Overview of QED/EW corrections benchmarking for DY

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LHC EW WG general meeting

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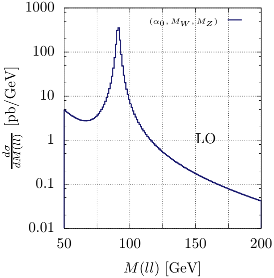
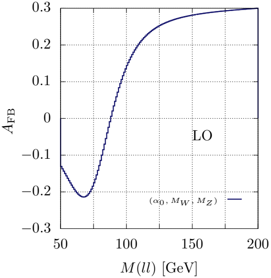
on behalf of the QED/EW precision subgroup WG

Main focus of the subgroup activity





- Tuned comparison/benchmarking of EW precision tools for neutral-current Drell-Yan
- Focus on $\frac{d\sigma}{dM(ll)}$ and $A_{\rm FB}$ distributions

Main motivation

- high-precision determination of $\sin^2\vartheta_l^{\rm eff}$ through template fits
- target accuracy: $\Delta\sin^2\vartheta_l^{\rm eff} \sim 1.6\cdot 10^{-4} \to \Delta A_{FB} \sim 5\cdot 10^{-4}$
- it is crucial to assess the precision of the available tools as well as theory uncertainties

Two classes of comparisons*

Weak corrections**

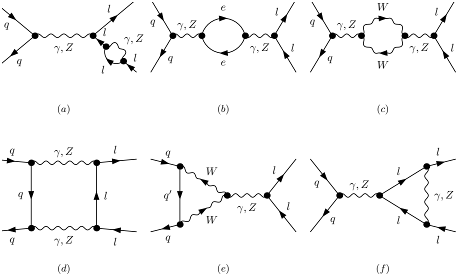
- at NLO and NLO+HO (universal leading fermionic)
- for several input parameter schemes
- for different strategies for the treatment of unstable particles

QED corrections**

- breakdown of IRS, FSR, and IFI contributions
- study of $ll\gamma$ and $ll\gamma\gamma$ production (see work by Scott Yost in the last few meetings)

* at QCD LO
** for NC DY the separation of weak and QED corrections is gauge-invariant

NLO EW and NLO EW +HO



NLO EW

NLO weak=

NLO EW - photonic corrs

HO= leading fermionic corrections

- from $\Delta\alpha(M_Z^2) \sim \log \frac{m_f^2}{M_Z^2}$ and $\Delta\rho \sim m_t^2$
- for instance, in the (α_0, M_W, M_Z) scheme appear in $|M|^2$ with

$$\alpha_0 \to \frac{\alpha_0}{1 - \Delta\alpha(M_Z^2)}, \qquad s_W^2 \to s_W^2 + \Delta\rho c_W^2$$

EW input parameter schemes

scheme choice = choice of the 3 independent EW params

- all choices formally equivalent at a given order in P.T.
- numerical differences in predictions from missing H.O. terms
- possible criteria for a scheme choice: parametric uncertainties, perturbative convergence, need to vary a specific independent parameter
- some examples:
  - (α_0, M_W, M_Z), $(\alpha(M_Z), M_W, M_Z)$, and (G_μ, M_W, M_Z) (LHC)
  - (α_0, G_μ, M_Z) (LEPI)
  - $(\alpha_0, M_Z, \sin^2\theta_W^{eff})$ and $(G_\mu, M_Z, \sin^2\theta_W^{eff})$

Adopted input parameter schemes

- (G_μ, M_W, M_Z)

  MCSANC, POWHEG_EW, RADY, WZGRAD2

- (α_0, M_W, M_Z)

  MCSANC, POWHEG_EW, RADY, WZGRAD2

- $(G_\mu, \sin^2\vartheta^\ell_{\rm eff}, M_Z)$, $(\alpha_0, \sin^2\vartheta^\ell_{\rm eff}, M_Z)$

  POWHEG_EW, RADY

- (α_0, G_μ, M_Z)

  DIZET (used in TAUSPINNER+DIZET and KKMC_HH)

