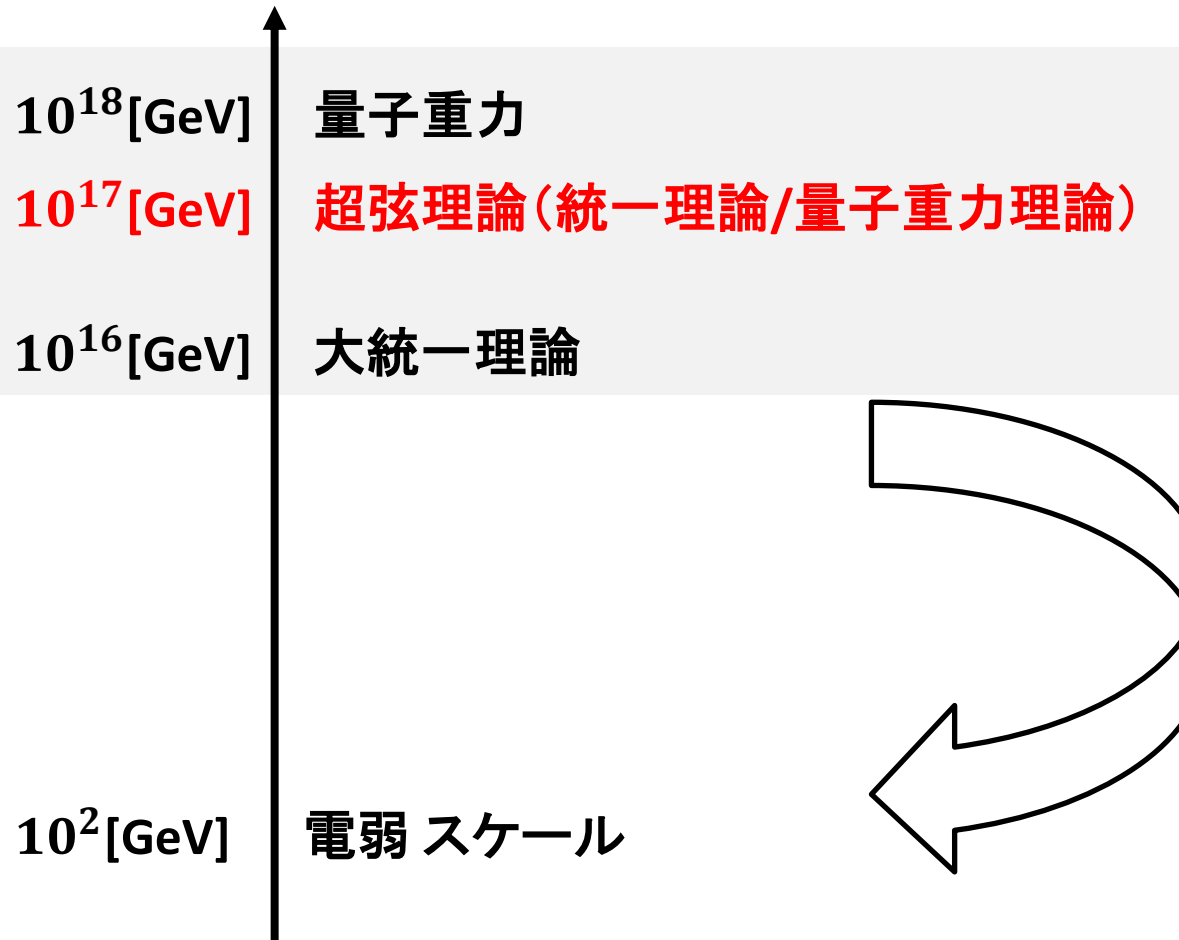


# Modular symmetry in the string EFT

大塚 啓 (九州大)

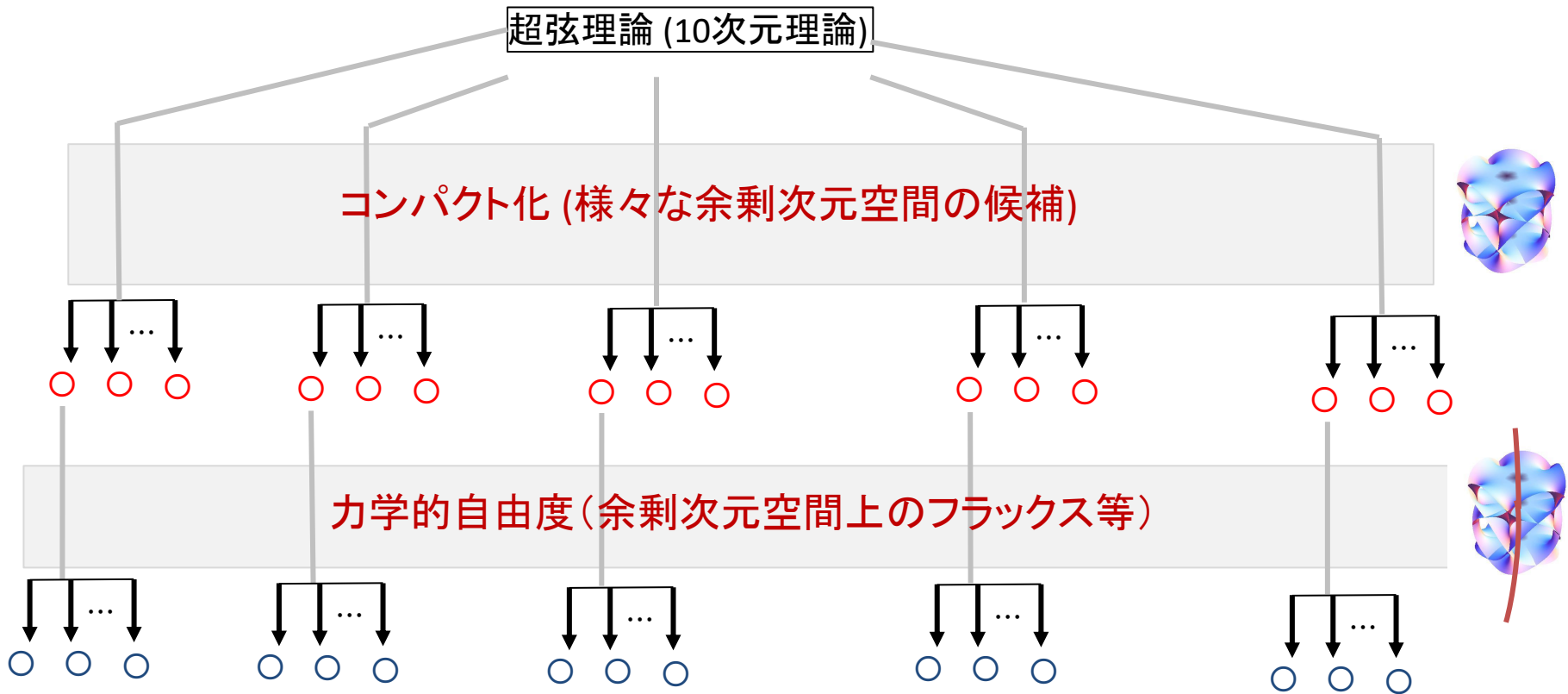
“Kagoshima Workshop on Particles, Fields and Strings 2023”  
2023年2月20日

# 弦理論の有効理論(string EFT)



弦理論の有効理論の性質？

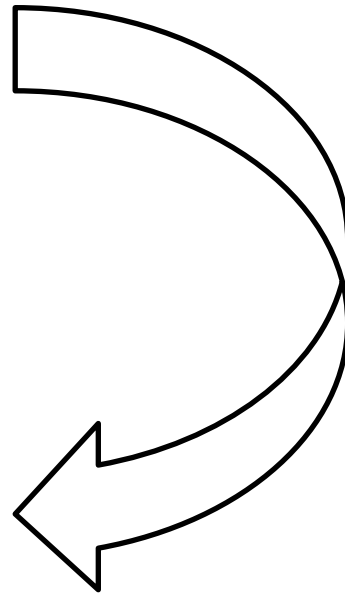
# 弦理論には無数の真空



- $O(10^{500})$  Type IIB flux vacua
- $O(10^{272,000})$  F-theory flux vacua

Ashok-Douglas ('04)  
Taylor-Wang ('15)

# 弦理論の有効理論(string EFT)

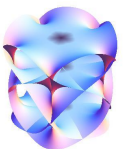


String EFTの性質？

- 沼地予想

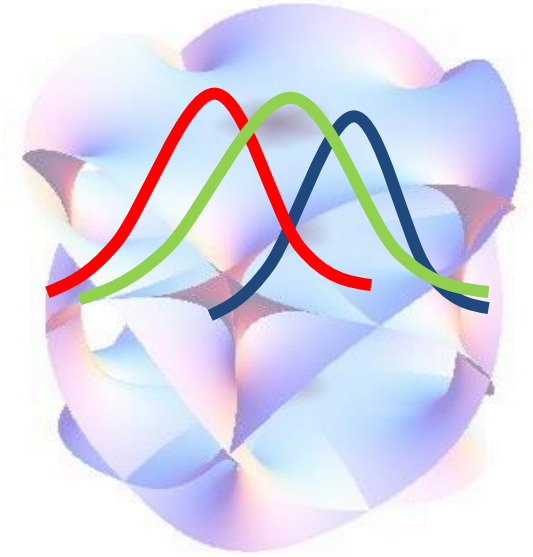


- 余剰次元空間の幾何学的対称性  
(フレーバー/CP 対称性)



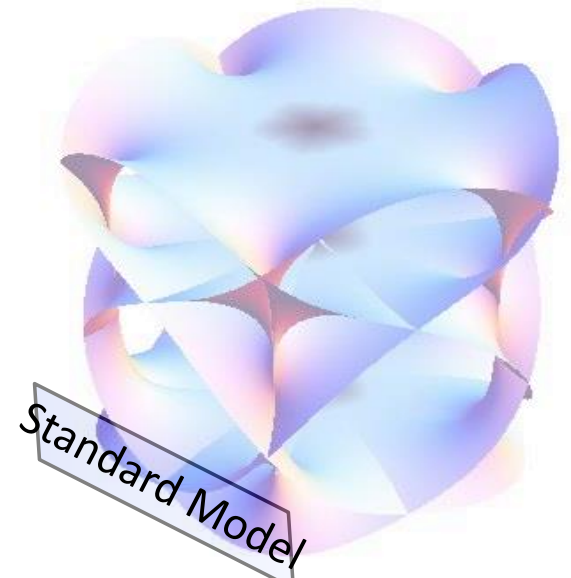
# 余剰次元空間の幾何学的対称性(フレーバー/CP 対称性)

余剰次元空間上に素粒子の波動関数  
 (“Global model”)



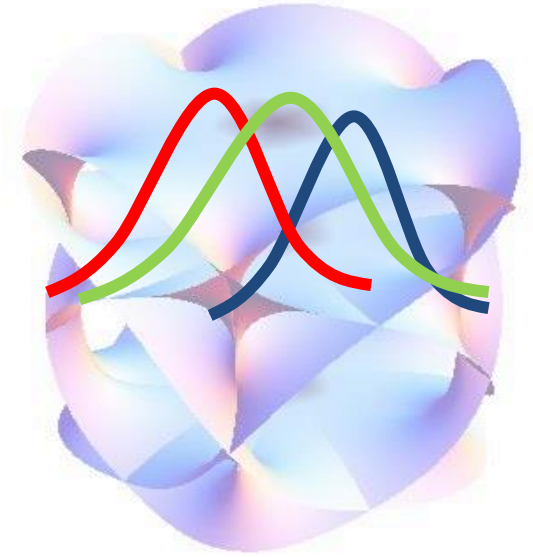
余剰次元空間の特異点にStandard Model  
 (“Local model”)

