High Energy Resummation for Jet Processes at the LHC

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CERN TH Seminar Dec 2022

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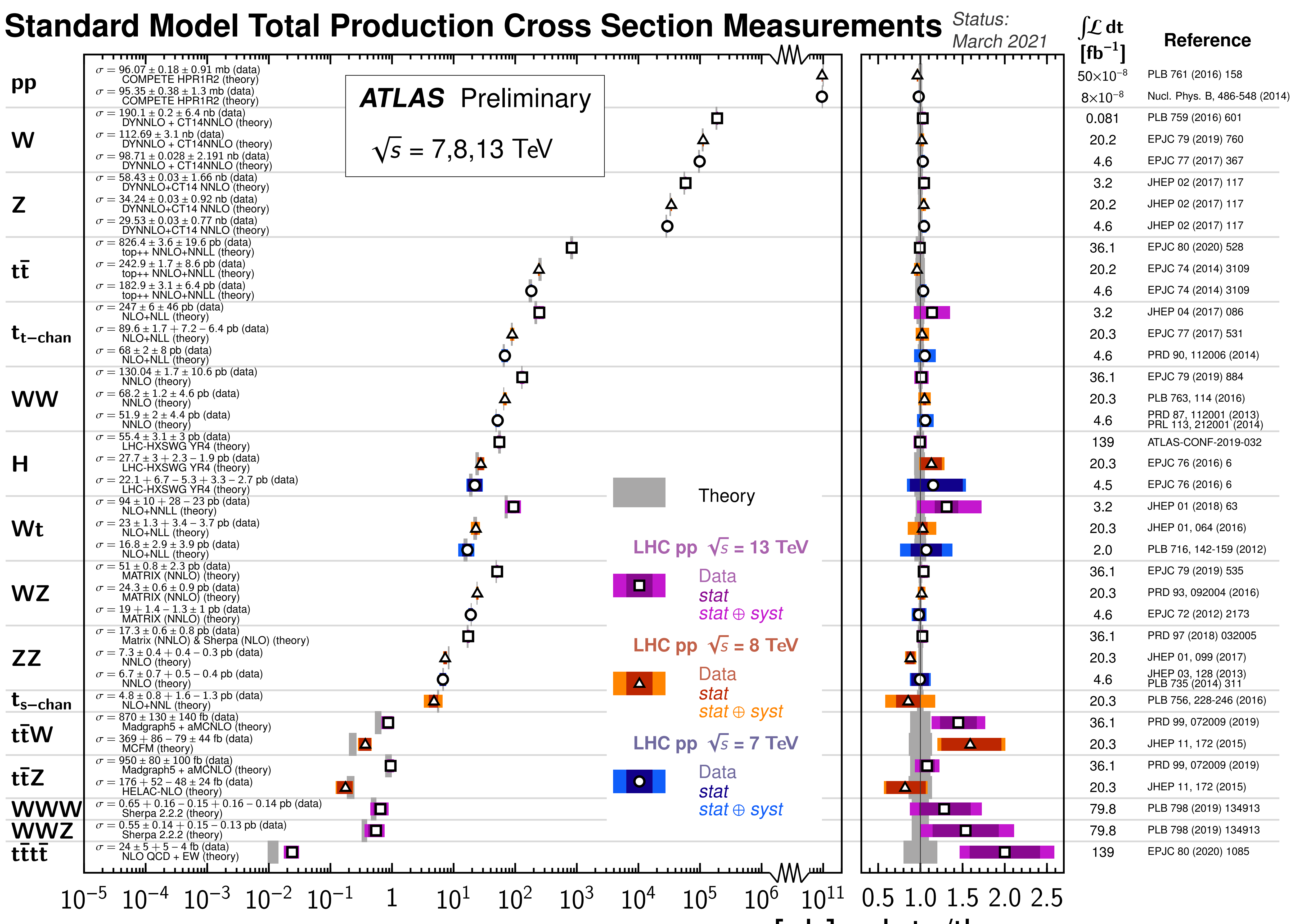
Established by the European Commission

Photo Credit: Kinrannoch Photography

LHC Summary So Far…

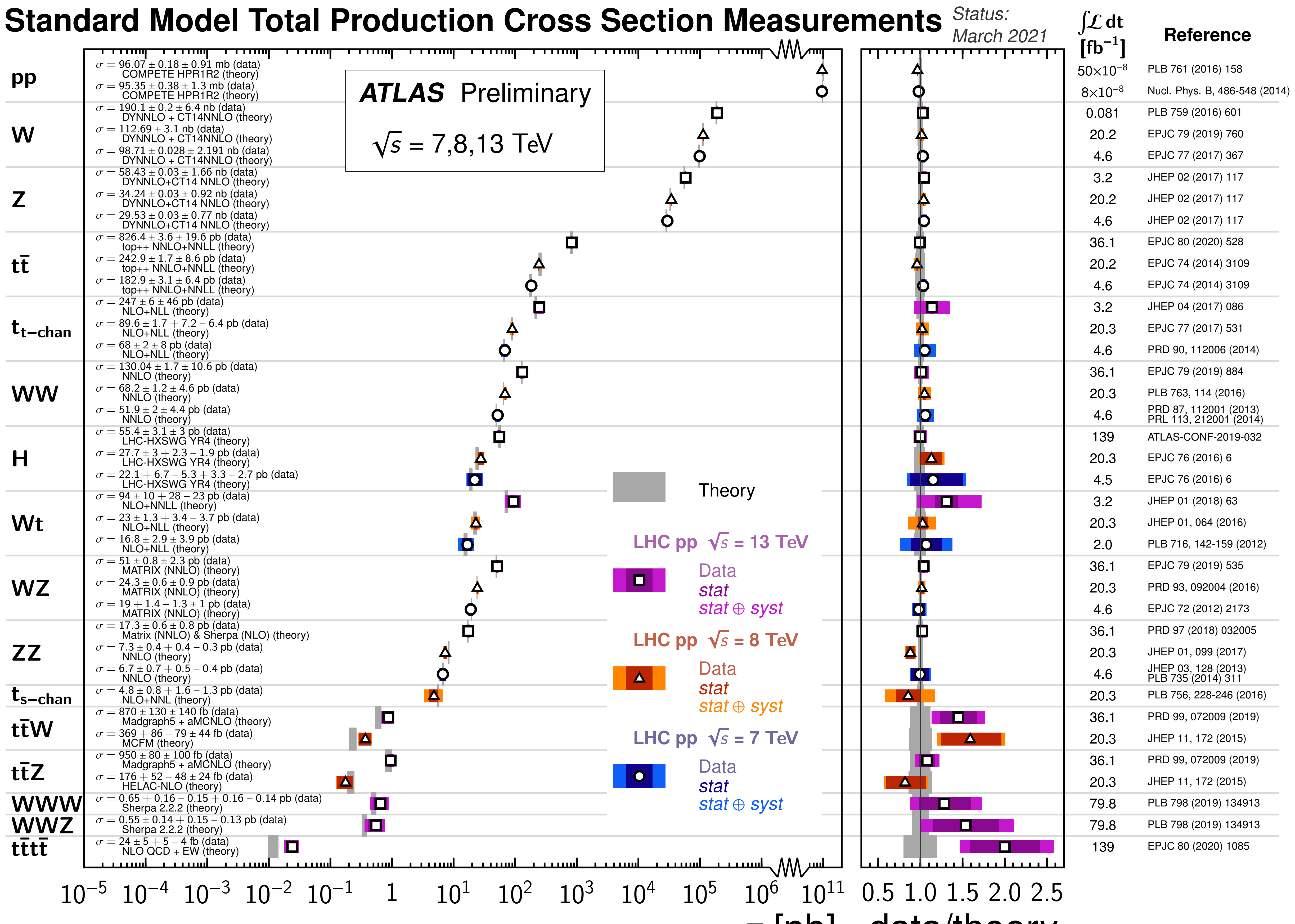


ATL-PHYS-PUB-2021-005

- Phenomenal agreement with theory so far
- Very sophisticated calculations, e.g. $t\bar{t}$ at NNLO+NNLL

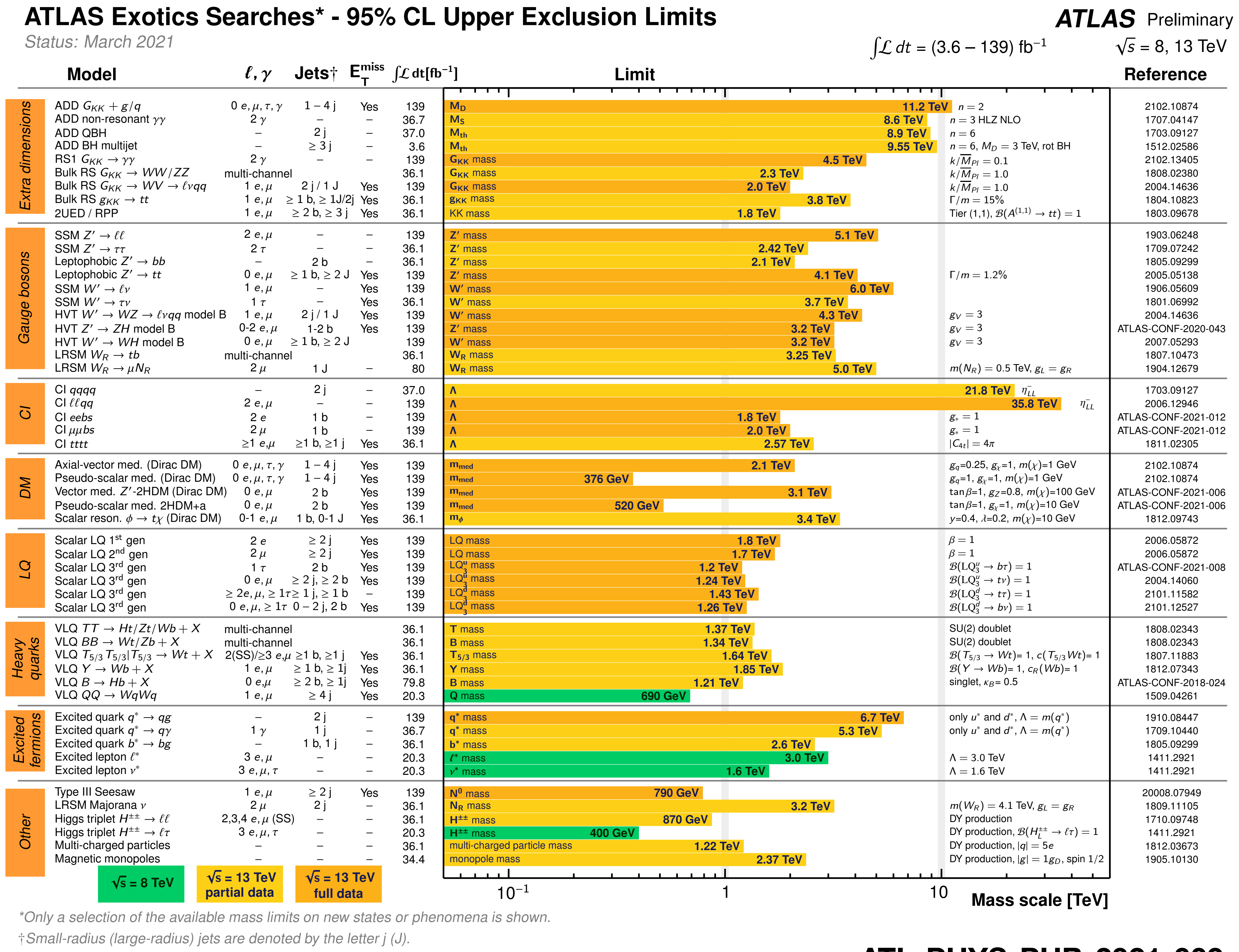
Czakon, Mitov arXiv:1112.5675

LHC Summary So Far…



ATL-PHYS-PUB-2021-005

New physics searches have generated many exclusion limits (huge range of models, very high limits)



ATL-PHYS-PUB-2021-009

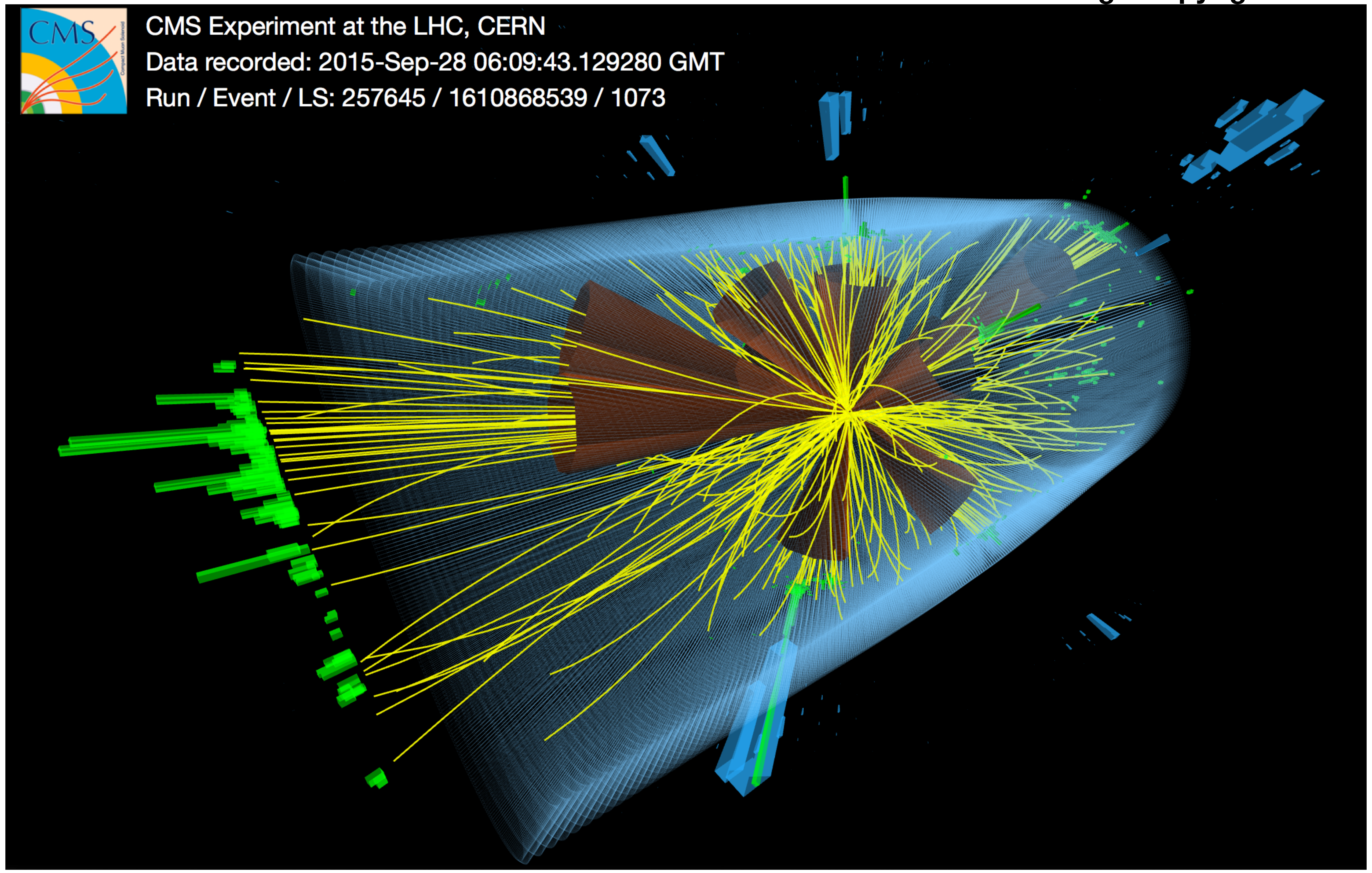
- Phenomenal agreement with theory so far
- Very sophisticated calculations, e.g. $t\bar{t}$ at NNLO+NNLL

Czakon, Mitov arXiv:1112.5675

A Challenge at High Centre of Mass

Image Copyright CERN

12 jets with p_T > **50 GeV** at CMS (13 TeV)



Many colour-charged, hard particles with p_T, s_{ij}, $\hat{s}$

Large logs in s_{ij}/p_T^2 damage convergence of pert. expansion

Which all-order?

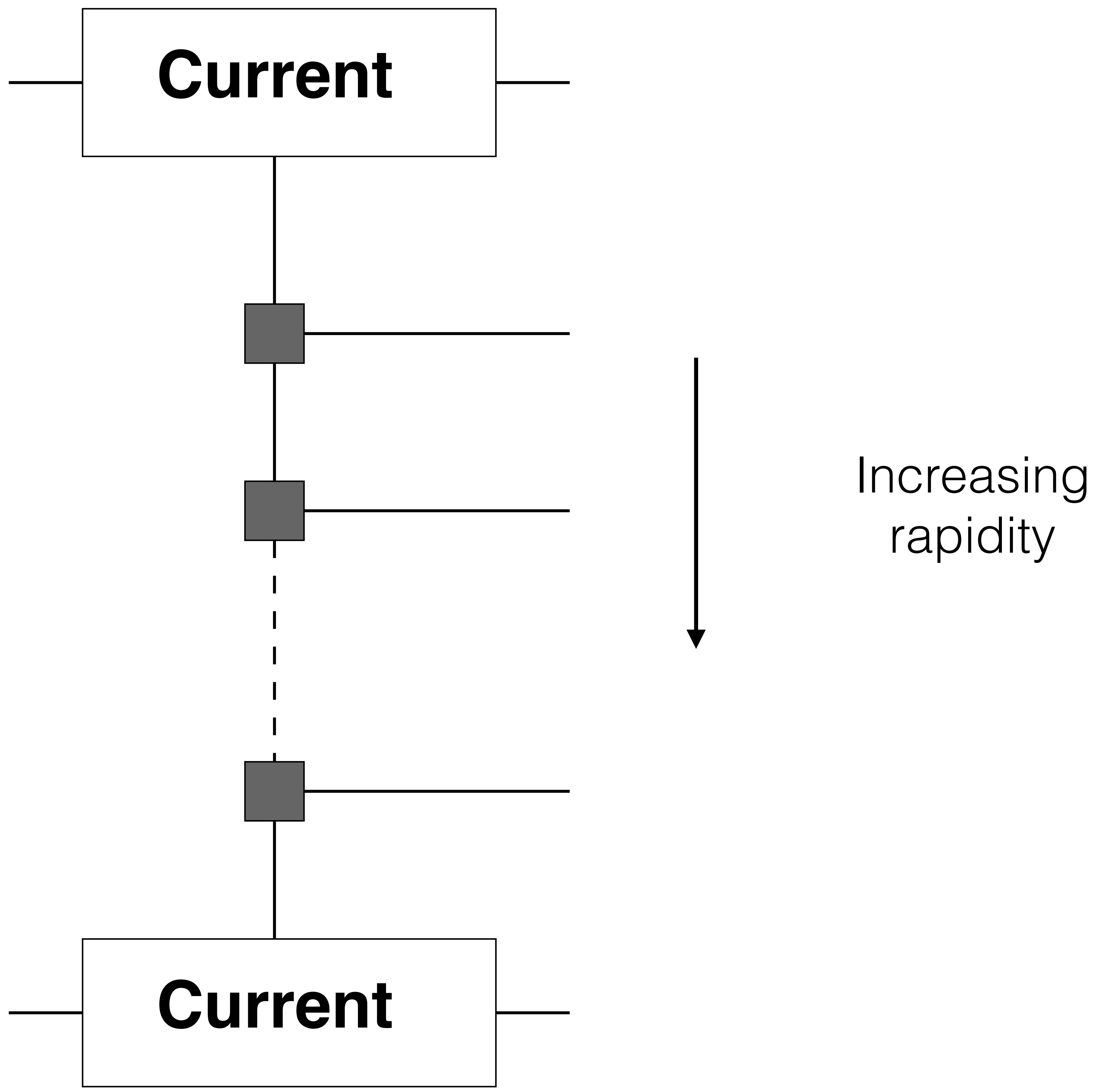
Inclusive 2-jet cross section given by $\int d\mathrm{PS}_2\, |M_{2j+}|^2$, with

$$
\begin{aligned}
|M_{2j+}|^2 = \;& \alpha_s^2 \left(a_2 (s^2/t^2) + b_2 \right) \\
&+ \alpha_s^3 \left(a_3 (s^2/t^2) \log(s/t) + b_3 (s^2/t^2) + c_3 \right) \\
&+ \alpha_s^4 \left(a_4 (s^2/t^2) \log^2(s/t) + b_4 (s^2/t^2) \log(s/t) + \ldots \right) \\
&+ \ldots
\end{aligned}
$$

