Fundamental constants, their numerical values & units

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Outline (as of 2014 – the last LSA of the old SI)

- *universality of the constants; variety of the experiments; overdetermined data set; correlated input data; need for the least-square evaluation;*
- *the progress and the state of the art; numerical values of c, e_0, m_0, M_e, M_p, m_e/m_p, R_∞, a, hN_A; h, N_A, e, m_e, m_p, k, R, G in various units;*
- *LSA procedure;*
- *structure of the data; hierarchy in the accuracy of the data; disentanglement of the sets;*
- *set-by-set analysis;*
- *exactly known values;*
- *determination of the Rydberg constant;*
- *atomic masses and AME by AMDC;*
- *determination of the fine structure constant;*
- ~~*determination of the Planck constant;*~~
- ~~*determination of the Boltzmann constant;*~~
- *determination of the Newton's constant of gravity;*
- *inconsistent data sets and the Birge ratio;*
- *Gaussian distribution, c^2 and c^2 distribution;*
- *multivariate LSA with correlations;*
- *correlated output;*
- *microscopic and macroscopic values;*
- *LSA and verification of fundamental laws*

The CODATA's constants

- *Universal*
- *Practical*
- *Not necessary fundamental*
- *Allow a [relatively] high accuracy*
- *Need high-precision experiments*
- *Involve standards*

- *Atomic*
  - R_∞, a, $A_r(e)$, m_e/m_p
- *Thermodynamic [values]*
  - N_A, k
- *Macroscopic [values]*
  - h, e, m_p
- *G*

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  - ~~N_A, k~~
- ~~Macroscopic [values]~~
  - ~~h, e, m_p~~
- G

The CODATA's constants

CODATA deals not with the constants, but with their <u>numerical</u> values.

The proton mass in the kilograms <u>was</u> the ratio of the proton mass (atomic scale) and the mass of the IKP (macroscopic).

- Atomic
  - R_∞, a, $A_r(e)$, m_e/m_p
- ~~Thermodynamic [values]~~
  - ~~N_A, k~~
- ~~Macroscopic [values]~~
  - ~~h, e, m_p~~
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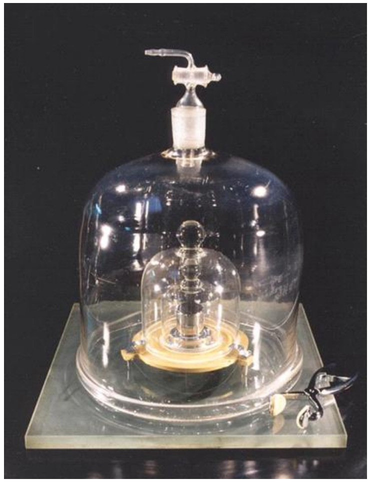
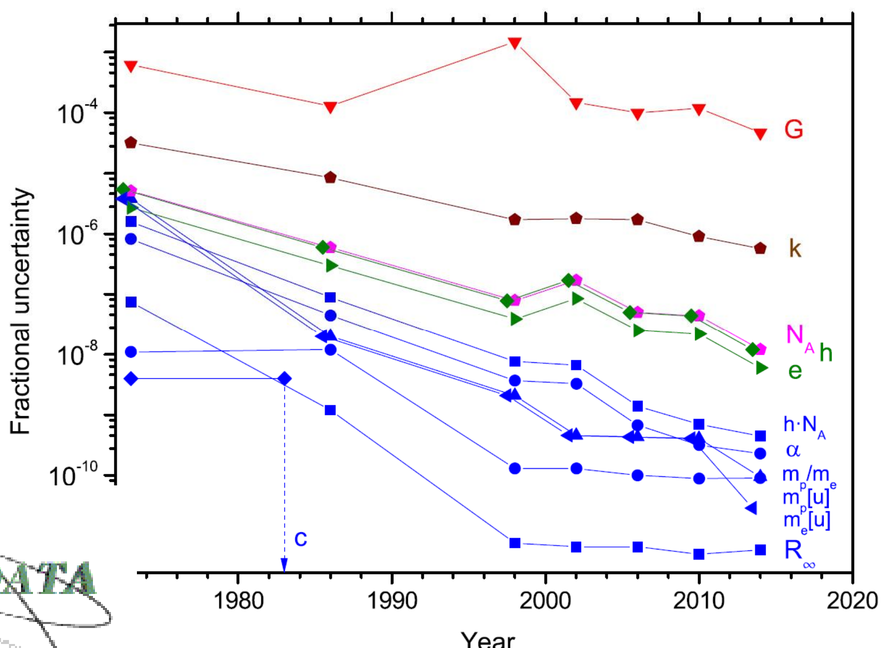
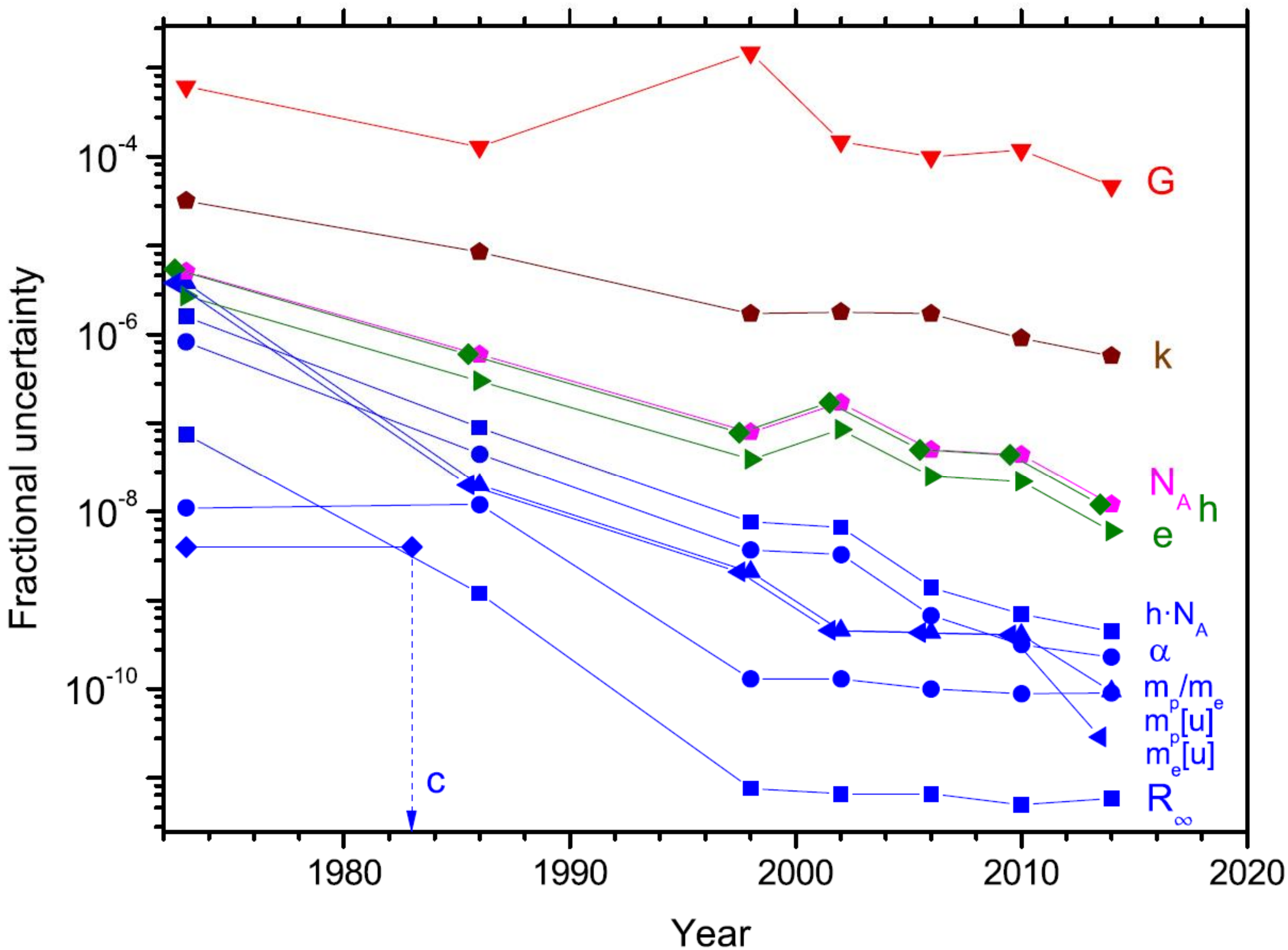


Variety of phenomena involved (dominant uncertainty)

- *Atomic and quantum values*
- ~~*Macroscopic experiments*~~
- ~~*Macroscopic experiments and isotopic composition*~~
- ~~*Thermodynamics and isotopic composition*~~



Progress (up to 2014)



Two-steps of the adjustment

- ***Set-by-set analysis***
- *Set-by-set pre-average*

- *Evaluation of the data altogether*

Input: disentanglement

- *In principle everything is correlated.*
- *How can we disentangle the sets?*

Input: disentanglement – basic equations

$$R_\infty = \nu_H \left[c_1 \left(1 - \frac{m_e}{m_p} \right) + c_2 \alpha^2 \right]$$

$$\alpha^2\big|_{\text{recoil}} = 2R_\infty \frac{h}{m_{\text{atom}} c} \frac{m_{\text{atom}}}{m_p} \frac{m_p}{m_e}$$

$$\frac{m_e}{m_p}\bigg|_{\text{ion}} = r_{\text{ion}} \left(1 + c_3 \alpha + c_4 (Z\alpha)^2\right)$$

$$\frac{m_e}{m_p}\bigg|_{\bar{p}\text{He}} = \frac{R_\infty c}{\nu_{\bar{p}\text{He}}} \left(1 + c_5 \alpha^2\right) \; ,$$

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Determination of R_∞

$$\frac{m_e}{m_p} \bigg|_{\text{ion}} = r_{\text{ion}} \left(1 + c_3 \alpha + c_4 (Z\alpha)^2 \right)$$

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$$\left. \frac{m_e}{m_p} \right|_{\bar{p}\mathrm{He}} = \frac{R_\infty c}{\dots} \left(1 \dots {}^2 \right)$$

