Future $\mathrm{e^+e^-}$ center-of-mass energy determinations with dilepton final states

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- Based on "Center-of-mass energy determination using $\mathrm{e^+e^-} \to \mu^+\mu^-(\gamma)$ events at future $\mathrm{e^+e^-}$ colliders" (2209.03281) with Brendon Madison.
- We use a muon **momenta** based estimator, $\sqrt{s}_p$, to measure precisely the absolute center-of-mass energy scale of actual **collisions** without assuming ISR is collinear.
- Needs great momentum resolution and exquisite control of tracker momentum scale.
- Uses all dimuon events. It can work well at all $\sqrt{s}$ and especially for $\sqrt{s} \approx M_{\mathrm{Z}}$.
- Relevant to linear and circular $\mathrm{e^+e^-}$ colliders: C^3, CLIC, ILC, ReLiC, FCC-ee.
- Applies also to Bhabhas, $\mathrm{e^+e^-} \to \mathrm{e^+e^-}(\gamma)$.

The Fine Print

Why not just use resonant depolarization (RDP) given that this has worked great in the past for measurements of the masses of the Z and the J/ψ?

Answers

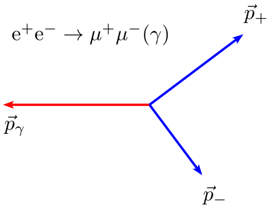
1. RDP not feasible for longitudinally polarized single-pass linear colliders.
2. RDP not achievable in circular colliders if the beam energy spread (BES) is too big[1]. This was the case at LEP for running at $\sqrt{s}$ above about 120 GeV.
3. Even for FCC-ee at the Z, it appears impossible to use RDP for the **colliding bunches** given the large beamstrahlung-induced BES.
4. RDP is only achievable for FCC-ee when measuring the non-colliding pilot bunches, and only for lower center-of-mass energies (91 GeV, 161 GeV). Maybe not even at $\sqrt{s} = 175$ GeV. Certainly not at $\sqrt{s}$ values consistent with ZH production Higgs-factory type operation like 240/250 GeV.
5. For energies/colliders with feasible RDP, inferring the correct $\sqrt{s}$ requires a trusted model for transporting the orbital beam energies to collision $\sqrt{s}$.

Message

For all colliders, measurements with collision events will give luminosity weighted estimates of center-of-mass energy for the bunches that do collide at each IP.

[1] Sufficient transverse polarization requires a BES less than 55 MeV (1909.12245)

$\sqrt{s}_p$ Method in a Nutshell



Measure $\sqrt{s}_p$ using,
$(|\vec{p}_+|, |\vec{p}_-|, |\vec{p}_+ + \vec{p}_-|)$

Assuming,

- **Equal** beam energies, E_b
- The lab **is** the CM frame, $(\sqrt{s} = 2\,E_\mathrm{b},\ \sum \vec{p}_i = 0)$
- The system recoiling against the dimuon is **massless**

$$\sqrt{s} = \sqrt{s}_p \equiv E_+ + E_- + |\vec{p}_+ + \vec{p}_-|$$

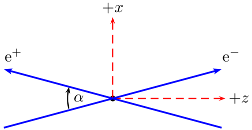
$$\sqrt{s}_p = \sqrt{p_+^2 + m_\mu^2} + \sqrt{p_-^2 + m_\mu^2} + |\vec{p}_+ + \vec{p}_-|$$

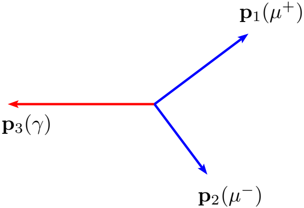
An estimate of $\sqrt{s}$ using only the (precisely measurable) muon momenta

- No assumption on the photon direction.
- With ILD detector at ILC - expect 0.14% momentum resolution for typical 71 GeV muons in Zγ events at $\sqrt{s} = 250$ GeV.
- Detector-level studies are with full simulation and reconstruction.

Essentials Explained

General case. 3 nuisance parameters: **crossing angle**, α, **recoil mass**, M_3, **event collision energy asymmetry**, $(E_\mathrm{b}^- - E_\mathrm{b}^+)/(E_\mathrm{b}^- + E_\mathrm{b}^+) = \overline{\Delta E_\mathrm{b}}/E_\mathrm{ave}$.





1. $\sqrt{s} = E_1^* + E_2^* + E_3^* = E_{12}^* + E_3^*$
2. $\sqrt{s} = E_{12}^* + \sqrt{(p_{12}^*)^2 + M_3^2}$ (general M_3)
3. $\sqrt{s} = E_{12}^* + |\mathbf{p}_{12}^*|$ (assuming $M_3 = 0$)

We have the measured dimuon 4-vector in the detector frame $(E_{12}, \mathbf{p}_{12})$. Need to apply the appropriate boost from lab back to the CM frame to obtain $(E_{12}^*, \mathbf{p}_{12}^*)$. The boost velocity (in the horizontal x-z plane) is

$$\boldsymbol{\beta} = (\beta_x, \beta_y, \beta_z) = \left(\sin(\alpha/2), 0, \frac{\overline{\Delta E_\mathrm{b}}}{E_\mathrm{ave}} \cos(\alpha/2)\right)$$

$\beta_x = 0.007/0.015$ (ILC/FCC-ee). β_z depends on the collision energy asymmetry.

Today

Recent talks given on same basic subject, Snowmass, FCC-EPOL, Hamburg
See those and the preprint for more details. Many of the basic ideas were presented in ECFA LC2013.
In today's talk, I'll try to both introduce this, and add a few new things that go beyond recent talks.
New in Hamburg:

1. **Measuring single beam energies in collision.**
2. Emphasize **order of magnitude** improvement with $\sqrt{s}_p$ method (2 ppm) over Zγ angles method (25 ppm) at $\sqrt{s} = 250$ GeV.
3. More on $(E_+, E_-, z_{\mathrm{collision}})$ luminosity spectrum. Important for simulations.
4. (Effects of E-z correlations in initial beam.)

New today:

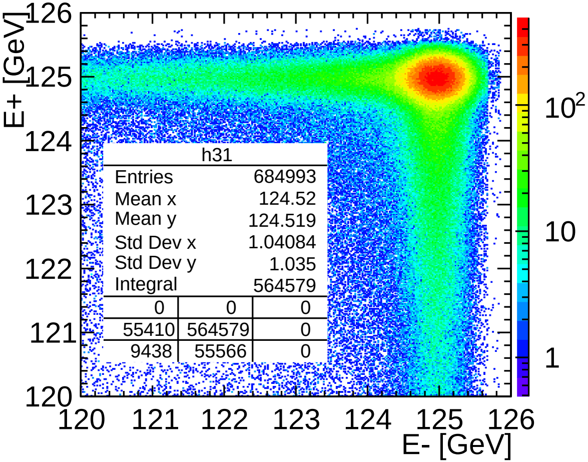
1. New Guinea-PIG studies with better treatment of BES.
2. More physically motivated parametrization for energy distribution modeling (removes/reduces observed bias).
3. Realization that the Bhabha channel, (only buys a factor of 2 for Zs) is **very important** also at wide angle for center-of-mass energies beyond the Z. (factor of 20 (s+t)/s enhancement for $\theta > 30^\circ$ at $\sqrt{s} = 250$ GeV).

Ideally, the 2-d distribution of the **absolute beam energies** after beamstrahlung. From this we would know the distribution of both $\sqrt{s}$ and the initial state momentum vector (especially the z component).

Backup has the related 1-d distributions ($E_+, E_-, \sqrt{s}, p_z$) with empirical fits.

Absolute energies of peak position (E) and shape (LS)

[dL/d$\sqrt{s}$: see work by Boogert, Frary, Miller, Moenig, Sailer, Poss]

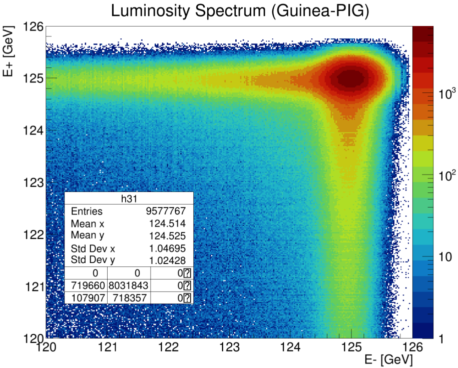


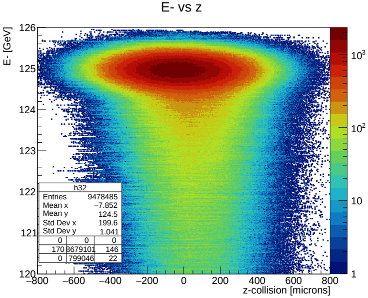
Whizard 250 GeV SetA $e^+e^- \to \mu^+\mu^-(\gamma)$ events

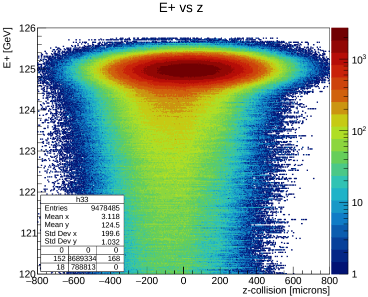
Luminosity Spectrum: $L(x_1, x_2)$

Defining $x_1 = 2E_- / \sqrt{s}_{\mathrm{nom}}$,
$x_2 = 2E_+ / \sqrt{s}_{\mathrm{nom}}$

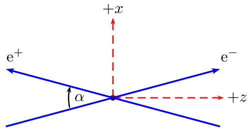
$$\sigma_{\mathrm{Eff}}^{\mathrm{Machine}} = \iint_0^{x_{\max}} \mathrm{d}x_1 \mathrm{d}x_2 \; \mathscr{L}(x_1, x_2) \sigma\left(\sqrt{x_1 x_2 s_{\mathrm{nom}}}\right)$$



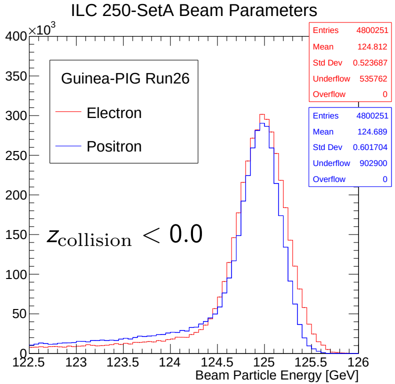


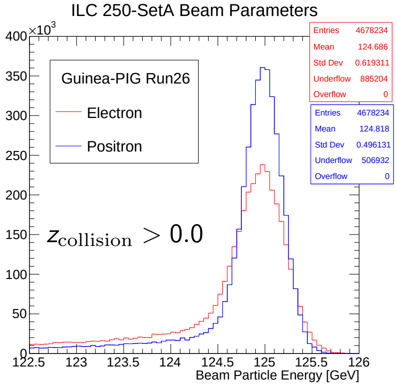


Beamstrahlung dependence on longitudinal collision point



ILC beams collide head-on (CC). FCC-ee beams cross at an angle.





Beam particle collisions happening before/after the respective beam reaches $z = 0$ tend to have higher/lower beam energies (after beamstrahlung).

