GPDs, Angular Momentum, and TMDs

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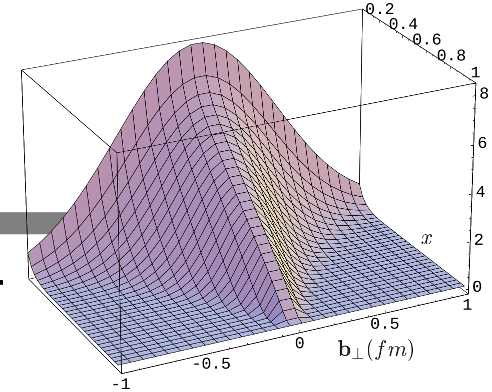
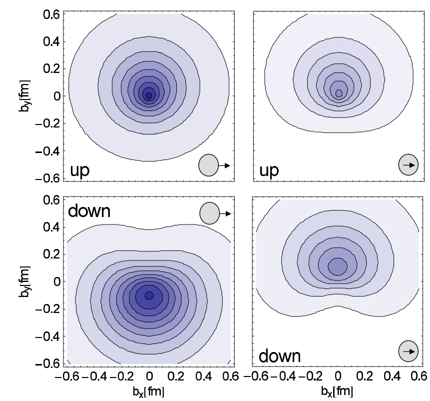
Outline



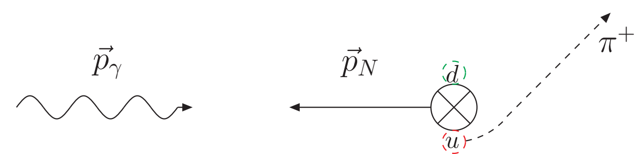
- Probabilistic interpretation of GPDs as Fourier trafo of impact parameter dependent PDFs
  - $H(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow q(x, \mathbf{b}_\perp)$
  - $E(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ distortion of PDFs when the target is $\perp$ polarized



- Q^2 evolution of DVCS $\longrightarrow$ GPDs

$\hookrightarrow$ SSA in SIDID/DY (Sivers & Boer-Mulders)

- 'Color decoherence' at large Q^2/small x



$\hookrightarrow$ twist-3 quark-gluon correlations:
$\int dx\, x^2 \bar{g}_2(x)$ & $\int dx\, x^2 \bar{e}(x)$

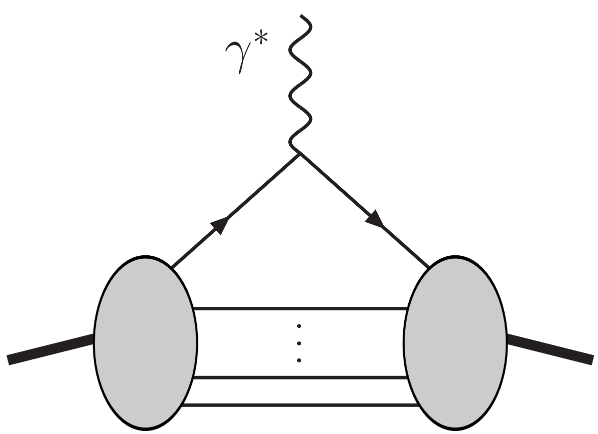
- Summary

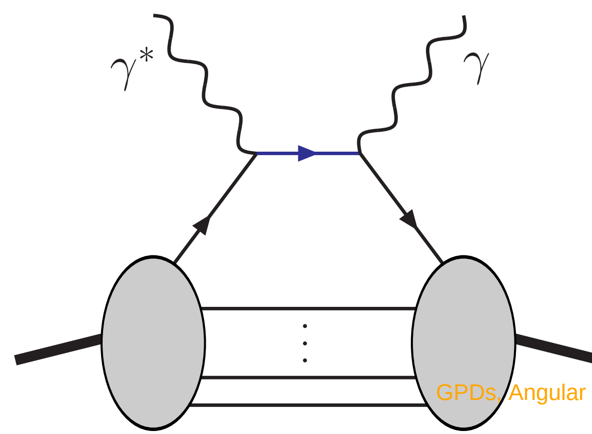
Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of t, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

$$\int dx H_q(x,\xi,t) = F_1^q(t) \qquad \int dx \tilde{H}_q(x,\xi,t) = G_A^q(t)$$
$$\int dx E_q(x,\xi,t) = F_2^q(t) \qquad \int dx \tilde{E}_q(x,\xi,t) = G_P^q(t),$$

  - x_i and x_f are the momentum fractions of the quark before and after the momentum transfer; $2\xi = x_f - x_i$
- GPDs can be probed in deeply virtual Compton scattering (DVCS) as well as deeply virtual meson production (DVMP)





Impact parameter dependent PDFs

- define $\perp$ localized state [D.Soper,PRD**15**, 1141 (1977)]

$$\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\lambda\right\rangle \equiv \mathcal{N}\int d^2\mathbf{p}_\perp \left|p^+,\mathbf{p}_\perp,\lambda\right\rangle$$

  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
  $\mathbf{R}_\perp \equiv \frac{1}{P^+}\int dx^- d^2\mathbf{x}_\perp\, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x,\mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi}\left\langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right|\bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp)\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

$\hookrightarrow$

$$\begin{aligned} q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} H(x,0,-\mathbf{\Delta}_\perp^2), \\ \Delta q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} \tilde{H}(x,0,-\mathbf{\Delta}_\perp^2), \end{aligned}$$

Impact parameter dependent PDFs

- No relativistic corrections (Galilean subgroup!)