Determination of a

Input: disentanglement – basic equations

$$R_\infty = \nu_H \left[c_1 \left(1 - \frac{m_e}{m_p} \right) + c_2 \alpha^2 \right]$$

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fractional uncertainty:

- $u(R_\infty) \sim 6 \times 10^{-12}$
- $u(a^2) \sim 4 \times 10^{-10}$
- $u(m_e/m_p) \sim 10^{-10}$

Input: disentanglement – additive terms

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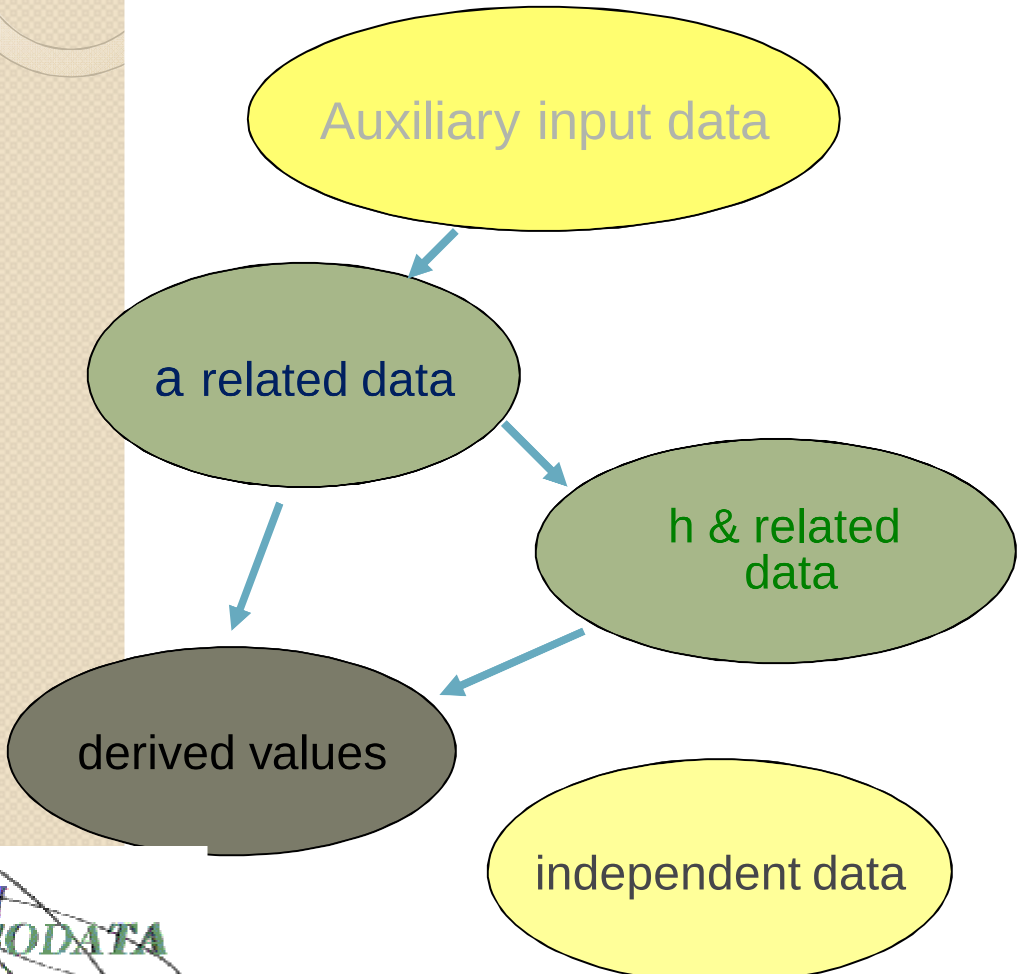
$$= \frac{R_\infty c}{\nu_{\bar{p}\text{He}}}\left(1 + c_5\alpha^2\right) ,$$

smallness of contributions

- $a \sim 10^{-2}$
- $a^2 \sim 10^{-4}$
- $m_e/m_p \sim 10^{-3}$

Structure of the input data and output values



- Auxiliary data = **exact** + the most accurate data which are to be evaluated <u>prior</u> the adjustment: $R_{¥}$, m_e/m_p, atomic masses.
- a related data: h/m, hN_A ...
- h related data: e, e/h, ...
- The lines (®) are equations: e.g., theoretical expressions for h/M, the Lamb shift, ...
- Some data are measured, a lot are derived: m_p [kg], m_e [Mev/c^2], ...
- G is uncorrelated,...

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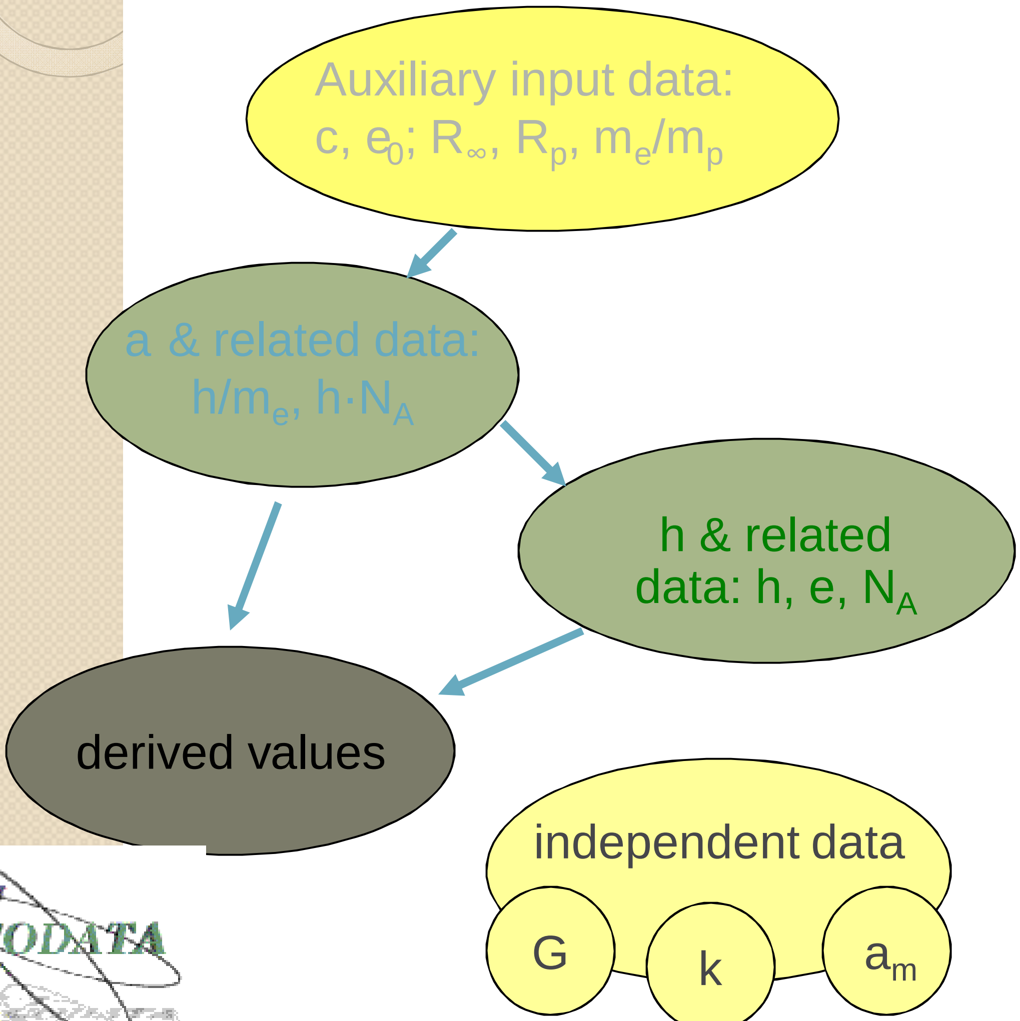

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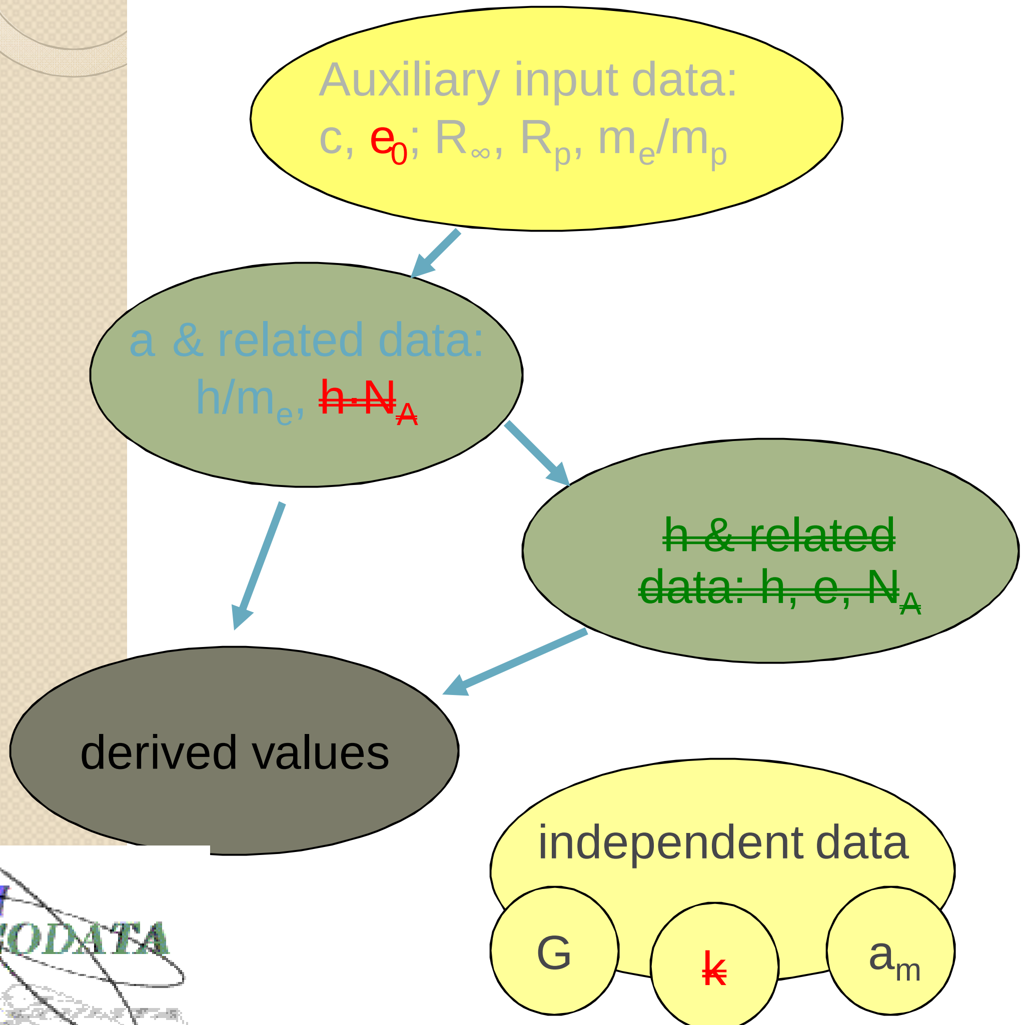