*M. Cicoli, A. Cotellucci, H.O., working in progress*



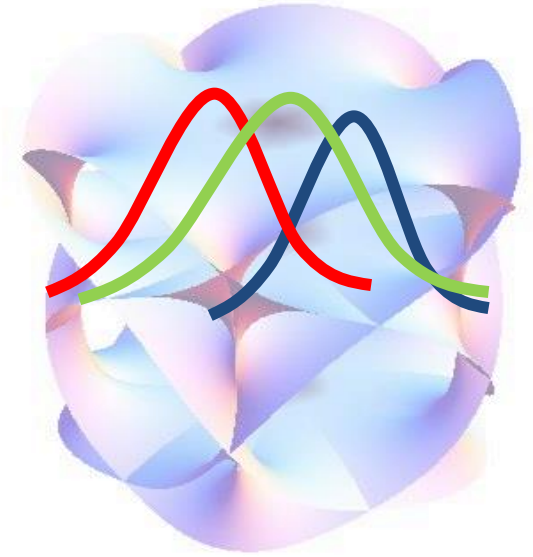
# 余剰次元空間の幾何学的対称性(フレーバー/CP 対称性)

余剰次元空間上に素粒子の波動関数  
 (“Global model”)



# 余剰次元空間の幾何学的対称性(フレーバー/CP 対称性)

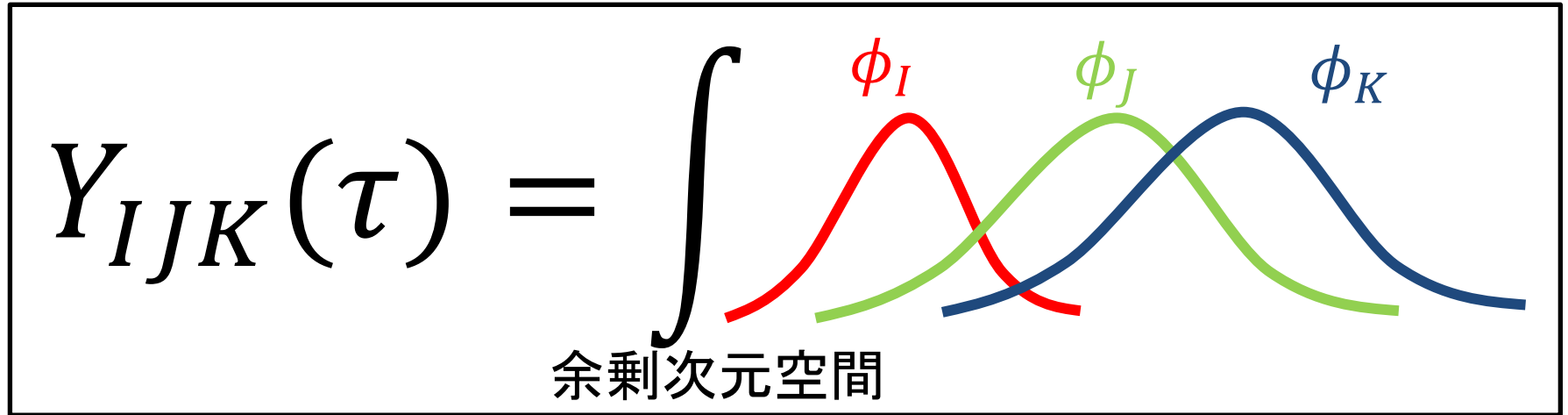
余剰次元空間上に素粒子の波動関数  
 (“Global model”)



- ・ (弦理論EFTにおける)4次元湯川結合  
= 3個の(ゼロモード)波動関数の重なり積分

$$Y_{IJK} = \int_{\text{余剰次元空間}} \phi_I \phi_J \phi_K$$

# 余剰次元空間の幾何学的対称性(フレーバー/CP 対称性)

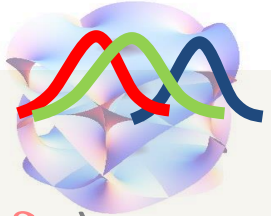
$$Y_{IJK}(\tau) = \int_{\text{余剰次元空間}} \phi_I \phi_J \phi_K$$


The diagram shows three overlapping curves representing fields  $\phi_I$  (red),  $\phi_J$  (green), and  $\phi_K$  (blue) plotted against a horizontal axis. The curves are arranged in a sequence from left to right, with  $\phi_I$  peaking first, followed by  $\phi_J$ , and then  $\phi_K$ . The integration is performed over the '余剰次元空間' (extra-dimensional space).



# 余剰次元空間の幾何学的対称性 (フレーバー/CP 対称性)

$$Y_{IJK}(\tau) = \int_{\text{余剰次元空間}} \phi_I \phi_J \phi_K$$



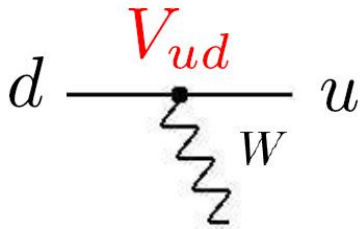
- モジュライ依存  $\tau$  (余剰次元空間 計量  $g$  の変形自由度:  $\delta g$ )  
—  $Y(\tau)$  は余剰次元空間の幾何学的対称性の下で非自明に変換

$$\begin{aligned} \tau &\rightarrow \gamma\tau \\ Y(\tau) &\rightarrow \rho(\gamma)Y(\tau) \end{aligned}$$

幾何学的対称性  $\approx$  クォーク/レプトンのフレーバー対称性

# クォークセクターのフレーバー構造

- Quark sector (Weak mixing):



$$L \simeq g \bar{u}_L \gamma_\mu d_L W_\mu^+ + \text{h.c.}$$

フレーバー固有状態

$$= g \bar{u}_L^{\text{mass}} \gamma_\mu (V_{uL}^+ V_{dL}) d_L^{\text{mass}} W_\mu^+ + \text{h.c.}$$

↓  
質量固有状態

$$\equiv V_{\text{CKM}}$$

PDG ('20)

$$V_{\text{CKM}} = \begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}$$

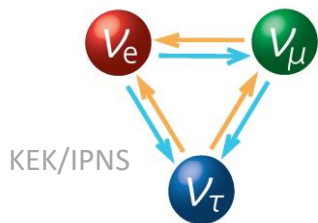
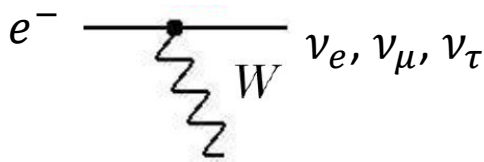
フレーバー混合: フレーバー固有状態と質量固有状態のずれ

- CP violation

$$J_{CP}(\text{quark}) \simeq 3 \times 10^{-5}$$

# レプトンセクターのフレーバー構造

## Lepton sector (Large mixing):



- 素粒子標準模型ではニュートリノ質量0
- フレーバー混合なし



- Super-Kamiokandeによるニュートリノ質量発見
- 3世代のニュートリノ、フレーバー混合有

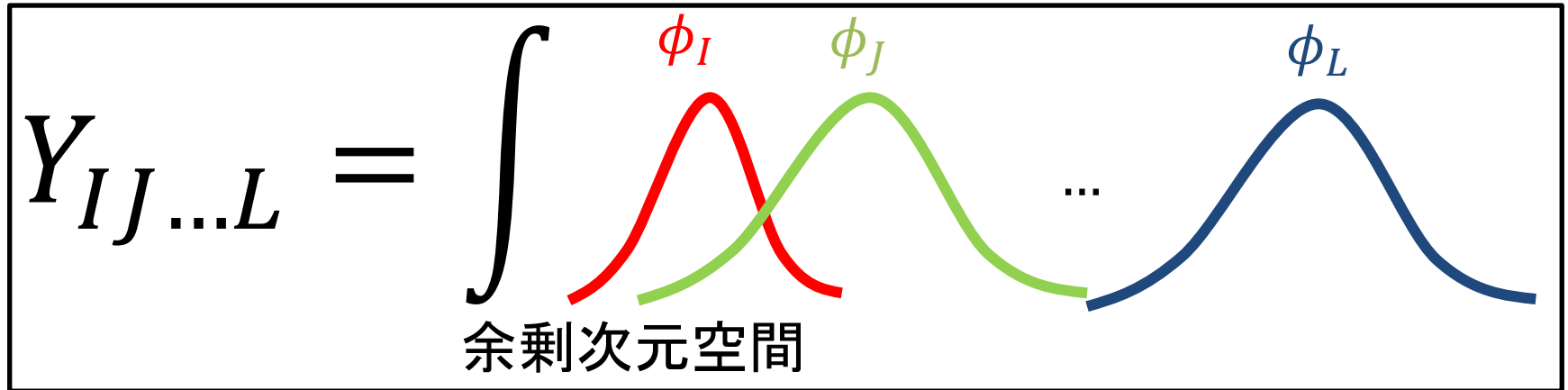
NuFIT 5.0 (2020)

$$|U|_{3\sigma}^{\text{w/o SK-atm}} = \begin{pmatrix} 0.801 \rightarrow 0.845 & 0.513 \rightarrow 0.579 & 0.143 \rightarrow 0.156 \\ 0.233 \rightarrow 0.507 & 0.461 \rightarrow 0.694 & 0.631 \rightarrow 0.778 \\ 0.261 \rightarrow 0.526 & 0.471 \rightarrow 0.701 & 0.611 \rightarrow 0.761 \end{pmatrix}$$

## CP violation

$J_{CP}(\text{lepton})$ : within reach @T2K, ハイパーカミオカンデ(2027~), DUNE(2026~)

# 余剰次元空間の幾何学的対称性(フレーバー/CP 対称性)

$$Y_{IJ\dots L} = \int_{\text{余剰次元空間}} \phi_I \phi_J \dots \phi_L$$


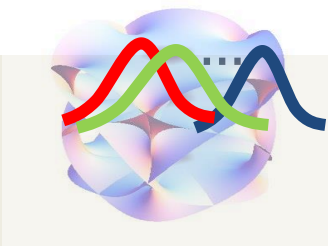
The diagram illustrates the integration of fields in extra dimensions. The equation shows  $Y_{IJ\dots L} = \int_{\text{余剰次元空間}} \phi_I \phi_J \dots \phi_L$ . The fields are represented by colored curves: red for  $\phi_I$ , green for  $\phi_J$ , and blue for  $\phi_L$ . Ellipses indicate intermediate fields. The integration is over the extra dimension space.