NLO weak corrs: (G_μ, M_W, M_Z), $d\sigma/dM(ll)$

Code:		$89 < M_{\ell\bar\ell}$[GeV] < 93	$60 < M_{\ell\bar\ell}$[GeV] < 81	$81 < M_{\ell\bar\ell}$[GeV] < 101	$101 < M_{\ell\bar\ell}$[GeV] < 150
		σ(LO) (pb)			
MCSANC		612.531(5)	46.870(2)	880.527(6)	-
POWHEG$_{\text{ew}}$ (FS)		612.529(8)	46.8697(8)	880.513(9)	30.8686(5)
RADY (FS)		612.526(1)	46.8708(1)	880.520(2)	30.86835(6)
WZGRAD2		612.521(7)	46.868(4)	880.520(10)	-
		σ(NLO)/σ(LO)			
MCSANC		0.99167(2)	1.02865(7)	0.99206(1)	-
POWHEG$_{\text{ew}}$ (FS)	no α resc.	0.99121(3)	1.02972(4)	0.99163(2)	0.98888(3)
	α resc.	0.99150(3)	1.02871(4)	0.99191(2)	0.98926(3)
RADY (FS)	no α resc.	0.99118(1)	1.02965(1)	0.99160(1)	0.98886(1)
	α resc.	0.99148(1)	1.02863(1)	0.99189(1)	0.98924(1)
WZGRAD2		0.99198(1)	1.02913(4)	0.99239(1)	-
		σ(NLO + HO)/σ(LO)			
MCSANC		0.99232(2)	1.02614(7)	0.99268(1)	-
POWHEG$_{\text{ew}}$ (FS)	α resc.	0.99216(3)	1.02603(4)	0.99253(2)	0.98968(3)
	no α resc.	0.99181(3)	1.02577(4)	0.99218(1)	0.98919(3)
RADY (FS) α no resc.		0.99179(1)	1.02589(1)	0.99216(1)	0.98915(1)
TauSpinner+DIZET (estimated)		0.99211(0)	1.02321(0)	0.99264(0)	0.98884(0)

- some entry still missing...
- overall agreement 0.01% level
- LO $\sim \alpha_{G_\mu}$, $\delta = \text{NLO}/\text{LO} - 1 \sim \alpha_{\text{loop}}$

- $\alpha_{\text{loop}} = \alpha_0$ (resc)
- $\alpha_{\text{loop}} = \alpha_{G_\mu}$ (nonresc)

NLO weak corrs: (G_μ, M_W, M_Z), A_{FB}

Code:		$89 < M_{\ell\bar\ell}[\mathrm{GeV}] < 93$	$60 < M_{\ell\bar\ell}[\mathrm{GeV}] < 81$	$81 < M_{\ell\bar\ell}[\mathrm{GeV}] < 101$	$101 < M_{\ell\bar\ell}[\mathrm{GeV}] < 150$
A_{FB}(LO)					
MCSANC		0.04654(1)	−0.20299(4)	0.04481(1)	-
POWHEG$_{\mathrm{ew}}$ (FS)		0.04655(2)	−0.202975(24)	0.04481(2)	0.22608(4)
RADY (FS)		0.046547(4)	−0.202955(4)	0.044812(3)	0.226090(4)
WZGRAD2		0.04654(1)	−0.20299(8)	0.04482(1)	-
A_{FB}(NLO) − A_{FB}(LO)					
MCSANC (FS)		−0.01717(2)	−0.01183(8)	−0.01715(2)	−0.00688(7)
POWHEG$_{\mathrm{ew}}$ (FS)	α resc.	−0.01718(3)	−0.01198(3)	−0.01718(3)	−0.00680(3)
	no α resc.	−0.01779(3)	−0.01239(3)	−0.01778(3)	−0.00705(5)
RADY (FS)	α resc.	−0.017166(5)	−0.011988(6)	−0.017156(5)	−0.006809(6)
	no α resc.	−0.017778(5)	−0.012399(6)	−0.017767(5)	−0.007052(6)
WZGRAD2		−0.01716(2)	−0.01186(11)	−0.01715(2)	−0.00686(14)
A_{FB}(NLO + HO) − A_{FB}(NLO)					
MCSANC		0.00137(2)	0.00111(8)	0.00137(2)	-
POWHEG$_{\mathrm{ew}}$ (FS)	α resc.	0.00136(3)	0.00113(3)	0.00137(3)	0.0004(2)
	no α resc.	0.00183(3)	0.00147(3)	0.00183(2)	0.00057(35)
RADY (FS) no α resc.		0.001829(5)	0.001437(6)	0.001830(5)	0.000582(6)
A_{FB}(NLO + HO) − A_{FB}(LO)					
MCSANC		−0.01551(2)	−0.01059(8)	−0.01551(1)	-
POWHEG$_{\mathrm{ew}}$ (FS)	α resc.	−0.01582(3)	−0.01085(3)	−0.01581(3)	−0.0064(2)
	no α resc.	−0.01597(3)	−0.01092(3)	−0.01596(3)	−0.0065(5)
RADY (FS) no α resc.		−0.015948(5)	−0.010962(6)	−0.015937(5)	−0.006470(6)
TauSpinner+DIZET (estimated)		−0.01507(0)	−0.01104(0)	−0.01514(0)	0.00684(0)

