Review on Fragmentation Functions

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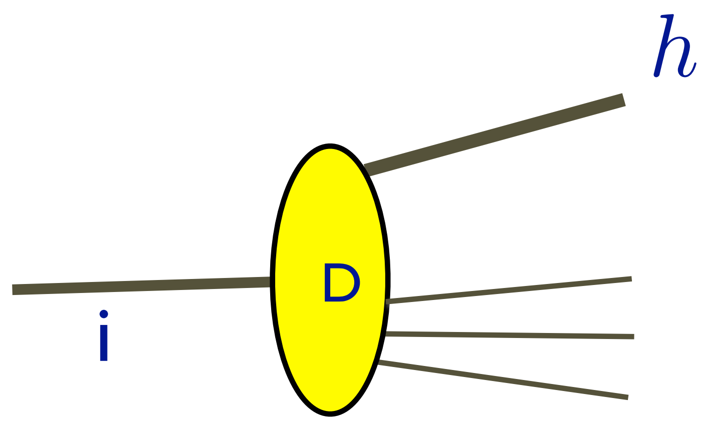
Fragmentation functions

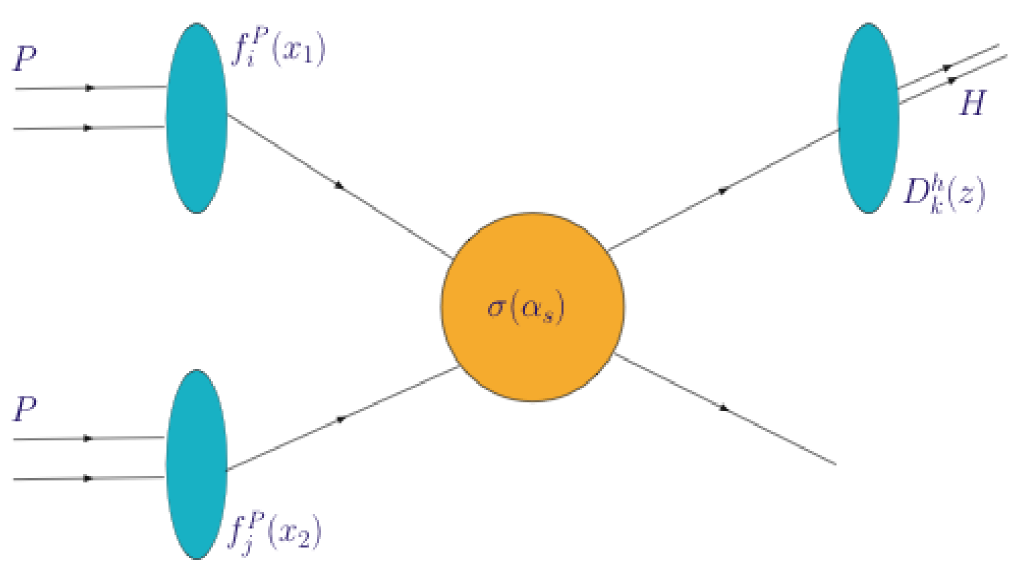
- Describe the collinear transition of a parton *i* into a massless hadron *h* carrying momentum fraction *z*



Represent the probability that a parton hadronizes

- Time-like version of PDFs : information on structure of the nucleon

- Relevant any time a hadron is produced in high energy collisions



Why care about hadrons?

- Hadrons : dominant final state in pp and ep collisions
- Heavy Ion Collisions : QGP by suppression in hadron-production
- The flavor-separation of polarized parton distributions depends crucially on SIDIS results (Hermes-SMC-Compass)
- Extraction of polarized gluon density depends much on PHENIX and STAR pion data

Different FFs → very different results for spin dependent distributions

DdeF, Navarro, Sassot (2005) wrong D_f → wrong Δf

- Look into "unconventional targets" : Lambda polarized fragmentation functions
- Key elements for Target Fragmentation: evolution of fracture functions
- Fundamental role in understanding single spin asymmetries, transversity...
- TH challenge: NLO pQCD fails to describe fixed target experiments!

Some properties of $D_i^h(z,\mu^2)$

- ***Non-perturbative*** objects but universal : *can be obtained from global analysis!*
- ***Depend*** on energy fraction $z = \frac{E_h}{E_i}$
- ***Depend*** on factorization scale μ^2 : described by AP equations

$$\frac{d}{d\ln\mu^2} D_q^h(z,\mu^2) = \left[P_{qq}\otimes D_q^h + P_{gq} D_g^h\right](z,\mu^2)$$

NLO
partial NNLO: Moch, Vogt

Kernels very singular at small z :negative FFs ! $P_{gg} \to \frac{2C_A}{z} - \frac{\alpha_s}{2\pi}\frac{4C_A^2}{z}\ln^2 z$

- Anyway, mass and higher twist effects not trivial : no systematic treatment

Limits use to z > 0.05 / 0.1

- ***Energy- Momentum conservation*** sum rule $\sum_h \int_0^1 z\, D_i^h(z,\mu^2)\, dz = 1$

without much practical use

Fragmentation Functions

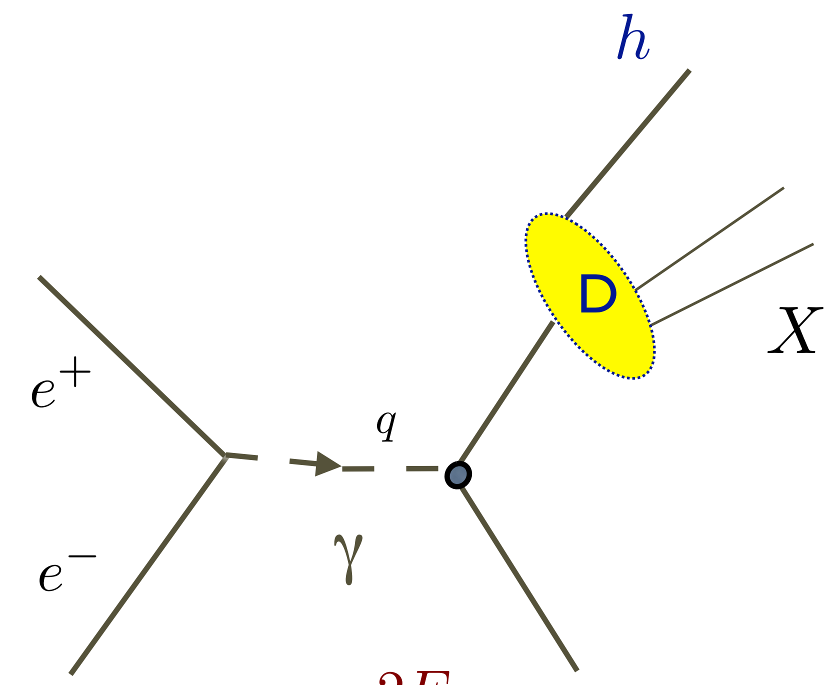
How to obtain them?

Global Analysis : look at different observables

e^+e^- *SIDIS*   *pp*

e+e- (SIA) single-inclusive annihilation

$$e^+e^- \to (\gamma,\, Z) \to H$$



$$z = \frac{2E_h}{Q}$$

- Cross section depends on two structure functions

$$\frac{1}{\sigma_{tot}}\frac{d\sigma^H}{dz} = \frac{\sigma_0}{\sum_q \hat{e}_q^2}\left[2\,F_1^H(z,Q^2) + F_L^H(z,Q^2)\right]$$

$$\sigma_{tot} = \sum_q \hat{e}_q^2\,\sigma_0\left[1 + \frac{\alpha_s(Q^2)}{\pi}\right]$$

- At NLO they can be written as

$$2F_1^H(z,Q^2) = \sum_q \hat{e}_q^2\left\{\left[D_q^H(z,Q^2) + D_{\bar{q}}^H(z,Q^2)\right] + \frac{\alpha_s(Q^2)}{2\pi}\left[C_q^1 \otimes (D_q^H + D_{\bar{q}}^H) + C_g^1 \otimes D_g^H\right](z,Q^2)\right\}$$

Known up to NNLO

Coeff. functions
Calculable in pQCD

- Notice that SIA can only give information on the sum $D_q^H(z,Q^2) + D_{\bar{q}}^H(z,Q^2)$

e+e- (SIA) single-inclusive annihilation

Advantages

- Very precise data from LEP/SLD
- Heavy quark tagged data
- Only fragmentation functions enter (clean process)

Disadvantages

- SIA data dominated by precise LEP/SLD measurements at M_Z
- weak scale dependence (bad resolution for g fragmentation)
- mostly determine "singlet" distribution (high precision)

$$\Sigma = D_u + D_{\bar{u}} + D_d + D_{\bar{d}} + D_s + D_{\bar{s}} + D_c + D_{\bar{c}} + D_b + D_{\bar{b}}$$

- OPAL offers "flavor separated data": no trivial interpretation at NLO
- not precise at large z (relevant for pp collisions)
- Can not separate $D_q^h(z,Q^2)$ from $D_{\bar{q}}^h(z,Q^2)$

SIDIS

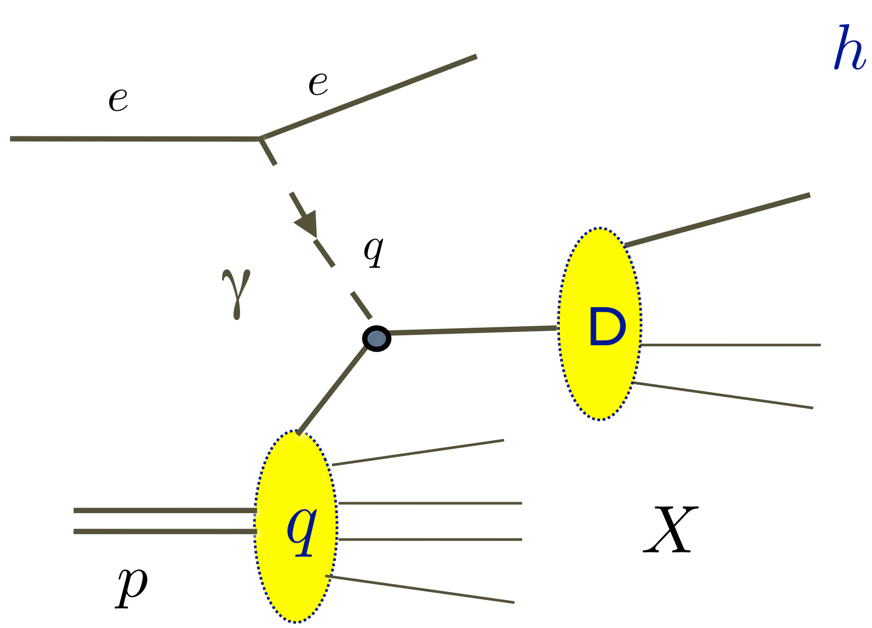
current fragmentation

- Distributions in *x* and *z*

$$\frac{d\sigma^H}{dx\, dQ^2\, dz_H} = \frac{2\pi\alpha^2}{xsQ^2}\left[\frac{1+(1-y)^2}{y} 2F_1^H(x, z_H, Q^2) + \frac{2(1-y)}{y} F_L^H(x, z_H, Q^2)\right]$$

$$x = \frac{Q^2}{2P^N \cdot q} \qquad y = \frac{Q^2}{sx}$$

"scaling variable" $\quad z_H \equiv \frac{P^H \cdot P^N}{P^N \cdot q}$



at LO $\quad 2F_1^H(x, z_H, Q^2) = \sum_{q,\bar{q}} e_q^2 \quad q(x, Q^2) D_q^H(z_H, Q^2)$ expression known up to NLO

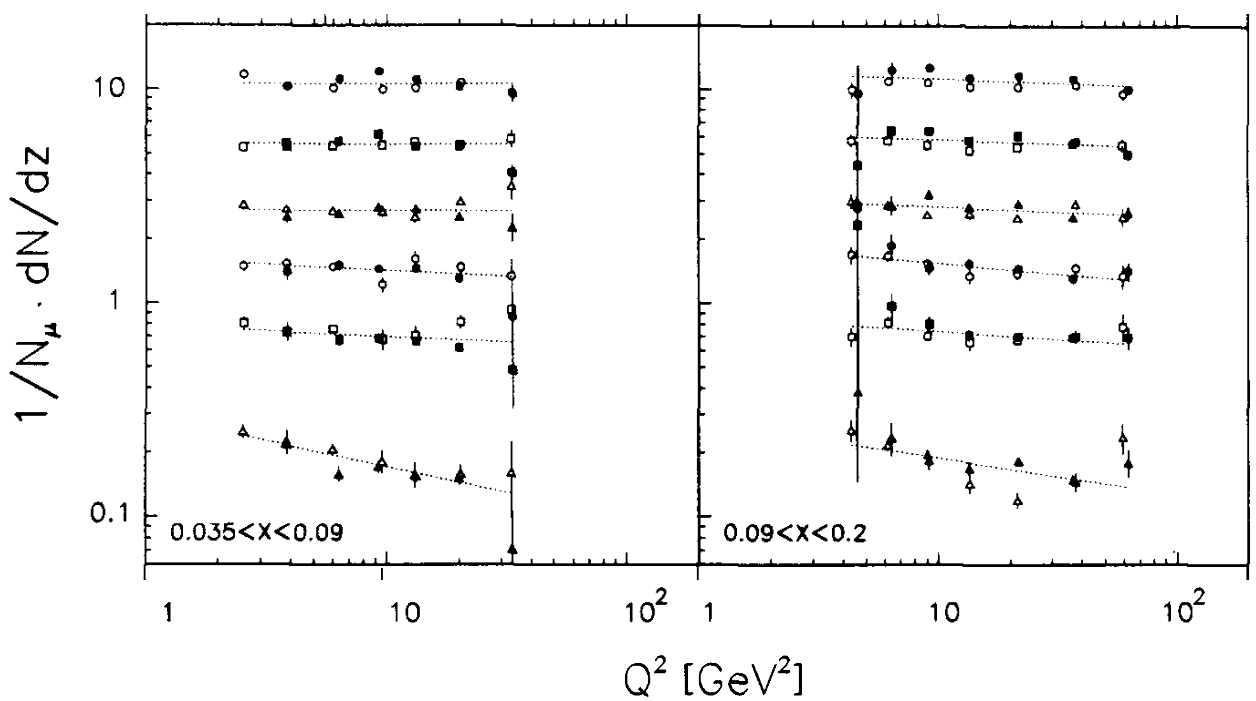
"effective charge" : different for quarks and antiquarks!
Most valuable observable for flavor/charge separation!
Different targets (and x!) can be used

SIDIS

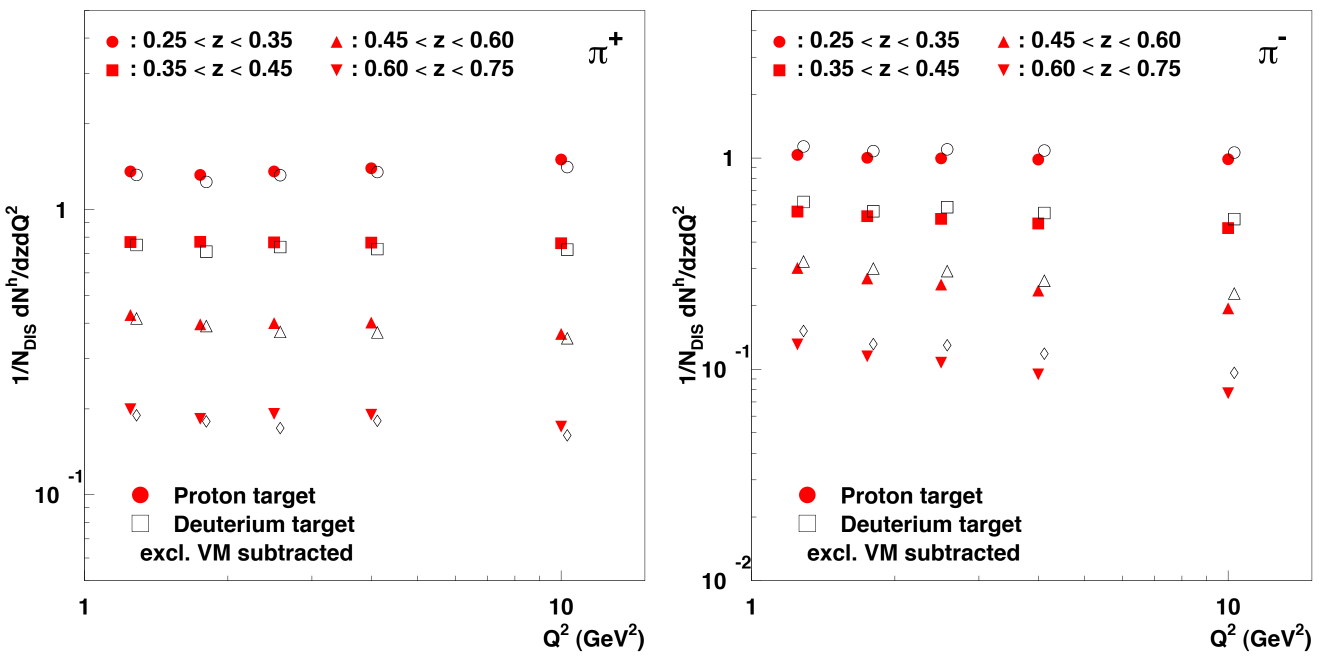
- charged separated data for π^+, π^-, K^+, K^- Hermes (yet preliminary)

h^+, h^- EMC

EMC (h)



Hermes



Other possible observables in DIS:

- photoproduction (involves photon pdfs)
- transverse momentum distributions (HERA, EXP+TH uncertainties)

} check

SIDIS

Advantages :

- **allows flavor/charge separation**
- larger z
- smaller Q^2, improves scale coverage in evolution : D_g
- relevant for Hermes and Compass kinematics (spin physics)
- almost no HQ fragmentation

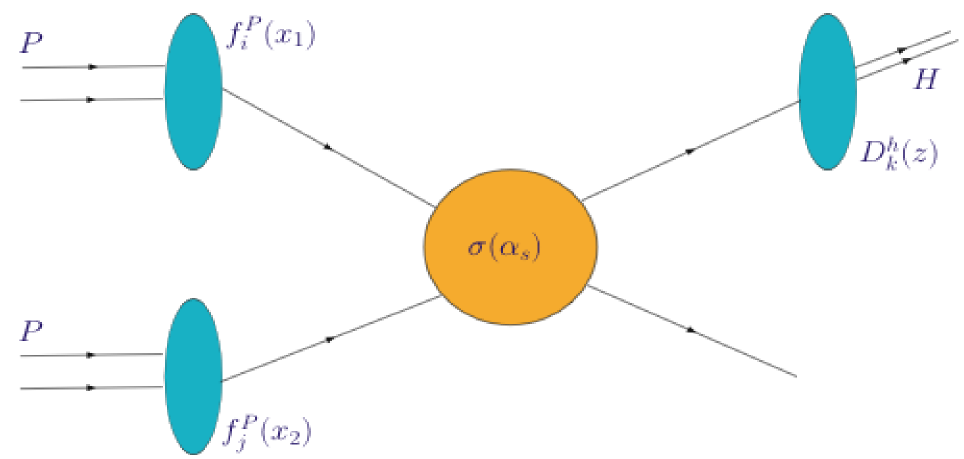
Disadvantages :

- introduce "dependence" on pdfs (but unpolarized pdfs well constrained from DIS in the same kinematical range, s?)
- non-perturbative corrections at small Q^2?
- almost no HQ fragmentation

Hadron-Hadron collisions

Transverse momentum distribution

hard scale



$q\bar{q} \longrightarrow g$

$qq \longrightarrow q$

$qg \longrightarrow g$

$qg \longrightarrow q$

$gg \longrightarrow q$

$gg \longrightarrow g$

$$d\sigma(pp \to h) = \sum_{i,j,k} \int_0^1 dx_1\, f_i^P(x_1, \mu_{FI}^2) \int_0^1 dx_2\, f_j^P(x_2, \mu_{FI}^2) \int_0^1 dz\, D_k^h(z, \mu_{FF}^2)\, d\hat{\sigma}(ij \to k)$$

