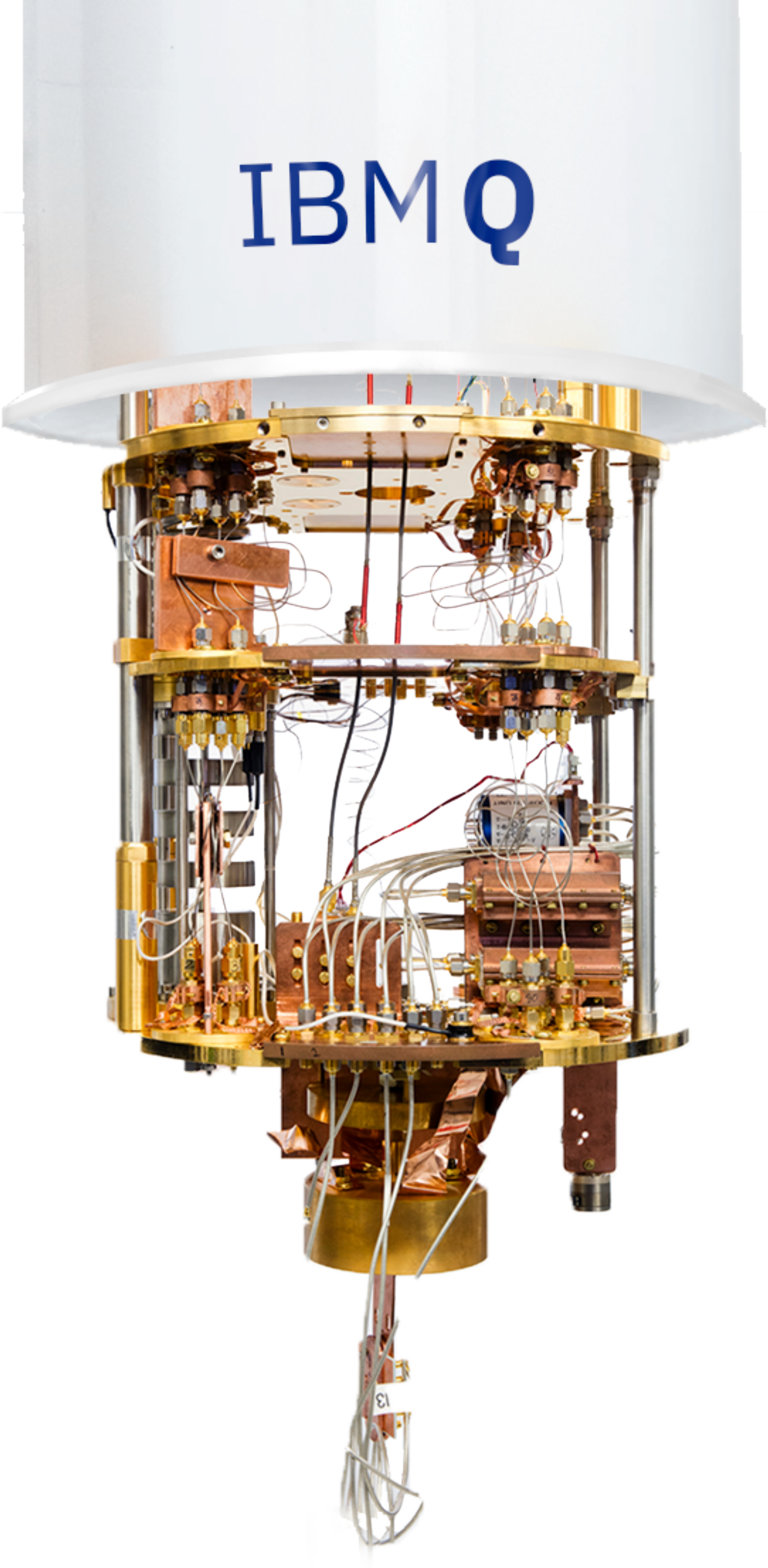
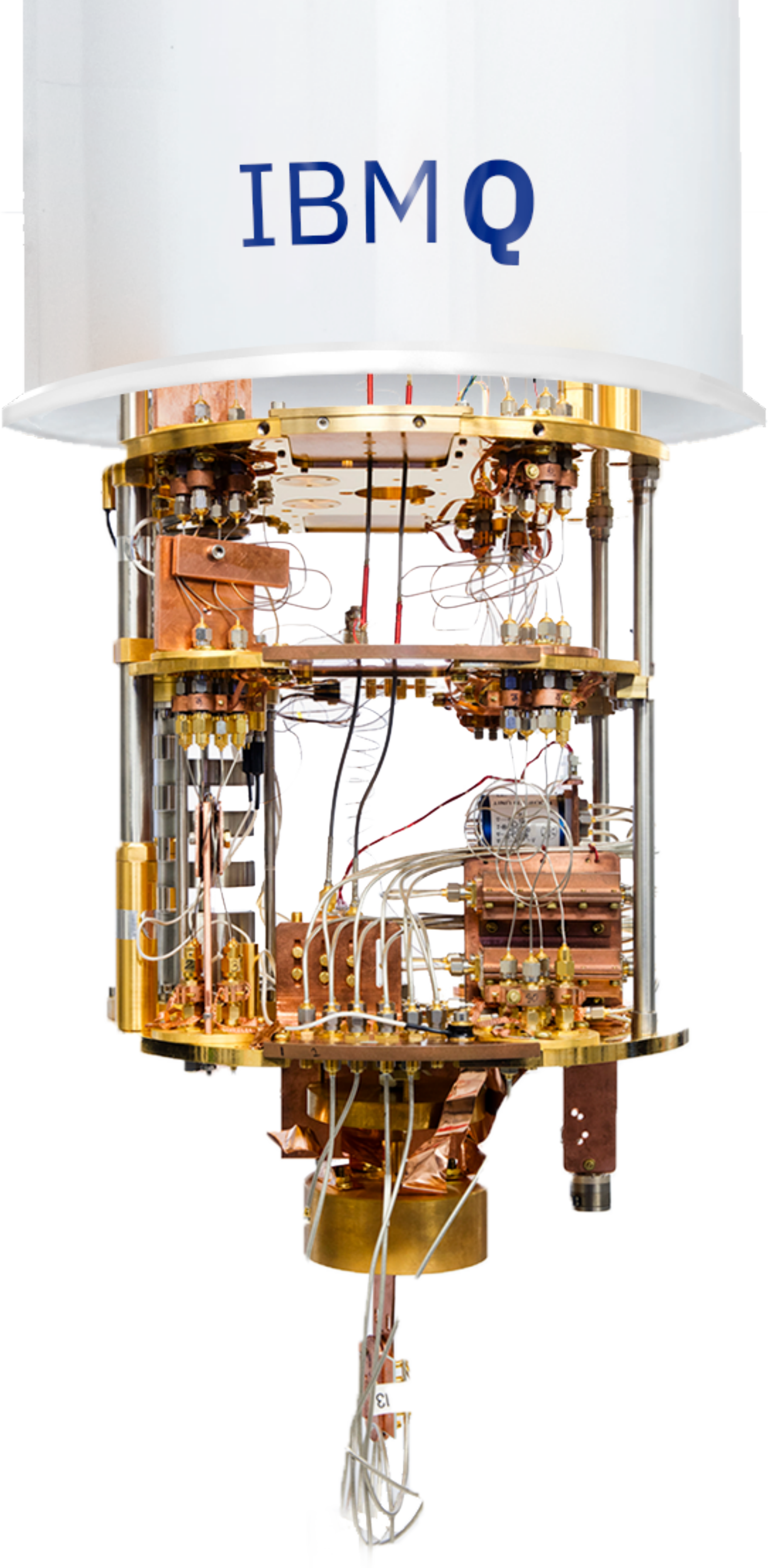


Imperial College London

Simulating high energy collision events on a Quantum Computer

Simon Williams

Milan Joint Phenomenology Seminars - 23rd January



Imperial College London

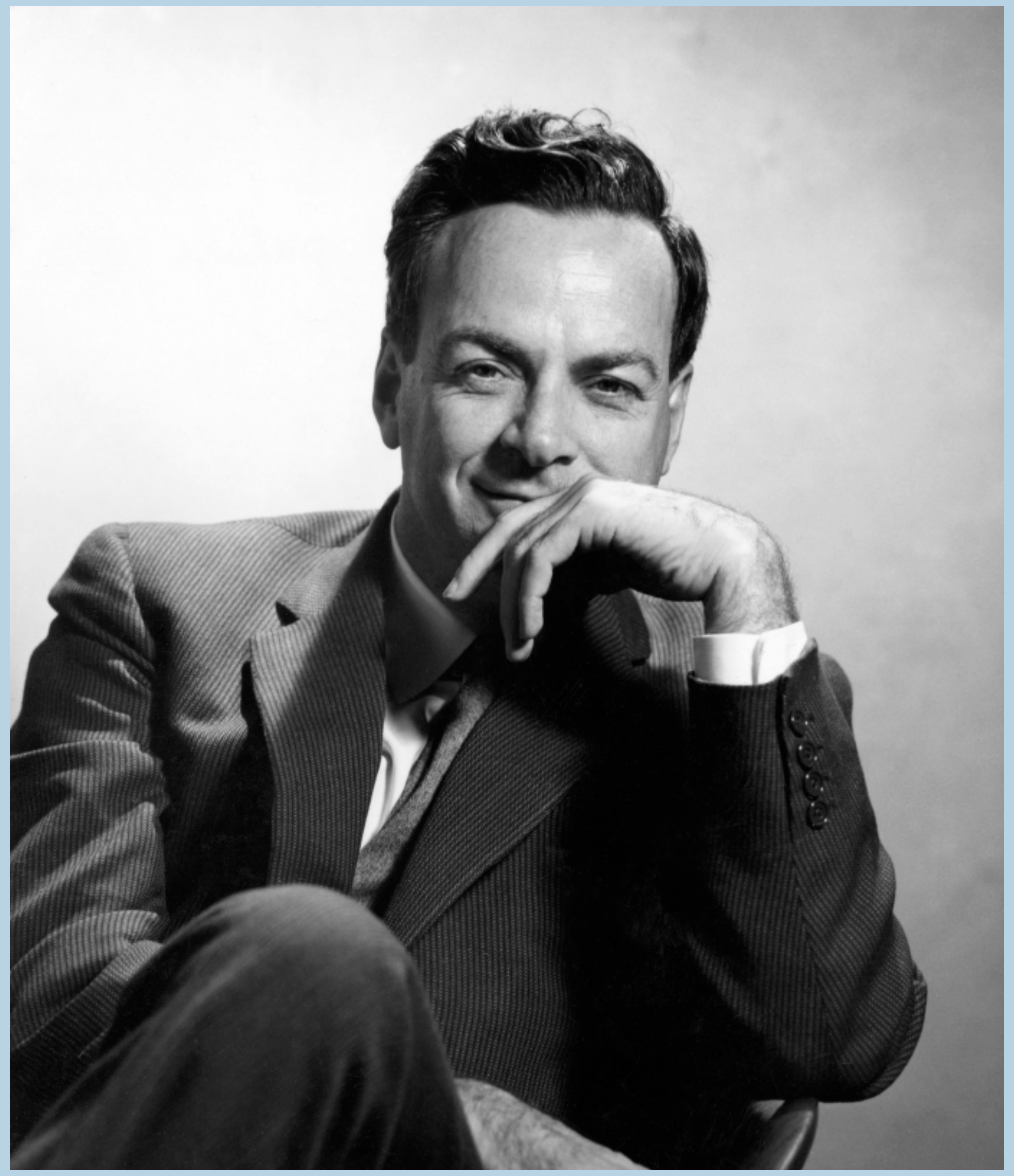
- Quantum Computing - The Power of the Qubit
- Why are we interested in High Energy Physics?
- The Parton Shower
    - Discretising QCD
- Collider Events on a Quantum Computer

G. Gustafson, S. Prestel, M. Spannowsky and S. Williams, Collider Events on a Quantum Computer, *JHEP* 11 (2022) 035, arXiv:2207.10694

LUND UNIVERSITY

IP3

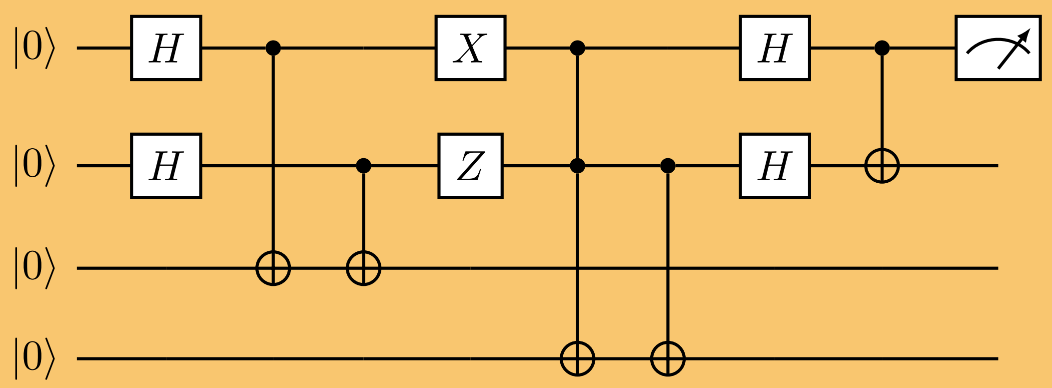
Quantum Computing - The Power of the Qubit!



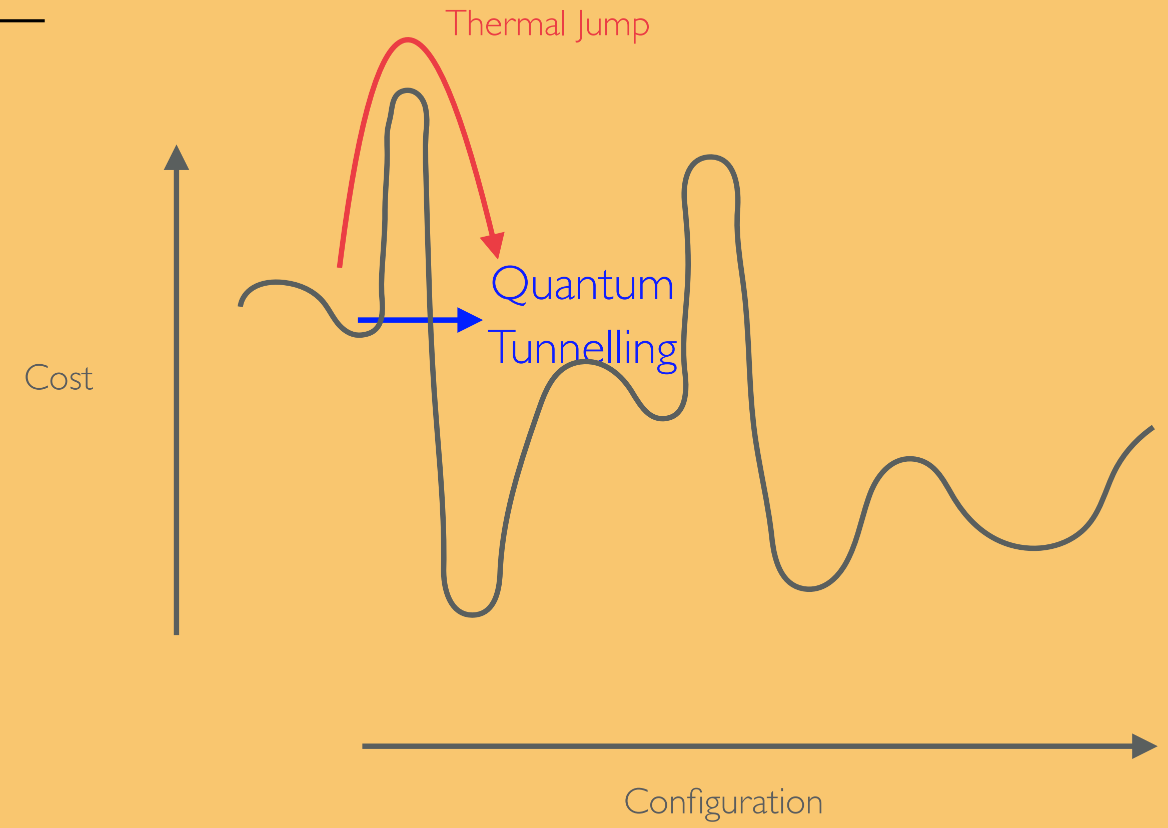
"Nature is quantum […] so if you want to simulate it, you need a quantum computer"
- Richard Feynman (1982)

Quantum Computing has had a lot of successes since - most recently with Shor and Deutsch winning the **Breakthrough Prize** and the **2022 Nobel Prize** going to Quantum Information

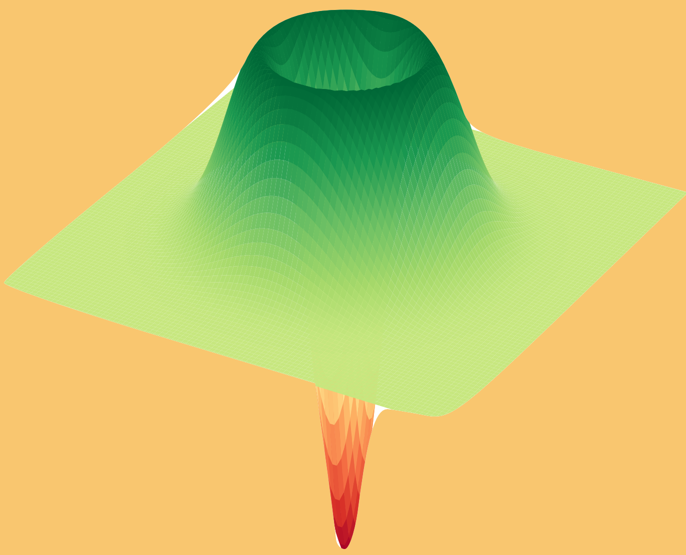
Types of Quantum Device:



Gate Quantum Computing

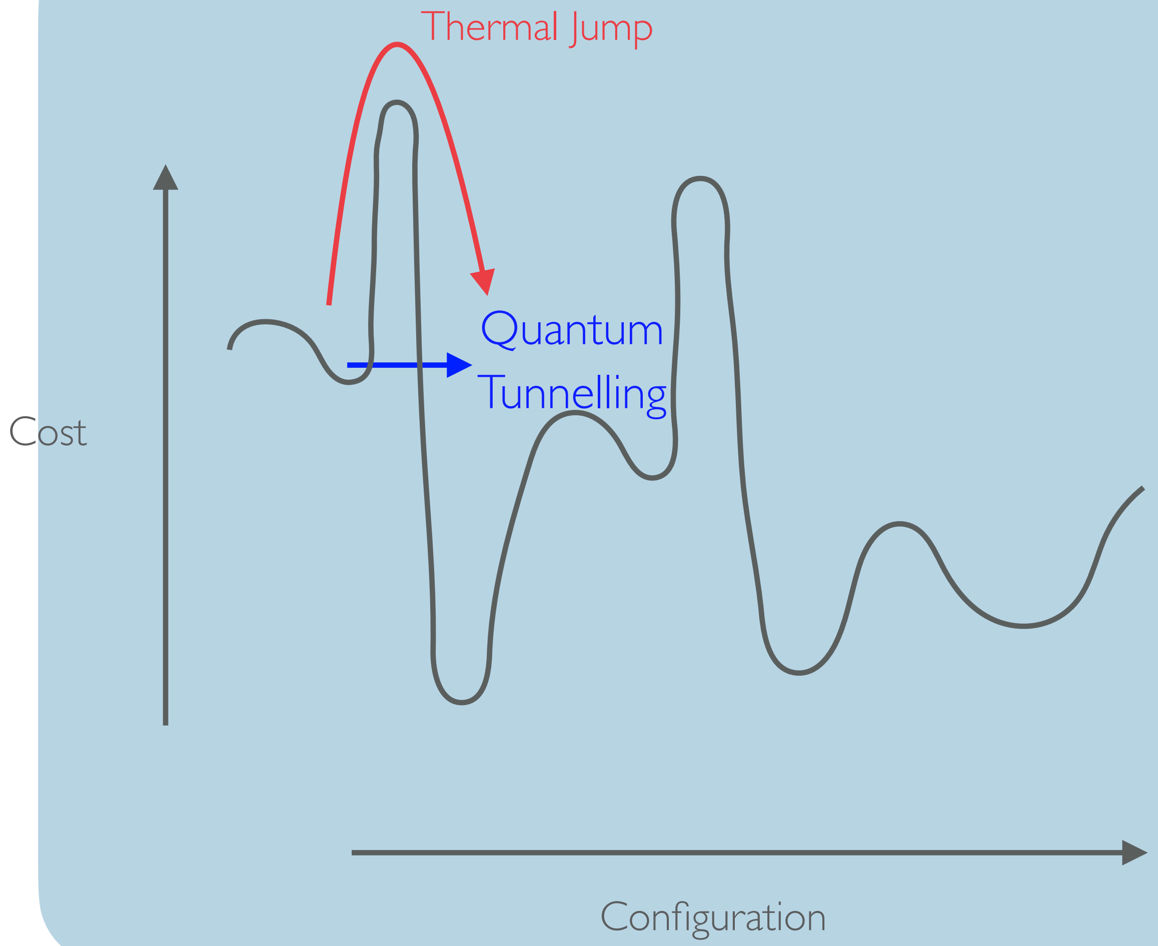


Quantum Annealing



Photonic Devices

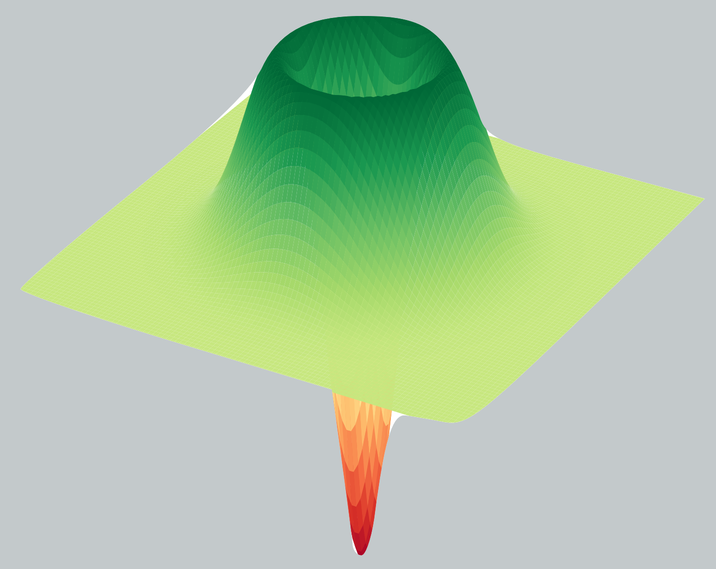
Types of Quantum Computing Devices



Quantum Annealing

$$H(\sigma) = -\sum_{i,j} J_{ij}\sigma_i\sigma_j - \mu \sum_j h_j\sigma_j$$

Photonic Quantum Devices



Type of gate quantum computing, manipulating photon states

Advantages:

- Well suited to optimisation problems

Disadvantages:

- Uncontrollable, noisy devices
- Not universal devices

Advantages:

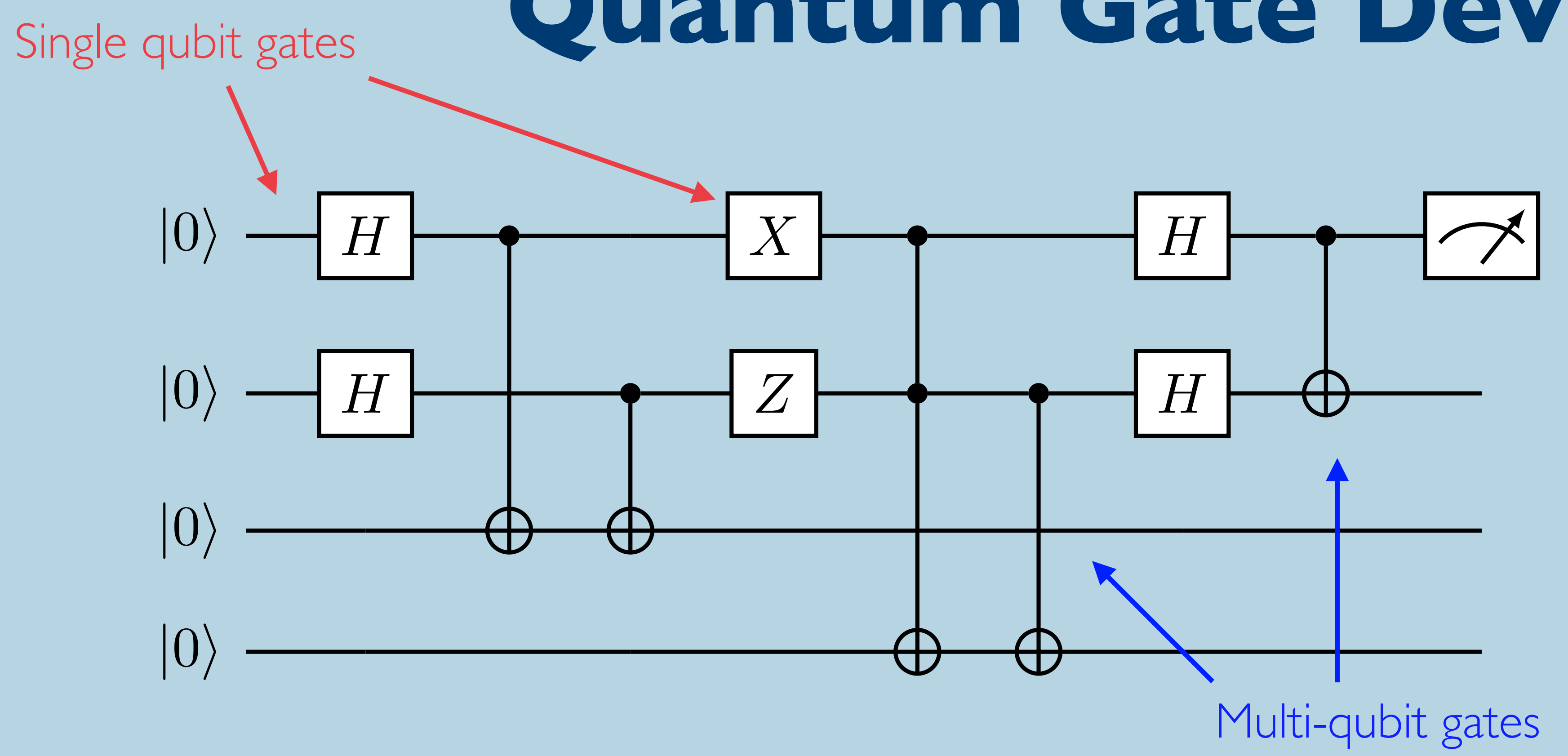
- Continuous variable devices
- Only weak interactions with environment

Disadvantages:

- All states must be Gaussian

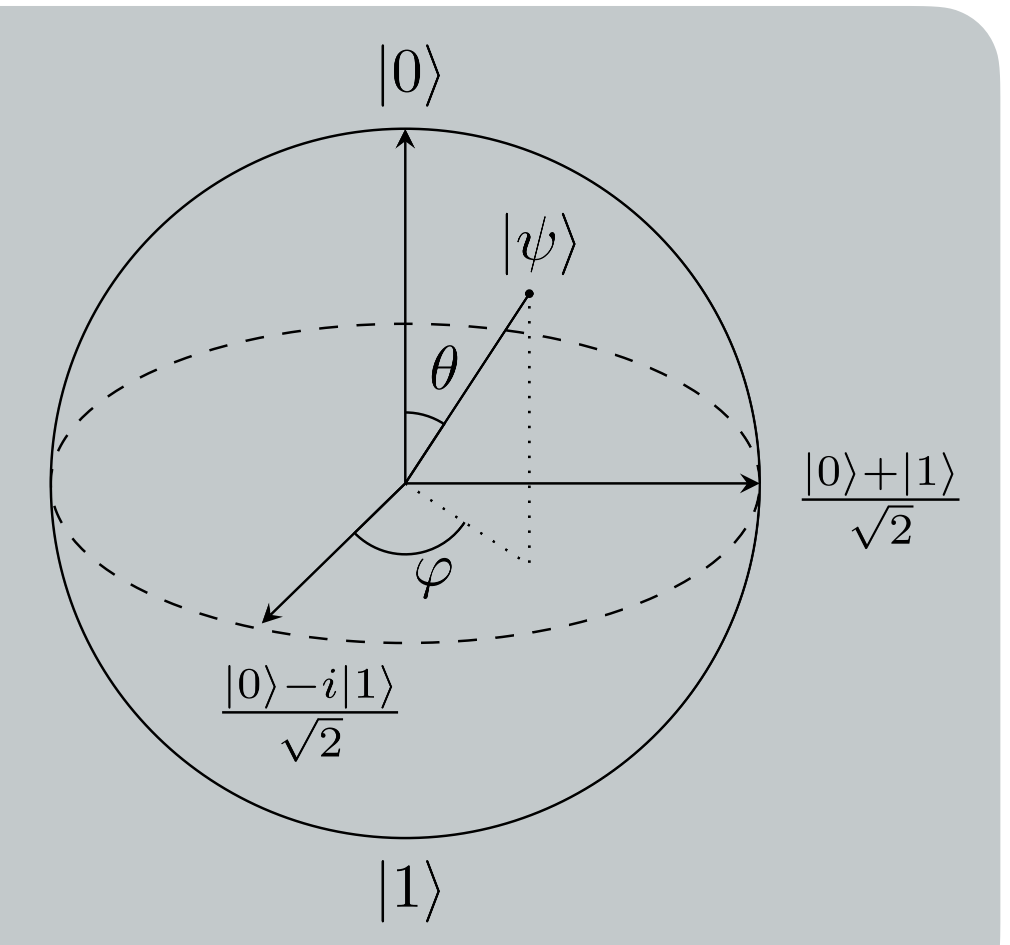
Types of Quantum Computing Devices

Quantum Gate Devices



Qubit model:

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$



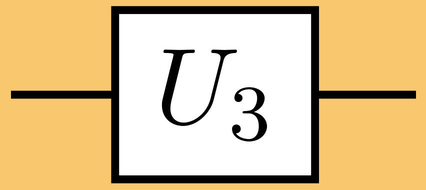
Advantages:

- Highly controllable qubits
- Universal computation

Disadvantages:

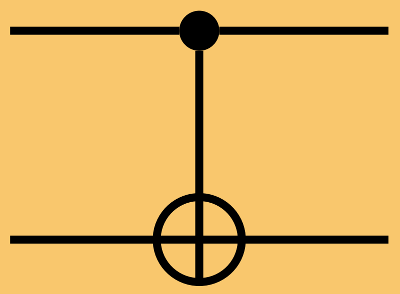
- Small number of qubits, not very fault tolerant

Single qubit gates:



$$U_3|0\rangle \rightarrow \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

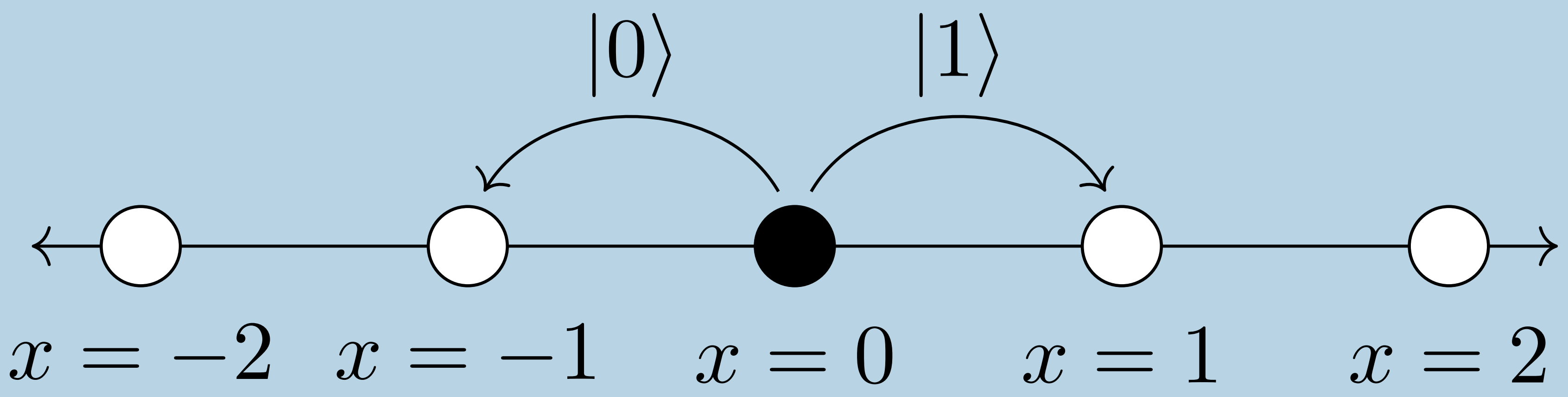
Multi-qubit gates:



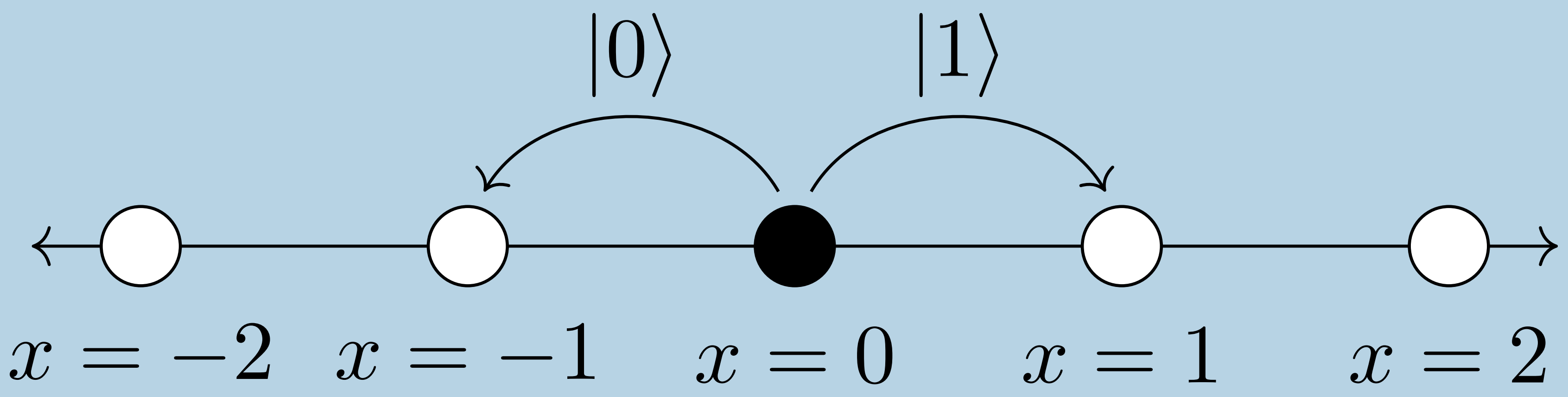
$$\text{CNOT}|00\rangle \rightarrow |00\rangle, \text{CNOT}|10\rangle \rightarrow |11\rangle,$$
$$\text{CNOT}|01\rangle \rightarrow |01\rangle, \text{CNOT}|11\rangle \rightarrow |10\rangle$$

Classical Random Walk

Classical Random Walk

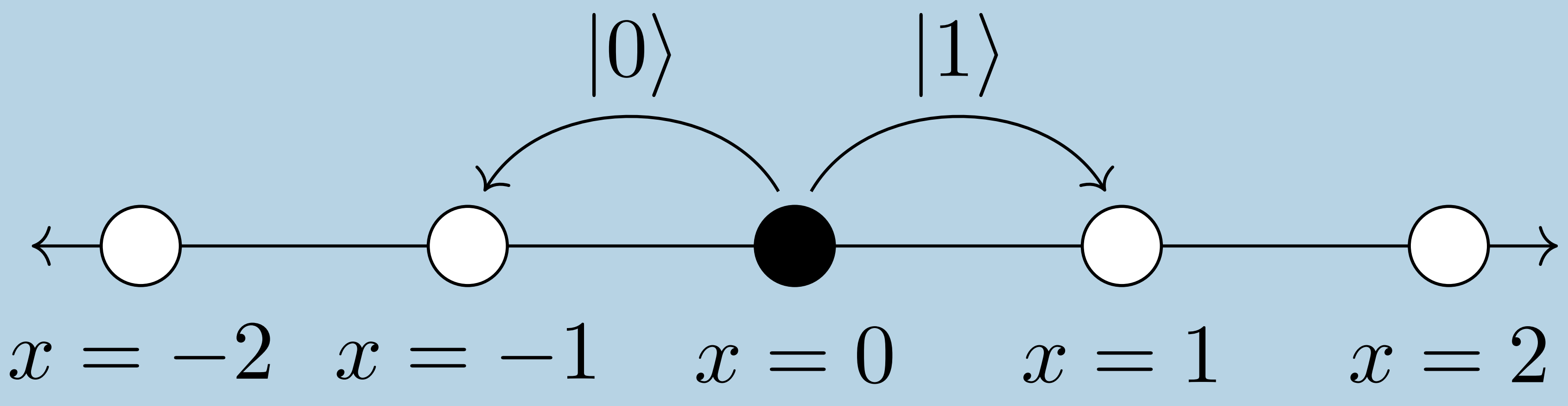


Classical Random Walk



$$\left.\begin{aligned}\mathscr{H}_P &= \{\,|i\rangle : i \in \mathbb{Z}\}\\ \mathscr{H}_C &= \{\,|0\rangle, |1\rangle\}\end{aligned}\right\}\mathscr{H} = \mathscr{H}_C \otimes \mathscr{H}_P$$

Classical Random Walk



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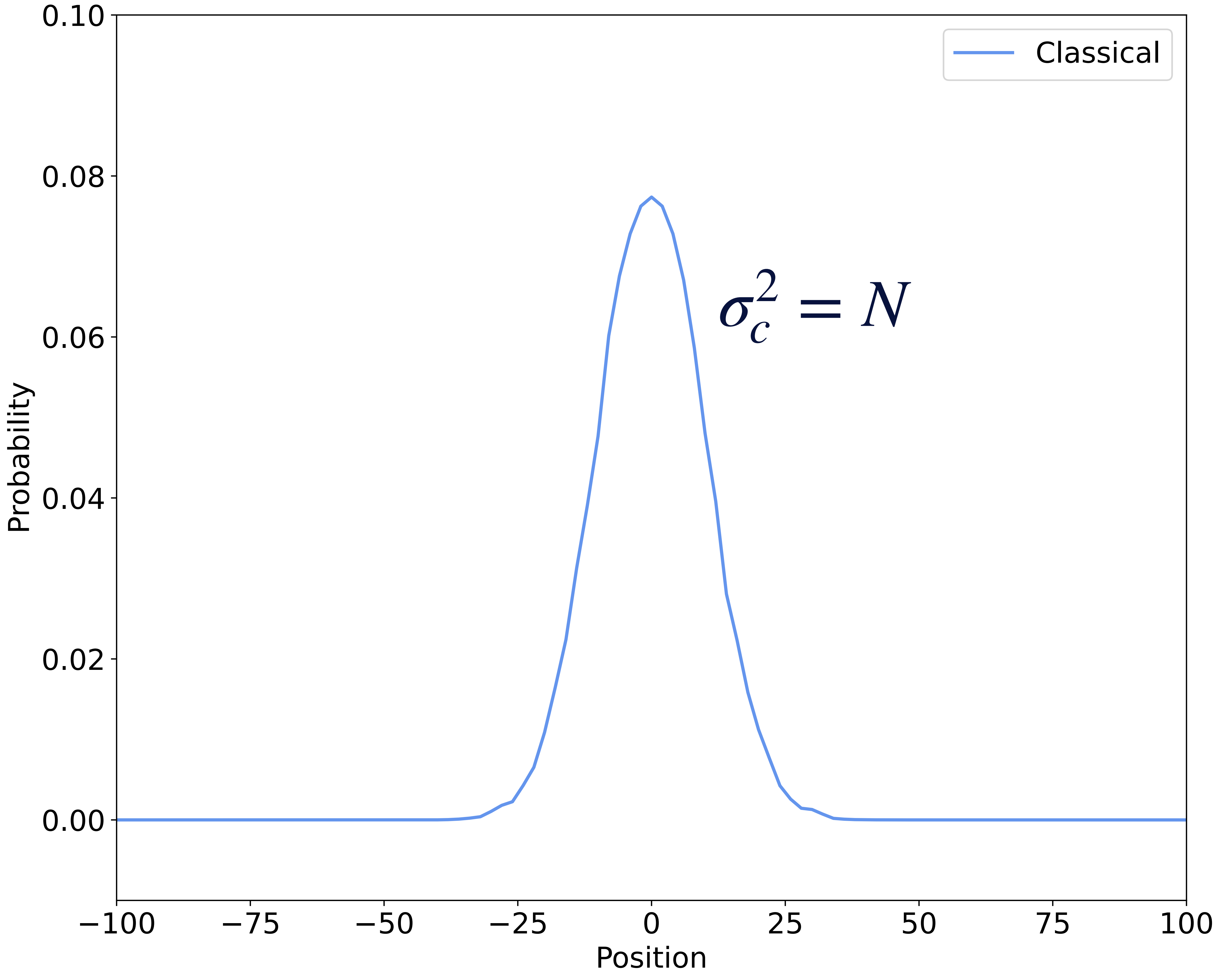
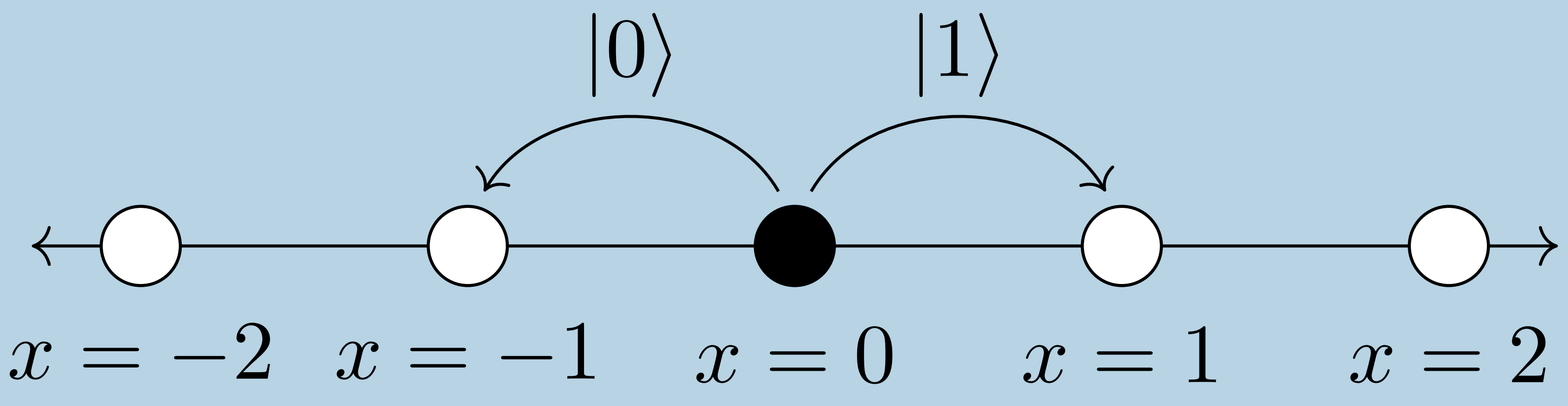


The Quantum Walk



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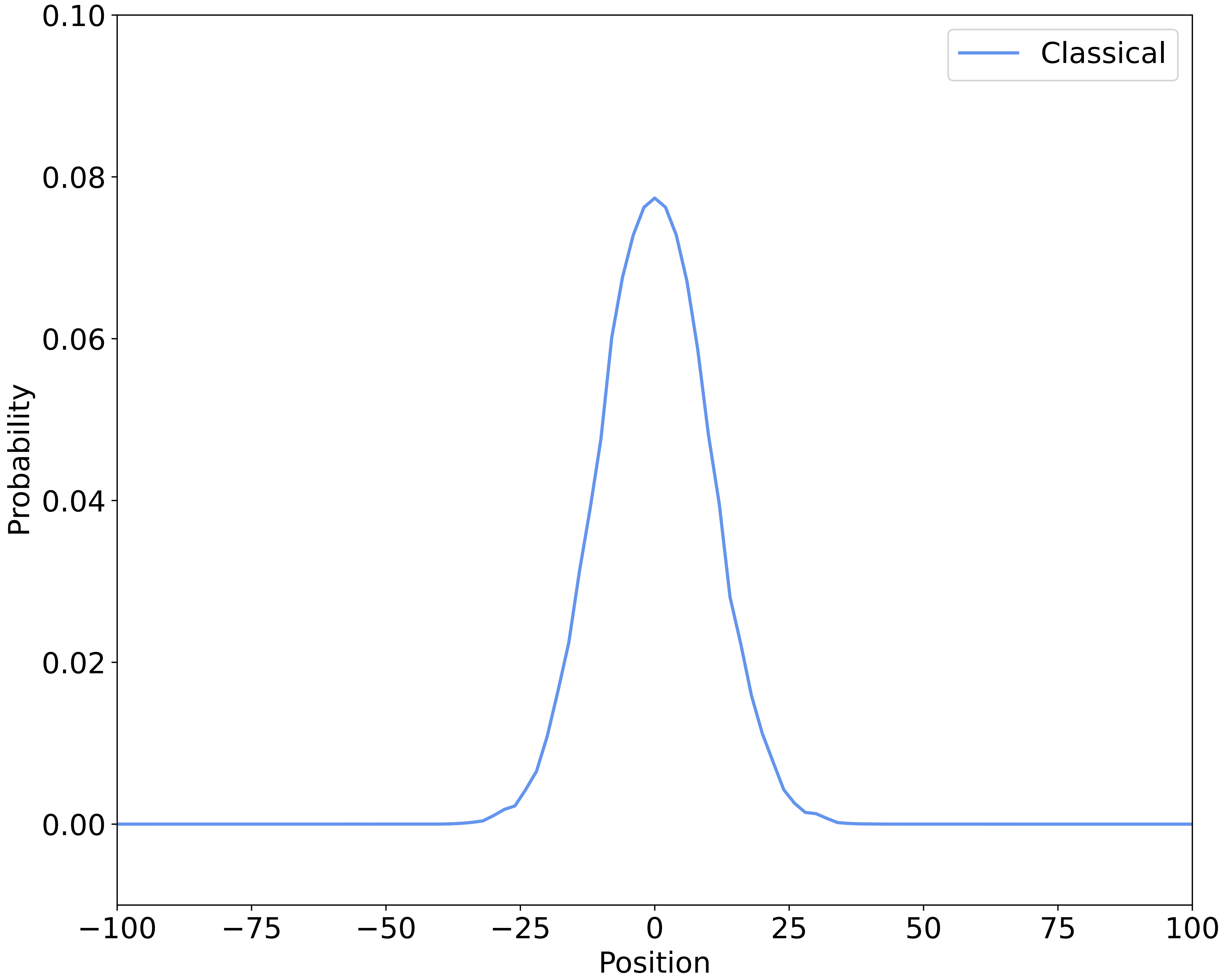

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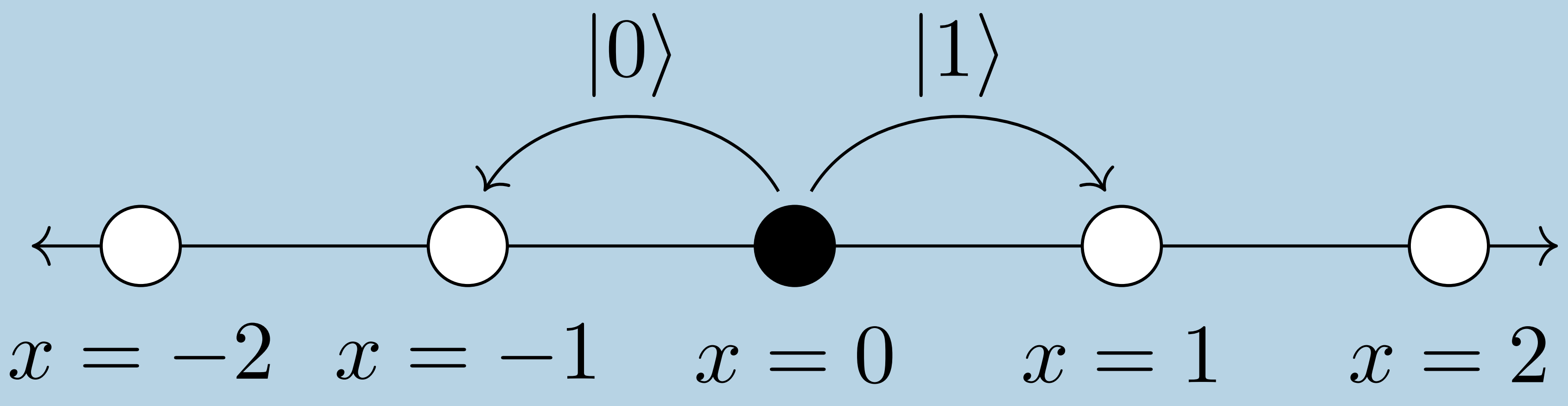

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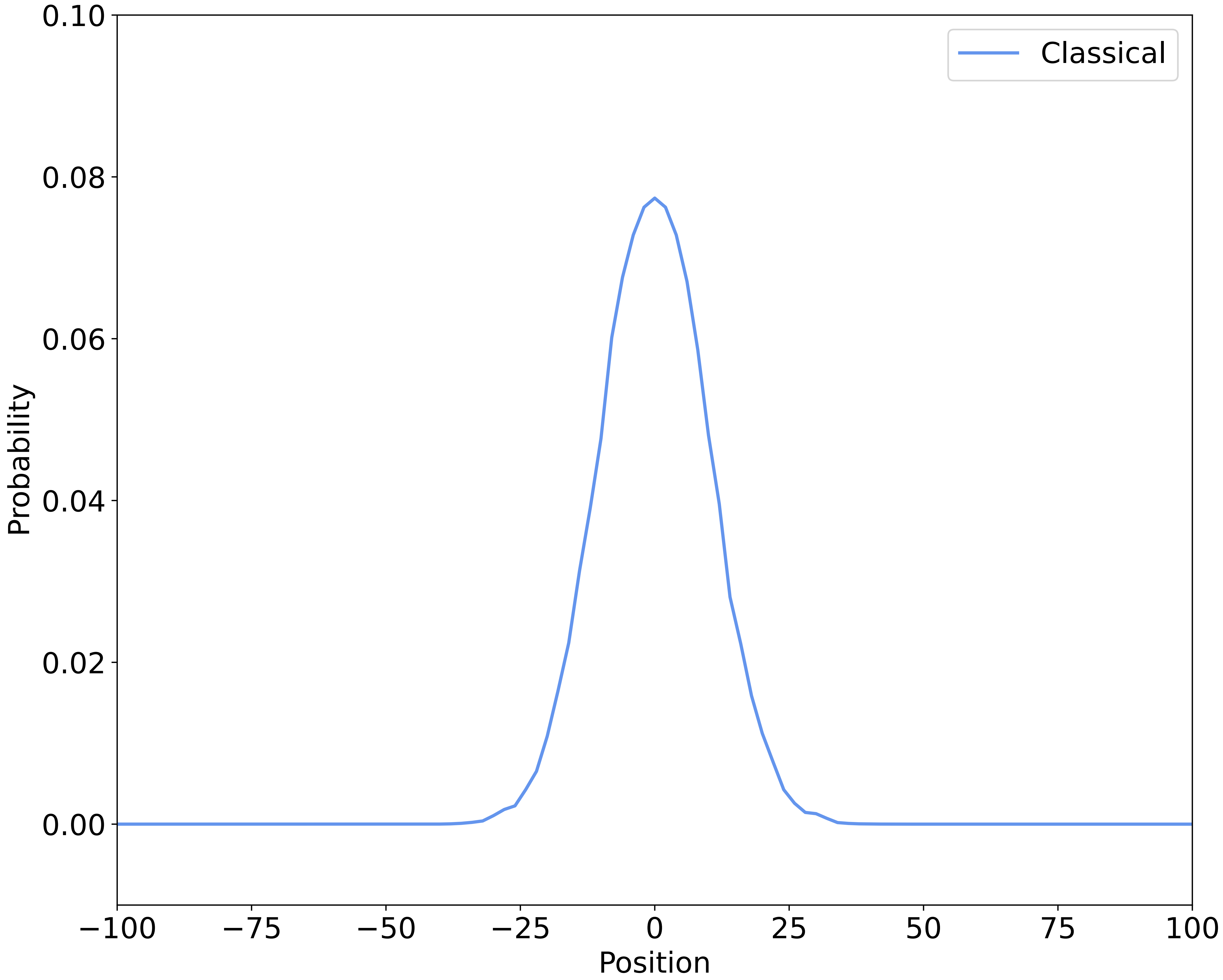
Unitary Transformation:

$$U = S\cdot(C \otimes I)$$



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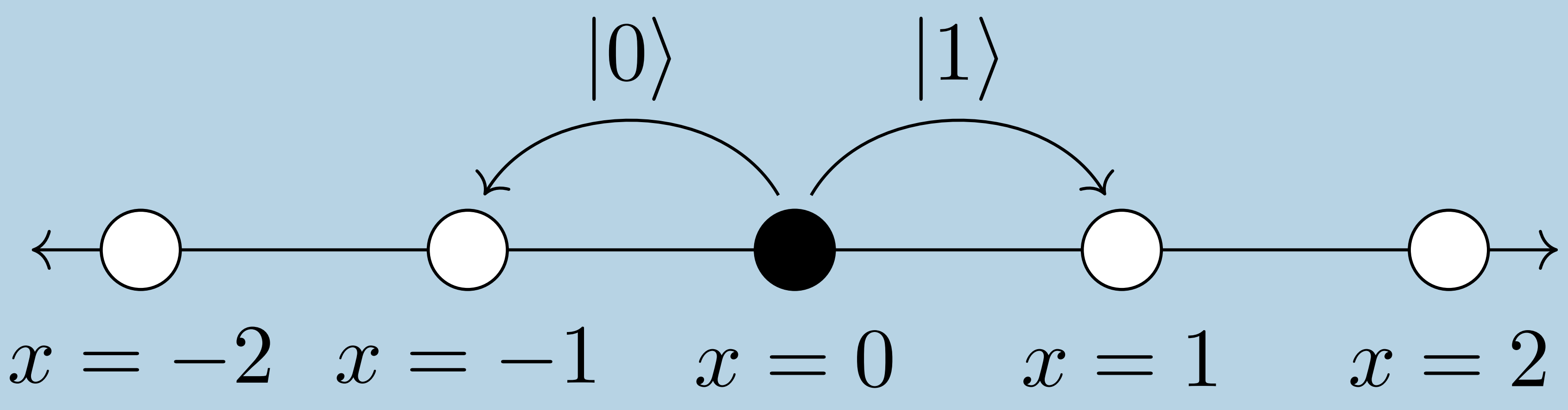

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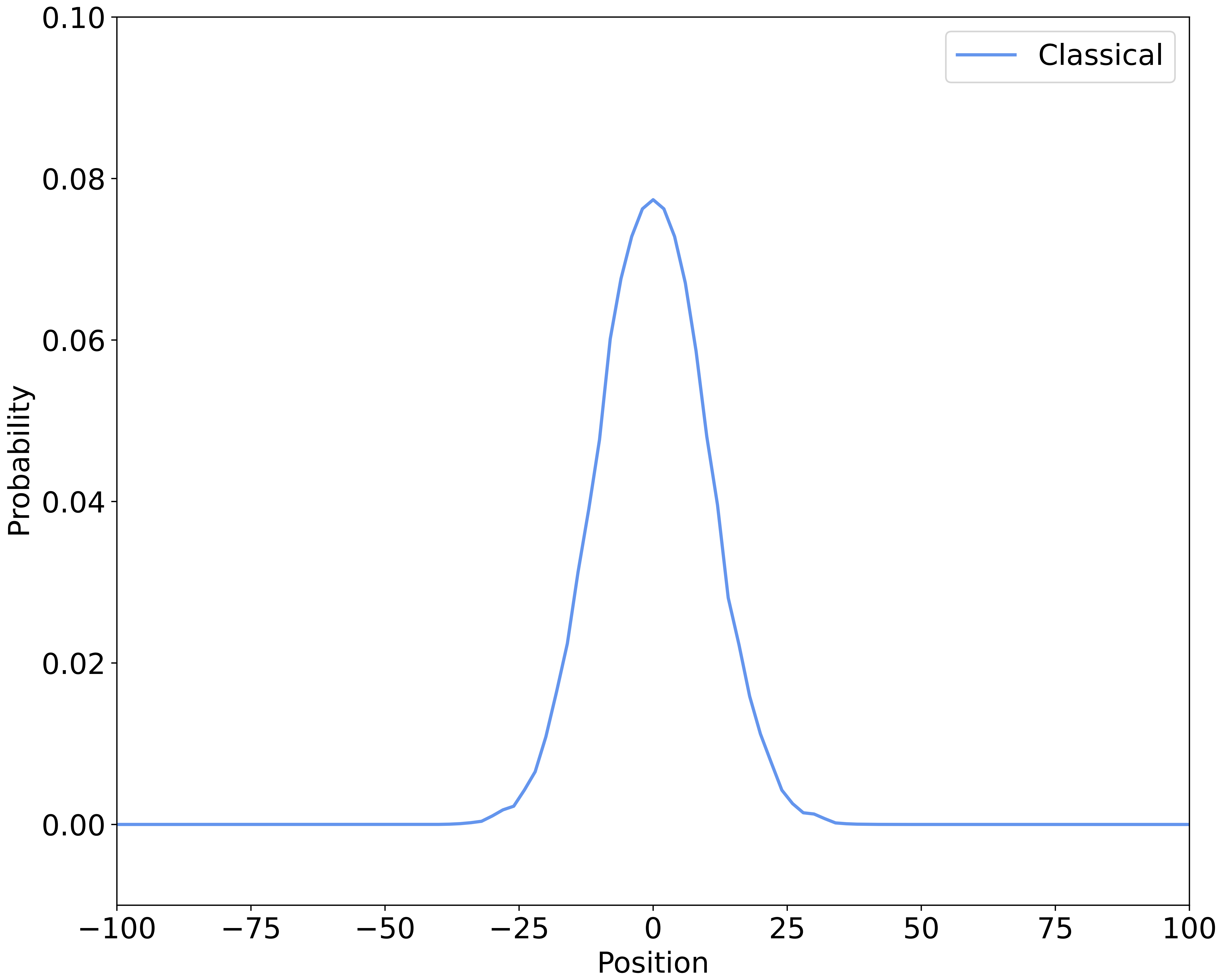
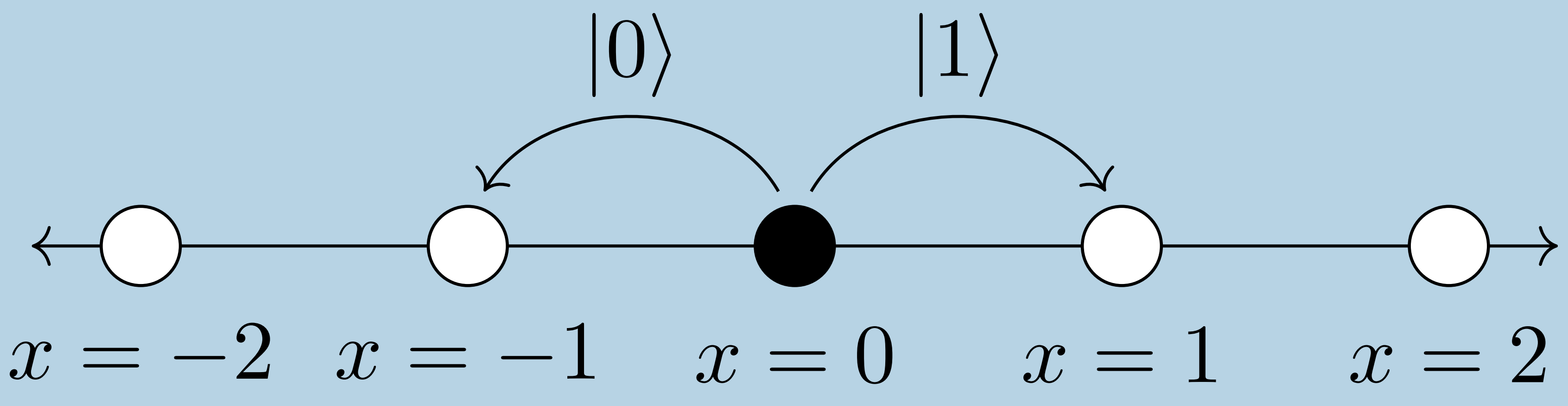
Coin Operation:

$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$



The Quantum Walk



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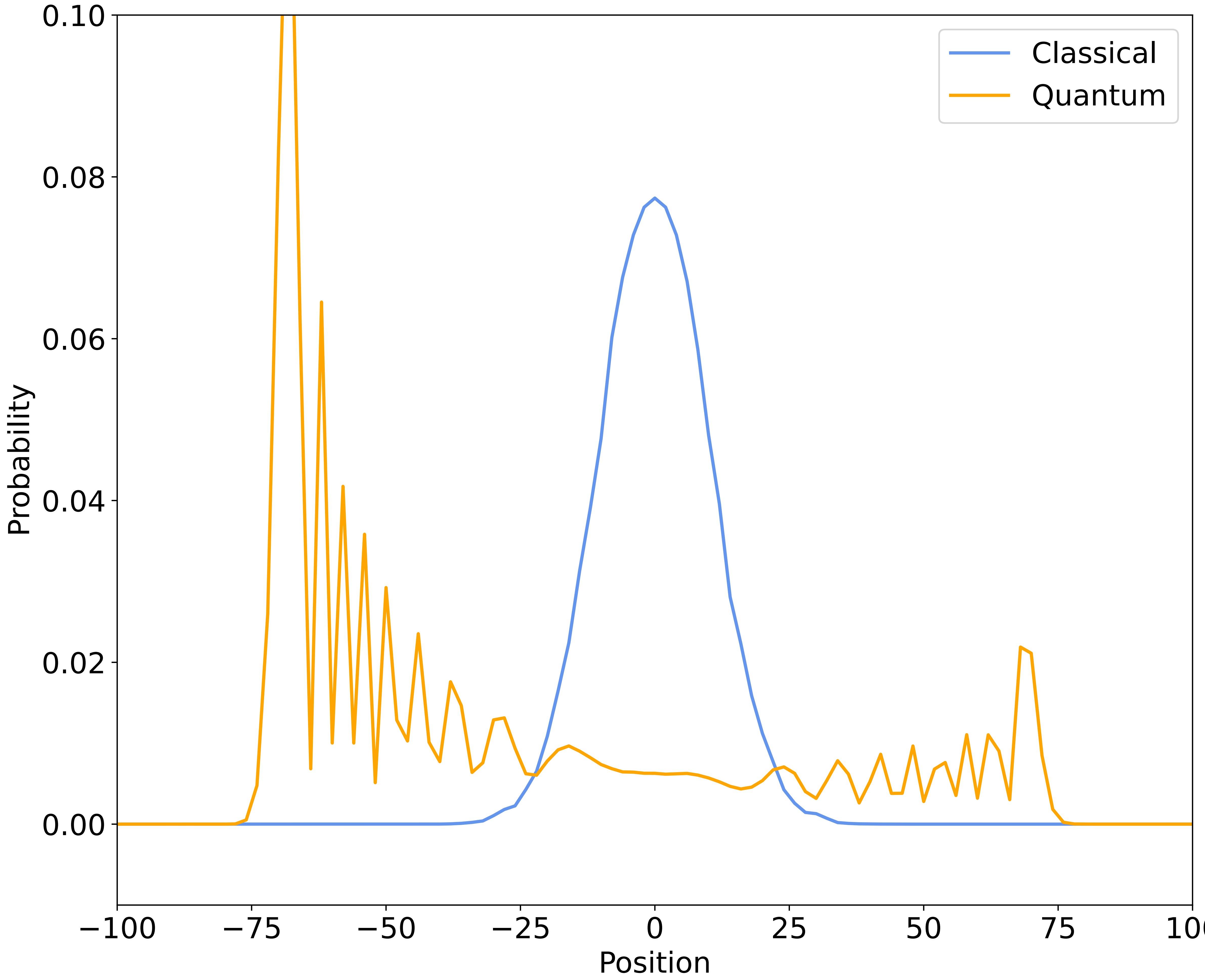
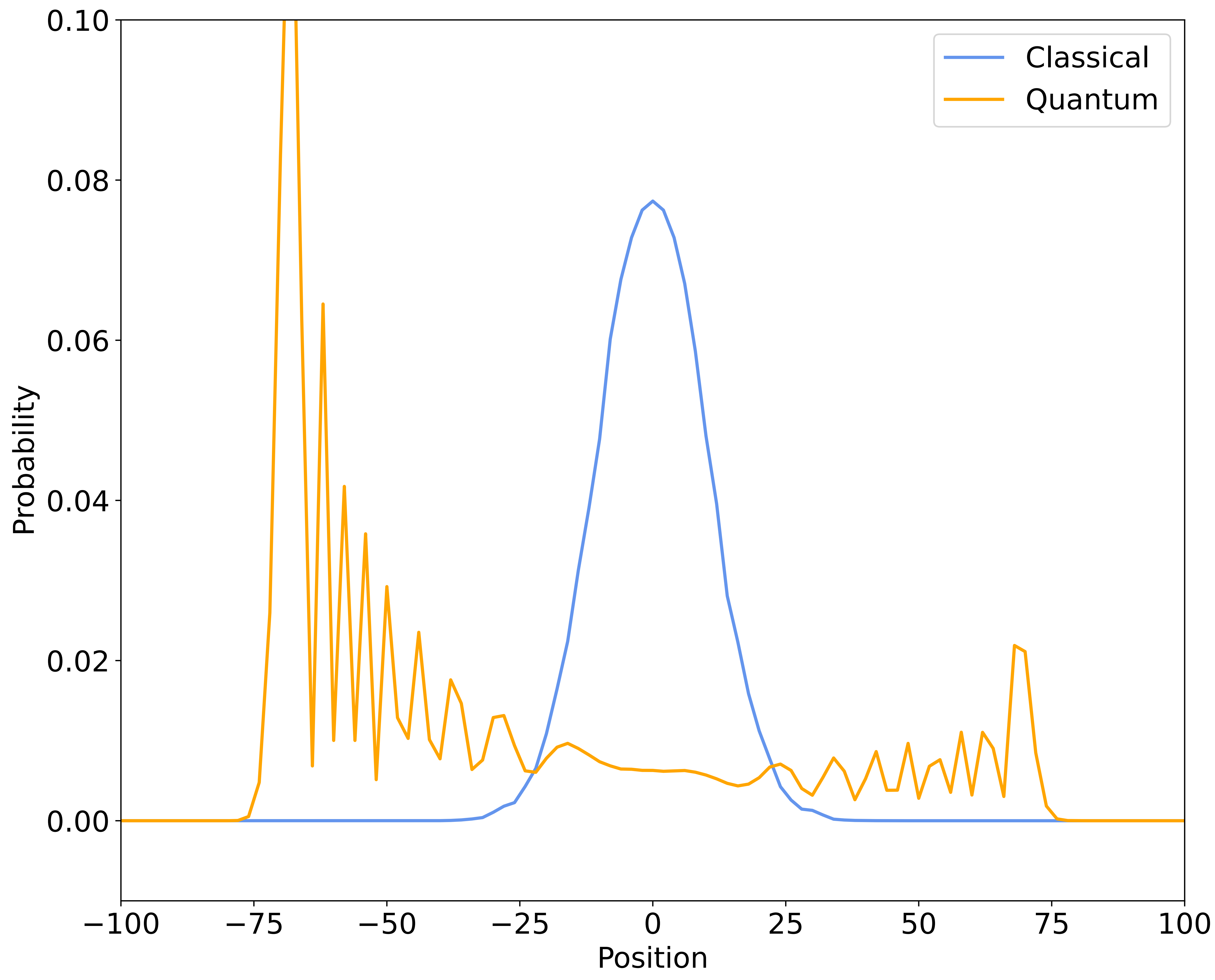