$\hookrightarrow$ corrolary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free from relativistic corrections (Soper 1977; MB 2003)

- $q(x, \mathbf{b}_\perp)$ has probabilistic interpretation as number density
- $\xi = 0$ essential for probabilistic interpretation

$$\left\langle p^{+\prime}, 0_\perp \right| b^\dagger(x, \mathbf{b}_\perp) b(x, \mathbf{b}_\perp) \left| p^+, 0_\perp \right\rangle \sim \left| b(x, \mathbf{b}_\perp) \rangle | p^+, 0_\perp \right|^2$$

works only for $p^+ = p^{+\prime}$

- Reference point for IPDs is transverse center of (longitudinal) momentum $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{i,\perp}$

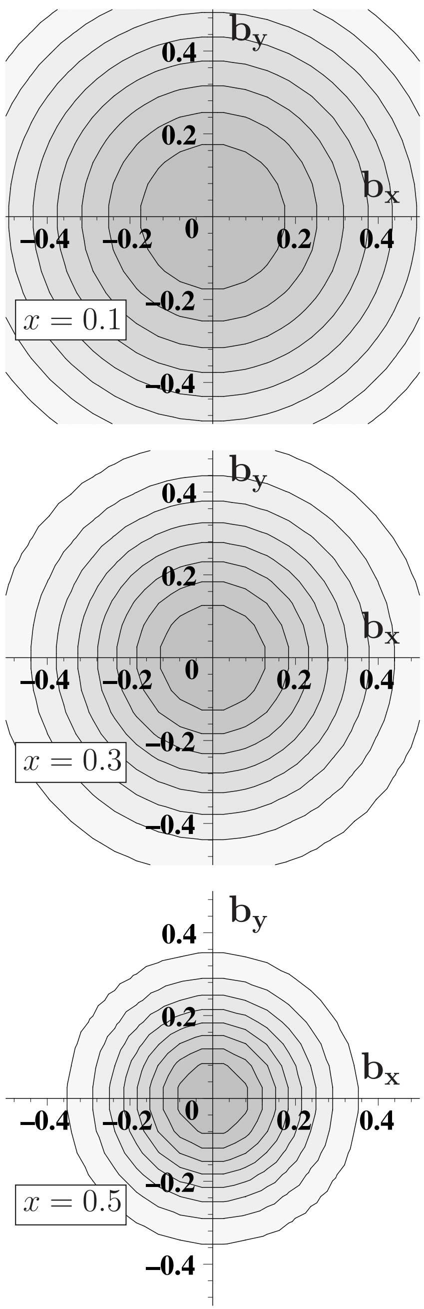
$\hookrightarrow$ for $x \to 1$, active quark 'becomes' COM, and $q(x, \mathbf{b}_\perp)$ must become very narrow (δ-function like)

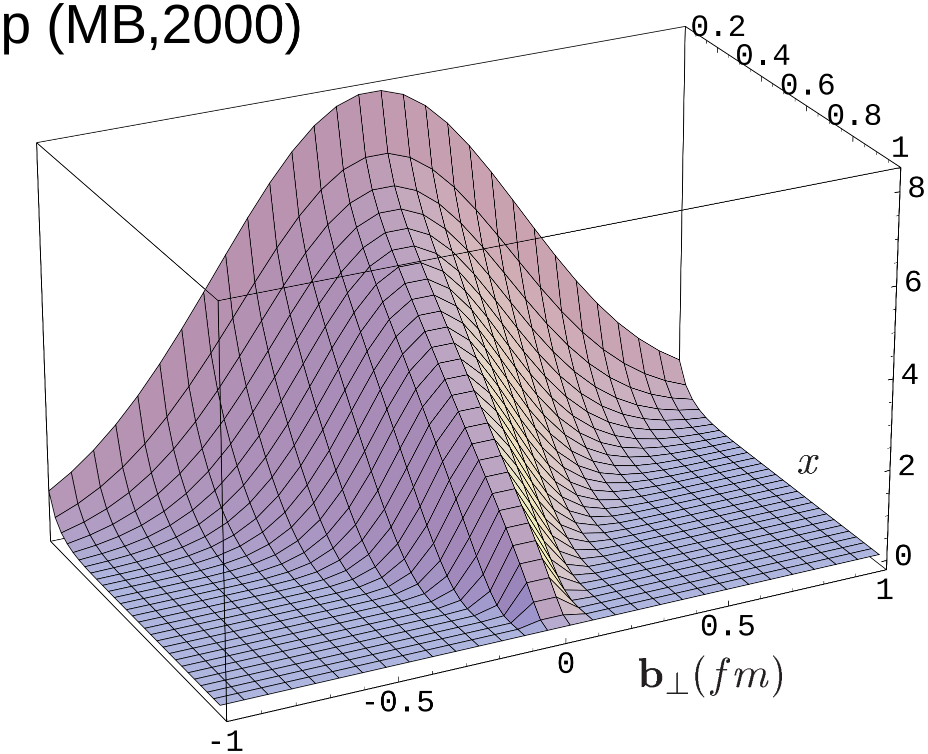
$\hookrightarrow$ $H(x, 0, -\mathbf{\Delta}_\perp^2)$ must become $\mathbf{\Delta}_\perp$ indep. as $x \to 1$ (MB, 2000)

$\hookrightarrow$ consistent with lattice results for first few moments

unpolarized p (MB,2000)

$q(x, \mathbf{b}_\perp)$ for unpol. p





$x =$ momentum fraction of the quark

$\vec{b} = \perp$ position of the quark

Transversely Deformed Distributions and $E(x,0,-\boldsymbol{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\uparrow\rangle = H(x,0,-\boldsymbol{\Delta}_\perp^2)$$
$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\downarrow\rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\boldsymbol{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF)
  $|X\rangle \equiv |p^+,\mathbf{R}_\perp = \mathbf{0}_\perp,\uparrow\rangle + |p^+,\mathbf{R}_\perp = \mathbf{0}_\perp,\downarrow\rangle$.