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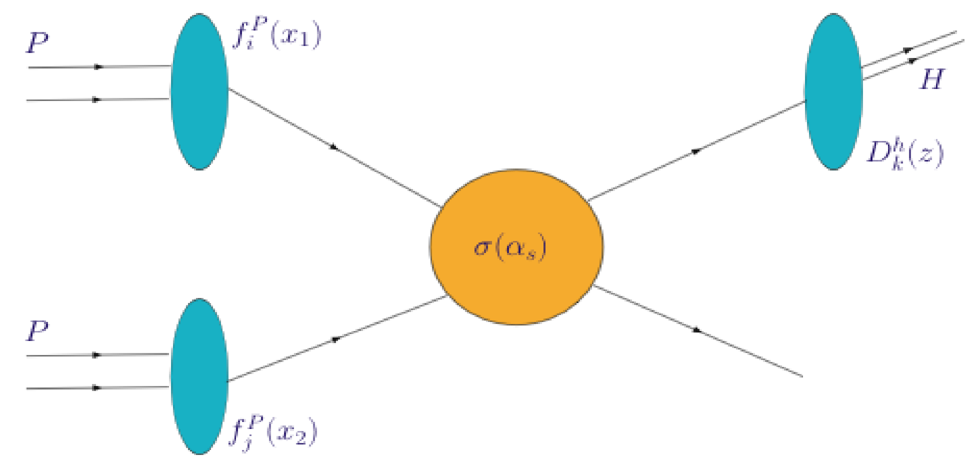

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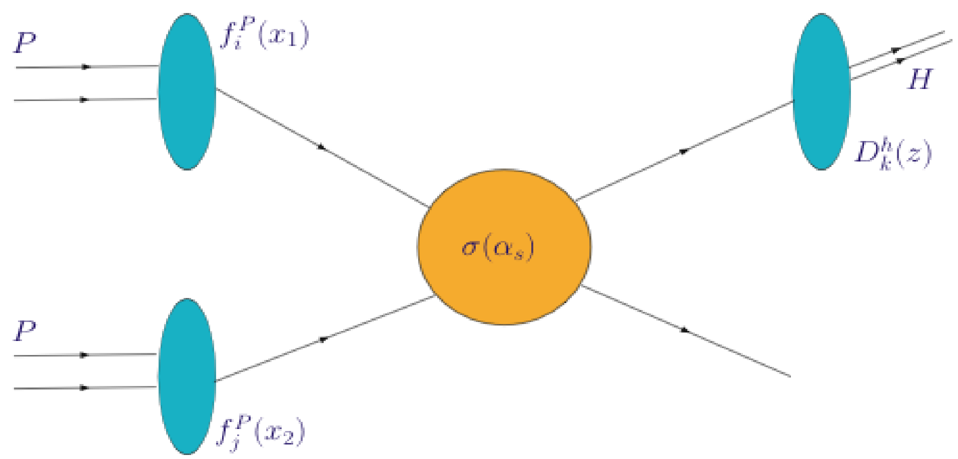
Advantages : allows (very little) flavor/charge separation (gluon dominated)
several subprocesses
much larger z
large (direct!) contribution from D_g
relevant for RHIC kinematics (polarized/heavy ion)
almost no HQ fragmentation

Hadron-Hadron collisions

Transverse momentum distribution

hard scale



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several subprocesses
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almost no HQ fragmentation

Disadvantages: problems for fixed target experiments, use only colliders
otherwise Threshold resummation needed (not in first approach)
larger TH uncertainty (scale dependence)
larger dependence on pdfs
almost no HQ fragmentation

DdeF, W.Vogelsang

NLO analyses for light hadrons

CGGRW(1994), BFGW(2000) — Bourhis et al (2001)

BKK(1995), KKP(2000), AKK(2005,2008)* — Albino, Kniehl, Kramer (1995-2008)

KRE(2000)** — Kretzer (2000)

HKNS(2007) — Hirai, Kumano, Nagai, Sudoh (2007)

use only SIA: no charge separation or ad-hoc assumption

$$D_{\bar{q}}^{h^+}(z,Q^2) = (1-z)\, D_q^{h^+}(z,Q^2)$$

* AKK2008 includes pp data (with some charge separation)

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First global analysis of fragmentation functions : DSS (2007)

DdeF, R.Sassot, M.Stratmann

Global FIT

Advantage : Constrain FF with almost all available data
Check of pQCD framework
Each experiment spans a different kinematical range
Precise determination of distributions and first estimation of uncertainties
No need for naive relations between observables and FF's

Disadvantage : more work required !
Feasible if Mellin technique used
Tension between different observables requires careful analysis

- Complicated observables:
  - without simple NLO interpretation
  - involving MC
  - "averaged" bins
  - large scale dependence
  - Weights

DSS global analysis

D.deF, M. Stratmann, R. Sassot

fragmentation functions for $\pi^+, \pi^-, \pi^0, K^+, K^-, K^0, \overline{K}^0, p, \bar{p}, n, \overline{n}, h^+, h^-$

residual

$$h^+ = \pi^+ + K^+ + p + res^+$$

LO and NLO global fits available

SIA data includes: TPC, TASSO, SLD, ALEPH, DELPHI, OPAL + "flavor" tag

SIDIS data from : HERMES, EMC(h) $Q > 1\,\mathrm{GeV}$

pp data from : PHENIX, STAR, BRAHMS, CDF, UA1, UA2 $p_T > 1\mathrm{GeV}$ (η)

colliders

cut

Estimation of uncertainties using Lagrange multipliers

Technical details

- Flexible parametrization

$$D_i^H(z, Q_0^2) = N_i\, z^{\alpha_i} (1-z)^{\beta_i} \left[1 + \gamma_i (1-z)^{\delta_i}\right]$$

at initial scale

$$\begin{aligned} Q_0^2 &= 1\,\mathrm{GeV}^2 \quad u, d, s, g \\ Q_0^2 &= m_Q^2 \quad c, b \end{aligned}$$

with

α_s and Λ_{QCD} from MRST **2002**

~23 parameters
+ normalizations

- Normalizations for different experiments (if not included in syst.)

- Try to avoid Isospin symmetry assumptions

- Allowing for possible breaking of SU(3) of sea and SU(2) in favored distributions

$$D_{d+\bar{d}}^{\pi^+} = N\, D_{u+\bar{u}}^{\pi^+}$$

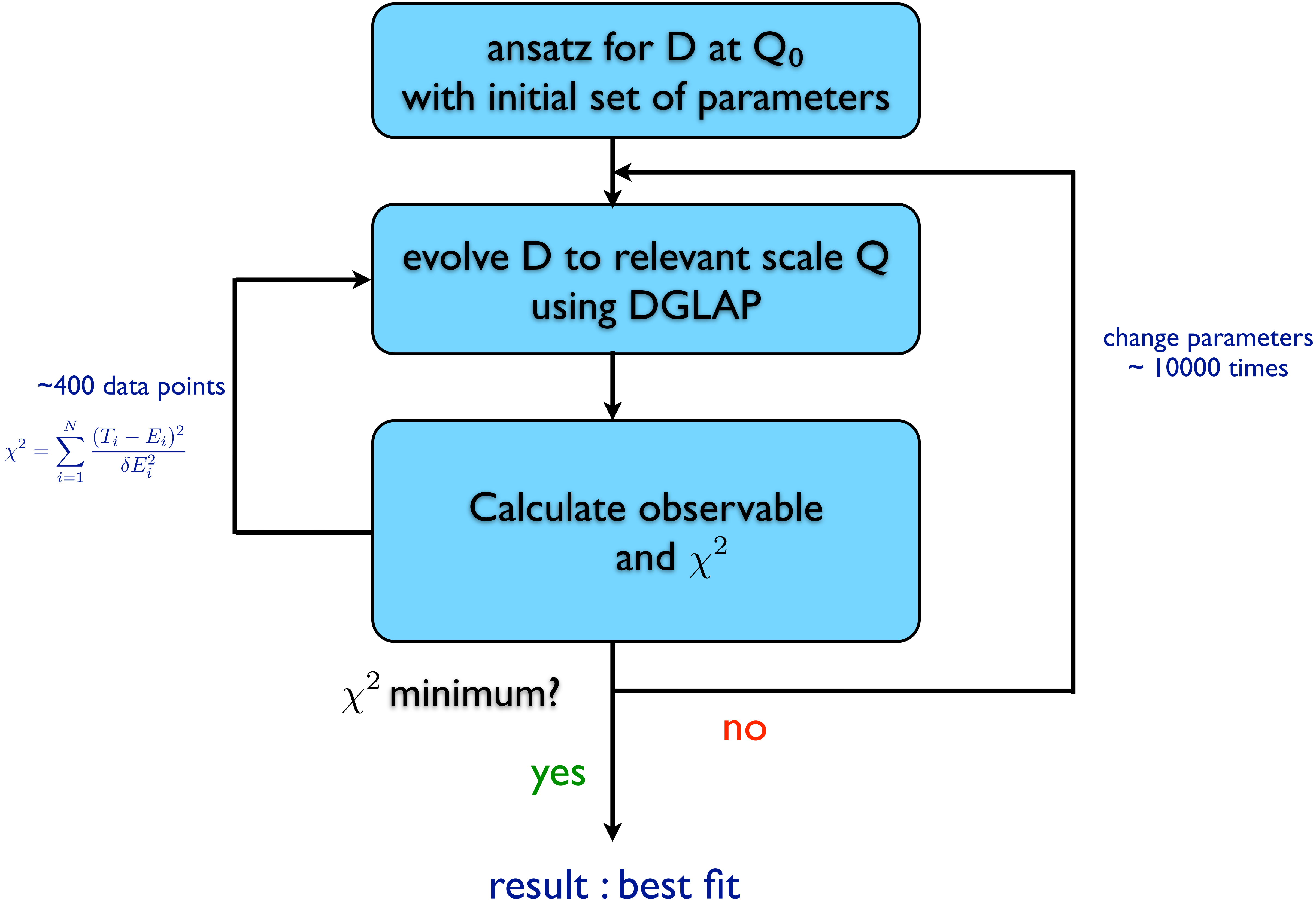
$$D_s^{\pi^+} = D_{\bar{s}}^{\pi^+} = N' D_{\bar{u}}^{\pi^+}$$

- unless data can not discriminate for unfavored fragmentations

$$D_{\bar{u}}^{\pi^+} = D_d^{\pi^+}$$

$$D_{\bar{u}}^{K^+} = D_s^{K^+} = D_d^{K^+} = D_{\bar{d}}^{K^+}$$

D's obtained by global fit : χ^2 minimization



Main problem: how to perform so many evaluations in short time

Solution: work in Mellin space $$D_i^n(\mu^2) = \int_0^1 dz\, z^{n-1}\, D_i(z,\mu^2)$$

- ✓ Convolutions become products : simple solution for evolution equations
- ✓ Back to "z" space can be done fast
- ✓ Rather natural for SIA and SIDIS (analytical)

$$d\sigma = \sum_{abc} \int\int\int f_a f_b\, d\hat{\sigma}_{ab\to cX}\, D_c\, dx_a dx_b dz_c$$

$$D_c \leftarrow \frac{1}{2\pi i}\int_{\mathcal{C}_n} dn\, z_c^{-n}\, D_c^n$$

$$d\sigma = \frac{1}{2\pi i}\sum_{abc} \int_{\mathcal{C}_n} dn \; D_c^n \; \int\int\int f_a f_b\, d\hat{\sigma}_{ab\to cX}\, dx_a dx_b dz_c$$

$\int_{\mathcal{C}_n} dn$	D_c^n	$\int\int\int f_a f_b\, d\hat{\sigma}_{ab\to cX}\, dx_a dx_b dz_c$
Mellin inversion	FIT	Precompute once and save

New technique developed to compute the grids: any observable

deF, Stratmann, Sassot, Vogelsang

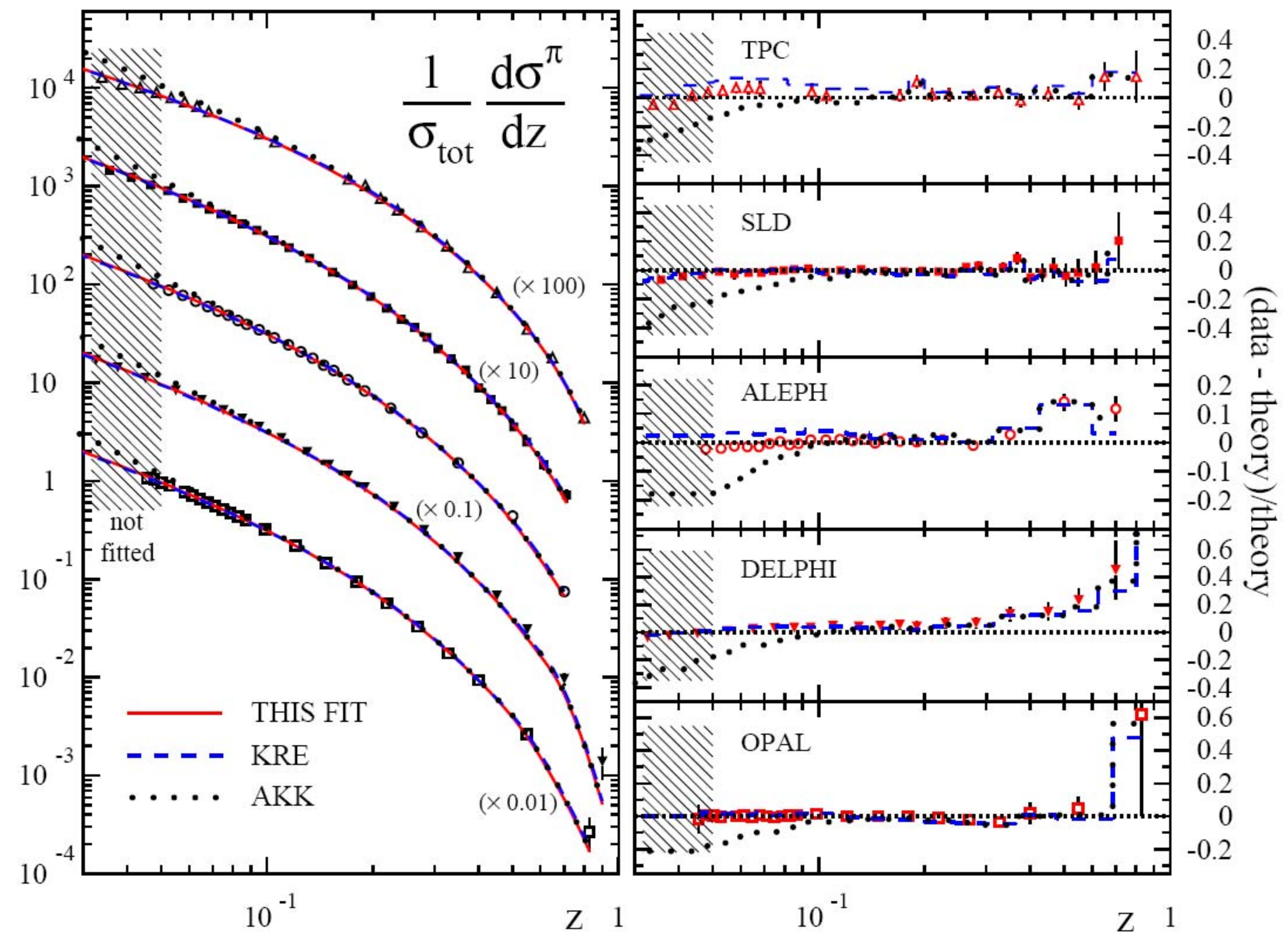
allows for >5000 ev./sec. !

Some plots

z>0.05 (0.1 for Kaons/Protons)

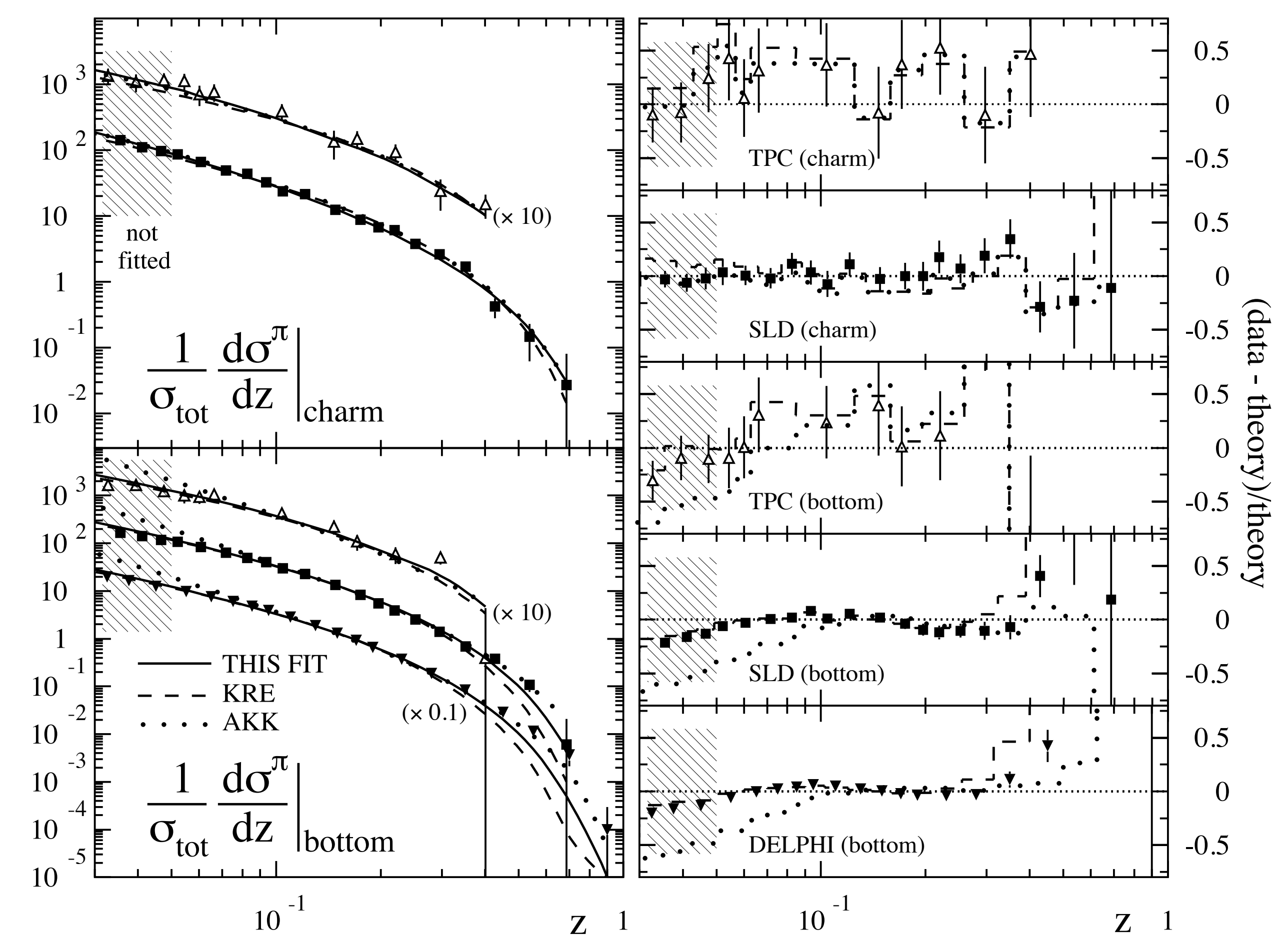
SIA: still works very well within the global fit

"inclusive"



Large errors at z>0.5

heavy quark tagged



✓ extrapolation to smaller z

Good description of SIDIS multiplicities

Hermes data (not final)

