UCLouvain

MadNIS

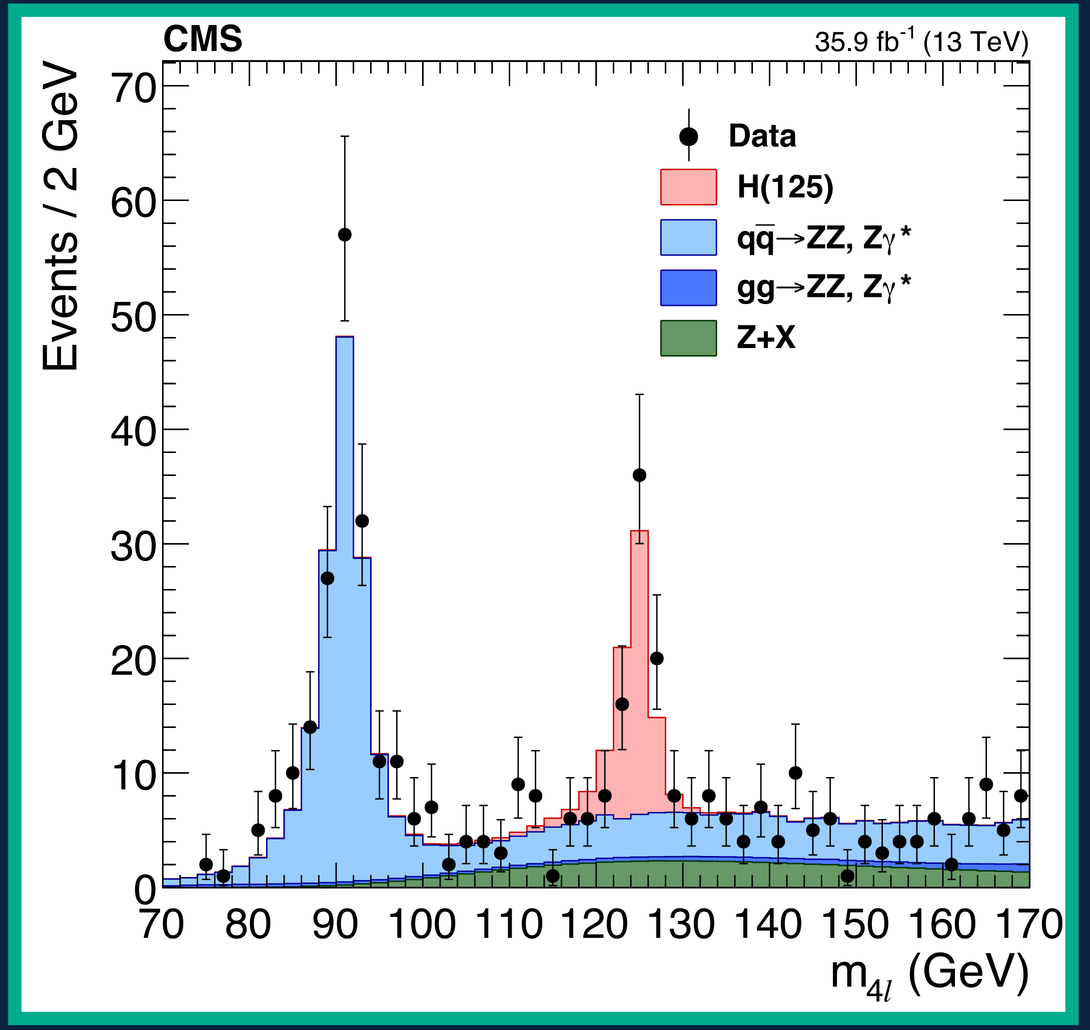
Neural networks for multi-channel importance sampling in MadGraph

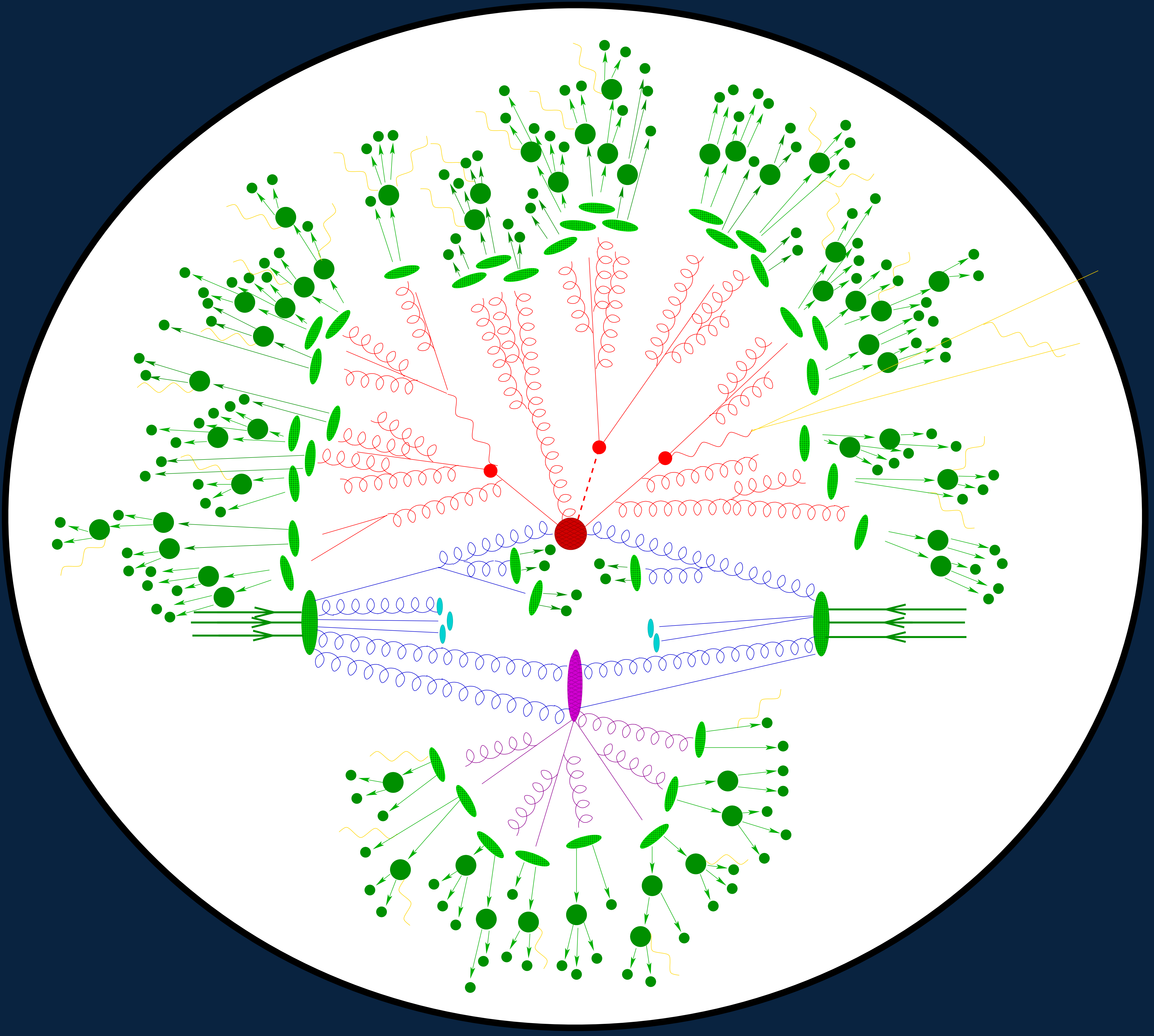
IML - Optimal transport and invertible algorithms

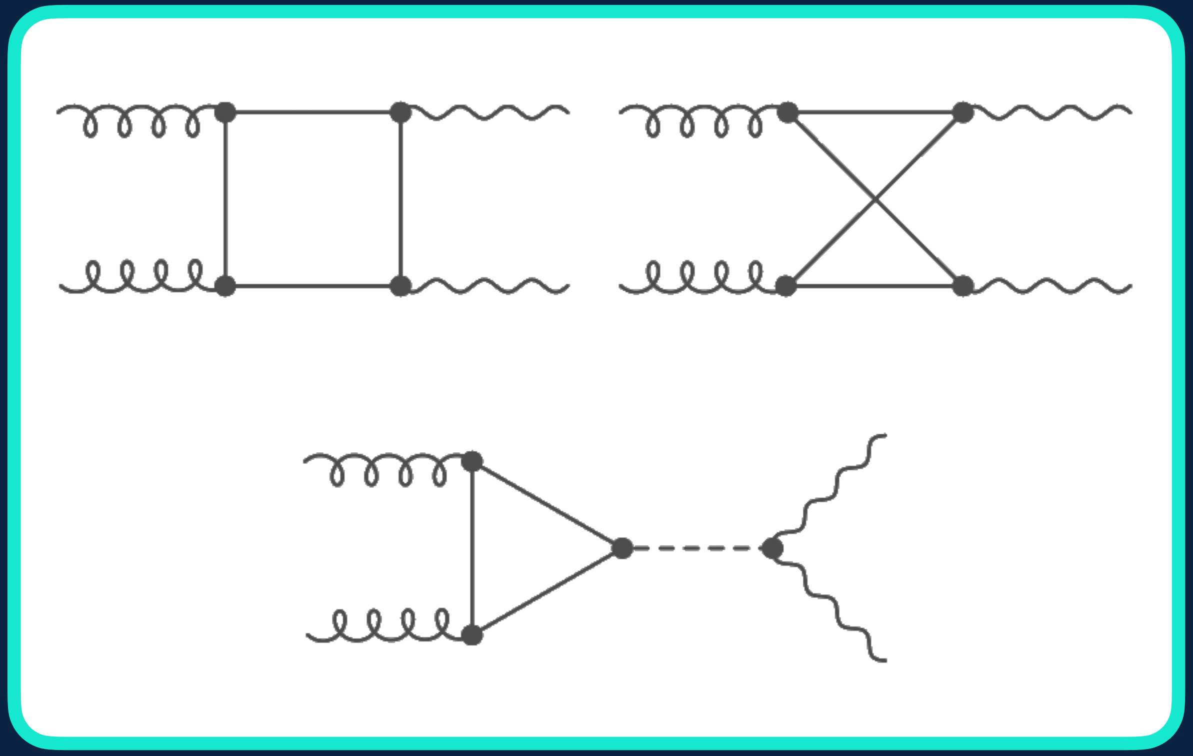
Ramon Winterhalder — UC Louvain

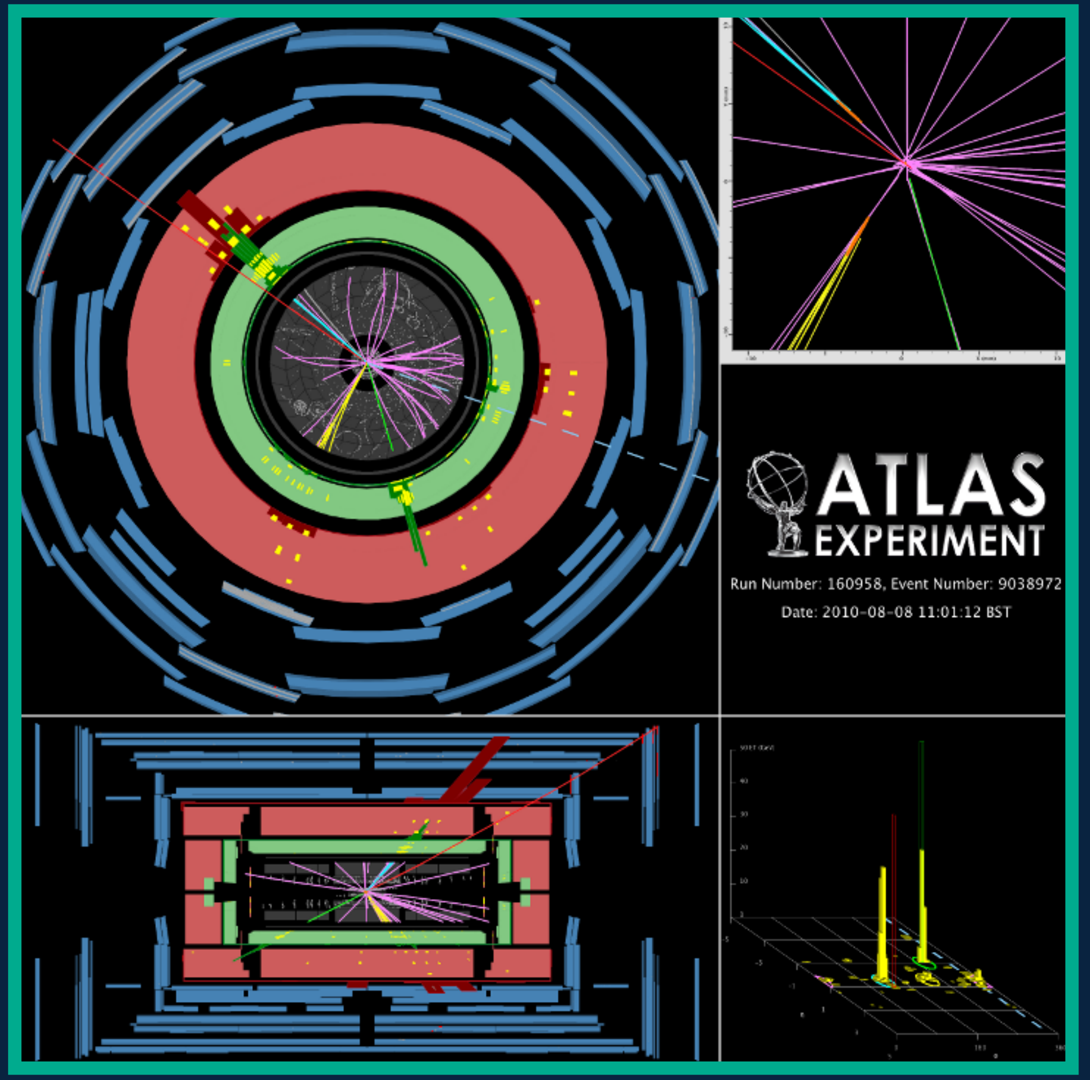
In collaboration with Anja Butter, Theo Heimel, Joshua Isaacson, Claudius Krause, Fabio Maltoni, Olivier Mattelaer and Tilman Plehn

