The Target Fragmentation Region of Polarized Leptoproduction

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[M. Anselmino, V.B. & A. Kotzinian, arXiv:1102.4214 [hep-ph], PLB in press]

Single-spin and azimuthal asymmetries in leptoproduction have been intensively explored both from an experimental and a theoretical viewpoint
[For a review: VB, Bradamante & Martin, arXiv:1011.0909 [hep-ph]]

The work has focused on the Current Fragmentation Region (CFR), which probes the transverse-momentum dependent distributions of the nucleon (TMDs)

But final hadrons are also found among the remnants of the struck target, i.e. in the Target Fragmentation Region (TFR)

The QCD formalism of TFR is based on the concept of Fracture Function
[Trentadue & Veneziano (1994)]

Fracture functions describe the structure of a nucleon when this fragments into a hadron (they represent the conditional probabilities to find a quark with a given momentum inside a nucleon which emits a hadron)

Collinear fracture functions have been studied by Trentadue, Veneziano & Grazzini (1994-98) and Graudenz (1994) for the unpolarized case, and by de Florian et al. (1996-97) for the longitudinally polarized case

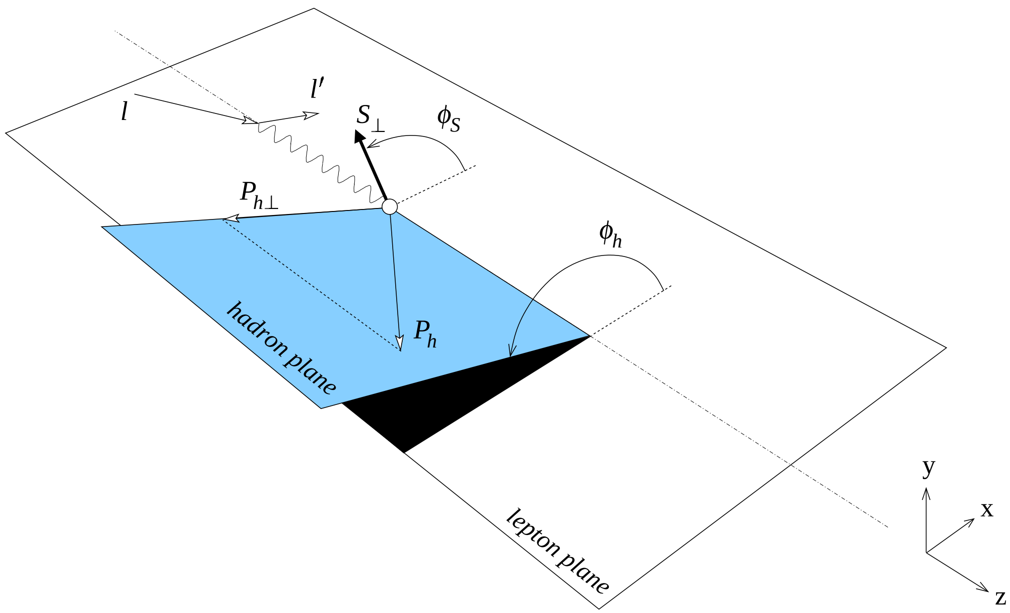
Unpolarized "extended" fracture functions have been considered by Grazzini, Trentadue & Veneziano (1998), and by Ceccopieri and Trentadue (2006)

What is needed is a complete (leading-twist) description of polarized and transverse-momentum dependent fracture functions

Some questions:

- What is the spectrum of angular modulations in the TFR?
- Do the CFR and the TFR exhibit the same asymmetries?
- Is the transverse polarization of quarks probed in the TFR?

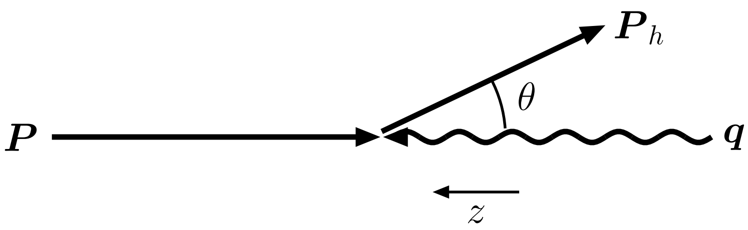
Semi-inclusive DIS: $l(\ell) + N(P) \to l(\ell') + h(P_h) + X(P_X)$



Invariants: $W^2 = (P+q)^2 \qquad x_B = \frac{Q^2}{2P\cdot q} \qquad y = \frac{P\cdot q}{P\cdot \ell} \qquad z_h = \frac{P\cdot P_h}{P\cdot q}$

Kinematic regions of leptoproduction

In the $\gamma^* N$ cm frame: $z_h \sim \frac{P_h^+}{q^+}$



The magnitude of P^+ determines the fragmentation region ($q^+ \sim Q$):