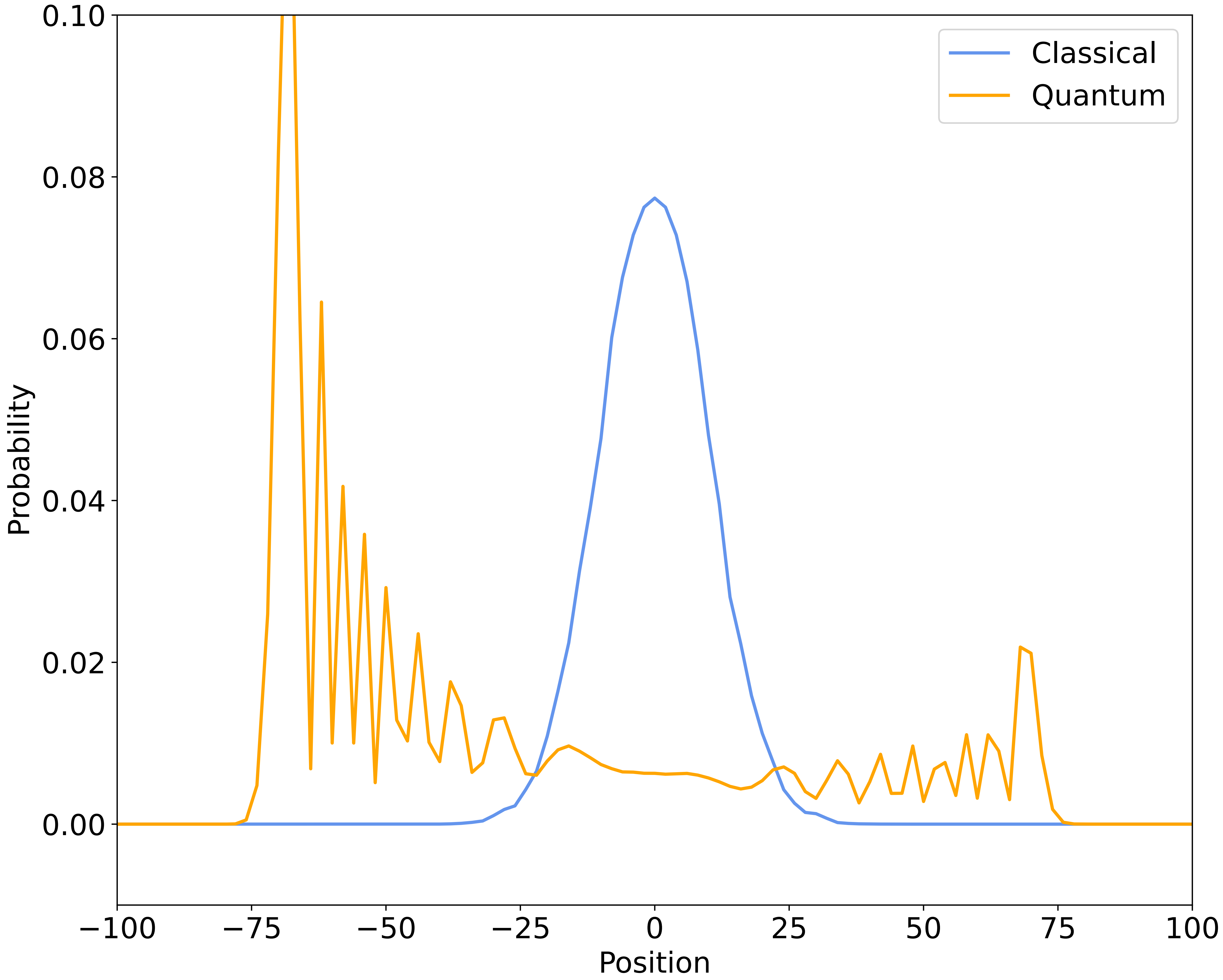
The Quantum Walk - Coin initialisation



The Quantum Walk - Coin initialisation

Initialising the coin in the $-|1\rangle$ state

$$H(-|1\rangle) = \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big)$$



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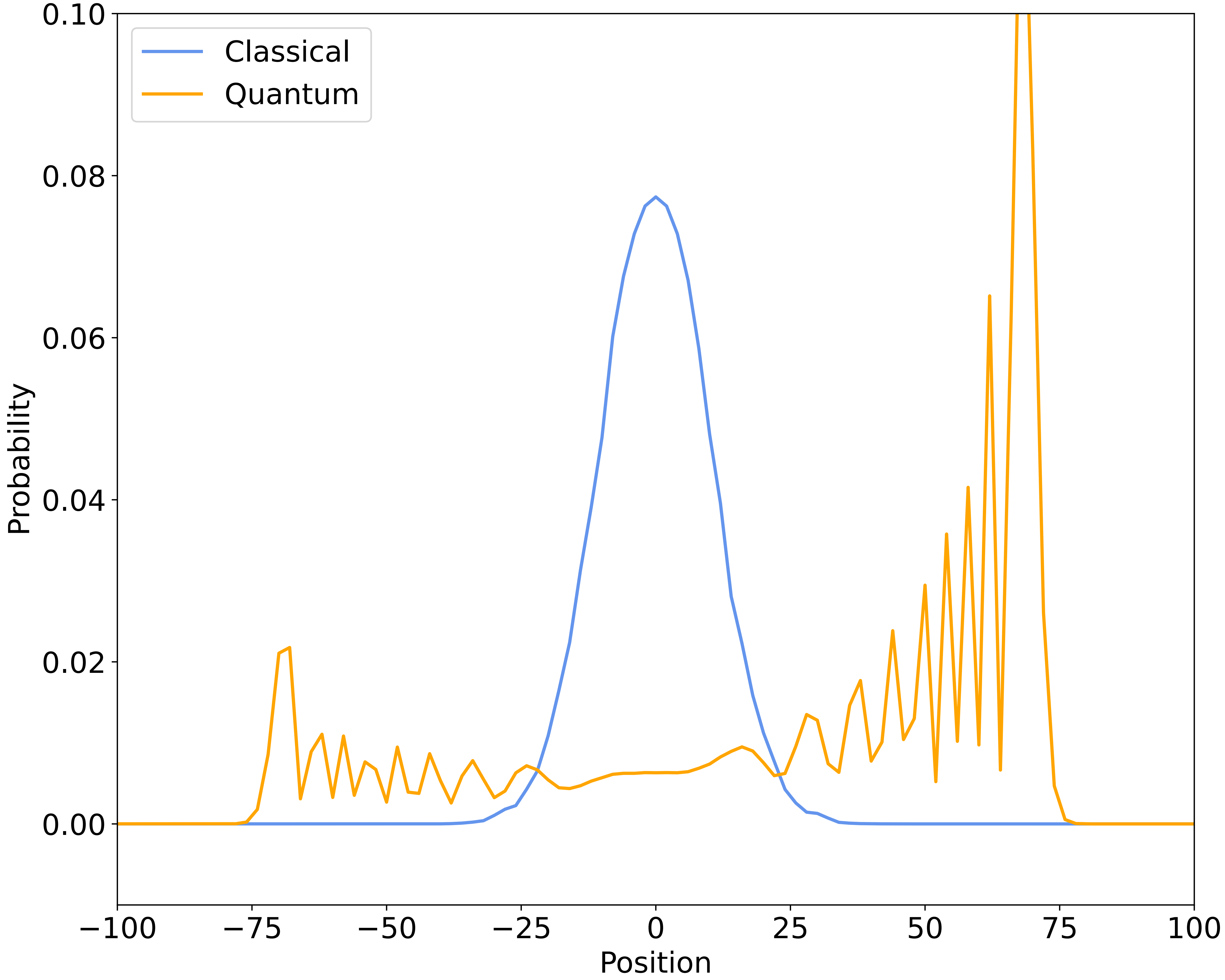

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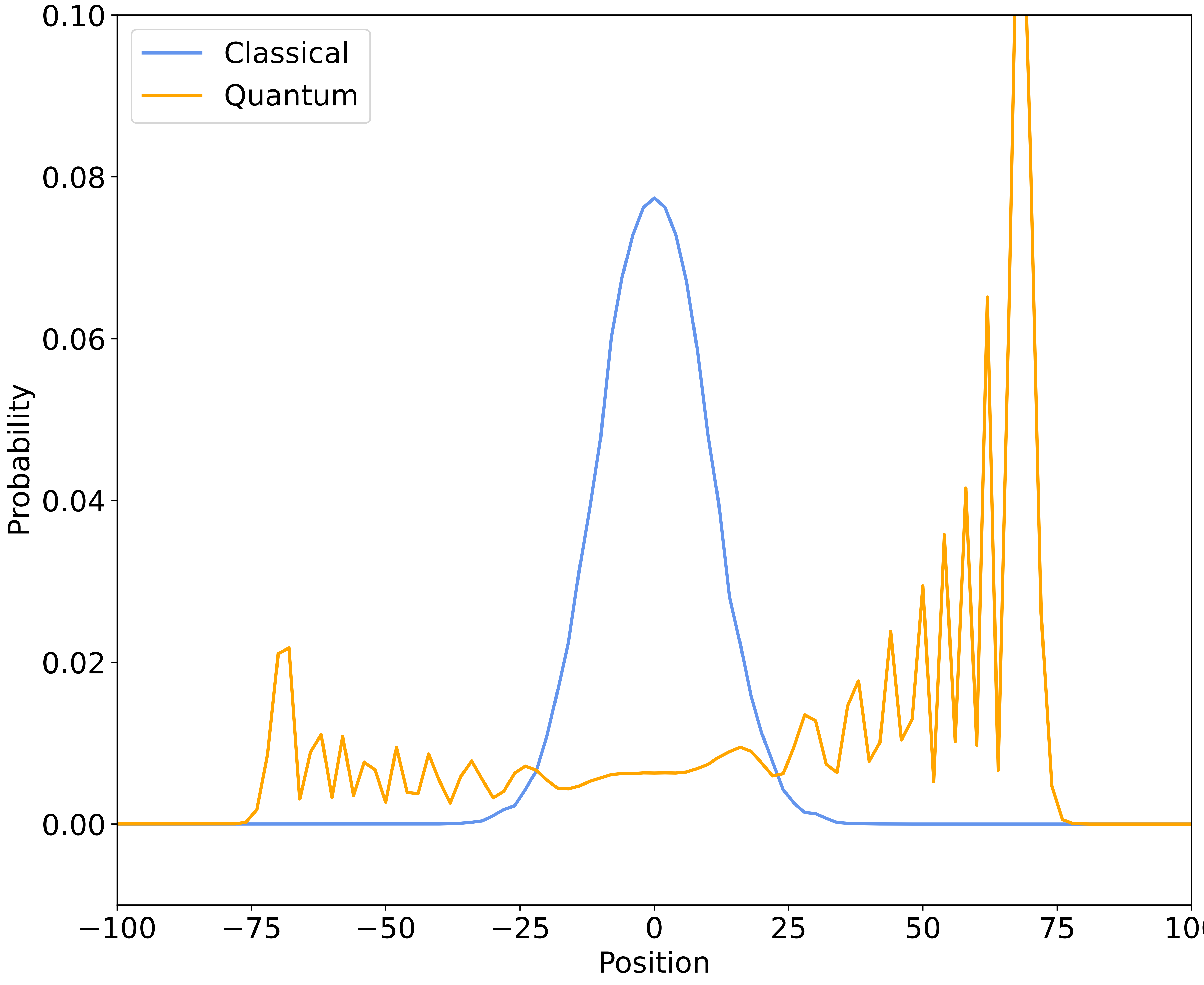
$$H(-|1\rangle) = \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big)$$

Removing the asymmetry:

$$|c\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle - i|1\rangle\big)$$



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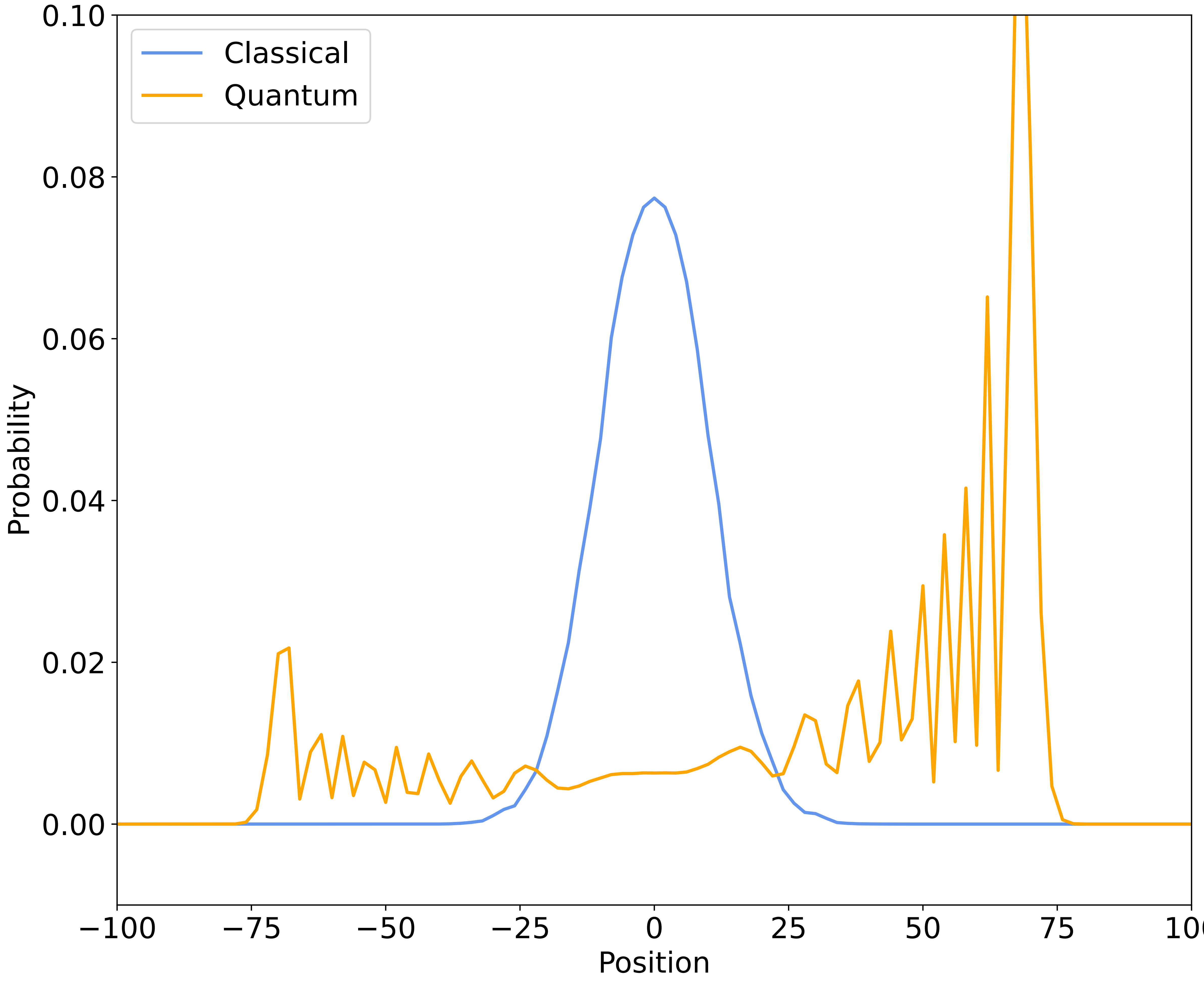
Left moving part ($|c\rangle = |0\rangle$) propagates in **real amplitudes**. **Right moving part** ($|c\rangle = |1\rangle$) propagates in **imaginary amplitudes**.



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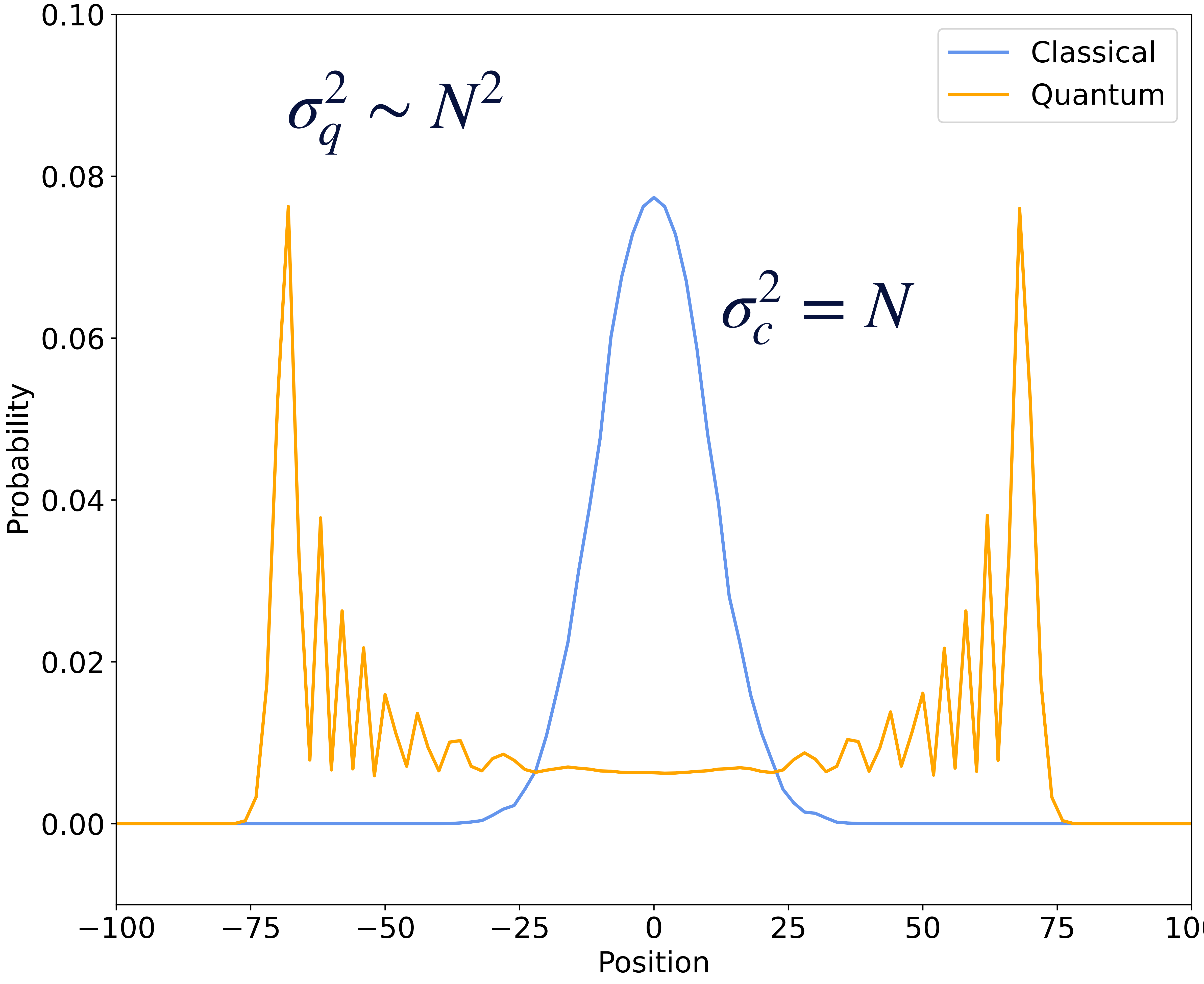
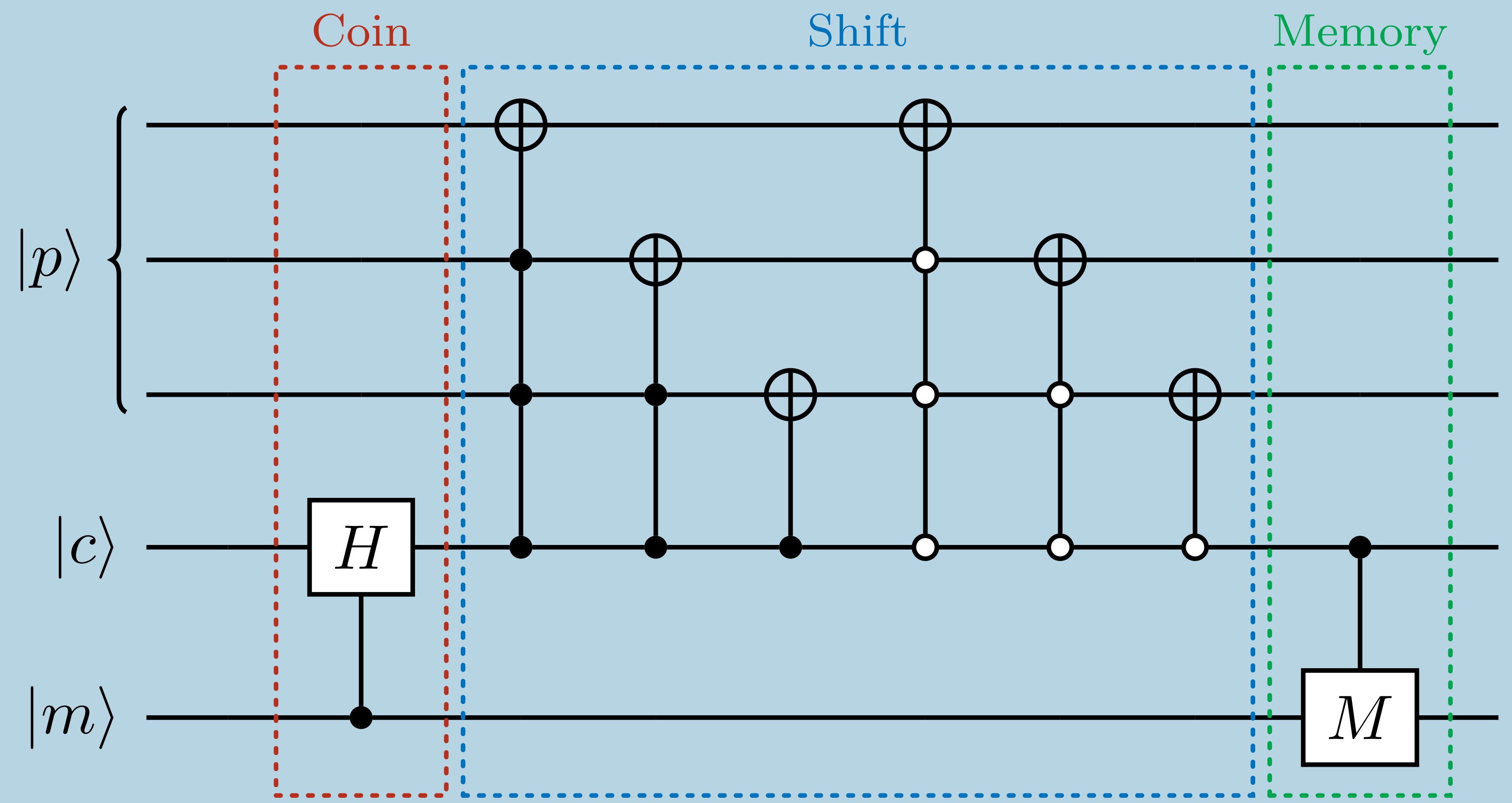


Quantum Walks with Memory



Qubit model:

Augment system further by adding an additional memory space

$$\mathscr{H} = \mathscr{H}_P \otimes \mathscr{H}_C \otimes \mathscr{H}_M$$

Advantages:

- Arbitrary dynamics
- Classical dynamics in unitary evolution

Disadvantages:

- Tight conditions on quantum advantage

Quantum Parton Showers:

Quantum Walks with memory have proven to be very useful for quantum parton showers.

Phys.Rev.D 106 (2022) 5, 056002

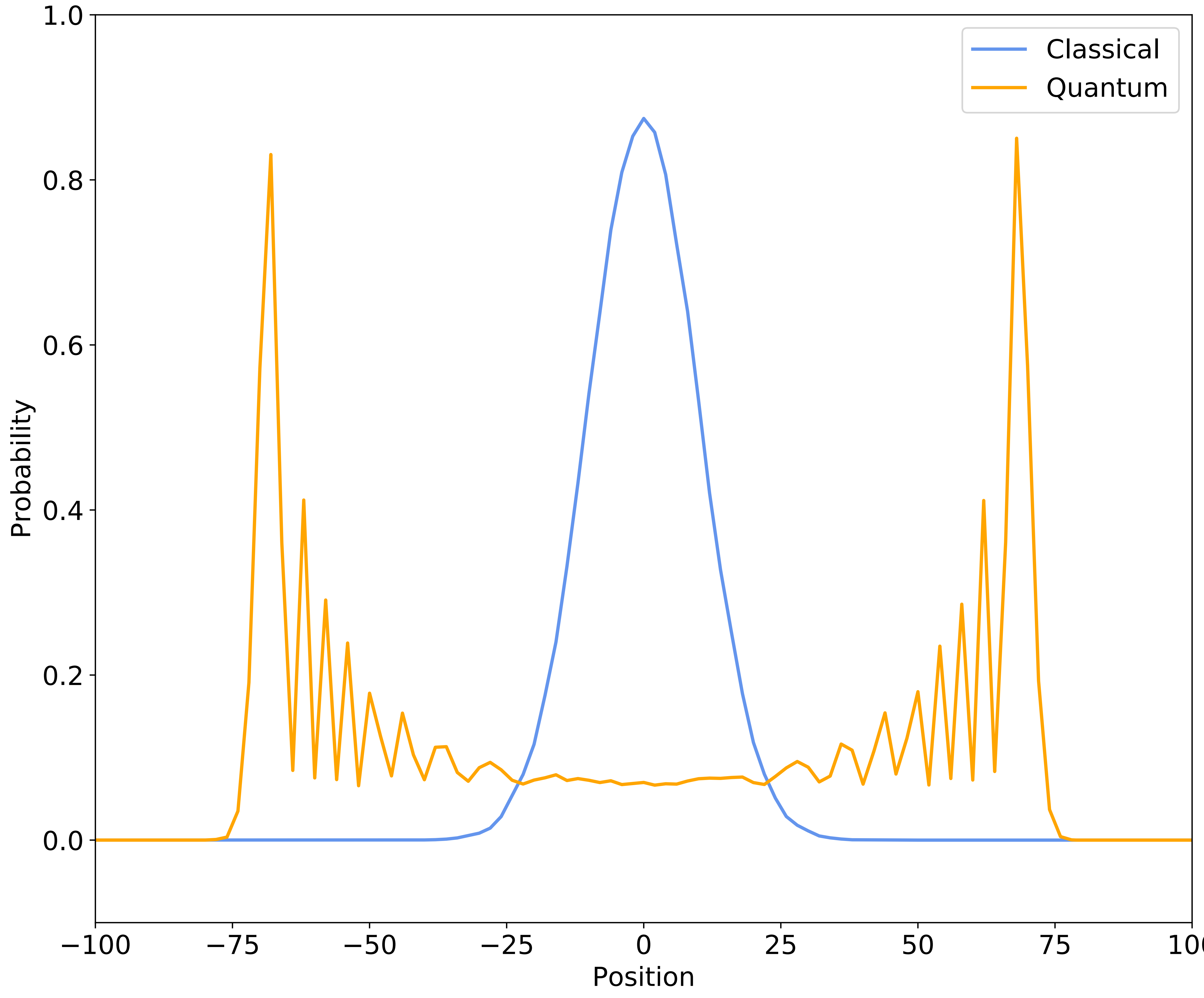
Speed up via Quantum Walks

Quantum Walks have long be conjectured to achieved at least **quadratic speed up**

Szegedy Quantum Walks have been proven to achieve quadratic speed up for **Markov Chain Monte Carlo**

This has been proven under the condition that the MCMC algorithm is **reversible and ergodic**

Work is ongoing to prove this is true for all QWs, but latest upper limits are on par with classical RW



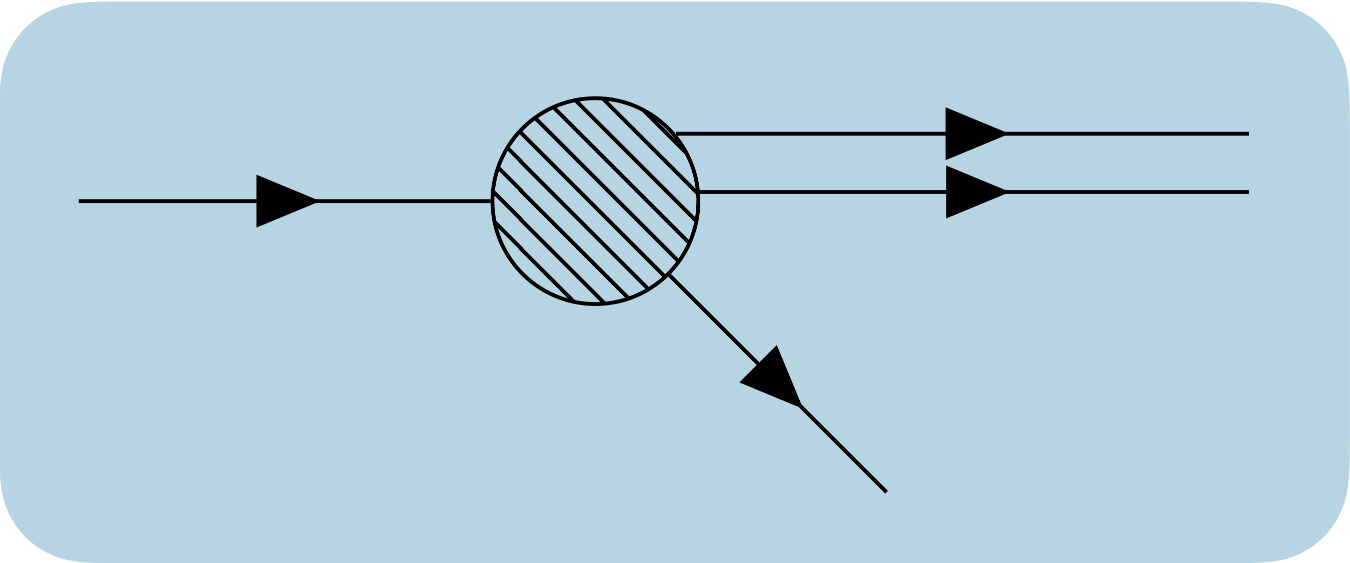
The Power of the Qubit! - Why are we interested in HEP?