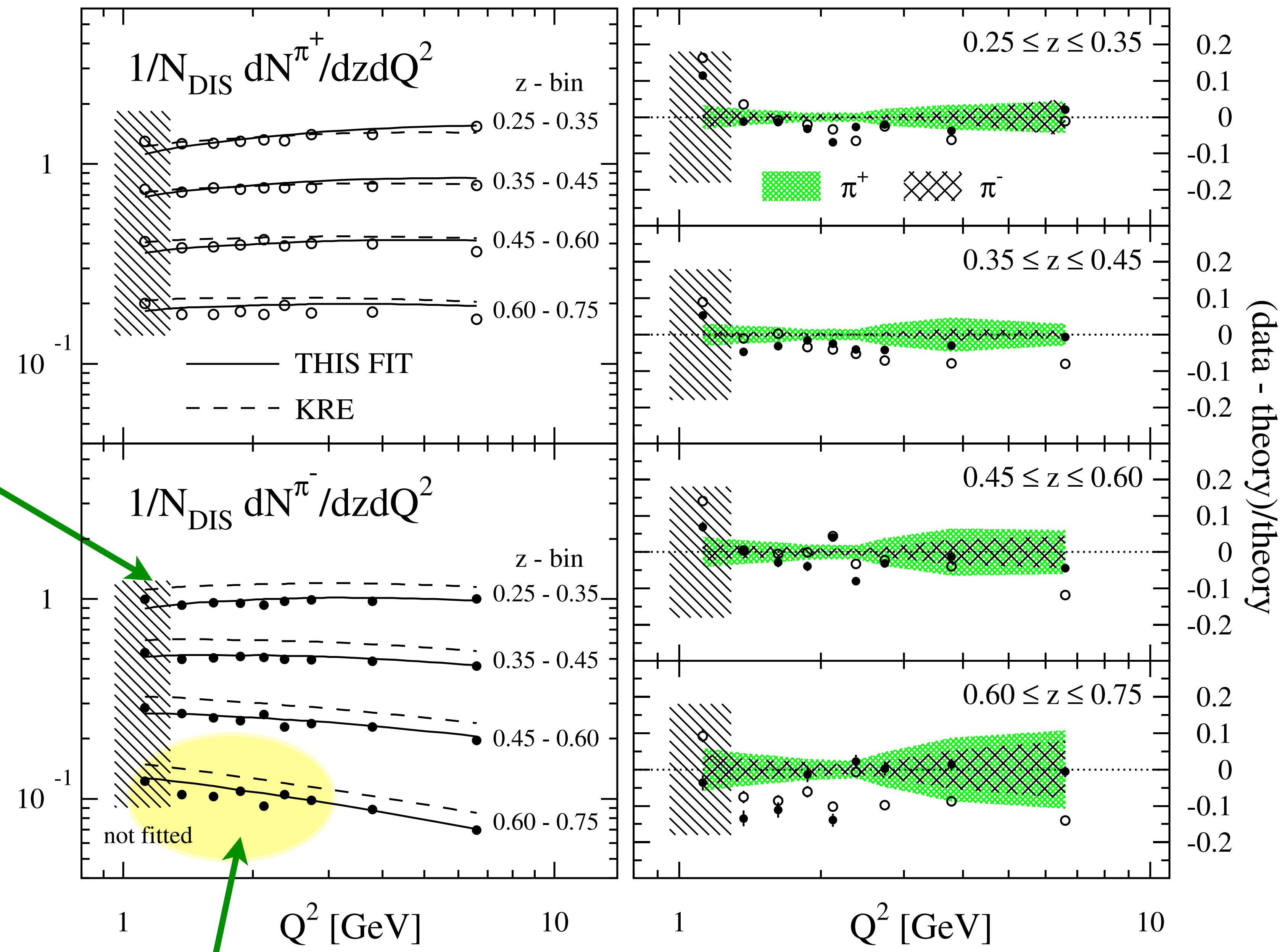
π^+

π^-

ad-hoc charge separation from Kretzer fails

$D_d^{\pi^+} \simeq (1-z) D_u^{\pi^+}$

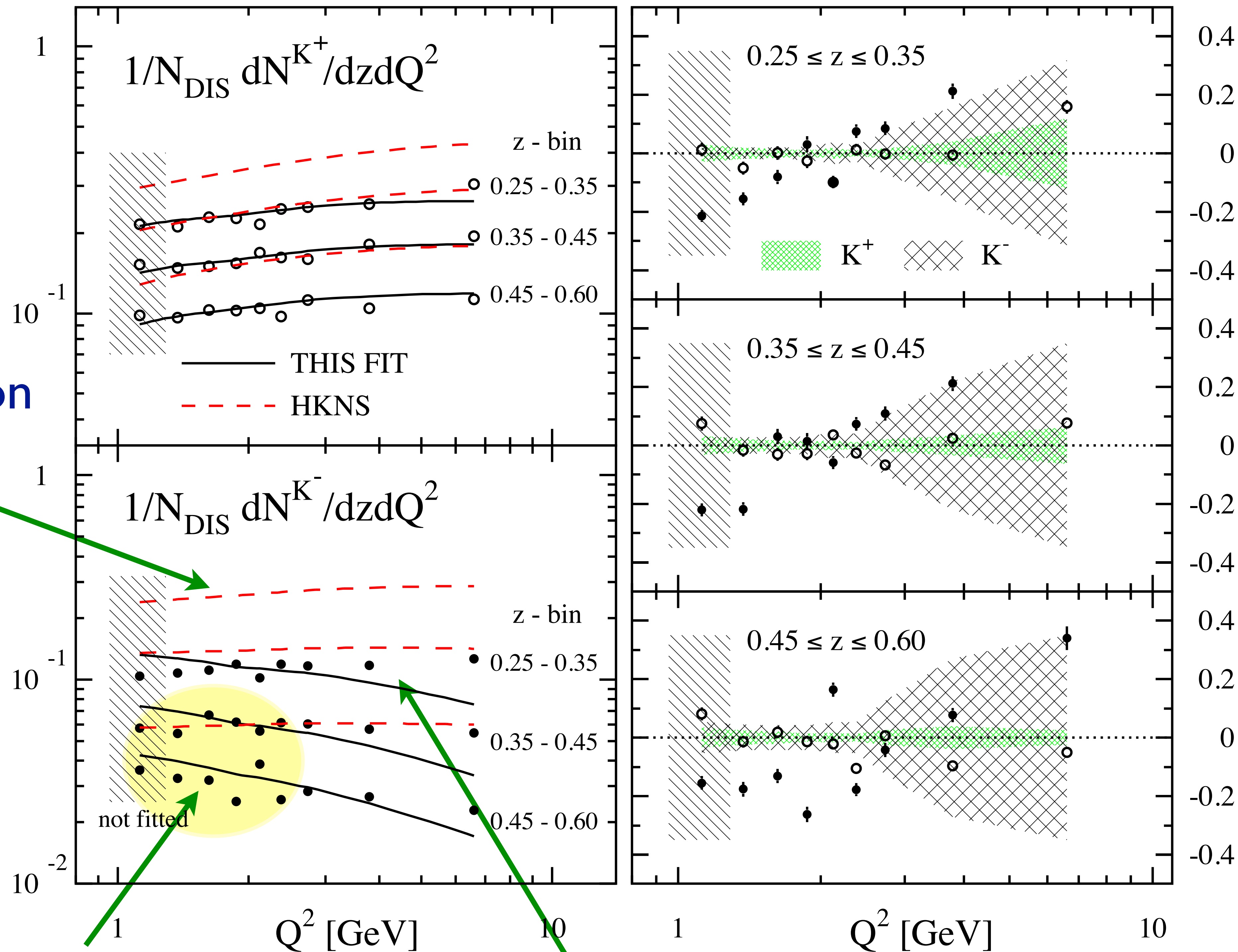
large z covered



hard to fit!

SIDIS
not so nice for K

ad-hoc charge separation from HKNS fails factor of 2 or 3!!



(EXP) even harder to fit!

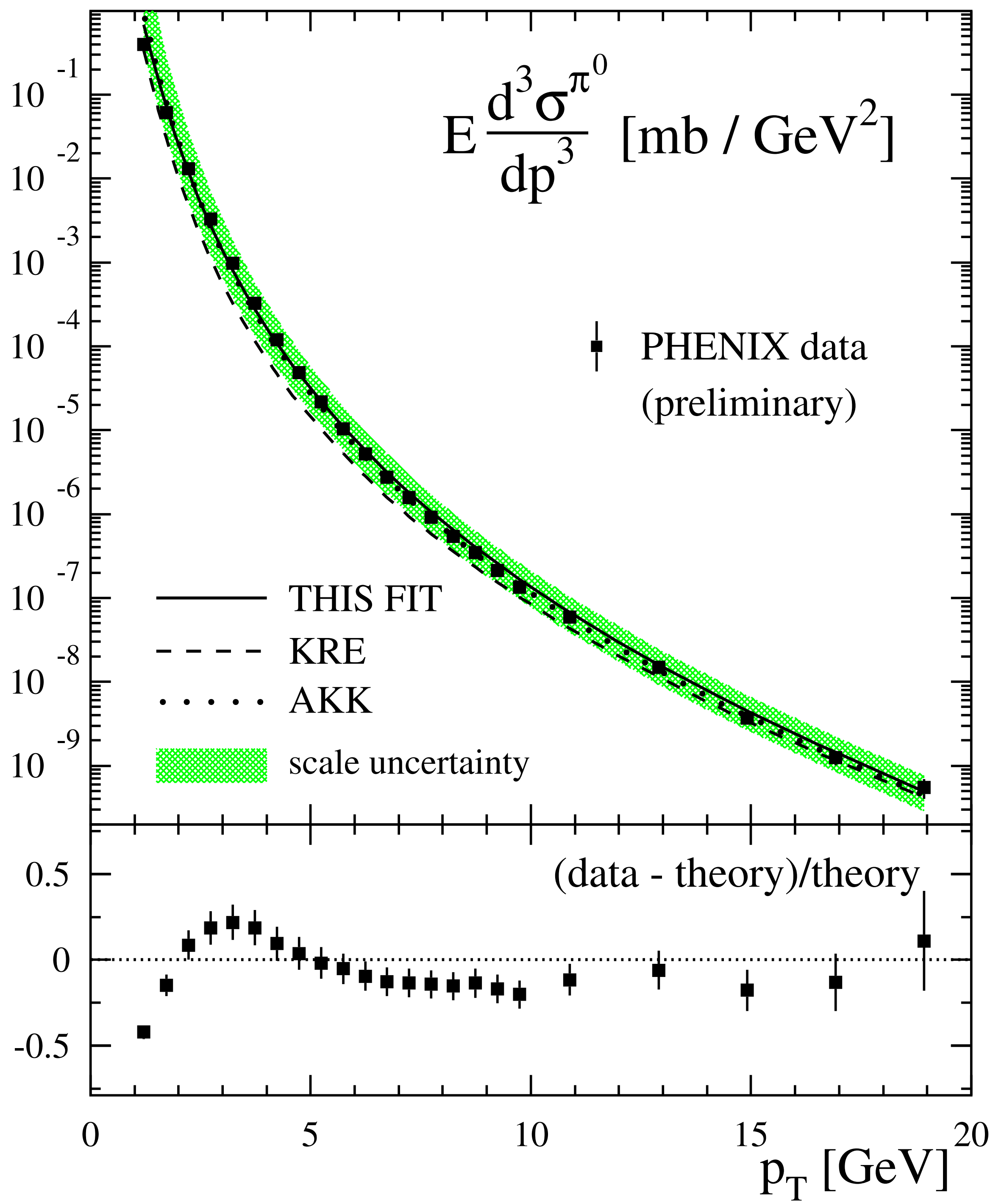
(TH) issues with the slope

HKNS, AKK08 and Kretzer can not be used for SIDIS !

More precise K data needed → essential for $\Delta s(x, Q^2)$ determination!

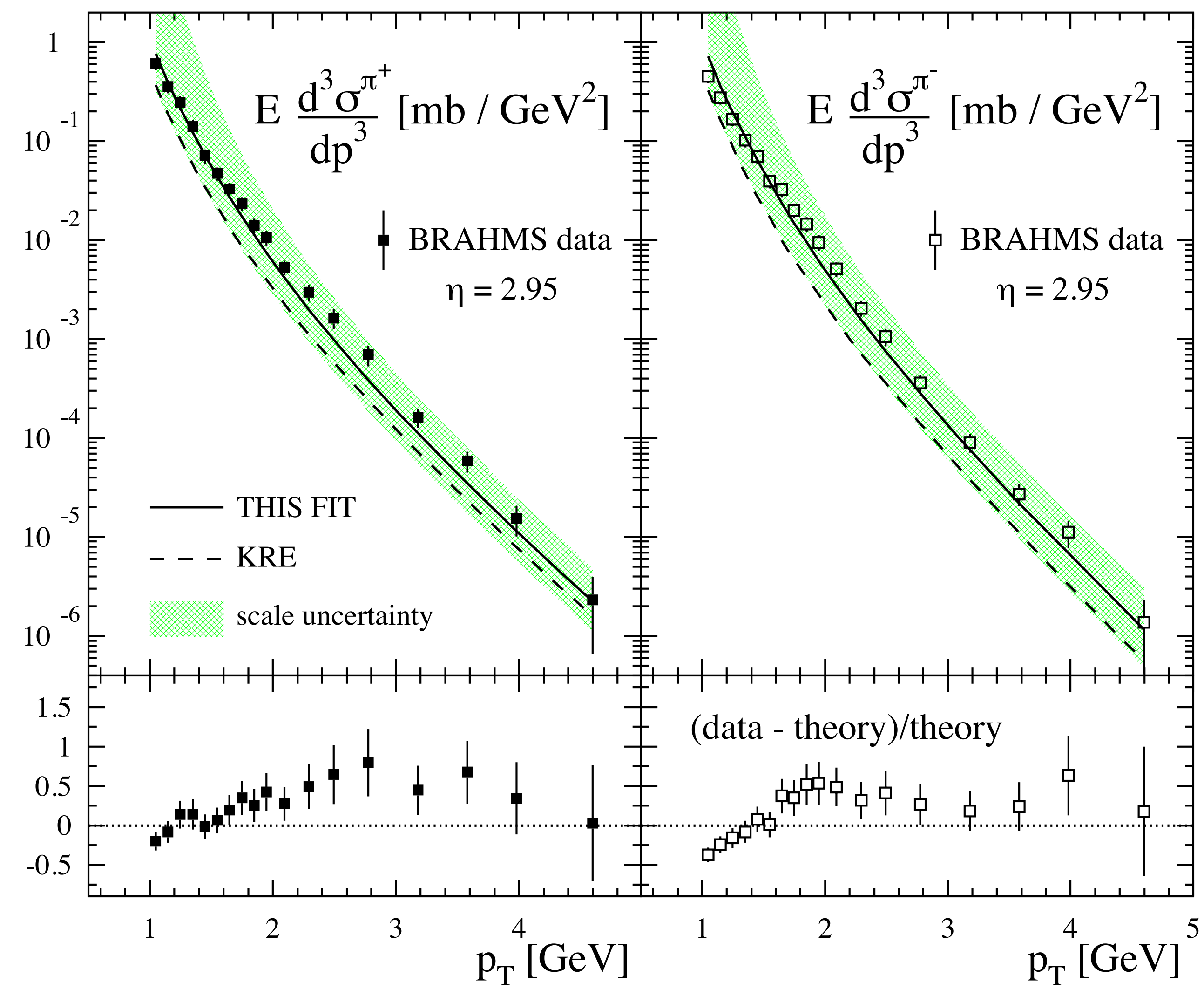
pp data well reproduced

- Large scale uncertainty
- Mainly samples z>0.5

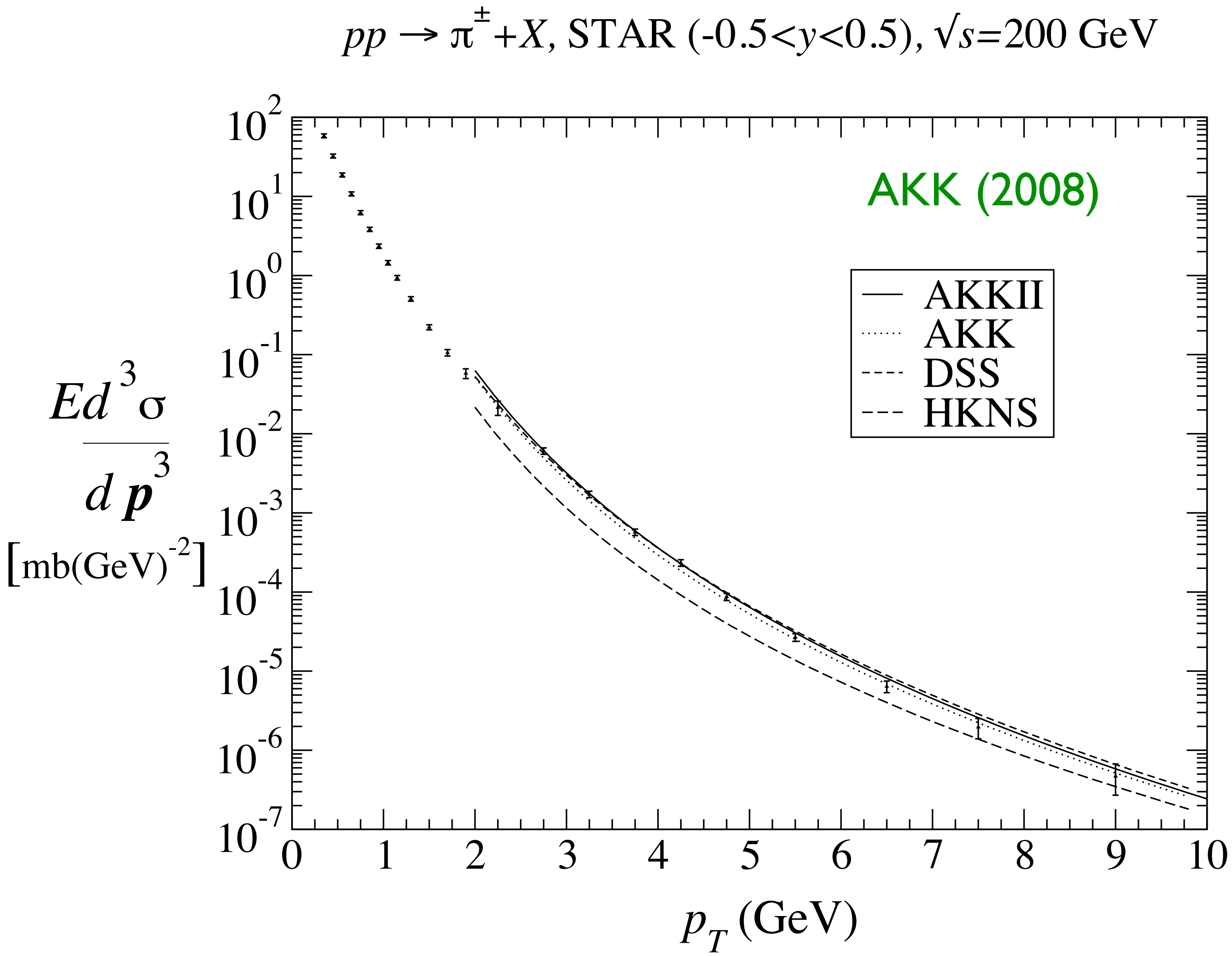


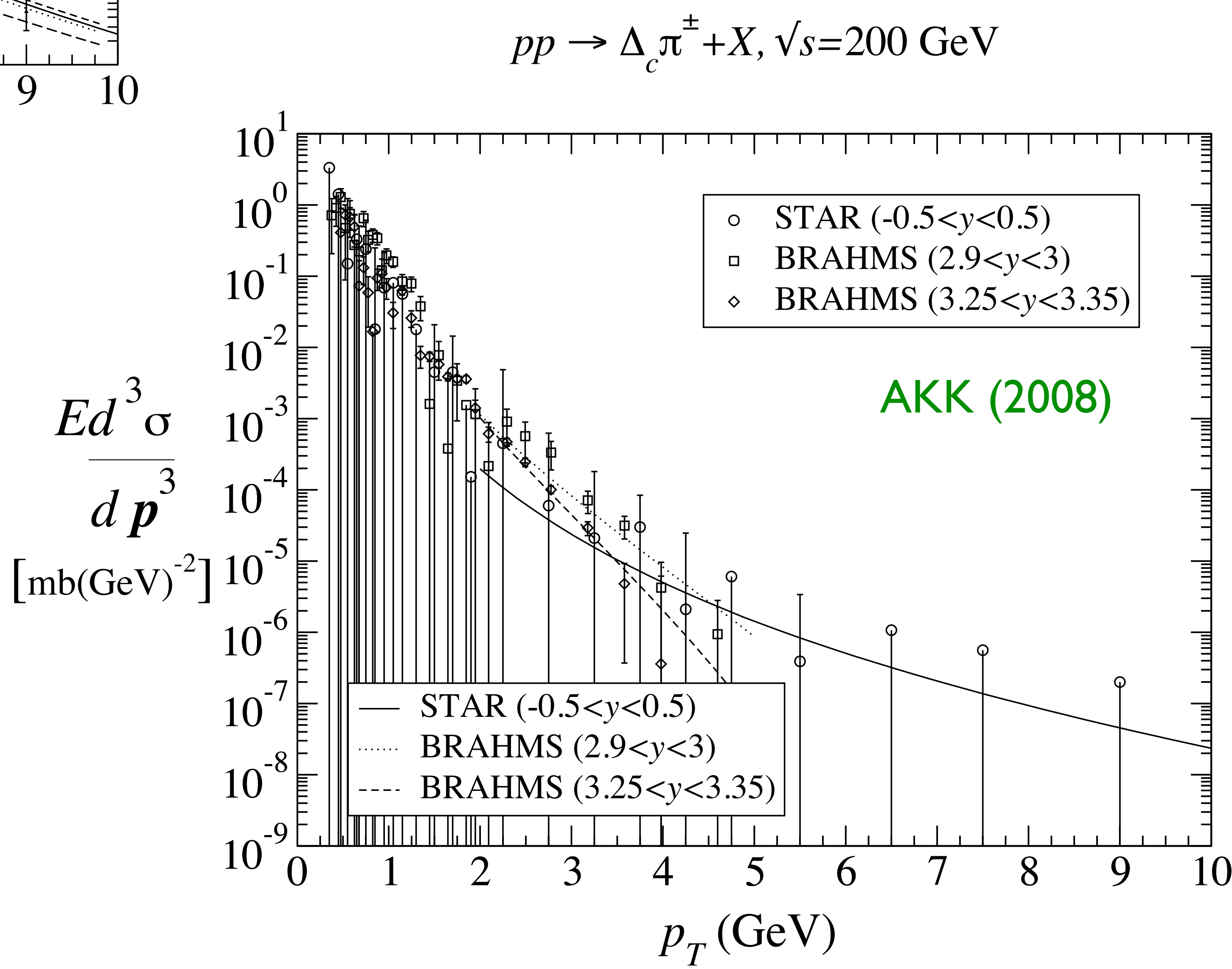
Neutral pions at Phenix

$\mu_F = \mu_R = p_T$



Charged pions (at large z)





$$\sigma(pp \to \pi^+) - \sigma(pp \to \pi^-)$$

"Charge-sign asymmetries" from pp
~ 1 or 2 orders of magnitude smaller
and "huge errors"