Current fragmentation region (CFR): $P_h^+ \sim Q \to z_h$ finite

Target fragmentation region (TFR): $P_h^+ \sim 0 \to z_h \to 0$

In terms of Feynman's variable $x_F = 2P_{h\|}/W$:

CFR corresponds to $x_F > 0$, TFR corresponds to $x_F < 0$ (γ^* in the $+z$ direction)

Explicitly, z_h is given by (in the $\gamma^* N$ cm frame):

$$z_h = \frac{E_h}{E(1 - x_B)} \frac{1 - \cos\theta}{2} \equiv z \frac{1 - \cos\theta}{2} \qquad \text{TFR} : \theta \to 0$$

Separation of CFR and TFR

Rapidity separation between current and target fragments [Berger (1987)]:

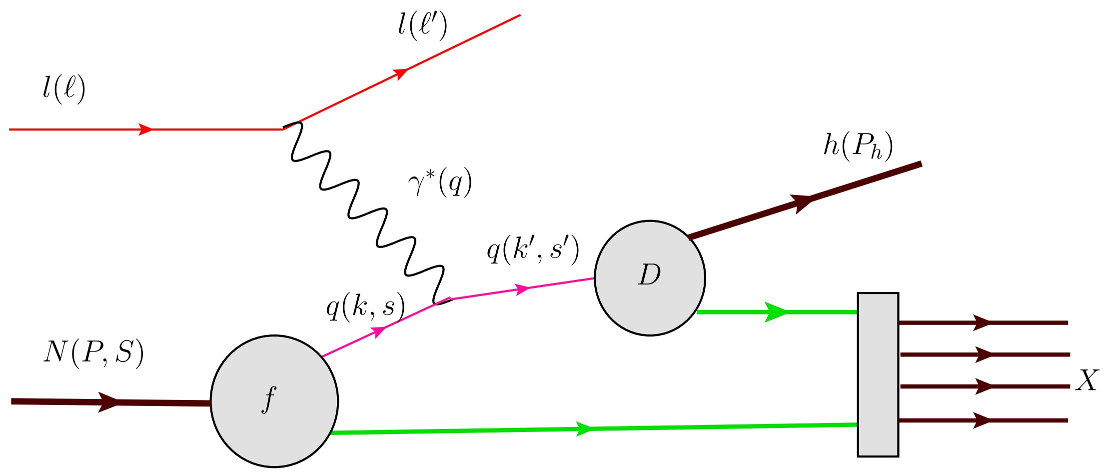
$$\Delta y = \ln \frac{W^2}{M^2} = \ln \frac{Q^2(1 - x_B)}{x_B\, M^2}$$

If $\Delta y >$ few units (~ 4, but this number is somehow a guess), hadrons in the whole z_h range belong to CFR (this corresponds to $W > 7.4$ GeV)

For smaller W, a lower cut on z_h is needed in order to exclude hadrons produced in the TFR

JLab, HERMES, COMPASS require W larger than 2, 3, 5 GeV, respectively

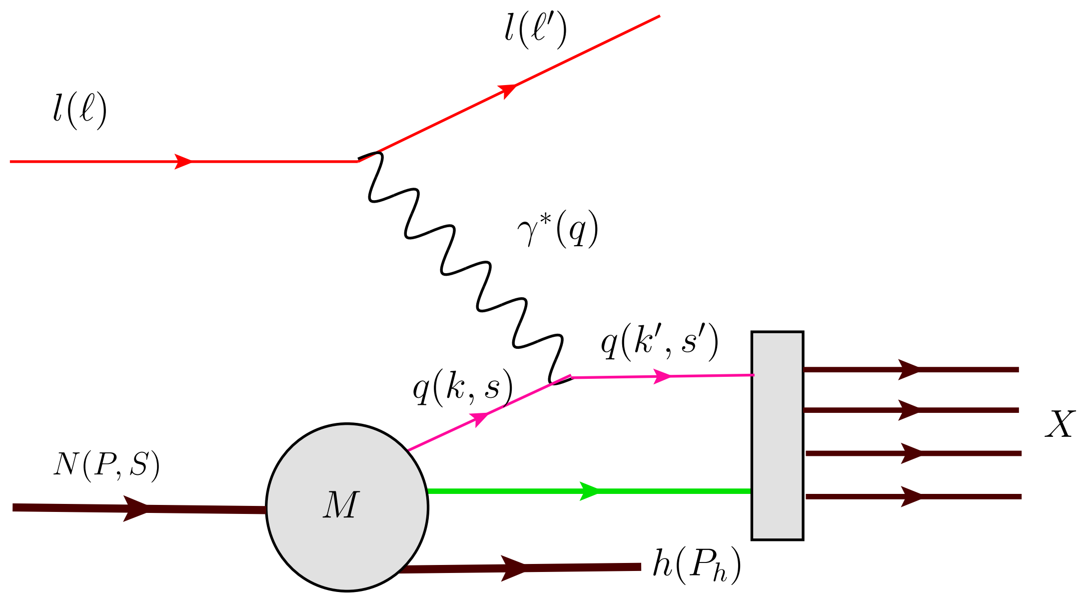
Current fragmentation region



Cross section : $$\frac{\mathrm{d}\sigma^{\mathrm{CFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z_h} = \sum_a e_a^2\, f_a(x_B)\, \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}\, D_a(z_h)$$