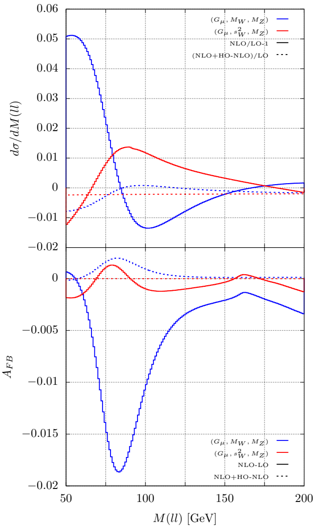
NLO weak corrs: (α_0, M_W, M_Z), $d\sigma/dM(ll)$

Code:	$89 < M_{\ell\bar{\ell}}$[GeV] < 93	$60 < M_{\ell\bar{\ell}}$[GeV] < 81	$81 < M_{\ell\bar{\ell}}$[GeV] < 101	$101 < M_{\ell\bar{\ell}}$[GeV] < 150
		σ(LO) (pb)		
MCSANC	571.412(5)	43.724(2)	821.414(6)	-
POWHEG$_{\text{ew}}$ (FS)	571.416(7)	43.7239(8)	821.414(9)	28.7967(4)
RADY (FS)	571.414(1)	43.725(1)	821.420(2)	28.7965(6)
WZGRAD2	571.409(7)	43.722(4)	821.419(9)	-
		σ(NLO)/σ(LO)		
MCSANC	1.05117(1)	1.08830(4)	1.05157(1)	-
POWHEG$_{\text{ew}}$ (FS)	1.05095(3)	1.08815(4)	1.05136(2)	1.04870(3)
RADY (FS)	1.05100(1)	1.08816(1)	1.05141(1)	1.0487685(7)
WZGRAD2	1.05151(1)	1.08854(9)	1.05191(1)	-
		σ(NLO + HO)/σ(LO)		
MCSANC	1.06452(1)	1.1004(4)	1.06491(1)	-
POWHEG$_{\text{ew}}$ (FS)	1.06381(3)	1.09911(4)	1.06420(2)	1.06175(3)
RADY (FS)	1.06387(1)	1.09979(1)	1.06426(1)	1.0614687(8)
TauSpinner+DIZET estimated	1.06558(0)	1.09892(0)	1.06613(0)	1.06202(0)

NLO weak corrs: (α_0, M_W, M_Z), $A_{\rm FB}$

Code:	$89 < M_{\ell\bar{\ell}}$[GeV] < 93	$60 < M_{\ell\bar{\ell}}$[GeV] < 81	$81 < M_{\ell\bar{\ell}}$[GeV] < 101	$101 < M_{\ell\bar{\ell}}$[GeV] < 150
$A_{\rm FB}$(LO)				
MCSANC	0.04655(1)	−0.20304(4)	0.04482(1)	-
POWHEG$_{\rm ew}$ (FS)	0.04655(2)	−0.20296(2)	0.04481(2)	0.226094(25)
RADY (FS)	0.046547(4)	−0.202955(4)	0.044812(3)	0.226090(4)
WZGRAD2	0.04654(1))	−0.20299(8)	0.04482(1)	-
$A_{\rm FB}$(NLO) − $A_{\rm FB}$(LO)				
MCSANC	−0.01618(1)	−0.01118(7)	−0.01618(1)	−0.00647(7)
POWHEG$_{\rm ew}$ (FS)	−0.01621(3)	−0.01134(3)	−0.01620(2)	−0.00643(4)
RADY (FS)	−0.016195(5)	−0.011332(6)	−0.016186(5)	−0.006423(6)
WZGRAD2	−0.01619(2)	−0.01121(12)	−0.01617(2)	−0.00650(14)
$A_{\rm FB}$(NLO + HO) − $A_{\rm FB}$(NLO)				
MCSANC	0.00077(1)	0.00068(6)	0.00078(1)	-
POWHEG$_{\rm ew}$ (FS)	0.00077(3)	0.00073(3)	0.00078(2)	0.000232(35)
RADY (FS)	0.000771(5)	0.000664(7)	0.000774(6)	0.000245(6)
$A_{\rm FB}$(NLO + HO) − $A_{\rm FB}$(LO)				
MCSANC	−0.01519(1)	−0.01035(6)	−0.01517(1)	-
POWHEG$_{\rm ew}$ (FS)	−0.01544(3)	−0.01061(3)	−0.01542(2)	−0.006100(35)
RADY (FS)	−0.015424(5)	−0.010668(6)	−0.015412(5)	−0.006178(6)
TauSpinner+DIZET (estimated)	−0.01508(0)	−0.01104(0)	−0.01515(0)	0.00684(0)

