Various outer radiative correction calculations to beta decays*

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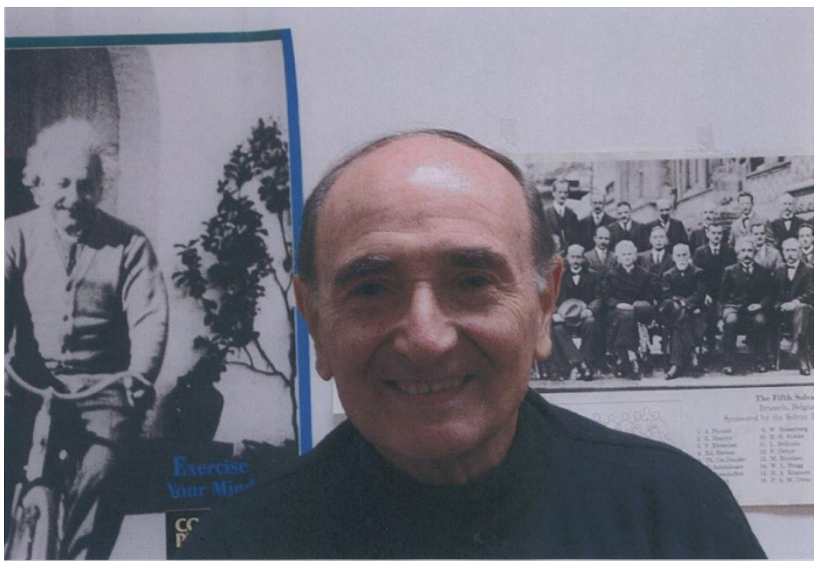
KIT IAP

VCES meeting, 24. Nov. 2022



A. Sirlin (1930-2022)

A. N. Ivanov (1945-2021)

* this talk is dedicated to the memory of A. Sirlin and A. N. Ivanov

For many details and citations missing in my talk see:

F. Glück, Radiative corrections to neutron and nuclear β decays: a serious kinematics problem in the literature, arXiv: 2205.05042v2 (2022).

Why are radiative corrections important?

They are small, but we are looking for small SM or non-SM effects:

- Test of CKM unitarity
- Right-handed, scalar, tensor couplings
- Weak magnetism, second class currents, TRV
- Neutrino mass measurements (tritium beta decay)

aSPECT experiment (proton energy spectrum in neutron decay),
M. Beck et al, Phys. Rev. C 101, 055506 (2020):

$|\lambda|=|G_A/G_V|=1.2677\ (28)$

PERKEO III experiment (electron asymmetry in neutron decay),
B. Märkisch et al, Phys. Rev. Lett. 122, 242501 (2019):

$|\lambda|=|G_A/G_V|=1.2764\ (6)$ → 3 σ difference !

Beyond SM?? E.g. scalar or tensor couplings?

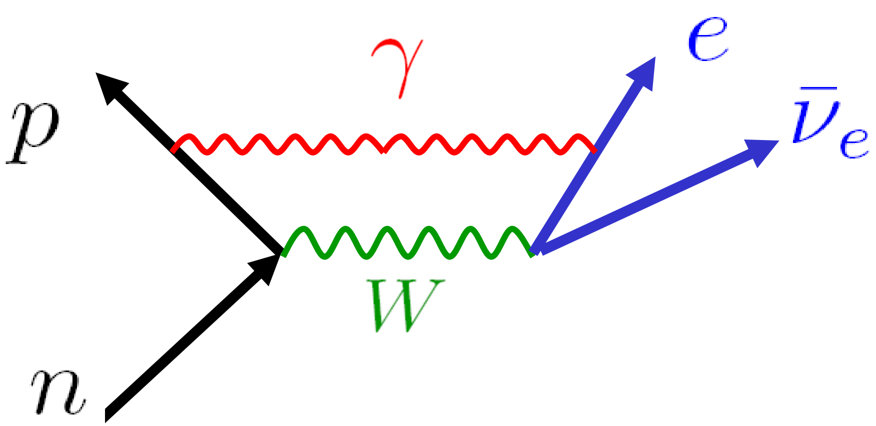
Fictitious aSPECT exp. analysis without outer radiative correction:

$|\lambda|=|G_A/G_V|=1.2796\ (28)$ → more than 4 σ change !

Outer radiative correction important also for aCORN experiment (electron-neutrino correlation in neutron decay), and for Nab experiment (electron and proton energy Dalitz distribution in neutron decay).

Virtual correction

Photon exchange between charged particles:



Virtual photon is off-shell: $k^2_{VIRT} \neq 0$

Energy (K) and momentum (k) of virtual photon are independent !

3-body decay kinematics, like at zeroth-order (without rad. corr.)

Order-α virtual amplitude by 4-dimensional integral:

$$\mathcal{M}_{VIRT} \sim \int d^4k \left\{ \begin{array}{l} \text{wave} \quad \text{functions} \\ \text{propagators} \\ \text{vertices} \end{array} \right\}$$

Interference between zeroth-order amplitude $\mathcal{M}_0$ and virtual correction amplitude $\mathcal{M}_{VIRT}$

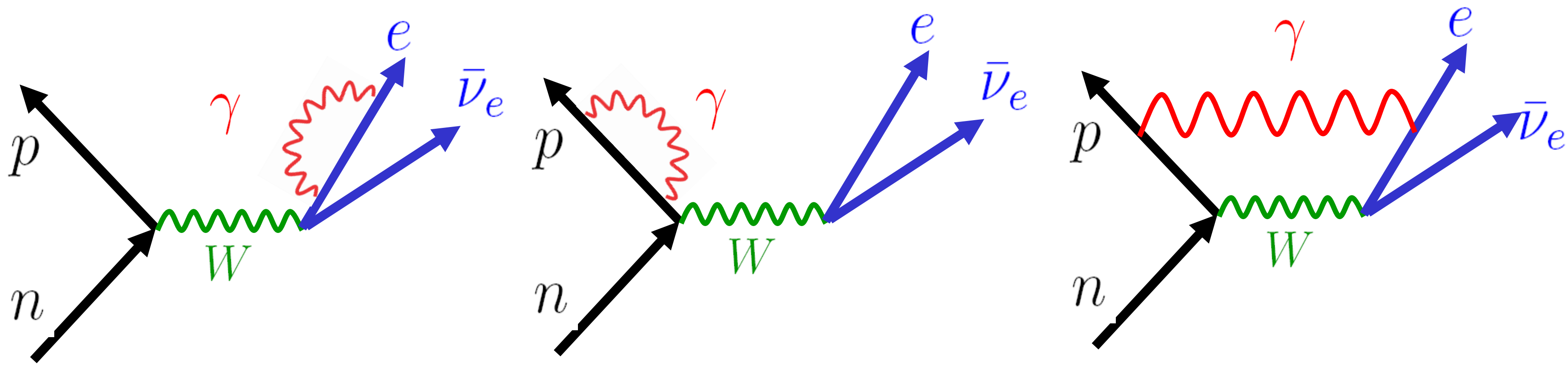
(virtual process indistinguishable from zeroth-order process)