Factorization in x_B (fraction of the nucleon momentum carried by the quark) and z_h (fraction of the momentum of the struck quark carried by the final hadron)

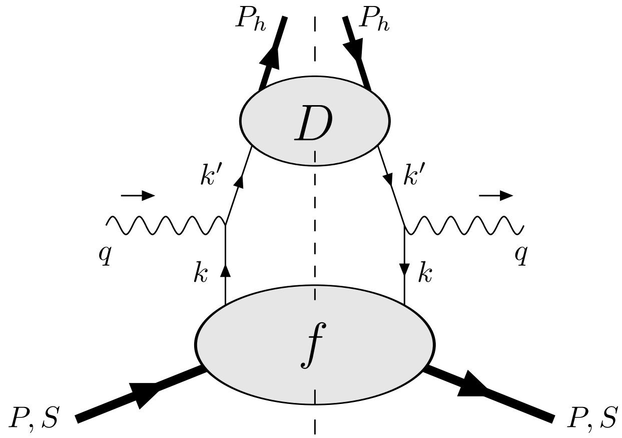
Target fragmentation region

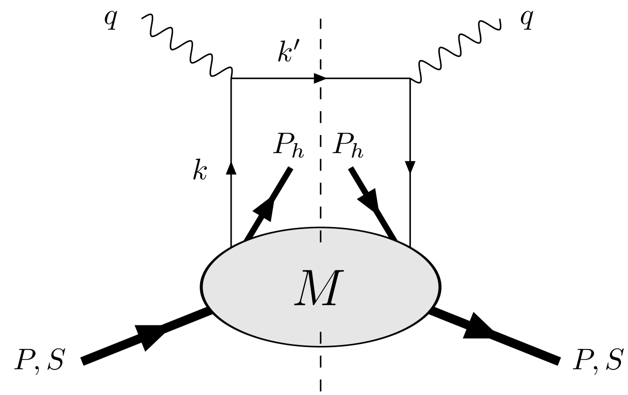


Cross section :
$$\frac{\mathrm{d}\sigma^{\mathrm{TFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z} = \sum_a e_a^2\,(1-x_B)\,M_a(x_B,(1-x_B)z)\,\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}$$

No factorization in x_B and $z \equiv E_h/E\,(1-x_B)$ (quark emission is not separated by hadron production)

Hadronic tensor in parton model





CFR: $W^{\mu\nu} \sim \int \mathrm{d}^4k\, \mathrm{Tr}\left[\mathcal{F}\gamma^\mu \mathcal{D}\gamma^\nu\right]$ TFR: $W^{\mu\nu} \sim \int \mathrm{d}^4k\, \mathrm{Tr}\left[\mathcal{M}\gamma^\mu (\not{k} + \not{q})\gamma^\nu\right]$

$\mathcal{F}$ Quark correlation matrix, $\mathcal{D}$ Fragmentation (decay) matrix, $\mathcal{M}$ Fracture matrix

The fracture functions represent the combined distributions of initial quarks and final hadrons in a nucleon

Fracture matrix

Parametrizations of quark and hadron momenta:

$$k^\mu = xP^\mu + k_\perp^\mu \qquad x = k^-/P^- = x_B$$
$$P_h^\mu = \zeta P^\mu + P_h^\mu \qquad \zeta = P_h^+/P^+ \simeq E_h/E = z\,(1-x_B)$$

The matrix containing all fracture functions is

$$\mathcal{M}(x_B, \boldsymbol{k}_\perp, \zeta, \boldsymbol{P}_{h\perp}) = \frac{1}{4\zeta} \int \frac{\mathrm{d}\xi^+\, \mathrm{d}^2\boldsymbol{\xi}_\perp}{(2\pi)^6}\, \mathrm{e}^{\mathrm{i}(x_B P^- \xi^+ - \boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp)}$$
$$\times \sum_X \int \frac{\mathrm{d}^3\boldsymbol{P}_X}{(2\pi)^3\, 2E_X} \langle P, S|\overline{\psi}(0)|P_h; X\rangle\langle P_h; X|\psi(\xi^+, 0, \boldsymbol{\xi}_\perp)|P, S\rangle$$

At leading twist (i.e. order $(P^-)^1$), $\mathcal{M}$ has only vector, axial and tensor components:

$$\mathcal{M} = \frac{1}{2}\left(\mathcal{V}_\mu \gamma^\mu + \mathcal{A}_\mu \gamma_5 \gamma^\mu + \mathrm{i}\, \mathcal{T}_{\mu\nu}\, \sigma^{\mu\nu}\gamma_5\right)$$

Time reversal has no implications due to the presence of the $|P_h; X\rangle$ out states.

