Pion Polarizability: Chiral Perturbation Theory vs. Dispersion Theory

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- GIS: J. Gasser, M.A. Ivanov, M.E. Sainio, Nucl. Phys. B 745, 84 (2006)
- Bürgi: U. Bürgi, Nucl. Phys. B 479, 392 (1996)
- FK: L.V. Fil'kov, V.L. Kashevarov, Phys. Rev. C 72, 035211 (2005)
- PDS: B. Pasquini, D.Drechsel, S.Scherer, Phys. Rev. C 77, 065211 (2008)

Introduction

- $\alpha =$ electric polarizability, $\beta =$ magnetic polarizability
  units: $10^{-4}\mathrm{fm}^3$
  forward polarizability of pion, $\alpha + \beta$
  backward polarizability of pion, $\alpha - \beta$
- Prediction of ChPT at $\mathcal{O}(p^6)$ [GIS]:
  $\alpha_{\pi^+} + \beta_{\pi^+} = 0.16 \pm 0.1, \quad \alpha_{\pi^+} - \beta_{\pi^+} = 5.7 \pm 1.0$
- Prediction of Ref. [FK]:
  $\alpha_{\pi^+} + \beta_{\pi^+} = 0.17 \pm 0.02, \quad \alpha_{\pi^+} - \beta_{\pi^+} = 13.60 \pm 2.15$

Experimental Information and Data Analysis Backward polarizability $\alpha_{\pi^+} - \beta_{\pi^+}$ in units of $10^{-4}\,\mathrm{fm}^3$

reaction	analysis [experiment]	$\alpha_{\pi^+} - \beta_{\pi^+}$
$\pi^- Z \to \gamma \pi^- Z$	Serpukhov (1983)	$15.6 \pm 6.4 \pm 4.4$
	COMPASS(201?)	??±??±??
$\gamma p \to \pi^+ n$	Lebedev (1984)	40 ± 24
	Mainz (2005)	$11.6 \pm 1.5 \pm 3.0 \pm 0.5$
$\gamma\gamma \leftrightarrows \pi^+\pi^-$	D. Babusci *et al.* (1992)	
	[PLUTO (1984)]	$38.2 \pm 9.6 \pm 11.4$
	[DM1 (1986)]	34.4 ± 9.2
	[DM2 (1987)]	52.6 ± 14.8
	[MARK II (1990)]	4.4 ± 3.2
	J.F. Donoghue & B. Holstein (1993) [MARK II (1990)]	5.4
	A. Kaloshin & V. Serebryakov (1994) [MARK II (1990), CBC (1990)]	5.25 ± 0.95
	L. Fil'kov (2005) [TPC/2γ (1986),MARK II (1990)] [CELLO (1992), VENUS (1995)] [ALEPH (2003), BELLE (2005)]	13.0 (+2.6, −1.9)

Compton Scattering: Kinematics

$$\gamma(k) + \pi(p) \to \gamma(k') + \pi(p')$$

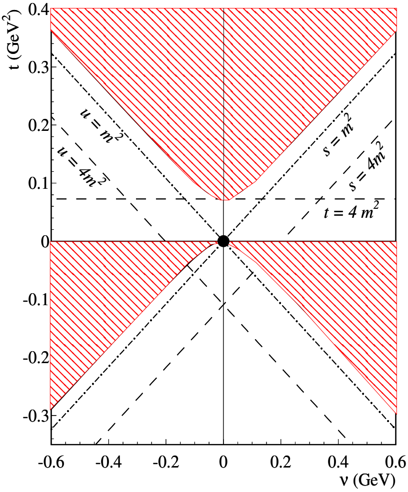
3 Mandelstam variables:
$s = (k+p)^2, \quad t = (k-k')^2, \quad u = (k-p')^2$
(constraint $s + t + u = 2m_\pi^2$)

Mandelstam plane: Xing-symmetric $\nu = (s-u)/(4m_\pi)$ and t

$(\nu,\ t) \Leftrightarrow$ photon lab energies E_γ and E'_γ and lab scattering angle θ:
$\nu = E_\gamma + t/(4m_\pi) = \frac{1}{2}(E_\gamma + E'_\gamma)$
$t \ = -4E_\gamma\, E'_\gamma \sin^2(\theta/2) = -2m_\pi(E_\gamma - E'_\gamma)$

Scattering matrix has 2 independent amplitudes:
$M^{+-}(\nu, t)$ helicity-flip, forward scattering, $\Rightarrow \alpha + \beta$
$M^{++}(\nu, t)$ NO helicity-flip, backward scattering, $\Rightarrow \alpha - \beta$

Physical regions in Mandelstam plane

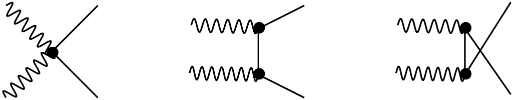


- red hatched: physical regions $\gamma + \gamma \to \pi + \pi$ $\gamma + \pi \to \gamma + \pi$
- two-pion thresholds at $s = 4m_\pi^2$, $u = 4m_\pi^2$, $t = 4m_\pi^2$
- DR integration paths $t = 0$ (forward), $\theta = 180^\circ$ (backward) $u = m_\pi^2$, $s = m_\pi^2$, ...