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Input: hierarchy – exact values

- *Exact values are adopted by definition.*
- *That is not necessary the SI definitions.*

Input:
hierarchy – exact values

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Quantity	Symbol	Value
speed of light [in vacuum]	c	$299\,792\,458\ \mathrm{m\,s^{-1}}$
magnetic constant [of vacuum] *aka.* permeability of free space	μ_0	$4\pi \times 10^{-7}\mathrm{N\,A^{-2}}$ $= 12.566\,370\,614\ldots \times 10^{-7}\ \mathrm{N\,A^{-2}}$
electric constant [of vacuum] *aka.* permittivity of free space	$\epsilon_0 = 1/(c^2\mu_0)$	$8.854\,187\,817\ldots \times 10^{-12}\ \mathrm{F\,m^{-1}}$
mass of an atom of ^{12}C	$m(^{12}\mathrm{C})$	12 u
relative atomic weight of ^{12}C	$A_r(^{12}\mathrm{C})$	12
molar mass of ^{12}C	$M(^{12}\mathrm{C})$	$12\ \mathrm{g\,mol^{-1}}$
temperature of the triple point of water	T_{triple}	273.16 K
HFS interval in Cs-133	$\nu_{\mathrm{HFS}}(^{133}\mathrm{Cs})$	9 192 631 770 Hz

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- *They are not necessary fundamental.*

Quantity	Symbol	Value
sp	c	$299\,792\,458\ \mathrm{m\,s^{-1}}$
m		
ak	μ_0	$4\pi \times 10^{-7}\mathrm{N\,A^{-2}}$
		$= 12.566\,370\,614... \times 10^{-7}\ \mathrm{N\,A^{-2}}$
el		
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h = 6.626 070 15 $\times 10^{-34}$ J Hz^{-1}
e = 1.602 176 634 $\times 10^{-19}$ C
N_A = 6.022 140 76 $\times 10^{23}$ mol^{-1}
k = 1.380 649 $\times 10^{-23}$ J K^{-1}

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magnetic constant [of vacuum] *aka.* permeability of free space	μ_0	
	$\epsilon_0 = 1/(c$	
	$m(^{12}C$	
	$A_r(^{12}C$	
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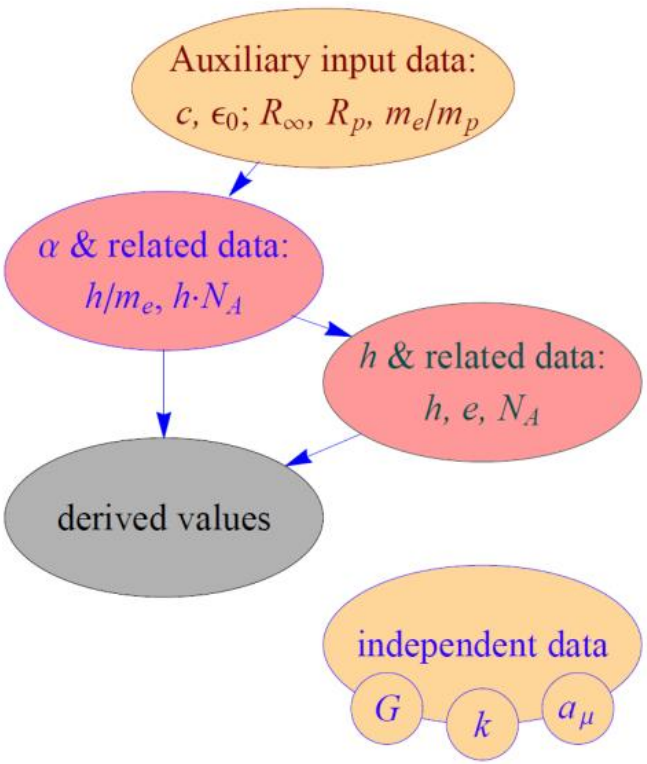
The most trickiest situation is with N_A and molar mass of ^{12}C.

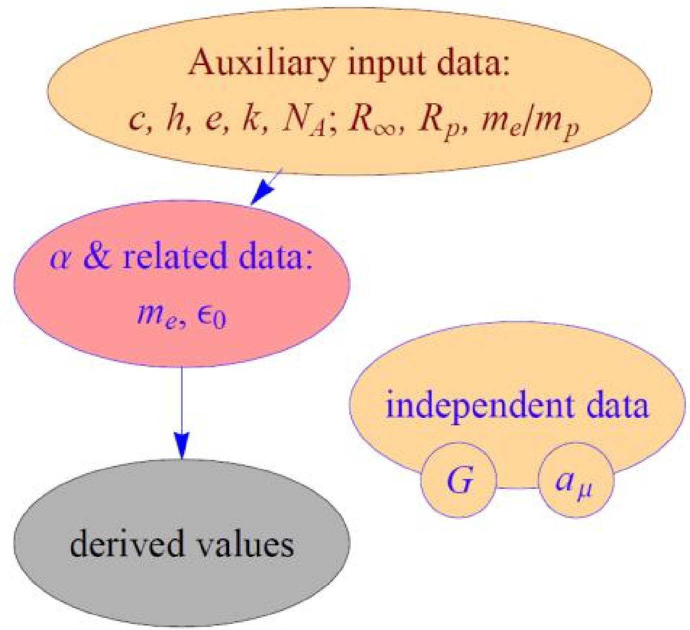
- In the old SI we knew the mass of a mole (12 g), but not the number of atoms.
- Now we know the number, but not the mass.
- In the old SI N_A set the ratio of 1 kg and 1 u.
- Not anymore.

The structure of LSA

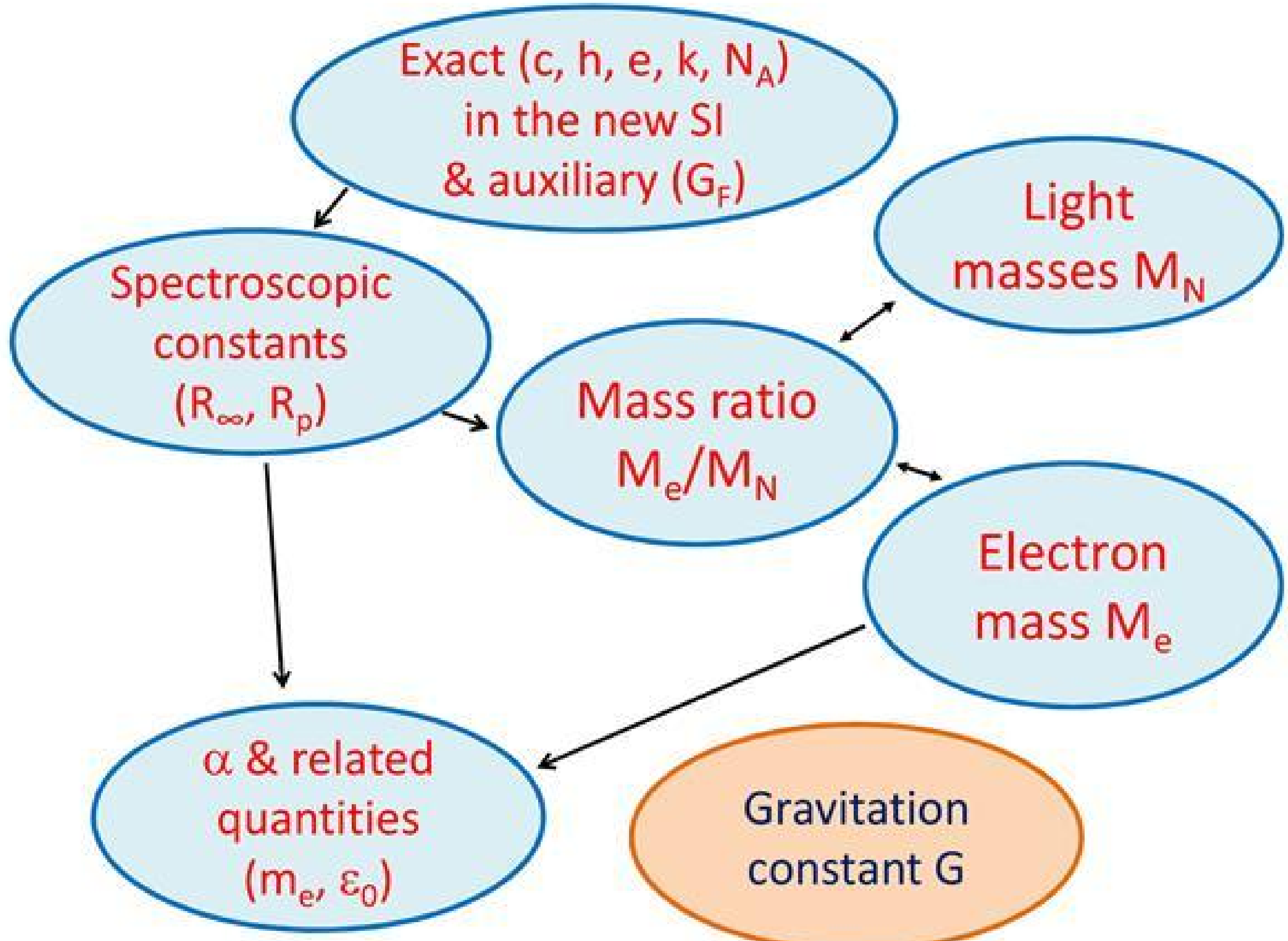
in the old SI



in the new SI



Detailed structure of LSA for 2022



Main [quantum] input (dominant)

