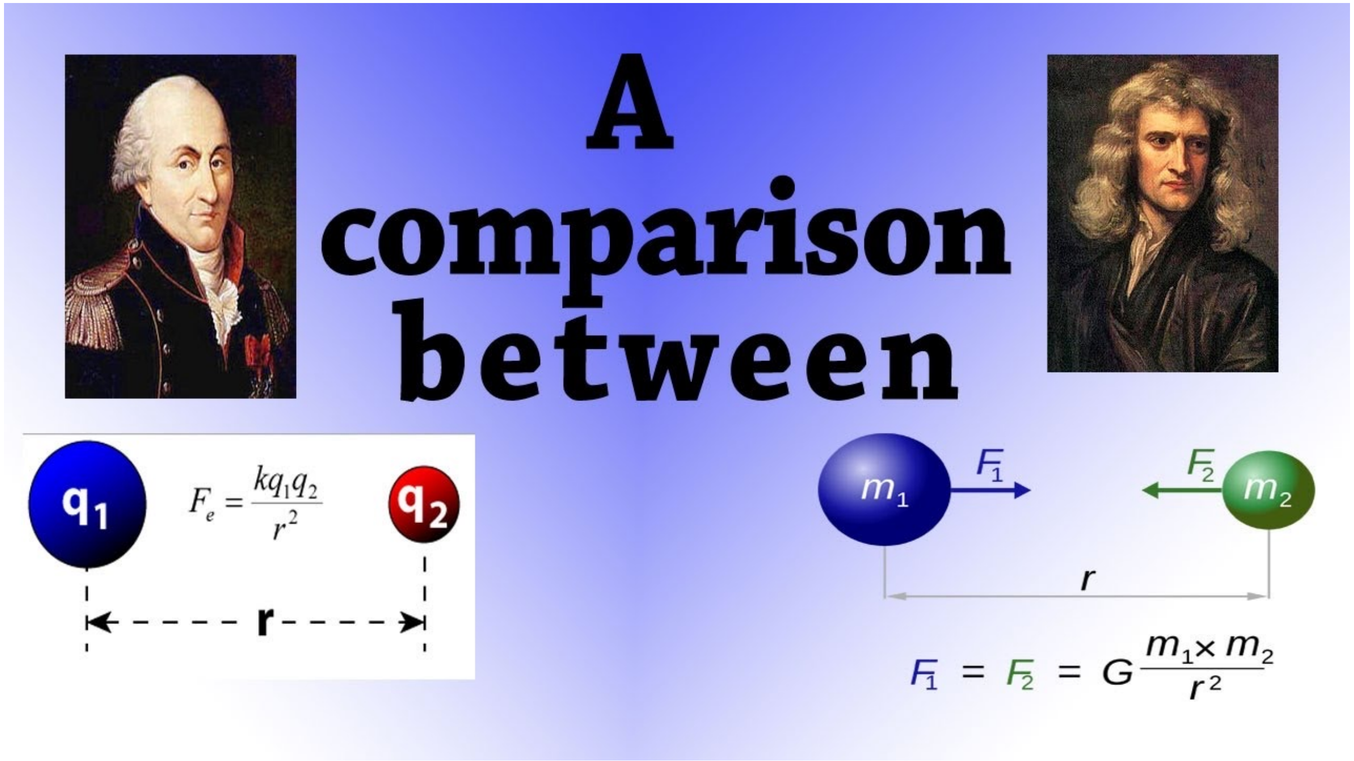


If true, baryon B and lepton L number can only be approximate symmetries of The Standard Model (even B-L).

- The weak gravity conjecture (WGC)

Arkani-Hamed, Motl,Nicolis, Vafa, 2006

Loose form: **GRAVITY IS THE WEAKEST FORCE.**



For a theory with a massless photon coupled to gravity, it implies that there should exist one charged particle with

$$|q| M_P \geq m$$

Some arguments in favor of WGC:

- Avoidance of stable charged black hole remnants

- A charged (RN) black hole has $|Q| M_P < M$
It can evaporate by emitting particles with $|q| M_P \geq m$

- Absence of global symmetries in string theory/quantum gravity

- In the limit $q \to 0$ gauge symmetry becomes global.
This should be forbidden, at least in string theory.