Chiral Perturbation Theory

- pion-pion interaction at low energies ruled by chiral symmetry of QCD
- χPT systematic expansion in small momenta and symmetry-breaking terms (quark/pion mass)
- chiral symmetry requires derivative coupling
- $\mathcal{L}_2 = \frac{F_\pi^2}{4}[D_\mu U D^\mu U^\dagger + m_\pi^2(U + U^\dagger)]$
- electroweak interaction added by covariant derivative: $D_\mu U = \partial_\mu U - ie(Q_\pi U - U Q_\pi)A_\mu$
- F_π=decay constant, m_π=mass, Q_π=charge

Tree Diagrams



Born contribution = contact + direct + exchange terms

invariant Born amplitudes

$$M_B^{++}(\nu,t) = m_\pi^2\, M_B^{+-}(\nu,t) = -\frac{e^2 Q^2}{2(\nu-\nu_B(t))(\nu+\nu_B(t))}$$

poles at $\nu(t) = \pm\nu_B(t) = \pm t/4m_\pi$

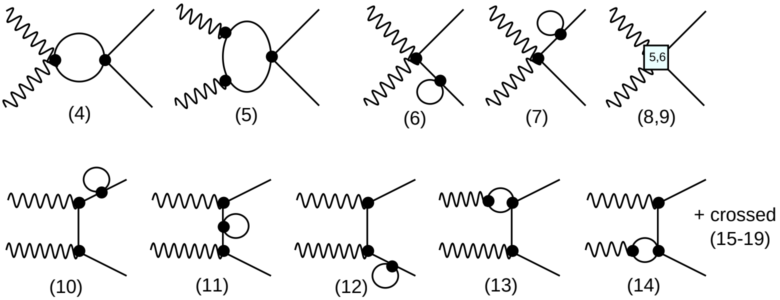
at this level, the pion has no internal degrees of freedom
there is no dispersion of the electromagnetic wave
the polarizabilities vanish

One-Loop Order

at $\mathcal{O}(p)^4$: 14 loop diagrams from $\mathcal{L}_2$ plus 10 counter terms

$$\mathcal{L}_4 = \sum_{i=1}^{7} l_i K_i + \sum_{i=1}^{3} h_i \bar{K}_i = \frac{1}{4} l_1 [D_\mu U D^\mu U^\dagger]^2 + \dots$$

7 LECs $l_1 \dots l_7$ + 3 LECs h_i = 10 LECs



However, at one-loop order, $\alpha \pm \beta$ depends on only 1 LEC:

$$l_\Delta = l_6 - l_5.$$

Two-Loop Order

At $\mathcal{O}(p)^6$:
78 two-loop diagrams from $\mathcal{L}_2 \bigotimes \mathcal{L}_2 \bigotimes \mathcal{L}_2$
38 one-loop diagrams from $\mathcal{L}_2 \bigotimes \mathcal{L}_4$
plus 57 new counter terms from $\mathcal{L}_6$

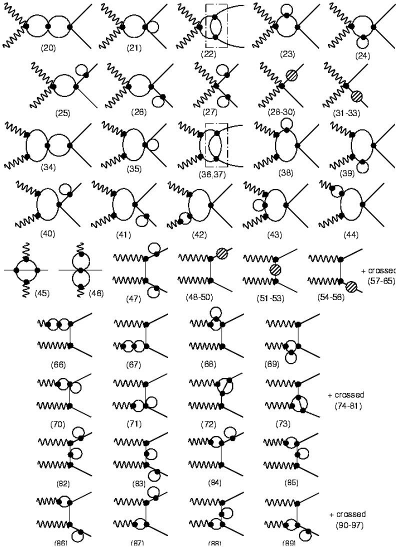
$$\mathcal{L}_6 = \sum_{i=1}^{57} c_i P_i = c_1 [D_\mu U D^\mu U^\dagger]^3 + \dots$$

However, at two-loop order, $\alpha \pm \beta$ depends on only 7 LECs:
5 LECs of $\mathcal{O}(p)^4$ ($l_1, l_2, l_3, l_4, l_\Delta = l_6 - l_5$) and
2 combinations of the 57 c_i (a and b)

