Flux Compactification and the Hierarchy Problem

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Problems in the Standard Model

- Hierarchy problem
- Unpredictable observables

  (masses of quarks, leptons & Higgs, CP phase, flavor mixing angles, etc)
- Dark matter, Dark energy
- Neutrino oscillation
- Unification
- Gravity
- Charge quantization
- Number of generations
- …

Extension of the SM required

Problems in the Standard Model

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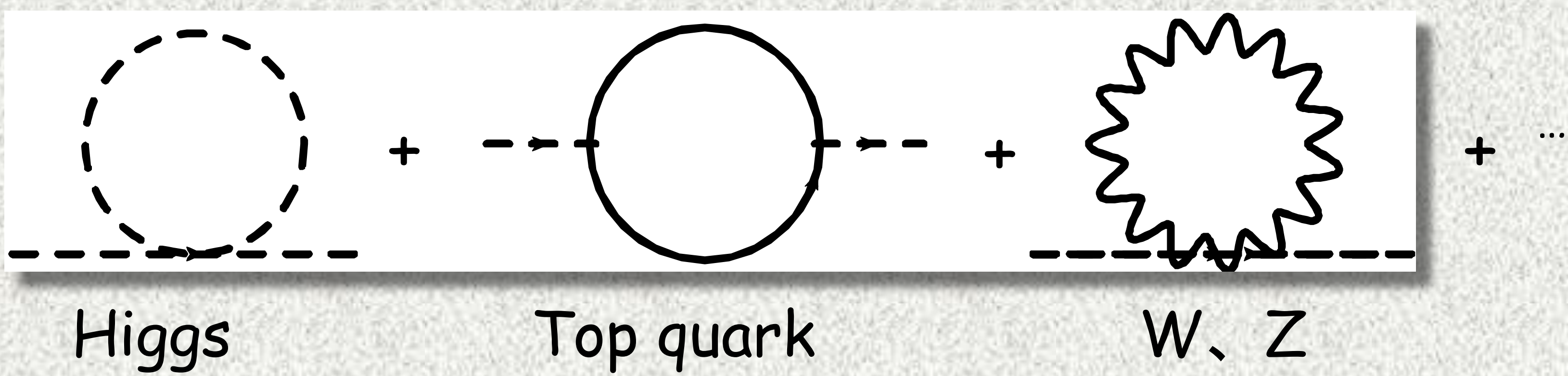
Extension of the SM required

Hierarchy problem

Unnatural fine-tuning of parameters
in the SM Higgs mass

$$m_H^2 = m_0^2 + \delta m^2 \approx (125GeV)^2$$

Physical Higgs mass — classical — Quantum corrections



$$m_H^2 = \mathcal{O}\left(M_P^2\right) - \mathcal{O}\left(M_P^2\right) \approx \left(125GeV\right)^2$$

1.0000000000000000000000000000000001
– 1.000000000000000000000000000000000
(32digits)

Unnatural fine-tuning!!

Can we explain naturally Higgs mass without unnatural fine-tuning??

$$m_H^2 = \mathcal{O}\left(M_P^2\right) - \mathcal{O}\left(M_P^2\right) \approx \left(125GeV\right)^2$$

1.000000000000000000000000000000001
– 1.000000000000000000000000000000000

(32digits)

New physics around 1TeV expected

$$m_H^2 = \mathcal{O}\left(1TeV^2\right) - \mathcal{O}\left(1TeV^2\right) \approx \left(125GeV\right)^2$$

However, no signature of new physics so far …

Good chance to reconsider??

SM might be correct at higher scale

- GUT ⇒ up to 10^{16}GeV
- or up to Planck scale 10^{18}GeV

m_{Higgs}^2 =0@10^{16} , 10^{18} GeV
including quantum corrections
Possible to generate m_{Higgs}^2 =125GeV
by some mechanism??

example

Magnetic Flux Compactification

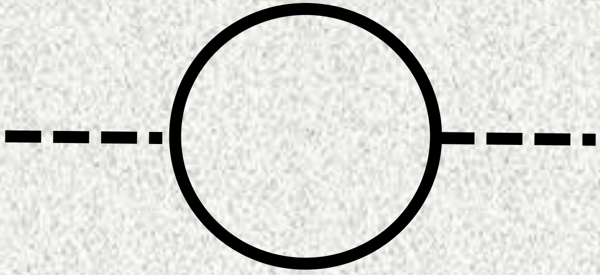
Motivations of Flux Compactification

- Chiral fermions
- 3 generations?
- Yukawa hierarchy
- SUSY breaking
- String phenomenology
- …

6D QED on T^2 with magnetic flux

Buchmuller, Dierigl and Dudas, JHEP04 (2018) 151 [hep-th:1804.07497]

Cancellation of 1-loop corrections to mass of 0 mode scalar $A_{5,6}$ (WL scalar)



= 0 @1-loop

0 mode scalar $A_{5,6}$
= NG boson of $x_{5,6}$ translation

We would like to identify scalar 0 modes $A_{5,6}$ with SM Higgs

Extension of Buchmuller et al's work to Non-abelian gauge theory is necessary

As a first step, extended to SU(2) YM

Hirose & Maru, JHEP1908 (2019) 054 [arXiv:1904.06028]

1. Introduction

2. Cancellation of WL scalar mass @1-loop in SU(2) YM

Hirose & Maru, JHEP08 (2019) 054

3. Nonvanishing finite scalar masses generation

4. Gauge symmetry breaking

5. Summary

6D SU(2) Yang-Mills compactified on T^2 with flux

$$\mathcal{L}_6 = -\frac{1}{4} F^a_{MN} F^{aMN} - \frac{1}{2\xi}\left(D_\mu A^{a\mu} + \xi \mathcal{D}_m A^{am}\right)^2 - \bar{c}^a \left(D_\mu D^\mu + \xi D_m \mathcal{D}^m\right) c^a$$

$$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M - ig\left[A_M, A_N\right]^a$$

$$D_M A^a_N = \partial_M A^a_N - ig\left[A_M, A_N\right]^a, \mathcal{D}_m A^{am} = \partial_m A^{am} - ig\left[\langle A_m \rangle, A^m\right]^a$$

M=0,1,2,3,5,6, m=5,6, a=1,2,3

flux

$$D^m \langle F_{mn} \rangle = 0 \Rightarrow \langle A^1_5 \rangle = -\frac{1}{2} f x_6, \langle A^1_6 \rangle = \frac{1}{2} f x_5, \langle A^{2,3}_5 \rangle = \langle A^{2,3}_6 \rangle = 0$$

$$\Rightarrow \langle F^1_{56} \rangle = f, \quad \frac{g}{2\pi} \int_{T^2} d^2 x f = \frac{g}{2\pi} f L_5 L_6 = N \in \mathbb{Z}$$

(4+2)-dim decomposition

$$
\begin{aligned}
\mathcal{L}_{total} = & -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{2\xi}D_\mu A^{a\mu}D_\nu A^{a\nu} - \partial_\mu\bar{\phi}^a\,\partial^\mu\phi^a \\
& -\frac{1}{2}\partial A^a_\mu\,\bar{\partial}A^{a\mu} + g^2\left[A_\mu,\phi\right]^a\left[A^\mu,\phi\right]^a - \frac{g}{\sqrt{2}}\left(-\partial A^a_\mu\left[A^\mu,\bar{\phi}\right]^a + \bar{\partial}A^a_\mu\left[A_\mu,\phi\right]^a\right) \\
& + ig\left(\partial_\mu\phi^a\left[A^\mu,\bar{\phi}\right]^a + \partial^\mu\bar{\phi}^a\left[A_\mu,\phi\right]^a\right) \\
& -\frac{1}{4}\left(D\bar{\phi}^a + \bar{D}\phi^a + \sqrt{2}g\left[\phi,\bar{\phi}\right]^a\right)^2 + \frac{\xi}{4}\left(\mathcal{D}\bar{\phi}^a - \bar{\mathcal{D}}\phi^a\right)^2 \\
& -\bar{c}^a\left(D_\mu D^\mu + \xi D_m\mathcal{D}^m\right)c^a
\end{aligned}
$$

$$
\partial \equiv \partial_z = \partial_5 - i\,\partial_6,\quad z \equiv \frac{1}{2}\left(x_5 + ix_6\right),\quad \phi = \frac{1}{\sqrt{2}}\left(A_6 + iA_5\right)
$$

Gauge boson mass terms

$$\mathcal{L}_{A^2} = -\frac{1}{2}\partial A_\mu^a \bar{\partial} A^{a\mu} + g^2 \left[A_\mu, \langle\phi\rangle\right]^a \left[A^\mu, \langle\bar{\phi}\rangle\right]^a$$

$$-\frac{g}{\sqrt{2}}\left(-\partial A_\mu^a \left[A^\mu, \langle\bar{\phi}\rangle\right]^a + \bar{\partial} A_\mu^a \left[A^\mu, \langle\phi\rangle\right]^a\right)$$

$$= -\frac{1}{2} A_\mu^a \left(-\mathcal{D}\bar{\mathcal{D}}\right) A^{a\mu}$$

$$\mathcal{D} = \begin{pmatrix} \partial & 0 & 0 \\ 0 & \partial & igf\bar{z} \\ 0 & -igf\bar{z} & \partial \end{pmatrix}$$

$$\bar{\mathcal{D}} = \begin{pmatrix} \bar{\partial} & 0 & 0 \\ 0 & \bar{\partial} & -igfz \\ 0 & igfz & \bar{\partial} \end{pmatrix}$$

$$\left[i\bar{\mathcal{D}}, i\mathcal{D}\right] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -2igf \\ 0 & 2igf & 0 \end{pmatrix} = 2igf\,\varepsilon^{a1c}$$

Gauge boson mass terms

$$\mathcal{L}_{A^2} = -\frac{1}{2}\partial A_\mu^a \bar{\partial} A^{a\mu} + g^2 \left[A_\mu, \langle\phi\rangle\right]^a \left[A^\mu, \langle\bar{\phi}\rangle\right]^a$$

$$-\frac{g}{\sqrt{2}}\left(-\partial A_\mu^a \left[A^\mu, \langle\bar{\phi}\rangle\right]^a + \bar{\partial} A_\mu^a \left[A^\mu, \langle\phi\rangle\right]^a\right)$$

$$= -\frac{1}{2} A_\mu^a \left(-\mathcal{D}\bar{\mathcal{D}}\right) A^{a\mu}$$

$$\mathcal{D}_{diag} = \begin{pmatrix} \partial & 0 & 0 \\ 0 & \partial - gf\bar{z} & 0 \\ 0 & 0 & \partial + gf\bar{z} \end{pmatrix}$$

$$\bar{\mathcal{D}}_{diag} = \begin{pmatrix} \bar{\partial} & 0 & 0 \\ 0 & \bar{\partial} + gfz & 0 \\ 0 & 0 & \bar{\partial} - gfz \end{pmatrix}$$

$$a = \frac{1}{\sqrt{2gf}} i\bar{\mathcal{D}}, \quad a^\dagger = \frac{1}{\sqrt{2gf}} i\mathcal{D}$$

$$\left[a, a^\dagger\right] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

creation, annihilation operators

Gauge boson mass terms

$$\mathcal{L}_{A^2} = -\frac{1}{2}\partial A_\mu^a \bar{\partial} A^{a\mu} + g^2 \left[A_\mu, \langle\phi\rangle\right]^a \left[A^\mu, \langle\bar{\phi}\rangle\right]^a$$

$$- \frac{g}{\sqrt{2}}\left(-\partial A_\mu^a \left[A^\mu, \langle\bar{\phi}\rangle\right]^a + \bar{\partial} A_\mu^a \left[A^\mu, \langle\phi\rangle\right]^a\right)$$

$$= -\frac{1}{2} A_\mu^a \left(-\mathcal{D}\bar{\mathcal{D}}\right) A^{a\mu}$$

$$\begin{cases} a_1 = \frac{1}{\sqrt{2gf}} i\bar{\partial} \\ a_2 = \frac{1}{\sqrt{2gf}} i\left(\bar{\partial} + gfz\right), \\ a_3 = \frac{1}{\sqrt{2gf}} i\left(\bar{\partial} - gfz\right) \end{cases} \quad \begin{cases} a_1^\dagger = \frac{1}{\sqrt{2gf}} i\partial \\ a_2^\dagger = \frac{1}{\sqrt{2gf}} i\left(\partial - gf\bar{z}\right) \\ a_3^\dagger = \frac{1}{\sqrt{2gf}} i\left(\partial + gf\bar{z}\right) \end{cases}$$

$$\left[a_1, a_1^\dagger\right] = 0$$

$$\left[a_2, a_2^\dagger\right] = 1$$

$$\left[a_3, a_3^\dagger\right] = -1$$

Gauge boson mass terms

$$\mathcal{L}_{A^2} = -\frac{1}{2}\partial A_\mu^a \bar{\partial} A^{a\mu} + g^2 \left[A_\mu, \langle\phi\rangle\right]^a \left[A^\mu, \langle\bar{\phi}\rangle\right]^a$$

$$- \frac{g}{\sqrt{2}}\left(-\partial A_\mu^a \left[A^\mu, \langle\bar{\phi}\rangle\right]^a + \bar{\partial} A_\mu^a \left[A^\mu, \langle\phi\rangle\right]^a\right)$$

$$= -\frac{1}{2} A_\mu^a \left(-\mathcal{D}\bar{\mathcal{D}}\right) A^{a\mu}$$

Diagonalized mass matrix

$$\left(m_A^2\right)_{diag} = -\mathcal{D}_{diag}\bar{\mathcal{D}}_{diag} = \begin{pmatrix} \left(l^2+m^2\right)/R^2 & 0 & 0 \\ 0 & 2g_6 f n_2 & 0 \\ 0 & 0 & 2g_6 f\left(n_3+1\right) \end{pmatrix}$$

l,m: integer, $n_{2,3}$: Landau level

Scalar mass & ghost mass

$$\mathcal{L}_{\varphi\varphi} = -\frac{1}{4}\left(\mathcal{D}\bar{\varphi}^a\mathcal{D}\bar{\varphi}^a + \mathcal{D}\bar{\varphi}^a\bar{\mathcal{D}}\varphi^a + \bar{\mathcal{D}}\varphi^a\mathcal{D}\bar{\varphi}^a + \bar{\mathcal{D}}\varphi^a\bar{\mathcal{D}}\varphi^a - 4gf\left[\varphi,\bar{\varphi}\right]^1\right)$$
$$-\frac{\xi}{4}\left(\mathcal{D}\bar{\varphi}^a\mathcal{D}\bar{\varphi}^a - \mathcal{D}\bar{\varphi}^a\bar{\mathcal{D}}\varphi^a - \bar{\mathcal{D}}\varphi^a\mathcal{D}\bar{\varphi}^a + \bar{\mathcal{D}}\varphi^a\bar{\mathcal{D}}\varphi^a\right)$$

$$\mathcal{L}_{cc} = -\bar{c}^a\xi\mathcal{D}_m\mathcal{D}^m c^a \qquad \phi^a = \langle\phi^a\rangle + \varphi^a$$

$$\left(m_\varphi^2\right)_{diag} = \begin{pmatrix} (1+\xi)\left(l^2+m^2\right)/2R^2 & 0 & 0 \\ 0 & g_6 f\left((1+\xi)n_2+1\right) & 0 \\ 0 & 0 & g_6 f\left((1+\xi)n_3+\xi\right) \end{pmatrix}$$

$$\left(m_c^2\right)_{diag} = \xi\begin{pmatrix} \left(l^2+m^2\right)/R^2 & 0 & 0 \\ 0 & g_6 f\left(2n_2+1\right) & 0 \\ 0 & 0 & g_6 f\left(2n_3+1\right) \end{pmatrix}$$

Scalar mass & ghost mass

$$\left(m_\varphi^2\right)_{diag} = \begin{pmatrix} (1+\xi)\left(l^2+m^2\right)/2R^2 & 0 & 0 \\ 0 & g_6 f\left((1+\xi)n_2+1\right) & 0 \\ 0 & 0 & g_6 f\left((1+\xi)n_3+\xi\right) \end{pmatrix}$$

$$\left(m_c^2\right)_{diag} = \xi \begin{pmatrix} \left(l^2+m^2\right)/R^2 & 0 & 0 \\ 0 & g_6 f\left(2n_2+1\right) & 0 \\ 0 & 0 & g_6 f\left(2n_3+1\right) \end{pmatrix}$$