The 16 leading-twist fracture functions (dependent on $x_B, \boldsymbol{k}_\perp^2, \zeta, \boldsymbol{P}_{h\perp}^2, \boldsymbol{k}_\perp \cdot \boldsymbol{P}_{h\perp}$)

Unpolarized fracture functions:

$$\mathcal{M}^{[\gamma^-]} = \hat{M} + \frac{\boldsymbol{P}_{h\perp} \times \boldsymbol{S}_\perp}{m_h} \hat{M}_T^h + \frac{\boldsymbol{k}_\perp \times \boldsymbol{S}_\perp}{m_N} \hat{M}_T^\perp + \frac{S_\parallel \, (\boldsymbol{k}_\perp \times \boldsymbol{P}_{h\perp})}{m_N \, m_h} \hat{M}_L^{\perp h}$$

Longitudinally polarized fracture functions:

$$\mathcal{M}^{[\gamma^- \gamma_5]} = S_\parallel \, \Delta \hat{M}_L + \frac{\boldsymbol{P}_{h\perp} \cdot \boldsymbol{S}_\perp}{m_h} \Delta \hat{M}_T^h + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{m_N} \Delta \hat{M}_T^\perp + \frac{\boldsymbol{k}_\perp \times \boldsymbol{P}_{h\perp}}{m_N \, m_h} \Delta \hat{M}^{\perp h}$$

Transversely polarized fracture functions:

$$\begin{aligned}
\mathcal{M}^{[\mathrm{i}\,\sigma^{i-} \gamma_5]} \quad = \quad & S_\perp^i \, \Delta_T \hat{M}_T + \frac{S_\parallel \, P_{h\perp}^i}{m_h} \Delta_T \hat{M}_L^h + \frac{S_\parallel \, k_\perp^i}{m_N} \Delta_T \hat{M}_L^\perp \\
& + \frac{(\boldsymbol{P}_{h\perp} \cdot \boldsymbol{S}_\perp) \, P_{h\perp}^i}{m_h^2} \Delta_T \hat{M}_T^{hh} + \frac{(\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp) \, k_\perp^i}{m_N^2} \Delta_T \hat{M}_T^{\perp\perp} \\
& + \frac{(\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp) \, P_{h\perp}^i - (\boldsymbol{P}_{h\perp} \cdot \boldsymbol{S}_\perp) \, k_\perp^i}{m_N m_h} \Delta_T \hat{M}_T^{\perp h} \\
& + \frac{\epsilon_\perp^{ij} P_{h\perp j}}{m_h} \Delta_T \hat{M}^h + \frac{\epsilon_\perp^{ij} k_{\perp j}}{m_N} \Delta_T \hat{M}^\perp
\end{aligned}$$

Nomenclature and notation

- $\hat{M}$: unpolarized quarks
  $\Delta\hat{M}$: longitudinally polarized quarks
  $\Delta_T\hat{M}$: transversely polarized quarks
- Subscripts L and T label the polarization of the target
- Superscripts h and $\perp$ signal the presence of factors $P_{h\perp}^i$ and $k_\perp^i$, respectively
- Fracture functions integrated over the transverse momentum of quarks do not have the hats

Unpolarized and longitudinally polarized fracture functions

Integrating over $\boldsymbol{k}_\perp$ only 4 fracture functions survive:

$$\int \mathrm{d}^2\boldsymbol{k}_\perp\, \mathcal{M}^{[\gamma^-]} = M(x_B,\zeta,\boldsymbol{P}^2_{h\perp}) + \frac{\boldsymbol{P}_{h\perp}\times\boldsymbol{S}_\perp}{m_h}\, M^h_T(x_B,\zeta,\boldsymbol{P}^2_{h\perp})$$

$$\int \mathrm{d}^2\boldsymbol{k}_\perp\, \mathcal{M}^{[\gamma^-\gamma_5]} = S_\parallel\, \Delta M_L(x_B,\zeta,\boldsymbol{P}^2_{h\perp}) + \frac{\boldsymbol{P}_{h\perp}\cdot\boldsymbol{S}_\perp}{m_h}\, \Delta M^h_T(x_B,\zeta,\boldsymbol{P}^2_{h\perp})$$

Sivers-type fracture function, analogous to $f_{1T}^\perp$ (emission of an unpolarized quark from a transversely polarized nucleon):