Guinea-PIG Configurations

Recently, experimenting with Guinea-PIG. Goal: more reliable model of the luminosity spectrum and the combined effects of BES and beamstrahlung.

Guinea-PIG File Configurations

Current ILC250 files use:
waist_y=1.1*sigma_z.1;
n_x=128; n_y=256; n_z=32; n_t=4;
cut_x=15.0*sigma_x.1; cut_y=30.0*sigma_y.1; cut_z=3.5*sigma_z.1;
NB. CPU needs scale as (n_m)^2.

Run #	n_m	seed	load_beam	do_espread	force_symmetric	Lee/BX (e30)	de1/de2 [GeV]	miss
27	160k	2777626648	3	0	0	1.99533	3.15/3.14	1.5%/0.0%
30	200k	1500	0	1 (3)	0	1.87891	3.26/3.27	1.7%/1.0%
31	200k	1501	0	1 (3)	1	2.10226	3.05/3.08	0/0
32	200k	1502	0	1 (3)	1	2.12344	3.06/3.12	0/0
33	200k	1502	0	0	1	2.12344	3.07/3.12	0/0
35	200k	1502	0	1 (4)	1	2.12344	3.06/3.12	0/0
36	320k	1502	0	0	1	2.11698	3.10/3.08	0/0
37	320k	1503	0	0	1	2.10929	3.06/3.06	0/0
40	500k	1640	0	0	1	2.11095	3.06/3.08	0/0

1. pre-smear
2. multiplicative BES
3. additive BES
4. no BES (post-smear)

- Switched to using force_symmetric = 1 (impose up-down (y) and left-right (x) symmetry of bunch charge densities), and no BES (post-smear).
- Main GP plots for 1M "Run4X" events, basically 10 runs like Run40 with independent seeds, and selecting the first 100k lumi. events in each.
- Fixes underlying issues in standard ILC files with re-use of same beam particles.

Fit parametrizations

Work in 2209.03281 used three empirical parametrizations to model energy distributions with beamstrahlung and energy spread:

1. Asymmetric Crystal Ball. Up to 7 parameters.
2. Double exponential tail convolved with Gaussian (6 parameters).
3. Single exponential tail convolved with Gaussian (4 parameters).

The double exponential tail was chosen mostly because it could be easily convolved analytically and it gave reasonably satisfactory fits.

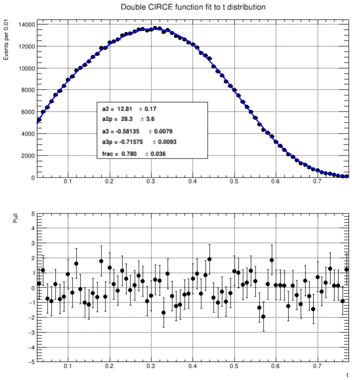
We now have a new fit parametrization using the CIRCE-like function (T. Ohl) convolved using numerical integration with a Gaussian (5 parameters), labeled "Beta tail" in the plots. The CIRCE function is the combination of a Beta distribution with parameters, α, β used to simulate the beamstrahlung tail, and a Dirac δ-function at $x = 1$ for the unperturbed beam. The Gaussian is for beam energy spread.
Here, $x \equiv E/E_0$, and,

$$\mathrm{Beta}(x; \alpha, \beta) \sim x^{\alpha-1}(1-x)^{\beta-1}$$

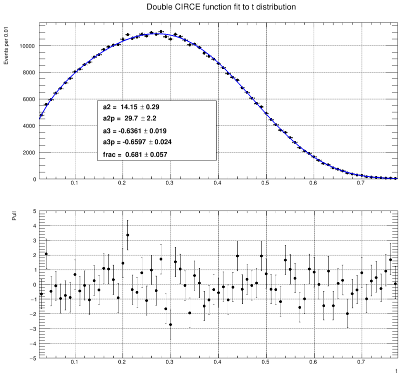
These fits have 5 parameters: α, β, E_0, σ, and f_{peak}.
Given the singularity at $x = 1$, the integration maps x to $t \equiv (1-x)^{1/\eta}$.

Run4X Circe Style Fits (no BES)

A "double Beta function" is needed to fully describe these "deconvolved" distributions at the 1M event level. Use $\eta = 4$ here. Cut at $x \leq 1 - 4 \times 10^{-7}$.



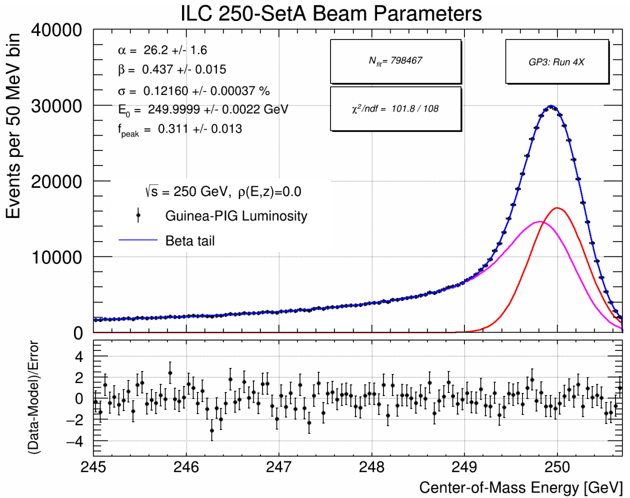
Body (χ^2/dof $= 58.8/70$)



Arms (χ^2/dof $= 89.6/70$)

Fits with 1M events are fine! 2-d event populations: 24.55% (peak), 29.80% (body), 45.65% (arms).

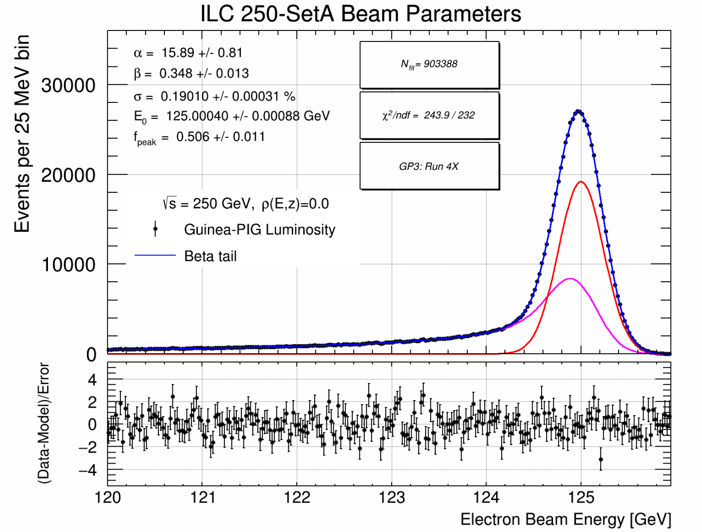
Center-of-Mass Energy (After Beamstrahlung)



$$\sigma/\sqrt{s} = 0.1216 \pm 0.0004\% \text{ (cf } 0.1217\% \text{ in TDR (} 0.190\% \oplus 0.152\%)/2)$$

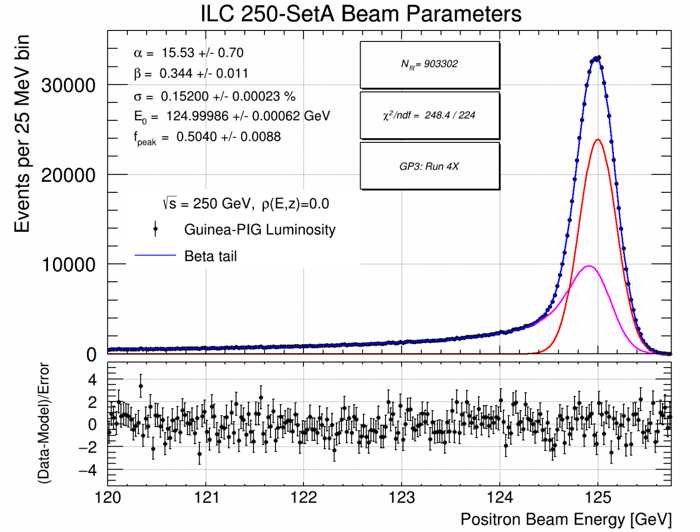
Negligible bias now with single Beta function in the convolution.

Electron Energy (After Beamstrahlung)



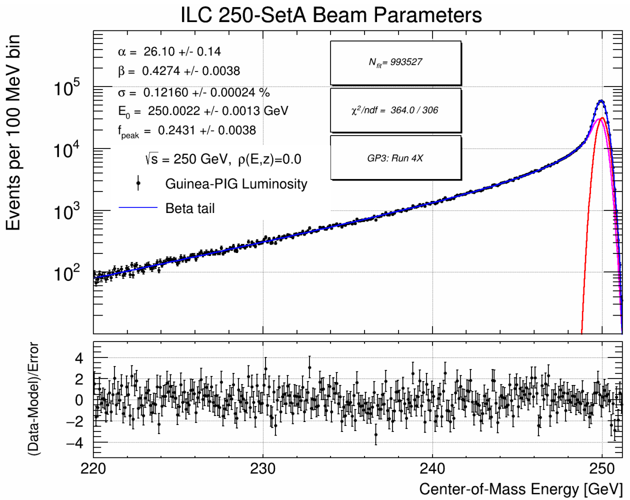
$$\sigma/E_{\mathrm{b}}^{-} = 0.1901 \pm 0.0003\% \text{ (cf } 0.190\% \text{ in TDR)}$$

Positron Energy (After Beamstrahlung)



$$\sigma/E_{\mathrm{b}}^{+} = 0.1520 \pm 0.0002\% \text{ (cf } 0.152\% \text{ in TDR)}$$

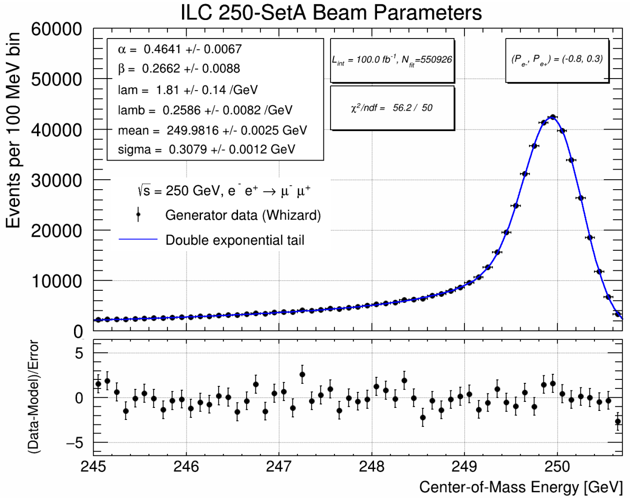
Center-of-Mass Energy (After Beamstrahlung)



$\sigma/\sqrt{s} = 0.1216 \pm 0.0002\%$ (cf 0.1217% in TDR ($0.190\% \oplus 0.152\%)/2$)

Single Beta function works reasonably over large range – reduced uncertainties!