7 LECS to be determined

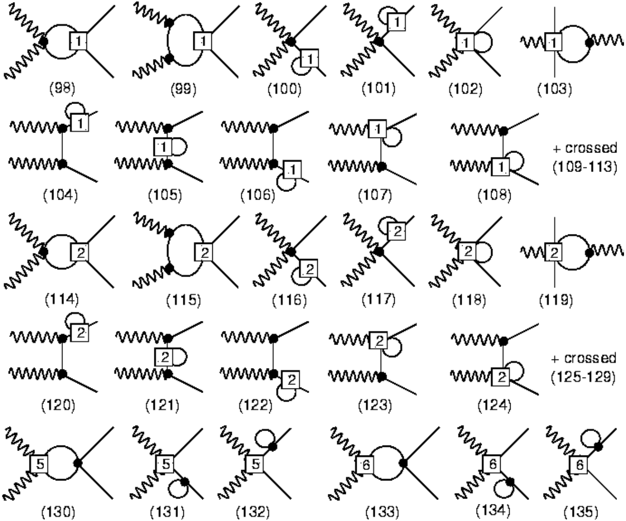
Diagrams Two-Loop Order (I)

78 Diagrams built from $\mathcal{L}_2 \bigotimes \mathcal{L}_2 \bigotimes \mathcal{L}_2$



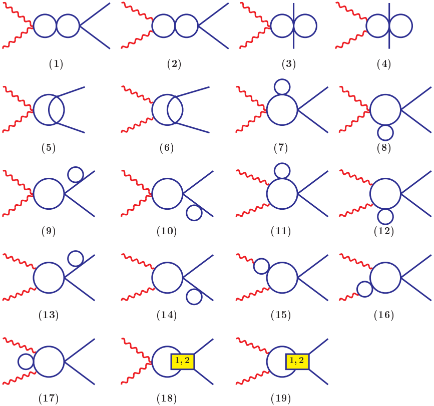
Diagrams Two-Loop Order (II)

38 Diagrams built from $\mathcal{L}_2 \bigotimes \mathcal{L}_4$

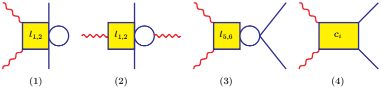


Diagrams (III)

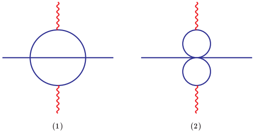
two-loop from $\mathcal{L}_2$ and
one-loop from $\mathcal{L}_4$



one-loop from $\mathcal{L}_4$ and
counterterms from $\mathcal{L}_6$



acnode and butterfly diagrams



Loop Integrals

$$
\begin{aligned}
P^{(1)}_{\mathrm{A;\,box;+}} = \;& 4s^2x_2^2x_3^3\Big\{6x_1\Big[9-2(23+8x_2)x_3-(67-405x_2+31x_2^2)x_3^2 \\
&\quad +(70+39x_2-808x_2^2+20x_2^3)x_3^3 \\
&\quad +9(6-61x_2+53x_2^2+60x_2^3)x_3^4-81x_2(3-10x_2+8x_2^2)x_3^5\Big] \\
&\quad +3x_1^2x_2x_3\Big[-92-171x_3+592x_2x_3 \\
&\quad +(231+4(194-339x_2)x_2)x_3^2 \\
&\quad +9(1-2x_2)(71-22x_2-60x_2^2)x_3^3 \\
& +27(1-2x_2)^2(11-16x_2)x_3^4\Big]-2x_1^3x_2^2x_3^2\Big[245+(1-2x_2)x_3 \\
& \times(470+27(1-2x_2)x_3(15+8(1-2x_2)x_3))\Big] \\
& +6x_3(-19+24x_3+35x_3^2-36x_3^3 \\
& +x_2^2x_3(-25+40x_3+27(5-6x_3)x_3^2) \\
& +x_2(10+44x_3-199x_3^2+9x_3^3(8+9x_3)))\Big\} \\
& +12\nu^2x_2^3x_3^4\Big\{-20-16x_1(2+7x_1) \\
& +\Big[48+50x_2+310x_1+101x_1^2+30x_1(3+15x_1-5x_1^2)x_2\Big]x_3 \\
& +\Big[-78+2x_1+411x_1^2-20(9+63x_1+57x_1^2+13x_1^3)x_2 \\
& +120x_1(1+x_1+5x_1^2)x_2^2\Big]x_3^2-3\Big[24+3x_1(94+73x_1) \\
& -2(69+351x_1+381x_1^2+19x_1^3)x_2 \\
& +4x_1(24+117x_1+34x_1^2)x_2^2\Big]x_3^3 \\
& +9(1-2x_2)\Big[18+54x_1-36x_1x_2+x_1^2(33-2(9+8x_1)x_2)\Big]x_3^4\Big\} \\
& -48sx_2x_3^2\Big\{-6+\Big[18+20x_2-31x_1x_2\Big]x_3 \\
& -\Big[29+49x_2+4x_2^2+4x_1x_2(9-(22-8x_1)x_2)\Big]x_3^2 \\
& +\Big[-6+202x_2-54x_2^2+164x_1x_2 \\
& +114x_1x_2^2-4x_1x_2^3-x_1^2x_2^2(149-58x_2)\Big]x_3^3 \\
& +\Big[65-333x_2-40x_2^2+189x_1x_2-4x_1(137+75x_2)x_2^2 \\
& +5x_1^2x_2^2(23+12x_2(5+x_2))\Big]x_3^4 \\
& -9\Big[4-46x_2^2-x_1^2x_2^2(1-2x_2)(41+8x_2) \\
& +x_1x_2(25+18x_2-76x_2^2)\Big]x_3^5 \\
& +81x_2(1-2x_2)\Big[2-3x_1+x_1(2+x_1)x_2-2x_1^2x_2^2\Big]x_3^6\Big\} \\
& +96x_2x_3^2\Big\{-21+\Big[36+65(1-x_1)x_2\Big]x_3 \\
& +\Big[23-8(23-x_1)x_2+176x_1x_2^2\Big]x_3^2 \\
& -2\Big[6-(83+37x_1)x_2+120x_1x_2^2\Big]x_3^3 \\
& -\Big[29+10(17-7x_1)x_2-100x_1x_2^2\Big]x_3^4 \\
& -9\Big[8-(31+x_1)x_2+32x_1x_2^2\Big]x_3^5+81(1-2x_2)(1-2x_1x_2)x_3^6\Big\}\,,
\end{aligned}
$$