Zeroth-order + order-α virtual correction for observable quantities:

$$\int |\mathcal{M}_0 + \mathcal{M}_{VIRT}|^2$$

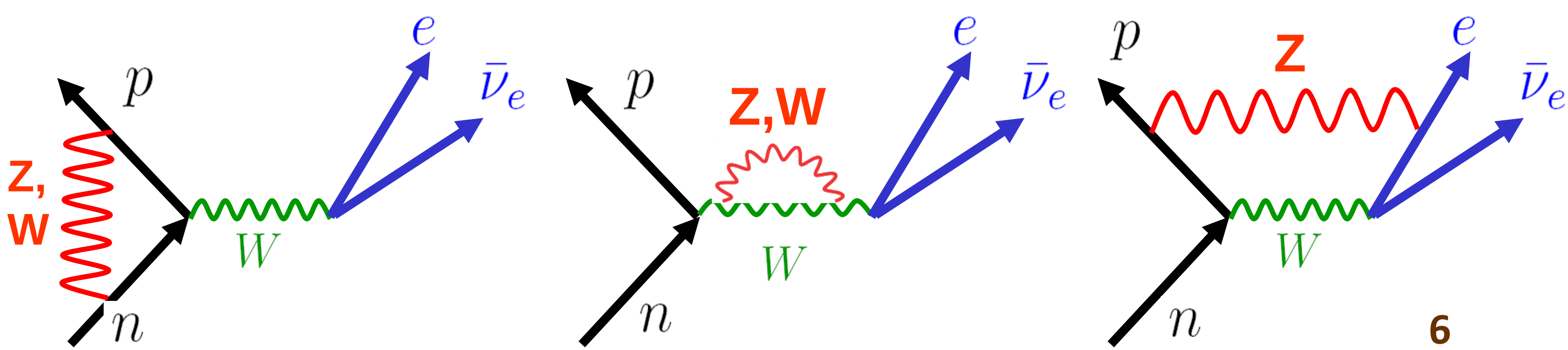
Sirlin (1974,1978)

Photonic diagrams (+3 WWγ graphs):



self-energy e **self-energy p** **box**

Non-photonic diagrams (examples):



Duplication of photonic self-energy integrals by photon propagator decomposition:

$$\frac{1}{k^2} = \underbrace{\frac{1}{k^2 - M_W^2}}_{\text{1. part}} - \underbrace{\frac{M_W^2}{k^2 - M_W^2}\frac{1}{k^2}}_{\text{2. part}}$$

weak correction: all non-photonic + WWγ graphs + 1. part ph. self-energy

photonic correction: photonic box + 2. part photonic self-energy

photonic corrections are UV finite

weak correction: asymptotoc freedom of QCD and electroweak renormalization → cancelation of UV divergences, finite rad. corr.; also IR finite

Weak correction to total beta decay rate:

r_{WEAK}=0.02 % (A. Sirlin, Rev. Mod. Phys. 50 (1978) 573)

Bremsstrahlung correction

Bloch-Nordsieck theorem (1937)

Charged particle processes: bremsstrahlung (BR) photons are always present; probability(no BR photons)=0

Finite energy resolution: only $K>K_{min}$ BR events can be distinguished from processes without any photons

neutron decay: K_{max}=780 keV

with K_{min}=1 keV:

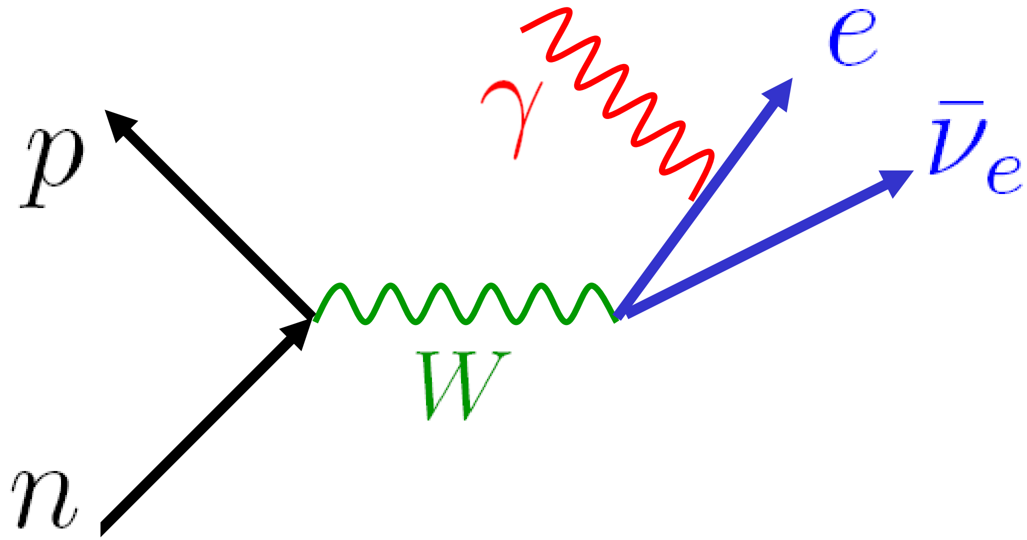
$P(1\ \gamma)=0.5\ \%$

$P(2\ \gamma)=0.001\ \%$

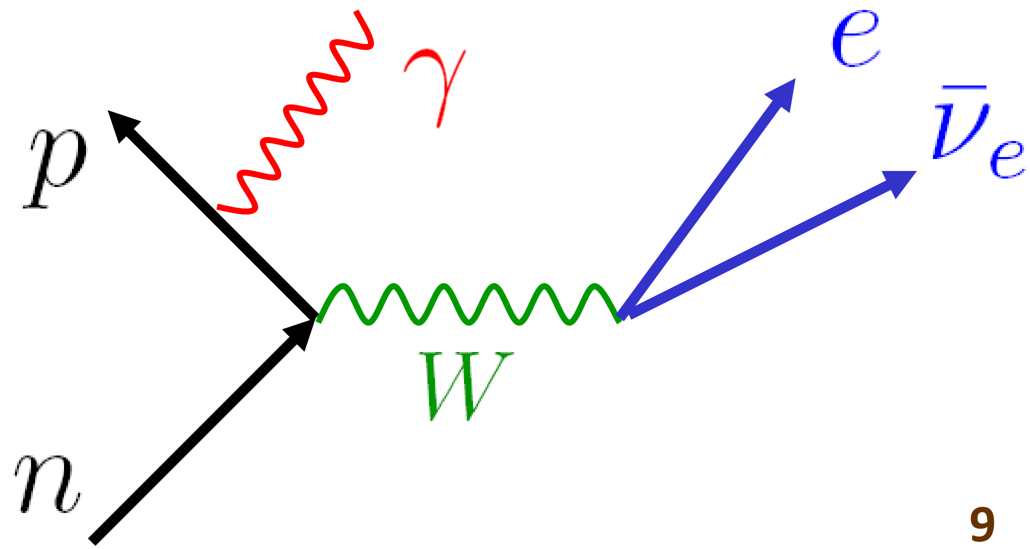
internal photon bremsstrahlung in neutron decay:

$$n \longrightarrow p e \bar{\nu}_e \gamma$$

$\mathcal{M}_l$

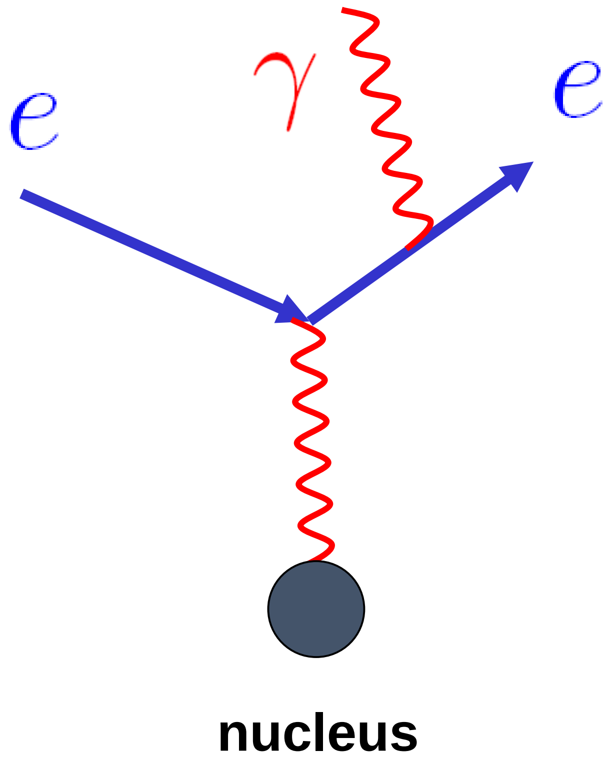


$\mathcal{M}_h$



Internal (inner) bremsstrahlung completely different from external BR. External BR is independent of the decay, internal BR occurs during the decay.

external BR of electron:



Possible confusion: inner and outer radiative correction.
The inner radiative correction (completely virtual process) has nothing to do with the inner bremsstrahlung !