# 余剰次元空間の幾何学的対称性 (フレーバー/CP 対称性)

$$Y_{IJ\dots L} = \int_{\text{余剰次元空間}} \phi_I \phi_J \dots \phi_L$$

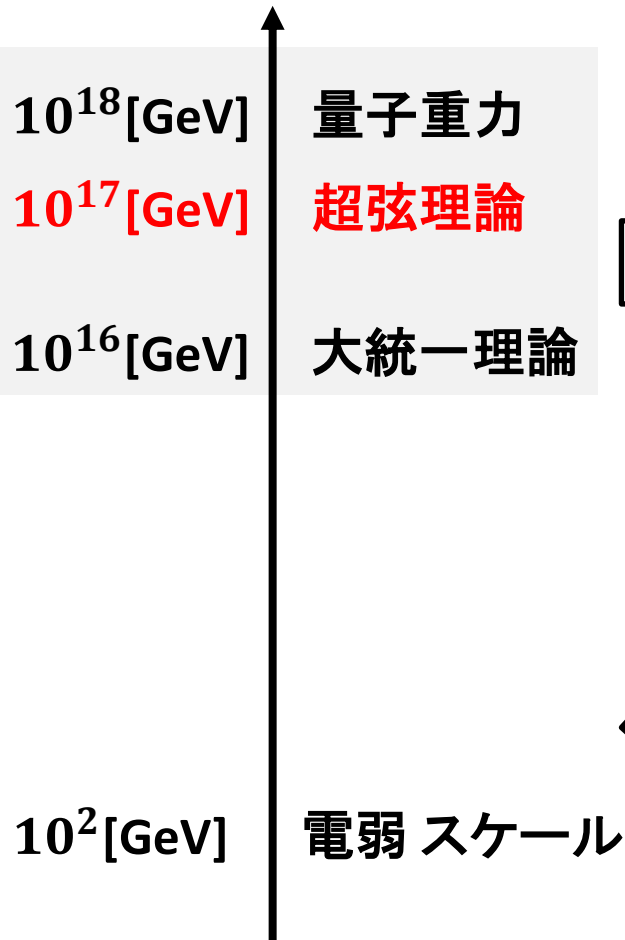
- 4次元 $n$ -点結合 (高次演算子)  
=  $n$ 個のゼロモード波動関数の重なり積分

- 湯川結合同様にモジュライ依存  $\tau$        $\tau \rightarrow \gamma\tau$   
 $Y(\tau) \rightarrow \rho(\gamma)Y(\tau)$

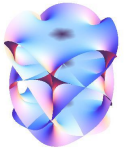


幾何学的対称性 = string EFTのもつ重要な特徴

# 今日のトーク



余剰次元空間のモジュラー対称性  
(フレーバー/CP 対称性)



弦理論のEFTに強い制限

(湯川結合/高次演算子:モジュライ場の関数)

現象論:

- (i) Minimal Flavor Violation
- (ii) Standard Model Effective Field Theory (SMEFT)
- (iii) B/L数を破る高次演算子を制限

# Outline

## 1. 導入

## 2. 幾何学的対称性とフレーバー対称性

(i) 2次元トーラス (ii) 6次元カラビヤウ空間

## 3. Landscape of modular flavor models

## 4. モジュラー対称性がもたらす現象論

(i) Minimal Flavor Violation

(ii) Standard Model Effective Field Theory (SMEFT)

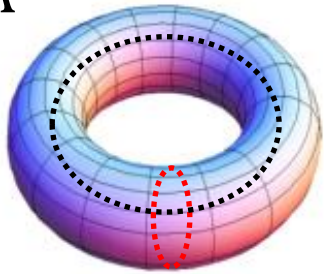
(iii) 陽子崩壊

## 5. 結論

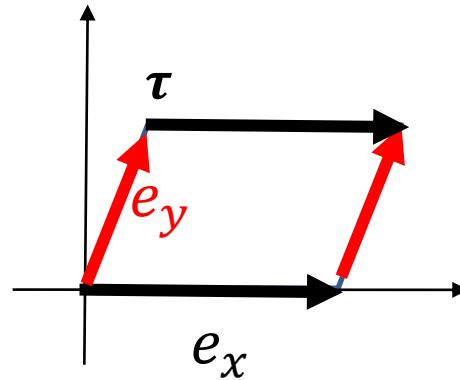
# モジュラー対称性 ～2次元トーラス～

- $SL(2, \mathbb{Z})$  geometric (modular) symmetry

$$T^2 = \mathbb{C}/\Lambda$$



=



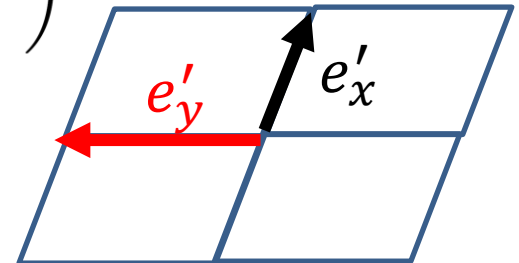
- Lattice vectorsの持つ対称性:

$$\begin{pmatrix} e'_y \\ e'_x \end{pmatrix} = \begin{pmatrix} p & q \\ s & t \end{pmatrix} \begin{pmatrix} e_y \\ e_x \end{pmatrix}$$

$p, q, s, t \in \mathbb{Z}$  satisfying  $pt - qs = 1$

Two generators :  $S$  and  $T$

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

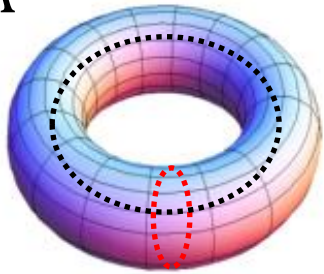




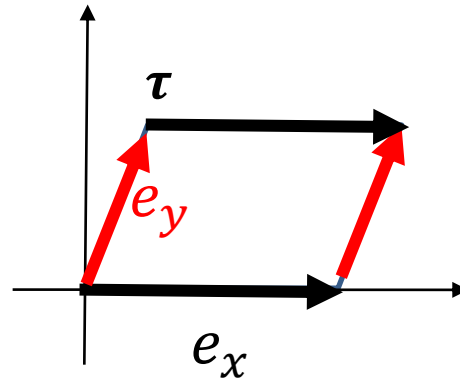
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=



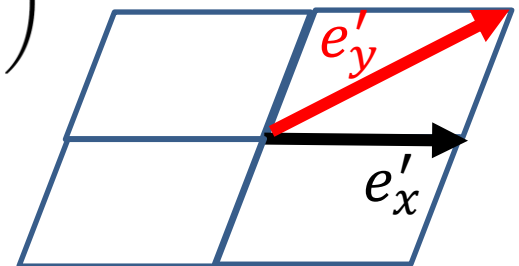
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$$p, q, s, t \in \mathbb{Z} \text{ satisfying } pt - qs = 1$$

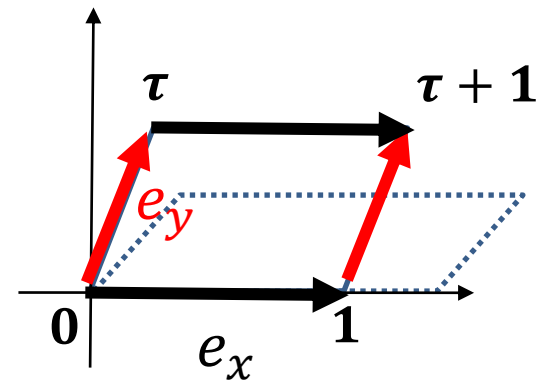
Two generators :  $S$  and  $T$

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

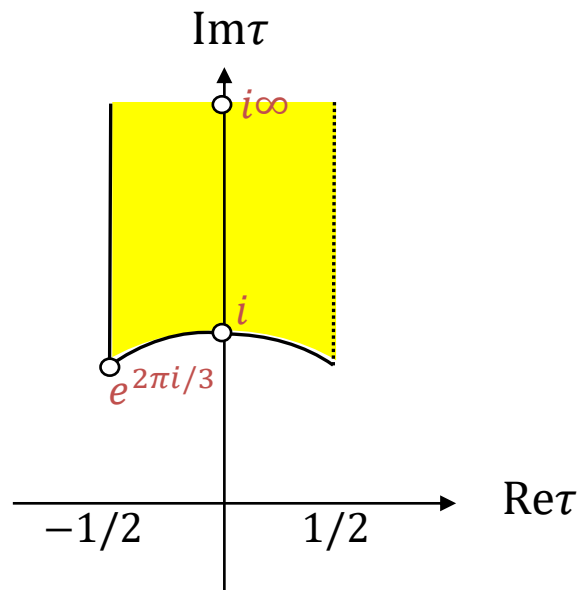


- Modulus  $\tau$

$$\tau \equiv \frac{e_y}{e_x} \rightarrow \tau' \equiv \frac{e'_y}{e'_x} = \frac{p\tau + q}{s\tau + t}$$



- $\tau$  の基本領域:



$$S : \tau \rightarrow -1/\tau$$

$$T : \tau \rightarrow \tau + 1$$

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

# Finite subgroups of modular group

Modular group

$$\bar{\Gamma} \simeq \{S, T \mid S^2 = 1, (ST)^3 = 1\}$$

Finite subgroups

$$\Gamma_N = \bar{\Gamma}/\bar{\Gamma}(N) \quad \bar{\Gamma}(N): \text{congruence subgroup}$$

$$\Gamma_N \simeq \{S, T \mid S^2 = 1, (ST)^3 = 1, T^N = 1\}$$

Non-abelian discrete groups :

$$\Gamma_3 \simeq S_3, \quad \Gamma_4 \simeq A_4, \quad \Gamma_5 \simeq S_4, \quad \Gamma_6 \simeq A_5, \dots$$

クォーク/レプトンのフレーバー対称性

# Finite subgroups of modular group

Modular group

$$\bar{\Gamma} \simeq \{S, T \mid S^2 = 1, (ST)^3 = 1\}$$

Finite subgroups

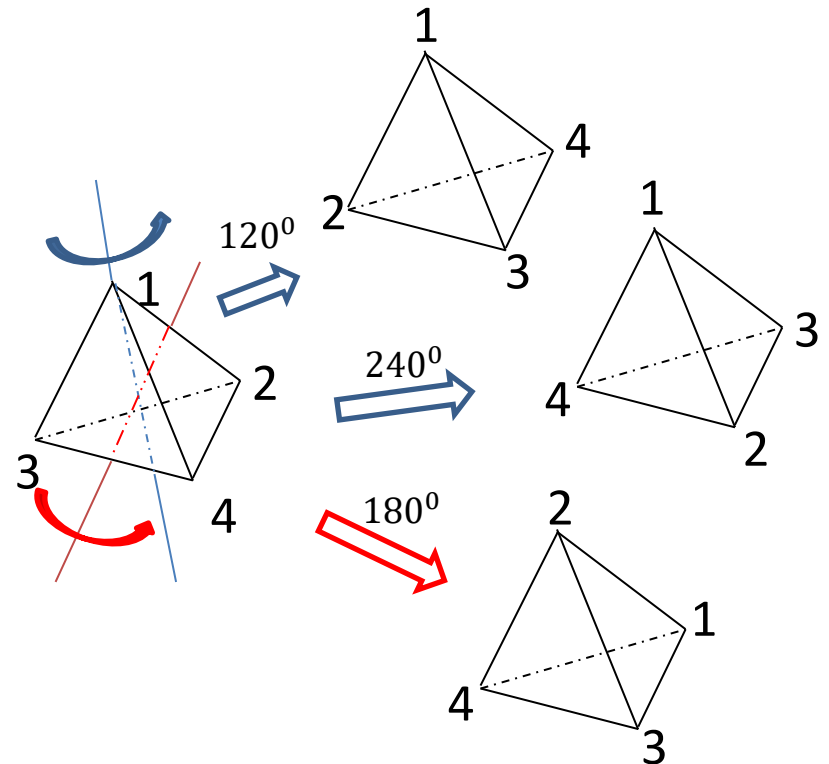
$$\Gamma_N = \bar{\Gamma}/\bar{\Gamma}(N)$$

$\bar{\Gamma}(N)$ : congruence subgroup

$\Gamma_3 = A_4$ : Tetrahedral symmetry

Generators :  $S$  and  $T$

$$S^2 = (ST)^3 = T^3 = 1$$



# Finite subgroups of modular group

Modular group

$$\bar{\Gamma} \simeq \{S, T \mid S^2 = 1, (ST)^3 = 1\}$$

Finite subgroups

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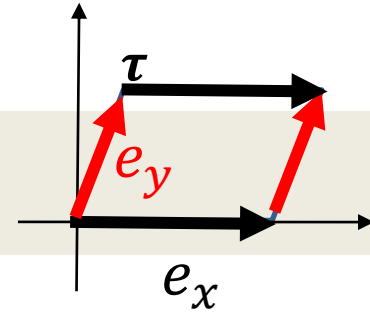
$$\Gamma_3 \simeq S_3, \quad \Gamma_4 \simeq A_4, \quad \Gamma_5 \simeq S_4, \quad \Gamma_6 \simeq A_5, \dots$$

