Link soliton in model with $U(1)_{B-L}$ and $U(1)_{PQ}$ symmetries

Yu Hamada (KEK)

arXiv: 2303.XXXXX (work in progress)

w/ M. Eto (Yamagata U.) and M. Nitta (Keio U.)

22nd Feb. 2023 @ Kagoshima workshop

$U(1)_{global} \times U(1)_{gauge}$ を自発的に破る模型では
変態的なソリトンができます

$U(1)_{PQ}$ $U(1)_{B-L}$

↑ ↑

$U(1)_{global} \times U(1)_{gauge}$ を自発的に破る模型では

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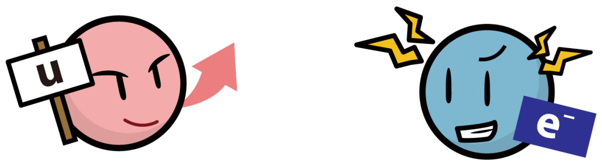
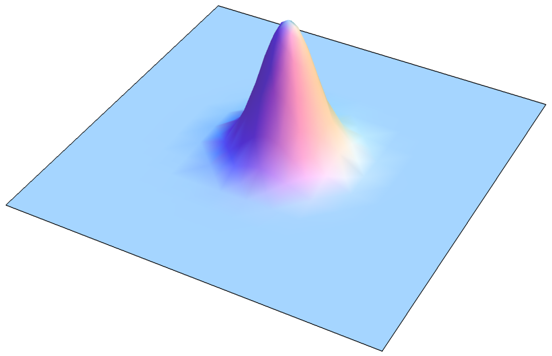
Introduction

Soliton

- **(素)粒子**：真空まわりの場のゆらぎ



- **ソリトン**：素粒子でない古典的励起 (エネルギーの``カタマリ")



津波

wikipedia
``神奈川沖浪裏"

``Collision of KdV solitons"
(from YouTube)

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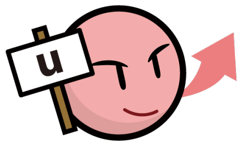

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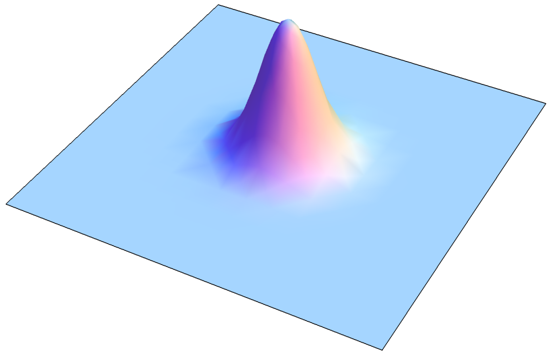

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Eg.) Abrikosov-Nielsen-Olesen string

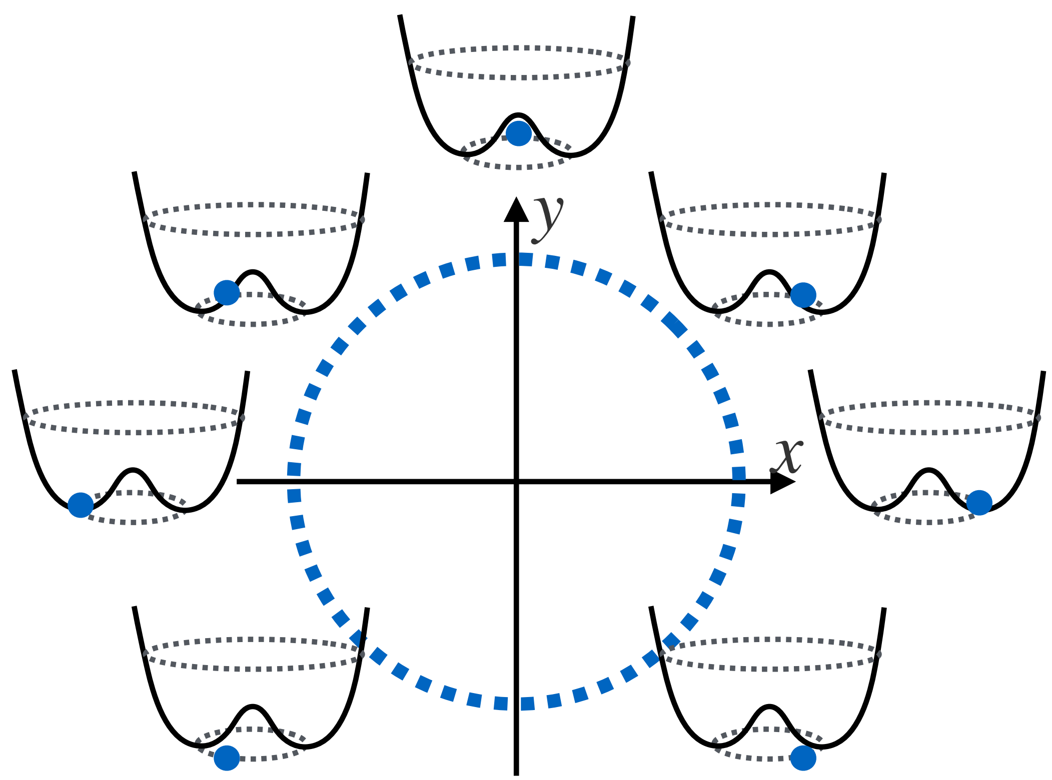
[Abrikosov '58]

[Nielsen-Olesen '73]

- 3+1 D Abelian-Higgs model $\langle \phi \rangle = v \ \rightarrow \ \cancel{U(1)}$

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu \phi|^2 + m^2|\phi|^2 - \lambda|\phi|^4$$

- z軸方向に一様な配位を仮定
- xy平面上で、各点で異なる真空の位相を選んでも良い



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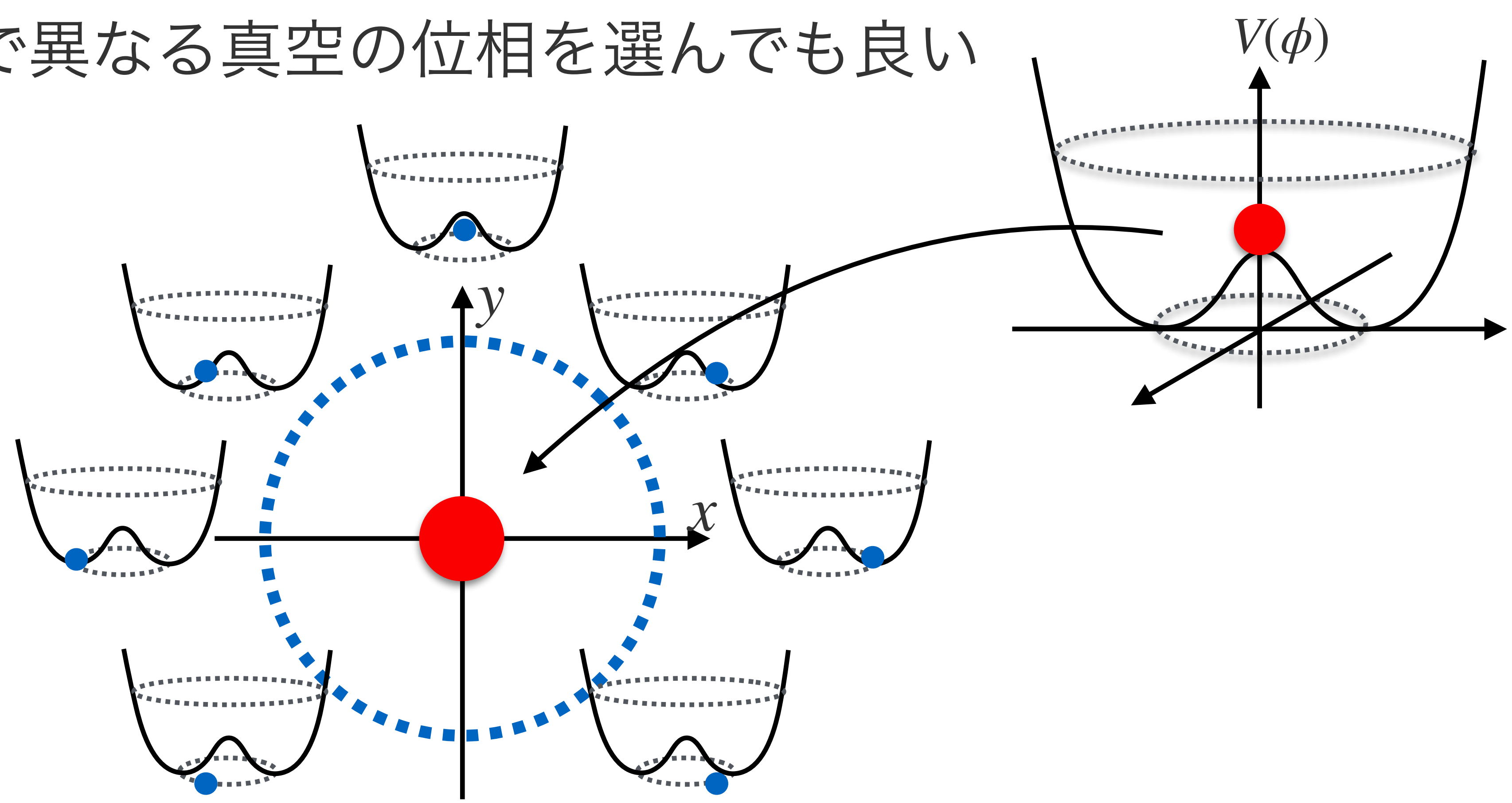

→ 中心は必ずpotentialの頂点にいる→**excitation (soliton)**

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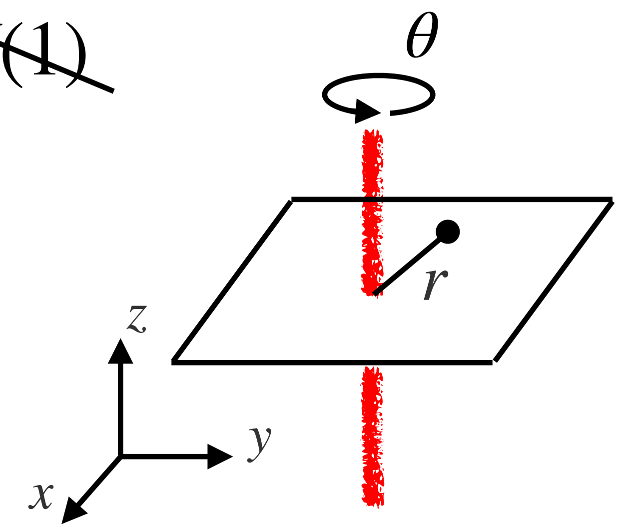

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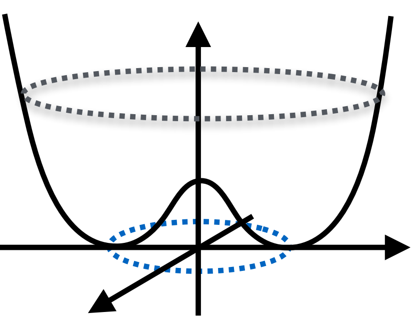
- **Field configuration:**



$$\phi(x) = v f(r) e^{i\theta} \qquad \vec{A}(x) = g^{-1} a(r)\, \vec{e}_\theta$$

ϕ's phase has winding # = 1

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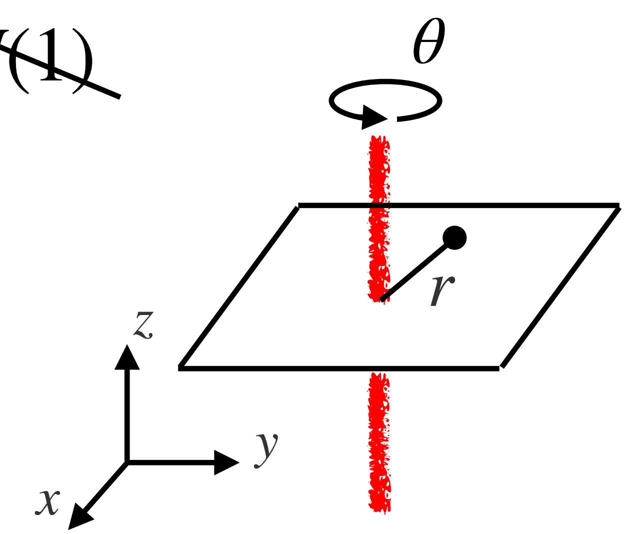

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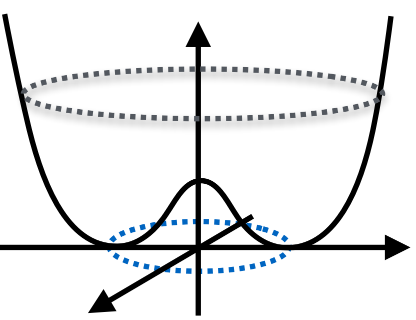

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$$f'' + \frac{1}{r}f' - \frac{(1-a)^2}{r^2}f - \frac{1}{2}\frac{\partial V}{\partial f} = 0$$

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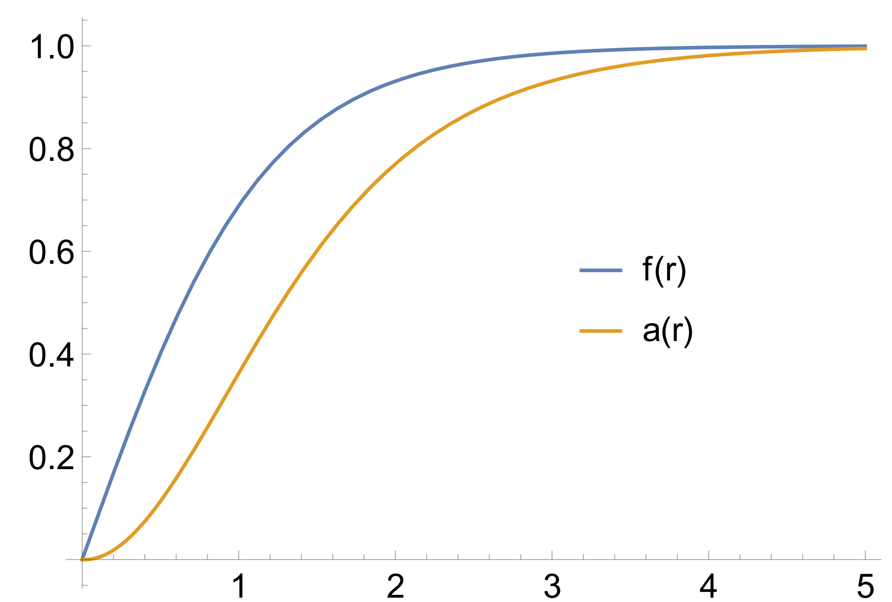

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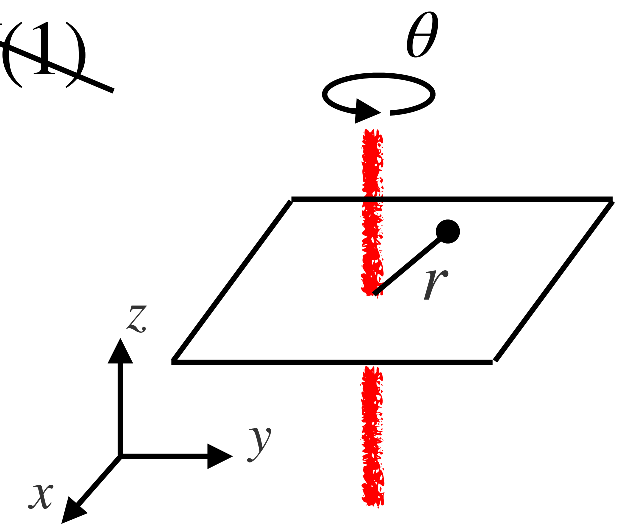

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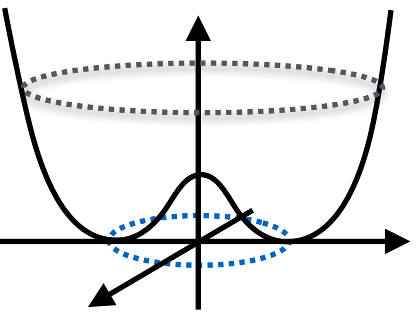

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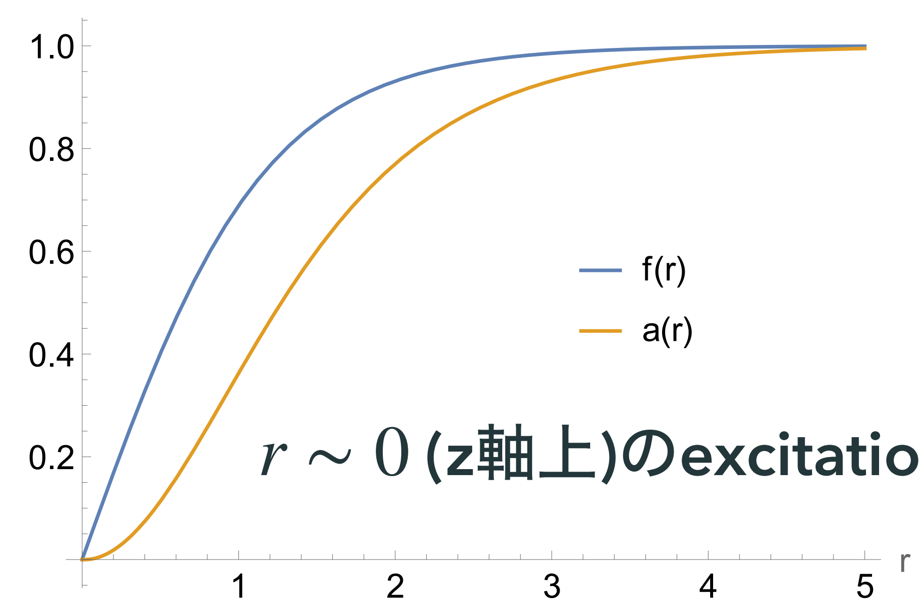