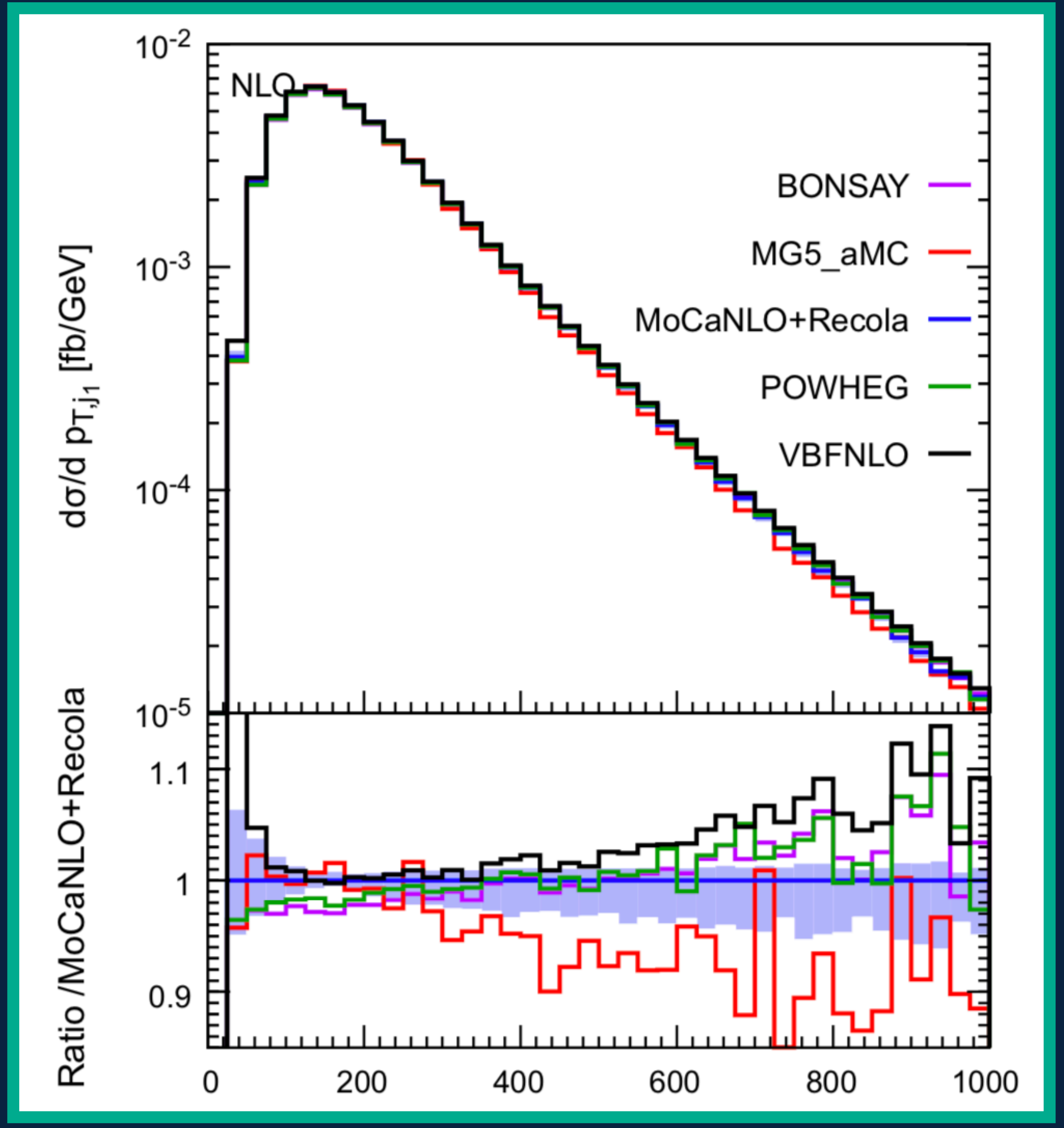
Data analysis in HEP



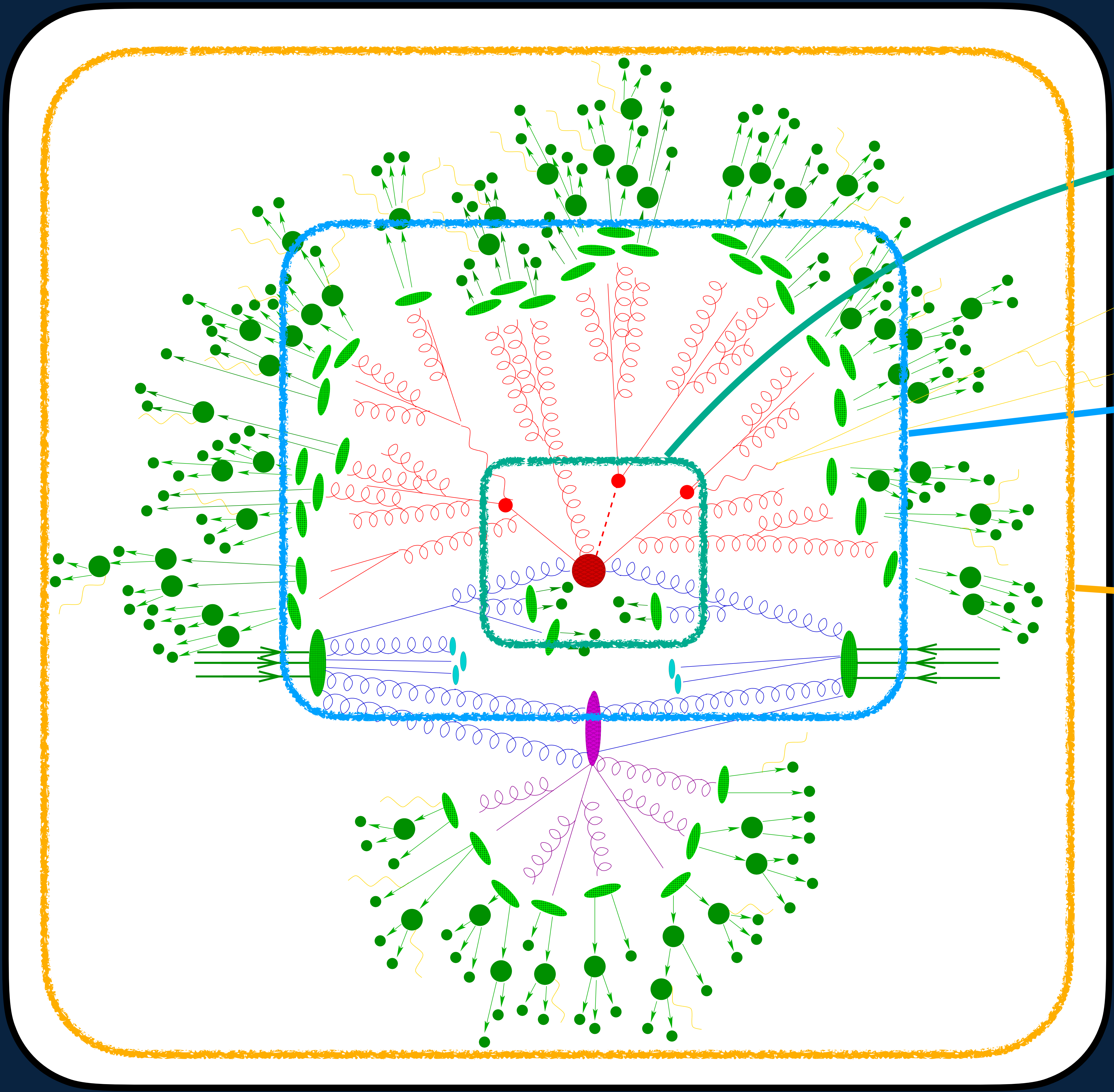








Simulations for the LHC



✦ **Hard Process**

✤ Depends on the Model (SM, SUSY,…)

✤ Perturbative QCD

✦ **Parton Showering**

✤ Universal (QCD)

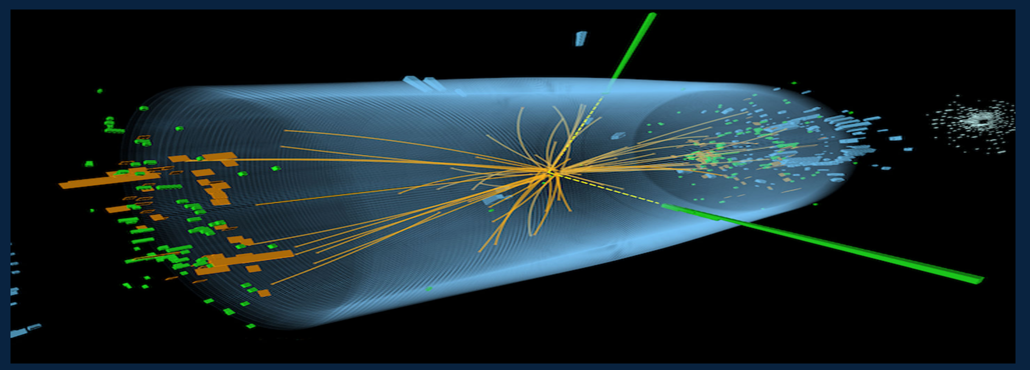
✦ **Hadronization**

✤Model-based, universal

✦ **Underlying Event**

✤Model-based, non-universal

✦ **Detector Simulation**



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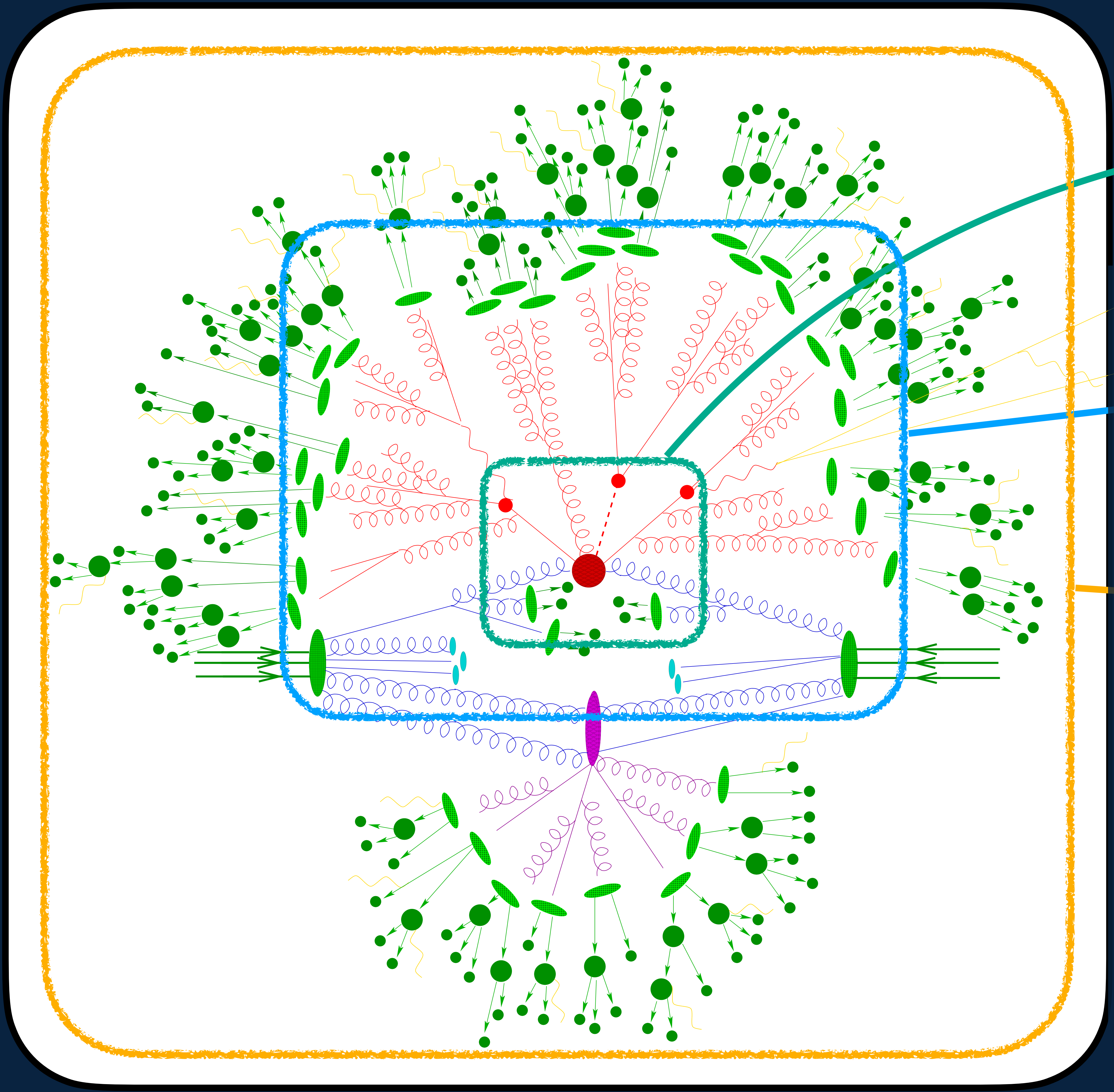

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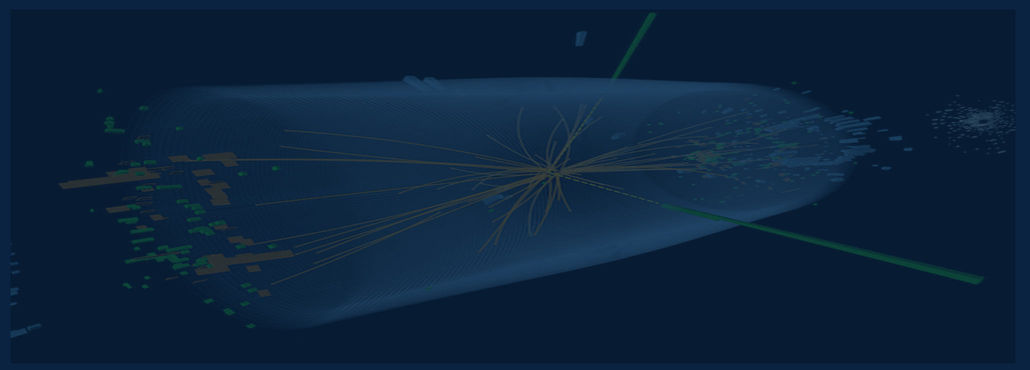

Theory predictions in HEP

$$M_{\lambda,c,\ldots}(p_a, p_b \,|\, p_1, \ldots, p_n) : \mathbb{M} \to \mathbb{C}$$

Quantum numbers: spin, colour charge etc.