- LO = first line, NLO = first two lines
- Leading logs = the 'a'-terms: $\alpha_s^{2+k} \log^k (s^2/t^2)$
- Logs arise from integrals over loop momenta in virtuals and from integrals over reals
- Our description = **LO + LL + ...**

Amplitudes in the High Energy Limit

Fortunately, the matrix elements of these processes simplify in the High Energy limit: $s_{ij} \to \infty$, $|p_{Ti}|$ finite



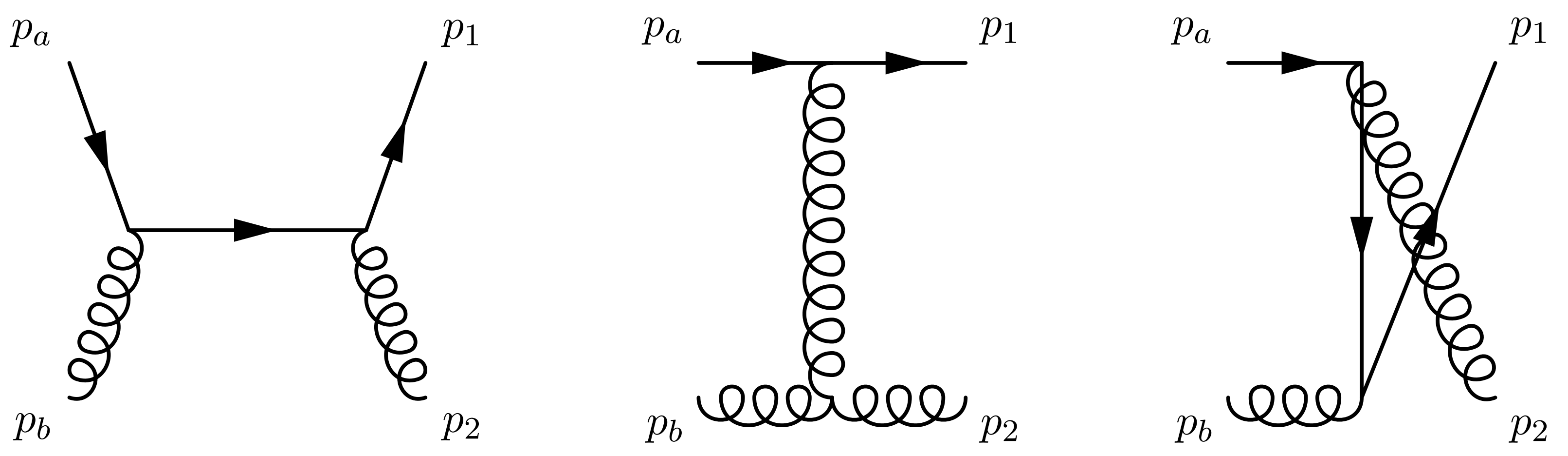
Local pieces, independent of the rest of the process

Applies to loop diagrams too, and generates leading logs in s_{ij}/p_T^2

Can use this simpler structure to make an efficient event generator for arbitrary numbers of quarks/gluons.

High Energy limit of amplitudes also many theory applications…

Examples of the HE Limit



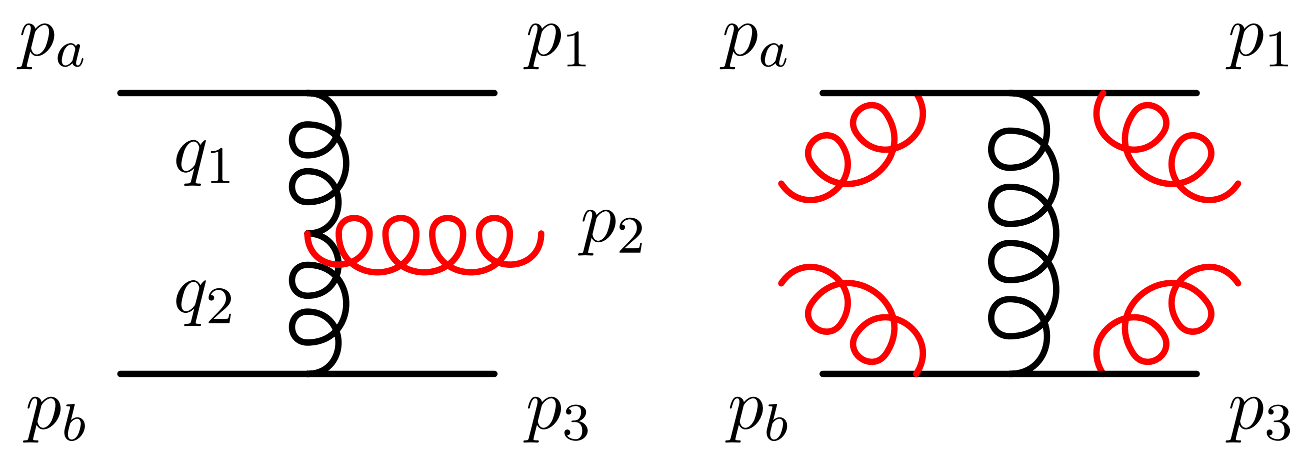
- Exact result:

$$\frac{g_s^4 C_{CAM}}{6}\frac{|j^\mu(p_a,p_1)\cdot j_\mu(p_b,p_2)|^2}{\hat{t}^2}$$

with

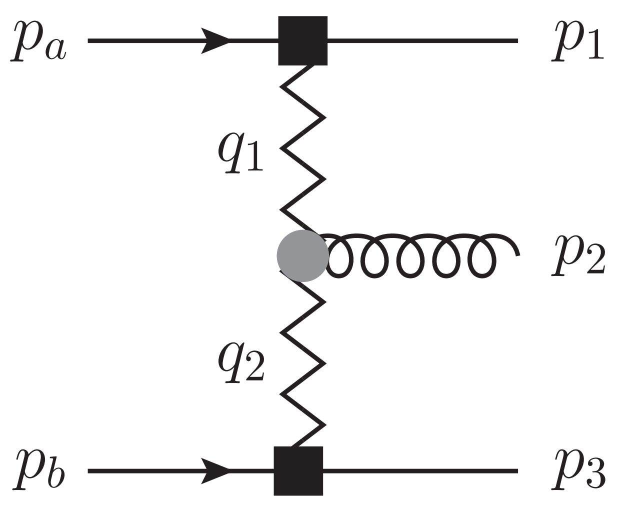
$$C_{CAM}=\frac{1}{2}\left(C_A-\frac{1}{C_A}\right)\left(\frac{p_b^-}{p_2^-}+\frac{p_2^-}{p_b^-}\right)+\frac{1}{C_A}$$

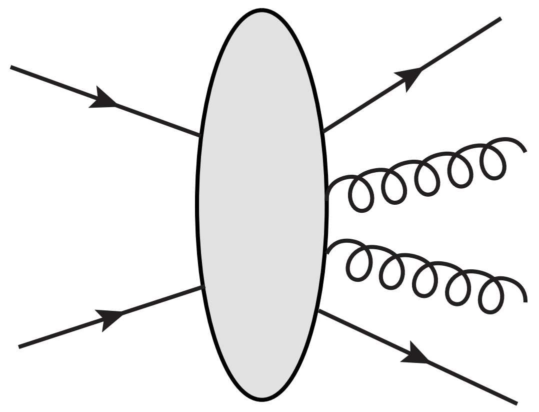
- Only t-pole remains explicitly



$$\mathcal{A}_{qQ\to qgQ}=g_s^3\mathcal{C}_g\varepsilon_\rho^*\frac{j^\mu(p_a,p_1)\cdot j_\mu(p_b,p_3)}{q_1^2q_2^2}V^\rho(q_1,q_2)$$

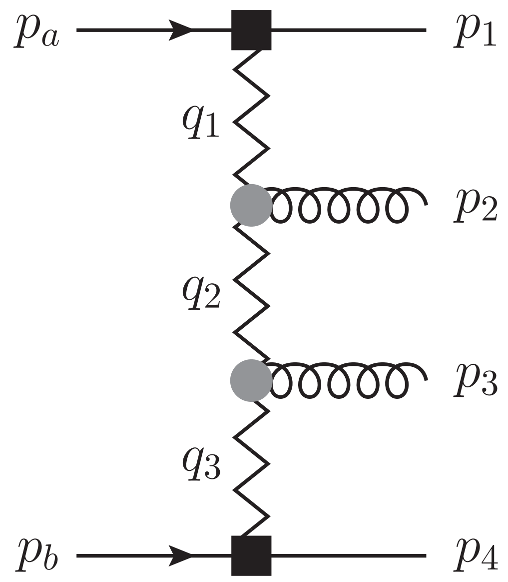
$$\begin{aligned}V^\rho(q_1,q_2)=&-(q_1+q_2)^\rho\\&+\frac{p_a^\rho}{2}\left(\frac{q_1^2}{p_2\cdot p_a}+\frac{p_2\cdot p_b}{p_a\cdot p_b}+\frac{p_2\cdot p_3}{p_a\cdot p_3}\right)+p_a\leftrightarrow p_1\\&-\frac{p_b^\rho}{2}\left(\frac{q_2^2}{p_2\cdot p_b}+\frac{p_2\cdot p_a}{p_b\cdot p_a}+\frac{p_2\cdot p_1}{p_b\cdot p_1}\right)-p_b\leftrightarrow p_3\end{aligned}$$





$$\mathcal{A}_{qQ\to qggQ}=g_s^4\mathcal{C}_g\mathcal{C}_{g'}\frac{j^\mu(p_a,p_1)\cdot j_\mu(p_b,p_4)}{q_1^2q_2^2q_3^2}\times\varepsilon_\rho^*V^\rho(q_1,q_2)\times\varepsilon_\sigma^*V^\sigma(q_2,q_3)$$

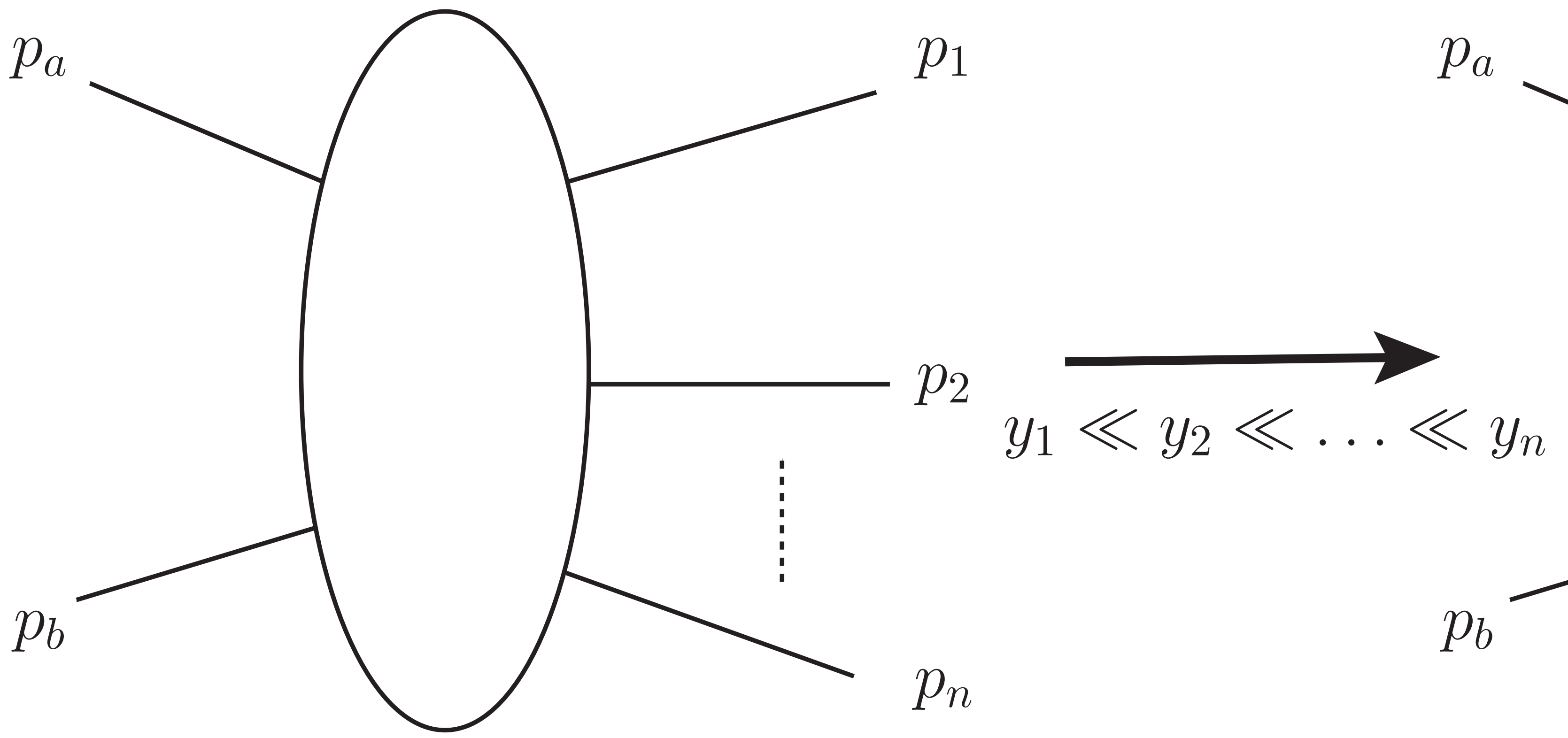
- Same functions $V^\rho(q_i,q_{i+1})$



Andersen & JMS arXiv:0908.2786, 0910.5113, Lipatov Sov. J. Nucl. Phys. 23:338-345, 1976

Regge Scaling of Amplitudes

Regge scaling dictates the scaling of the amplitude with s_{ij} for a given process

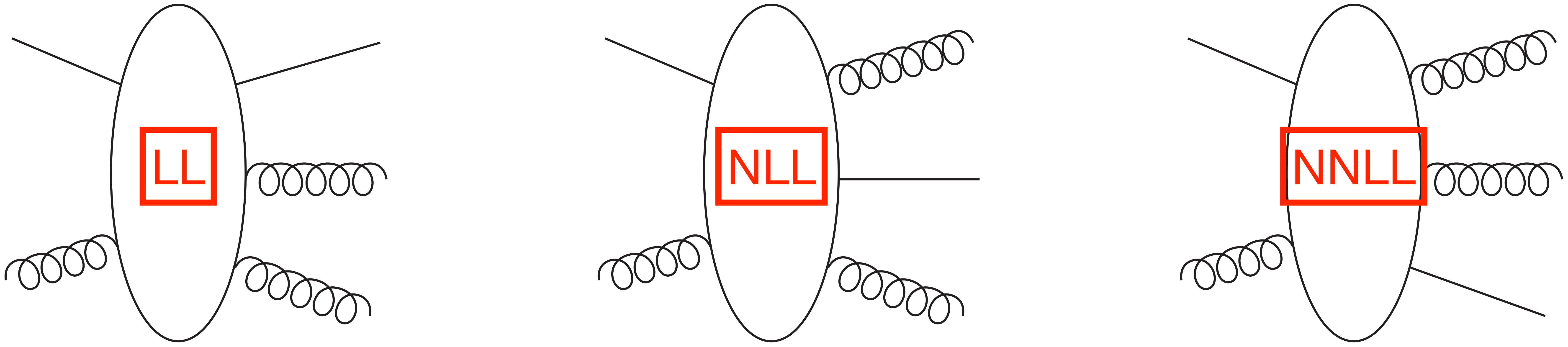


$$\mathcal{M} \propto s_{12}^{\alpha_{12}} \ldots s_{n-1,n}^{\alpha_{n-1,n}} \; f(\{p_{T,i}\})$$

where α_{ij} is the spin of that particle in effective t-channel

Brower, DeTar, Weis Phys Rept. 14:257, 1974

Powers of s_{ij} in the real matrix elements match powers of log in the inclusive matrix element.



Main Equations

Squared Matrix Element

$$\overline{\left|\mathcal{M}_{\text{HEJ}}^{\text{reg}}(\{p_i\})\right|}^2 = \frac{1}{4\,(N_C^2-1)}\;\left\|S_{f_1 f_2\to f_1 f_2}\right\|^2$$
$$\cdot\left(g^2\;K_{f_1}\;\frac{1}{t_1}\right)\cdot\left(g^2\;K_{f_2}\;\frac{1}{t_{n-1}}\right)$$
$$\cdot\prod_{i=1}^{n-2}\left(g^2 C_A\;\left(\frac{-1}{t_i t_{i+1}}V^\mu(q_i,q_{i+1})V_\mu(q_i,q_{i+1})-\frac{4}{\mathbf{p}_i^2}\;\theta\left(\mathbf{p}_i^2<\lambda^2\right)\right)\right)$$
$$\cdot\prod_{j=1}^{n-1}\exp\left[\omega^0(q_j,\lambda)(y_j-y_{j+1})\right],$$

Andersen & JMS arXiv:0908.2786, 0910.5113

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$$\cdot \prod_{j=1}^{n-1} \exp\left[\omega^0(q_j,\lambda)(y_j-y_{j+1})\right],$$

Skeleton/Born Process

Resolved Real Emissions

Virtual + Unresolved Real (finite)

Andersen & JMS arXiv:0908.2786, 0910.5113

Main Equations

Squared Matrix Element

$$\overline{\left|\mathcal{M}^{\rm reg}_{\rm HEJ}(\{p_i\})\right|}^2 = \frac{1}{4\,(N_C^2-1)}\,\|S_{f_1 f_2\to f_1 f_2}\|^2 \cdot \left(g^2\,K_{f_1}\,\frac{1}{t_1}\right)\cdot\left(g^2\,K_{f_2}\,\frac{1}{t_{n-1}}\right) \cdot \prod_{i=1}^{n-2}\left(g^2 C_A\left(\frac{1}{t_i t_{i+1}}V^\mu(q_i,q_{i+1})V_\mu(q_i,q_{i+1}) - \frac{4}{\mathbf{p}_i^2}\,\theta\left(\mathbf{p}_i^2<\lambda^2\right)\right)\right) \cdot \prod_{j=1}^{n-1}\exp\left[\omega^0(q_j,\lambda)(y_j-y_{j+1})\right],$$