*F. Feruglio, 1706.08749*

クォーク/レプトンの**フレーバー対称性**として盛んに議論されてきたが  
従来のフレーバー模型とは何が違うのか？

# 従来のフレーバーモデルとの違い

- 湯川結合  $Y(\tau)$  はモジュライ  $\tau (= e_y/e_x)$  依存



—  $Y(\tau)$  は  $\tau$  のモジュラー変換のもとで、非自明に変換

E.g.,  $A_4$  triplet with modular weight  $k = 2$

$$Y_1(\tau) = \frac{i}{2\pi} \left( \frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - \frac{27\eta'(3\tau)}{\eta(3\tau)} \right)$$

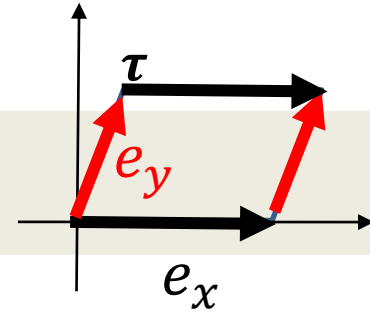
$$Y_2(\tau) = \frac{-i}{\pi} \left( \frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega^2 \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} \right),$$

$$Y_3(\tau) = \frac{-i}{\pi} \left( \frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega^2 \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} \right),$$

$\eta$  : Dedekind eta-function

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E.g.,  $A_4$  triplet with modular weight  $k = 2$

$$\begin{aligned} Y_1(\tau) &= 1 + 12q + 36q^2 + 12q^3 + \dots, & q &= e^{2\pi i\tau} \\ Y_2(\tau) &= -6q^{1/3}(1 + 7q + 8q^2 + \dots), & \text{Im}\tau &\gg 1 \\ Y_3(\tau) &= -18q^{2/3}(1 + 2q + 5q^2 + \dots). \end{aligned}$$

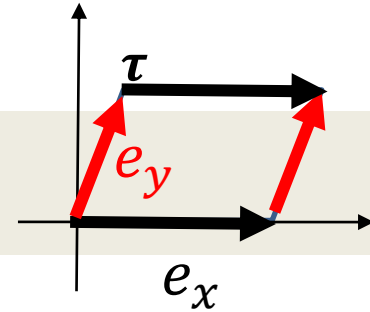
$\eta$  : Dedekind eta-function

F. Feruglio, 1706.08749

- モジュライ依存の湯川結合
  - クォーク/レプトンセクターの質量/混合と整合的

# 従来のフレーバーモデルとの違い

- 湯川結合  $Y(\tau)$  はモジュライ  $\tau (= e_y/e_x)$  依存



	$L$	$\{e^c, \mu^c, \tau^c\}$	$N^c$	$H_u$	$H_d$
$SU(2)_L$	<b>2</b>	<b>1</b>	<b>1</b>	<b>2</b>	<b>2</b>
$U(1)_Y$	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$
$A_4$	<b>3</b>	$\{1, 1', 1'\}$	<b>3</b>	<b>1</b>	<b>1</b>
$-k_I$	-2	$\{-2, -4, -4\}$	-2	0	0

**Table 1.** Charge assignments under  $SU(2)_L \times U(1)_Y \times A_4$  in the lepton and Higgs sectors, where  $k_I$  denotes the modular weight of matter superfields  $\Phi_I$ .

$\eta$  : Dedekind eta-function

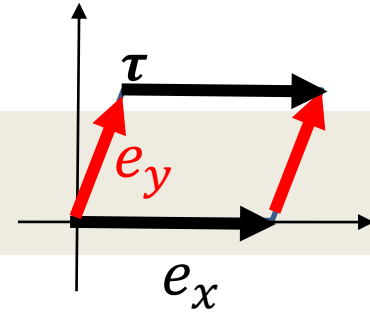
F. Feruglio, 1706.08749

- モジュライ依存の湯川結合
  - クォーク/レプトンセクターの質量/混合と整合的



# 従来のフレーバーモデルとの違い

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- $Y(\tau)$  は  $\tau$  のモジュラー変換のもとで、非自明に変換

E.g.,  $A_4$  triplet with modular weight  $k = 2$

$$Y_1(\tau) = 1 + 12q + 36q^2 + 12q^3 + \dots,$$

$$Y_2(\tau) = -6q^{1/3}(1 + 7q + 8q^2 + \dots),$$

$$Y_3(\tau) = -18q^{2/3}(1 + 2q + 5q^2 + \dots).$$

$$q = e^{2\pi i \tau}$$

$$\text{Im} \tau \gg 1$$

$\eta$  : Dedekind eta-function

- $\text{Re} \tau$  (axion) が **CP phase** を決定

F. Feruglio, 1706.08749

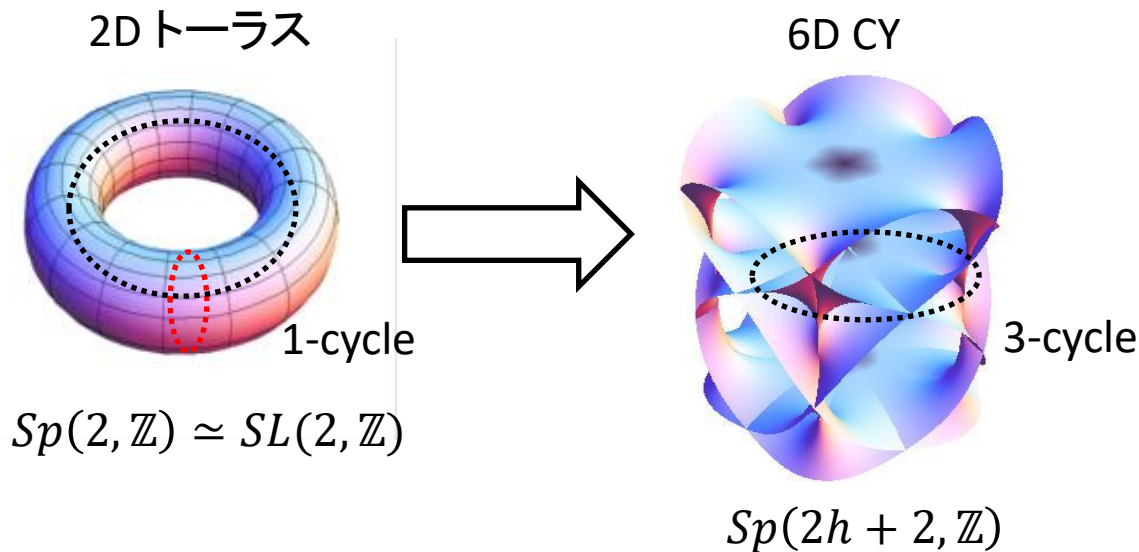
- $\langle \tau \rangle$  がフレーバー/CP対称性とその破れを決定

- レプトンセクターのCPの破れを予言

# 6次元 Calabi-Yau (CY) threefolds

- 弦理論の真空解 (少なくとも  $O(10^8)$  存在)
- $Sp(2h + 2, \mathbb{Z})$  symplectic modular symmetry

A. Strominger ('90),  
P. Candelas, X. de la Ossa ('91)



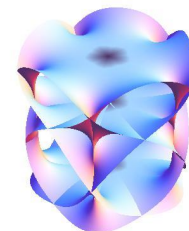
- $Sp(2h + 2, \mathbb{Z})$  : 3-cycleの(dual)基底ベクトル  $\{\alpha^I, \beta_I\}$ の変換

# 4D SUSY $E_6$ GUT from Heterotic string on 6D CY

Candelas-Horowitz-Strominger-Witten ('85)

- 4D ゲージ対称性 :

$$E_6 \times E_8^{(\text{hidden})}$$



- 物質場 ( $E_6 : 27$  or  $\overline{27}$ )  $\approx$  モジュライ

$$\overline{27}^i \approx \text{モジュライ } \tau^i$$

(3-cycle volumeを決定)

- 湯川結合 ( $\overline{27}^3$ )

$$W = F_{ijk} \overline{27}^i \overline{27}^j \overline{27}^k$$

$$F_{ijk} = \partial_{\tau^i} \partial_{\tau^j} \partial_{\tau^k} F \quad (F(\tau) : \text{prepotential})$$

— Modular symmetry ( $\tau \rightarrow \gamma\tau$ ) の下でテンソル表現

Calabi-Yau空間のSymplectic modular symmetry  
~ フレーバー対称性

▪ Symplectic 変換 :

モジュライ:  $\tau^i = X^i \rightarrow \tilde{\tau}^i \simeq \frac{\partial \tilde{X}^i}{\partial X^j} \tau^j$

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e.g.,  $S_4$  対称性 Ishiguro-Kobayashi-Otsuka, 2107.00487

- $S_4$  flavor symmetry

$$W = 27^1 27^2 27^3$$

$$F_{123} = 1, \text{ otherwise } 0$$

- Invariant under two generators

$$P: 27^1 \rightarrow 27^3, 27^2 \rightarrow 27^1, 27^3 \rightarrow 27^2$$

$$Q: 27^1 \rightarrow -27^1, 27^2 \rightarrow -27^3, 27^3 \rightarrow 27^2$$

- $S_4$  triplet :  $\{27^1, 27^2, 27^3\}$

- $S_4 \subset Sp(2 \times 3 + 2, \mathbb{Z}) = \boxed{Sp(8, \mathbb{Z})}$        $\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow \boxed{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

- realized in some classes of 6D Calabi-Yaus and toroidal orbifolds

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*Ishiguro-Kobayashi-Otsuka, 2010.10782*



# Geometrical origin of 4D CP

- Assumption : 10D = 4D + 6D Calabi-Yau (CY)

Consider simultaneous transformations of

— 4D parity

— 6D orientation reversing :  $z_i \rightarrow -\bar{z}_i$  ( $i = 1, 2, 3$ )

( $z_i$ : local coordinates of 6D space)

(Volume form :  $dV \rightarrow -dV$ )

$$dV \propto dz_1 \wedge dz_2 \wedge dz_3 \wedge d\bar{z}_1 \wedge d\bar{z}_2 \wedge d\bar{z}_3$$

*Strominger-Witten ('85)*

*Dine-Leigh-MacIntire ('92)*

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10D Majorana-Weyl spinor under  $SO(1,9) = SO(1,3) \times SO(6)$  :

$$16 = (2, 4_+) \oplus (2', \bar{4}_-)$$

$2, 2'$  : left- and right-handed spinors of  $SL(2, \mathbb{C})$

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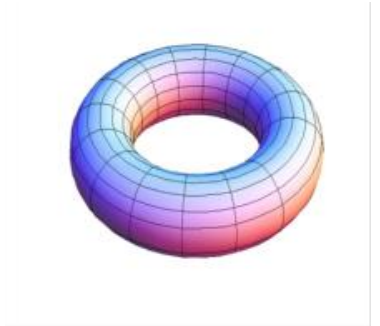
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- Such transformations correspond to 4D CP (axion  $\rightarrow$  - axion)

# 4D CP and modular symmetry

— 6D orientation reversing :  $z_i \rightarrow -\bar{z}_i$  ( $i = 1,2,3$ )

$$\tau_i \rightarrow -\bar{\tau}_i$$



- 4D CP is unbroken at  $\text{Re}\tau_i = 0, \pm\frac{1}{2}$

(分配関数が  $\theta = \pi$  でCP不変)

$$\text{Re}\tau_i = \pm\frac{1}{2} \xrightarrow{\text{CP}} \mp\frac{1}{2} \xrightarrow{T} \text{Re}\tau_i = \pm\frac{1}{2}$$