↪ unpolarized quark distribution for this state:

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} E(x,0,-\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\boldsymbol{\Delta}_\perp}$$

- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3 !
  [X.Ji, PRL **91**, 062001 (2003)]

Transversely Deformed PDFs and $E(x,0,-\boldsymbol{\Delta}_\perp^2)$

- $q(x,\mathbf{b}_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons !

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} E(x,0,-\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$$

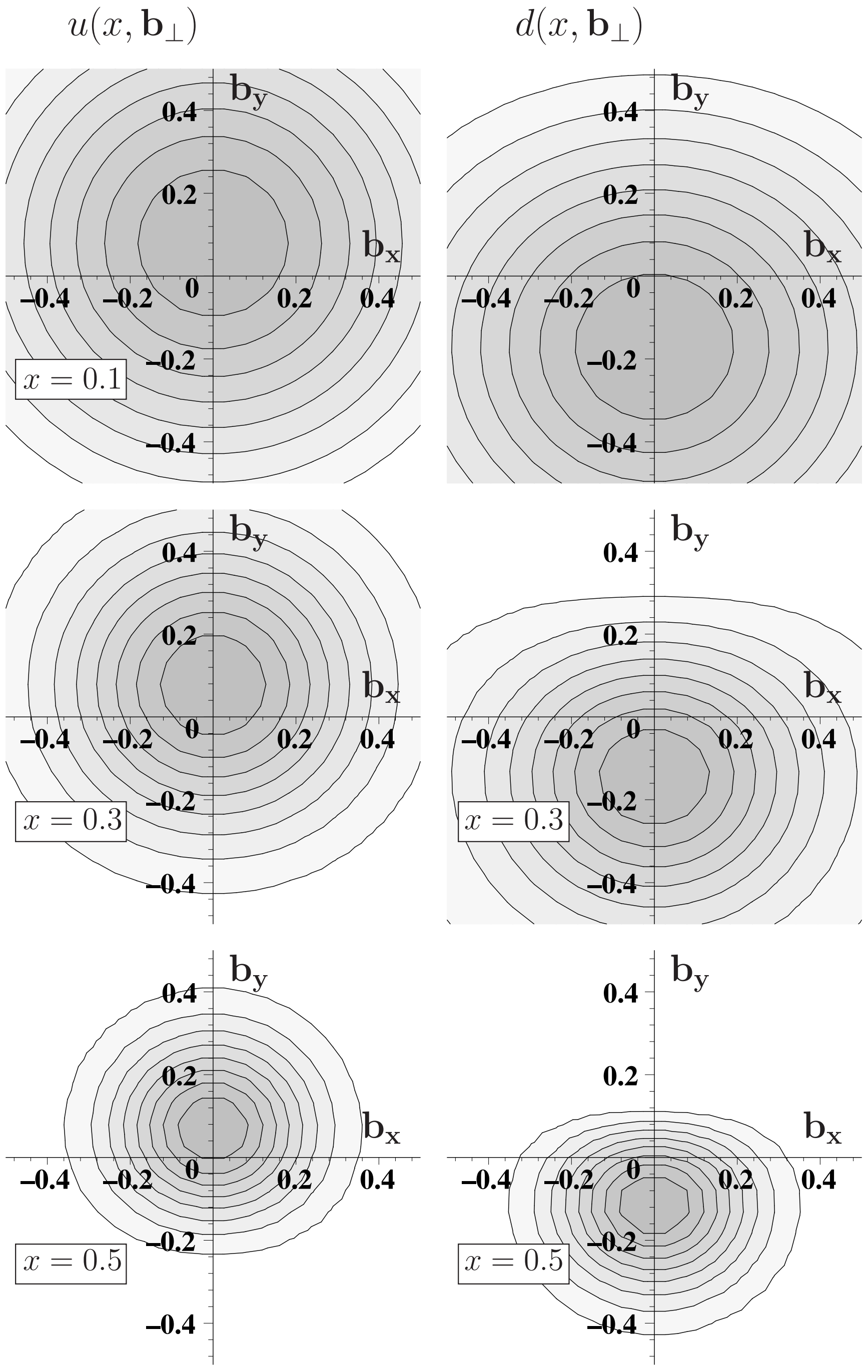
- mean $\perp$ deformation of flavor q ($\perp$ flavor dipole moment)

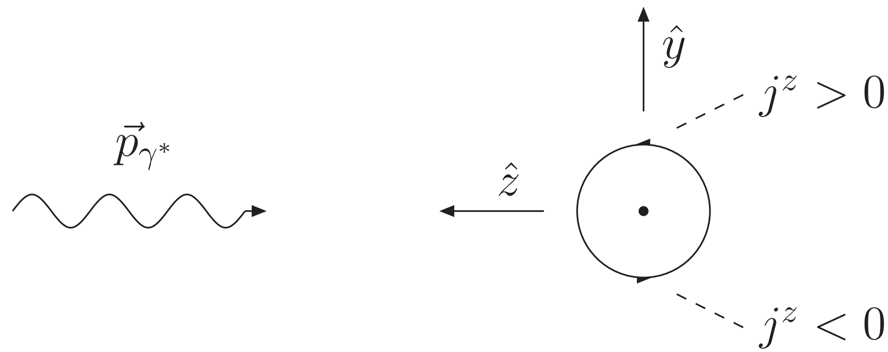
$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp) b_y = \frac{1}{2M}\int dx E_q(x,0,0) = \frac{\kappa_q^p}{2M}$$