$r \sim 0$ **(z軸上)のexcitation**

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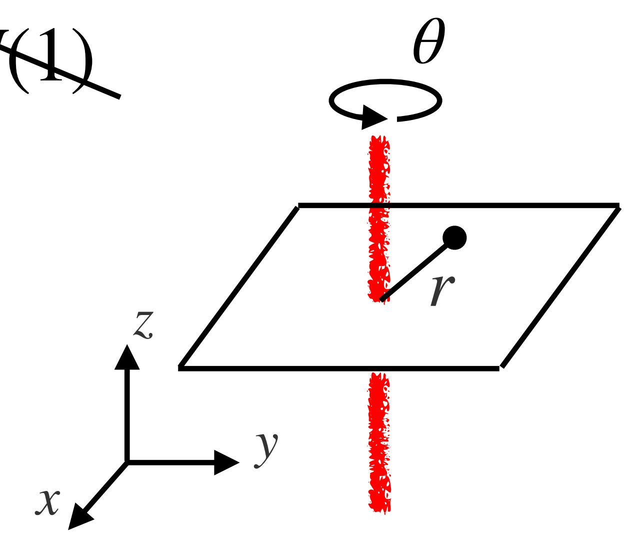

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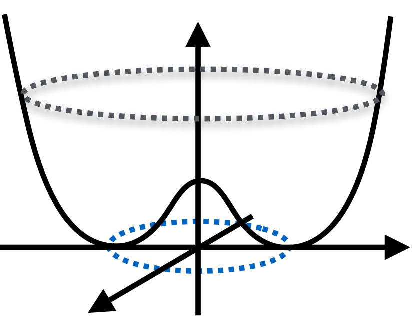

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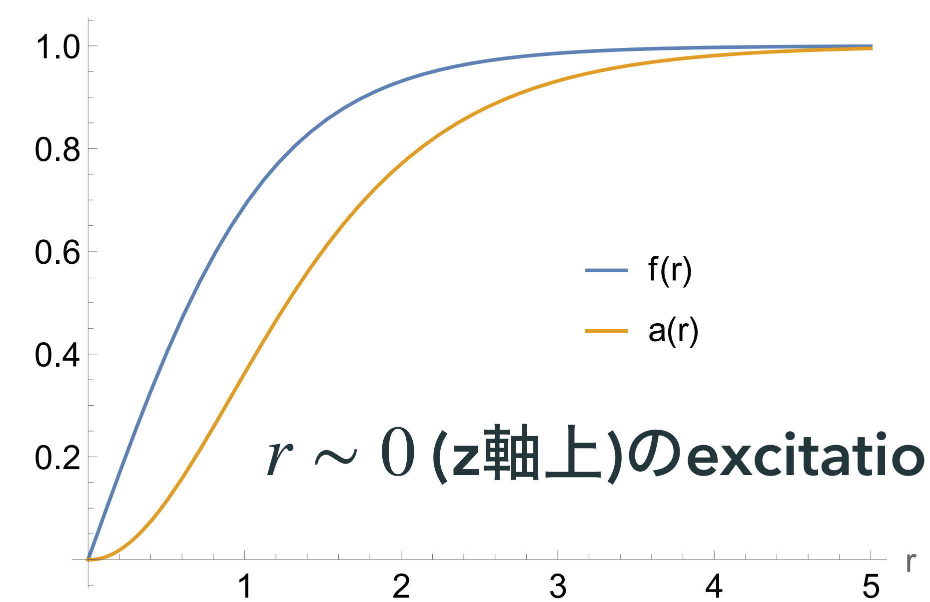
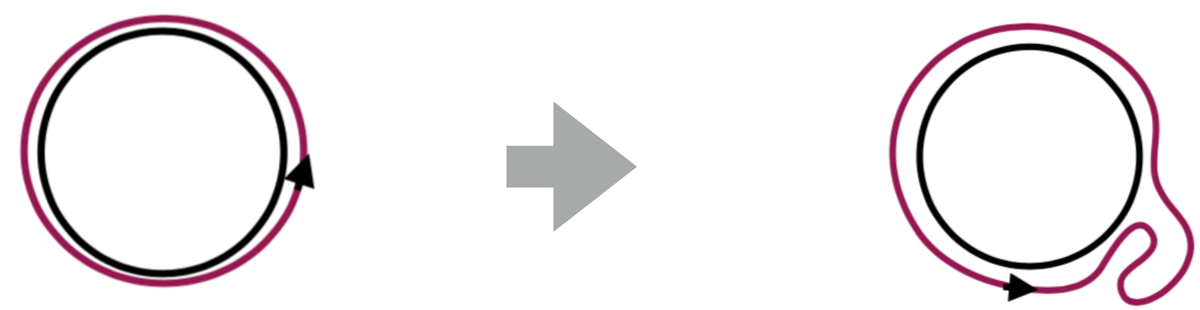


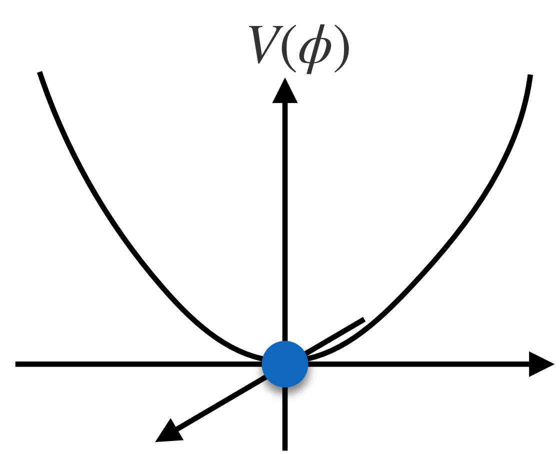
Quantized magnetic flux: $\int d^2x\, B = \oint d\vec{l}\cdot\vec{A} = 2\pi/g$

Topology of vacuum

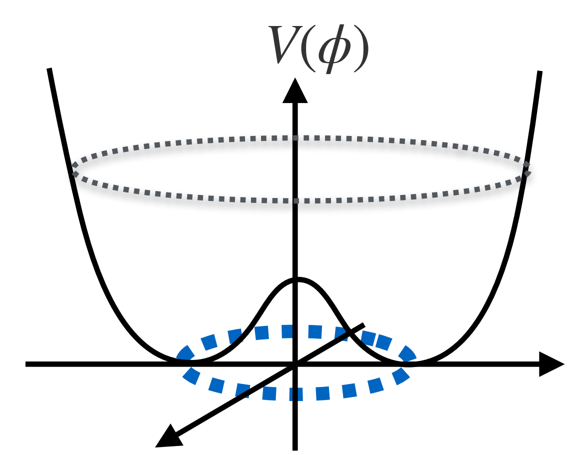
- このwindingはいかなる連続変形でも取り除けない →安定



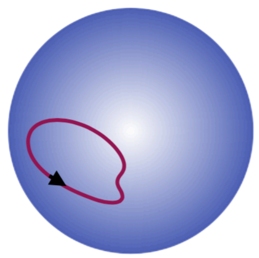
- 真空が ``**circle structure**'' であることが安定性を保証



point

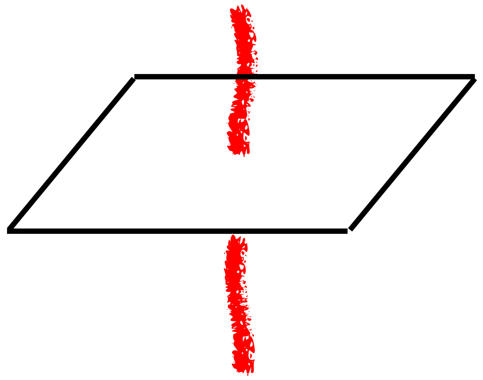


circle S^1



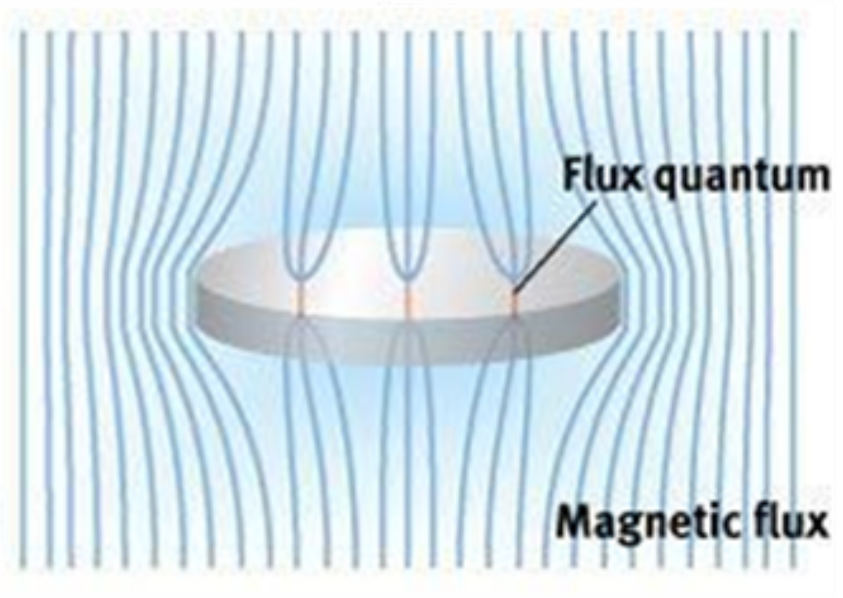
S^2

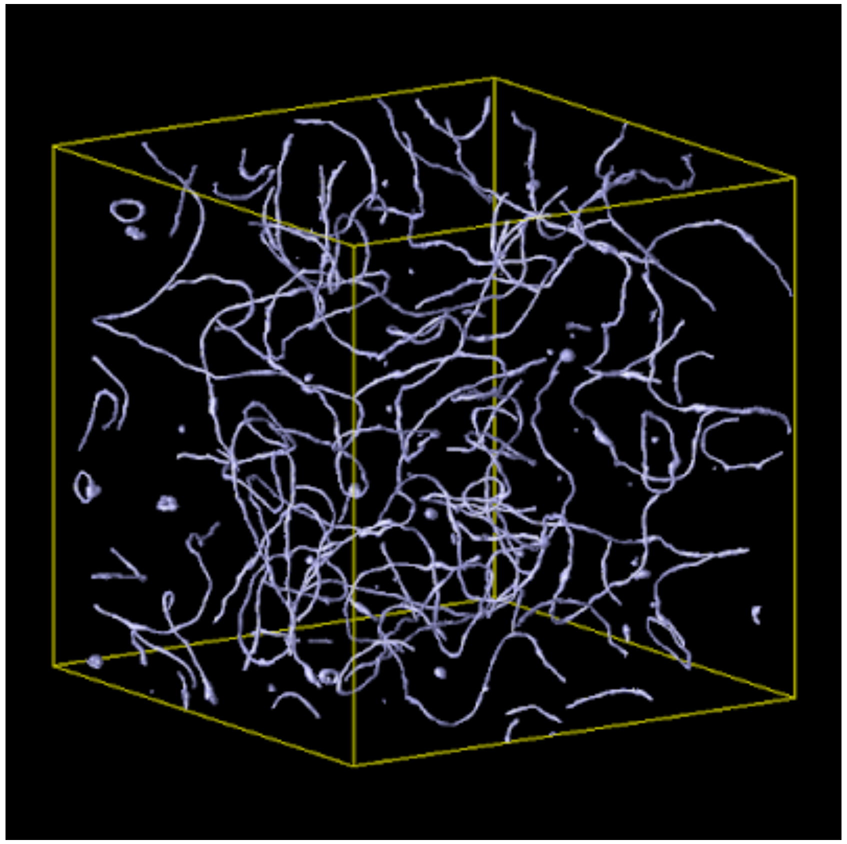
→このようなsolitonを**vortex string**と呼ぶ



Vortex string in many systems

- **Magnetic flux tube in superconductor**
    - characterize phases of supercond.
- Vortex string in the universe: **Cosmic string**
    - CMB observation
    - Gravitational wave
    - **strong evidence of new physics**, but haven't yet been discovered.
    - 昔からよく調べられている
    ('90sに流行った->最近また流行ってる?)





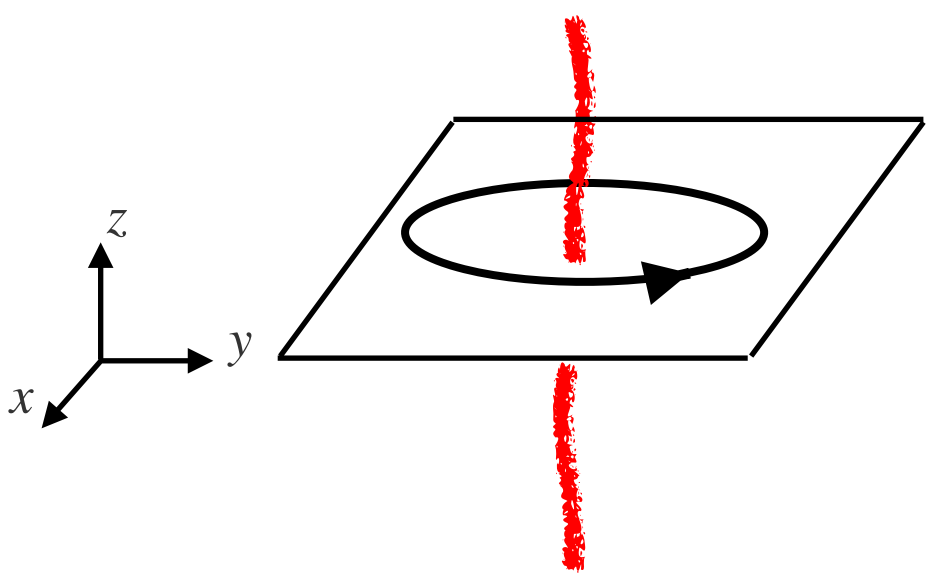
https://motls.blogspot.com/2017/12/crackpots-lies-about-cosmic-string.html

Global vs Local strings

- SSB of **gauged** $U(1)$ sym → **local** vortex string

→ **quantized magnetic flux:** $$\int d^2x\, B = 2\pi/g$$

- SSB of **global** $U(1)$ sym → **global** vortex string



→ **w/o magnetic flux**

$$\phi(x) = v\, f(r) e^{i\theta}$$

string周りでNG boson phaseが

0 から 2πに変化

EOM: $f'' + \frac{1}{r} f' - \frac{(1-\alpha)^2}{r^2} f - \frac{1}{2}\frac{\partial V}{\partial f} = 0$

Global vs Local strings

https://www.youtube.com/@PIKOTARO

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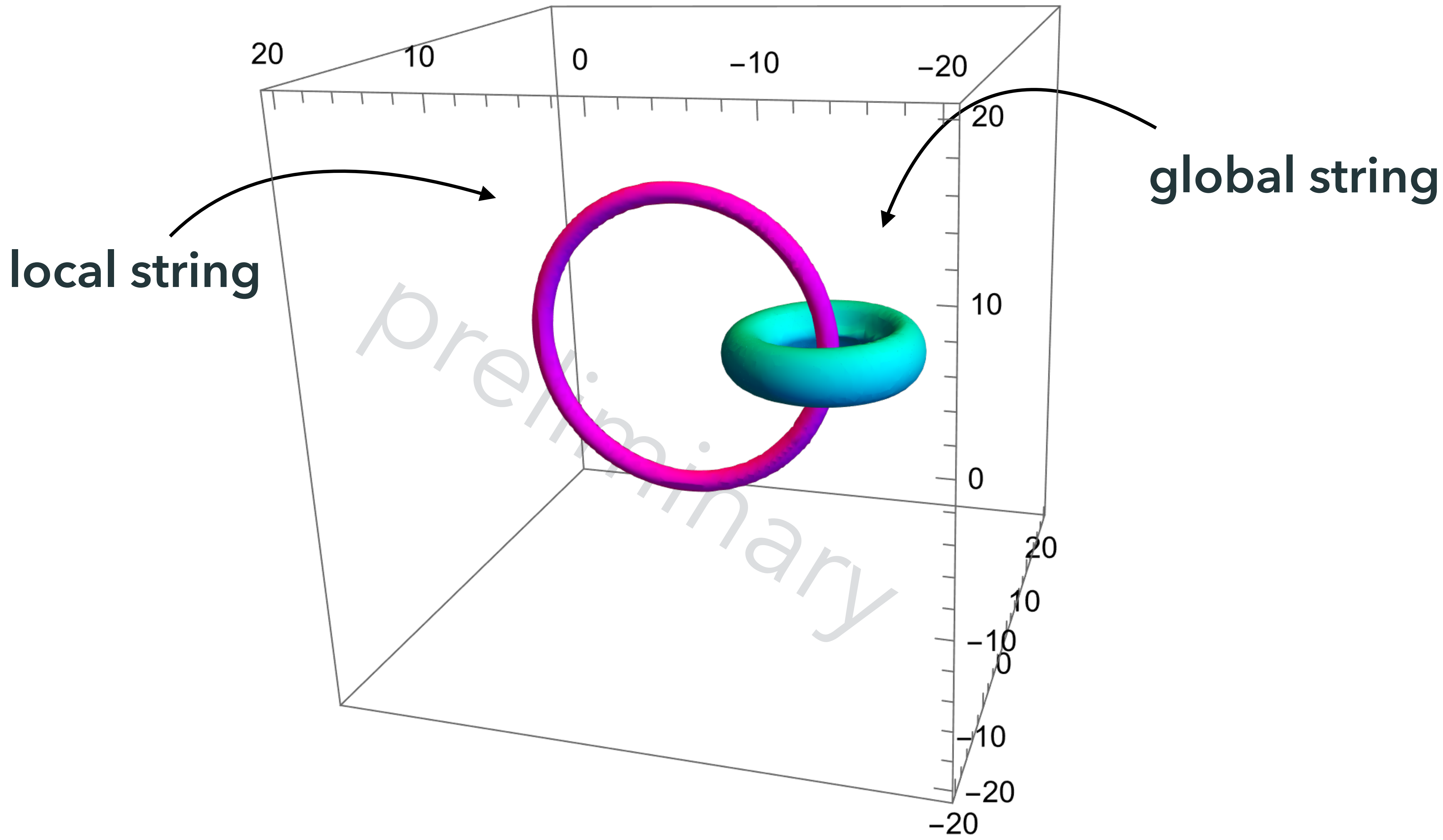
Link soliton



link soliton made of local & global strings!