There are potential intriguing connections between WGC and

- The hierarchy problem (Cheung-Remmen) : quadratically div. contributions to a charged scalar could violate WGC

$$|q| M_P \geq m$$

log divergent

$$\delta q \sim \log(\Lambda / m)$$

quadratically div. (Higgs scalar)

$$\delta m_h^2 \sim \Lambda^2$$

⟹ the UV cutoff Λ cannot be too high ?

- Cosmic censorship (Horowitz et al.) : bad singularities in geometries violating CC are forbidden by WGC

If there are other light (scalar) mediators , the mass of a field can depend on the scalar (i.e. Yukawa interactions) $m(\phi^i)$ ⇒

The WGC can be refined into a Repulsive Force Condition (RFC)

$$F_{\text{gauge}} \geq F_{\text{gravity}} + F_{\text{scalar}}$$

leading to

$$q^2 M_P^2 \geq m^2 + g^{ij}(\partial_{\phi^i} m)(\partial_{\phi^j} m) M_P^2$$

Light scalars are often present in string theory constructions.
WGC and RFC were checked in many string examples.

People are also considering more radical conjectures, like the Scalar Weak Gravity Conjecture :
scalar mediation should be stronger than gravity one.

$$g^{ij}(\partial_{\phi^i} m)(\partial_{\phi^j} m) M_P^2 \geq m^2$$

Not enough tested by now. It could be true for (at least) one state/particle, (very) probably not for all states of a theory.

- UV/IR connexions ?

Bekenstein,1981: « The entropy in a region of space is bounded by the BH entropy that can be stored in a region of the same size »

Field theory with UV cutoff Λ_{UV} , $S_{EFT} \sim L^3 \Lambda_{UV}^3$

Sphere of radius L , $\Lambda_{IR} = \frac{1}{L}$, $S_{BH} \sim L^2 M_P^2$

$$S_{EFT} \leq S_{BH} \quad \rightarrow \quad \Lambda_{UV}^3 \leq \Lambda_{IR} M_P^2$$

(Cohen,Kaplan,Nelson, 1999)

The energy density in our universe provides a natural IR cutoff

$$\Lambda_{IR} \sim V_0^{1/4} \sim 10^{-3}\ eV \quad \Longrightarrow$$

$$\Lambda_{UV} \leq V_0^{1/2} M_P^{2/3} \sim 2.4 \times 10^8\ GeV \ \ll M_P$$

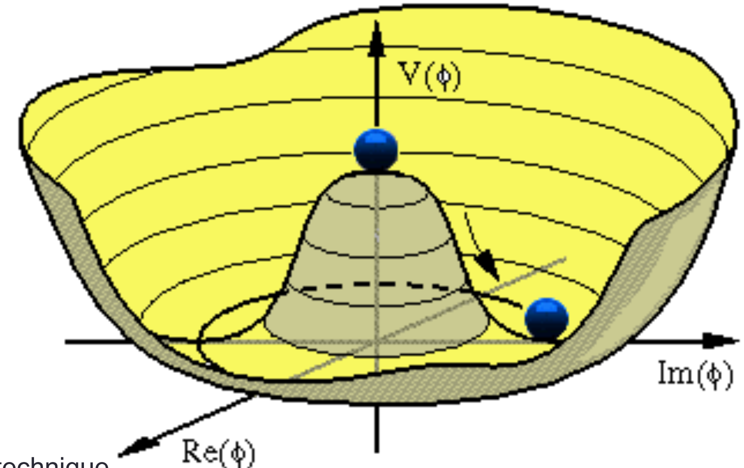
- The dS conjecture

It seems hard to obtain a de Sitter space in string theory. This led to one of the most controversial Swampland conjecture:

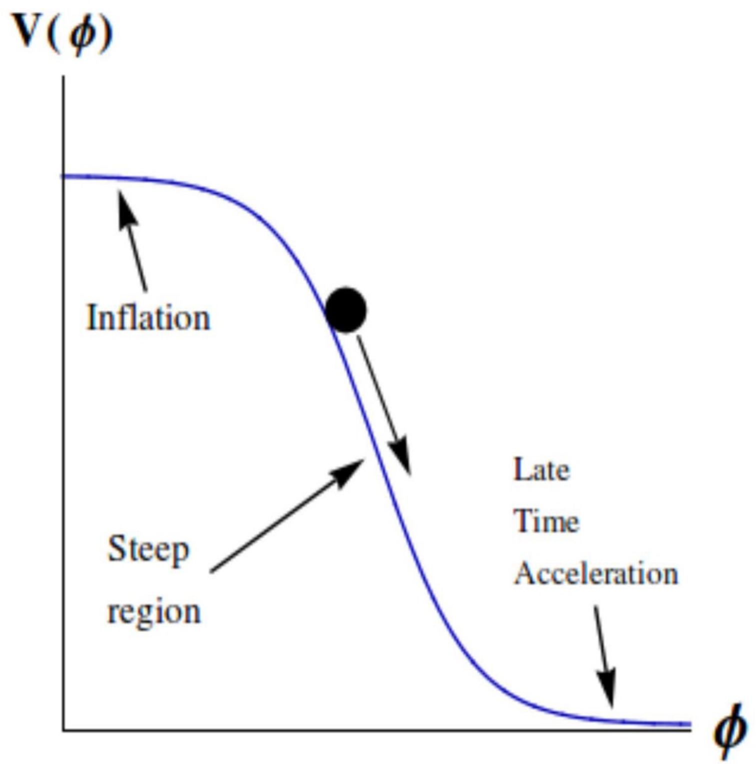
An EFT coupled to gravity must satisfy

$$M_P \frac{|\nabla V|}{V} \geq c \qquad \text{or} \qquad \min\left(\nabla_i \nabla_j V\right) \leq \frac{-c' V}{M_P^2}$$

where $c, c' \sim O(1)$



The dS conjecture rules out the existence of de Sitter minima. We live however in an expanding Universe, we need a positive dark energy ! The conjecture favors therefore dynamical dark energy models like quintessence.



- Distance conjecture

- DC: There is an infinite tower of states that become exponentially light at any infinite field distance limit

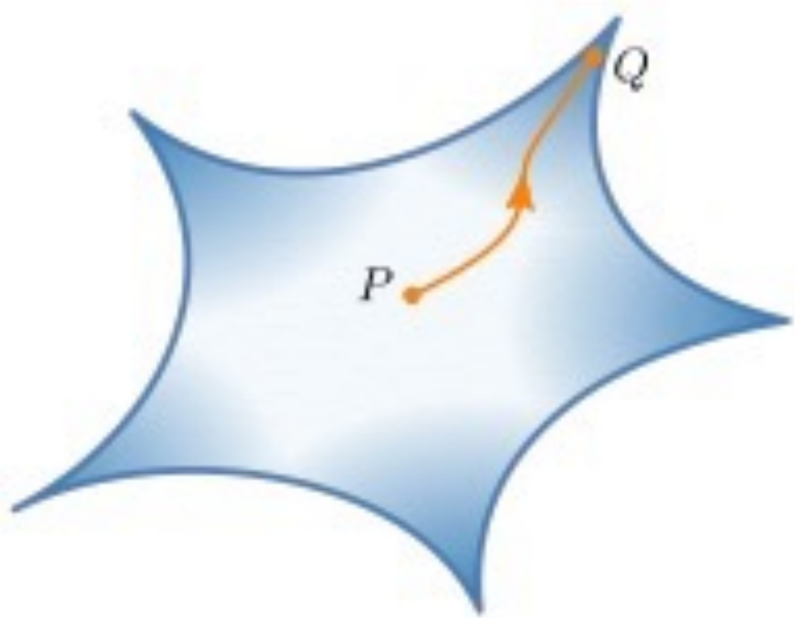
$$M(Q) \sim M(P)\, e^{-\lambda \Delta\phi} \quad \text{when } \Delta\phi \to \infty,$$