- $\kappa^p = 1.913 = \frac{2}{3}\kappa_u^p - \frac{1}{3}\kappa_d^p + \ldots$

$\hookrightarrow$ neglecting strange (and heavier) quarks:
  - $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \quad \Rightarrow$ shift in $+\hat{y}$ direction
  - $\kappa_d^p = 2\kappa_n + \kappa_p = -2.033 \quad \Rightarrow$ shift in $-\hat{y}$ direction
  - for proton polarized in $+\hat{x}$ direction
  - $d_y^q = \mathcal{O}(\pm 0.2 fm)$

p polarized in $+\hat{x}$ direction (MB,2003)





- virtual photon ‘sees’ enhancement when quark currents point in direction opposite to photon momentum

$\hookrightarrow$ sideways shift of quark distributions

- sign & magnitude of shift (model-independently) predicted to be related to the proton/neutron anomalous magnetic moment!

The Ji-relation (poor man's derivation)

- What distinguishes the Ji-decomposition from other decompositions is the fact that L_q can be constrained by experiment:

$$\langle \vec{J}_q \rangle = \vec{S} \int_{-1}^{1} dx\, x \left[H_q(x,\xi,0) + E_q(x,\xi,0) \right]$$

(nucleon at rest; $\vec{S}$ is nucleon spin)

$\hookrightarrow$ $L_q^z = J_q^z - \frac{1}{2}\Delta q$

- derivation (MB-version):
  - consider nucleon state that is an eigenstate under rotation about the $\hat{x}$-axis (e.g. nucleon polarized in $\hat{x}$ direction with $\vec{p} = 0$ (wave packet if necessary)
  - for such a state, $\langle T_q^{00} y \rangle = 0 = \langle T_q^{zz} y \rangle$ and $\langle T_q^{0y} z \rangle = -\langle T_q^{0z} y \rangle$

  $\hookrightarrow$ $\langle T_q^{++} y \rangle = \langle T_q^{0y} z - T_q^{0z} y \rangle = \langle J_q^x \rangle$

  $\hookrightarrow$ relate 2^{nd} moment of $\perp$ flavor dipole moment to J_q^x

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  - effect sum of two effects:
    - $\langle T^{++}y\rangle$ for a point-like transversely polarized nucleon
    - $\langle T_q^{++}y\rangle$ for a quark relative to the center of momentum of a transversely polarized nucleon
  - 2^{nd} moment of $\perp$ flavor dipole moment for point-like nucleon

$$\psi = \begin{pmatrix} f(r) \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} f(r) \end{pmatrix} \chi \qquad \text{with} \qquad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The Ji-relation (poor man's derivation)

- derivation (MB-version):
  - $T_q^{0z} = i\bar{q}\left(\gamma^0\partial^z + \gamma^z\partial^0\right)q$
  - since $\psi^\dagger\partial_z\psi$ is even under $y \rightarrow -y$, $i\bar{q}\gamma^0\partial^z q$ does not contribute to $\langle T^{0z}y\rangle$

  $\hookrightarrow$ using $i\partial_0\psi = E\psi$, one finds

$$\begin{aligned}\langle T^{0z}b_y\rangle &= E\int d^3r\psi^\dagger\gamma^0\gamma^z\psi y = E\int d^3r\psi^\dagger\begin{pmatrix}0 & \sigma^z\\ \sigma^z & 0\end{pmatrix}\psi y\\ &= \frac{2E}{E+M}\int d^3r\chi^\dagger\sigma^z\sigma^y\chi f(r)(-i)\partial^y f(r)y = \frac{E}{E+M}\int d^3r f^2(r)\end{aligned}$$

  - consider nucleon state with $\vec{p}=0$, i.e. $E=M$ & $\int d^3r f^2(r) = 1$

  $\hookrightarrow$ 2^{nd} moment of $\perp$ flavor dipole moment $\langle T_q^{++}y\rangle = \langle T^{0z}b_y\rangle = \frac{1}{2}$

  $\hookrightarrow$ 'overall shift' of nucleon COM yields contribution $\frac{1}{2}\int dx\, xH_q(x,0,0)$ to $\langle T_q^{++}y\rangle$

The Ji-relation (poor man's derivation)

- spherically symmetric wave packet for Dirac particle with $J_x = \frac{1}{2}$ centered around the origin has $\perp$ center of momentum $\frac{1}{M}\langle T_q^{++} b_y\rangle$ not at origin, but at $\frac{1}{2M}$!
- consistent with

$$\frac{1}{2} = \langle J_x \rangle = \langle \left(T^{0z}b^y - T^{0y}b^z\right)\rangle = 2\langle T^{0z}b^y\rangle = \langle T^{++}b^y\rangle$$

  - 'overall shift of $\perp$ COM yields $\langle T_q^{++} b_y\rangle = \frac{1}{2}\int dx\, xH_q(x,0,0)$
  - intrinsic distortion adds $\frac{1}{2}\int dx\, xE_q(x,0,0)$ to that

$\hookrightarrow$

$$\langle \vec{J}_q \rangle = \vec{S}\int_{-1}^{1} dx\, x\left[H_q(x,\xi,0) + E_q(x,\xi,0)\right]$$