Link soliton

- Massage of this talk:

$U(1)_{global} \times U(1)_{gauge}$ を自発的に破る模型では link solitonが存在する！

- Key: Chern-Simons coupling $\frac{c}{16\pi^2}\int d^4x\, aF\tilde{F}$
- motivativeなセットアップ:

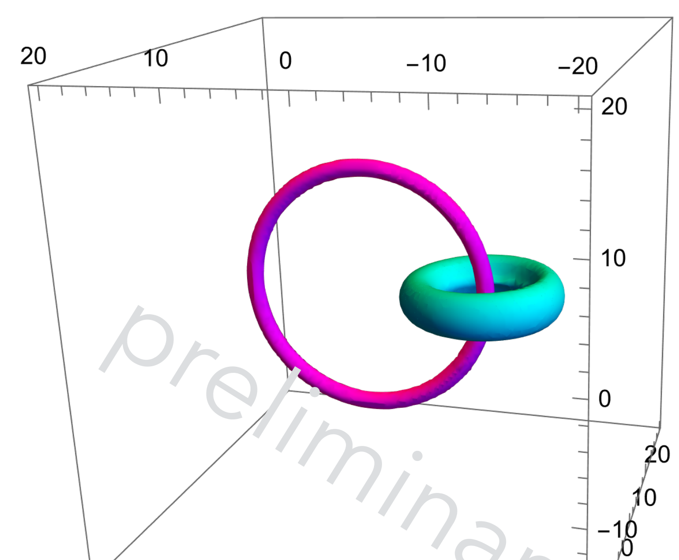
$$\begin{cases} U(1)_{global} = U(1)_{PQ} \\ U(1)_{gauge} = U(1)_{B-L} \end{cases}$$

QCD axion → strong CP & DM

RHν → Type-I seesaw, GUT

axion stringとB-L stringからなるlink

→バリオジェネシス、原子重力波での検出、GUTとの関係？ etc.



Plan of talk

- Introduction
- Link soliton
- Application to Baryogenesis
- Summary

Link soliton

The model

3+1D theory:

$$\mathscr{L} = |D_\mu \phi_1|^2 + |\partial_\mu \phi_2|^2 - \frac{1}{4g^2} F_{\mu\nu}^2 - V(\phi_1, \phi_2)$$

$$D_\mu \phi_1 = (\partial_\mu - igA_\mu)\phi_1$$

$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

- Symmetries:

$$U(1)_{gauge} : \phi_1 \to e^{i\theta_1} \phi_1 \qquad U(1)_{global} : \phi_2 \to e^{i\theta_2} \phi_2$$

- For $\kappa > 0$ & $\lambda > 0$, both symmetries are broken at the vacuum:

$$\langle \phi_1 \rangle = v_1,\ \langle \phi_2 \rangle = v_2$$

→ local string (ϕ_1 string) & global string (ϕ_2 string)

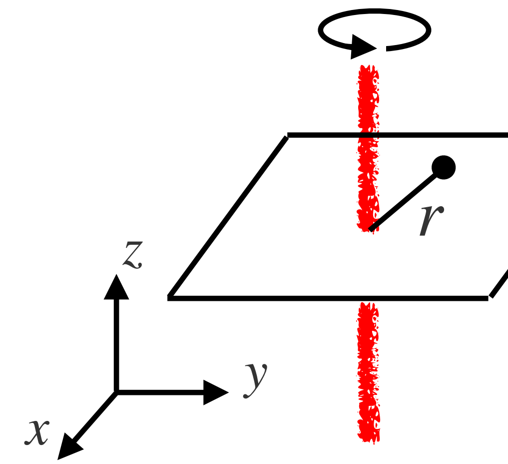
ϕ_1 string & ϕ_2 string

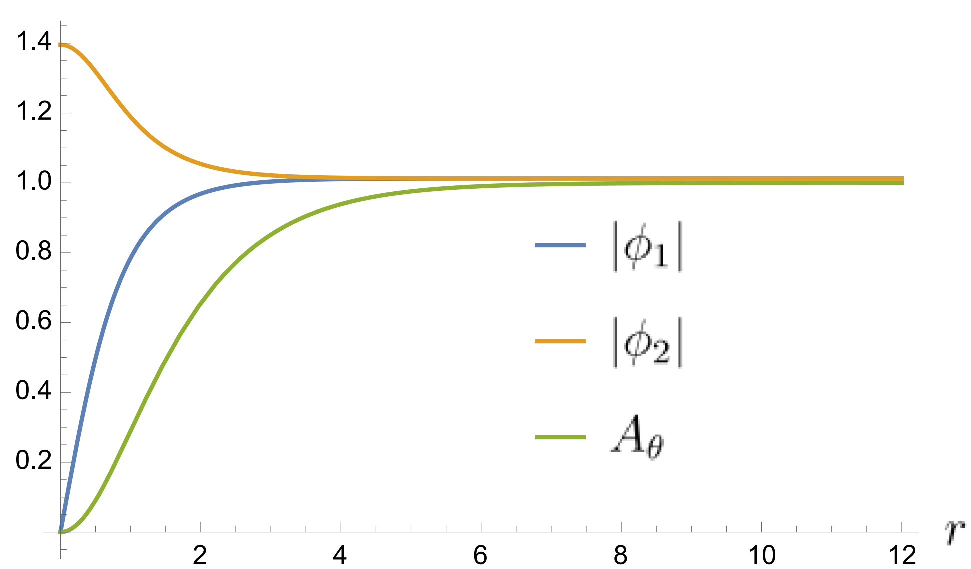
- **Field configuration for ϕ_1 string (local):**

$$\phi_1(x) = v_1 f_1(r) e^{i\theta} \qquad \phi_2(x) = v_2 f_2(r) \qquad \vec{A}(x) = g^{-1} a(r) \, \vec{e_\theta}$$

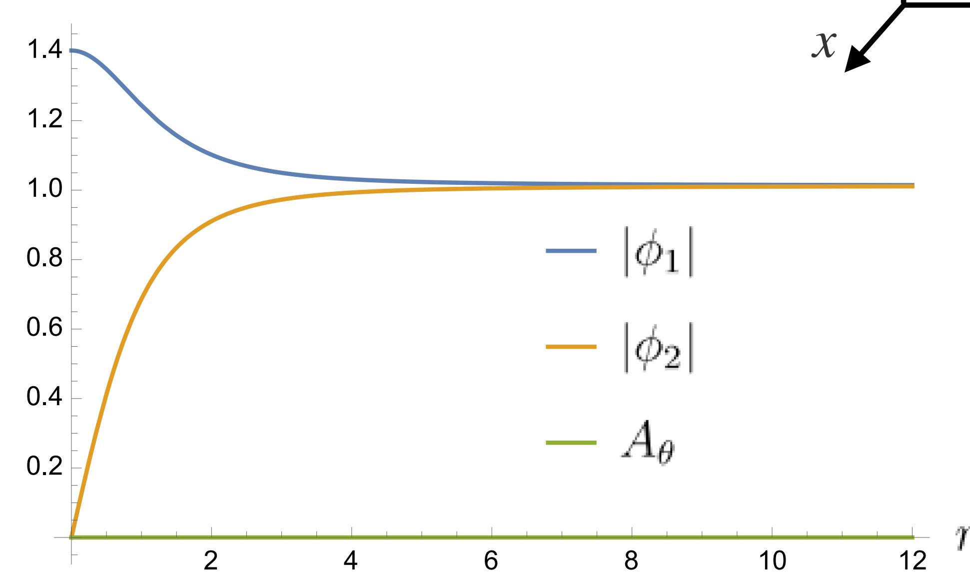
- **Field configuration for ϕ_2 string (global):**

$$\phi_1(x) = v_1 h_1(r) \qquad \phi_2(x) = v_2 h_2(r) e^{i\theta} \qquad A_\mu(x) = 0$$





ϕ_1 string



ϕ_2 string

Chern-Simons coupling

3+1D theory:

Chern-Simons coupling

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$$a \equiv -i \arg(\phi_2)$$

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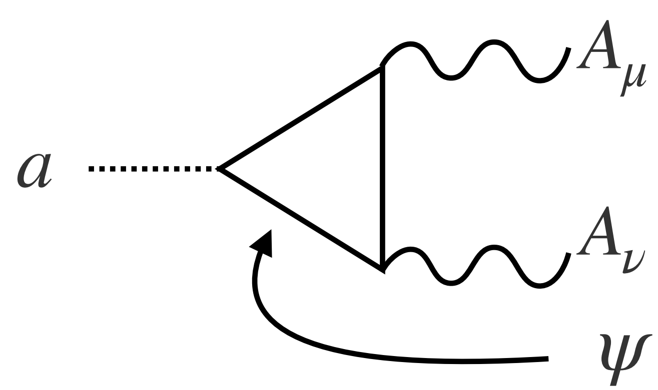
- At the broken phase, CS coupling is induced by triangle anomaly.



dependent on matter sector

$$\Rightarrow c = \sum_f Q^f_{global} (Q^f_{gauge})^2$$

taken as free parameter

Chern-Simons coupling

3+1D theory:

Chern-Simons coupling

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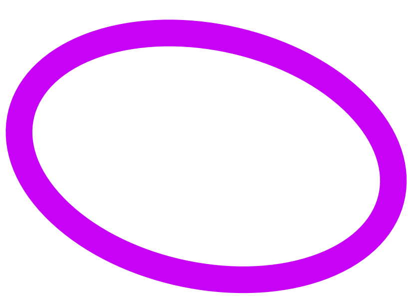
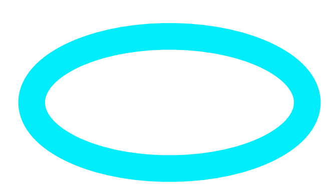
- CS couplingは単独のstringには効かない

ϕ_1 **string**



ϕ_2 **string**

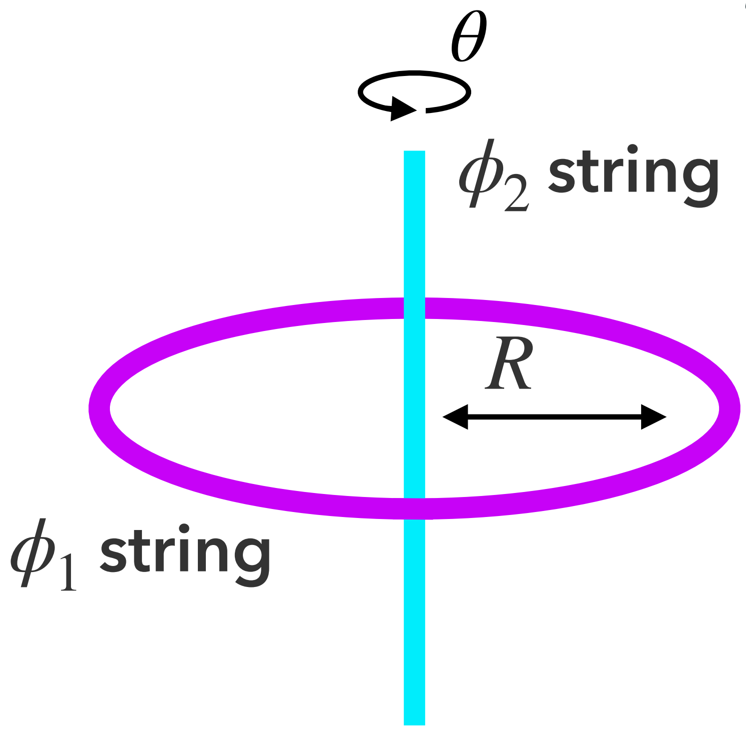


→ こういうループは縮んで消える

Charged string

Rewriting CS coupling: $$\frac{c}{16\pi^2} aF_{\mu\nu}\tilde{F}^{\mu\nu} \longrightarrow -\frac{c}{16\pi^2}\left(\partial_i a\right) A_0 B^i$$

linkしてるとき、$\partial_i a$ と B_i は同じ向き $\Rightarrow \left(\partial_i a\right) A_0 B^i = \frac{1}{R} A_0 |\vec{B}|$



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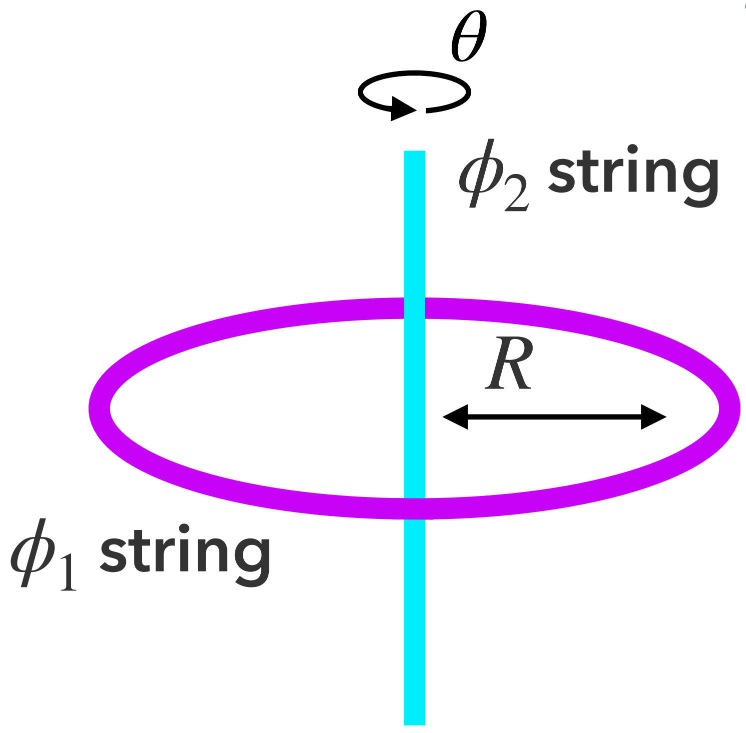

Gauss law:

$$\frac{\delta \mathscr{L}}{\delta A_0} = \partial_i E_i - g^2 J^0 + \frac{g^2 c}{16\pi^2 R} |\vec{B}| = 0$$

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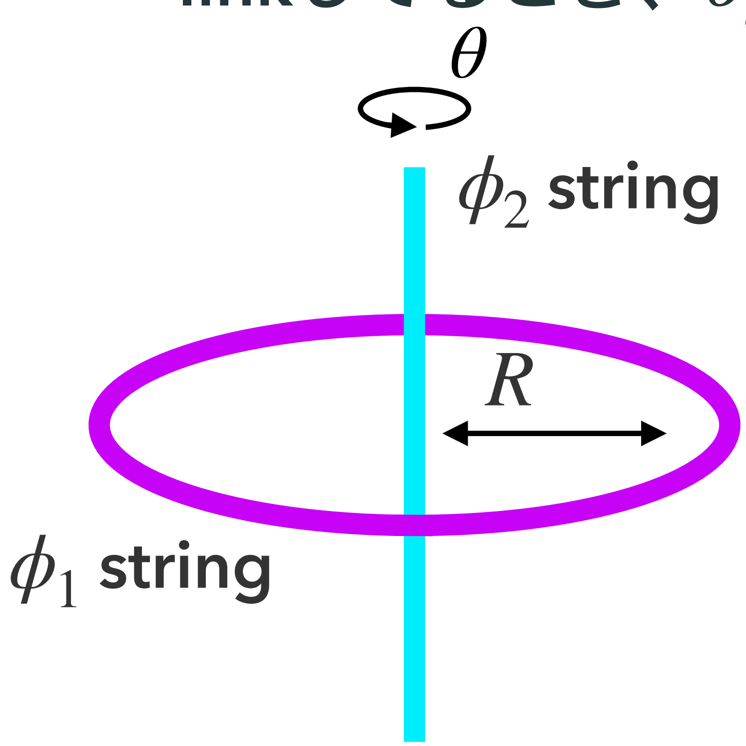

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$$\Rightarrow \int d^3x\, J^0 = 2\pi R \int d^2x \frac{c}{16\pi^2 R} |\vec{B}| = \frac{c}{4g}$$

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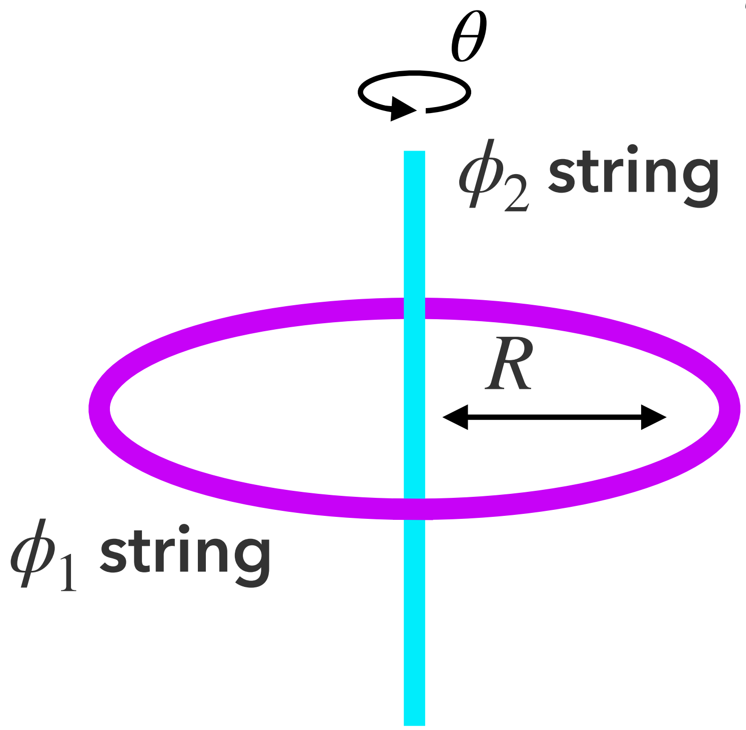

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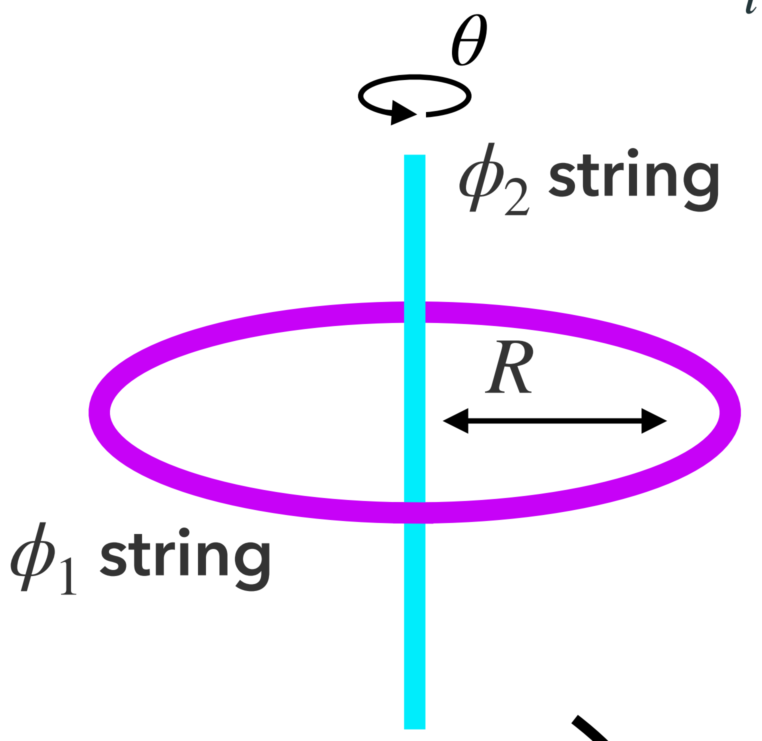
$$\Rightarrow \int d^3x\, J^0 = 2\pi R \int d^2x \frac{c}{16\pi^2 R}|\vec{B}| = \frac{c}{4g}$$