Photon bremsstrahlung amplitude (gauge invariant):

$$\mathcal{M}_{BR} = \mathcal{M}_l + \mathcal{M}_h$$

$\mathcal{M}_l \longrightarrow$ QED (accurate,reliable calc.)

$\mathcal{M}_h \longrightarrow$ generally model (strong int.) dependent

BUT !

BR photon energy in neutron decay < 0.78 MeV

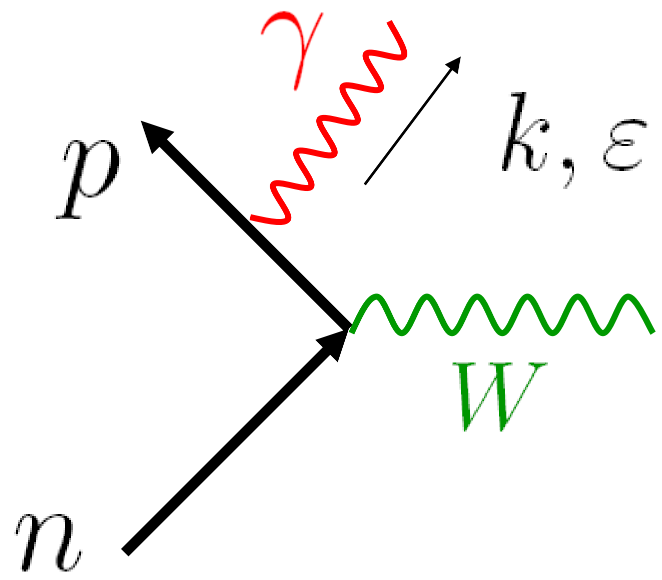
$\longrightarrow$ BR photon wavelength > 1500 fm

BR photons in neutron decay can see only the proton charge (and slightly the nucleon magnetic moment), but not the inner structure of the nucleons !

Order-K^{-1} part of the hadronic BR amplitude:

$$\mathcal{M}_h[K^{-1}] = e\frac{(p\varepsilon)}{(pk)}\mathcal{M}_0$$

$$k = (K, \mathbf{k}), \quad K = |\mathbf{k}|$$



$\mathcal{M}_0$ **: zeroth-order amplitude (without radiative corr.)**

⟹ 1/K behaviour of low energy BR photon spectrum

Low theorem **(F. E. Low, Phys. Rev. 110 (1958) 974)**

From EM current conservation (gauge invariance) the order-K^0 part (next order, subleading) of the hadronic BR amplitude can also be reliably (model independently) computed

(depends on magnetic moments of the nucleons)

Many experimental tests of Low theorem in high energy decay and scattering processes

From Low theorem: only the order-K part of the BR photon amplitude is model dependent

$$O(K^0) \sim \frac{K}{m_n} O(K^{-1}) \sim 10^{-3} \cdot O(K^{-1})$$ **(K=1 MeV)**

$$O(K) \sim \left(\frac{K}{m_n}\right)^2 O(K^{-1}) \sim 10^{-6} \cdot O(K^{-1})$$ **(K=1 MeV)**

→ **10^{-6} accuracy of photon BR calc. in neutron decay (for K=100 keV: 10^{-8} accuracy)**

No information about strong interaction dynamics from photon bremsstrahlung in neutron decay !

Photon BR measurement in neutron decay: test of QED and Low theorem in a low energy weak decay process

Photon bremsstrahlung: no interference with zeroth-order amplitude (BR photon is in principle detectable)

Order-α radiative correction calculation of observable quantities:

$$\int |\mathcal{M}_0 + \mathcal{M}_{VIRT}|^2 + \int |\mathcal{M}_{BR}|^2$$

Order-α terms:

$$\int |\mathcal{M}_{BR}|^2 = -C_1 \ln m_\gamma + C_2$$

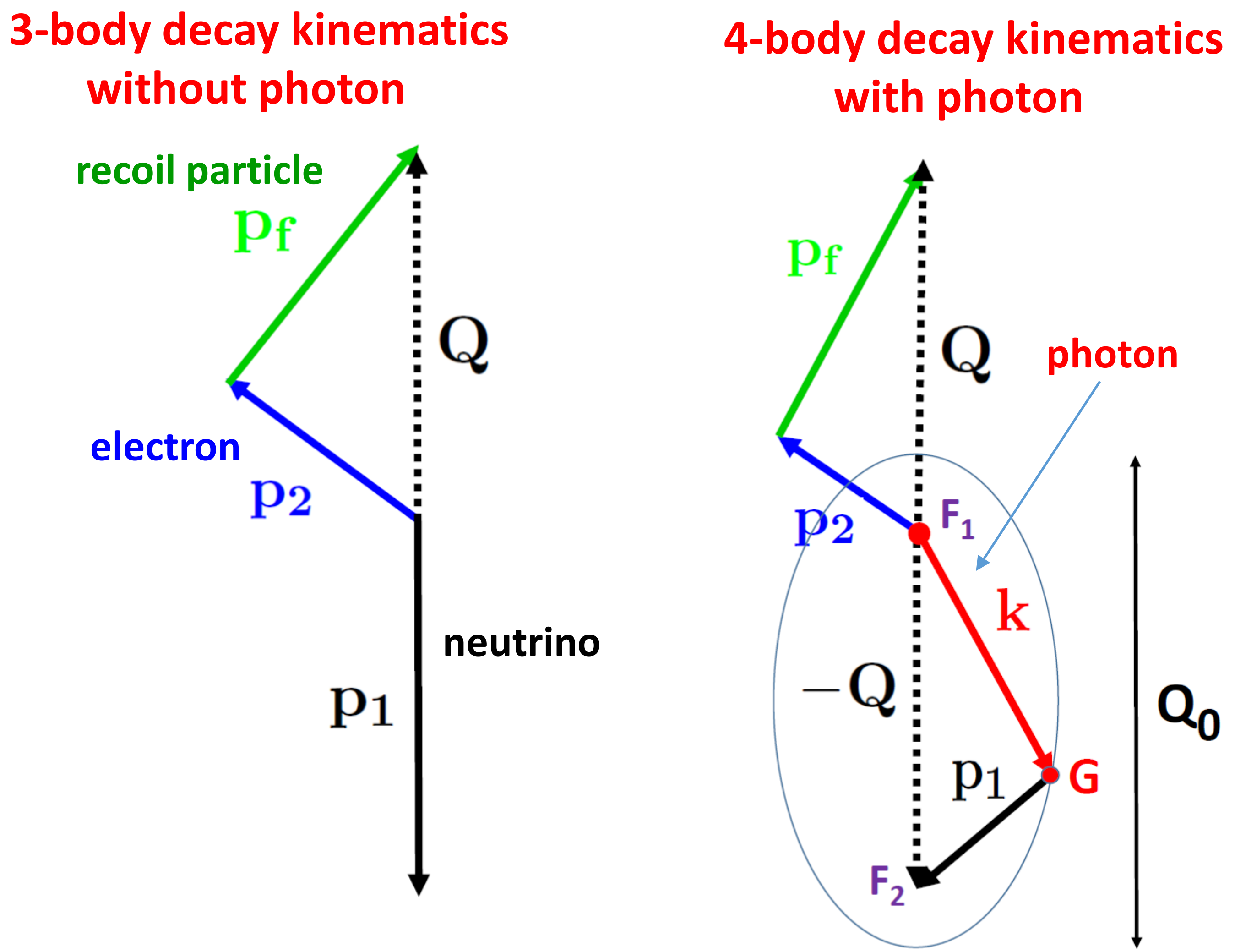
$$2 \int \Re(\mathcal{M}_0 \mathcal{M}^*_{VIRT}) = C_1 \ln m_\gamma + C_3$$

(m_γ: IR regulator photon mass)

Infrared divergent terms cancel in the VIRTUAL+BR sum

Radiative correction: virtual + bremsstrahlung correction

BR photon changes the decay kinematics !