In Feynman gauge $\xi=1$,

both mass spectrum coincide

⇒ Calculation of 1-loop effective potential

is greatly simplified

4D effective theory

$$\mathcal{L}_{total} = -\frac{1}{4}\tilde{F}^a_{\mu\nu}\tilde{F}^{a\mu\nu} - \partial_\mu \bar{\tilde{\varphi}}^a \partial^\mu \tilde{\varphi}^a - \bar{\tilde{c}}^a \mathcal{D}_\mu \mathcal{D}^\mu \tilde{c}^a \qquad \phi^a = \langle \phi^a \rangle + \varphi^a$$

$$-\frac{1}{2}\tilde{A}^a_\mu m_A^2 \tilde{A}^{a\mu} - \bar{\tilde{\varphi}}^a m_\varphi^2 \tilde{\varphi}^a - \bar{\tilde{c}}^a m_c^2 \tilde{c}^a$$

Interaction terms

$$+ig\left(\partial_\mu \varphi^a \left[A^\mu, \varphi\right]^a + \partial^\mu \bar{\varphi}^a \left[A_\mu, \varphi\right]^a\right) + g^2 \left[A_\mu, \varphi\right]^a \left[A^\mu, \varphi\right]^a$$

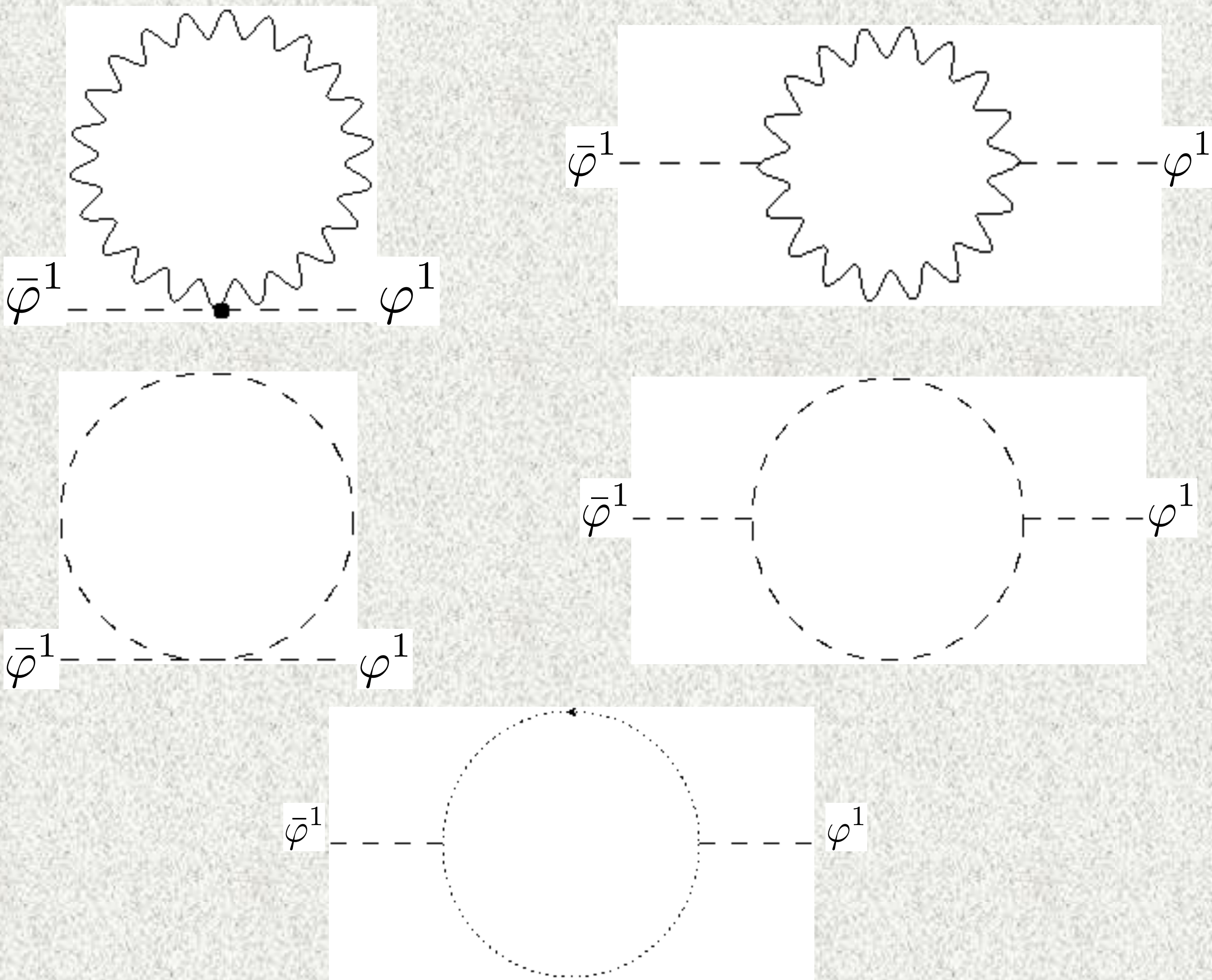
$$+\frac{g}{\sqrt{2}}\left(\mathcal{D}\bar{\varphi}^a - \bar{\mathcal{D}}\varphi^a\right)[\varphi, \bar{\varphi}]^a - \frac{1}{2}g^2[\varphi, \bar{\varphi}]^a[\varphi, \bar{\varphi}]^a$$

$$-\frac{g\xi}{\sqrt{2}}\left([\varphi, \bar{c}]^a \bar{\partial} c^a - [\bar{\varphi}, \bar{c}]^a \partial c^a\right)$$

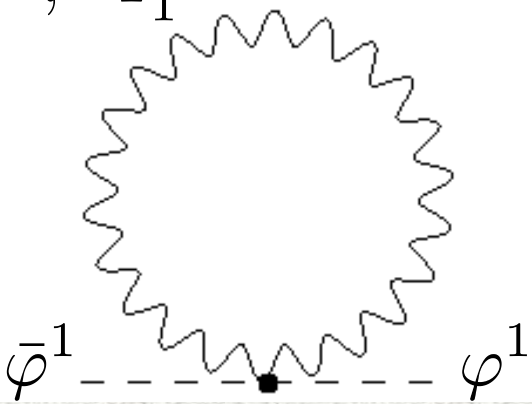
$$\tilde{A}^a_\mu = U A^a_\mu, \quad \tilde{A}^{a\mu} = U^{-1} A^{a\mu},$$

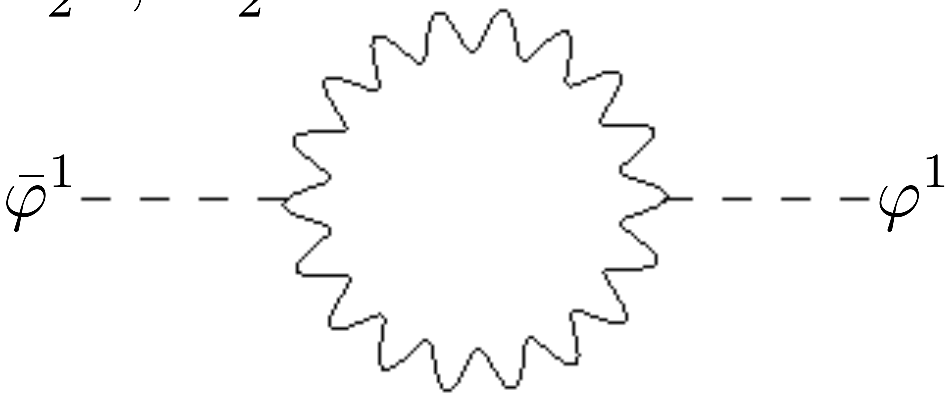
$$\tilde{\varphi} = U^{-1}\varphi, \quad \bar{\tilde{\varphi}} = U\bar{\varphi}, \quad \bar{\tilde{c}}^a = U\bar{c}^a, \quad \tilde{c}^a = U^{-1}c^a \qquad U = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & i \\ 0 & i & 1 \end{pmatrix}$$

Diagrams to be calculated



Gauge boson loop





$$I_1^{(2)} + I_2^{(2)} = -6ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\alpha n} - \frac{\alpha(n+1)}{(p^2+\alpha n)(p^2+\alpha(n+1))}\right)$$

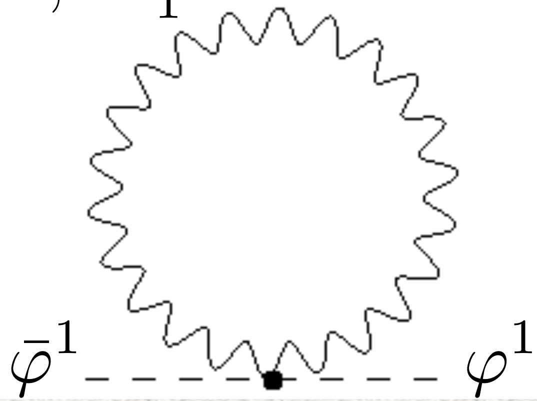
$$-2ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{\xi}{p^2+\alpha n} - \frac{\alpha(n+1)\xi^2}{(p^2+\alpha n\xi)(p^2+\alpha(n+1)\xi)}\right)$$

$$I_1^{(3)} + I_2^{(3)} = -6ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\alpha(n+1)} - \frac{\alpha(n+1)}{(p^2+\alpha(n+1))(p^2+\alpha(n+2))}\right)$$

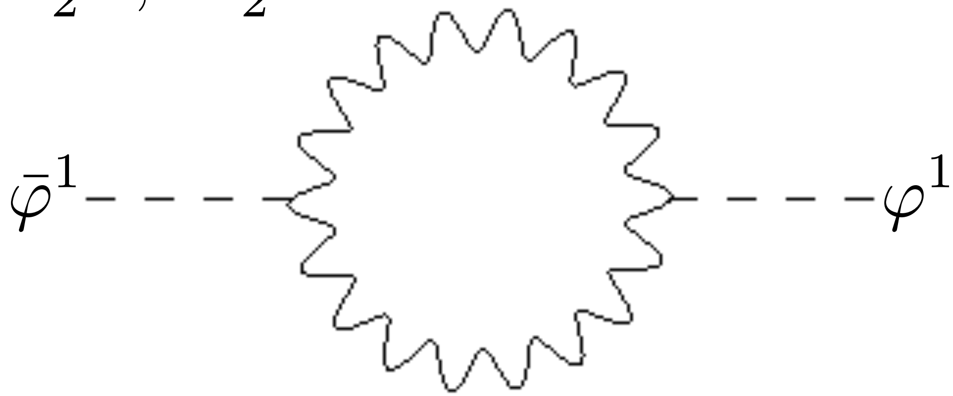
$$-2ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{\xi}{p^2+\alpha(n+1)} - \frac{\alpha(n+1)\xi^2}{(p^2+\alpha(n+1)\xi)(p^2+\alpha(n+2)\xi)}\right)$$

Gauge boson loop

$I_1^{(2)}, \quad I_1^{(3)}$



$I_2^{(2)}, \quad I_2^{(3)}$



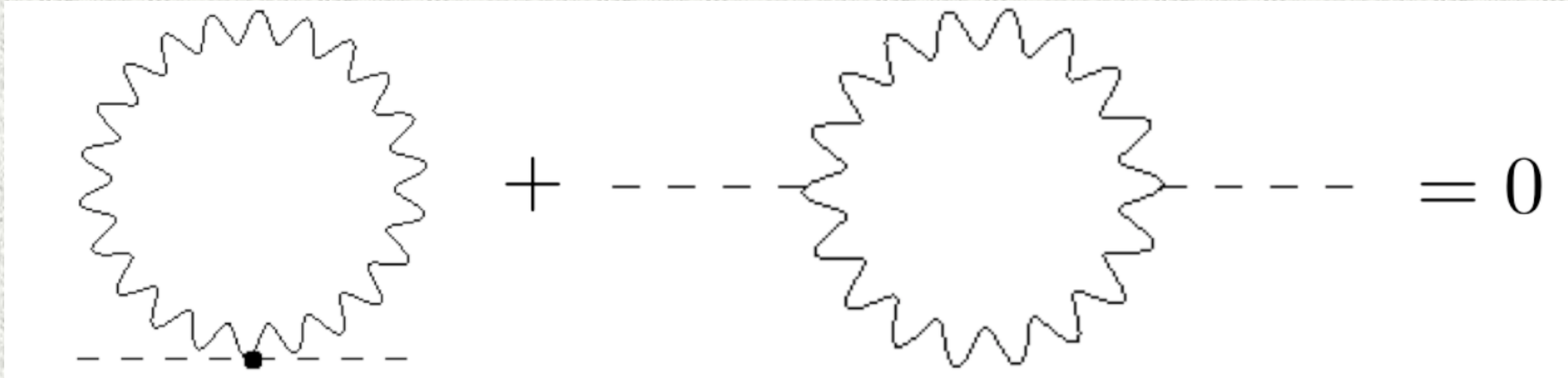
$$I_1^{(2)} + I_2^{(2)} = -6ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\alpha n} - \frac{\alpha(n+1)}{(p^2+\alpha n)(p^2+\alpha(n+1))}\right)$$

$$-2ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{\xi}{p^2+\alpha n\xi} - \frac{\alpha(n+1)\xi^2}{(p^2+\alpha n\xi)(p^2+\alpha(n+1)\xi)}\right)$$

$$\sum_{n=0}^{\infty}\left(\frac{1}{p^2+\alpha n} - \frac{\alpha(n+1)}{(p^2+\alpha n)(p^2+\alpha(n+1))}\right) = \sum_{n=0}^{\infty}\left(-\frac{n}{p^2+\alpha n} + \frac{n+1}{p^2+\alpha(n+1)}\right) = 0$$

2nd line is also 0 after $p^2 \to p^2\xi$ $I_1^{(3)}+I_2^{(3)}$ is applied, too

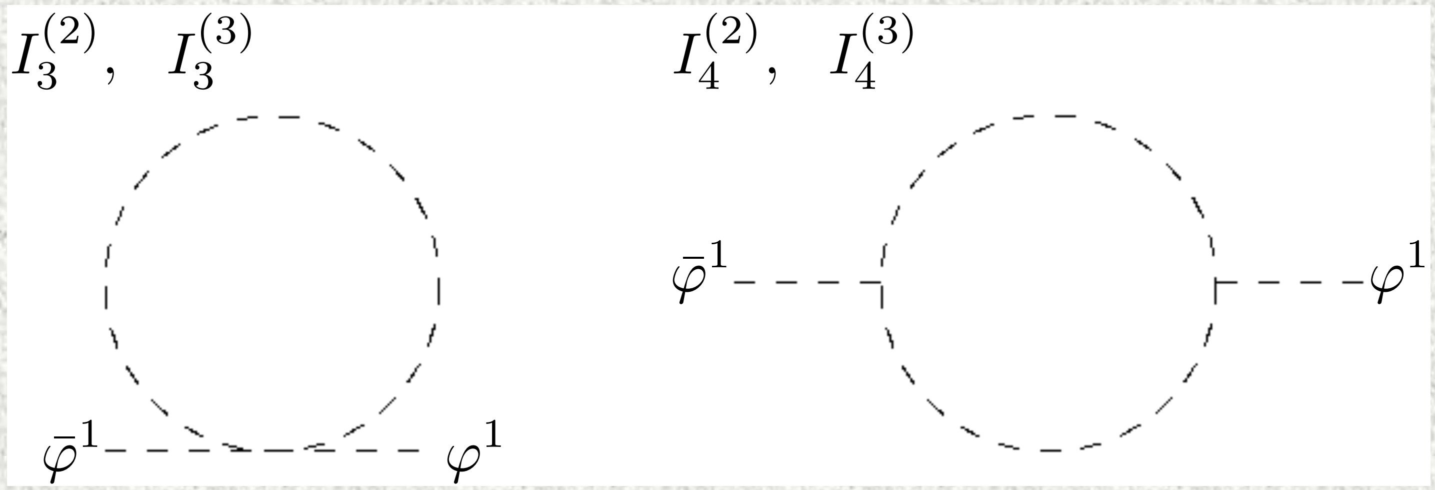
Gauge boson loop



$$I_1^{(2)} + I_2^{(2)} = 0, \quad I_1^{(3)} + I_2^{(3)} = 0$$

Cancelled in arbitrary gauge ξ !!