$\mathcal{A}_{DVCS} \overset{?}{\rightsquigarrow} GPDs$

- Ji relation $J_q = \int_0^1 dx x \left[H(x,\xi,0) + E(x,\xi,0)\right]$ requires $GPDs(x,\xi,0)$ for (common) fixed ξ for all x
- transverse imaging requires GPDs for $\xi = 0$
- $\mathcal{A}_{DVCS}(\xi,t) \longrightarrow \int_{-1}^1 dx \frac{GPD^{(+)}(x,\xi,t)}{x-\xi+i\varepsilon}$
  - ξ longitudinal momentum transfer on the target $\xi = \frac{p^{+\prime}-p^+}{p^{+\prime}+p^+}$
  - x (average) momentum fraction of the active quark $x = \frac{k^{+\prime}+p^+}{p^{+\prime}+p^+}$
- $\Im\mathcal{A}_{DVCS}(\xi,t) \longrightarrow GPD^{(+)}(\xi,\xi,t)$
  - only sensitive to 'diagonal' $x = \xi$
  - limited ξ range, e.g. $-t = \frac{4\xi^2 M^2 + \mathbf{\Delta}_\perp^2}{1-\xi^2}$ implies $\xi > \xi_{min}$ for fixed t
- $\Re\mathcal{A}_{DVCS}(\xi,t) \longrightarrow \int_{-1}^1 dx \frac{GPD^{(+)}(x,\xi,t)}{x-\xi}$ probes GPDs off the diagonal, but ...

$\mathcal{A}(\xi,t) \longleftrightarrow GPD^{(+)}(\xi,\xi,t),\ \Delta(t)$

- (Anikin, Teryaev, Diehl, Ivanov, Brodsky, Szczepaniak, ...): dispersion relation for DVCS amplitude

$$\Re\mathcal{A}(\nu,t,Q^2) = \frac{\nu^2}{\pi}\int_0^\infty \frac{d\nu'^2}{\nu'^2}\frac{\Im\mathcal{A}(\nu',t,Q^2)}{\nu'^2-\nu^2} + \Delta(t,Q^2)$$

- In combination with LO factorization ($\mathcal{A} = \int_{-1}^1 dx \frac{H(x,\xi,t,Q^2)}{x-\xi+i\varepsilon}$)

$$\Re\mathcal{A}(\xi,t,Q^2) = \int_{-1}^1 dx \frac{H(x,\xi,t,Q^2)}{x-\xi} = \int_{-1}^1 dx \frac{H(x,x,t,Q^2)}{x-\xi} + \Delta(t,Q^2)$$

- Earlier derived from polynomiality (Goeke,Polyakov,Vanderhaeghen)

$\hookrightarrow$ 'Condense' information contained in $\mathcal{A}_{DVCS}$ (fixed Q^2) into $GPD(x,x,t,Q^2)$ & $\Delta(t,Q^2)$

$$\mathcal{A}(\xi,t,Q^2) \leftrightarrow \begin{cases} GPD(\xi,\xi,t,Q^2) \\ \Delta(t,Q^2) \end{cases}$$

$\mathcal{A}(\xi,t) \longleftrightarrow GPD(\xi,\xi,t)$, $\Delta(t)$

- $\Re\mathcal{A}(\xi,t) = \int_{-1}^{1} dx \frac{H(x,\xi,t)}{x-\xi}$ probes GPDs for $x \neq \xi$, but new information can be 'projected back' onto diagonal plus D-term!
- remaining 'new' (not in $\Im\mathcal{A}$) info on GPDs after 'projecting back' onto diagonal:
  - D-form factor
  - constraints from $\int dx \frac{GPD(x,x,t)}{x-\xi}$ on $GPD(\xi,\xi,t)$ in kinematically inaccessible range $\xi < \xi_{min}$ & $\xi > \xi_{max}$
- Information away from diagonal ($x = \xi$): Q^2 evolution: changes x distribution in a known way for fixed ξ

DVCS $\rightsquigarrow GPD(x,\xi,t)$ (a mathematical exercise)

$$GPD(x,\xi,t,Q^2) = (1-x^2)\sum_{n=0}^{\infty} C_n^{3/2}(x) \sum_{m=0(even)}^{n} a_{nm}(\xi)\mathcal{C}_{n-m}(\xi,t,Q^2)$$

- $C_n^{3/2}(x)$ Gegenbauer polynomials; $a_{nm}(\xi)$ known polynomial
- $\mathcal{C}_k(\xi,t,Q^2)$ unknown, but evolve with known power $\sim \gamma_k$ of $\alpha_s(Q^2)$
- consider $x=\xi$ (relabel: $k=n-m$)

$$GPD(\xi,\xi,t,Q^2) = (1-\xi^2)\sum_{k=0}^{\infty}\mathcal{C}_k(\xi,t,Q^2) f_k(\xi) \qquad (1)$$

with $f_k(\xi) = \sum_{m=0(even)}^{\infty} a_{m+k,m}(\xi) C_{m+k}^{3/2}(\xi)$ known function.

- for fixed ξ, each term in (1) evolves with different γ_k

$\hookrightarrow$ from Q^2-dependence of $GPD(\xi,\xi,t,Q^2)$ (fixed ξ and t) over 'wide' range of Q^2, in principle possible to determine $\mathcal{C}_k(\xi,t,Q^2)$