ϕ_1 stringが電荷を持つ → 電場による反発で縮まない

Charged string

Rewriting CS coupling: $\frac{c}{16\pi^2} a F_{\mu\nu}\tilde{F}^{\mu\nu} \longrightarrow -\frac{c}{16\pi^2}\left(\partial_i a\right) A_0 B^i$

linkしてるとき、$\partial_i a$ と B_i は同じ向き $\Rightarrow \left(\partial_i a\right) A_0 B^i = \frac{1}{R} A_0 |\vec{B}|$



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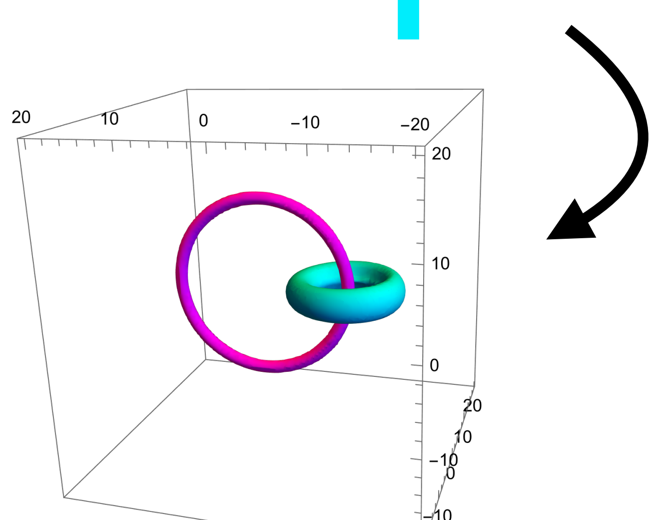


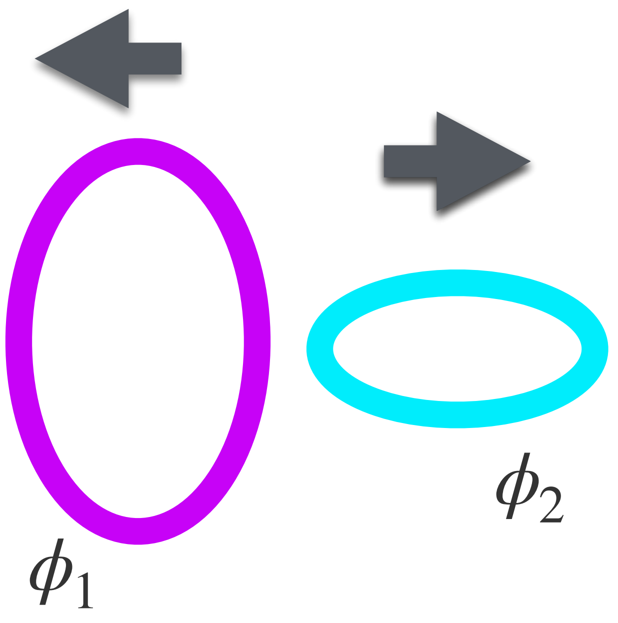
ϕ_1 stringが電荷を持つ → 電場による反発で縮まない

Link stability

- linkを外してdecayできる？

  → $\lambda \gg g^2, \kappa, \chi$ と取っておけば外れない

$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

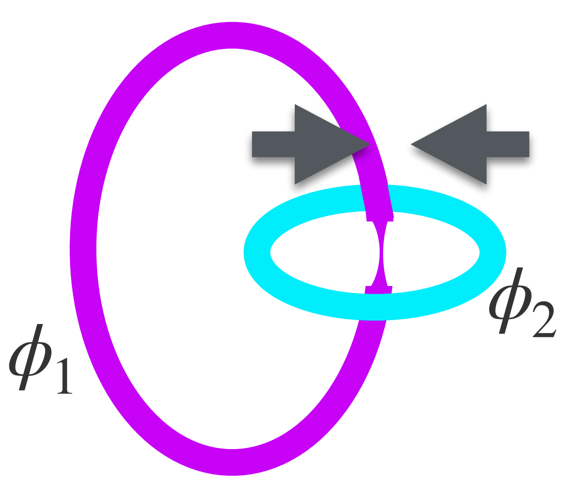


→ non-linear σ-model w/ $O(4)$ sym. → $O(3)$ sym.

link = skyrmion [Gudnason-Nitta '20]

- ϕ_2 stringは縮んでdecayできる？

  → $v_2/v_1 \ll 1$ と取っておけば縮まない



この2つの条件のもとで古典的に安定

ただし量子効果で崩壊できる (後述)

Numerical calculation

Energy:

$$\mathcal{E} = |D_i\phi_1|^2 + |\partial_i\phi_2|^2 + V(\phi_1, \phi_2) + \frac{1}{2g^2}(\partial_i A_j)^2$$

$$-g^2|\phi_1|^2 A_0^2 - \frac{1}{2g^2}(\partial_i A_0)^2 - \frac{c}{16\pi^2}\, aF_{\mu\nu}\tilde{F}^{\mu\nu}$$

- Not positive definite → remove A_0 by solving Gauss law:

$$\frac{\delta\mathcal{L}}{\delta A_0} = \partial^2 A_0 - 2g^2|\phi_1|^2 A_0 + \frac{g^2 c}{16\pi^2}(\vec{\nabla} a)\cdot\vec{B} = 0$$

Numerical calculation

Energy:

$$\mathcal{E} = |D_i\phi_1|^2 + |\partial_i\phi_2|^2 + V(\phi_1, \phi_2) + \frac{1}{2g^2}(\partial_i A_j)^2$$

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$\sim M_c^2 A_0 \sim \mathcal{O}(g^2 v_1^2/c^2)\, A_0$ for large c

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$\sim M_c^2 A_0 \sim \mathcal{O}(g^2 v_1^2/c^2)\, A_0$ for large c

$$\therefore A_0 \approx \frac{g^2 c}{16\pi^2}\frac{(\vec{\nabla} a)\cdot\vec{B}}{2g^2|\phi_1|^2}$$

Numerical calculation

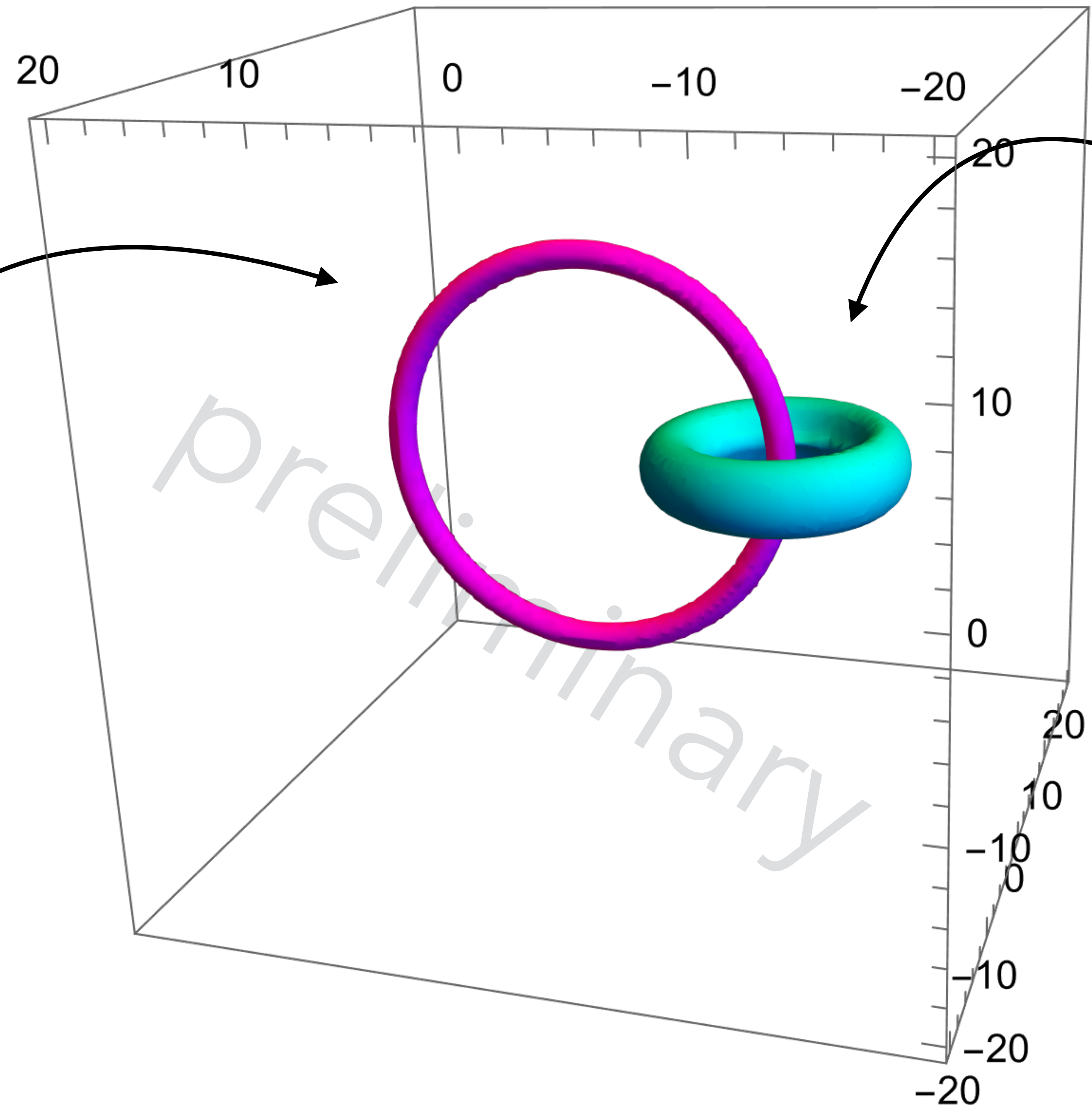
Energy:

$$\mathcal{E} = |D_i\phi_1|^2 + |\partial_i\phi_2|^2 + V(\phi_1, \phi_2) + \frac{1}{2g^2}(\partial_i A_j)^2 + \left(\frac{g^2 c}{16\pi^2}\right)^2 \frac{\left((\vec{\nabla} a)\cdot \vec{B}\right)^2}{2g^2|\phi_1|^2}$$

- positive definite -> no obstacle
- Minimizing energy via non-linear conjugate gradient method
- CPU 3584-cores parallelizing on YITP computer cluster
- lattice spacing =$0.2/gv_1$, $N = 200^3$, converged w/ O(1) days

Numerical solution

ϕ_1 string (local)

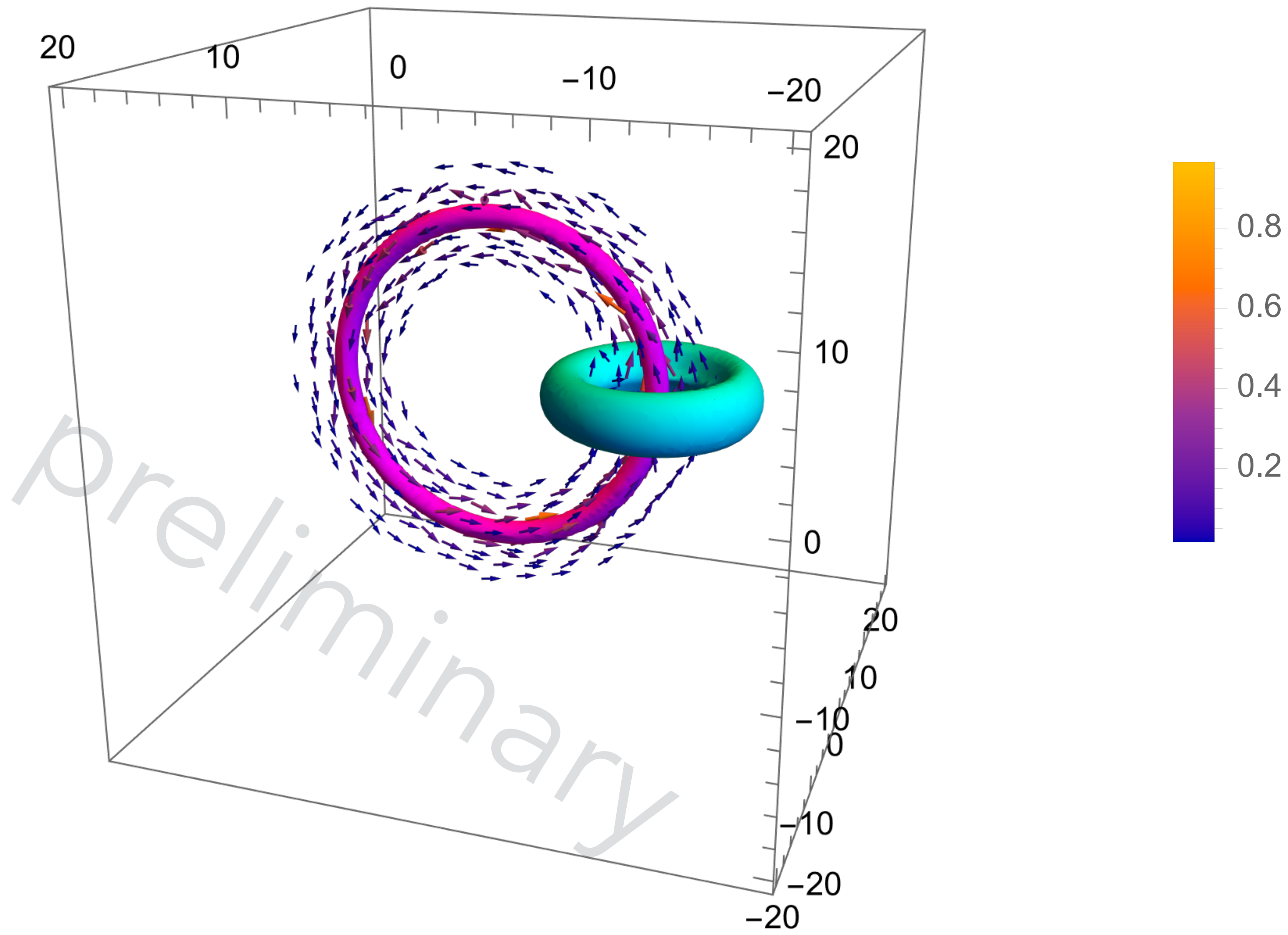


ϕ_2 string (global)

$\lambda/g^2 = 100,\ \chi/g^2 = 19.5,\ \kappa/g^2 = 0.1,\ v_2/v_1 = 0.05 \qquad g^2 c/(16\pi^2) = 16$