$(G_\mu, \sin^2\vartheta^\ell_{\rm eff}, M_Z)$ and $(\alpha_0, \sin^2\vartheta^\ell_{\rm eff}, M_Z)$



- $\sin^2\vartheta_l^{\rm eff}$ independent parameter, can be used as fit variable
- typically small weak corrections
- at present available in POWHEG and RADY

Work in progress

- tuned POWHEG-RADY comparison
- checks against DIZET (flvour dependence of corrections/corrections to $\sin^2\vartheta_{u/d}^{\rm eff}$)

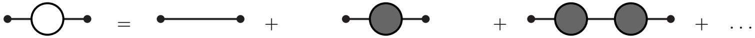
$(G_\mu, \sin^2\vartheta^\ell_{\rm eff}, M_Z)$: $A_{\rm FB}$ preliminary

Code/scheme:	$89 < M_{\ell\ell}[GeV] < 93$	$60 < M_{\ell\ell}[GeV] < 81$	$81 < M_{\ell\ell}[GeV] < 101$	$101 < M_{\ell\ell}[GeV] < 150$
		$A_{\rm FB}$(LO)		
RADY/CMS	0.030552(3)	−0.214572(4)	0.028815(4)	0.220793(5)
Powheg/CMS	0.03056(2)	−0.21459(2)	0.02881(2)	0.22077(35)
RADY/PS	0.030552(3)	−0.214572(4)	0.028815(4)	0.220793(5)
Powheg/PS	0.03056(2)	−0.21459(2)	0.02881(2)	0.22077(35)
RADY/FS	0.030552(3)	−0.214572(4)	0.028815(4)	0.220793(5)
Powheg/FS	0.03056(2)	−0.21459(2)	0.02881(2)	0.22077(35)
|X−CMS|	0	0	0	0
		$A_{\rm FB}$(NLO weak)		
RADY/CMS	0.030459(3)	−0.214082(4)	0.028738(4)	0.219509(5)
Powheg/CMS	0.03046(2)	−0.21408(2)	0.02873(2)	0.219506(25)
RADY/PS	0.030376(3)	−0.214136(4)	0.028658(4)	0.219475(5)
Powheg/PS	0.03038(2)	−0.21413(2)	0.02865(2)	0.219472(25)
RADY/FS	0.030589(3)	−0.213854(4)	0.028871(4)	0.219573(5)
Powheg/FS	0.03059(2)	−0.21385(2)	0.02886(2)	0.219571(25)
|PS−CMS|	0.00008	0.00005	0.00008	0.00003
|FS−CMS|	0.0001	0.0002	0.0001	0.00006

Gauge invariance and treatment of resonances

for more details, see Denner, SD, 1912.06823 and refs. therein

Dyson summation of propagators mixes perturbative orders.



$$G(p^2) = \frac{\mathrm{i}}{p^2-M^2} + \frac{\mathrm{i}}{p^2-M^2}\,\mathrm{i}\Sigma_\mathrm{R}(p^2)\,\frac{\mathrm{i}}{p^2-M^2} + \cdots$$

$$= \frac{\mathrm{i}}{p^2 - M^2 + \Sigma_\mathrm{R}(p^2)}, \qquad \Sigma_\mathrm{R}(M^2) = \mathrm{i}M\Gamma$$

But:

Consistency of pert. calculations often requires complete fixed orders.

$\hookrightarrow$ Consistency jeopardized if no special care is taken!