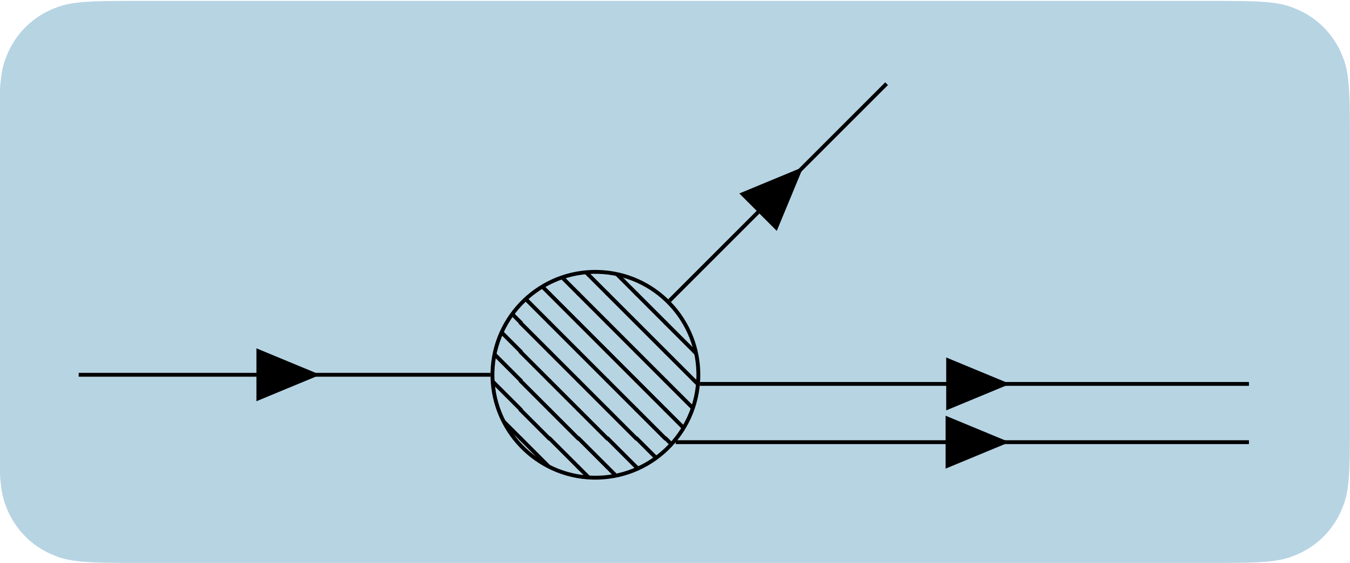

CMS
CMS Experiment at the LHC, CERN
Data recorded: 2021-Oct-19 13:01:24.690432 GMT
Run / Event / LS: 345881 / 17244 / 734


CMS
CMS Experiment at the LHC, CERN
Data recorded: 2016-Oct-11 10:44:24.059904 GMT
Run / Event / LS: 282842 / 47118579 / 25

The Power of the Qubit! - Why are we interested in HEP?

Parton Density Functions





Phys. Rev. D 103, 034027

The Power of the Qubit! - Why are we interested in HEP?

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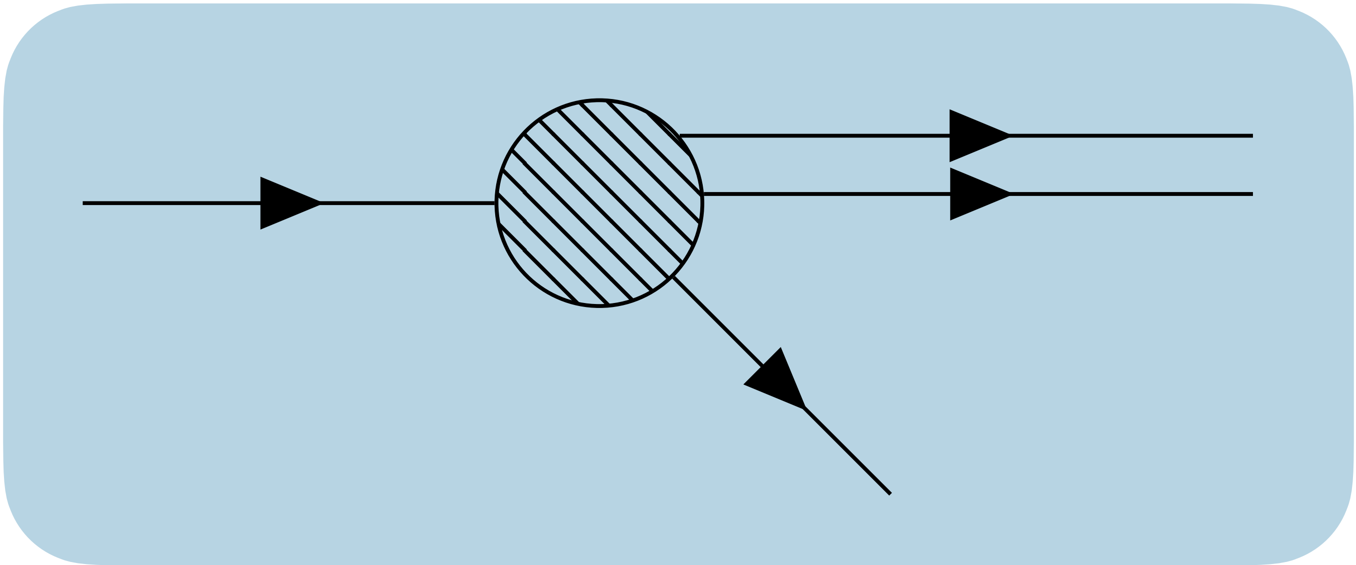
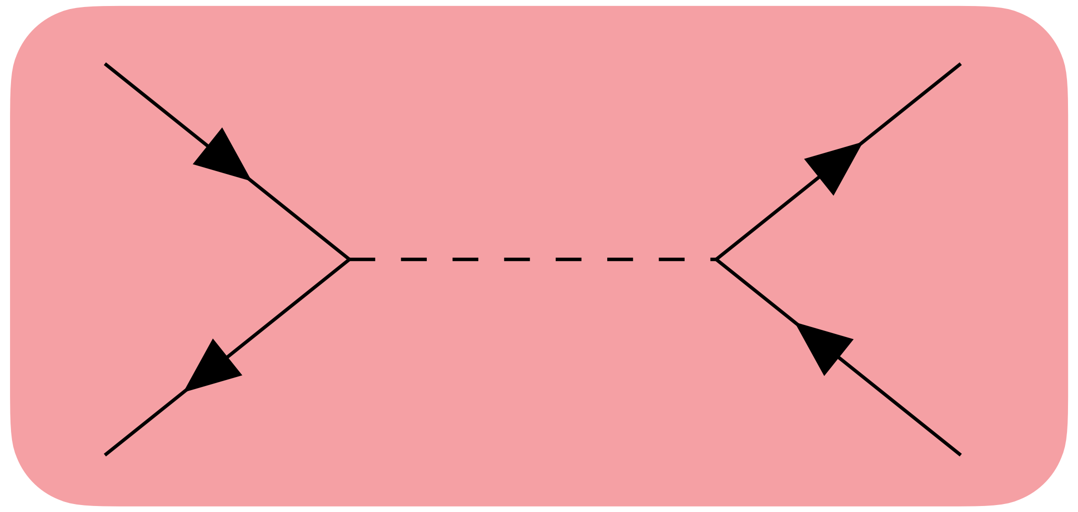
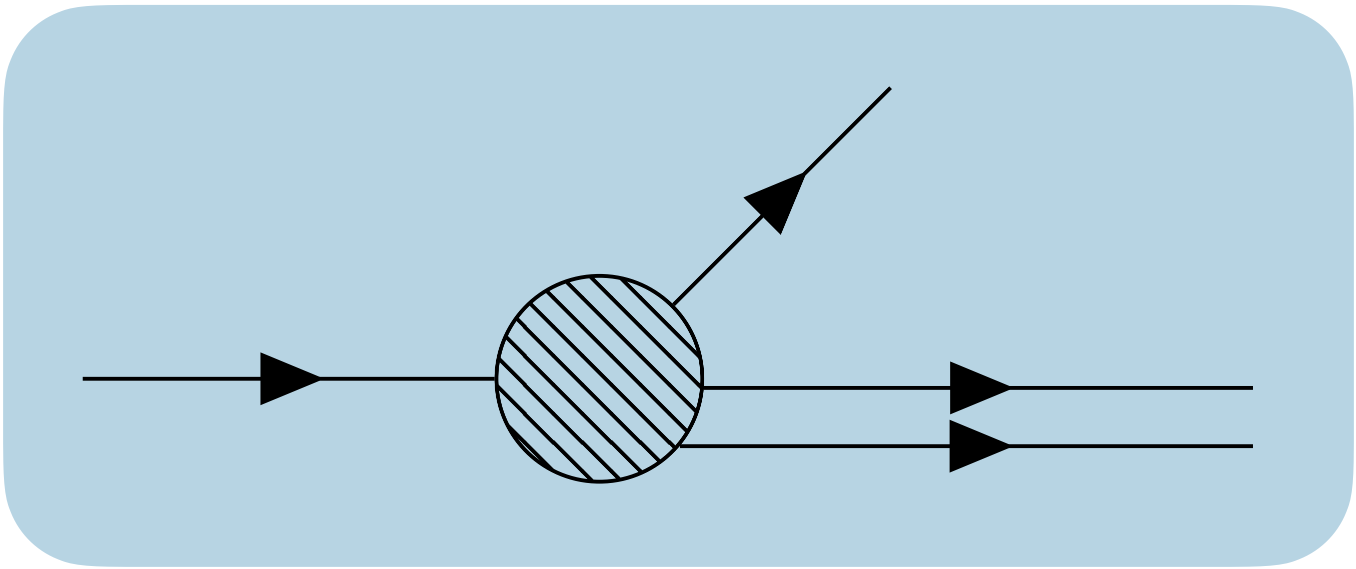


Hard Process



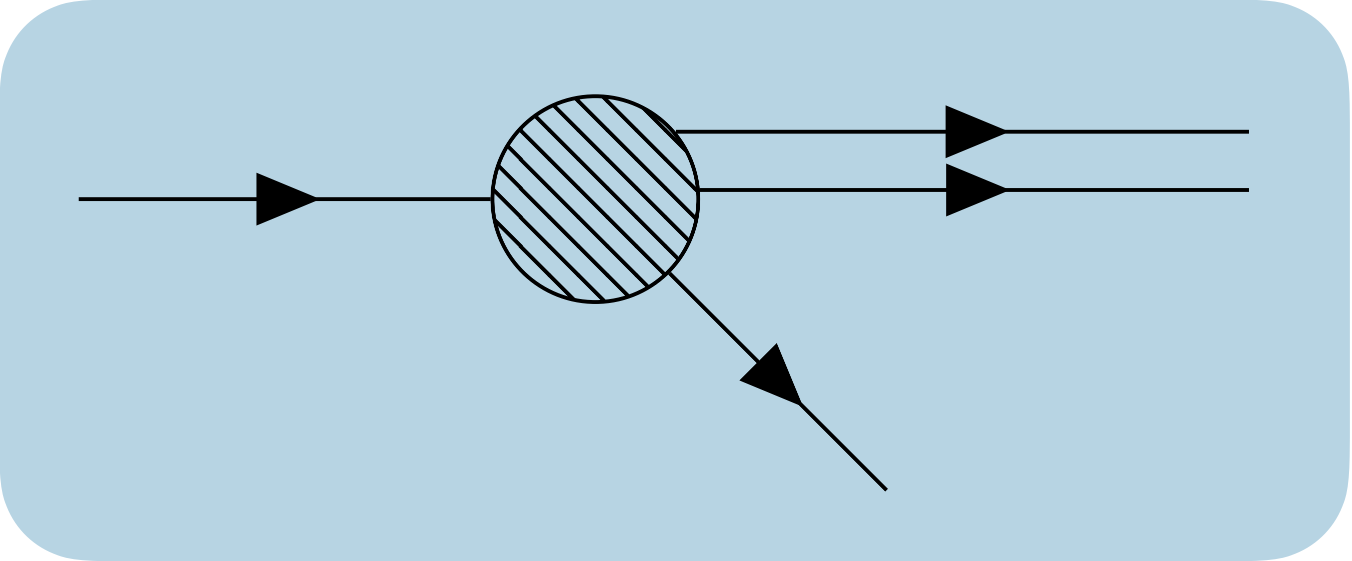
Phys. Rev. D 103, 076020



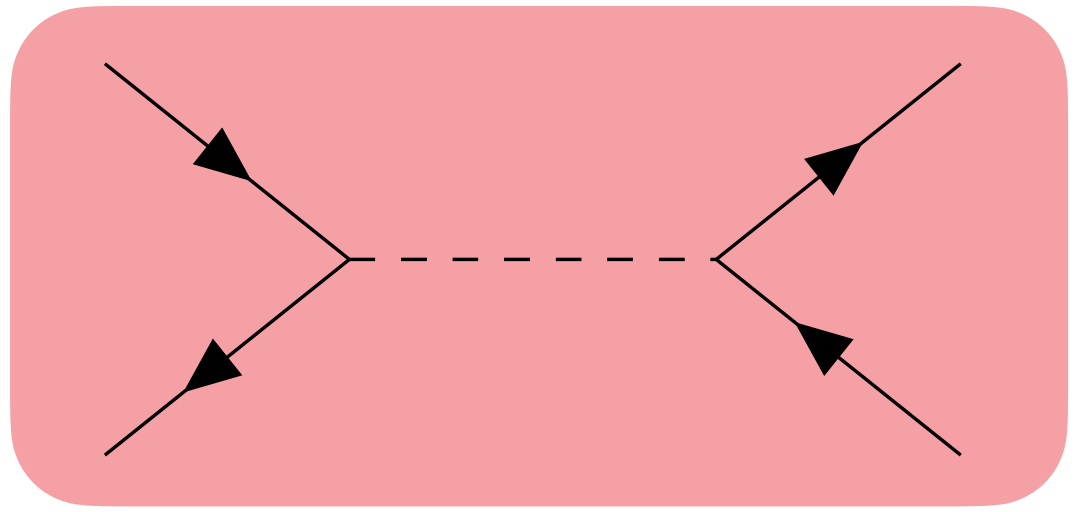
Phys. Rev. D 103, 034027

The Power of the Qubit! - Why are we interested in HEP?

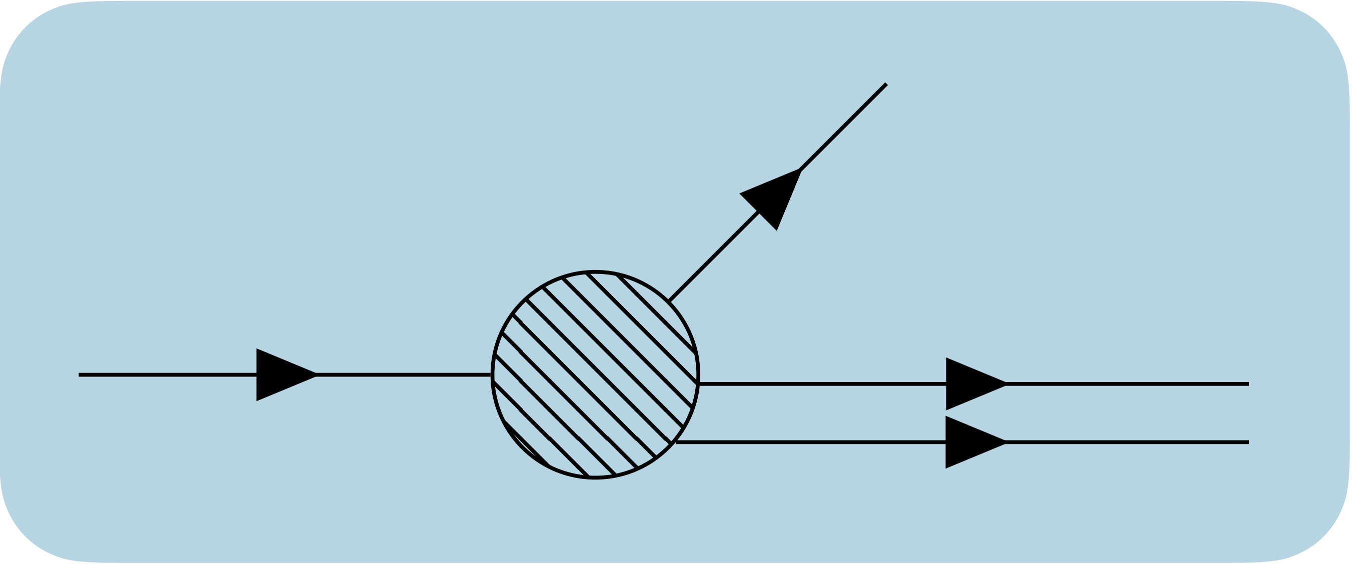
Parton Density Functions



Hard Process

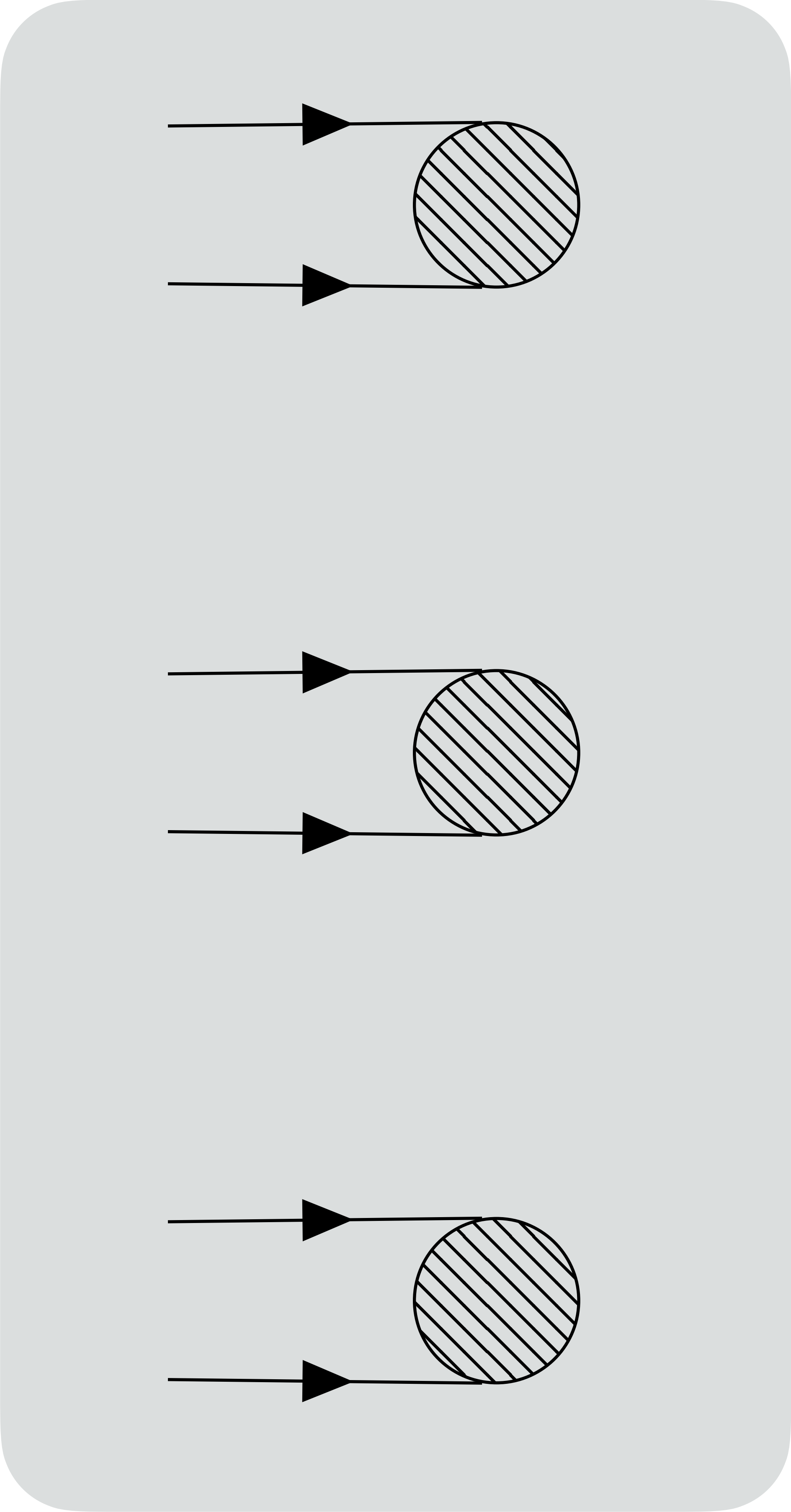


Phys. Rev. D 103, 076020



Phys. Rev. D 103, 034027

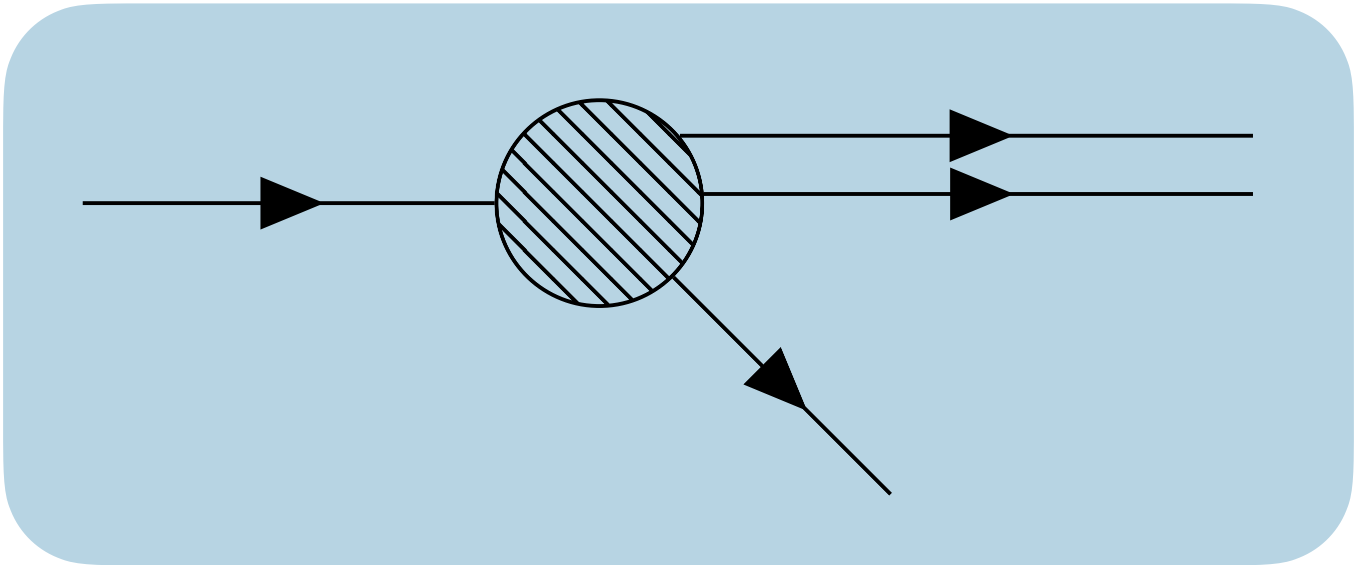
Hadronisation

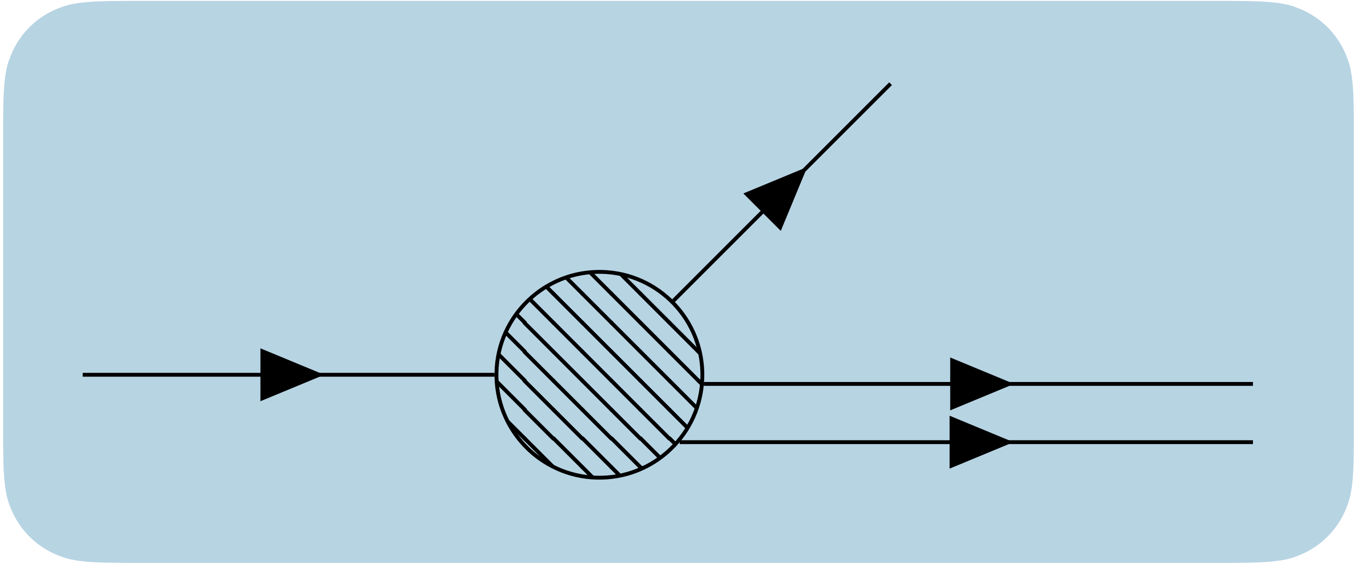


JHEP 11 (2022) 035

The Power of the Qubit! - Why are we interested in HEP?

Parton Density Functions





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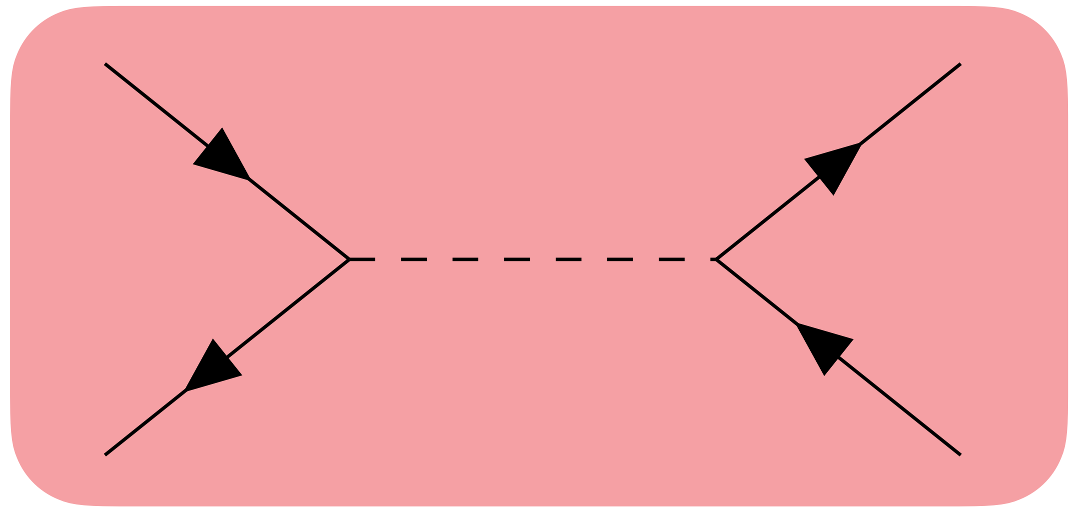
Hard Process