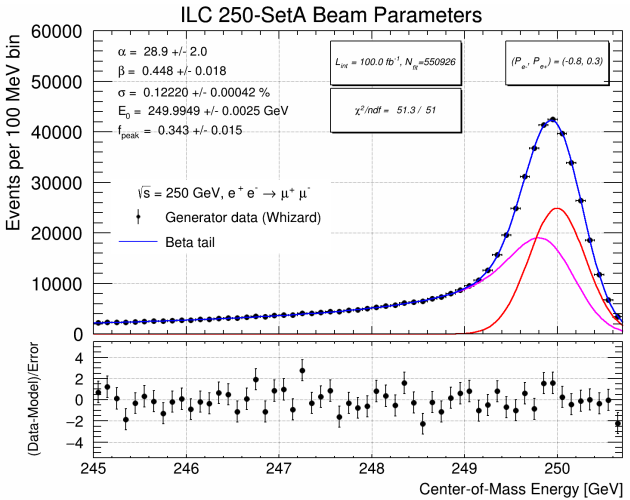
Center-of-Mass Energy (After Beamstrahlung)



$$\sigma/\sqrt{s} = 0.1232 \pm 0.0005\% \text{ (cf } 0.1217\% \text{ in TDR (} 0.190\% \oplus 0.152\%)/2)$$

Significiant bias in fitted energy scale parameter for double exponential tail model.

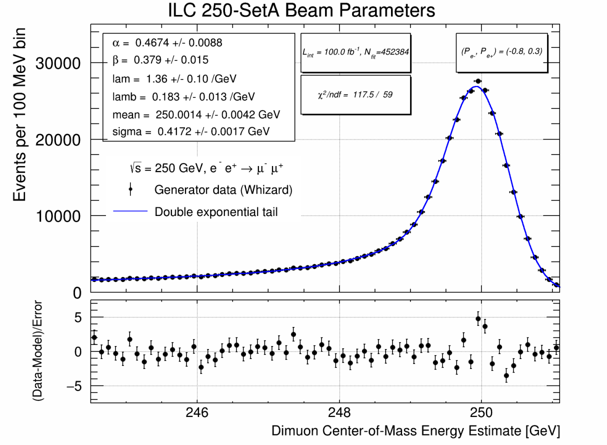
Center-of-Mass Energy (After Beamstrahlung)



$$\sigma/\sqrt{s} = 0.1222 \pm 0.0004\% \text{ (cf } 0.1217\% \text{ in TDR (} 0.190\% \oplus 0.152\%)/2)$$

Bias is reduced. But still (1/s) cross-section bias effect? Aim to incorporate too.

Assumes $M_3 = 0$ and $\overline{\Delta E_{\rm b}} = 0$. Same as $\boldsymbol{\beta} = (\beta_x, 0, 0) = (\sin(\alpha/2), 0, 0)$.



$\sigma/\sqrt{s} = 0.1669 \pm 0.0007\%$ (cf 0.1232% with true $\sqrt{s}$)

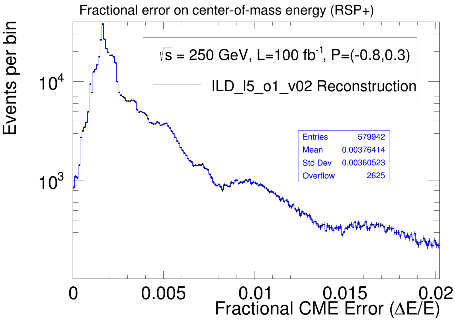
- This is the generator-level $\sqrt{s}_p$ calculated from the 2 muons
- Why so broad? Why fewer events?
- Because some events violate the assumptions that $\overline{\Delta E_{\rm b}} = 0$ and $M_3 = 0$
- The former is no surprise given the p_z distribution
- The latter is associated with events with 2 or more non-collinear ISR/FSR photons

Event Selection Requirements

Currently rather simple.

Use latest full ILD simulation/reconstruction at 250 GeV.

- Require exactly two identified muons
- Opposite sign pair
- Require uncertainty on estimated $\sqrt{s}_p$ of the event of less than 0.8% of $\sqrt{s}_{\mathrm{nom}}$ based on propagating track-based error matrices
- Categorize reconstruction quality as **gold** (<0.15%), **silver** ([0.15, 0.30]%), **bronze** ([0.30, 0.80]%)
- Require the two muons pass a vertex fit with p-value > 1 %

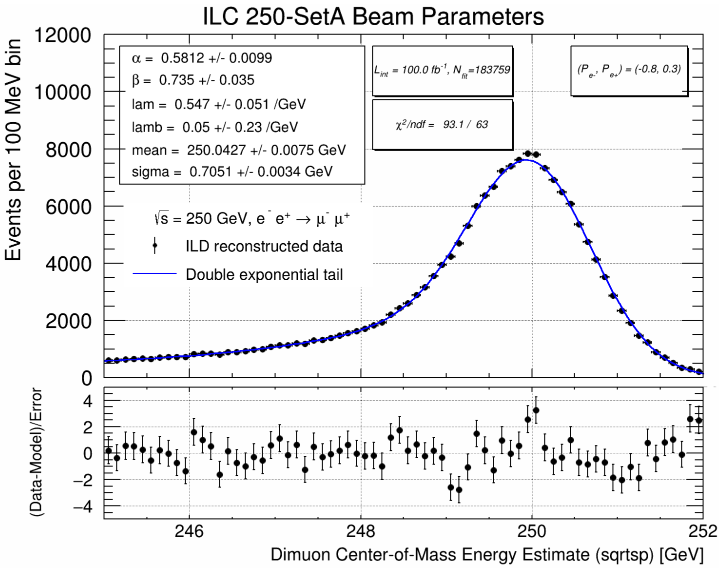


Selection efficiencies for (80%/30%) beam polarizations:

- $\varepsilon_{-+} = 70.4 \pm 0.1$ %
- $\varepsilon_{+-} = 68.0 \pm 0.1$ %
- $\varepsilon_{--} = 70.1 \pm 0.1$ %
- $\varepsilon_{++} = 68.3 \pm 0.1$ %

Backgrounds not yet studied in detail, ($\tau^+\tau^-$ is small:0.15%, of no import for the $\sqrt{s}$ peak region).

Silver Quality Dimuon PFOs (After BS)



Peak width 1.69 ± 0.01 wider than $\sqrt{s_p}$ (gen).

$\sqrt{s}$ Sensitivity Estimates at $\sqrt{s} = 250$ GeV

Statistical uncertainties in ppm on $\sqrt{s}$ for $\mu^+\mu^-$ channel

L_{int} [ab^{-1}]	Poln [%]	ε [%]	Gold	Silver	Bronze	All categories
2.0	$0, 0$	69.3	5.1	2.4	6.1	2.1
0.9	$-80, +30$	70.4	6.4	3.1	7.7	2.6
0.9	$+80, -30$	68.0	7.5	3.4	8.7	2.9
0.1	$-80, -30$	70.1	25	12	30	10
0.1	$+80, +30$	68.3	28	13	33	11
2.0	Combined	-	4.7	2.2	5.6	**1.9**

Fractional errors on μ parameter (mode of peak) when fitting with 6-parameter double exponential tail function with all 5 shape parameters fixed to their best-fit values. (4/3 for bronze). The $\mathrm{e}^+\mathrm{e}^-$ channel should also be used. Much larger statistics from t-channel enhanced Bhabhas (also at wide angle!).

Bottom-line

Statistical uncertainty at $\sqrt{s} = 250$ GeV of 2 ppm with momentum-based estimator. This far exceeds the 25 ppm stat. uncertainty (Hinze 2005) of the angles-based estimator used at LEP2.

New approach to tracker momentum scale

See LCWS2021 talk for details. Use Armenteros-Podolanski kinematic construction for 2-body decays (AP).

1. Explore AP method using mainly $\mathrm{K_S^0} \to \pi^+\pi^-$, $\Lambda \to \mathrm{p}\pi^-$ (inspired by Rodríguez et al.). **Much higher statistics than J/ψ alone.**
2. If proven realistic, **enables precision Z program** (polarized lineshape scan)
3. Bonus: potential for **large improvement in** parent and child particle **masses**

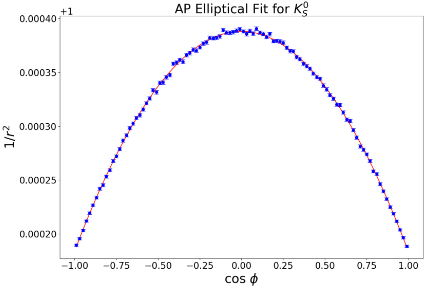
For a "V-decay", $M^0 \to m_1^+ m_2^-$, decompose the child particle lab momenta into components transverse and parallel to the parent momentum. The distribution of (child p_T, $\alpha \equiv \frac{p_L^+ - p_L^-}{p_L^+ + p_L^-}$) is a semi-ellipse with parameters relating the CM decay angle, θ^*, β, and the masses, (M, m_1, m_2), that determine, p^*.

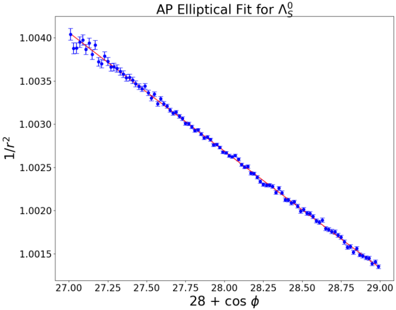
By obtaining sensitivity to both the parent and child masses, and positing improving ourselves the measurements of more ubiquitous parents ($\mathrm{K_S^0}$ and Λ), can obtain high sensitivity to the momentum scale

Proving the feasibility of sub-10 ppm momentum-scale uncertainty needs much work when typical existing experiments reach at best 100 ppm with the notable exception of CDF (30 ppm).

Tracker momentum scale sensitivity estimate

Used sample of 250M hadronic Z's at $\sqrt{s} = 91.2$ GeV. Fit $\mathrm{K}^0_\mathrm{S}, \Lambda, \overline{\Lambda}$ in various momentum bins.





1. $m_{\mathrm{K}^0_\mathrm{S}}$: 0.48 ppm
2. m_Λ: 0.072 ppm
3. m_π: 0.46 ppm
4. S_p: 0.57 ppm

See backup for tracker linearity remarks.

- Fit fixes proton mass
- Factors of (54, 75, 3) improvement over PDG for $(\mathrm{K}^0_\mathrm{S}, \Lambda/\overline{\Lambda}, \pi^\pm)$
- Momentum-scale to **2.5 ppm stat.** per 10M hadronic Z. ILC Z (250 GeV) run: 400 (≈ 10) such samples.