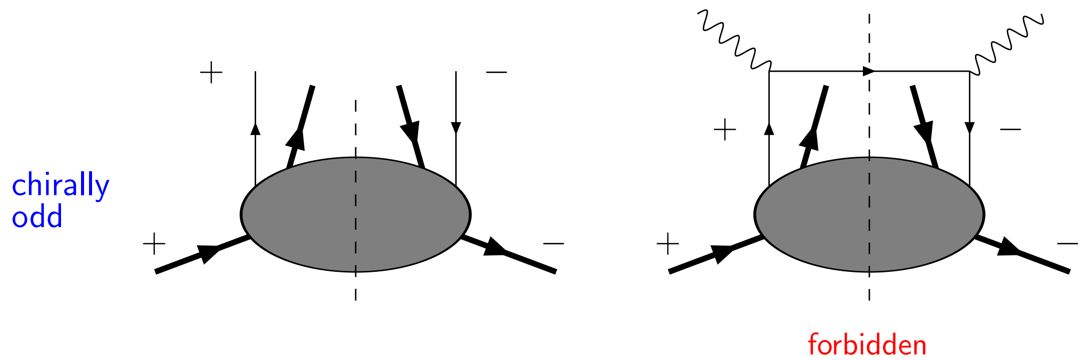
$$M^h_T(x_B,\zeta,\boldsymbol{P}^2_{h\perp}) = \int \mathrm{d}^2\boldsymbol{k}_\perp\, \left\{\hat{M}^h_T + \frac{m_h}{m_N}\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{P}_{h\perp}}{\boldsymbol{P}^2_{h\perp}}\,\hat{M}^\perp_T\right\} \qquad [\sin(\phi_S-\phi_h)]$$

Fracture function analogous to $g_{1T}^\perp$ (emission of a longitudinally polarized quark from a transversely polarized nucleon)

$$\Delta M^h_T(x_B,\zeta,\boldsymbol{P}^2_{h\perp}) = \int \mathrm{d}^2\boldsymbol{k}_\perp\, \left\{\Delta\hat{M}^h_T + \frac{m_h}{m_N}\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{P}_{h\perp}}{\boldsymbol{P}^2_{h\perp}}\,\Delta\hat{M}^\perp_T\right\} \qquad [\cos(\phi_S-\phi_h)]$$

Transversely polarized fracture functions

Chirally-odd fracture functions not probed in single-particle leptoproduction



To observe the $\Delta_T M$'s we need a hadron in the CFR (see later)

Momentum sum rules

Relate integrals of fracture functions over the hadronic variables to quark distribution functions, e.g.:

$$\sum_h \int_0^{1-x_B} \mathrm{d}\zeta\, \zeta\, M(x_B, \zeta) = (1 - x_B)\, f_1(x_B) \quad \text{[Trentadue \& Veneziano]}$$

This means that, if the target emits a quark with momentum fraction x_B, the total momentum fraction available for the produced hadron is $1 - x_B$

Generalization for transverse-momentum dependent fracture functions:

$$\sum_h \int_0^{1-x_B} \mathrm{d}\zeta\, \zeta \int \mathrm{d}^2\boldsymbol{P}_{h\perp}\, \mathcal{M}(x_B, \boldsymbol{k}_\perp, \zeta, \boldsymbol{P}_{h\perp}) = (1 - x_B)\, \mathcal{F}(x_B, \boldsymbol{k}_\perp)$$

where $\mathcal{F}$ is the correlation matrix containing the TMDs:

$$\mathcal{F}(x_B, \boldsymbol{k}_\perp) = \frac{1}{2} \int \frac{\mathrm{d}\xi^+\, \mathrm{d}^2\boldsymbol{\xi}_\perp}{(2\pi)^3}\, \mathrm{e}^{\mathrm{i}(x_B P^- \xi^+ - \boldsymbol{k}_\perp \cdot \boldsymbol{\xi}_\perp)}\, \langle P, S|\overline{\psi}(0)\, \psi(\xi^+, 0, \boldsymbol{\xi}_\perp)|P, S\rangle$$

The momentum sum rules involving f_1, $f_{1T}^{\perp}$, $h_1^{\perp}$, h_1 are:

$$\sum_h \int d\zeta\,\zeta \int d^2\boldsymbol{P}_{h\perp}\,\hat{M} = (1-x_B)\,f_1(x_B,\boldsymbol{k}_\perp^2)$$

$$\sum_h \int d\zeta\,\zeta \int d^2\boldsymbol{P}_{h\perp}\left\{\hat{M}_T^{\perp} + \frac{m_N}{m_h}\,\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{P}_{h\perp}}{\boldsymbol{k}_\perp^2}\,\hat{M}_T^{h}\right\} = -(1-x_B)\,f_{1T}^{\perp}(x_B,\boldsymbol{k}_\perp^2)$$

$$\sum_h \int d\zeta\,\zeta \int d^2\boldsymbol{P}_{h\perp}\left\{\Delta_T\hat{M}^{\perp} + \frac{m_N}{m_h}\,\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{P}_{h\perp}}{\boldsymbol{k}_\perp^2}\,\Delta_T\hat{M}^{h}\right\} = -(1-x_B)\,h_1^{\perp}(x_B,\boldsymbol{k}_\perp^2)$$

$$\sum_h \int d\zeta\,\zeta \int d^2\boldsymbol{P}_{h\perp}\left\{\Delta_T\hat{M}_T + \frac{\boldsymbol{k}_\perp^2}{2m_N^2}\,\Delta_T\hat{M}_T^{\perp\perp} + \frac{\boldsymbol{P}_{h\perp}^2}{2m_h^2}\,\Delta_T\hat{M}_T^{hh}\right\} = (1-x_B)\,h_1(x_B,\boldsymbol{k}_\perp^2)$$