- CP as an outer automorphism of  $SL(2, \mathbb{Z})$  ( $\gamma$ )

$$\tau_i \xrightarrow{\text{CP}} -\bar{\tau}_i \xrightarrow{\gamma} -\frac{p\bar{\tau}_i + q}{s\bar{\tau}_i + t} \xrightarrow{\text{CP}^{-1}} \frac{p\tau_i - q}{-s\tau_i + t}$$

*H. P. Nilles, M. Ratz, A. Trautner, P. K. S. Vaudrevange ('18),  
P. P. Novichkov, J. T. Penedo, S. T. Petcov, A. V. Titov ('19)*

$$\gamma = \begin{pmatrix} p & q \\ s & t \end{pmatrix} \rightarrow u(\gamma) = CP\gamma CP^{-1} = \begin{pmatrix} p & -q \\ -s & t \end{pmatrix} \quad CP = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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*Ishiguro-Kobayashi-Otsuka, 2010.10782*

*Kobayashi-Nishimura-Otsuka, Working in progress*

— 湯川結合はモジュラー形式 Prepotential :  $F = F_{\text{cubic polynomial}} + F_{\text{instanton}}$

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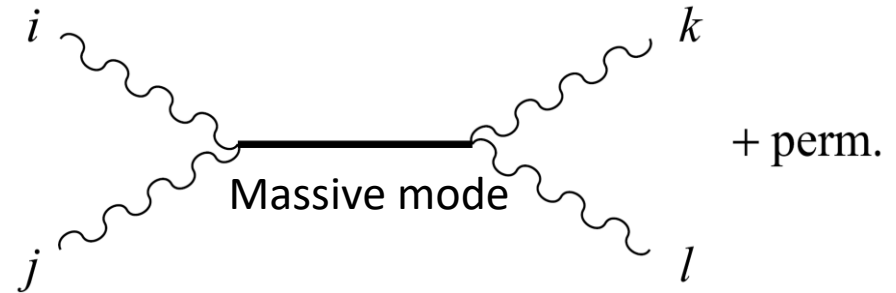
Ishiguro-Kobayashi-Otsuka, 2010.10782

# $n$ -点結合 in SUSY $E_6$ GUT

Bershadsky-Cecotti-Ooguri-Vafa ('93)

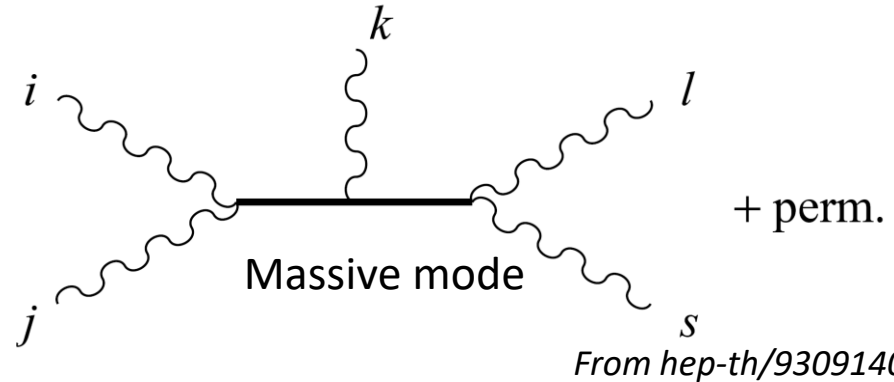
- Dimension-5

$$\frac{F_{ijkl}}{\Lambda} 27_i 27_j 27_k 27_l$$



- Dimension-6

$$\frac{F_{ijkl s}}{\Lambda^2} 27_i 27_j 27_k 27_l 27_s$$



- $n$ -点結合 :  $F_{ij\dots n} = \partial_i \partial_j \dots \partial_n F$

$F$  : prepotential

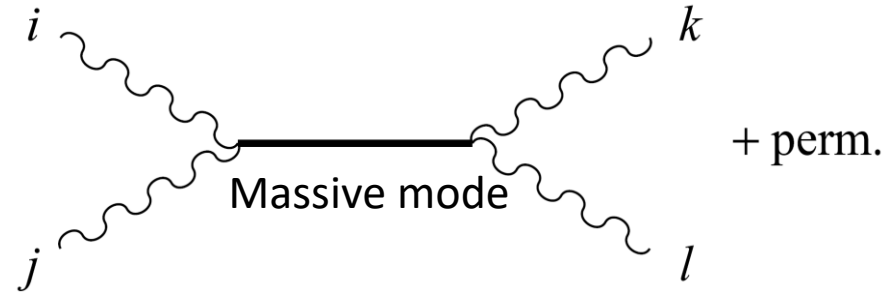
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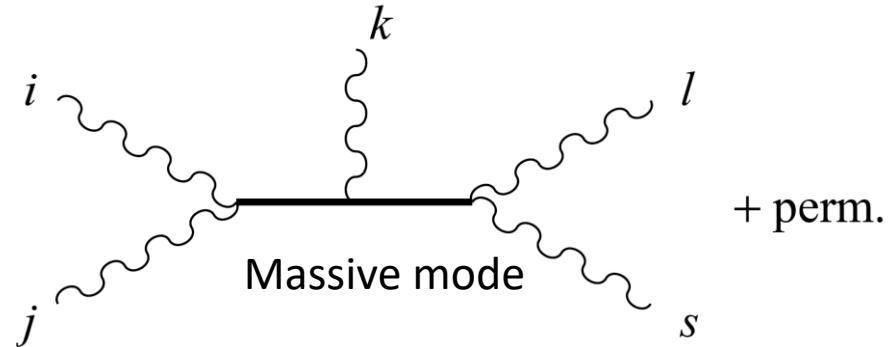
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From hep-th/9309140

- Prepotential :  $F = F_{\text{cubic polynomial}} + F_{\text{instanton}}$

E.g.,  $F_{ijkl} = \partial_i \partial_j \partial_k \partial_l F_{\text{instanton}}$  は,  $\exp(2\pi i \tau)$  の関数 ( $\tau = \text{モジュライ}$ )

-> Flavor の破れは  $\text{Im} \tau \gg 1$  で抑えられる



# Outline

## 1. 導入

## 2. 幾何学的対称性とフレーバー対称性

(i) 2次元トーラス (ii) 6次元カラビヤウ空間

## 3. Landscape of modular flavor models

## 4. モジュラー対称性がもたらす現象論

(i) Minimal Flavor Violation

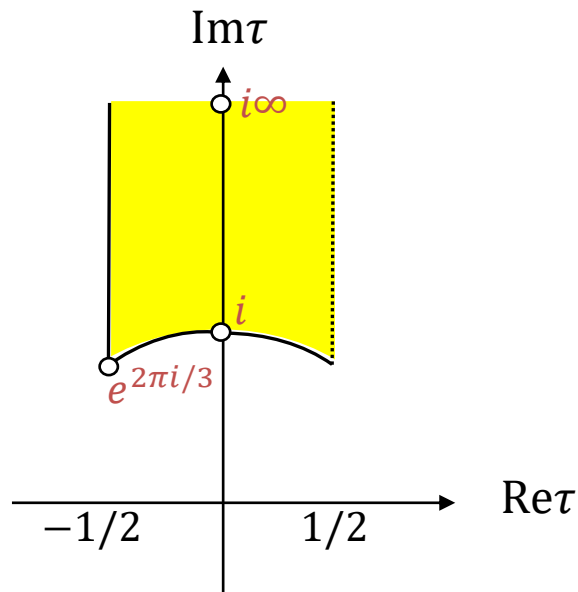
(ii) Standard Model Effective Field Theory (SMEFT)

(iii) 陽子崩壊

## 5. 結論

問 湯川結合  $Y(\tau)$  はモジュライ  $\tau$  依存  $\langle \tau \rangle$  をどのようにして決めるか？

- トーラスのModulus  $\tau$  の基本領域:



$$S : \tau \rightarrow -1/\tau$$

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

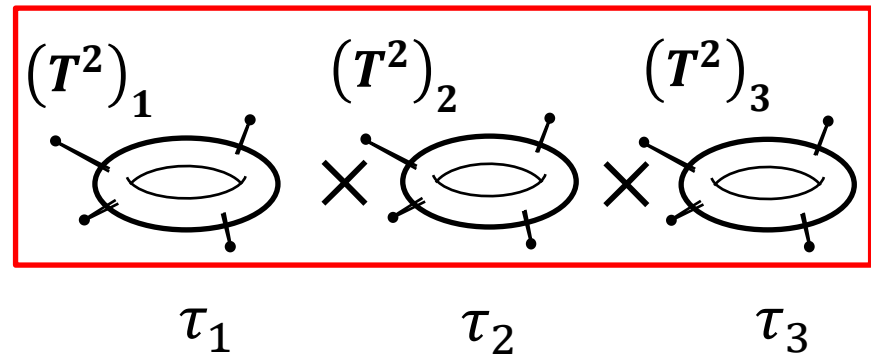
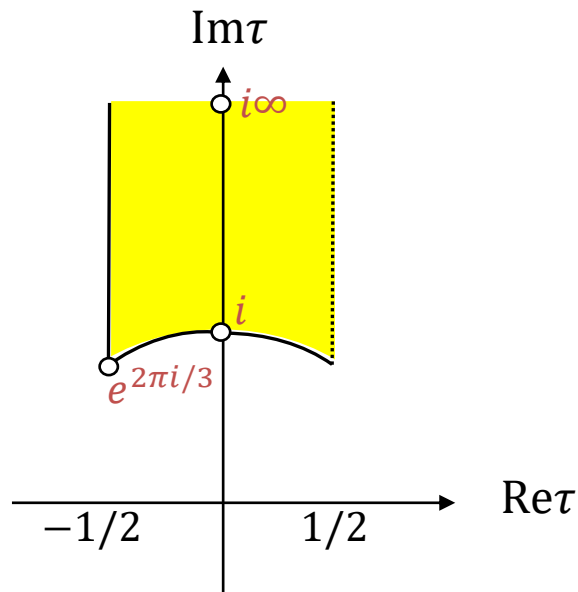
$$T : \tau \rightarrow \tau + 1$$

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→ Type IIB弦理論 on  $T^6 / (\mathbb{Z}_2 \times \mathbb{Z}'_2)$  フラックスコンパクト化において、 $\langle \tau \rangle$  を決定

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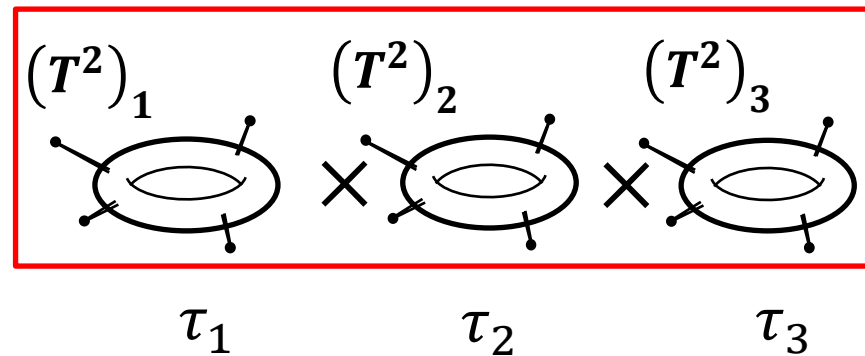
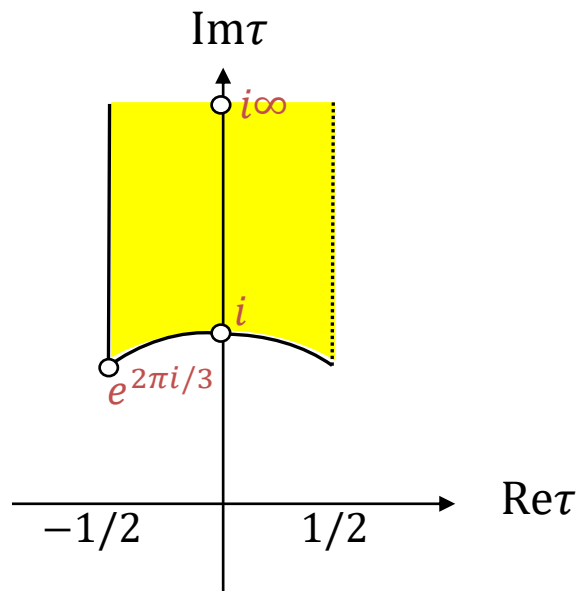
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## Introduction — フラックスコンパクト化

- Flux compactification is useful to stabilize the moduli fields.