Skeleton/Born Process

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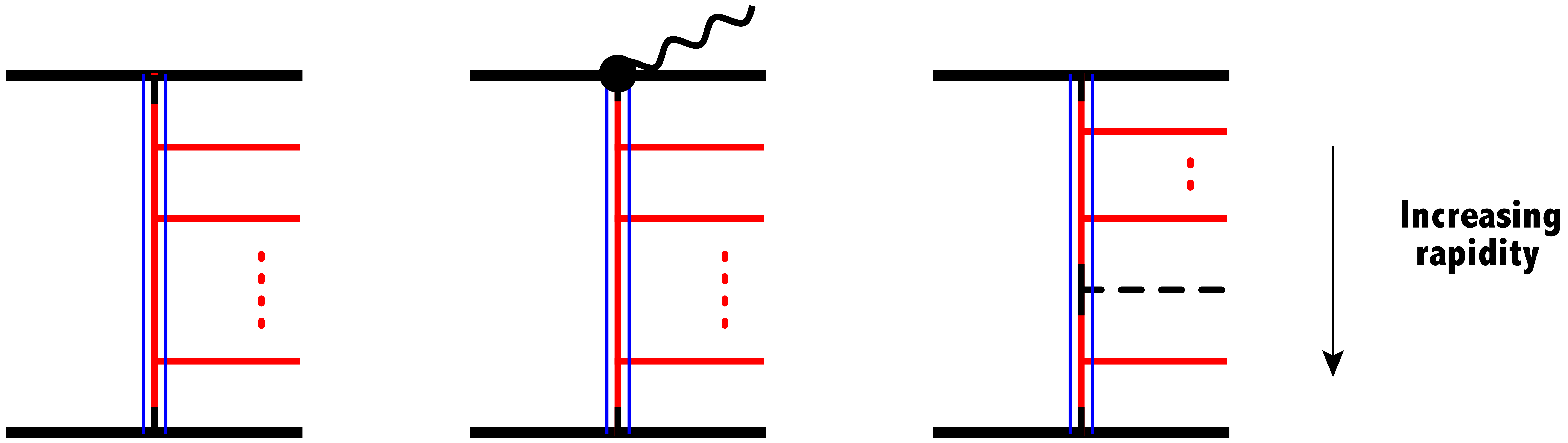
Black = Skeleton/Born function

Red = Range of resummation

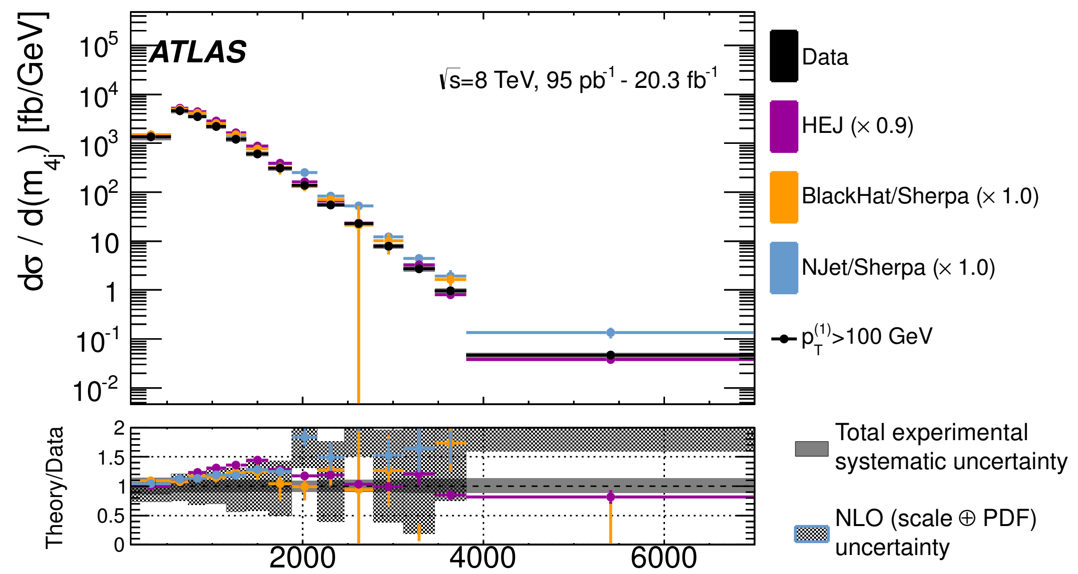


$\geq 2j,$ $\quad W, Z/\gamma^* + \geq 2j,$ $\quad H + \geq 2j$

"HEJ 2"

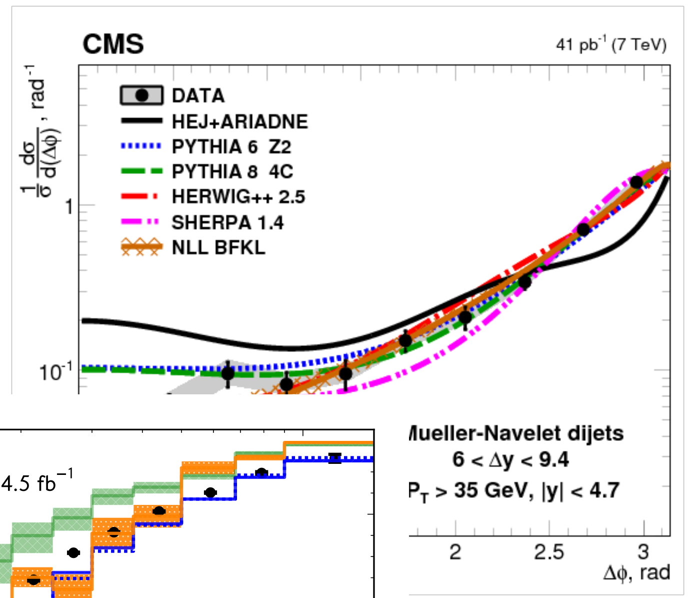
The High Energy Jets (HEJ) framework is

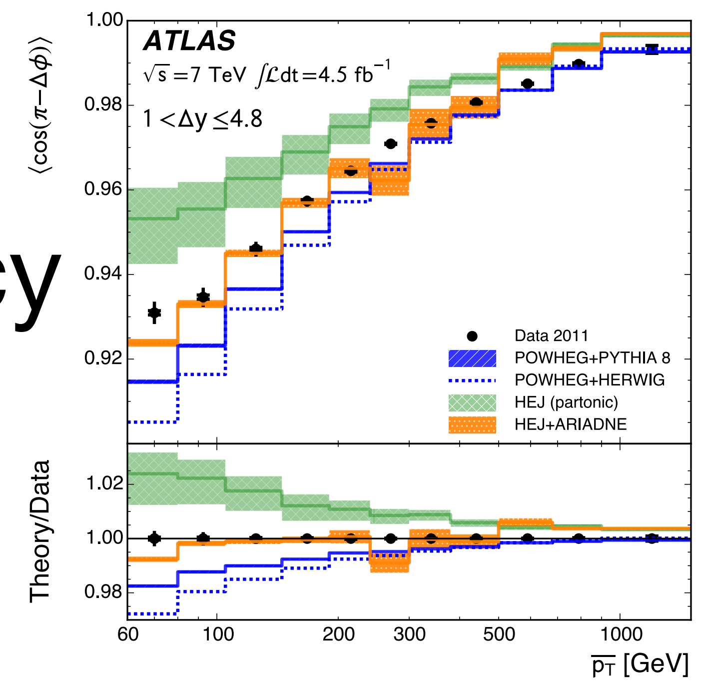
- exact for simple processes (2 to 2 (+X))
- accurate to leading logarithm in s/t
- constructed event-by-event
- takes LO samples as input
- gauge invariant in all phase space
- sufficiently fast for numerical integration (up to 30 gluons)



HEJ2 event generator: https://hej.hepforge.org

Andersen, Hapola, Heil, Maier & JMS arXiv:1902.08430



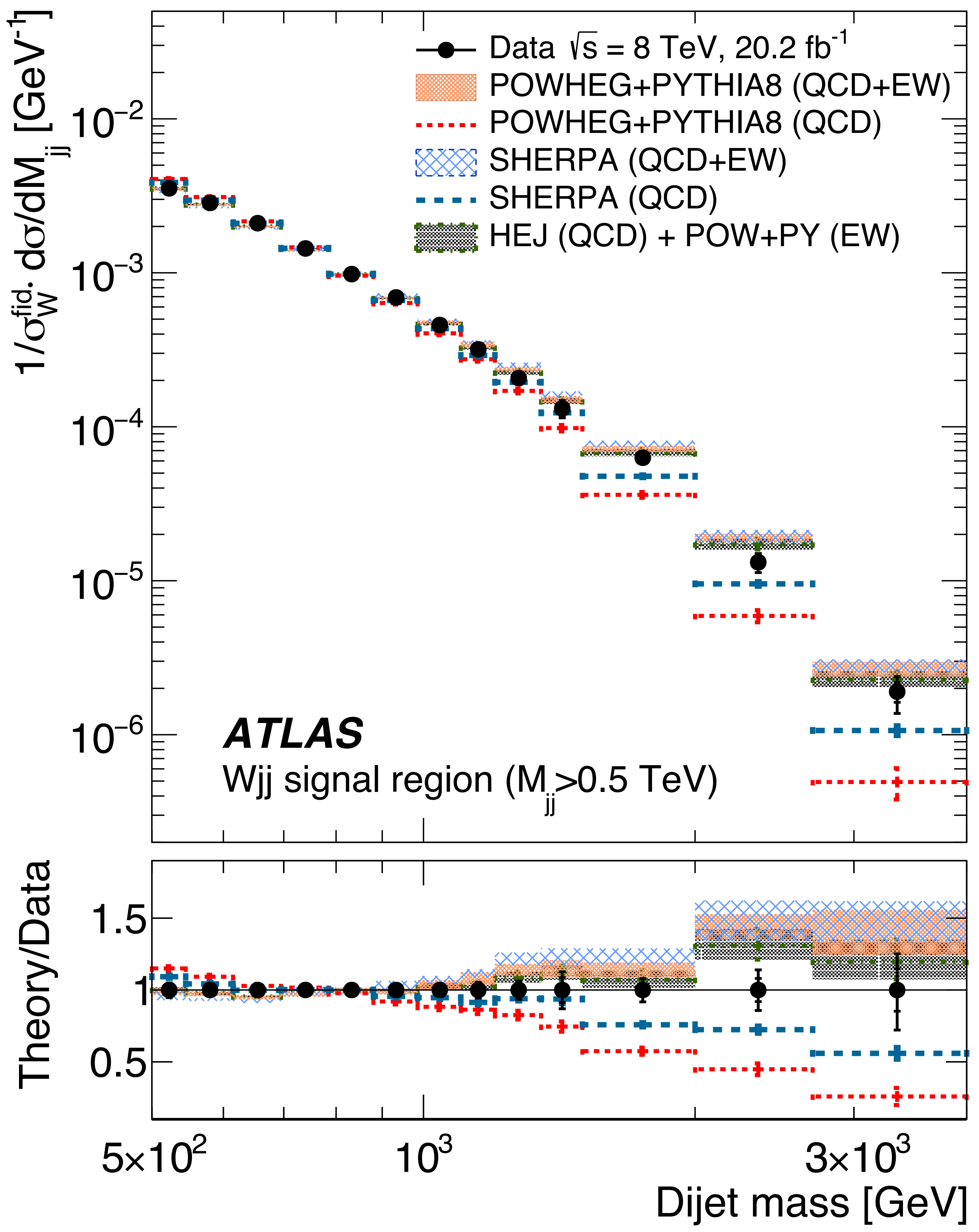


Extra colour-neutral bosons can be added without affecting the logarithmic accuracy

HEJ2.1 includes: $\geq 2j, H+\geq 2j, W(\to \ell\nu)+\geq 2j, Z/\gamma^*(\to \ell\bar{\ell})+\geq 2j$

Andersen, Black, Brooks, Ducloué, Heil, Maier & JMS arXiv:2110.15692

2017 ATLAS W+2j



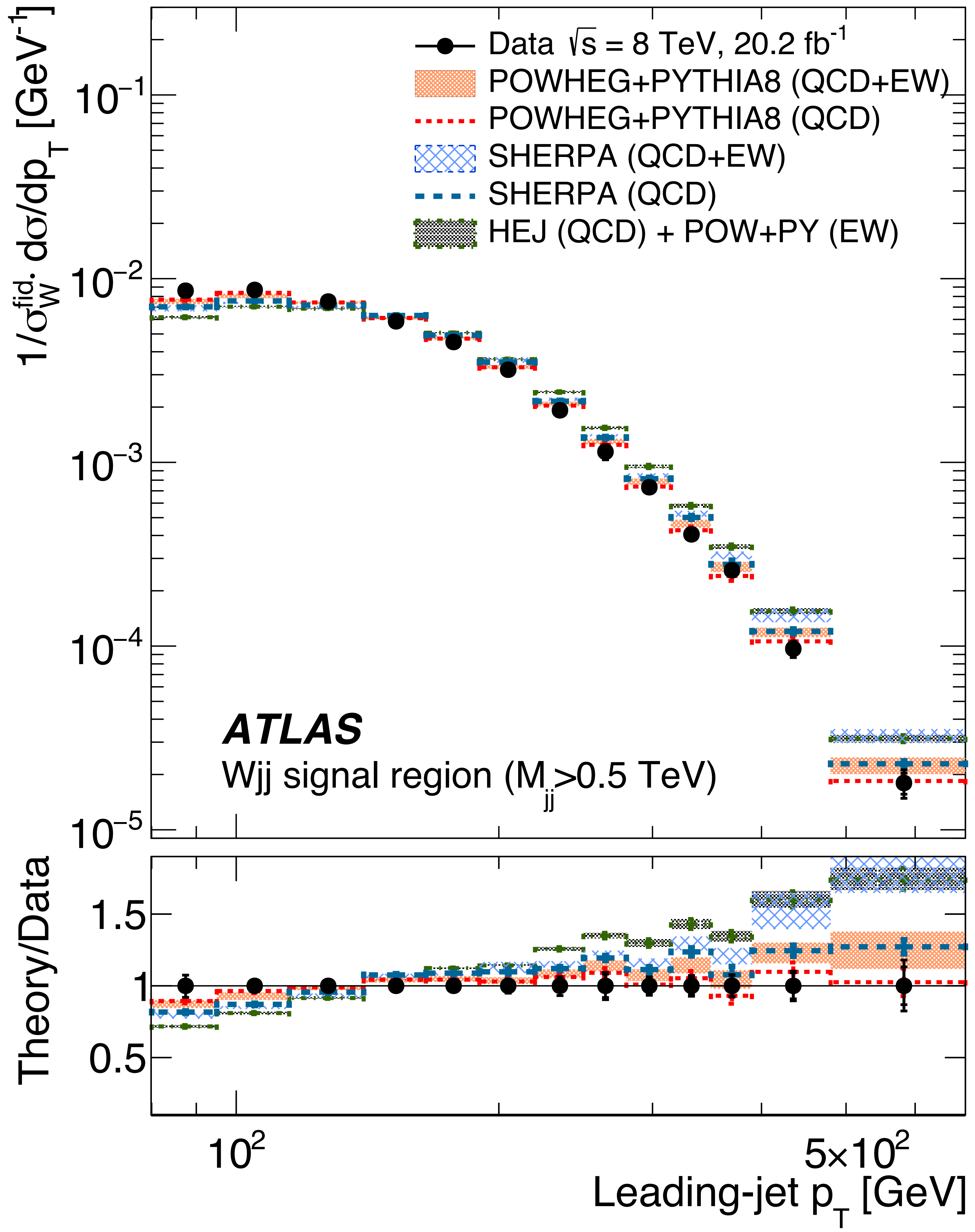
W+2j study to investigate separation of QCD/EW contributions compared to NLO+PS (Powheg/Sherpa) and HEJ+EW from Powheg

- 8 TeV data probing out to 3 TeV already
- QCD contribution decreases at large dijet mass, but remains significant
- NLO+PS slightly overshoot, and increasing

Similar for e.g. delta-y

ATLAS arXiv:1703.04362

2017 ATLAS W+2j

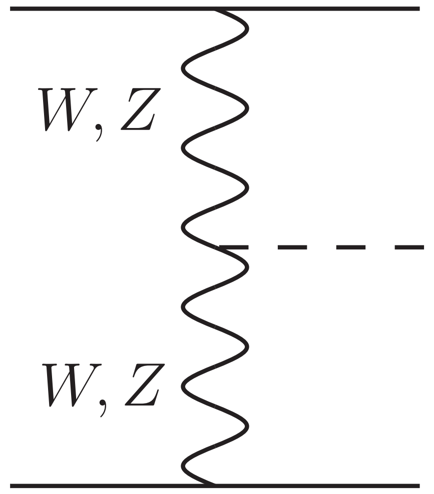


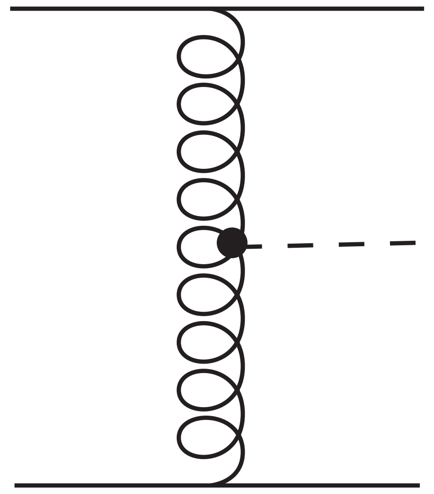
- Different picture when plotted versus p_T as no systematic evolution in pT in HEJ.
- QCD contribution no longer suppressed compared to EW
- Subleading corrections and NLO matching improve this (see later)
- Can also combine with a parton shower

Andersen, Brooks & Lönnblad arXiv:1712.00178
Andersen, Hassan, Jaskiewicz arXiv:2210.06898

ATLAS arXiv:1703.04362

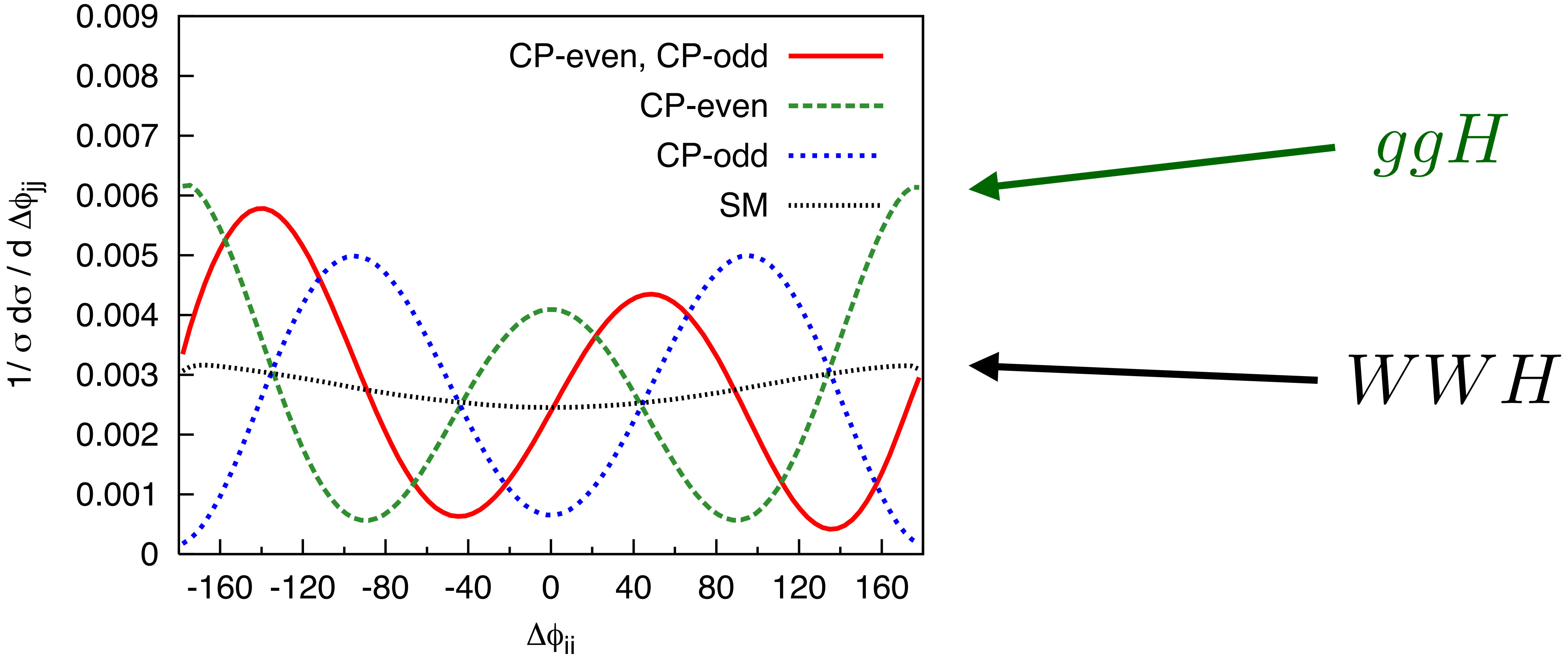
Higgs + Dijets





Higgs boson looks like SM so far, but critical to check CP structure of couplings to bosons