In bremsstrahlung part of rad. corr. calc.:
3-body decay kinematics is not suitable to use!!

Bremsstrahlung correction calculations

- theoretically simple and reliable
- technically complicated

Integration in many dimensional phase space:

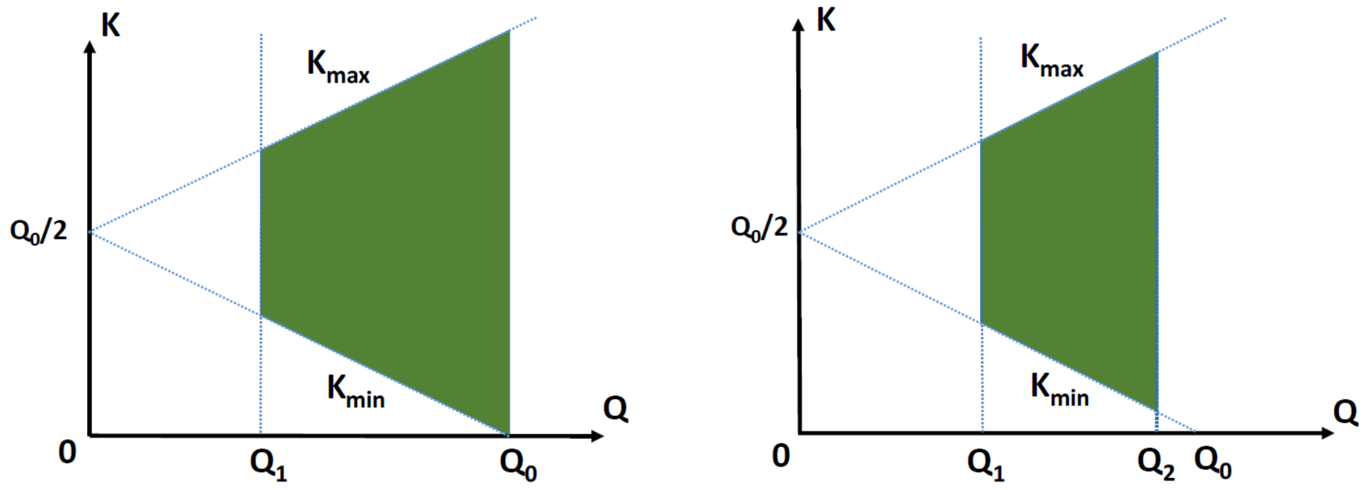
$$\int \frac{d^3\mathbf{p}_p}{E_p} \frac{d^3\mathbf{p}_e}{E_e} \frac{d^3\mathbf{p}_\nu}{E_\nu} \frac{d^3\mathbf{k}}{K} \delta^4(p_n - p_p - p_e - p_\nu - k) \cdot \sum_{spin} |\mathcal{M}_{BR}|^2$$

$\sum_{spin} |\mathcal{M}_{BR}|^2$: Dirac matrix algebra, Lorentz-indices

Computation by symbolic algebra code (Reduce, Mathematica), or by hand.

Double energy Dalitz distribution BR correction:

$$W_{BR}(E_2,T) = \frac{1}{2^{10}\pi^6 m_i}\int_{Q_1}^{Q_{\max}} dQ \int_{K_{\min}}^{K_{\max}} dK \frac{K}{\sqrt{K^2+m_\gamma^2}} \int_0^{2\pi} d\phi_k \,\bar{M}_{BR}$$



(a) IN region (b) OUT region

K=|**k**| → photon energy

BR phase space integration methods

A, Analytical (Q, K, φ_k) integrations
(F. Glück, diploma and PhD works;
F. Glück and K, Tóth, 1990; E. S. Ginsberg, 1967, etc.)

B, Semianalytical (Q, K, φ_k) integrations
(F. Glück, diplome work; F. Glück,1993; F. Glück and K, Tóth, 1992;
C-Y. Seng et al, 2021; F. Glück, SANDI: Semi-Analytical Neutron Decay Integrator
C++ code for unpolarized neutron and nuclear beta decays (to be published))

C, Monte Carlo integrations
(F. Glück, 1997; F. Glück and I. Joó, 1997; F. Glück, GENDER: Generation of Neutron
Decay Events with Radiative and recoil corrections (to be published))

The Monte Carlo method has several advantages (e.g. much simpler than the (semi)analytical integrations; same code for many different quantities; experimental details can be included, etc.)

Inner and outer corrections

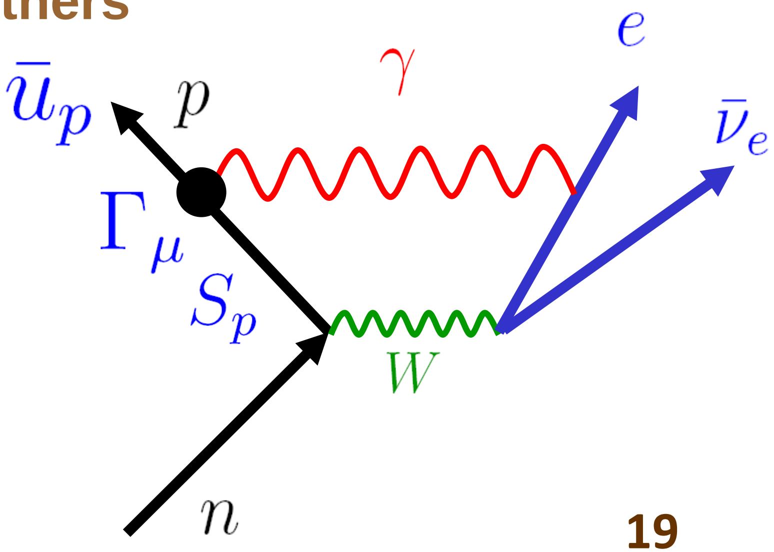
Photonic virtual correction: **- IR divergent**
- strong interaction dependent

(1 GeV photons disturb the nucleon inner structure)

Radiative correction contribution with small photon energy (BR + virtual): IR divergent, no strong interaction dependence, depends on particle momenta (changes the spectrum shapes)
→ should be separated from the others

Sirlin, 1967:

$$Z_\mu = \bar{u}_p \Gamma_\mu S_p$$



Point-like hadron model:

$$Z_\mu = \bar{u}_p \gamma_\mu \frac{\not{p} + \not{k} + m}{(p+k)^2 - m^2}$$

Convective term – spin term separation (Yennie, Frautschi, Suura,1961; Meister, Yennie,1962):

$$\bar{u}_p\gamma_\mu(\not{p}+\not{k}+m) = \bar{u}_p\left\{\underbrace{2p_\mu + k_\mu}_{\text{convective term}} \quad + \quad \underbrace{\frac{1}{2}[\gamma_\mu, \not{k}]}_{\text{spin term}}\right\}$$

Outer (model independent) virtual correction:

photonic virtual integrals with convective term

$$Z_\mu = \bar{u}_p\Gamma_\mu S_p = Z_\mu^{\mathrm{MI}} + Z_\mu^{\mathrm{MD}}$$

$$Z_\mu^{\mathrm{MI}} = \bar{u}_p\frac{2p_\mu + k_\mu}{(p+k)^2 - m^2}$$

Z_μ^{MD} : **precise calculation difficult, but its general properties are similar to spin term**

Outer radiative correction= outer virtual + bremsstrahlung

Properties of outer correction:

i, no (or small) strong interaction dependence → reliable

ii, sensitive to experimental details (f.e.: photon bremsstrahlung changes the kinematics)

iii, changes the spectrum shapes and asymmetries

Outer radiative corrections are important for the experimental analyses !