Kinematics: Momenta in Minkowski space, masses, etc.

$$\sigma = \frac{1}{\text{flux}} \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b\, f(x_a) f(x_b) \int \mathrm{d}\Phi_n \left\langle \left| M_{\lambda,c,\ldots}(p_a, p_b \,|\, p_1, \ldots, p_n) \right|^2 \right\rangle$$

Cross section: more generally, differential observables*

PDFs: convolution over all possible initial state configurations

Phase-space integral: over final state kinematics

Squared amplitude: summed over final states, averaged over initial states

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Standard Monte Carlo integration

$$I = \int_V \mathrm{d}^d x\, f(x) \simeq \frac{1}{N} \sum_{j=1}^{N} f(x_j) = \langle f \rangle_x \qquad \sigma_I \simeq \sqrt{\frac{\langle f^2 \rangle_x - \langle f \rangle_x^2}{N-1}}$$

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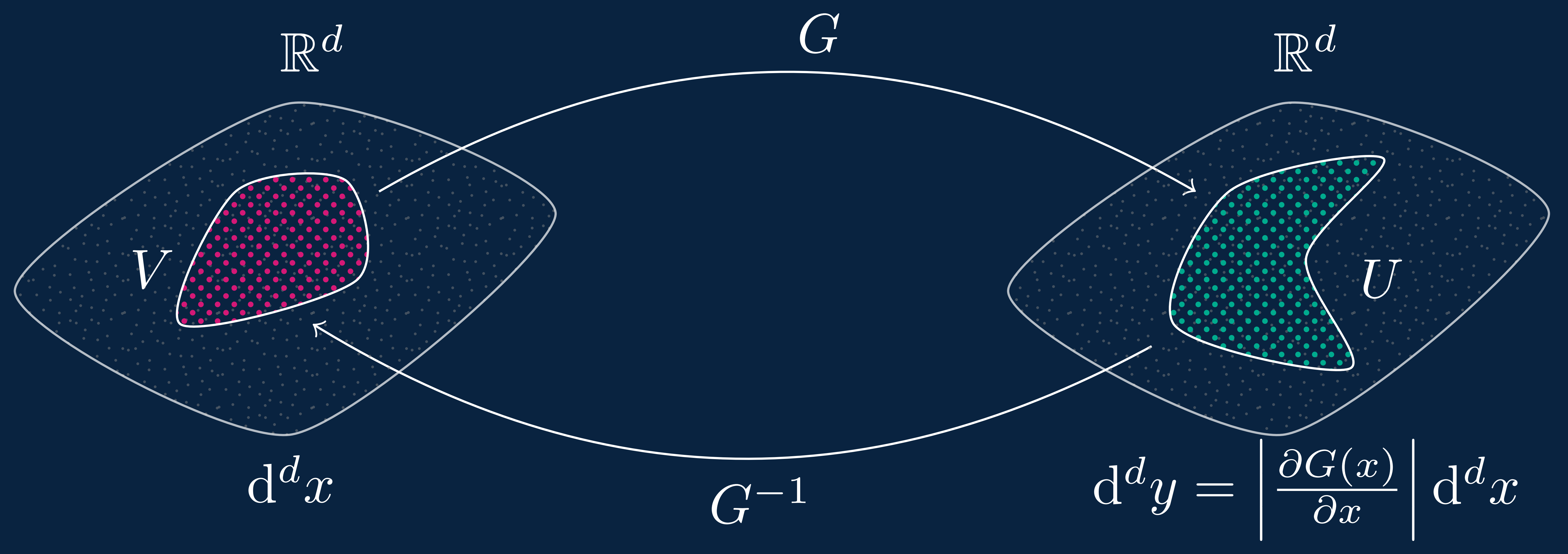
$$y = G(x) \qquad g(x) = \left|\frac{\partial G(x)}{\partial x}\right|$$



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Multi-Channel Integration → $\sum \alpha_i(x) = 1$

$$I = \int_V \mathrm{d}^d x\, f(x) = \sum_i \int_V \mathrm{d}^d x\, \alpha_i(x) f(x) = \sum_i \int_{U_i} \mathrm{d}^d y_i\, \alpha_i(x) \frac{f(x)}{g_i(x)}\bigg|_{x\equiv x(y_i)}$$

Channel Mappings

$$y_i = G_i(x) \qquad g_i(x) = \left|\frac{\partial G_i(x)}{\partial x}\right|$$

Neural Multi-Channel Monte Carlo

Multi-Channel Importance Sampling

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n-dimensional remapping

- tractable Jacobian with **normalizing flow**

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$$\alpha_i(x) \to \alpha_i(x|\theta), \quad \sum_i \alpha_i(x|\theta) = 1$$

k-dimensional regression

- with boundary condition

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Neural Multi-Channel Monte Carlo

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Further Improvements?

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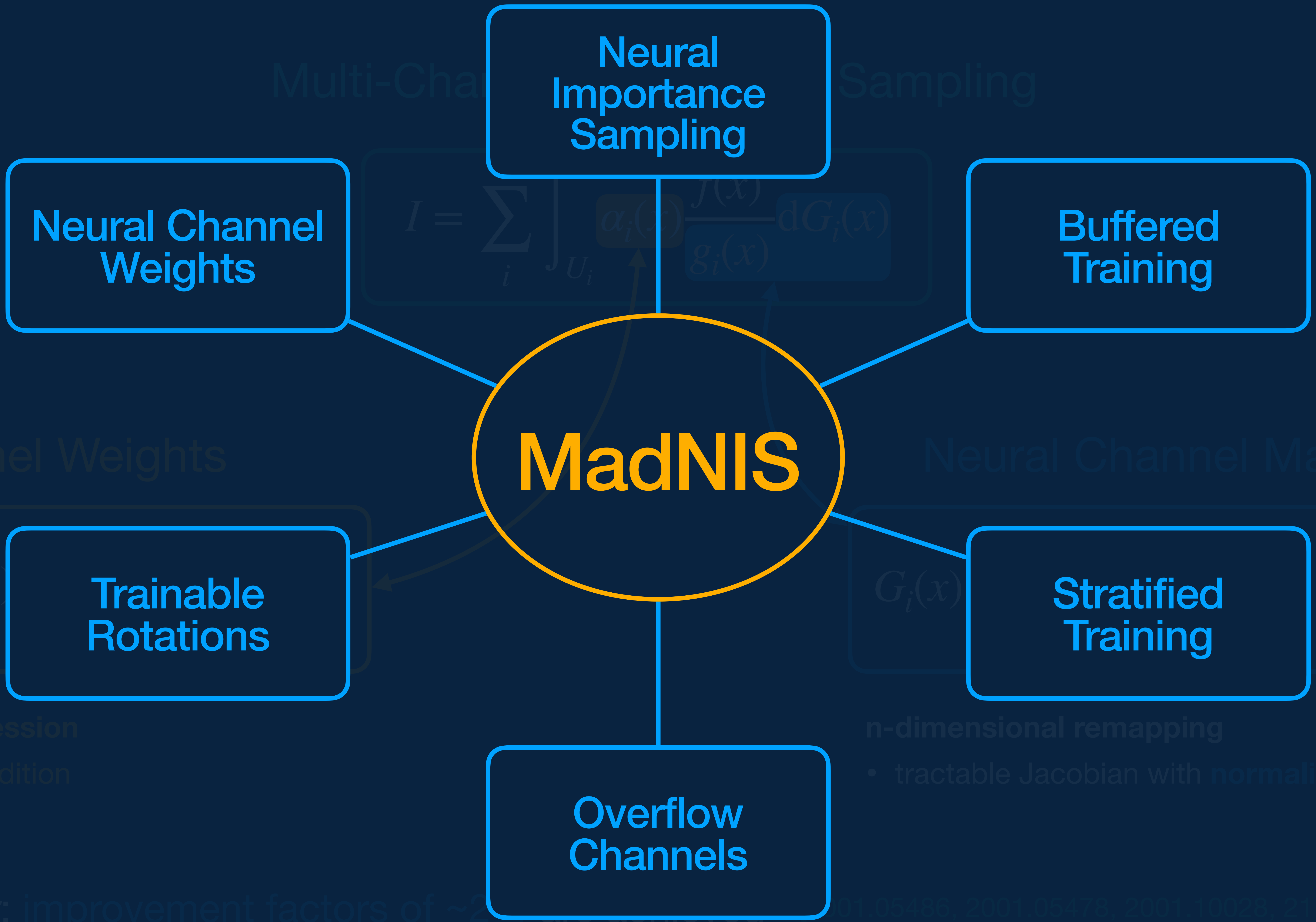

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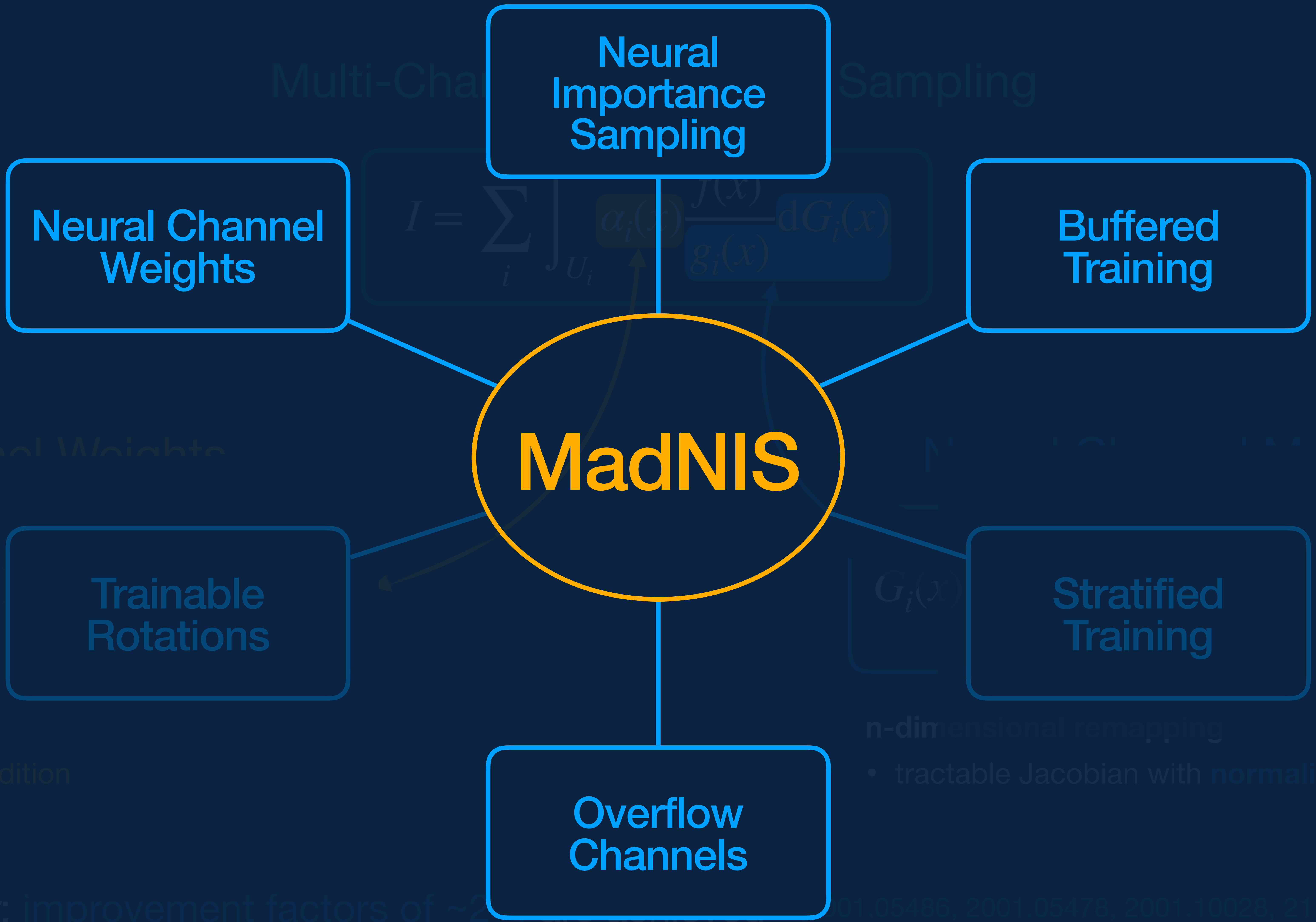

MadNIS

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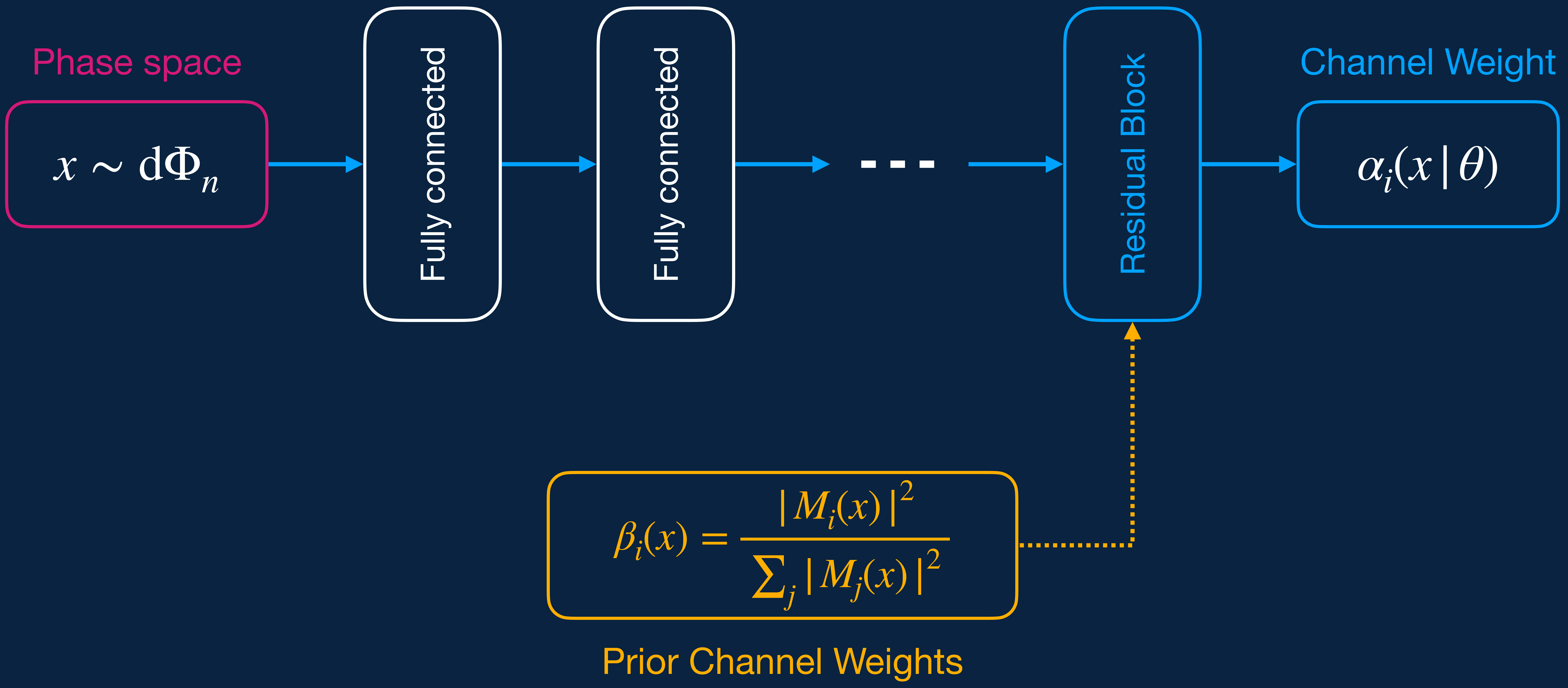

