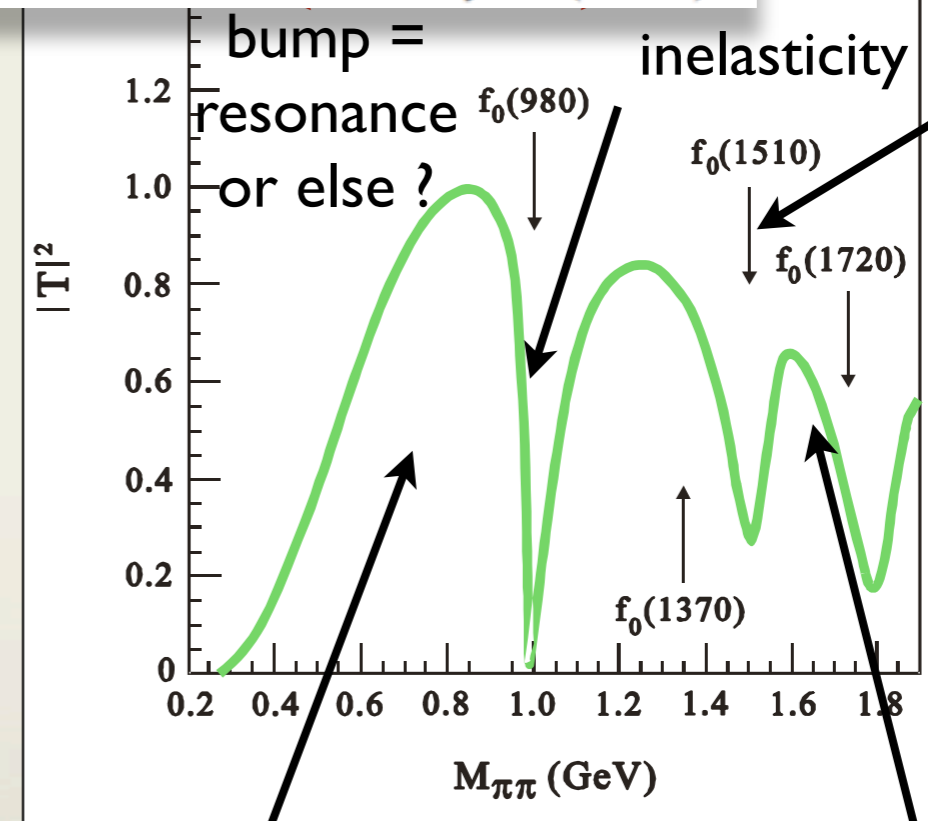
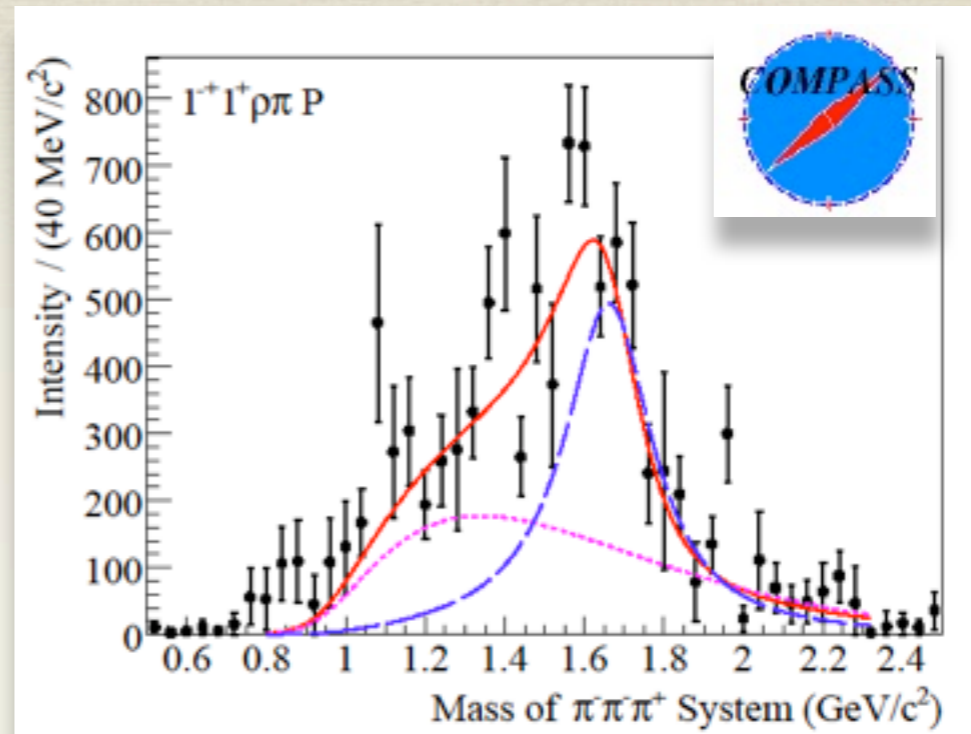
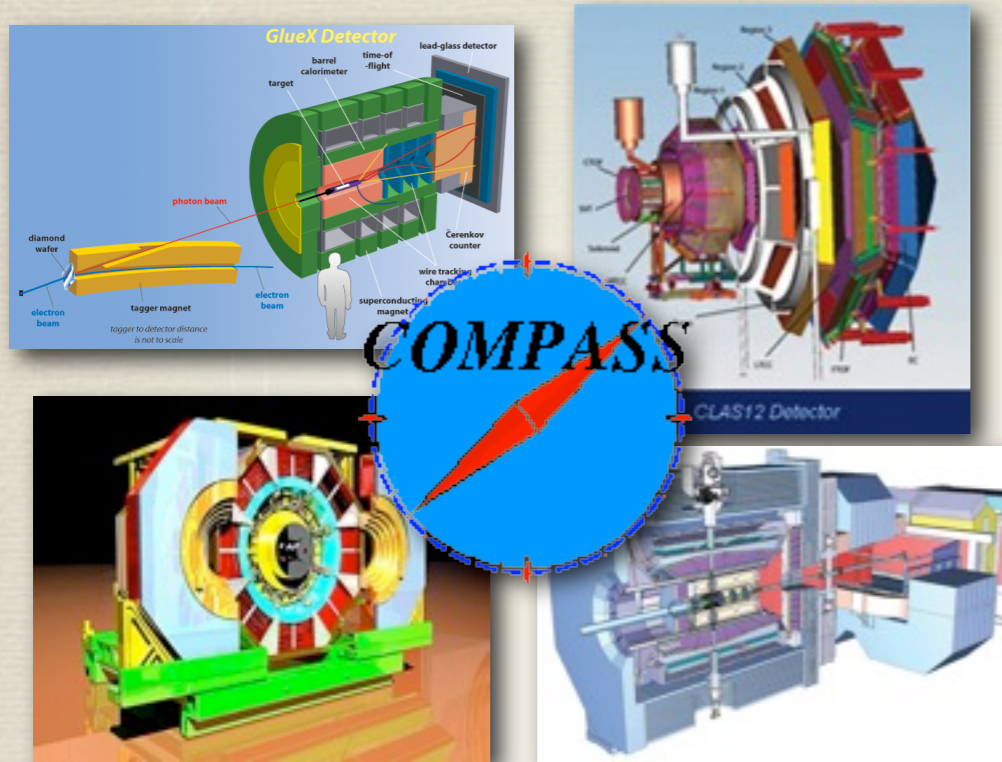


Key issues:

I What is the connection between data and physical amplitudes ?



2. What is the connection amplitudes (real world) and resonances (“unphysical sheets”) between resonances and QCD ?

3. What is the connection between resonances and QCD ?

>> amplitude analysis <<
(analytic properties, dispersion relations, QCD and model input)

dynamically generated σ ?

quark model (nn, ss) ?

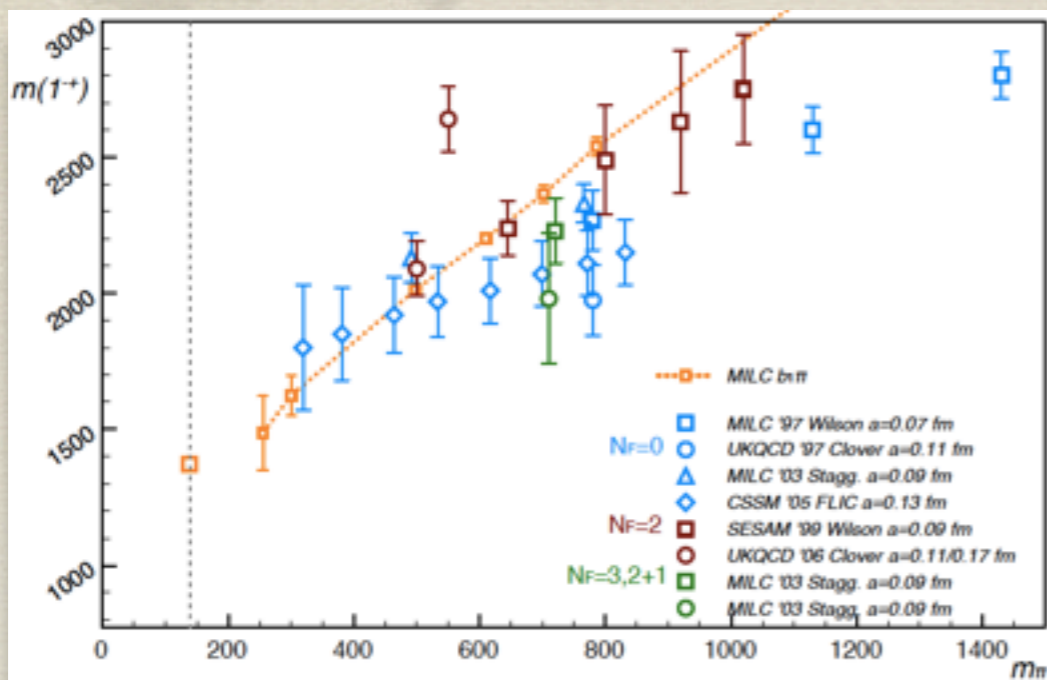
Phenomenology overview of exotics and spectroscopy

Adam Szczepaniak
Indiana University

Outline:

- * (Selected) aspects on theory and phenomenology of gluonic excitations
- * Experimental status
- * (Selected) aspects of PWA

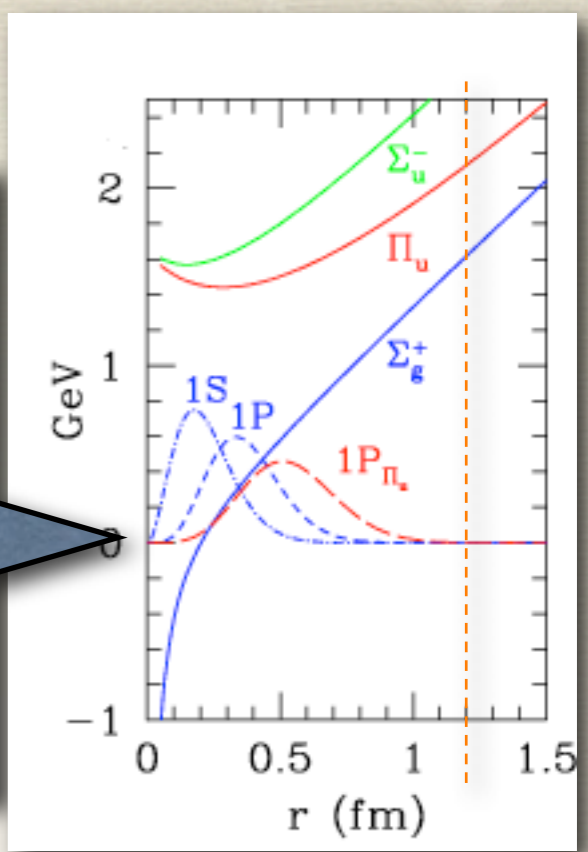
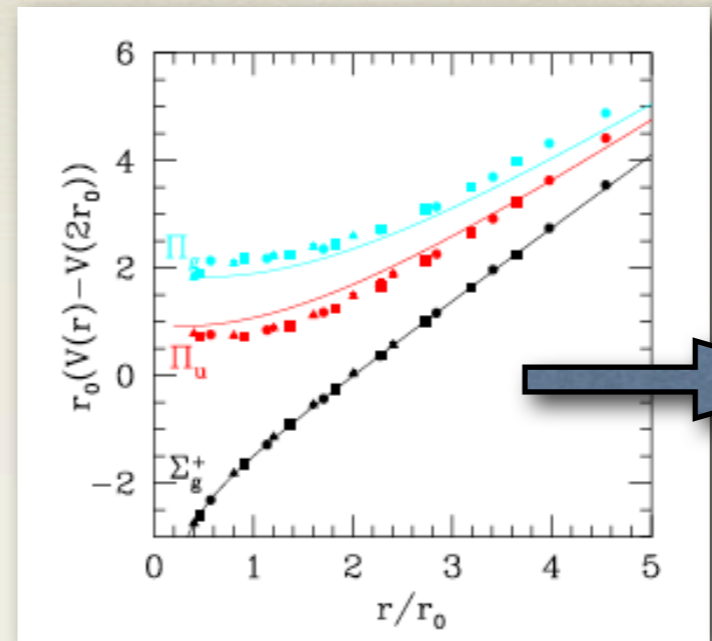
* $1_{S_{Q\bar{Q}=1}}^{-+} = \frac{0^{++}}{2} + \rho \sim 0.8 \text{ GeV} + 0.77 \text{ MeV} \sim 1.6 \text{ GeV}$



Charmonium 1^{-+}

Ref.	Method	ΔM (GeV)
MILC 97	W	1.34(8)(20)
MILC 99	SW	1.22(15)
CP-PACS 99	NR	1.323(13)
JKM 99	LBO	1.19

Excitations in excess of 1GeV



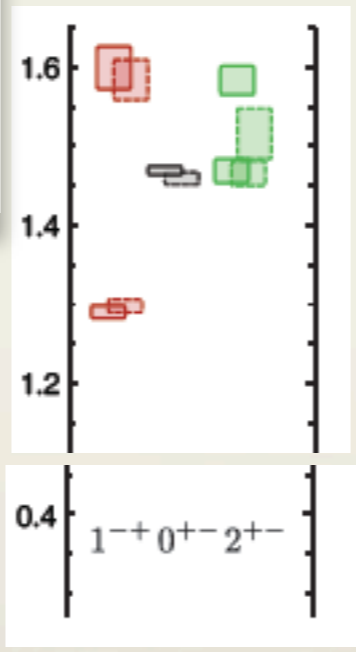
* $J^{PC} = 1^{-+}$ lowest state

Higher masses have also been resolved

Chiral extrapolations 100-200 MeV (Thomas,APS)

In large- N_c same as for ordinary mesons $O(1/N_c)$ (Cohen)

Preliminary (toy) lattice computation of widths agrees with models (Michael,McNeile) (Burns,Close)



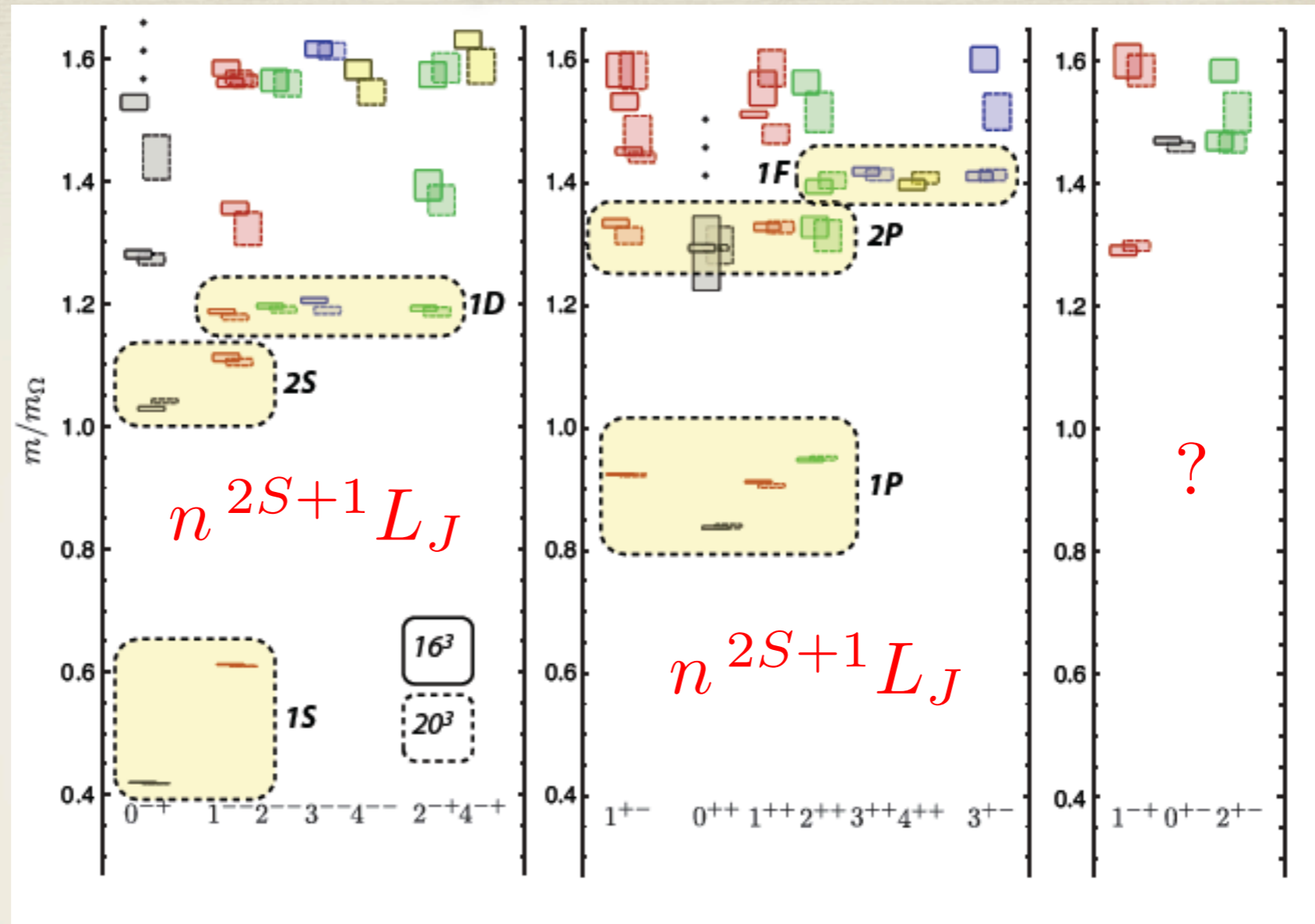
1^{-+} (1.8 GeV)	$b_1 \pi$	$f_1 \pi$	$\rho \pi$	
PSS	S 73 D 1	S 9 D 0.04	P 13	Γ MeV
IKP	S 51 D 11	S 14 D 7	P 12	

Isgur, Kokosky, Paton (85)
Close, Page (95)
Page, Swanson, Szczepaniak (99)
Close, Dudek (04)

Structure

* normal meson spectrum seems to be very quark model-like !

(J.Dudek et al. (2010))

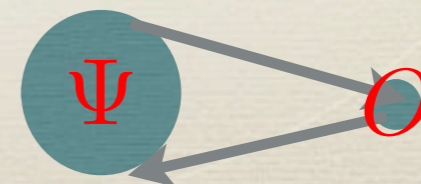


to determine structure study

$$\langle \text{Vacuum} | O[q, g] | \text{Meson} \rangle$$

$$\bar{q}(x) \Gamma^i q(x) \sim b^\dagger(k) \sigma^i d(-k)$$

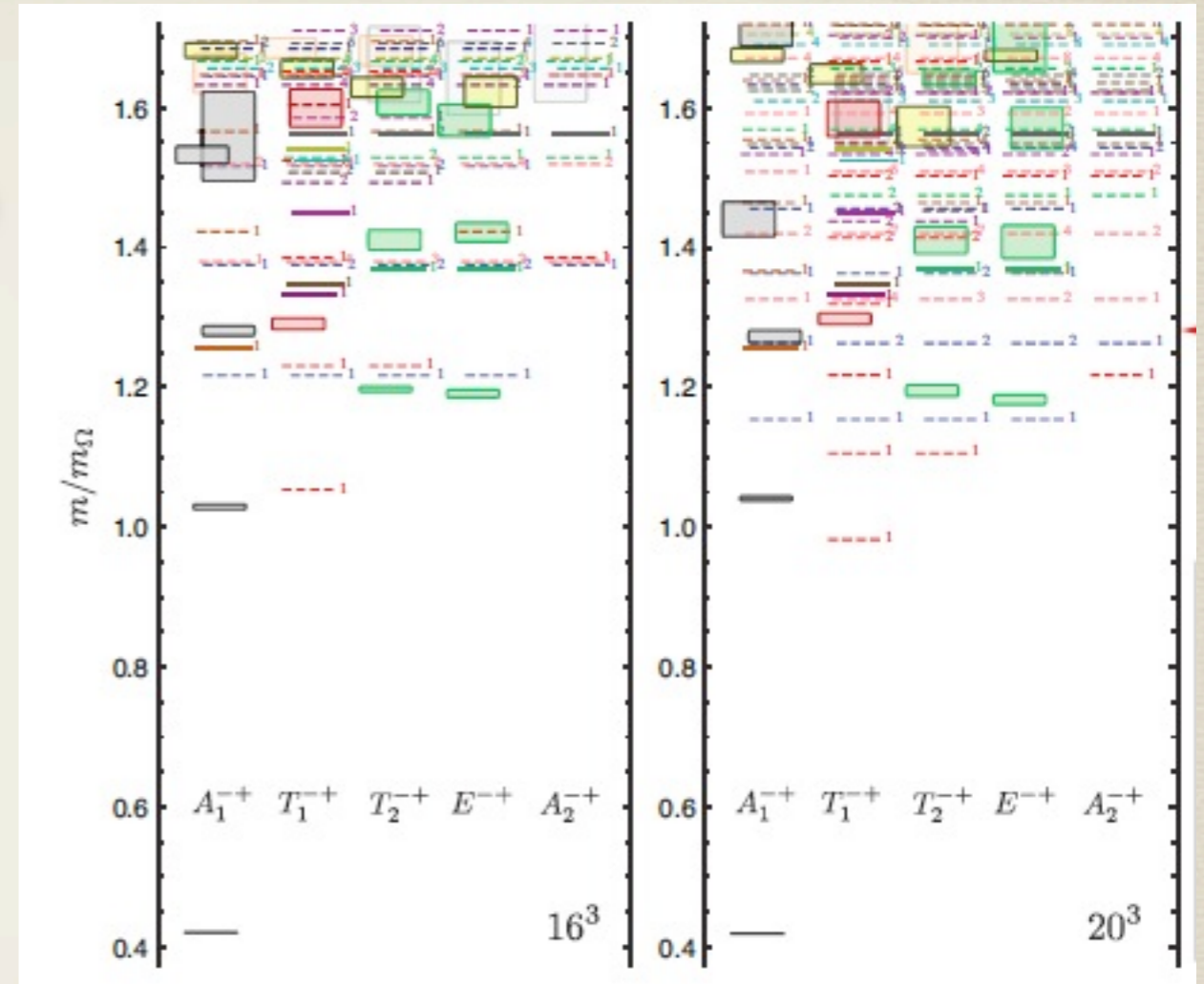
$$\bar{q}(x) F_{ij}(x) q(x) \sim b^\dagger(k) \vec{k} \times \vec{a}(q) d(-q - k)$$