Experimental details are important for the outer radiative correction calculations!

Radiative corr.= BR + virtual = inner (MD) +outer (MI)

Inner (model dependent) = weak + inner part of photonic virtual corr.

⟶ inner correction is pure virtual (no IR divergence)

outer virtual: main contribution from small energy virtual photons (small energy = much smaller than nucleon mass)

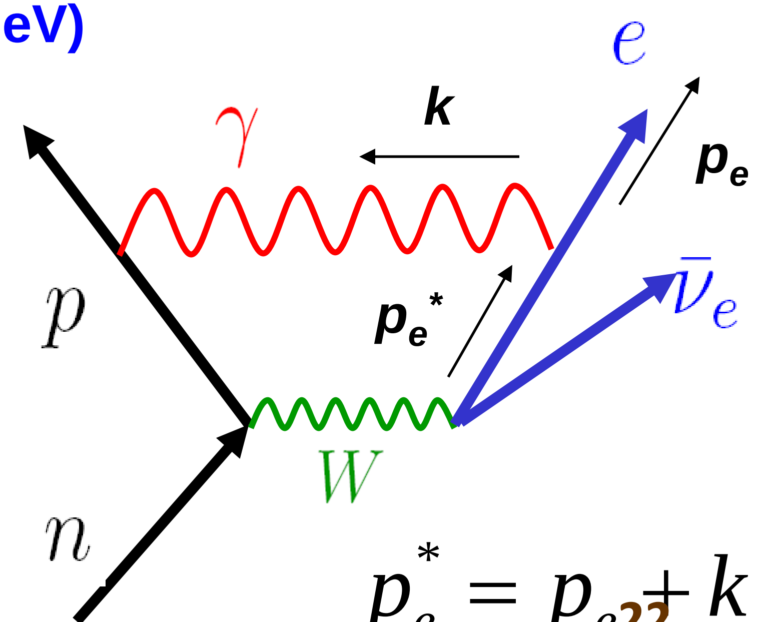
inner: main contribution from intermediate and high energy virtual photons (intermediate energy: not far from 1 GeV; high energy: much larger than 1 GeV)

Small photon energy (momentum):

Propagator momenta are sensitive to external momenta

Large photon energy (momentum):

Propagator momenta depend mainly on virtual photon momentum, they are not sensitive to the external momenta



$$p_e^* = p_e + k$$

→ **no change of spectrum shapes and angular distributions due to the inner correction**

A. Sirlin, Phys. Rev. 164 (1967) 1767

for neutron decay

Neglecting terms of order $\sim \alpha \frac{E_e}{m_n} \ln\left(\frac{m_n}{E_e}\right) \sim 10^{-5},$

the inner correction can be absorbed (approximately) into the dominant form factors f_1 and g_1

Effective form factors:

$$f_1' := f_1\left(1 + \frac{\alpha}{2\pi}c\right), \quad g_1' := g_1\left(1 + \frac{\alpha}{2\pi}d\right)$$

Inner corr.: 2 numbers (c, d)

Redefinition of G_V and λ :

$$G_V := G_\mu V_{ud} f_1', \quad \lambda := g_1'/f_1'$$

All measureable quantities in neutron decay depend on these effective parameters (c and d are the same for all quantities)

SM tests by comparison of λ from different types of experiments (like electron asymmetry and electron-neutrino correlation) are independent of the inner correction !

Inner correction to the vector coupling constant is important for V_{ud} determination and for CKM unitarity test !

Recoil-type and neutrino-type outer rad. corrections

Outer rad. corr. to electron energy spectrum in nuclear beta decays (Sirlin 1967) and to electron asymmetry (Shann 1971):
simple analytical formulas.

Recoil particle is not observed → analytical integration of bremsstrahlung amplitude squared is possible.

Y. Yokoo and M. Morita (1976), K. Fujikawa and M.Igarashi (1976),
A. Garcia and M. Maya (1978), A. Garcia (1982):
fixed electron-neutrino angle → analytical integration is still possible!

Rad. corr. calc. to electron-neutrino correlation with these analytical formulas: are approriate if neutrino is explicitly observed (measured).

But it is not observed (in most beta decay experiments) !

Electron-neutrino angle: connected to observable recoil particle by 3-body kinematics → wrong in bremsstrahlung calculation !!!

(K. Toth, KFKI-1984-52, K. Toth et al., Phys. Rev. D33 (1986) 3306, Phys. Rev. D 40 (1989) 119)

Neutrino-type outer rad. corr.: using fixed neutrino direction, and 3-body kinematics connection to recoil particle.

Recoil-type outer rad. corr.: using only the electron and recoil particle momenta as fixed parameters, but not the neutrino and the photon (the latter are integrated over the allowed phase space).

Fixed neutrino direction, integration with respect to photon: recoil particle momentum follows the photon momentum due to momentum conservation →
the neutrino-type correction calculations are not appropriate for observables where the recoil particle is measured
(e.g. recoil energy spectrum, electron-recoil Dalitz distr. etc.), because the recoil momentum has to be fixed during the integration.

The neutrino-type and the recoil-type correction results are completely different (see plots later).

For quantities where the recoil particle is not observed:
the two calculations agree with each other.

Neutrino-type outer rad. corr.:

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Recoil-type outer rad. corr.:

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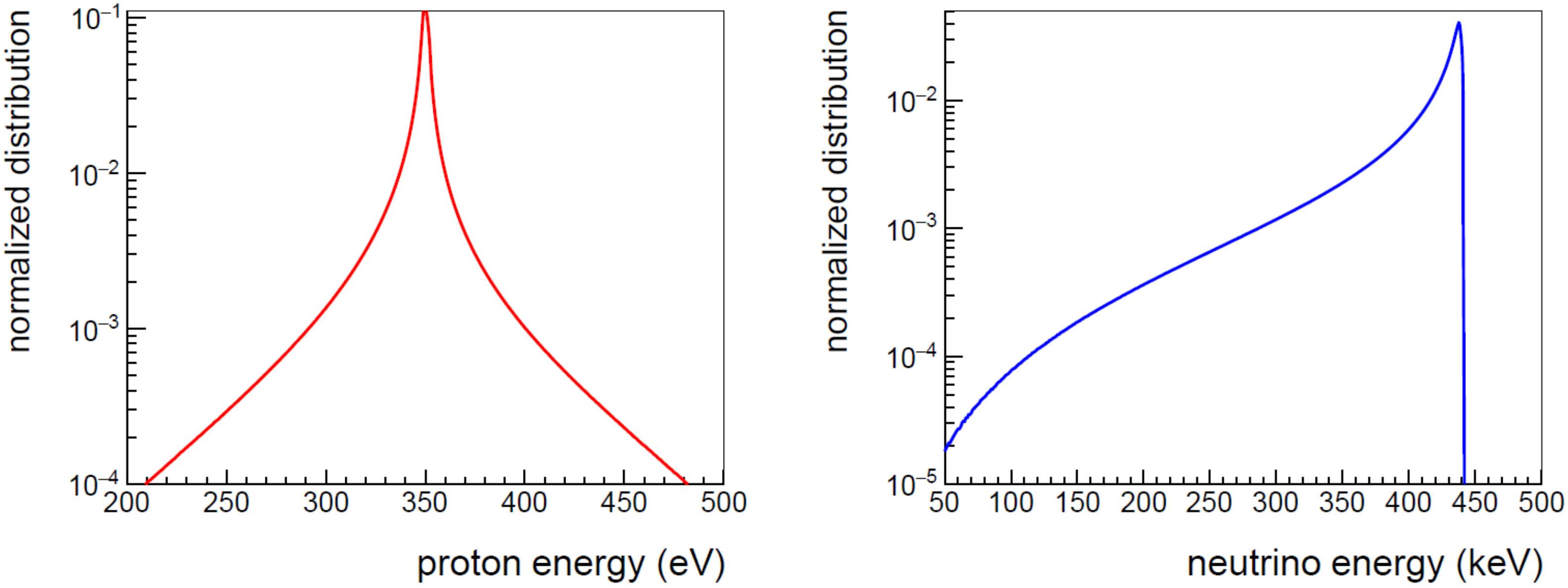
Proton and neutrino energy distributions in neutron decay for fixed electron energy and electron-neutrino angle (computed by new MC code GENDER)



(a) proton energy distribution

(b) neutrino energy distribution

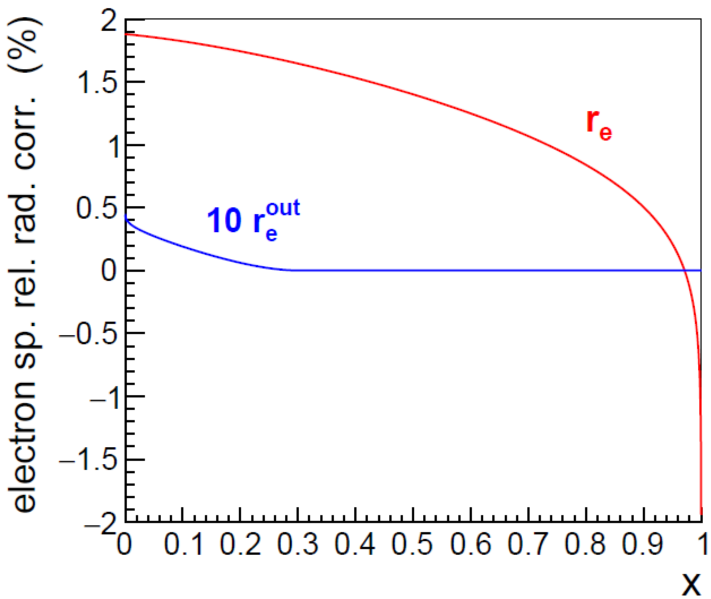
Figure 4: Proton and neutrino energy (T and E_1) distributions in neutron decay with fixed $E_2 - m_2 = 340$ keV electron kinetic energy and 90° electron-neutrino angle. With $K = 0$ bremsstrahlung photon energy: $T = T_0 = 350.87$ eV, $E_1 = E_{10} = 441.98$ keV. Only the $|T - T_0| > 1$ eV events (with 0.57 % probability) are plotted.

From: F. Glück, arXiv:2205.05042v2

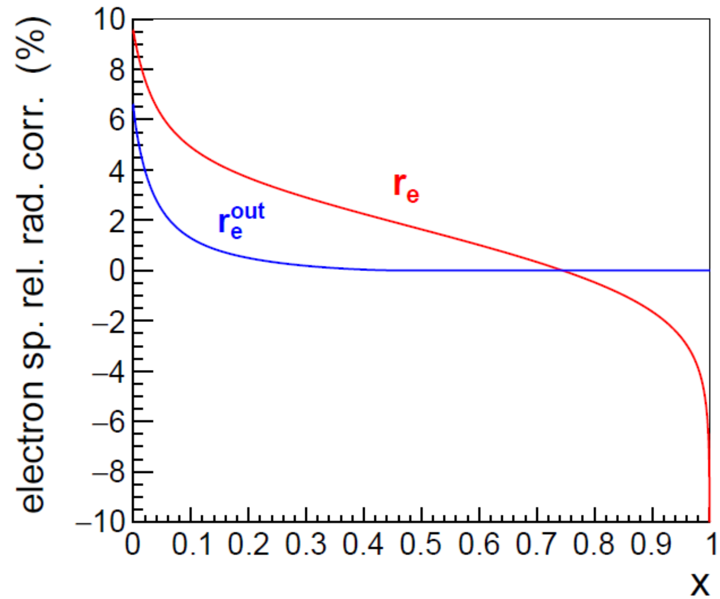
With 3-body decay kinematics (used by neutrino-type rad. corr.): proton and neutrino energy are both fixed.

Relative outer radiative correction r_e to electron energy spectrum

From: F. Glück, arXiv:2205.05042v2



(a) neutron decay



(b) 10 MeV nuclear decay

$$E_2 = m_2 + (E_{2m} - m_2)x$$

(E_2, m_2: electron total energy and mass)

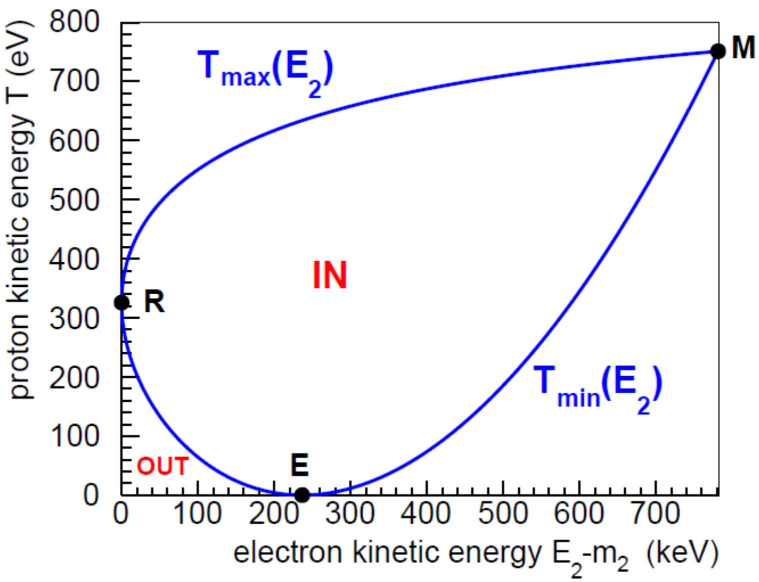
$$r_e = 100 \left(\int_{T'_{min}(E_2)}^{T_{max}(E_2)} dT \cdot W_\gamma(E_2, T) \right) / w_{e0}(E_2)$$

No difference between recoil-type and neutrino-type corrections.