Experiment	Constant	QED	Others
Lamb shift in mH	R_p	*	
e-p scattering	input for mH theory	*	
1s-2s in H	$R_{¥}$	*	R_p
1s-2s in H-D (isotopic)	R_d	*	R_p
Lamb shift in mD	R_d	*	
bound electron's g (H-like C)	M_e	*	
[absolute] mass spectrometry	M_p, M_d, M_h, M_a		
[relative] mass spectrometry	M_p/M_d, $M_h/(M_p+M_d)$, $M_h – M_t$		
HD^+ spectroscopy	$m_e/(m_p+m_d)$	*	$R_{¥}$, R_p, R_d
$g_e – 2$	a	*	
Raman spectroscopy	a		$R_{¥}$

The Lamb shift in muonic hydrogen: experiment

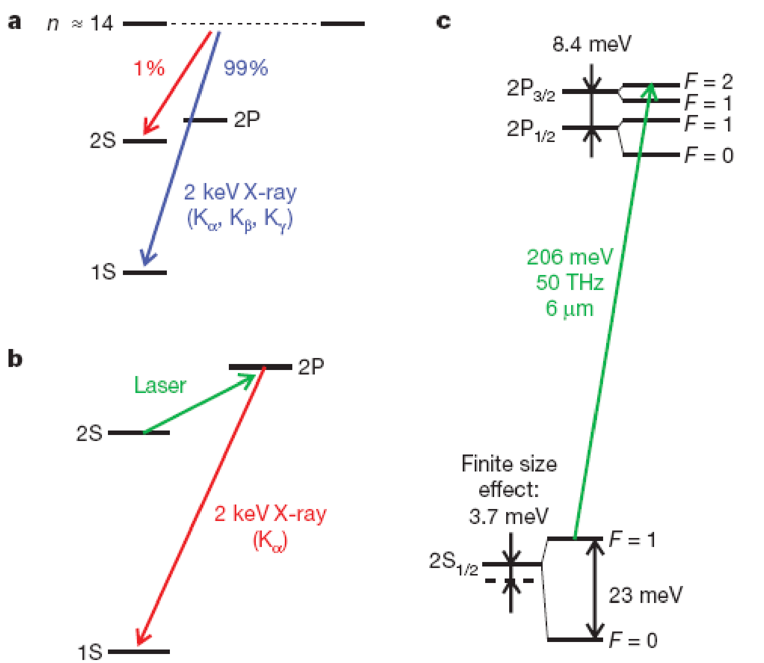


Figure 1 | Energy levels, cascade and experimental principle in muonic hydrogen. **a**, About 99% of the muons proceed directly to the 1S ground state during the muonic cascade, emitting 'prompt' K-series X-rays (blue). 1% remain in the metastable 2S state (red). **b**, The μp(2S) atoms are illuminated by a laser pulse (green) at 'delayed' times. If the laser is on resonance, delayed K_α X-rays are observed (red). **c**, Vacuum polarization dominates the Lamb shift in μp. The proton's finite size effect on the 2S state is large. The green arrow indicates the observed laser transition at $\lambda = 6\ \mu$m.

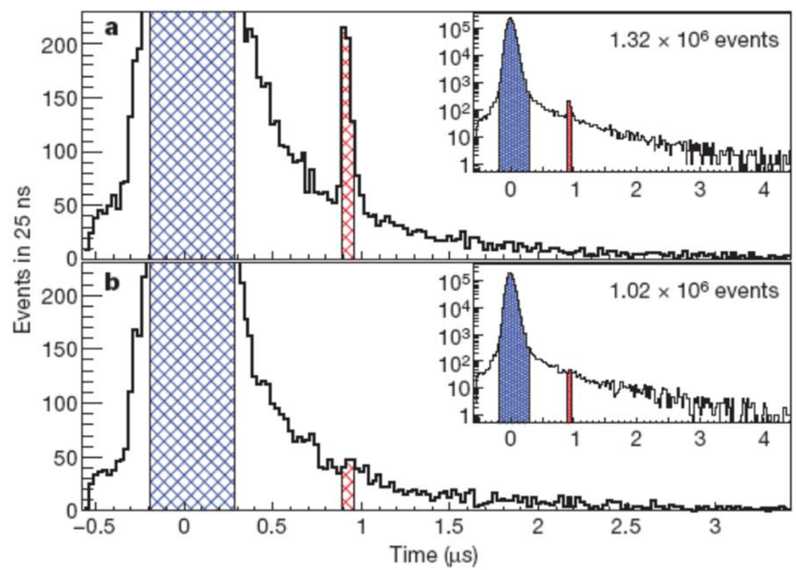


Figure 4 | Summed X-ray time spectra. Spectra were recorded on resonance (**a**) and off resonance (**b**). The laser light illuminates the muonic atoms in the laser time window $t \in [0.887, 0.962]$ μs indicated in red. The 'prompt' X-rays are marked in blue (see text and Fig. 1). Inset, plots showing complete data; total number of events are shown.

Proton radius: the puzzle

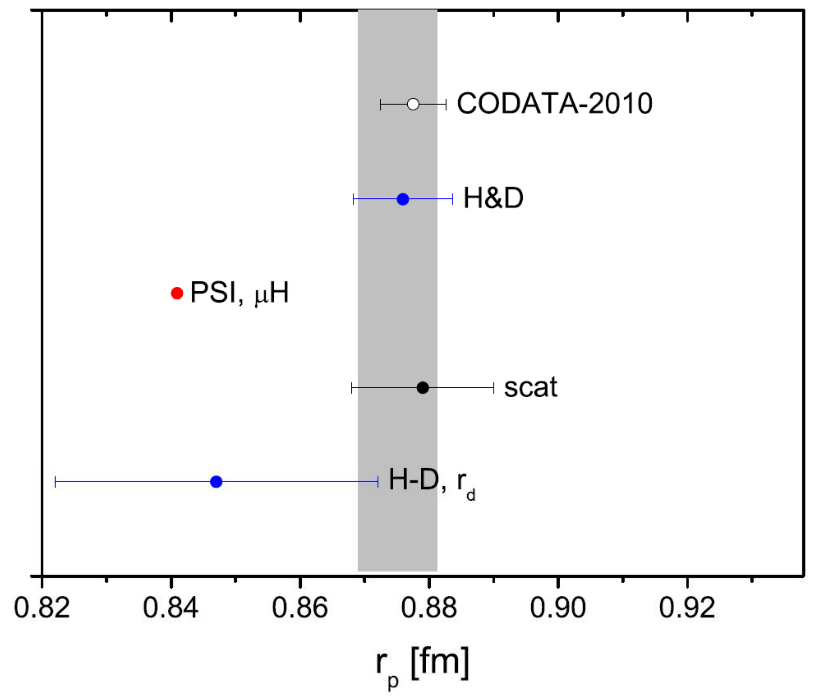
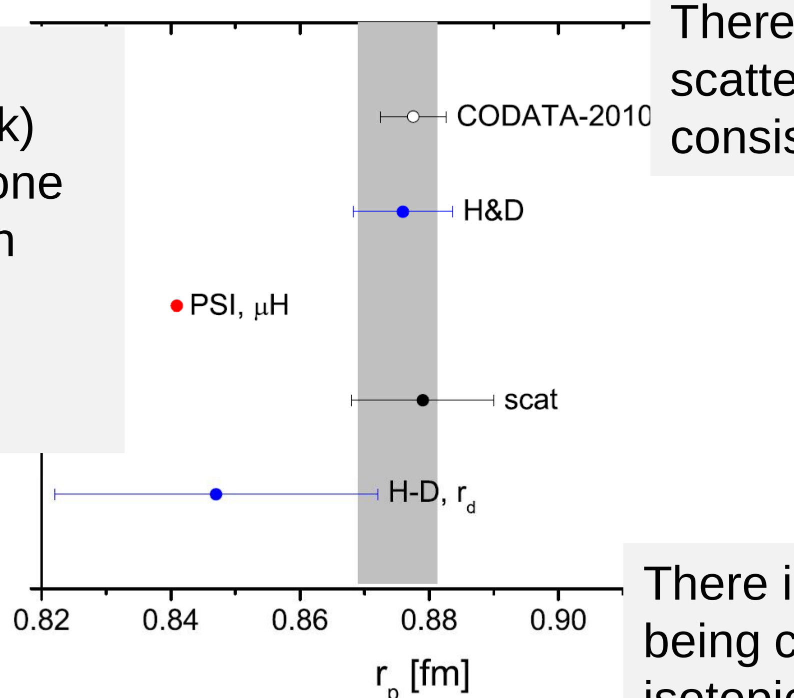


FIG. 5: Determination of the proton rms charge radius r_p. We follow the notation of [2]. The gray vertical belt is for the CODATA recommended value [2].

Proton radius: the puzzle

There are several H results (2×MPQ, York) consistent with mH, one (LKB) consistent with \`old’ H, and one in between (Colorado State).



There is a new e-p scattering result (Prad) consistent with mH.

There is a mD result; being combined with H-D isotopic of 1s-2s it is consistent with mH.

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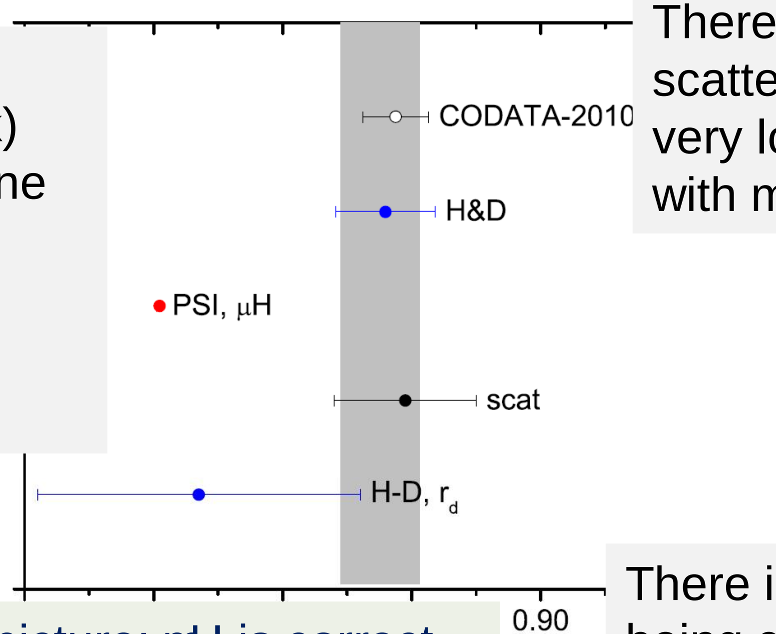
Proton radius: the puzzle

There are several H results (2×MPQ, York) consistent with mH, one (LKB) consistent with `old' H, and one in between (Colorado State).