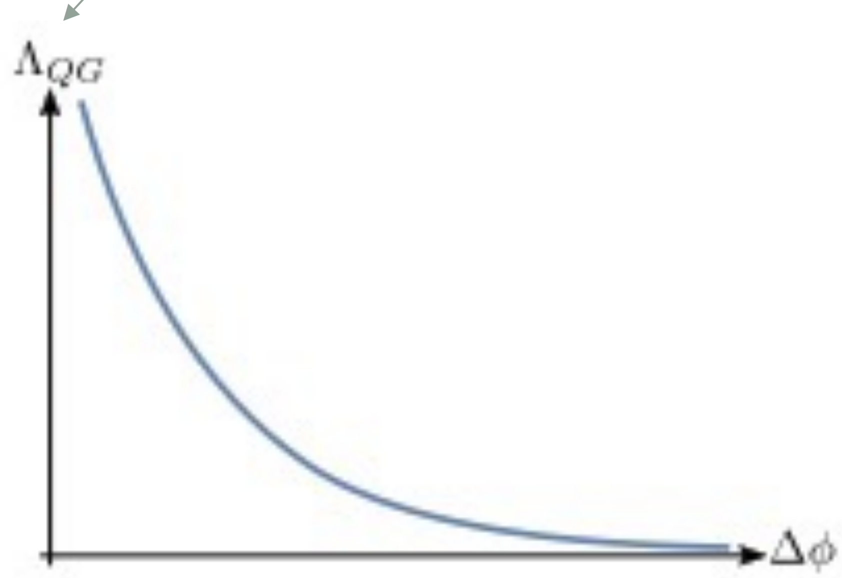
where $\Delta\phi \equiv d(P,Q)$ is the geodesic field distance between points P and Q, and $\lambda \sim O(1)$

The infinite tower of states signals of breakdown of the EFT and the lowering of the UV cutoff that decreases exponentially in terms of the proper field distance

$$\Lambda_{QG} = \Lambda_0\, e^{-\lambda \Delta\phi}$$

Quantum gravity scale





Quantum gravity moduli space with asymptotic limits at infinite distance

Quantum gravity cutoff falling exponentially with the distance, due to the tower of light states from the distance conjecture

Since $\Lambda_0 \leq M_P$, one finds that field variations are limited

$$\Delta\phi \leq \frac{1}{\lambda} \log \frac{M_p}{\Lambda}$$

This limit has direct implications: in inflation, $H \leq \Lambda$ so

$$\Delta\phi \leq \frac{1}{\lambda} \log \left(\frac{M_p}{H} \right)$$

On the other hand, during inflation

$$\frac{M_p}{H} = \sqrt{\frac{2}{\pi^2 A_s r}}$$

amplitude of scalar perturbations   tensor/scalar ratio

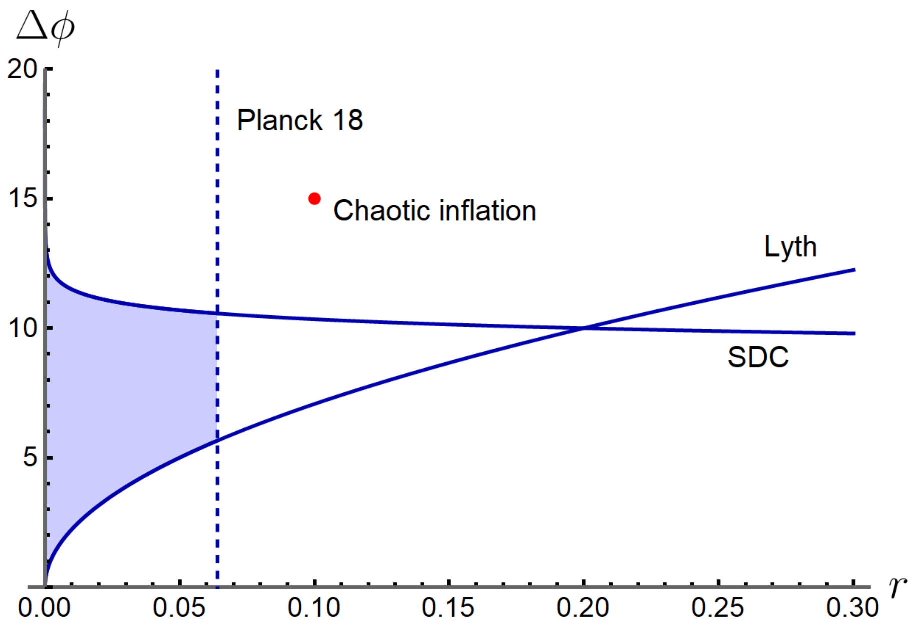
One finds

$$\Delta\phi \leq -\frac{1}{2\lambda}\left(\log\left(\frac{\pi^2 A_s}{2}\right) + \log r\right)$$

