Lattice Hadron Spectroscopy

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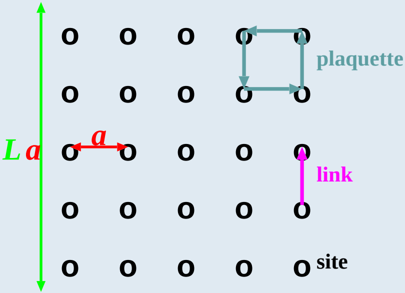
Outline

- Present status of Lattice QCD
- The light hadron spectrum
- Strong decays and mixing
- Heavy hadrons
- Summary

Lattice QCD



typical values:

$a^{-1} = 1.5\text{–}4$ GeV, $La = 1.5\text{–}6$ fm

continuum limit: $a \to 0$, La fixed

infinite volume: $La \to \infty$

$$\langle O \rangle = \frac{1}{Z} \int [dU]\,[d\psi][d\bar{\psi}]\,O[U] e^{-S[U,\psi,\bar{\psi}]}$$

"Measurement": average over a *representative* ensemble of gluon configurations $\{U_i\}$ with probability $P(U_i) \propto \int [d\psi][d\bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$

$$\langle O \rangle = \frac{1}{n} \sum_{i=1}^{n} O(U_i) + \Delta O \qquad \Delta O \propto \frac{1}{\sqrt{n}} \xrightarrow{n\to\infty} 0$$

Input: $\mathcal{L}_{QCD} = -\frac{1}{16\pi\alpha_L} FF + \bar{q}_f(\not{D} + m_f)q_f$

$$m_N^{\text{latt}} = m_N^{\text{phys}} \longrightarrow a$$
$$m_\pi^{\text{latt}}/m_N^{\text{latt}} = m_\pi^{\text{phys}}/m_N^{\text{phys}} \longrightarrow m_u \approx m_d$$
$$\cdots$$

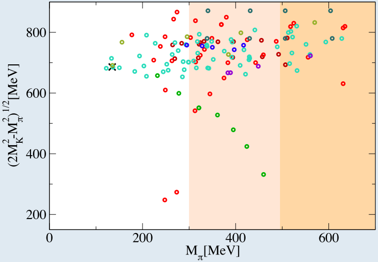
Output: hadron masses, matrix elements, decay constants, etc...

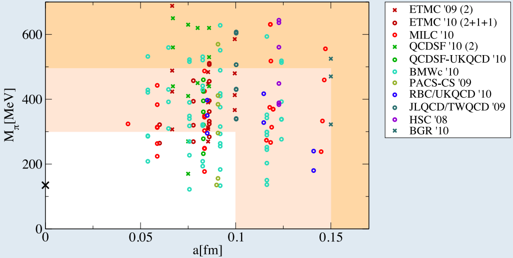
Extrapolations:

1. $a \to 0$: functional form known.
2. $L \to \infty$: harmless but often computationally expensive.
3. $m_q^{\text{latt}} \to m_q^{\text{phys}}$: chiral perturbation theory (χPT) but m_q^{latt} must be sufficiently small to start with.

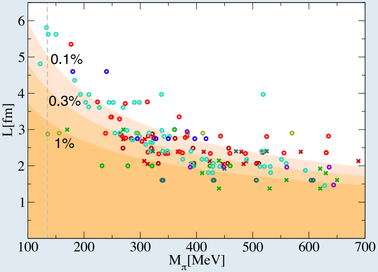
   ($m_{\text{PS}}^{\text{latt}} = m_\pi^{\text{phys}}$ has only very recently been realized.)

Landscape of current lattice simulations





Figures taken from C Hoelbling 11



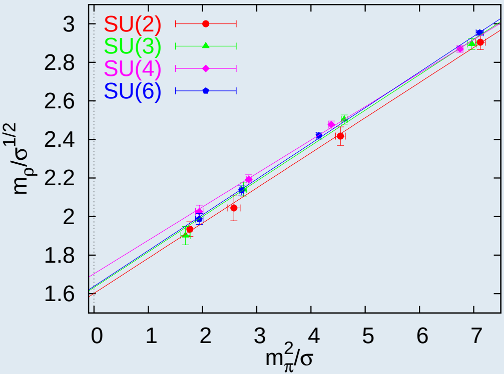
Bug or feature?