Azimuthal angle between the dijets is sensitive to this
Figy et al hep-ph/0609075



Use distinctive event shape to separate channels with "VBF cuts"

$$\text{e.g.}\quad \Delta y_{jj} > 2.8,\ m_{jj} > 400\ \text{GeV}$$

BUT this precisely enhances higher orders in pert. expansion

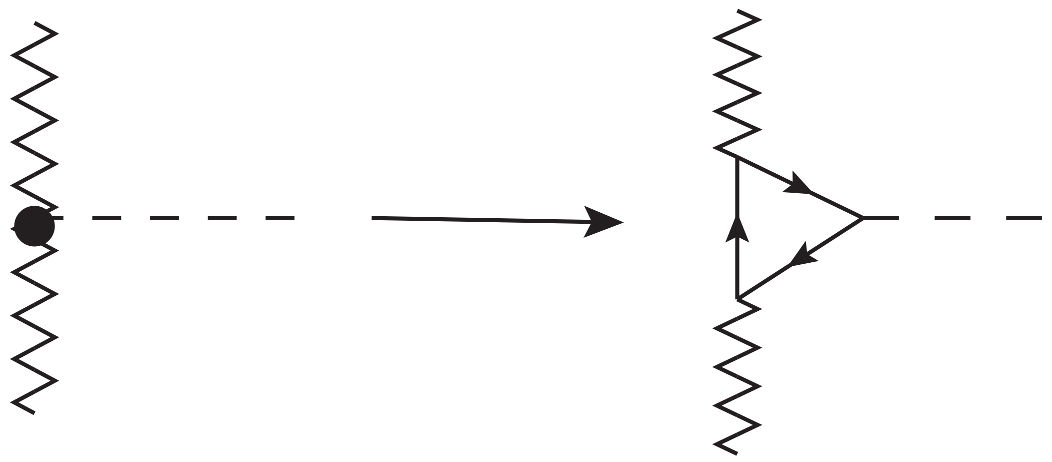
Finite Quark Masses in H+2j

Fixed-order stalled for full quark mass effects because LO = 1-loop.
LO results only for 2 and 3 jets (no NLO for 2j+)

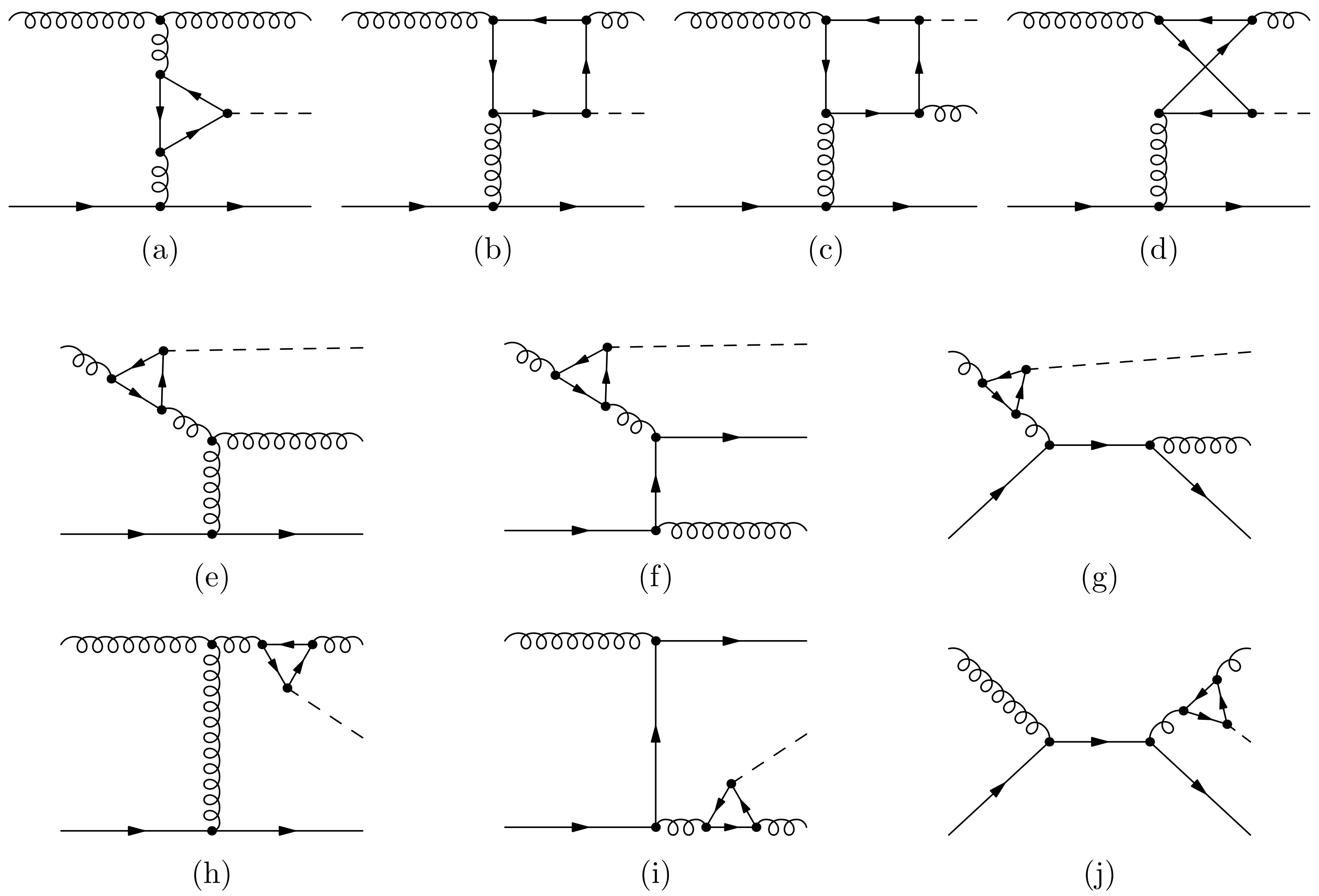
Del Duca et al hep-ph/0105129, hep-ph/0108030
Greiner et al arXiv:1608.01195

In HEJ, factorised structure removes complexity from increasing number of jets

Del Duca, Kilgore, Oleari, Schmidt & Zeppenfeld hep-ph/0301013
Andersen, Cockburn, Heil, Maier & JMS arXiv:1812.08072



Straight-forward
e.g. $qQ \to qHQ$



Outer Higgs more involved but calculated

Finite Quark Masses in HEJ

HEJ can include finite quark mass and loop propagator effects for *any* number of jets

Performed at amplitude level so we include mass effects from top quark, bottom quark *and* the interference between the two

Fixed-order matching performed to highest-available accuracy

Here use Sherpa and OpenLoops

Gleisberg et al arXiv:0811.4622; Cascioli, Maierhöfer, Pozzorini arXiv:1111.5206

Highest available =

finite m_t $H+2j$ at LO ($3j$ results exist, but events not available)

infinite m_Q $H+2j$ at NLO

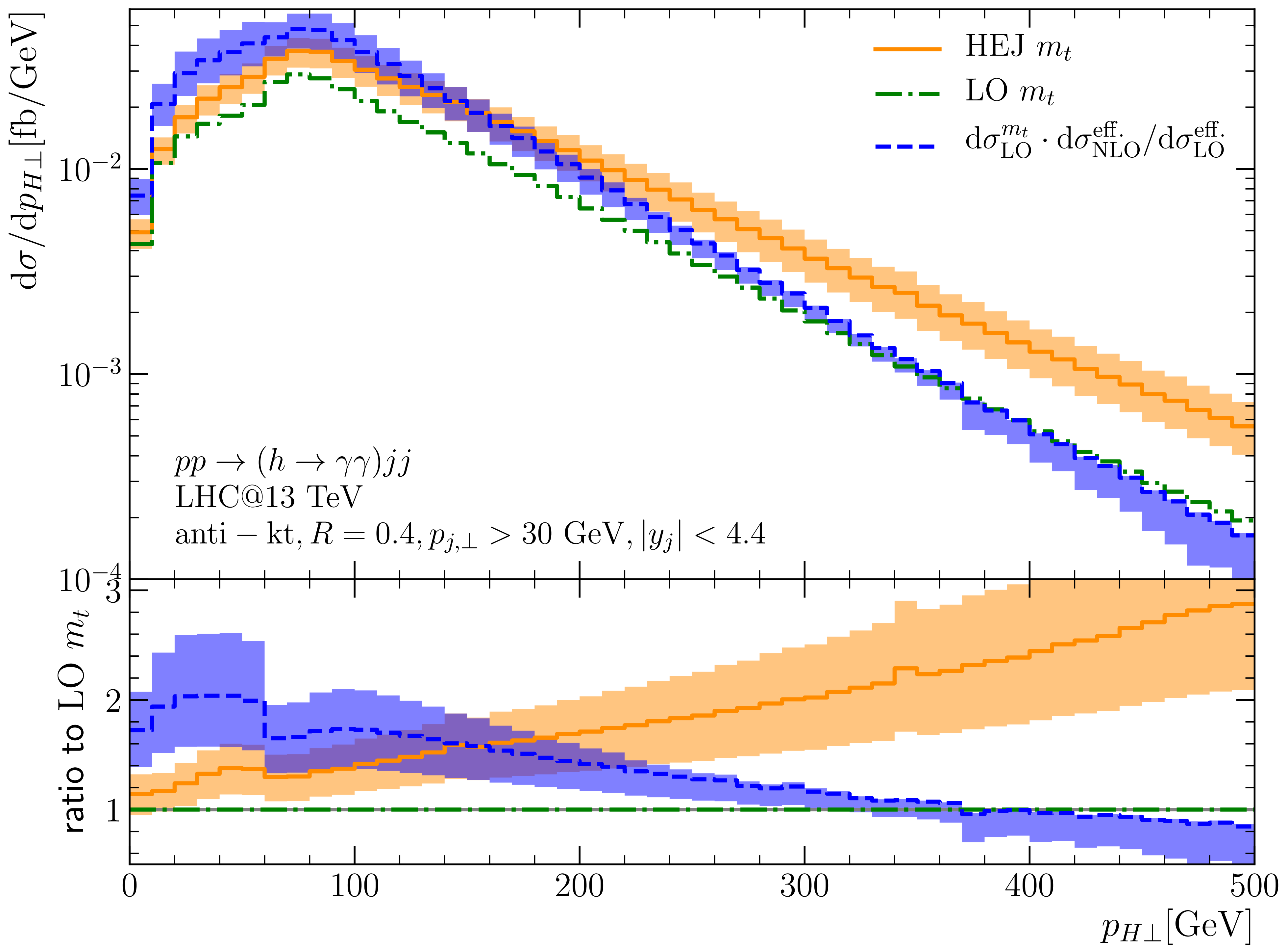
$H+5j$ at LO

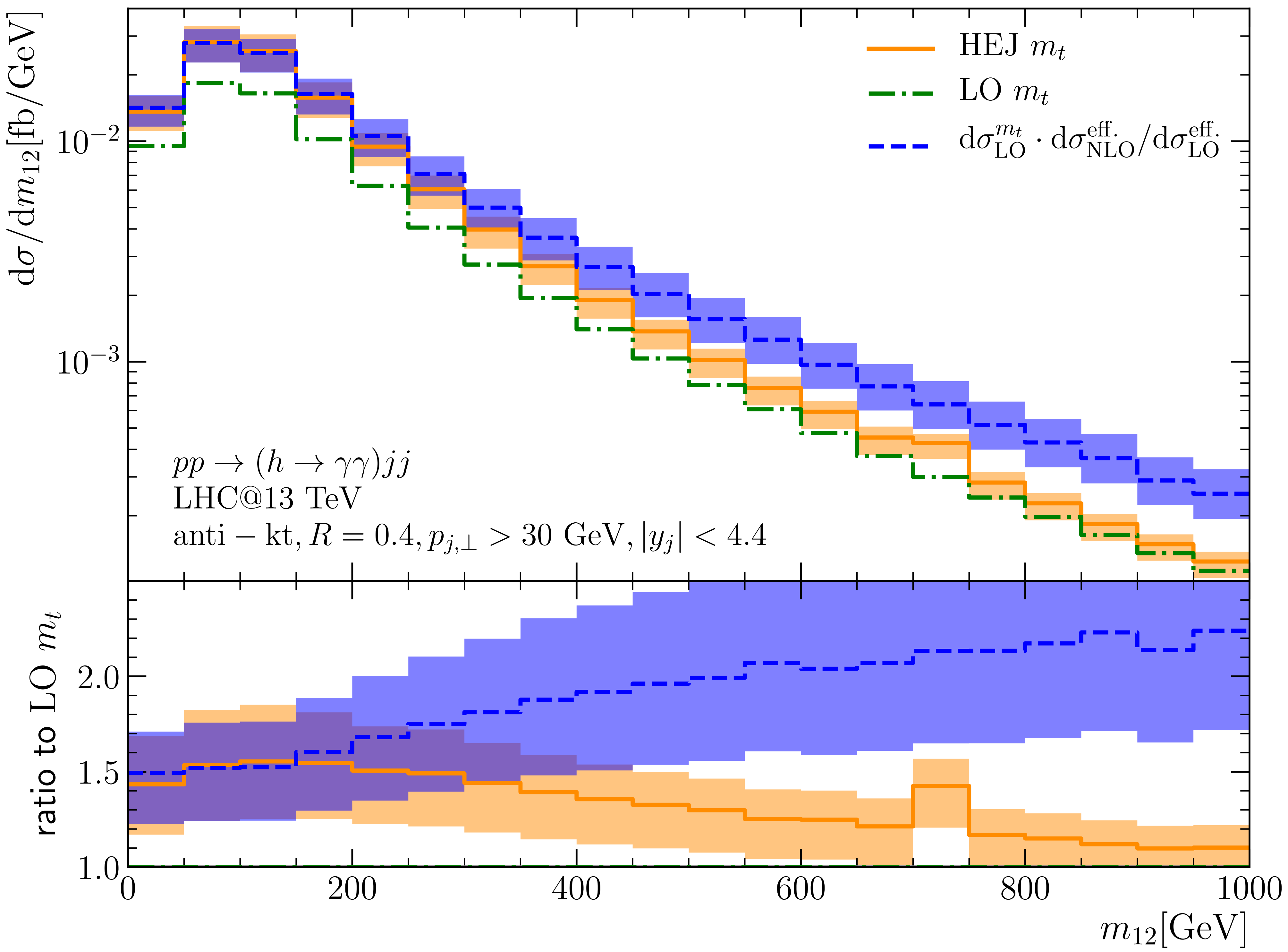
All predictions shown with $\mu_F=\mu_R=\max(m_H,m_{12})$ with indt variations by 1/2,2

Finite Quark Mass Results

First probe the impact of higher orders in α_s

HEJ here temporarily without m_b





NLO K-factors clearly not flat, very scale-dependent, all choices have problems

HEJ harder $p_{H\perp}$ spectrum

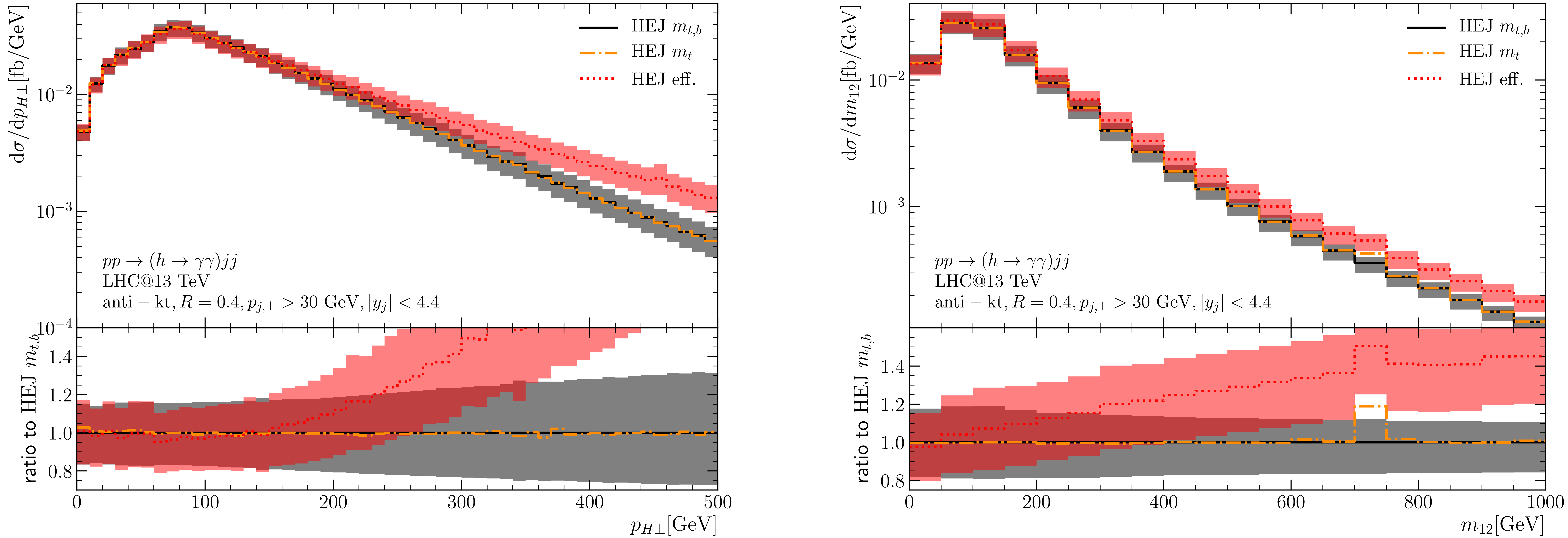
HEJ much steeper drop with m_{12}

Andersen, Cockburn, Heil, Maier & JMS arXiv:1812.08072

Finite Quark Mass Results

Now probe the impact of quark masses

Andersen, Cockburn, Heil, Maier & JMS arXiv:1812.08072



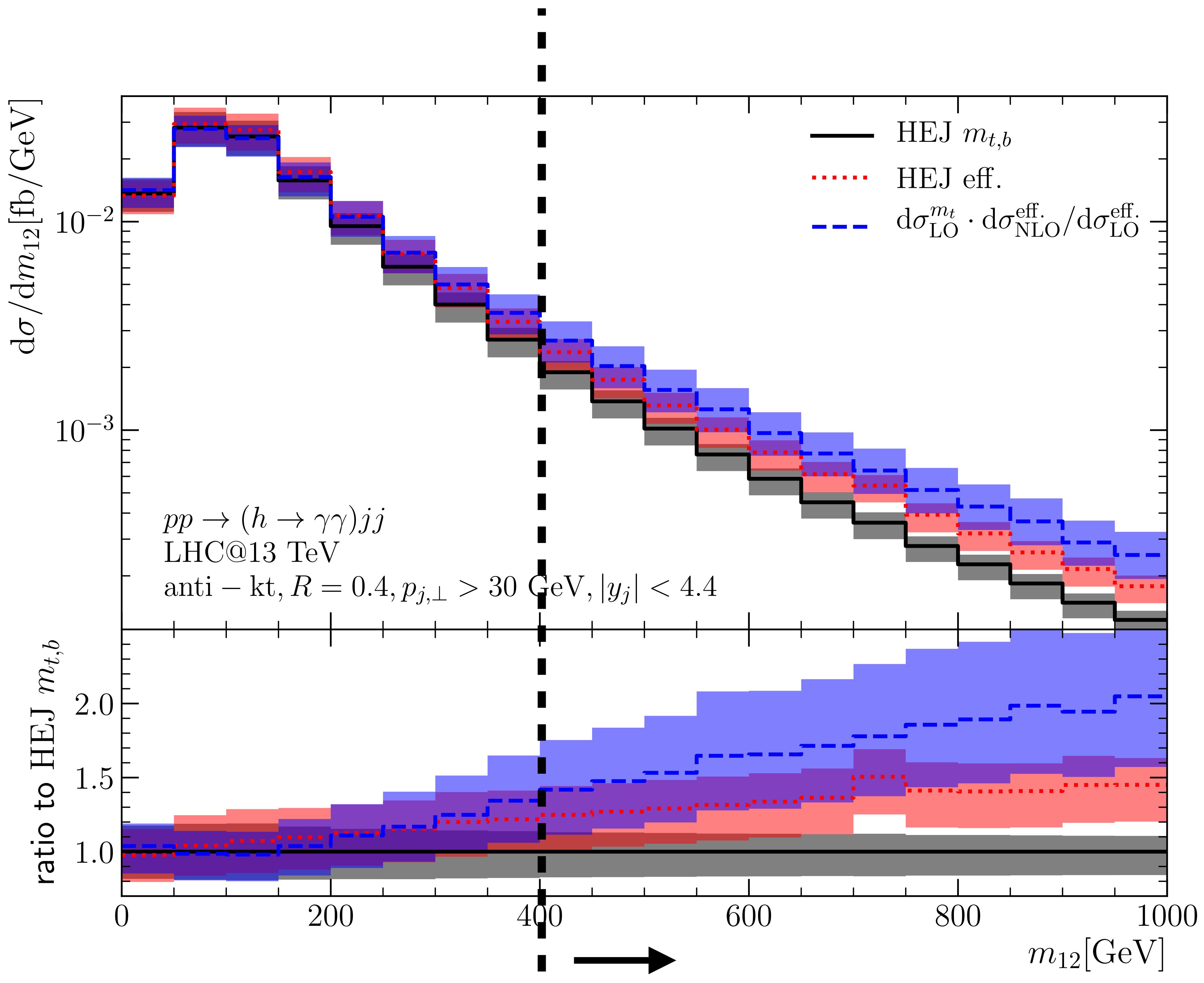
- Importance of finite quark mass increases with $p_{H\perp}$
- Relatively small impact of m_b, finite m_t lowers predictions at large m_{12}
- Therefore finite quark mass effects make VBF cuts more effective