This can be combined with the Lyth bound:

$$\Delta\phi \geq \left(\frac{r}{0.002}\right)^{1/2}$$

Ex: ($\lambda = 1$; result sensitive to the value of λ)



- Festina Lente (Montero,van Riet,Venke+Vafa, 2019)

- In de Sitter spacetime with an abelian gauge field, there are charged extremal (Nariai) black holes.
- Imposing that these black holes be able to discharge through Schwinger pair production ⟹

any charged particle of mass m and charge q should satisfy

$$m^4 > 2q^2 g^2 V_0$$

Fulfilled by the Standard Model

$$m_e^4 \sim 10^{-12}\ GeV^4 \gg 2e^2 V_0 \sim 10^{-48}\ GeV^4$$

Some implications of FL:

- It improves significantly the c.c. problem:

$$V_0 < \frac{m_e^4}{2e^2} \sim (MeV)^4 \sim 10^{-84}\ M_P^4$$

- Any non-abelian theory must be confining or broken (off-diagonal generators are charged particles) ! It fits with the Standard Model.

- Strongly constrain inflation ; Higgs needs to have large vev's during inflation !

- All tests of swampland conjectures were done essentially on supersymmetric strings.

In this case, due to SUSY some conjectures are trivial:

- WGC, since attraction exactly cancel repulsion (BPS condition)
- dS conjecture, since SUSY implies AdS or Minkowski vacua

- String vacua with broken supersymmetry are a more natural framework to study swampland conjectures (my activity), situation considerably more difficult.

2) Causality and unitarity conditions

- A low-energy effective action has an expansion in the dimension of operators.
- Higher dimension/derivative operators are generated by heavy particles, new interactions. Some of them are constrained by general consistency conditions

Ex:

$$\mathcal{L} = (\partial\Pi)^2 + (\partial\phi)^2 - F_{\mu\nu}^2 +$$
$$\frac{1}{\Lambda^4}\left[c_1(\partial\Pi)^4 + c_2(\partial_\mu\phi)(\partial_\nu\phi)(\partial^\mu\Pi)(\partial^\nu\Pi) + c_3(F_{\mu\nu}^2)^2\right] + \cdots$$

It turns out that $c_i \geq 0$ for the effective theory to respect general consistency conditions :

1) Causality (for some operators). Ex: consider time-dependent solutions for ϕ (inflation…) ⇒

$$\mathcal{L} = (1 + \frac{c_2}{\Lambda^4} \dot{\phi}^2)(\dot{\Pi})^2 - (\partial_i \Pi)^2 + \cdots$$

If $c_2 \leq 0$, the field Π has super-luminal propagation

2) Unitarity, analyticity. Ex:

Amplitudes $A(1+2 \to 1+2) > 0$ in the forward limit. One can show in our ex. that

$$A(\Pi + \Pi \to \Pi + \Pi)_{t=0} > 0 \ \to \ c_1 \geq 0$$

$$A(\Pi + \phi \to \Pi + \phi)_{t=0} > 0 \ \to \ c_2 \geq 0$$

$$A(A_\mu + A_\nu \to A_\mu + A_\nu)_{t=0} > 0 \ \to \ c_3 \geq 0$$

where $t = (p_1 - p_3)^2$ is the Mandelstam parameter

- Such positivity constraints are widely used nowadays in Standard Model Effective Field Theory (SMEFT)

It was shown recently in various situations that mild gravity violations of positivity conditions are possible/consistent due to time delay in GR

$$\frac{c_i}{\Lambda^2} \sim - \frac{1}{M_P^2}$$

Conclusions and Perspectives

- No complete theory of quantum gravity, string theory the best candidate to date
- Swampland conjectures do impose Quantum Gravity constraints on BSM and cosmological models.
- Important to test more these conjectures. Some of them (ex. WGC) well tested, others rather conjectural for now.
  - Strings with broken SUSY, not many checks.
- Causality and unitarity/analyticity provide more traditional consistency conditions.

Thank You !