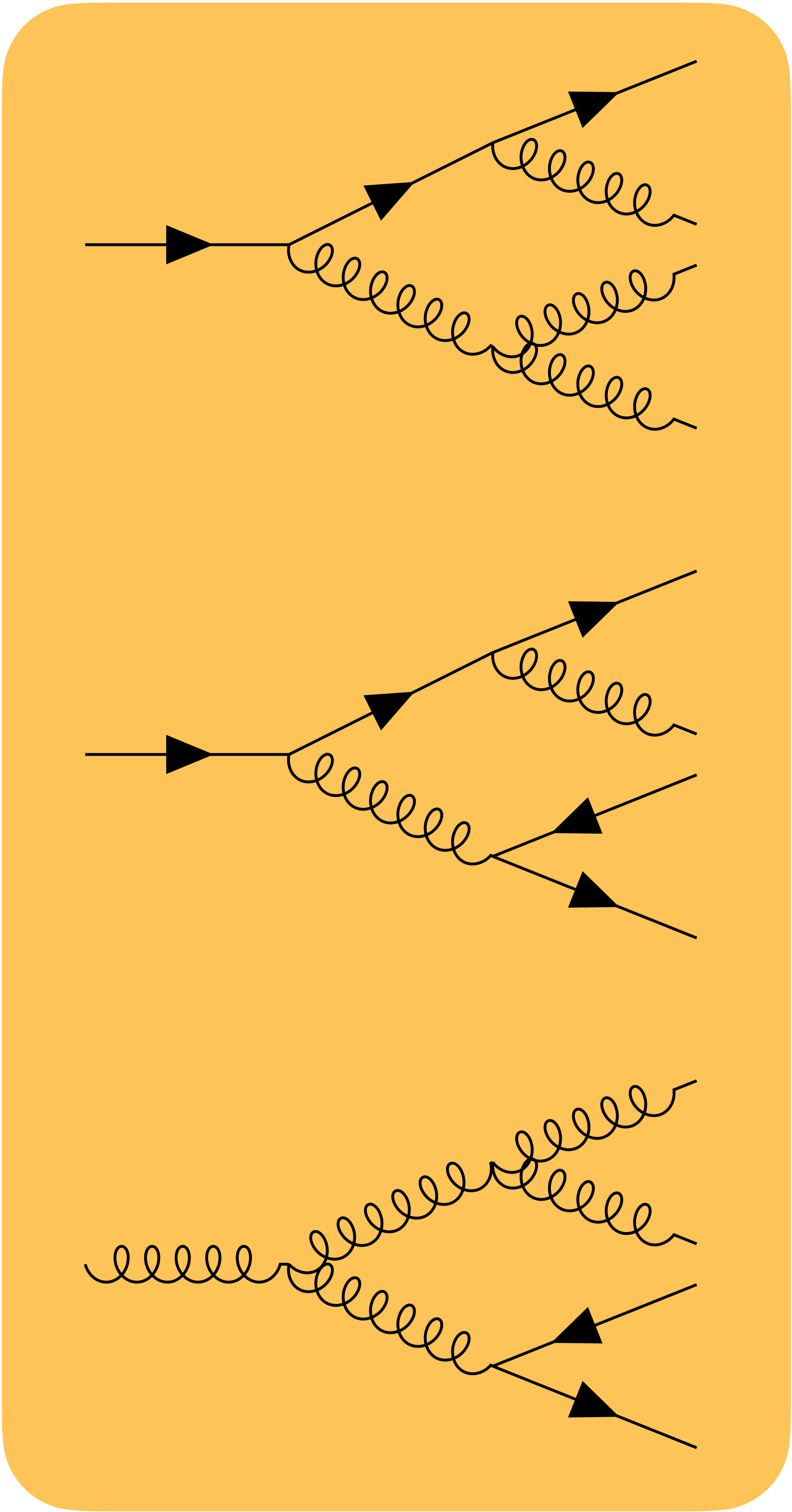


Phys. Rev. D 103, 076020

Phys. Rev. D 106, 056002

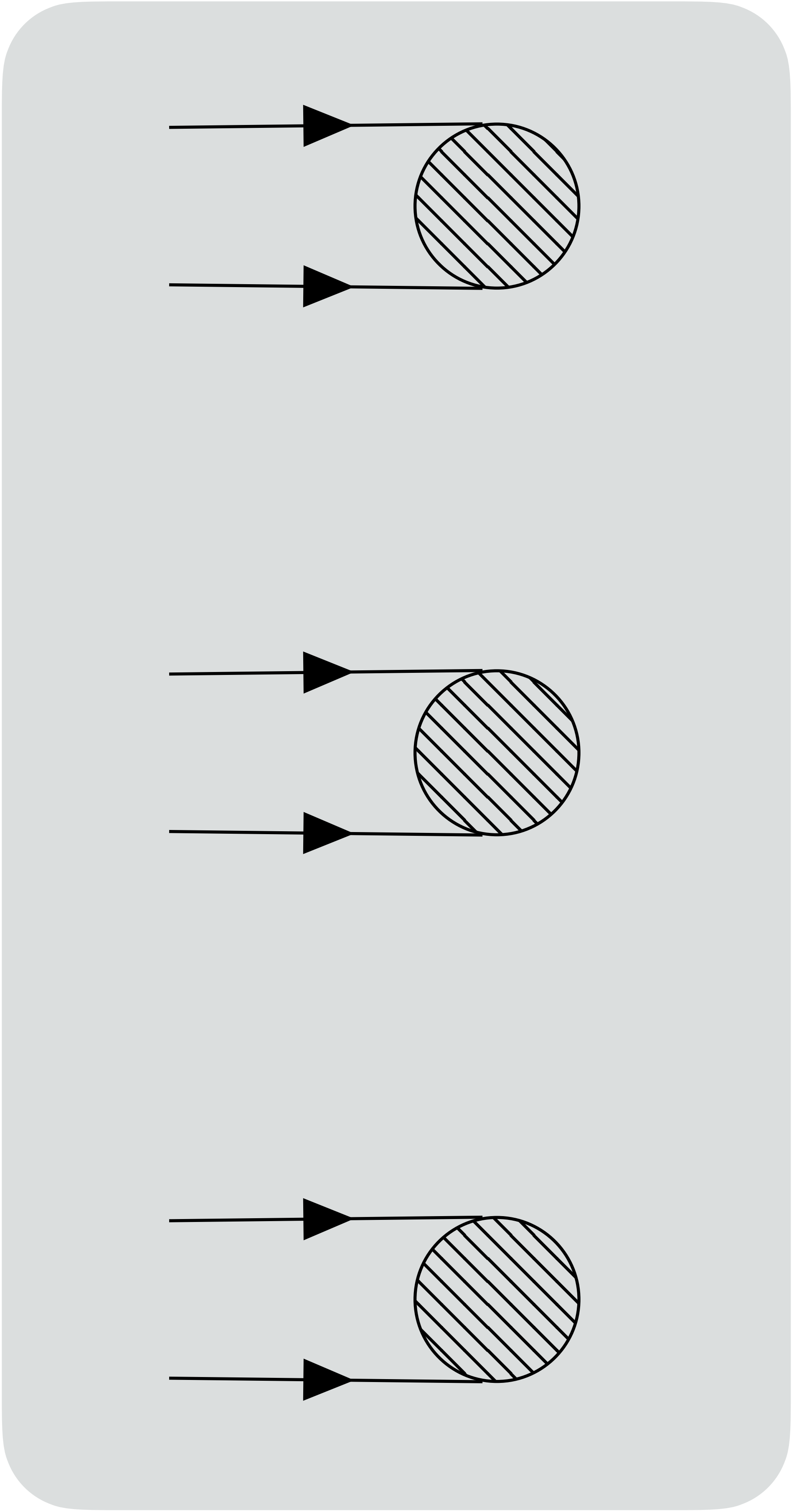
Phys. Rev. Lett. 126, 062001

Parton Shower



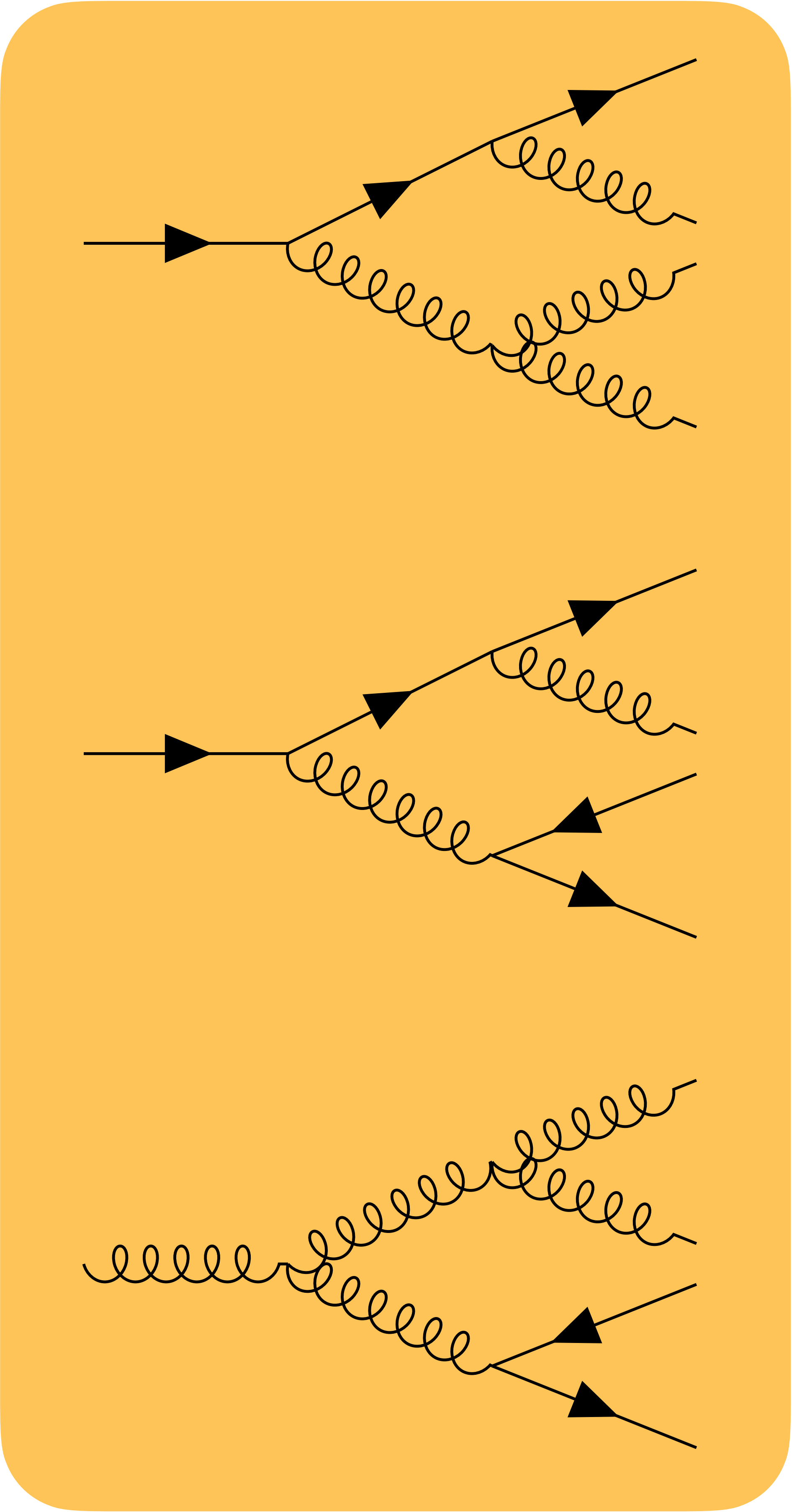
JHEP 11 (2022) 035

Hadronisation

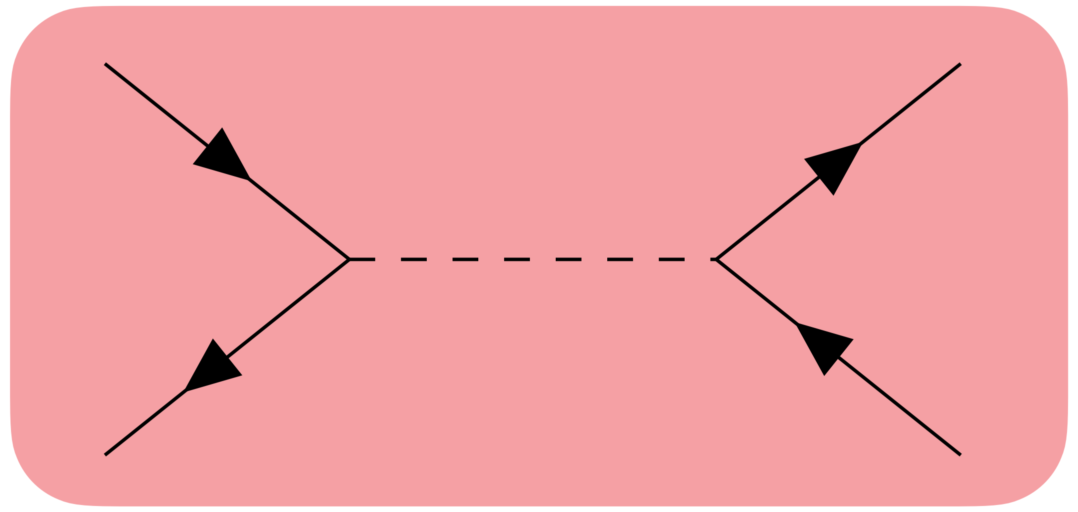


The Power of the Qubit! - Why are we interested in HEP?

Parton Shower



Hard Process



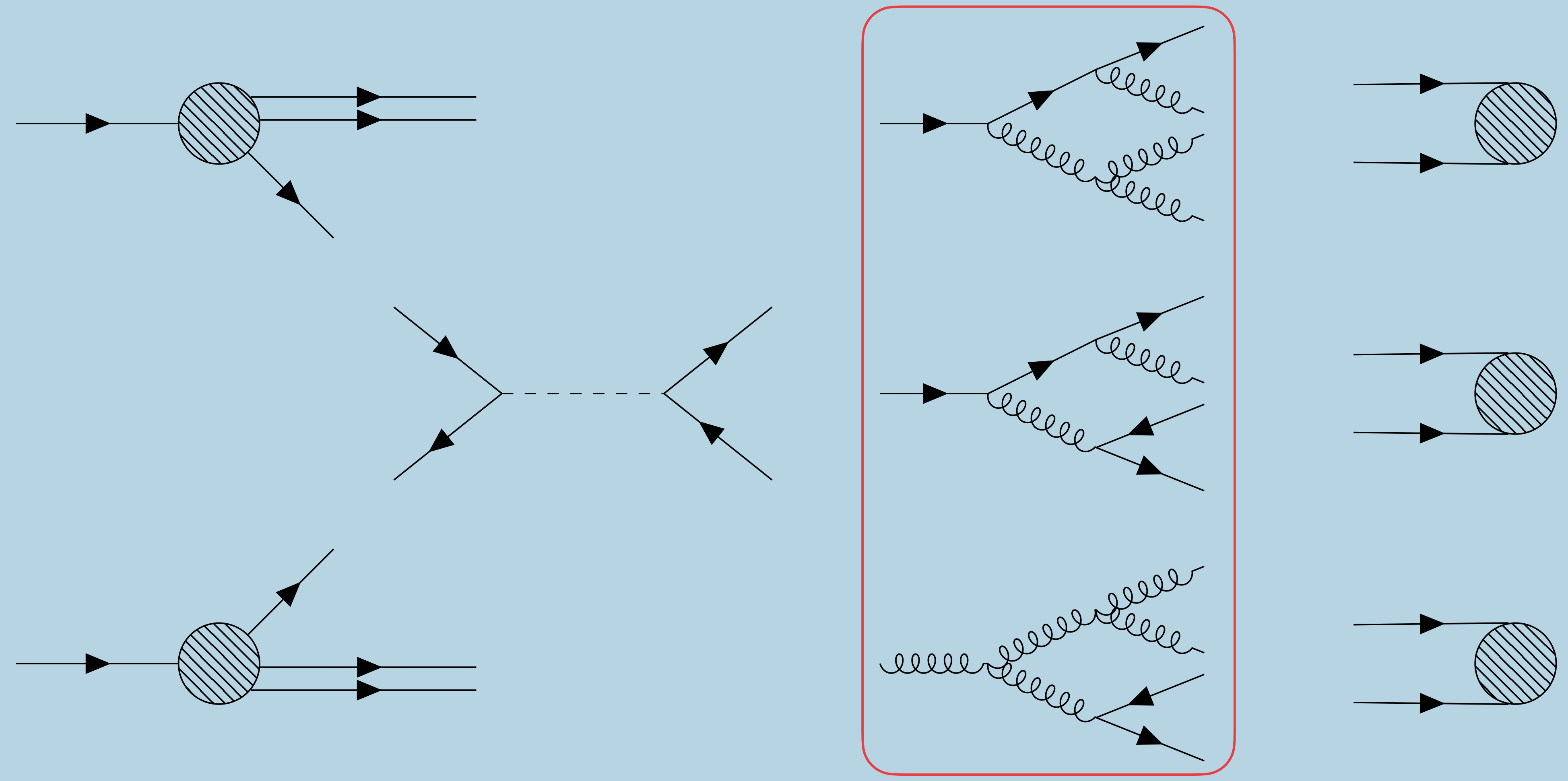
Phys. Rev. D 103, 076020

Phys. Rev. D 106, 056002

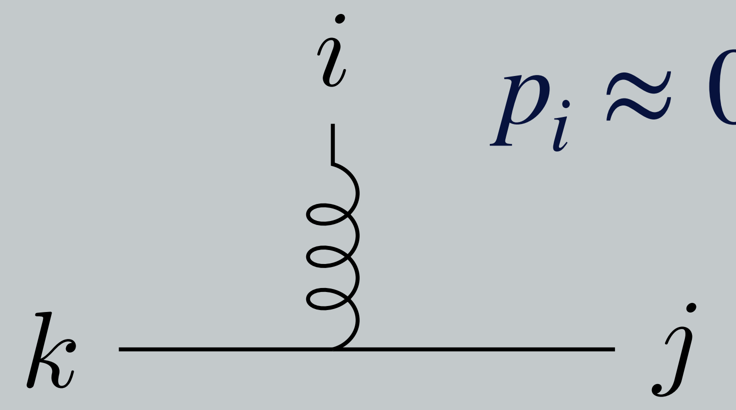
Phys. Rev. Lett. 126, 062001

JHEP 11 (2022) 035

The Parton Shower



Soft mode:

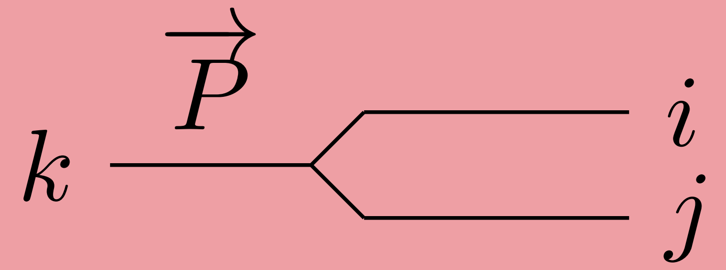


$p_i \approx 0$

Interference effects only allow for partial factorisation

Leading contributions to the decay rate in the collinear limit are included in the soft limit

Collinear mode:



$p_i = zP, \quad p_j = (1 - z)P$

Successive decay steps factorise into independent quasi-classical steps

In this limit, the decay from high energy to low energy proceeds as a **colour-dipole cascade**.

This interpretation allows for straightforward interference patterns and momentum conservation

The Parton Shower - The Veto Algorithm

The choice of the variables ξ and t is known as the **phase space parameterisation**

Non-Emission Probability

$$\Delta(t_n, t) = \exp\left(-\int_t^{t_n} dt d\xi \frac{d\phi}{2\pi} C \frac{\alpha_s}{2\pi} \frac{2 s_{ik}(t,\xi)}{s_{ij}(t,\xi) s_{jk}(t,\xi)}\right)$$

Master Equation

$$\mathcal{F}_n(\Phi_n, t_n, t_c; O) = \Delta(t_n, t_c) O(\Phi_n) + \int_{t_c}^{t_n} dt d\xi \frac{d\phi}{2\pi} C \frac{\alpha_s}{2\pi} \frac{2 s_{ik}(t,\xi)}{s_{ij}(t,\xi) s_{jk}(t,\xi)} \Delta(t_n, t) \mathcal{F}_n(\Phi_{n+1}, t, t_c; O)$$

Inclusive Decay Probability

$$d\mathcal{P}\left(q(p_{\mathrm{I}})\bar{q}(p_{\mathrm{K}}) \to q(p_i) g(p_j) \bar{q}(p_k)\right) \simeq \frac{ds_{ij}}{s_{\mathrm{IK}}} \frac{ds_{jk}}{s_{\mathrm{IK}}} C \frac{\alpha_s}{2\pi} \frac{2 s_{\mathrm{IK}}}{s_{ij} s_{jk}}$$

Current interpretations of the veto algorithm treat the phase space variables ξ and t as **continuous**

Collider Events on a Quantum Computer

Gösta Gustafson,[a] **Stefan Prestel,**[a] **Michael Spannowsky,**[b] **Simon Williams**[c]

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[b] *Institute for Particle Physics Phenomenology, Department of Physics, Durham University, Durham DH1 3LE, U.K.*
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ABSTRACT: High-quality simulated data is crucial for particle physics discoveries. Therefore, Parton shower algorithms are a major building block of the data synthesis in event generator programs. However, the core algorithms used to generate parton showers have barely changed since the 1980s. With quantum computers' rapid and continuous development, dedicated algorithms are required to exploit the potential that quantum computers provide to address problems in high-energy physics. This paper presents a novel approach to synthesising parton showers using the Discrete QCD method. The algorithm benefits from an elegant quantum walk implementation which can be embedded into the classical toolchain. We use the `ibm_algiers` device to sample parton shower configurations and generate data that we compare against measurements taken at the ALEPH, DELPHI and OPAL experiments. This is the first time a Noisy Intermediate-Scale Quantum (NISQ) device has been used to simulate realistic high-energy particle collision events.

Discrete QCD - Abstracting the Parton Shower Method

1. Parameterise phase space in terms of gluon transverse momentum and rapidity:

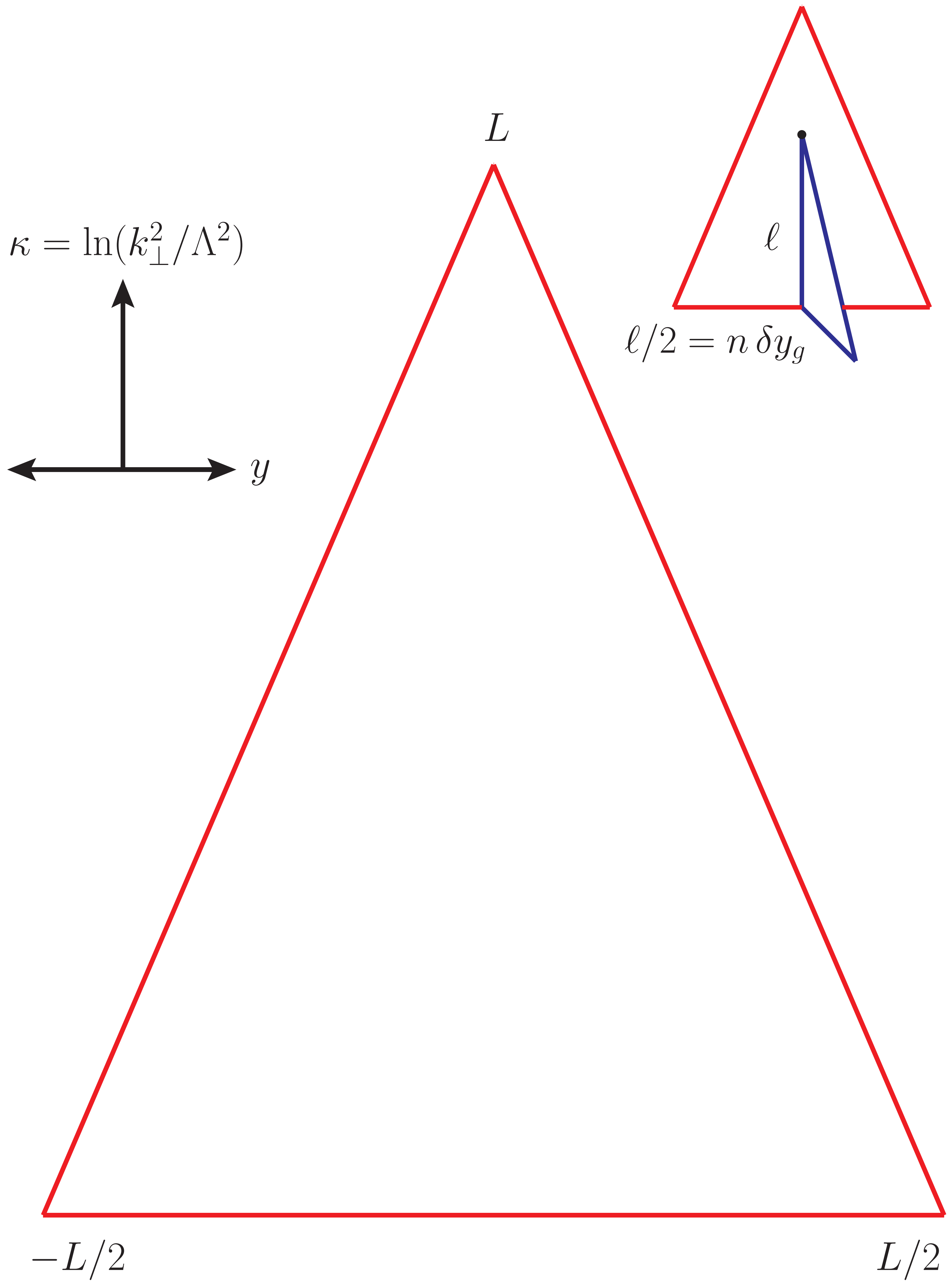
$$k_\perp^2 = \frac{s_{ij}s_{jk}}{s_{\mathrm{IK}}} \quad \text{and} \quad y = \frac{1}{2}\ln\left(\frac{s_{ij}}{s_{jk}}\right)$$

which leads to the inclusive probability:

$$d\mathcal{P}\left(q(p_{\mathrm{I}})\bar{q}(p_{\mathrm{K}}) \to q(p_i)g(p_j)\bar{q}(p_k)\right) \simeq = \frac{C\alpha_s}{\pi}d\kappa dy$$

where $\kappa = \ln\left(\frac{k_\perp^2}{\Lambda^2}\right)$ and Λ is an arbitrary mass scale

Due to the colour charge of emitted gluons, the rapidity span for subsequent dipole decays is increased. This is interpreted as **"folding out"**



Discrete QCD - Abstracting the Parton Shower Method

2. Neglect $g \to q\bar{q}$ splittings and examine transverse-momentum-dependent running coupling

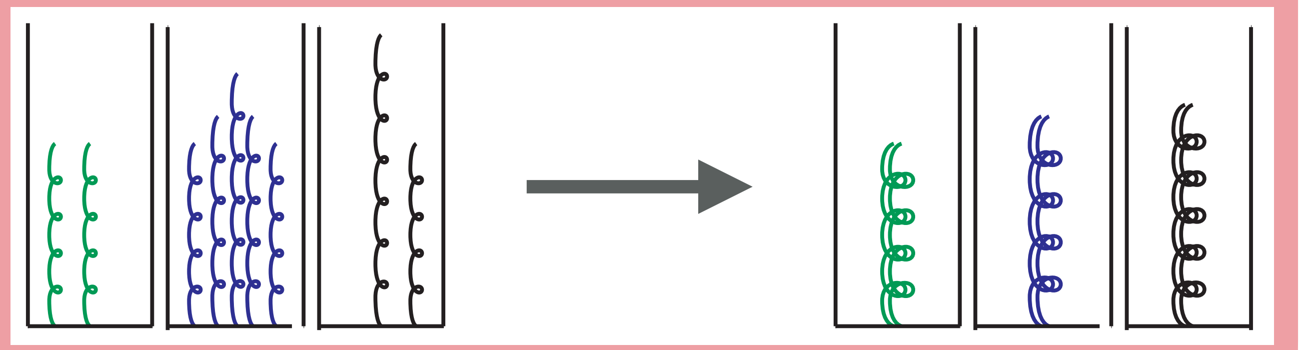
$$\alpha_s(k_\perp^2) = \frac{12\pi}{33 - 2n_f} \frac{1}{\ln(k_\perp^2/\Lambda_{\rm QCD}^2)}$$

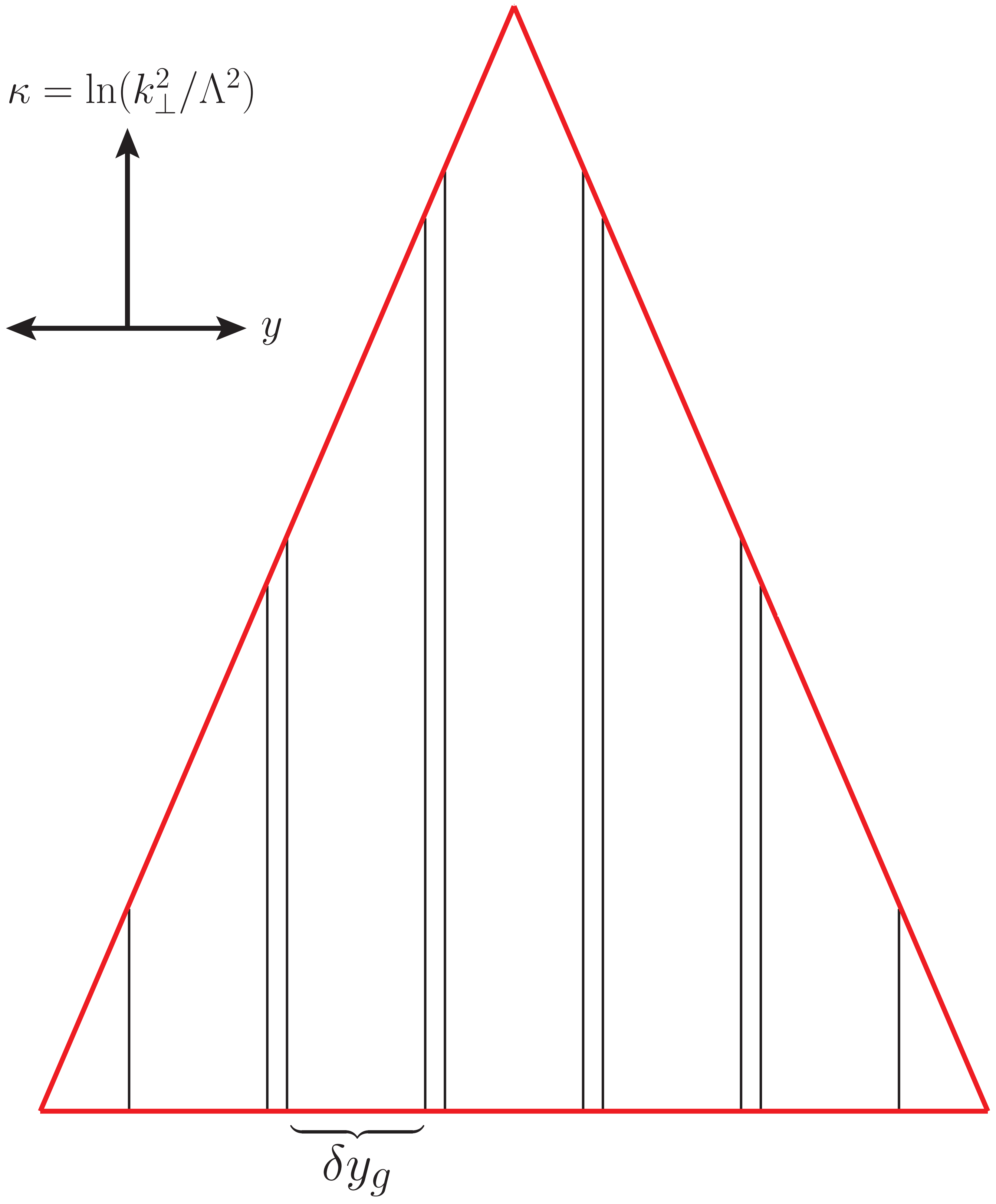
leads to the inclusive probability

$$d\mathcal{P}\left(q(p_{\rm I})\bar{q}(p_{\rm K}) \to q(p_i)g(p_j)\bar{q}(p_k)\right) \simeq = \frac{d\kappa}{\kappa}\frac{dy}{\delta y_g} \quad \text{with} \quad \delta y_g = \frac{11}{6}$$

Interpreting the running coupling renormalisation group as a gain-loss equation:

Gluons within δy_g act coherently as one effective gluon





Discrete QCD - Abstracting the Parton Shower Method

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$$\alpha_s(k_\perp^2) = \frac{12\pi}{33 - 2n_f} \frac{1}{\ln(k_\perp^2/\Lambda_{\rm QCD}^2)} = \frac{\text{const.}}{\kappa}$$

leads to the inclusive probability

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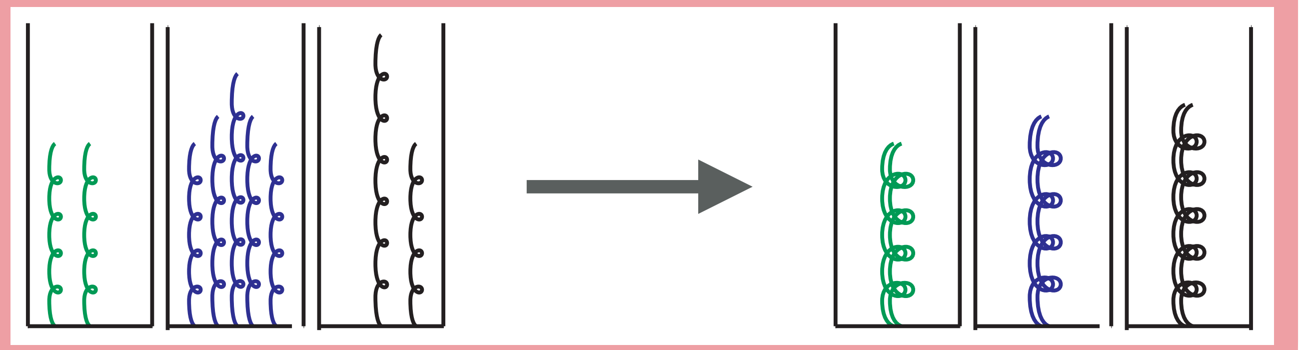
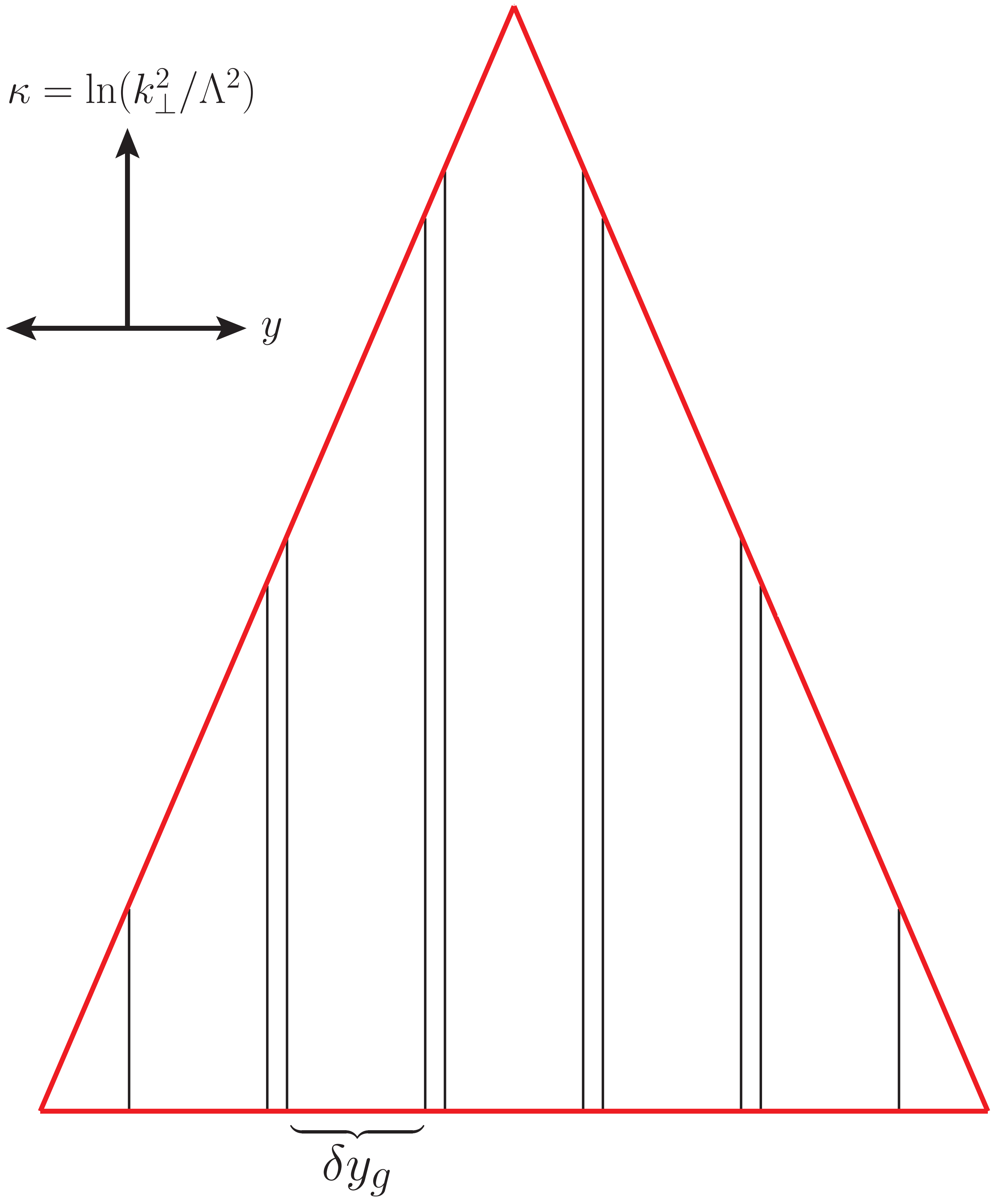
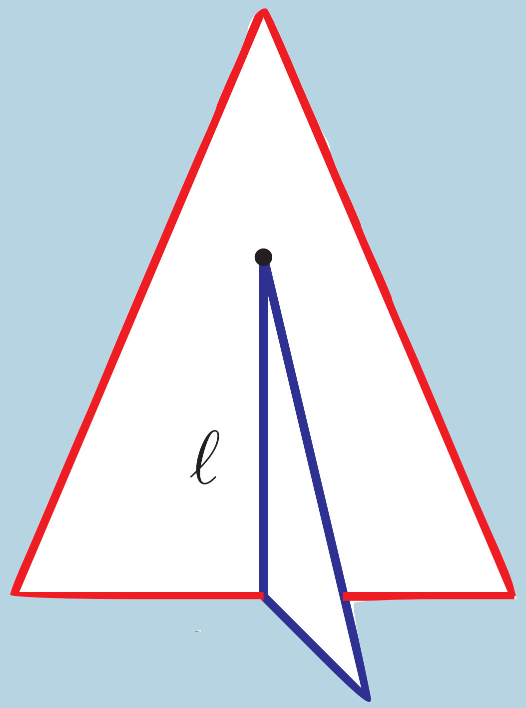