Let us consider higher-dimensional Maxwell's theory on  $R^{1,3} \times M$ ,

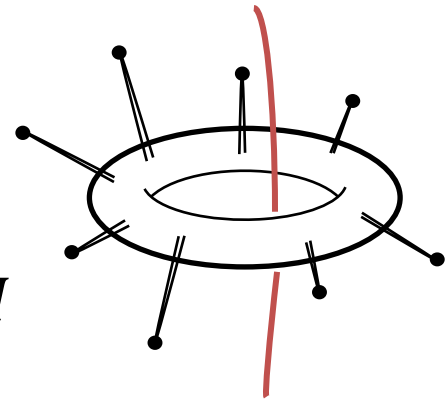
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When there exists a magnetic flux  $F_p$  in a cycle  $\Sigma_p$  of  $M$

$$\int_{\Sigma_p} F_p = n \in \mathbb{Z}$$

induces the moduli potential through “\*”, depending on the metric

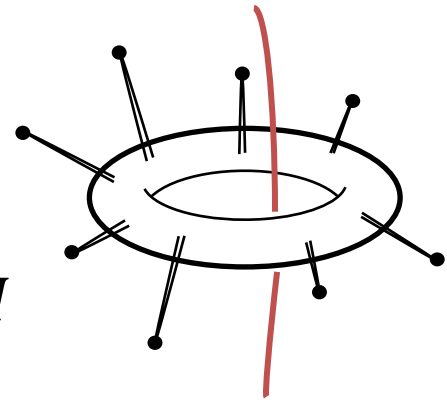
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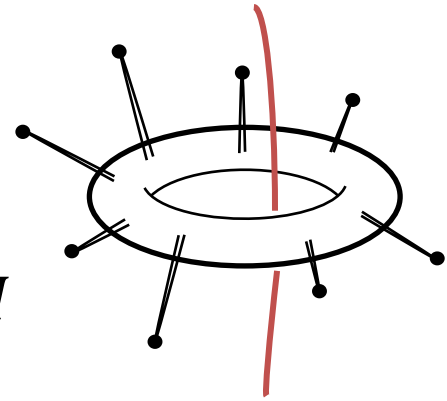
Type IIB 弦理論:  
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4D N=1 effective superpotential on  $T^6 / (\mathbb{Z}_2 \times \mathbb{Z}'_2)$  :

$$W_{\text{flux}} = a^0 \tau^3 - 3a\tau^2 - 3b\tau - b_0 - S(c^0 \tau^3 - 3c\tau^2 - 3d\tau - d_0)$$

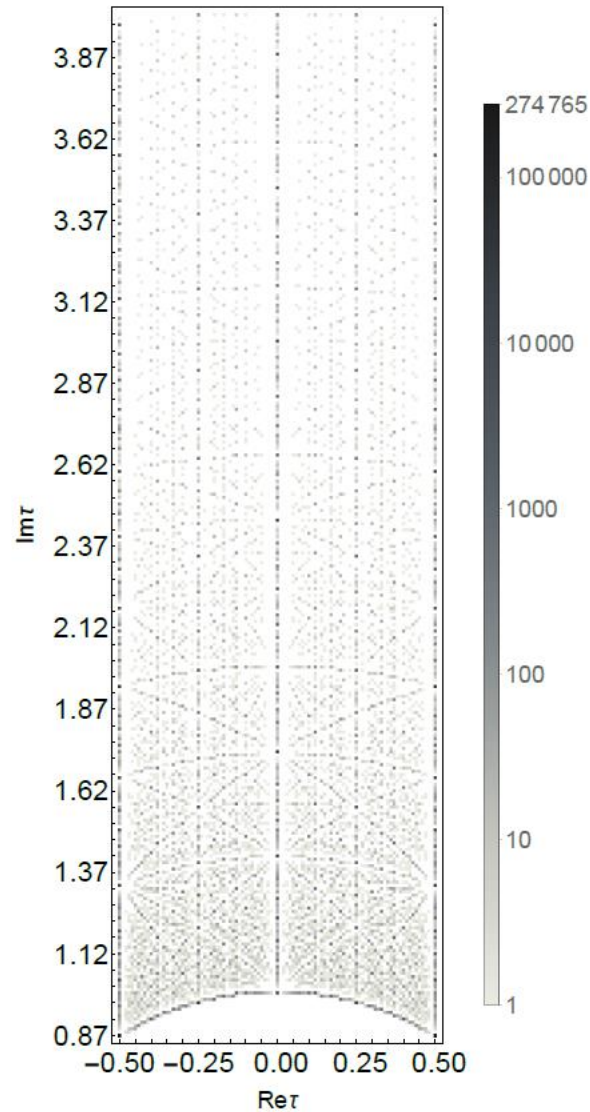
Flux quanta :  $\{a^0, a, b, b_0, c^0, c, d, d_0\}$



# SUSY stable vacua on $(\text{Re}\tau, \text{Im}\tau)$ -plane

- ・ フラックスの総和 =  $O(10^3)$   
(真空の数:  $O(2 \times 10^6)$ )

Ishiguro-Kobayashi-Otsuka,  
2011.09514



$(\text{Re } \tau, \text{Im } \tau)$	$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$	$(0, \sqrt{3})$	$(-\frac{1}{2}, \frac{\sqrt{15}}{2})$	$(-\frac{1}{4}, \frac{\sqrt{15}}{4})$	$(0, 1)$	$(-\frac{1}{2}, \frac{\sqrt{7}}{2})$	$(0, \sqrt{6})$	$(0, \sqrt{\frac{3}{2}})$	$(0, \sqrt{2})$	$(-\frac{1}{2}, \frac{3\sqrt{3}}{2})$
Probability (%)	40.3	7.55	4.85	4.85	3.79	2.43	1.88	1.88	1.88	1.49

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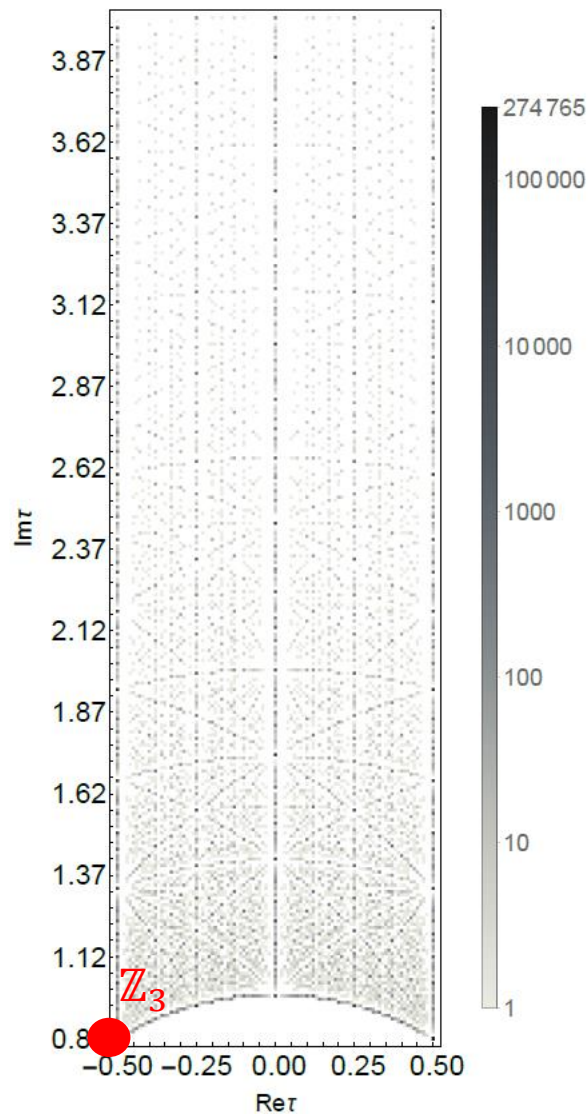
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$$\gamma_0 \tau_{\text{fix}} = \tau_{\text{fix}}$$

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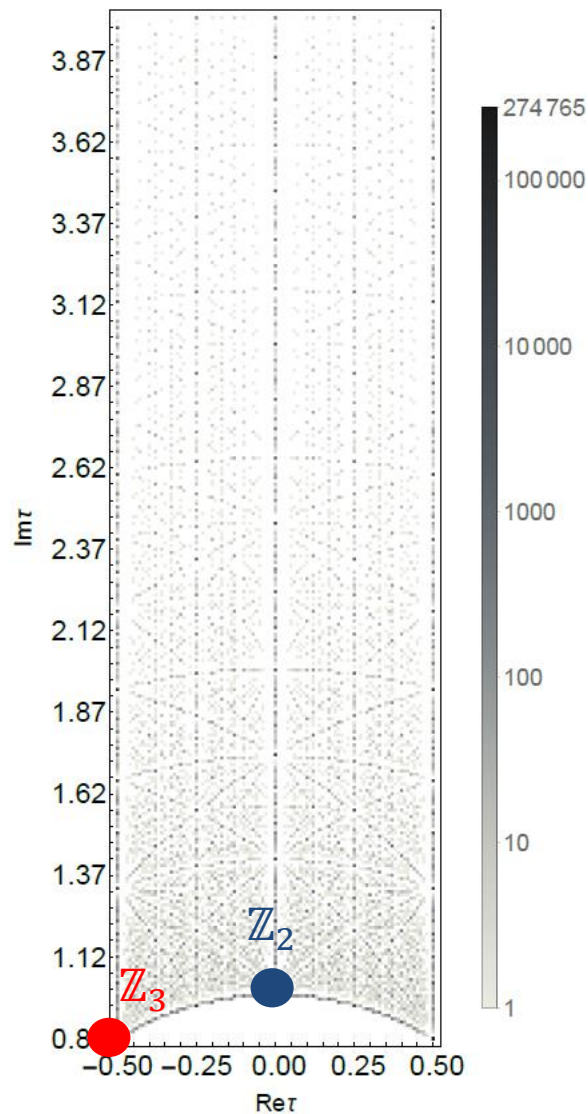
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$$\mathbb{Z}_2 : \{1, S\}$$



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Ishiguro-Kobayashi-Otsuka,  
2011.09514

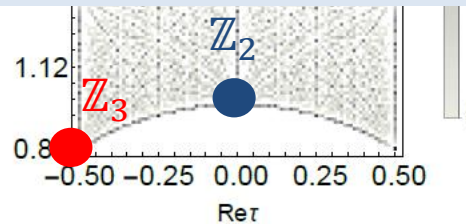
- 弦理論のFlux Landscapeでは、 $\mathbb{Z}_3$ 固定点が好まれる

- $\text{Re}\tau = 0, \pm 1/2$  が好まれ、CPが破れる真空  $O(1)\%$

$$\text{Re}\tau_i = \pm \frac{1}{2} \xrightarrow{\text{CP}} \mp \frac{1}{2} \xrightarrow{T} \text{Re}\tau_i = \pm \frac{1}{2}$$

- 固定点近傍は実験結果と整合的  
(レプトンセクターの質量・混合角)

Bottom-up approach :  
Novichkov-Penedo-Petcov-Titov ('18),  
Novichkov-Petcov-Tanimoto ('18),  
Okada-Tanimoto ('19,'20),  
Ding-King-Liu-Lu ('19),...