Final results expressed by many loop integrals and polynomials. Simple loop integrals $I_k(t)$ given by analytic expressions for all values of t, complex for $t \geq 4m_\pi^2$. In some cases, single or double integrals to be evaluated by numerical integration. These involve rather messy polynomials (see left!)

Polarizabilities of Charged Pion

$$\alpha_{\pi^+} \pm \beta_{\pi^+} = \alpha_{fs}/(16\pi^2 F_\pi^2 m_\pi)\{\mathbf{c_{1\pm}} + m_\pi^2/(16\pi^2 F_\pi^2)\,\mathbf{d_{1\pm}} + \mathcal{O}(m_\pi^4)\}$$

$\mathbf{c_{1+}} = 0, \quad \mathbf{c_{1-}} = l_\Delta$

$$\mathbf{d_{1+}} = -\frac{4}{9}\ell^2 + \frac{53}{54}\ell - \frac{2}{9}\ell l_1 - \frac{2}{3}\ell l_2 - \frac{2}{9} l_1 - \frac{2}{3} l_2 - \frac{91}{162} - \frac{8105}{576} + \frac{135}{64}\pi^2 + 8b$$

$$\mathbf{d_{1-}} = -\frac{4}{3}\ell l_1 + \frac{4}{3}\ell l_2 - \frac{4}{3}\ell l_\Delta + \frac{4}{9} l_1 + \frac{4}{9} l_2 - \frac{1}{3} l_3 + \frac{4}{3} l_4 l_\Delta - \frac{187}{81} + \frac{41}{432} - \frac{53}{64}\pi^2 + a + 8b$$

$\ell = ln(m_\pi^2/\mu^2)$, with μ the scale of the renormalization

$\mathcal{O}(p^4)$: exact cancelation of all loops in forward direction
in backward direction remarkable reduction of 16 diagrams
to ONE LEC (l_Δ)

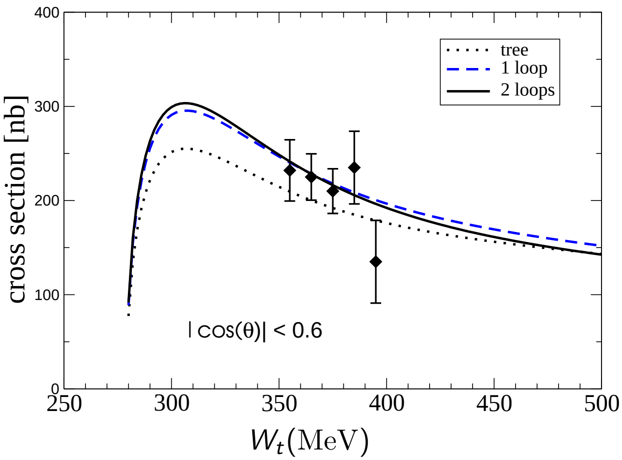
$\mathcal{O}(p^6)$: large cancelations among scale dependent log terms,
LECs $\mathcal{O}(p^4)$, LECs $\mathcal{O}(p^6)$, and constants

How to Determine the LECs??

- $\mathcal{O}(p^4)$:
  only ONE LEC, $l_\Delta = l_6 - l_5 = 3.0 \pm 0.3$
  $l_6 \leftrightarrow$ vector form factor ($< r^2 >_V^\pi$)
  $l_5 \leftrightarrow$ radiative pion decay ($\pi \to e + \nu + \gamma$)
  pion polarizability and weak current $V - A$ related
- $\mathcal{O}(p^6)$
  4 more LECs $\mathcal{O}(p^4)$:
  $l_1 = -0.4 \pm 0.6$ and $l_2 = 4.3 \pm 0.1 \leftrightarrow$
  $\pi\pi$ scattering and K_{e4} decay ($K^+ \to \pi^+\pi^- e^+ \nu_e$)
  $l_3 = 2.9 \pm 2.4 \leftrightarrow SU(3)$ mass formula,
  $l_4 = 4.4 \pm 0.2 \leftrightarrow F_K/F_\pi$
  2 new LECs $\mathcal{O}(p^6)$:
  $a = -5 \pm 5$ and $b = 0.4 \pm 0.4 \leftrightarrow$ resonance saturation
  $(\rho, a_1, b_1, \ldots)$.
- LECs have large error bars, but their contributions are small