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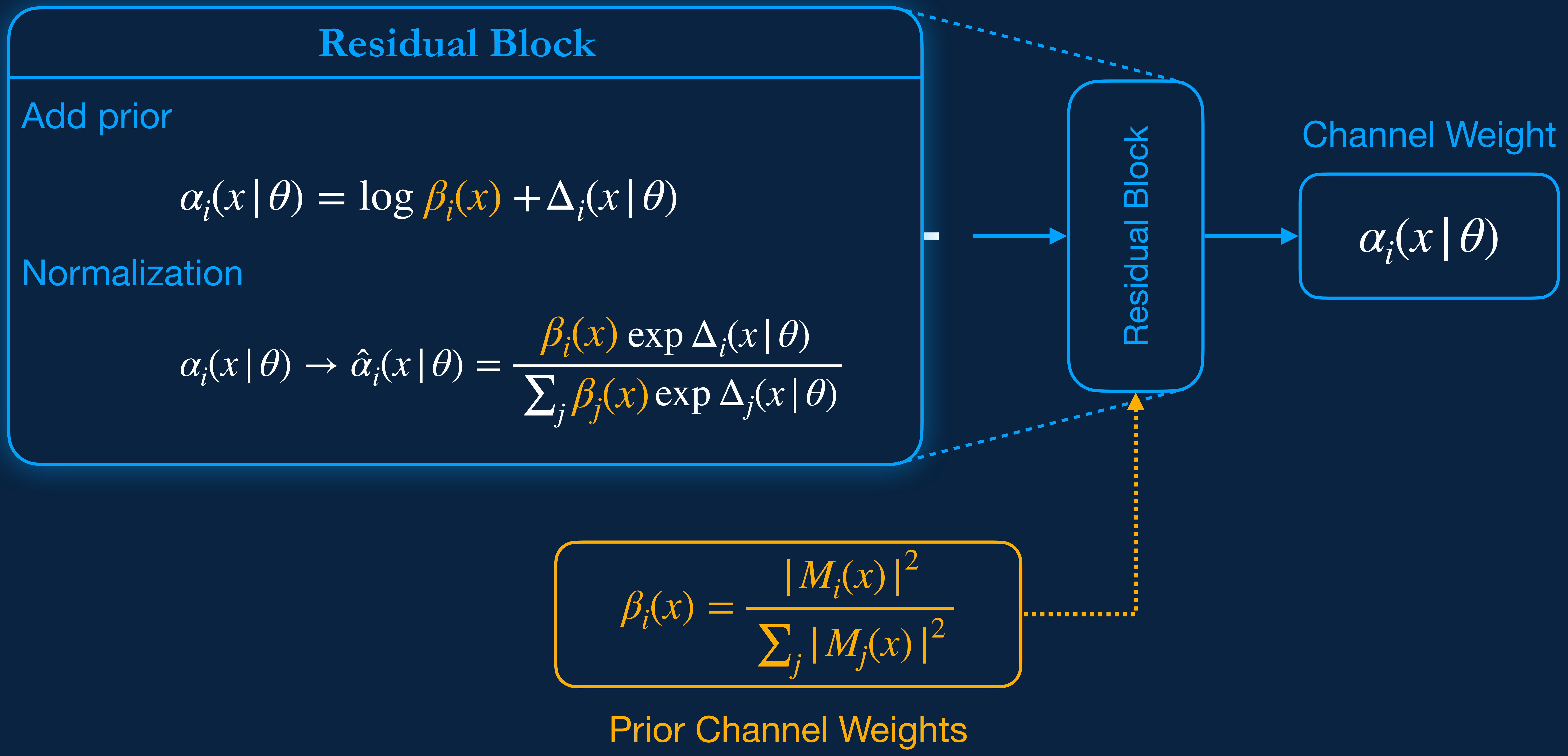

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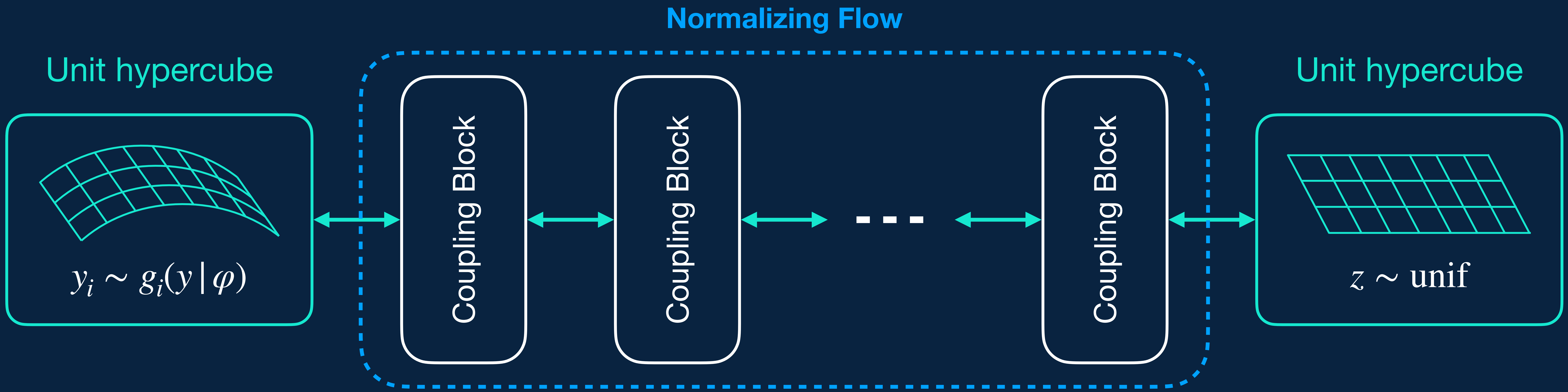

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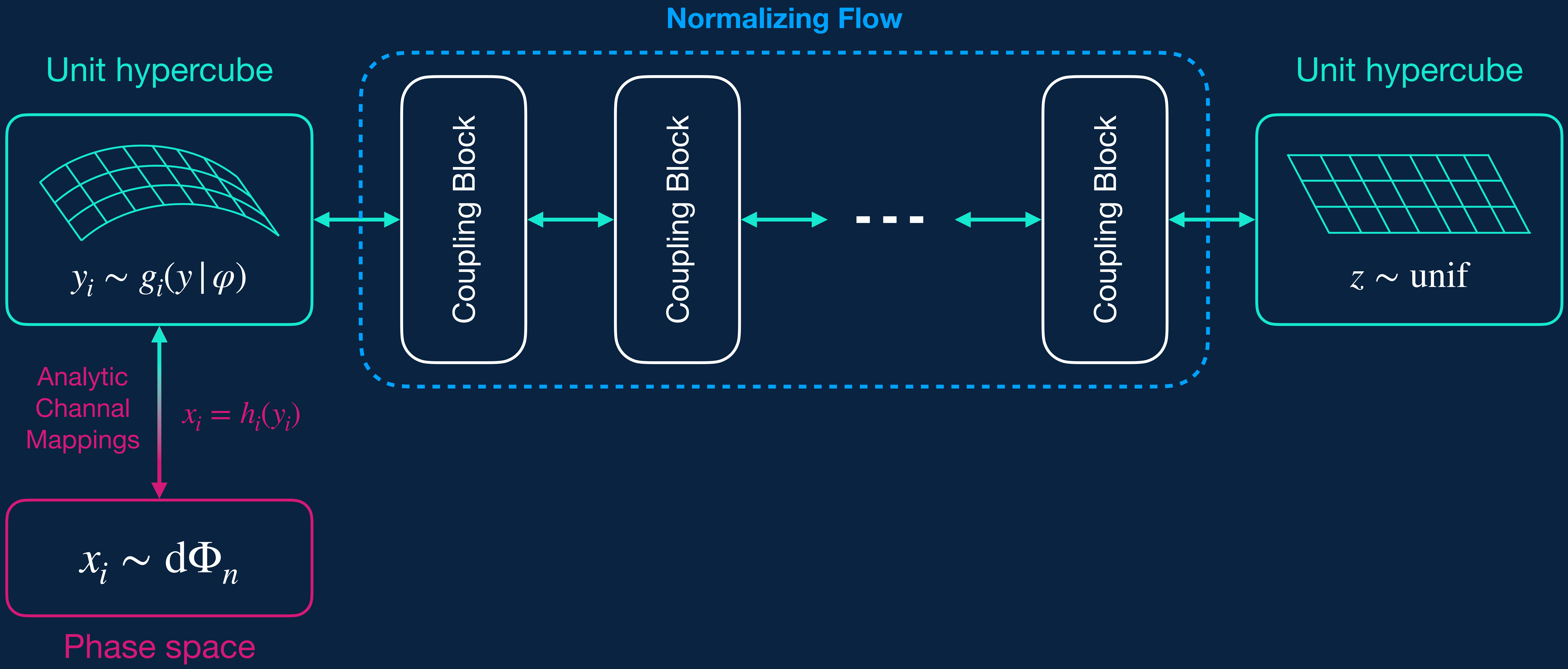
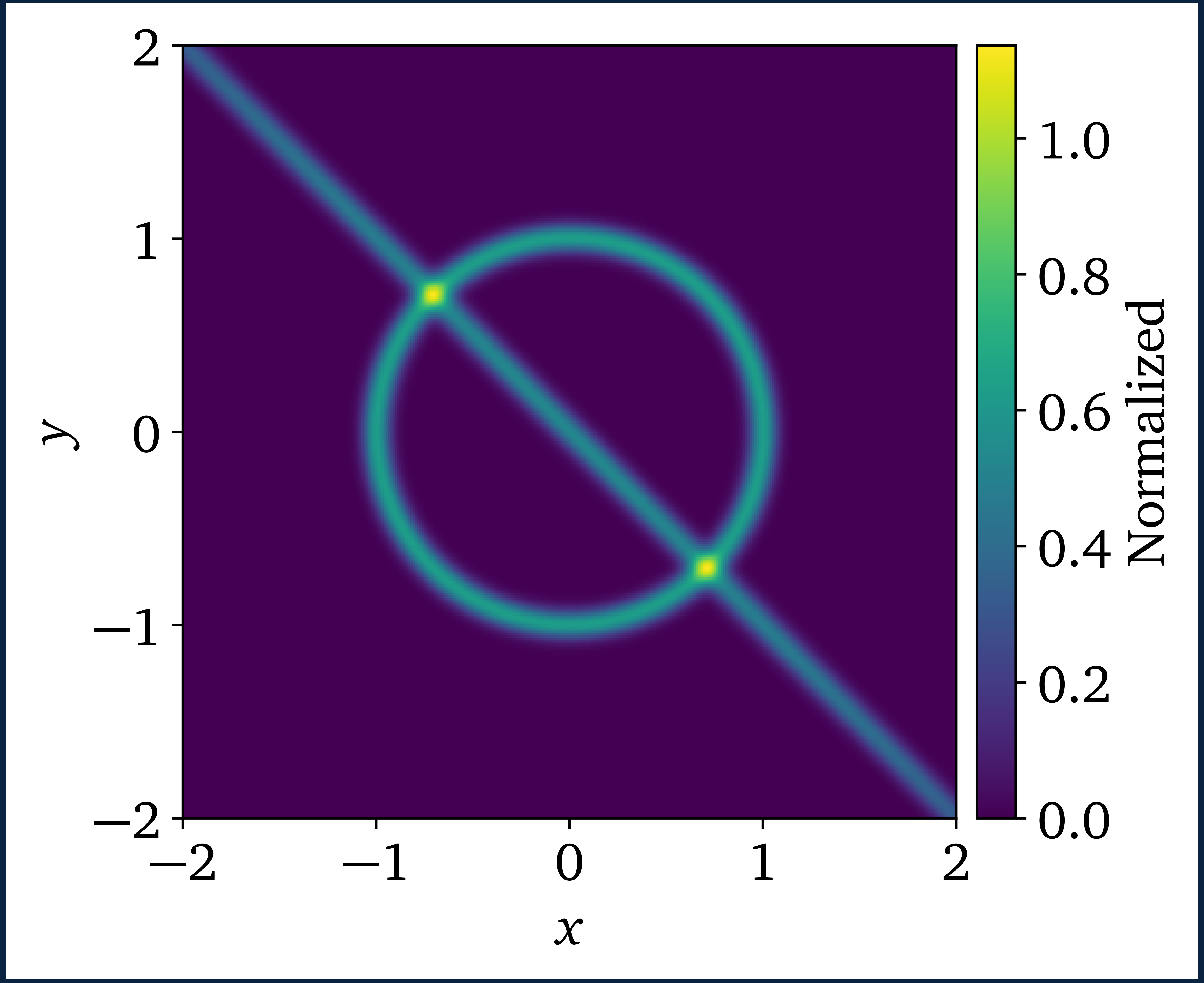


Example I

Crossed Ring

Toy Example: Crossed Ring



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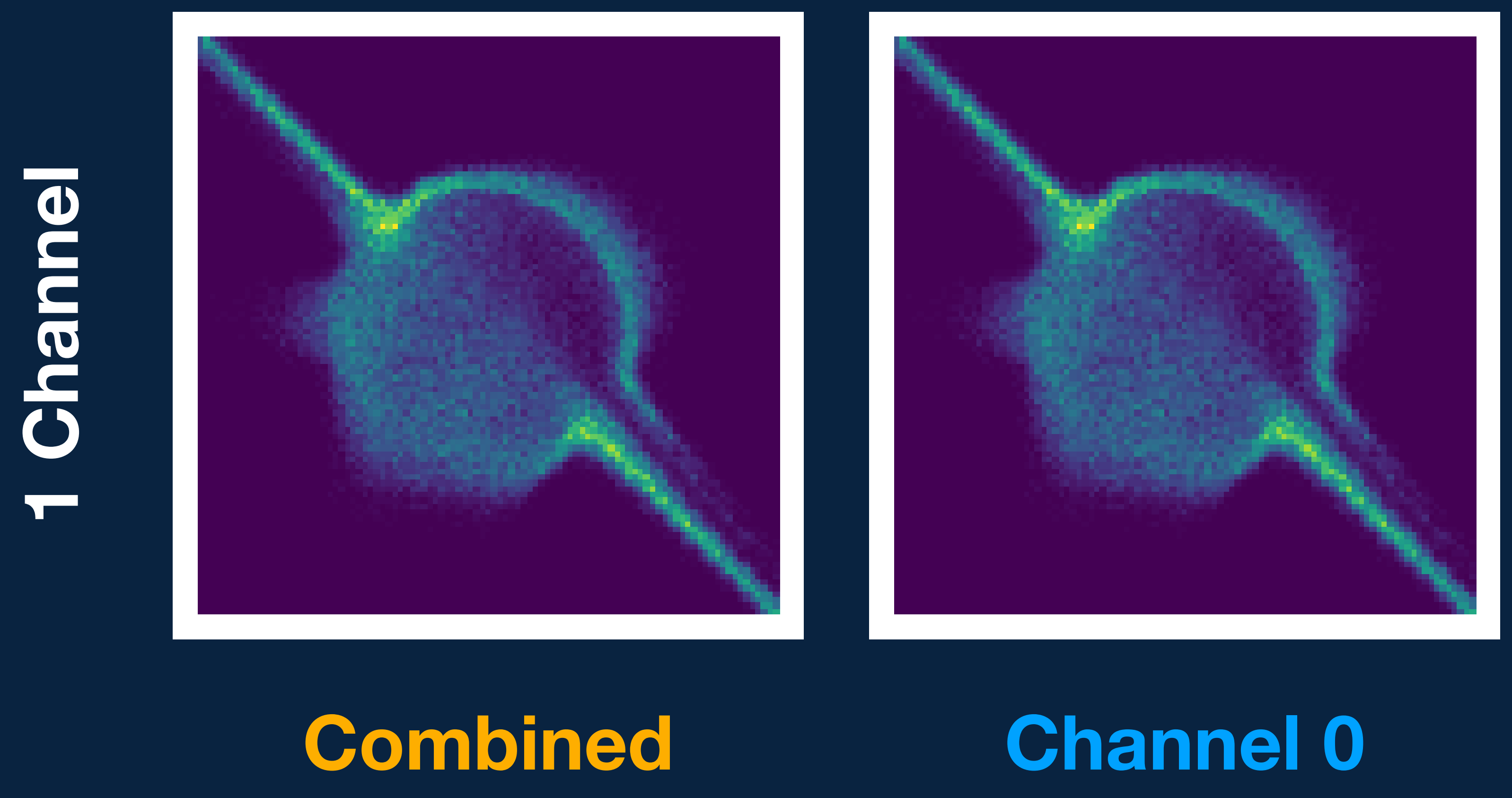