Gauge-invariance requirements:

- proper cancellation of gauge-parameter dependences
  (relations between self-energies, vertex corrections, boxes, etc.)
- validity of (internal) Ward identities
  (e.g. ruling cancellations for forward scattering of $\mathrm{e}^{\pm}$ or at high energies)

Required: schemes to introduce width Γ

- without breaking gauge invariance
- maintaining (at least) NLO accuracy everywhere in phase space

Adopted width schemes within the WG

- complex mass scheme (CMS)

  complex M_W and M_Z

$$\mu_V^2 = M_V^2 - i\Gamma_V M_V \Longrightarrow \cos^2\vartheta = \frac{\mu_W^2}{\mu_Z^2}$$

- factorization scheme (FS):

  global correction factor in the limit $\Gamma \to 0$

$$d\sigma_{\text{weak}} = \delta_{\text{weak}}^{\Gamma=0} \times d\sigma_{LO}^{\Gamma\neq 0}$$

- pole scheme (PS):

  amplitude organized in resonant g.i. contributions

$$\begin{aligned}\mathcal{M} &= \frac{R(p^2)}{p^2-M^2} + N(p^2) = \frac{R(M^2)}{p^2-M^2} + \frac{R(p^2)-R(M^2)}{p^2-M^2} + N(p^2) \\ &\to \frac{\bar{R}(M^2-i\Gamma M)}{p^2-M^2+i\Gamma M} + \frac{R(p^2)-R(M^2)}{p^2-M^2} + \bar{N}(p^2)\end{aligned}$$

Comparison of width schemes

A_{FB} in (G_μ, M_W, M_Z) scheme - comparison with RADY

Code/scheme:	$89<M_{\ell\ell}[\text{GeV}]<93$	$60<M_{\ell\ell}[\text{GeV}]<81$	$81<M_{\ell\ell}[\text{GeV}]<101$	$101<M_{\ell\ell}[\text{GeV}]<150$
		$A_{\rm FB}(\rm LO)$		
RADY/CMS	0.046551(4)	−0.202894(5)	0.044817(4)	0.226101(5)
Powheg/CMS	0.04655(2)	−0.20292(4)	0.04481(2)	0.22608(4)
RADY/PS	0.046547(4)	−0.202955(4)	0.044812(3)	0.226090(4)
Powheg/PS	0.04655(2)	−0.202975(24)	0.04481(2)	0.22608(4)
RADY/FS	0.046547(4)	−0.202955(4)	0.044812(3)	0.226090(4)
Powheg/FS	0.04655(2)	−0.202975(24)	0.04481(2)	0.22608(4)
|X−CMS|	<0.00001	0.00006	<0.00001	0.00001
		$A_{\rm FB}(\rm NLO\ weak)$		
RADY/CMS	0.028568(4)	−0.215645(5)	0.026846(4)	0.218966(5)
Powheg/CMS	0.02855(2)	−0.21566(2)	0.02682(2)	0.21895(2)
RADY/PS	0.028574(4)	−0.215576(4)	0.026852(4)	0.218938(4)
Powheg/PS	0.02856(2)	−0.21559(2)	0.02683(2)	0.218933(25)
RADY/FS	0.028768(4)	−0.215354(4)	0.027044(4)	0.219037(4)
Powheg/FS	0.02875(2)	−0.21537(2)	0.02702(2)	0.219032(25)
|PS−CMS|	$\lesssim 0.00001$	0.0001	$\lesssim 0.00001$	$\lesssim 0.00002$
|FS−CMS|	0.0002	0.0003	0.0002	0.0001

S. Dittmaier, EWWG 14-5-20

Towards uncertainty estimates in the pure weak sector

On Z resonance (leading pole term):

$$A_4 = \frac{\sum_q X_q \, 8 \, \frac{v_\ell}{a_\ell} \frac{v_q}{a_q}}{\sum_q X_q \left(1 + \frac{v_\ell^2}{a_\ell^2}\right)\left(1 + \frac{v_q^2}{a_q^2}\right)} \qquad X_q = f_q(x_1) f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1) f_q(x_2)$$

$$\frac{v_\ell}{a_\ell} = 1 - 4 s_\ell^2, \qquad s_\ell^2 \equiv \sin^2 \theta_{\rm eff}^\ell$$

$$\frac{v_q}{a_q} = 1 - 4|e_q|(s_\ell^2 + \Delta_q) \qquad \Delta_q = \underbrace{\Delta_{q(1)}}_{\text{implemented}} + \underbrace{\Delta_{q(2)}}_{\text{missing}}$$

$$\frac{\delta A_4}{A_4} \approx \frac{\sum_q X_q \, (-4|e_q|\Delta_{q(2)})}{\sum_q X_q \, (1 - 4|e_q| s_\ell^2)} + \frac{\sum_q X_q \, 8|e_q| \, (1 - 4|e_q| s_\ell^2) \Delta_{q(2)}}{\sum_q X_q \, [1 + (1 - 4|e_q| s_\ell^2)^2]}$$