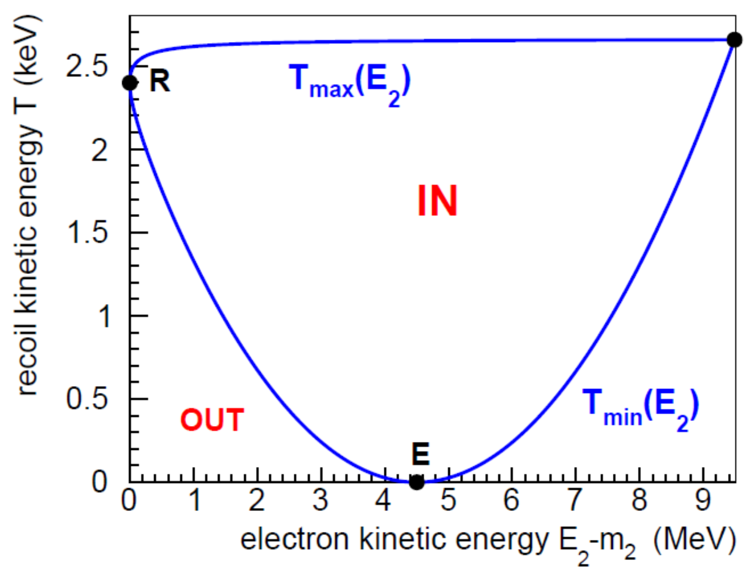
Logarithmic singularity at x=1 can be removed by exponentiation (not important for experimental analyses).

Double energy Dalitz plots

3-body kinematics: region IN.
4-body kinematics: regions IN + OUT.



(a) neutron decay



(b) 10 MeV nuclear decay

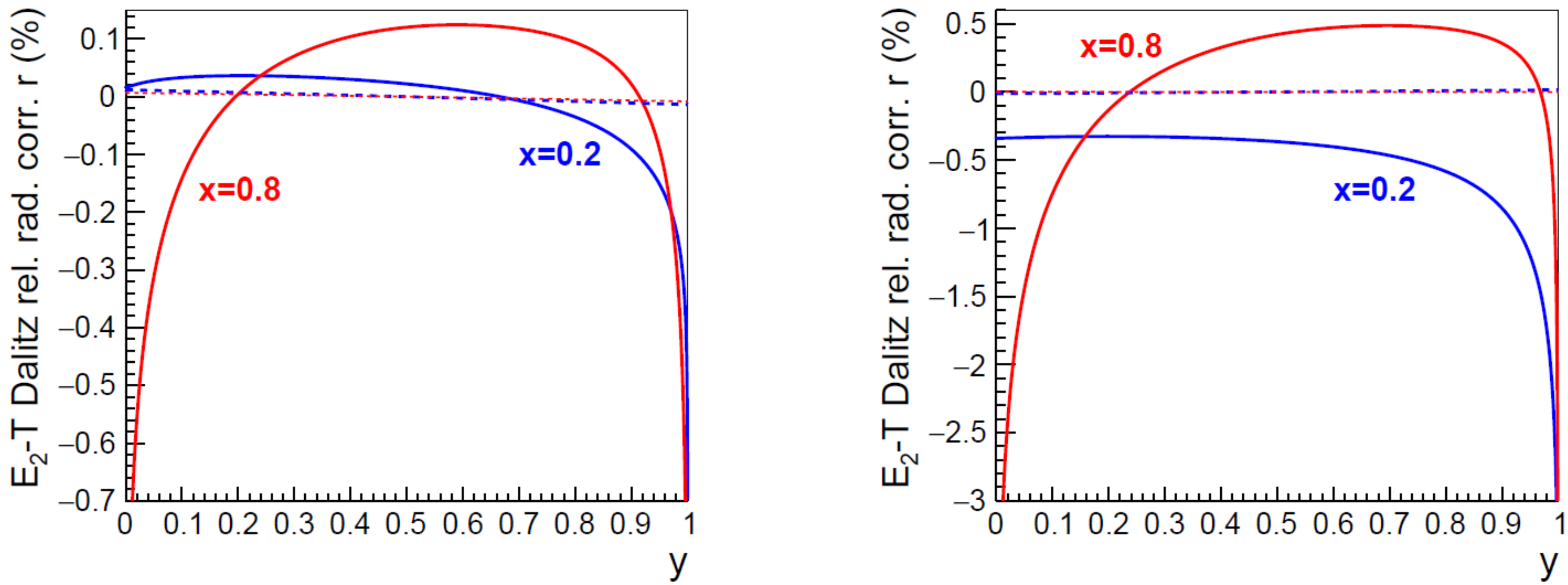
From: F. Glück, arXiv:2205.05042v2

$$T'_{min}(E_2) = T_{min}(E_2) \text{ for } E_2 > E_{2h}$$ (point E)

$$T'_{min}(E_2) = 0 \text{ for } E_2 < E_{2h}$$

Relative outer radiative correction *r* to the (E_2,T) double energy Dalitz distribution

From: F. Glück, arXiv:2205.05042v2



(a) neutron decay (b) 10 MeV nuclear decay

$$T = T_{min}(E_2) + (T_{max}(E_2) - T_{min}(E_2))y$$

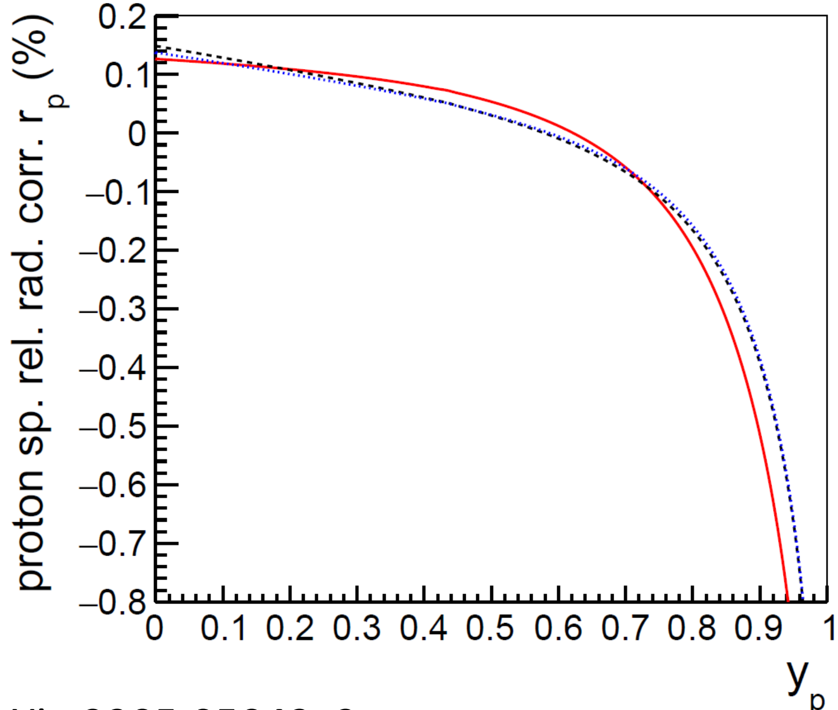
(T: recoil particle kinetic energy)

Solid curves: recoil-type correction,
dashed curves: neutrino-type correction.

$$r = r(x,y) = 100\frac{W_\gamma(E_2,T)}{W_0(E_2,T)} - r_e(x)$$

This correction is important for the Nab experiment analysis.

Relative outer radiative correction r_p to the proton energy spectrum in neutron decay



From: F. Glück, arXiv:2205.05042v2

$$T = y_p T_m$$

Red curve: recoil-type correction,
dashed (black) curve: neutrino-type correction,
dotted (blue) curve: electron-type correction.