Discrete QCD - Abstracting the Parton Shower Method

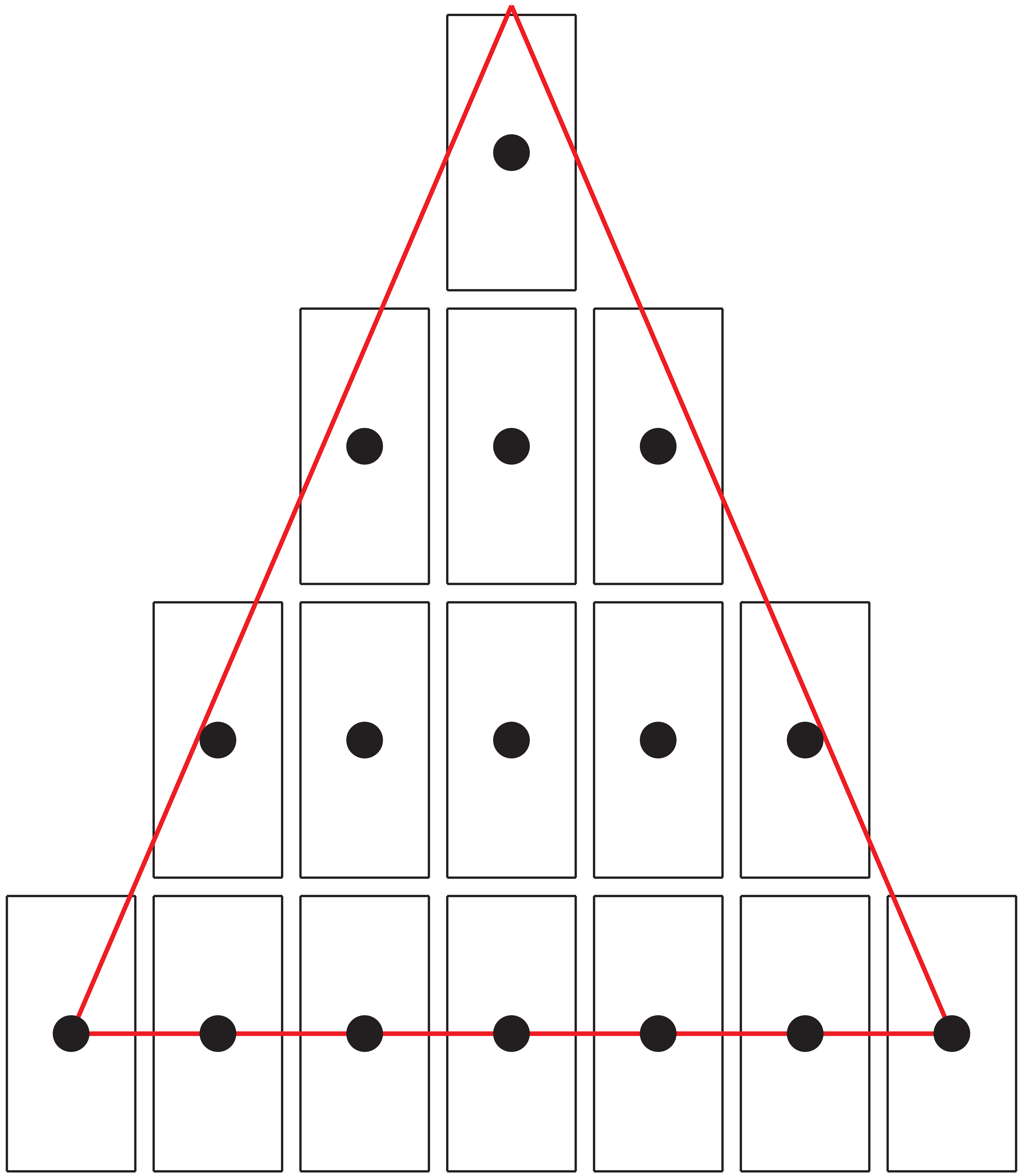
Folding out extends the baseline of the triangle to positive y by $\frac{l}{2}$, where l is the height at which to emit effective gluons



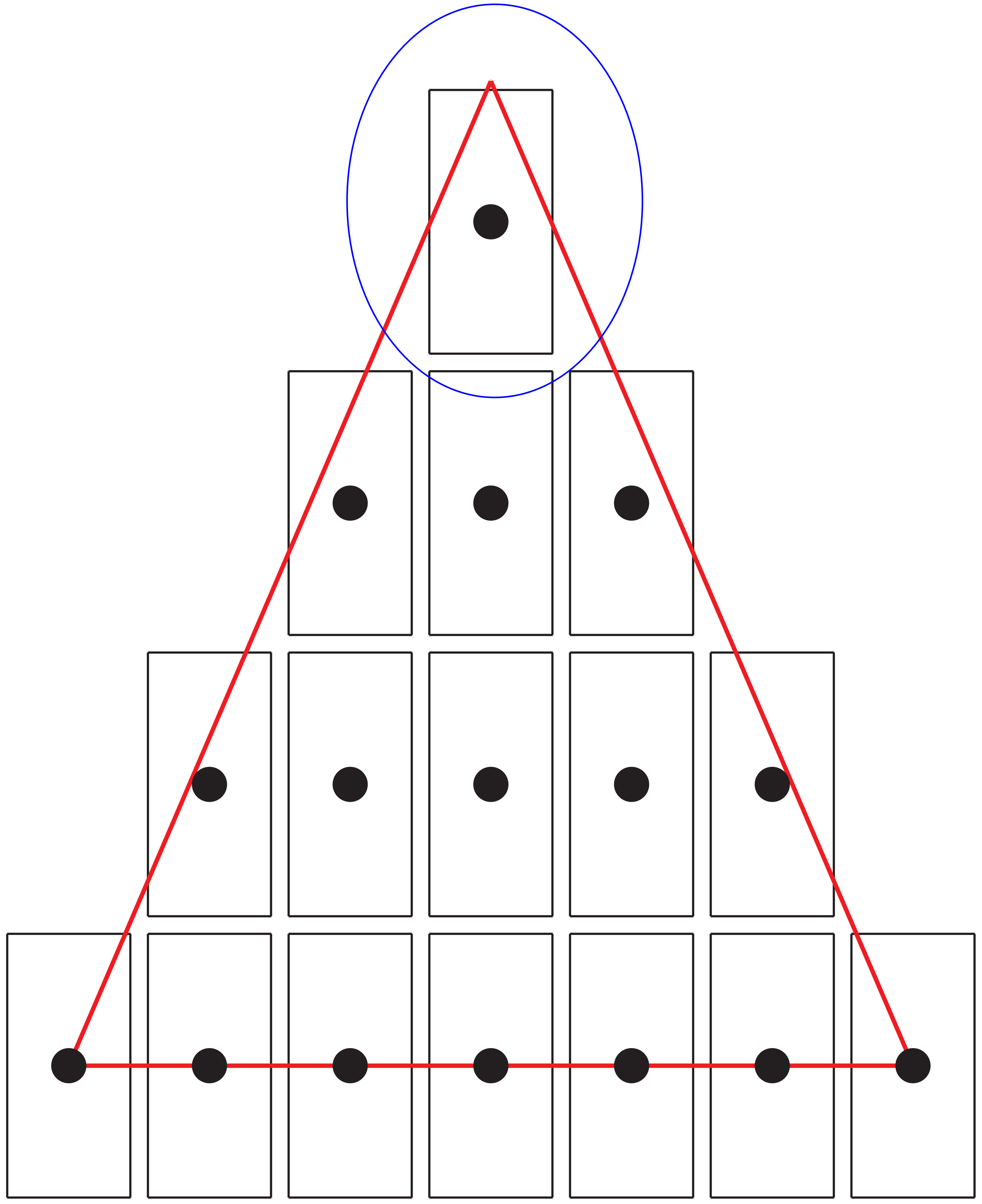
A consequence of folding is that the κ axis is quantised into multiples of $2\delta y_g$

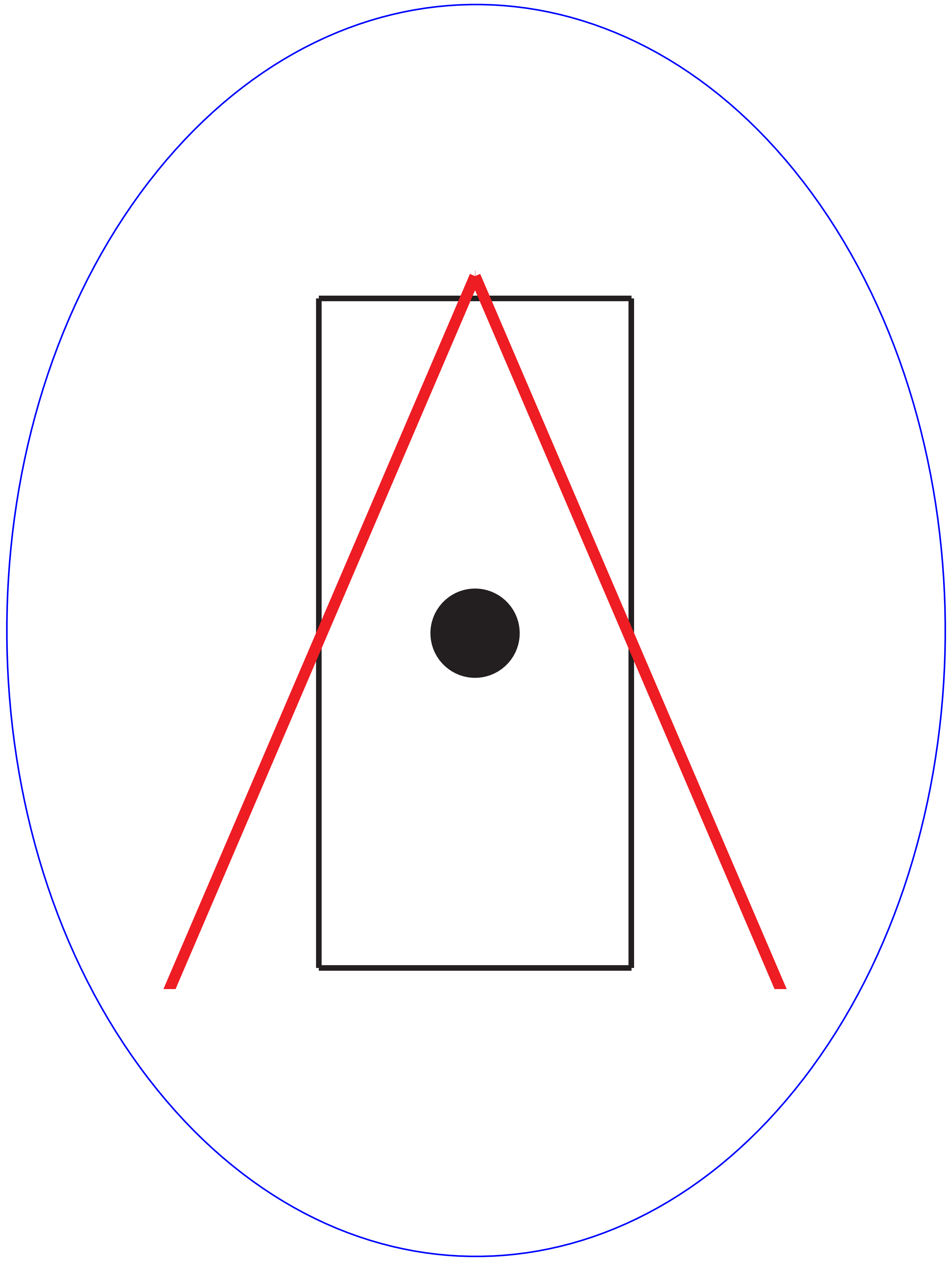
Each rapidity slice can be treated independently of any other slice. The exclusive rate probability takes the simple form:

$$\frac{d\kappa}{\kappa}\exp\left(-\int_{\kappa}^{\kappa_{max}}\frac{d\bar{\kappa}}{\bar{\kappa}}\right)=\frac{d\kappa}{\kappa_{max}}$$

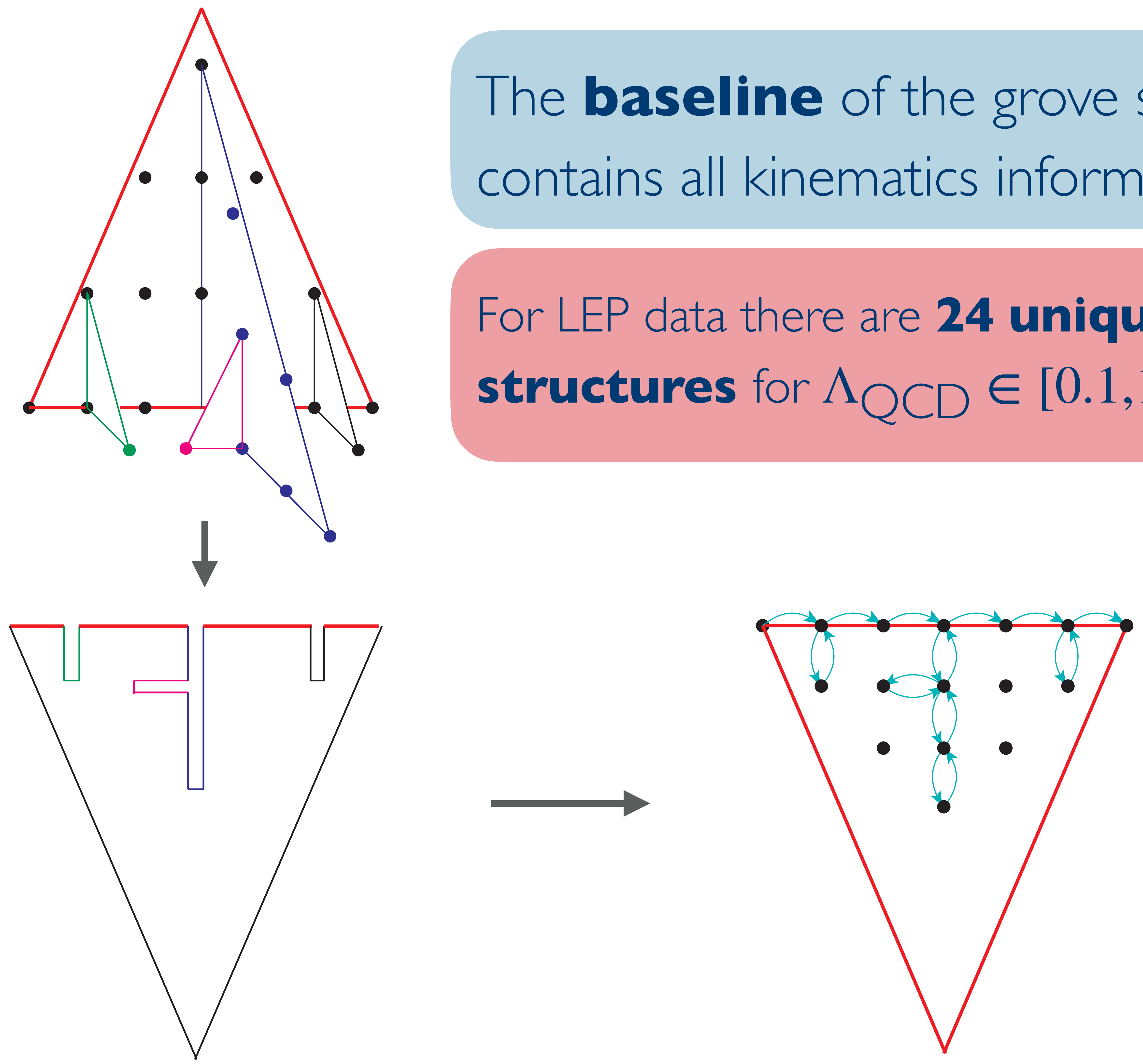


Collider Events on a Quantum Computer





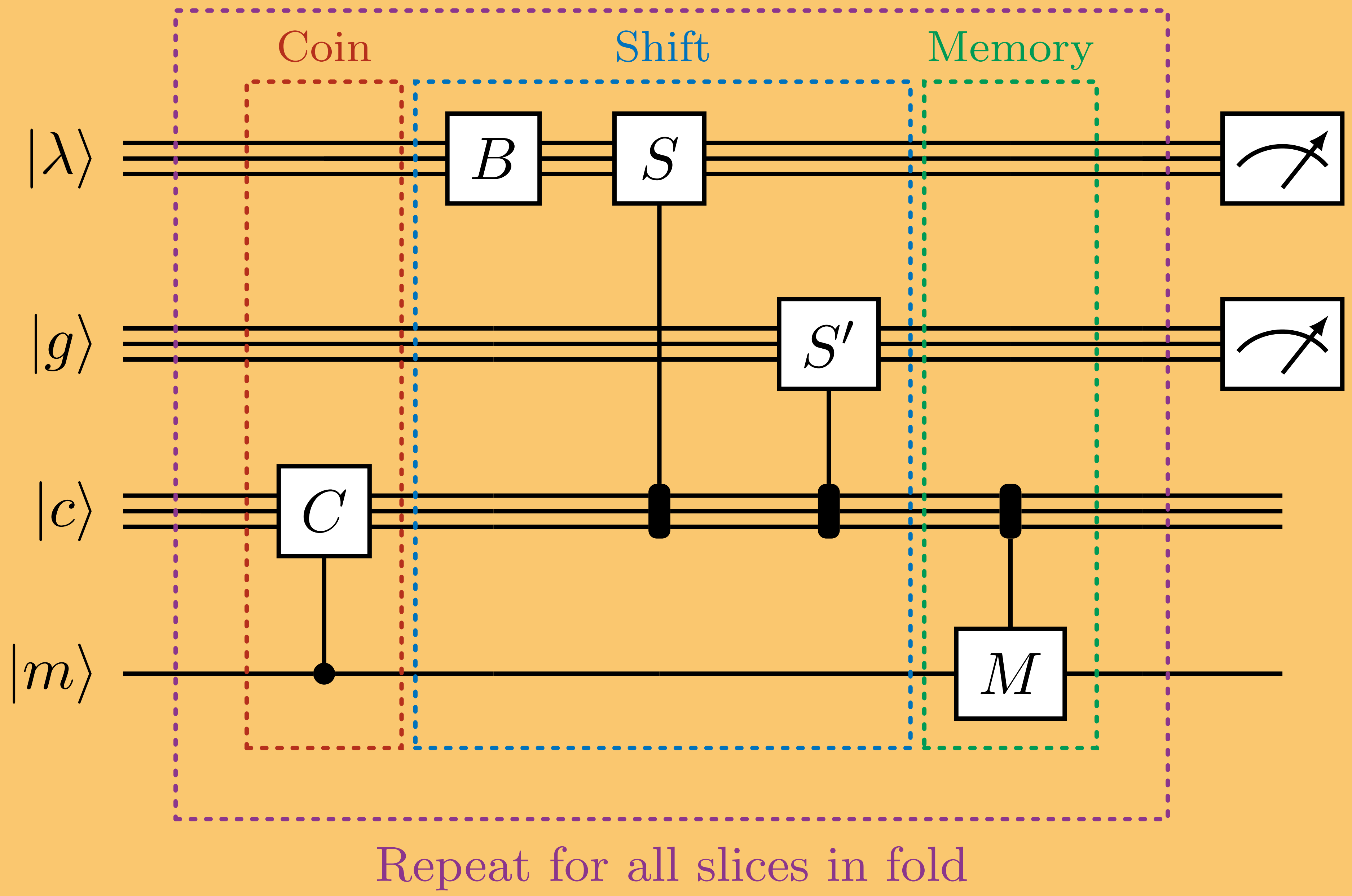
Discrete QCD as a Quantum Walk



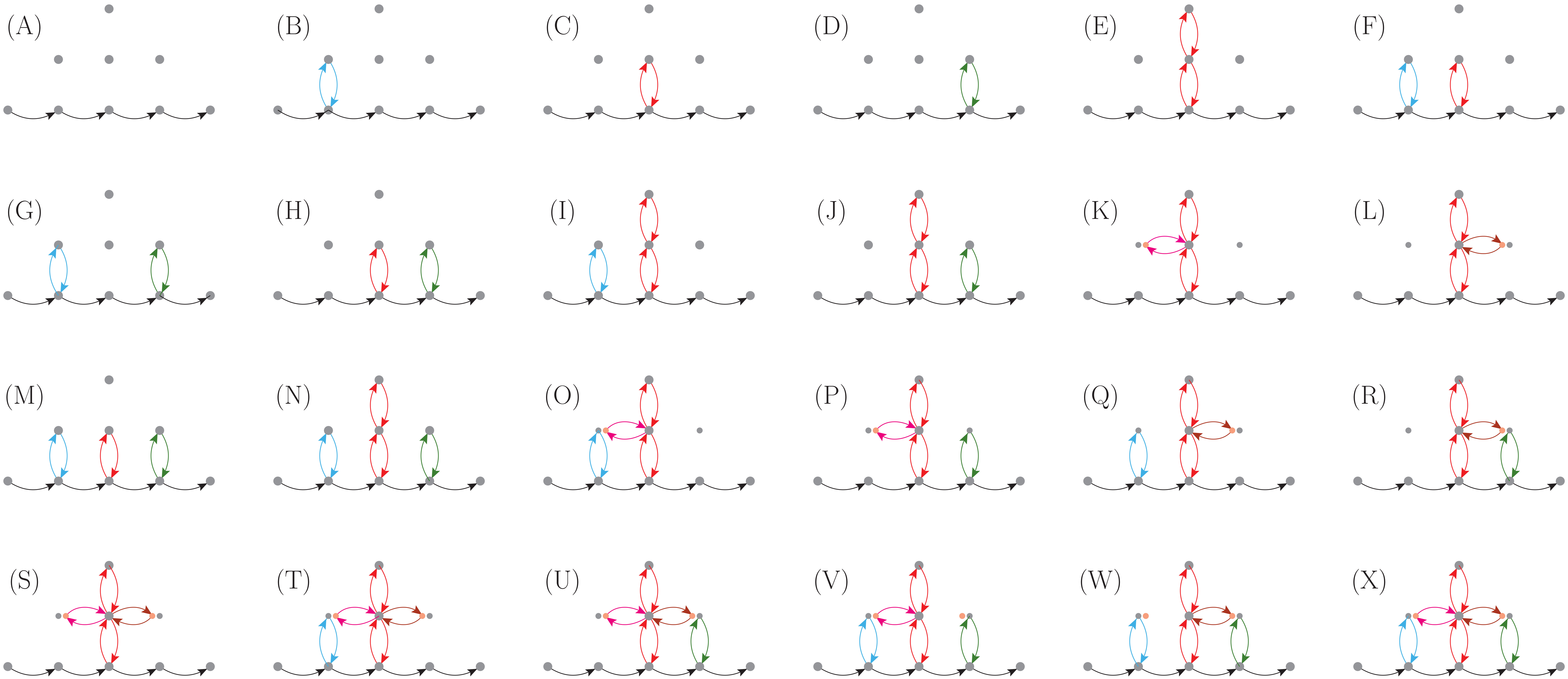
The **baseline** of the grove structure contains all kinematics information

For LEP data there are **24 unique grove structures** for $\Lambda_{\mathrm{QCD}} \in [0.1,1]$ GeV

The Discrete-QCD dipole cascade can therefore be implemented as a simple **Quantum Walk**



Discrete QCD - Grove Structures



Generating Scattering Events from Groves

Once the grove structure has been selected, event data can be synthesised in the following steps using the baseline:

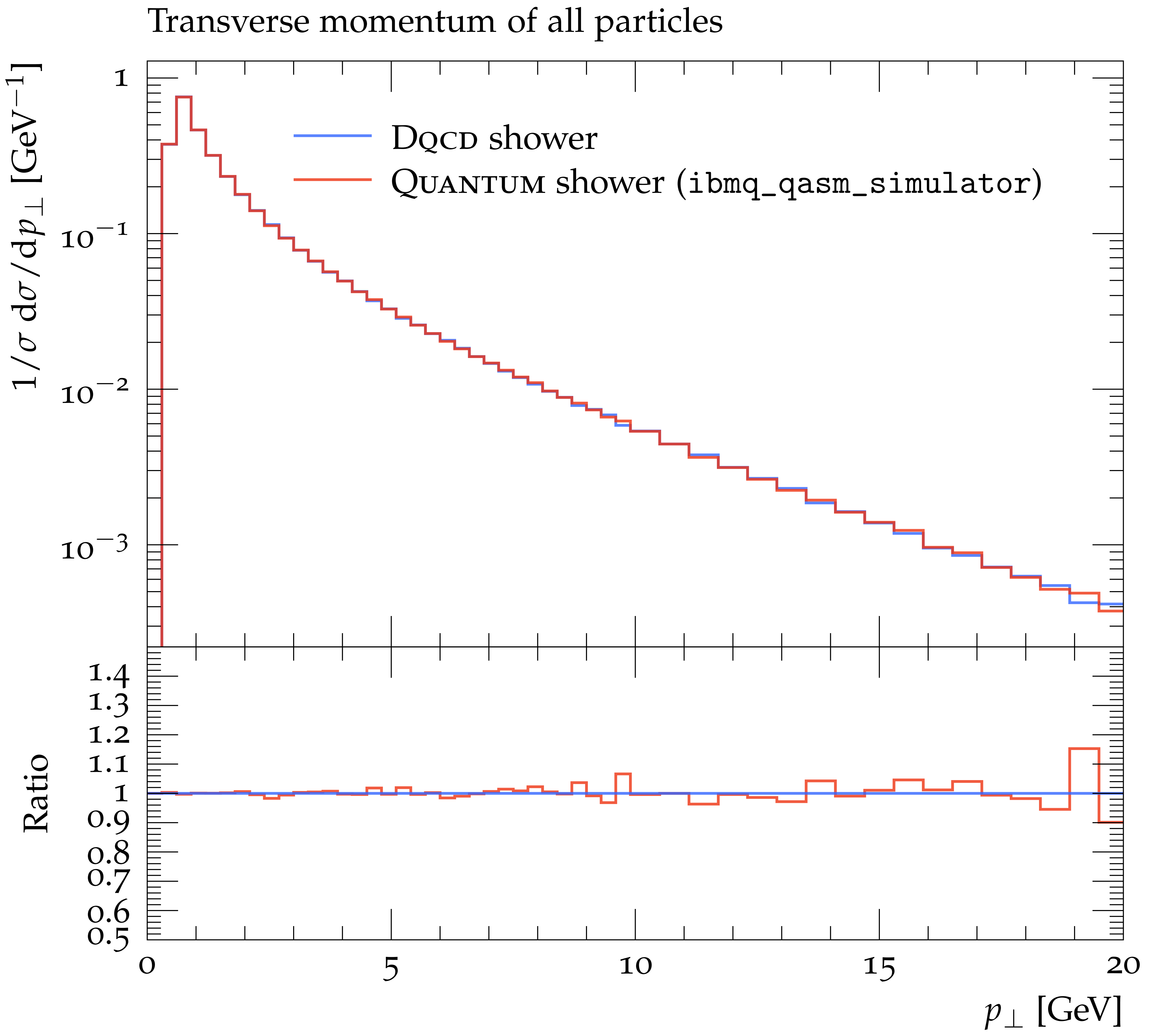
1. Create the highest κ effective gluons first (i.e. go from top to bottom in phase space)

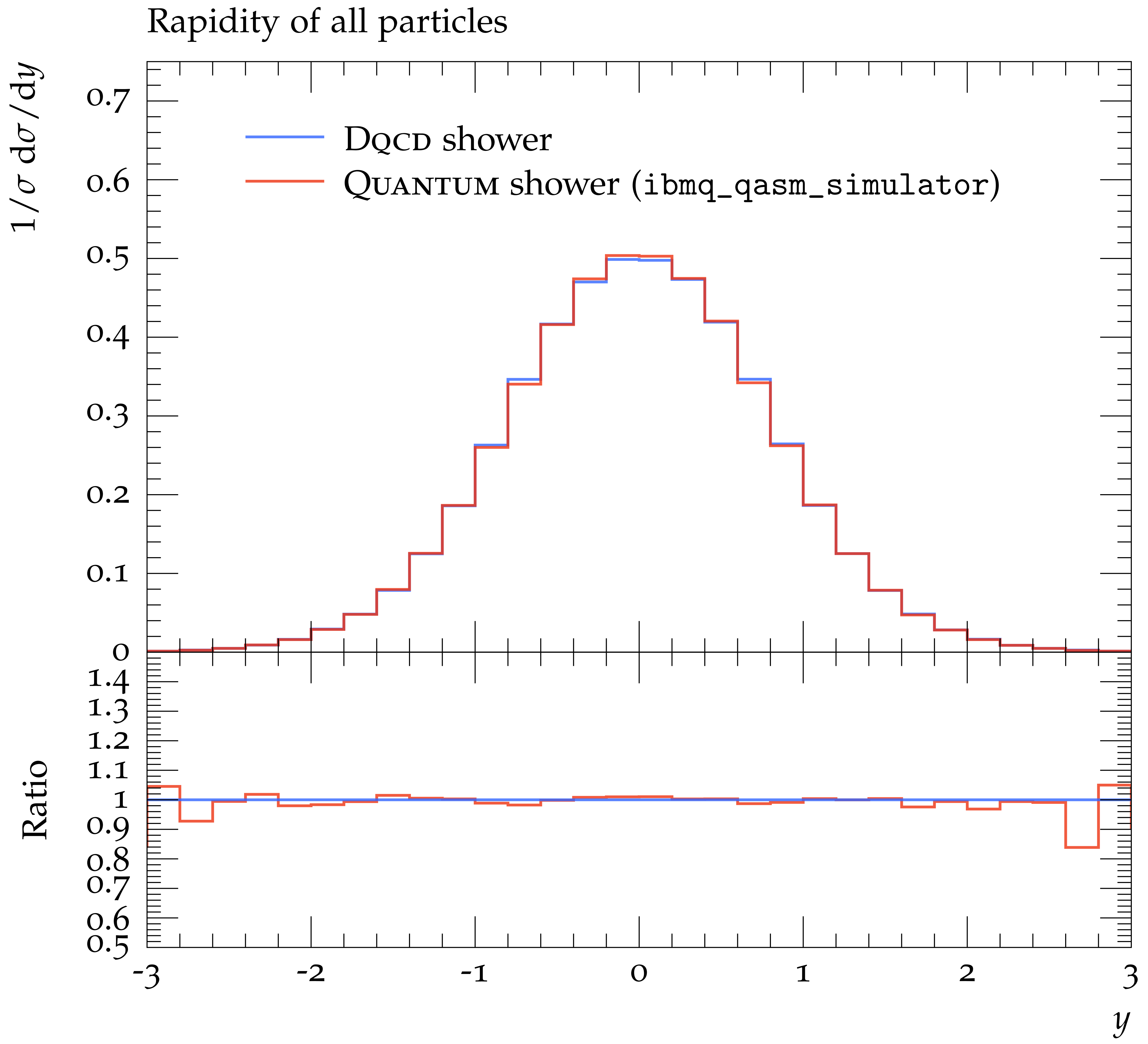
2. For each effective gluon j that has been emitted from a dipole IK, read off the values s_{ij}, s_{jk} and s_{IK} from the grove

3. Generate a uniformly distributed azimuthal decay angle ϕ, and then employ momentum mapping (here we have used Phys. Rev. D 85, 014013 (2012), 1108.6172) to produce post-branching momenta

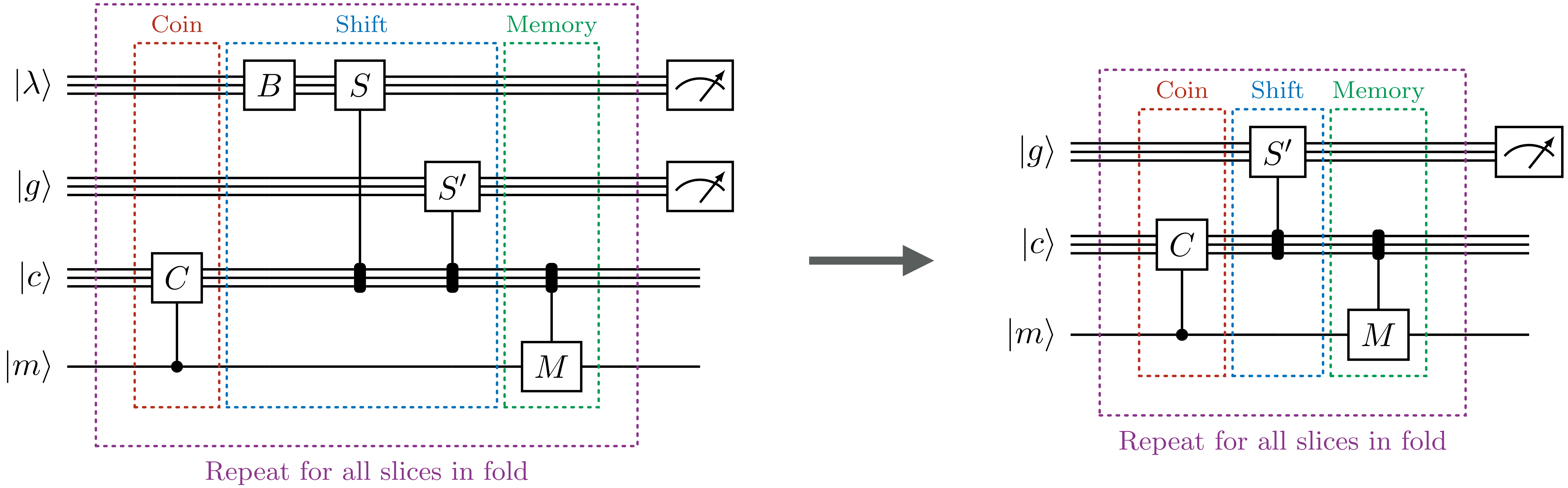
This has been done using the ibm_cloud 27 qubit device ibm_algiers, with 20,000 shots on the device. A comparison with a like-for-like classical parton shower algorithm has been made.