Typically $\chi^2/dof \sim 2$

Large χ^2 from a few isolated data points (small z in SIA, and some SIDIS and pp)

very precise data

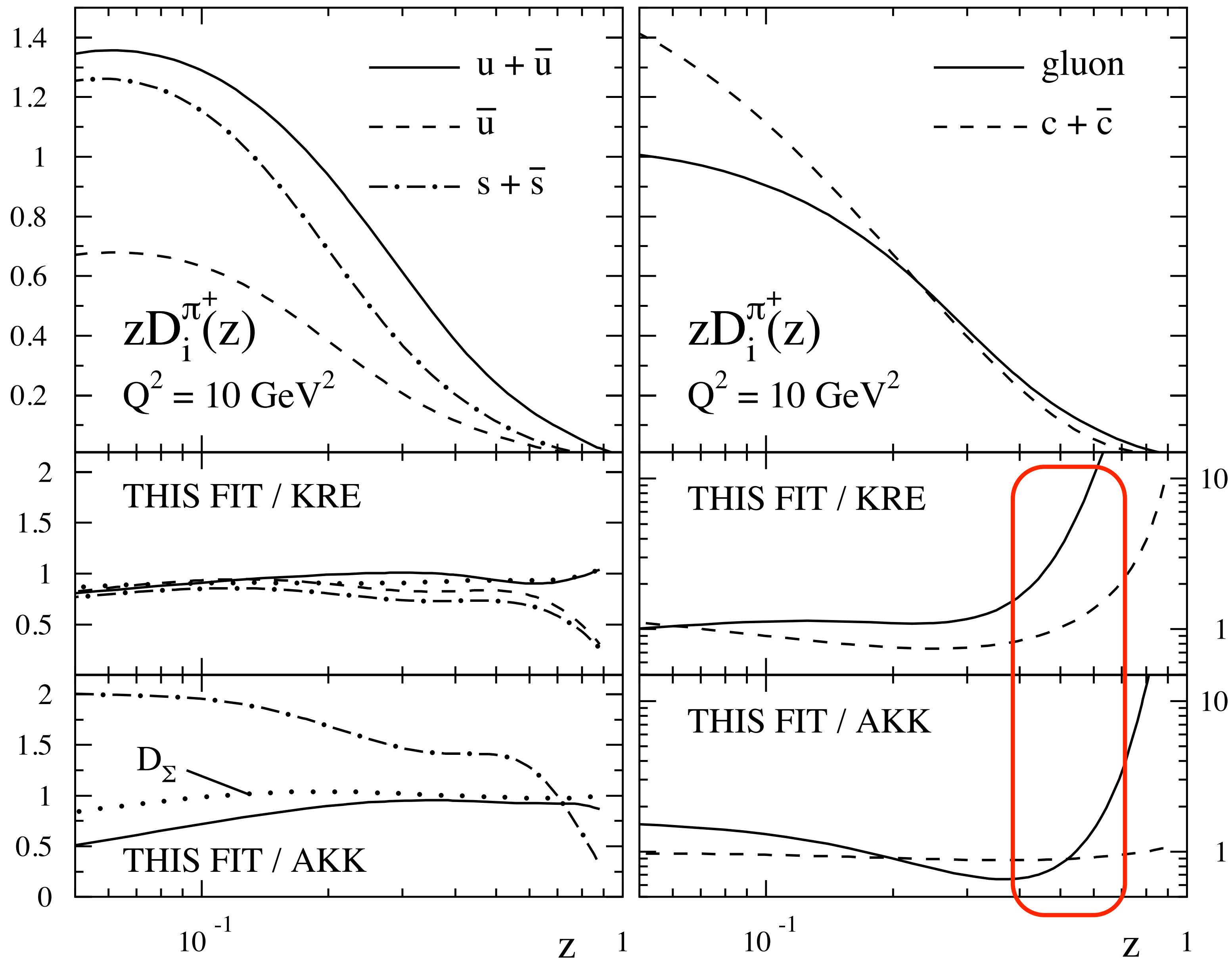
Also tension between experiments (like Delphi at large z)

χ^2 grows (~25%) for LO fits : mostly from pp data where NLO corrections are very large

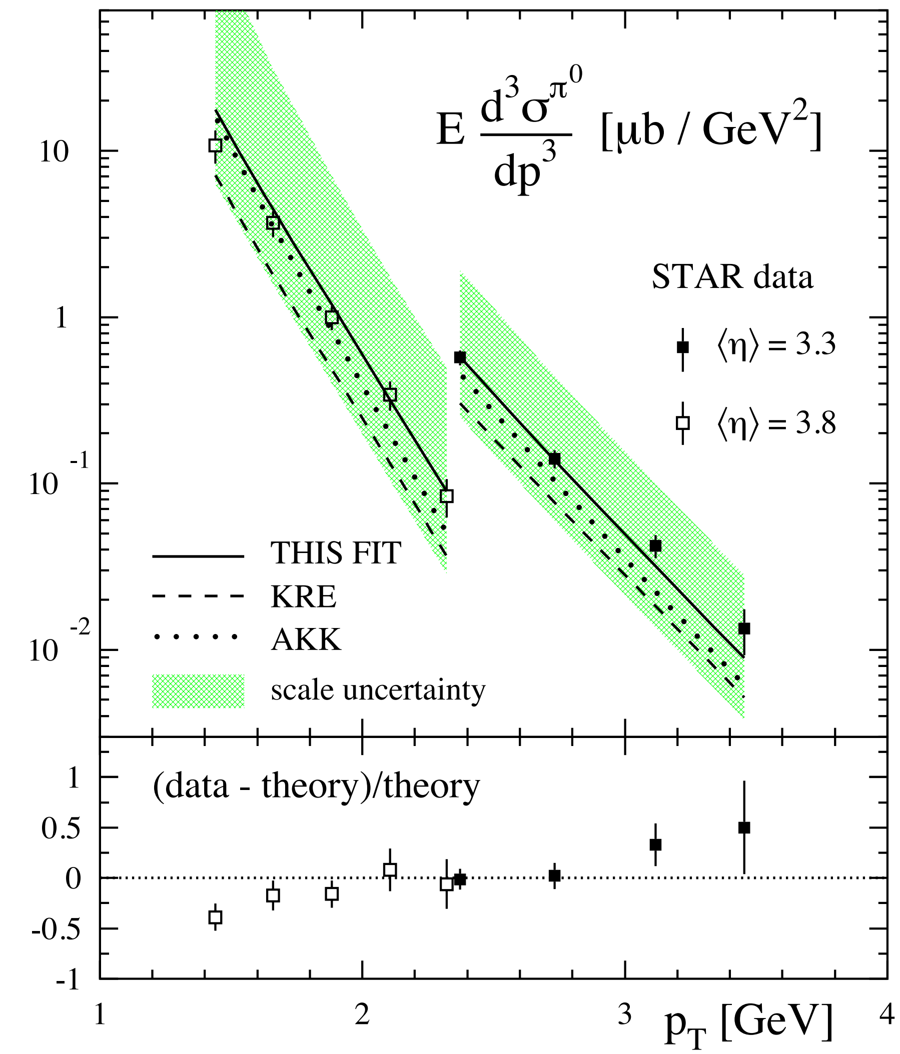
Pions + kaons + protons almost saturate charged hadrons: Mainly a prediction

Experiment	Data type	Relative normalization in fit	Data points fitted	χ^2
TPC [15]	Inclusive	0.94	17	18.5
	"uds tag"	0.94	9	1.9
	"c tag"	0.94	9	5.7
	"b tag"	0.94	9	7.4
TASSO [38]	Inclusive (34 GeV)	0.94	11	30.1
	Inclusive (44 GeV)	0.94	7	20.5
SLD [16]	Inclusive	1.008	28	14.0
	"uds tag"	1.008	17	11.6
	"c tag"	1.008	17	11.1
	"b tag"	1.008	17	33.2
ALEPH [11]	Inclusive	0.97	22	38.3
DELPHI [12]	Inclusive	1.0	17	42.3
	"uds tag"	1.0	17	26.4
	"b tag"	1.0	17	42.8
OPAL [13,14]	Inclusive	1.0	21	9.2
	"u tag"	1.10	5	11.8
	"d tag"	1.10	5	9.0
	"s tag"	1.10	5	49.8
	"c tag"	1.10	5	38.3
	"b tag"	1.10	5	73.0
HERMES [17]	π^+	1.03	32	67.4
	π^-	1.03	32	120.8
PHENIX [18]	π^0	1.09	23	76.4
STAR [22]	$\pi^0, \langle\eta\rangle = 3.3$	1.05	4	3.4
	$\pi^0, \langle\eta\rangle = 3.7$	1.05	5	9.8
BRAHMS [21]	$\pi^+, \langle\eta\rangle = 2.95$	1.0	18	28.2
	$\pi^-, \langle\eta\rangle = 2.95$	1.0	18	43.0
Total			392	843.7

Meet the Distributions : pions



large rapidity at STAR



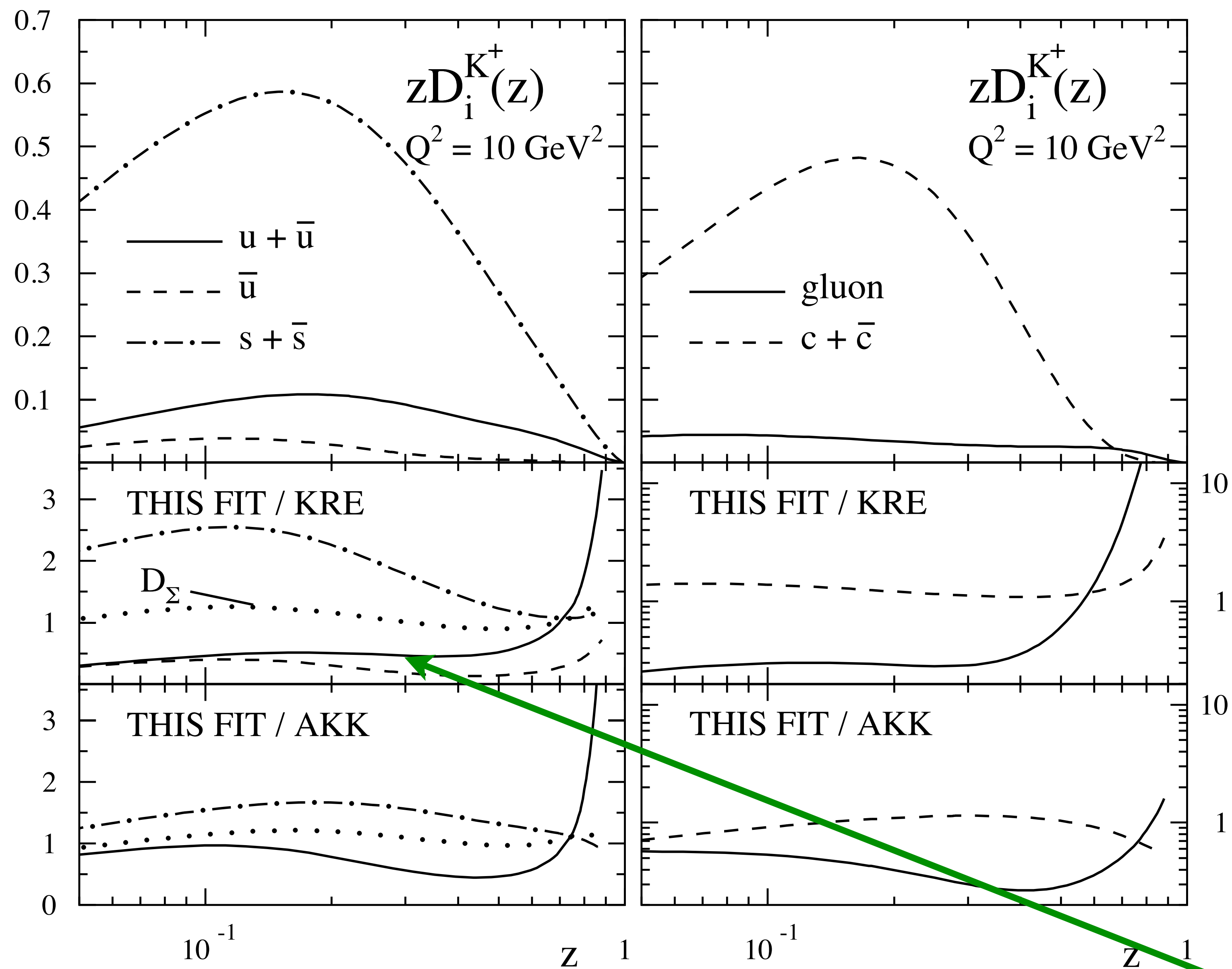
Large differences visible in the gluon at large z : explains pp

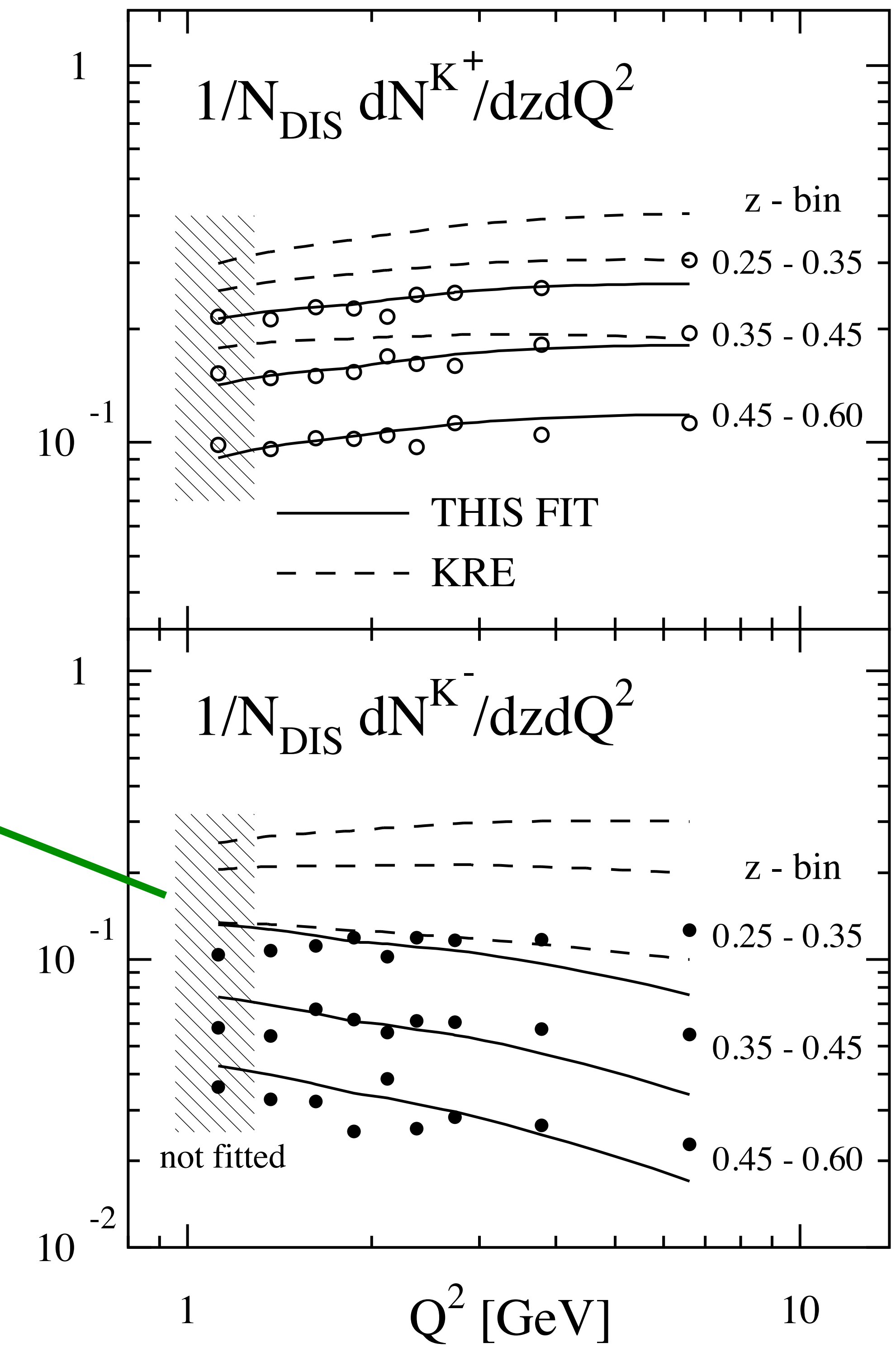
Large differences in unfavored distributions : explains SIDIS and pp

Similar singlet

For pions u fragmentation smaller than AKK : required by SIDIS

Meet the Distributions : Kaons





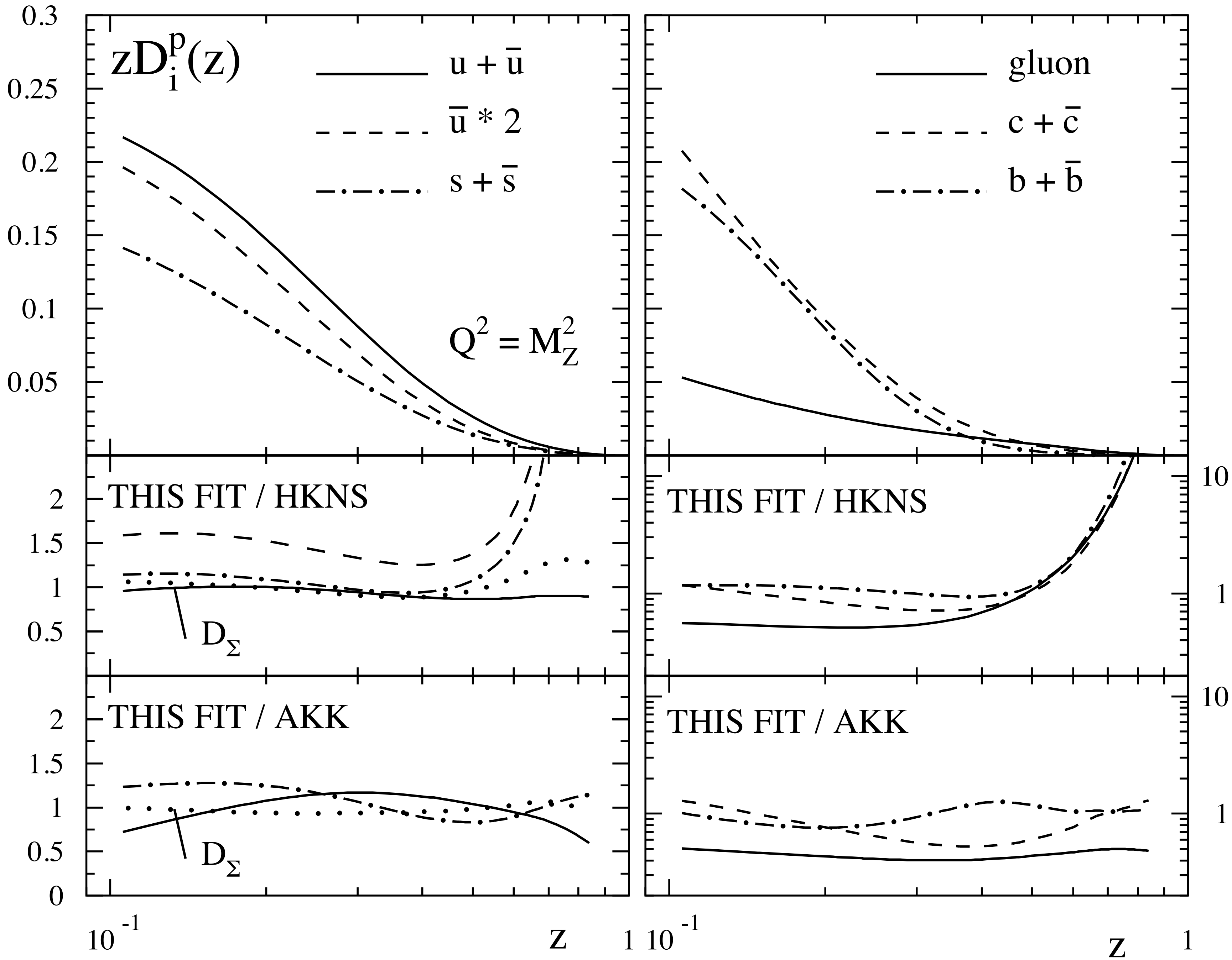
Smaller u required by SIDIS

VERY different gluon by pp

Similar singlet (larger dominant s)