Unrealistic parameters or virtual worlds for theoreticians?

Real experiments can only be performed in one world.

Lattice "experiments" allow us to vary T, n_f, N, m_q, V etc.

Example: $SU(N)$ mesons ($N = 2, 3, 4, 6$) F Bursa, GB 07/08

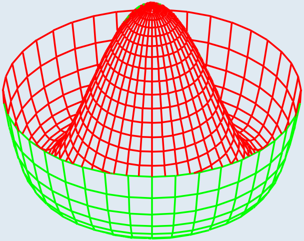


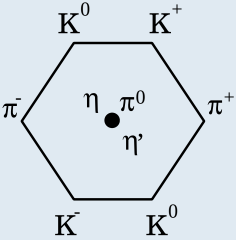
Chiral symmetry (breaking)

Global symmetry of the $m = 0$, $n_f = 3$ QCDlite™:

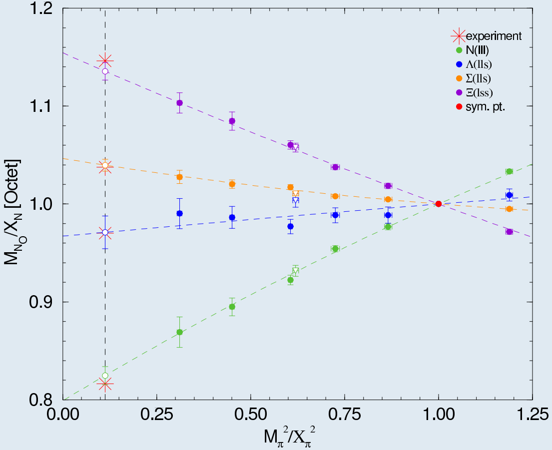
Ignore $U_V(1)$ symmetry (baryon number conservation)
$U_A(1)$ anomaly: $\partial_\mu j^5_\mu = -\frac{1}{16\pi^2} F * F \longrightarrow$ heavy η'
$m = 0$ χ-symmetry *spontaneously* broken:

$SU_L(3) \otimes SU_R(3) \longrightarrow SU_V(3)$ 8 Nambu-Goldstone bosons!



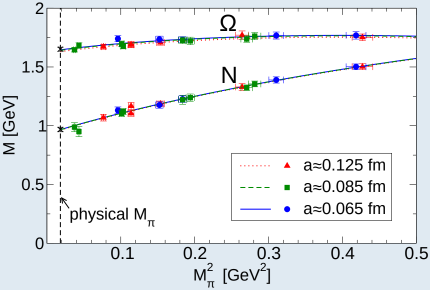


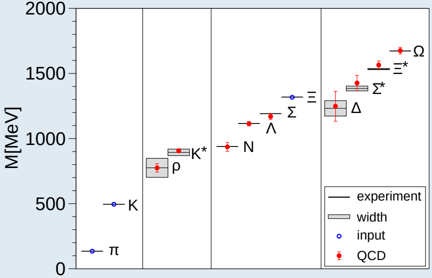
QCDSF: $m_u + m_d + m_s \propto m_\pi^2 + 2m_K^2 \approx$ const. = physical



QCDSF + UKQCD: W Bietenholz et al. 10

BMW: chiral extrapolation and the continuum limit





Marseille-Wuppertal-Budapest: S Dürr et al. 09/10

Similar results by PACS-CS and HPQCD/MILC

Problems:

- Excited states decay strongly (in experiment and on the lattice when sea quark masses become light).
- $\rightarrow$ Difficult to differentiate resonances from the continuum of multi-particle scattering states.
- Decay channels also effect the position of the real part of the pole.

When sea quarks are light (as in experiment), in particular the scalar meson sector may be distorted by tetraquarks/molecules.