Leptoproduction of a spinless hadron, $l + N \to l' + h + X$

$$\frac{\mathrm{d}\sigma^{\mathrm{TFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}\zeta\,\mathrm{d}^2\boldsymbol{P}_{h\perp}\,\mathrm{d}\phi_S} = \frac{2\alpha_{\mathrm{em}}^2}{Q^2 y} \left\{ \left(1 - y + \frac{y^2}{2}\right) \right.$$
$$\times \sum_a e_a^2 \left[M(x_B, \zeta, \boldsymbol{P}_{h\perp}^2) - |\boldsymbol{S}_\perp| \frac{|\boldsymbol{P}_{h\perp}|}{m_h} M_T^h(x_B, \zeta, \boldsymbol{P}_{h\perp}^2) \sin(\phi_h - \phi_S) \right]$$
$$+ \lambda_l\, y \left(1 - \frac{y}{2}\right) \sum_a e_a^2 \left[S_\parallel\, \Delta M_L(x_B, \zeta, \boldsymbol{P}_{h\perp}^2) \right.$$
$$\left.\left. + \ |\boldsymbol{S}_\perp| \frac{|\boldsymbol{P}_{h\perp}|}{m_h} \Delta M_T^h(x_B, \zeta, \boldsymbol{P}_{h\perp}^2) \cos(\phi_h - \phi_S) \right] \right\}$$

Only four fracture functions involved, and two modulations. No Collins-type $\sin(\phi_S + \phi_h)$ term.

SIDIS cross section: 18 structure functions (at LT, 4 in TFR, 4 + 4 in CFR)

$$\frac{d^6\sigma}{dx_B\,dy\,dz_h\,d\phi_h\,dP_{h\perp}^2\,d\phi_S} = \frac{\alpha_{\rm em}^2}{x_B y Q^2}\Big\{(1-y+\frac{1}{2}y^2)\,F_{UU,T} + (1-y)\,F_{UU,L}$$

$$+(2-y)\sqrt{1-y}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + (1-y)\,\cos 2\phi_h\,F_{UU}^{\cos 2\phi_h} + \lambda_\ell\,y\sqrt{1-y}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}$$

$$+S_\parallel\Big[(2-y)\sqrt{1-y}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + (1-y)\,\sin 2\phi_h\,F_{UL}^{\sin 2\phi_h}\Big]$$

$$+S_\parallel\,\lambda_\ell\,\Big[y(1-\frac{1}{2}y)\,F_{LL} + y\sqrt{1-y}\cos\phi_h\,F_{LL}^{\cos\phi_h}\Big]$$

$$+S_\perp\,\Big[\sin(\phi_h-\phi_S)\,\Big((1-y+\frac{1}{2}y^2)\,F_{UT,T}^{\sin(\phi_h-\phi_S)} + (1-y)\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big)$$

$$+(1-y)\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + (1-y)\,\sin(3\phi_h-\phi_s)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\,(2-y)\sqrt{1-y}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + (2-y)\sqrt{1-y}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

$$+S_\perp\,\lambda_\ell\,\Big[y(1-\frac{1}{2}y)\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + y\sqrt{1-y}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}$$

$$+\,y\sqrt{1-y}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}$$

Structure functions in the TFR:

$$\left[F_{UT,T}^{\sin(\phi_h-\phi_S)}\right]_{\rm TFR} = -\sum_a e_a^2\, x_B\, \frac{|\boldsymbol{P}_{h\perp}|}{m_h}\, M_T^h(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)$$

$$\left[F_{LT}^{\cos(\phi_h-\phi_S)}\right]_{\rm TFR} = \sum_a e_a^2\, x_B\, \frac{|\boldsymbol{P}_{h\perp}|}{m_h}\, \Delta M_T^h(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)$$

For comparison, the corresponding structure functions in CFR are:

$$\left[F_{UT,T}^{\sin(\phi_h-\phi_S)}\right]_{\rm CFR} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{m_N}\, f_{1T}^\perp\, D_1\right] \qquad \left[F_{LT}^{\cos(\phi_h-\phi_S)}\right]_{\rm CFR} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{m_N}\, g_{1T}\, D_1\right]$$

Transverse–momentum convolution

$$\mathcal{C}\,[w\,f\,D] \equiv \sum_a e_a^2\, x_B \int \mathrm{d}^2\boldsymbol{k}_\perp \int \mathrm{d}^2\boldsymbol{\kappa}_\perp\, \delta^2(\boldsymbol{k}_\perp - \boldsymbol{\kappa}_\perp - \boldsymbol{P}_{h\perp}/z) \times w(\boldsymbol{k}_\perp,\boldsymbol{\kappa}_\perp)\, f(x_B,\boldsymbol{k}_\perp^2)\, D(z_h,\boldsymbol{\kappa}_\perp^2)$$