Full HEJ Prediction vs “best” fixed-order

- Resummation alone reduces cross section at large values of m_{12}

- Finite quark mass/loop effects reduce x-section in VBF cuts by *further* 11%

Prediction	xs after VBF cuts
Fixed order	9%
HEJ	4%



Typical VBF cut

Andersen, Cockburn, Heil, Maier, JMS arXiv:1812.08072

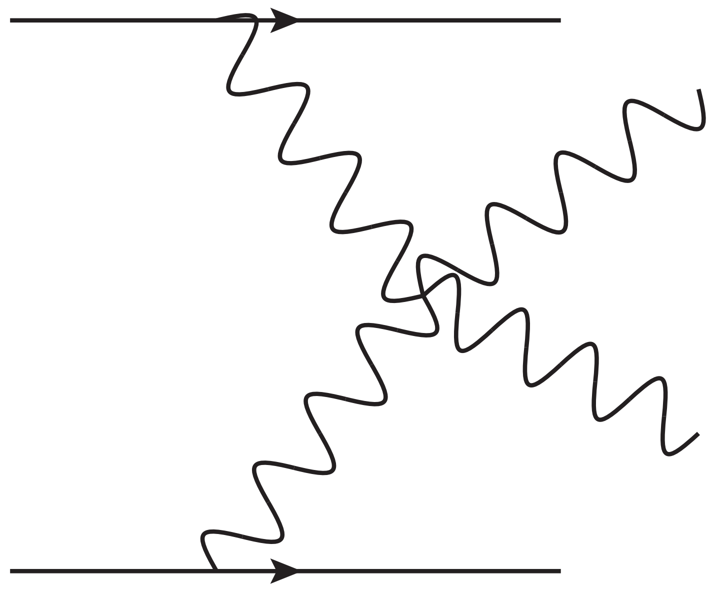
Vector Boson Scattering

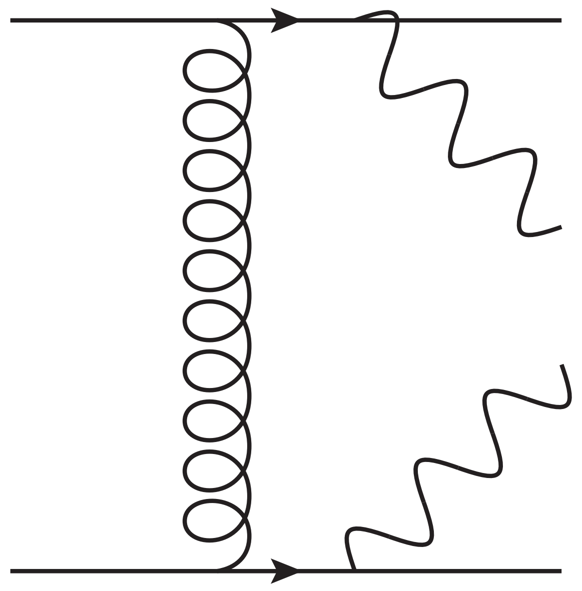
New in HEJ 2.2!

Vector Boson Scattering (VBS) sensitive probe of EWSB

$pp \to W^+W^+jj$ proceeds through various diagrams including

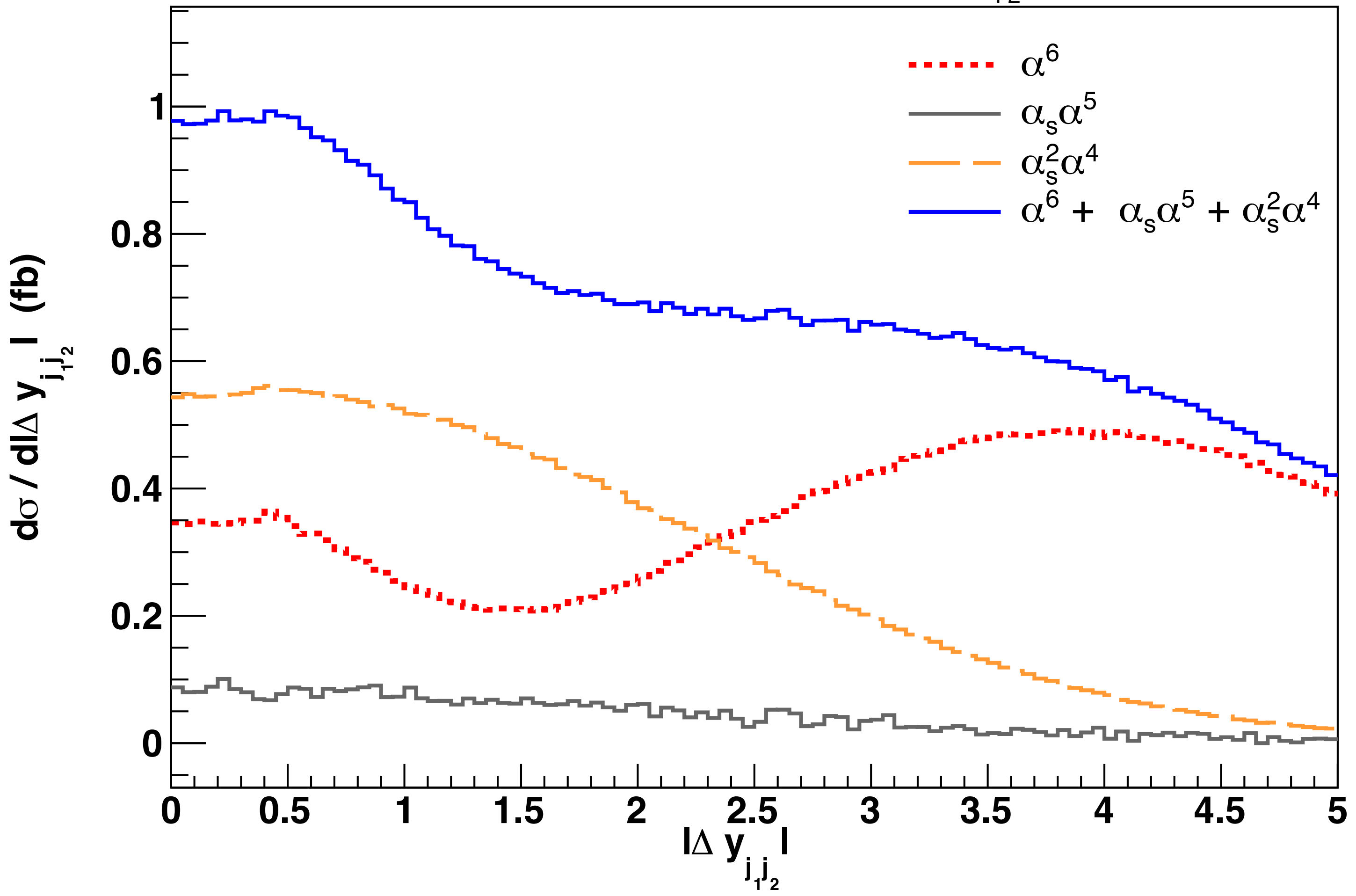
$$\mathrm{EW} = O(\alpha_W^4)$$





$$\mathrm{QCD} = O(\alpha_W^2 \alpha_s^2)$$

Inclusive study at LO: $d\sigma / d|\Delta y_{j_1 j_2}|$ (fb)



Ballestrero et al arXiv:1803.07943

We would like to separate the EW and QCD channels, justified by assessing interference between the two to be small

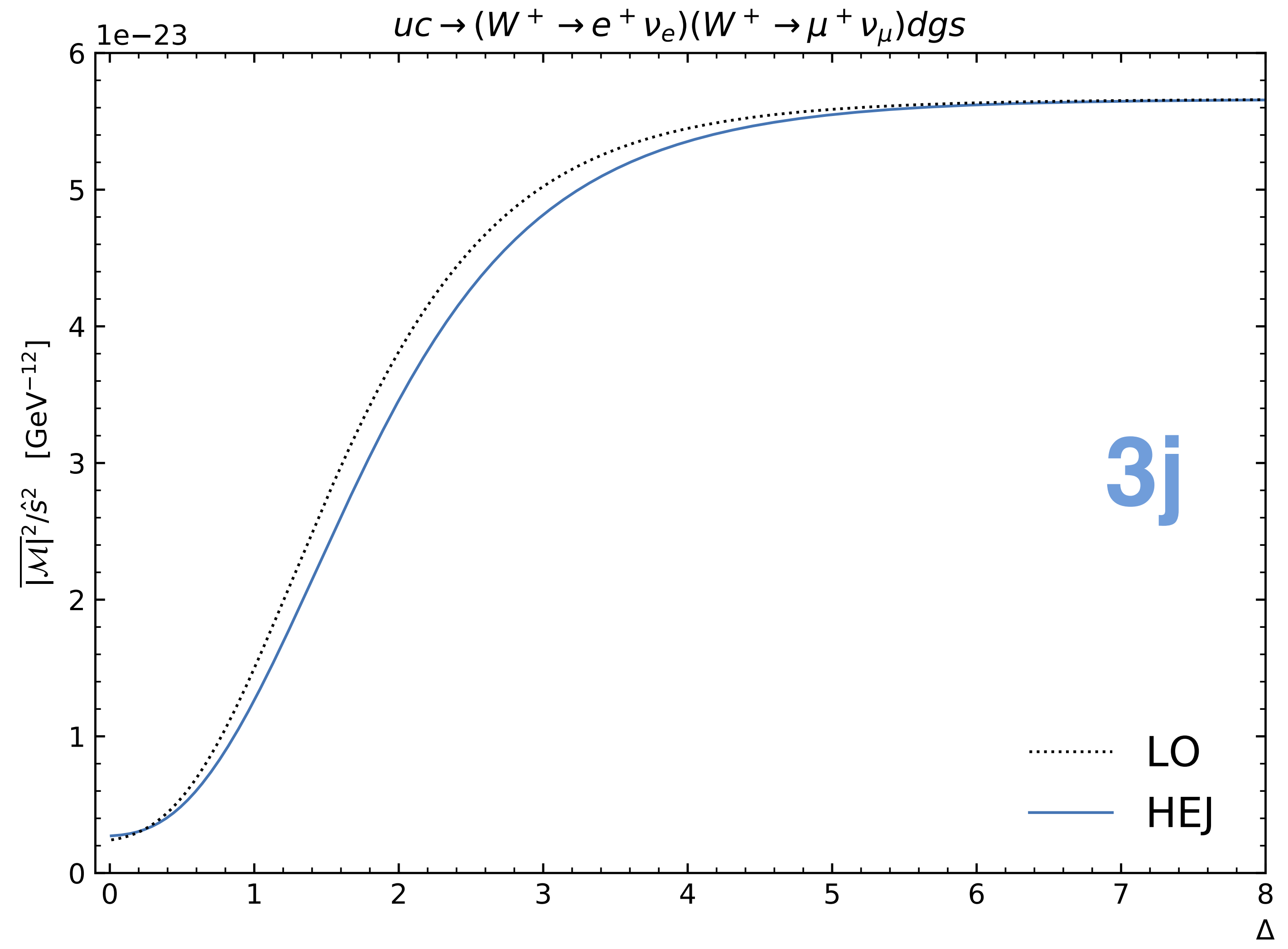
To isolate EW component, typically apply VBS cuts of large rapidity and/or large invariant mass on the jets.

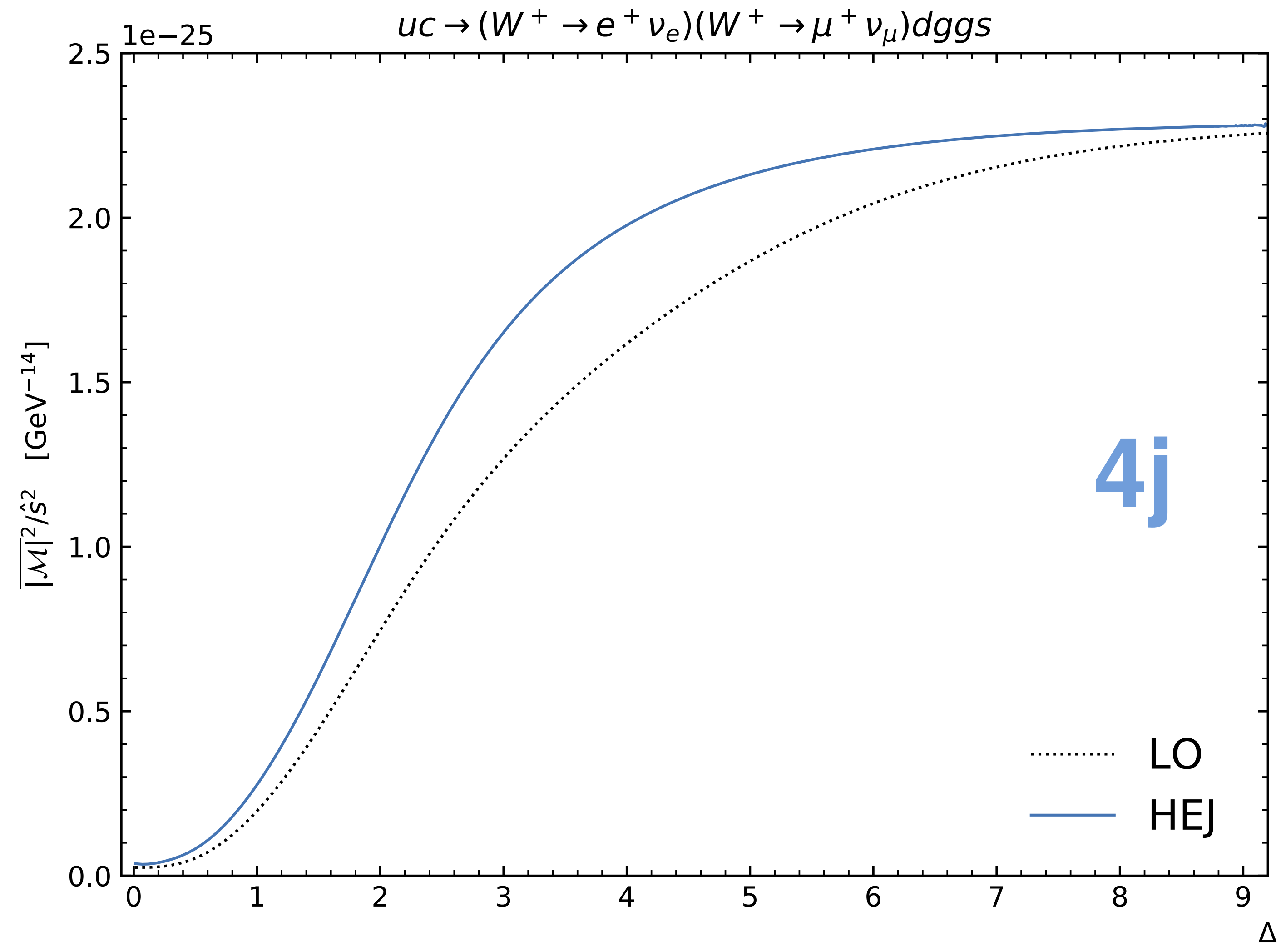
Very similar to $pp \to Hjj$

Vector Boson Scattering

Now included in HEJ, where LL effects obtained by combining two single-W pieces

PLUS interference





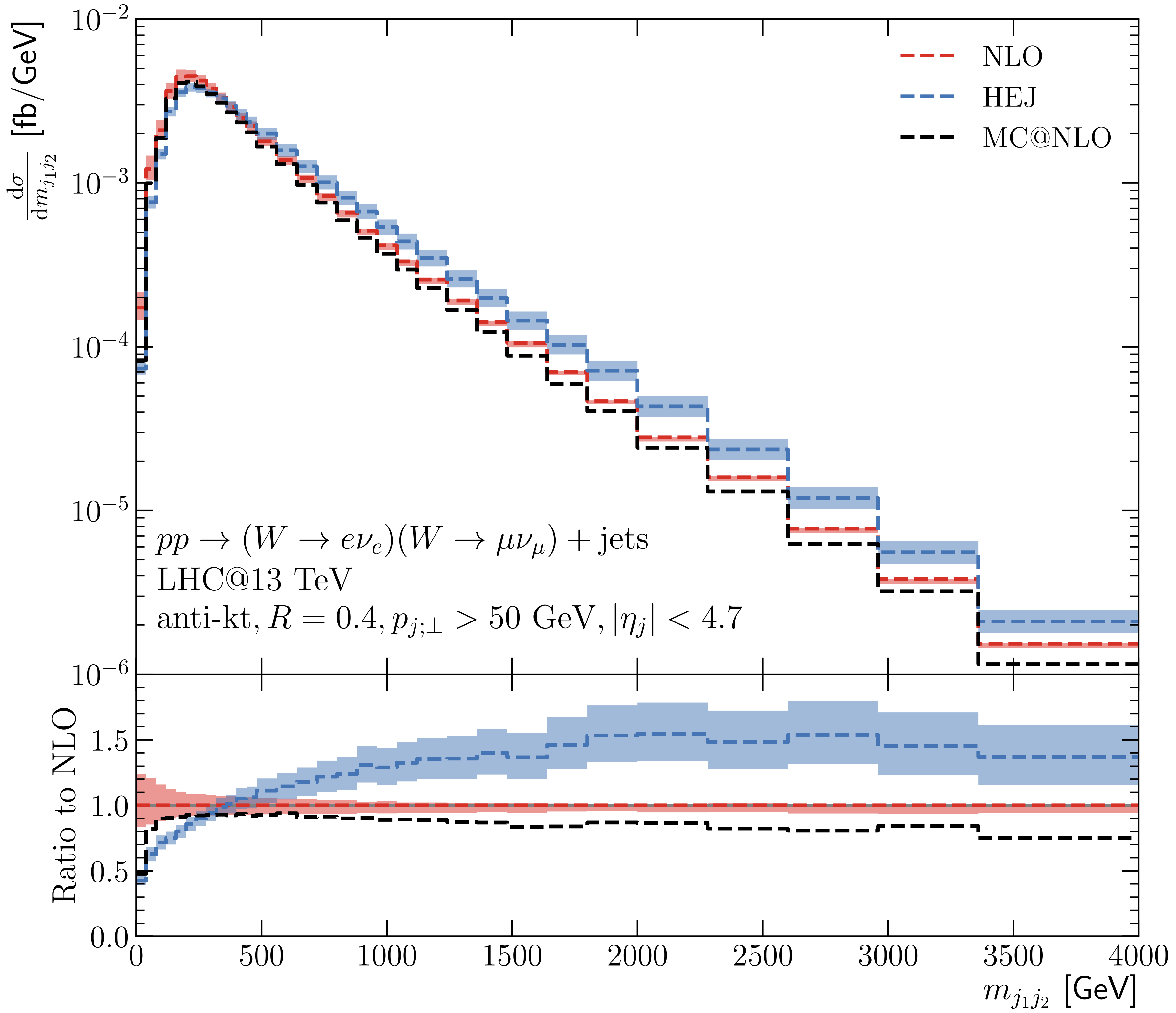
Impact on cross sections much reduced here (central scale choice)