Issues for K^- (s and $\bar{u}$ in proton at large x)

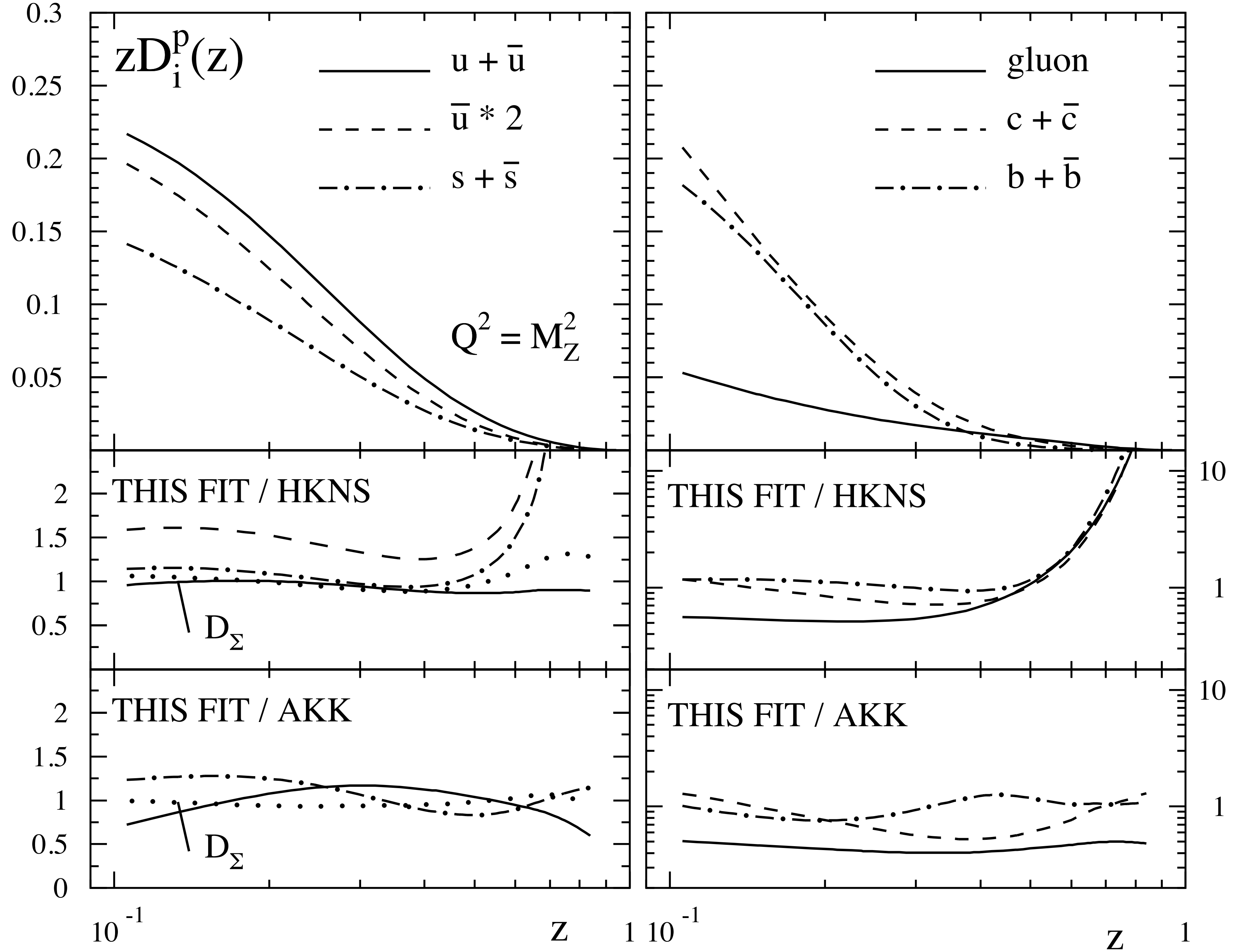
Meet the Distributions : protons



Differences with HKNS also sizeable : gluons, unfavored and large z (pp)

No SIDIS : favored/unfavored separation not that reliable (assumptions)

Meet the Distributions : protons



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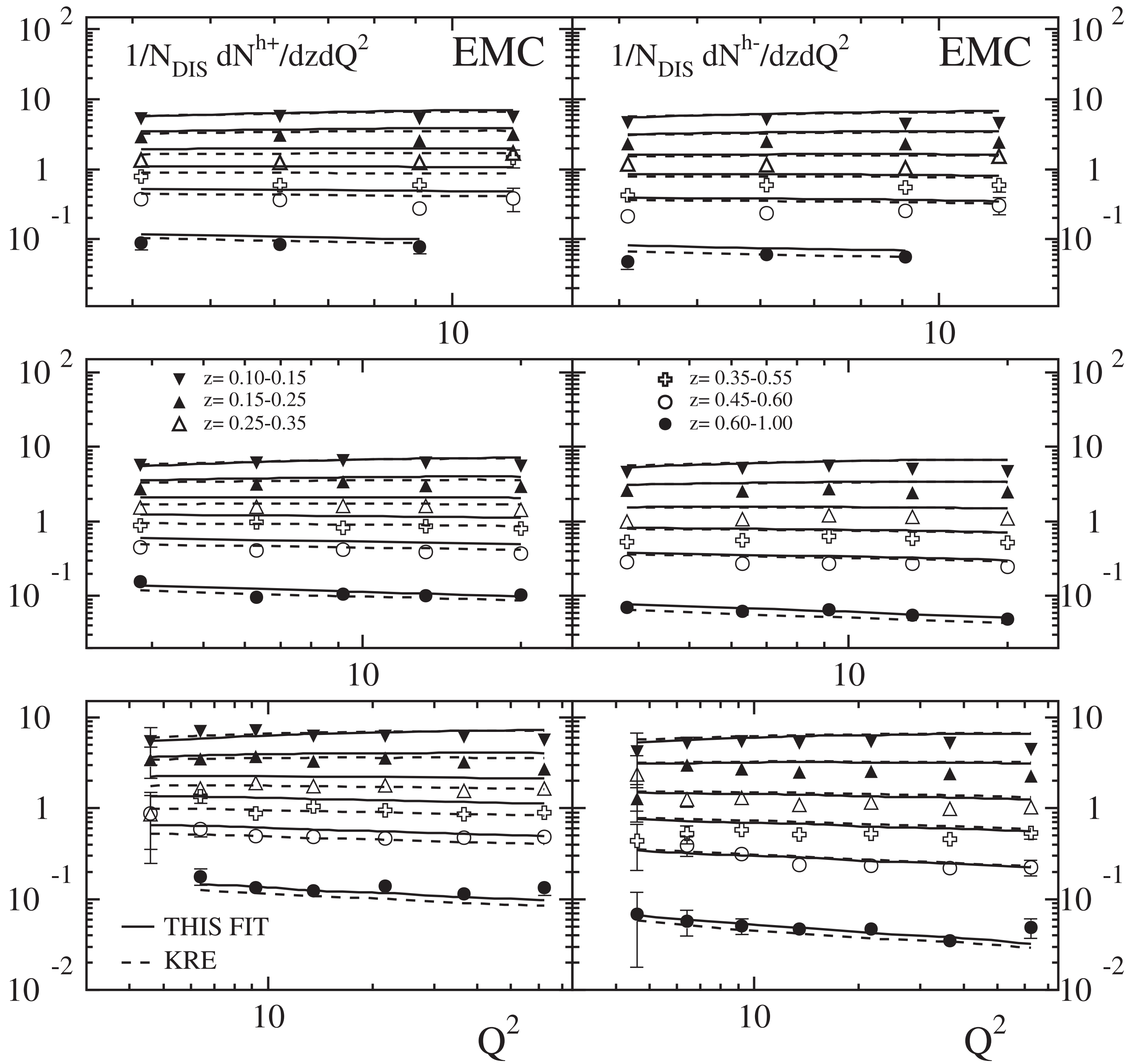
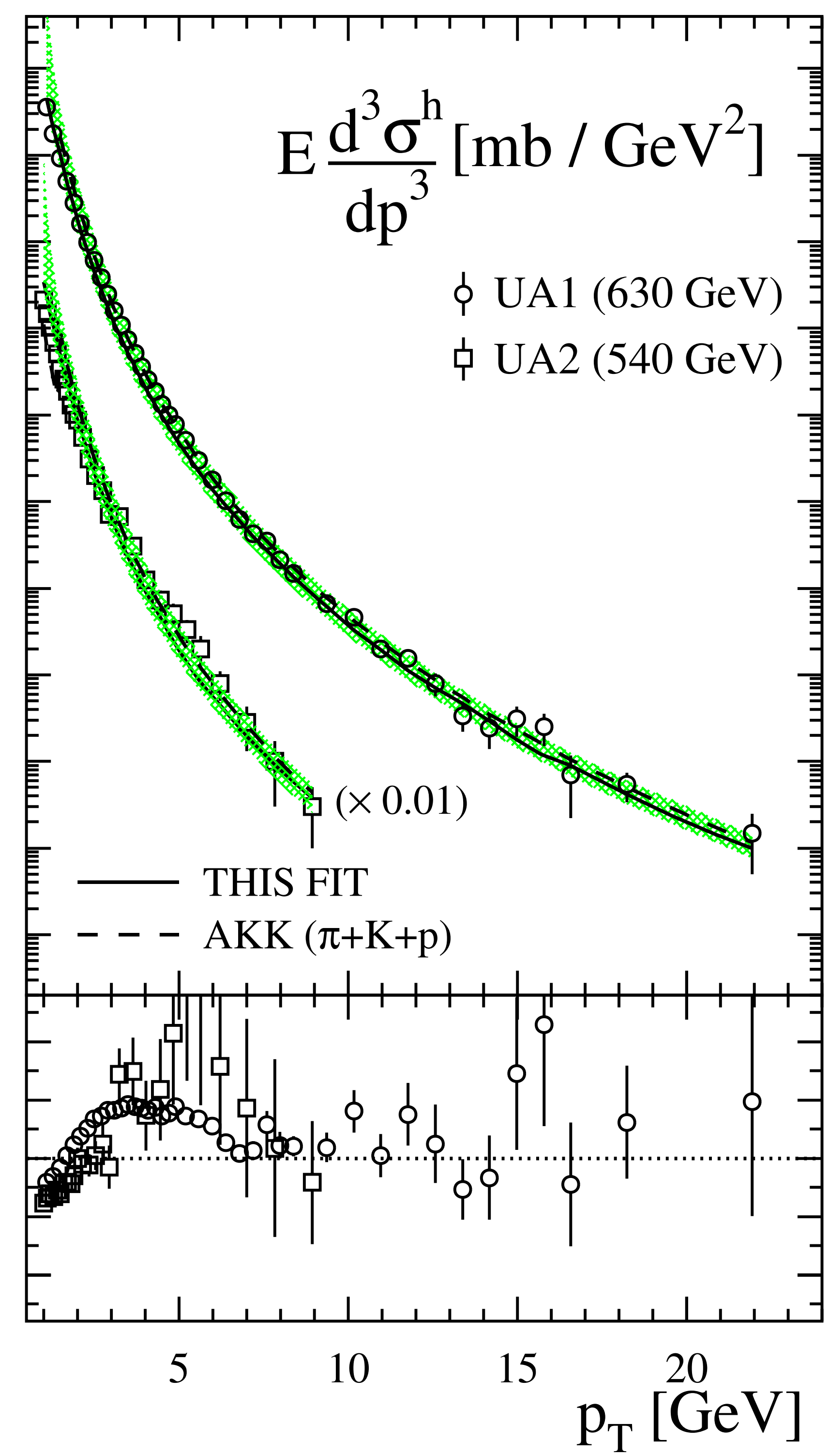
In general differences look much larger than for pdf fits : comparison between GLOBAL fits

Charged hadrons: Pions + kaons + protons almost saturate charged hadrons:
residual only sizable for HQ
Mainly a prediction/check : OK

SIDIS



PP



Uncertainties on fragmentation functions

✓ HKNS uncertainties using Hessian Method M.Hirai et al

only e^+e^- data so uncertainties are not well defined for gluons and flavor/charge separation

Hessian method might fail when far from quadratic dependence

✓ DSS uncertainties using Lagrangian multipliers D.deF, M. Stratmann, R. Sassot

study uncertainties for truncated moments but does not translate easily into uncertainties for z-dependence fragmentation functions

More work (and data!) needed for a precise estimate of uncertainties

Improvements on description of fragmentation functions over the last few years

Future:

EXP

- SIDIS: COMPASS
- SIDIS: Hermes Final? } "next summer"
- Belle/Babar: precision at lower energies } "next summer"
- more pp data (RHIC, LHC?)

TH

- SIDIS at NNLO
- Splitting at NNLO
- Threshold resummation for several processes
- More global fits to compare!

- More precise determination of fragmentation functions

- Realistic estimate of uncertainties and several sets to compute uncertainties for other observables (like pdf's)

Improvements on description of fragmentation functions over the last few years

Future:

EXP

SIDIS: COMPASS

"next summer" { SIDIS: Hermes Final?
Belle/Babar: precision at lower energies

more pp data (RHIC, LHC?)

TH

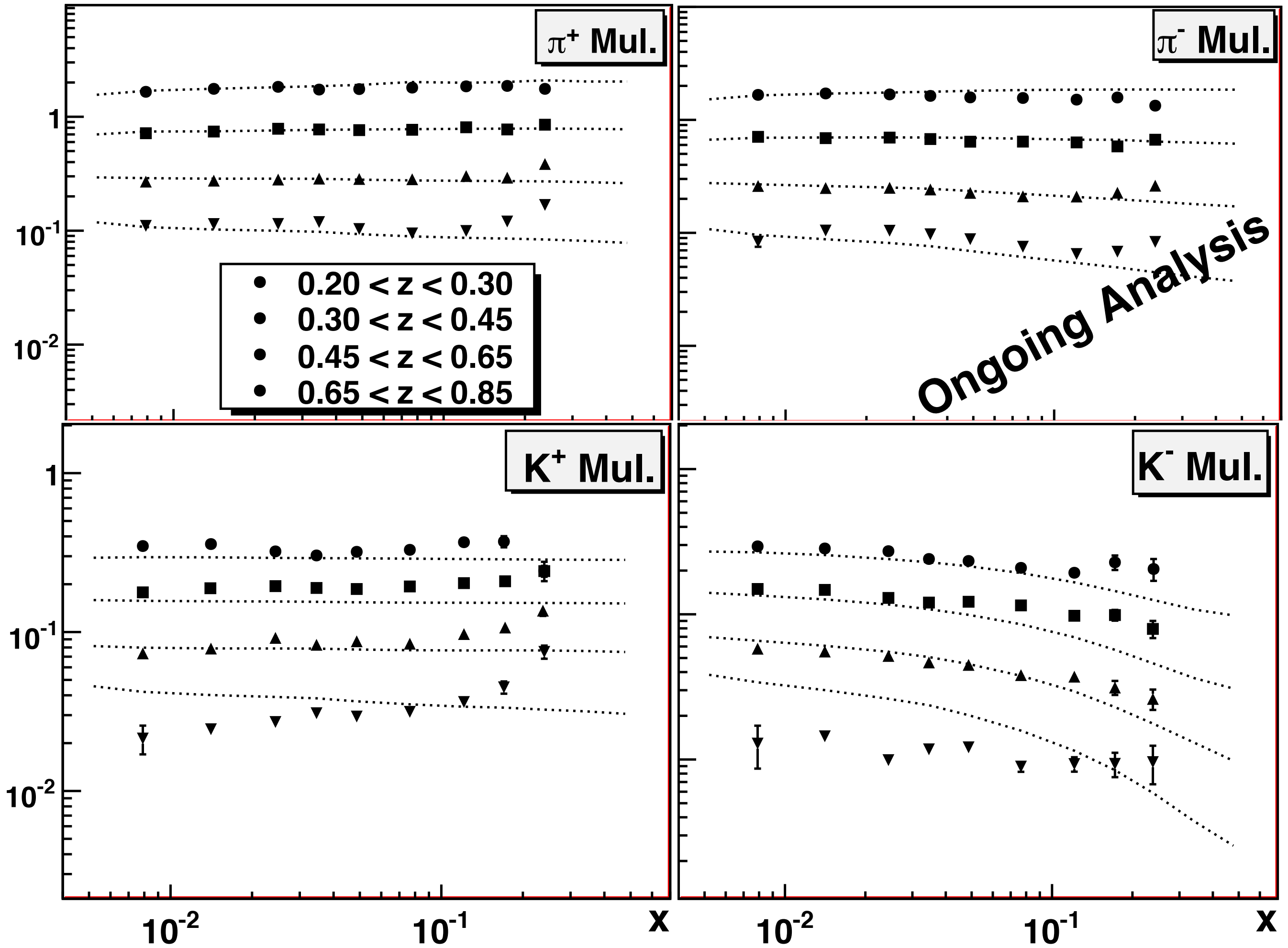
SIDIS at NNLO
Splitting at NNLO
Threshold resummation for several processes
More global fits to compare!