$\Delta_{q(2)}$ is known (in SM) for leading Z pole term

Off Z pole: need to include non-res. terms, *estimate* their missing 2-loop terms

A. Freitas, 14/05/2021

Including photon exchange and photon form factor estimate:

(neglecting boxes and s-dependence of Z form factors)

$$A_4 = \frac{\sum_q X_q\, 4\left(\frac{v_\ell}{a_\ell}\frac{v_q}{a_q} + \frac{v_{\ell q}(s)}{a_\ell a_q}\right)}{\sum_q X_q\left(1 + \frac{v_\ell^2}{a_\ell^2} + \frac{v_q^2}{a_q^2} + \frac{v_{\ell q}^2(s)}{a_\ell^2 a_q^2}\right)}$$

$$X_q = f_q(x_1) f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1) f_q(x_2)$$

$$v_{\ell q}(s) = v_\ell v_q + \frac{s - M_Z^2 - iM_Z\Gamma_Z}{s}\, e^2\, e_q\,(1 + \overline{\Delta}_q)$$

$$\frac{v_\ell}{a_\ell} = 1 - 4s_\ell^2, \qquad s_\ell^2 \equiv \sin^2\theta^\ell_{\text{eff}}$$

$$\frac{v_q}{a_q} = 1 - 4|e_q|(s_\ell^2 + \Delta_q)$$

$$\Delta_q = \Delta_{q(1)} + \Delta_{q(2)}$$

$$\overline{\Delta}_q = \underbrace{\overline{\Delta}_{q(1)}}_{\text{known}} + \underbrace{\overline{\Delta}_{q(2)}}_{\text{unknown}}$$

$\Delta_{q(2)}$ is known (in SM) for leading Z pole term

$$\overline{\Delta}_{q(2)} = \pm\overline{\Delta}_{q(1)} \times \frac{g^2}{16\pi^2} n_f, \qquad n_f = 6 + 6N_c$$

(maybe underestimate?)

A. Freitas, 10/12/2021

Including photon exchange and photon form factor estimate:

Impact of EW 2-loop contributions (without EW×QCD):

$\delta A_4/A_4$: $[10^{-4}]$

$m_{\ell\ell}$ [GeV]	Scheme:	α'	α	G_μ
60		0.37	0.35	15.50
70		0.52	0.60	8.99
80		1.53	1.61	37.37
M_Z-2		17.54	10.27	208.5
M_Z-1		2.14	1.97	27.6
M_Z		0.58	0.59	0.57
M_Z+2		0.45	0.46	10.61
M_Z+1		0.55	0.55	16.15
100		0.84	0.83	24.85
110		0.80	0.81	21.71
130		0.53	0.56	12.34
150		0.34	0.38	6.04

- dominated by photon form factor unc. $\overline{\Delta}_q$
- artifically large corrections for G_μ scheme

  [same for (G_μ, s_ℓ, M_Z) scheme?]

A. Freitas, 10/12/2021

towards higher perturbative accuracy:
new $\mathcal{O}(\alpha\alpha_s)$ calculation

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Exact mixed NNLO QCD-EW corrections to the Neutral Current Drell-Yan process

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in collaboration with: R.Bonciani, F.Buccioni, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano,

based on arXiv:2106.11953, arXiv:2007.06518, arXiv:2111.12694

Estimate of the residual uncertainties

The impact of the NNLO QCD-EW corrections is twofold: more accurate predictions (additional higher orders)
reduced uncertainties (scale, inputs, matching)

Ongoing phenomenological studies for full NC DY

A representative example from the results for the on-shell Z production total cross section R.Bonciani, F.Buccioni, N.Rana, AV, arXiv:2007.06518
→ dependence on the EW input-scheme choice

comparison of (G_μ, M_W, M_Z) and $(\alpha(0), M_W, M_Z)$ (very conservative choice that maximises the spread of the results)

order	G_μ	$\alpha(0)$	$\delta(G_\mu-\alpha(0))$ (%)
NNLO-QCD	55787	53884	3.53
NNLO-QCD+NLO-EW	55501	55015	0.88
NNLO-QCD+NLO-EW+ NNLO QCD-EW	55469	55340	0.23

the LO + NLO-EW result would suffer of only 0.55% spread;
the NLO-QCD and NNLO-QCD corrections are only LO-EW and reintroduce a dependence (→0.88%)
which is reduced by the NNLO QCD-EW (→0.23%)

The availability of N3LO-QCD and NNLO QCD-EW results can bring the study of EW gauge bosons in the per mille arena !!!
Is the full NNLO-EW calculation negligible at this level ?