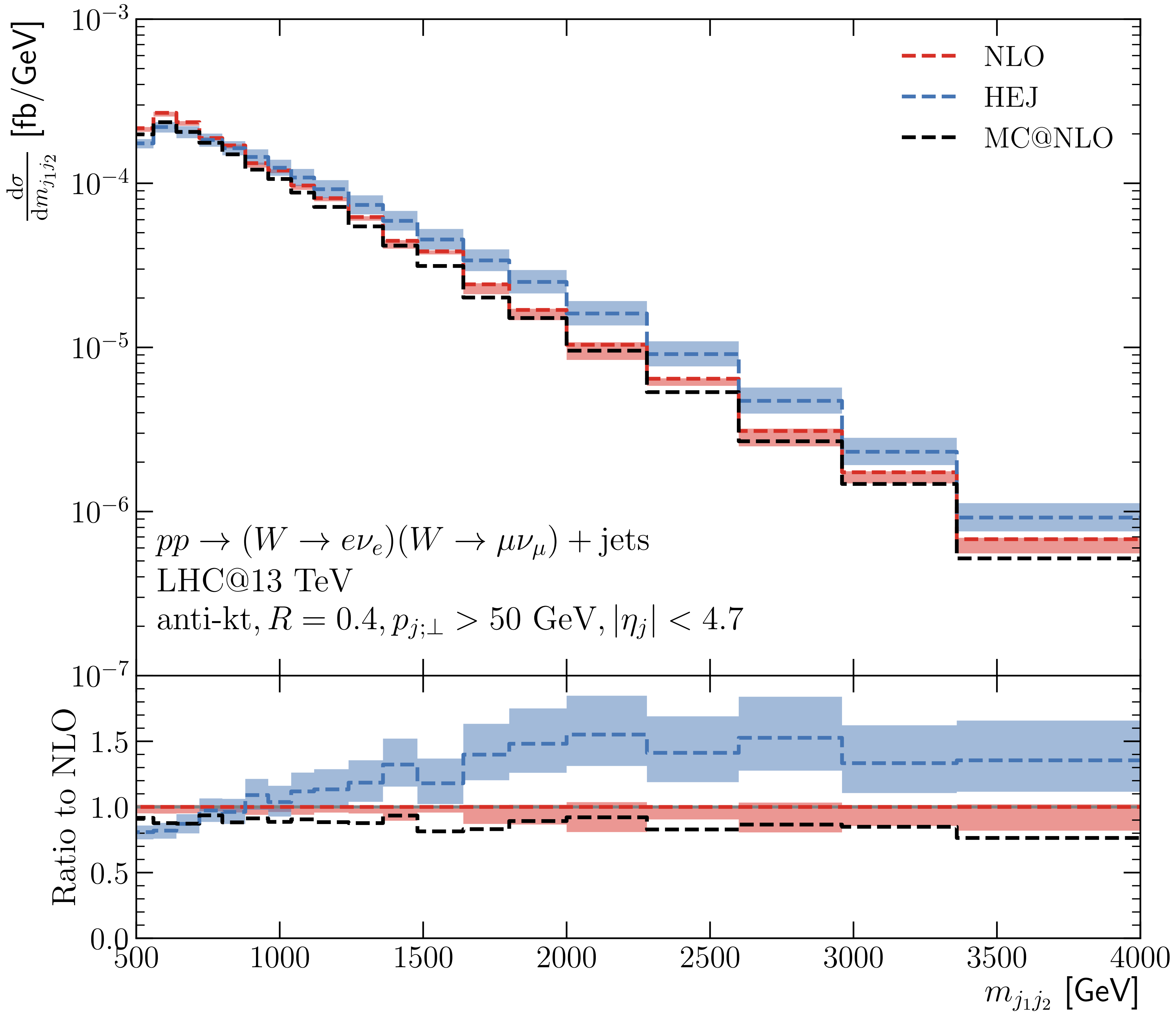
Cross Section (fb)	*without* VBS cuts, σ_{incl}	*with* VBS cuts, σ_{VBS}	$\sigma_{\text{VBS}}/\sigma_{\text{incl}}$
HEJ2 W^+W^+	1.428 ± 0.002	0.1219 ± 0.0004	0.0854 ± 0.0003
NLO W^+W^+	1.41 ± 0.05	0.12 ± 0.07	0.08 ± 0.02
HEJ2 W^-W^-	0.6586 ± 0.0003	0.0402 ± 0.0001	0.0610 ± 0.0002
NLO W^-W^-	0.68 ± 0.02	0.04 ± 0.01	0.06 ± 0.02

Andersen, Ducloué, Elrick, Nail, Maier, JMS arXiv:2107.06818

Vector Boson Scattering — Distributions

Andersen, Ducloué, Elrick, Nail, Maier, JMS arXiv:2107.06818

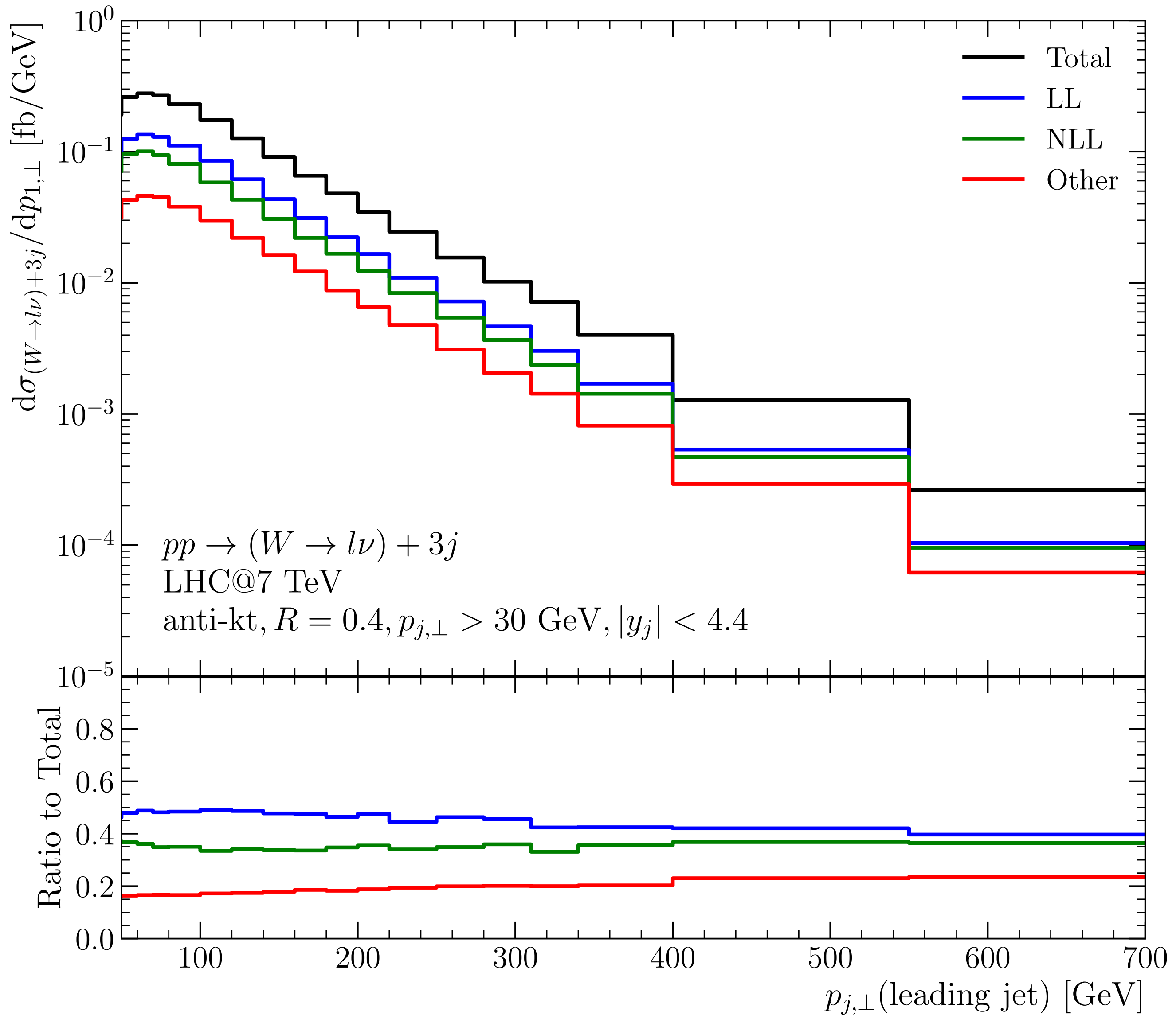




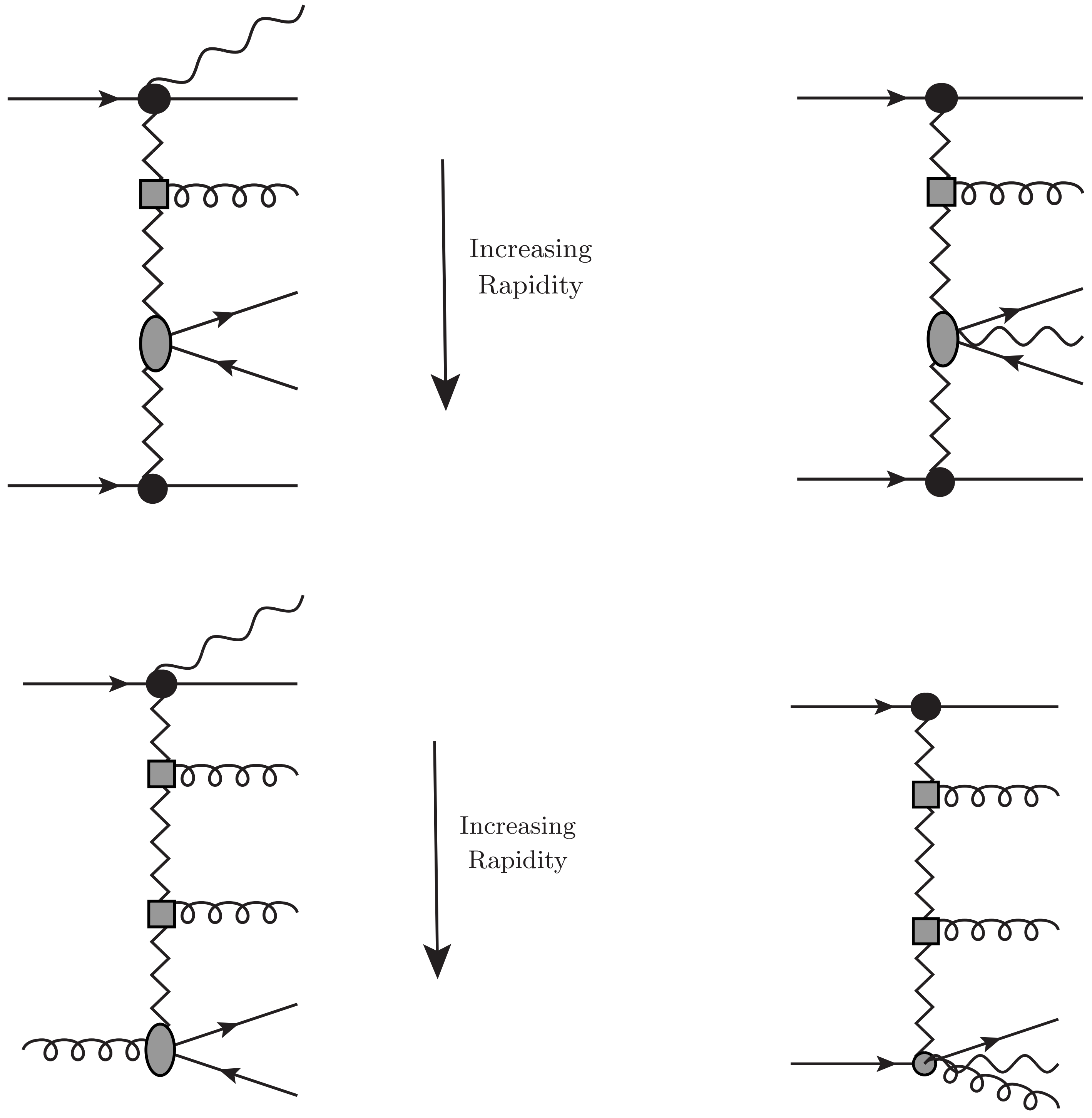
Shape of distributions changed by the all-order resummation

Agreement of total cross sections is a cancellation across phase space, not agreement throughout

Improvements at Large p_T: NLL



Observed that particle channels which are formally next-to-leading log, contribute significantly at large p_T



Can consistently apply resummation to all such channels (part of full NLL, and step towards it)

Andersen, Black, Brooks, Byrne, Maier, JMS arXiv:2012.10310

Improvements at Large p_T: NLO Matching

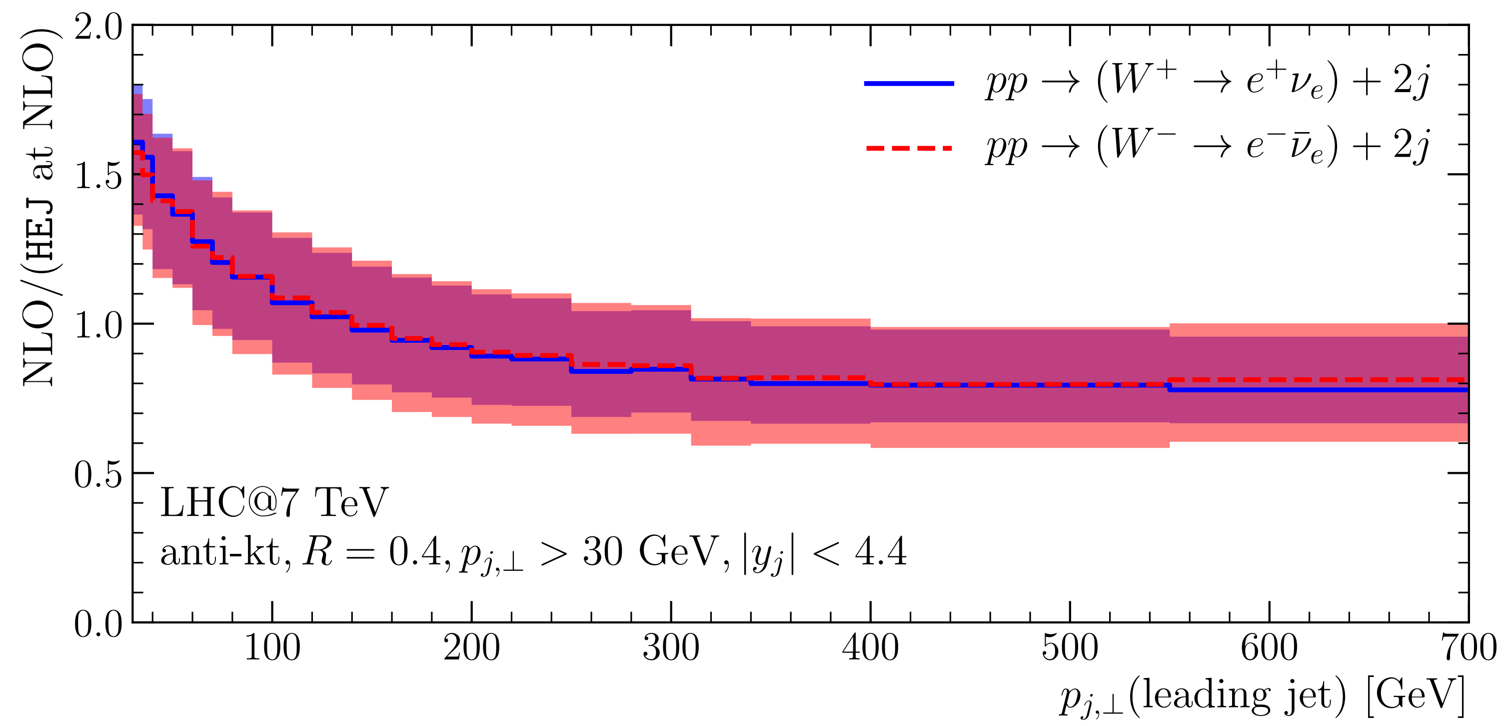
Not able yet to match to NLO event-by-event, but can do better than a k-factor by matching bin-by-bin

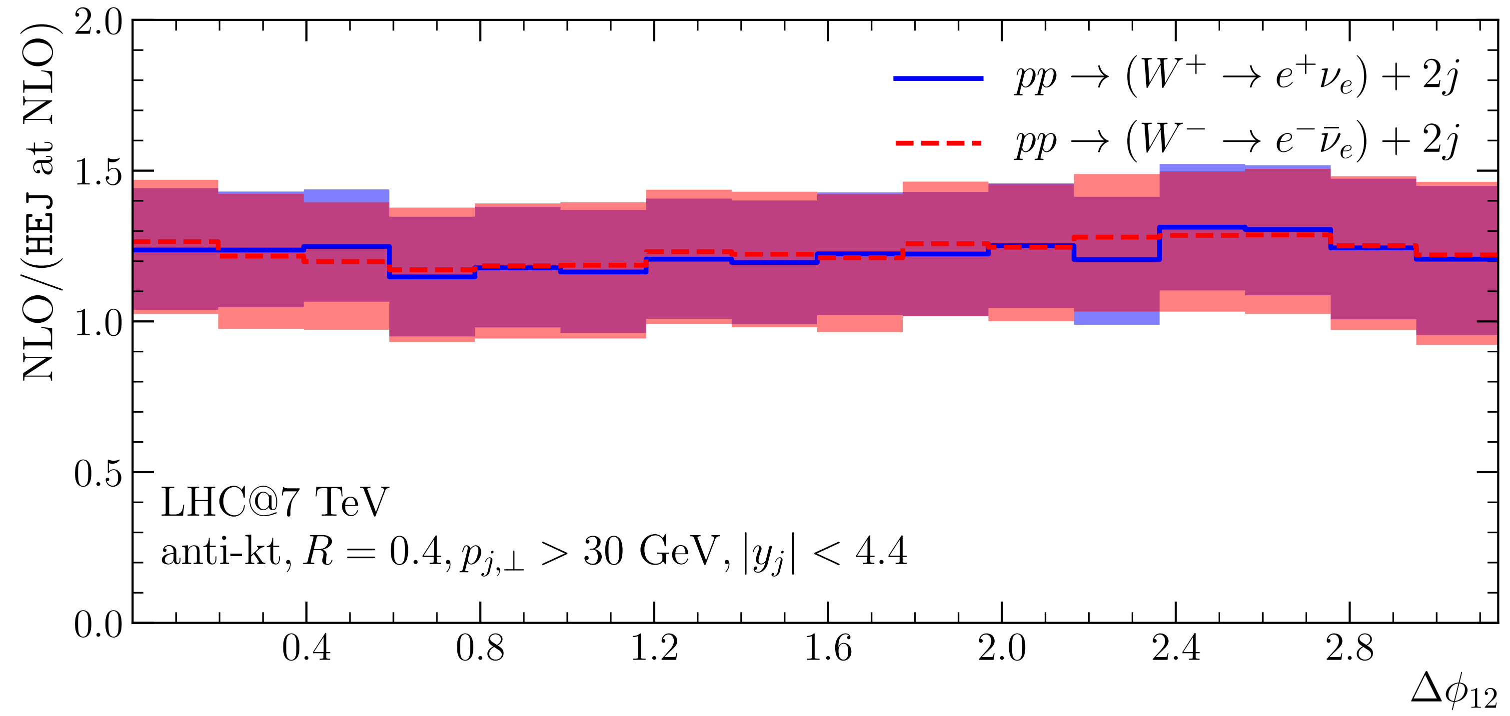
We derive predictions from HEJ, truncated to NLO and take the ratio to full NLO for each distribution.