- More precise determination of fragmentation functions

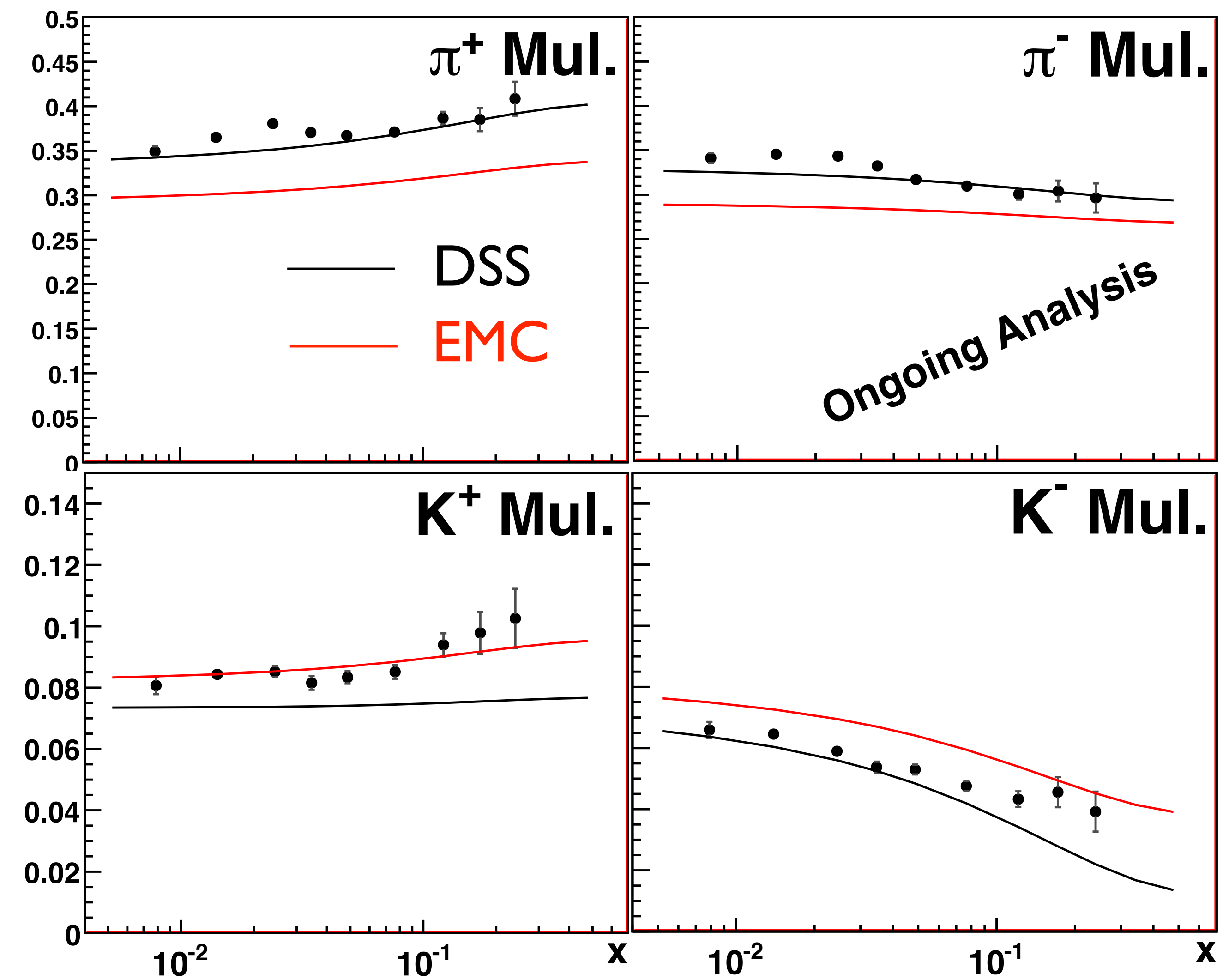
- Realistic estimate of uncertainties and several sets to compute uncertainties for other observables (like pdf's)

TH effort usually triggered by precise data !

COMPASS-II proposal : Q&A



Extensive measurements of multiplicities in several variables (and bins)



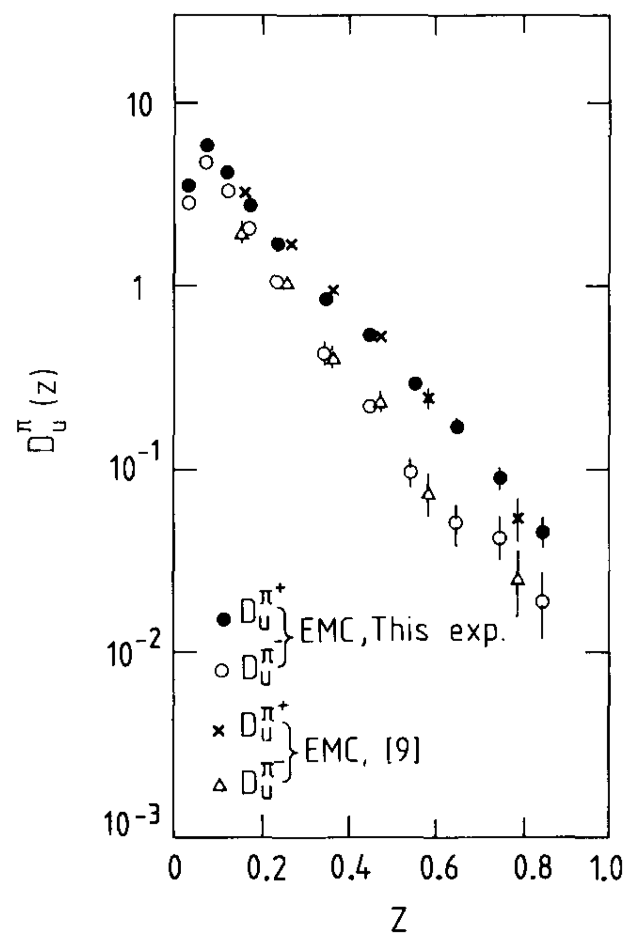
We can be very optimistic !!

Older data not very well documented (no multiplicities, only FFs)

MEASUREMENTS OF THE u VALENCE QUARK DISTRIBUTION FUNCTION IN THE PROTON AND u QUARK FRAGMENTATION FUNCTIONS

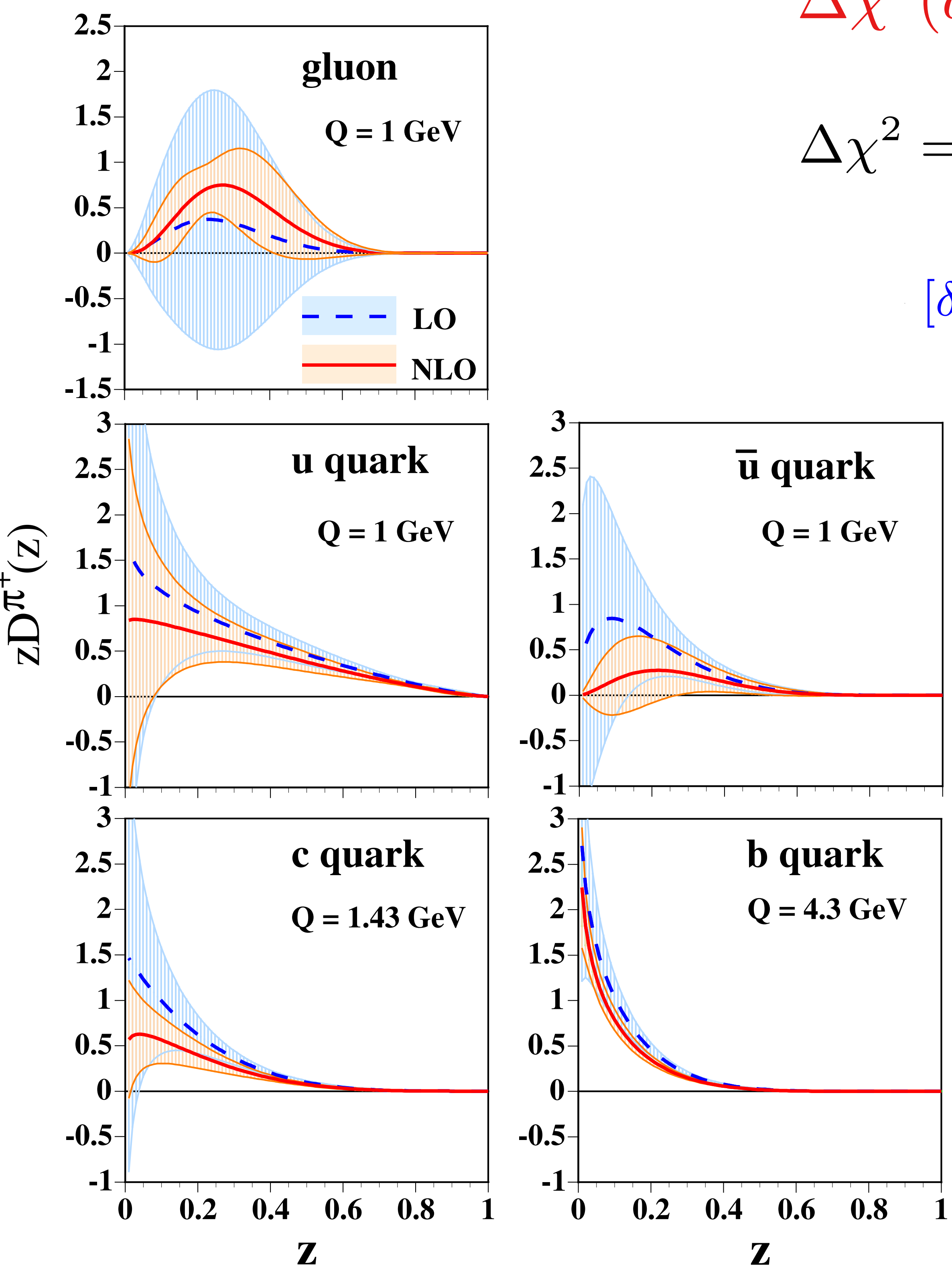
The European Muon Collaboration

Nuclear Physics B321 (1989) 541–560
North-Holland, Amsterdam



Uncertainties on fragmentation functions

HKNS uncertainties using Hessian Method



$$\Delta\chi^2(a) = \chi^2 - \chi_0^2 = \sum_{i,j} H_{ij}\delta a_i \delta a_j + \cdots$$

$$\Delta\chi^2 = 15.94$$

$$[\delta D_i^h(z)]^2 = \Delta\chi^2 \sum_{j,k} \left(\frac{\partial D_i^h(z,a)}{\partial a_j}\right)_{\hat{a}} H_{jk}^{-1} \left(\frac{\partial D_i^h(z,a)}{\partial a_k}\right)_{\hat{a}}$$

Main issue : only SIA data

Gluon: uncertainties overestimated due to lack of other data?

Charge separation: uncertainties could be underestimated due to assumptions

For some parameters, χ^2 profile VERY different from quadratic even in global analysis

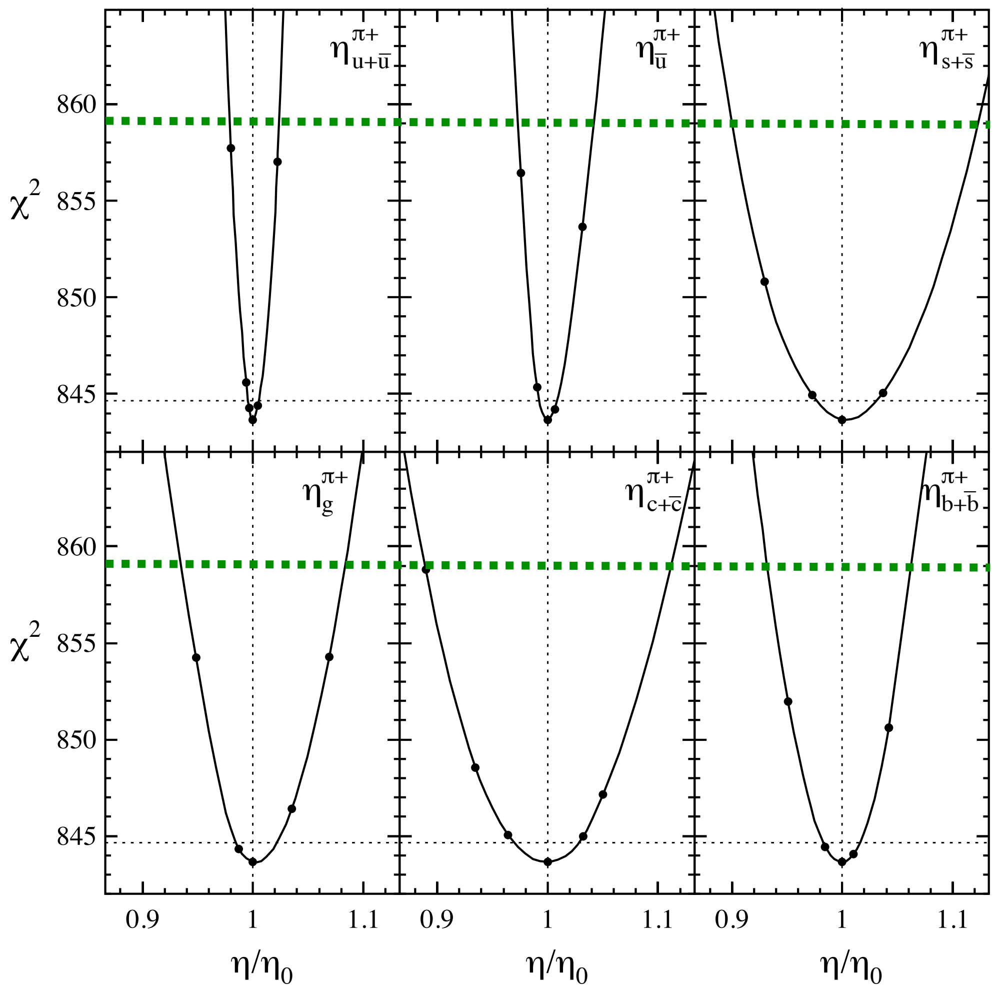
DSS uncertainties using Lagrangian multipliers

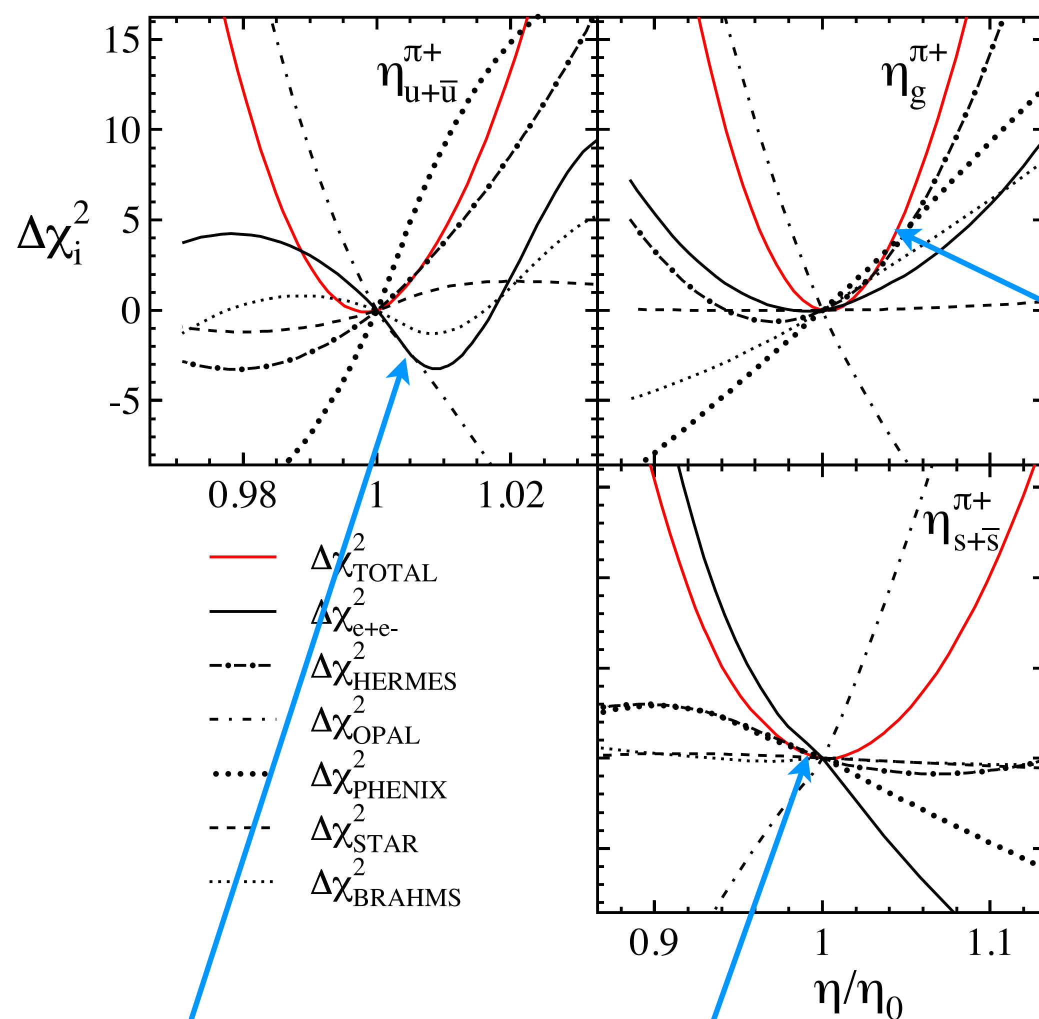
$$\Phi(\lambda_i, \{a_j\}) = \chi^2(\{a_j\}) + \sum_i \lambda_i \, \mathcal{O}_i(\{a_j\})$$

$$\int_{0.2}^{1} z \, D_i^H(z, Q = 5\,\mathrm{GeV}) \, dz$$

See how fit deteriorates when FFs forced to give different prediction for $\mathcal{O}_i$