Collision Beam Energies using $\mathrm{e^+e^-} \to \mu^+\mu^-(\gamma)$

Infer, the e^- and e^+ beam energies from the muons alone under the *assumption* of one collinear **undetected** ISR photon. (E, p_z) conservation equations:

$$E_- + E_+ = E_1 + E_2 + |p_\gamma^z| / \cos(\alpha/2) \quad (1)$$

$$(E_- - E_+) \cos(\alpha/2) = p_1^z + p_2^z + p_\gamma^z \quad (2)$$

Solve for E_- and E_+,

$$E_- = \frac{1}{2}\left((E_1 + E_2) + \frac{(p_1^z + p_2^z)}{\cos(\alpha/2)} + \frac{(|p_\gamma^z| + p_\gamma^z)}{\cos(\alpha/2)} \right) \quad (3)$$

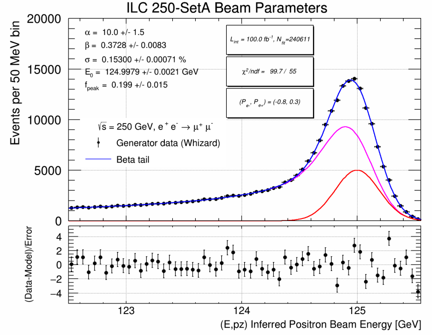
$$E_+ = \frac{1}{2}\left((E_1 + E_2) - \frac{(p_1^z + p_2^z)}{\cos(\alpha/2)} + \frac{(|p_\gamma^z| - p_\gamma^z)}{\cos(\alpha/2)} \right) \quad (4)$$

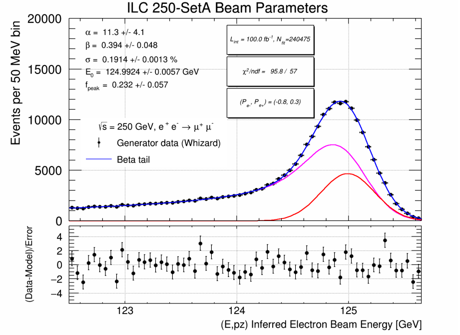
- If $p_\gamma^z \leq 0$, there is NO p_γ^z induced error for the muons-only E_- equation[1]
- If $p_\gamma^z \geq 0$, there is NO p_γ^z induced error for the muons-only E_+ equation[1]

Exact for one of the beams with one collinear ISR photon present! But really wrong for the other beam - especially for $\mathrm{Z}\gamma$ events ($E_\pm$ error is $\frac{-|p_\gamma^z|}{\cos(\alpha/2)}$).

[1]Obtained by neglecting the **unmeasured** red p_γ^z dependent terms.

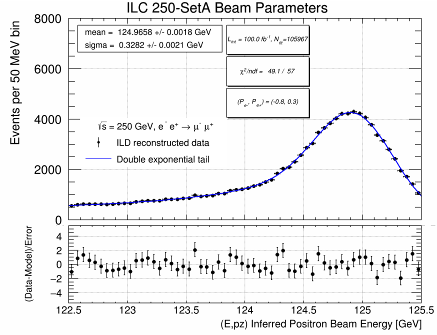
Collision Beam Energy (Generator level)

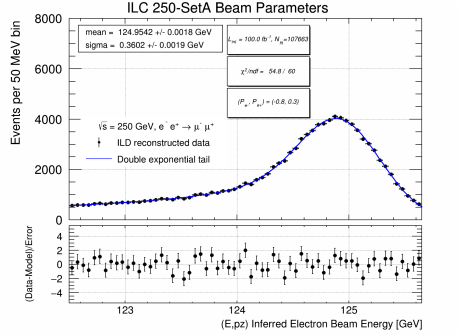




Generator-level rms of peak very similar to intrinsic expectation from beam energy spread alone of 0.152% (e^+) and 0.190% (e^-).

Here for "silver" quality dimuons.





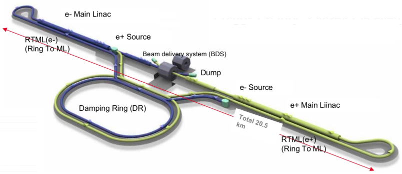
- Also relevant for luminosity spectrum extraction. (Note. T. Barklow also discussed these estimators in the past).
- Precision degraded by detector resolution as expected, but can still resolve well the differences.
- Likely complementary to $\sqrt{s_p}$ approach. Although the advantage of a more direct single beam measurement is diluted statistically by the wrong hemisphere feature.

ILC

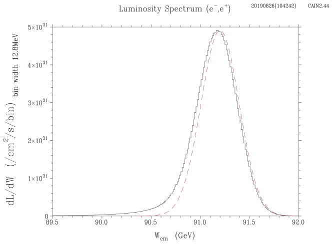
The ILC linear e^+e^- collider has been designed with an emphasis on an **initial-stage Higgs factory** that starts at $\sqrt{s} = 250$ GeV and is **expandable in energy** to run at higher energies for pair production of top quarks and Higgs bosons, and potentially to 1 TeV and more.

Particular strengths: **Longitudinally polarized electron and positron beams** and **higher energies**. Many new measurement possibilities. Very complementary to those feasible with unpolarized & lower energy reach e^+e^- circular colliders.

The ILC is designed primarily to explore the 200 – 1000 GeV energy frontier regime. This has been the focus in making the case for the project.
It is also capable of running at the **Z** and **WW** threshold.



See B. List's talk for ILC details (p22)



Z running – see Yokoya, Kubo, Okugi

Snowmass LoI Studies

Studies were undertaken to :

1. understand ILC capabilities for a precision measurement of the Z lineshape observables with a **scan** using longitudinally polarized beams,
2. further explore an experimental strategy for $\sqrt{s}$ determination using di-leptons, and
3. further explore M_{W} capabilities synergistic with a concurrent Higgs program.

Focus of this talk: reporting progress on experimental issues associated with **center-of-mass energy** (item 2) which is a pre-requisite for fully exploiting a polarized Z scan (item 1) and underpin M_{W} prospects (item 3).

Key Issue: Systematic control for the absolute scale of (**in collision...**) **center-of-mass energy** at **all** C-o-M energies

Note: 10^{10} hadronic Z's - 0.001% uncertainties - already a big challenge for absolute observables. Less so for asymmetries and relative cross-sections vs $\sqrt{s}$.

ILC A_{LR} Prospects from Z Running (Updated)

Use 4 cross-section measurements ($\sigma_{\pm\pm}$) to measure simultaneously:

$$A_{\mathrm{LR}},\ |P(e^-)|,\ |P(e^+)|,\ \sigma_u$$

L (fb^{-1})	N_Z^{had}	$\lvert P(e^-)\rvert$	$\lvert P(e^+)\rvert$	ΔA_{LR} (stat.)	ΔA_{LR} (syst).
100	3.3×10^9	80%	30%	2.3×10^{-5}	1.9×10^{-5}
100	4.2×10^9	80%	60%	2.0×10^{-5}	1.7×10^{-5}
250	8.4×10^9	80%	30%	1.4×10^{-5}	1.3×10^{-5}
250	1.1×10^{10}	80%	60%	1.3×10^{-5}	1.3×10^{-5}

Estimated uncertainties on A_{LR} for 4 different scenarios of Z-pole running with data-taking fractions at $\sqrt{s} = 91.2$ GeV in each of the 4 helicity configurations $(-+), (+-), (--), (++)$ chosen to minimize the statistical uncertainty on the asymmetry. The quoted statistical uncertainty includes Bhabha statistics for relative luminosity. The systematic uncertainty includes 5 ppm uncertainty on the absolute center-of-mass energy and a 1% understanding of beamstrahlung effects.

Total uncertainty on A_{LR} of 3.0×10^{-5} (scenario 1) to 1.8×10^{-5} (scenario 4). Corresponds to uncertainty on $\sin^2 \theta_{\mathrm{eff}}^{\ell}$ of 3.8×10^{-6} (1) to 2.3×10^{-6} (4).

Initial State Kinematics with Crossing Angle

Define the two beam energies (after beamstrahlung) as E_{b}^{-} and E_{b}^{+} for the electron beam and positron beam respectively.
Initial-state energy-momentum 4-vector (neglecting m_{e})

$$E = E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}$$
$$p_x = (E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}) \sin(\alpha/2)$$
$$p_y = 0$$
$$p_z = (E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+}) \cos(\alpha/2)$$

The corresponding center-of-mass energy is

$$\sqrt{s} = 2\sqrt{E_{\mathrm{b}}^{-} E_{\mathrm{b}}^{+}} \cos(\alpha/2)$$

Hence if α is known (14 mrad for ILC), evaluation of the collision center-of-mass energy amounts to measuring the two beam energies. Introducing,

$$E_{\mathrm{ave}} \equiv \frac{E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}}{2} \;, \overline{\Delta E_{\mathrm{b}}} \equiv \frac{E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+}}{2}$$

then with this notation,

$$\sqrt{s} = 2\sqrt{E_{\mathrm{ave}}^{2} - (\overline{\Delta E_{\mathrm{b}}})^{2}} \cos(\alpha/2)$$

Final State Kinematics and Equating to Initial State

Let's look at the final state of the $\mathrm{e^+e^-} \to \mu^+\mu^-(\gamma)$ process. Denote the μ^+ as particle 1, the μ^- as particle 2, and the rest-of-the event (RoE) as system 3. We can write this final-state system 4-vector as

$$(E_1 + E_2 + E_3,\ \vec{p_1} + \vec{p_2} + \vec{p_3})$$

Applying $(E, \vec{p})$ conservation we obtain,

$$E_1 + E_2 + \sqrt{p_3^2 + M_3^2} = 2\ E_{\mathrm{ave}} \tag{5}$$

$$\vec{p_1} + \vec{p_2} + \vec{p_3} = (2\ E_{\mathrm{ave}} \sin(\alpha/2), 0, 2\ \overline{\Delta E_{\mathrm{b}}} \cos(\alpha/2)) \equiv \vec{p}_{\mathrm{initial}} \tag{6}$$

The RoE is often not fully detected and needs to be inferred using $(E, \vec{p})$ conservation. We have 4 equations and 6 unknowns:
the 3 components of the RoE momentum ($\vec{p_3}$), E_{ave}, $\overline{\Delta E_{\mathrm{b}}}$, and M_3.
Our approach is to solve for E_{ave} for various assumptions on ($\overline{\Delta E_{\mathrm{b}}}$, M_3).
Specifically we then focus on using the simplifying assumptions of the original $\sqrt{s}_p$ method that $M_3 = 0$ and $\overline{\Delta E_{\mathrm{b}}} = 0$. Note: latter is often a poor assumption for the p_z conservation component on an event-to-event basis.