Running on a Quantum Simulator





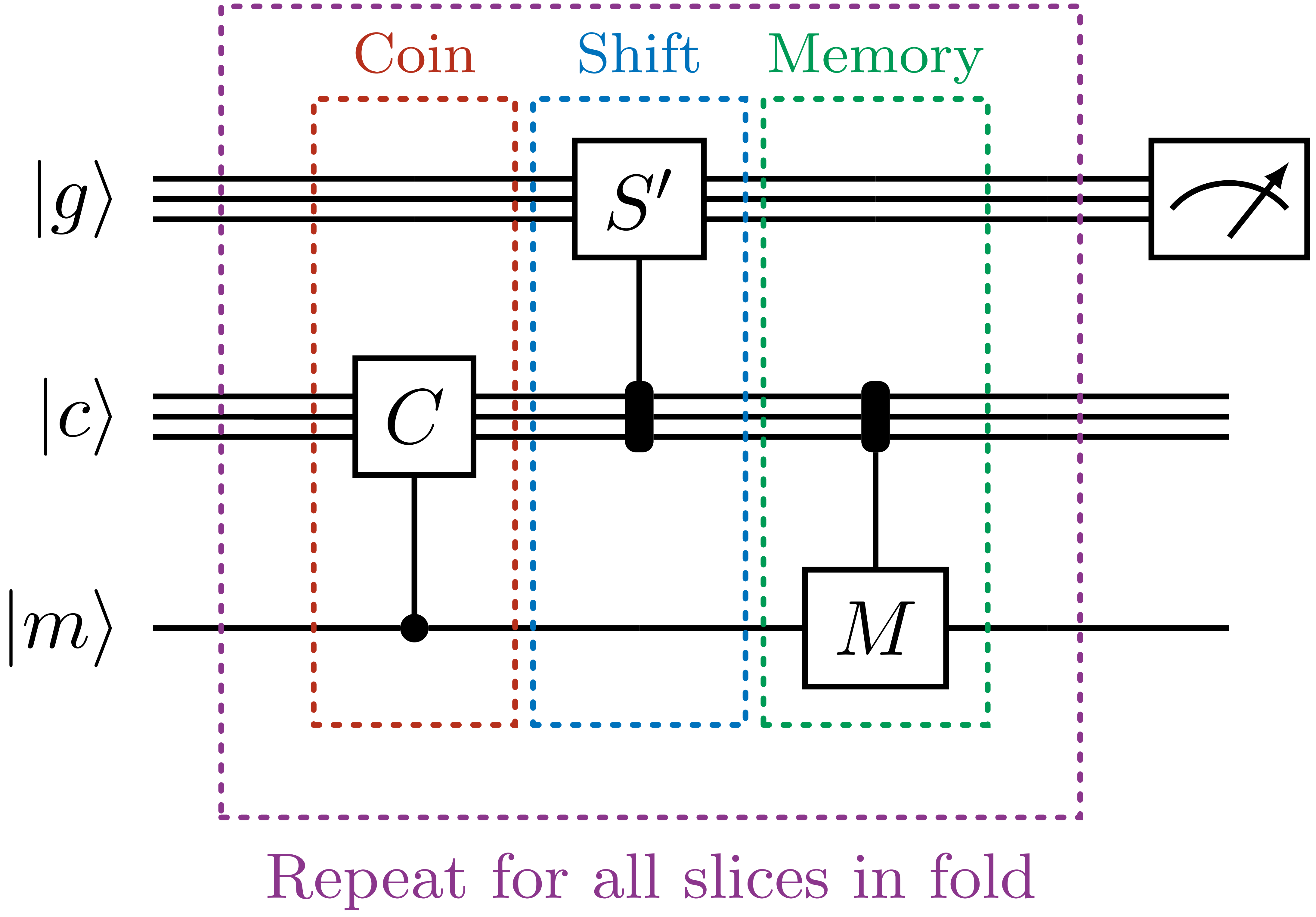
Running on a Quantum Device - Streamlined Circuit



15 qubits
116 gate operations
(102 multi-qubit, 14 single qubit)

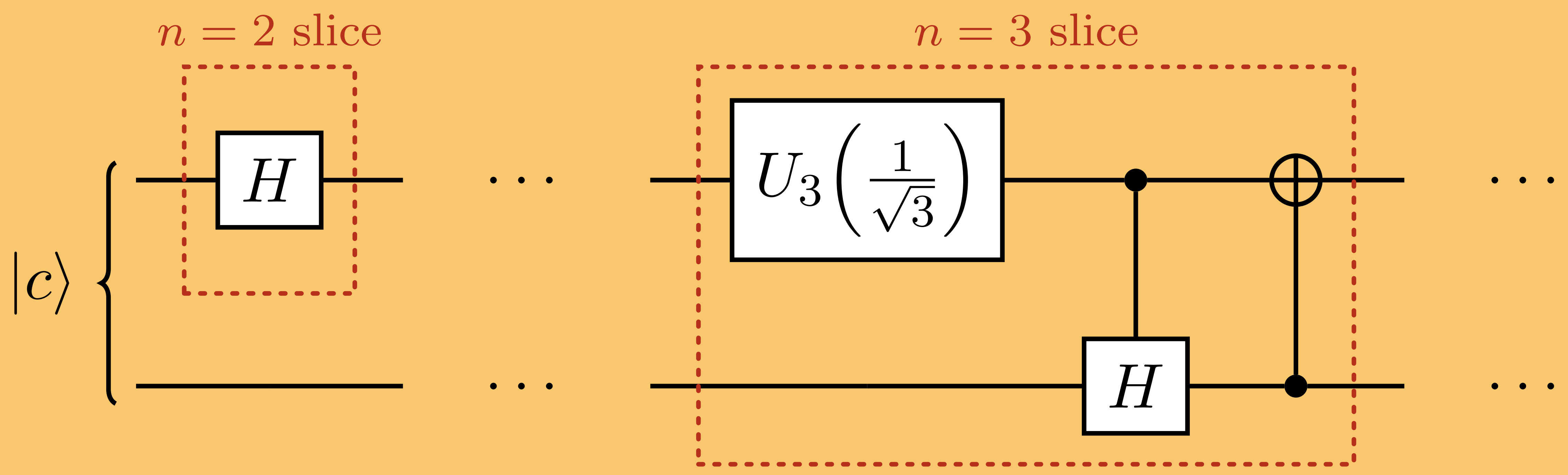
10 qubits
21 gate operations
(12 multi-qubit, 9 single qubit)

Running on a Quantum Device - Streamlined Circuit

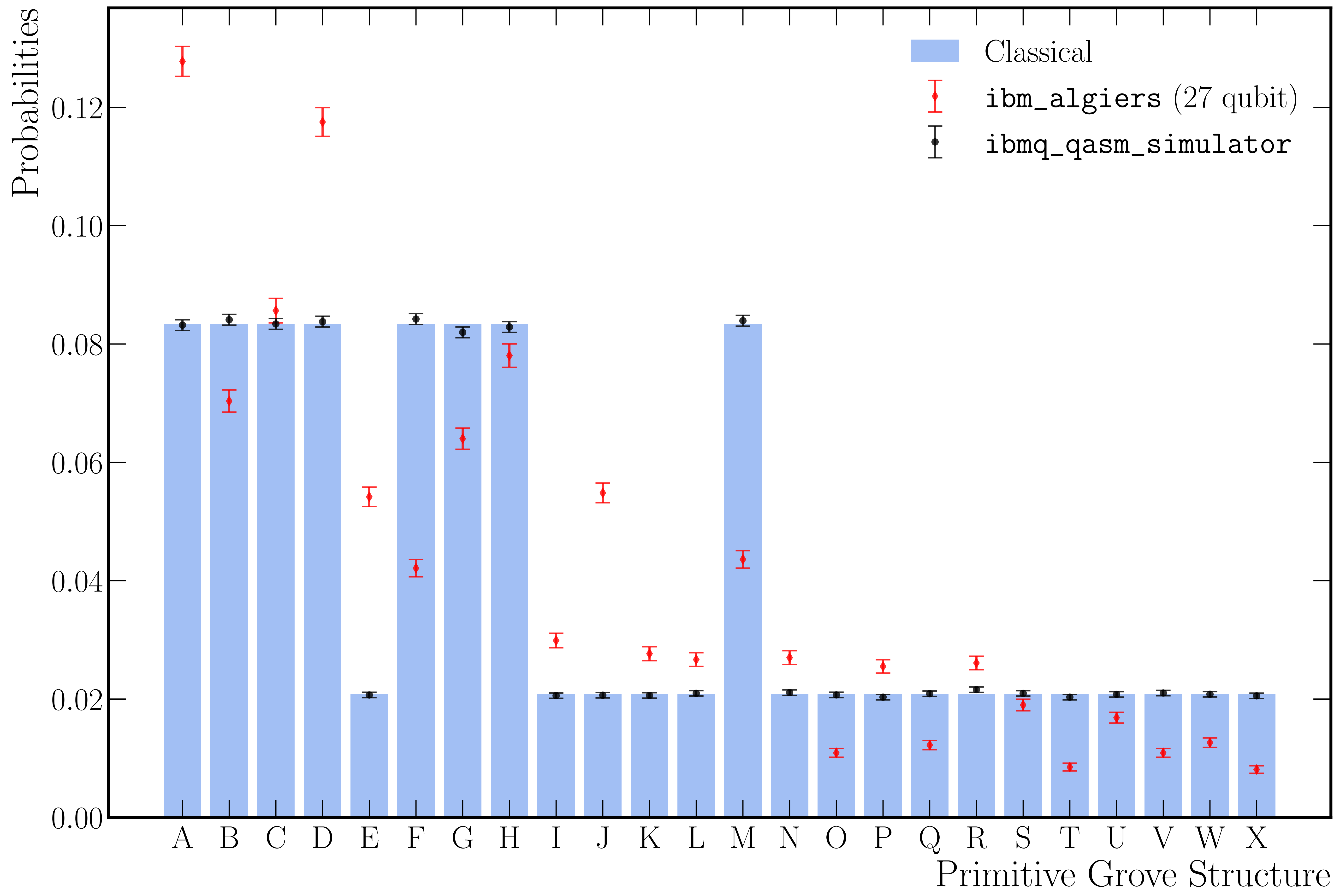


Coin operation constructs an equal superposition of n states on a coin register of m qubits:

$$C|0\rangle^{\otimes m} = \frac{1}{\sqrt{n}}\big(|0\rangle + |1\rangle + \ldots + |n-1\rangle\big)$$



Discrete QCD as a Quantum Walk - Raw Grove Simulation



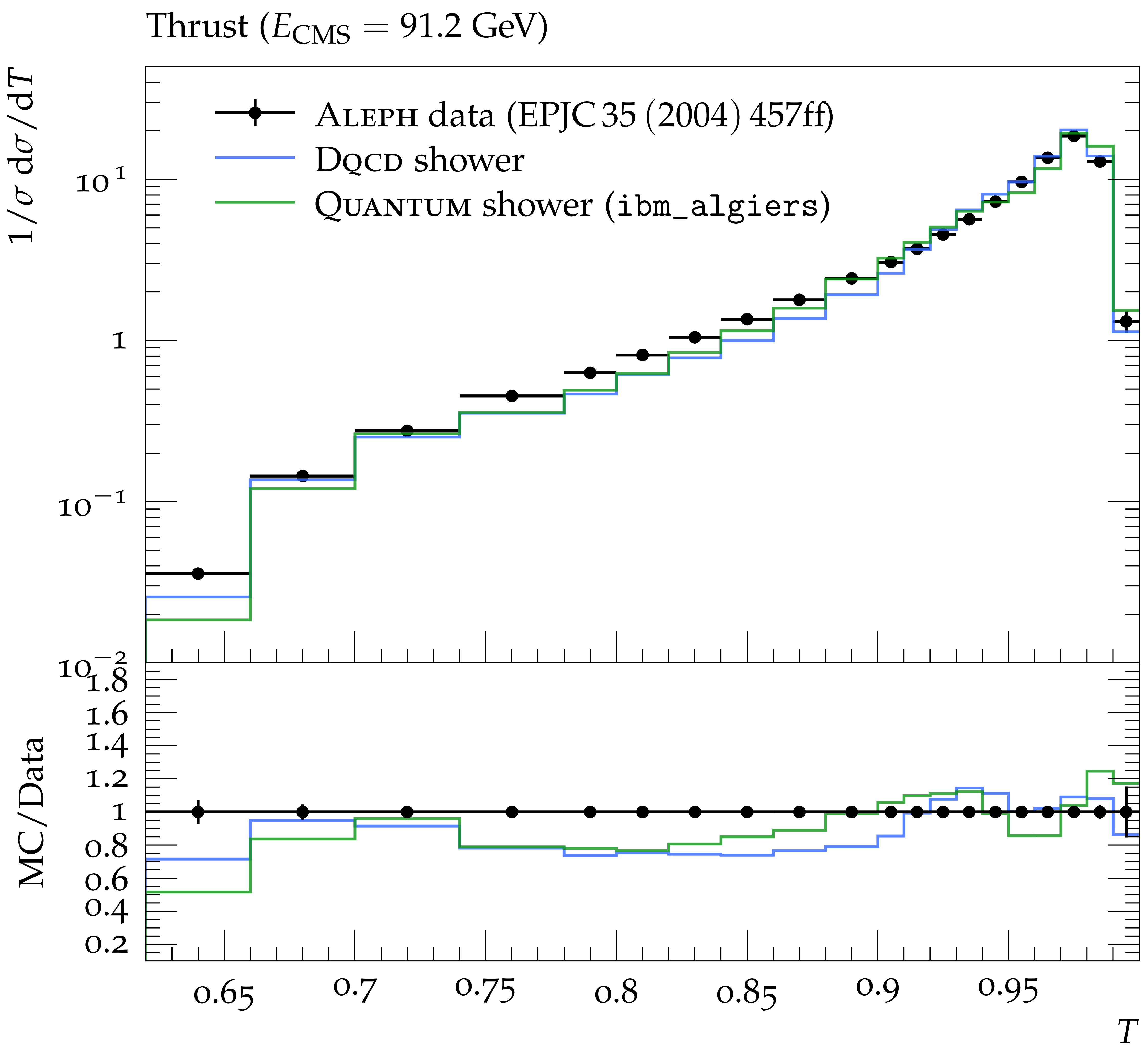
The algorithm has been run on the **IBM Falcon 5.11r chip**

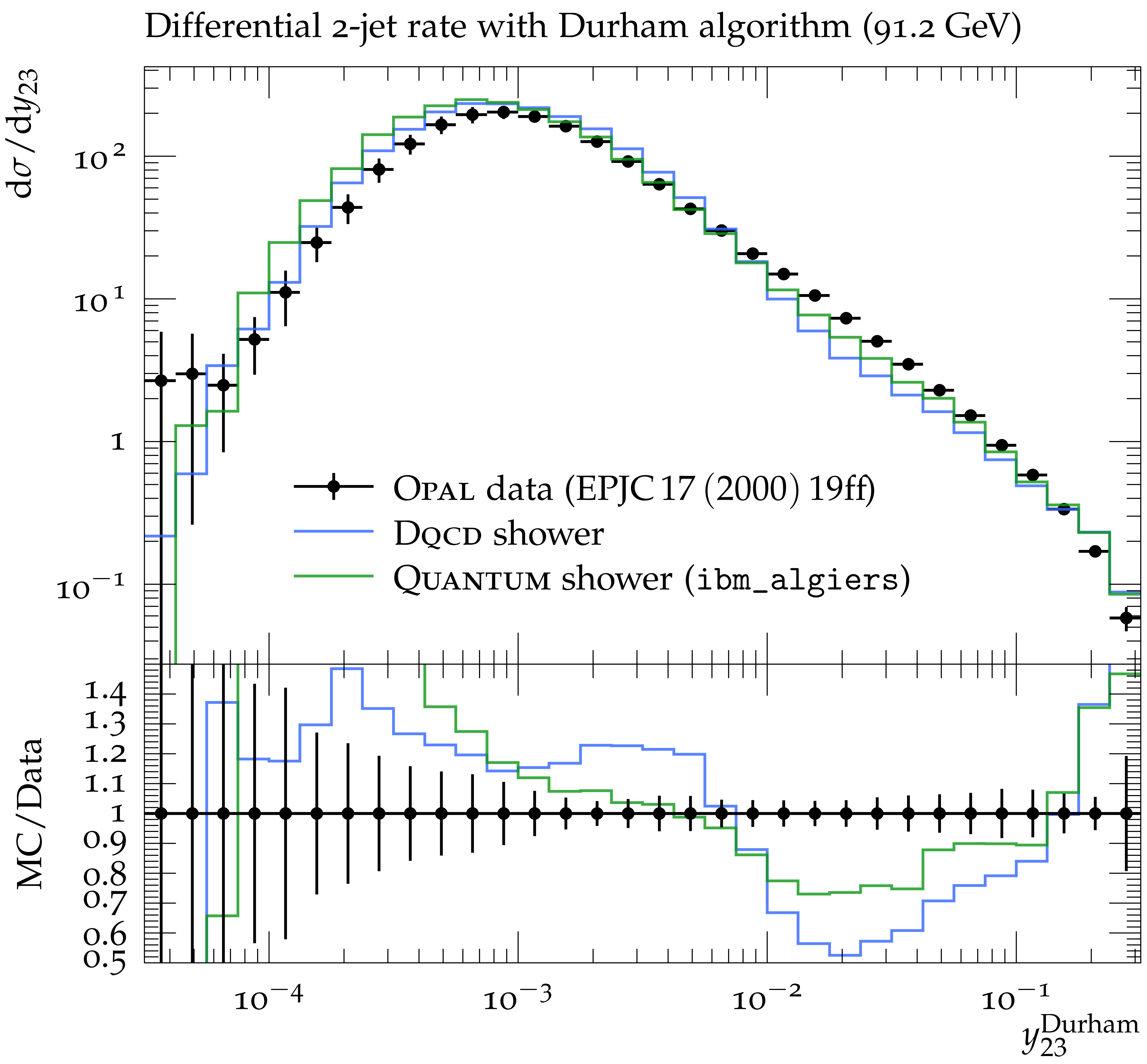
The figure shows the uncorrected performance of the **ibm_algiers** device compared to a simulator

The 24 grove structures are generated for a $E_{CM} = 91.2$ GeV, corresponding to typical collisions at LEP.

Main source of error from CNOT errors from large amount of SWAPs

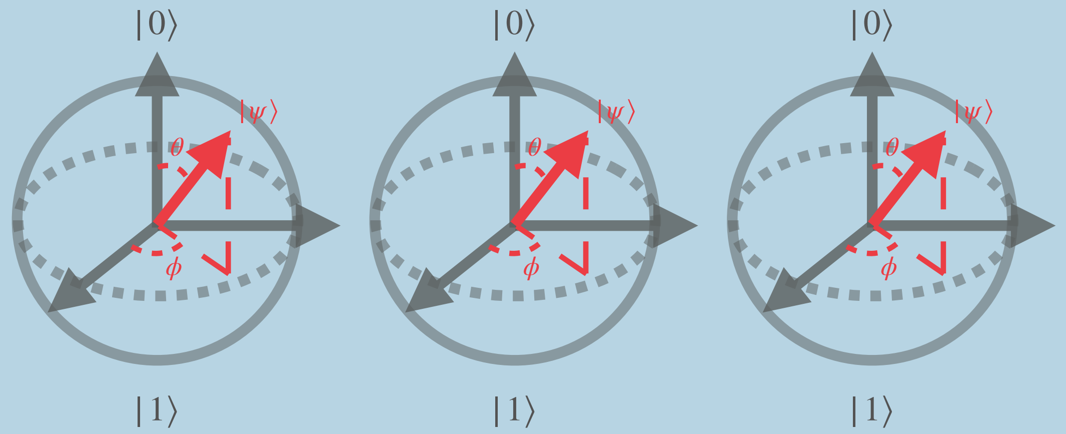
Collider Events on a Quantum Computer





The Future of Quantum Computing

More qubits?



A lot of emphasis on more qubits, but without fault tolerance, large qubit devices become **impractical**

Be better architects?

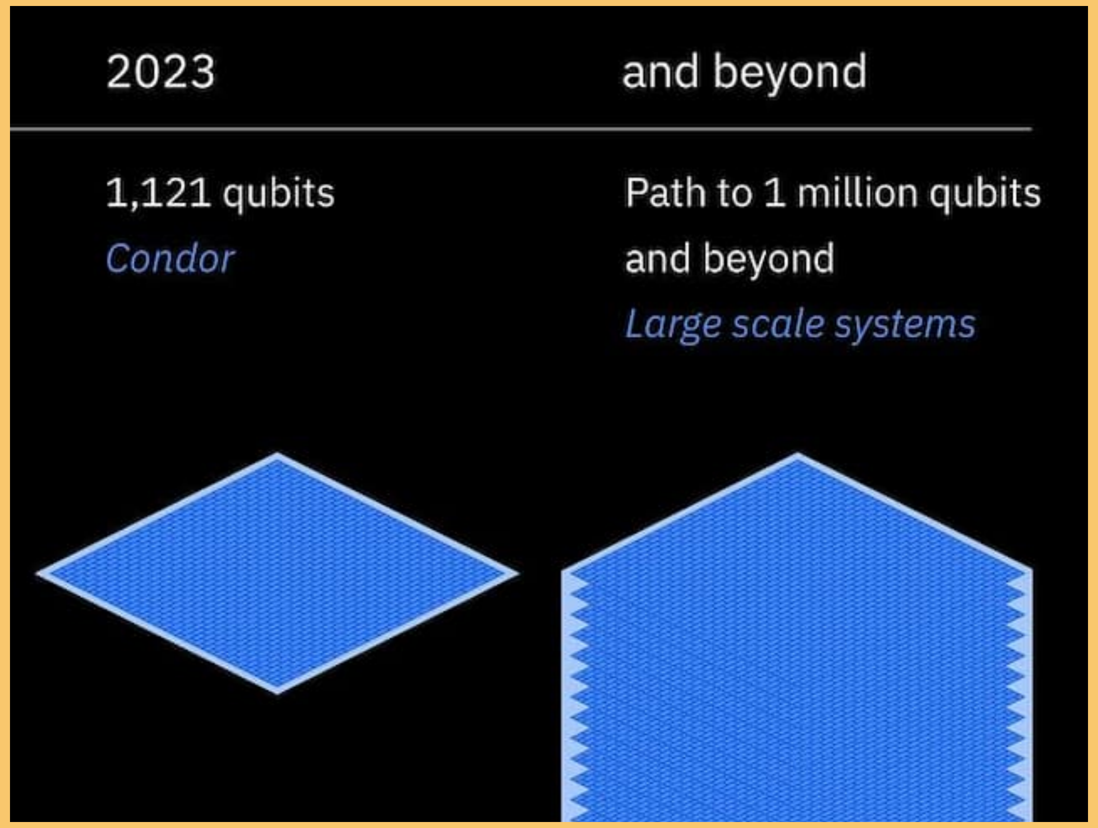
Realistic algorithms are already being created for NISQ devices. Efficient architectures allow for **practical algorithms** on NISQ devices.

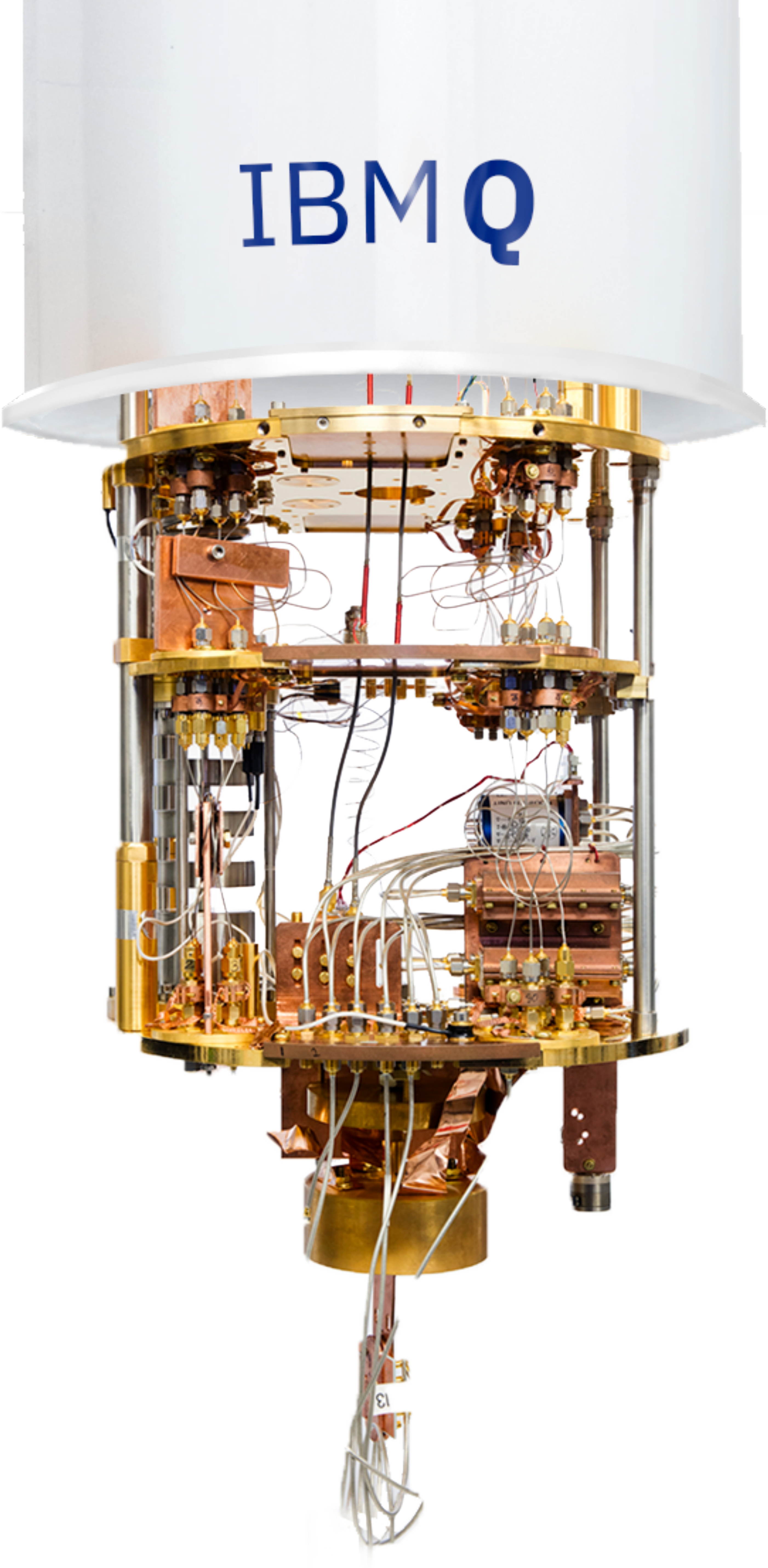
Better technology?