There is a new e-p scattering result (Prad) at very low q^2 consistent with mH.



There is a mD result; being combined with H-D isotopic of 1s-2s it is consistent with mH.

In terms of a big picture: mH is correct and dominates because of it high accuracy,
while the data from scattering and `old' H are a kind of compromised and a possible discrepancy with them is unimportant.

on rms charge radius r_p.
ray vertical belt is for the

Friar contribution and needs of the e-p scattering data for μH evaluation

Contribution/Uncertainty [to the μH Lamb shift]	Value
U: expt	0.004 meV
U: QED	< 0.001 meV
C: proton polarizability	0.009(2) meV
C: Friar term	~ 0.020 meV

$$I_{\rm Fr} = \int_0^\infty \frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1 - 2G_E'(0)\, q^2\right]$$

Friar contribution and needs of the e-p scattering data for mH eva

- To find the Friar term we need a fit in a broad area.
- It should have a controlled accuracy.
- It should be consistent with the R_p from mH.
- It should have a very good accuracy for low q^2.

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C: proton polarizability	0.009(2) meV
C: Friar term	~ 0.020 meV

$$I_{\rm Fr} = \int_0^\infty \frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1 - 2G'_E(0)\,q^2\right]$$

Friar contribution and needs of the e-p scattering data for mH eva

- To find the Friar term we need a fit in a broad area.
- It should have a controlled accuracy.
- It should be consistent with the R_p from mH.
- It should have a very good accuracy for low q^2.
- Standard fits performed as usual over all the data but Prad disagrees with mH.
- Standard fits have the best absolute accuracy for the data in the middle of the fitted area.
- The accuracy is comparable for $G(q^2)$, but we need $(G(q^2))^2 – 1 – 2G'(0)\times q^2$.
- Prad with a result, consistent with mH, is based on the data at very low end of the required area of integration.

Contribution/U [to the mH Lam
U: expt
U: QED
C: proton polariz
C: Friar term

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- Prad with a result, consistent with mH, is based on the data at very low end of the required area of integration.

The puzzle is now a more practical one. We need to find a way to appropriately evaluate an integral over the electric form factor.

$$I_{Fr} = \int_0^\infty \ldots \Big]^2\Big]$$

Other related data and constants

The Rydberg constant

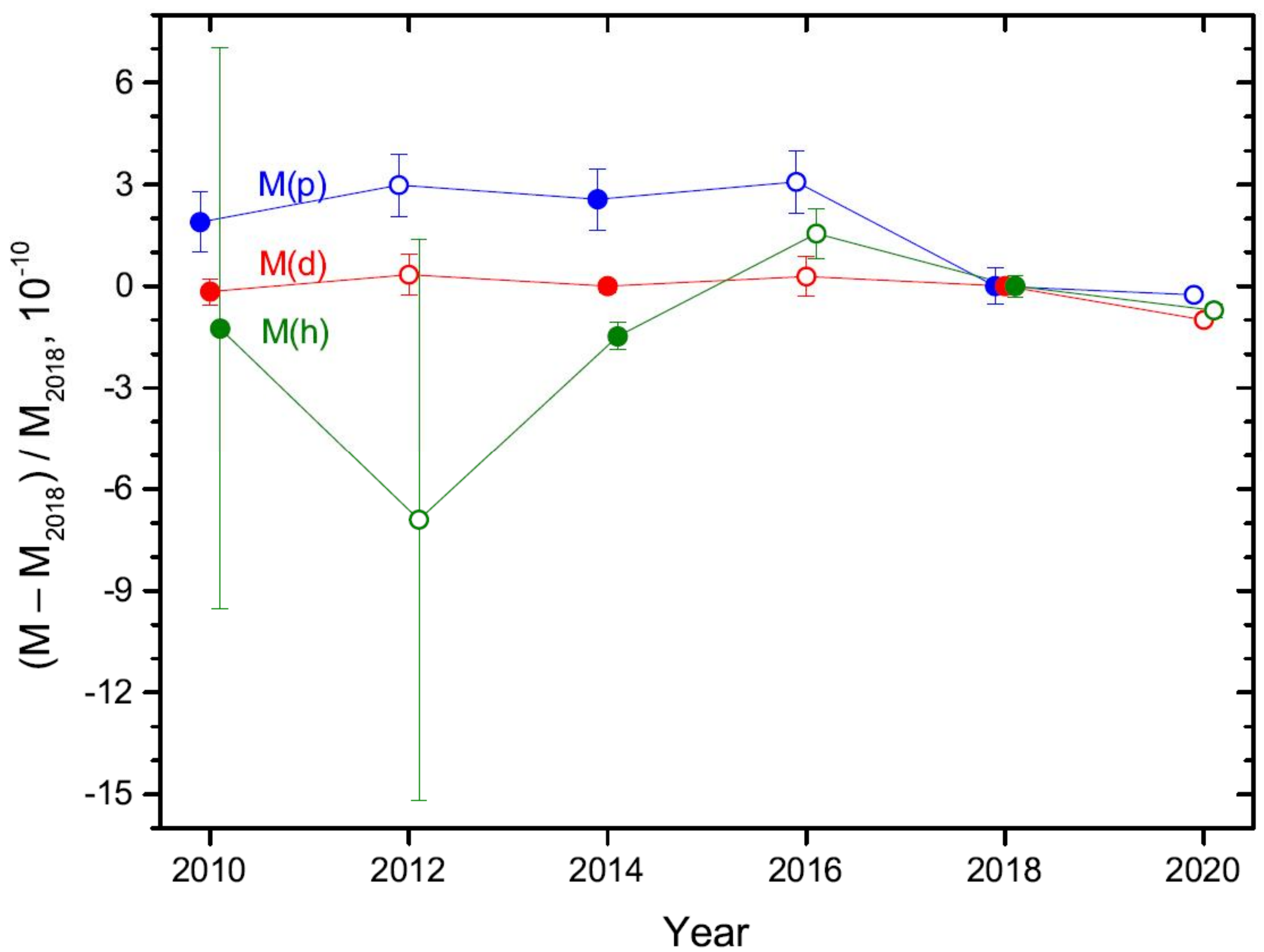
– Once we have QED theory and R_p, we find $R_{¥}$. The dominant contribution is from 1s-2s in H.

The deuteron radius

– Once we have QED theory and R_p, we find R_d. The dominant contribution is from isotopic shift H-D for 1s-2s.

The rest of H and D data are for a cross check of theory.
The mD result is mostly for a cross check.

Progress in determination of nuclear masses (p, d, h)



Zoo of ions (uncertainty: parts in 10^{-11})

A	Absolute	Relative	Absolute
1			H^+
2	D^+	H_2^+, D^+	D^+
3	$^3He^+$	HD^+, $^3He^+$, T^+, H_3^{+*}	HD^+
4	$^4He^{++}$		

Absolute = comparison to an ^{12}C ion (often @ **the same Z/A**)

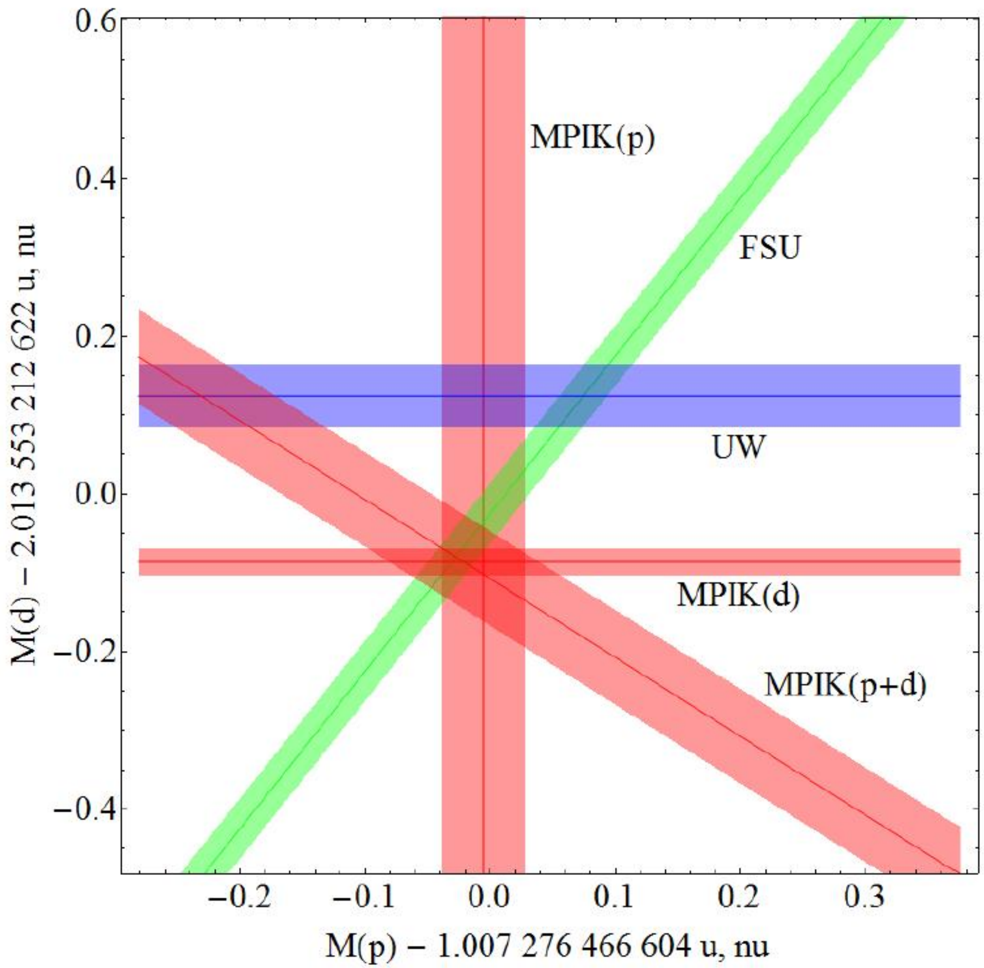
Relative = comparison of ions$^+$ @ the same A