Scalar loop

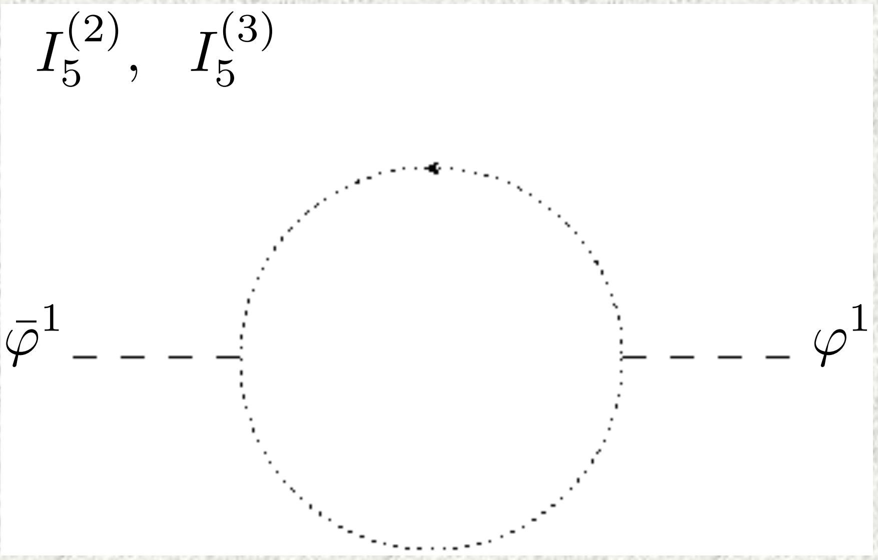


$$I_3^{(2)} = -ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+\frac{\alpha}{2}\left((1+\xi)n+1\right)}, \quad I_3^{(3)} = -ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+\frac{\alpha}{2}\left((1+\xi)n+\xi\right)}$$

$$I_4^{(2)} = \frac{ig^2|N|}{2}\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)}{\left(p^2+\frac{\alpha}{2}\left((1+\xi)n+1\right)\right)\left(p^2+\frac{\alpha}{2}\left((1+\xi)(n+1)+1\right)\right)}$$

$$I_4^{(3)} = \frac{ig^2|N|}{2}\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)}{\left(p^2+\frac{\alpha}{2}\left((1+\xi)n+\xi\right)\right)\left(p^2+\frac{\alpha}{2}\left((1+\xi)(n+1)+\xi\right)\right)}$$

Ghost loop



$$I_5^{(2)} = I_5^{(3)} = \frac{ig^2|N|\xi^2}{2}\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)}{\left(p^2+\alpha(n+1/2)\right)\left(p^2+\alpha(n+3/2)\right)}$$

Cancellation between scalar loop & ghost loop

Landau gauge: ξ=0 ⇒ ghost loop trivially zero

Scalar loop

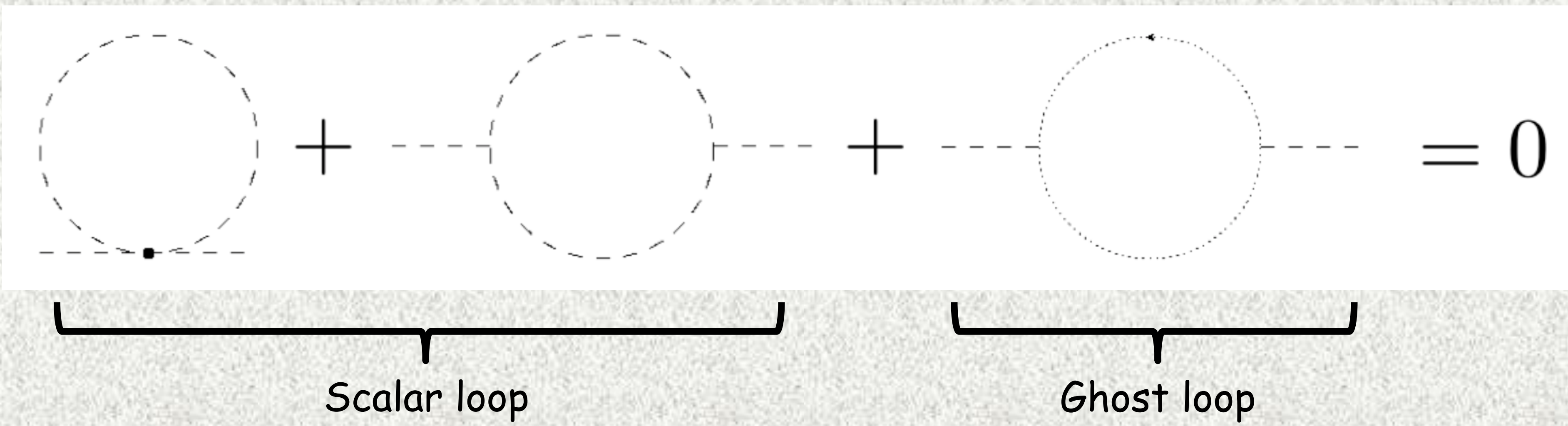
$$I_3^{(2)} + I_4^{(2)} = -ig^2|N|\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\frac{\alpha}{2}(n+1)} - \frac{\frac{\alpha}{2}(n+1)}{\left(p^2+\frac{\alpha}{2}(n+1)\right)\left(p^2+\frac{\alpha}{2}(n+2)\right)} \right)$$

$$= -ig^2|N|\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\left(-\frac{n}{p^2+\frac{\alpha}{2}(n+1)} + \frac{n+1}{p^2+\frac{\alpha}{2}(n+2)} \right) = 0$$

$$I_3^{(3)} + I_4^{(3)} = -ig^2|N|\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\frac{\alpha}{2}n} - \frac{\frac{\alpha}{2}(n+1)}{\left(p^2+\frac{\alpha}{2}n\right)\left(p^2+\frac{\alpha}{2}(n+1)\right)} \right)$$

$$= -ig^2|N|\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\left(-\frac{n}{p^2+\frac{\alpha}{2}n} + \frac{n+1}{p^2+\frac{\alpha}{2}(n+1)} \right) = 0$$

Feynman gauge: $\xi=1$



$$I_3^{(2)}+I_4^{(2)}+I_5^{(2)}=I_3^{(3)}+I_4^{(3)}+I_5^{(3)}$$

$$=-ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\alpha\left(n+\frac{1}{2}\right)}-\frac{\frac{\alpha}{2}(n+1)}{\left(p^2+\alpha\left(n+\frac{1}{2}\right)\right)\left(p^2+\alpha\left(n+\frac{3}{2}\right)\right)}-\frac{\frac{\alpha}{2}(n+1)}{\left(p^2+\alpha\left(n+\frac{1}{2}\right)\right)\left(p^2+\alpha\left(n+\frac{3}{2}\right)\right)}\right)$$

$$=-ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\alpha\left(n+\frac{1}{2}\right)}-\frac{\alpha(n+1)}{\left(p^2+\alpha\left(n+\frac{1}{2}\right)\right)\left(p^2+\alpha\left(n+\frac{3}{2}\right)\right)}\right)$$

$$=-ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(-\frac{n}{p^2+\alpha\left(n+\frac{1}{2}\right)}+\frac{n+1}{p^2+\alpha\left(n+\frac{3}{2}\right)}\right)=0$$

Physical reason of cancellation

Translations in compactified space

$$\delta_T A_5^a = \left(\varepsilon_5 \partial_5 + \varepsilon_6 \partial_6\right) \tilde{A}_5^a - \frac{f}{2} \varepsilon_6 \delta^{a1}$$

$$\delta_T A_6^a = \left(\varepsilon_5 \partial_5 + \varepsilon_6 \partial_6\right) \tilde{A}_6^a + \frac{f}{2} \varepsilon_5 \delta^{a1}$$

$$\delta_T \phi^a = \left(\varepsilon \partial + \bar{\varepsilon} \bar{\partial}\right) \varphi^a + \frac{f}{\sqrt{2}} \bar{\varepsilon} \delta^{a1}$$

a=1

0 mode

$$\delta_T \phi^1 = \frac{f}{\sqrt{2}} \bar{\varepsilon}$$

constant shift

$$\partial \equiv \partial_5 - i \partial_6, \quad \phi^a = \frac{1}{\sqrt{2}}\left(A_6^a + i A_5^a\right) = \frac{f}{\sqrt{2}} \bar{z} \delta^{a1} + \varphi^a$$

$$\delta_T \phi^1 = \frac{f}{\sqrt{2}} \bar{\varepsilon}$$

constant shift

⇒ ϕ^1 NG boson of spontaneously broken translational symmetry in 5,6 directions

⇒ Only derivative interactions are allowed
(Not only mass terms, but also potential forbidden)

Analogy: π meson

NG boson of spontaneously broken chiral symmetry ⇒ chiral Lagrangian

1. Introduction
2. Cancellation of WL scalar mass @1-loop in SU(2) YM
3. Nonvanishing finite scalar masses generation

Hirose & Maru, JHEP06 (2021) 159

4. Gauge symmetry breaking
5. Summary

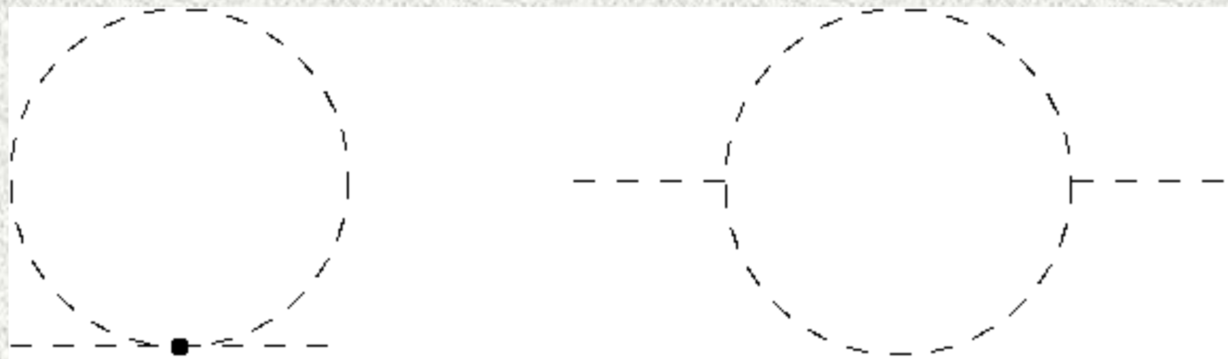
Pion has a mass since it is a pseudo NG boson of explicitly broken chiral symmetry by quark mass terms

To obtain WL scalar mass, it must be a pseudo NG boson of explicitly broken translational symmetry in compactified space, too

↓

What are the explicit breaking terms, corresponding to quark mass terms in pion case ??

Divergence structure of the loop integral & mode sum



$$I(x;a,b) = \sum_{n=0}^{\infty} \int \frac{d^4k}{(2\pi)^4} \frac{\left(k^2\right)^a}{\left(k^2+\alpha(n+x)\right)^b}$$

$$= \frac{1}{\alpha^{b-a}} \left(\frac{4\pi}{\alpha}\right)^{\varepsilon-2} \frac{\Gamma(a+2-\varepsilon)\Gamma(\varepsilon+b-a-2)}{\Gamma(b)\Gamma(2-\varepsilon)} \zeta[\varepsilon+b-a-2,x]$$

x: parameter specifying KK mass spectrum
(x=0(KK gauge), ½(KK scalar), 1(KK fermion))
2a: number of derivatives
b: number of propagators (only b=1,2 cases considered)

$$J(x;a,b)=\frac{\Gamma(a+2-\varepsilon)\Gamma(\varepsilon+b-a-2)}{\Gamma(b)\Gamma(2-\varepsilon)}\zeta[\varepsilon+b-a-2,x]$$

$$=\begin{cases}(-1)^a\Gamma(\varepsilon-1)\zeta[\varepsilon-a-1,x]\ (b=1)\\ (-1)^a(\varepsilon-a-1)\Gamma(\varepsilon-1)\zeta[\varepsilon-a,x]\ (b=2)\end{cases}$$

$$\Gamma(\varepsilon-1)=\frac{\Gamma(\varepsilon)}{\varepsilon-1}=-\left(\frac{1}{\varepsilon}-\gamma_E+1+\mathcal{O}(\varepsilon)\right)$$

$$\zeta[\varepsilon-p,x]=\zeta[-p,x]+\frac{\partial\zeta[-p,x]}{\partial(-p)}\varepsilon+\mathcal{O}(\varepsilon^2)$$

$$\zeta[-2n,1]=\zeta[-2n,1/2]=\zeta[-2n,0]=0$$

$$\Gamma(\varepsilon-1)\zeta(\varepsilon-p,x)=finite(p=even)$$

J(x;a,1) for odd a, J(x;a,2) for even a are finite

1-loop finite corrections by 4-point interaction

Scalar loop

$$J\left(1/2;a,1\right)\to\bar{\varphi}\varphi\,\partial_{\mu_1}\cdots\partial_{\mu_a}\bar{\Phi}\,\partial^{\mu_1}\cdots\partial^{\mu_a}\Phi$$

Fermion loop

$$J\left(1;a,1\right)\to\bar{\varphi}\varphi\bar{\psi}\left(\gamma^{\mu}\partial_{\mu}\right)^{2a-1}\psi$$

SU(2) gauge loop

$$J\left(1/2;a,1\right)\to\bar{\varphi}\varphi\,\partial_{\mu_1}\cdots\partial_{\mu_a}A^{a}_{\nu}\,\partial^{\mu_1}\cdots\partial^{\mu_a}A^{a\nu}$$

1-loop finite corrections by 3-point interactions

Scalar loop

$$J\left(1/2;0,2\right)\to\bar{\varphi}\bar{\Phi}\Phi+\varphi\bar{\Phi}\Phi$$

$$J\left(1/2;a,2\right)\to\bar{\varphi}\partial_{\mu_1}\cdots\partial_{\mu_{a/2}}\bar{\Phi}\partial^{\mu_1}\cdots\partial^{\mu_{a/2}}\Phi+\varphi\partial_{\mu_1}\cdots\partial_{\mu_{a/2}}\bar{\Phi}\partial^{\mu_1}\cdots\partial^{\mu_{a/2}}\Phi$$

Fermion loop

$$J\left(1;a,2\right)\to\bar{\varphi}\bar{\psi}\left(\gamma^{\mu}\partial_{\mu}\right)^{a-1}\psi+\varphi\bar{\psi}\left(\gamma^{\mu}\partial_{\mu}\right)^{a-1}\psi$$

SU(2) gauge loop

$$J\left(1/2;a,2\right)\to\bar{\varphi}\partial_{\mu_1}\cdots\partial_{\mu_{a/2}}A^{a}_{\nu}\partial^{\mu_1}\cdots\partial^{\mu_{a/2}}A^{a\nu}+\varphi\partial_{\mu_1}\cdots\partial_{\mu_{a/2}}A^{a}_{\nu}\partial^{\mu_1}\cdots\partial^{\mu_{a/2}}A^{a\nu}$$

Illustration of finite WL scalar mass

6D scalar QED on T^2 with flux

$$\mathcal{L} = -\frac{1}{4}F_{MN}F^{MN} - D_M\bar{\Phi}D^M\Phi + \kappa\left(\phi\bar{\Phi}\Phi + \bar{\phi}\bar{\Phi}\Phi\right)$$

$$\phi = \langle\phi\rangle + \varphi$$

New contribution from κ interactions

$$\delta m^2 = \frac{|N|\ln 2}{32\pi^2}\frac{\kappa^2}{L^2}$$

Finite as expected!!