Scattering phase

Scattering amplitude for partial wave ℓ:

$$a_\ell = sin\delta_\ell e^{i\delta_\ell} = -\frac{\Gamma_{\mathrm{R}}(E)E}{E^2 - m_{\mathrm{R}}^2 + iE\Gamma_{\mathrm{R}}(E)}\,.$$

Breit-Wigner (Lorentz curve) cross section: a_ℓ^2.
Scattering phase: $\delta_\ell(E)$, $\tan\delta_\ell(E) = E\Gamma_{\mathrm{R}}(E)/(m_{\mathrm{R}}^2 - E^2)$
E: energy in the CM frame.
Amplitudes like a_ℓ cannot directly be calculated in Euclidean spacetime.
However, m_{R} and δ_ℓ are accessible.
Example $\rho \to \pi\pi$. Effective Range formula (expansion about small p):

$$\tan\delta_1 = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p^3}{E(m_\rho^2 - E^2)}\,, \quad p^2 = \left(\frac{E}{2}\right)^2 - m_\rho^2\,,$$

where $m_\rho = m_{\mathrm{R}}$, $\Gamma_\rho = \Gamma_{\mathrm{R}}(m_\rho) = g_{\rho\pi\pi}^2 p_\rho^3/(6\pi m_\rho^2)$.

Finite size formula

Two non-interacting pions in the CM frame:

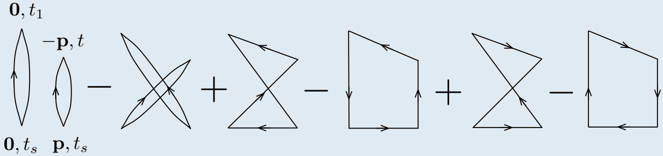
$$E_0 = 2\sqrt{m_\pi^2 + p_0^2} \quad \text{where} \quad p_0^2 = \left(\frac{2\pi}{La}\right)^2 \mathbf{n}^2 \,, n_i = -L/2{+}1, -L/2{+}2, \cdots, L/2$$

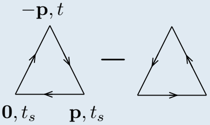
Interacting case (CM frame), depends on the volume:

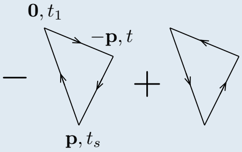
$$E = 2\sqrt{m_\pi^2 + p^2} \quad \text{where} \quad p^2 = \left(\frac{2\pi}{La}\right)^2 q^2 \,,$$

where q is a real number. Then
M Lüscher 91, K Rummukainen, S Gottlieb 95

$$\tan\delta_1(E) = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1,q)} \quad \text{where} \quad \mathcal{Z}_{00}(r,q) = \frac{1}{\sqrt{4\pi}} \sum_{\mathbf{n}\in\mathbb{Z}^3} \left(\mathbf{n}^2 - q^2\right)^{-r} .$$

$G_{\pi\pi\to\pi\pi}(t) =$ 

$G_{\pi\pi\to\rho}(t) =$ 

$G_{\rho\to\pi\pi}(t) =$ 

PACS-CS: S Aoki et al 10

Also:

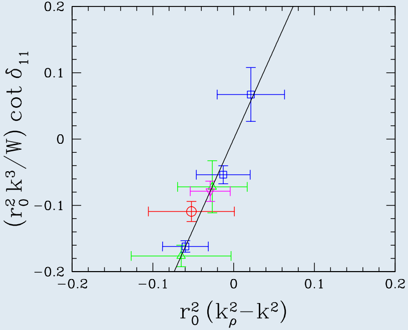
BMW: J Fison et al 10

ETMC: X Feng, K Jansen, D Renner 10

QCDSF: M Göckeler et al 08

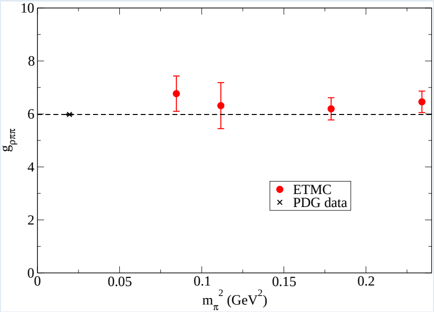
Benchmark: $\rho \longrightarrow \pi\pi$

QCDSF



Slope is $g_{\rho\pi\pi}$.