$$\Delta\chi^2 = 15(\sim 2\%)$$





Precision

Tension

Complementarity

Constrained parabola as a result of global fit

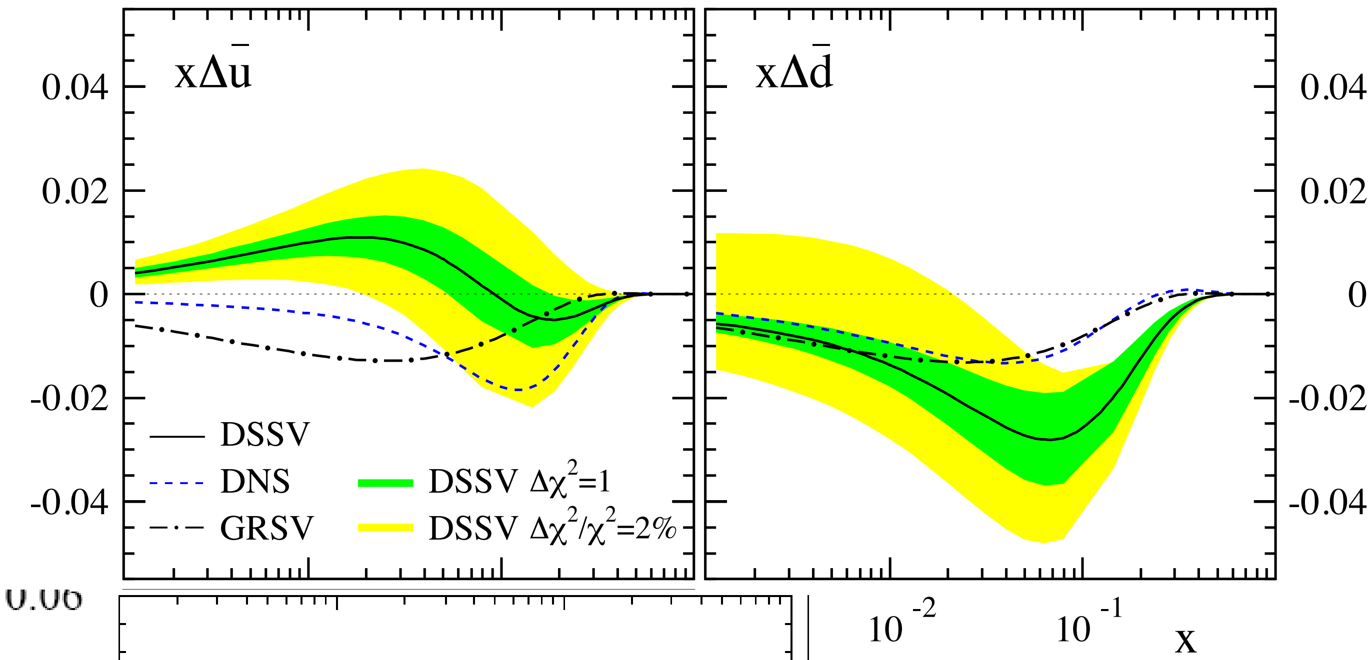
u < 5% dominated by z~0.2

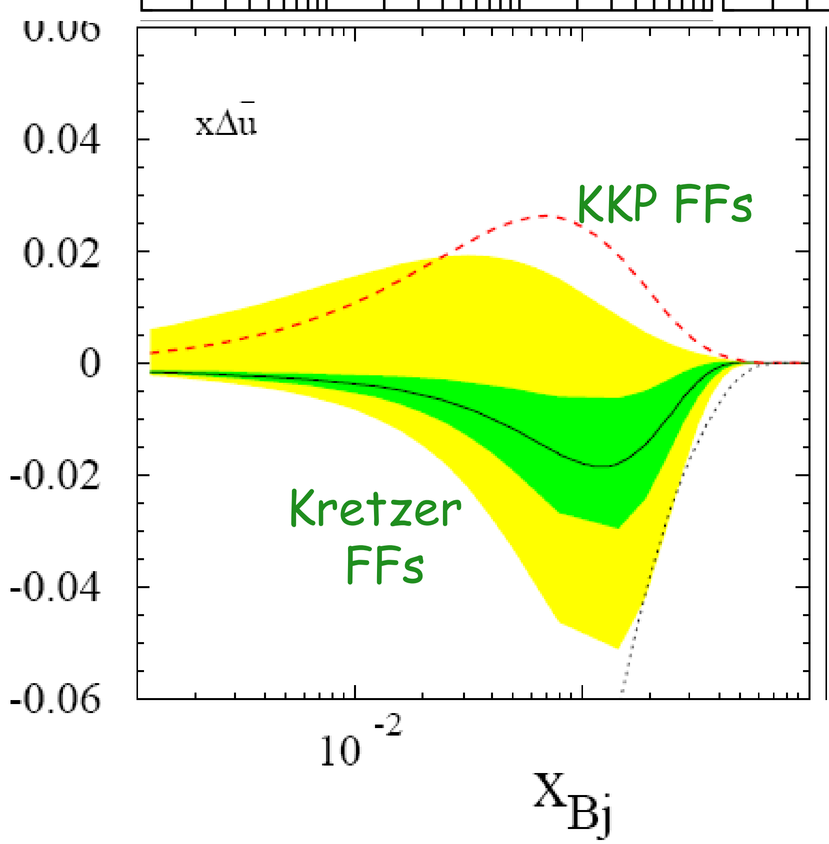
s ~ 10 %

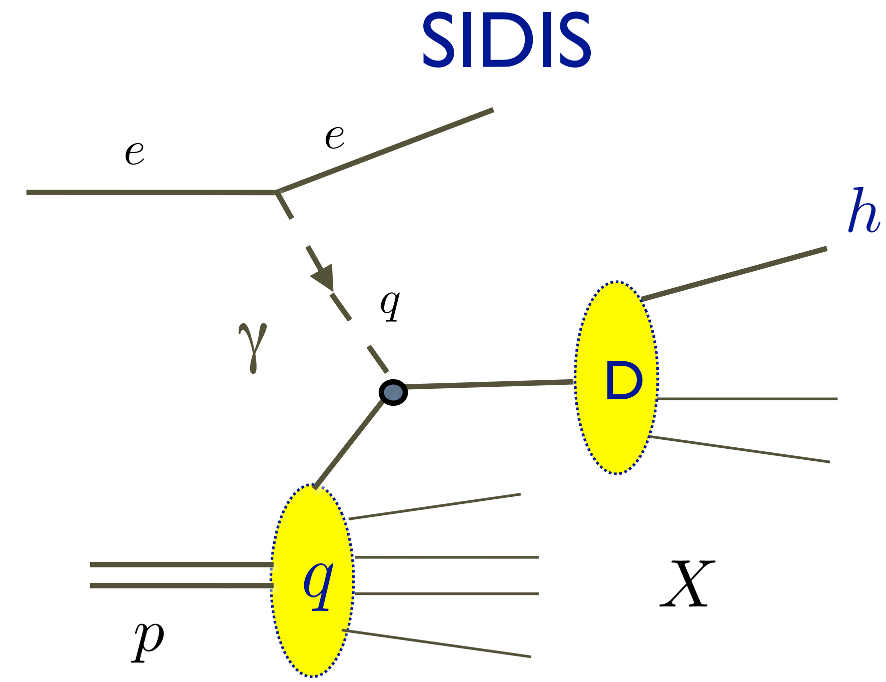
uncertainties : factor of 2 larger for Kaons

~ 20-25% for protons

• The flavor-separation of polarized parton distributions depends crucially on SIDIS results (Hermes-SMC-Compass)



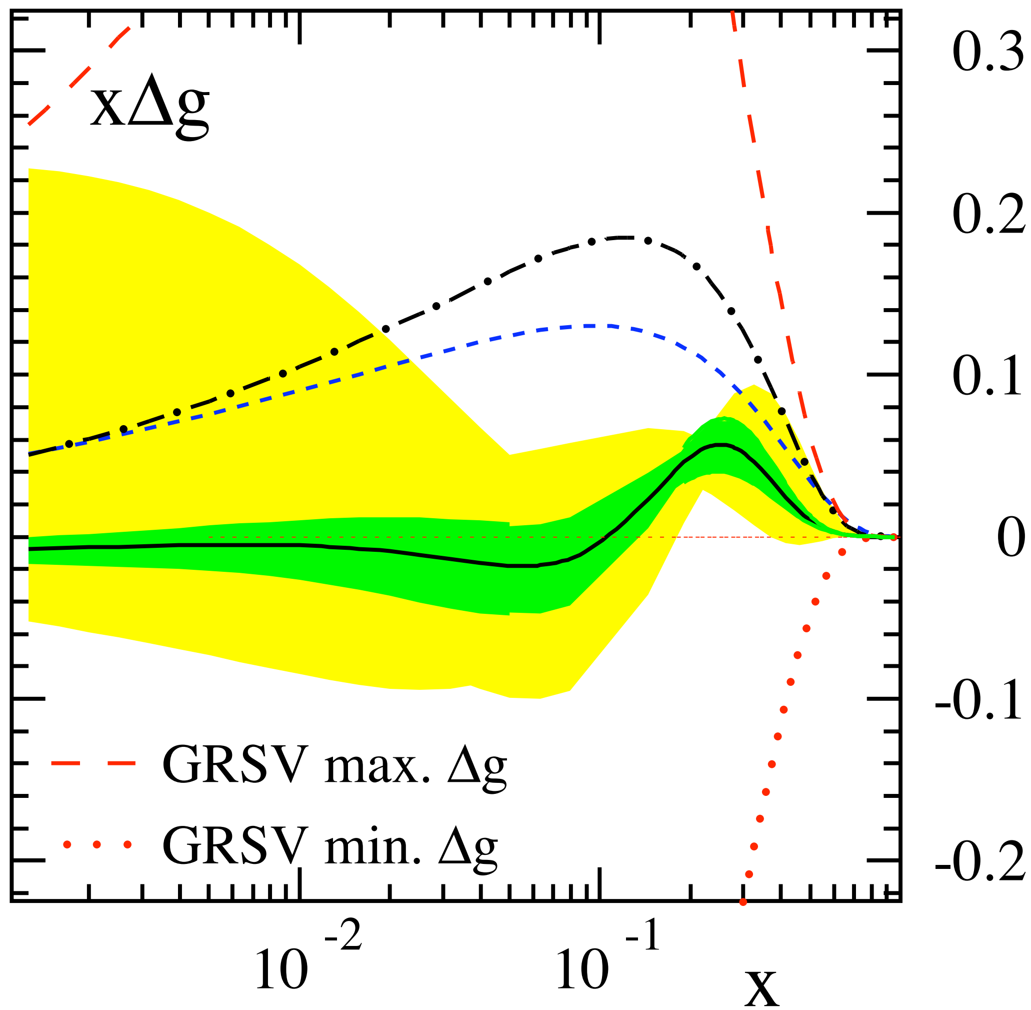


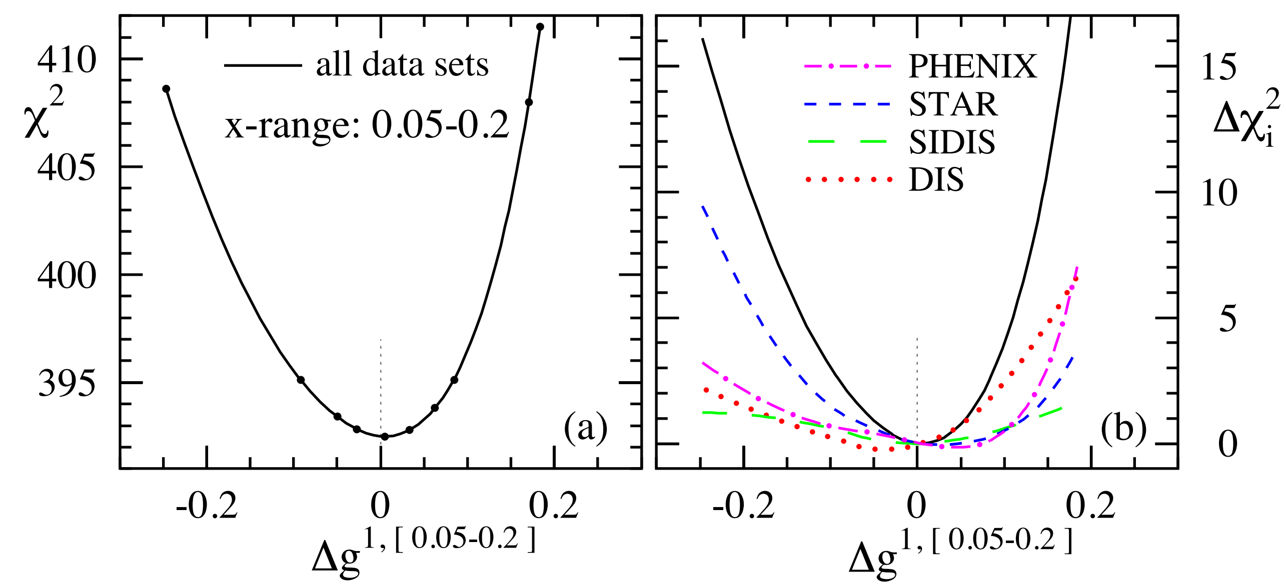


DSSV (deF, Stratmann, Sassot, Vogelsang)

Different FFs ➡ very different results for spin dependent distributions

- Extraction of polarized gluon density depends much on PHENIX and STAR pion data

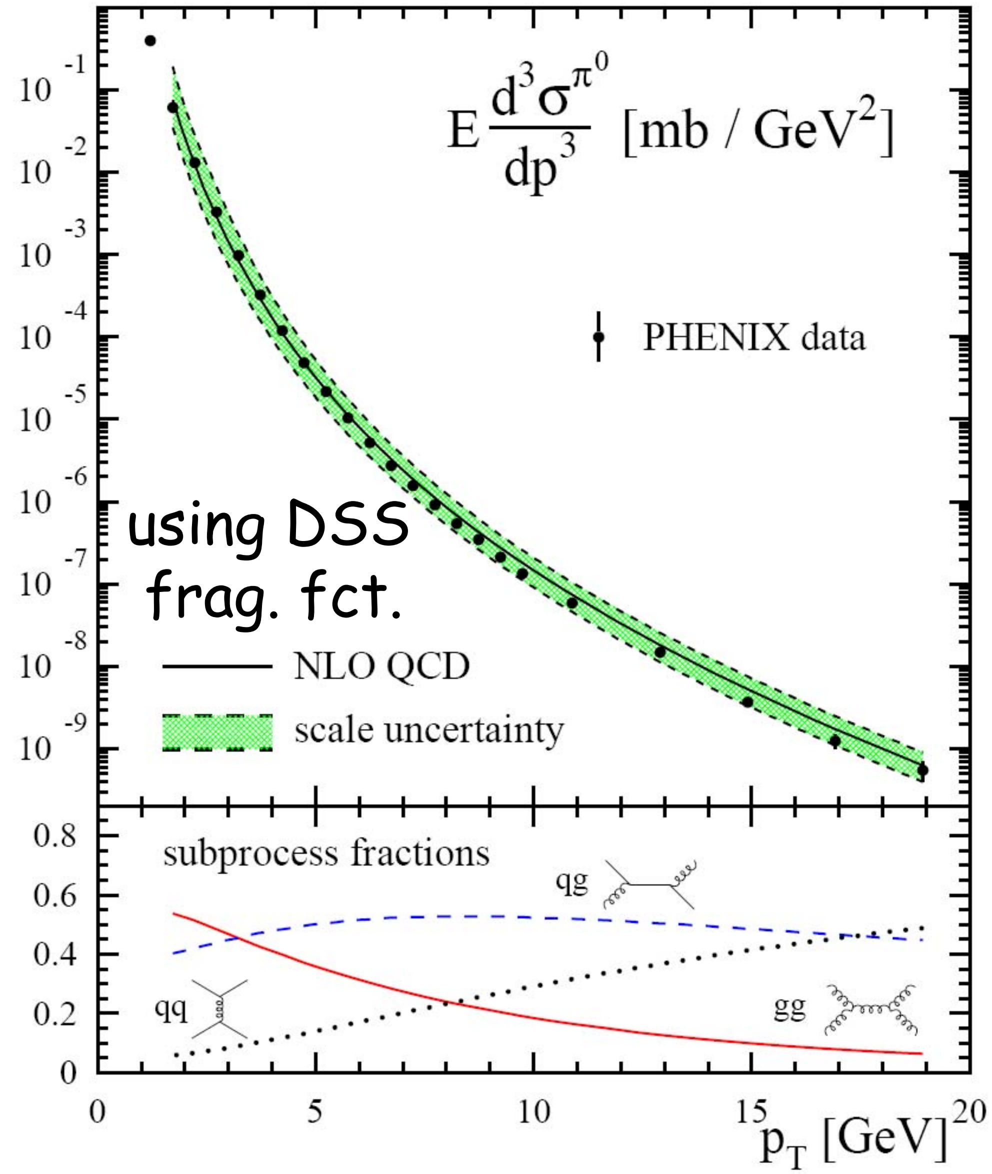




DSSV (deF, Stratmann, Sassot, Vogelsang)

wrong D_g → wrong Δg

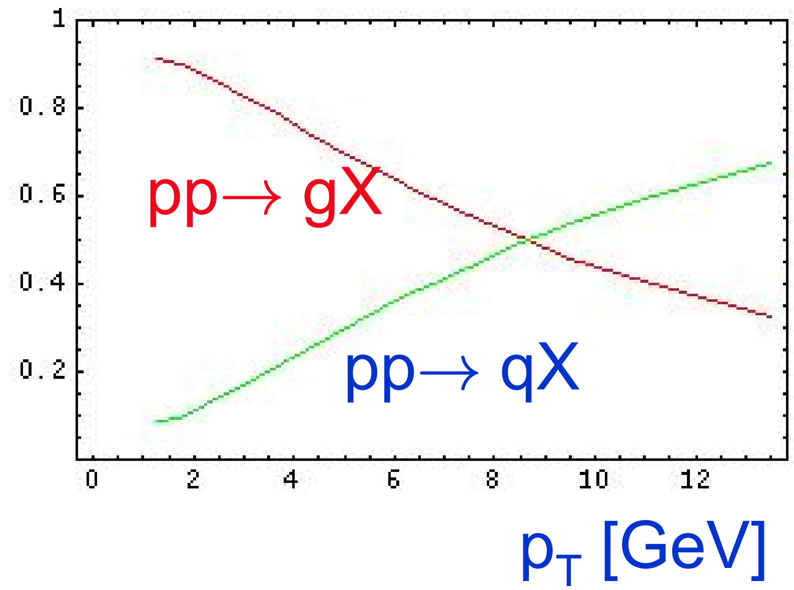
Example @ Phenix

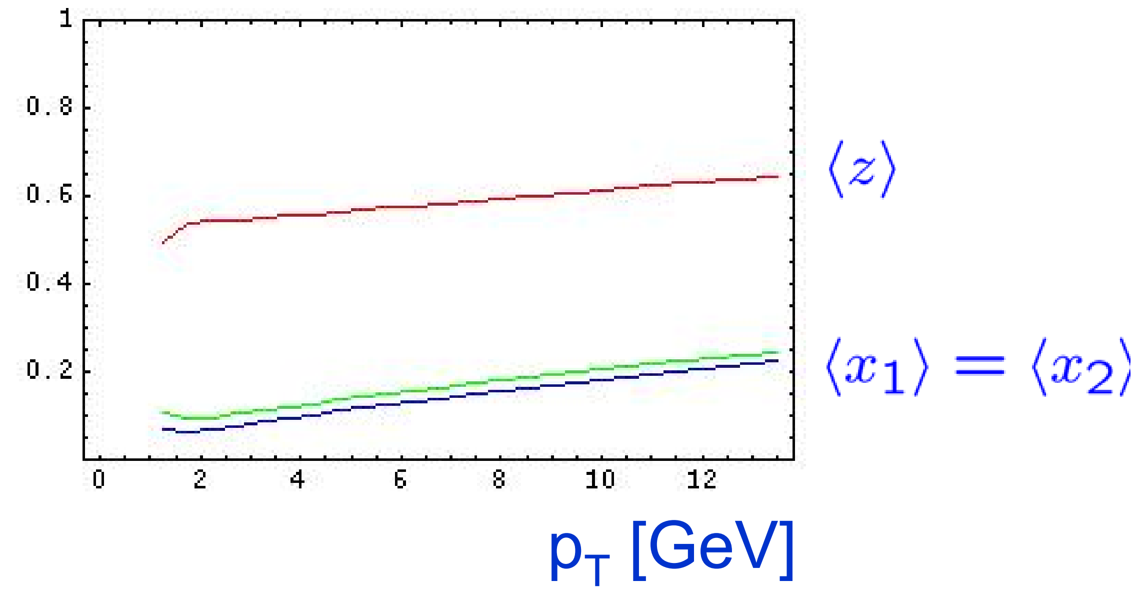


Lot of "initial gluon"
(gg and qg)
"flavor blind"

- low transverse momentum probe
  gluon fragmentation

central
rapidity
PHENIX





17. FRAGMENTATION FUNCTIONS IN e^+e^-, ep AND pp COLLISIONS

Revised October 2009 by O. Biebel (Ludwig-Maximilians-Universität, Munich, Germany), D. de Florian (Dep. de Física, FCEyN-UBA, Buenos Aires, Argentina), D. Milstead (Fysikum, Stockholms Universitet, Sweden), and A. Vogt (Dep. of Mathematical Sciences, University of Liverpool, UK).