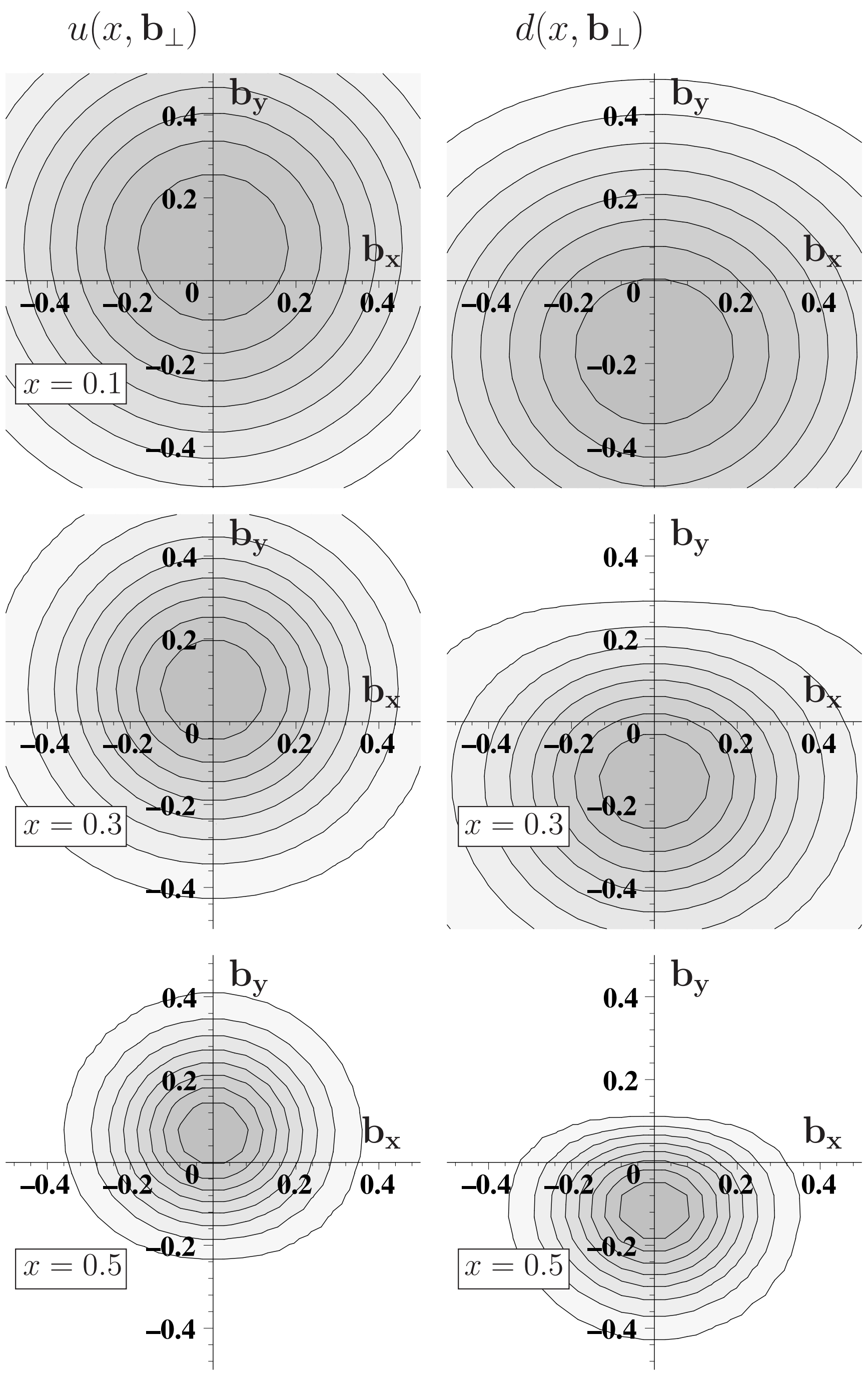
$\hookrightarrow$ $GPD(x,\xi,t,Q^2)$ for $x\neq\xi$ model-independently!

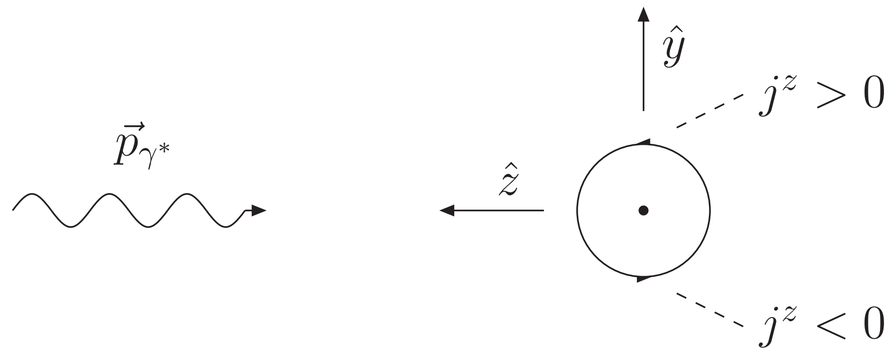
DVCS $\rightsquigarrow GPD(x,\xi,t)$ (a mathematical exercise)

$\hookrightarrow$ from Q^2-dependence of $GPD(\xi,\xi,t,Q^2)$ (fixed ξ and t) over 'wide' range of Q^2, in principle possible to determine $GPD(x,\xi,t,Q^2)$ for $x \neq \xi$ model-independently!

- issues:
  - higher twist 'contamination'
  - higher order evolution kernel
  - limited coverage in Q^2 (here, an EIC would be a giant leap!) and ξ
  - singular shape of GPDs (cusp at $x = \xi$) requires many polynomials in Gegenbauer expansion

p polarized in $+\hat{x}$ direction (MB,2003)