The Averaged Beam Energy Quadratic

This approach results in a quadratic equation in $E_{\rm ave}$, ($AE_{\rm ave}^2 + BE_{\rm ave} + C = 0$), with coefficients of

$$A = \cos^2(\alpha/2)$$

$$B = -E_{12} + p_{12}^x \sin(\alpha/2)$$

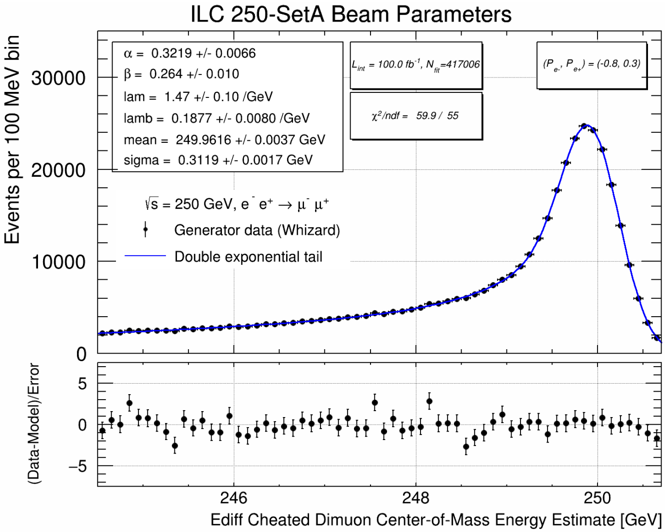
$$C = (M_{12}^2 - M_3^2)/4 + p_{12}^z \overline{\Delta E_{\rm b}} \cos(\alpha/2) - \overline{\Delta E_{\rm b}}^2 \cos^2(\alpha/2)$$

Based on this, there are a number of cases of interest to solve for $E_{\rm ave}$:

1. Zero crossing angle, $\alpha = 0$, $\overline{\Delta E_{\rm b}} = 0$, $M_3 = 0$.
2. Crossing angle and $\overline{\Delta E_{\rm b}} = 0$, $M_3 = 0$.
3. Crossing angle and $\overline{\Delta E_{\rm b}}$ non-zero, $M_3 = 0$.
4. Crossing angle and M_3 non-zero, $\overline{\Delta E_{\rm b}} = 0$.
5. Crossing angle and $\overline{\Delta E_{\rm b}}$ and M_3 non-zero.

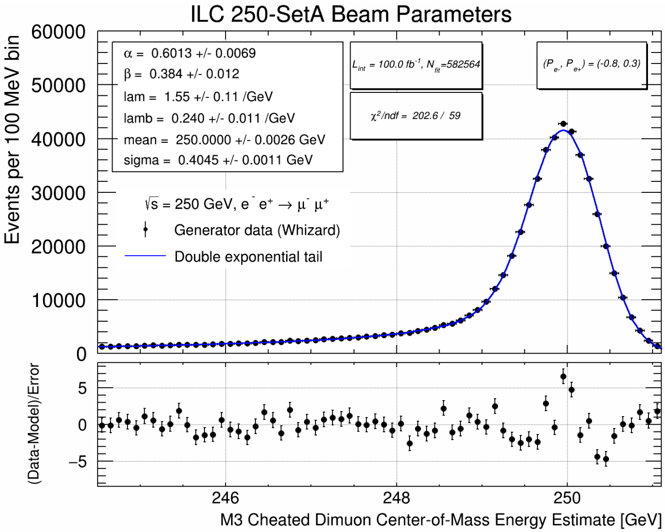
The original formula, $\sqrt{s} = E_1 + E_2 + |\vec{p}_{12}|$, arises trivially in the first case. In the rest of this talk the $\sqrt{s}$ estimate from the largest positive solution of the second case is what I now mean by $\sqrt{s}_p$. Obviously it is also a purely muon momentum dependent quantity.

Cheated $\overline{\Delta E_b}$ Center-of-Mass Energy Estimate (After BS)



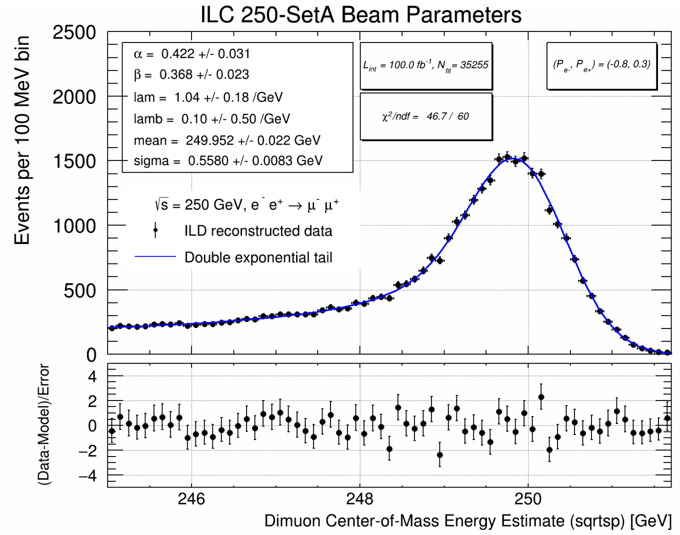
$$\sigma/\sqrt{s} = 0.1248 \pm 0.0007\% \text{ (cf } 0.1232 \pm 0.0005\% \text{ for } \sqrt{s})$$

Cheated M_3 Center-of-Mass Energy Estimate (After BS)



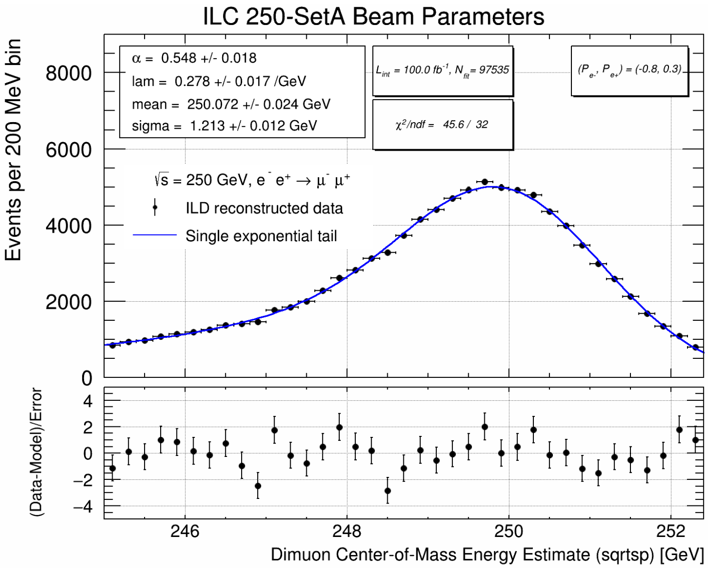
$$\sigma/\sqrt{s} = 0.1618 \pm 0.0004\% \text{ (cf } 0.1232 \pm 0.0005\% \text{ for } \sqrt{s})$$

Gold Quality Dimuon PFOs (After BS)



Peak width 1.34 ± 0.02 wider than $\sqrt{s_p}$ (gen).

Bronze Quality Dimuon PFOs (After BS)



Peak width 2.91 ± 0.03 wider than $\sqrt{s_p}$ (gen).

Outlook and Future Work

Lots of opportunities to improve this:

1. Constrained kinematic fits. For example one can test the consistency with the pure 2-body hypothesis of $e^+e^- \to \mu^+\mu^-$ while fitting for the two unmeasured parameters of $E_{\rm ave}$ and $\overline{\Delta E_{\rm b}}$, and also perform fits with the $e^+e^- \to \mu^+\mu^-\gamma$ hypothesis.

2. Extend the techniques to the $e^+e^- \to e^+e^-$ channel.

3. Exploit fully events with detected photons.

4. Implement complete end-to-end measurement scheme and understand how best to use different kinematic regimes and correct/mitigate observed biases.

5. Characterize better the intrinsic limitations associated with beam energy spread, beamstrahlung, ISR, FSR, backgrounds, and detector acceptance and resolution. This includes studies with more specialized physics event generators such as KKMCee [29].

6. Tracker momentum scale studies using $J/\psi \to \mu^+\mu^-$, $K^0_S \to \pi^+\pi^-$, $\Lambda^0 \to p\pi^-$. We have some preliminary results [30] further applying the technique advocated in [31] based on the Armenteros-Podolanski [32] reconstruction technique. A more novel aspect is that one can aspire to simultaneously improve the measurements of the K^0_S and Λ masses and the momentum scale given that the masses of their decay products are very well known.

7. Understand the relative merit of dimuons for luminosity spectrum determination compared with Bhabhas and integrate both techniques in a global analysis.

8. Characterize further the scope for measuring accelerator parameters such as the crossing angle and beamstrahlung-induced correlations including the observed dependence of the beam energy spectrum on the longitudinal collision vertex. The latter has been shown to be easily measurable with vertex fits in $e^+e^- \to \mu^+\mu^-(\gamma)$ events.

Summary of Progress

Progress

- New high precision method for momentum-scale using especially $\mathrm{K^0_S}$ and Λ. Promises 2.5 ppm uncertainty per 10M hadronic Zs.
- More detailed investigation of dimuons for $\sqrt{s}$ and dL/d$\sqrt{s}$ reconstruction. Capable of 2 ppm stat. uncertainty for ILC at $\sqrt{s} = 250$ GeV and 2 ppm for every $1\mathrm{fb}^{-1}$ of the standard 100 fb^{-1} ILC run at the Z.
- Baseline ILC250 can make precision measurements at the Z and if needed at the WW threshold. Use the actual colliding beams for center-of-mass energy measurement. Opens up capabilities for high precision A_{LR}, M_{W}, M_{Z}, Γ_{Z}.

Conclusions

- Tracking detectors designed for ILC have the potential to measure beam energy related quantities with precision similar to the intrinsic energy spread using dimuon events (and also especially wide-angle Bhabha events).
- At $\sqrt{s} = 250$ GeV, dimuon estimate of 1.9 ppm statistical precision on $\sqrt{s}$. More than sufficient (10 ppm needed) to not limit M_{W} measurements.
- Applying the same $\sqrt{s}$ techniques to running at the Z-pole enables a high precision electroweak measurement program for ILC that takes advantage of absolute center-of-mass energy scale knowledge.

Backup Slides

The tracker momentum scale linearity question

Are you really sure that you can use low momentum tracks to infer the momentum scale of high momentum (100 GeV) tracks?

- I have not yet investigated this aspect carefully.
- I have talked to experts on CDF and CMS about this, who did not tell me to my face that this is crazy.
- CDF has long claimed that J/ψ calibration can work for M_{Z}.
- There are indeed a number of subtle effects that need to be correctly accounted for, but fundamentally in a low mass tracker it should be much easier than in CMS.

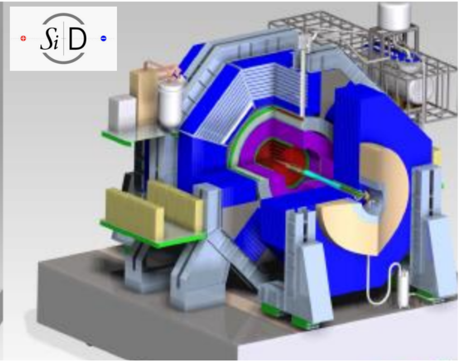
Note that for electrons, the best statistical precision would be if one can use the momentum measurement of electrons. The electron momentum response will of course be different from the muon momentum response.