Rel. error:
1.17 ± 0.13

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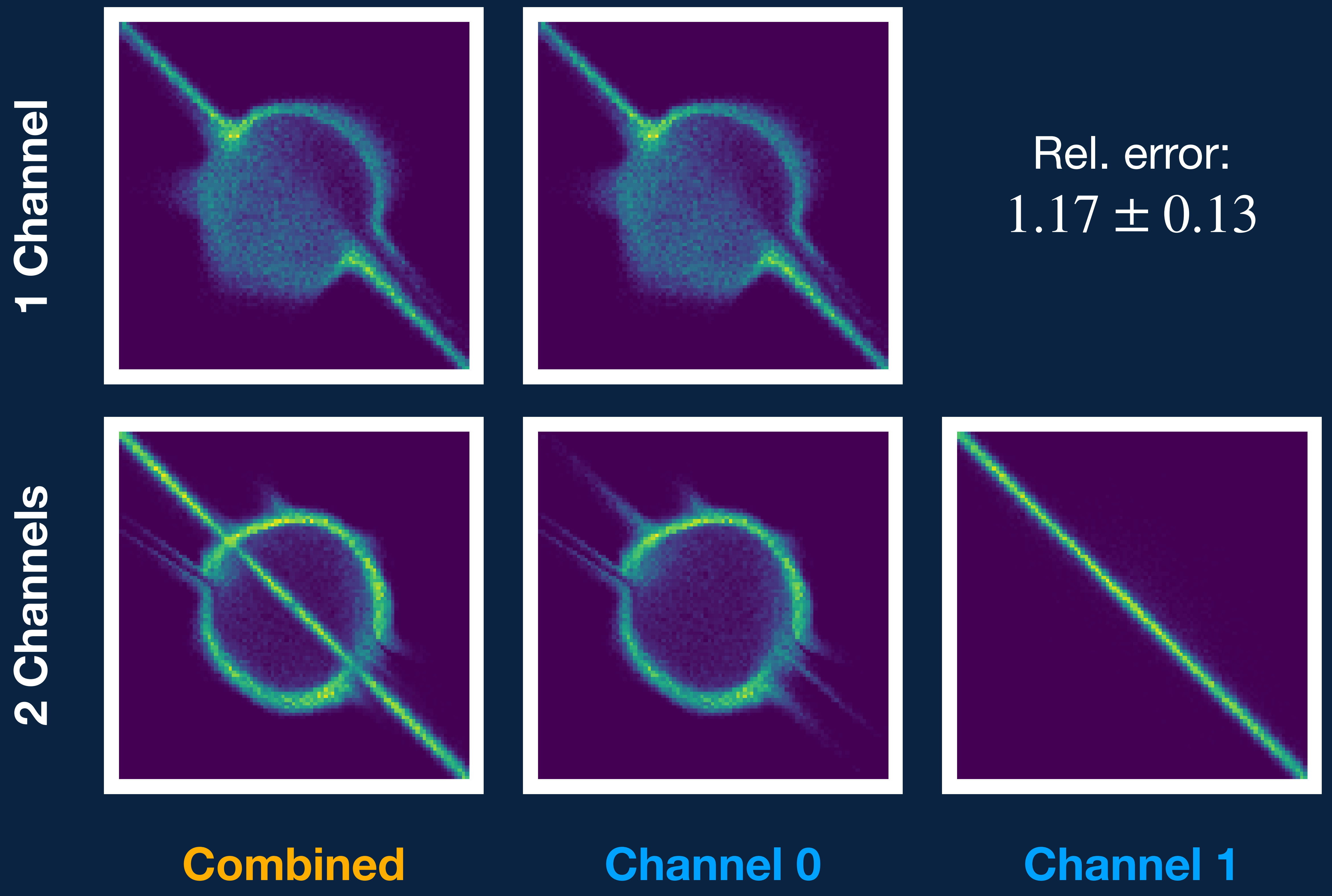

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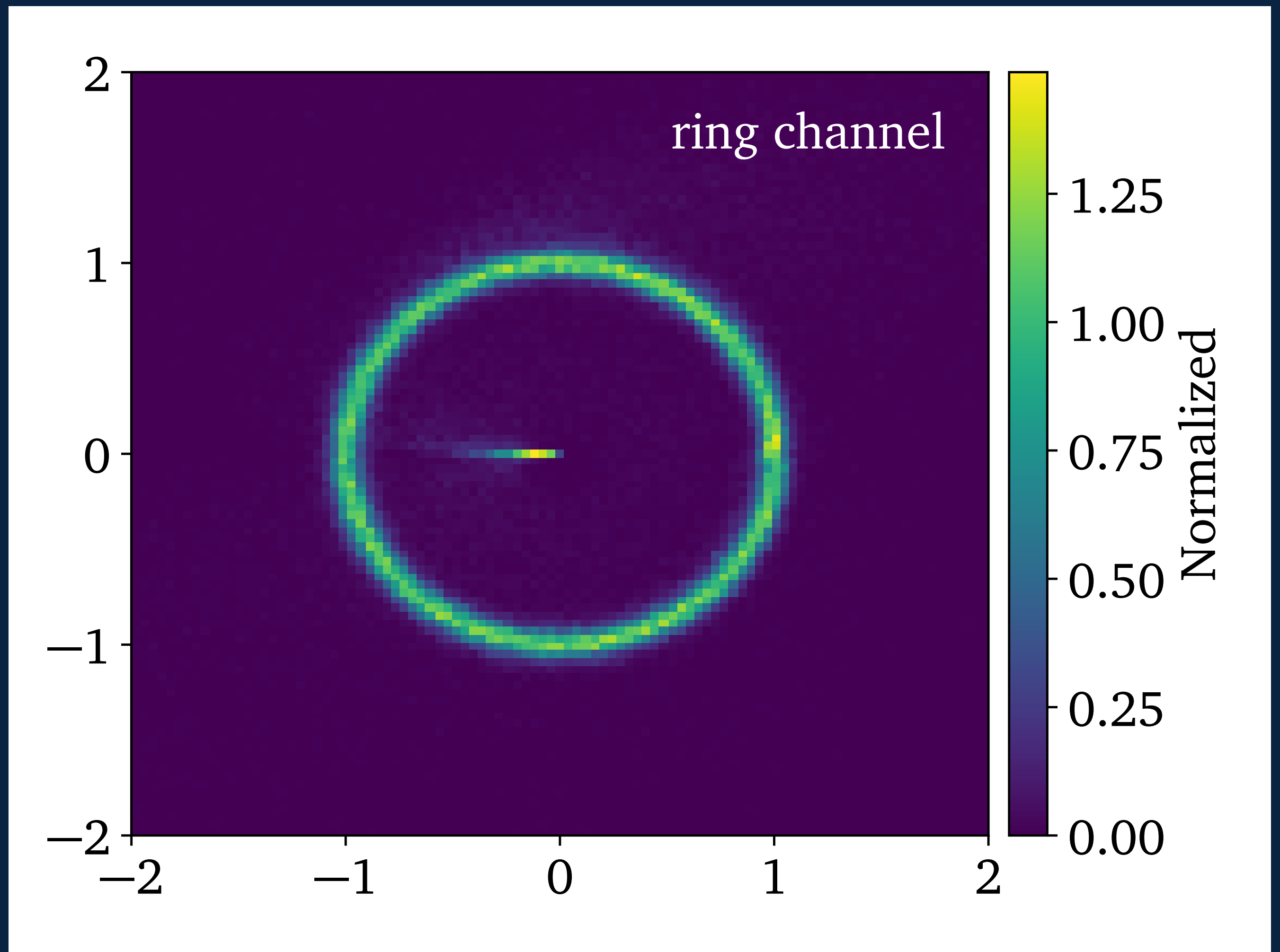

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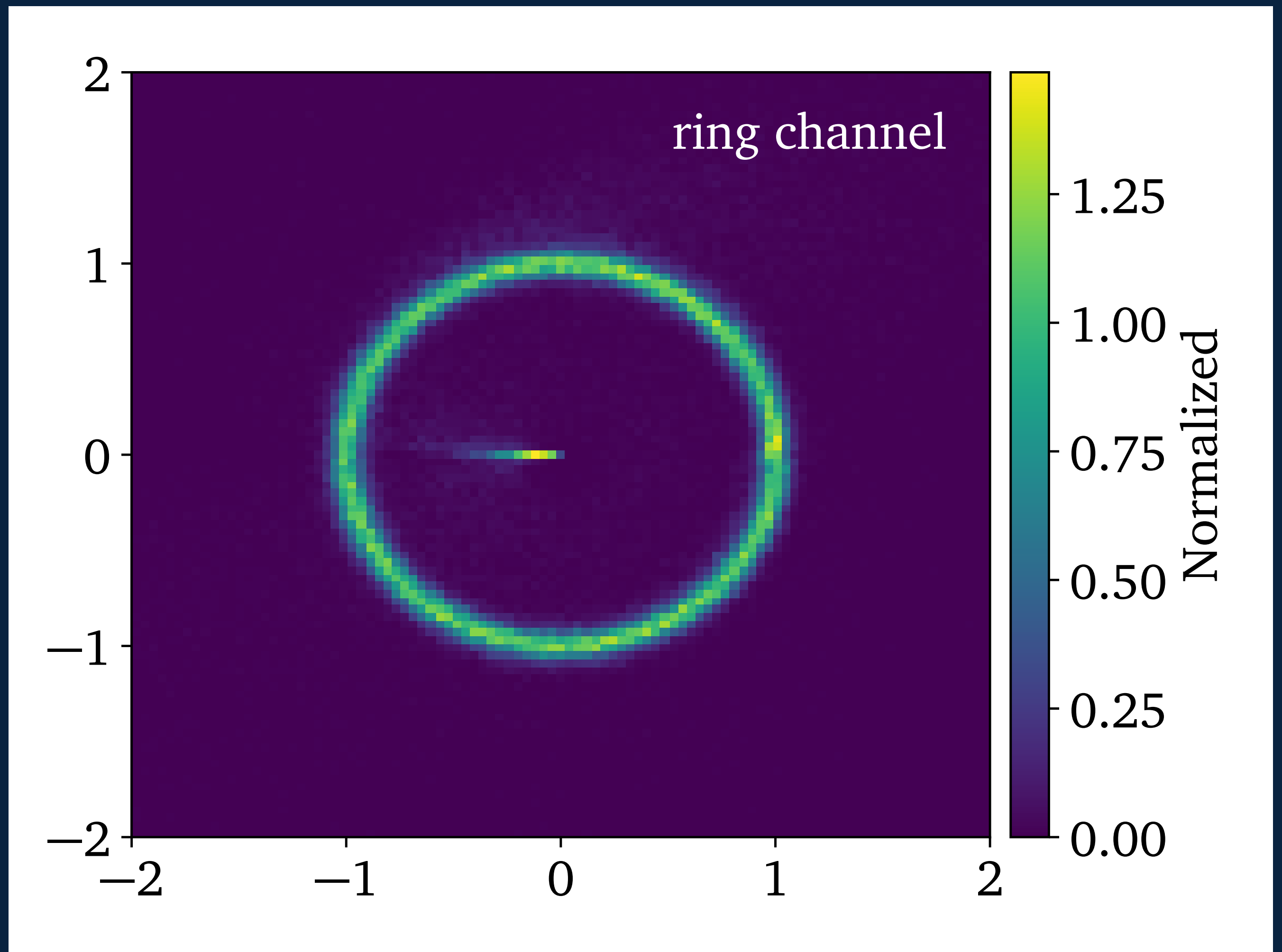