Cross Section

$\sigma(W_t)$ for $|\cos\theta| \leq 0.6$

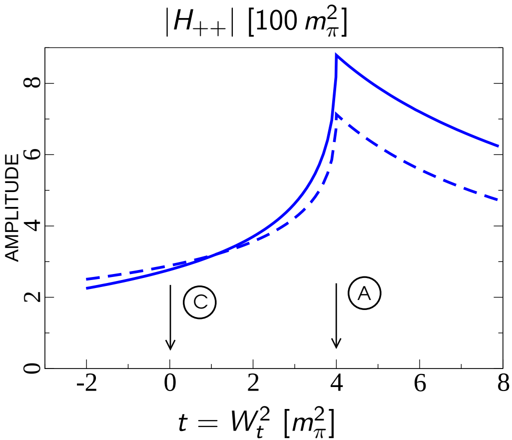


Ref. GIS, data from J. Boyer et al. (MARK II, SLAC), PRD 42 (1990)

♠ polarizabilities related to DIFFERENCE between tree and loop

♡ two-loop corrections $\ll$ experimental errors

Extrapolation of Helicity Non-Flip Amplitude



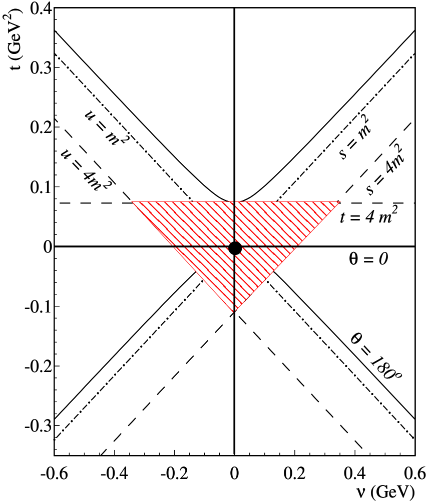
solid: 2-loop, dashed: 1-loop (ChPT prediction of Ref. GIS)

A: $(\gamma\gamma \to \pi\pi)_{\rm thr}$ at $t = 4\, m_\pi^2$, C: $(\gamma\pi \to \gamma\pi)_{\rm thr}$ at $t = 0$, $s = m_\pi^2$

♡ two-loop corrections drop from cusp effect at A to near zero values at C (Compton threshold)

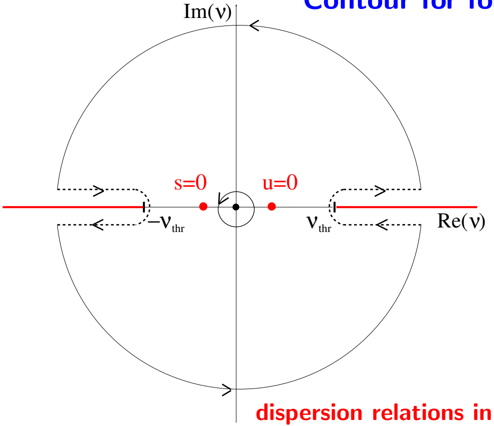
♡ scale-dependence through chiral logs very small at C

Forward dispersion relations



- forward DR (at $t = 0$ or $\theta = 0^\circ$) in complex plane $\nu = E_\gamma^{\rm lab}$ $\Rightarrow$ Compton amplitude at threshold ($\nu = 0, t = 0$)
- within red hatched triangle, amplitude real, no singularities (except for pole terms)
- within circle about origin and fitting into triangle, amplitude $F(\nu^2, t)$ given by Taylor series
- $F(0,0) \Rightarrow$ Thomson scattering $\partial/\partial\nu^2\, F(\nu^2, 0)|_{\nu=0}$ $\Rightarrow$ forward polarizability

Contour for forward DR



- **Cauchy integral**

  $F(0) = \frac{1}{2\pi i} \oint F(\nu') \frac{d\nu'}{\nu'}$
  along small circle with radius ε about $\nu = 0$
- Step 2: extend contour until it hits non-analytic structures or infinity
- Step 3: add contributions from cuts, poles, or infinity