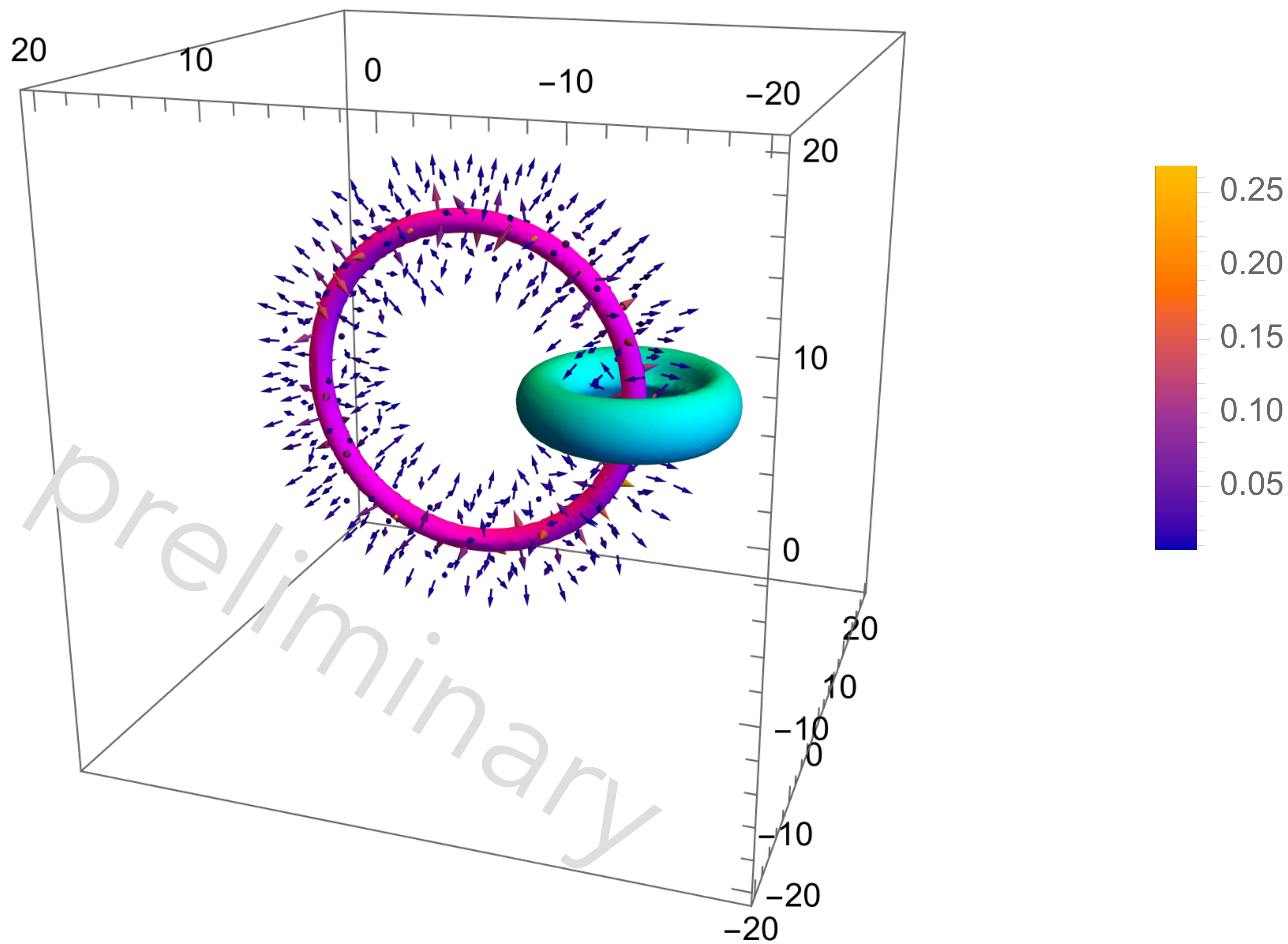
Magnetic field

$\vec{B}$



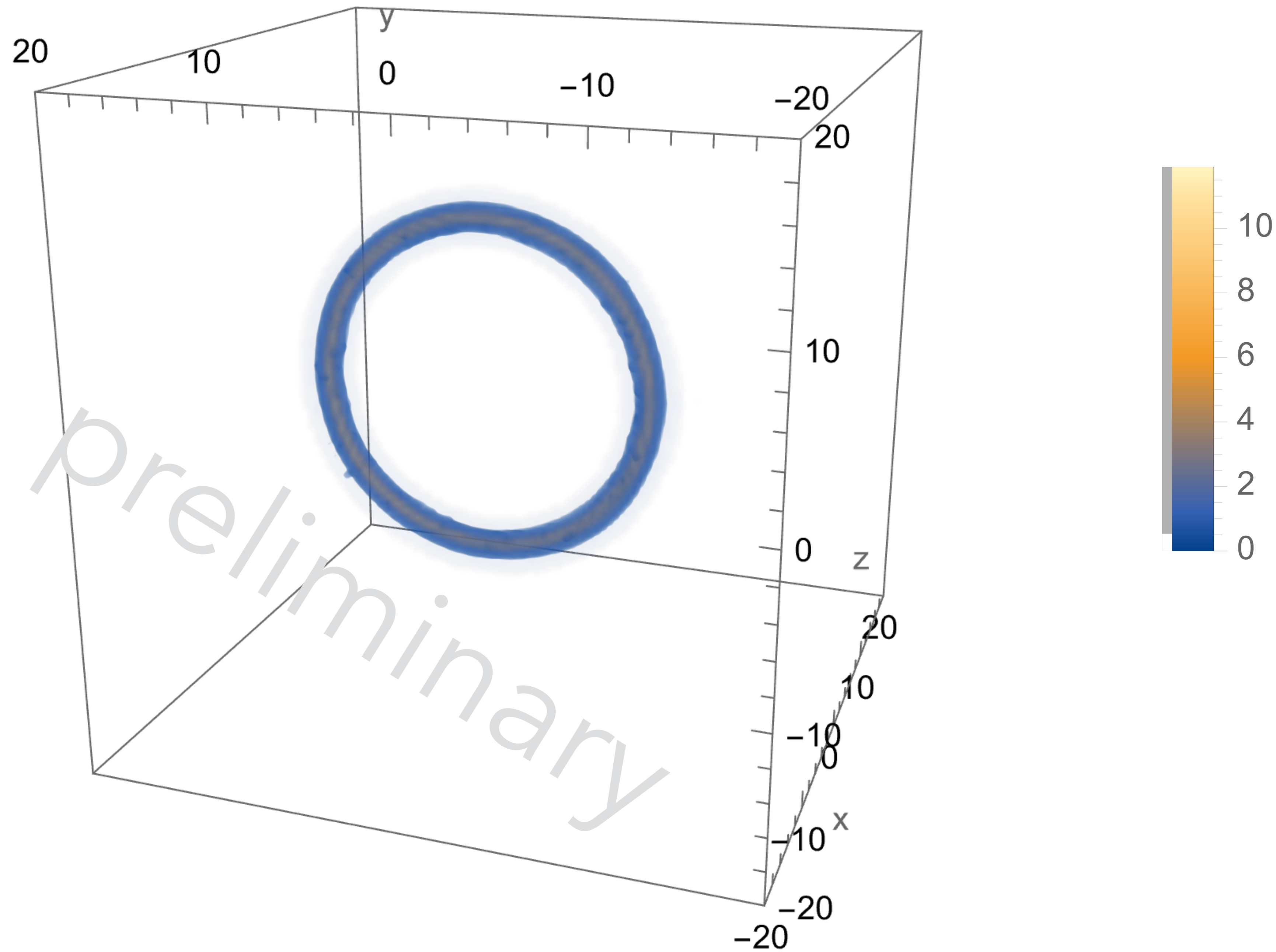
Electric field

$$\vec{E} = \vec{\nabla} A_0$$



Energy

Energy is dominated by ϕ_1 string



total energy: $E \sim 504 g v_1$

Plan of talk

- Introduction
- Link soliton
- Application to Baryogenesis
- Summary

Application to Baryogenesis

General setup

More general charge assignment:

$$\mathscr{L} = |D_\mu\phi_1|^2 + |D_\mu\phi_2|^2 - \frac{1}{4g^2}F_{\mu\nu}^2 - V(\phi_1, \phi_2)$$

$$D_\mu\phi_1 = (\partial_\mu - igq_1A_\mu)\phi_1 \qquad D_\mu\phi_2 = (\partial_\mu - igq_2A_\mu)\phi_2 \qquad q_2/q_1 \ll 1$$

$$V(\phi) = \lambda\left(|\phi_1|^2 + |\phi_2|^2 - \mu^2\right)^2 - \kappa|\phi_1|^2|\phi_2|^2 + \chi|\phi_2|^4$$

- Symmetries:

$$U(1)_{gauge} : \begin{cases} \phi_1 \to e^{iq_1\theta_1}\phi_1 \\ \phi_2 \to e^{iq_2\theta_1}\phi_2 \end{cases} \qquad U(1)_{global} : \begin{cases} \phi_1 \to e^{-iq_2\theta_1}\phi_1 \\ \phi_2 \to e^{iq_1\theta_1}\phi_2 \end{cases}$$

General setup

More general charge assignment:

$$\mathcal{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 - \frac{1}{4g^2} F_{\mu\nu}^2 - V(\phi_1, \phi_2)$$

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$$U(1)_{gauge} : \begin{cases} \phi_1 \to e^{iq_1\theta_1}\phi_1 \\ \phi_2 \to e^{iq_2\theta_1}\phi_2 \end{cases} \qquad U(1)_{global} : \begin{cases} \phi_1 \to e^{-iq_2\theta_1}\phi_1 \\ \phi_2 \to e^{iq_1\theta_1}\phi_2 \end{cases}$$

→ Also ϕ_2 string contains magnetic flux, but the solution is almost the same.

General setup

More general charge assignment:

$$\mathscr{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 - \frac{1}{4g^2} F_{\mu\nu}^2 - V(\phi_1, \phi_2) + \frac{c}{16\pi^2} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$D_\mu \phi_1 = (\partial_\mu - i g q_1 A_\mu)\phi_1 \qquad D_\mu \phi_2 = (\partial_\mu - i g q_2 A_\mu)\phi_2 \qquad q_2/q_1 \ll 1$$

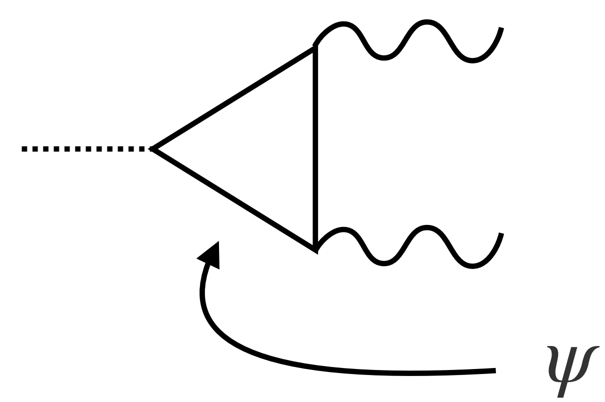
$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

- Definition of "axion" is more complicated

$$a \equiv \frac{1}{i\sqrt{q_2^2 v_1^2 + q_1^2 v_2^2}} \left(-q_2 v_1 \arg(\phi_1) + q_1 v_2 \arg(\phi_2) \right)$$

- Triangle anomaly is also complicated

  → c will be taken as free parameter.



The model

$$\mathcal{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 - \frac{1}{4g^2} F_{\mu\nu}^2 - V(\phi_1, \phi_2) \ + \frac{c}{16\pi^2} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

- Natural setup: $U(1)_{gauge} = U(1)_{B-L}$ & $U(1)_{global} = U(1)_{PQ}$

  Type-I seesaw → ν-mass

  QCD axion → strong CP & Dark matter

  $\Rightarrow v_1 \sim v_2 \sim 10^{9-12}\,\mathrm{GeV}$

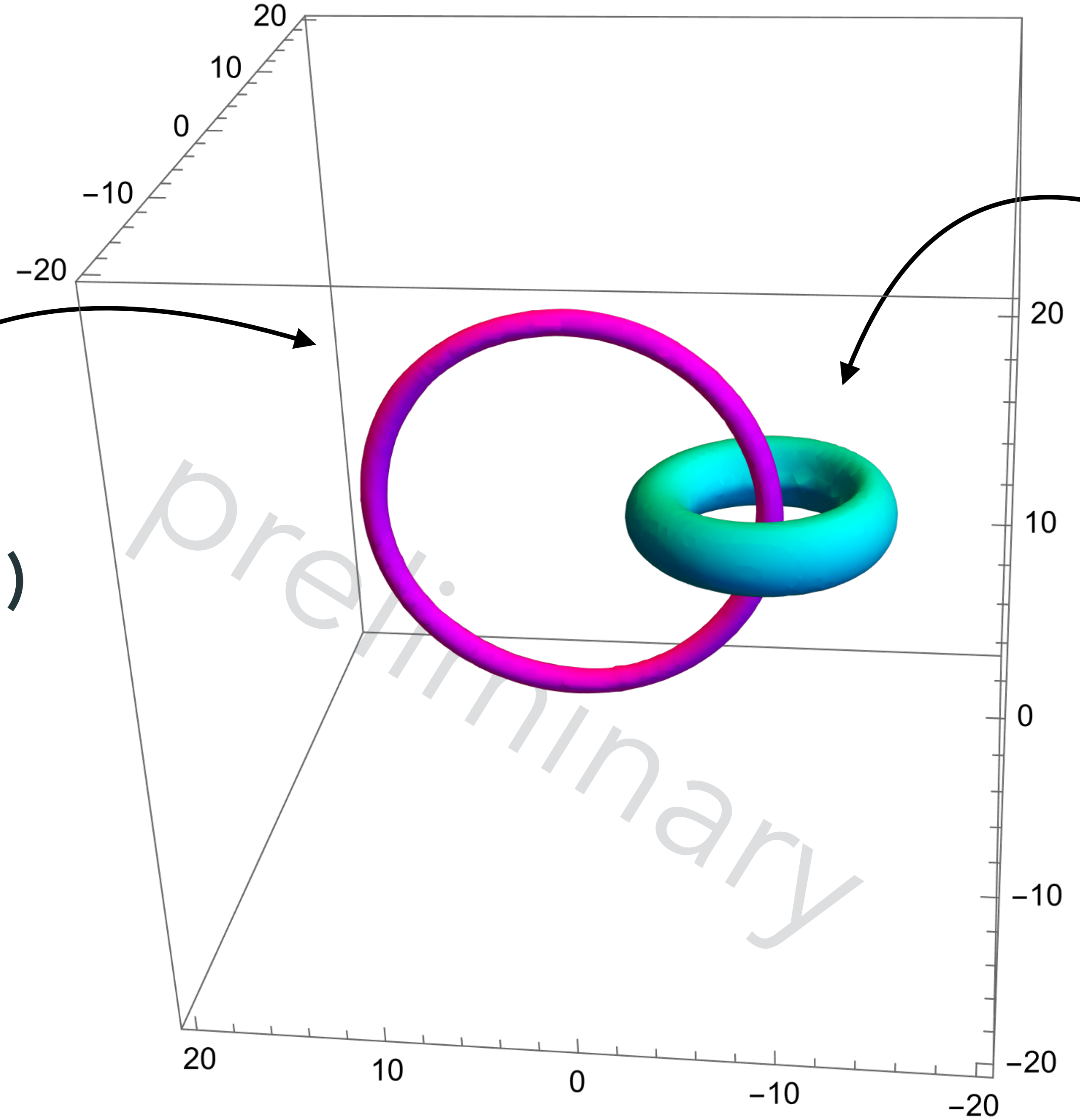
- Axion quality problem can be avoided by gauged PQ mechanism.

  $\Rightarrow q_1 = 1,\ q_2 = 0.1$ [Fukuda-Ibe-Suzuki-Yanagida '17]

- Assume kinetic mixing with $U(1)_Y$ in SM: $\mathcal{L} \supset \frac{\epsilon}{2} Y_{\mu\nu} F^{\mu\nu}$

Numerical solution

Solution is almost same as that w/ $q_2 = 0$.



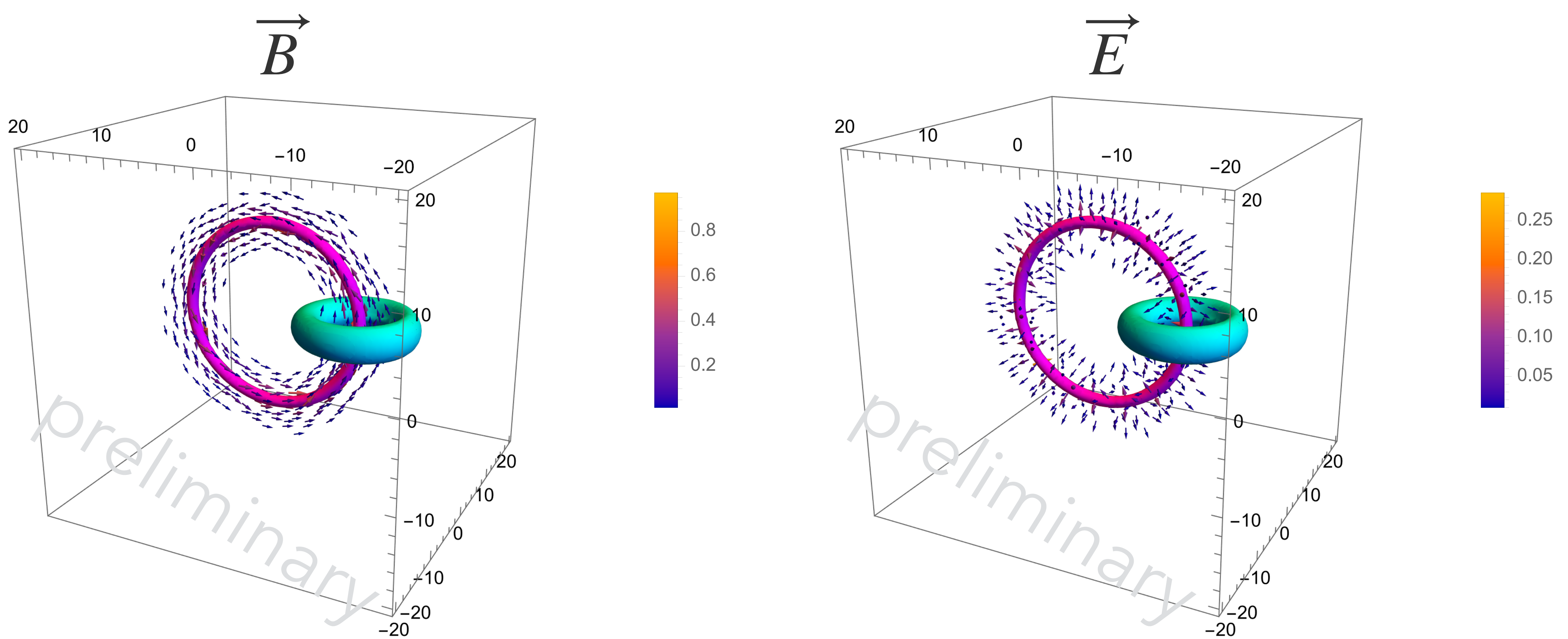
ϕ_1 **string**
(almost local)

ϕ_2 **string**
(almost global)

$\lambda/g^2 = 100,\ \chi/g^2 = 19.5,\ \kappa/g^2 = 0.1,\ v_2/v_1 = 0.05 \qquad g^2 c/(16\pi^2) = 16 \qquad q_2/q_1 = 0.1$

Magnetic & electric field

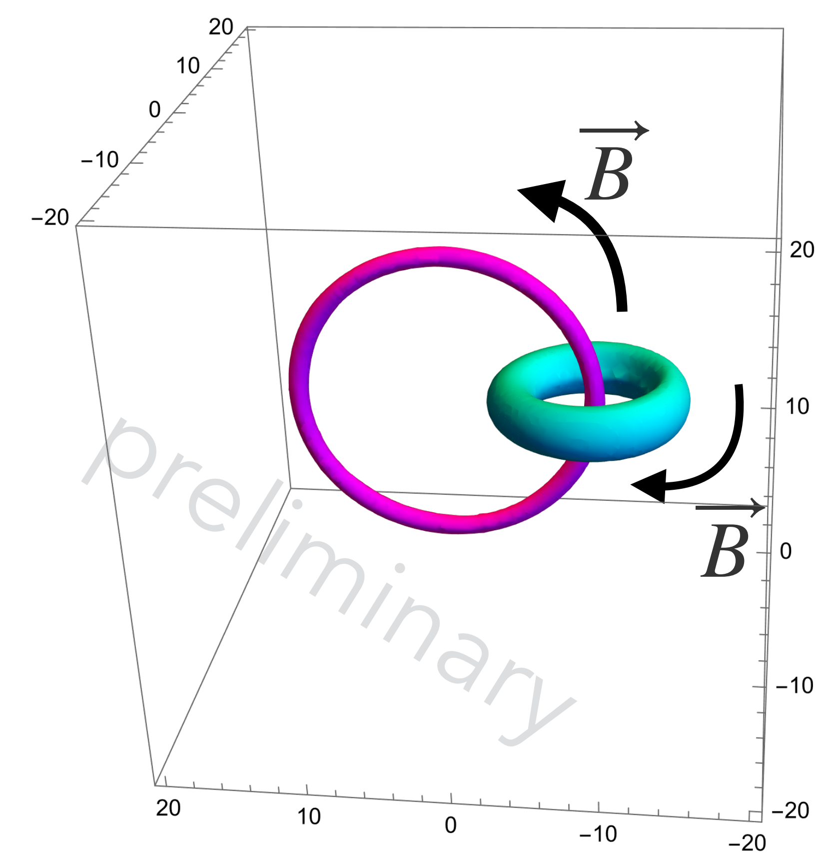
- Solution is almost same as that w/ $q_2 = 0$.
- But note that ϕ_2 string also contains small B & E fluxes (cannot be seen).