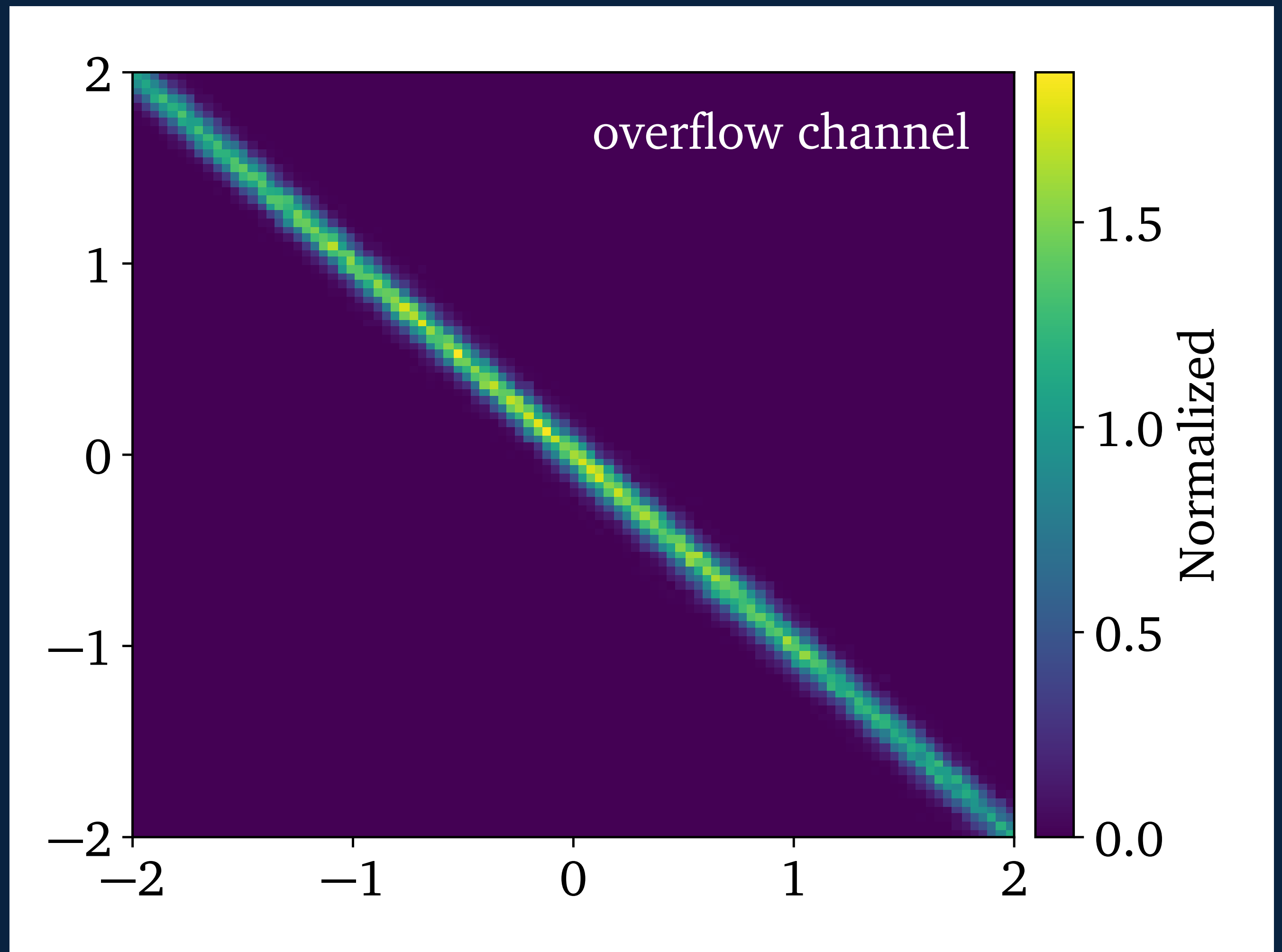

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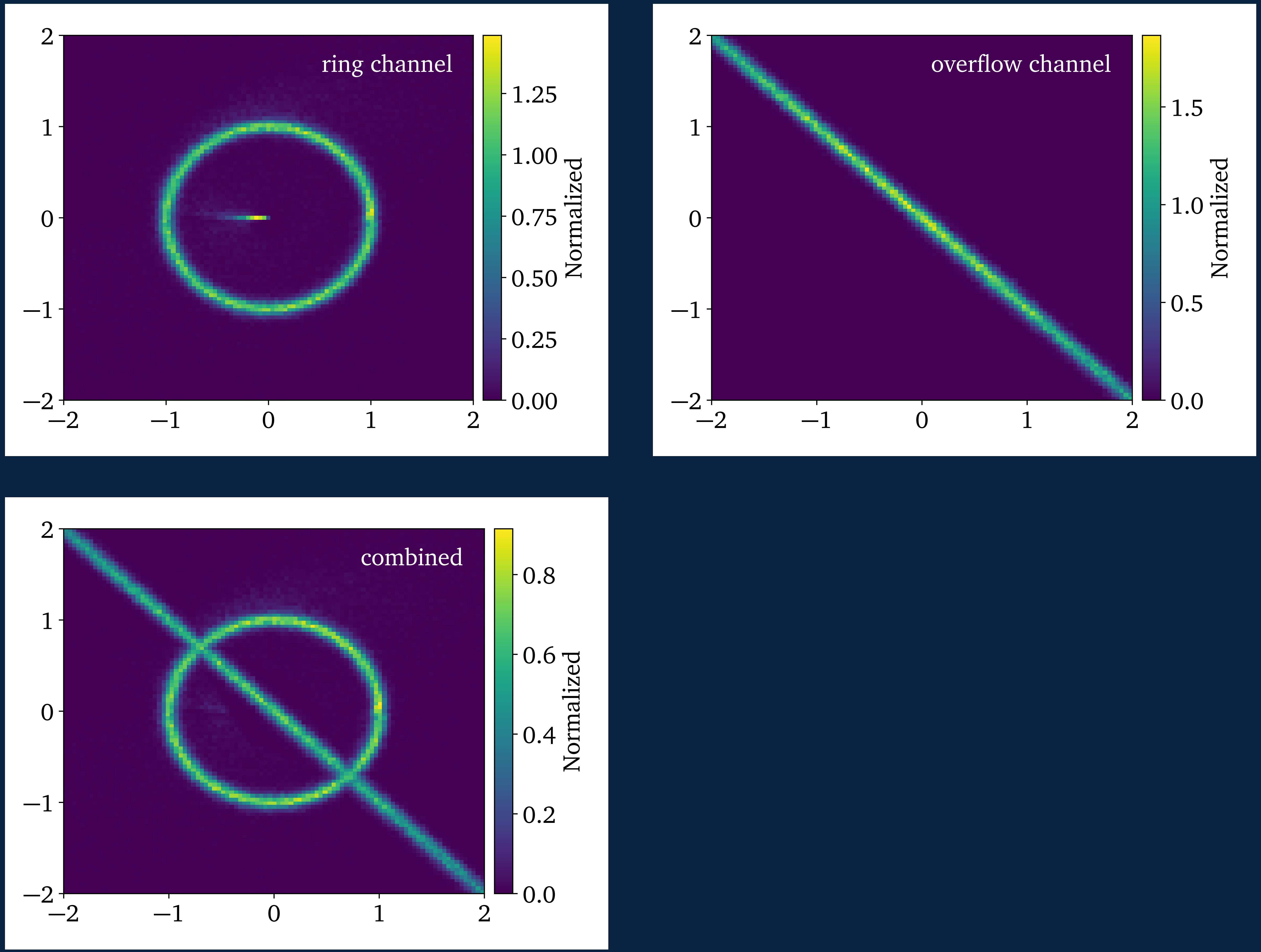

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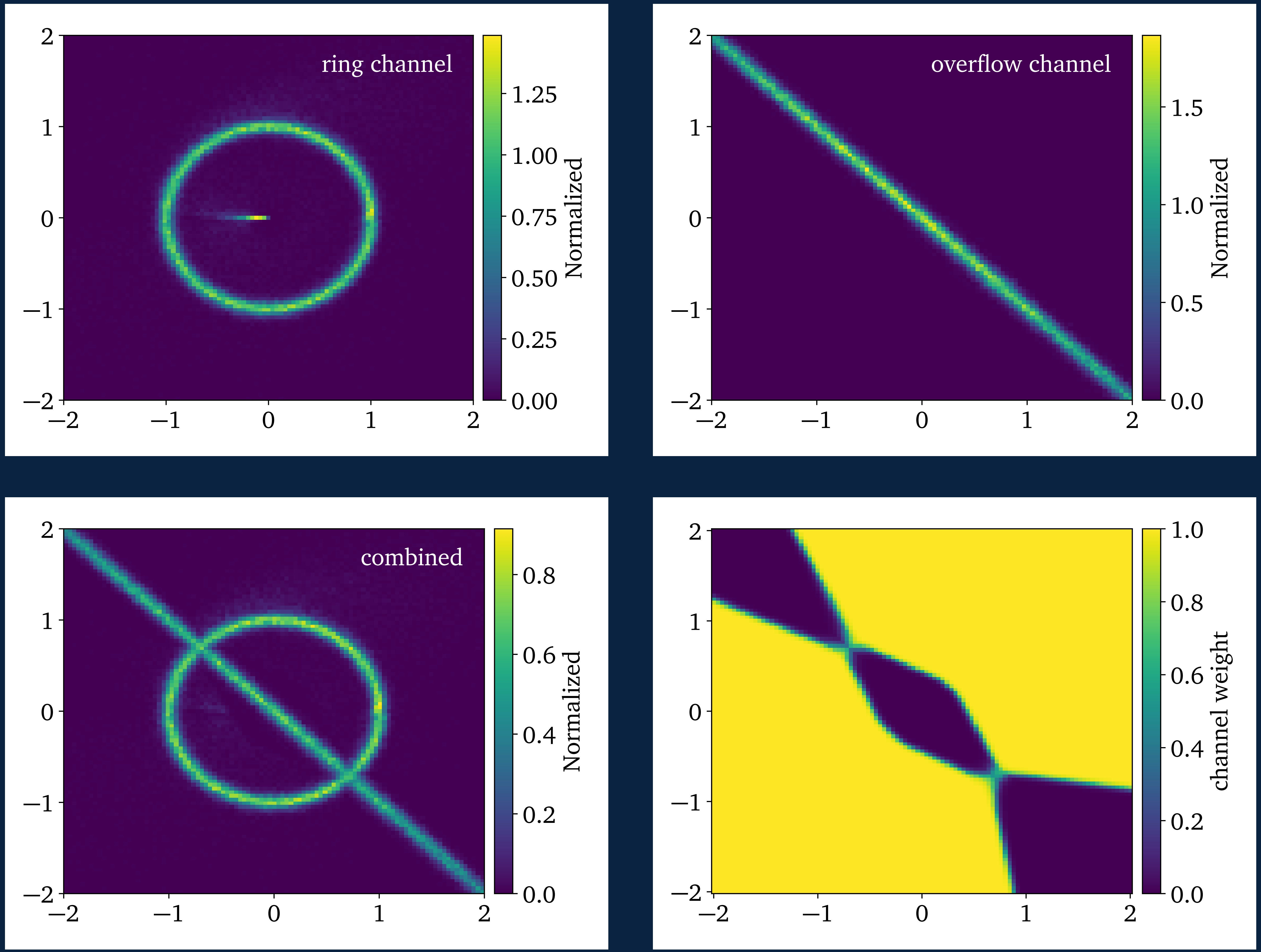

Rel. error:
0.37 ± 0.05

Example II

Drell-Yan + Z’

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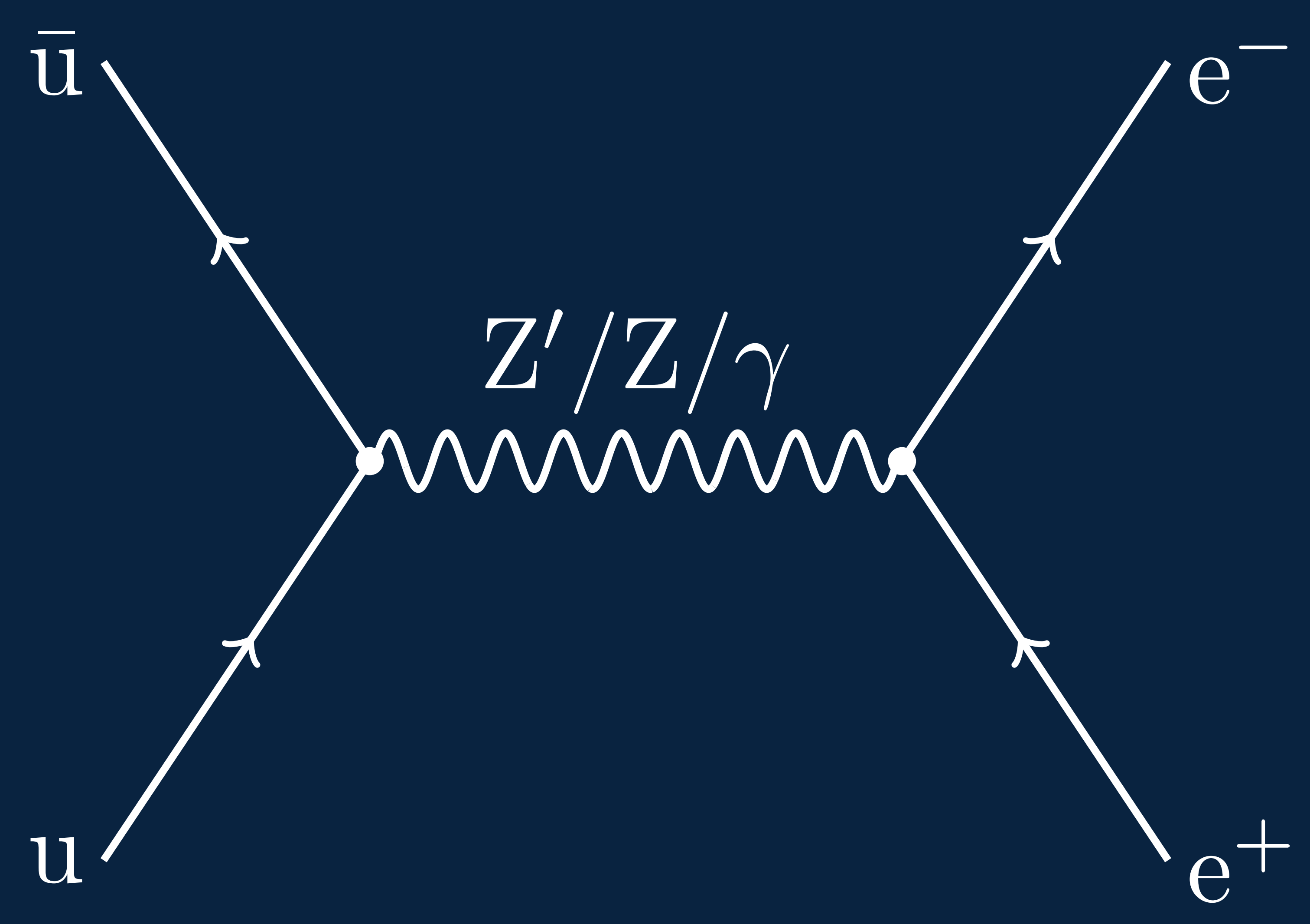
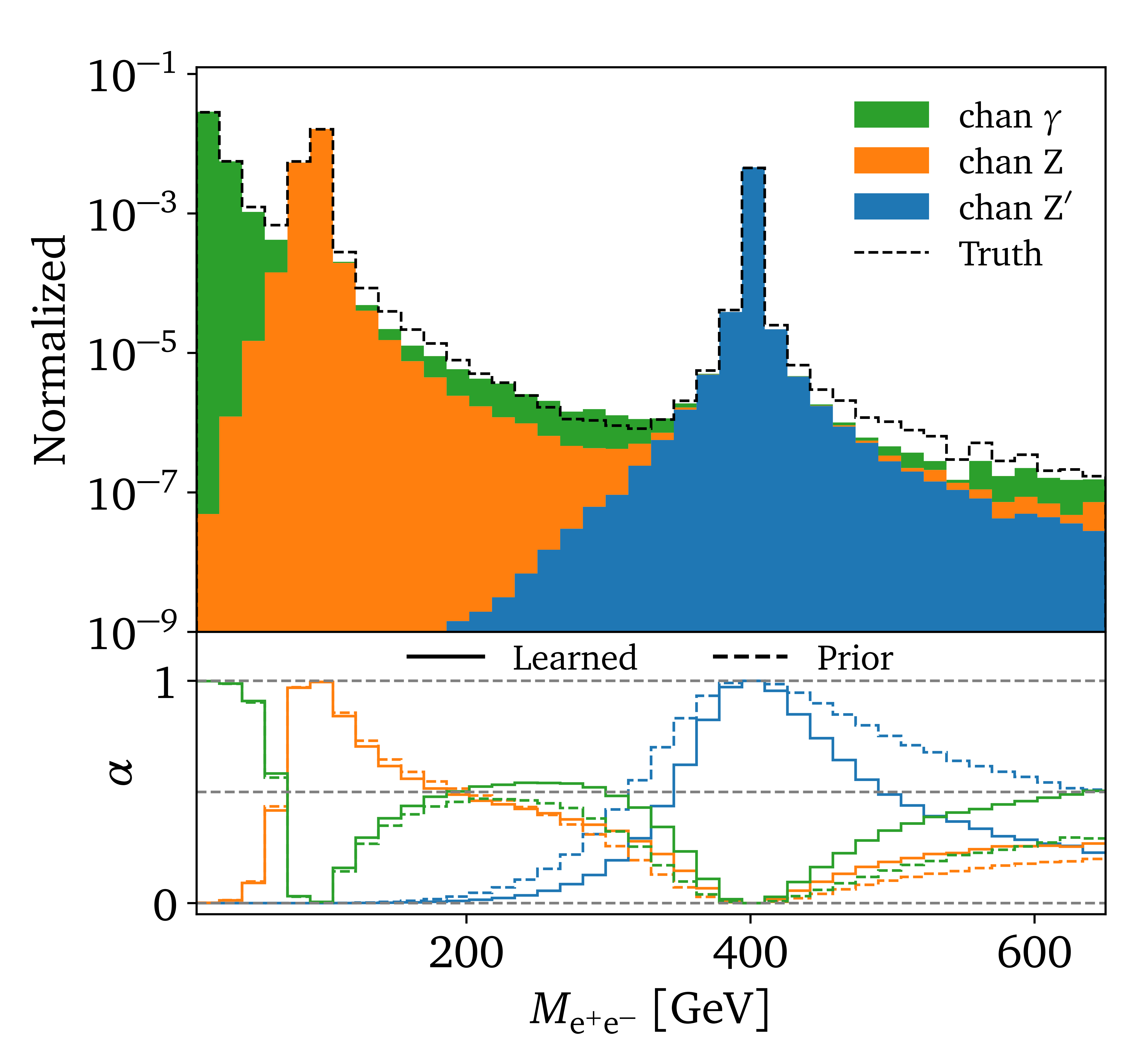
Implementation