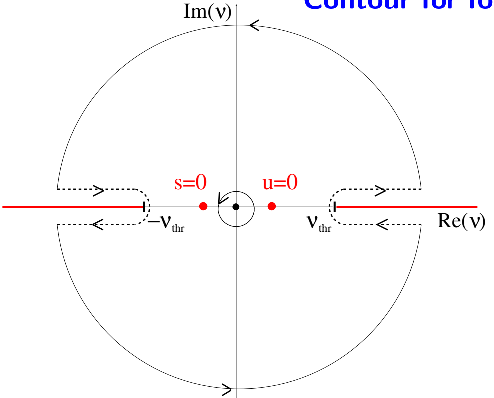
  ("big circle" $R \to \infty$)

dispersion relations in particle physics

- (i) $|F|^2$ should be integrable along Im ν=const

  $\Rightarrow$ no contribution from "infinity"
- (ii) F should be an analytic function on "physical sheet" except for square-root singularities at particle production thresholds:
  s-channel cut $\nu_{\rm thr} \leq {\rm Re}(\nu) < \infty$, u-channel cut $-\infty < {\rm Re}(\nu) \leq -\nu_{\rm thr}$

Contour for forward DR



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  s-channel cut $\nu_{\rm thr} \leq \mathrm{Re}(\nu) < \infty$,   u-channel cut $-\infty < \mathrm{Re}(\nu) \leq -\nu_{\rm thr}$
- Because FK ansatz yields unphysical cuts starting at $s = 0$ and $u = 0$,

  FK ignore respective imaginary parts. Result: non-analytic amplitudes.

Model of Fil'kov and Kashevarov

spectral function $\Rightarrow$ mesons $(\rho, \omega, \Phi, a_1, a_2, b_1, \sigma, f_0, f_2)$.

vector (V) and axial vector (A) contributions to amplitudes:
(1) $M_V^{++}(s) = -s\, M_V^{+-}(s), \quad M_A^{++}(s) = +s\, M_A^{+-}(s)$.

polarizabilities defined at Compton threshold ($s = m_\pi^2$):
(2) $\alpha + \beta = m_\pi/(2\pi)\, M^{+-}(m_\pi^2), \quad \alpha - \beta = 1/(2\pi m_\pi) M^{++}(m_\pi^2)$

Combining Eqs. (1) and (2) we find:
(3) $(\alpha_V - \beta_V)/(\alpha_V + \beta_V) = -1, \quad (\alpha_A - \beta_A)/(\alpha_A + \beta_A) = +1$

$$\Rightarrow \alpha_V = 0, \quad \beta_A = 0$$

as expected from any diagrammatic approach:
pion (0^-) + Photon (M1 $= 1^+$) $\rightarrow$ vector meson (1^-)
pion (0^-) + Photon (E1 $= 1^-$) $\rightarrow$ axial vector meson (1^+)
M1 transition $\Rightarrow$ magnetic polarizability β
E1 transition $\Rightarrow$ electric polarizability α

Results of Fil'kov and Kashevarov

Table: Polarizabilities according to Ref. FK, in units of $10^{-4}\,\mathrm{fm}^3$

			$\alpha-\beta$	$\alpha+\beta$	α	β	$\frac{\alpha-\beta}{\alpha+\beta}$
π^+	ρ	M1	-1.15	0.063	-0.54	0.61	-18.3
	a_1	E1	2.26	0.051	1.16	-1.10	44.3
	b_1	E1	0.93	0.021	0.48	-0.45	44.3
	a_2	M2	1.51	0.031	0.77	-0.74	48.7
π^0	ρ	M1	-1.58	0.080	-0.75	0.84	-19.8
	ω	M1	-12.56	0.721	-5.92	6.64	-17.4

Energy dependence of width and coupling

vector (V) and axial vector (A) contributions to amplitudes:

$$M^{+-}(s) = \frac{4\, g(s)^2}{M^2 - s - iM\Gamma(s)}$$

$$M_V^{++}(s) = -s\, M_V^{+-}(s), \quad M_A^{++}(s) = +s\, M_A^{+-}(s).$$

¶ energy-dependent width (P wave $\sim q^3$ @ threshold)

$\Gamma(s) = \left(\frac{s - 4\,m^2}{M^2 - 4\,m^2}\right)^{3/2} \Gamma_0$

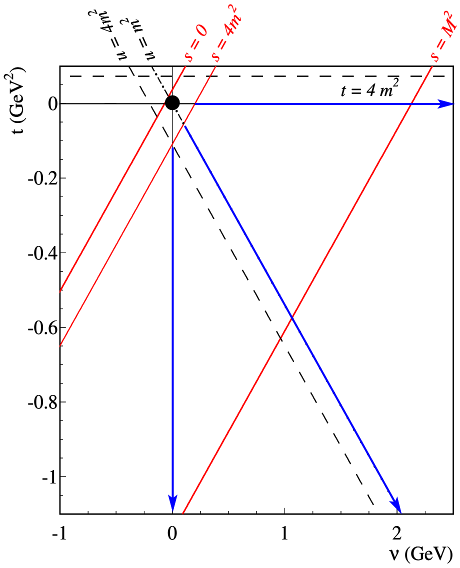
Γ_0 width at resonance, $s = M^2$

¶ energy-dependent coupling constant ($s^{-1/2}$ singularity @ $s = 0$)