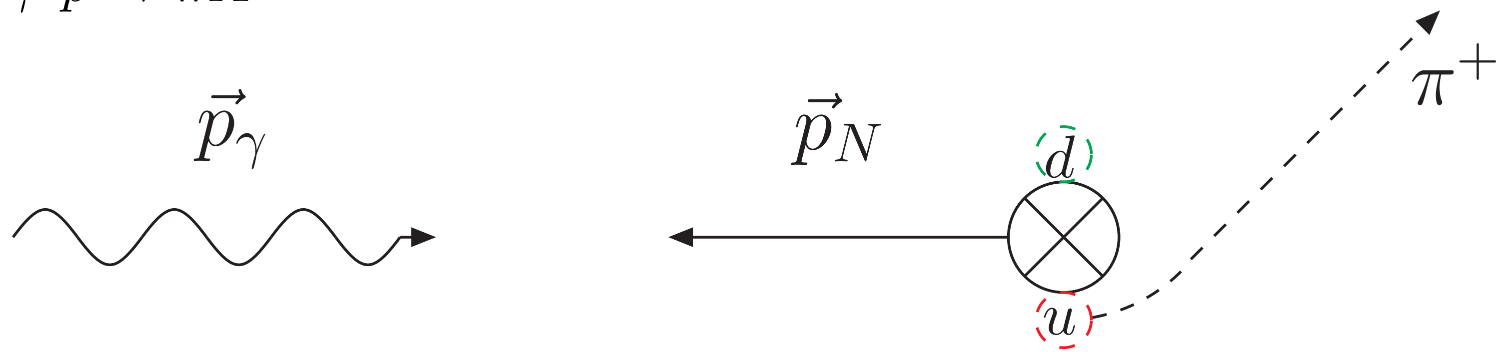
- virtual photon 'sees' enhancement when quark currents point in direction opposite to photon momentum

$\hookrightarrow$ sideways shift of quark distributions

- sign & magnitude of shift (model-independently) predicted to be related to the proton/neutron anomalous magnetic moment!

'Chromodynamik Lensing: GPD $\longleftrightarrow$ SSA

- example: $\gamma^* p \to \pi X$



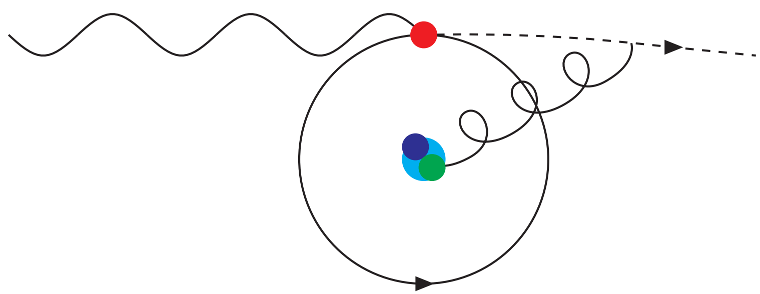
- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

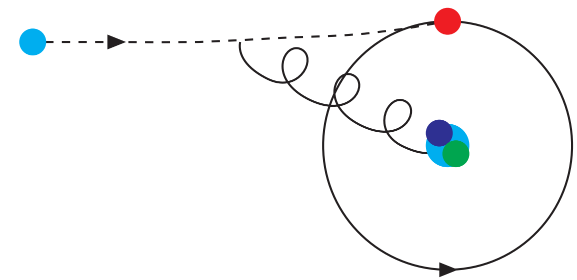
$\hookrightarrow$ correlation between sign of κ_q^p and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q^p$

- $f_{1T}^{\perp q} \sim -\kappa_q^p$ confirmed by HERMES data (also consistent with COMPASS deuteron data $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$)

$f_{1T}^{\perp}(x,\mathbf{k}_\perp)_{DY} = -f_{1T}^{\perp}(x,\mathbf{k}_\perp)_{SIDIS}$



a)



b)

- time reversal: FSI $\leftrightarrow$ ISI

SIDIS: compare FSI for 'red' q that is being knocked out with ISI for an anti-red $\bar{q}$ that is about to annihilate that bound q

$\hookrightarrow$ FSI for knocked out q is attractive

DY: nucleon is color singlet $\rightarrow$ when to-be-annihilated q is 'red', the spectators must be anti-red

$\hookrightarrow$ ISI with spectators is repulsive

- test of this relation is a test of TMD factorization

Color Decoherence and Evolution of SSAs

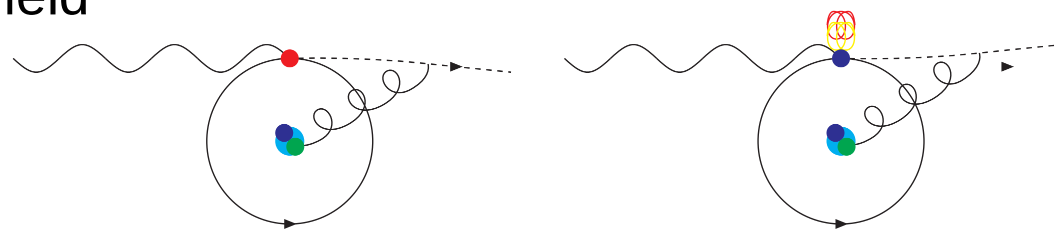
- LO Evolution equations for SSAs known (Qiu & Sterman + others), but evolution of Sivers function $f_{1T}^{\perp}$ requires 'off-diagonal' quark-gluon correlations ($f_{1T}^{\perp}$ related to 'diagonal' quark-gluon correlations

$\hookrightarrow$ Measurement of $f_{1T}^{\perp}$ at one Q^2 not sufficient to predict $f_{1T}^{\perp}$ at higher Q^2!

- What to expect?

Color Decoherence and Evolution of SSAs

- 'Chromodynamic lensing' mechanism for $\perp$ SSA requires long range coherence of color field



- QCD-evolution destroys color coherence:
  - consider 'red' quark
  - $\hookrightarrow$ attracted to 'anti-red' spectators
  - after 'dressing' itself with a gluon, previously 'red' quark more likely to be 'blue' or 'green'
  - $\hookrightarrow$ after dressing, attraction to 'far away' spectators mostly gone
  - only attracted to very close by (at high Q^2) gluon from dressing
  - $\hookrightarrow$ expect no $\perp$ SSA from long-range color fields after dressing!!!
- at high Q^2, quarks at low x likely to have dressed themselves with perturbative gluon

$\hookrightarrow$ fraction of quarks at low x/high Q^2 that still 'sees' long range coherent color field significantly decreased

$\hookrightarrow$ 'Chromodynamic lensing' mechanism for $\perp$ SSA suppressed!