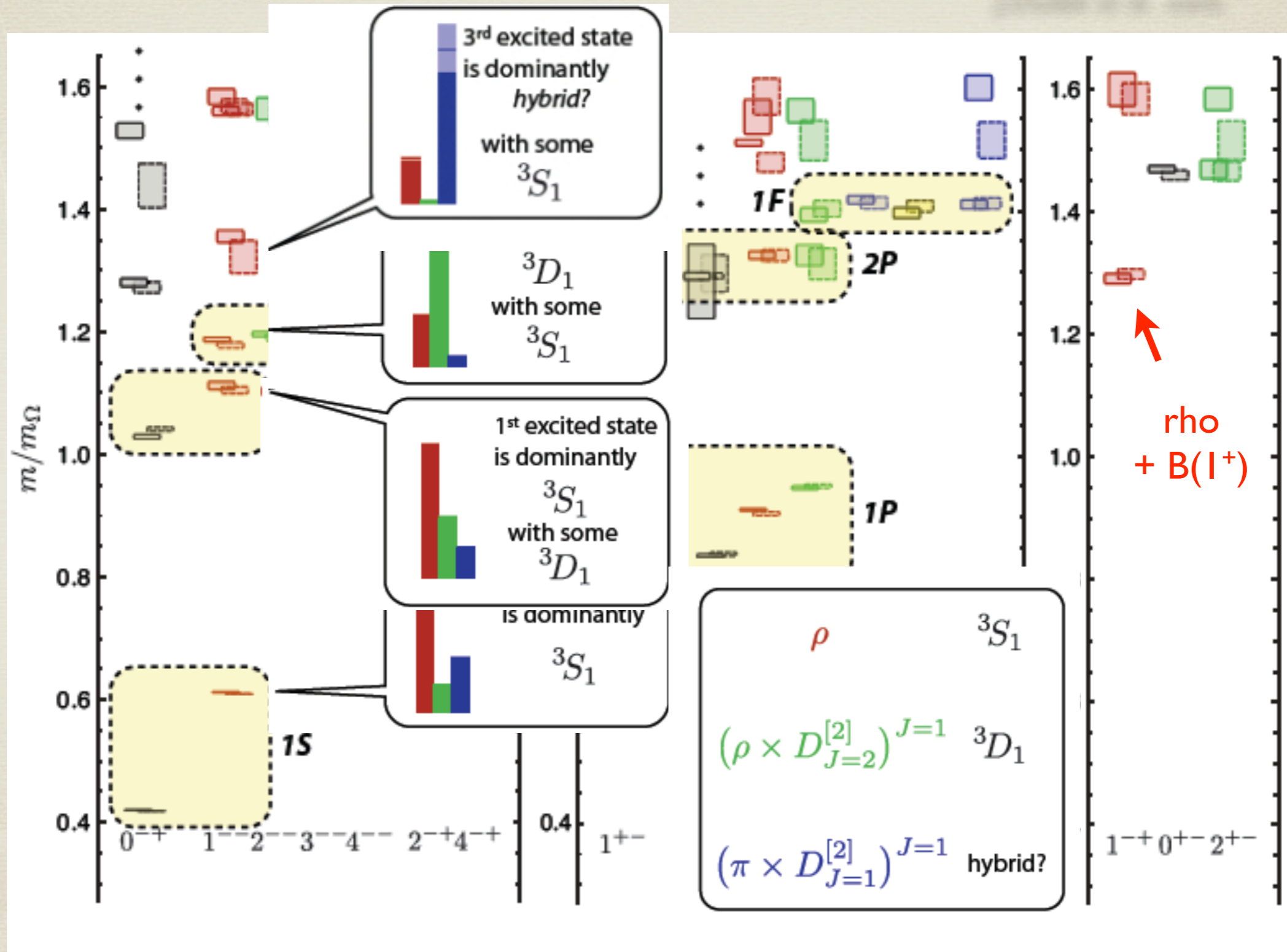
if single hadron states exist: lattice is the place to find them

J.Dudek et al. cont.

- * On finite volume multi-meson state and single hadron states are discrete.
- * If there are single hadron states, use volume dependence to disentangle
- * Continuum states can have any J,P,C but not single hadron states
- * The choice of operators minimizes overlap with multi-meson states
- * In the continuum these these states should disappear through cuts onto unphysical sheets (as CDD poles)



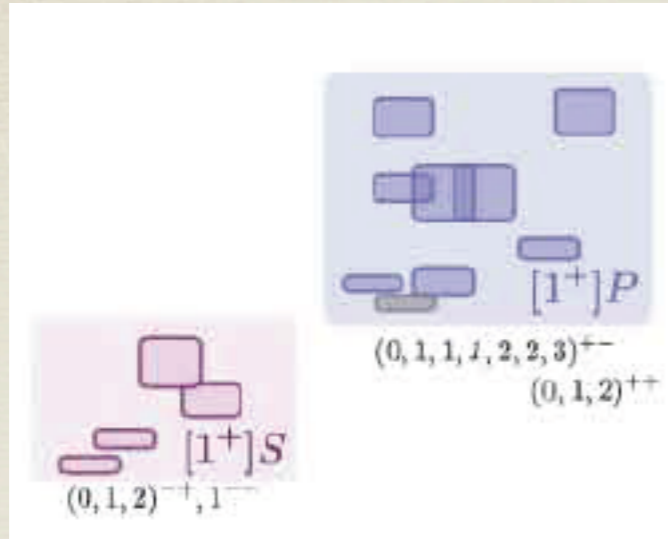
>> there is evidence for single hadron states <<
(no surprising, quark model, CDD poles, etc.)



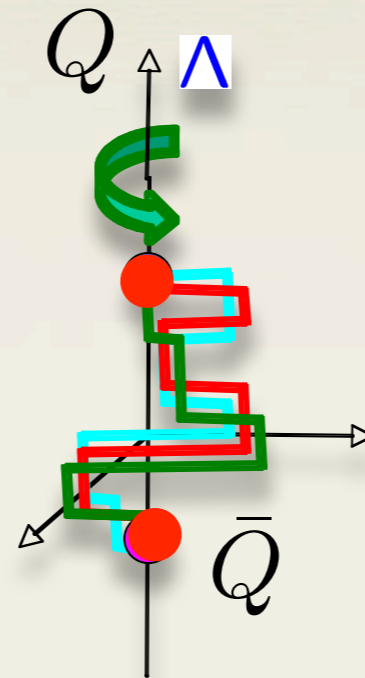
A "good" gluon structure model should describe all these :



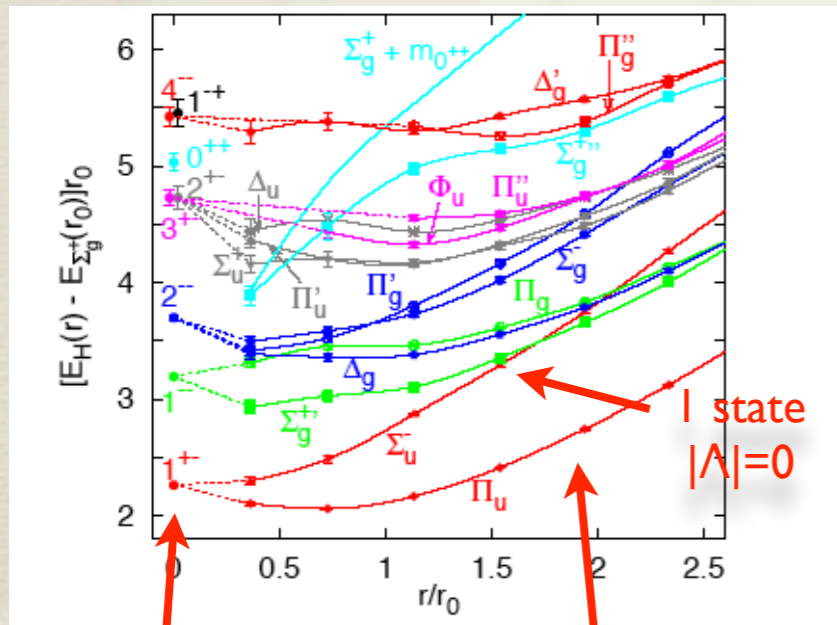
* exotic and crypto-exotic mesons



* adiabatic potentials (axial symmetry)



* glue lump (rotational symmetry)

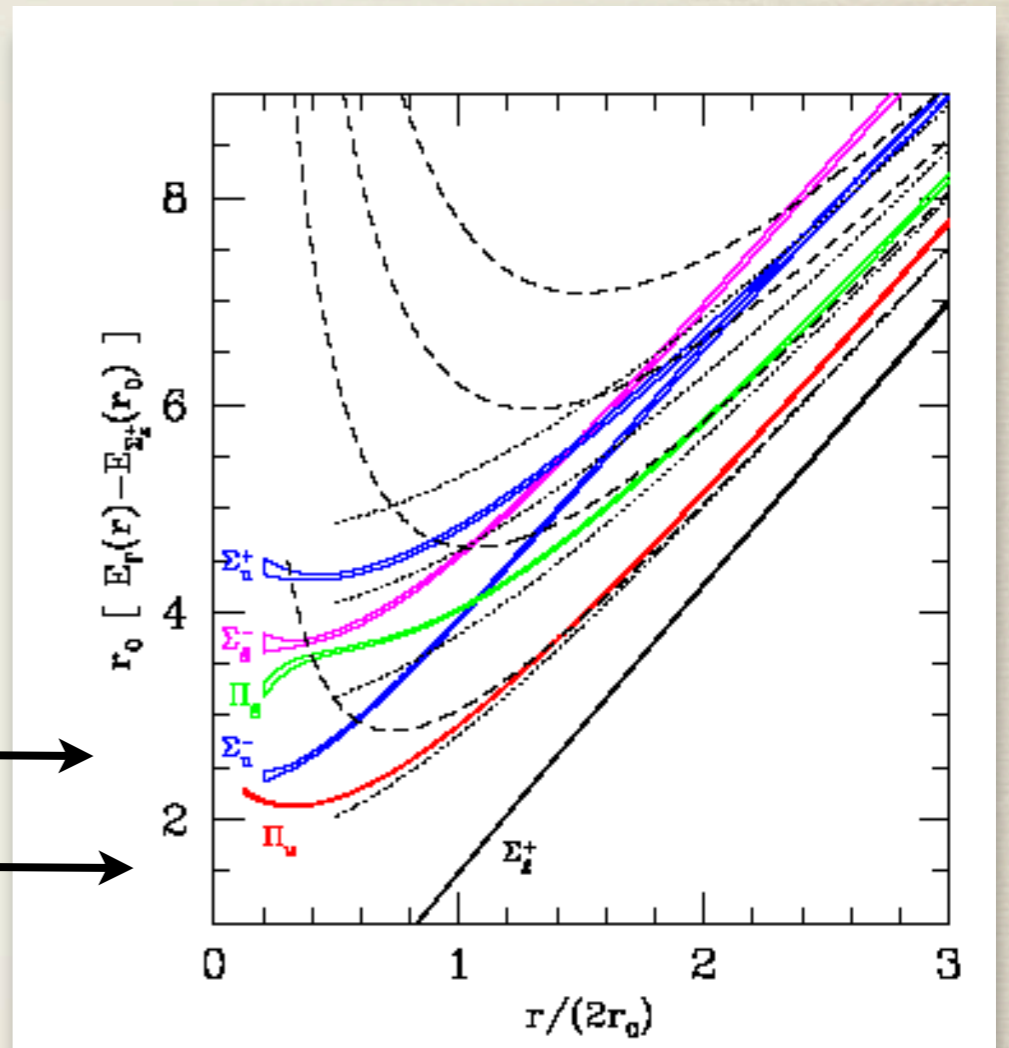


3 states
 $M = \pm 1, 0$!

2 states
 $|\Lambda| = \pm 1$

G.Bali, A.Pineda (2003)

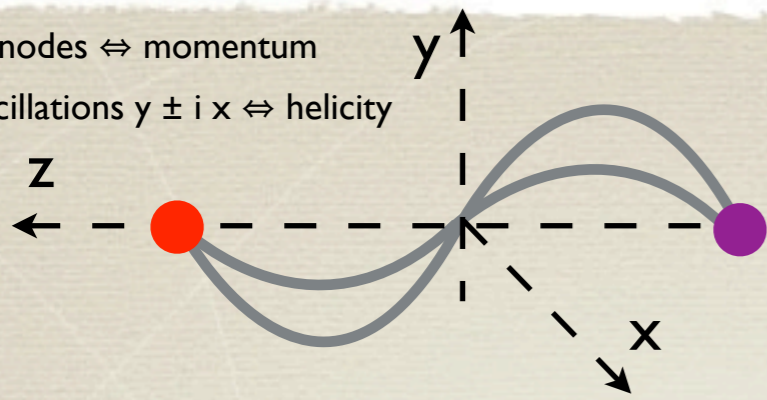
$PC = +1$
 $PC = -1$



$$\Lambda_{PC}^Y = (\Sigma, \Pi, \Delta, \dots)_{u,g}^{\pm}$$

C.Morningstar et al. (1999)

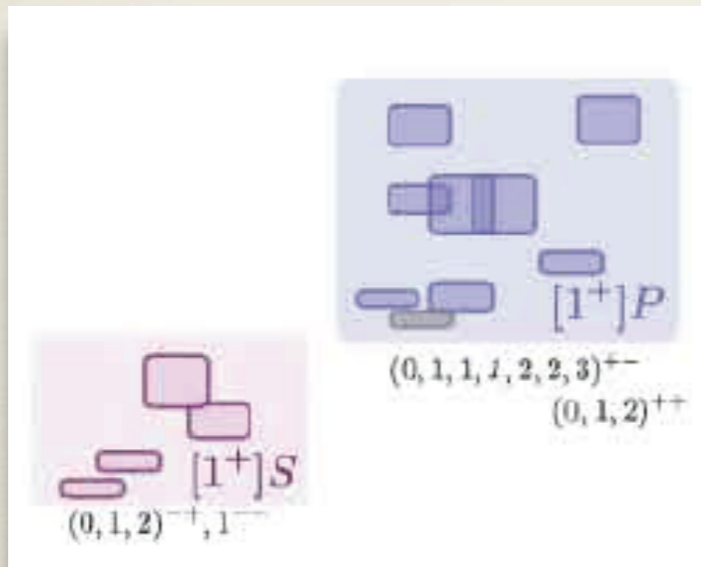
m nodes \Leftrightarrow momentum
 oscillations $y \pm i x \Leftrightarrow$ helicity



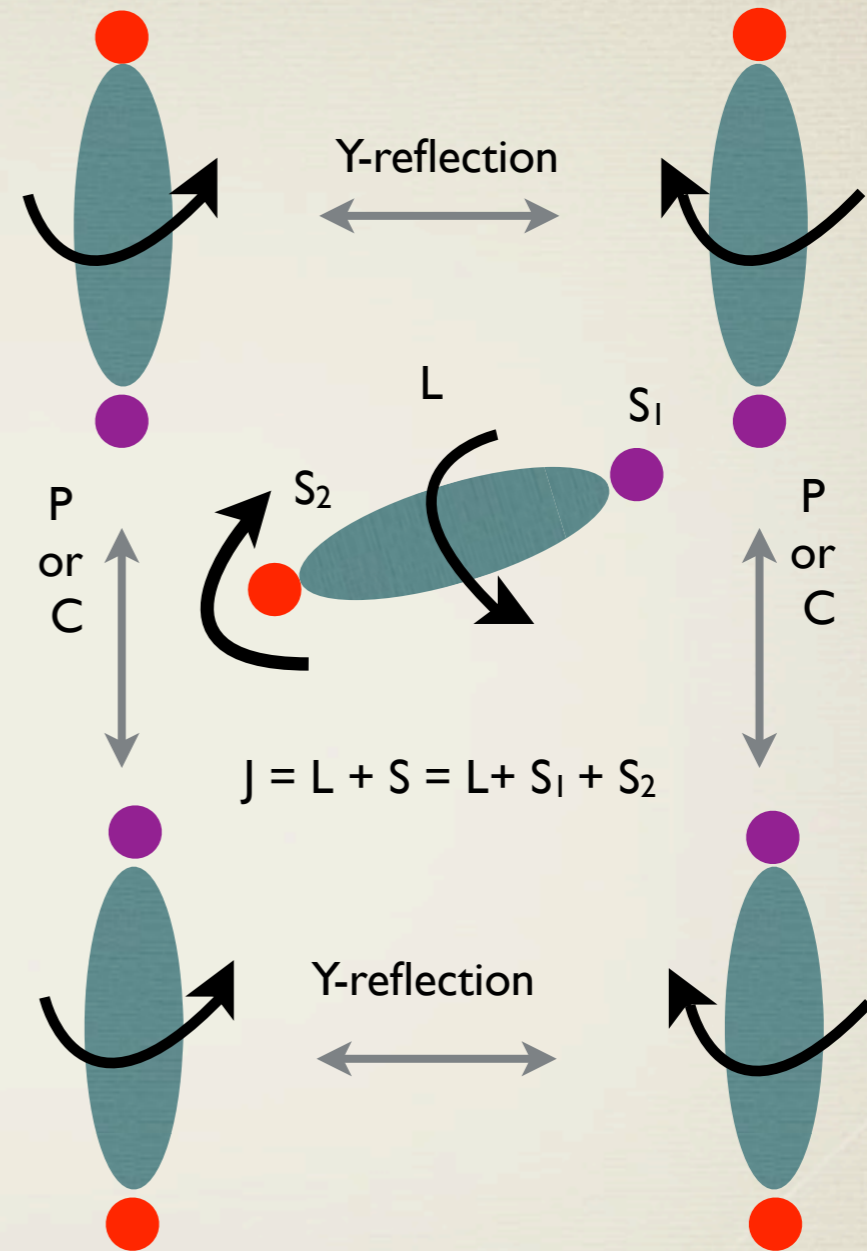
Strong coupling Hamiltonian: product of links (non-relativistic beads)
 \gg chain of coupled harmonic oscillators

$$\begin{array}{l}
 S = 0 \\
 S = 1
 \end{array}
 \quad
 J^{PC} = \begin{array}{cc}
 1^{--} & 1^{++} \\
 (0, 1, 2)^{-+} & (0, 1, 2)^{+-}
 \end{array}$$

degenerate in FT



Flux tube does not agree with lattice !



problem with the rigid rotor

non-relativistic hybrids

$$H = H_D + H_{YM} + H_C$$

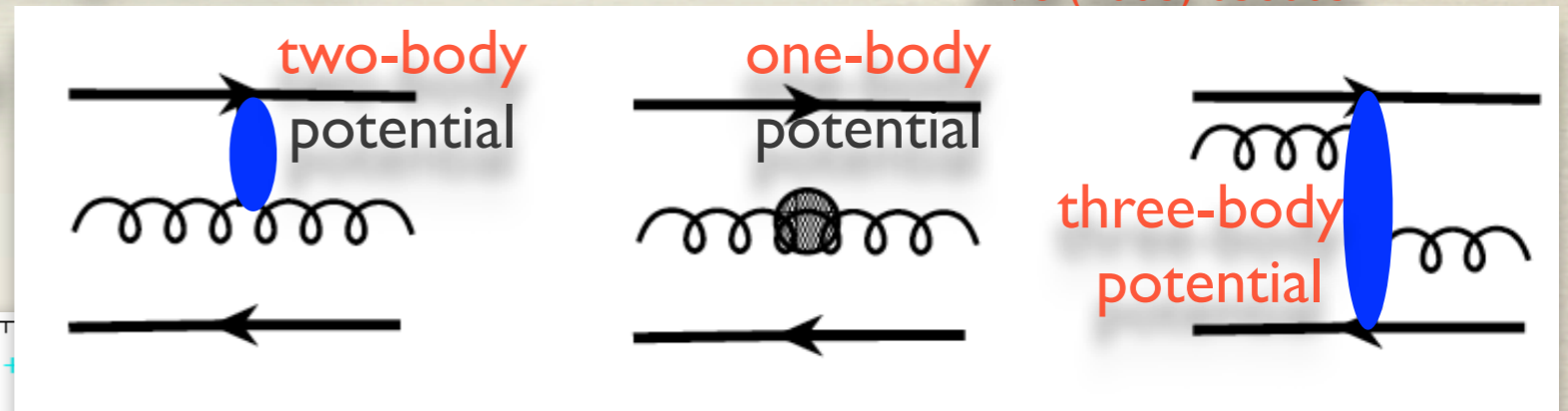
Coulomb Gauge (variational)

P.Guo et al., Phys.Rev. D78 (2008) 056003

* $Q\bar{Q}$ in $L=0, S=0, 1$

expected degeneracies

* coupled to 1^- glue in the relative $L=1$ state $\Rightarrow J_g^{PC} = 1^{+-}$