Leptoproduction of a spinless hadron and a current jet, $l + N \to l' + h + \text{jet} + X$

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^{\mathrm{TFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}\zeta\,\mathrm{d}^2\boldsymbol{P}_{h\perp}\,\mathrm{d}^2\boldsymbol{k}_\perp\,\mathrm{d}\phi_S} &= \frac{2\alpha_{\mathrm{em}}^2}{Q^2 y}\left\{\left(1-y+\frac{y^2}{2}\right)\right. \\
&\times \sum_a e_a^2 \left[\hat{M} + S_\parallel \frac{|\boldsymbol{P}_{h\perp}||\boldsymbol{k}_\perp|}{m_h\,m_N}\,\hat{M}_L^{\perp h}\,\sin(\phi_h-\phi_j)\right. \\
&\left. -|\boldsymbol{S}_\perp|\,\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\,\hat{M}_T^h\,\sin(\phi_h-\phi_S) - |\boldsymbol{S}_\perp|\,\frac{|\boldsymbol{k}_\perp|}{m_N}\,\hat{M}_T^\perp\,\sin(\phi_j-\phi_S)\right] \\
&+\lambda_l\,y\left(1-\frac{y}{2}\right)\sum_a e_a^2\left[\frac{|\boldsymbol{P}_{h\perp}||\boldsymbol{k}_\perp|}{m_h\,m_N}\,\Delta\hat{M}^{\perp h}\,\sin(\phi_h-\phi_j) + S_\parallel\,\Delta\hat{M}_L\right. \\
&\left.\left. +\ |\boldsymbol{S}_\perp|\,\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\,\Delta\hat{M}_T^h\,\cos(\phi_h-\phi_S) + |\boldsymbol{S}_\perp|\,\frac{|\boldsymbol{k}_\perp|}{m_N}\,\Delta\hat{M}_T^\perp\,\cos(\phi_j-\phi_S)\right]\right\}
\end{aligned}
$$

A richer structure, involving the 8 unintegrated fracture functions of unpolarized and longitudinally polarized quarks, but the process is hard to investigate experimentally

Leptoproduction of a polarized hadron, $l + N \to l' + h^{\uparrow} + X$

Introduce the hadron spin S_h and integrate over all transverse momenta:

$$\int d^2\boldsymbol{P}_{h\perp} \int d^2\boldsymbol{k}_{\perp}\, \mathcal{M}^{[\gamma^-]} = M(x_B,\zeta) + S_{\parallel}\, S_{h\parallel}\, M_L^L(x_B,\zeta) + (\boldsymbol{S}_{\perp}\cdot\boldsymbol{S}_{h\perp})\, M_T^T(x_B,\zeta)$$

$$\int d^2\boldsymbol{P}_{h\perp} \int d^2\boldsymbol{k}_{\perp}\, \mathcal{M}^{[\gamma^-\gamma_5]} = S_{\parallel}\, \Delta M_L(x_B,\zeta) + S_{h\parallel}\, \Delta M^L(x_B,\zeta) + (\boldsymbol{S}_{\perp}\times\boldsymbol{S}_{h\perp})\, \Delta M_T^T(x_B,\zeta)$$

A transversely polarized target can produce a transversely polarized hadron in two different ways:

- via the $\boldsymbol{S}_{\perp}\cdot\boldsymbol{S}_{h\perp}$ correlation, emitting an unpolarized quark
- via the $\boldsymbol{S}_{\perp}\times\boldsymbol{S}_{h\perp}$ correlation, emitting a longitudinally polarized quark

Cross section integrated over transverse momenta

$$\frac{d\sigma^{\mathrm{TFR}}}{dx_B\,dy\,d\zeta\;d\phi_S\,d\phi_{S_h}} = \frac{\alpha_{\mathrm{em}}^2}{\pi\,Q^2\,y}\,\Bigg\{\left(1-y+\frac{y^2}{2}\right)$$

$$\times \sum_a e_a^2\,\Big[M(x_B,\zeta)+S_\parallel\,S_{h\parallel}\,M_L^L(x_B,\zeta)+|\boldsymbol{S}_\perp|\,|\boldsymbol{S}_{h\perp}|\,M_T^T(x_B,\zeta)\,\cos(\phi_{S_h}-\phi_S)\Big]$$

$$+\lambda_l\,y\left(1-\frac{y}{2}\right)\sum_a e_a^2\,\Big[S_\parallel\,\Delta M_L(x_B,\zeta)+S_{h\parallel}\,\Delta M^L(x_B,\zeta)$$

$$+\;|\boldsymbol{S}_\perp|\,|\boldsymbol{S}_{h\perp}|\,\Delta M_T^T(x_B,\zeta)\,\sin(\phi_{S_h}-\phi_S)\Big]\Bigg\}\,.$$