- $\kappa = 0 \Rightarrow \delta m^2 = 0$
- same result is derived from V_{eff}@1-loop

If we regard $\delta m^2 = \frac{|N| \ln 2}{32\pi^2} \frac{\kappa^2}{L^2}$ as M_{Higgs}^2

$$\Rightarrow \quad \frac{\kappa}{L} \approx TeV$$

Even if 1/L is Planck scale,

$\kappa\left(\phi\bar{\Phi}\Phi + \bar{\phi}\bar{\Phi}\Phi\right)$ is generated as κ～TeV/Planck

by some dynamics, M_{Higgs}^2 is obtained

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Akamatsu, Hirose & Maru, PRD106(2022) 3 035035
5. Summary

6D SU(N) Yang-Mills compactified on T^2

$$\mathcal{L} = -\frac{1}{4} F^a_{MN} F^{aMN} + \mathcal{L}_{g-f} + \mathcal{L}_{ghost}$$

$$= -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2} D A^a_\mu \bar{D} A^{a\mu} - \partial_\mu \bar{\phi}^a \partial^\mu \phi^a - \frac{g}{\sqrt{2}} \left(\partial \bar{\phi}^a - \bar{\partial} \phi^a \right) \left[A_\mu, A^\mu \right]^a$$

$$+ ig \left(\partial_\mu \phi^a \left[A^\mu, \bar{\phi} \right]^a + \partial_\mu \bar{\phi}^a \left[A^\mu, \bar{\phi} \right]^a \right) - \frac{1}{4} \left(D \bar{\phi}^a + \bar{D} \phi^a + \sqrt{2} g \left[\phi, \bar{\phi} \right]^a \right)^2$$

$$- \frac{1}{2\xi} D_\mu A^{a\mu} D_\nu A^{a\nu} + \frac{\xi}{4} \left(\mathcal{D} \bar{\phi}^a - \bar{\mathcal{D}} \phi^a \right)^2 - \bar{c}^a \left(D_\mu D^\mu + \xi D_m D^m \right) c^a$$

Hereafter, Feynman gauge $\xi=1$ is considered

T^2 case ⇒ $V(\varphi)=0$

Explicit breaking
⇒ nonzero $V(\varphi)$??

Following simple cases are considered

SU(2) YM

$$\left\langle A^3 \right\rangle \equiv \frac{1}{\sqrt{2}} f\bar{z}, \left\langle A_6^1 \right\rangle \equiv v$$

$$SU(2) \xrightarrow[f]{} U(1) \xrightarrow[v]{} ?$$

SU(3) YM

(1)

$$\left\langle \phi^8 \right\rangle \equiv \frac{1}{\sqrt{2}} f\bar{z}, \left\langle \phi^1 \right\rangle \equiv \frac{1}{\sqrt{2}} w$$

$$SU(3) \xrightarrow[f]{} SU(2) \times U(1) \xrightarrow[w]{} ?$$

(2)

$$\left\langle \phi^8 \right\rangle \equiv \frac{1}{\sqrt{2}} f\bar{z}, \left\langle \phi^6 \right\rangle \equiv \frac{1}{\sqrt{2}} v$$

$$SU(3) \xrightarrow[f]{} SU(2) \times U(1) \xrightarrow[v]{} ?$$

Mass spectrum in SU(2) YM (vL<<1)

gauge field

$$m^2_{A_\mu,1} = \frac{1}{2} g^2 v^2 \left(n_1 = 0\right),\ m^2_{A_\mu,1'} = \alpha_2 \left(n_1 + 1\right)$$

$$m^2_{A_\mu,2'} = \alpha_2 \left(n_2 + 1\right) + g^2 v^2,\ m^2_{A_\mu,3} = 4\pi^2 \left(l_3^2 + m_3^2\right) + g^2 v^2$$

If v≠0 ⇒ SU(2) is completely broken

scalar & ghost

$$m^2_{\varphi,1} = \alpha_2 \left(n_1 + \frac{1}{2}\right),\ m^2_{\varphi,2'} = \alpha_2 \left(n_2 + \frac{1}{2}\right) + g^2 v^2$$

$$m^2_{\varphi,3} = 4\pi^2 \left(l_3^2 + m_3^2\right) + g^2 v^2$$

Same mass spectrum of ghost in Feynman gauge

Potential analysis

$$V_{tree} = \int_{T^2} dx_5\, dx_6\, \frac{g^2}{2}\left(\left[\langle\phi\rangle,\langle\bar{\phi}\rangle\right]^2\right)^2 = \frac{f^2}{24}(gv)^2$$

CW potential

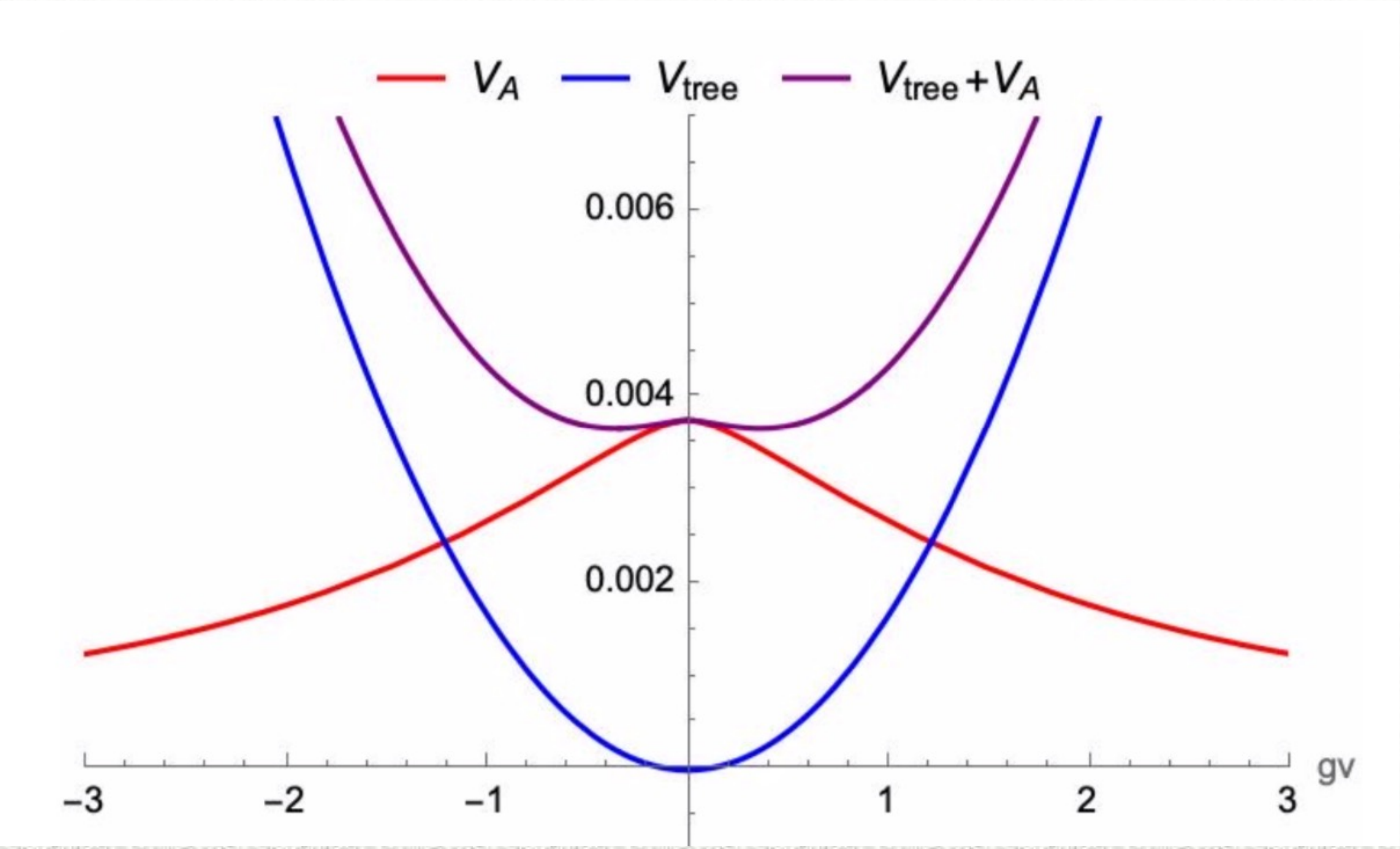
$$V_{1-loop} = \frac{(-1)^F}{8\pi^2} N \sum_n \int \frac{d^4 p_E}{(2\pi)^4} \ln\left(p_E^2 + m_n^2\right)$$

$$= \frac{\left(g^2v^2\right)^2}{6144\pi^4}\left(\frac{9\zeta(3)}{\pi} - 1\right) - \frac{3\left(g^2v^2\right)^{3/2}}{32\pi^5}\sum_{r,s\neq 0}\left(\frac{1}{r^2+s^2}\right)^{3/2} K_3\left(gv\sqrt{r^2+s^2}\right) + \frac{\left(g^2v^2\right)^3}{12288\pi^5}\left(\frac{9\zeta(3)}{\pi} - 1\right)$$

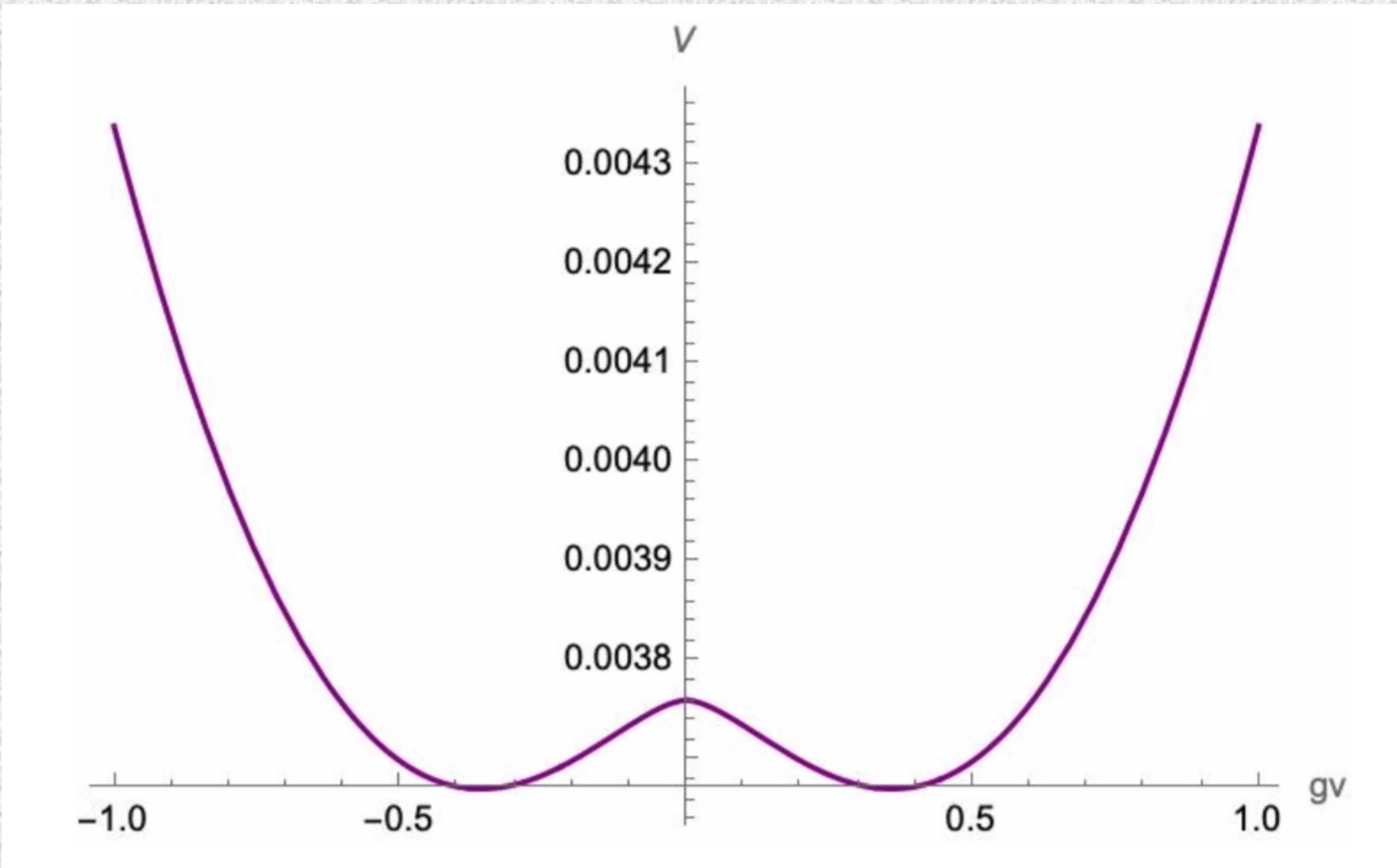
$$-\frac{3N^2}{16\pi^2}\left[\zeta^{(1,0)}\left(-2, \frac{g^2v^2}{4\pi N}\right) - \frac{1}{12}\left(\frac{g^2v^2}{4\pi N}\right)\left\{2\ln\left(\frac{g^2v^2}{4\pi N}\right) + 1\right\} + \frac{1}{2}\left(\frac{g^2v^2}{4\pi N}\right)^2 \ln\left(\frac{g^2v^2}{4\pi N}\right)\right.$$

$$\left. - \frac{1}{9}\left(\frac{g^2v^2}{4\pi N}\right)^3\left\{3\ln\left(\frac{g^2v^2}{4\pi N}\right) - 1\right\}\right]$$

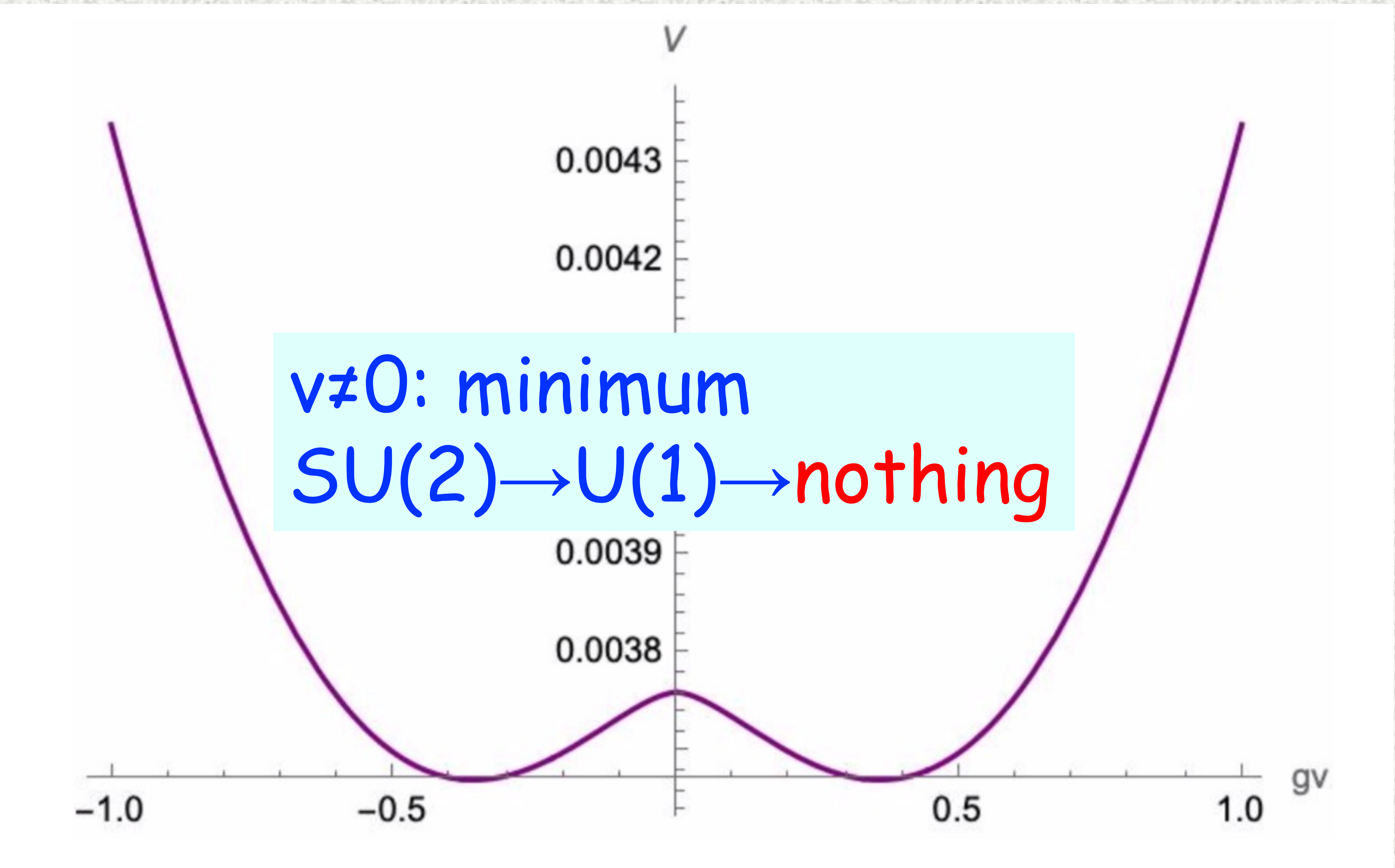
Potential analysis



Potential analysis



Potential analysis



v≠0: minimum

SU(2)⟶U(1)⟶nothing

SU(3) YM

(1) $$\left\langle \phi^{8} \right\rangle \equiv \frac{1}{\sqrt{2}} fz, \left\langle \phi^{1} \right\rangle \equiv \frac{1}{\sqrt{2}} w$$