- an important step would be estimating the size of the effects for A_4
- comparison with the approximation implemented in event generators featuring (QCD $\times$ EW)NLOPS

QED corrections (at QCD LO)

- important to disentangle on A_4: ISR, FSR, IFI

- A_4 defined as $8/3 A_{FB}$ or $4\langle\cos\vartheta\rangle$

  equivalent options definitions at LO, but real γ radiation breaks factorization formula

- IFI contributions defined as

$$\begin{aligned} A_{FB}^{\mathrm{IFI}} &= \frac{(\sigma_F-\sigma_B)^{\mathrm{NLO}}-(\sigma_F-\sigma_B)^{\mathrm{ISR}}-(\sigma_F-\sigma_B)^{\mathrm{FSR}}+2(\sigma_F-\sigma_B)^{\mathrm{LO}}}{(\sigma_F+\sigma_B)^{\mathrm{NLO}}-(\sigma_F+\sigma_B)^{\mathrm{ISR}}-(\sigma_F+\sigma_B)^{\mathrm{FSR}}+2(\sigma_F+\sigma_B)^{\mathrm{LO}}} \\ &= \\ \langle\cos\vartheta\rangle^{\mathrm{IFI}} &= \frac{\int\cos\vartheta\, d\sigma_{\mathrm{NLO}}-\int\cos\vartheta\, d\sigma_{\mathrm{ISR}}-\int\cos\vartheta\, d\sigma_{\mathrm{FSR}}+-\int\cos\vartheta\, d\sigma_{\mathrm{LO}}}{\int d\sigma_{\mathrm{NLO}}-\int d\sigma_{\mathrm{ISR}}-\int d\sigma_{\mathrm{FSR}}+2\int d\sigma_{\mathrm{LO}}} \end{aligned}$$

- tuned comparison at fixed order (NLO) level for all codes except for KKMC-HH which produces only exponentiated results for both ISR and FSR

- input parameter scheme: (α_0, M_W, M_Z)

Comparison at NLO QED: $A_4 = 8/3 A_{\mathrm{FB}}$

Code:		$89 < M_{\ell\bar{\ell}}[\mathrm{GeV}] < 93$	$60 < M_{\ell\bar{\ell}}[\mathrm{GeV}] < 81$	$81 < M_{\ell\bar{\ell}}[\mathrm{GeV}] < 101$	$101 < M_{\ell\bar{\ell}}[\mathrm{GeV}] < 150$
$8/3 \cdot [\mathrm{A_{FB}}\,(\text{NLO QED ISR}) - \mathrm{A_{FB}}\,(\text{LO})]/10^{-4}$					
MCSANC		0.2(3)	−5(2)	0.2(3)	5(2)
WZGRAD2		0.2(5)	−5(3)	0.3(5)	6(4)
KKMC-hh		−1.0(6)	0(1)	−0.5(5)	−8(2)
KKMC-hh (NISR)		−1(2)	0(4)	0(1)	6(8)
RADY (CMS)		0.16(4)	−4.05(3)	0.12(3)	4.90(3)
A. Huss		0.17(1)	−4.07(1)	0.11(1)	4.94(4)
$\mathrm{POWHEG_{ew}}$		0.1(1)	−4.0(4)	0.1(1)	4.5(7)
$8/3 \cdot [\mathrm{A_{FB}}\,(\text{NLO QED IFI}) - \mathrm{A_{FB}}\,(\text{LO})]/10^{-4}$					
MCSANC		−2.8(5)	−34(2)	−4.0(4)	−60(3)
WZGRAD2		−1.1(5)	−37(3)	−2.3(5)	−51(4)
KKMC-hh		−3.8(6)	−25(1)	−2.1(1)	−53(1)
KKMC-hh (NISR)		−3.1(6)	−17(1)	−3.2(5)	−60(3)
RADY (CMS)		−1.5(1)	−33.6(4)	−2.49(7)	−59.5(1)
A. Huss		−1.42(6)	−33.9(6)	−2.57(7)	−58.7(3)
$\mathrm{POWHEG_{ew}}$	$\mu_F = M_{\ell\bar{\ell}\gamma}$	−1.2(3)	−62(1)	−2.5(4)	−59(2)
	$\mu_F = M_{\ell\bar{\ell}}$	−1.3(6)	−34(2)	−2.6(7)	−59(3)