The transverse spin terms are proportional to $\cos(\phi_{S_h}-\phi_S)$ and $\sin(\phi_{S_h}-\phi_S)$

For comparison, the transverse spin term in the CFR is

$$\frac{d\sigma^{\mathrm{CFR}}}{dx_B\,dy\,dz_h\;d\phi_S\,d\phi_{S_h}} \sim (1-y)\sum_a e_a^2\,|\boldsymbol{S}_\perp|\,|\boldsymbol{S}_{h\perp}|\,h_1(x_B)H_1(z_h)\,\cos(\phi_S+\phi_{S_h})$$

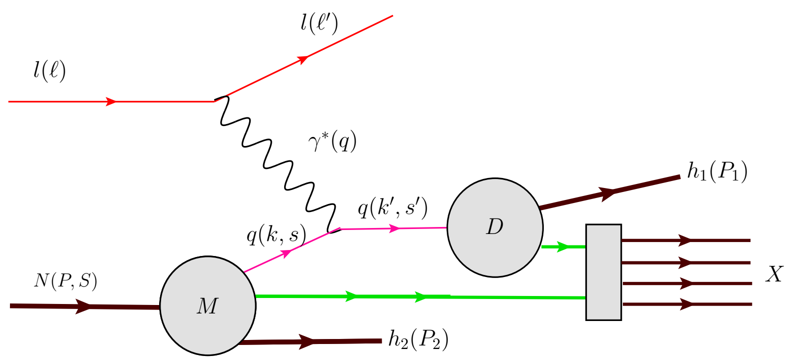
and involves the transversity distribution and fragmentation functions

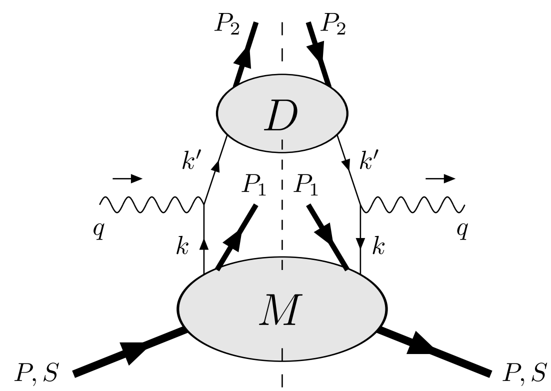
Extension to other processes

[Anselmino, VB & Kotzinian, in preparation]

Double hadron leptoproduction, with one hadron in the CFR, the other in the TFR

Probes the fracture functions of transversely polarized quarks

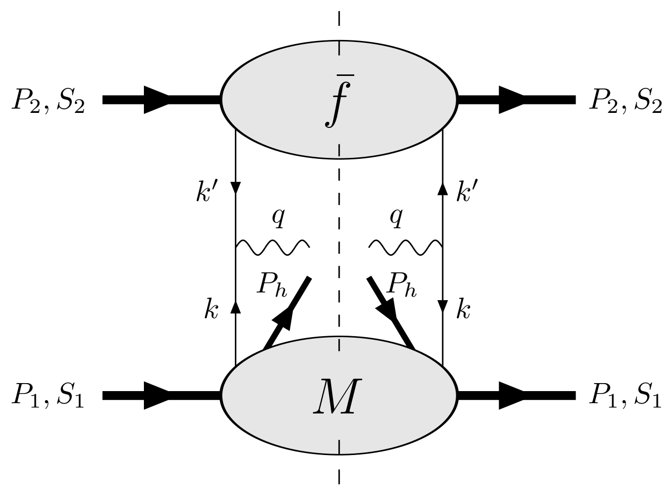




Fracture functions couple to the fragmentation functions D_1 and $H_1^\perp$ (Collins)

$$d\sigma \sim M \otimes D_1 + \Delta_T M \otimes H_1^\perp + \lambda_l \, \Delta M \otimes D_1$$

Associated hadron production in Drell-Yan processes



Fracture functions couple to quark (or antiquark) distribution functions

Conclusions and perspectives

- The production of spinless hadrons in the target fragmentation region is described by 16 polarized and transverse-momentum dependent fracture functions
- When a single hadron is detected in the TFR, only 4 fracture functions of unpolarized and longitudinally polarized quarks are probed.

  There is a Sivers-type modulation $\sin(\phi_h - \phi_S)$, no Collins-type $\sin(\phi_h + \phi_S)$
- Double hadron production in the CFR and in the TFR allows probing the transverse polarization of quarks
- To be studied:
  - Implications of the results on the TFR for present experiments
  - Phenomenology: simple models of fracture functions and predictions of TFR asymmetries
  - Possibilities to explore polarization and tranverse-momentum phenomena in the TFR at $p - p(\bar{p})$ machines and electron–ion collider