(2) $$\left\langle \phi^{8} \right\rangle \equiv \frac{1}{\sqrt{2}} fz, \left\langle \phi^{6} \right\rangle \equiv \frac{1}{\sqrt{2}} v$$

Potential@tree level

$$V_{tree} = \frac{1}{2} f^{2} + \frac{1}{32} \left(fgv \right)^{2}$$

Mass spectrum of gauge field in 1-8 case

$$m^2_{A_\mu,1} = 4\pi^2\left(l_1^2 + m_1^2\right), m^2_{A_\mu,2} = 4\pi^2\left\{l_2^2 + \left(m_2 - \frac{gw}{2\pi}\right)^2\right\} + \frac{1}{4}g^2v^2$$

$$m^2_{A_\mu,3} = 4\pi^2\left\{l_2^2 + \left(m_2 + \frac{gw}{2\pi}\right)^2\right\} + \frac{1}{4}g^2v^2,\ m^2_{A_\mu,4} = \alpha_3 n_4$$

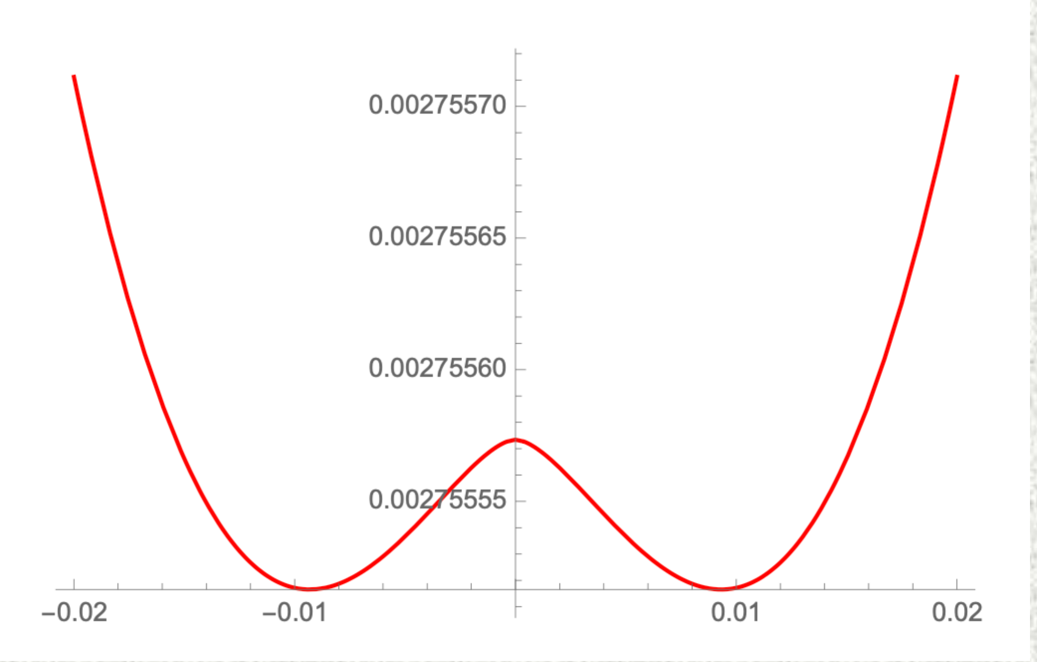
$$m^2_{A_\mu,5} = \alpha_3\left(n_5 + 1\right),\ m^2_{A_\mu,6} = \alpha_3 n_6$$

$$m^2_{A_\mu,7} = \alpha_3\left(n_7 + 1\right),\ m^2_{A_\mu,8} = 4\pi^2\left(l_8^2 + m_8^2\right)$$

$$\alpha_3 = \sqrt{3}gf$$

1-loop effective potential

$$V = V_{reg,A} \supset -\frac{3}{\pi^5} \sum_{r=-\infty}^{\infty} \sum_{s=1}^{\infty} \frac{1}{(r^2+s^2)^3} \cos(gvs)$$
$$- \frac{9N^2}{16\pi^2} \left[\zeta^{(1,0)}\left(-2, \frac{g^2v^2}{8\sqrt{3}\pi N}\right) - \frac{1}{12}\left(\frac{g^2v^2}{8\sqrt{3}\pi N}\right)\left(2\ln\frac{g^2v^2}{8\sqrt{3}\pi N} + 1\right) \right.$$
$$+ \frac{1}{36}\left(\frac{g^2v^2}{8\sqrt{3}\pi N}\right)^2 \left\{18\ln\frac{g^2v^2}{8\sqrt{3}\pi N} - \frac{9\zeta(3)}{2\pi^2} + 1\right\}$$
$$\left. - \frac{1}{9}\left(\frac{g^2v^2}{8\sqrt{3}\pi N}\right)^3 \left(3\ln\frac{g^2v^2}{8\sqrt{3}\pi N} - 1\right)\right]. \quad (105)$$

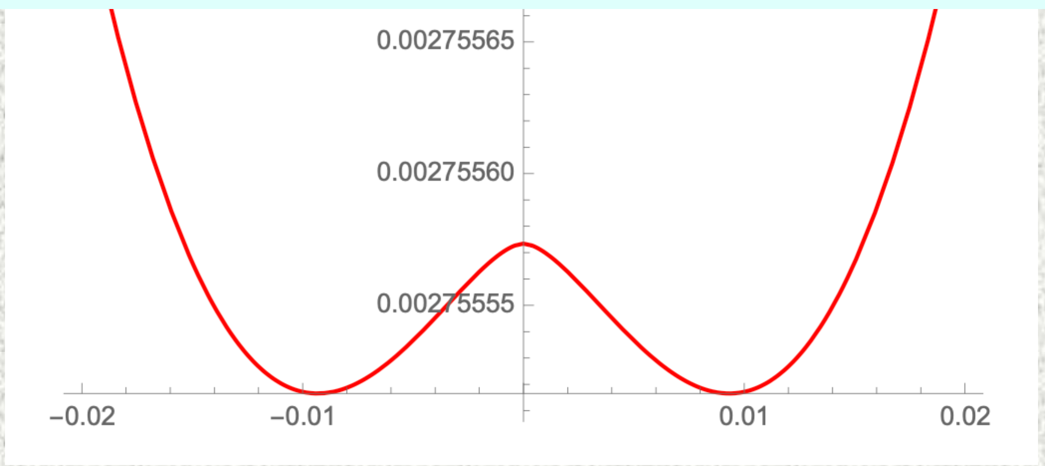


1-loop effective potential

$$V = V_{reg,A} \supset -\frac{3}{\pi^5}\sum_{r=-\infty}^{\infty}\sum_{s=1}^{\infty}\frac{1}{(r^2+s^2)^3}\cos(gvs)$$
$$-\frac{9N^2}{16\pi^2}\left[\zeta^{(1,0)}\left(-2,\frac{g^2v^2}{8\sqrt{3}\pi N}\right)-\frac{1}{12}\left(\frac{g^2v^2}{8\sqrt{3}\pi N}\right)\left(2\ln\frac{g^2v^2}{8\sqrt{3}\pi N}+1\right)\right.$$
$$+\frac{1}{36}\left(\frac{g^2v^2}{8\sqrt{3}\pi N}\right)^2\left\{18\ln\frac{g^2v^2}{8\sqrt{3}\pi N}-\frac{9\zeta(3)}{2\pi^2}+1\right\}$$
$$\left.-\frac{1}{9}\left(\frac{g^2v^2}{8\sqrt{3}\pi N}\right)^3\left(3\ln\frac{g^2v^2}{8\sqrt{3}\pi N}-1\right)\right]. \quad (105)$$

w≠0: minimum

SU(3)→SU(2)xU(1)→U(1)xU(1)



Mass spectrum of gauge field in 6-8 case (vL<<1)

$$m_{A_\mu,1}^2 = 4\pi^2\left(l_1^2 + m_1^2\right) + \frac{1}{4}g^2v^2,\ m_{A_\mu,2}^2 = 4\pi^2\left(l_2^2 + m_2^2\right) + \frac{1}{4}g^2v^2$$

$$m_{A_\mu,3'}^2 = 4\pi^2\left(l_{3'}^2 + m_{3'}^2\right) + g^2v^2,\ m_{A_\mu,4}^2 = \alpha_3 n_4 + \frac{1}{4}g^2v^2$$

$$m_{A_\mu,5}^2 = \alpha_3\left(n_5 + 1\right) + \frac{1}{4}g^2v^2,\ m_{A_\mu,6'}^2 = \frac{1}{2}g^2v^2\left(n_6 = 0\right), \alpha_3\left(n_{6'} + 1\right)\left(n_{6'} \geq 0\right)$$

$$m_{A_\mu,7'}^2 = \alpha_3\left(n_{7'} + 1\right) + g^2v^2,\ m_{A_\mu,8'}^2 = 4\pi^2\left(l_{8'}^2 + m_{8'}^2\right) \qquad \alpha_3 = \sqrt{3}gf$$

Unbroken U(1) in 8′ component

⇒ linear combination of 3 & 8 components

Same pattern of EW symmetry breaking in SM

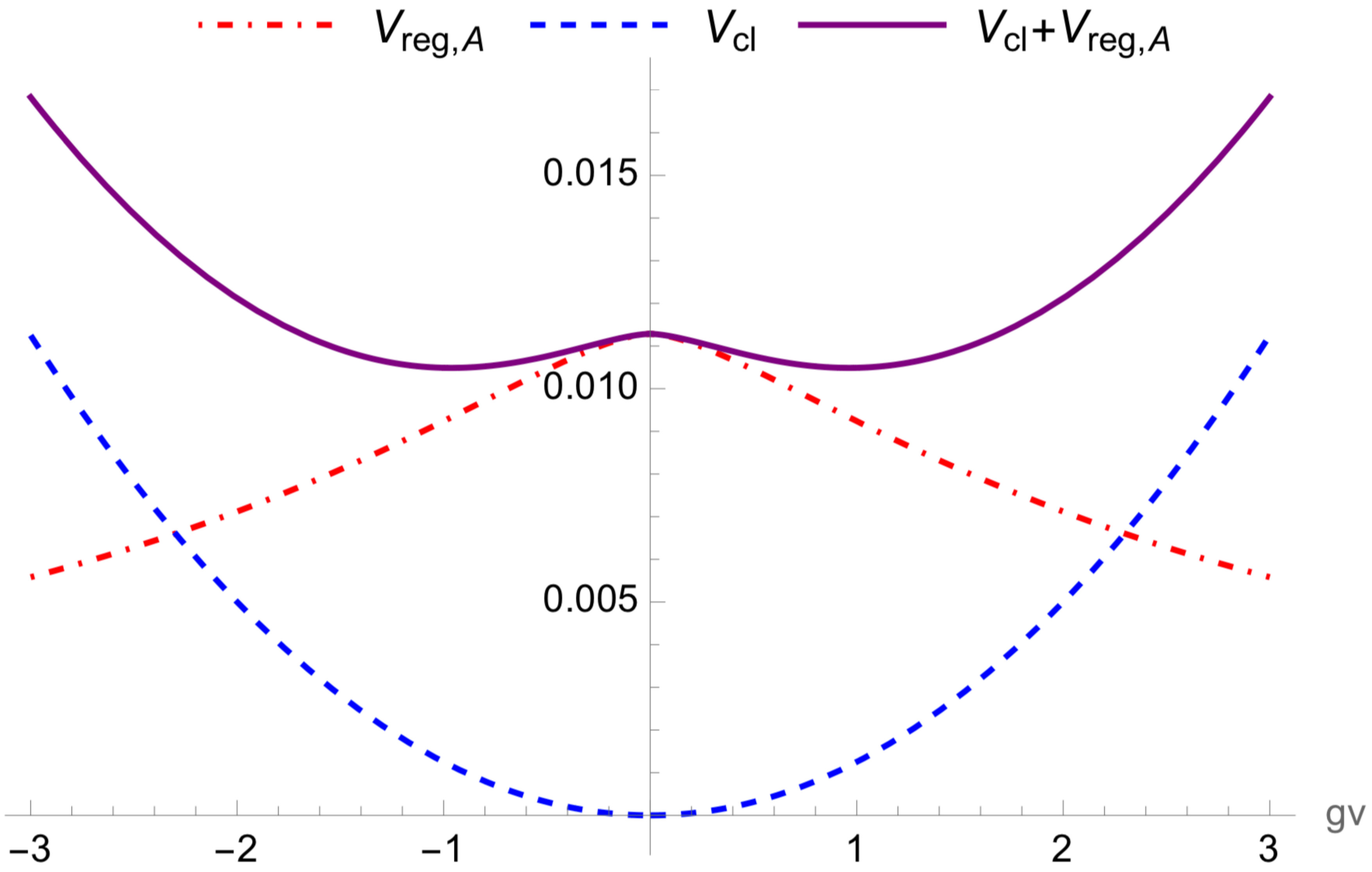
Potential analysis

$$V=\frac{1}{32}\left(fgv\right)^2+\frac{3}{8\pi^2}\Big(2V_{1-loop,l,m}\left(1/4\right)+V_{1-loop,l,m}\left(1\right)+V_{1-loop,n}\left(\alpha,1/4;0\right)$$
$$+V_{1-loop,n}\left(\alpha,1/4;1\right)+V_{1-loop,n}\left(\alpha,1;1\right)+V_{1-loop,0}\left(1/2\right)\Big)$$