$(\text{Re}\tau, \text{Im}\tau)$	$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$	$(0, \sqrt{3})$	$(-\frac{1}{2}, \frac{\sqrt{15}}{2})$	$(-\frac{1}{4}, \frac{\sqrt{15}}{4})$	$(0, 1)$	$(-\frac{1}{2}, \frac{\sqrt{7}}{2})$	$(0, \sqrt{6})$	$(0, \sqrt{\frac{3}{2}})$	$(0, \sqrt{2})$	$(-\frac{1}{2}, \frac{3\sqrt{3}}{2})$
Probability (%)	40.3	7.55	4.85	4.85	3.79	2.43	1.88	1.88	1.88	1.49

# MSSM-like string models on $T^6 / (\mathbb{Z}_2 \times \mathbb{Z}'_2)$ orientifold

Ishiguro-Okada-Otsuka., working in progress

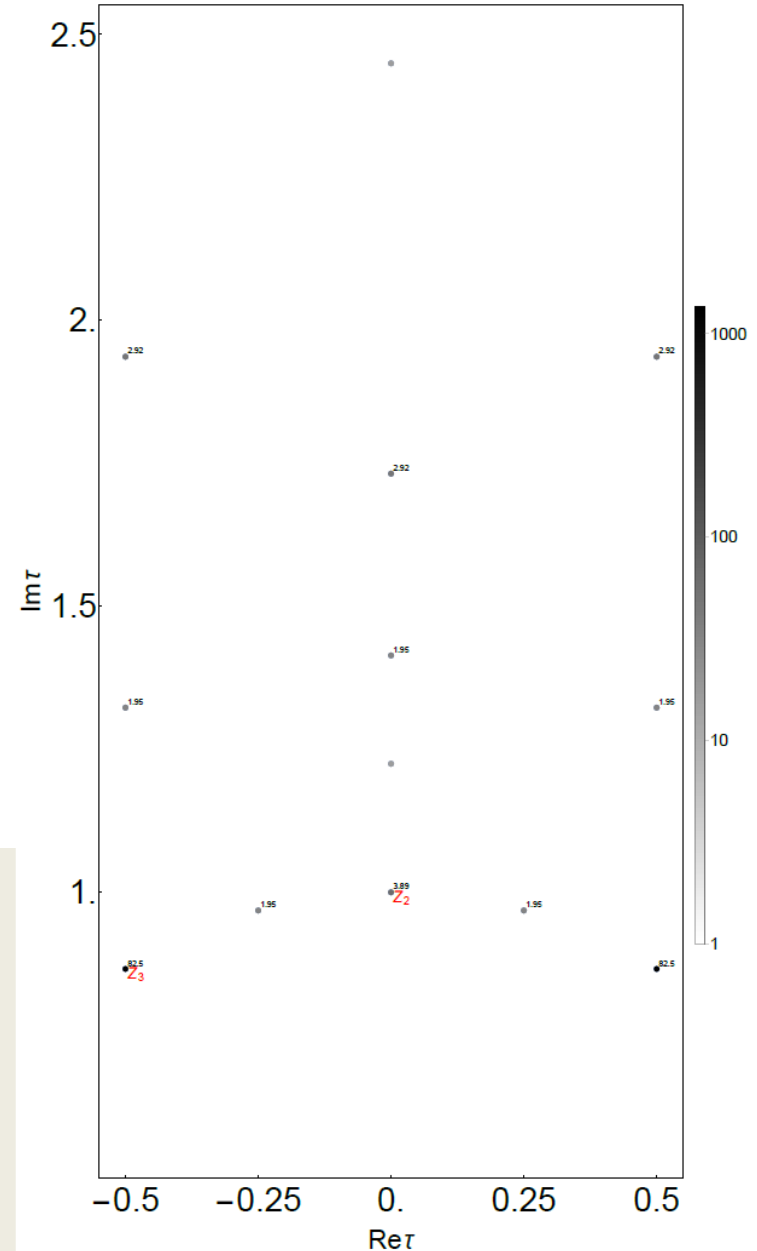
- Tadpole cancellation condition:  
(Charge conservation of D3-brane charge)

$$N_{\text{flux}} + N_{D3} - \frac{1}{2} N_{O3} = 0$$

- 3世代 string models:

$$SU(3) \times SU(2) \times SU(2) \times U(1)_{B-L} \times [U(1)' \times USp(8N_f)]$$

は先程の  $A_4$  模型 (NH) 同様に  $\mathbb{Z}_3$  固定点付近に局在



# ここまでのまとめ

- モジュラー対称性に基づくフレーバー対称性

$$SL(2, Z) \supset S_3, A_4, S_4, A_5, \dots, Sp(2h, Z) \supset S_4$$

$$G_{\text{flavor}} \rtimes \mathbb{Z}_2^{CP}$$

- フレーバー対称性及びその離散対称性( $\mathbb{Z}_2, \mathbb{Z}_3$ )

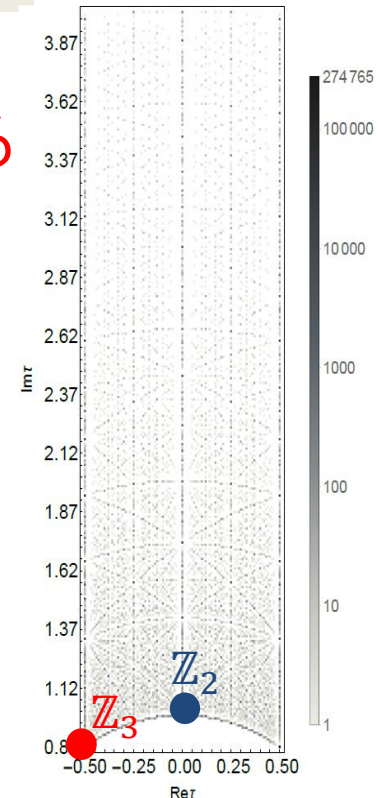
- 弦理論のランドスケープで、 $\mathbb{Z}_3$ 固定点が好まれる

$$\tau = \tau_{\mathbb{Z}_3}$$

- CPが破れる真空0(1)%
- 現象論的模型とも整合的

## モジュラー対称性に基づく他の現象論:

- 陽子崩壊, SMEFT, Minimal Flavor Violation, ...



# Outline

## 1. 導入

## 2. 幾何学的対称性とフレーバー対称性

(i) 2次元トーラス (ii) 6次元カラビヤウ空間

## 3. Landscape of modular flavor models

## 4. モジュラー対称性がもたらす現象論

(i) Minimal Flavor Violation

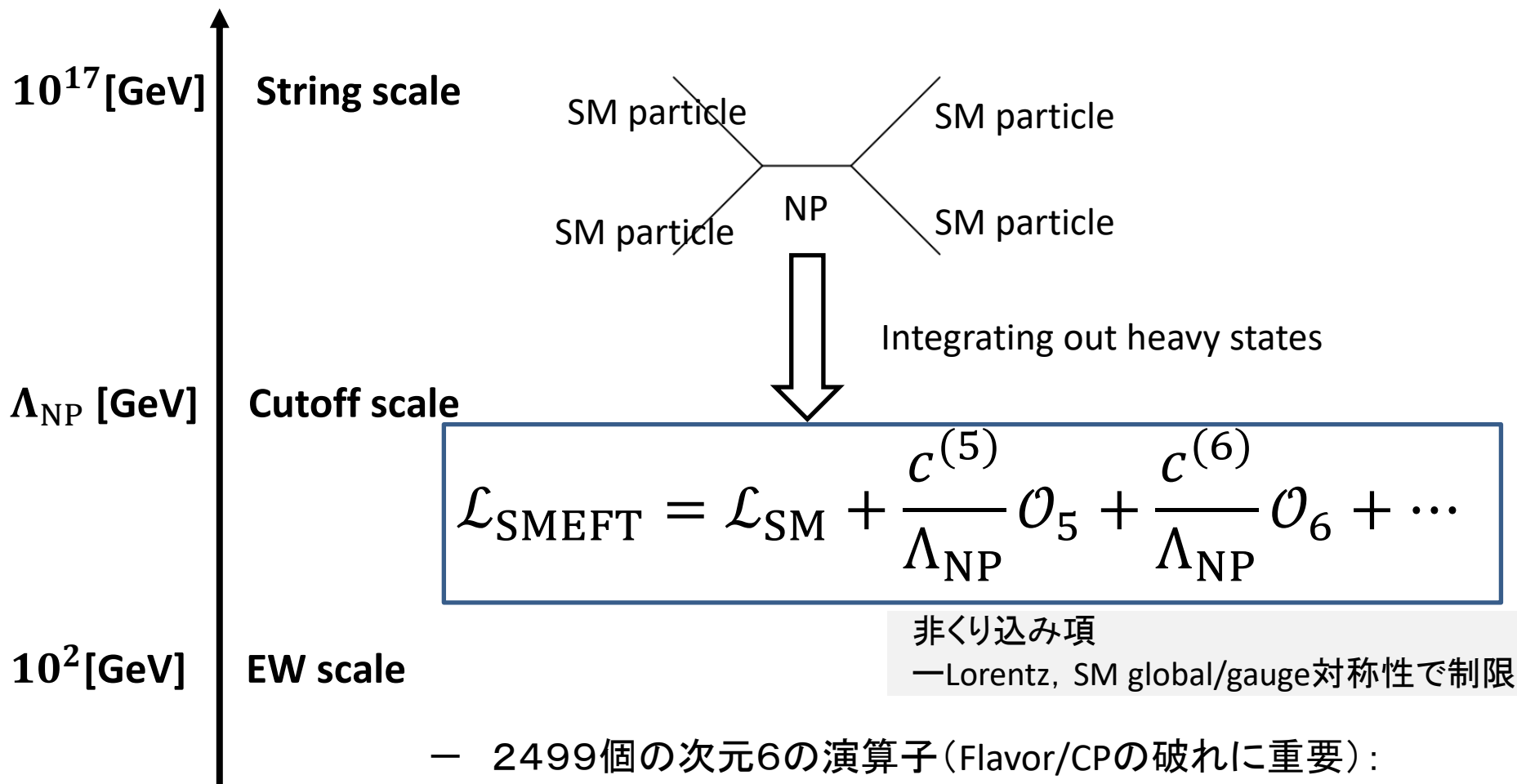
(ii) Standard Model Effective Field Theory (SMEFT)

(iii) 陽子崩壊

## 5. 結論

# SMEFT (Standard Model Effective Field Theory)

— 新物理の兆候を探る重要なBottom-up approach



— 2499個の次元6の演算子 (Flavor/CPの破れに重要):

e.g.,  $\Delta F = 1$  semi-leptonic operators:  $(\bar{L}\gamma^\mu L)(\bar{Q}\gamma_\mu Q), \dots$



## (i) Minimal Flavor Violation (MFV) hypothesis(1/2)

- SMのFlavor対称性 ( $\bar{\psi} i \gamma^\mu D_\mu \psi$ )

$$G_F = SU(3)_{Q_L} \times SU(3)_{u_R} \times SU(3)_{d_R} \times SU(3)_{L_L} \times SU(3)_{e_R}$$

Quarks/Leptonsの湯川結合の存在で破れる:  $\bar{Q}_L Y_U^{ij} u_R^j \bar{H}, \dots$

- Minimal Flavor Violation (MFV) hypothesis :

*D' Ambrosio-Giudice-Isidori-Strumia,  
hep-ph/0207036*

i) 湯川結合 = Flavor対称性の下で非自明な表現

$$\begin{array}{ll} E.g., & Q_L: (3, 1, 1, 1, 1) \\ & u_R: (1, 3, 1, 1, 1) \end{array} \quad Y_U: (3, \bar{3}, 1, 1, 1)$$

ii) 全ての高次演算子はFlavor対称性の下で不変

$$E.g., \quad \bar{Q}_L Y_U Y_U^\dagger Q_L \quad Y_U Y_U^\dagger: (8, 1, 1, 1, 1)$$

- #次元6の演算子:  $2499 - > 111$

## (i) Minimal Flavor Violation (MFV) hypothesis(1/2)

- SMのFlavor対称性 ( $\bar{\psi} i \gamma^\mu D_\mu \psi$ )

$$G_F = SU(3)_{Q_L} \times SU(3)_{u_R} \times SU(3)_{d_R} \times SU(3)_{L_L} \times SU(3)_{e_R}$$

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- #次元6の演算子: 2499  $\rightarrow$  111

UVとの関係は？

# (i) Minimal Flavor Violation (MFV) hypothesis(2/2)

- 弦理論EFTにおける高次演算子の構造？

- 様々な弦理論のEFTsでは,

$$n\text{-点結合} = (\text{湯川結合})^{n-2}$$

$$\text{e.g., 4点} = (\text{湯川結合})^2 \sum_m y_{ijm} y_{mkl}$$

- トーラス :

(i) Type IIA intersecting D6-brane models: Cvetic-Papadimitriou ('03),..