Quark-Gluon Correlations (Introduction)

- (longitudinally) polarized polarized DIS at leading twist $\longrightarrow$ 'polarized quark distribution' $g_1^q(x) = q^\uparrow(x) + \bar{q}^\uparrow(x) - q_\downarrow(x) - \bar{q}_\downarrow(x)$
- $\frac{1}{Q^2}$-corrections to X-section involve 'higher-twist' distribution functions, such as $g_2(x)$

$$\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$$

- $g_2(x)$ involves quark-gluon correlations and does not have a parton interpretation as difference between number densities
- for $\perp$ polarized target, g_1 and g_2 contribute equally to σ_{LT}

$$\sigma_{LT} \propto g_T \equiv g_1 + g_2$$

$\hookrightarrow$ 'clean' separation between higher order corrections to leading twist (g_1) and higher twist effects (g_2)

- what can one learn from g_2?

Quark-Gluon Correlations (QCD analysis)

- $g_2(x) = g_2^{WW}(x) + \bar{g}_2(x)$, with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$
- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

- $\sqrt{2} G^{+y} \equiv G^{0y} + G^{zy} = -E^y + B^x$
- sometimes called color-electric and magnetic polarizabilities
  $2M^2 \vec{S} \chi_E = \left\langle P,S \left| \vec{j}_a \times \vec{E}_a \right| P,S \right\rangle$ & $2M^2 \vec{S} \chi_B = \left\langle P,S \left| j_a^0 \vec{B}_a \right| P,S \right\rangle$
  with $d_2 = \frac{1}{4}\left(\chi_E + 2\chi_M\right)$ — but these names are misleading!

Quark-Gluon Correlations (Interpretation)

- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

- QED: $\bar{q}(0)eF^{+y}(0)\gamma^+q(0)$ correlator between quark density $\bar{q}\gamma^+q$ and ($\hat{y}$-component of the) Lorentz-force

$$F^y = e\left[\vec{E} + \vec{v} \times \vec{B}\right]^y = e\left(E^y - B^x\right) = -e\left(F^{0y} + F^{zy}\right) = -e\sqrt{2}F^{+y}.$$

for charged paricle moving with $\vec{v} = (0,0,-1)$ in the $-\hat{z}$ direction

$\hookrightarrow$ matrix element of $\bar{q}(0)eF^{+y}(0)\gamma^+q(0)$ yields γ^+ density (density relevant for DIS in Bj limit!) weighted with the Lorentz force that a charged particle with $\vec{v} = (0,0,-1)$ would experience at that point

$\hookrightarrow$ d_2 a measure for the **color Lorentz force** acting on the struck quark in SIDIS in the instant **after being hit by the virtual photon**

$$\langle F^y(0) \rangle = -2M^2 d_2 \qquad \text{(rest frame; } S^x = 1\text{)}$$

Quark-Gluon Correlations (Interpretation)

- x^2-moment of twist-4 polarized PDF $g_3(x)$

  $\int dx x^2 g_3(x) \rightsquigarrow \left\langle P,S \middle| \bar{q}(0) g\tilde{G}^{\mu\nu}(0)\gamma_\nu q(0) \middle| P,S \right\rangle \sim f_2$

↪ different linear combination $f_2 = \chi_E - \chi_B$ of χ_E and χ_M

↪ combine with $d_2 \Rightarrow$ disentangle electric and magnetic force

- What should one expect (sign/magnitude)?
  - $\kappa_q^p \longrightarrow$ signs of deformation (u/d quarks in $\pm\hat{y}$ direction for proton polarized in $+\hat{x}$ direction $\longrightarrow$ expect force in $\mp\hat{y}$

  ↪ d_2 positive/negative for u/d quarks in proton

  - large N_C: $d_2^{u/p} = -d_2^{d/p}$ (consistent with $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$)
  - $F^y = -2M^2 d_2 = -10\frac{\text{GeV}}{fm} d_2$ $\Rightarrow$ expect $|d_2| \ll 1$

- lattice (Göckeler et al., 2005): $d_2^u \approx 0.010$ and $d_2^d \approx -0.0056$

↪ $\langle F_u^y(0)\rangle \approx -100\frac{\text{MeV}}{fm}$ $\langle F_d^y(0)\rangle \approx 56\frac{\text{MeV}}{fm}$

- x^2-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ **transverse force on transversely polarized quark in unpolarized target** ($\leftrightarrow$ Boer-Mulders $h_1^\perp$)

Summary

- GPDs $\overset{FT}{\longleftrightarrow}$ IPDs (impact parameter dependent PDFs)
- $E^q(x,0,-\mathbf{\Delta}_\perp^2) \leftrightarrow \kappa_{q/p}$ (contribution from quark flavor q to anom magnetic moment)
- $E^q(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow$
- GPDs for $x \neq \xi$ from Q^2 evolution $\perp$ deformation of PDFs for $\perp$ polarized target
- $\perp$ deformation $\leftrightarrow$ (sign of) SSA (Sivers; Boer-Mulders)
- Q^2 evolution $\longrightarrow$ Color decoherence
- $\perp$ deformation $\leftrightarrow$ (sign of) quark-gluon correlations ($\int dx\, x^2 \bar{g}_2(x)$, $\int dx\, x^2 \bar{e}(x)$)