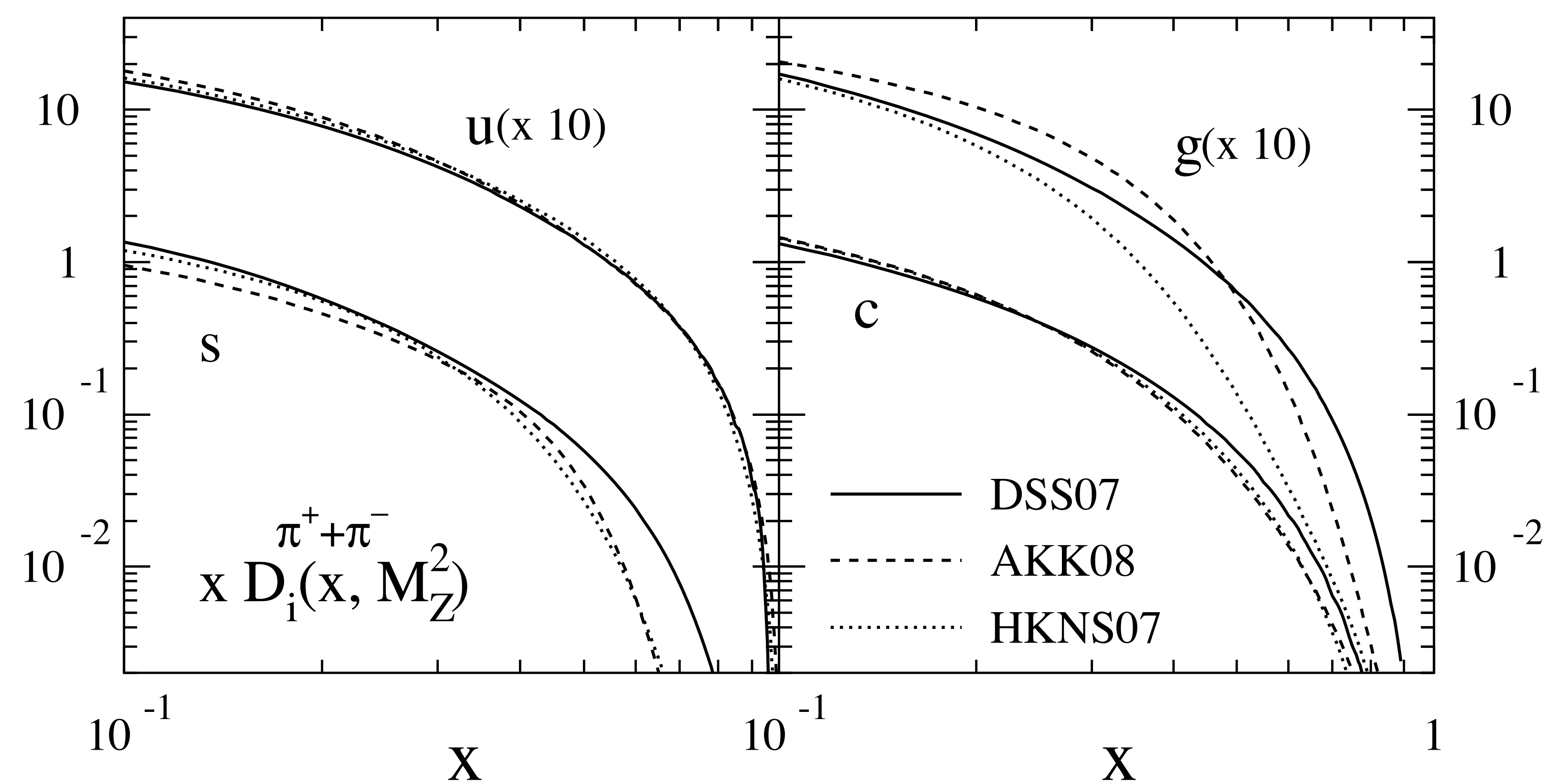
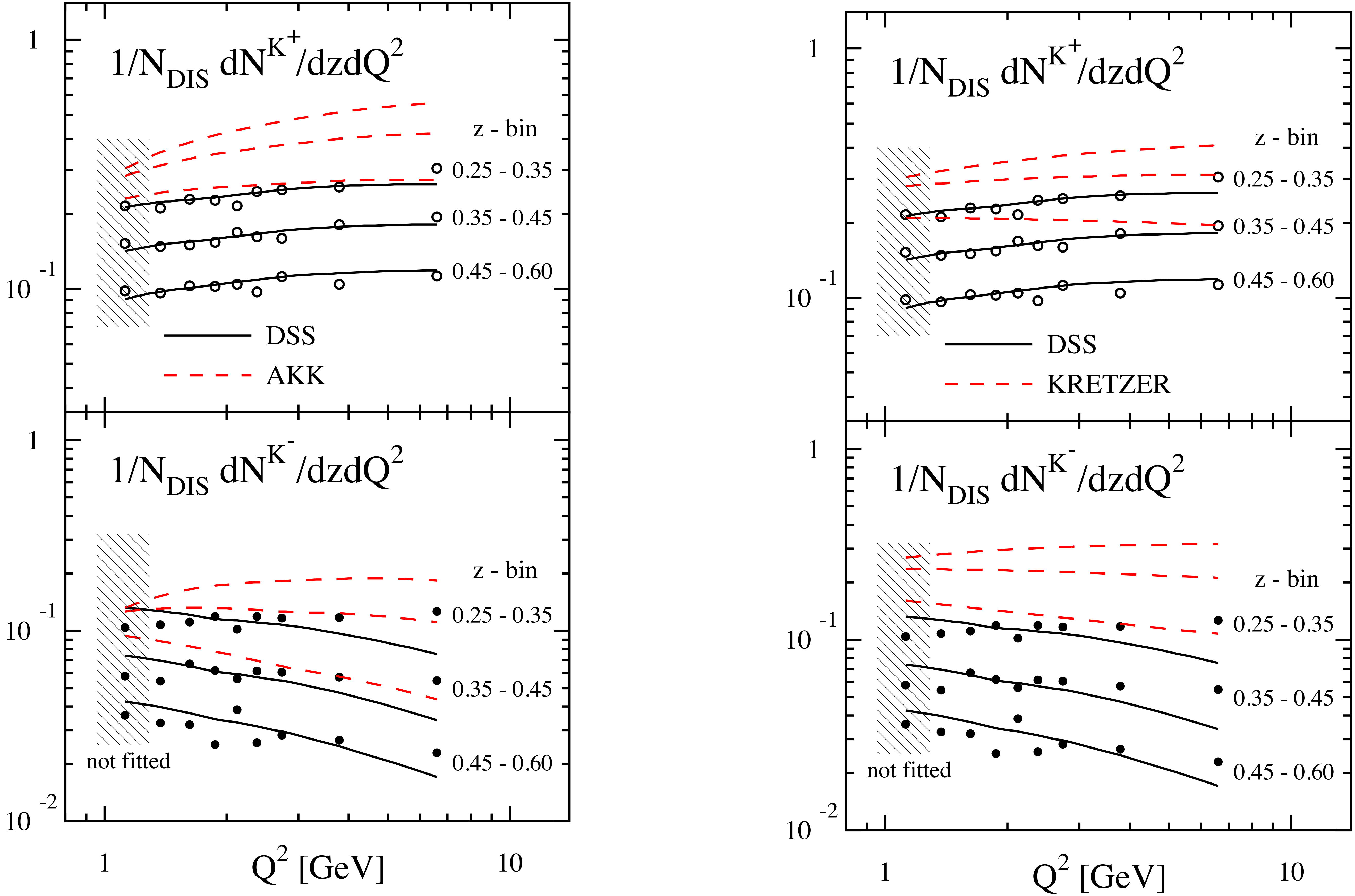


Figure 17.6: Comparison of up, strange, charm and gluon NLO fragmentation functions for $\pi^+ + \pi^-$ at the mass of the Z. The different lines correspond to the result of the most recent analyses performed in Refs. [93,94,98].

More K multiplicities AKK(08) and Kretzer



Spin-off

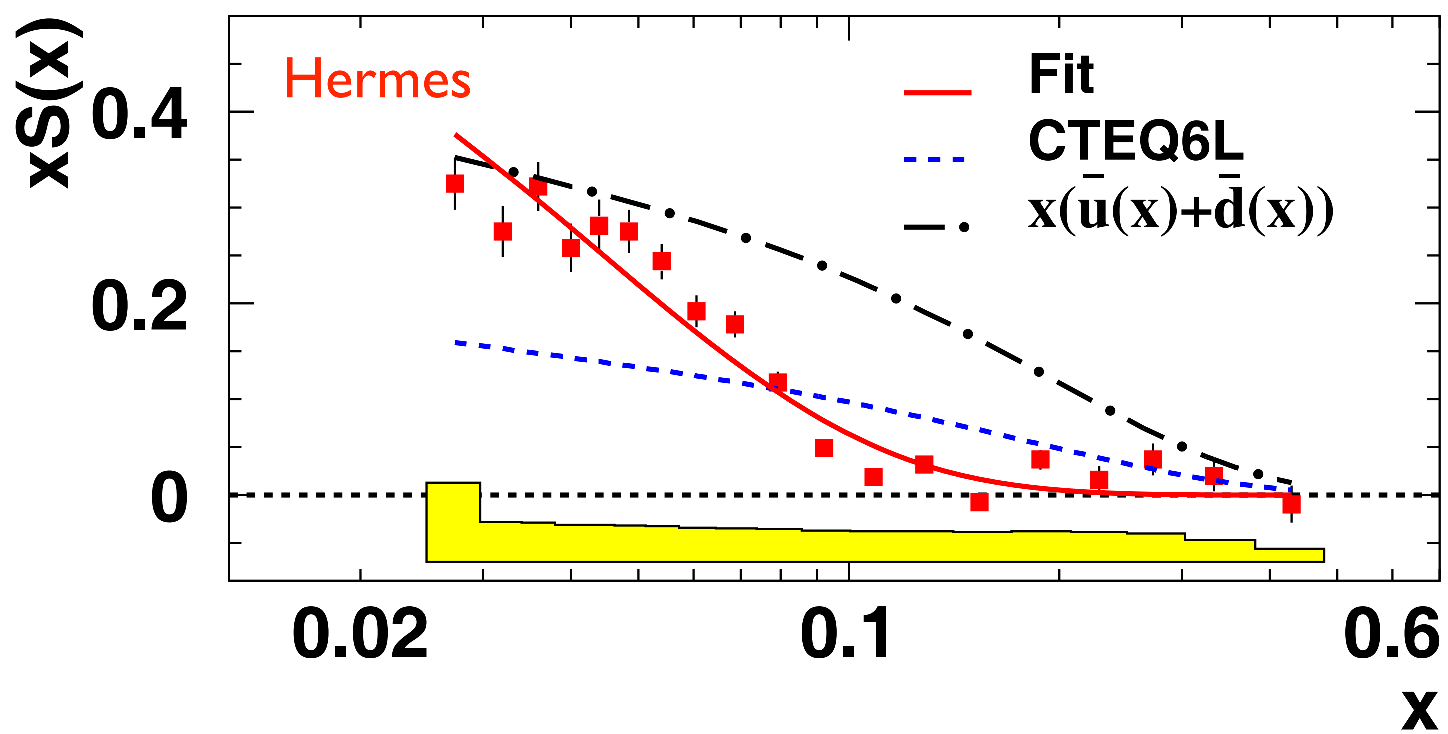
Can we say something about the strange pdf?

May be, first make sure fragmentation is right!

Careful : only true at LO!!!

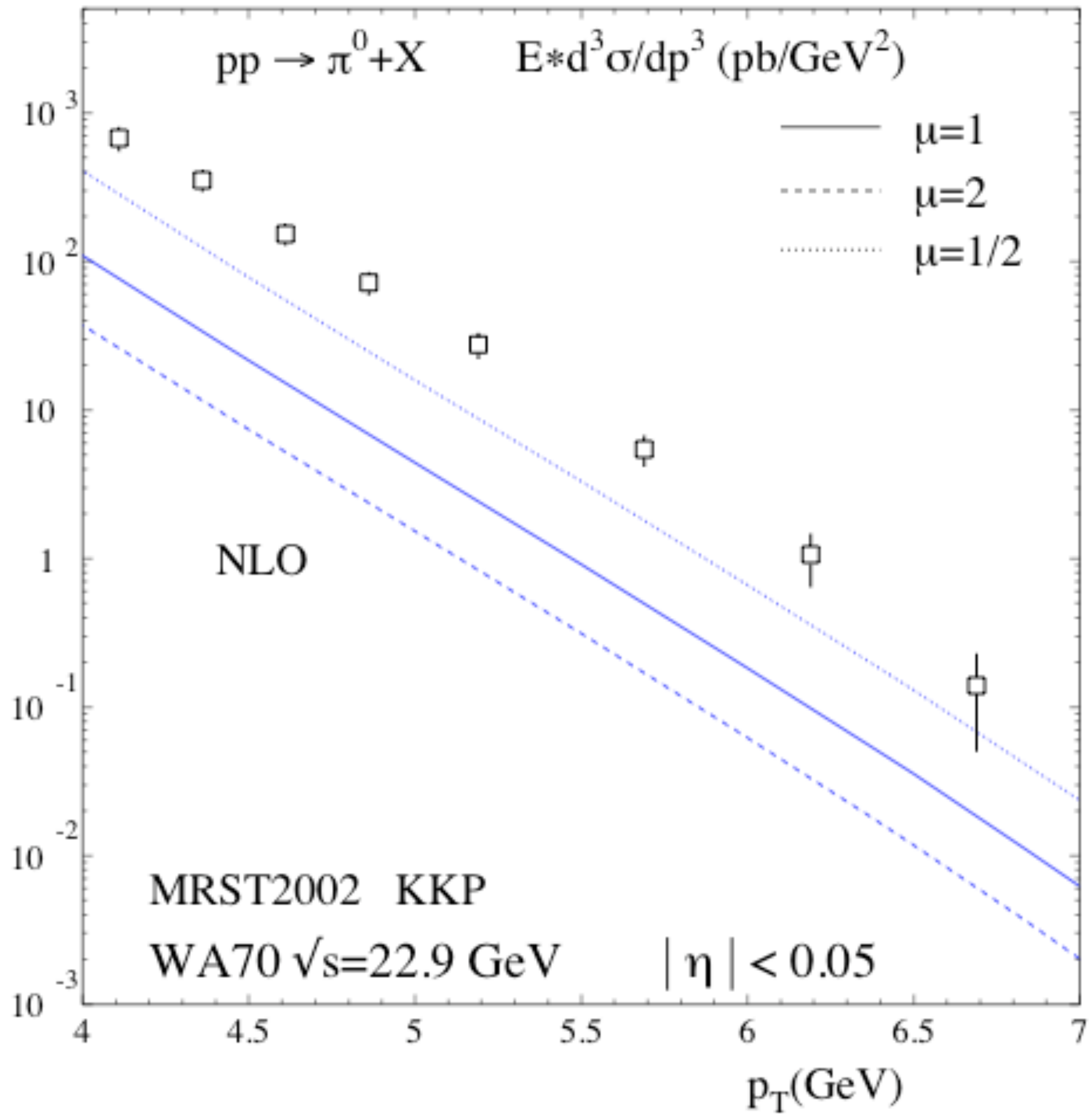
from fit of same data

$$\frac{dN^K(x)}{dN^{DIS}(x)} = \frac{Q(x)\int D^K_{1,Q}(z)dz + S(x)\int D^K_{1,S}(z)dz}{5Q(x)+2S(x)}$$



Better: check directly multiplicities with different strange distributions (even possible to fit them at NLO)

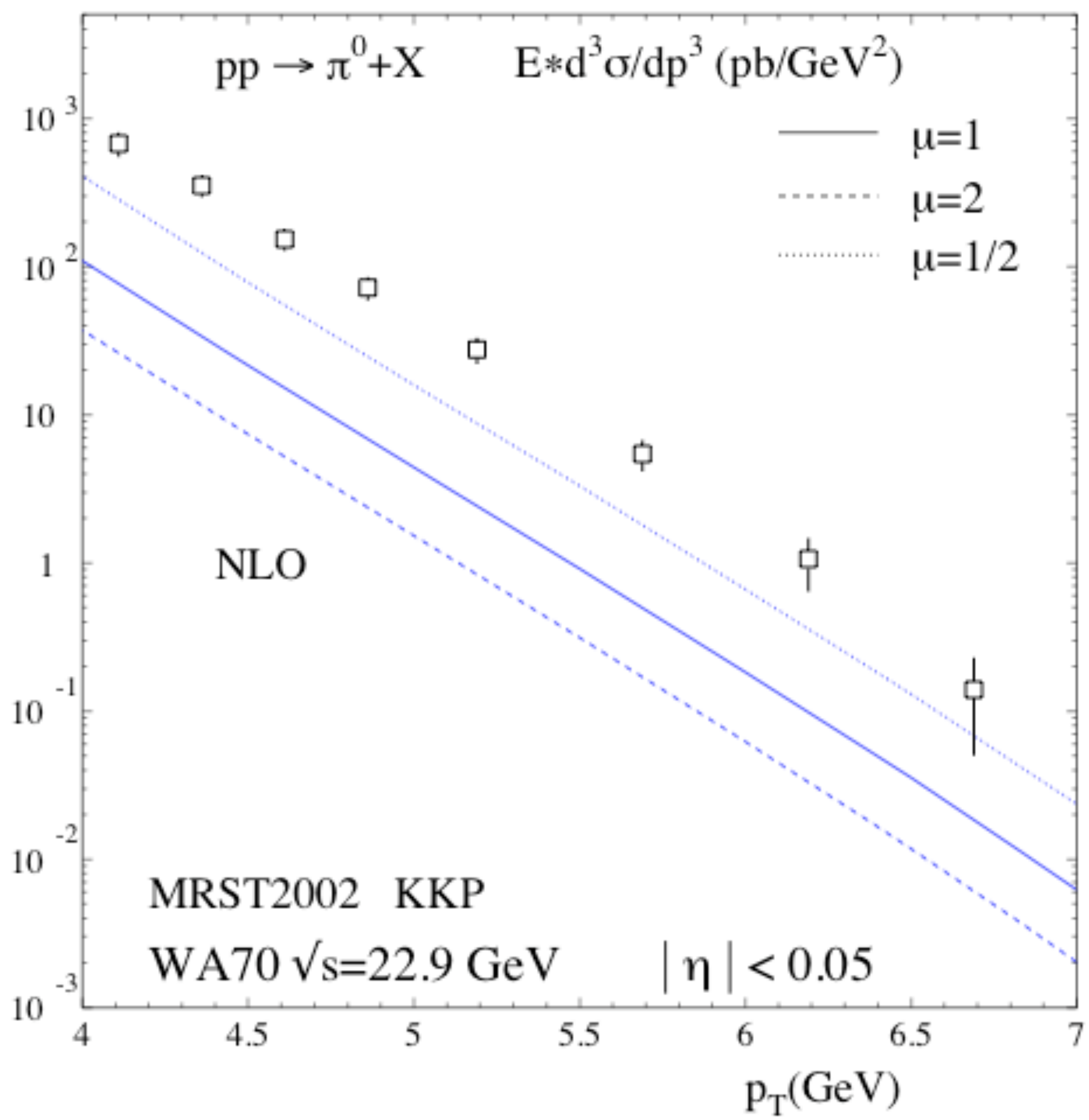
- Present TH challenge: Serious disagreement between TH and EXP for fixed target kinematics in pp collisions

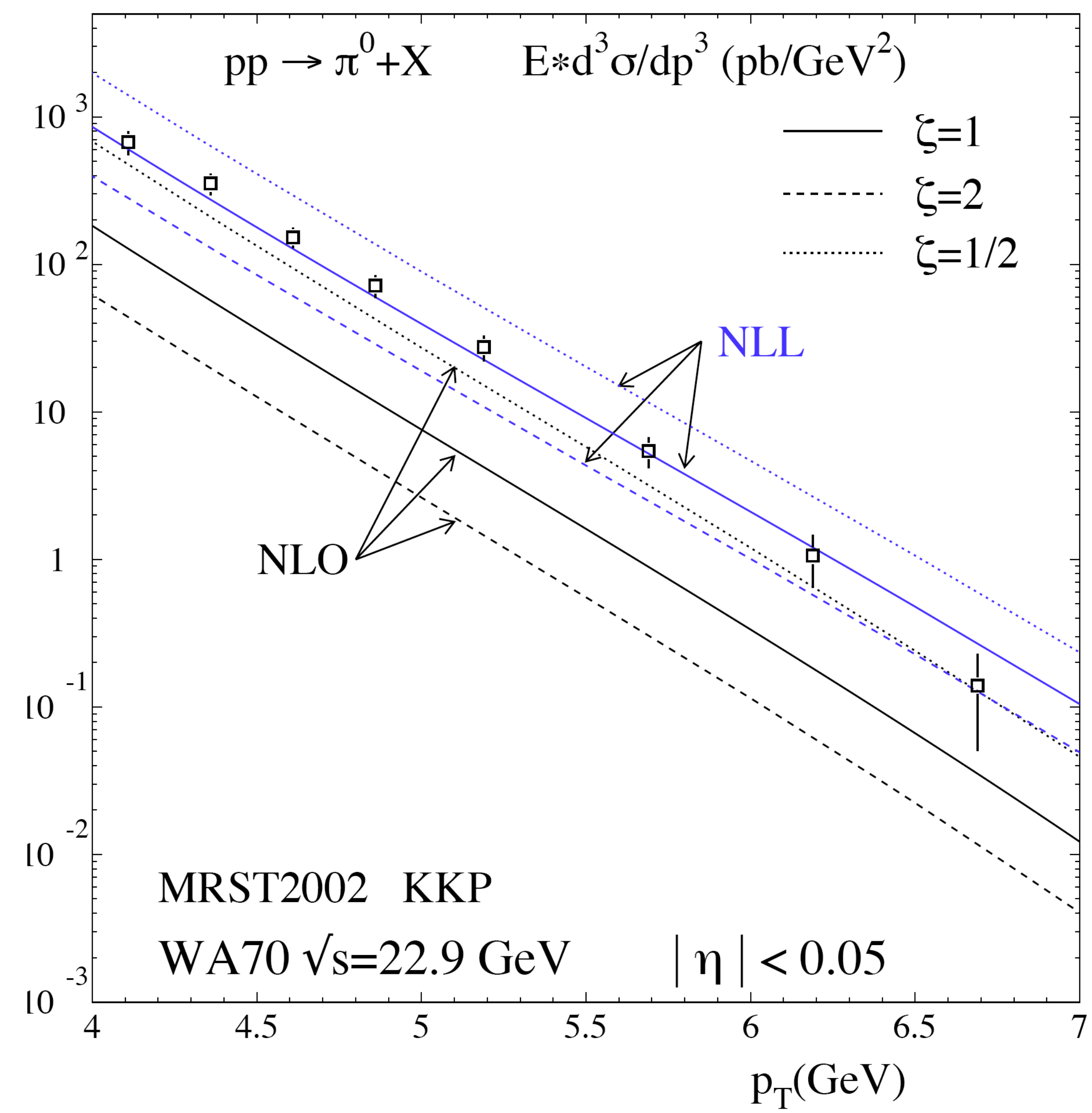


Ad-hoc intrinsic transverse momentum sometimes introduced.
But pQCD can solve the problem : resummation

D.deF, W.Vogelsang

- Present TH challenge: Serious disagreement between TH and EXP for fixed target kinematics in pp collisions





Ad-hoc intrinsic transverse momentum sometimes introduced.
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D.deF, W.Vogelsang