$g(s)^2 = \frac{6\pi M}{s^{1/2}} \left(\frac{M}{M^2 - m^2}\right)^3 \Gamma_\gamma$

Γ_γ partial decay width for meson $\to \pi\gamma$

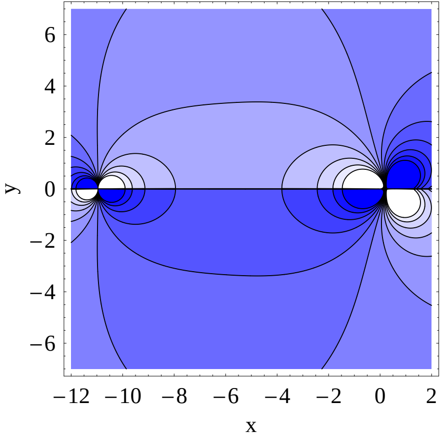
Singularities of FK model



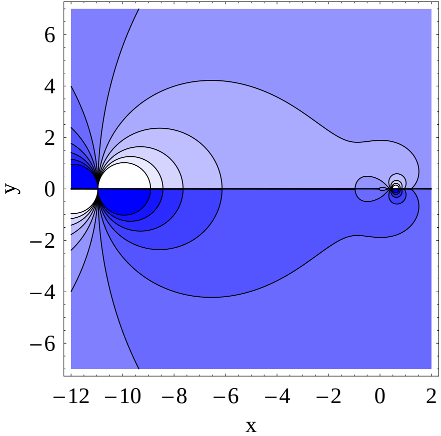
- unphysical singularities at $s = 0$, $t = 0$, $u = 0$, very close to Compton threshold.
  singularities are introduced to ensure square integrability of amplitudes
  Titchmarsh theorem $\Rightarrow$ Re and Im are Hilbert transforms
- singularities lead to unphysical cuts
  imaginary parts "below physical threshold" set zero
- result: non-analytic function,
  Titchmarsh theorem annulled

Polemics: contour plots for ρ contribution

$\mathrm{Im}[M^{+-}(s = x + iy)] \Rightarrow \alpha + \beta$



$\mathrm{Im}[M^{++}(s = x + iy)] \Rightarrow \alpha - \beta$

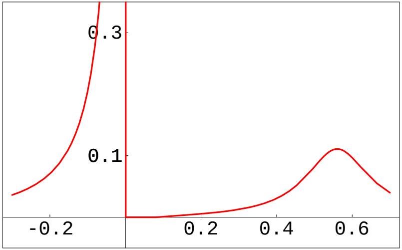


- physical cut with maximum near $x = M^2 \approx 0.55\mathrm{GeV}^2$
- unphysical (left-hand) cut with "bound state" near $-11\ \mathrm{GeV}^2$
- $\alpha \pm \beta$ determined at $\{x = m^2, y = 0\}$, "squeezed in between cuts"
- $\alpha - \beta$ (right plot): physical ρ dwarfed by unphysical phenomena

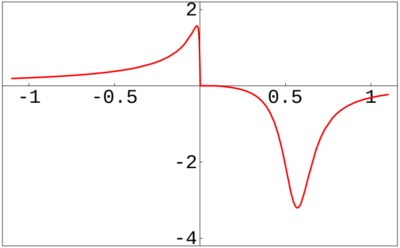
Integrands for $\alpha \pm \beta$

as function of $x = \mathrm{Re}(s)$ in units GeV ^{2}

Integrand for $\alpha+\beta$



Integrand for $\alpha-\beta$



$$\alpha + \beta = 0.17\ (\mathrm{l.c.}) + 0.03\ (\mathrm{r.c.}) = +0.20$$

$$\alpha - \beta = 0.98\ (\mathrm{l.c.}) - 1.18\ (\mathrm{r.c.}) = -0.20$$

With all contributions taken care of, the amplitude has the properties of a ρ meson, $\alpha = 0$, $\beta = 0.20$. Neglect of left cut violates the spin-parity properties of ρ meson.

Contributions to electric (α) and magnetic (β) polarizabilities

for ρ meson with several resonance models

A $\hat{=}$ pole at $M - i\Gamma_0/2$, B $\hat{=}$ pole at $M - i\Gamma(s)/2$, C $\hat{=}$ $\Gamma(s)^2 \to 0$

$\{A0, B0, C0\} \hat{=} g(M^2)$, $\{A, B, C\} \hat{=} g(s)$

$\alpha + \beta$

	real	r.c.	l.c.	rest
$A0$	0.04	0.04	0.00	–
$B0$	0.04	0.03	–	0.01
$C0$	0.04	0.03	–	0.00
A	0.20	0.05	0.15	–
B	0.20	0.03	0.17	0.00
C	0.20	0.03	0.17	–

$\alpha - \beta$

	real	r.c.	l.c.	rest
$A0$	−0.04	−1.04	−0.08	1.08
$B0$	−0.04	−1.15	–	1.11
$C0$	−0.04	−1.93	–	1.89
A	−0.20	−1.06	0.86	–
B	−0.20	−1.02	0.81	0.01
C	−0.20	−1.18	0.98	–

Numbers in units of 10^{-4} fm^3.
First column: Model, second column: values from real part of amplitudes
further columns: contributions from Cauchy integral (r.c. and l.c. = integrals along right and left cuts, rest= residues of poles and "big circle").