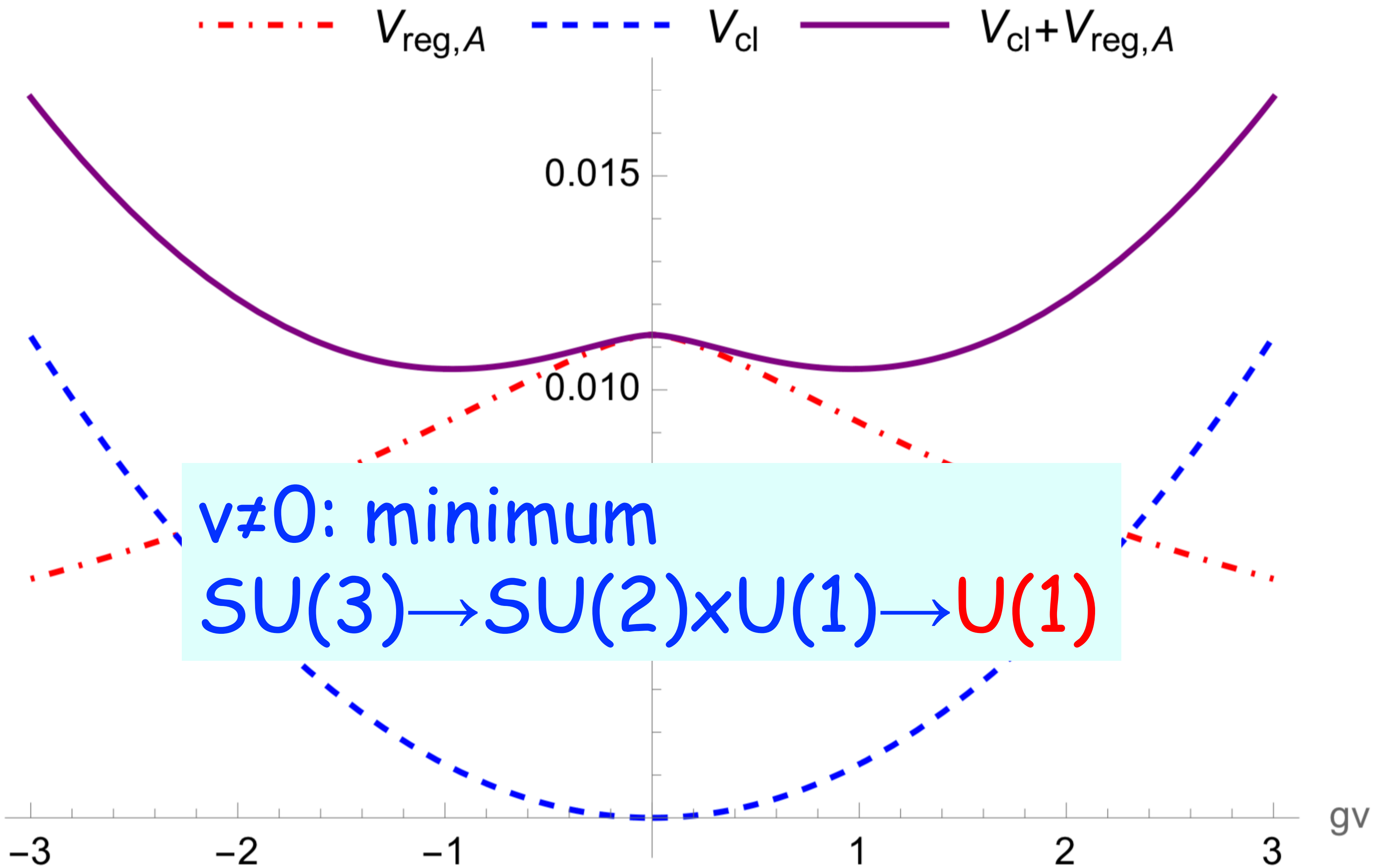
$$V_{1-loop,n}\left(\alpha,B;x<1\right)=-\frac{\alpha^2}{32\pi^2}\left[\zeta^{(1,0)}\left(-2,x+\frac{Bg^2v^2}{\alpha}\right)-\frac{1}{12}\left(x+\frac{Bg^2v^2}{\alpha}\right)\left\{2\ln\left(x+\frac{Bg^2v^2}{\alpha}\right)+1\right\}\right.$$
$$+\frac{1}{18}\left(x+\frac{Bg^2v^2}{\alpha}\right)^2\left\{9\ln\left(x+\frac{Bg^2v^2}{\alpha}\right)-\frac{9\zeta(3)}{\pi^2}+1\right\}$$
$$\left.-\frac{1}{9}\left(x+\frac{Bg^2v^2}{\alpha}\right)^3\left\{3\ln\left(x+\frac{Bg^2v^2}{\alpha}\right)-1\right\}\right]$$

$$V_{1-loop,l,m}\left(C\right)=-\frac{\left(Cg^2v^2\right)^{3/2}}{32\pi^2}\sum_{r,s\neq 0}\left(\frac{1}{r^2+s^2}\right)^{3/2}K_3\left(gv\sqrt{C\left(r^2+s^2\right)}\right)+\frac{\left(Cg^2v^2\right)^3}{4608\pi^3}\left(\frac{9\zeta(3)}{\pi^2}-1\right)$$

Potential analysis



Potential analysis



v≠0: minimum

SU(3)→SU(2)xU(1)→U(1)

Summary

- No signature of New physics
  → SM might be correct up to Planck scale
  → reconsider the hierarchy problem??
- Scalar mass = 0@M_P is favorable
  → flux compactification
- In 6D SU(2)YM with flux compactification,
  WL scalar mass@1-loop=0 was shown
- Gauge loop → in arbitrary gauge
  Scalar loop + ghost loop →
  shown in Landau & Feynman gauge

Summary

- Masslessness
  $\Rightarrow$ WL scalar = NG boson of translation
  in compactified space
- To identify WL scalar with SM Higgs
  $\Rightarrow$ WL scalar should be a pseudo NG boson
  $\Rightarrow$ mass & potential are generated
- Classification of explicit breaking terms
  of translational symmetry providing
  finite WL scalar mass@ 1-loop

Summary

- Finite WL scalar mass@1-loop was obtained in 6D scalar QED
- Gauge symmetry breaking analyzed in coexisting both flux & const VEV
  - SU(2) → U(1) → nothing (3-1)
  - SU(3) → SU(2)xU(1) → SU(2)xU(1) (8-1)
  - SU(3) → SU(2)xU(1) → U(1) (8-6)

Summary

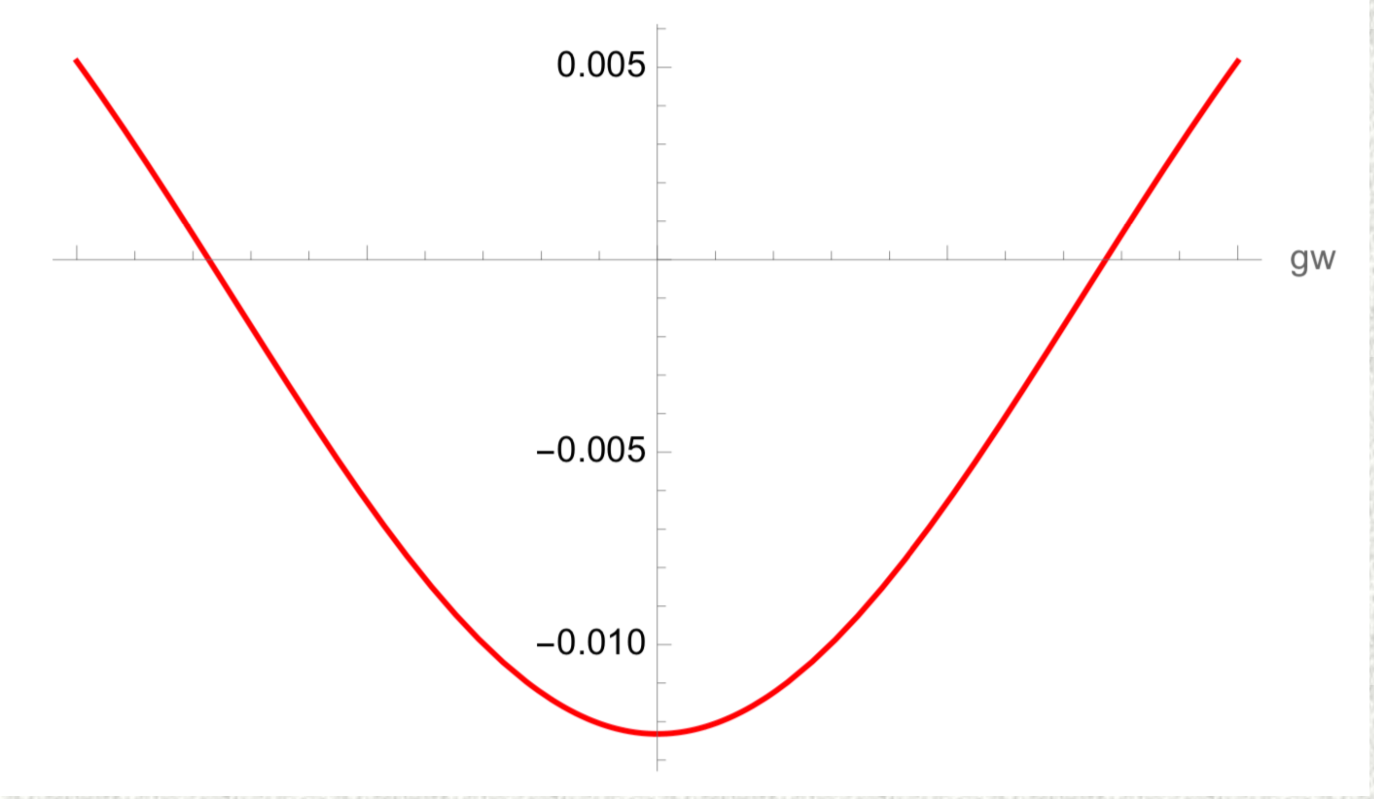
- Origin of explicit breaking terms??
  small effects are required
  - Quantum gravity or stringy effects?
  - Nontrivial background as vortex?
  - Dynamical SUSY breaking??
- As for gauge symmetry breaking,
  fermion loop contribution,
  EW symmetry breaking, Higgs mass
  must be studied

Thank you!!

1-loop effective potential

$$V_{1-loop} = \frac{3}{8\pi^2}\left(V_+ + V_-\right) = -\frac{3g^3v^3}{64\pi^5}\sum_{r=-\infty}^{\infty}\sum_{s=1}^{\infty}\cos(gws)\left(\frac{1}{r^2+s^2}\right)^{3/2} K_3\left(\frac{gv}{2}\sqrt{r^2+s^2}\right)$$

$$V_\pm = \sum_{l,m}\int\frac{d^4p}{(2\pi)^4}\ln\left[p^2+4\pi^2\left\{l^2+\left(m\pm\frac{gw}{2\pi}\right)^2\right\}+\frac{1}{4}g^2v^2\right]$$



1-loop effective potential

$$V_{1-loop} = \frac{3}{8\pi^2}\left(V_+ + V_-\right) = -\frac{3g^3v^3}{64\pi^5}\sum_{r=-\infty}^{\infty}\sum_{s=1}^{\infty}\cos(gws)\left(\frac{1}{r^2+s^2}\right)^{3/2} K_3\left(\frac{gv}{2}\sqrt{r^2+s^2}\right)$$

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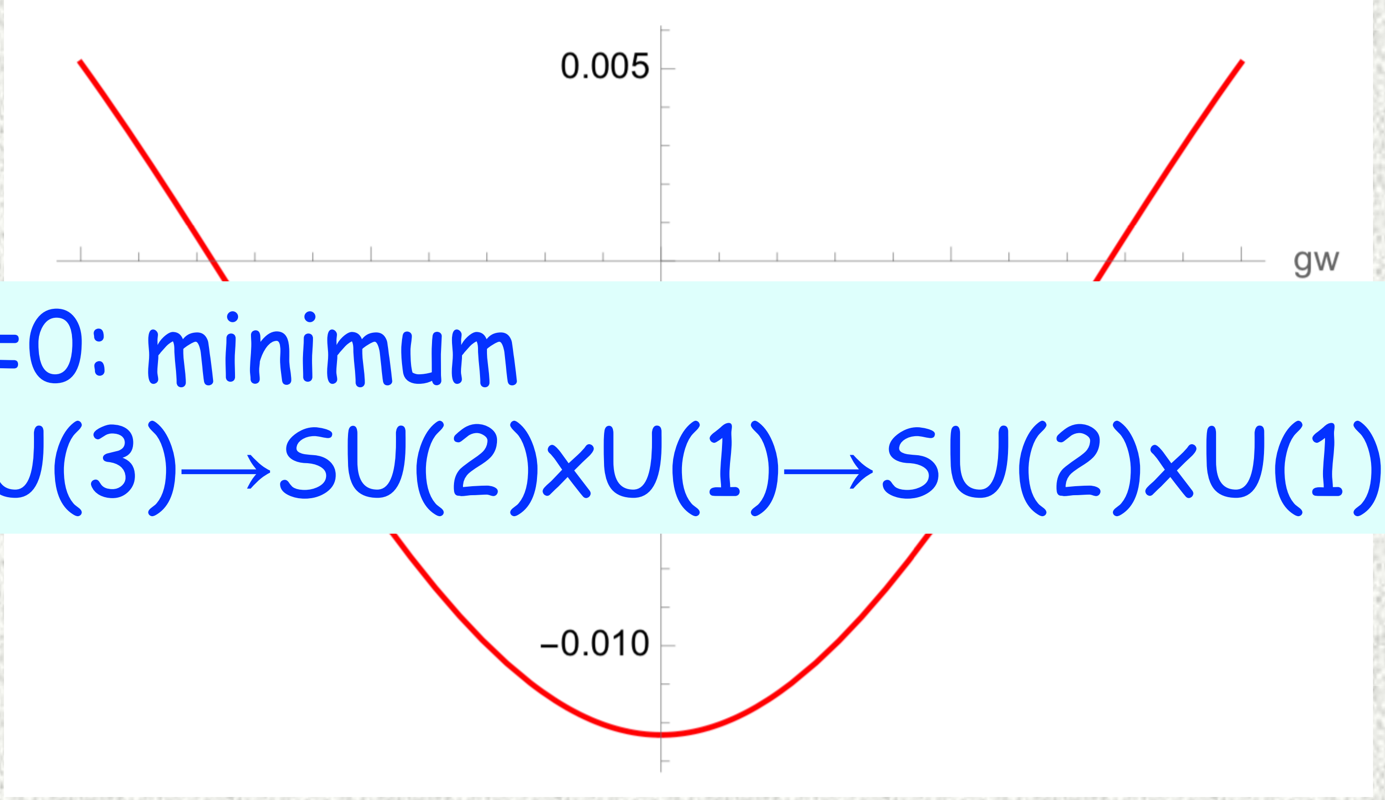
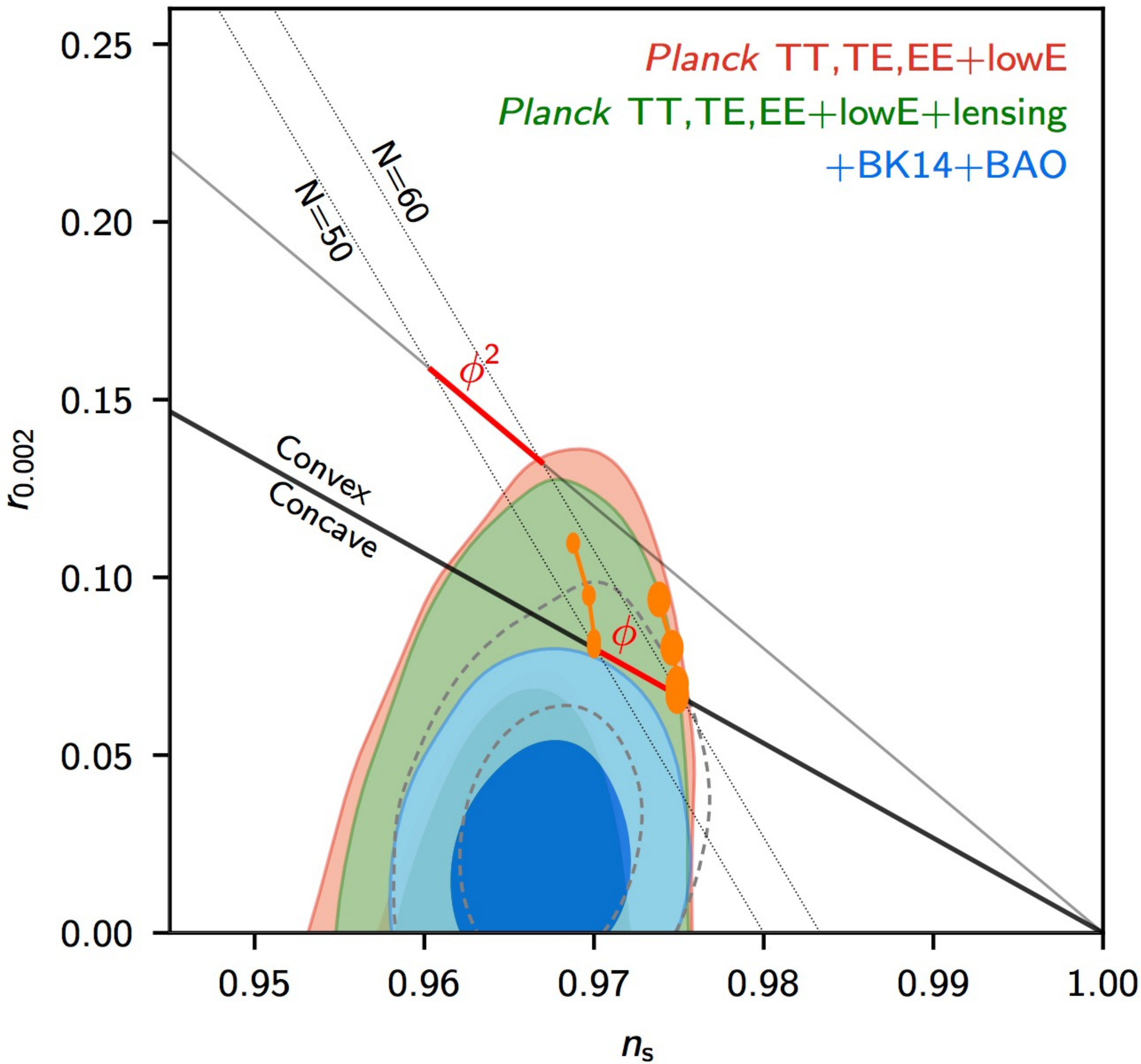


w=0: minimum

SU(3)→SU(2)xU(1)→SU(2)xU(1)

