Quantum Computing

Lecture 1

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On the origins of chess

"Just one grain of wheat on the first square of a chessboard. Then put two on the second square, four on the next, then eight, and continue, doubling the number of grains on each successive square, until every square on the chessboard is reached."

"Sure thing fam." – famous last words

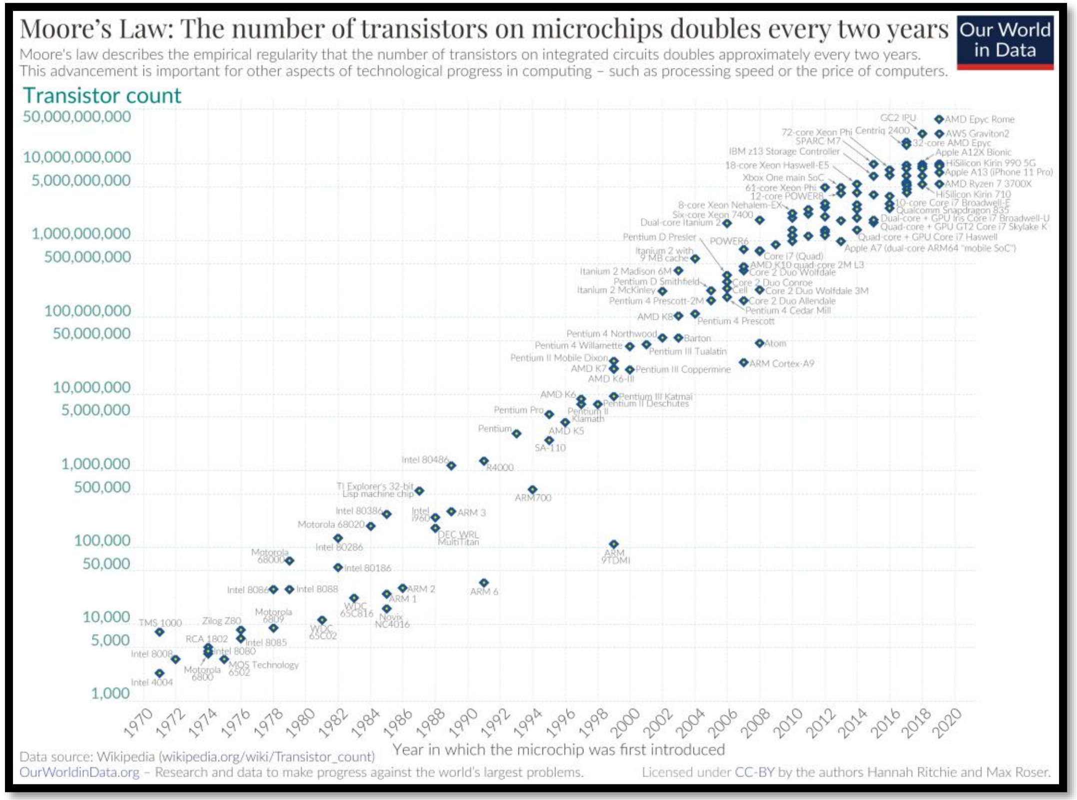
18,446,744,073,709,551,615 grains (2^{64})

Over 2,000 times the annual world production of wheat!

Moore's "Law"

- More of an observation by Gordon Moore of Intel (in 1965)
- Processing powers of (classical) computers double every 2 years
- Production cost correspondingly halve
- Number of transistors on a CPU chip double every two years (18 months)
- Moore's Law estimated to end sometime in the 2020s. Many experts agree that Moore's Law is no longer true
- Transistors can't go any smaller without adverse temperature and quantum effects