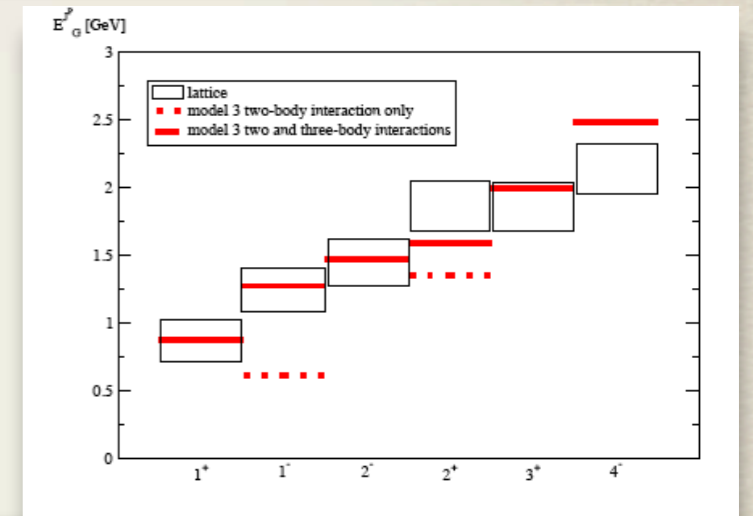
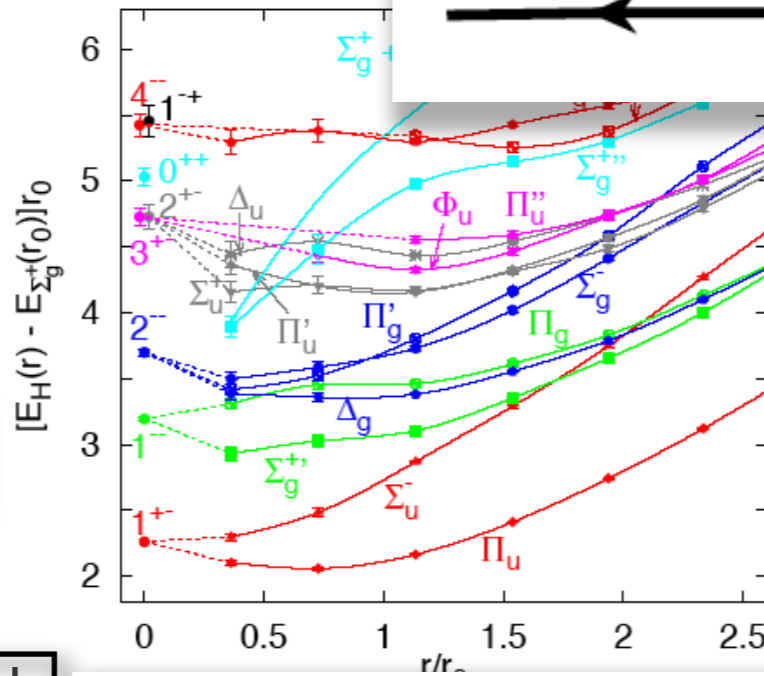
J^{PC} glue

$J^{PC} Q\bar{Q}$

$$1^{+-} \times 0_{S_{Q\bar{Q}}}^{-+} = 1^{--}$$

$$1^{+-} \times 1_{S_{Q\bar{Q}}=1}^{--} =$$

$$0^{-+}, 1^{-+}, 2^{-+}$$

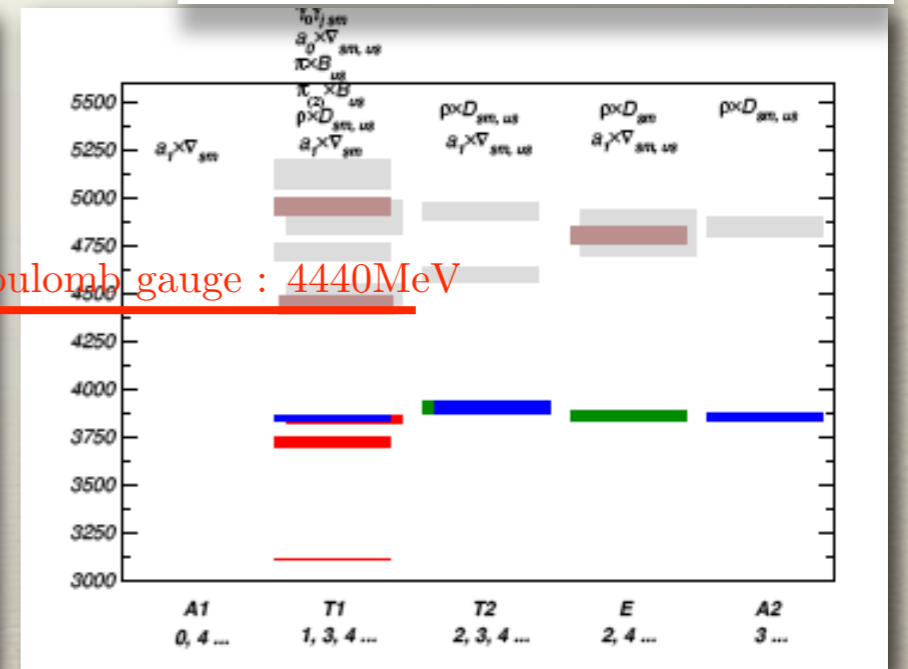
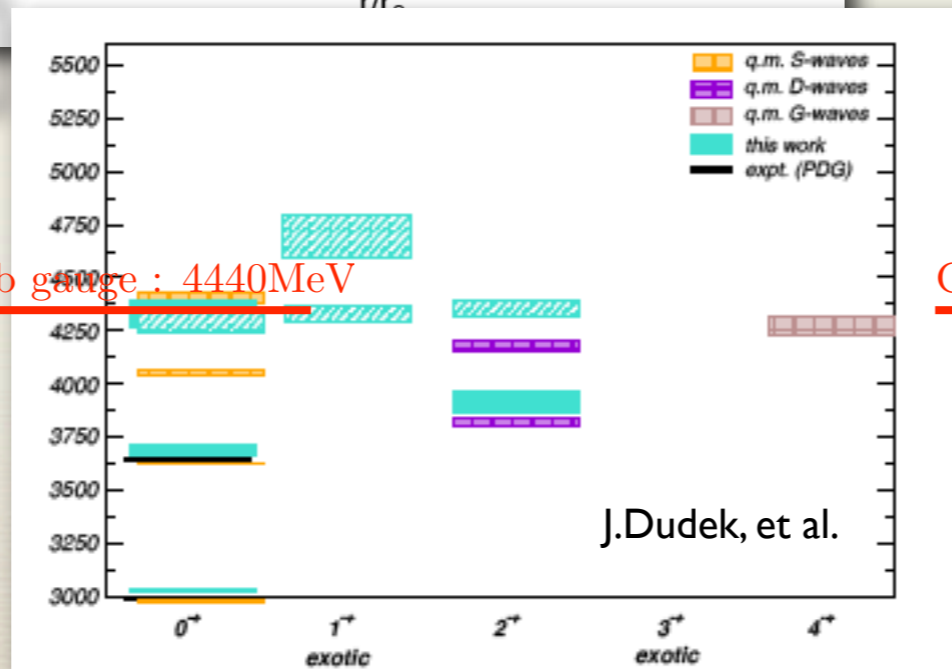


Coulomb gauge : 4440MeV

Coulomb gauge : 4440MeV

* experiment

Y(4260) (Belle, BaBar)



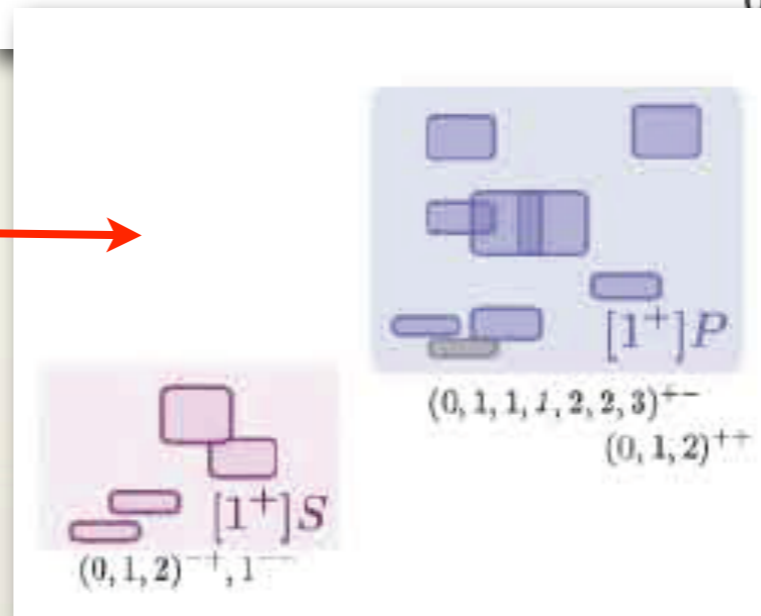
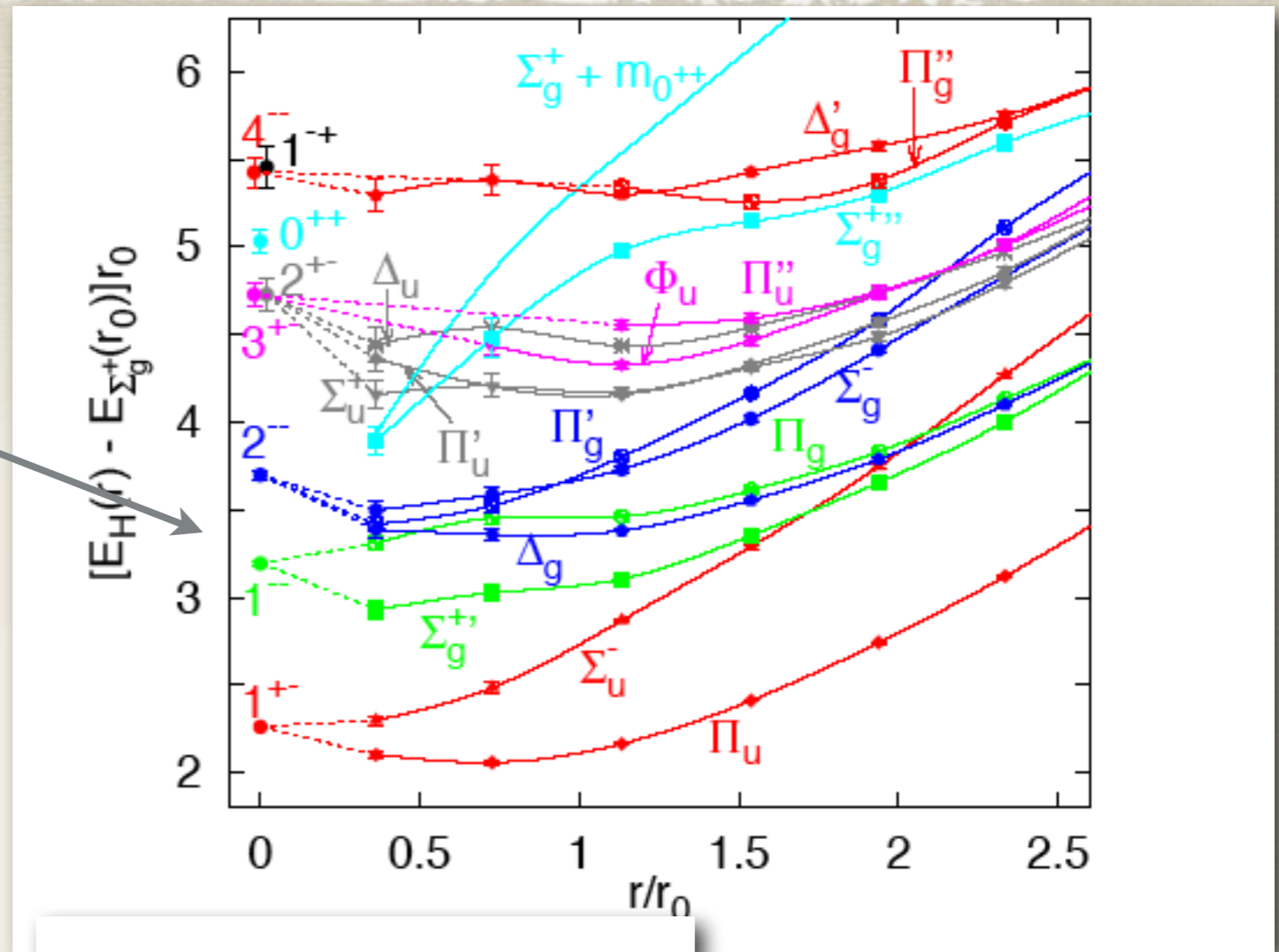
$$|hybrid\rangle = |JJ_g L_{Q\bar{Q}} S\rangle$$

$$J_g^{PC} = |^{--}$$

	$S = 0$	$S = 1$
$L = 0$	1^{+-}	$(0, 1, 2)^{++}$
$L = 1$	$(0, 1, 2)^{-+}$	$(0, 1^3, 2^2, 3)^{--}$

$$J_g^{PC} = |^{+-}$$

	$S = 0$	$S = 1$
$L = 0$	1^{--}	$(0, 1, 2)^{-+}$
$L = 1$	$(0, 1, 2)^{++}$	$(0, 1^3, 2^2, 3)^{+-}$



Exotic story $\pi^- p \rightarrow \eta \pi^0 N$ ($\eta\pi^0$) in P-wave has $J^{PC}=1^-+$!
 $\rightarrow \eta \pi^- p$

$$\pi^- p \rightarrow \eta \pi^- p$$

$$M = 1370 \pm 16_{-30}^{+50} \text{ MeV} / c^2$$

$$\Gamma = 385 \pm 40_{-105}^{+65} \text{ MeV} / c^2$$

$$\pi^- p \rightarrow \eta \pi^0 n$$

New results: No consistent B-W interpretation possible but a weak $\eta\pi$ interaction exists and can reproduce the exotic wave

$$\pi^- p \rightarrow \eta' \pi^- p$$

$$M = 1597 \pm 10_{-10}^{+45} \text{ MeV} / c^2$$

$$\Gamma = 340 \pm 40_{-50}^{+50} \text{ MeV} / c^2$$

$$\pi^- p \rightarrow \rho^0 \pi^- p$$

$$M = 1593 \pm 8_{-47}^{+29} \text{ MeV} / c^2$$

$$\Gamma = 168 \pm 20_{-12}^{+150} \text{ MeV} / c^2$$

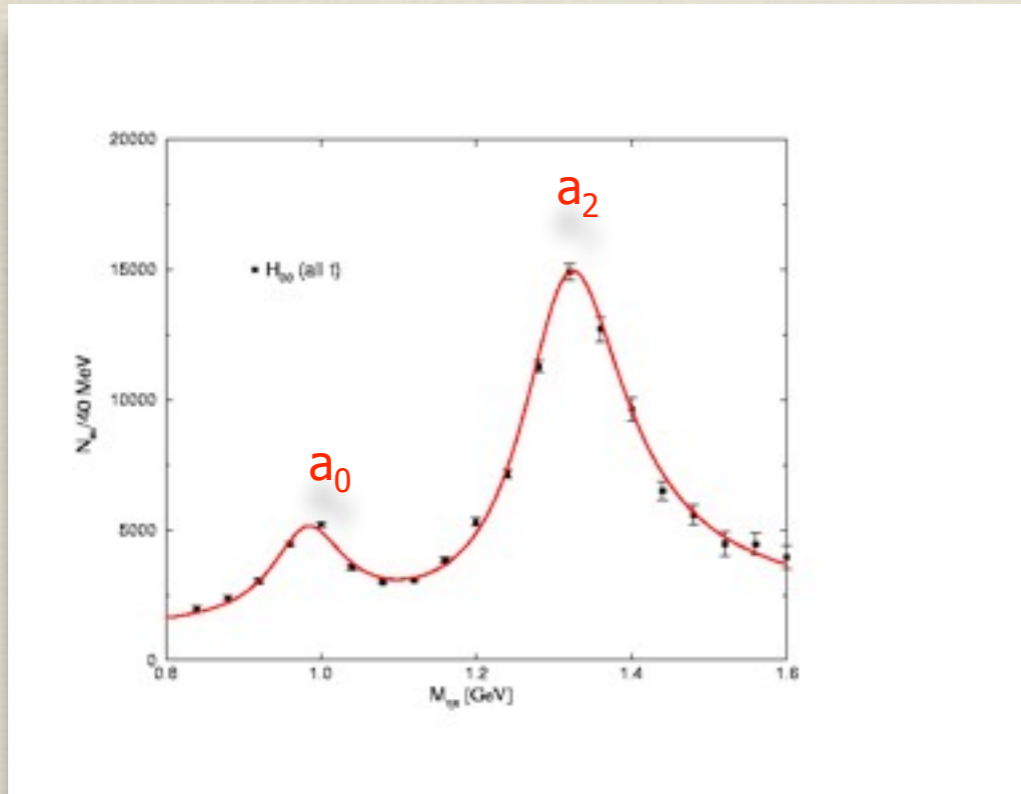
$$\pi^- p \rightarrow b_1 \pi p$$

$$\pi^- p \rightarrow f_1 \pi p$$

BNL (E852)

new analysis reduces the strength but COMPASS find the signal again, JLab large uncertainties

Example $\pi^- p \rightarrow \pi^0 \eta n$ (from E852)
 (Dzierba, et al.)

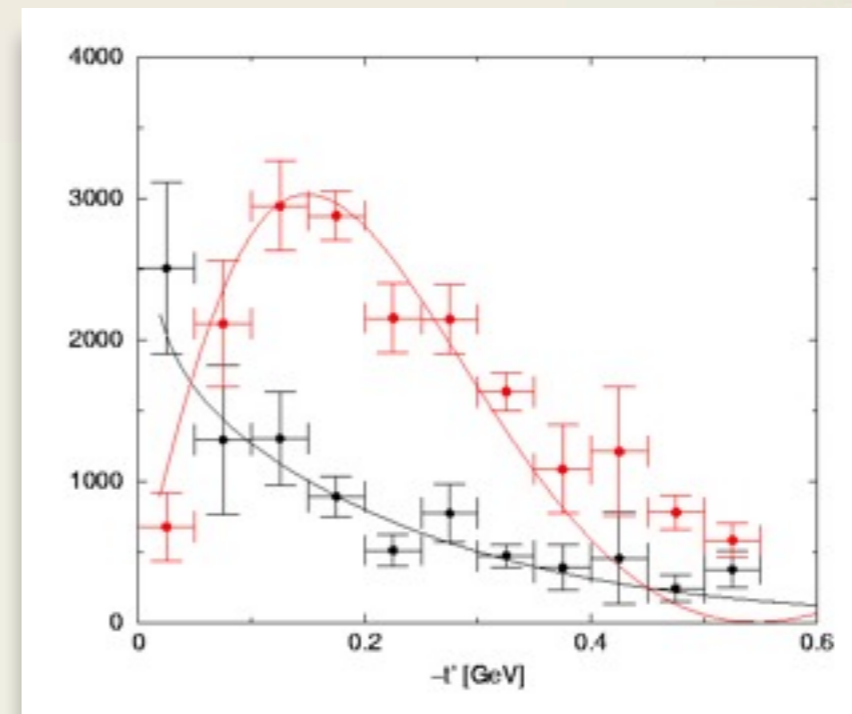
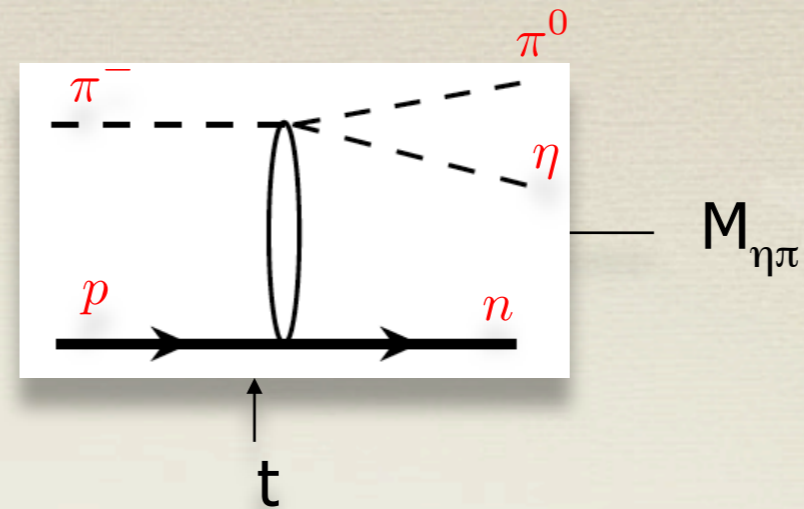


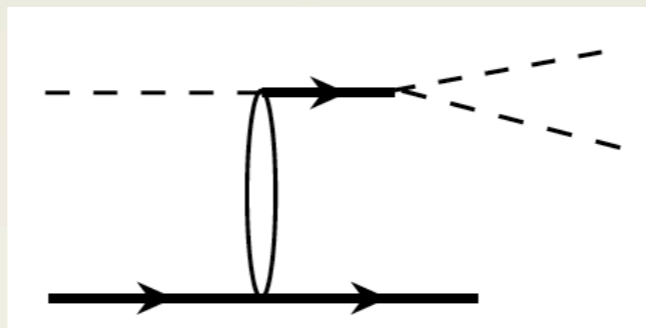
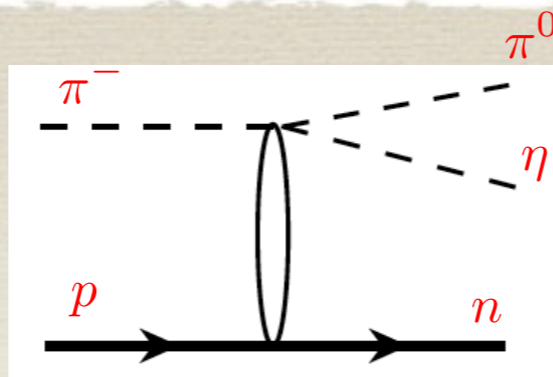
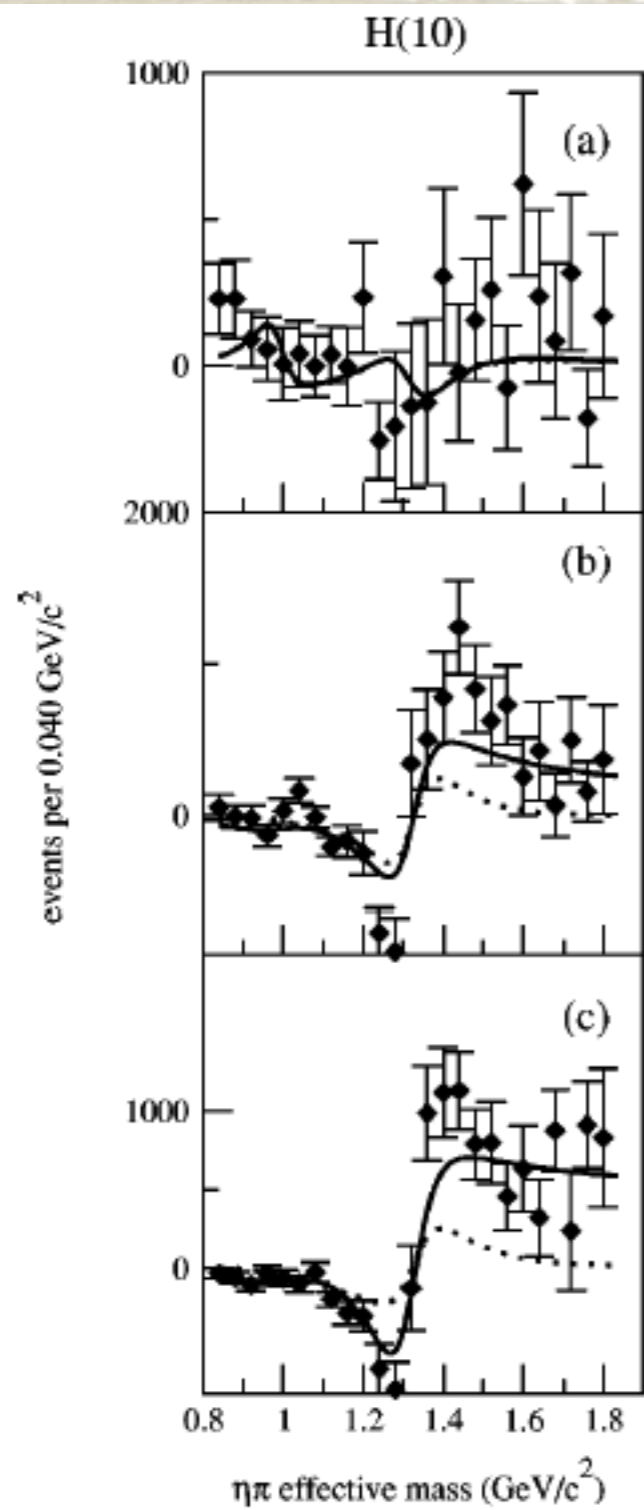
Relevant partial waves :