Backup



Planck TT,TE,EE+lowE
Planck TT,TE,EE+lowE+lensing
+BK14+BAO
N=50
N=60
ϕ^2
ϕ
Convex
Concave
$r_{0.002}$
n_s
0.25
0.20
0.15
0.10
0.05
0.00
0.95
0.96
0.97
0.98
0.99
1.00

On the gauge invariance of interaction $\varphi\bar{\Phi}\Phi$

Gauge invariance?? $\because \quad \delta\varphi \propto \partial\Lambda\left(z,\bar{z}\right)$

$\Rightarrow$ $\varphi,\bar{\varphi}$ should be expressed by gauge invariant non-local Wilson line operators

$$U_5 = \exp\left[ig\oint dx^5 A_5\right] = \exp\left[\frac{g}{\sqrt{2}}\oint\left(\varphi dz + \varphi d\bar{z} - \bar{\varphi} dz - \bar{\varphi} d\bar{z}\right)\right]$$

$$U_6 = \exp\left[ig\oint dx^6 A_6\right] = \exp\left[\frac{g}{\sqrt{2}}\oint\left(\varphi dz - \varphi d\bar{z} + \bar{\varphi} dz - \bar{\varphi} d\bar{z}\right)\right]$$

$\varphi\bar{\Phi}\Phi, \bar{\varphi}\bar{\Phi}\Phi$ can be obtained by expansion

$$i\left(U_5 - U_5^\dagger\right)\bar{\Phi}\Phi - i\left(U_6 - U_6^\dagger\right)\bar{\Phi}\Phi \supset ig_4\varphi\bar{\Phi}\Phi - ig_4\bar{\varphi}\bar{\Phi}\Phi$$

$\varphi\bar{\Phi}\Phi, \bar{\varphi}\bar{\Phi}\Phi$ can be obtained by expansion

$$i\left(U_5 - U_5^\dagger\right)\bar{\Phi}\Phi - i\left(U_6 - U_6^\dagger\right)\bar{\Phi}\Phi \supset ig_4\varphi\bar{\Phi}\Phi - ig_4\bar{\varphi}\bar{\Phi}\Phi$$

$$U_5 - U_5^\dagger = 2i\sin\left[g\oint dx^5 A_5\right], U_6 - U_6^\dagger = 2i\sin\left[g\oint dx^6 A_6\right]$$

are not obviously invariant

under the constant shift symmetry

$$A_5 \to A_5 - f\varepsilon_6/2,\ A_6 \to A_6 + f\varepsilon_5/2$$

We have also shown a cancellation
in a presence of higher dimensional operators

Hirose & Maru, J.Phys.G 48 (2021) 5 [2012.03494]

$$\mathcal{L} = -\frac{1}{4} F_{MN} F^{MN} + \frac{1}{\Lambda^2}\mathcal{O}_1(D,F) + \frac{1}{\Lambda^4}\mathcal{O}_2(D,F) + \frac{1}{\Lambda^6}\mathcal{O}_3(D,F) + \cdots$$

Only dim-6 operators are considered in our paper

$$\mathcal{O}_1 = Tr\left[D_L D^L D_M D_N F^{MN}\right] + 2Tr\left[D_L F_{MN} D^L F^{MN}\right] + \varepsilon^{M_1N_1M_2N_2M_3N_3} Tr\left[F_{M_1N_1} F_{M_2N_2} F_{M_3N_3}\right]$$

Reason of cancellation is very simple:
WL scalar appears in a commutator

$[\quad ,\varphi]$ invariant under a constant shift

$$\mathcal{L} = \frac{1}{\Lambda^2}\left(\mathcal{L}_{\varphi\varphi AA} + \mathcal{L}_{\varphi\varphi\varphi\varphi} + \mathcal{L}_{\varphi AA} + \mathcal{L}_{\varphi\varphi\varphi}\right)$$

$$\begin{aligned}
\mathcal{L}_{\varphi\varphi AA} &= 8g^2\overline{\varphi}^1\varphi^1 \sum_{n,j} \partial_\mu \widetilde{A}^2_{\nu,n,j}\partial^\mu \widetilde{A}^{2\nu}_{n,j} + 8g^2\overline{\varphi}^1\varphi^1 \sum_{n,j} \partial_\mu \widetilde{A}^3_{\nu,n,j}\partial^\mu \widetilde{A}^{3\nu}_{n,j} \\
&\quad + 16g^2\overline{\varphi}^1\varphi^1 \sum_{n,j} \alpha n \widetilde{A}^2_{\mu,n,j}\widetilde{A}^{2\mu}_{n,j} + 16g^2\overline{\varphi}^1\varphi^1 \sum_{n,j} \alpha(n+1)\widetilde{A}^3_{\mu,n,j}\widetilde{A}^{3\mu}_{n,j}, \\
\mathcal{L}_{\varphi\varphi\varphi\varphi} &= 8g^2\overline{\varphi}^1\varphi^1 \sum_{n,j} \partial_\mu \overline{\widetilde{\varphi}}^2_{n,j}\partial^\mu \widetilde{\varphi}^2_{n,j} + 8g^2\overline{\varphi}^1\varphi^1 \sum_{n,j} \partial_\mu \overline{\widetilde{\varphi}}^3_{n,j}\partial^\mu \widetilde{\varphi}^3_{n,j} \\
&\quad + 16g^2\overline{\varphi}^1\varphi^1 \sum_{n,j} \alpha n \widetilde{\varphi}^2_{n,j}\overline{\widetilde{\varphi}}^2_{n,j} + 16g^2\overline{\varphi}^1\varphi^1 \sum_{n,j} \alpha(n+1)\widetilde{\varphi}^3_{n,j}\overline{\widetilde{\varphi}}^3_{n,j},
\end{aligned}$$

$$\mathcal{L} = \frac{1}{\Lambda^2}\left(\mathcal{L}_{\varphi\varphi AA} + \mathcal{L}_{\varphi\varphi\varphi\varphi} + \mathcal{L}_{\varphi AA} + \mathcal{L}_{\varphi\varphi\varphi}\right)$$

$$\begin{aligned}
\mathcal{L}_{\varphi AA} = &+4\sqrt{2}ig\sum_{n,j}\sqrt{\alpha\,(n+1)}\partial_\mu \widetilde{A}^2_{\nu,n,j}\partial^\mu \widetilde{A}^{2\nu}_{n+1,j}\overline{\varphi}^1 - 4\sqrt{2}ig\sum_{n,j}\sqrt{\alpha(n+1)}\partial_\mu \widetilde{A}^3_{\nu,n+1,j}\partial^\mu \widetilde{A}^{3\nu}_{n,j}\overline{\varphi}^1 \\
&- 4\sqrt{2}ig\sum_{n,j}\sqrt{\alpha(n+1)}\partial_\mu \widetilde{A}^2_{\nu,n+1,j}\partial^\mu \widetilde{A}^{2\nu}_{n,j}\varphi^1 + 4\sqrt{2}ig\sum_{n,j}\sqrt{\alpha(n+1)}\partial_\mu \widetilde{A}^3_{\nu,n,j}\partial^\mu \widetilde{A}^{3\nu}_{n+1,j}\varphi^1 \\
&+ 4\sqrt{2}ig\sum_{n,j}\alpha\left(n+\frac{1}{2}\right)\sqrt{\alpha(n+1)}\widetilde{A}^2_{\mu,n+1,j}\widetilde{A}^{2\mu}_{n,j}\overline{\varphi}^1 \\
&- 4\sqrt{2}ig\sum_{n,j}\alpha\left(n+\frac{3}{2}\right)\sqrt{\alpha(n+1)}\widetilde{A}^3_{\mu,n,j}\widetilde{A}^{3\mu}_{n+1,j}\overline{\varphi}^1 \\
&- 4\sqrt{2}ig\sum_{n,j}\alpha\left(n+\frac{1}{2}\right)\sqrt{\alpha(n+1)}\widetilde{A}^2_{\mu,n+1,j}\widetilde{A}^{2\mu}_{n,j}\varphi^1 \\
&+ 4\sqrt{2}ig\sum_{n,j}\alpha\left(n+\frac{3}{2}\right)\sqrt{\alpha(n+1)}\widetilde{A}^3_{\mu,n,j}\widetilde{A}^{3\mu}_{n+1,j}\varphi^1, \qquad (45)
\end{aligned}$$

$$\mathcal{L} = \frac{1}{\Lambda^2}\left(\mathcal{L}_{\varphi\varphi AA} + \mathcal{L}_{\varphi\varphi\varphi\varphi} + \mathcal{L}_{\varphi AA} + \mathcal{L}_{\varphi\varphi\varphi}\right)$$

$$\begin{aligned}
\mathcal{L}_{\varphi\varphi\varphi} = &+4\sqrt{2}ig\sum_{n,j}\sqrt{\alpha(n+1)}\partial_\mu\overline{\widetilde{\varphi}}^2_{n+1,j}\partial^\mu\widetilde{\varphi}^2_{n,j}\overline{\varphi}^1 - 4\sqrt{2}ig\sum_{n,j}\sqrt{\alpha(n+1)}\partial_\mu\overline{\widetilde{\varphi}}^3_{n,j}\partial^\mu\widetilde{\varphi}^3_{n+1,j}\overline{\varphi}^1 \\
&- 4\sqrt{2}ig\sum_{n,j}\sqrt{\alpha(n+1)}\partial_\mu\widetilde{\varphi}^2_{n+1,j}\partial^\mu\overline{\widetilde{\varphi}}^2_{n,j}\varphi^1 + 4\sqrt{2}ig\sum_{n,j}\sqrt{\alpha\,(n+1)}\partial_\mu\widetilde{\varphi}^3_{n,j}\partial^\mu\overline{\widetilde{\varphi}}^3_{n+1,j}\varphi^1 \\
&+ 4\sqrt{2}ig\sum_{n_2,j}\alpha\left(n-\frac{1}{4}\right)\sqrt{\alpha(n+1)}\overline{\widetilde{\varphi}}^2_{n_2+1,j}\widetilde{\varphi}^2_{n_2,j}\overline{\varphi}^1 \\
&- 4\sqrt{2}ig\sum_{n_3,j}\alpha\left(n+\frac{9}{4}\right)\sqrt{\alpha(n+1)}\overline{\widetilde{\varphi}}^3_{n_3,j}\widetilde{\varphi}^3_{n_3+1,j}\overline{\varphi}^1 \\
&- 4\sqrt{2}ig\sum_{n_2,j}\alpha\left(n-\frac{1}{4}\right)\sqrt{\alpha(n+1)}\overline{\widetilde{\varphi}}^2_{n_2,j}\widetilde{\varphi}^2_{n_2+1,j}\varphi^1 \\
&+ 4\sqrt{2}ig\sum_{n_3,j}\alpha\left(n+\frac{9}{4}\right)\sqrt{\alpha(n+1)}\overline{\widetilde{\varphi}}^3_{n_3+1,j}\widetilde{\varphi}^3_{n_3,j}\varphi^1,
\end{aligned} \tag{46}$$

Divergence structure of the loop integral and mode sum: part 2

More generalization is possible

$$I'(x;a,b)=\sum_{n=0}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{(k^2)^a}{\left(k^2+\alpha(n+x)\right)^b}$$

$$=\frac{1}{\alpha^{b-a-1}}\left(\frac{4\pi}{\alpha}\right)^{\varepsilon-2}\frac{\Gamma(a+2-\varepsilon)\Gamma(\varepsilon+b-a-2)}{\Gamma(b)\Gamma(2-\varepsilon)}\zeta[\varepsilon+b-a-3,x]$$

$$K(x;a,b)=\frac{\Gamma(a+2-\varepsilon)\Gamma(\varepsilon+b-a-2)}{\Gamma(b)\Gamma(2-\varepsilon)}\zeta[\varepsilon+b-a-3,x]$$

$$=\begin{cases}(-1)^a\,\Gamma(\varepsilon-1)\zeta[\varepsilon-a-2,x]\,(b=1)\\(-1)^a(\varepsilon-a-1)\Gamma(\varepsilon-a-1)\zeta[\varepsilon-a-1,x]\,(b=2)\end{cases}$$

K(x;a,1) for even a, K(x;a,2) for odd a are finite

1-loop finite corrections by 4-point interaction

Scalar loop

$$K\left(1/2;0,1\right)\rightarrow\bar{\varphi}\varphi\bar{\Phi}\left(a^{\dagger}a+\frac{1}{2}\right)\Phi$$

$$K\left(1/2;a,1\right)\rightarrow\bar{\varphi}\varphi\,\partial_{\mu_1}\cdots\partial_{\mu_a}\bar{\Phi}\left(a^{\dagger}a+\frac{1}{2}\right)\partial^{\mu_1}\cdots\partial^{\mu_a}\Phi$$

Fermion loop

$$K\left(1;a,1\right)\rightarrow\bar{\varphi}\varphi\bar{\psi}\left(\gamma^{\mu}\partial_{\mu}\right)^{2a-1}\left(a^{\dagger}a+1\right)\psi$$

SU(2) gauge loop

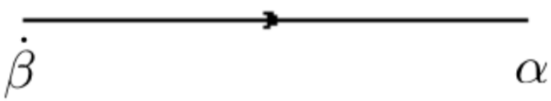
$$K\left(0;a,1\right)\rightarrow\bar{\varphi}\varphi\,\partial_{\mu_1}\cdots\partial_{\mu_a}A^{a}_{\nu}\left(a^{\dagger}a\right)\partial^{\mu_1}\cdots\partial^{\mu_a}A^{a\nu}$$

3-point interaction terms are hard to guess
because the interactions cannot be expressed by mass squared operators

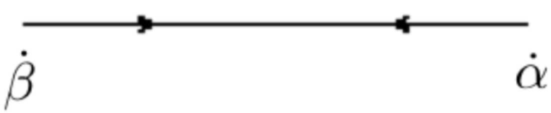
propagators

$$= \frac{-i}{p^2 + m^2}$$

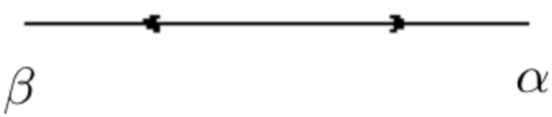
$$= \frac{-i}{p^2 + m^2} \left[g^{\mu\nu} - (1 - \xi) \frac{p^\mu p^\nu}{p^2 + \xi m^2} \right]$$



$$= \frac{i p \cdot \sigma_{\alpha\dot{\beta}}}{p^2 + m^2} \quad \text{or} \quad \frac{-i p \cdot \bar{\sigma}^{\dot{\beta}\alpha}}{p^2 + m^2}$$



$$= \frac{-im}{p^2 + m^2} \delta^{\dot{\alpha}}{}_{\dot{\beta}}$$



$$= \frac{-im}{p^2 + m^2} \delta_{\alpha}{}^{\beta}$$

6D SU(2) Yang-Mills theory compactified on T^2 with magnetic flux

$$\mathcal{L}_6 = -\frac{1}{4} F^a_{MN} F^{aMN} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2} F^a_{\mu 5} F^{a\mu 5} - \frac{1}{2} F^a_{\mu 6} F^{a\mu 6} - \frac{1}{2} F^a_{56} F^{a56}$$

$$\mathcal{L}_{gf} = -\frac{1}{2\xi}\left(D_\mu A^{a\mu} + \xi \mathcal{D}_m A^{am} \right)^2$$

$$\mathcal{L}_{ghost} = -\overline{c}^a \left(D_\mu D^\mu + \xi D_m \mathcal{D}^m \right) c^a$$

$$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M - ig\left[A_M, A_N \right]^a$$

$$D_M A^a_N = \partial_M A^a_N - ig\left[A_M, A_N \right]^a$$

$$\mathcal{D}_m A^{am} = \partial_m A^{am} - ig\left[\langle A_m \rangle, A^m \right]^a$$