Inconsistencies

Quantity	Value UW	Value FSU	Value MPIK
$M(d)$ [u]	2.013 553 212 **746**(40)	—	2.013 553 212 **535**(17)
			2.013 553 212 **521**(67)
$M(h)$ [u]	3.014 932 246 668(43)	—	—
$M(h)/M(d)$	1.497 319 379 **286**(41)	1.497 319 379 **566**(32)	—
$M(d)/M(p)$	—	1.999 007 501 **273**(38)	1.999 007 501 **225**(34)
$M(p)$ [u]	—	—	1.007 276 466 597(33)

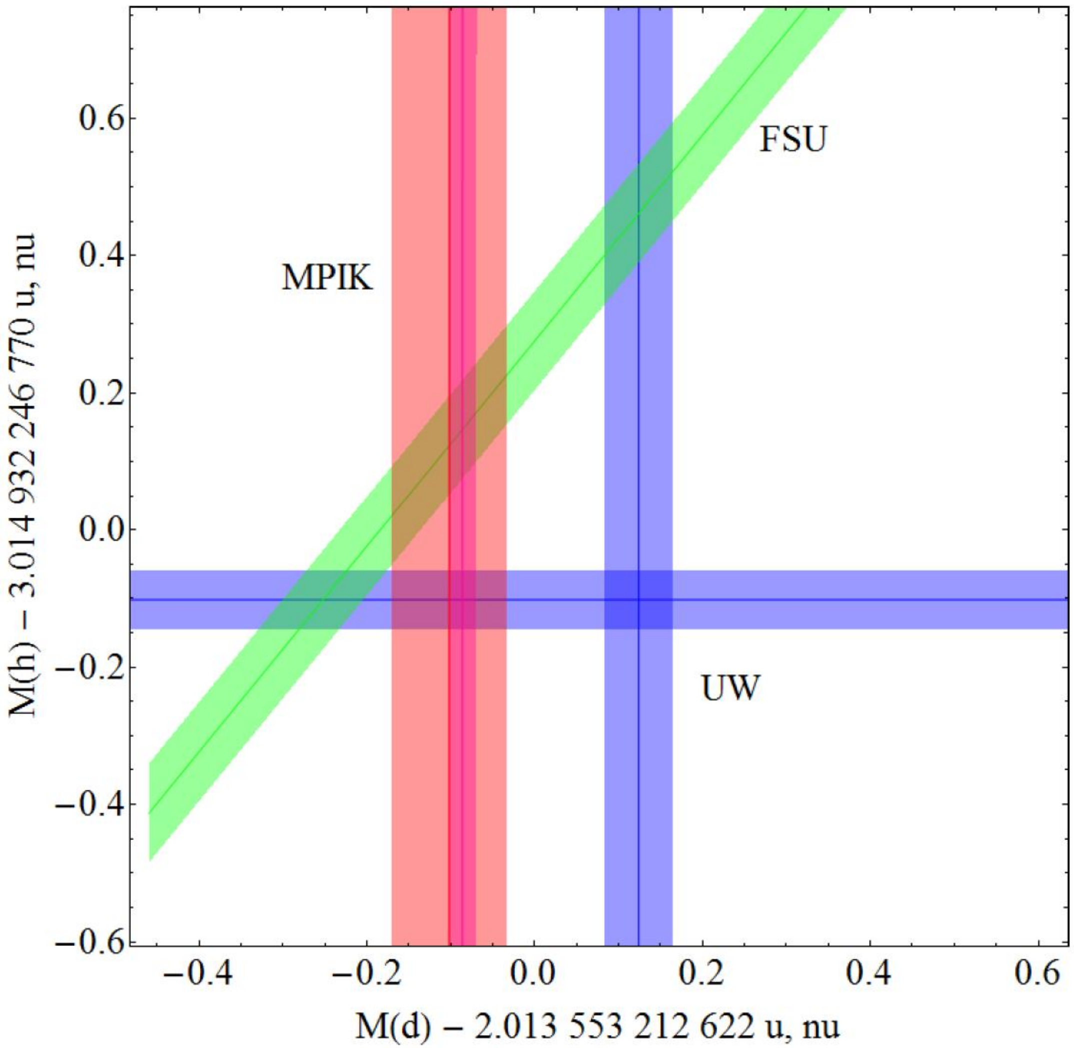
Inconsistencies

Quantity		Value MPIK
$M(d)$ [u]	2.013 5	3 212 **535**(17)
		3 212 **521**(67)
$M(h)$ [u]	3.014 9	—
$M(h)/M(d)$	1.497 3	—
$M(d)/M(p)$		7 501 **225**(34)
$M(p)$ [u]		6 466 597(33)



Inconsistencies

Quantity		Value
		MPIK
$M(d)$ [u]	2.013 5	212 **535**(17)
		212 **521**(67)
$M(h)$ [u]	3.014	—
$M(h)/M(d)$	1.497 3	—
$M(d)/M(p)$		01 **225**(34)
$M(p)$ [u]		466 597(33)

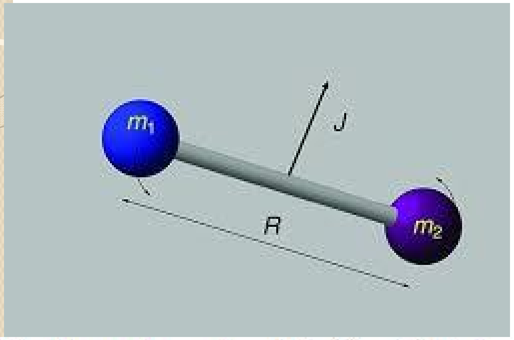


Players (outsiders)

- HD^+ spectroscopy
  - g factor of bound electron
  - R_∞, R_p, R_d
  - antiprotonic helium
- g spectroscopy

- electron spectrum of b decay of tritium
  - neutrino physics
    - oscillations (matrix)
    - oscillations (Dm^2)
  - Cosmology
    - Sm_n
    - CnB

QED contribution to the problem: rotational transitions in HD^+



S. Alighanbari, G.S. Giri, F.L. Constantin, V.I. Korobov, and S. Schiller, Nature **581**, 152 (2020).

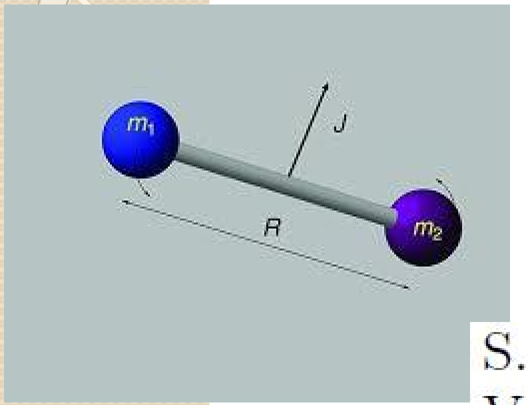
$$R_\infty \cdot \frac{M(e)}{M_r} = 8966.205\,150\,50(18)\ \mathrm{m}^{-1}\,.$$

$$\frac{M_r}{M(e)} = 1223.899\,228\,711(24)\,,$$

$$M_r = 0.671\,406\,527\,591(24)\ \mathrm{u}$$

Quantity	Contribution to the uncertainty [ppt]						
	exp.	th.	R_p	R_d	R_∞	$M(e)$	tot
$M_r/M(e)$	14	14	+2.2	+2.2	−1.9	–	20
M_r	14	14	+2.2	+2.2	−1.9	29	35

QED contribution to the problem: vibrational transitions in HD^+

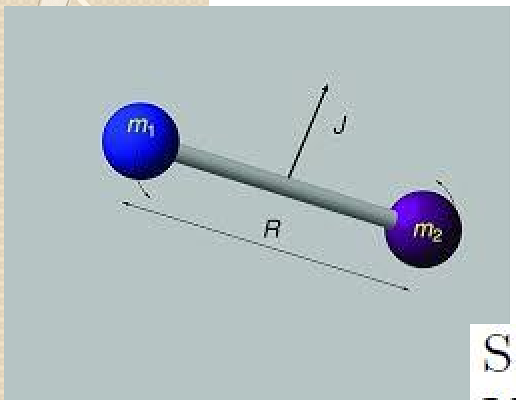


S. Patra, M. Germann, J.-Ph. Karr, M. Haidar, L. Hilico, V.I. Korobov, F.M.J. Cozijn, K.S.E. Eikema, W. Ubachs, and J.C.J. Koelemeij, Science, 10.1126/science.aba0453 (2020).

$$\frac{M_r}{M(e)} = 1223.899\,228\,686(28)$$

Quantity	Contribution to the uncertainty [ppt]						
	exp.	th.	R_p	R_d	R_∞	$M(e)$	tot
$M_r/M(e)$	8.2	21	+3.5	+3.4	−3.8	–	23
M_r	8.2	21	+3.5	+3.4	−3.8	29	37

QED contribution to the problem: vibrational transitions in HD^+



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S. Patra, M. Germann, J.-Ph. Karr, M. Haidar, L. Hilico, V.I. Korobov, F.M.J. Cozijn, K.S.E. Eikema, W. Ubachs, and J.C.J. Koelemeij, Science, 10.1126/science.aba0453 (2020).