$\eta\pi$ in S : a_0

$\eta\pi$ in D : a_2

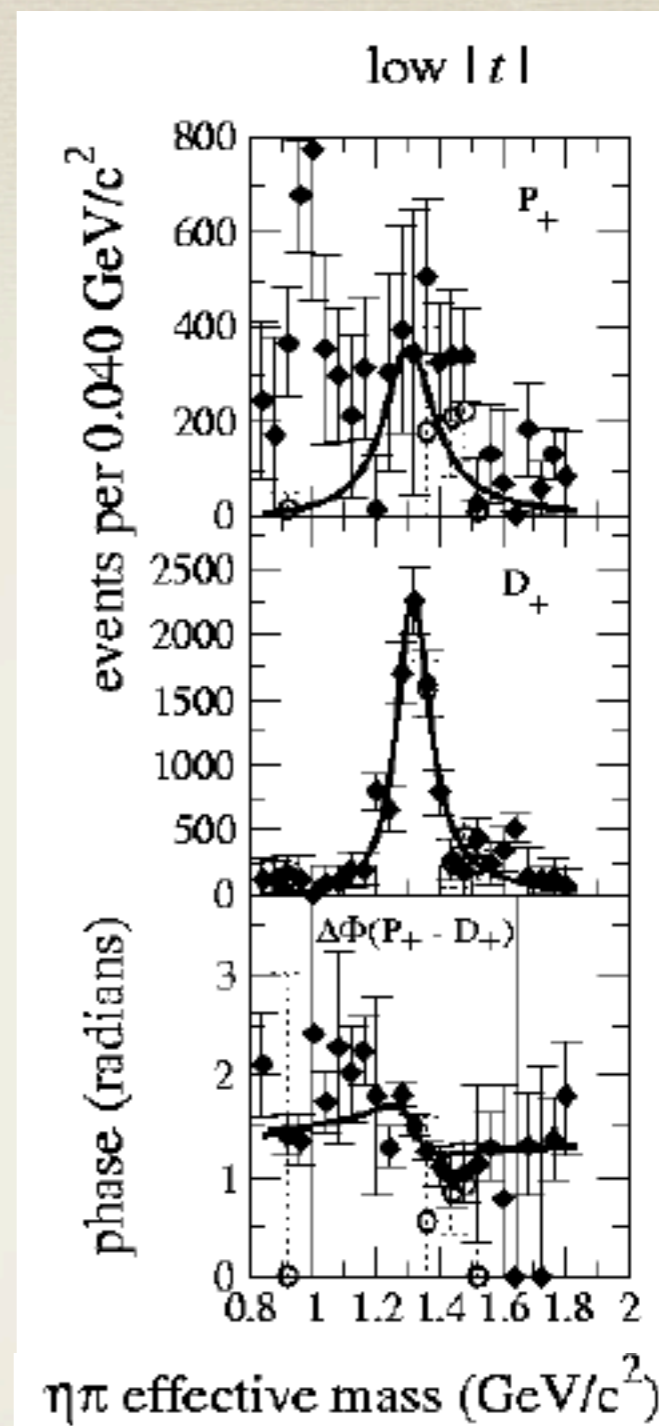
Is there an $\eta\pi$ P-wave : exotic I^{-+} quantum numbers ?



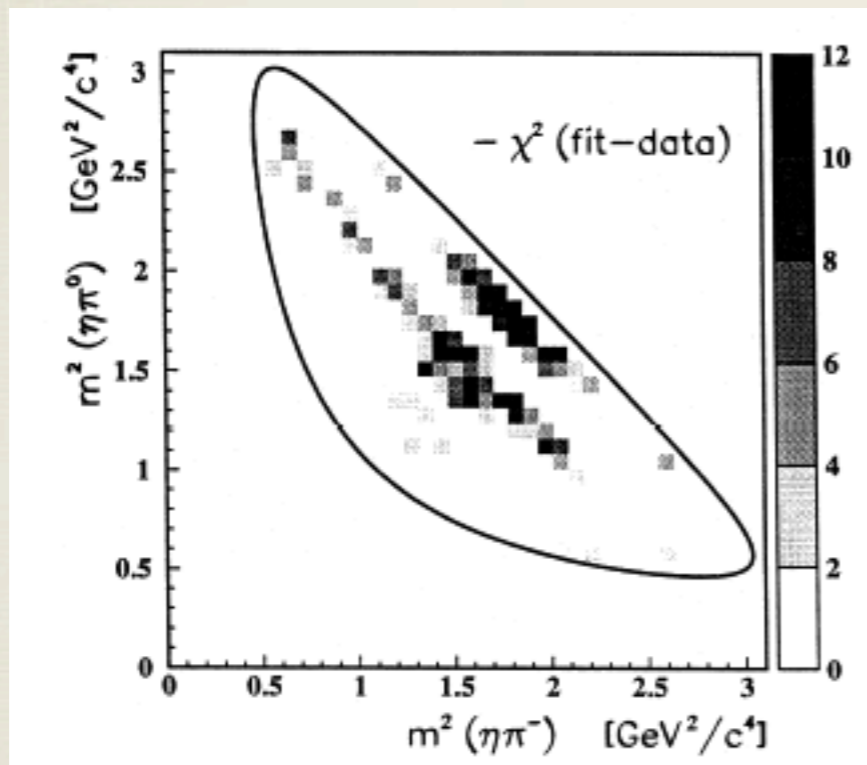
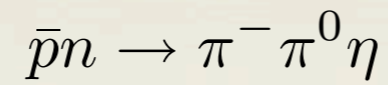


Hard to find a
common set of BW
parameters for all
three helicity states of
the resonance

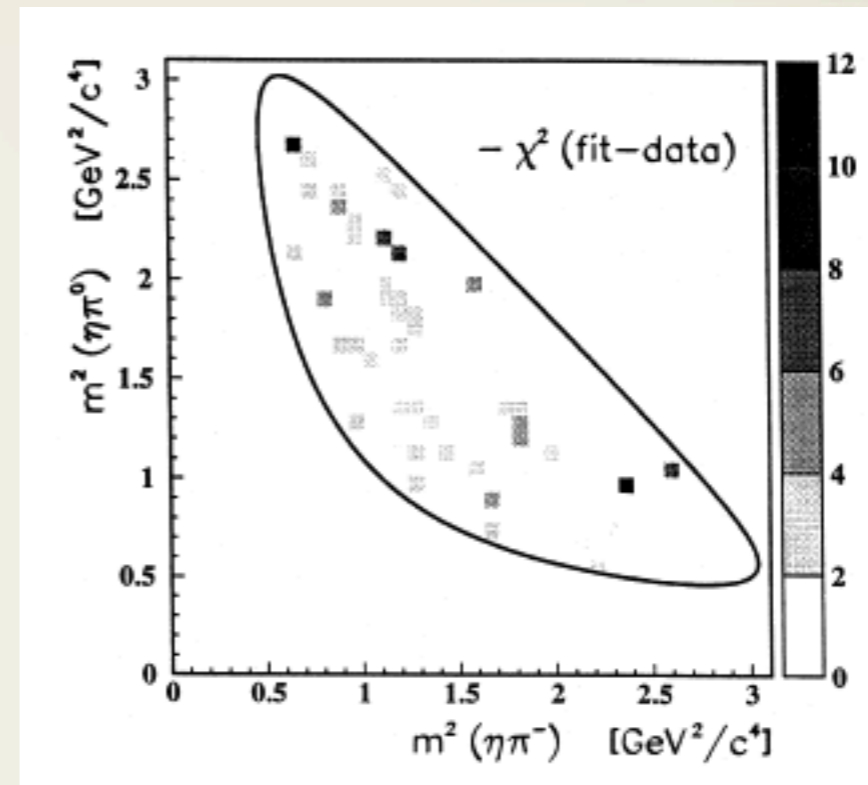
$\pi_1(1400) \rightarrow \pi_1(900 - 5000)!$



Crystal Barrel

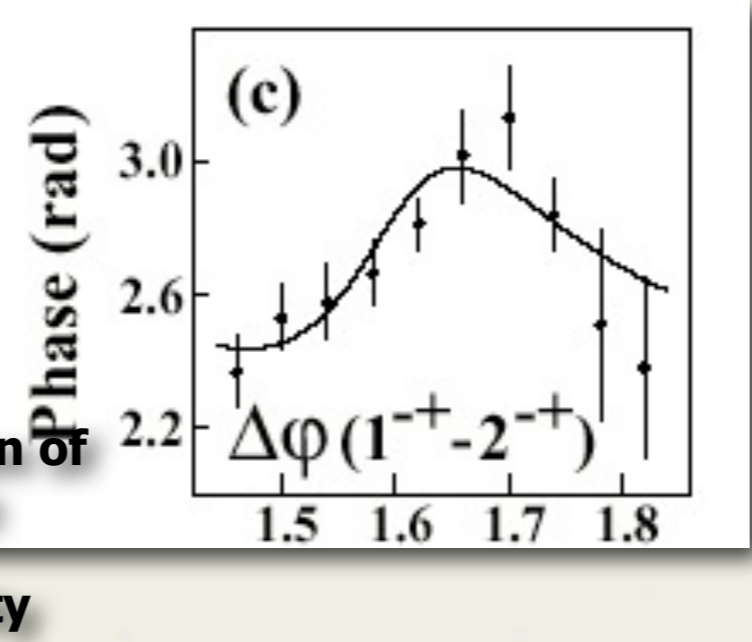
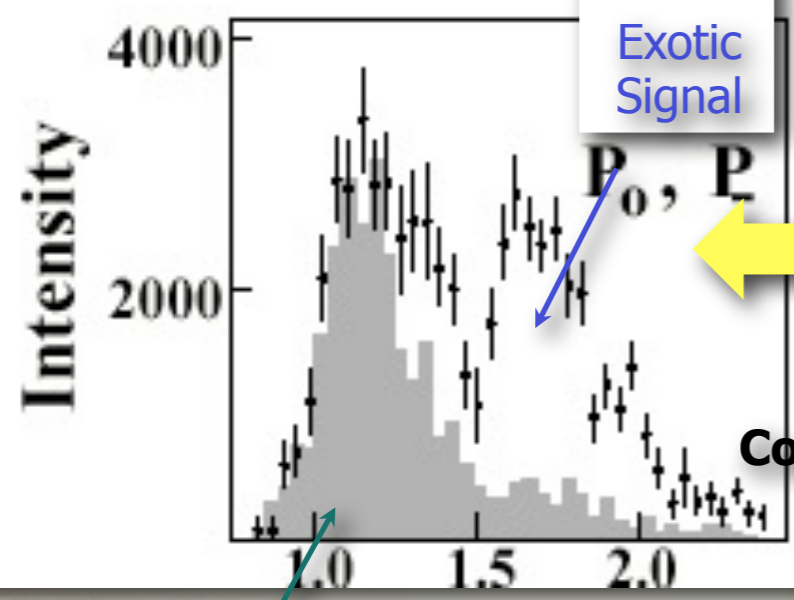


ρ, a_0, a_2



ρ, a_0, a_2, π_1

$\pi_1(1600)$ in $\pi^+\pi^-\pi^-$

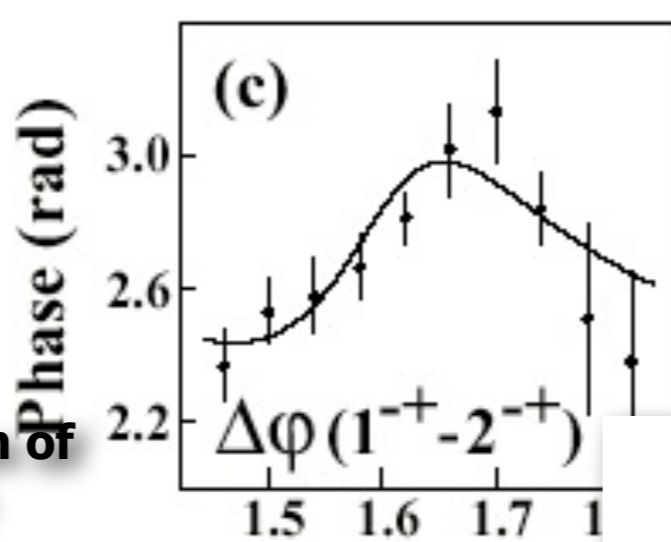
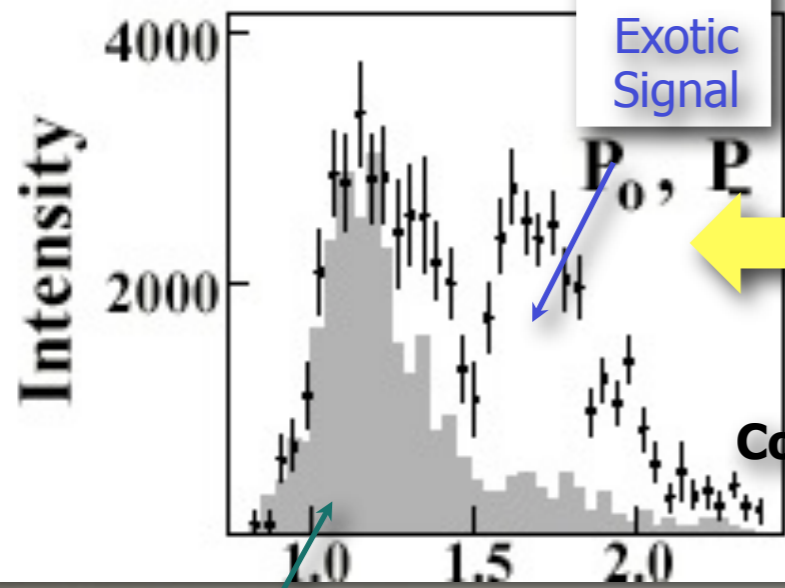


Leakage From
Non-exotic Wave
due to imperfectly
understood acceptance

Correlation of
Phase
&
Intensity

Based on 250K events

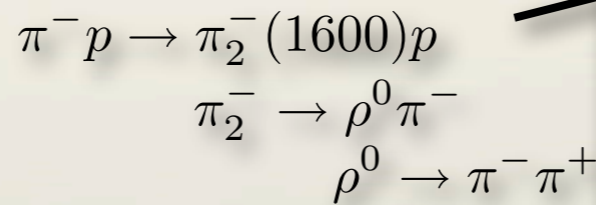
$\pi_1(1600)$ in $\pi^+\pi^-\pi^-$



Correlation of Phase & Intensity

Leakage From Non-exotic Wave due to imperfectly understood acceptance

Based on 250K events



E852 result

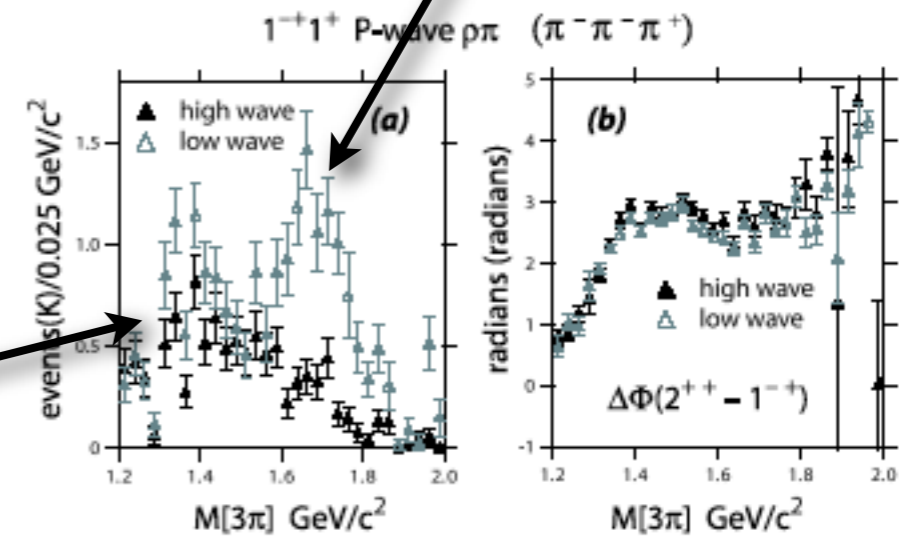


FIG. 25: (a) The $1^{-+}1^+$ P-wave $\rho\pi$ partial wave in the charged mode ($\pi^-\pi^-\pi^+$) for the high-wave set PWA and the low-wave set PWA and (b) the phase difference $\Delta\Phi$ between the 2^{++} and 1^{-+} for the two wave sets.

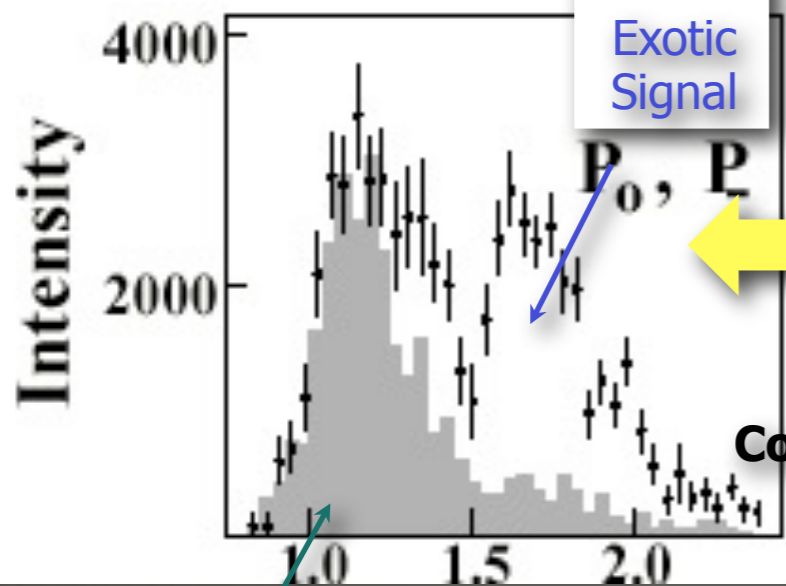
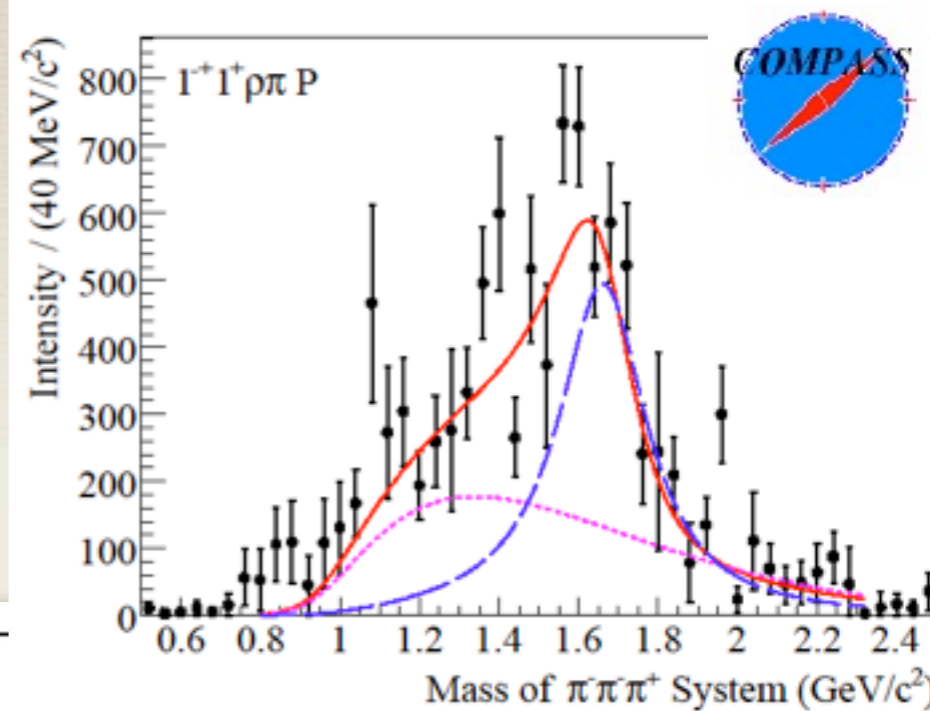
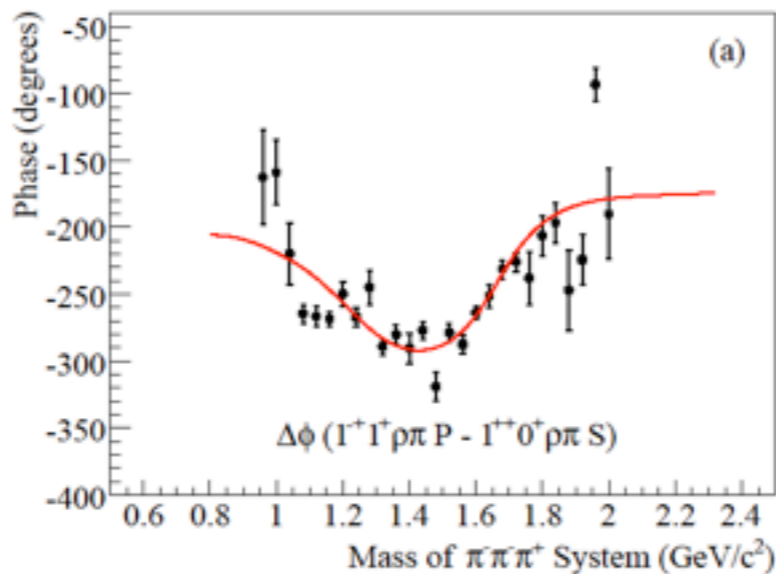
$\pi_1(1600)$ in $\pi^+\pi^-\pi^-$