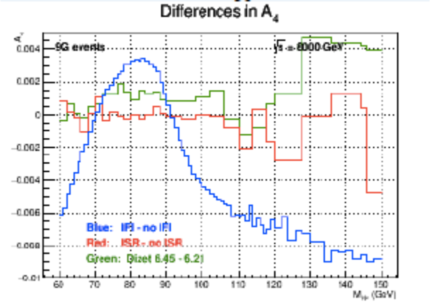
μ_F only enter as PDF factorization scale and the variations in the tab are

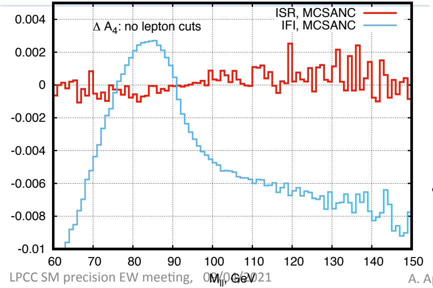
- α_S effects
- expected to become smaller when QCD corrs are included

Comparison at NLO QED: $\overline{A}_4 = 4 < \cos\theta >$

Code:		$89 < M_{\ell\bar{\ell}}[\text{GeV}] < 93$	$60 < M_{\ell\bar{\ell}}[\text{GeV}] < 81$	$81 < M_{\ell\bar{\ell}}[\text{GeV}] < 101$	$101 < M_{\ell\bar{\ell}}[\text{GeV}] < 150$
$[\text{A}_4\,(\text{NLO QED ISR}) - \text{A}_4\,(\text{LO})]/10^{-4}$					
RADY (CMS)		$0.15(3)$	$-4.05(3)$	$0.10(2)$	$4.89(2)$
A. Huss		$0.16(1)$	$-4.07(1)$	$0.11(1)$	$4.87(2)$
$\text{POWHEG}_{\text{ew}}$		$0.07(9)$	$-4.0(3)$	$0.10(7)$	$4.8(4)$
$[\text{A}_4\,(\text{NLO QED IFI}) - \text{A}_4\,(\text{LO})]/10^{-4}$					
RADY (CMS)		$-1.7(1)$	$-42.3(4)$	$-2.97(6)$	$-71.6(2)$
A. Huss		$-1.68(6)$	$-42.4(6)$	$-3.05(8)$	$-71.2(3)$
$\text{POWHEG}_{\text{ew}}$	$\mu_F = M_{\ell\bar{\ell}\gamma}$	$-1.5(5)$	$-70(1)$	$-3.0(4)$	$-71(3)$
	$\mu_F = M_{\ell\bar{\ell}}$	$-1.5(5)$	$-43(1)$	$-3.0(4)$	$-71(3)$

Shape of QED IFI and ISR corrections to A4





- **Shown here from KKMC-hh (top) and MC-SANC (bottom) in full phase space of decay leptons**
- **Bare muons, A4 = 8/3 AFB, AFB computed as $(\sigma_F - \sigma_B) / (\sigma_F + \sigma_B)$**
- **ISR is small, < 1 in units of 10^{-4} and flat versus m_{ll}**
- **IFI is small around Z pole. 1-3 in units of 10^{-4}, but has strong shape vs m_{ll}, with values between -100 and +50 in units of 10^{-4}. It follows roughly shape of asymmetry.**
- **Note that ΔA4 = 1 10^{-4} corresponds to ~ 1 10^{-5} for $\sin^2\theta^{eff}_l$, which means that ΔAFB = 1 10^{-4} → ~ 3 10^{-5}**

Summary and outlook

- tuned comparison of different EW precision tools in very good shape
- different codes by different groups now converging to the required accuracy for future LHC measurements
- fist steps toward a sound evaluation of residual theoretical uncertainties
- validation of calculations which can be used by our exp. colleagues for complete MC simulations