ETMC

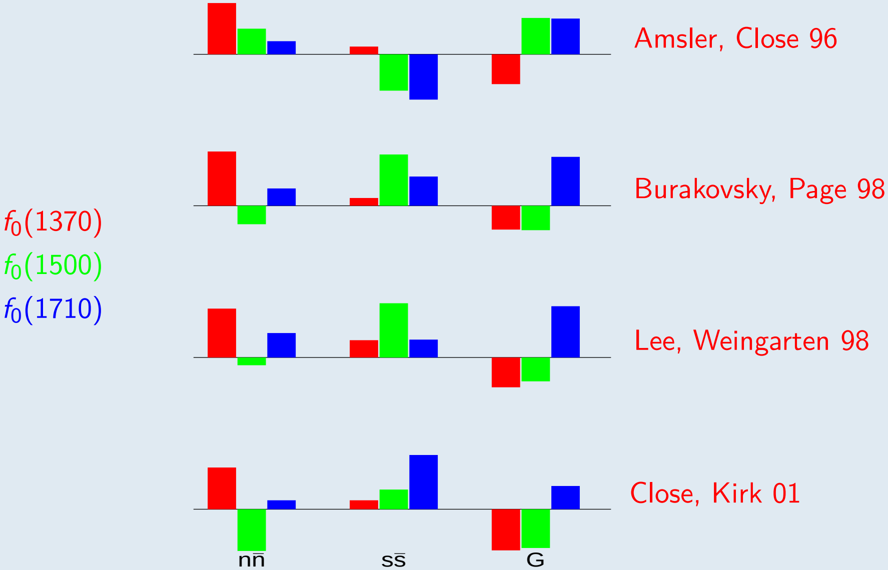


Indeed: $\Gamma \approx (170 \pm 50)$ MeV.

Scalar mesons $J^{PC} = 0^{++}$ in experiment

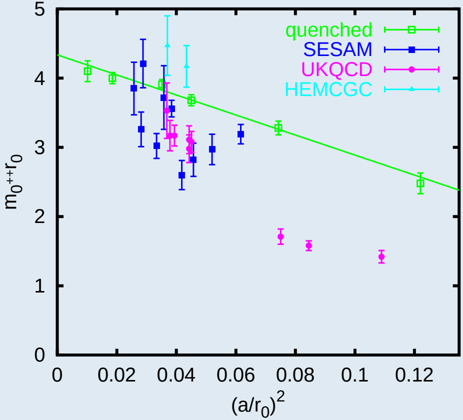
$I \neq 0$	$I = 0$	
$\kappa(800\ ??)$	$\sigma(600\ ??)$	$\pi\pi$, $q\bar{q}q\bar{q}$ tetraquark or $\pi\pi$ FSI ?
$a_0^-(980)$ $a_0^0(980)$ $a_0^+(980)$	$f_0(980)$	$K\bar{K}$, $q\bar{s}\bar{q}s$ tetraquark ?
$K_0^*(1430)$ $K_0^{*+}(1430)$	$f_0(1370)$	$u\bar{u} + d\bar{d} - 2s\bar{s}$?
$a_0^-(1450)$ $a_0^0(1450)$ $a_0^-(1450)$	$f_0(1500)$	$u\bar{u} + d\bar{d} + s\bar{s}$?
$K_0^{*-}(1430)$ $K_0^*(1430)$	$f_0(1710)$	glueball ?

Mixing? Coupling to $\pi\pi$, KK, $\eta\eta$?



Amsler, Close 96
Burakovsky, Page 98
Lee, Weingarten 98
Close, Kirk 01
$f_0(1370)$
$f_0(1500)$
$f_0(1710)$
$n\bar{n}$
$s\bar{s}$
G

This all uses prejudice from ancient lattice calculations!