ourworldindata.org



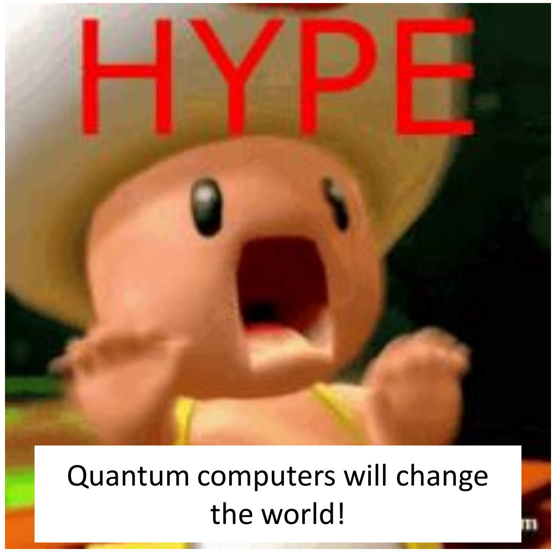
What is a classical computer?

- A classical computer utilizes processing chips
- A chip utilizes transistors
- A transistor is like a light switch (on or off)
- First computer by Intel had around **2300 transistors** (1971)
- Apple's M1 Max chip in the latest Macbook pro has **57 billion transistors**
- Cerebras supercomputing processor has **2.6 trillion transistors**

What is a quantum computer?

Let’s discuss what it is not first:

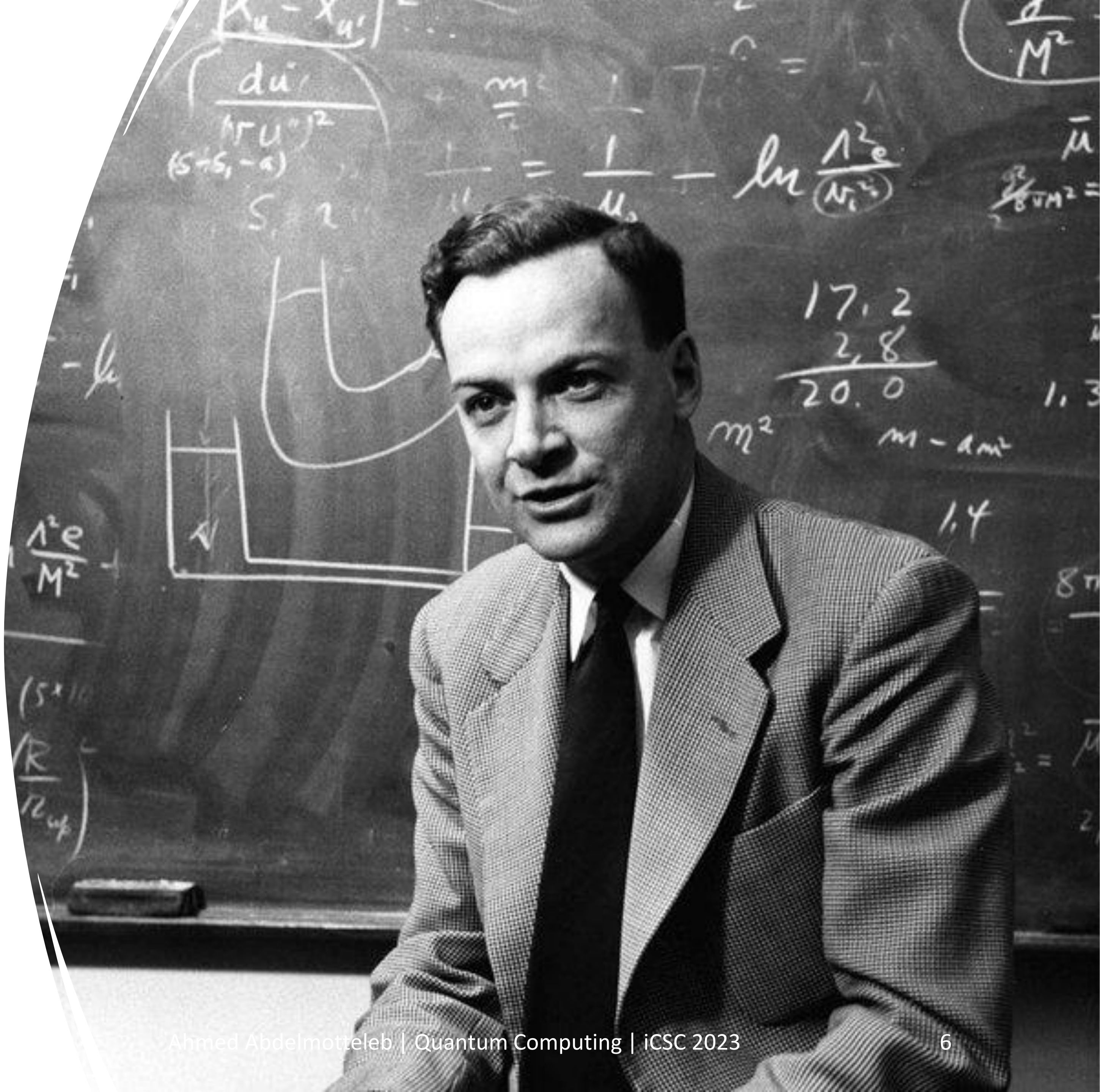
- It is not the end of classical, silicon-based computers
- It is not the savior of Moore’s law
- It is not the destroyer of the world of finance, cryptography and world order as we know it
- It cannot work without classical computers