- BW parameters for $\pi_1(1600)$

$$M = (1660 \pm 10^{+0}_{-64}) \text{ MeV}/c^2$$

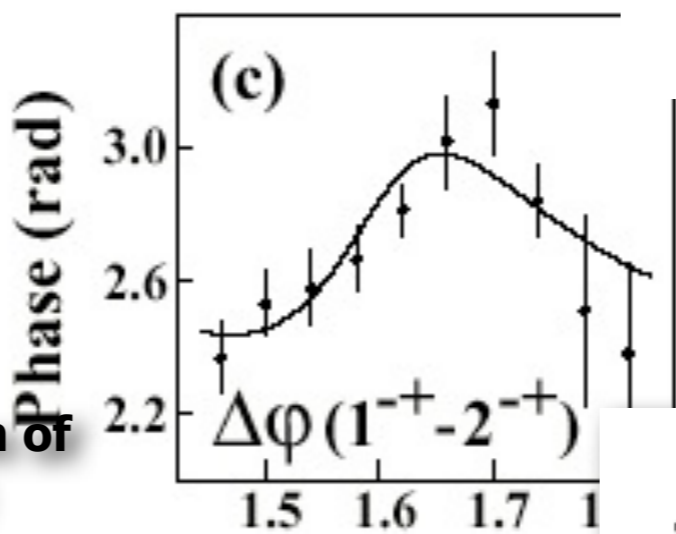
$$\Gamma = (269 \pm 21^{+42}_{-64}) \text{ MeV}/c^2$$

- Leakage negligible: <5%



Exotic Signal

Correlation of Phase & Intensity



E852 result

Leakage From Non-exotic Wave due to imperfectly understood acceptance

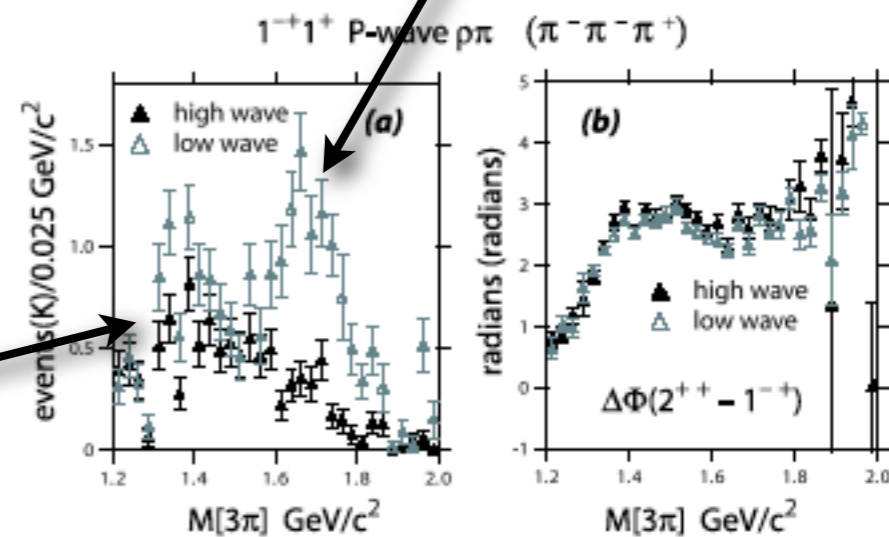
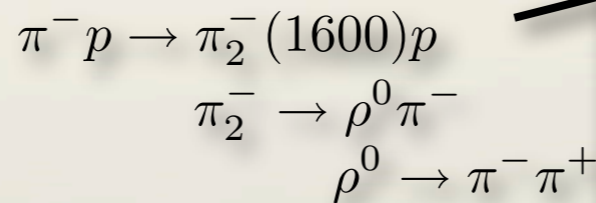
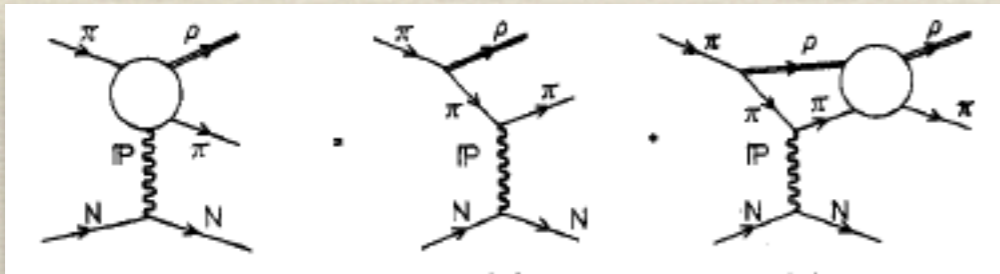


FIG. 25: (a) The 1^-+1^+ P -wave $\rho\pi$ partial wave in the charged mode ($\pi^-\pi^-\pi^+$) for the high-wave set PWA and the low-wave set PWA and (b) the phase difference $\Delta\Phi$ between the 2^{++} and 1^{-+} for the two wave sets.

Based on 250K events

$$e^{i\delta(E)} \cos \delta(E)$$



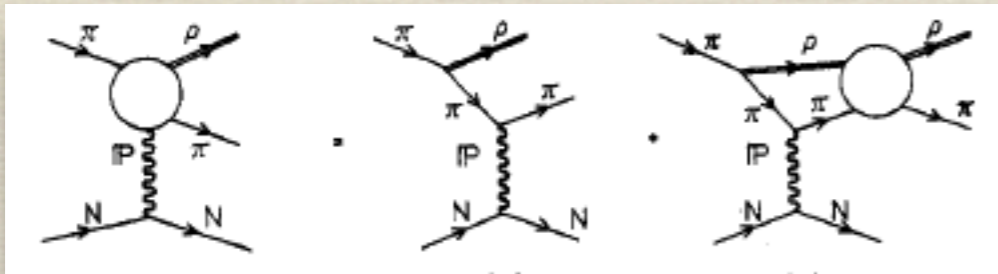
force=OPE

Extended source

Compact source

$$e^{i\delta(E)} \frac{\sin \delta(E)}{k}$$

$$e^{i\delta(E)} \cos \delta(E)$$

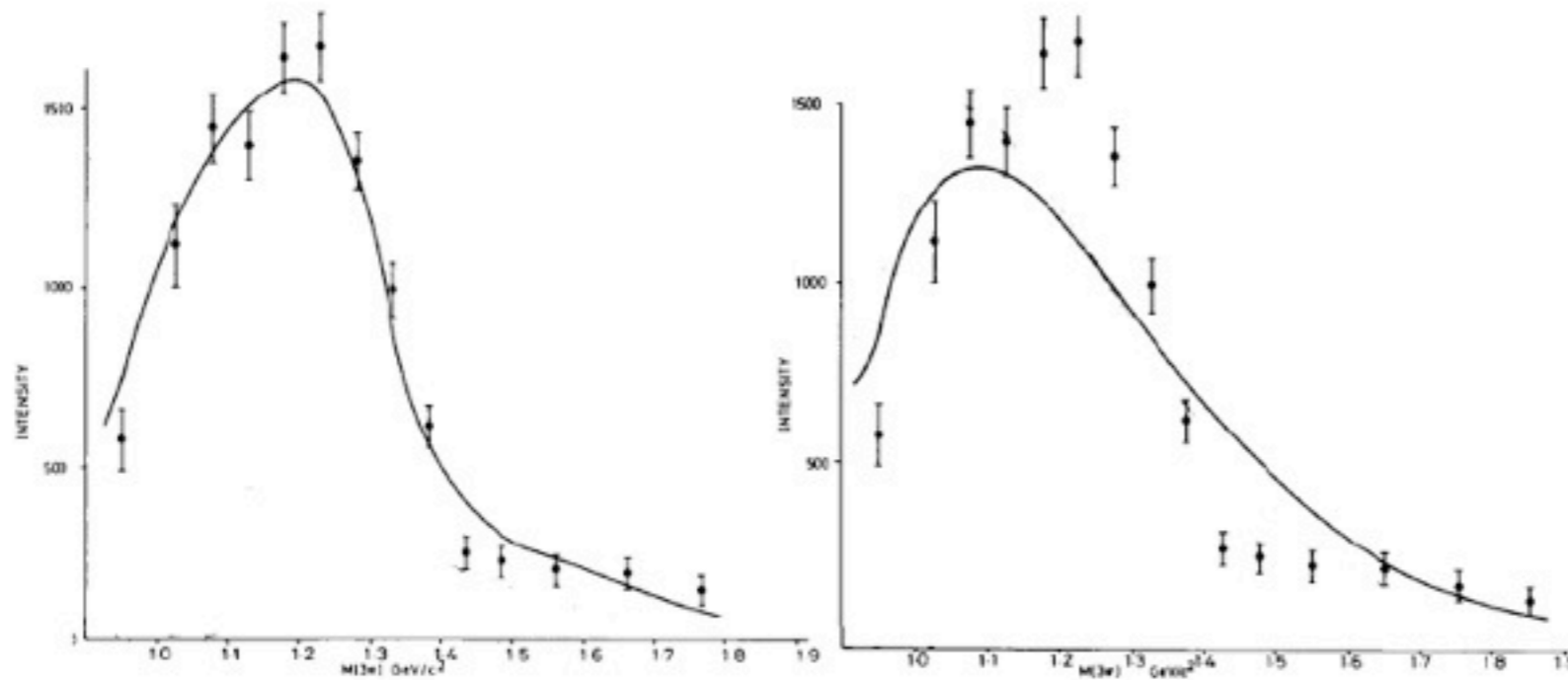


force=OPE

Compact source

$$e^{i\delta(E)} \frac{\sin \delta(E)}{k}$$

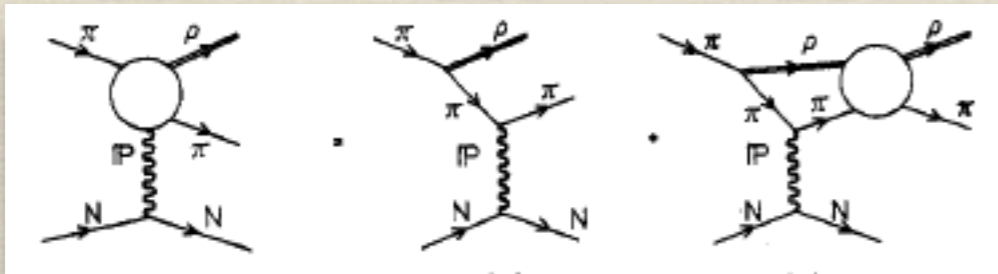
Extended source



M.G.Bowler,(1975)

Figure 11: Fit to the $1^+ \rho\pi$ intensity from $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$ at $E_\pi = 25$ and $E_\pi = 40$ GeV, CERN data [70], with (left) both long-range production from one pion exchange and short-range direct production and (right) short-range direct production only [63].

$$e^{i\delta(E)} \cos \delta(E)$$

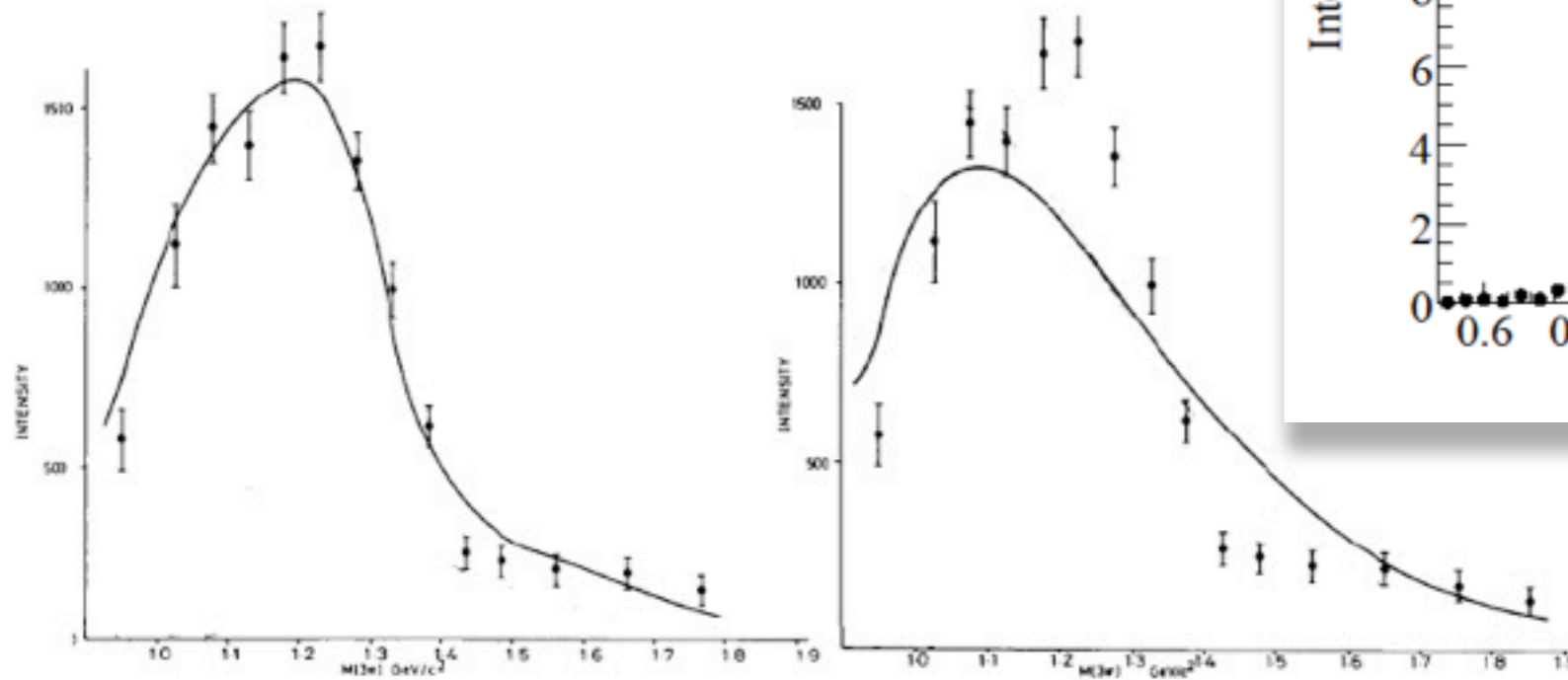
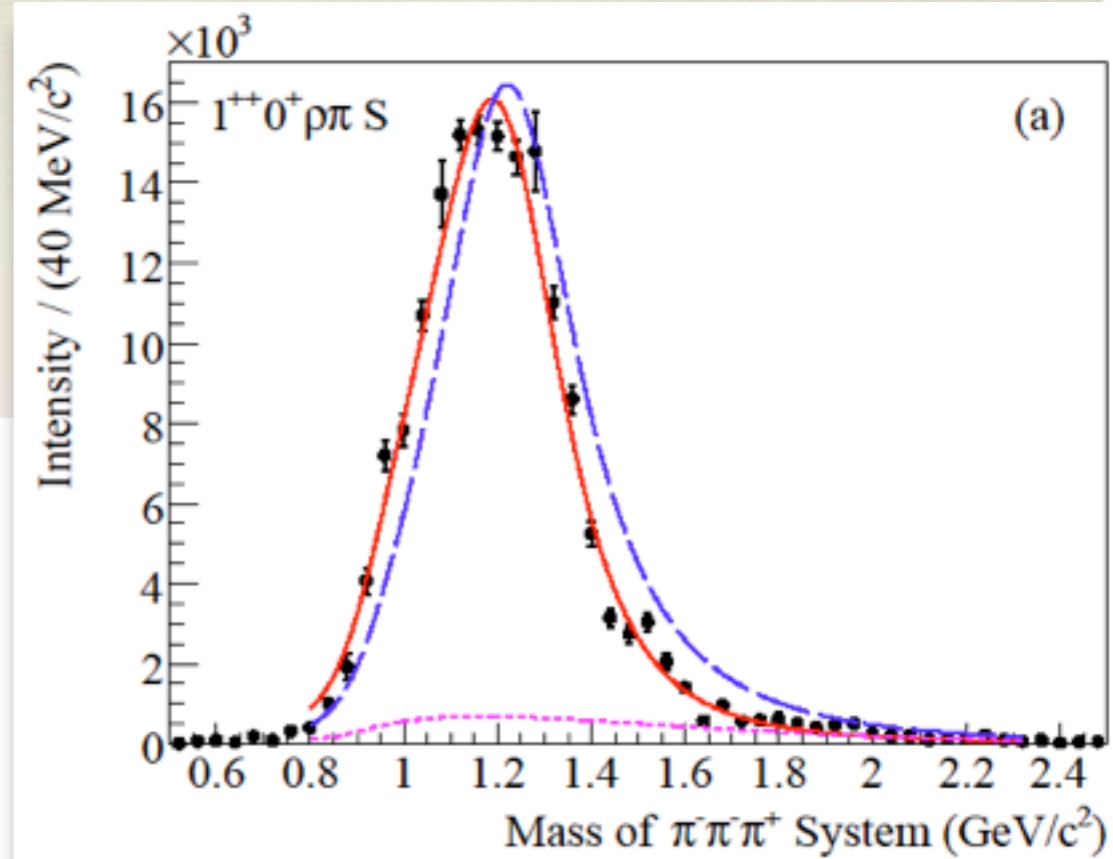


force=OPE

Extended source

Compact source

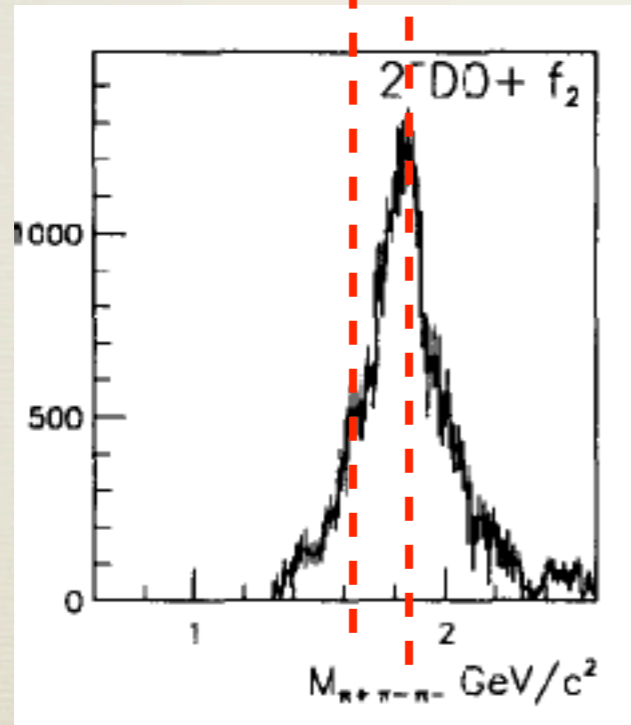
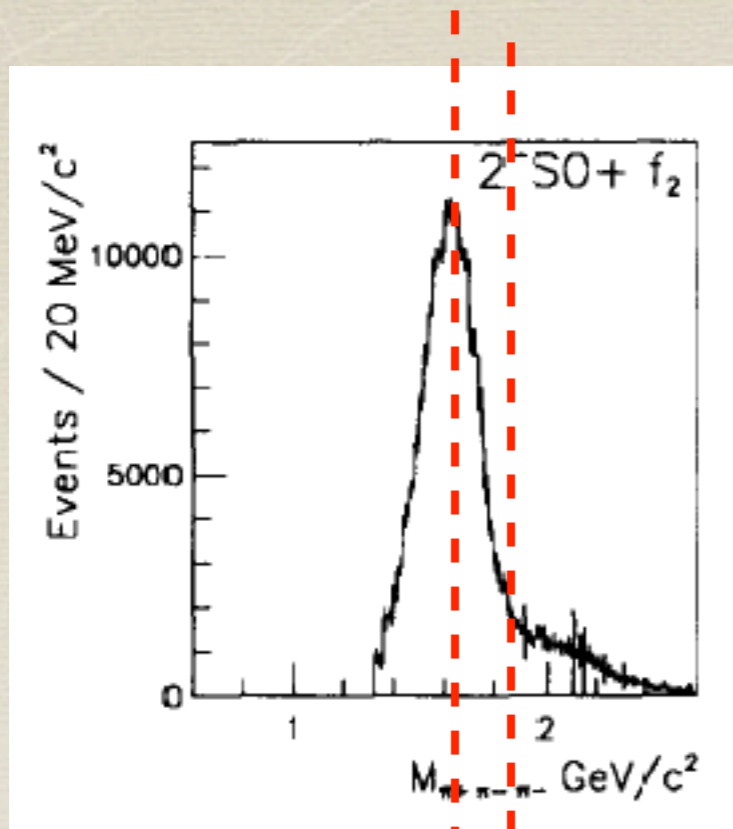
$$e^{i\delta(E)} \frac{\sin \delta(E)}{k}$$



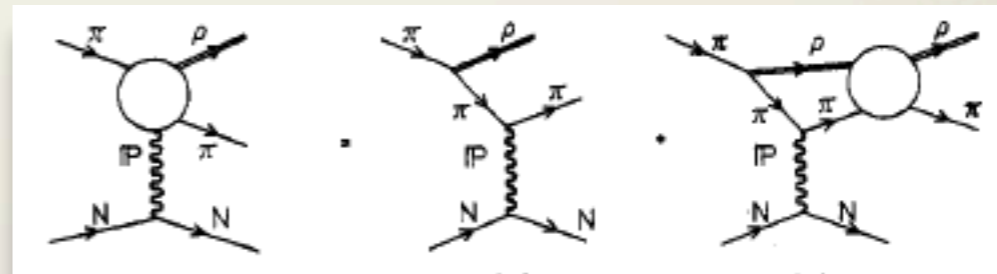
M.G.Bowler,(1975)

Figure 11: Fit to the $1^+ \rho\pi$ intensity from $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$ at $E_\pi = 25$ and $E_\pi = 40$ GeV, CERN data [70], with (left) both long-range production from one pion exchange and short-range direct production and (right) short-range direct production only [63].