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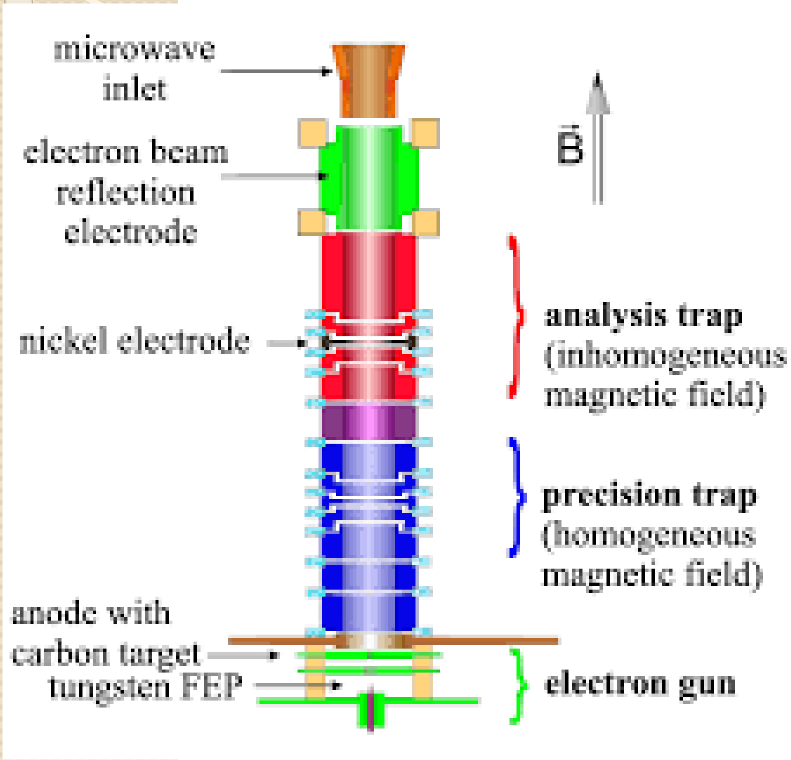
Electron's mass in atomic mass units: the bound *g* factor experiment



$$M(e) = 5.485\,799\,090\,65(16) \times 10^{-4}\ \mathrm{u}$$

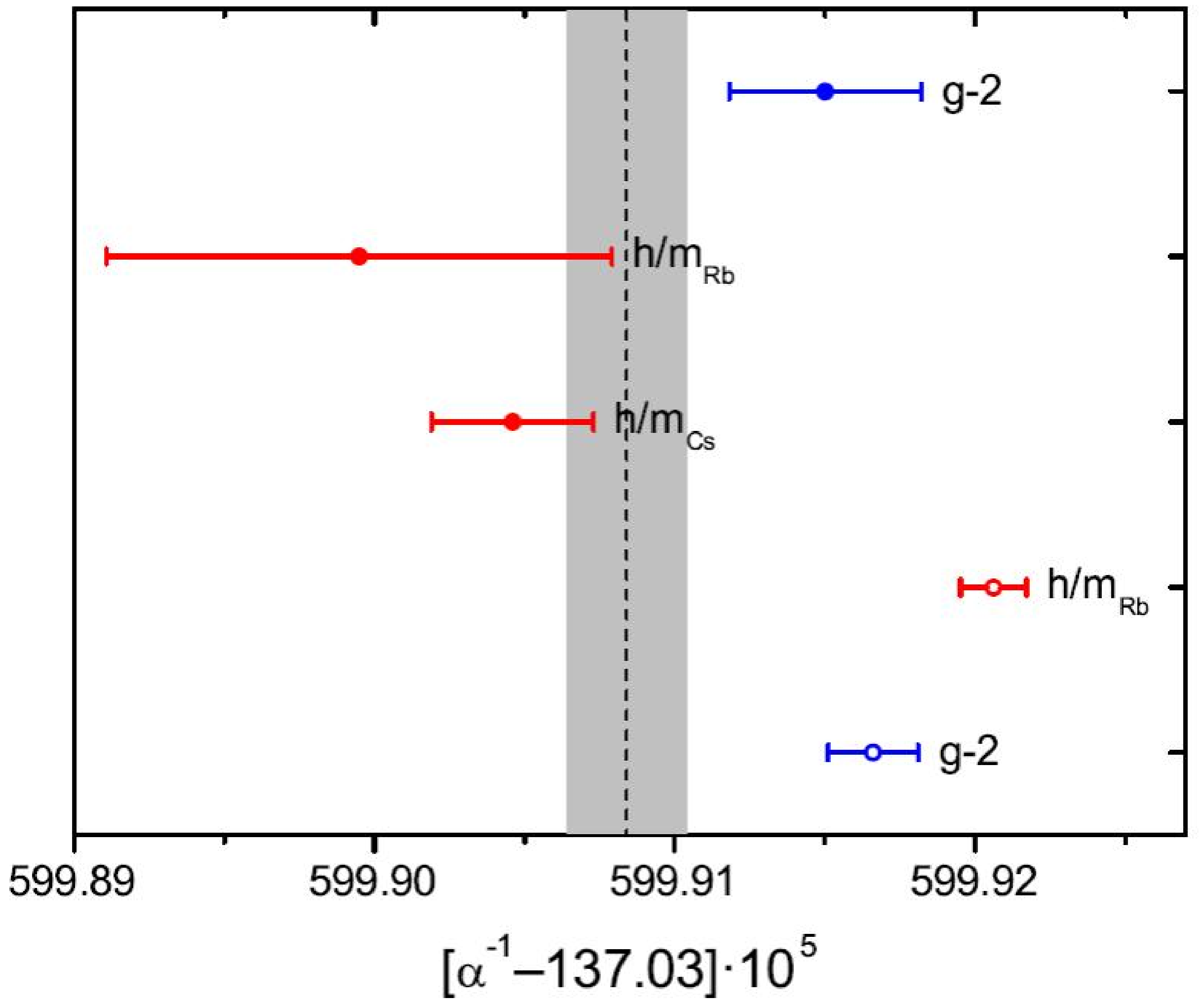
$g(^{12}C^{5+})$

$u_{exp} = 2.8 \times 10^{-11}$

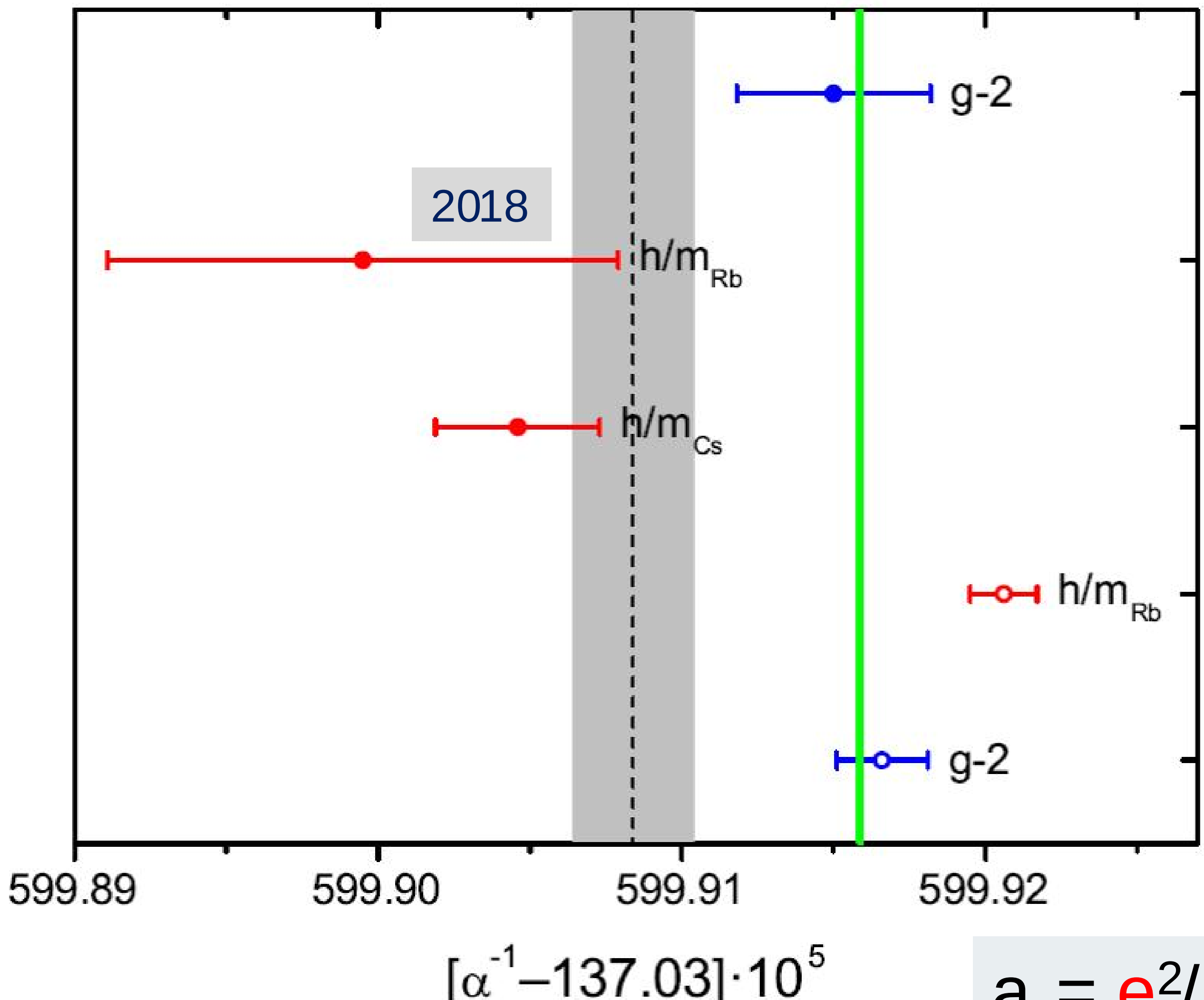
$u_{th} = 1.2 \times 10^{-11}$

S. Sturm, F. Köhler, J. Zatorski, A. Wagner, Z. Harman, G. Werth, W. Quint, C.H. Keitel, and K. Blaum, Nature **506**, 467 (2014).