We’ll address some more misconceptions in detail as we go along

“Let the computer itself be built of quantum mechanical elements which obey quantum mechanical laws.”

-Richard Feynman

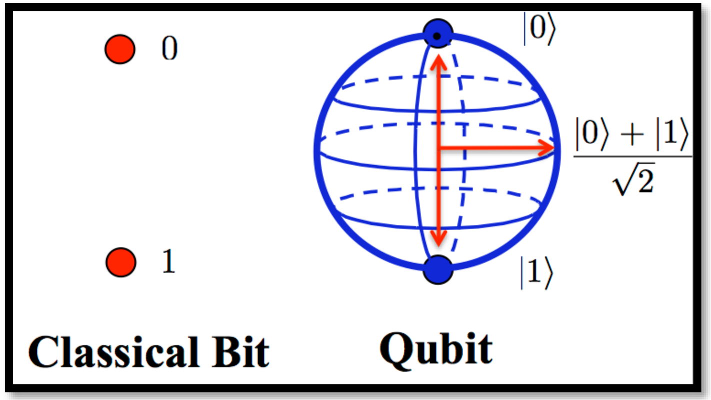


What is a quantum computer? - continued

- A system that utilizes atomic or subatomic particles and quantum mechanics to do computations
- In contrast to classical computing bits, which can be either 0 or 1, quantum bits or “qubits” can take an arbitrary combination of 0 and 1 (more on that in a “bit”)
- Many types of qubits utilized, including superconducting (IBM, Google), photonics (Xanadu, PsiQuantum), trapped ions (Honeywell, IONQ), annealing (D-Wave), and many more
- Utilises 3 main principles of quantum mechanics: **superposition**, **entanglement** and **interference**