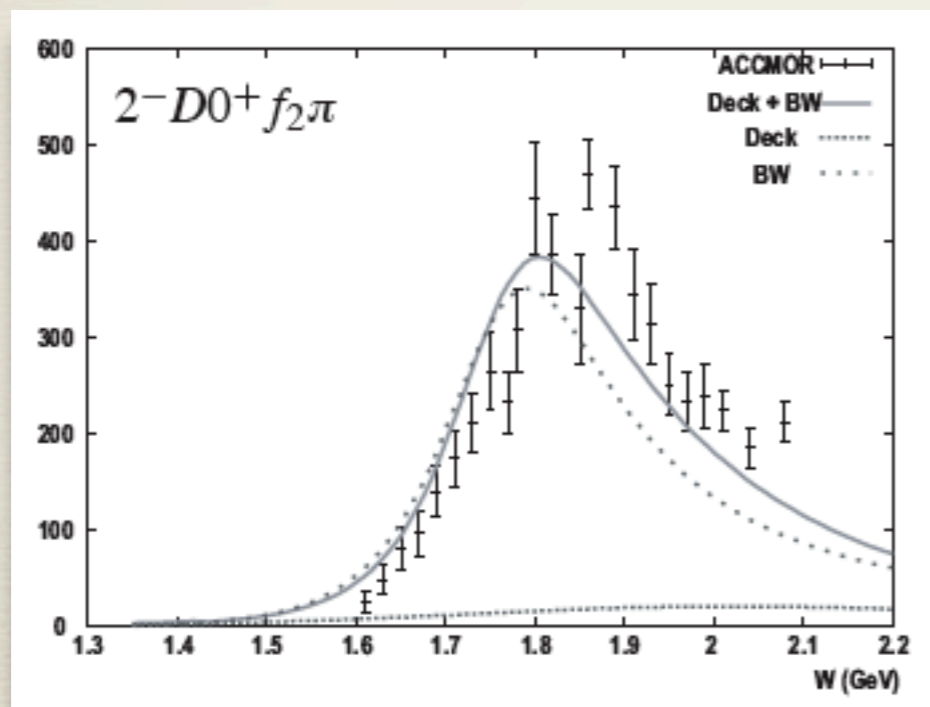
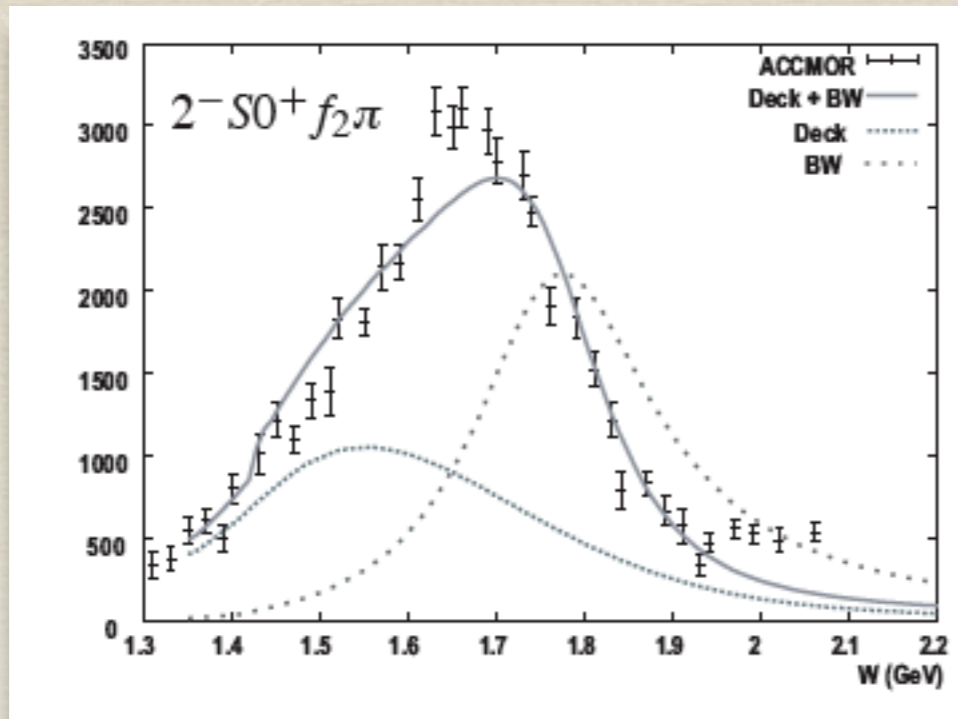
Moving $\pi_2(1670)$ peak



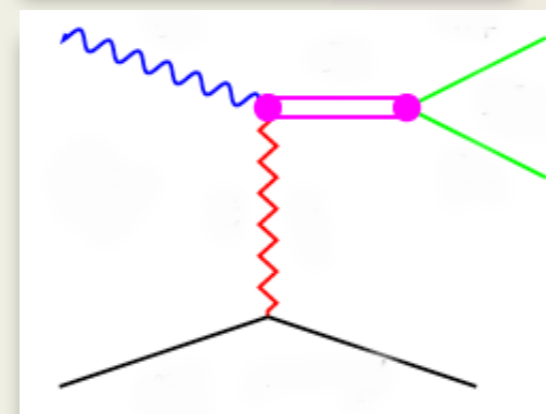
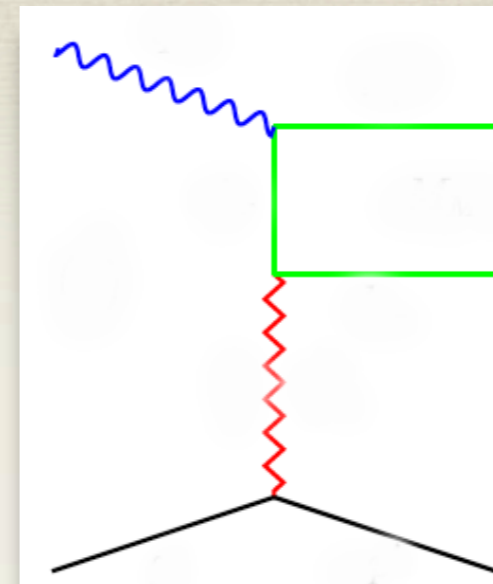
VALUE (MeV)	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
1672.4 ± 3.2 OUR AVERAGE		Error includes scale factor of 1.4. See the ideogram below.			
1749 ± 10 ± 100	145k	LU	05	E852	18 $\pi^- p \rightarrow \omega \pi^- \pi^0 p$
1676 ± 3 ± 8		1 CHUNG	02	E852	18.3 $\pi^- p \rightarrow \pi^+ \pi^- \pi^- p$
1685 ± 10 ± 30		2 BARBERIS	01		450 $p p \rightarrow p_f 3\pi^0 p_s$
1687 ± 9 ± 15		AMELIN	99	VES	37 $\pi^- A \rightarrow \omega \pi^- \pi^0 A^+$
1669 ± 4		BARBERIS	98B		450 $p p \rightarrow p_f p \pi p_s$
1670 ± 4		BARBERIS	98B		450 $p p \rightarrow p_f f_2(1270) \pi p_s$
1730 ± 20		3 AMELIN	95B	VES	36 $\pi^- A \rightarrow \pi^+ \pi^- \pi^- A$
1690 ± 14		4 BERDNIKOV	94	VES	37 $\pi^- A \rightarrow K^+ K^- \pi^- A$
1710 ± 20	700	ANTIPOV	87	SIGM	- 50 $\pi^- \text{Cu} \rightarrow \mu^+ \mu^- \pi^- \text{Cu}$
1676 ± 6		4 EVANGELISTA	81	OMEG	- 12 $\pi^- p \rightarrow 3\pi$
1657 ± 14		4,5 DAUM	80D	SPEC	- 63-94 $\pi p \rightarrow 3\pi X$
1662 ± 10	2000	4 BALTAY	77	HBC	+ 15 $\pi^+ p \rightarrow p 3\pi$
*** We do not use the following data for averages, fits, limits, etc. ***					
1742 ± 31 ± 49		ANTREASYAN	90	CBAL	$e^+ e^- \rightarrow e^+ e^- \pi^0 \pi^0 \pi^0$
1624 ± 21		1 BELLINI	85	SPEC	40 $\pi^- A \rightarrow \pi^- \pi^+ \pi^- A$
1622 ± 35		6 BELLINI	85	SPEC	40 $\pi^- A \rightarrow \pi^- \pi^+ \pi^- A$
1693 ± 28		7 BELLINI	85	SPEC	40 $\pi^- A \rightarrow \pi^- \pi^+ \pi^- A$
1710 ± 20		8 DAUM	81B	SPEC	- 63,94 $\pi^- p$
1650 ± 10		4 ASCOLI	73	HBC	- 5-25 $\pi^- p \rightarrow p \pi_2$



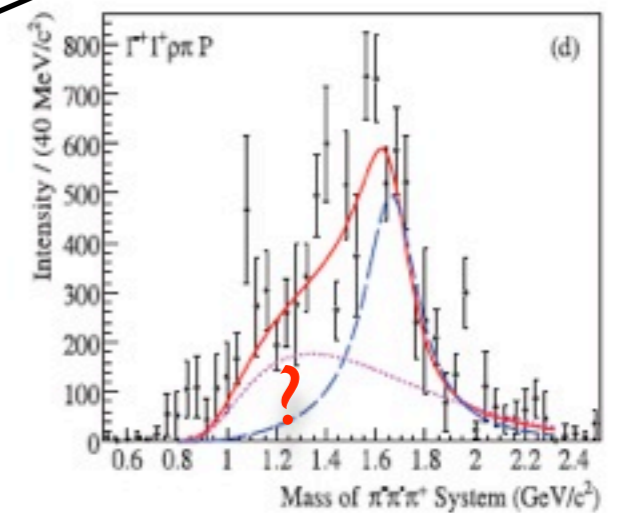
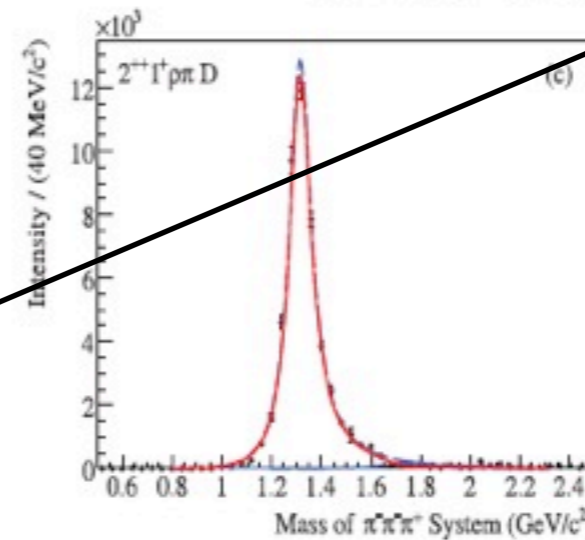
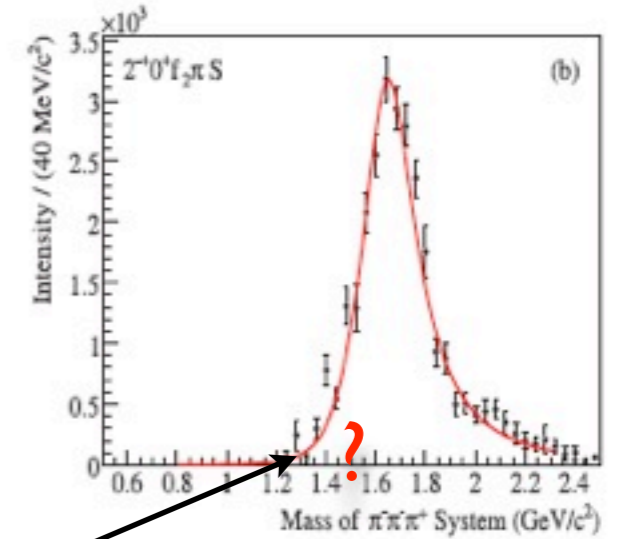
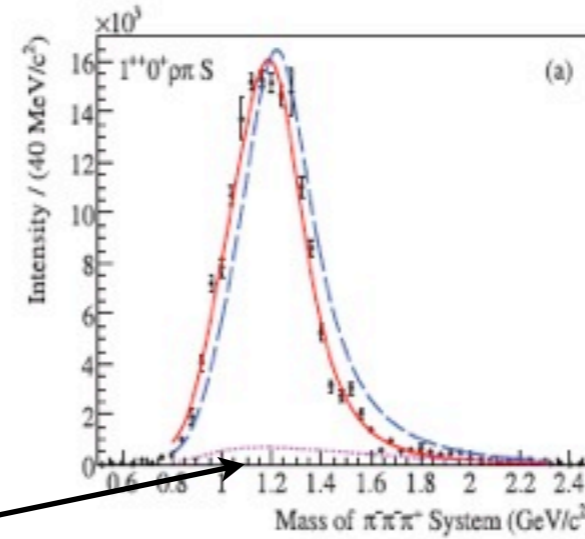
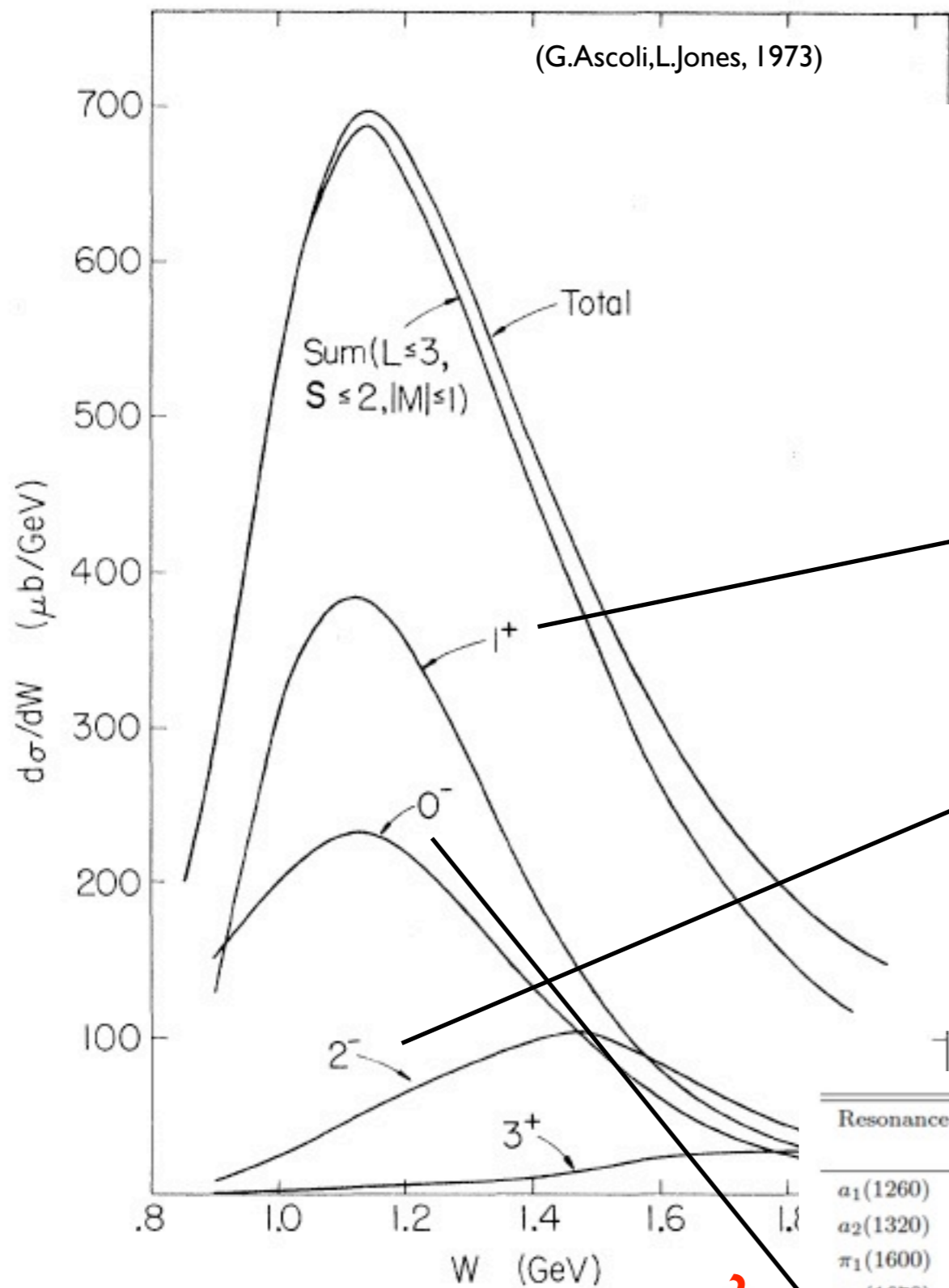
Duality @ work ?



J,Dudek,AS (2006)



Isobar-type fits could involve spurious resonances



M.G. Alekseev, et al., COMPASS (2010)

Resonance	Mass (MeV/c ²)	Width (MeV/c ²)	Intensity (%)	Channel $J^{PC} M^{\epsilon}[\text{isobar}]L$	Mass [26] (MeV/c ²)	Width [26] (MeV/c ²)
$a_1(1260)$	$1255 \pm 6_{-17}^{+7}$	$367 \pm 9_{-25}^{+28}$	$67 \pm 3_{-20}^{+4}$	$1^{+}0^{+}\rho\pi S$	1230 ± 40	250 – 600
$a_2(1320)$	$1321 \pm 1_{-7}^{+0}$	$110 \pm 2_{-15}^{+2}$	$19.2 \pm 0.6_{-2.2}^{+0.3}$	$2^{+}1^{+}\rho\pi D$	1318.3 ± 0.6	107 ± 5
$\pi_1(1600)$	$1660 \pm 10_{-64}^{+0}$	$269 \pm 21_{-64}^{+42}$	$1.7 \pm 0.2_{-0.1}^{+0.9}$	$1^{-}1^{+}\rho\pi P$	1662_{-11}^{+15}	234 ± 50
$\pi_2(1670)$	$1658 \pm 3_{-8}^{+24}$	$271 \pm 9_{-24}^{+22}$	$10.0 \pm 0.4_{-0.7}^{+0.7}$	$2^{-}0^{+}f_2\pi S$	1672.4 ± 3.2	259 ± 9
$\pi(1800)$	$1785 \pm 9_{-6}^{+12}$	$208 \pm 22_{-37}^{+21}$	$0.8 \pm 0.1_{-0.1}^{+0.3}$	$0^{-}0^{+}f_0\pi S$	1816 ± 14	208 ± 12
$a_4(2040)$	$1885 \pm 13_{-2}^{+50}$	$294 \pm 25_{-19}^{+46}$	$1.0 \pm 0.3_{-0.1}^{+0.1}$	$4^{+}1^{+}\rho\pi G$	2001 ± 10	313 ± 31

Q: How important are multi-particle
correlations
at low-energies
i.e. re-scattering in the isobar model?

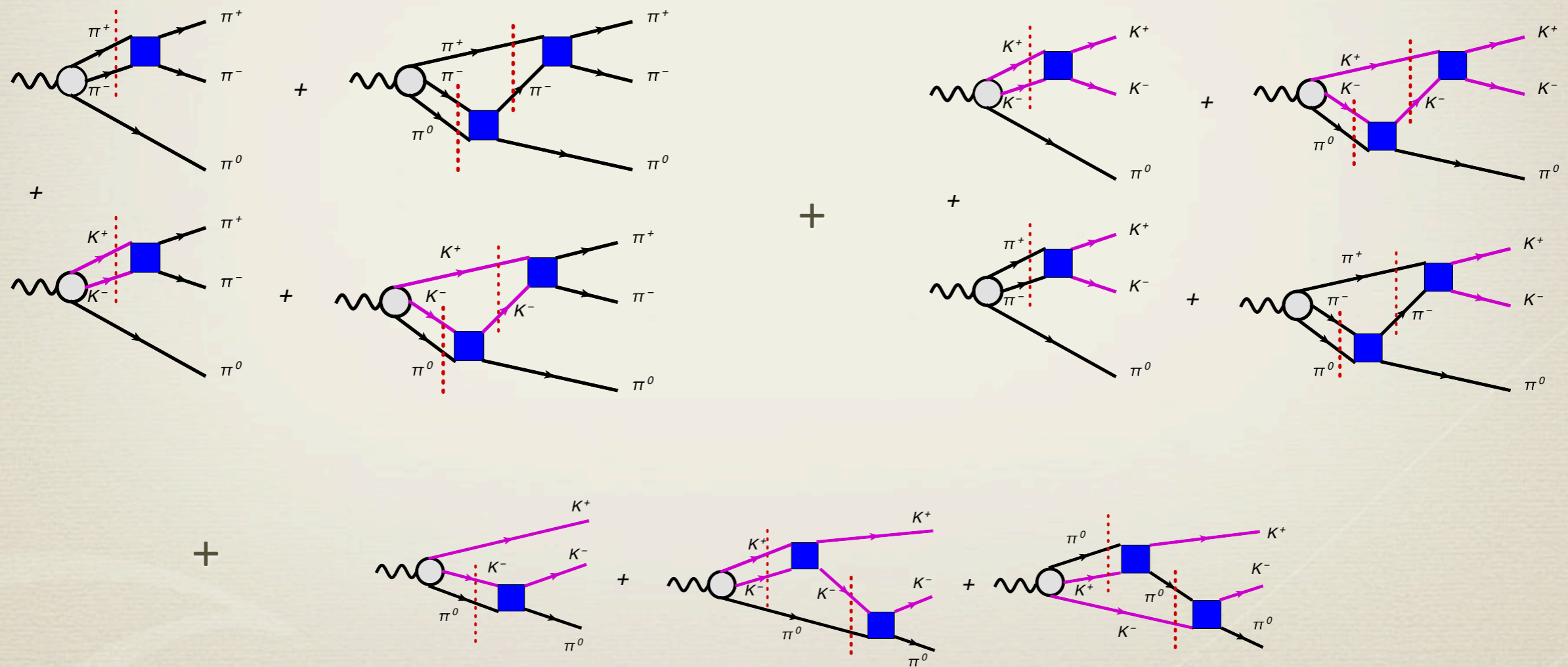
Khuri-Treiman (1960)
Pasquier-Pasquier (1968-1970)
Aitchison, Brehm (late 70's)

A: Apparently very small

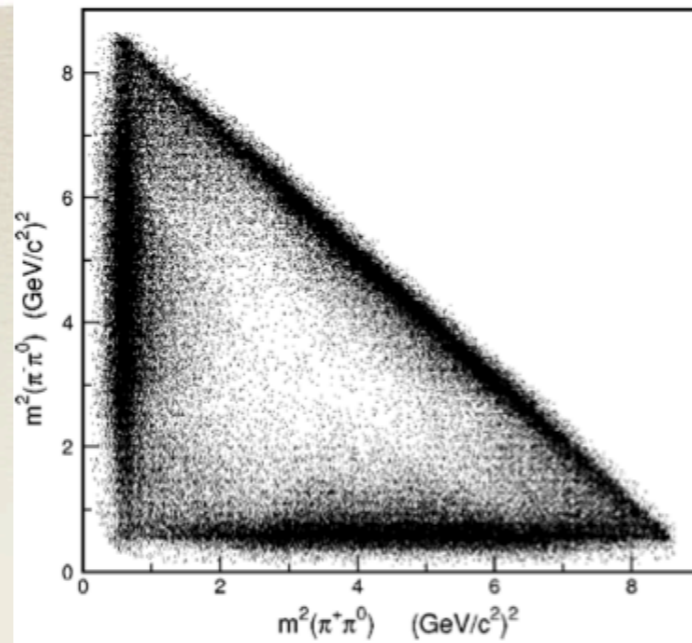
In three body final states ... one should consider ...