Helical magnetic field

- Since the magnetic fluxes are linked, this soliton has finite helicity (Chern-Simons number):

$$N_{CS}[A] \equiv \frac{1}{16\pi^2}\int d^3x\, AdA = \frac{1}{16\pi^2}\int d^3x\, \vec{A}\cdot\vec{B}$$



For $q_1 = 1,\ q_2 = 0.1, \quad N_{CS}[A] \simeq 0.28$

→ contains $U(1)_Y$ helicity: $N_{CS}[Y] \simeq \epsilon^2 N_{CS}[A]$

can be used for baryogenesis!

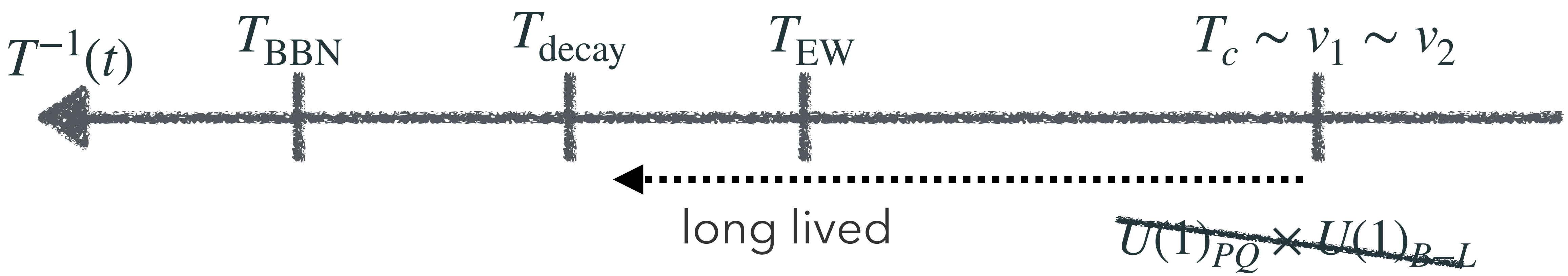
(cf: baryogenesis by helical $U(1)_Y$ field) [Kamada-Long '16]

Fate of link soliton

~~$U(1)_{PQ} \times U(1)_{B-L}$~~

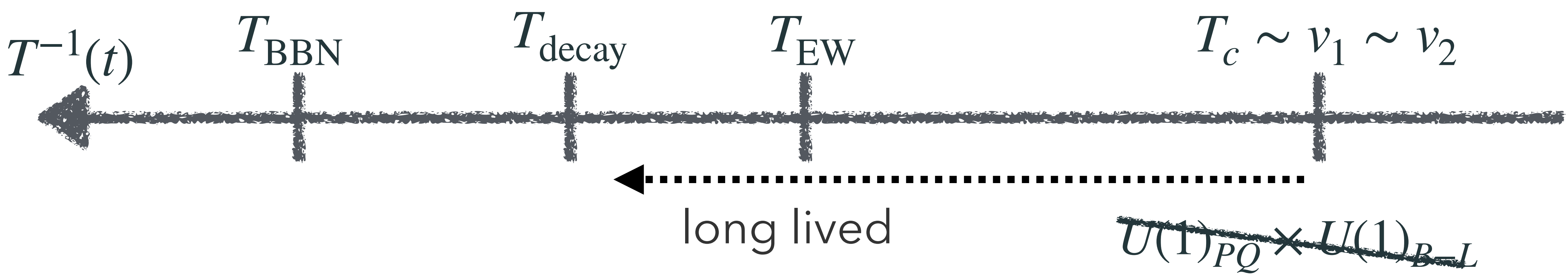
→ **production of links**

Fate of link soliton



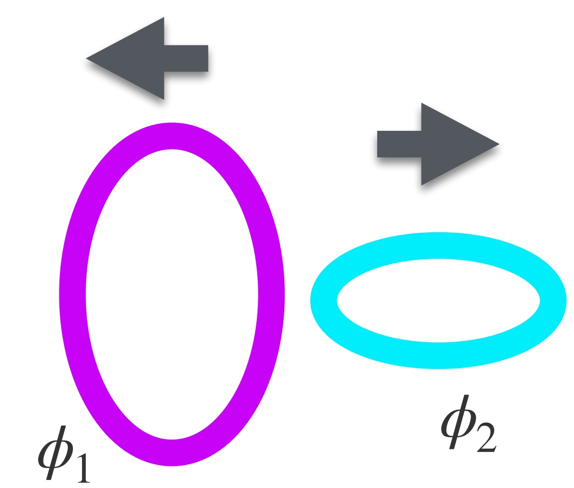
→ **production of links**

Fate of link soliton

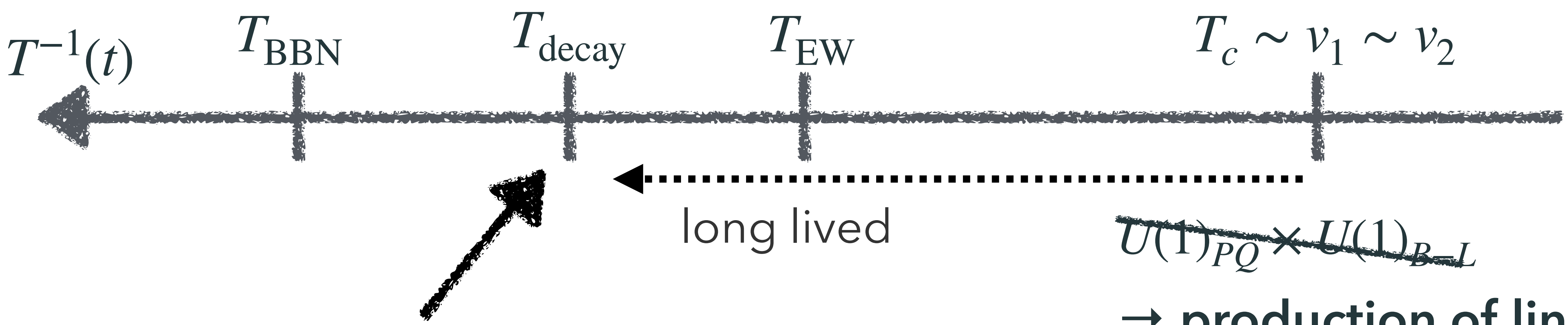


→ **production of links**

Links decay by tunneling effect after EW phase transition and before BBN.



Fate of link soliton



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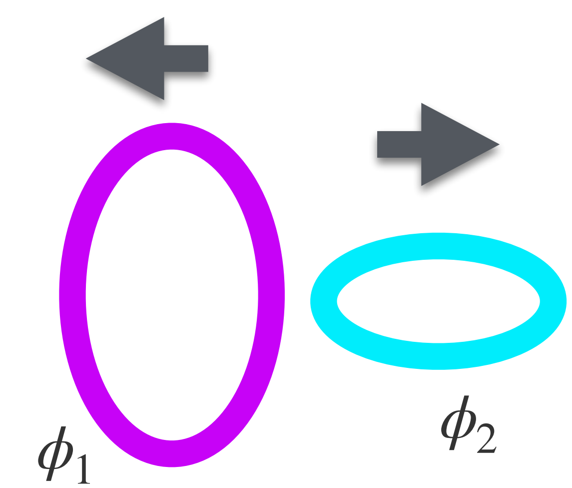


→ change of helicity: $\Delta N_{CS}[Y] \simeq \epsilon^2 N_{CS}[A]$

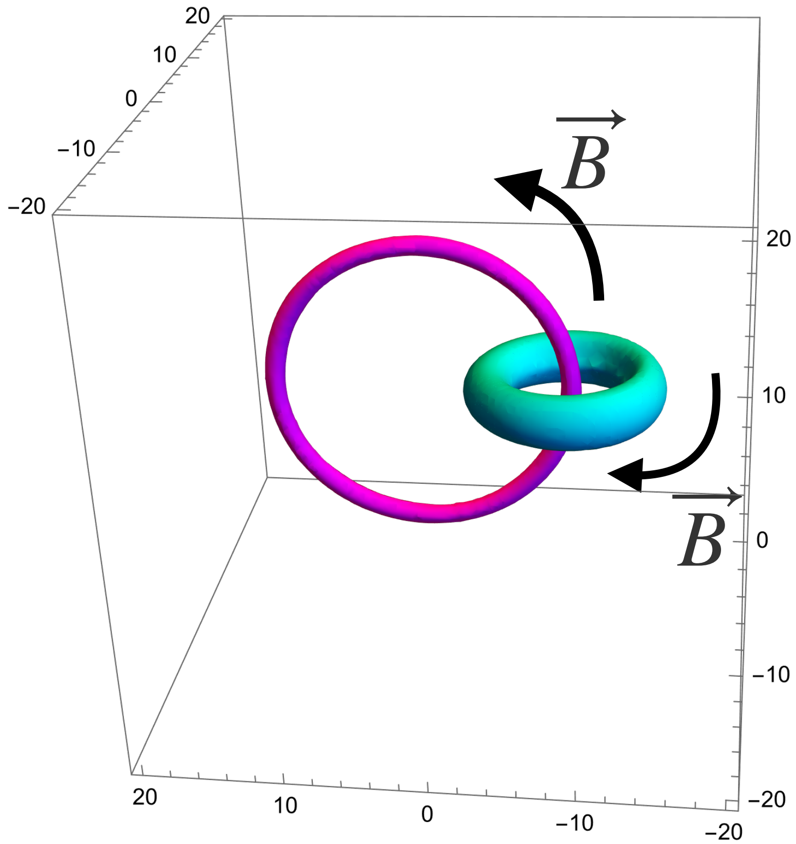
→ baryon # is generated through chiral anomaly:

$$\Delta B = \epsilon^2 N_{CS}[A] \text{ per link}$$

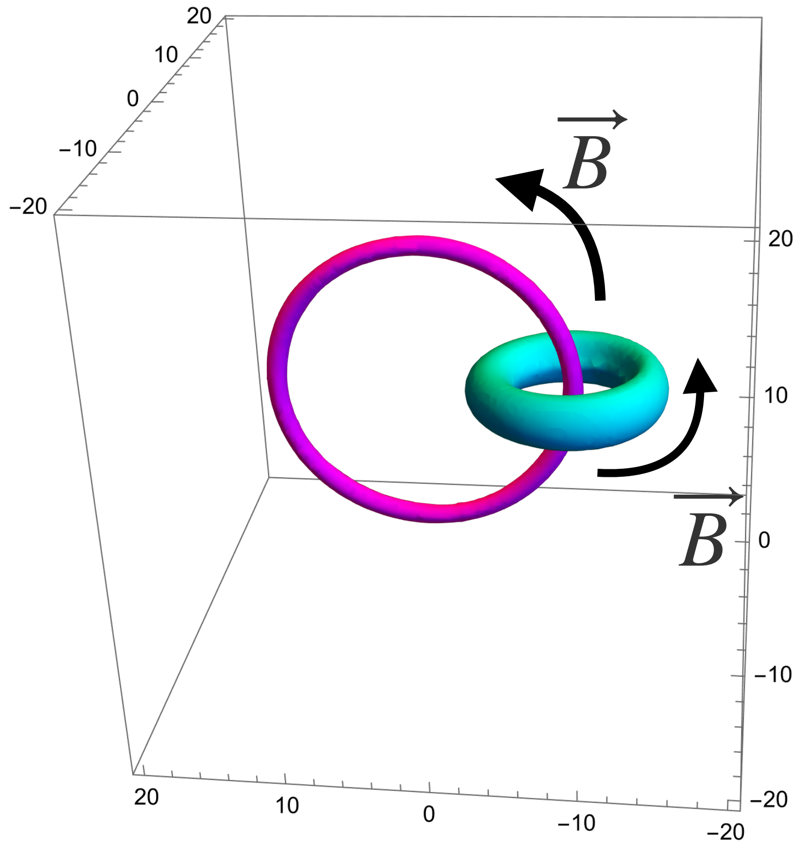
$$(\partial_\mu J^\mu_B \sim Y\tilde{Y} + \mathrm{tr}\, W\tilde{W})$$

Link and anti-link

- Anti-link solution also exists and its decay gives opposite baryon #.



link



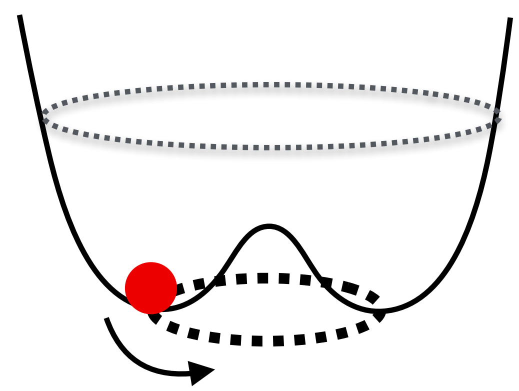
anti-link

- Net baryon # is zero unless there is "chemical potential" μ btw link and anti-link at the production $T \sim T_c$.

Origin of chemical potential?

- Choice (1): rotating pseudo scalar (axion-like particle)

$$\Delta\mathscr{L} = \frac{c}{16\pi^2} a' F_{\mu\nu}\tilde{F}^{\mu\nu} = -\frac{c}{16\pi^2} \underbrace{(\partial_0 a')}_{\equiv \mu_{eff}} AdA$$



(cf: Affleck-Dine mechanism, axiogenesis [Co-Harigaya '19])

- Choice (2): chiral asymmetry Q_5 from SO(10) GUT etc.

(cf: Chiral magnetic effect, Wash-in leptogenesis [Domcke+ '20])

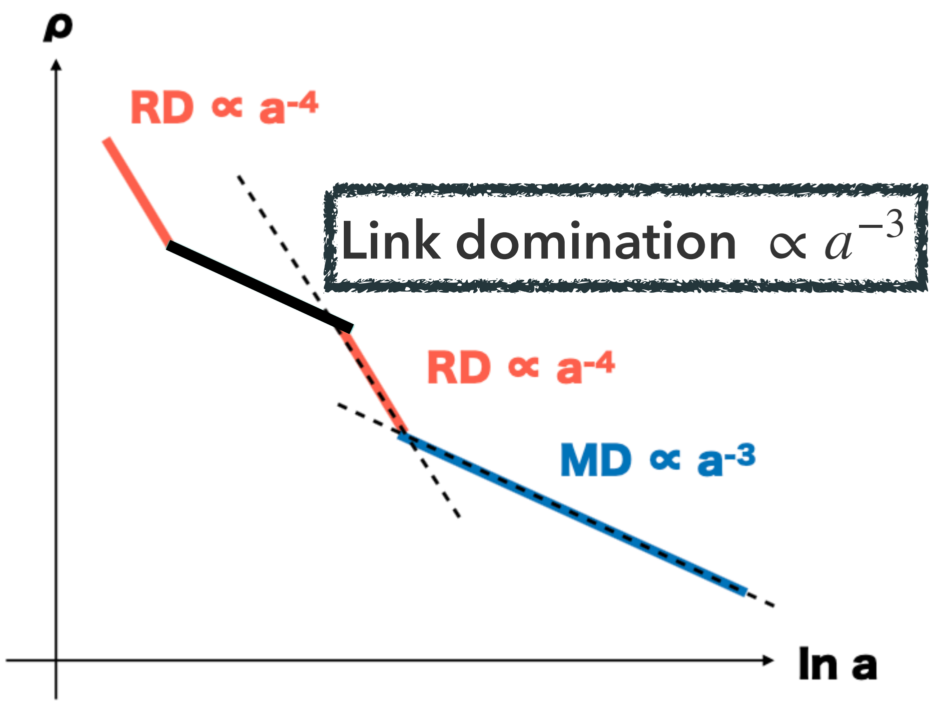
$$\eta_B = \frac{n_B}{s} \sim 10^{-10} \left(\frac{\epsilon}{0.1}\right)^2 \left(\frac{\mu_{eff}|_{T\sim v_1}}{0.1 v_1}\right)$$

Testability

- Before the links decay, they dominate the energy density of universe.

$$\frac{\rho_{link}}{\rho_\gamma}\bigg|_{T\sim T_{EW}} \simeq \frac{M_{link}n_{link}}{g_* T^4}\bigg|_{T\sim T_{EW}}$$

$$\simeq 10^{-4}\frac{v_1}{v_{EW}} \gg 1$$



- The entropy production due to decay cannot be ignored.