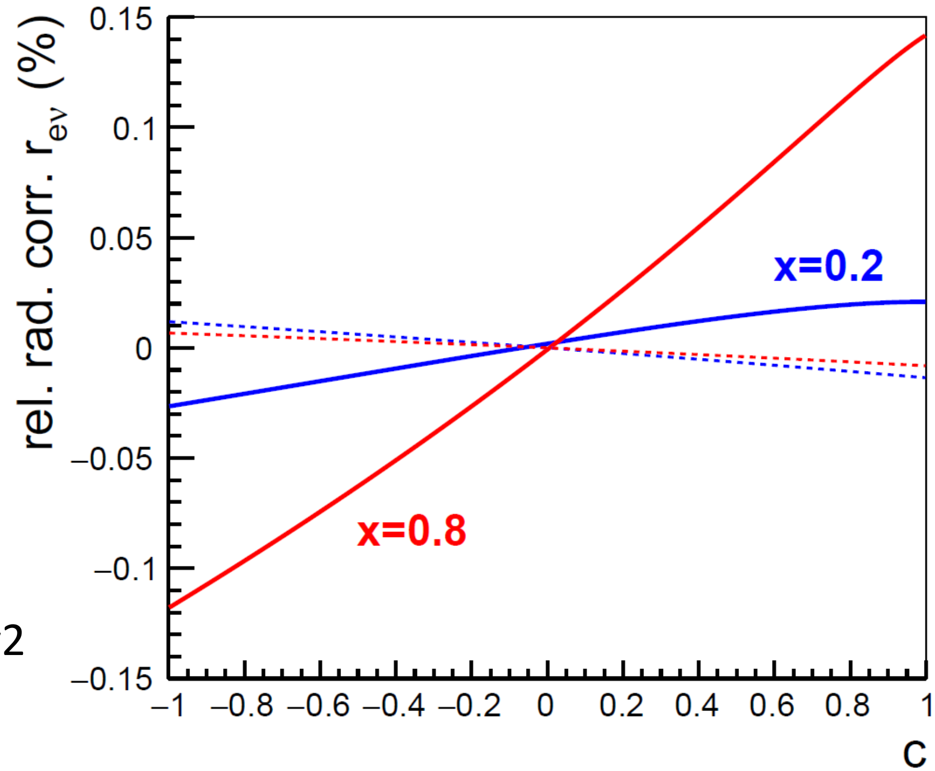
$$r_p = 100 \frac{w_{p\gamma}(T)}{w_{p0}(T)} - r_\rho$$

This correction is important for the aSPECT experiment analysis.

Relative outer radiative correction $r_{e\nu}$ to the (E_2, c) electron-neutrino correlation Dalitz distribution in neutron decay

$$r_{e\nu} = 100\frac{W_\gamma^{e\nu}(E_2, c)}{W_0^{e\nu}(E_2, c)} - r_e(x)$$

From: F. Glück, arXiv:2205.05042v2



Solid curves: recoil-type correction
(e-ν angle defined by electron and recoil p.),

$$c = \cos\theta_{e\nu} = -\frac{\mathbf{p_2} \cdot (\mathbf{p_2} + \mathbf{p_f})}{P_2|\mathbf{p_2} + \mathbf{p_f}|}$$

dashed curves: neutrino-type correction
(e-ν angle defined by electron and neutrino).

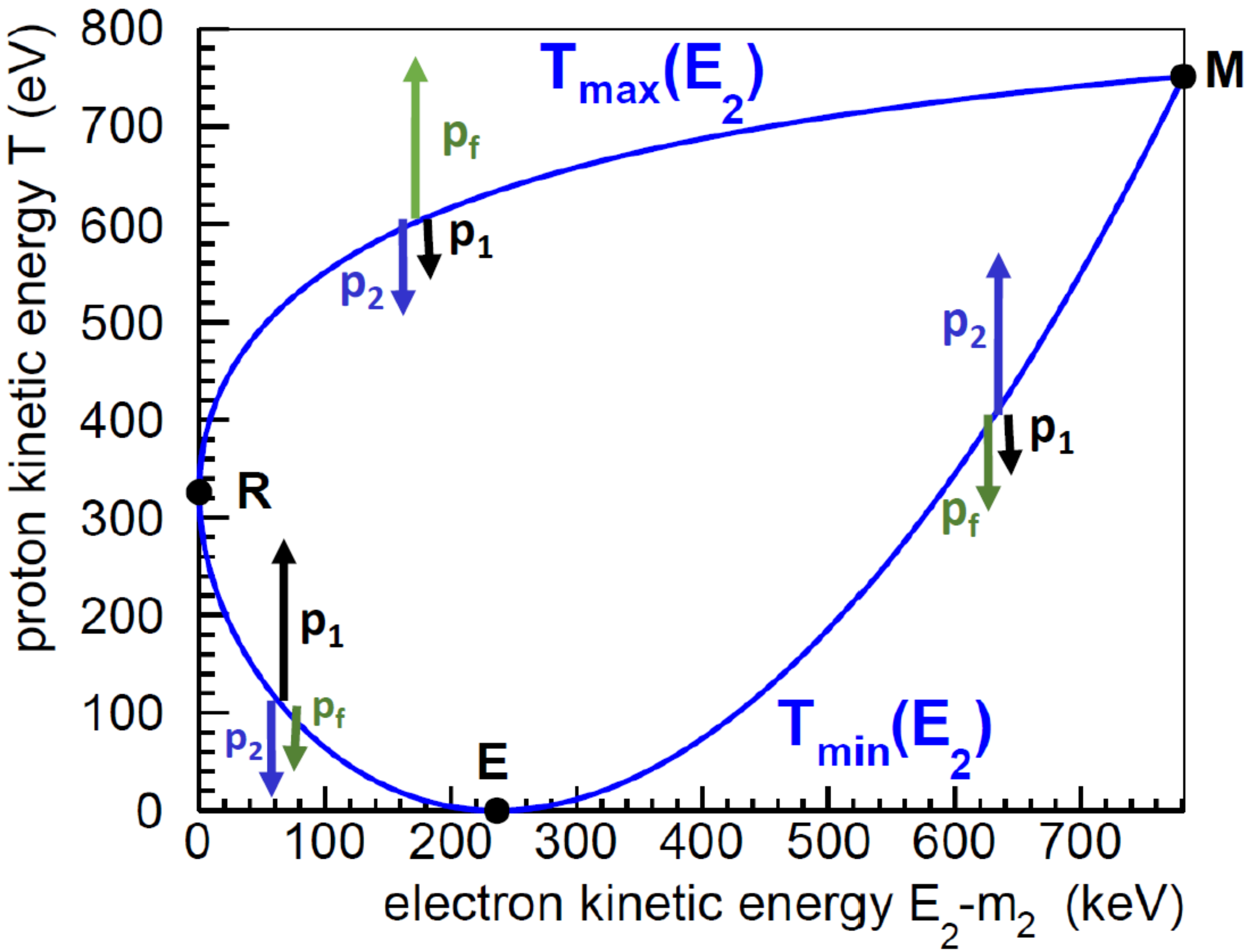
$$c = c_{12} = \cos\theta_{e\nu} = \frac{\mathbf{p_2} \cdot \mathbf{p_1}}{P_2 E_1}$$

This correction is important for the aCORN experiment analysis.

Summary

- Outer radiative corrections are important for precision analyses of beta decays.
- Presence of bremsstrahlung photon changes the kinematics from 3-body to 4-body → using 3-body kinematics in the bremsstrahlung calculations is not allowed.
- Various bremsstrahlung integration methods in the literature (analytical, semianalytical, Monte Carlo); the MC method has several important advantages.
- The neutrino-type calculation method uses 3-body kinematics in order to connect the unobserved neutrino momentum with the recoil particle.
- The neutrino-type and the recoil-type radiative correction results are completely different for observables with explicit recoil particle detection
  → the neutrino-type corrections are not suitable for the analyses of these measurements
  → the recoil-type calculations have to be used (although here the analytical bremsstrahlung integrations are much more difficult)

Backup slides



proton kinetic energy T (eV)
electron kinetic energy E_2-m_2 (keV)
$T_{max}(E_2)$
$T_{min}(E_2)$
M
R
E
p_f
p_1
p_2
0
100
200
300
400
500
600
700
800