Three-body interaction
+
Coupled-channels
+
Unitarity & Analyticity

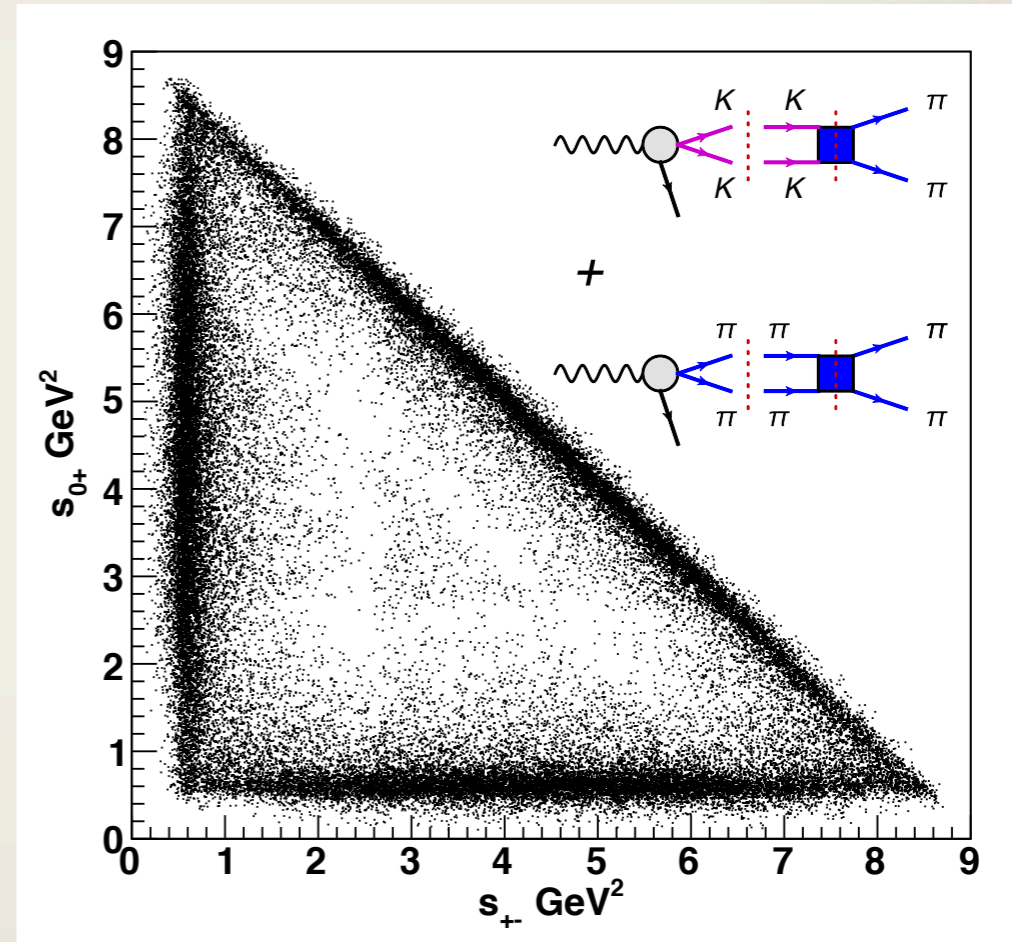
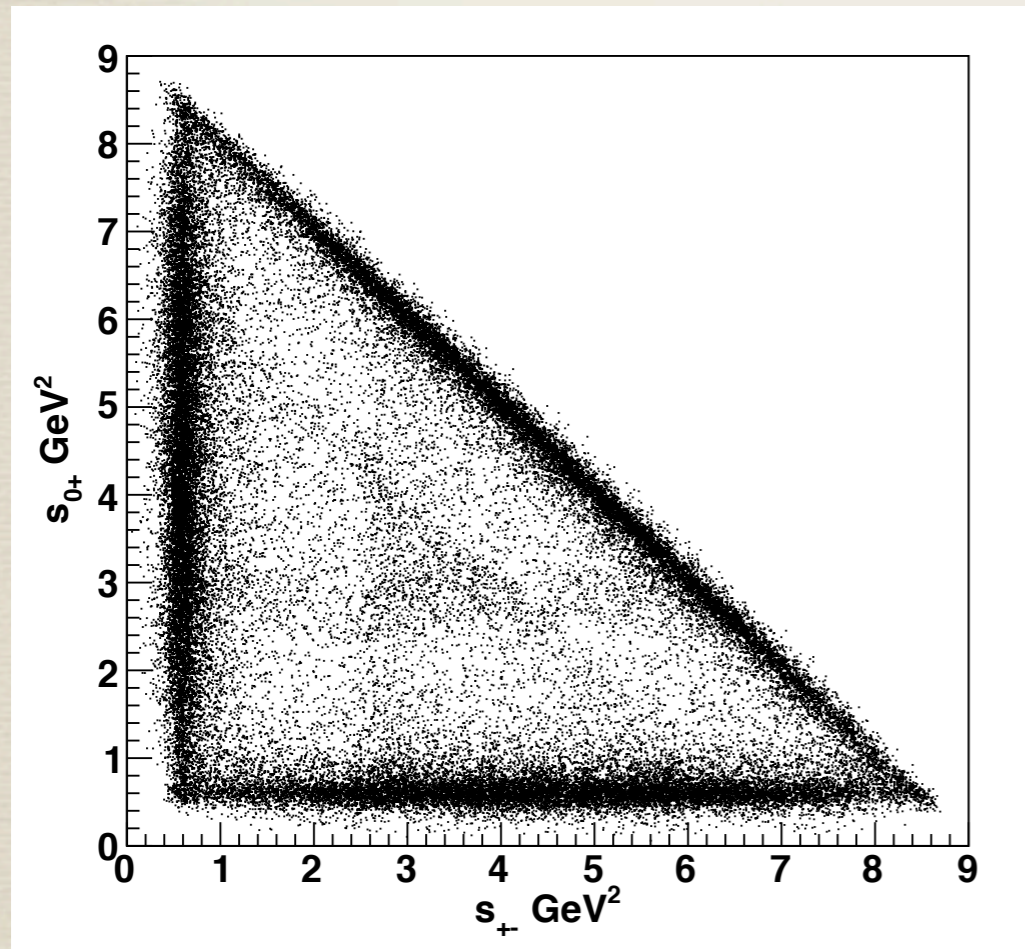
$$J/\Psi \rightarrow 3\pi + K\bar{K}\pi$$



$$J/\psi \rightarrow 3\pi$$



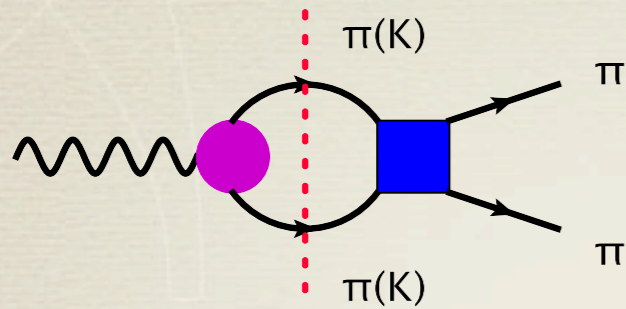
BES Collaboration
Phys.Rev.D70:012005,2004



$$F(s) \sim \frac{1}{D_{\pi\pi \rightarrow \pi\pi}(s)}$$

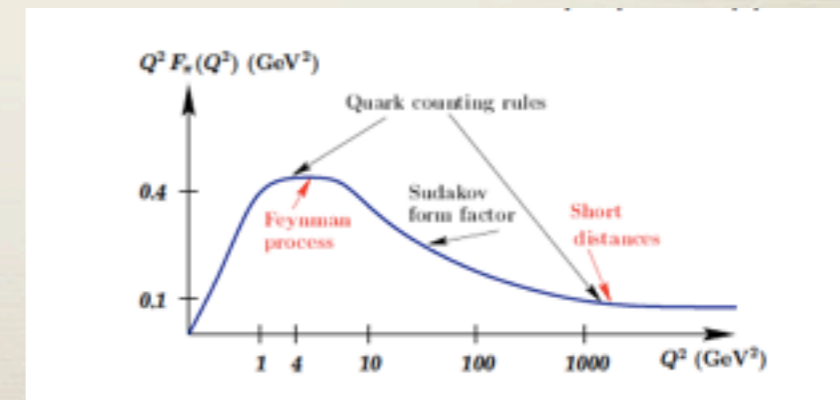
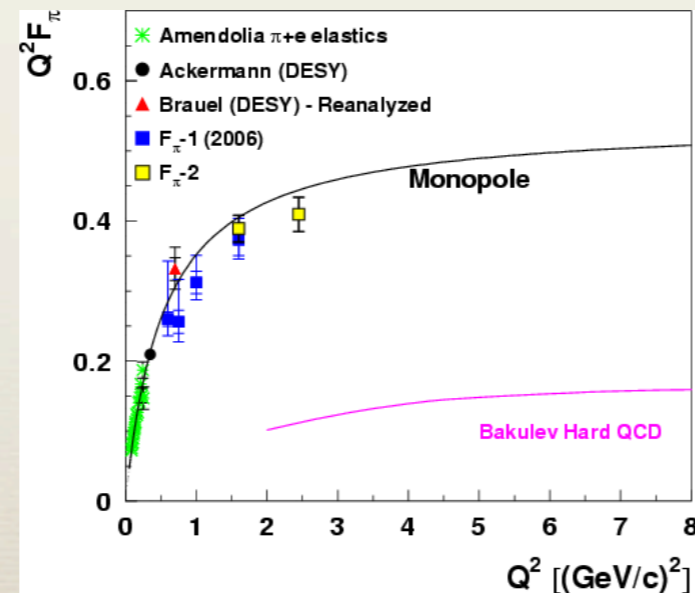
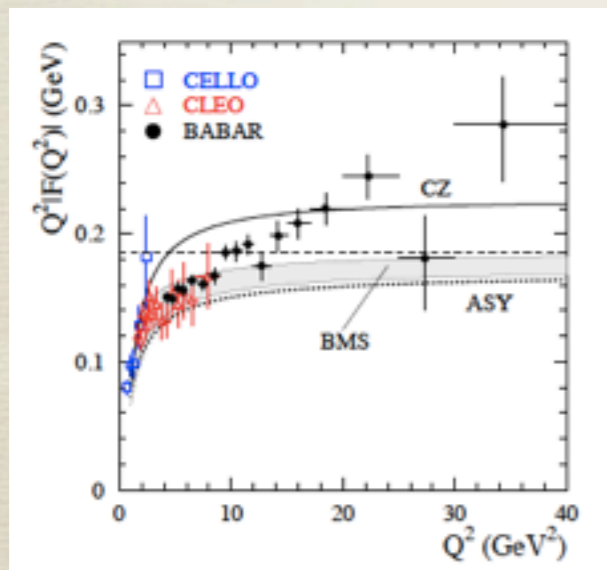
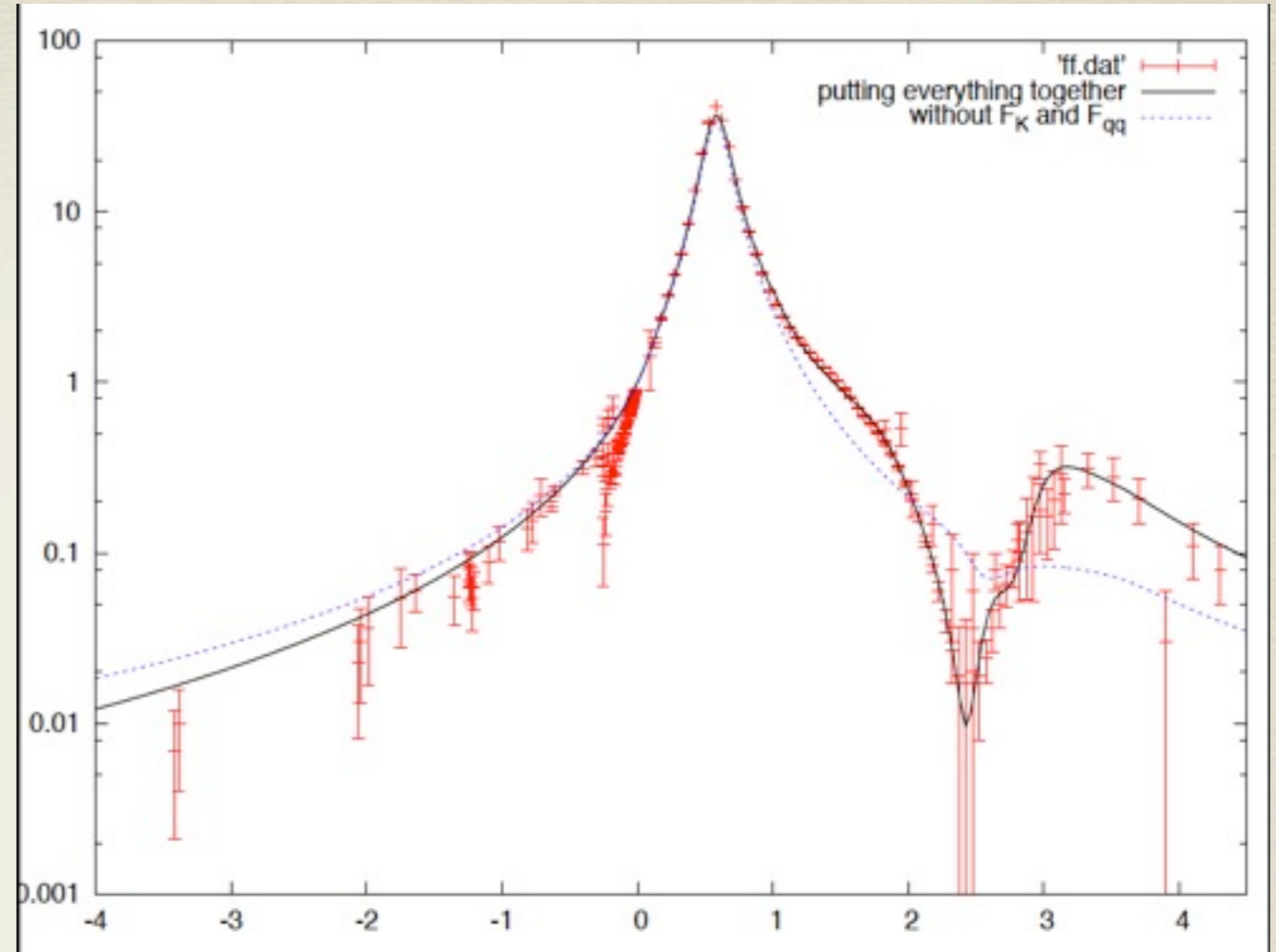
$$F(s) \sim \frac{1 + c_1 s}{D_{\pi\pi \rightarrow \pi\pi}(s)} + \frac{c_0}{D_{K\bar{K} \rightarrow \pi\pi}(s)}$$

Pion formfactor: $|F_{\pi\pi}(s)|^2$

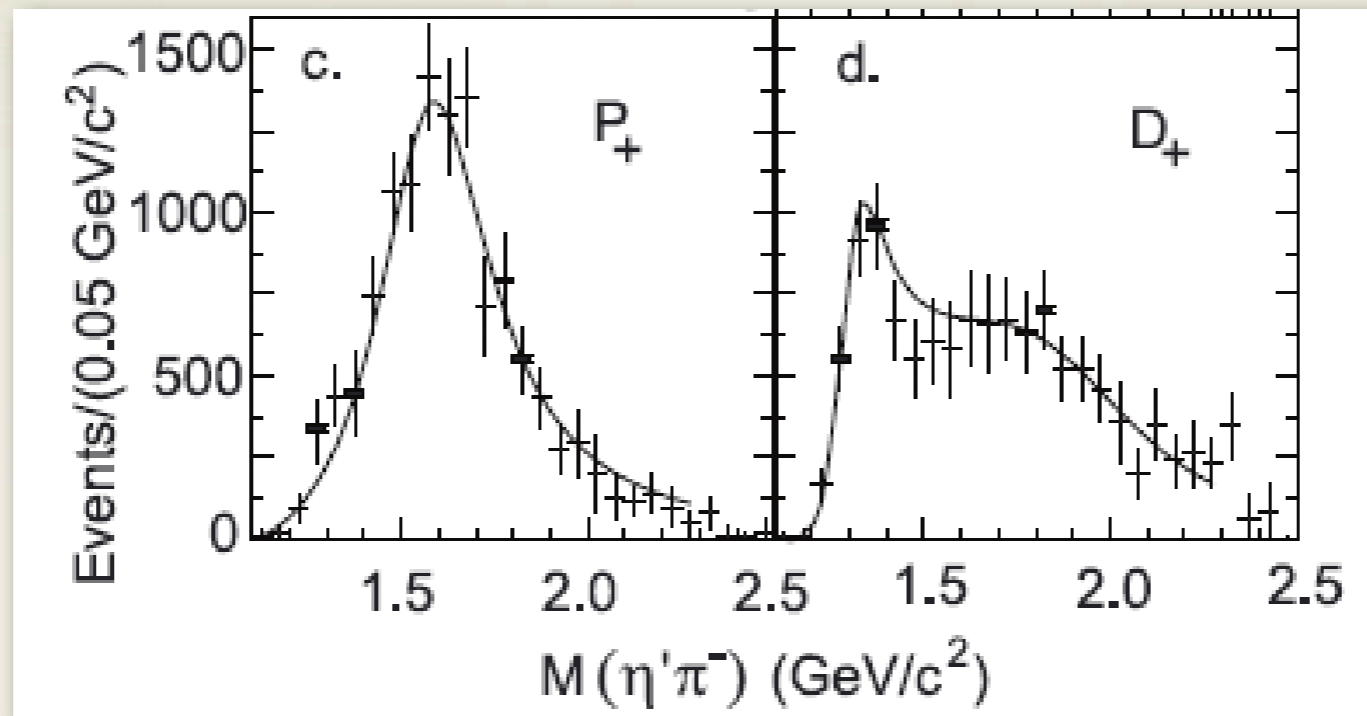
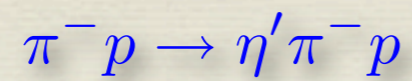


$$F(s) \sim \frac{1 + c_1 s}{D_{\pi\pi \rightarrow \pi\pi}(s)} + \frac{c_0}{D_{K\bar{K} \rightarrow \pi\pi}(s)}$$

Novel interpretation of asymptotic behavior
(M.Gorshteyn, P.Guos, AS (2011))

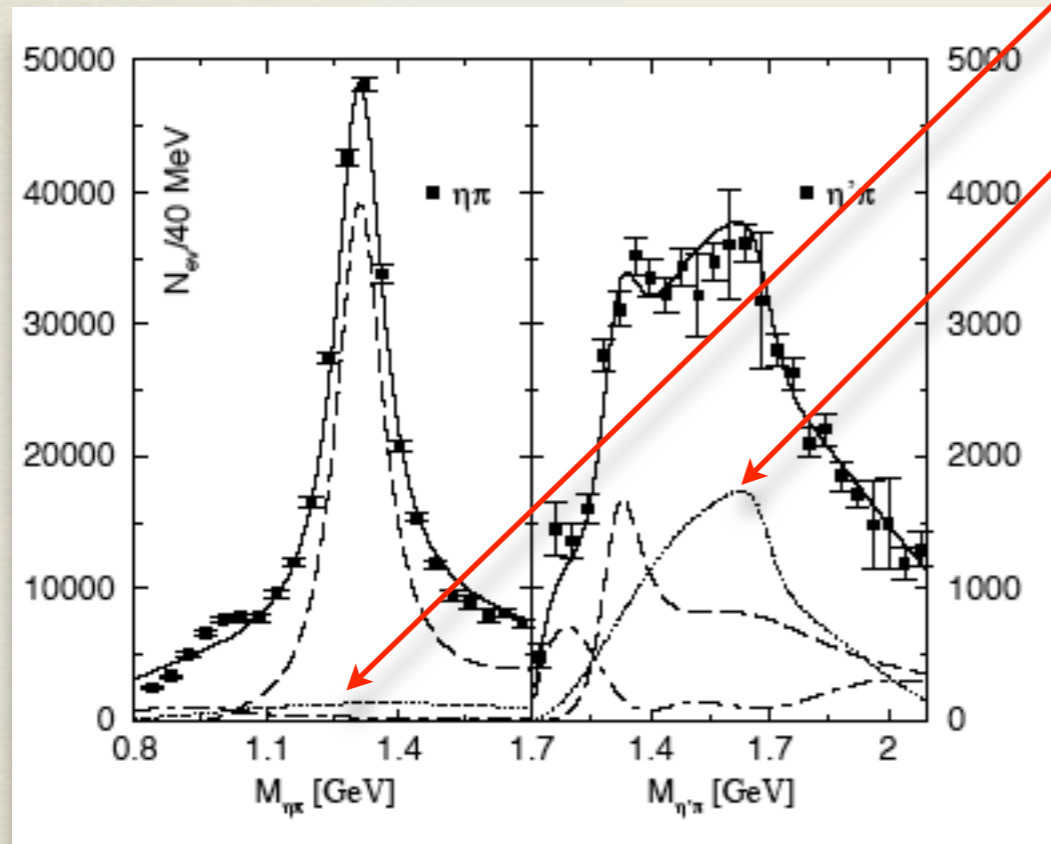


$J^{PC} = 1^{-+}$ exotic wave signals (E852 data)



(other signals
identified by E852,
CB,VES)

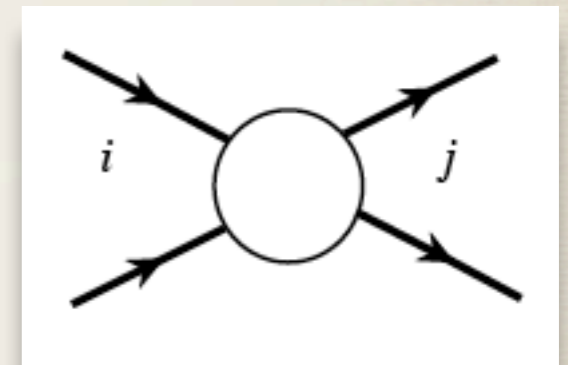
Fitting the E852 the $\eta\pi$ and $\eta'\pi$ spectra using efit give a good description of the exotic wave (APS et al.)