(ii) Type IIB D7-brane models:

Cremades-Ibanez-Marchesano ('04), Abe-Choi-Kobayashi-Ohki ('09),..

- 他の曲がった時空 :

Honda-Kobayashi-Otsuka, 1812.03357

$\phi_m$ : light modes or heavy superpartners

- 高次演算子に伴うFlavor/CP破れは湯川結合で決定
- 湯川結合は幾何学的対称性の下で非自明に変換 (spurions)

$$\tau \rightarrow \gamma\tau$$

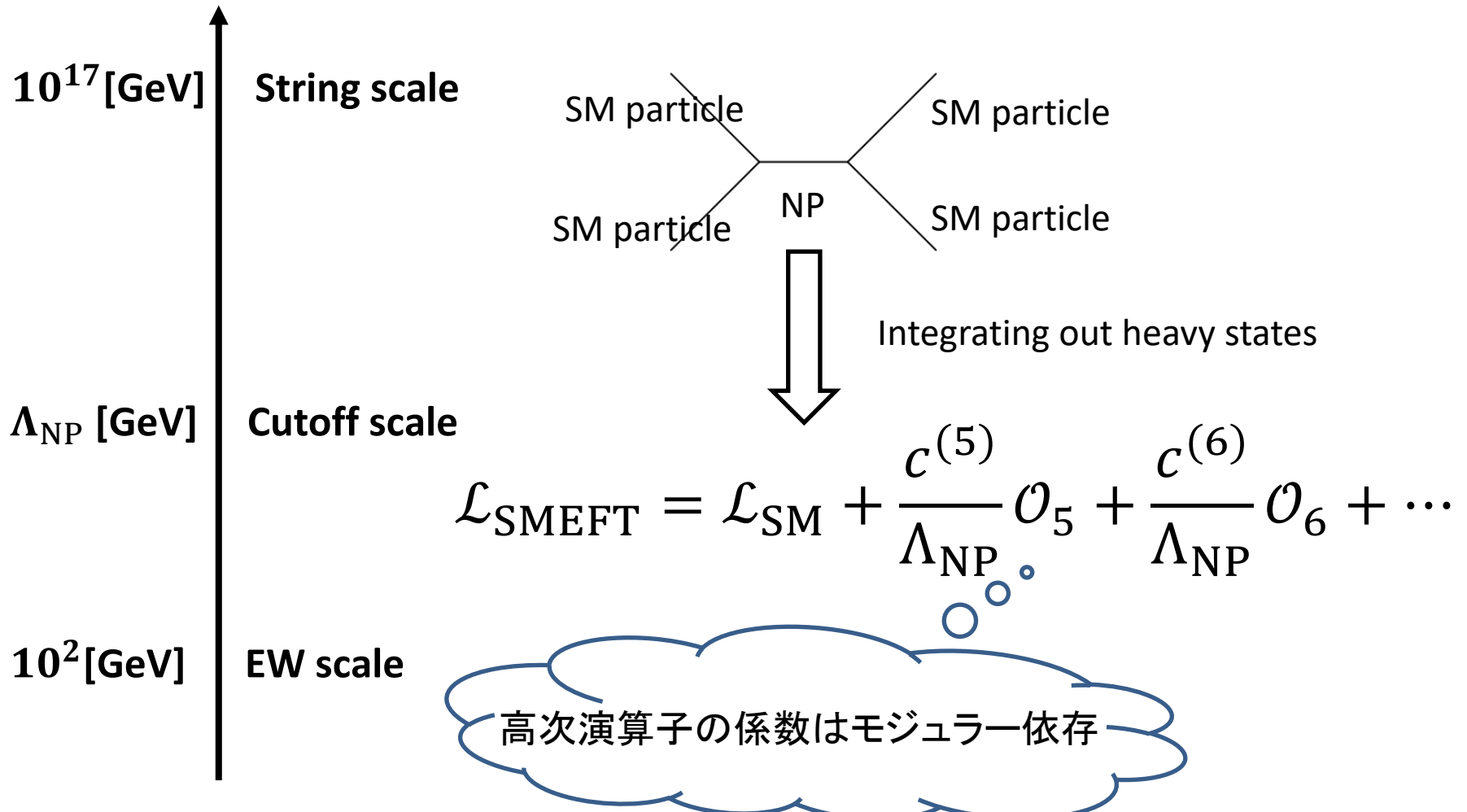
$$Y(\tau) \rightarrow \rho(\gamma)Y(\tau)$$

弦理論のEFTsにおけるMFV hypothesis!

## (ii) SMEFT with モジューラー対称性(1/2)

SMEFT(Standard Model Effective Field Theory)

— 新物理の兆候を探る重要なBottom-up approach



## (ii) SMEFT with モジューラー対称性(2/2)

### Lepton Flavor Violationsへの予言

- U(2) case

$$\text{BR}(\tau \rightarrow \mu\gamma) \gg \text{BR}(\mu \rightarrow e\gamma) \gg \text{BR}(\tau \rightarrow e\gamma)$$

*G. Isidori, J. Pages and F. Wilsch, 2111.13724*

- Modular  $A_4$  ( $\subset SL(2, Z)$ ) case

$$\text{BR}(\tau \rightarrow \mu\gamma) \gg \text{BR}(\mu \rightarrow e\gamma) \sim \text{BR}(\tau \rightarrow e\gamma)$$

*T. Kobayashi, H.O., M. Tanimoto, K. Yamamoto, 2204.12325, 2112.00493*

	$L_L$	$(e_R^c, \mu_R^c, \tau_R^c)$	$H_d, H_u$	$Y_{\mathbf{r}}^{(2)}$	$Y_{\mathbf{r}}^{(4)}$
$SU(2)$	<b>2</b>	<b>1</b>	<b>2</b>	<b>1</b>	
$A_4$	<b>3</b>	<b>(1, 1'', 1')</b>	<b>1</b>	<b>3</b>	<b>{3, 1, 1'}</b>
$k$	2	(0, 0, 0)	0	2	4

### (iii) Baryon/Lepton数を破る演算子(1/2)

Kobayashi-Nishimura-Otsuka-Tanimoto-Yamamoto, 2207.14014

- モジュラー対称性を持つ超対称模型では、  
湯川結合/高次演算子は偶数のモジュラーウェイト( $k_Y \in 2\mathbb{Z}$ )を持つ

$$\sum_n Y_{i_1 \dots i_n}(\tau) \phi_{i_1} \cdots \phi_{i_n}$$

モジュラー変換:  $\tau \rightarrow \gamma\tau = \frac{p\tau + q}{s\tau + t}$

湯川結合/高次演算子:  $Y(\tau) \rightarrow (s\tau + t)^{k_Y} \rho(\gamma) Y(\tau)$

物質場  $\phi_i$ :  
(モジュラーウェイト  $(k_i)$ )  $\phi_i \rightarrow (s\tau + t)^{-k_i} \rho_i(\gamma) \phi_i$

- モジュラー対称性は、  
モジュラーウェイト:  $k_Y = \sum_i k_i$  を要求

# (iii) Baryon/Lepton数を破る演算子(1/2)

Kobayashi-Nishimura-Otsuka-Tanimoto-Yamamoto, 2207.14014

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✓ : MSSM with finite modular symmetries  
で許される演算子

物質場  $\phi_i$  のモジュラーウェイト ( $k_i$ ):

- Higgs = 偶数
- その他 = 奇数

R-parity( $\mathbb{Z}_2$ )の実現

SU(5), SO(10) GUTでも同様

	(i)	(ii)	(iii)	(iv)
Yukawa	✓	✓	✓	✓
$H_u H_d$	✓	✓	✓	✓
$L H_u$	✓	✓		
$LL\bar{E}$	✓	✓		
$LQ\bar{D}$	✓	✓		
$\bar{U}\bar{D}\bar{D}$	✓		✓	
$QQQL$	✓			✓
$\bar{U}\bar{U}\bar{D}\bar{E}$	✓			✓
$QQQH_d$	✓		✓	
$Q\bar{U}\bar{E}H_d$	✓	✓		
$LH_u LH_u$	✓	✓	✓	✓
$LH_u H_d H_u$	✓	✓		
$\bar{U}\bar{D}^*\bar{E}$	✓	✓		
$H_u^* H_d \bar{E}$	✓	✓		
$Q\bar{U}L^*$	✓	✓		
$QQ\bar{D}^*$	✓		✓	

# (iii) Baryon/Lepton数を破る演算子(2/2)

Kobayashi-Nishimura-Otsuka-Tanimoto-Yamamoto, 2207.14014

- モジュラー対称性を持つ超対称模型では、  
湯川結合/高次演算子は偶数のモジュラーウェイト( $k_Y \in 2\mathbb{Z}$ )を持つ

✓ : MSSM with finite modular symmetries  
で許される演算子

物質場  $\phi_i$  のモジュラーウェイト ( $k_i$ ):

- すべての場 = 有理数

(II) Baryon triality ( $\mathbb{Z}_3$ )

(III) Proton hexality ( $\mathbb{Z}_6$ )

の実現

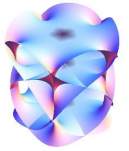
	(II)	(III)
Yukawa	✓	✓
$H_u H_d$	✓	✓
$L H_u$	✓	
$LL\bar{E}$	✓	
$LQ\bar{D}$	✓	
$\bar{U}\bar{D}\bar{D}$		
$QQQL$		
$\bar{U}\bar{U}\bar{D}\bar{E}$		
$QQQH_d$		
$Q\bar{U}\bar{E}H_d$	✓	
$LH_u LH_u$	✓	✓
$LH_u H_d H_u$	✓	
$\bar{U}\bar{D}^*\bar{E}$	✓	
$H_u^* H_d \bar{E}$	✓	
$Q\bar{U}L^*$	✓	
$QQ\bar{D}^*$		



# Conclusion



余剰次元空間のモジュラー対称性  
(フレーバー/CP 対称性)



弦理論のEFTに強い制限

(湯川結合/高次演算子:モジュライ場の関数)

現象論:

- (i) Minimal Flavor Violation
- (ii) Standard Model Effective Field Theory (SMEFT)
- (iii) B/L数を破る高次演算子を制限

展望:

- SMEFT (Quark sectorへの応用)
- 宇宙論 (work in progress)