- Custom amplitude in TensorFlow2
- Custom PS mappings in TensorFlow2
- PDFs from LHAPDF [1412.7420]





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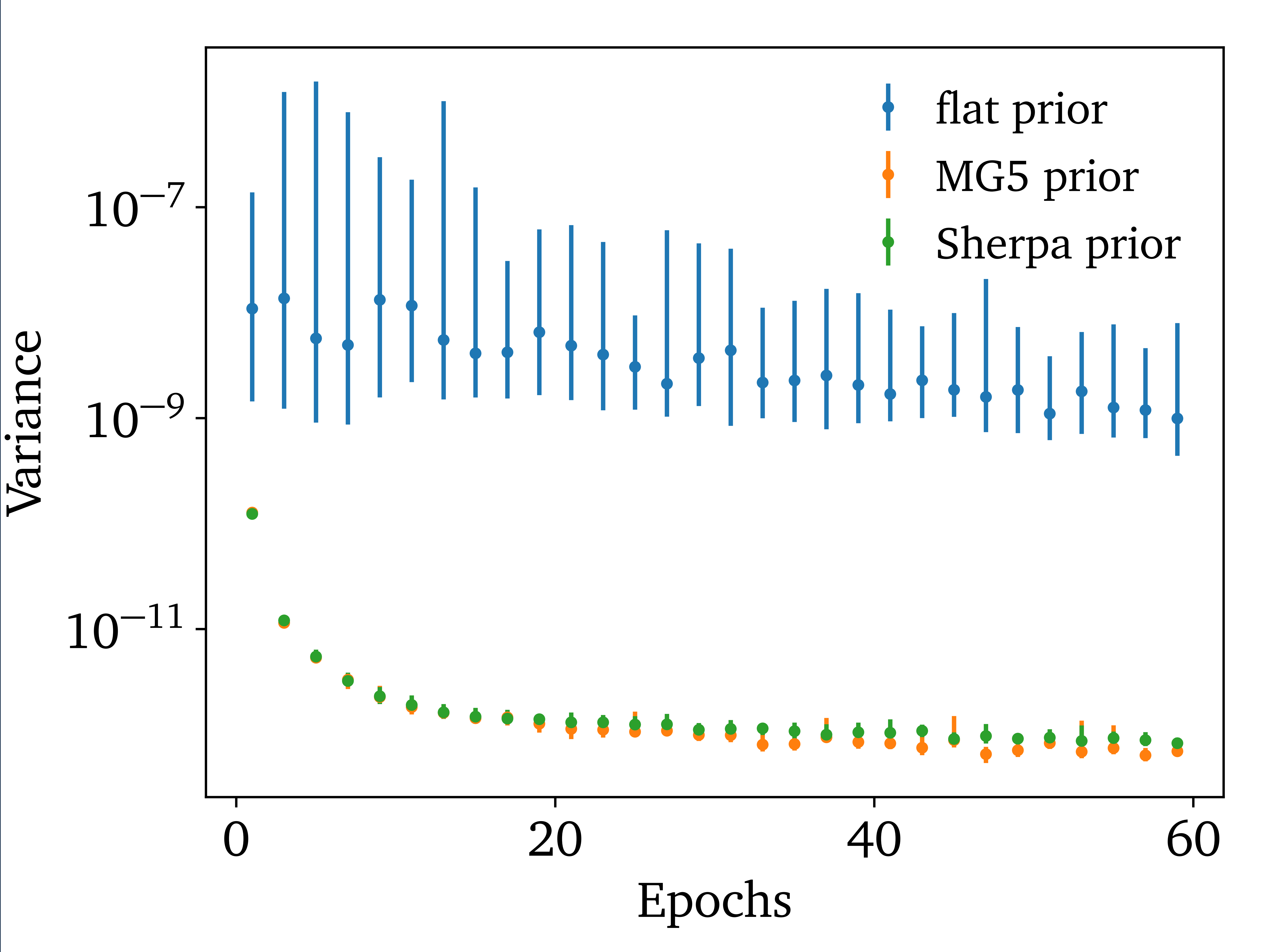

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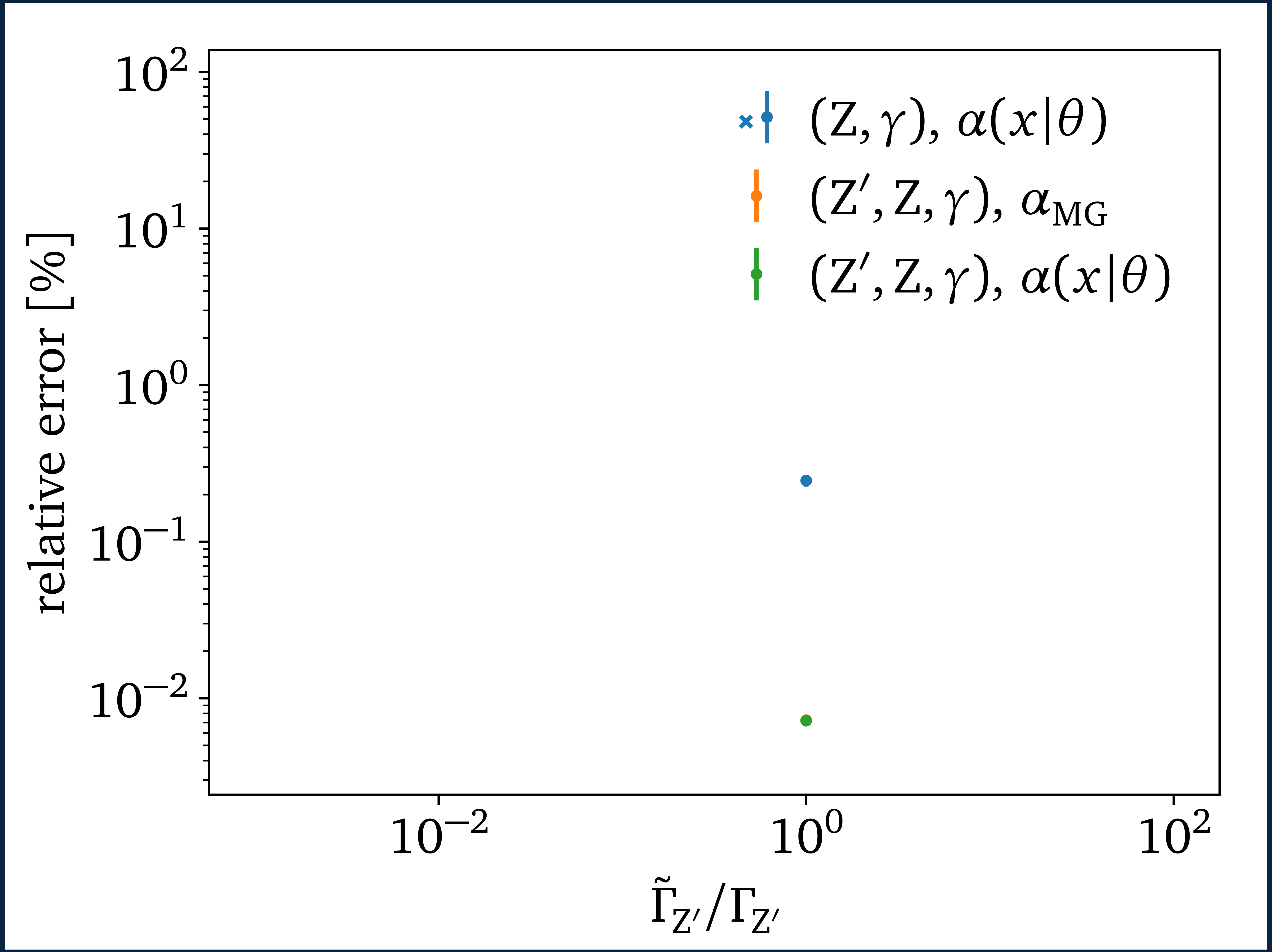

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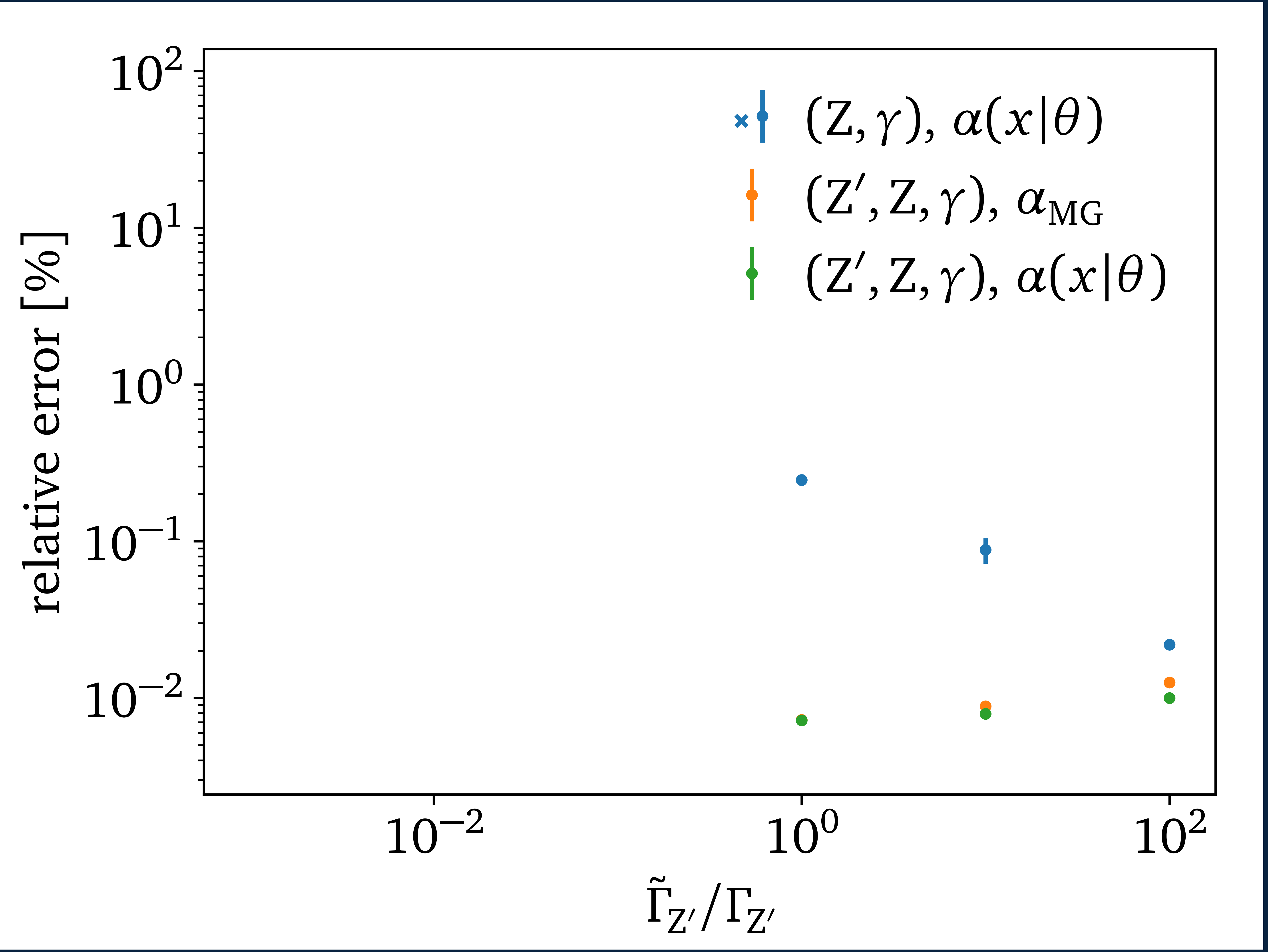