New technology could be the answer - will new qubit hardwares be more **fault tolerant?**

IBM Roadmap

On track to deliver **1000 qubits by 2023**





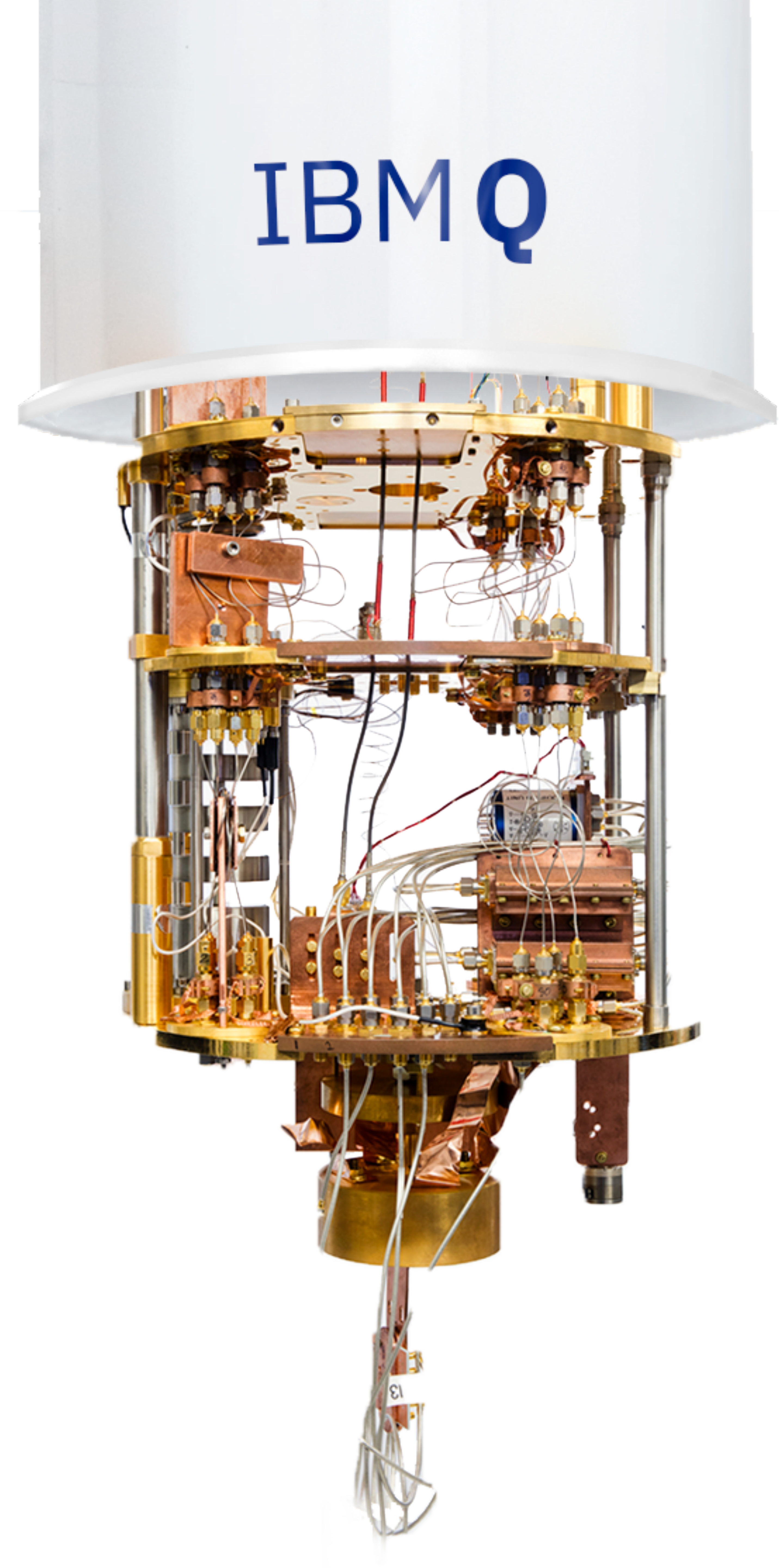
Summary

High Energy Physics is on the edge of a **computational frontier,** the High Luminosity Large Hadron Collider and FCC will provide **unprecedented amounts of data**

Quantum Computing offers an impressive and powerful tool to **combat computational bottlenecks**, both for theoretical and experimental purposes

The **first realistic simulation** of a **high energy collision** has been presented using a compact **quantum walk** implementation, allowing for the algorithm to be run on a **NISQ device**

Future Work: A dedicated research effort is required to fully evaluate the **potential** of **quantum computing** applications in **HEP**



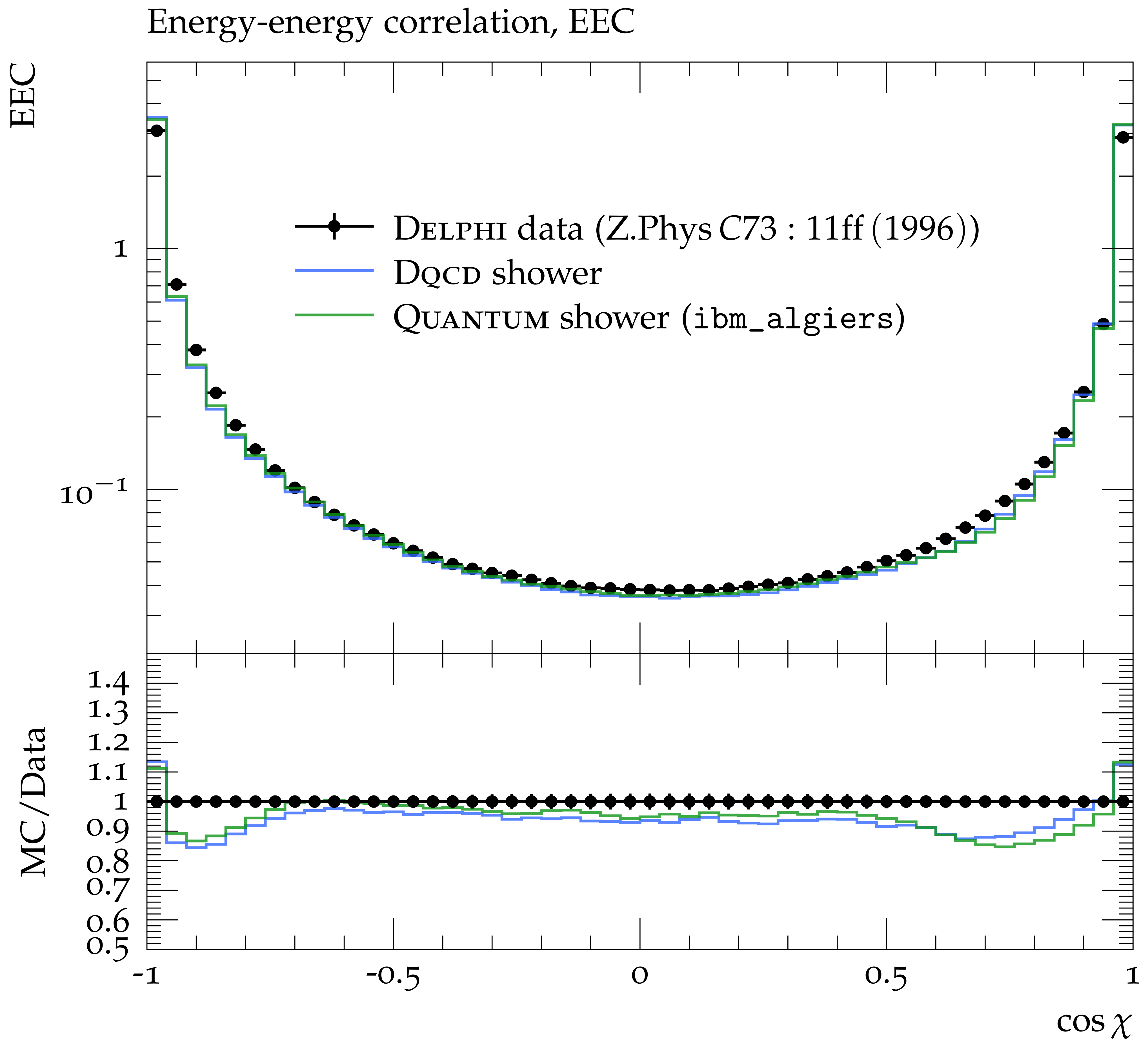
Imperial College London

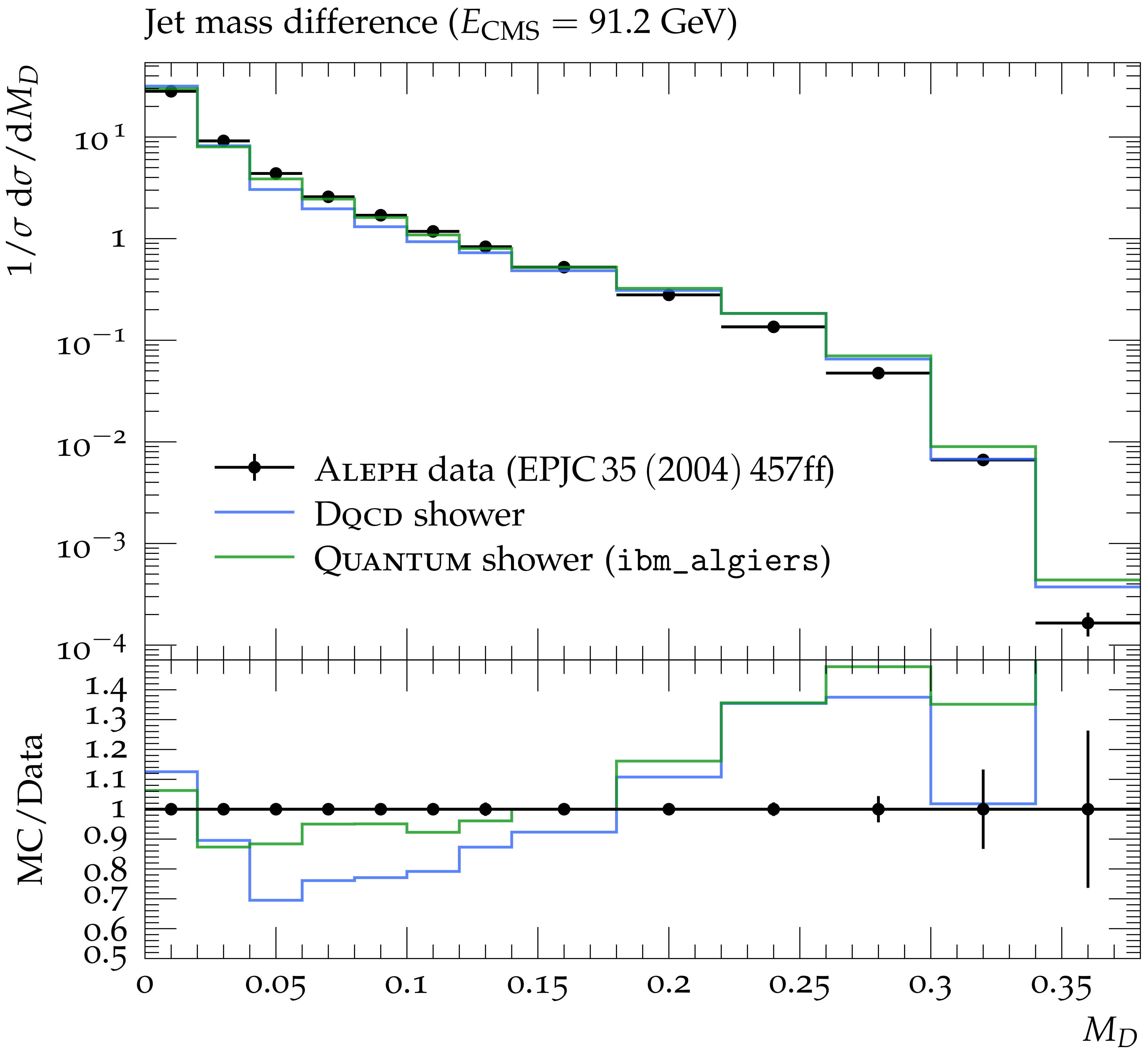
Backup Slides

Simon Williams

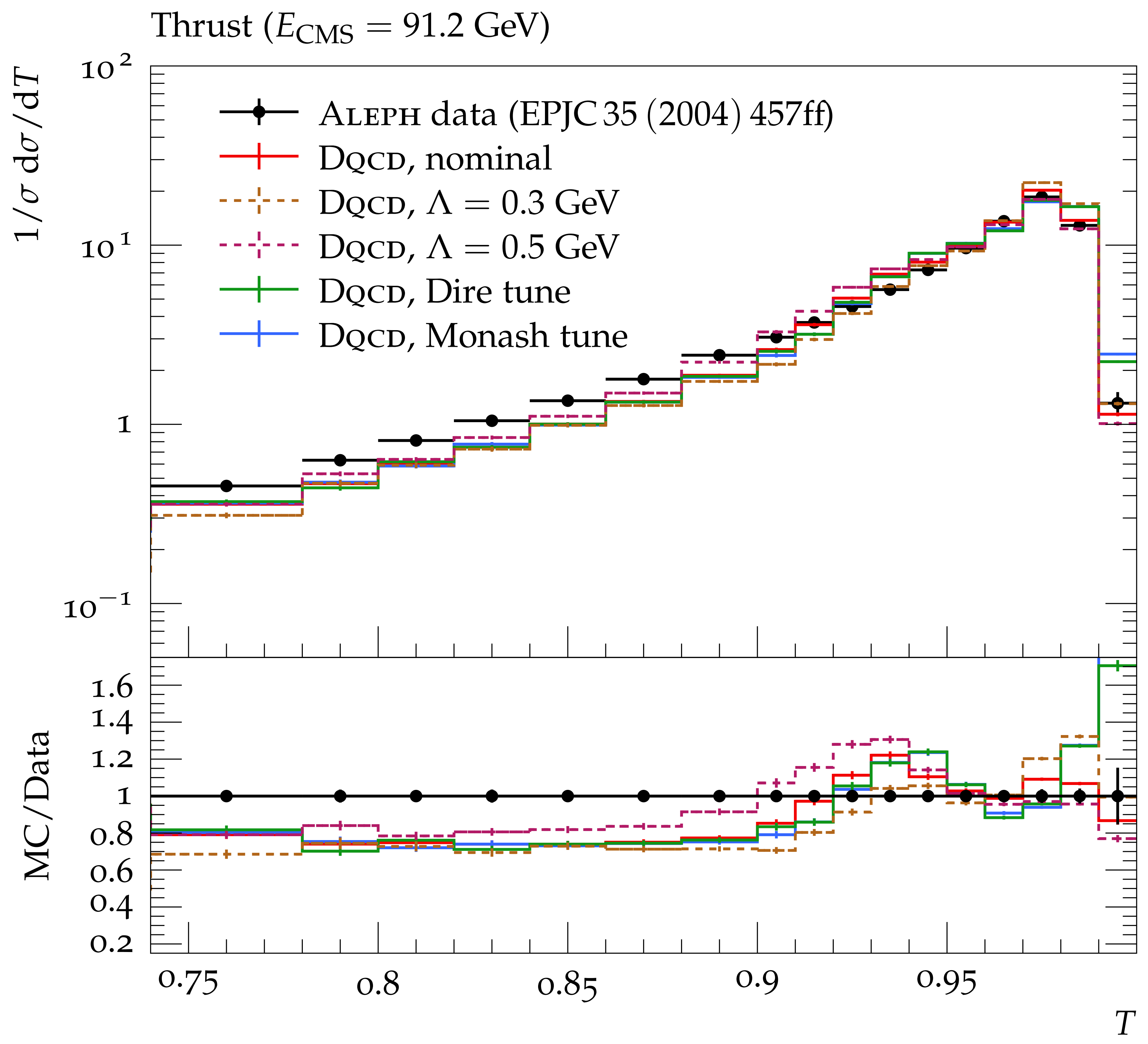
Milan Joint Phenomenology Seminars - 23rd January

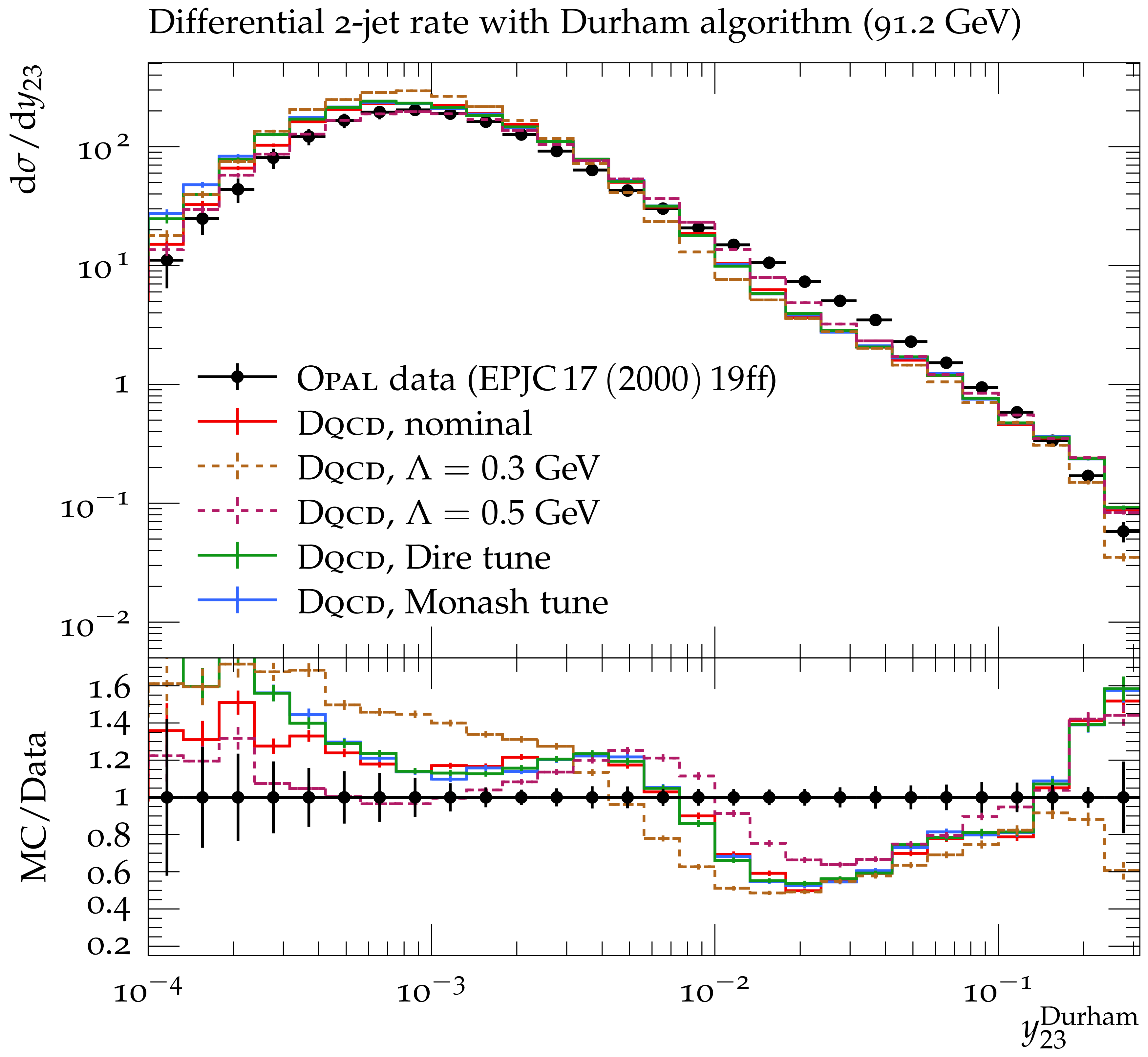
Collider Events on a Quantum Computer





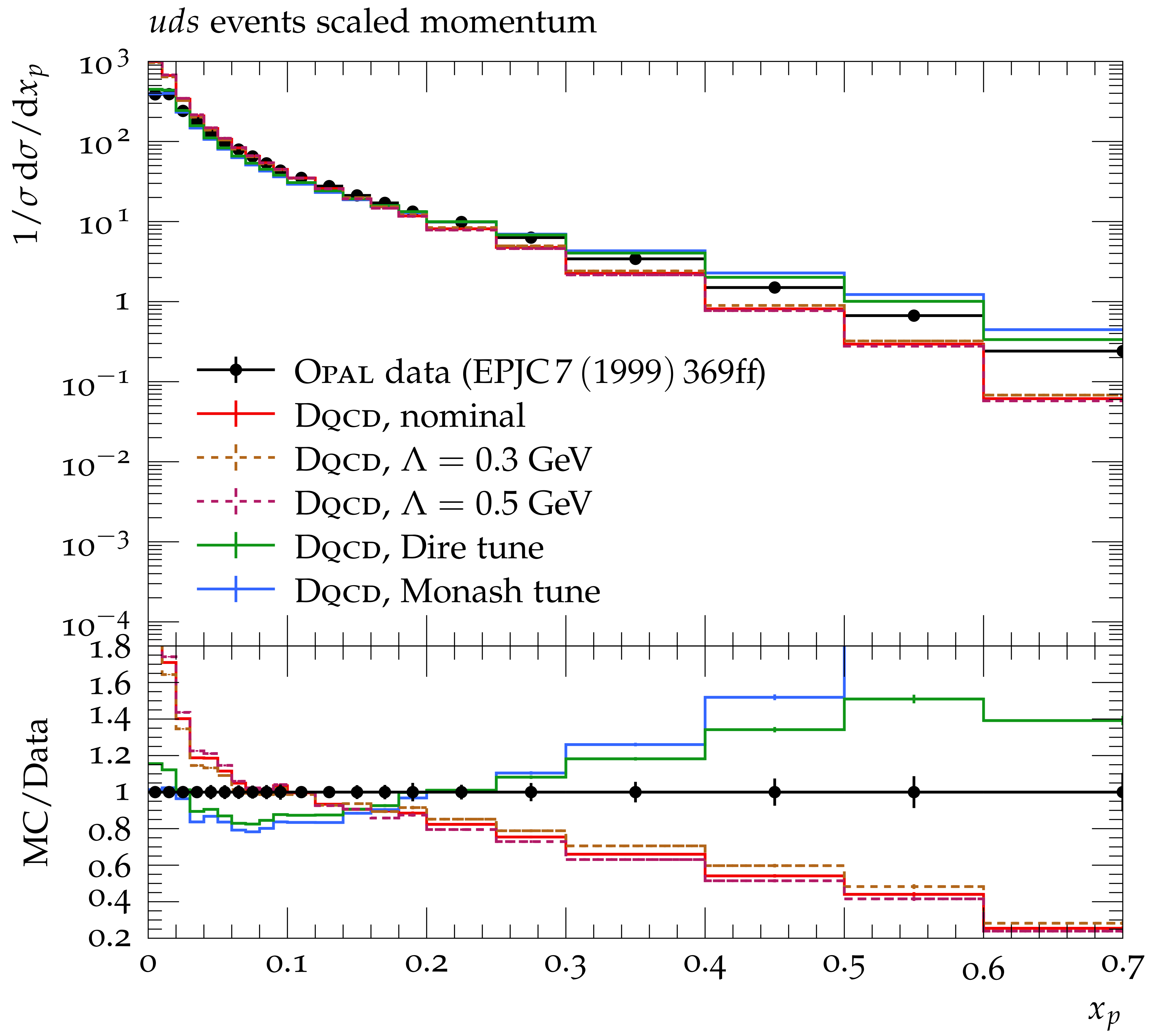
Collider Events on a Quantum Computer - Varying Λ

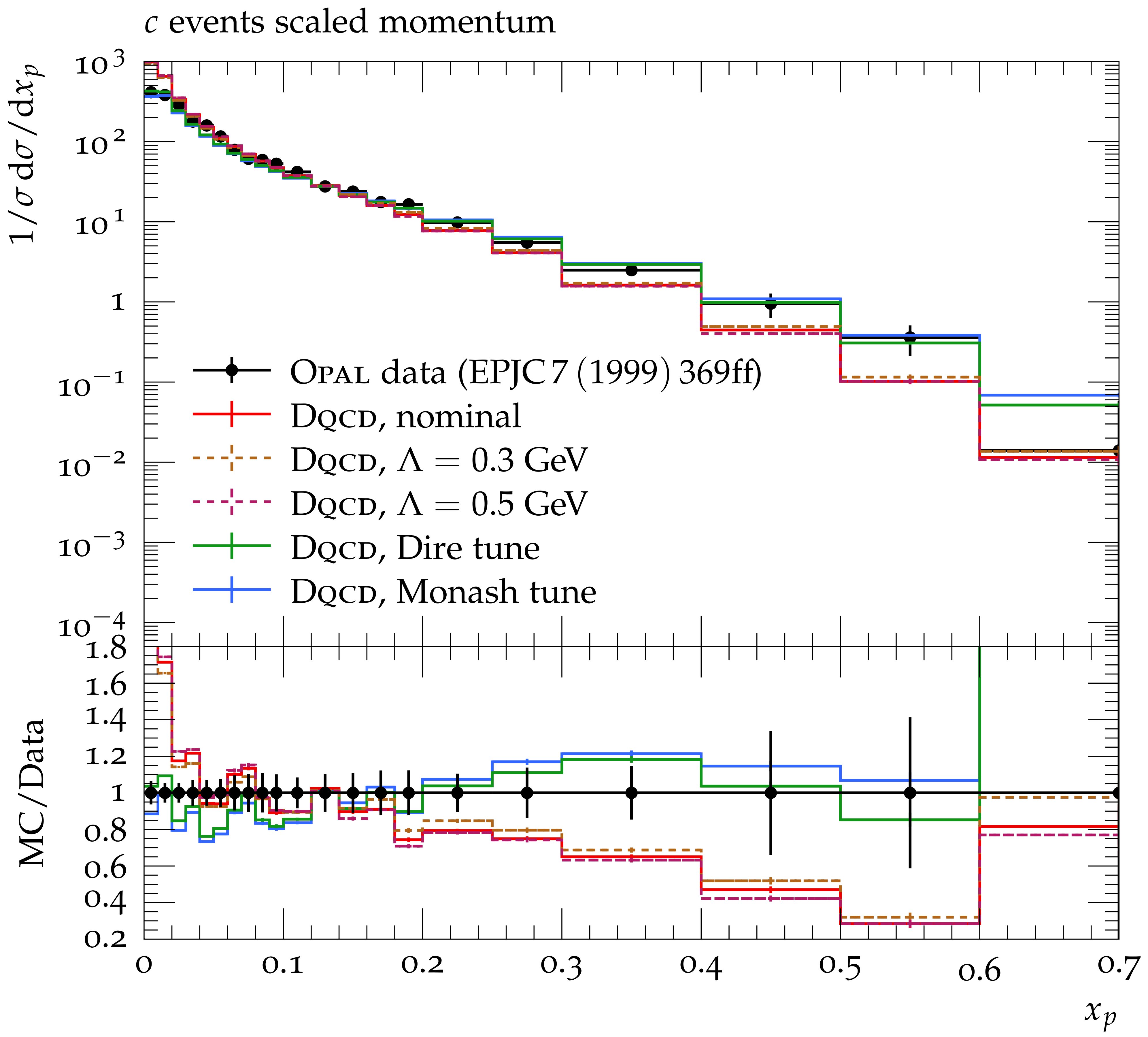




Varying values for the mass scale **Λ**. This leads to non-negligible uncertainties, however this is expected from a leading logarithm model.

Collider Events on a Quantum Computer - Varying Λ





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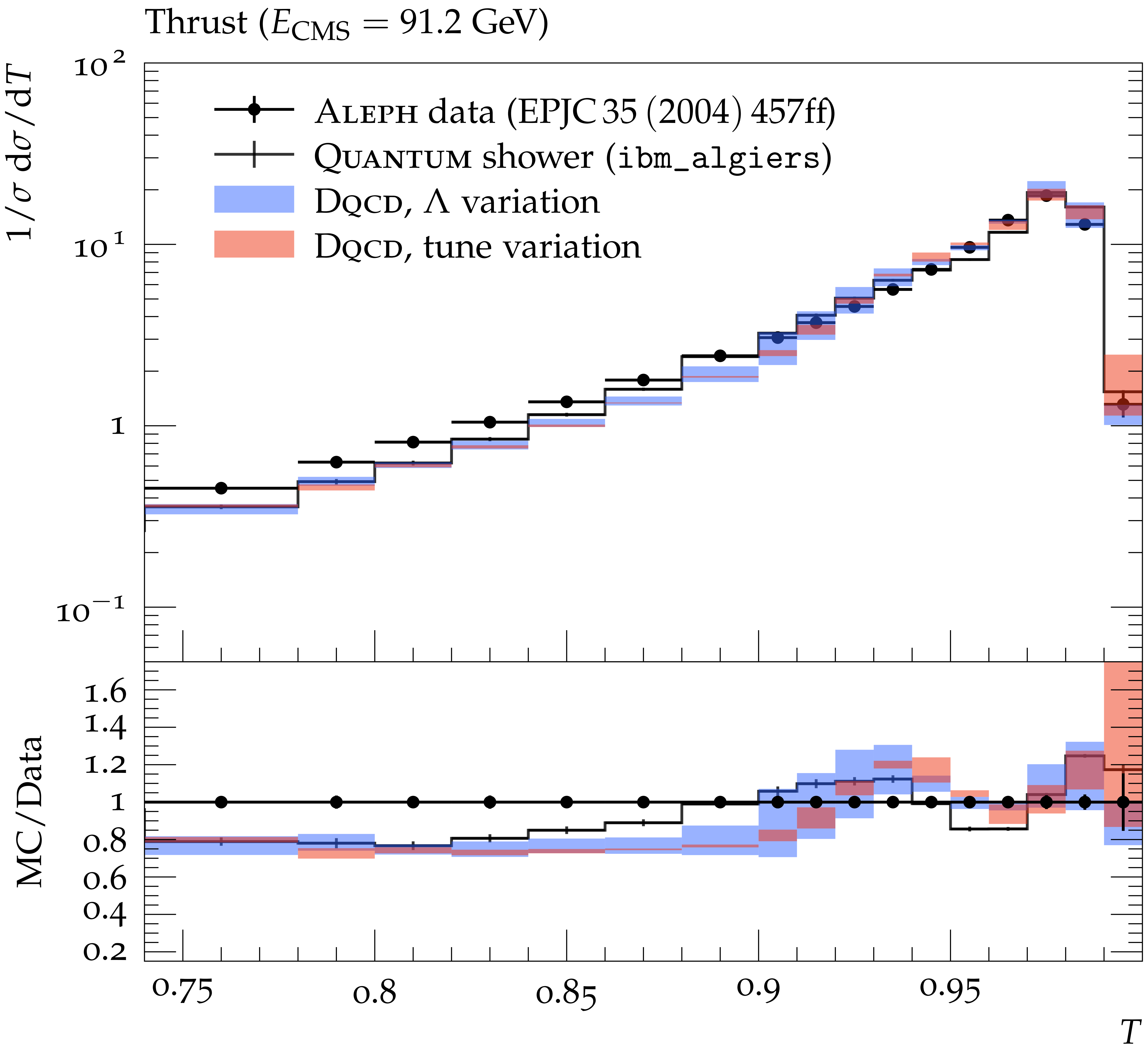
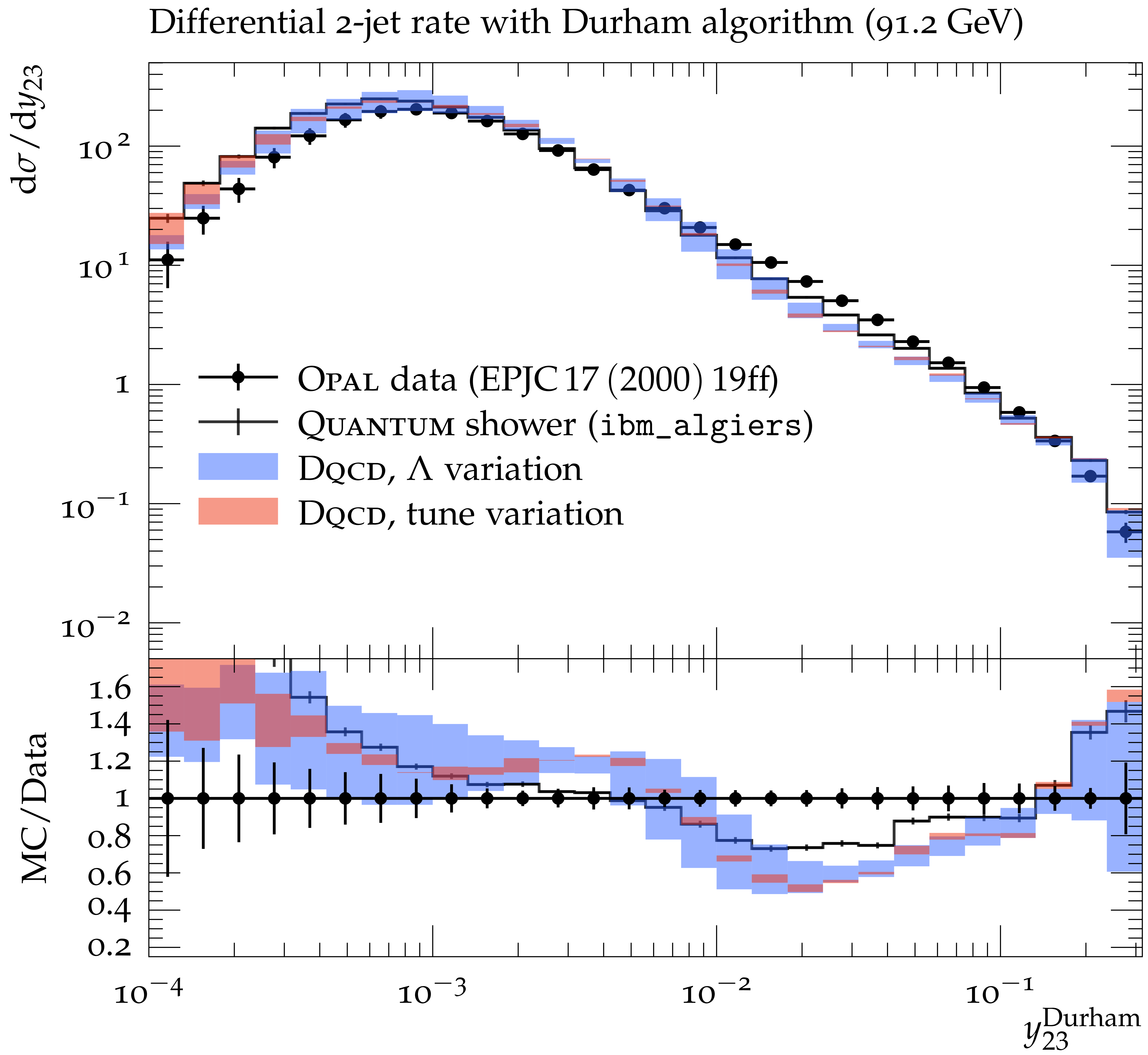
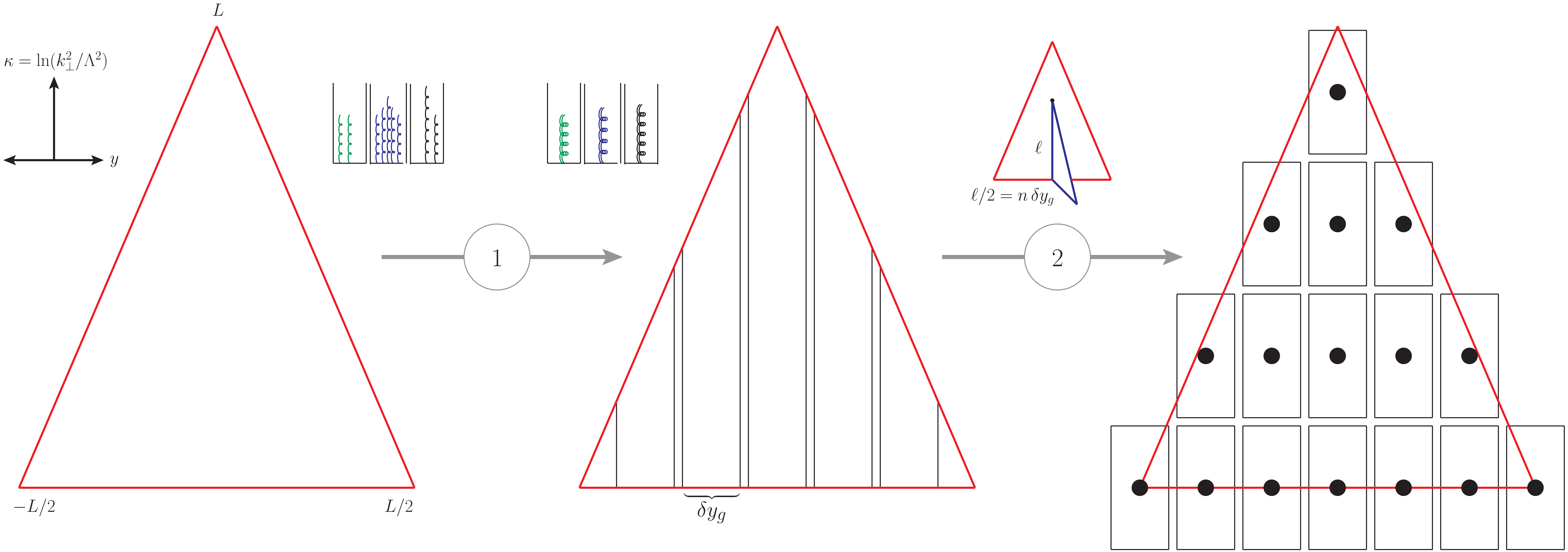




Collider Events on a Quantum Computer - Changing tune

Observables dominated by non-perturbative dynamics show mild dependence on the mass scale Λ, but are highly sensitive to changes in the tune.

Collider Events on a Quantum Computer



Looking to the Future of Quantum Computers