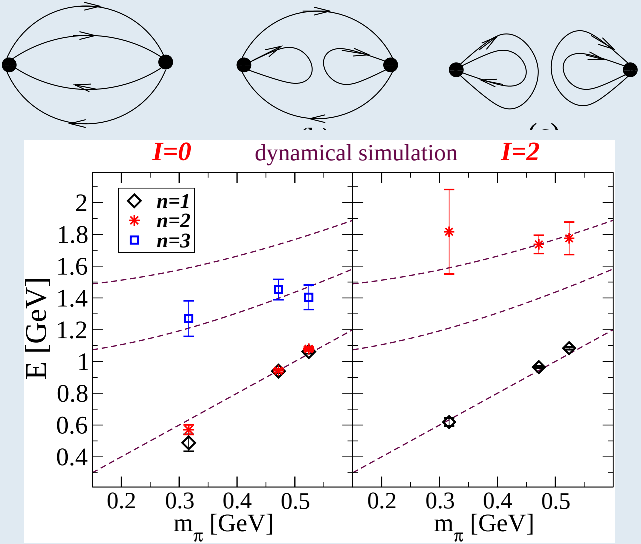
quenched
GB *et al.* 93, Teper/Lucini 01

SESAM/TχL $n_f = 2$ Wilson
GB *et al.* 99

UKQCD $n_f = 2$ clover
A. Hart *et al.* 01

HEMCGC $n_f \approx 2$ staggered
K. Bitar *et al.* 94

What about light scalar mesons/tetraquarks/molecules?



S Prelovsek et al 10 (also recent work at $I = 2$ by Dudek et al. (HSC))

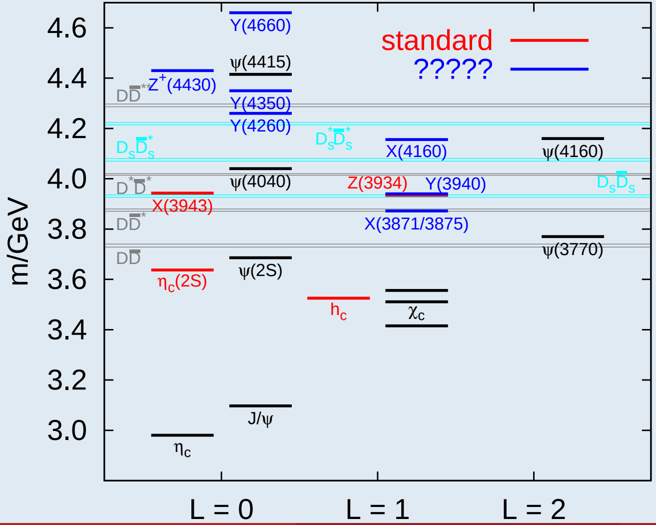
Hadron spectroscopy: exciting times

First time in >20 years: several new narrow(ish) resonances!

- ★ Υ D wave(s), η_b
- ★ B_c
- ★ η_c', h_c
- ★ $X(3872), X(3943), Y(4260), \ldots$
- ★ $D_{s0}^*(2317)$, $D_{s1}^*(2460)$, $\ldots$
- ★ >10 new charmed baryons

1974 – 1977: 10 $c\bar{c}$ resonances, 1978 – 2001: 0 $c\bar{c}$'s

2002 – 2008: $\leq$ 12 new $c\bar{c}$'s found by BaBar, Belle, CLEO-c, CDF, D0



new detectors

higher luminosity

new channels:

B decays

$\gamma\gamma$

$\psi\psi$-production

gg in $p\bar{p}$ collisions.

$c\bar{q}q\bar{c}$ in $c\bar{c}$?

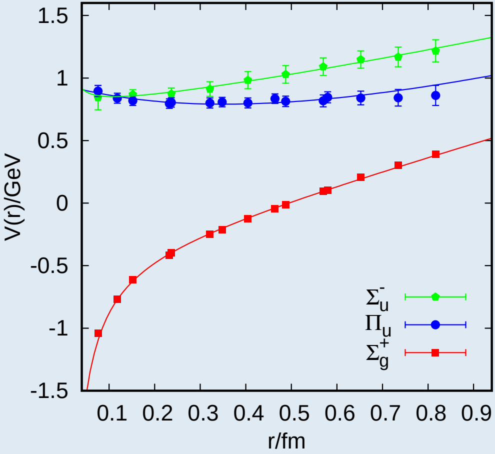
$cg\bar{c}$ hybrids ?