→ distorts spectrum of primordial gravitational wave

→ **probed by primordial gravitational wave?**

Summary

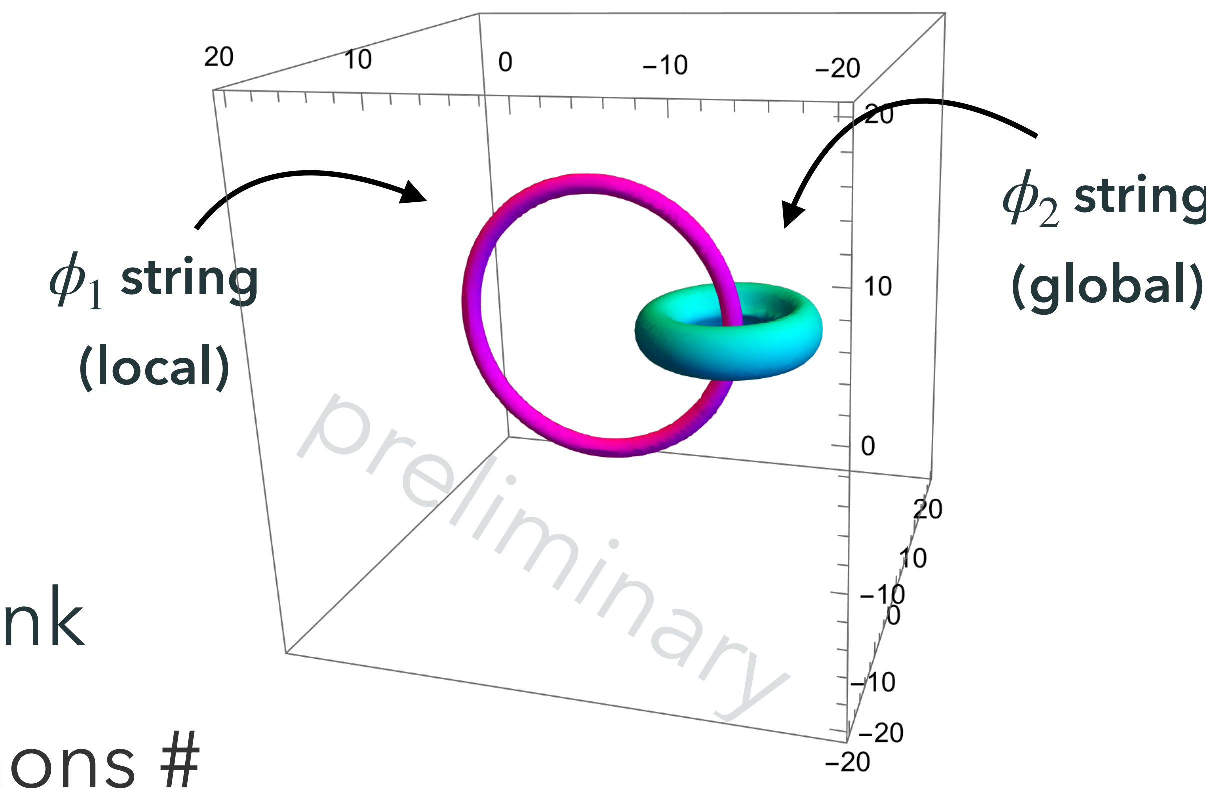
- Massage of this talk:

$U(1)_{global} \times U(1)_{gauge}$ を自発的に破る模型では link solitonが存在する！

- Key: Chern-Simons coupling $\frac{c}{16\pi^2}\int d^4x\, aF\tilde{F}$
- motivativeなセットアップ:

$$\begin{cases} U(1)_{global} = U(1)_{PQ} \\ U(1)_{gauge} = U(1)_{B-L} \end{cases}$$

axion stringとB-L stringからなるlink



→ Links carry non-zero Chern-Simons #

→ Baryon can be generated by decay of links

link = "origin of baryon"

Backup

Abelian-Higgs w/ Chern-Simons

Let's start from 2+1D Abelian-Higgs w/ CS term:

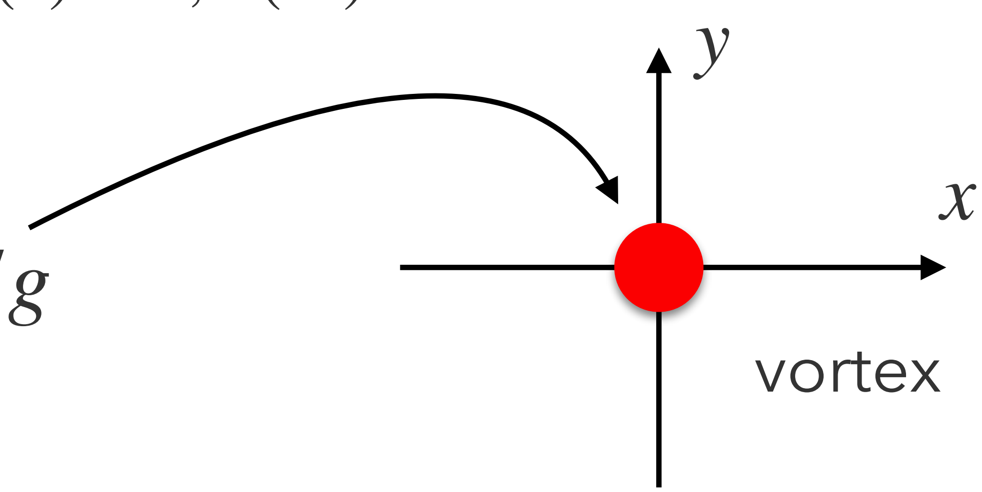
$$\mathscr{L} = |D_\mu \phi|^2 - \frac{1}{4}F_{\mu\nu}^2 - V(\phi) + g^2 c\, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

$$V(\phi) = -m^2|\phi|^2 + \lambda|\phi|^4$$

- For $c = 0$, A_0 is decoupled from static configurations.

→ static solution: $\begin{cases} \phi = v f(r)\, e^{i\theta} \\ A_\theta = a(r)/g \\ A_0 = A_r = 0 \end{cases}$ $\begin{cases} f(0) = 0,\ f(\infty) = 1 \\ a(0) = 0,\ a(\infty) = 1 \end{cases}$

→ quantized magnetic flux $\int d^2x B = 2\pi/g$



Chern-Simons vortex

Let's start from 2+1D Abelian-Higgs w/ CS term:

$$\mathscr{L} = |D_\mu \phi|^2 - \frac{1}{4}F_{\mu\nu}^2 - V(\phi) + g^2 c\, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

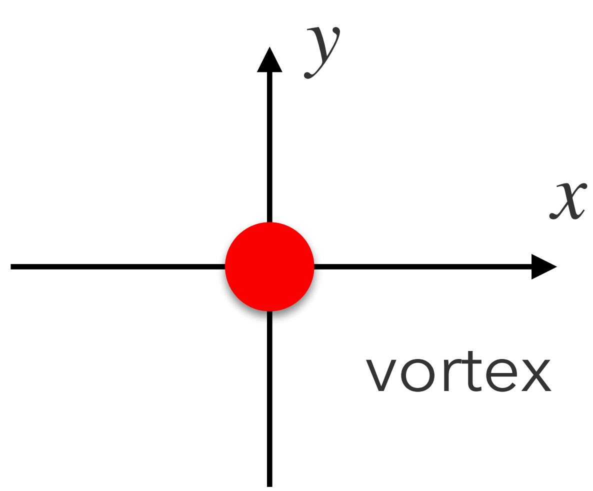
- For $c \neq 0$, A_0 is NOT, due to Gauss law constraint:

$$\frac{\delta \mathscr{L}}{\delta A_0} = \partial_i E_i - g^2 J^0 + g^2 c B = 0$$

$$E_i = \partial_i A_0$$

$$J^0 \equiv \phi^\dagger i D^0 \phi + (h.c.)$$

→ **magnetic flux sauces electric field!**



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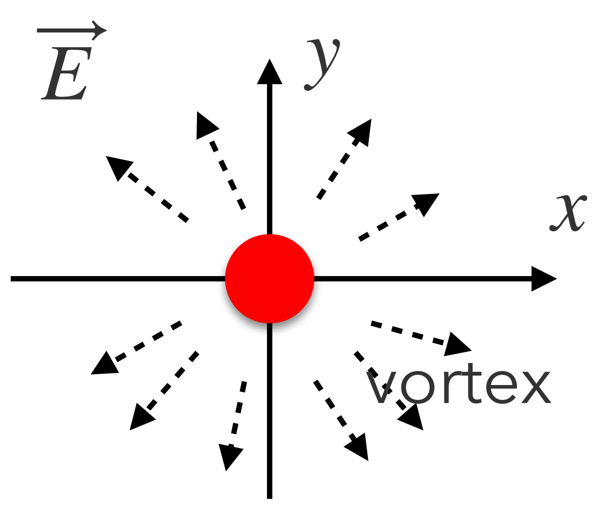

- quantized magnetic flux & electric charge

$$\int d^2x\, B = 2\pi / g \qquad \int d^2x\, J^0 = 2\pi c / g$$

Chern-Simons vortex

Let's start from 2+1D Abelian-Higgs w/ CS term:

$$\mathscr{L} = |D_\mu \phi|^2 - \frac{1}{4}F_{\mu\nu}^2 - V(\phi) + g^2 c\, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

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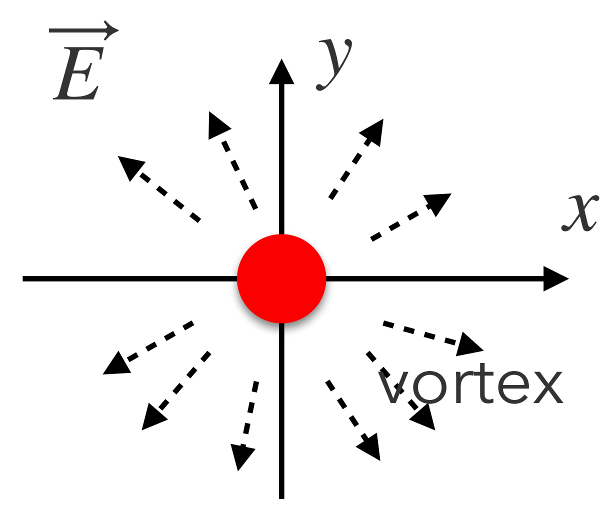


- quantized magnetic flux & electric charge

$$\int d^2x\, B = 2\pi/g \qquad \int d^2x\, J^0 = 2\pi c/g$$

called Chern-Simons vortex

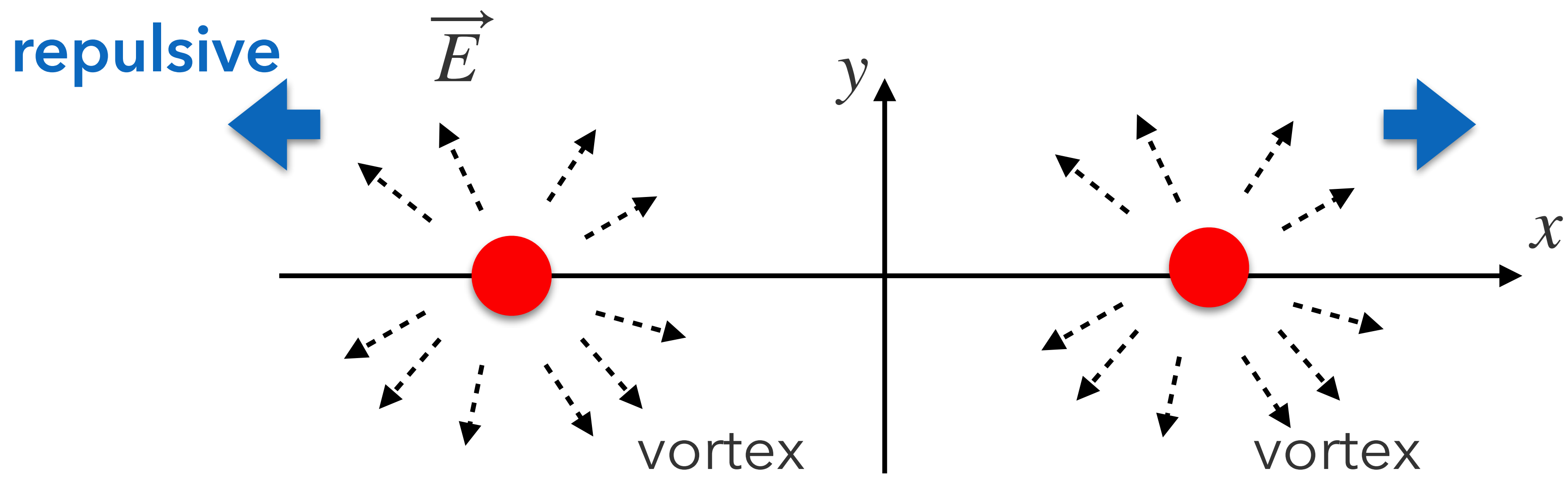
Interaction of Chern-Simons vortices

- typical length scale of $\vec{E}$: l_E

w/ $l_E^{-1} \equiv gv \left(\frac{1}{2}\sqrt{4+c^2} - \frac{c}{2} \right) \simeq \mathcal{O}(gv/c)$ for $c \to \infty$

Interaction of Chern-Simons vortices

- Interaction btw two CS vortices

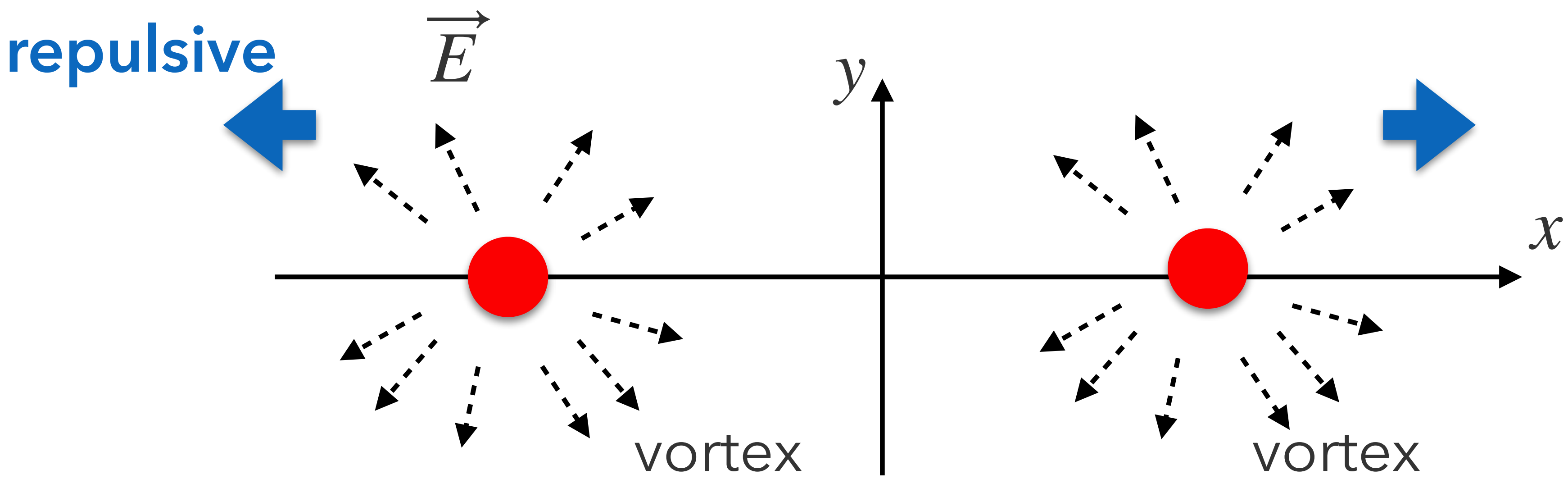


- typical length scale of $\overrightarrow{E}$: l_E

$$\text{w/ } l_E^{-1} \equiv gv\left(\frac{1}{2}\sqrt{4+c^2}-\frac{c}{2}\right) \simeq \mathcal{O}(gv/c) \text{ for } c \to \infty$$

Interaction of Chern-Simons vortices

- Interaction btw two CS vortices



- typical length scale of $\vec{E}$: l_E

$$\text{w/ } l_E^{-1} \equiv gv\left(\frac{1}{2}\sqrt{4+c^2}-\frac{c}{2}\right) \simeq \mathcal{O}(gv/c) \text{ for } c \to \infty$$

- For large c, **long-range repulsive force!**

Relation to Skyrmion

For $\lambda \gg g^2, \kappa, \chi$,

$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

$$\to \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2$$

→ non-linear sigma model w/ $O(4)$ symmetry,

which breaks into $O(3)$

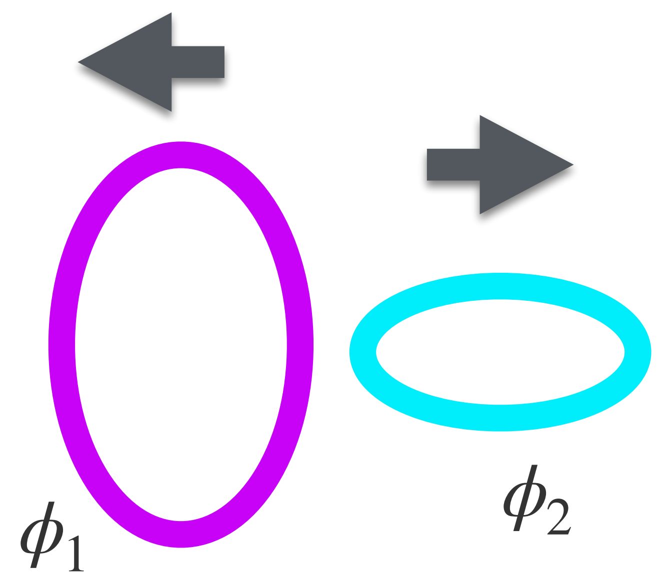
There exists Skyrmion defined by winding number:

$$N_{sk} = \int d^3x\, \epsilon^{ijk}\, \mathrm{Tr}\left[U^\dagger \partial_i U U^\dagger \partial_j U U^\dagger \partial_k U \right] \qquad U \equiv \begin{pmatrix} \mathrm{Re}\,\phi_1 & \mathrm{Im}\,\phi_2 \\ -\mathrm{Im}\,\phi_1 & \mathrm{Re}\,\phi_2 \end{pmatrix}$$

The link is nothing but the Skyrmion!