Magnetic flux

$$D^m \langle F_{mn} \rangle = 0 \Rightarrow \langle A^1_5 \rangle = -\frac{1}{2} f x_6, \langle A^1_6 \rangle = \frac{1}{2} f x_5, \langle A^{2,3}_5 \rangle = \langle A^{2,3}_6 \rangle = 0$$

$$\Rightarrow \langle F^1_{56} \rangle = f$$

$$\left[a_1, a_1^\dagger\right] = 0,\ \left[a_2, a_2^\dagger\right] = 1,\ \left[a_3, a_3^\dagger\right] = -1$$

Ground state

$$a_2 \psi^2_{0,j} = 0,\ a_3^\dagger \psi^3_{0,j} = 0$$

j = 0, …, |N|−1: degeneracy

Excited states

$$\psi^2_{n_2,j} = \frac{1}{\sqrt{n_2!}} \left(a_2^\dagger\right)^{n_2} \psi^2_{0,j}$$

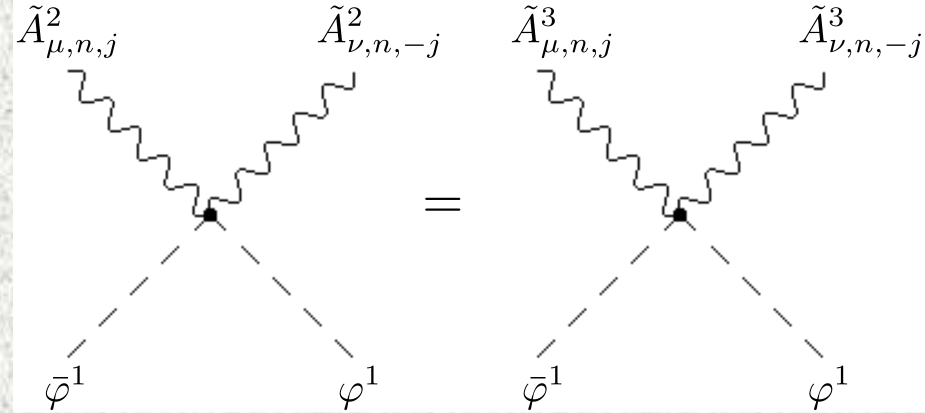
$$\psi^3_{n_3,j} = \frac{1}{\sqrt{n_3!}} \left(a^3\right)^{n_3} \psi^3_{0,j}$$

Orthonomality

$$\int_{T^2} d^2x \left(\psi^{a'}_{n'_a,j'}\right)^* \psi^a_{n_a,j} = \delta^{a'a} \delta_{n'_a n_a} \delta_{j'j}$$

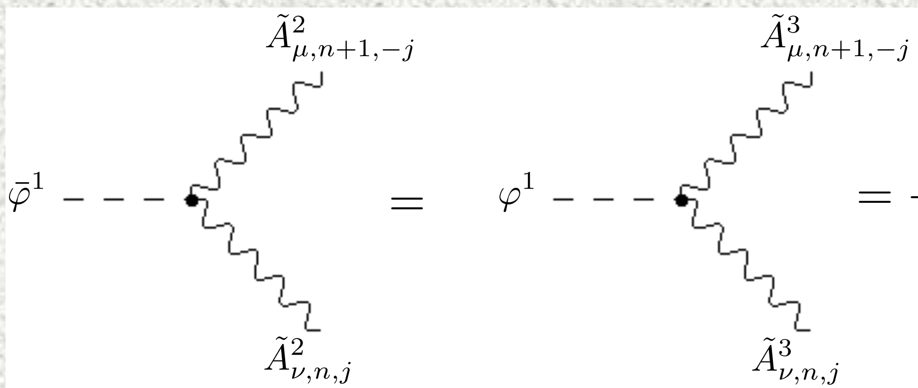
Feynman rules for interaction vertices

Gauge-gauge
-scalar-scalar

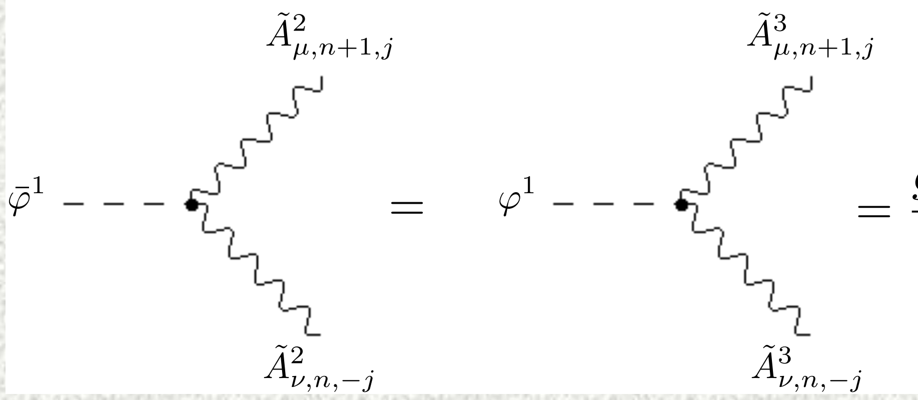


$$= -2ig^2\eta^{\mu\nu}$$

Gauge-gauge
-scalar



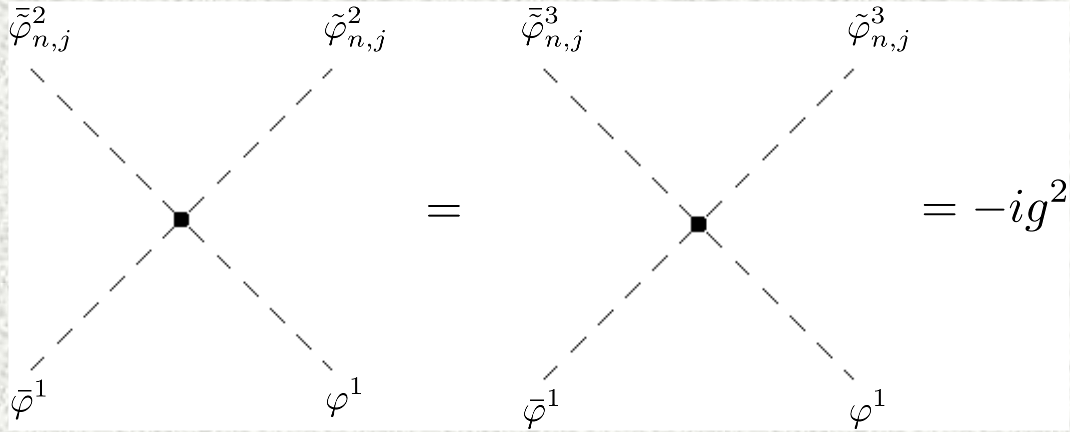
$$= -\frac{g\sqrt{\alpha(n+1)}}{\sqrt{2}}\eta^{\mu\nu}$$



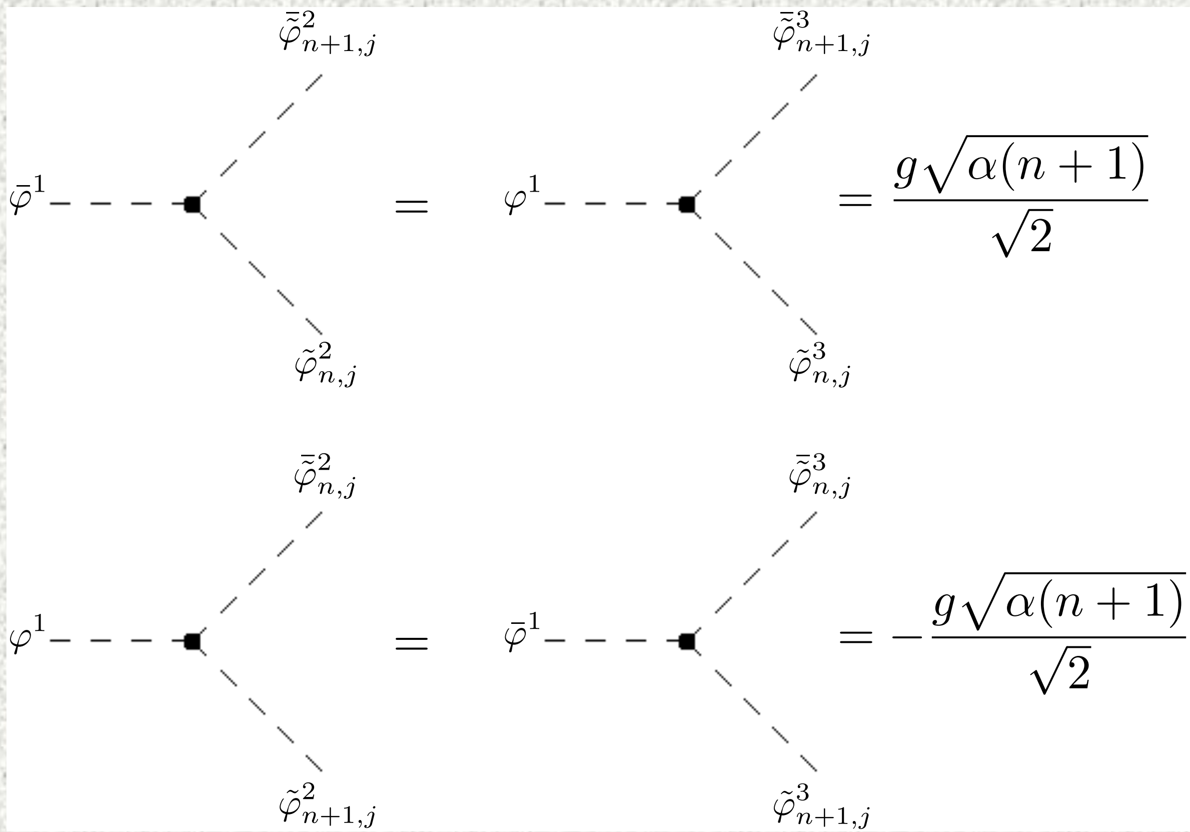
$$= \frac{g\sqrt{\alpha(n+1)}}{\sqrt{2}}\eta^{\mu\nu}$$

$\alpha = 2gf$

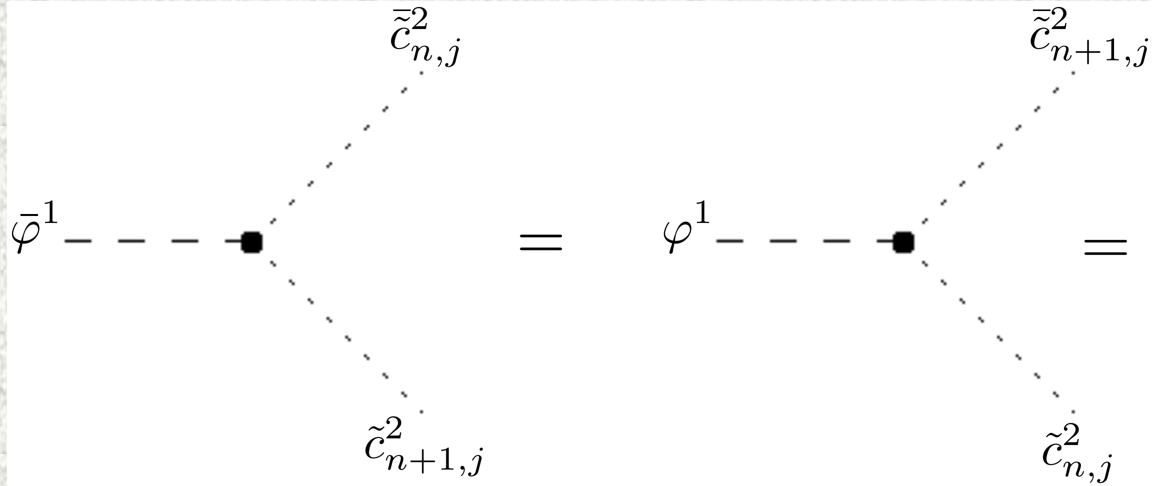
Scalar 4-point



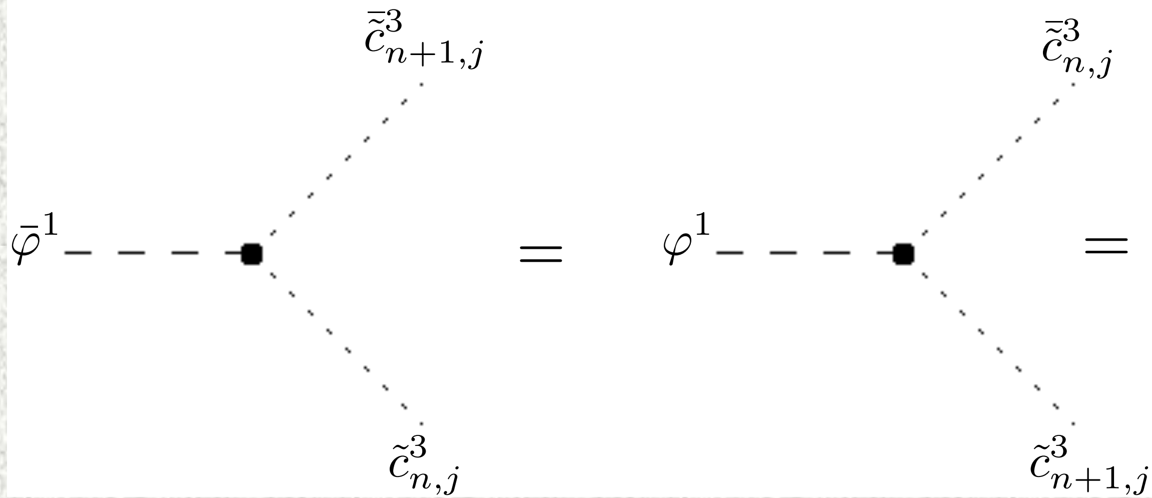
Scalar 3point



Ghost-ghost-scalar vertices



$$= \frac{g\xi\sqrt{\alpha(n+1)}}{\sqrt{2}}$$



$$= -\frac{g\xi\sqrt{\alpha(n+1)}}{\sqrt{2}}$$

$$\delta_T \phi^1 = \frac{f}{\sqrt{2}} \bar{\varepsilon}$$

constant shift

⇒ φ^1 NG boson of spontaneously broken translational symmetry in 5,6 directions

⇒ Only derivative interactions are allowed (Not only mass terms, but also potential forbidden)

Analogy: π meson

NG boson of spontaneously broken chiral symmetry ⇒ chiral Lagrangian

4D effective theory of 6D scalar QED on T^2 with flux

$$
\begin{aligned}
\mathcal{L}_{4D} = &-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \partial^\mu\bar{\varphi}\partial_\mu\varphi \\
&+ \sum_{n,j}\left(-D_\mu\overline{\Phi}_{n,j}D^\mu\Phi_{n,j} - \alpha\left(n+\frac{1}{2}\right)\overline{\Phi}_{n,j}\Phi_{n,j}\right. \\
&- ig\sqrt{2\alpha(n+1)}\overline{\varphi}\overline{\Phi}_{n+1,j}\Phi_{n,j} + ig\sqrt{2\alpha(n+1)}\varphi\overline{\Phi}_{n,j}\Phi_{n+1,j} - 2g^2\bar{\varphi}\varphi\overline{\Phi}_{n,j}\Phi_{n,j} \\
&\left.+ \kappa\bar{\varphi}\overline{\Phi}_{n,j}\Phi_{n,j} + \kappa\varphi\overline{\Phi}_{n,j}\Phi_{n,j} + \kappa\left\langle\phi\right\rangle_I\overline{\Phi}_{n,j}\Phi_{n,j} + \kappa\left\langle\bar{\phi}\right\rangle_I\overline{\Phi}_{n,j}\Phi_{n,j}\right),
\end{aligned}
$$