ILC Detectors

Modern detectors designed for ILC

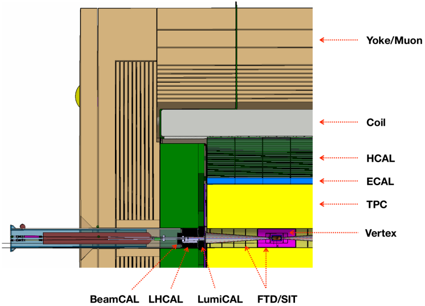
ILD = International Large Detector
(also ILD Interim Design Report (IDR))

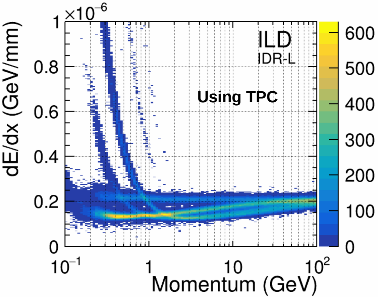
SiD = Silicon Detector

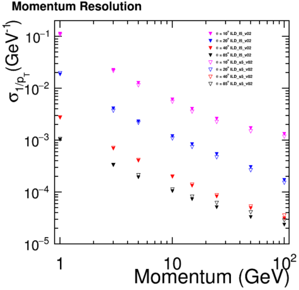


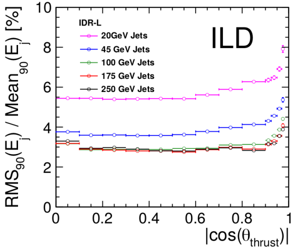
- B=3.5–5T. Particle-flow for hadronic jets. **Very hermetic**.
- Low material. Precision vertexing.
- ILD tracking centered around a Time Projection Chamber (TPC).

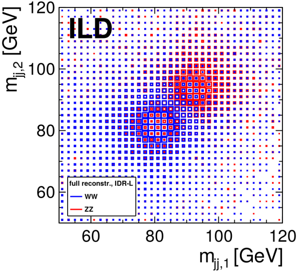
ILD Detector (See IDR: 2003.01116)





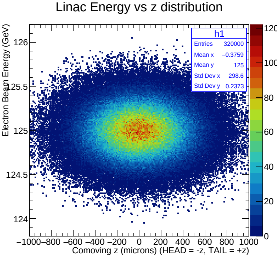


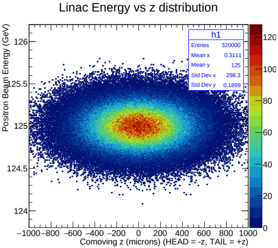




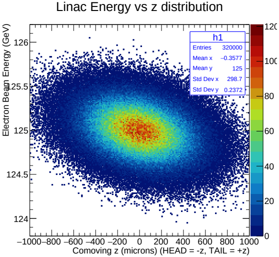
Linac Beam Energy, z Correlations

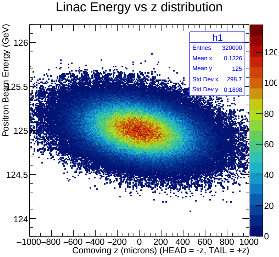
$\rho(E, z) = 0.0$



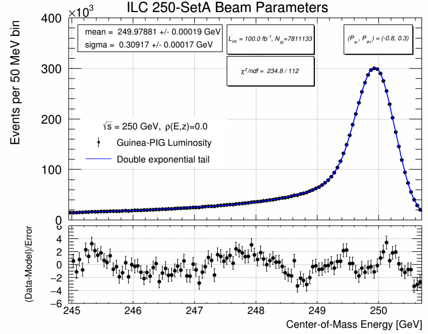


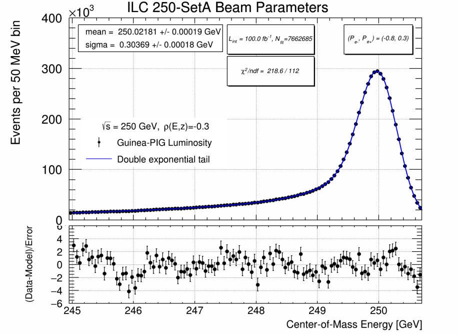
$\rho(E, z) = -0.30$





Effect on absolute $\sqrt{s}$ from beam-beam simulations including beamstrahlung.





$$\text{Find dependence of } \frac{1}{\sqrt{s}}\frac{d\sqrt{s}}{d\rho(E,z)} = \; -5 \text{ ppm}/\%.$$

- -30% is an initial guess (TESLA).
- From the direct in situ measurement perspective, exact value not important.
- However, when using upstream/downstream energy spectrometer measurements, it will be essential to measure/constrain well the correlation.

Generator-level Examples

Event	1	2	3	4	5	6
E_{b}^{-}	125.34	114.55	125.32	124.87	124.75	122.77
E_{b}^{+}	124.82	124.64	121.08	124.49	116.24	110.12
$\overline{\Delta E_{\mathrm{b}}}$	$+0.26$	-5.04	$+2.12$	$+0.19$	$+4.26$	$+6.33$
M_{12}	92.55	238.97	94.62	249.30	82.34	92.26
p_{12}	108.41	10.22	104.74	1.73	101.66	105.43
p_{12}^{x}	$+18.82$	$+1.67$	$+1.25$	$+1.70$	$+0.92$	$+1.03$
p_{12}^{y}	-14.54	0.00	$+0.21$	-0.01	0.00	-0.25
p_{12}^{z}	$+105.77$	-10.08	$+104.73$	$+0.35$	-101.65	$+105.43$
p_3	107.62	0.00	100.49	0.06	110.17	92.78
M_3	0.00	0.00	31.27	0.00	0.55	0.00
$\sqrt{s}$	250.15	238.97	246.35	249.35	240.84	232.53
E_{12}^{*} (β_x)	142.41	239.18	141.15	249.30	130.82	140.10
p_{12}^{*} (β_x)	108.24	10.08	104.73	0.35	101.65	105.43
$\sqrt{s_p}$	250.65	249.26	245.88	249.65	232.47	245.53
E_{12}^{*} $(\boldsymbol{\beta})$	142.20	238.97	139.36	249.30	134.49	134.57
p_{12}^{*} $(\boldsymbol{\beta})$	107.96	0.00	102.32	0.06	106.34	97.96
$\sqrt{s_p}$ (true $\overline{\Delta E_{\mathrm{b}}}$)	250.15	238.97	241.60	249.35	240.84	232.53
$\sqrt{s_p}$ (true M_3)	250.65	249.26	250.45	249.65	232.47	245.53

Makes use of radiative-return ($Z\gamma$) events too.

Introduction to Center-of-Mass Energy Issues

- Proposed $\sqrt{s}_p$ method uses only the momenta of leptons in dilepton events.
- Critical issue for $\sqrt{s}_p$ method: calibrating the **tracker momentum scale**.
- Can use $\mathrm{K^0_S}$, Λ, $J/\psi \to \mu^+\mu^-$ (mass known to 1.9 ppm).

For more details see studies of $\sqrt{s}_p$ from ECFA LC2013, and of momentum-scale from AWLC 2014. Recent $\mathrm{K^0_S}$, Λ studies at LCWS 2021 – much higher precision feasible ... few **ppm** (not limited by parent mass knowledge or J/ψ statistics). More in depth talks on $\sqrt{s}$: ILC physics seminar and ILC MDI/BDS/Physics talk

Today,

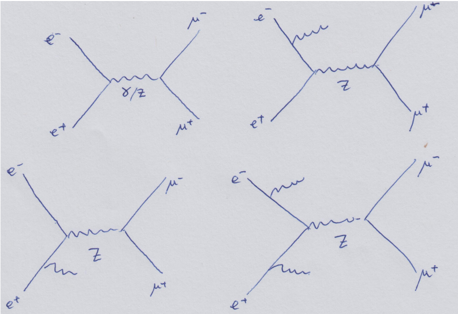
- Overview of the $\sqrt{s}_p$ method prospects with $\mu^+\mu^-$
- Brief overview of the "new" concept in recent tracker momentum scale studies (LCWS2021 talk).
- Bonus. Physics: M_{Z}. Beam knowledge: **luminosity spectrum**, $\mathrm{dL}/\mathrm{d}\sqrt{s}$.

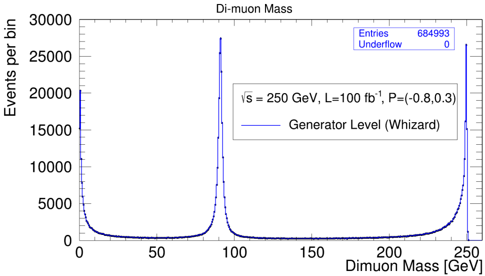
Dimuons

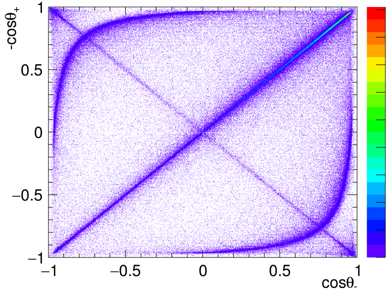
Three main kinematic regimes.

1. **Low** mass, $m_{\mu\mu} < 50$ GeV
2. **Medium** mass, $50 < m_{\mu\mu} < 150$ GeV
3. **High** mass, $m_{\mu\mu} > 150$ GeV

- Back-to-back events in the full energy peak.
- Significant radiative return (ISR) to the Z and to low mass.

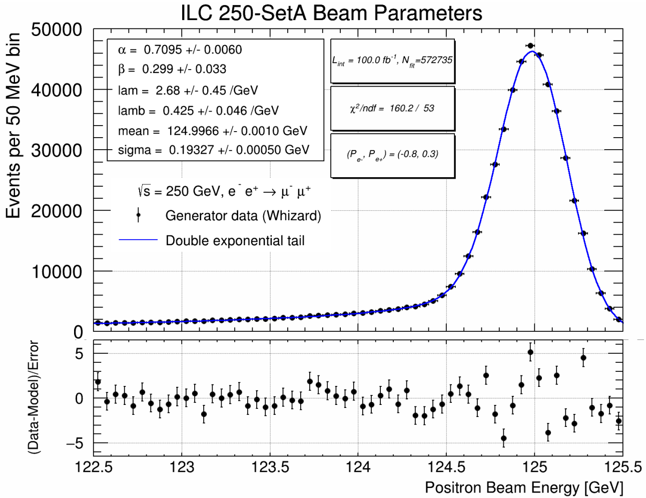






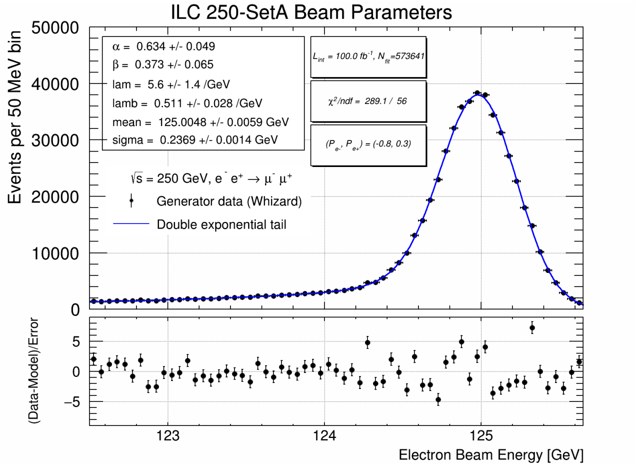
Positron Beam Energy (After Beamstrahlung)

Fits with (double-exponential tail + delta-function) convolved with Gaussian beam energy spread (6 parameters).