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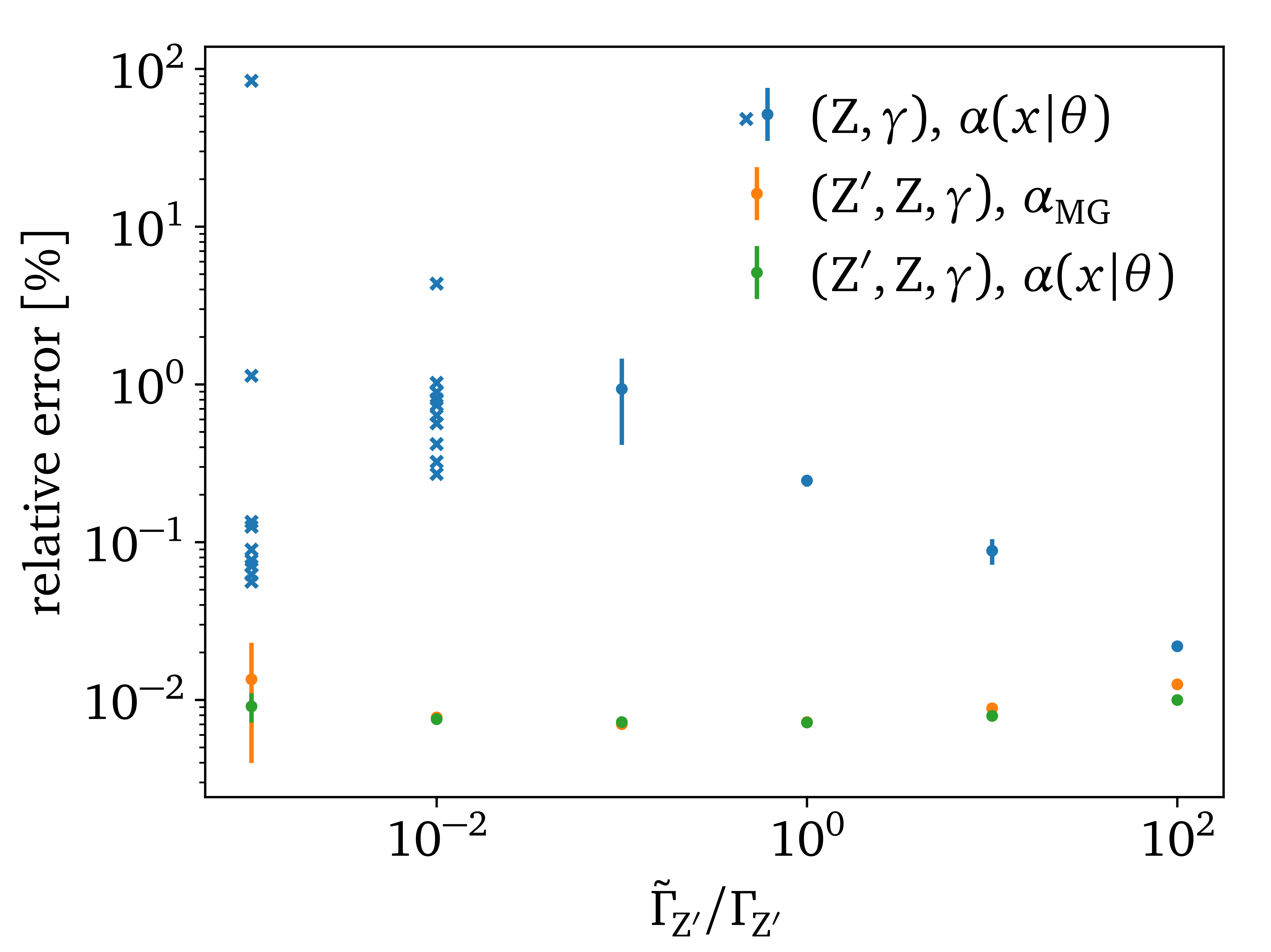
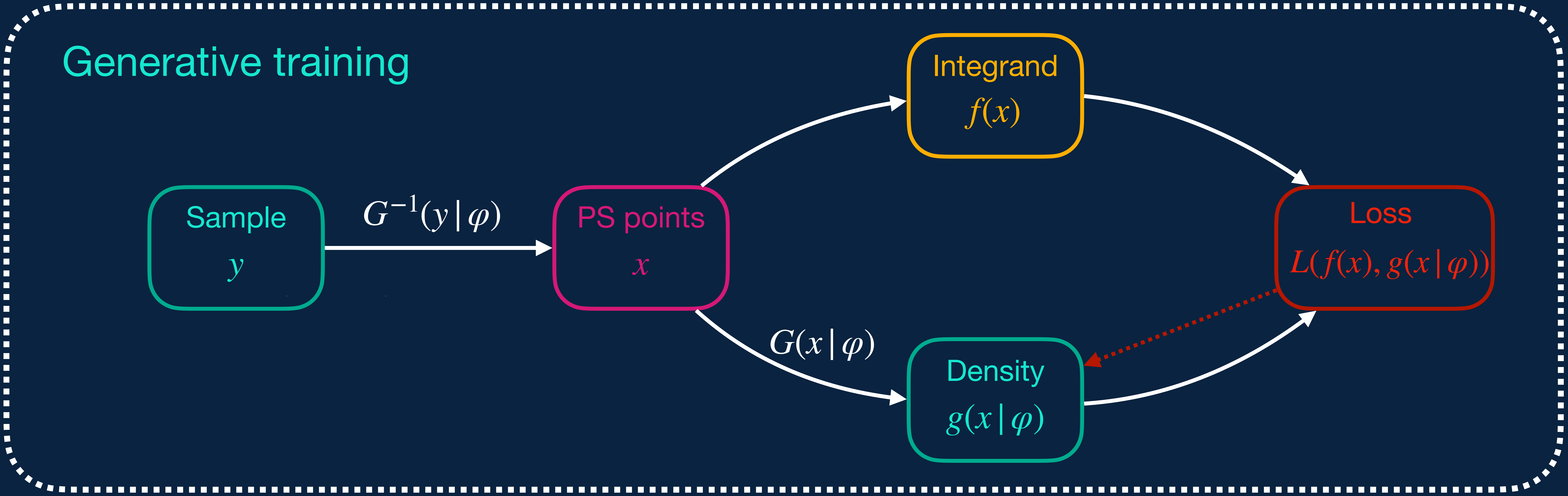
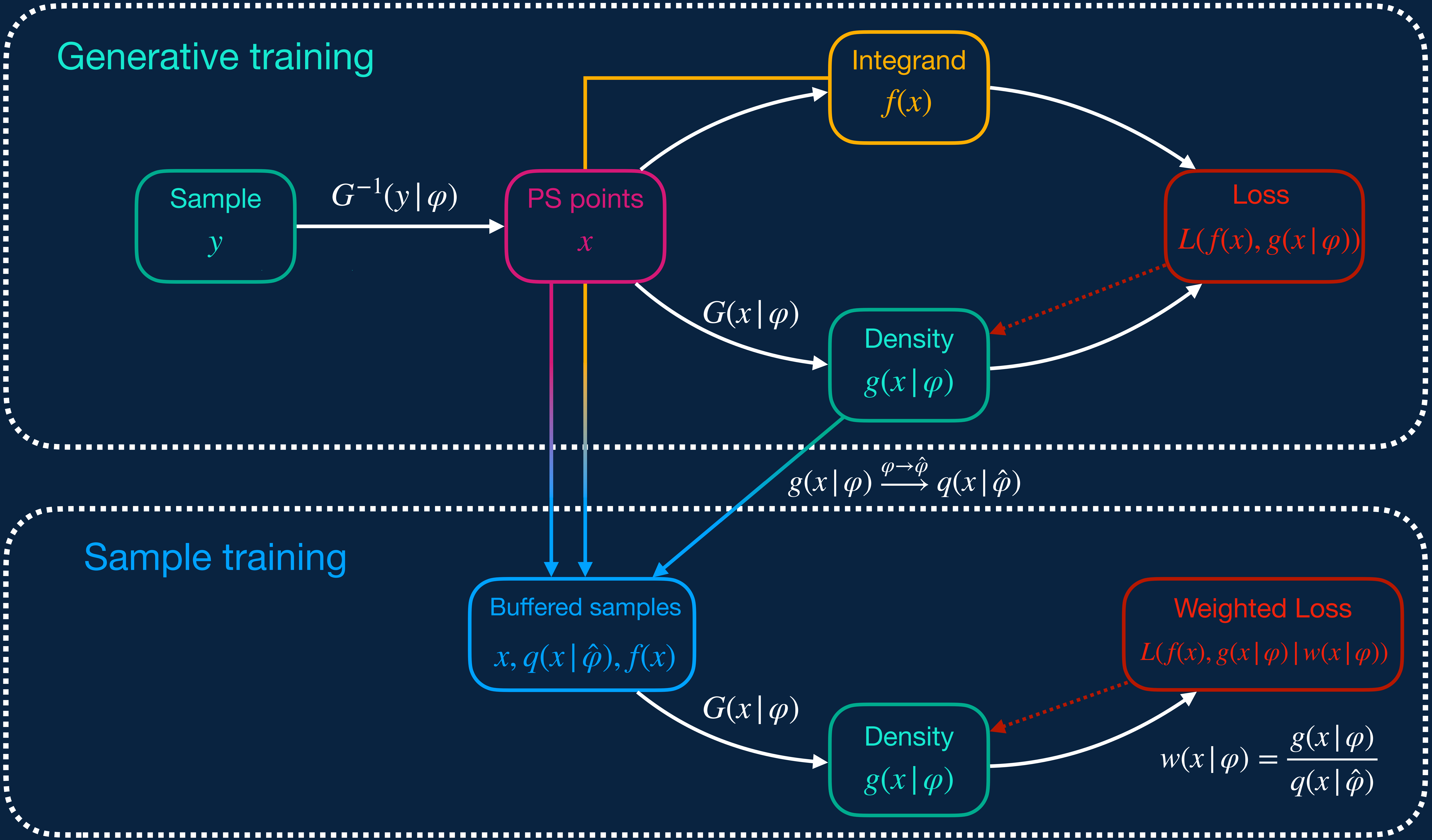


MadNIS

Buffered Training



Generative training
Sample
y
$G^{-1}(y|\varphi)$
PS points
x
Integrand
$f(x)$
$G(x|\varphi)$
Density
$g(x|\varphi)$
Loss
$L(f(x), g(x|\varphi))$



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$f(x)$
Loss
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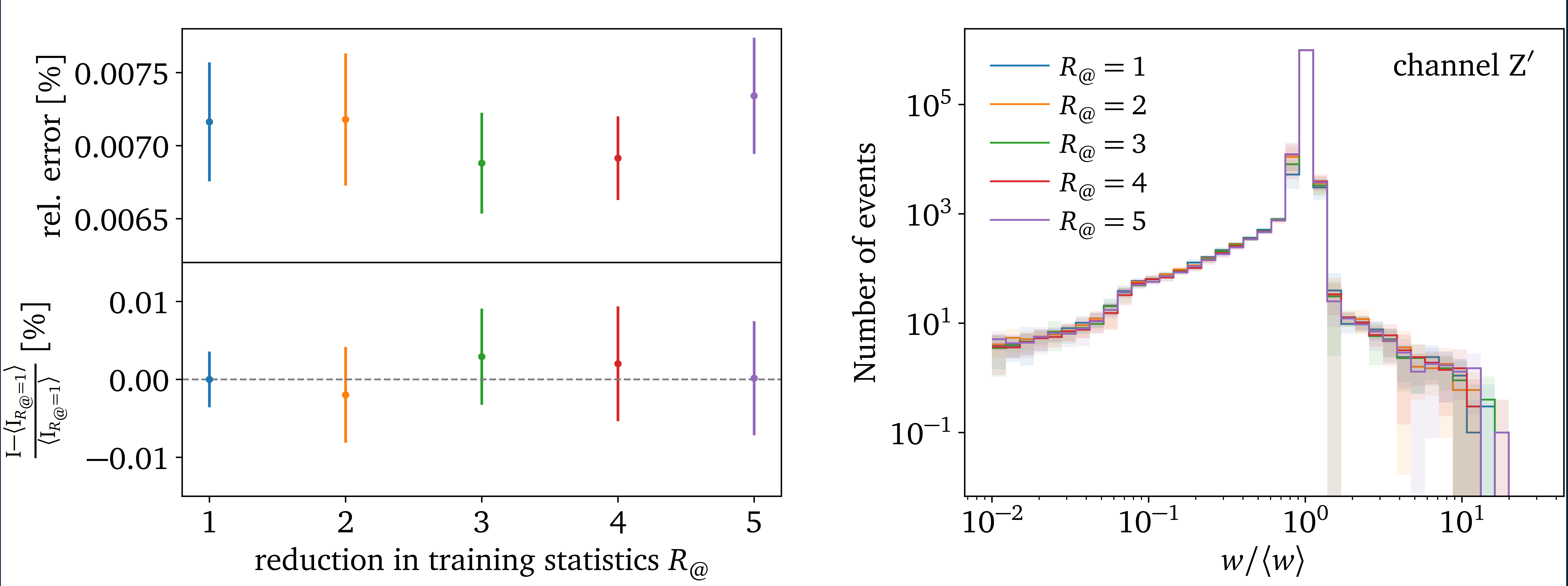
$G(x\,|\,\varphi)$
Density
$g(x\,|\,\varphi)$
$g(x\,|\,\varphi) \xrightarrow{\varphi \to \hat{\varphi}} q(x\,|\,\hat{\varphi})$
Sample training
Buffered samples
$x, q(x\,|\,\hat{\varphi}), f(x)$
Weighted Loss
$L(f(x), g(x\,|\,\varphi)\,|\,w(x\,|\,\varphi))$
$G(x\,|\,\varphi)$
Density
$g(x\,|\,\varphi)$
$w(x\,|\,\varphi) = \frac{g(x\,|\,\varphi)}{q(x\,|\,\hat{\varphi})}$

Buffered Training



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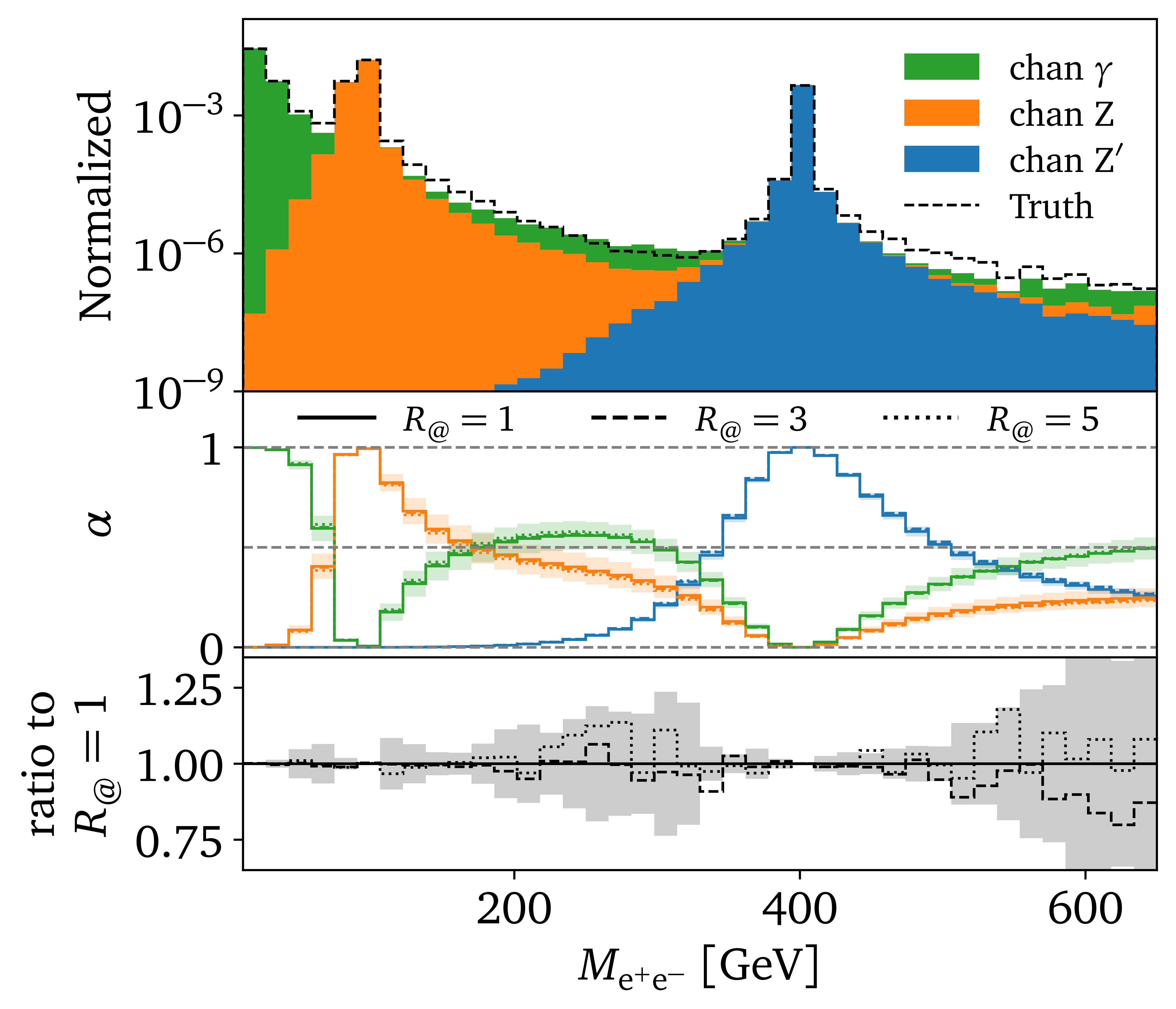

Summary and Outlook

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Stay tuned: Paper on arxiv tomorrow!! [14.12.2022]