Final predictions are then given by

$$w_{\mathrm{HEJ2\,NLO}} = w_{\mathrm{HEJ2}} \frac{w_{\mathrm{NLO}}}{w_{\mathrm{HEJ\,at\,NLO}}} + w_{\mathrm{FO\,W+\geq 4j}}$$

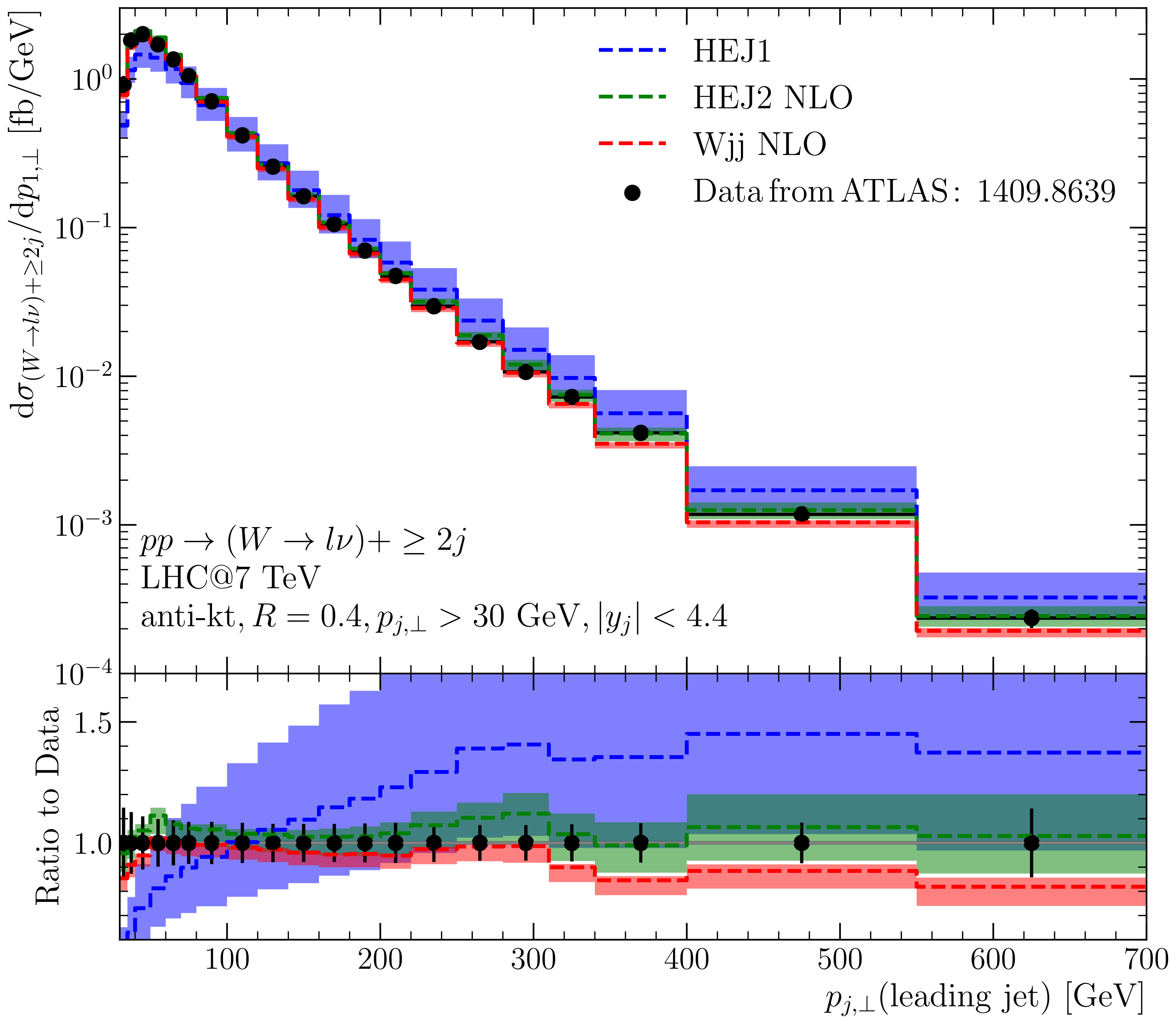
Can check by expansion that each bin is accurate to NLO+LL

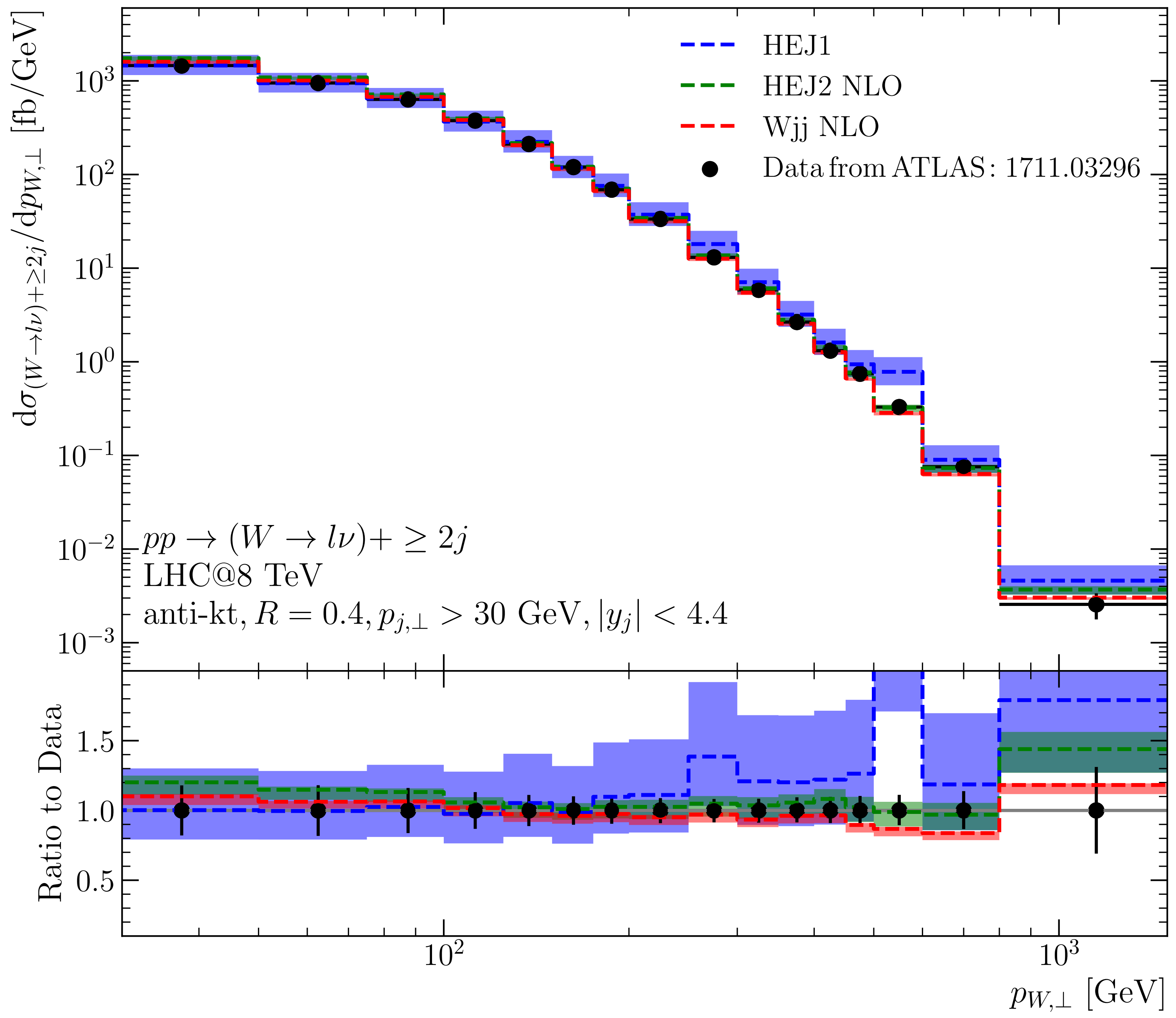




Andersen, Black, Brooks, Byrne, Maier, JMS arXiv:2012.10310

Improved W-Plus-Dijets





- HEJ2 NLO prediction lies between the previous two
- Scale variation reduced — larger than NLO due to higher multiplicities

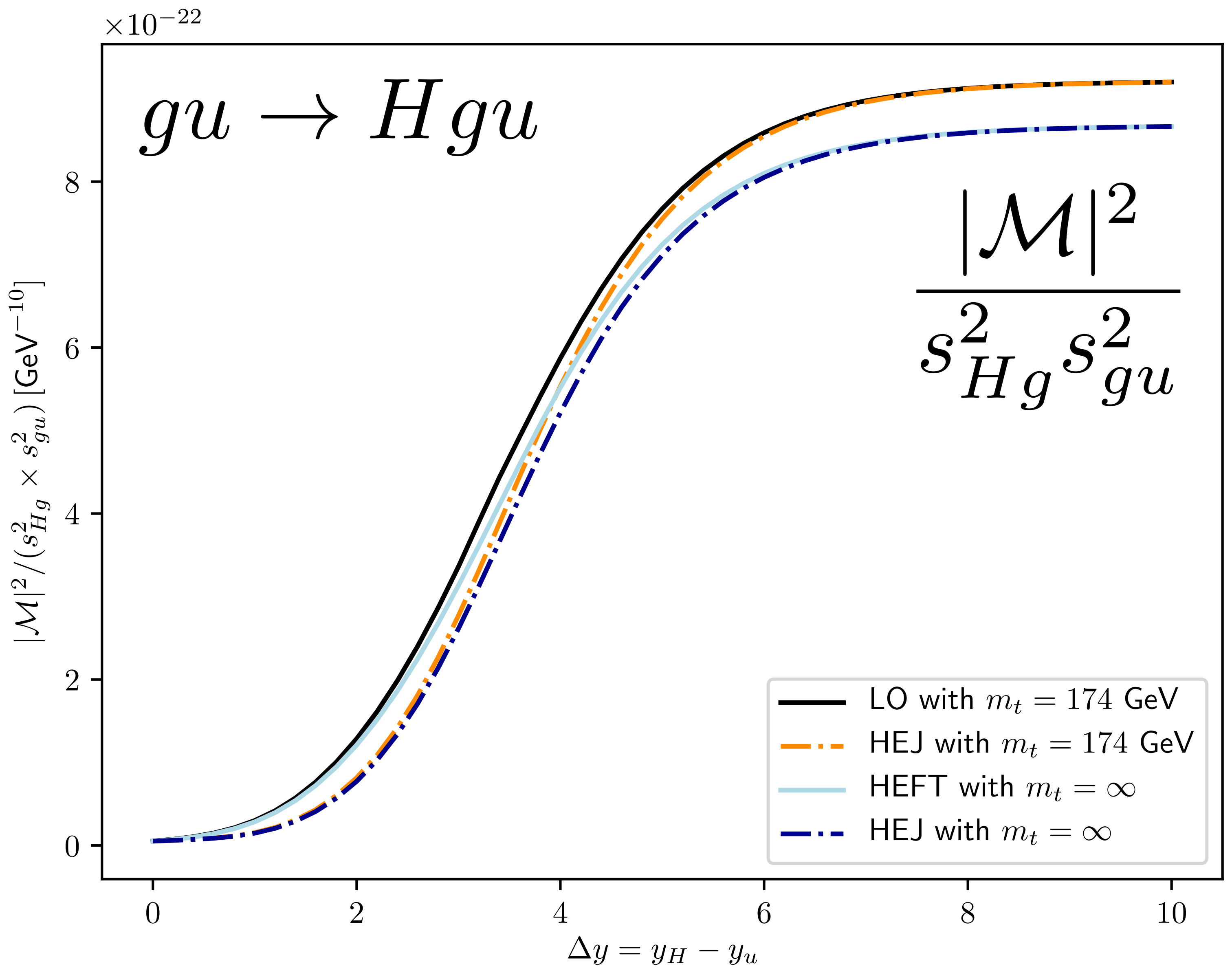
- At large p_T values, require ≥ 4j events to obtain good agreement

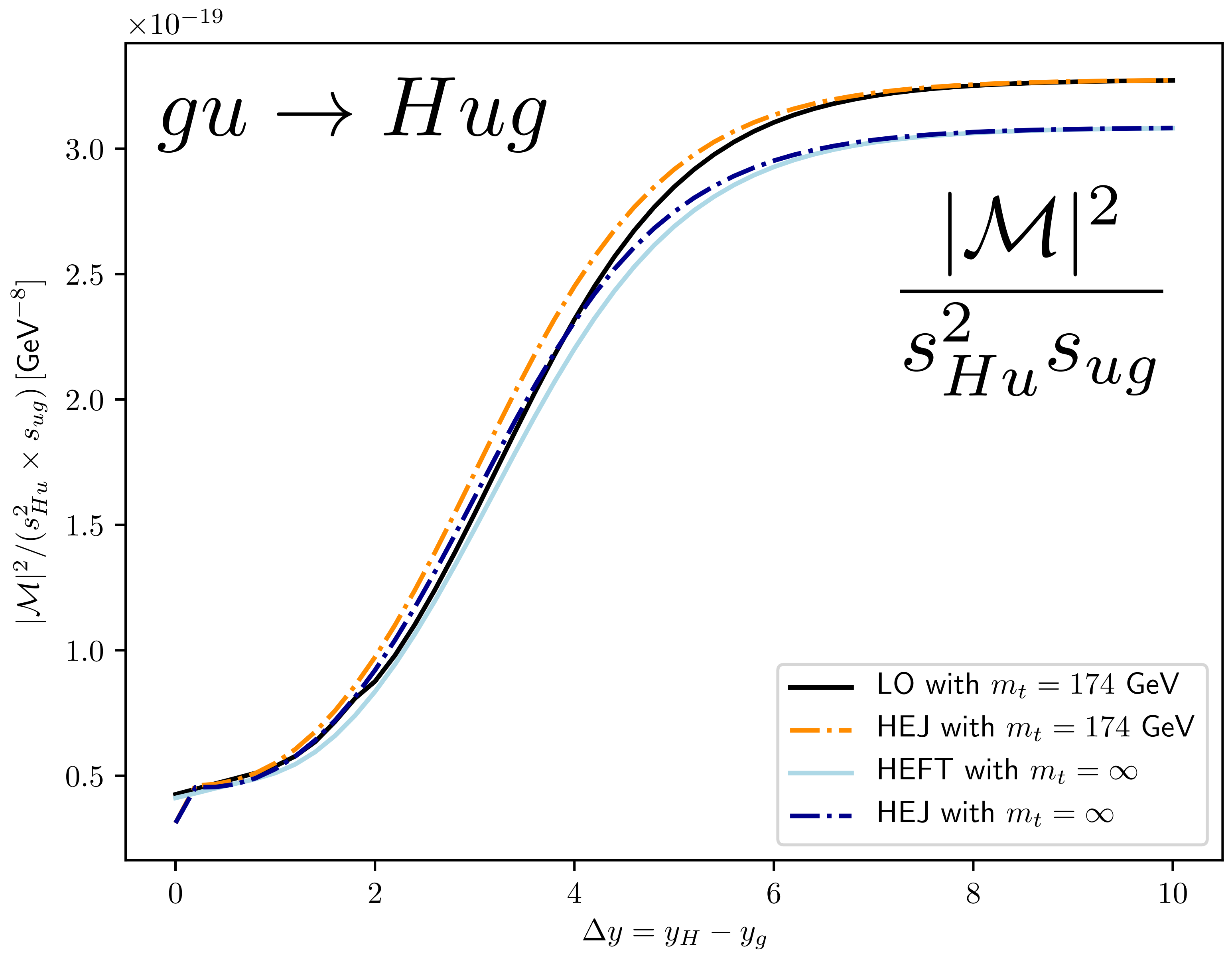
Andersen, Black, Brooks, Byrne, Maier, JMS arXiv:2012.10310

Higgs + 1j in HEJ

- HEJ has always resummed logarithms in the region between the outer jets in rapidity, hence always for processes with at least two jets
- Observed in H+2j studies, that scaling with an intermediate Higgs boson was as in QCD

Andersen, Hapola, Maier, JMS arXiv:1706.01002





- The same (Regge) scaling applies in the amplitude if the Higgs boson is external in rapidity
- Hence the same framework can be applied to H+1j

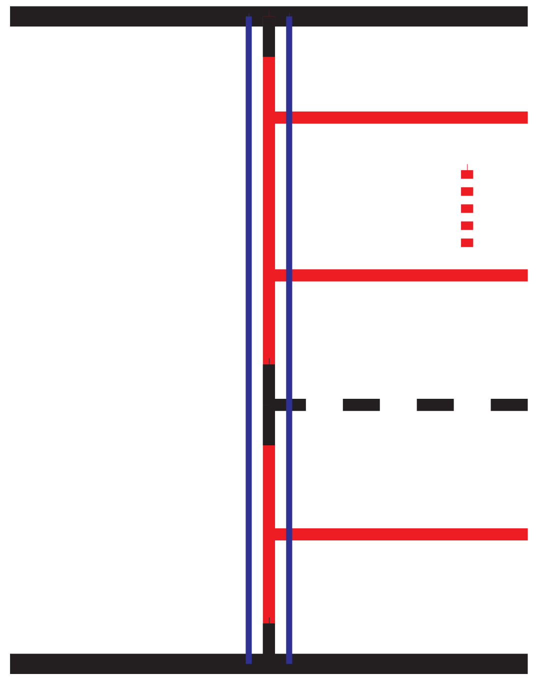
Andersen, Hassan, Maier, Paltrinieri, Papaefstathiou, JMS arXiv:2210.10671