Hybrid mesons

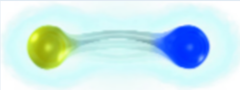
$m_c \gg \Lambda_{\mathrm{QCD}} \longrightarrow$ Adiabatic and non-relativistic approximations:

$H\psi_{nlm} = E_{nl}\psi_{nlm} \quad , \quad H = 2m_c + \frac{p^2}{m_c} + V(r)$

Lattice:

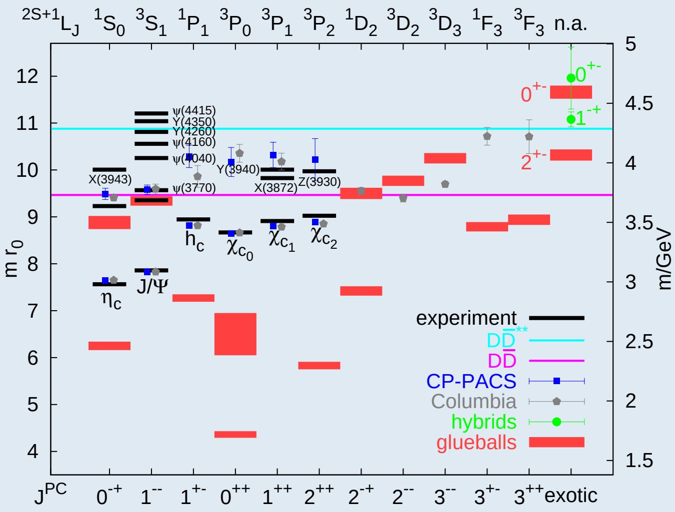


hybrid potential:

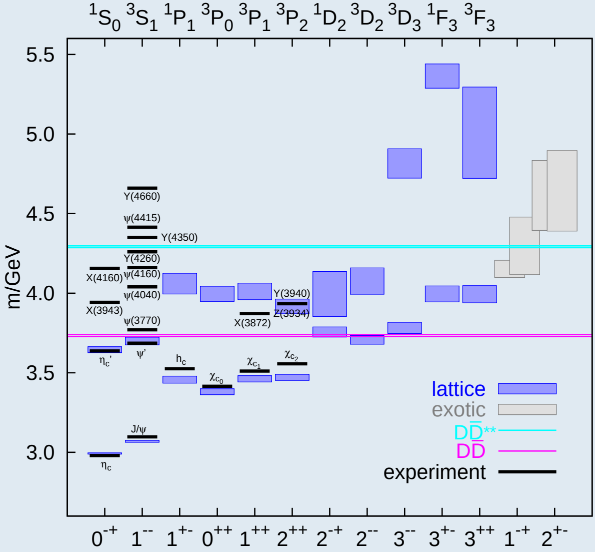


Quenched Lattice: glueballs, charmonia and hybrids

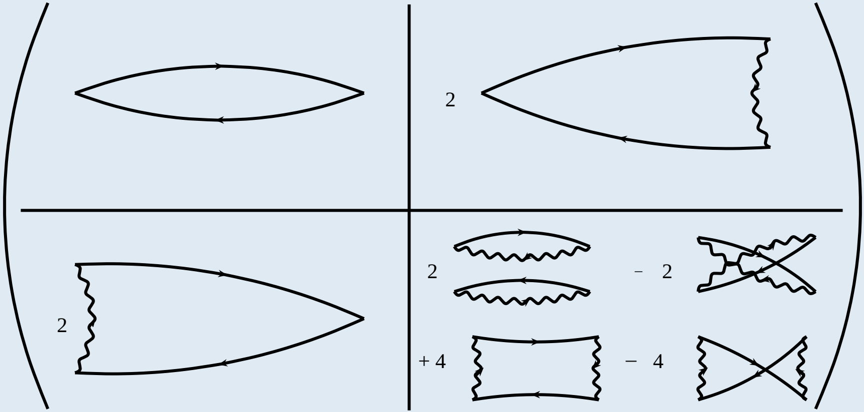
(No "disconnected diagrams" and no sea quarks → no mixing $G, c\bar{c}, c\bar{q}q\bar{c}$, no decay !)