$$\sigma/E = 0.1546 \pm 0.0004\% \text{ (cf } 0.152\% \text{ in TDR)}$$

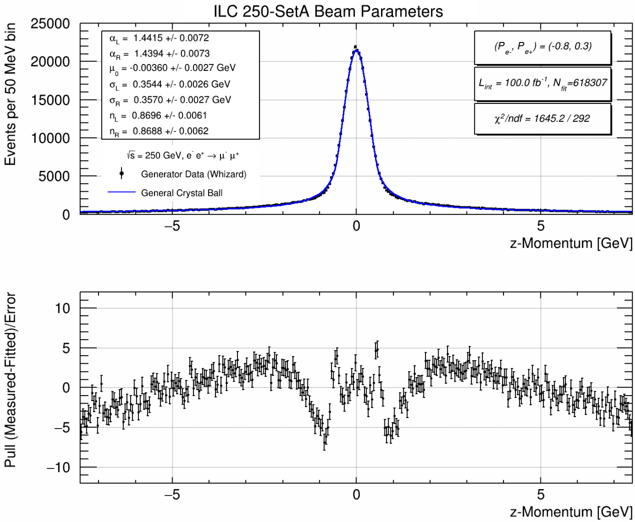
Electron Beam Energy (After Beamstrahlung)



$$\sigma/E = 0.1895 \pm 0.0011\% \text{ (cf 0.190\% in TDR)}$$

Note an undulator bypass could reduce this spread when one e^- cycle is used purely for e^+ production.

z-Momentum of e^+e^- system (After Beamstrahlung)



$\sigma/\sqrt{s} = 0.1416 \pm 0.0007\%$ (cf 0.122% from beam energy spread alone)

Generator-level $\sqrt{s}_p$ Biases

$M_{\mu^+\mu^-}$ range [GeV]	$\mu(\sqrt{s})$ [GeV]	$\mu(\sqrt{s}_p)$ [GeV]	$\mu(\sqrt{s}_p)$ - $\mu(\sqrt{s})$ [MeV]
$M > 150$	249.9792 ± 0.0011	250.0337 ± 0.0013	$+54.5 \pm 1.7$
$50 < M < 150$	249.9813 ± 0.0010	249.9602 ± 0.0017	-21.1 ± 2.0
$M < 50$	249.9871 ± 0.0015	249.9633 ± 0.0028	-23.8 ± 3.2
All	249.9816 ± 0.0008	250.0014 ± 0.0010	$+19.8 \pm 1.2$

Results of the 1-parameter fits for the μ parameter to the generator-level distributions of $\sqrt{s}$ and $\sqrt{s}_p$ for three different dimuon mass ranges for the 80%/30% LR helicity mixture. The statistical uncertainties of these tests reflect an integrated luminosity of 100 fb^{-1}. The last column gives the difference in MeV of the fit parameters for the two distributions.

Strong evidence that high mass events tend to be over-measured (addition of a fictitious photon in genuine 2-body $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-$ events), and that lower mass events are under-measured (multiple radiation more important).

Bias associated with neglecting M_3

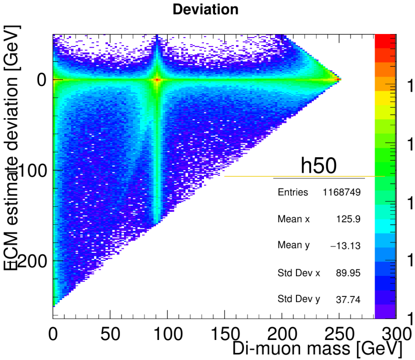
Naively with a mean value of M_3 of around 25 GeV, one imagines large biases for $\sqrt{s}_p$, but the median M_3 value is much lower, and examining the relevant equation, IF the boost is correct, the M_3 related bias goes as:

$$\Delta\sqrt{s} = |\mathbf{p}^*_{12}| - \sqrt{(p^*_{12})^2 + M_3^2}$$

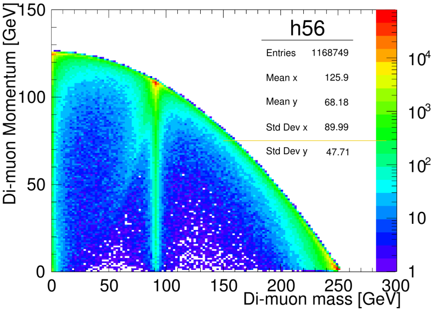
So for $p_{12} = 100$ GeV, the bias for a 10 GeV M_3 is only -0.50 GeV.

2d Generator Level Plots

Plot of $(\sqrt{s}_p - \sqrt{s})$ vs $M_{\mu\mu}$



Plot of $|p_{\mu\mu}|$ vs $M_{\mu^+\mu^-}$



Most events consistent with $M_3 \approx 0$

In most events, $\sqrt{s}_p$, is a reasonable estimator. But also can be off by a lot. WIP on identifying problematic events (eg. kinematic fits). It may be feasible to find alternative estimators/methods in those cases, or at least reject them.

Strategy for Absolute $\sqrt{s}$ and Estimate of Precision

Prior Estimation Method

- Guesstimate how well the peak position of the Gaussian can be measured using the observed $\sqrt{s}_p$ distributions in bins of fractional error

Current Thinking

- The luminosity spectrum and absolute center-of-mass energy are the same problem or at least very related. How well one can determine the absolute scale depends on knowledge of the shape (input also from Bhabhas).
- Beam energy spread likely to be well constrained by spectrometer data
- Likely need either a convolution fit (CF) or a reweighting fit
- Work is in progress on a CF by parametrizing the underlying (E_-, E_+) distribution, and modeling quantities related to $\sqrt{s}$ and p_z after convolving with detector resolution (and ISR, FSR and cross-section effects)

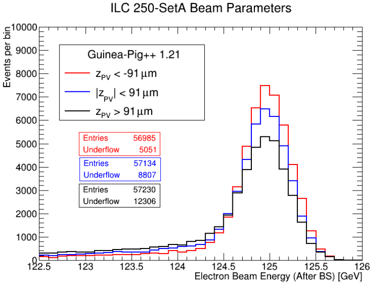
Current Estimation Method

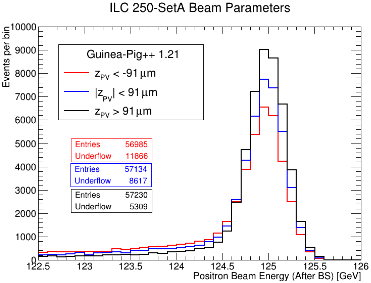
- Use estimates of the statistical error on the peak position for 6-parameter convolved double exponential tail fits to fully simulated data with the 5 shape parameters fixed to their best fit values.
- Fits are done in the 3 resolution categories.
- Next slide has these estimates

Beamstrahlung / z-Vertex Effects Explained

Divide interactions in 3 equi-probability parts according to z_{PV}. Preferentially

1. $\mathrm{e}^+\mathrm{e}^-$ collisions occurring more on the initial e^- side ($z < 0$)
2. $\mathrm{e}^+\mathrm{e}^-$ collisions mostly central
3. $\mathrm{e}^+\mathrm{e}^-$ collisions preferentially on the initial e^+ side ($z > 0$)

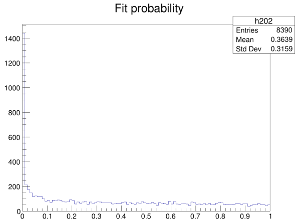


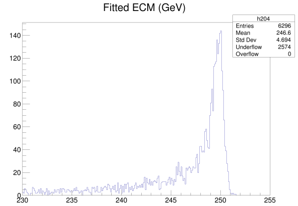


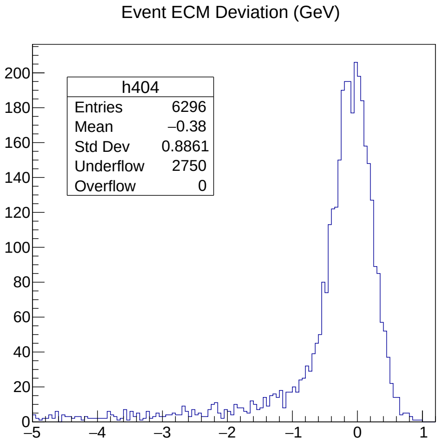
The beamstrahlung tail grows and the peak shrinks for e^- as z increases, and, for e^+ as z decreases. In both cases, the largest beamstrahlung tail occurs when the interacting e^- or e^+ has on average traversed more of the opposing bunch.

Thus both $\sqrt{s}$ and $p_z = E_- - E_+$ distributions depend on z. Likely needs to be taken into account for $\sqrt{s}$, $\mathrm{dL}/\mathrm{d}\sqrt{s}$, Higgs recoil, kinematic fits ...

Test consistency with $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-$ (no photons) by fitting for E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$ as unmeasured parameters (4C/2U/2dof). So measure $\sqrt{s}$ and collision asymmetry.

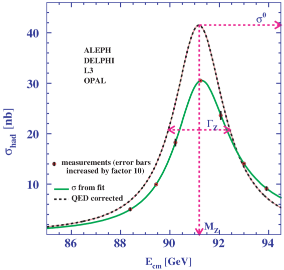




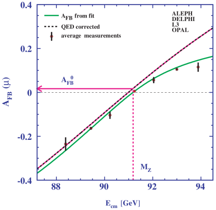


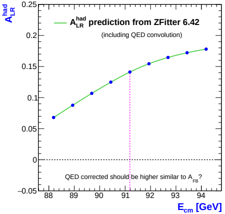
Plots require $p_{\mathrm{fit}} > 0.05$ (26% of all events). See backup for details. Use 0.15% momentum resolution. Peak width is 0.3 GeV (same as energy spread).

Essentially, perform LEP/SLC-style measurements in all channels but also with $\sqrt{s}$ dependence of the polarized asymmetries, A_{LR} and $A^f_{FB,LR}$, in addition to A_{FB}. (Also polarized $\nu\bar{\nu}\gamma$ scan.) Not constrained to LEP-style scan points.



LEP: $\Delta M_{\mathrm{Z}} = 2100$ MeV, $\Delta\Gamma_{\mathrm{Z}} = 2300$ MeV





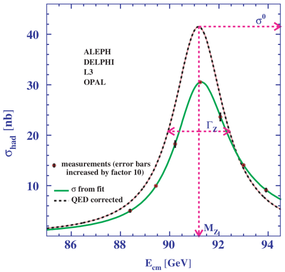
With 0.1 ab^{-1} polarized scan around M_{Z}, find **statistical** uncertainties of 35 keV on M_{Z}, and 80 keV on Γ_{Z}, from LEP-style fit to $(M_{\mathrm{Z}}, \Gamma_{\mathrm{Z}}, \sigma^0_{\mathrm{had}}, R^0_e, R^0_\mu, R^0_\tau)$ using ZFITTER for QED convolution.

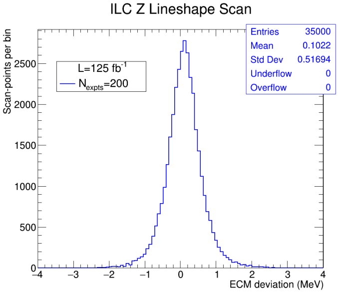
Exploiting this fully needs in-depth study of $\sqrt{s}$ **calibration systematics**

ILC $\mathcal{L}$ is sufficient for M_{Z} to be systematics limited

Γ_{Z} systematic uncertainty depends on $\Delta(\sqrt{s}_+ - \sqrt{s}_-)$, so expect $\Delta\Gamma_{\mathrm{Z}} \ll \Delta M_{\mathrm{Z}}$

Initial line-shape study (all 4 channels). Use unpolarized cross-sections for now.