Higgs + 1j in HEJ

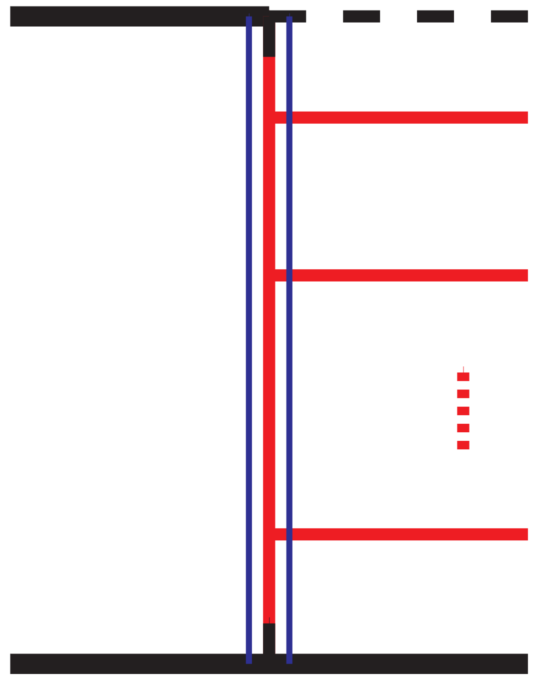
Black = Born/skeleton function

Red = Range of resummation

Previous 2jet+



Increasing rapidity

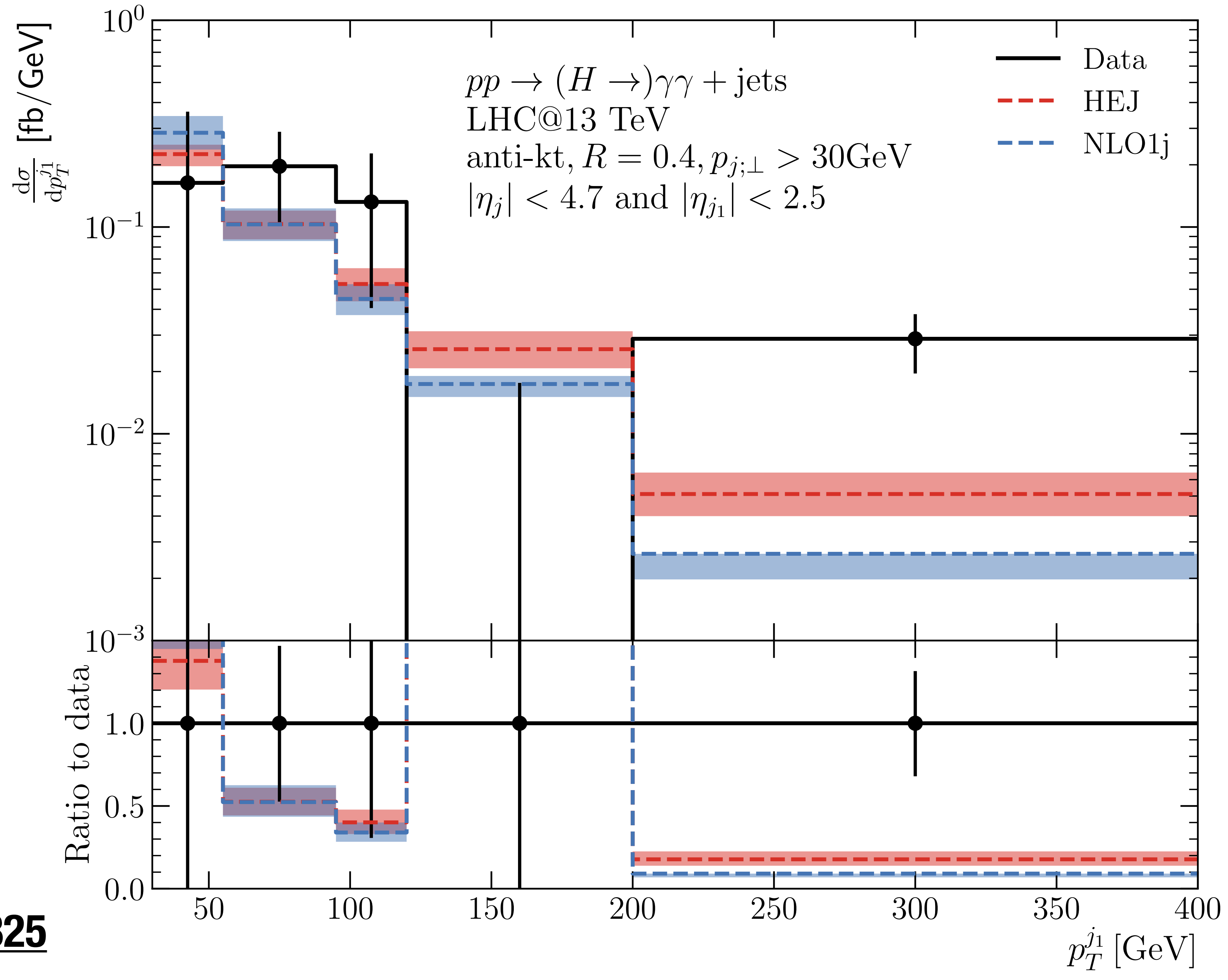


New 1jet+

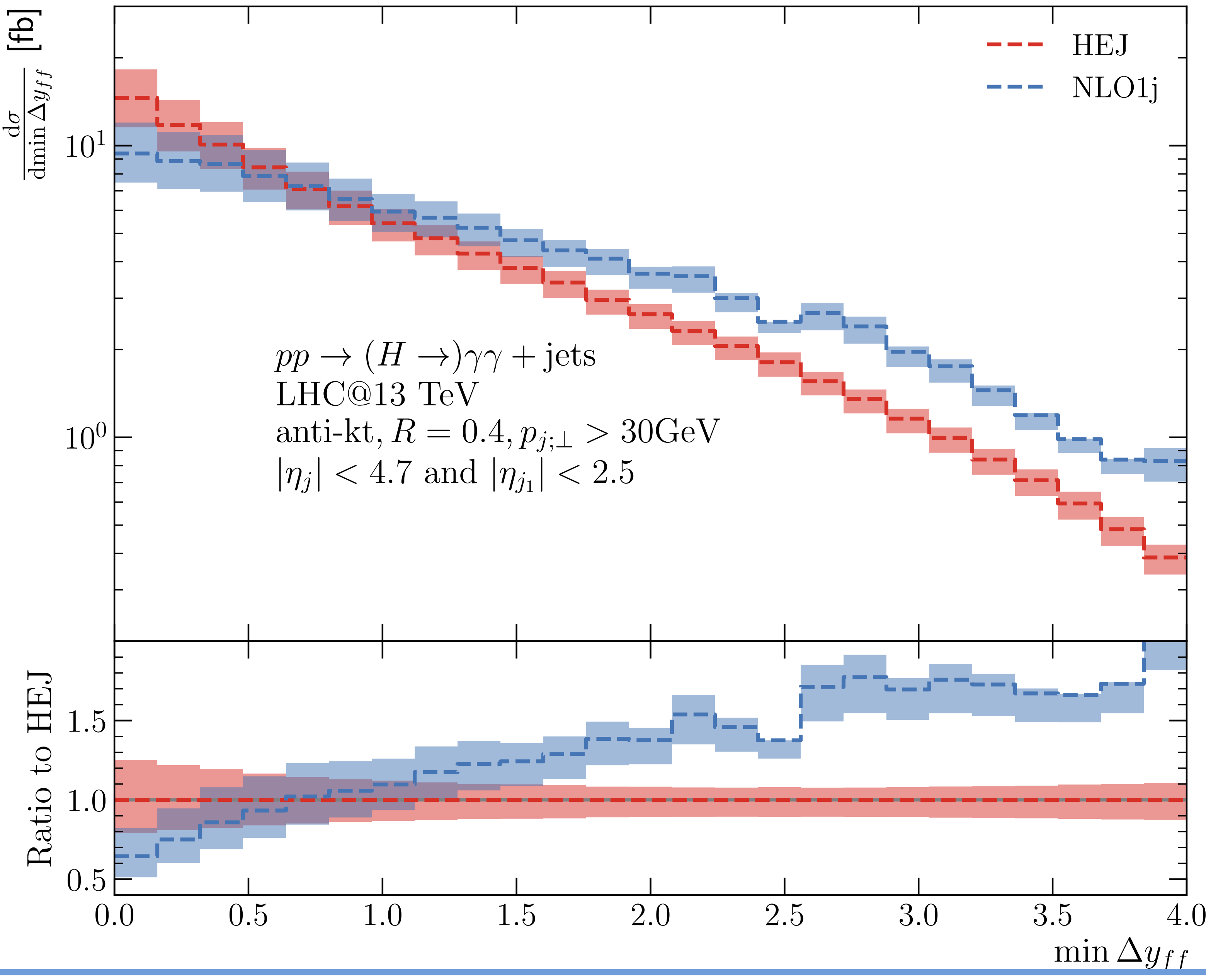
Similar effects on distributions

Andersen, Hassan, Maier, Paltrinieri, Papaefstathiou, JMS arXiv:2210.10671

Data includes “HX” not in HEJ or NLO

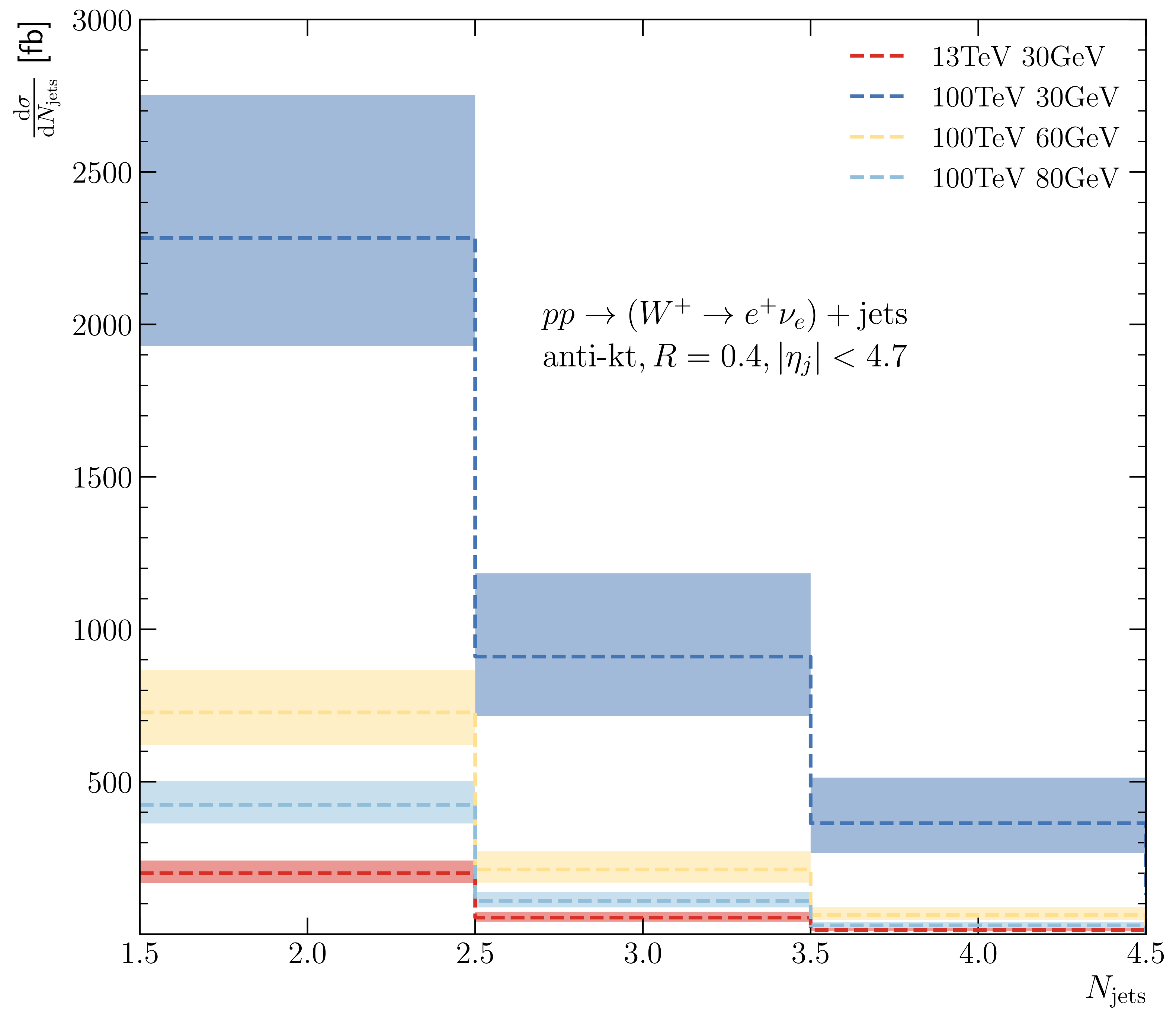


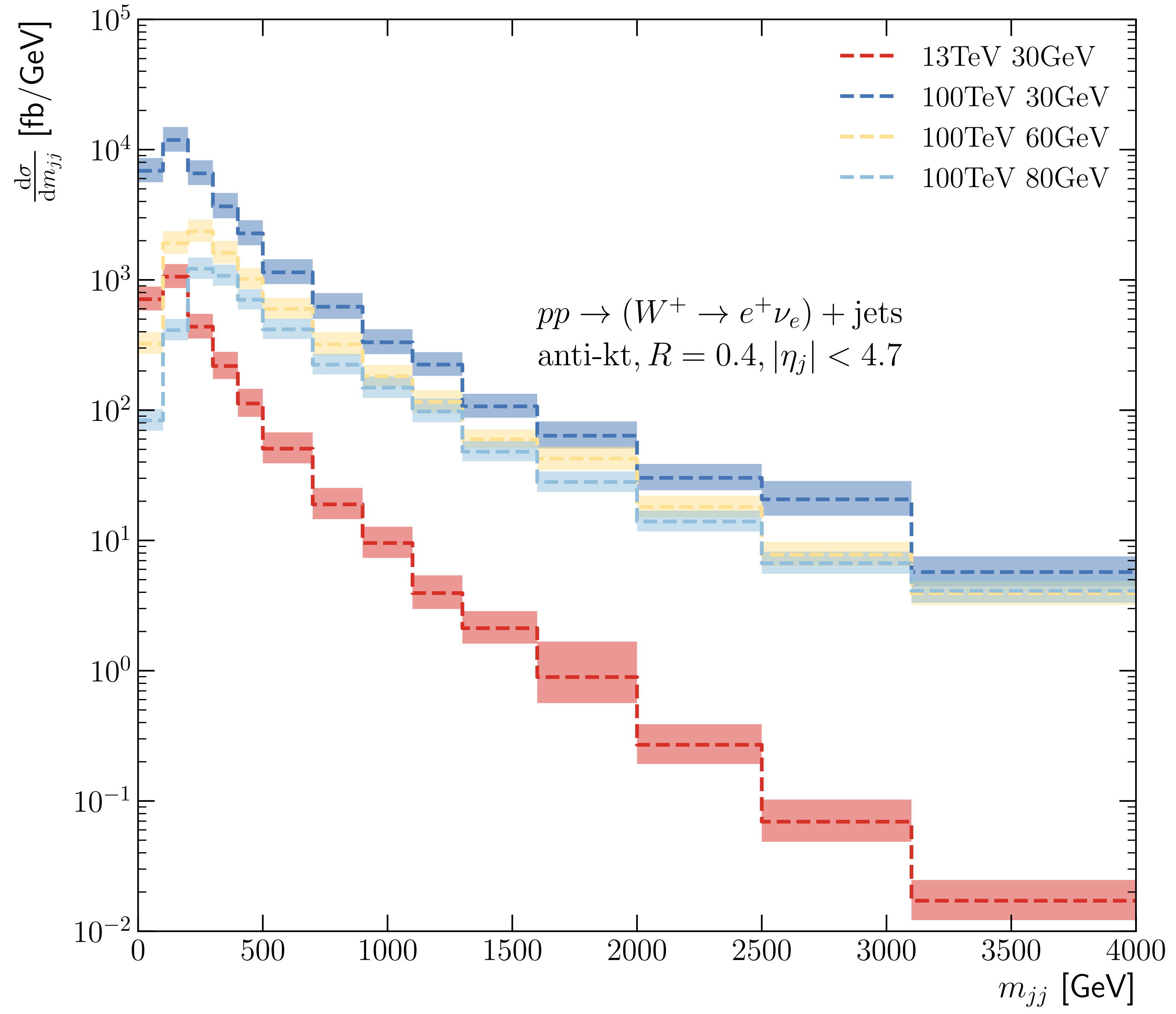
CMS data arXiv:1807.03825



Looking Forward

What about a 100 TeV collider? Even larger centre-of-mass energy will give even larger logs!



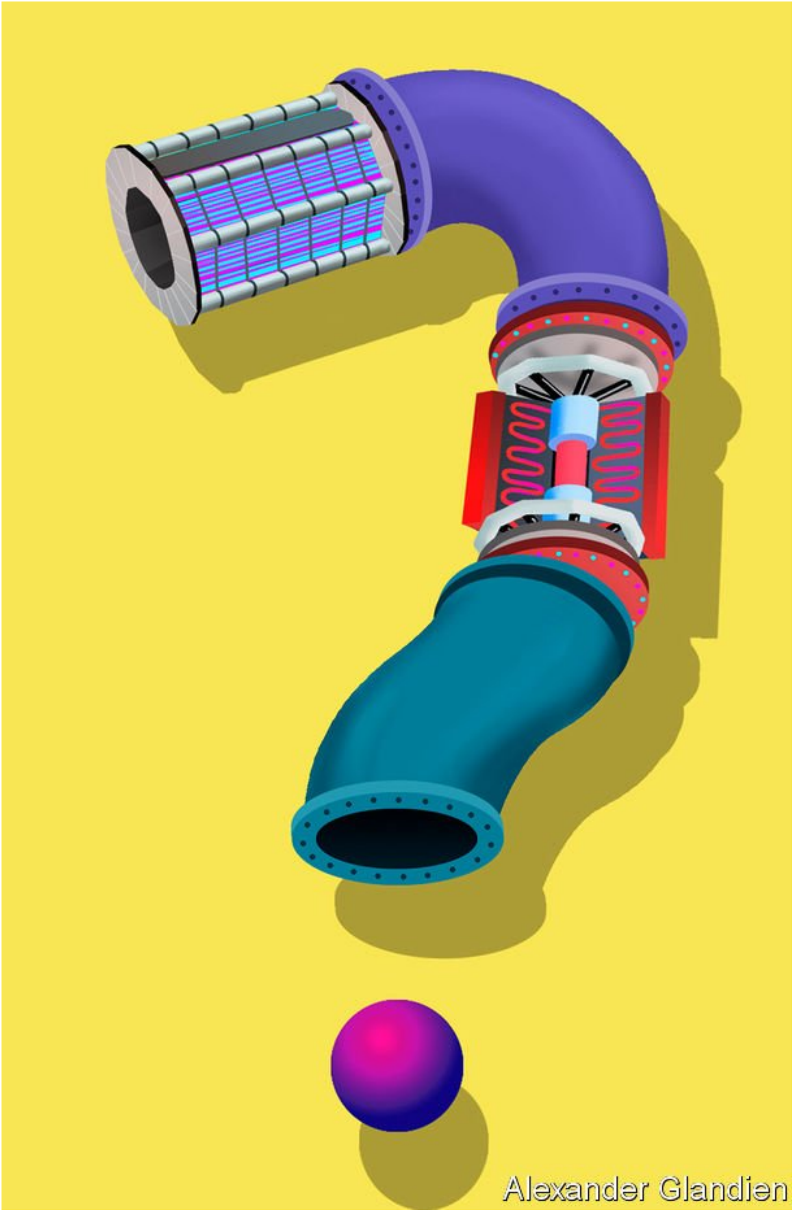


Higher pT cuts can control the jet rates, but impact of logs on distributions will be large

Plots: Conor Elrick

Summary & Outlook

- Current and future data demand higher precision predictions
- High Energy Jets allows the description of high energy logs in a fully flexible framework
- High Energy Jets provides alternative way to include finite quark mass effects
- Recent improvements improve the description of data away from the strict limit
- Ongoing work to increase accuracy to full NLL and to full NLO



HEJ2 event generator: https://hej.hepforge.org