C Ehmann, GB ($n_f = 2$, $a^{-1} \approx 1.73$ GeV from m_N)

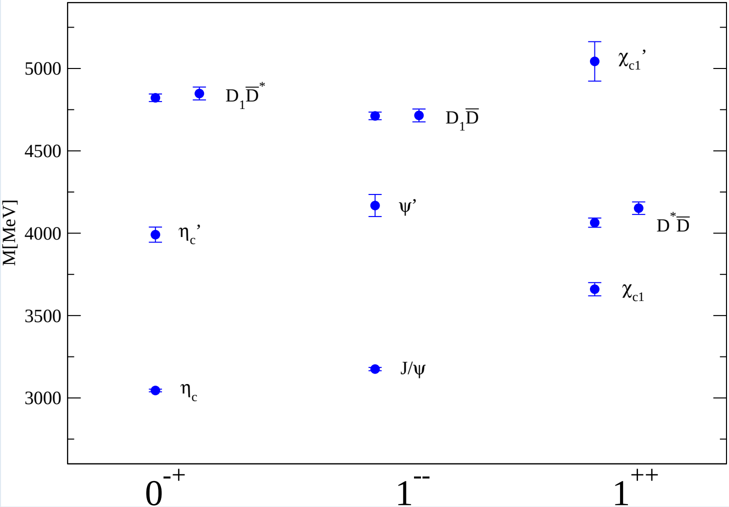


$c\bar{c} \leftrightarrow \overline{D}D$ mixing (for $n_f = 2$) GB, C Ehmann 09/10:

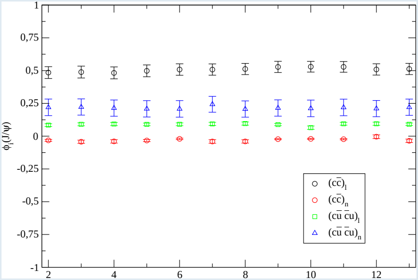


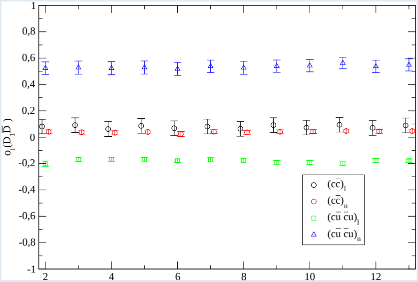
($c\bar{c}$ annihilation diagrams negelected.)

$n_f = 2,\ a^{-1} \approx 2.59\,\text{GeV},\ La \approx 1.83\,\text{fm},\ m_{\text{PS}} \approx 290\,\text{MeV}$

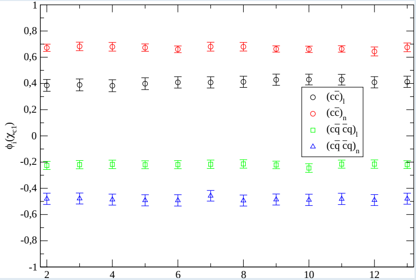


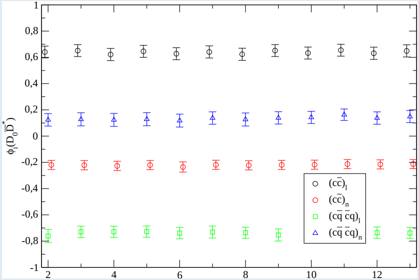
Eigenvector components of the J/ψ. Components of the $D_1\overline{D}$.





Eigenvector components of the χ_{c1}. Components of the $D^*\overline{D}$.





Summary and outlook

- $\exists$ first simulations at the physical point for $a^{-1} > 2$ GeV, $La > 3.5\, m_\pi^{-1} \approx 5\,\text{fm}$.
- The experimental "standard" light hadron spectrum has been reproduced.
- Tetraquarks and molecules present a challenge, in particular in the light scalar sector.
- Calculations of phase shifts for decays into two particle final states have been demonstrated to be feasible. Benchmarks: $\rho \to \pi\pi$, $\Delta \to N\pi$.
- Studies of $c\bar{c} \leftrightarrow c\bar{q}q\bar{c}$ are on their way.
- What about electromagnetic effects and $SU(2)$ isospin breaking?