Note: Sum of dispersive contributions $\equiv$ real part.

Dispersion relations à la Omnès

In region of elastic pion scattering, final-state interaction described by pion-pion phase shift δ^I_J with I=isospin, J=angular momentum

Omnès function $\Omega^I_J(t) = exp\left\{ \frac{t}{\pi} \int_{4m_\pi^2}^{\infty} dt' \frac{\delta^I_J(t')}{t'(t'-t-i\epsilon)} \right\}$

To determine $\alpha - \beta$, need helicity-conserving dispersive amplitude, full amplitude A minus Born amplitude B.

$(A^I_J - B^I_J)/\Omega^I_J$ has only right cut $4m_{\pi^2} < t < \infty$ and fulfills DR (simplified for S wave, J=0):

$$A^I_0(t) = \Omega^I_0(t)\left\{ B^I_0(t)\mathrm{Re}[(\Omega^I_0(t))^{-1}] - \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} dt' \frac{B^I_0(t')\mathrm{Im}[(\Omega^I_0)^{-1}(t')]}{t'-t} \right\}$$

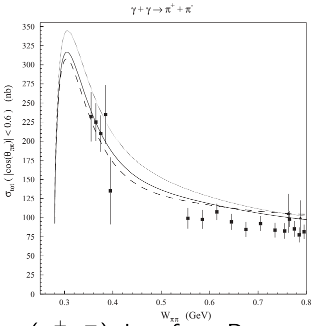
Connected with polarizability at $t = 0$ (Compton threshold:

$$\alpha - \beta = -\frac{1}{4\pi m_\pi} \left(A^V(m_\pi^2, 0) - \frac{1}{\pi} \int_{4m_{\pi^2}}^{\infty} dt' \frac{\mathcal{H}^I_{00}(t')\mathrm{Im}\big[(\Omega^I_0)^{-1}(t')\big]}{t'} \right)$$

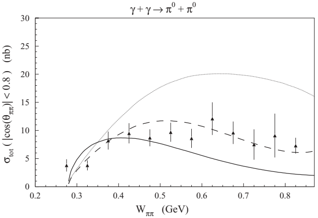
Note: δ^0_0 (S wave, Isospin 0) is positive and large compared to other partial waves.

Dispersion integral over region up to 800 MeV yields $\alpha - \beta \approx 5.5$ for both pions (Ref. PDS).

Cross sections



$(\pi^+\pi^-)$ data from Boyer



$(\pi^0\pi^0)$ data from Marsiske

solid: unsubtracted DR à la Omnès (integration up to 800 MeV)

dashed: subtracted DR
subtraction constant 5.7/-1.9 (GIS)

dotted: subtracted DR
subtraction constant 13.6/-3.5 (FK)

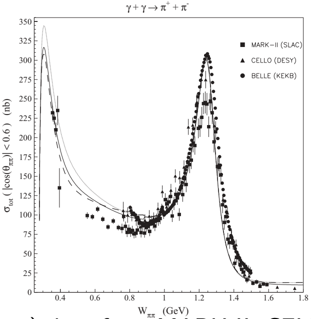
♠ prediction of ChPT in reasonable agreement with the data

♠ $(\pi^+\pi^-)$:
large Born term, dispersive contribution less than 10% of cross section, comparable to experimental errors

♠ $(\pi^0\pi^0)$:
unsubtracted DR does not work well

no good prediction for neutral pion

What about higher energies?



$(\pi^+\pi^-)$ data from MARK II, CELLO, BELLE

above 800 MeV cross section dominated by f_2 $[0^+(2^{++})]$
M2 transition:
contribution to DIPOLE polarizability negligible/zero

variations of dipole polarizability obscured under f_2

dispersion integral for polarizability weighted with $1/W_t^2$

Polarizability of the pion

- The polarizabilities are a fundamental property of a particle, and the pion is a basic building block of hadronic physics.
- A wide-spread range of values has been found by several analyses of different experiments.
- Within the framework of ChPT, the polarizabilities have been calculated at the 2-loop order. At least for the charged pion, the convergence of the loop expansion looks very good.
- We do not find a discrepancy between ChPT and DR.
- COMPASS claims to come up with a precise determination of the pion backward polarizability $(\alpha_\pi - \beta_\pi)$ from data taken in 2009, with values to be released soon. In particular, these data may challenge the earlier findings of the Serpukhov experiment. It is encouraging to learn that future COMPASS-II work is planned with significantly increased statistics, in order to (i) determine α_π and β_π independently, and (ii) get a first look at the Kaon polarizability.