[Gudnason-Nitta '20]

Decay of link soliton



$$\tau^{-1} \sim \Gamma \sim g v_1 \exp\left[-\frac{4}{3}\sqrt{\frac{\lambda v_1}{g v_2}}\right]$$

$$\frac{M_{\rm pl}}{v_{EW}^2} < \tau < \frac{M_{\rm pl}}{(1{\rm MeV})^2}$$

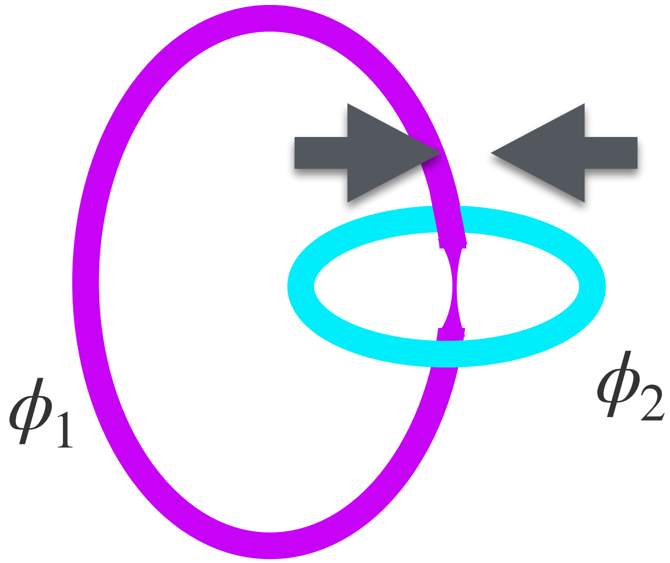
$$\lambda < 4\pi$$

$$g < \sqrt{24\pi}$$

$$\Leftrightarrow \log\frac{g v_1}{10^{11}{\rm GeV}} + 60 \lesssim \frac{4}{3}\sqrt{\frac{\lambda v_1}{g v_2}} \lesssim \log\frac{g v_1}{10^{11}{\rm GeV}} + 82$$

$$\therefore 100 \lesssim \frac{\lambda}{g} \lesssim 190$$

Decay of link soliton



$$\tau^{-1} \sim \Gamma \sim g v_1 \exp\left[-\frac{1}{g^2}\frac{v_1}{v_2}\#\right]$$

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$$\Leftrightarrow \log\frac{g v_1}{10^{11}\mathrm{GeV}} + 60 \lesssim \frac{v_1}{g^2 v_2}\# \lesssim \log\frac{g v_1}{10^{11}\mathrm{GeV}} + 82$$

our parameter choice: $\frac{v_1}{v_2} = 20$ $\quad \rightarrow 0.49 \lesssim g \lesssim 0.58$

Dimensionless unit

$$\mathscr{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 - \frac{1}{4g^2} F_{\mu\nu}^2 - V(\phi_1, \phi_2)$$

$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

$$\mathscr{L} = \frac{1}{g^2} \left[g^2 |D_\mu \phi_1|^2 + g^2 |D_\mu \phi_2|^2 - \frac{1}{4} F_{\mu\nu}^2 - g^2 V(\phi_1, \phi_2) \right]$$

- field redefinition: $g\phi_1 \to \phi$, $g\phi_2 \to \phi_2$

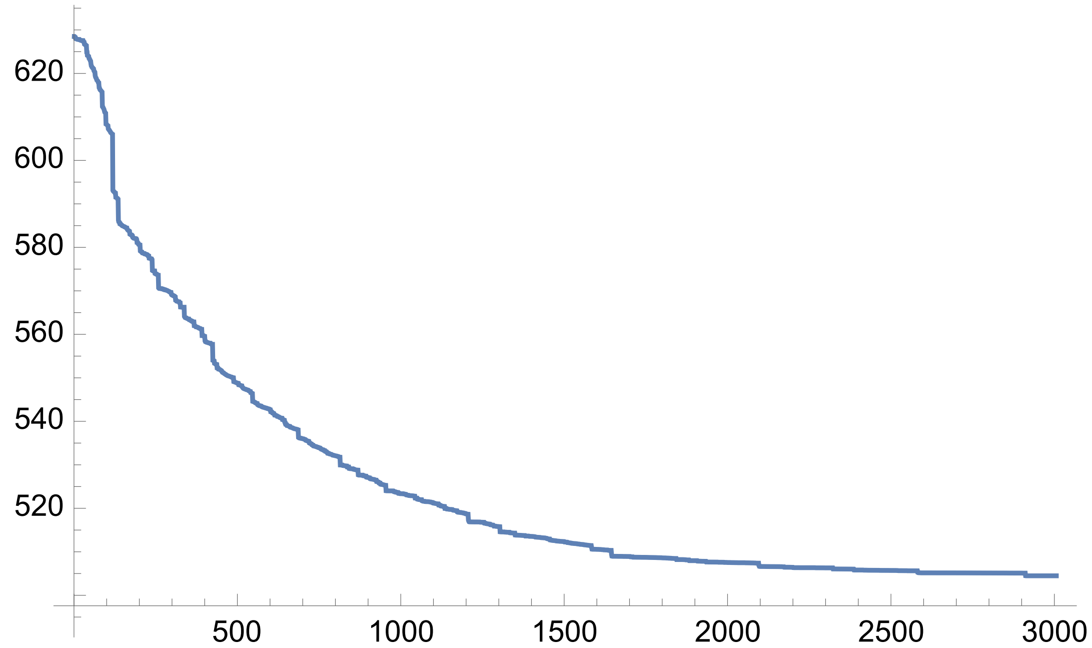
$$\mathscr{L} = \frac{1}{g^2} \left[|D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 - \frac{1}{4} F_{\mu\nu}^2 - \tilde{V}(\phi_1, \phi_2) \right]$$

$$\tilde{V}(\phi) = \frac{\lambda}{g^2} \left(|\phi_1|^2 + |\phi_2|^2 - g^2\mu^2 \right)^2 - \frac{\kappa}{g^2} |\phi_1|^2 |\phi_2|^2 + \frac{\chi}{g^2} |\phi_2|^4$$

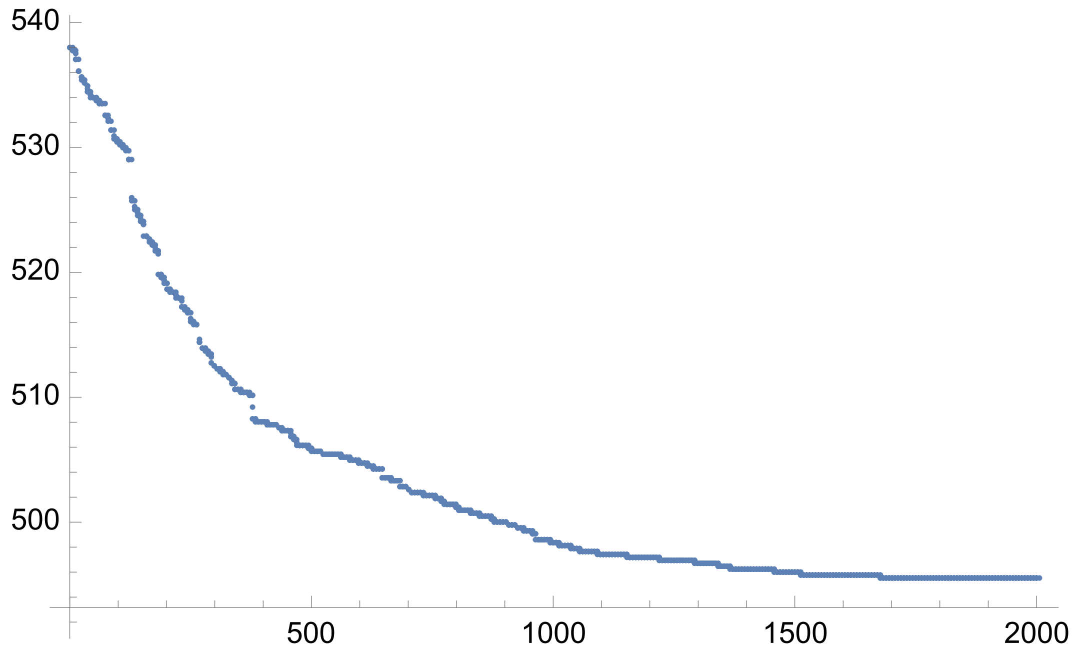
only dimensionful parameter $g\mu$

Energy convergence

$q_2/q_1 = 0$



$q_2/q_1 = 0.1$



The model

3+1D theory:

CS coupling

$$\mathcal{L} = |D_\mu \phi_1|^2 + |\partial_\mu \phi_2|^2 - \frac{1}{4g^2} F_{\mu\nu}^2 - V(\phi_1, \phi_2) + \frac{c}{16\pi^2} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

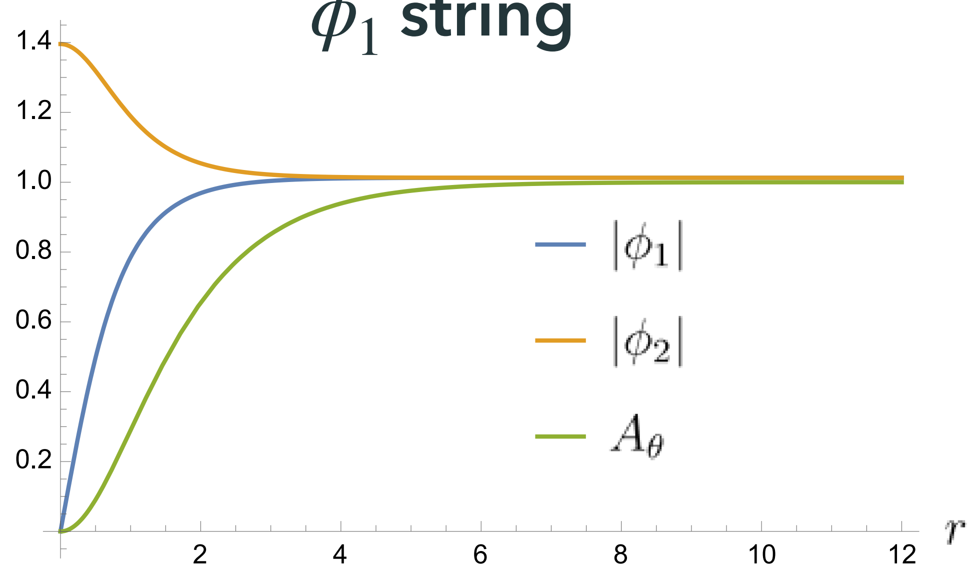
$$D_\mu \phi_1 = (\partial_\mu - igA_\mu)\phi_1$$

$$a \equiv -i \arg(\phi_2)$$

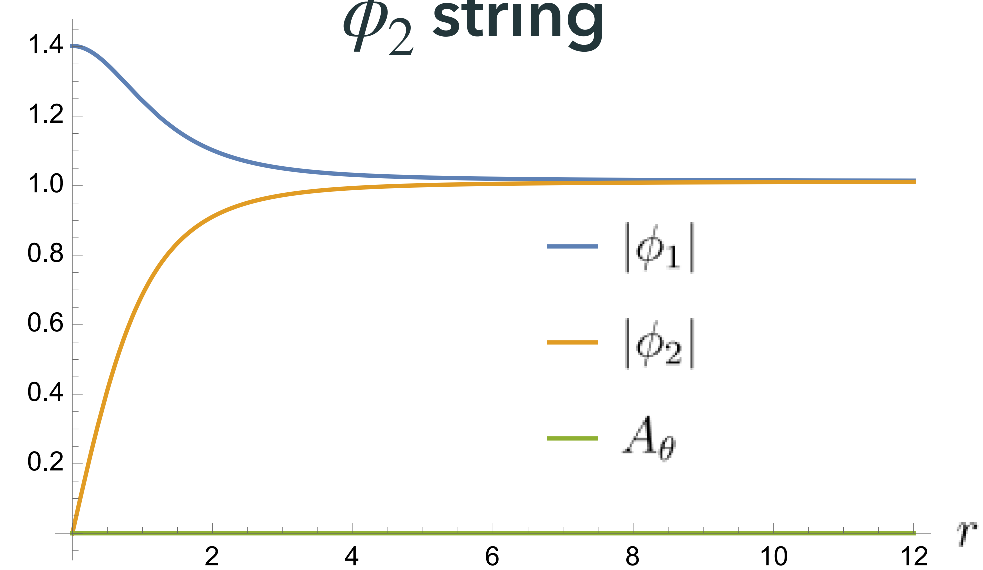
$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

- CS coupling does not affect single strings.

ϕ_1 string



ϕ_2 string



Interaction of Chern-Simons vortices

- Single static solution: $\begin{cases} \phi = v\, f(r)\, e^{i\theta} \\ A_\theta = a(r)/g \\ A_0 = b(r)/g \end{cases}$ $\begin{cases} f(0) = 0,\ f(\infty) = 1 \\ a(0) = 0,\ a(\infty) = 1 \\ b(0) = 0,\ b(\infty) = 0 \end{cases}$

- Asymptotic behavior at $r \to \infty$

$$1 - f(r) \sim e^{-M_\phi r} \qquad b(r) \sim 1 - a(r) \sim e^{-M_c r}$$

w/ $M_c \equiv gv \left(\frac{1}{2}\sqrt{4 + c^2} - \frac{c}{2} \right) \simeq \mathcal{O}(gv/c)$ for $c \to \infty$

Interaction of Chern-Simons vortices

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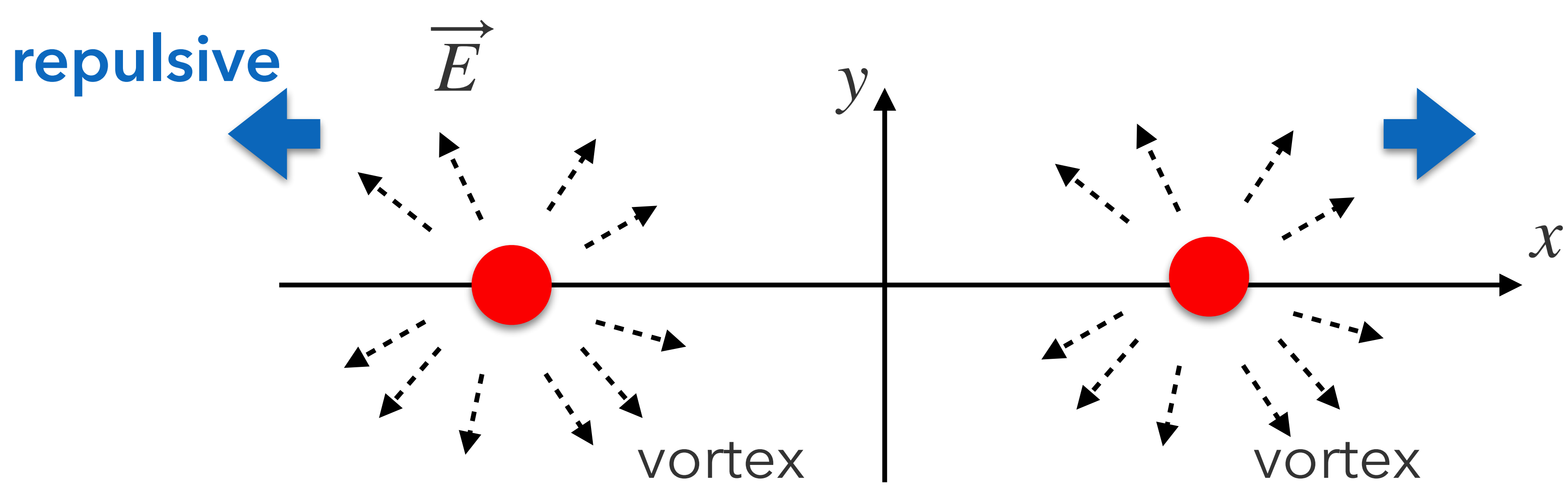
w/ $M_c \equiv gv\left(\frac{1}{2}\sqrt{4+c^2} - \frac{c}{2}\right) \simeq \mathcal{O}(gv/c)$ for $c \to \infty$

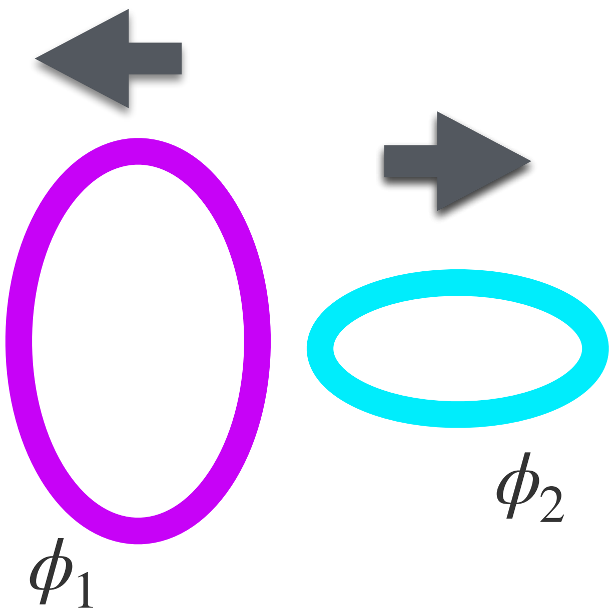
- For large c, **long-range repulsive force!**



Classical stability

- Delinking by passing through each other?

  → prevented by taking $\lambda \gg g^2, \kappa, \chi$



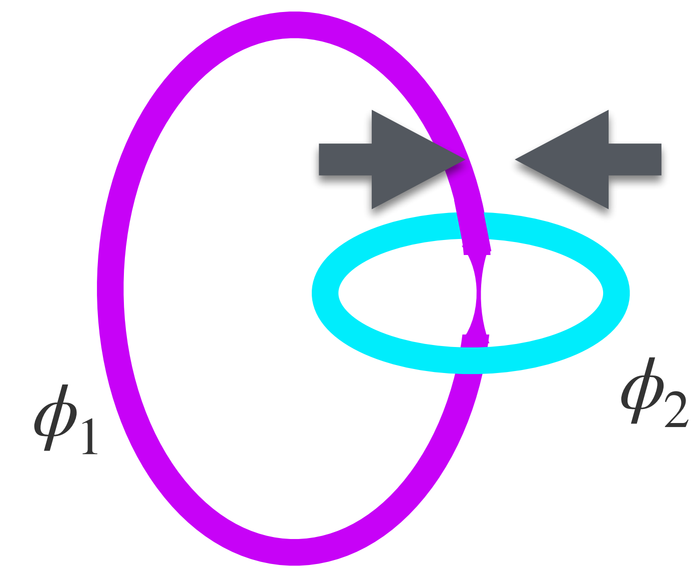
Overlap of strings ($\phi_1 = \phi_2 = 0$) cost large energy

$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

- ϕ_2 string is not charged and thus can shrink ?

  → prevented by taking $v_2/v_1 \ll 1$

ϕ_2 string is too light to pinch ϕ_1 string



➡ **classically stable (but decay by tunneling effect)**

The full Lagrangian

$$\mathscr{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 - \frac{1}{4g^2} F_{\mu\nu}^2 - V(\phi_1, \phi_2) + \frac{c}{16\pi^2} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$+\mathscr{L}_{mat} + \mathscr{L}_{SM} - V_{portal}(\phi_1, \phi_2, H) + \mathscr{L}_{kin.mix.}$$

- Natural setup: $U(1)_{gauge} = U(1)_{B-L}$ & $U(1)_{global} = U(1)_{PQ}$

  Type-I seesaw → ν-mass

  QCD axion → strong CP & Dark matter

  $\Rightarrow v_1 \sim v_2 \sim 10^{9-12}\,\mathrm{GeV}$

- Assume kinetic mixing with $U(1)_Y$ in SM: $\mathscr{L} \supset \frac{\epsilon}{2} Y_{\mu\nu} F^{\mu\nu}$

Baryon # from link

- Naively, anti-link is also produced → $n_{link} - n_{\overline{link}} = 0$?
- need "chemical potential" μ (discussed later) when produced:

$$\frac{n_{link} - n_{\overline{link}}}{s} \simeq \frac{\mu}{T}\frac{n_{link}}{s} \simeq \frac{\mu}{T}10^{-6}$$

We have used $n_{link} \sim 10^{-4}\,T^3$

[Vachaspati '84]

→ generated total baryon # due to decay:

$$\eta_B = \frac{n_B}{s} \sim 10^{-10}\left(\frac{\epsilon}{0.1}\right)^2\left(\frac{\mu/v_1}{0.1}\right)$$