Determination of the fine structure constant a

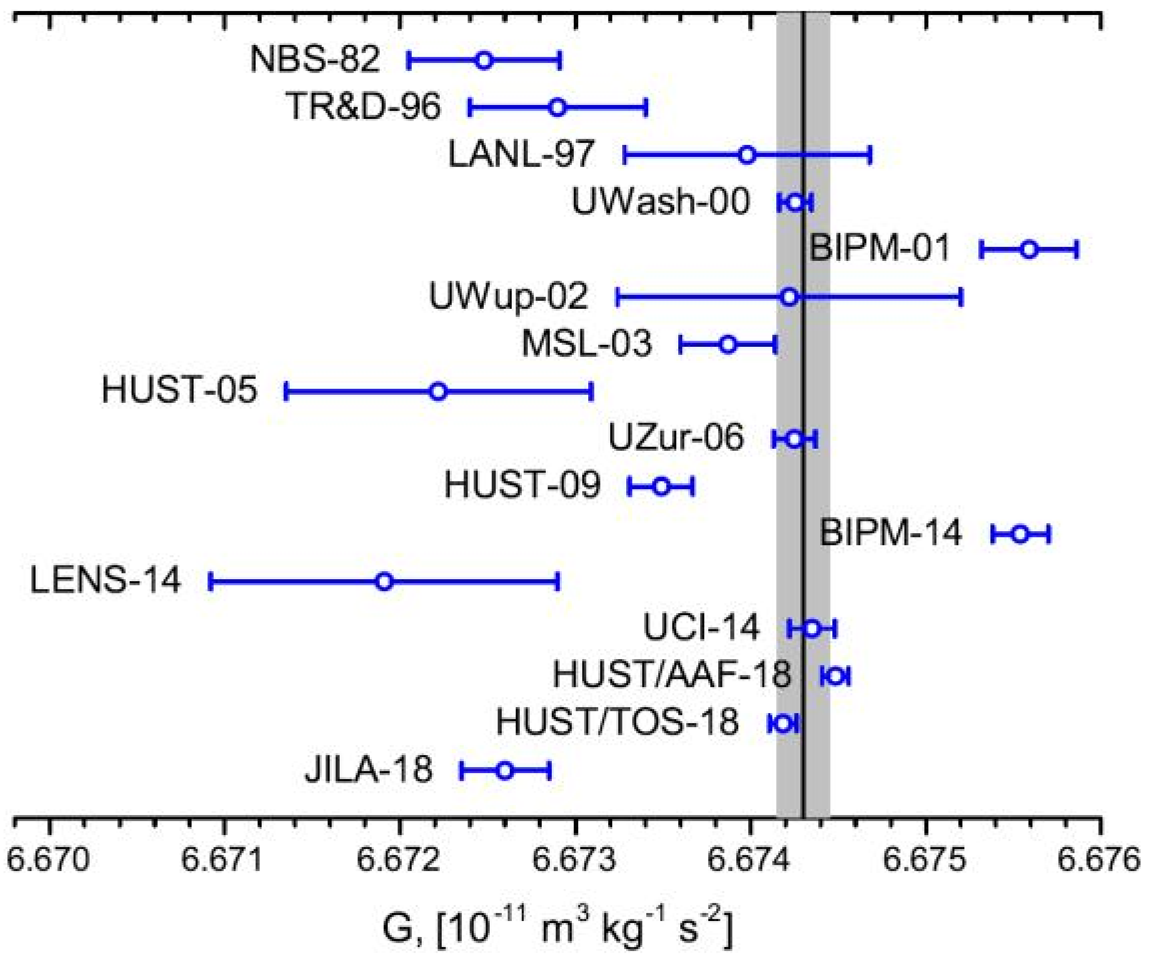


and e_0 and continuity of W and F

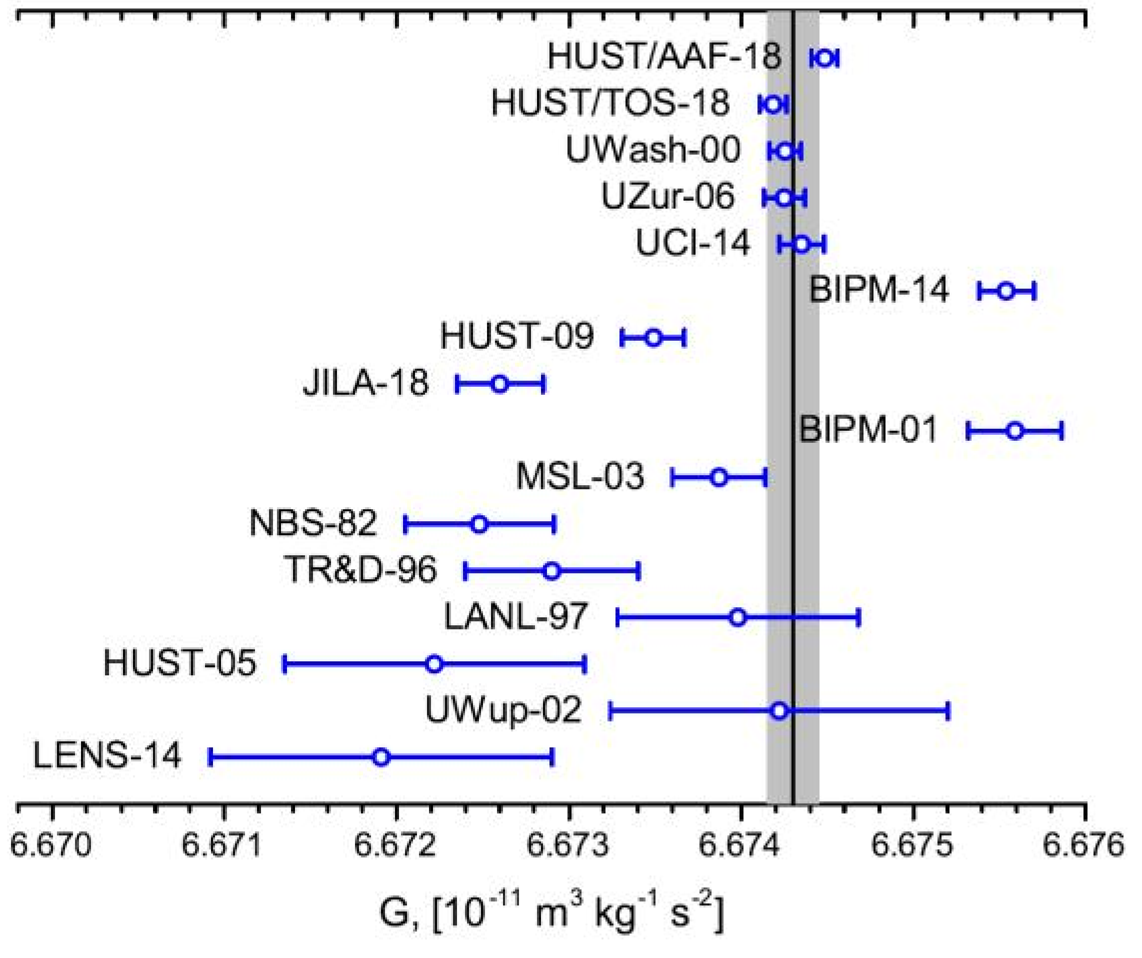


$a = e^2/(4pe_0\hbar c)$

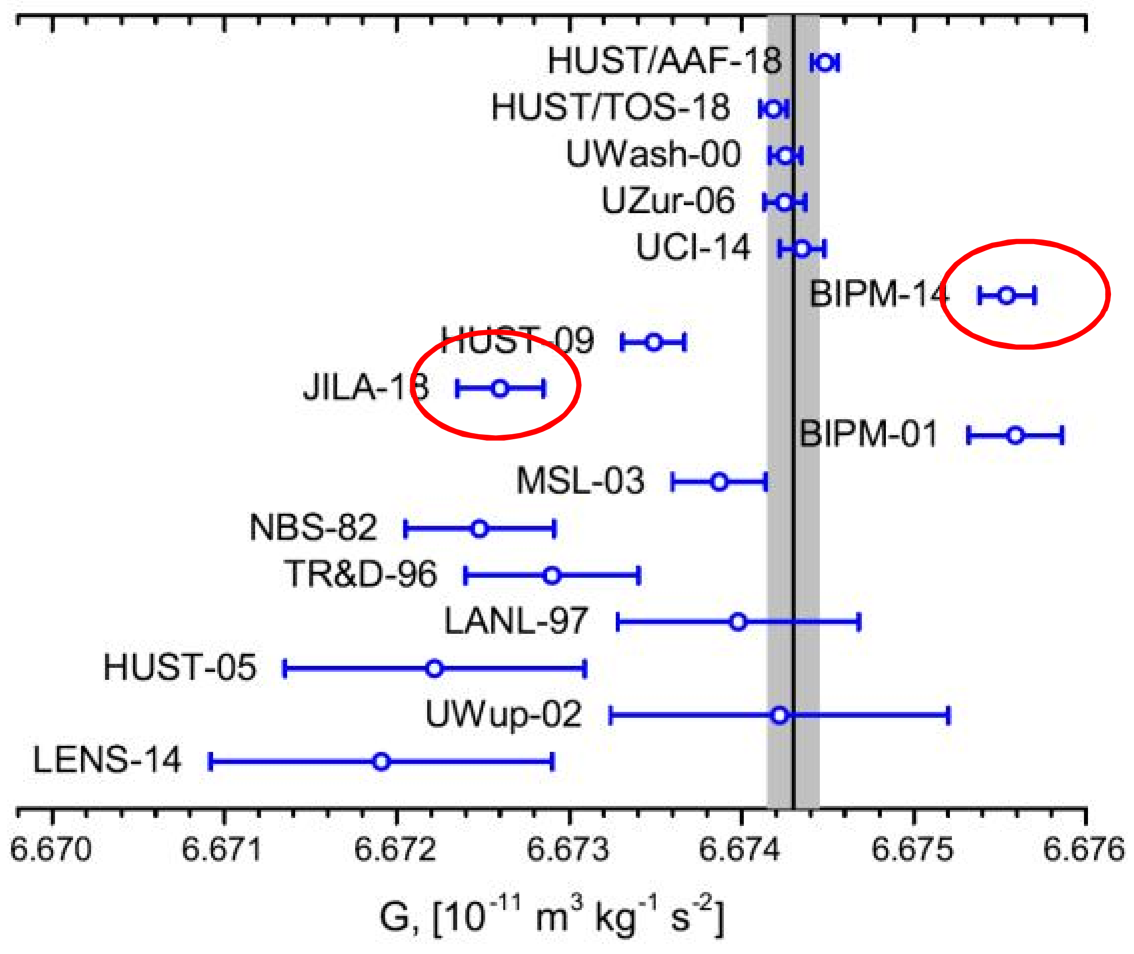
Determination of G



Determination of G



Determination of G: outliners



Thank you!