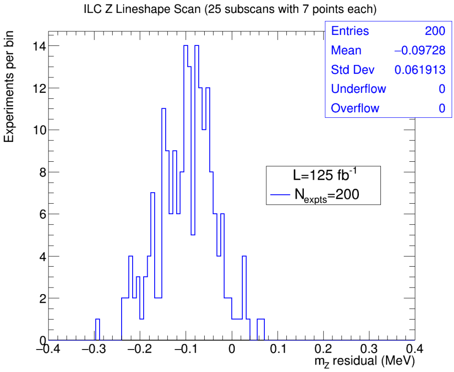
Uses $\sigma_{stat}/\sqrt{s}$ (%) $= 0.25/\sqrt{N_{\mu\mu}} \oplus\ 0.8/\sqrt{N_{\mathrm{h}}}$

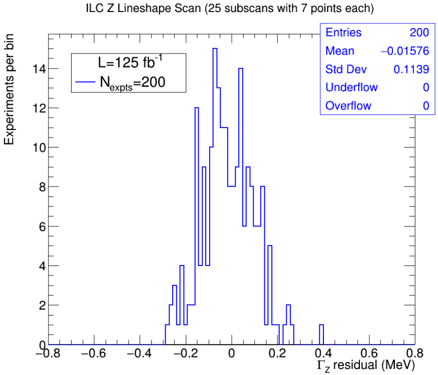
- Scan has 7 nominal $\sqrt{s}$ points, (peak,$\pm\Delta, \pm 2\Delta \pm 3\Delta$) with $\Delta = 1.05$ GeV
- 25 scans of 5 fb^{-1} per "experiment". $7 \times 25 \times 4 = 700$ σ_{tot} measurements.
- Assign luminosity per scan point in (2:1:2:1) ratio. (1 or 0.5 fb^{-1} each).
- Do LEP-style fit to $(M_{\mathrm{Z}}, \Gamma_{\mathrm{Z}}, \sigma^0_{\mathrm{had}}, R^0_e, R^0_\mu, R^0_\tau)$ using ZFITTER
- Model center-of-mass energy systematics and int. lumi syst. of 0.064%.
- Each scan-point (175 per expt.) shifted from $\sqrt{s}_{\mathrm{nominal}}$ by a 100% **correlated** overall scale systematic (here +100 keV) and by stat. component driven by stat. uncertainty of $\sqrt{s}$ measurement (typically 0.4 MeV/4.4 ppm).

Polarized Beams Z Scan for Z LineShape Study: WIP II

Ensemble tests with 200 experiments.

Currently, fit the 700 measured cross-sections (actually occuring at shifted $\sqrt{s}$) using assumed nominal $\sqrt{s}$. Ensemble mean χ^2 of 790 for 693 dof.

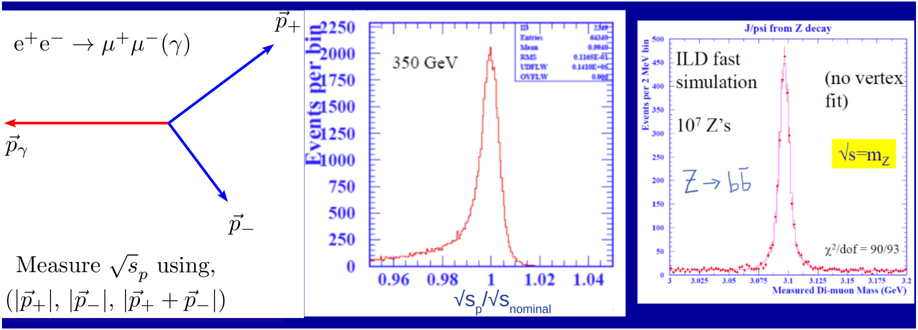




- As expected M_Z biased down by assumed scale error (here +100 keV) with stat. error of 50–60 keV.
- As expected Γ_Z bias small with stat. dominated error of 100–120 keV.
- Such an experiment has 1.9B hadronic Zs.

$\sqrt{s}_p$ Method for Center-of-Mass Energy

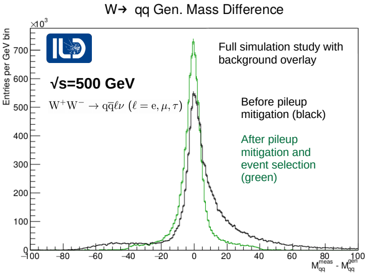
Use dilepton **momenta**, with $\sqrt{s}_p \equiv E_+ + E_- + |\vec{p}_{+-}|$ as $\sqrt{s}$ estimator.



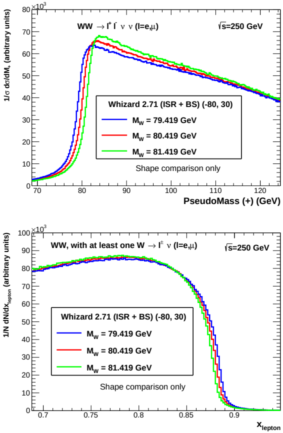
Tie detector p-scale to particle masses (know J/ψ, π^+, p to 1.9, 1.3, 0.006 ppm)

Measure $< \sqrt{s} >$ and luminosity spectrum with same events. Expect statistical uncertainty of 1.0 ppm on p-scale per 1.2M $J/\psi \to \mu^+\mu^-$ (4×10^9 hadronic Z's).

- excellent tracker momentum resolution - can resolve beam energy spread.
- feasible for $\mu^+\mu^-$ and e^+e^- (and ... 4l etc).



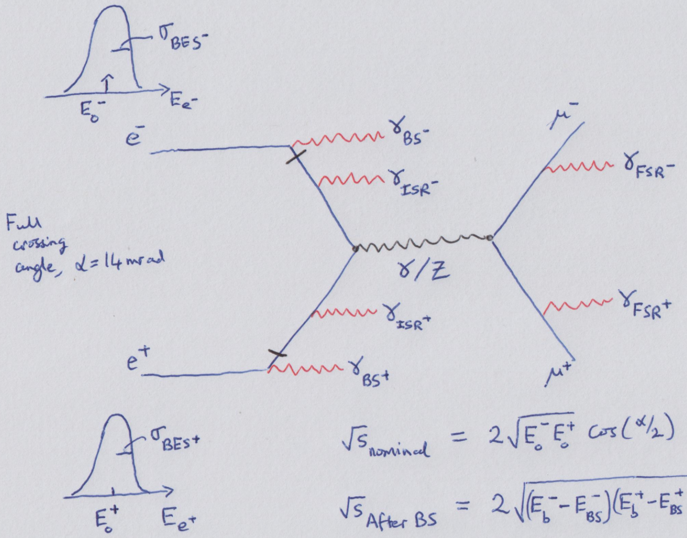
- **Hadronic mass study**, J. Anguiano (KU).
- Stat. $\Delta M_W = 2.4$ MeV for $1.6\ \mathrm{ab}^{-1}$ (-80%, +30%).
- Can be improved, but $m_{\rm had}$-only measurement likely limited by JES systematic
- Expect improvements with **constrained fit** and $\sqrt{s} = 250$ GeV data set



Sensitivity to M_W with lepton distributions: **dilepton pseudomasses**, lepton **endpoints**

- Stat. $\Delta M_W = 4.4$ MeV for $2\ \mathrm{ab}^{-1}$ (45,45,5,5) at $\sqrt{s} = 250$ GeV
- **Leptonic observables** (shape-only): M_+, M_-, $x_\ell \equiv E_\ell/E_b$. Exptl. systematics small.

Returning to $\sqrt{s}_p$ and Adding More Realism

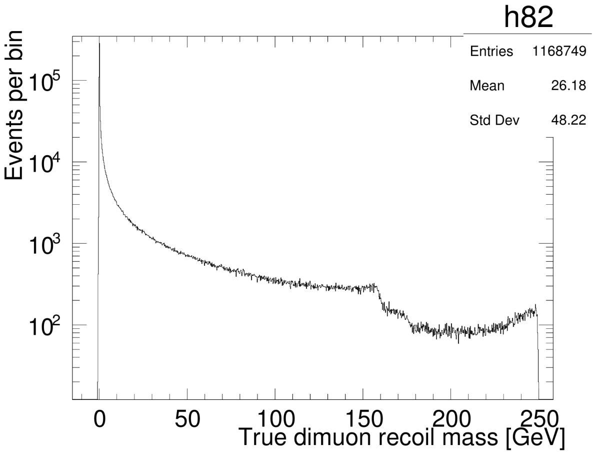


$$\sqrt{s}_{\text{nominal}} = 2\sqrt{E_0^- E_0^+}\cos(\alpha/2)$$

$$\sqrt{s}_{\text{After BS}} = 2\sqrt{(E_b^- - E_{BS}^-)(E_b^+ - E_{BS}^+)}\cos(\alpha/2)$$

Recoil Mass (at generator level)

Distribution of M_3.



Events in the tails will be from multiple non-collinear radiation (example ISR from both beams)

Kinematic Fits for $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-(\gamma)$

Inspired by revisiting some of the LEP2 techniques for M_{W} measurement, one can also cast the whole problem as a constrained fit problem. Promises to be very useful in event selection, hypothesis identification, and parameter measurement, but needs excellent object calibration and measurement uncertainties.

Two body fits

Test the hypothesis of $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-$ with no additional photons.

1. * Specify E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$ and fit with the 4 constraints of (E,p) conservation. (4C/4dof fit)
2. * Fit for E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$ as unmeasured fit parameters with the 4 constraints. (4C/2U/2dof fit).

Initial test implementation uses easily adaptable constrained fitting code of V. Blobel with toy MC based smearing and uncertainties.

1. Find 10.7% of events satisfy the 2-body hypothesis ($p_{\mathrm{fit}} > 0.01$) IF the correct E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$ are specified (Fit 1). For these events, $M_{\mu\mu}$ is synonymous with $\sqrt{s}$.
2. Find 26% of events satisfy fit 2 ($p_{\mathrm{fit}} > 0.05$). Note often the fitted $\sqrt{s}$ is near M_{Z} ... with large $|\overline{\Delta E_{\mathrm{b}}}|$.

Kinematic Fits for $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-(\gamma)$

Three particle collinear ISR fits

Test the $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-\gamma$ hypothesis where the γ is an undetected ISR photon collinear with one of the beams with z-hemisphere signed energy, E_{ISR}.

1. Specify E_{ave}, $\overline{\Delta E_{\mathrm{b}}}$, E_{ISR} and fit with 4 constraints. (4C/4dof fit)
2. * Specify E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$. Fit E_{ISR} as unmeasured parameter and fit with 4 constraints. (4C/1U/3dof fit)
3. Fit for E_{ave}, $\overline{\Delta E_{\mathrm{b}}}$, E_{ISR} as unmeasured fit parameters with the 4 constraints. (4C/3U/1dof fit).