P-wave

P -wave $\eta\pi, \eta'\pi$
2 coupled channels

S, D -wave $K\bar{K}, \eta\pi, \eta'\pi$
3 coupled channels



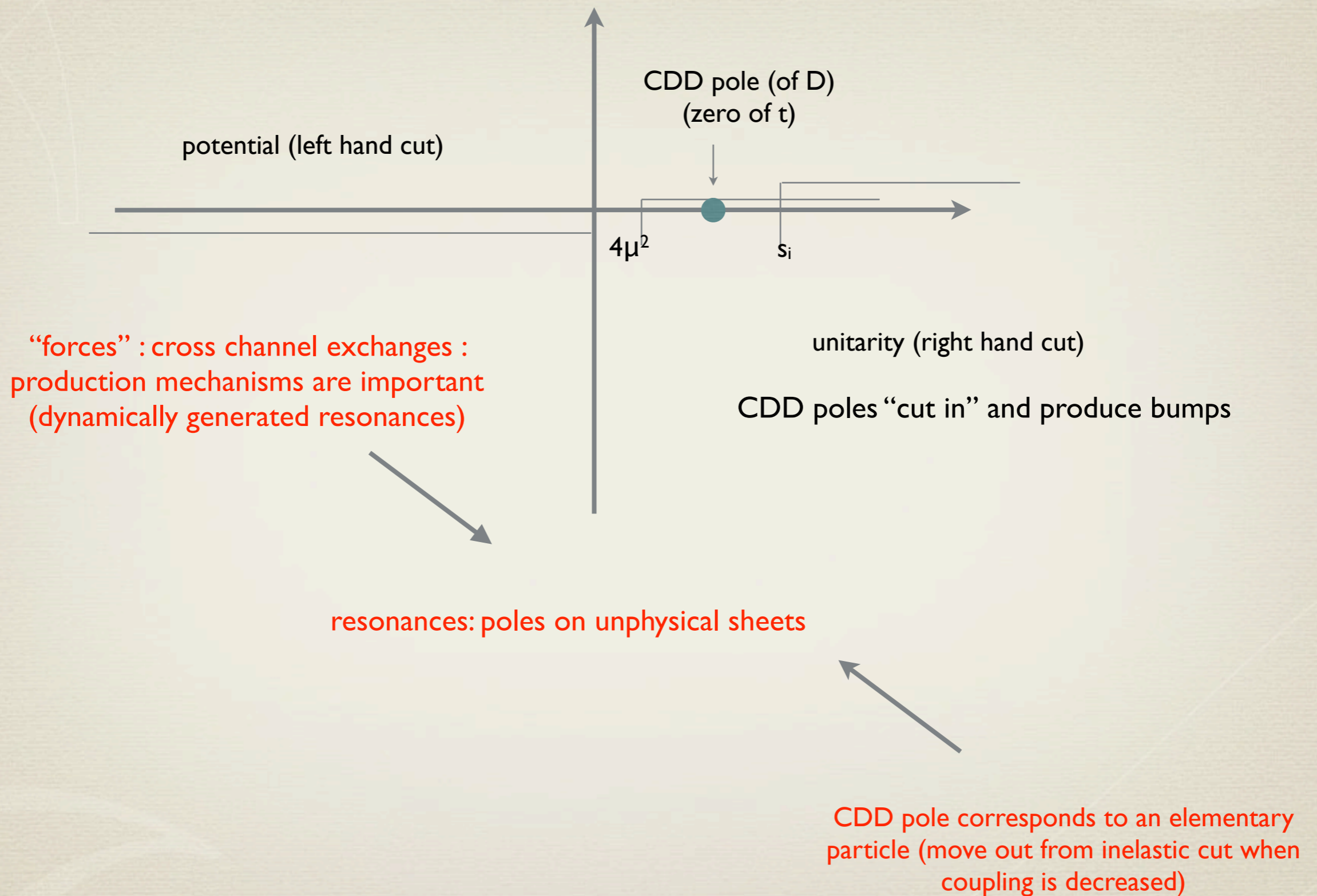
$$t(s) = \frac{1}{\text{Re } V^{-1} - i\rho}$$

long range forces between η and π
are expected to be weak

to fit the data V needs to have short
range interactions

Dynamical resonance or CDD poles ?

Towards a connection between data and resonances



* General idea

$$\text{Im}A(s) = R(s)\rho(s)|A(s)|^2$$

$$A(s) = \frac{1}{\pi} \int_{-\infty}^0 ds' \frac{\text{Im}A(s')}{s' - s} + \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\text{Im}A(s')}{s' - s}$$

integral equation for
the amplitude

* output : through unitarity related to
measured x-section

* input (“potential”) : through crossing lhc is
related to other physical amplitudes

caveats

* potential not known everywhere

* in principle many (∞) channels contribute

* x-sections known over limited energy
range

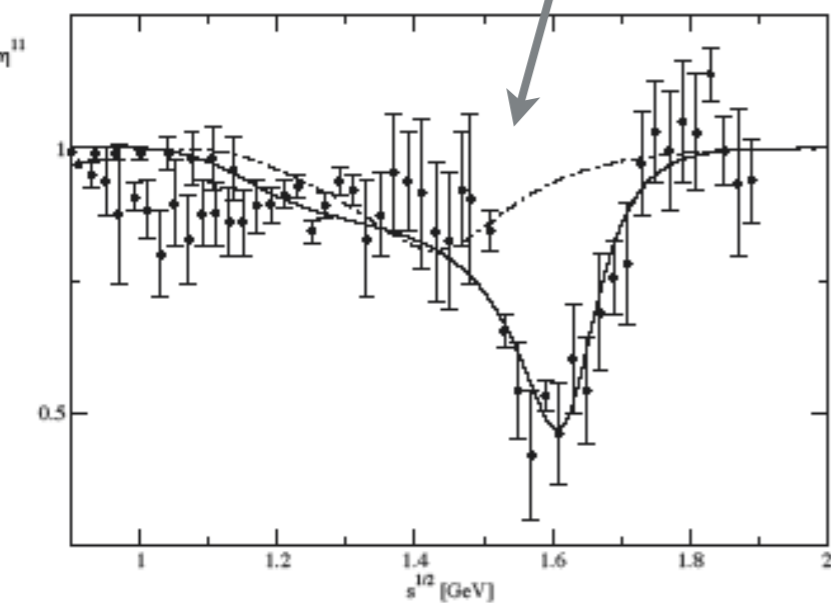
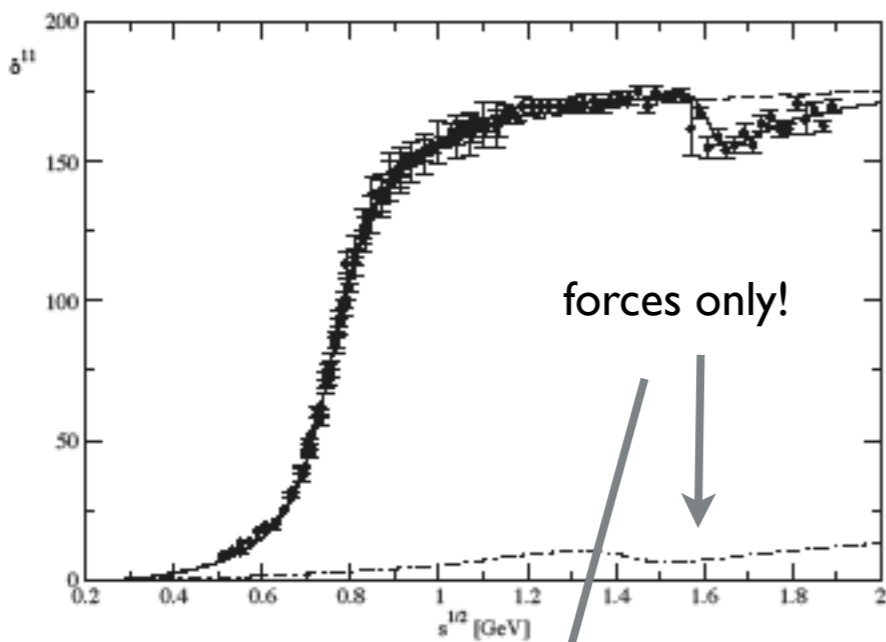
* solutions are not unique (CDD)

recent
improvements and
(1960's vs 2000)

* QCD: interpretation of the ambiguities

* chiral symmetry: low energy constraints

From dispersion relations

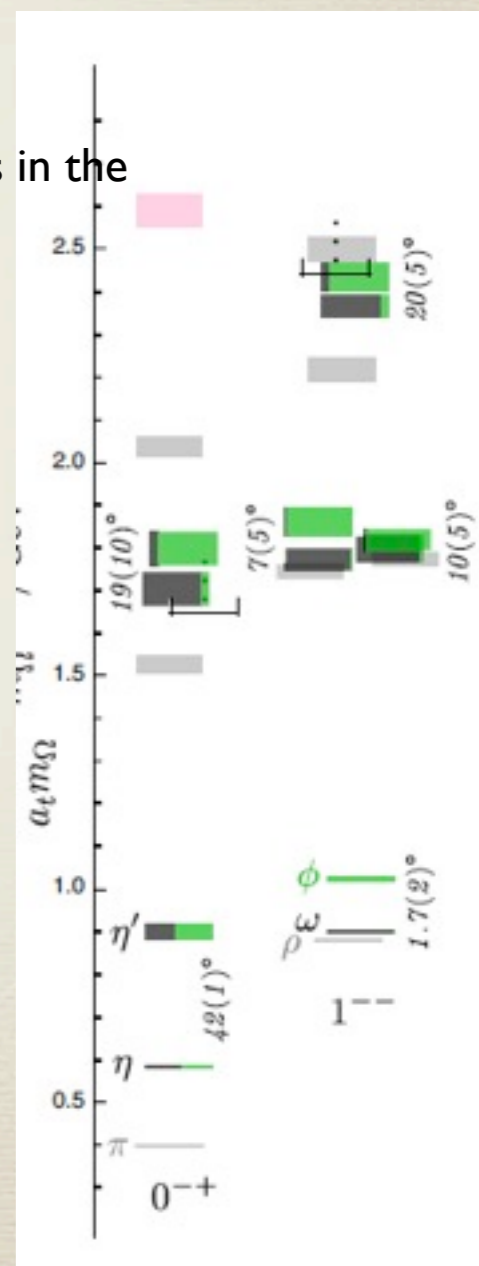


* CDD pole required

FIG. 1: P -wave phase shift (upper panel) and inelasticity (lower panel). Data from [34–36], dashed-dotted (solid) line solution of dispersion relation without (with) a CDD pole. Dashed line is the fit of the quark model from Eq.(33).

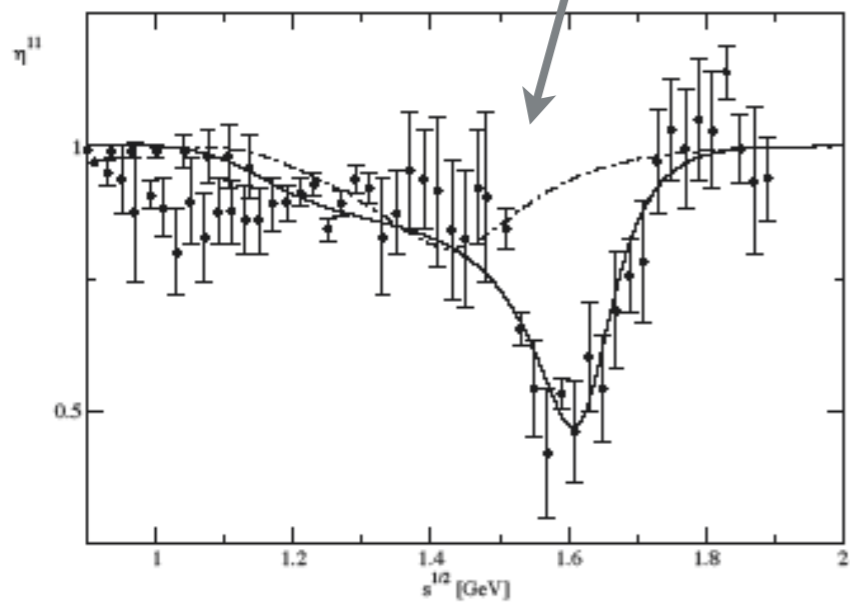
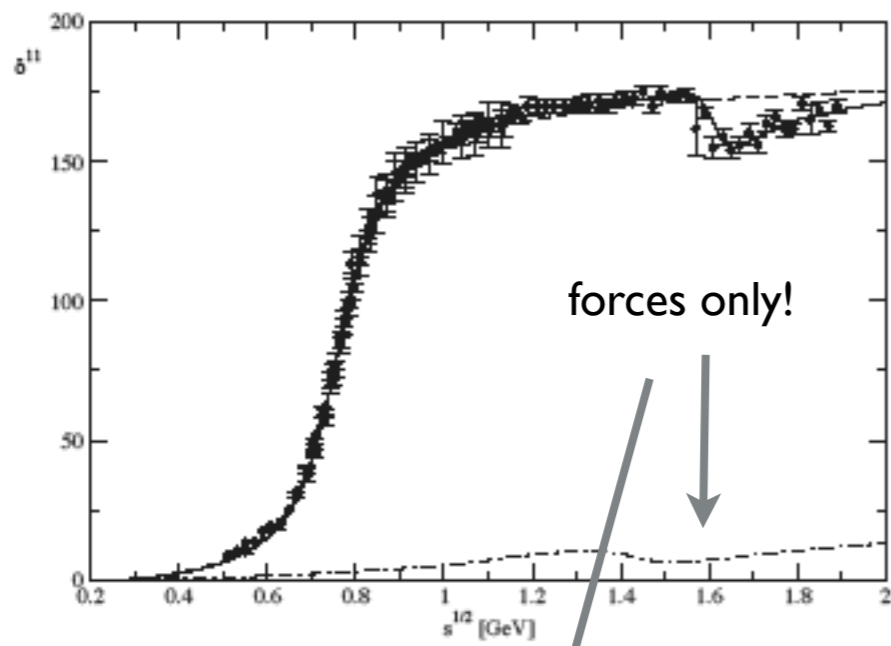
M.Battaglieri,R.de Vita,P.Guo,AS

- * bootstrap failed
- * resonances are not generated dynamically from interactions between other resonances
- * or as lattice suggests there are single hadron states in the spectrum



J.Dudek et al. 2011

From dispersion relations



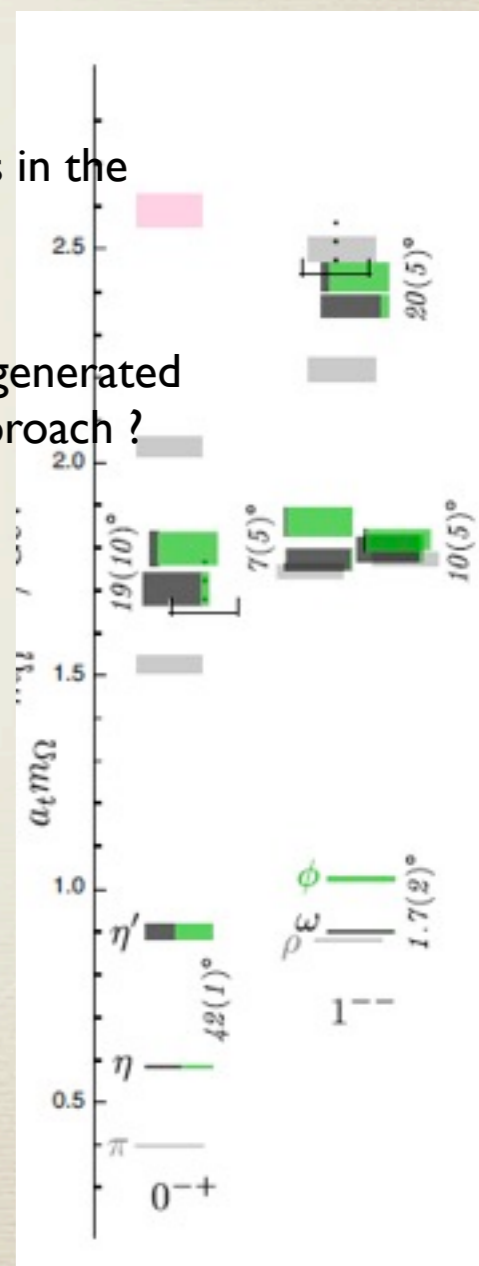
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FIG. 1: P -wave phase shift (upper panel) and inelasticity (lower panel). Data from [34–36], dashed-dotted (solid) line solution of dispersion relation without (with) a CDD pole. Dashed line is the fit of the quark model from Eq.(33).

M.Battaglieri,R.de Vita,P.Guo,AS

- * bootstrap failed
- * resonances are not generated dynamically from interactions between other resonances
- * or as lattice suggests there are single hadron states in the spectrum
- * how does it fit in with the success of dynamically generated resonance program from a unitarized chi-PT approach?

do the Uch-PT poles move?



J.Dudek et al. 2011

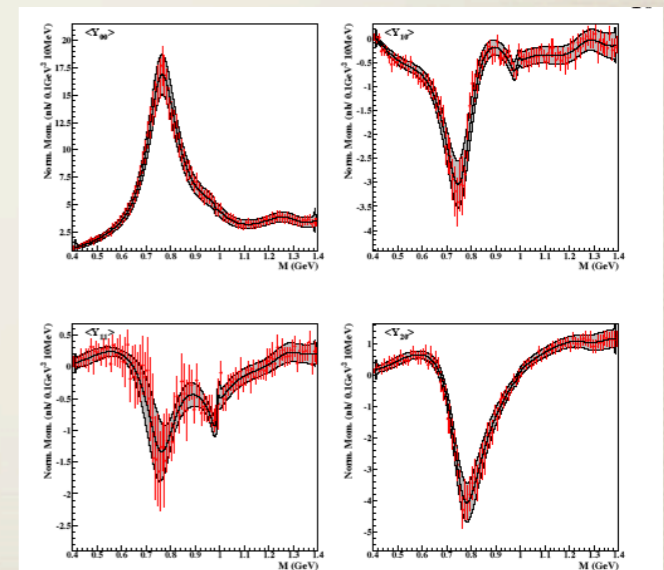
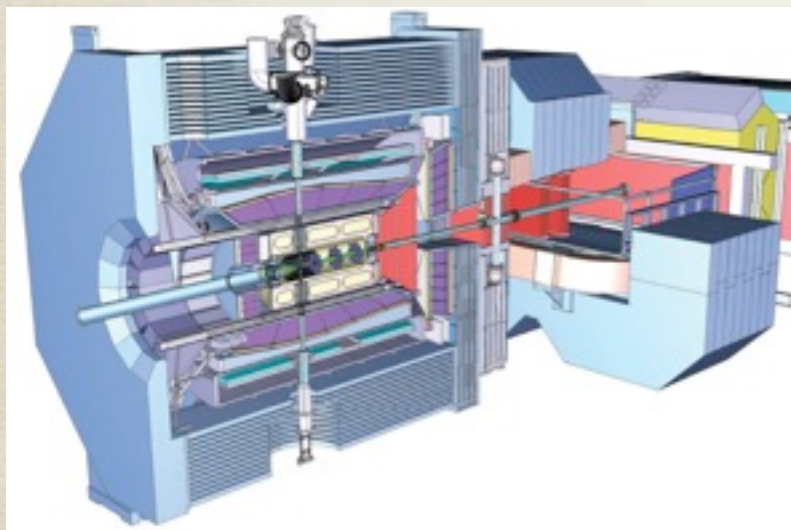
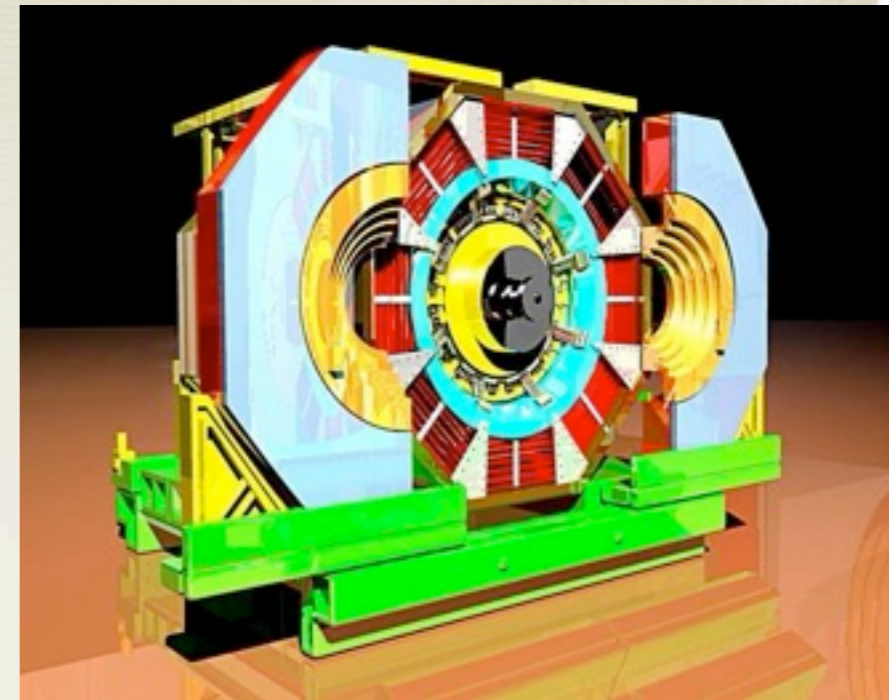
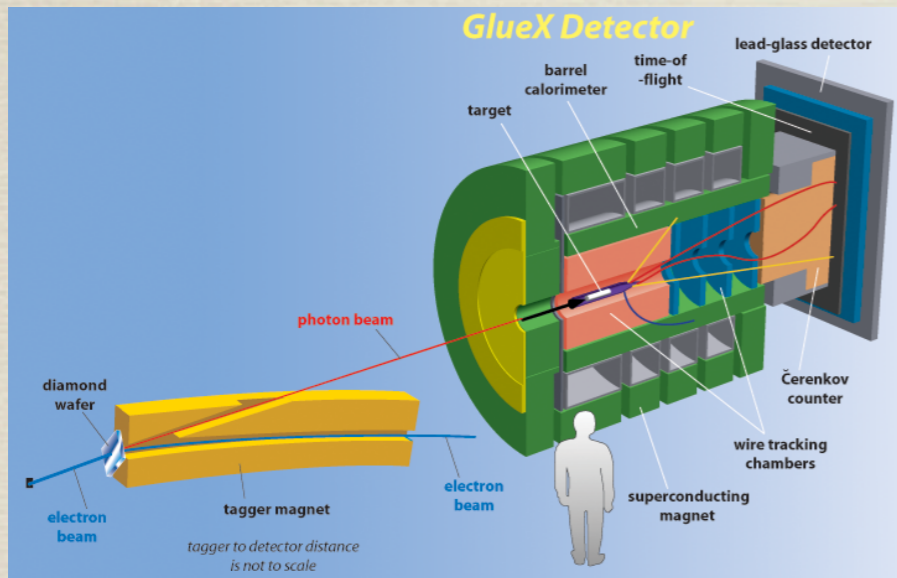


FIG. 6: Fit result (black line) of the final experimental moments (in red) in $3.2 < E < 3.4$ GeV and $0.5 < -t < 0.6$ GeV² bins.

CLAS PWA
M.Battaglieri et al. (2009)