Classical bit vs quantum bit

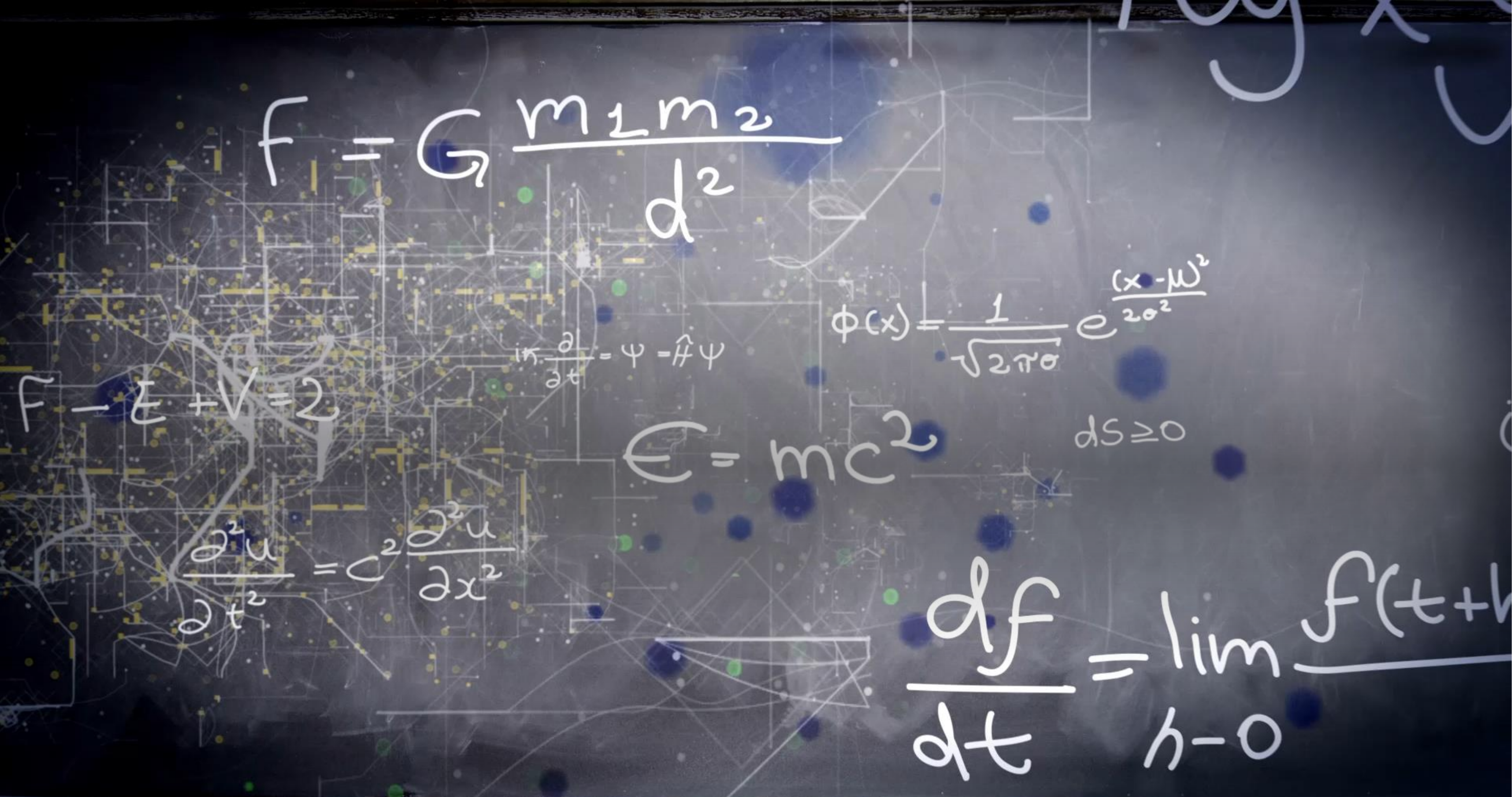
- Classical computers operate using strings of zeros and ones, such as 01100111 01101111 01110100 01100011 01101000 01100001
- Each option (0 or 1) is distinguishable and represents a physical state (e.g. light switch on or off, magnet up or down)
- Command issued by a certain string of "bits" that corresponds to specific physical states
  → leads to the triggering of the command
- A quantum bit (qubit) can exist in either one of the two discrete states (0 or 1), or it can exist in both states simultaneously
- Thanks to the principles of **superposition** and **entanglement**, we can have parallel processing (carrying out many processes simultaneously) – more on that shortly



Classical bit vs quantum bit - continued

- Coming back to exponentials, a classical computer's computational power increases as $\mathbf{2n}$, where n is the number of bits (linear)
- However, a quantum computer's computational power increases as $\mathbf{2^n}$ due to superposition, entanglement and the existence of many different outcomes for each combination of qubits (exponential)
- Therefore, if you have a classical chip with 1 million transistors, you can theoretically substitute it for a quantum chip with just 20 qubits*
- A classical chip with 1 billion transistors = quantum chip with just 30 qubits*
- A classical chip with 1 trillion transistors = quantum chip with just 40 qubits*

*in real life, we should also consider noise effects


F = G m1m2 / d2
ϕ(x) = 1/√2πσ e
dS ≥ 0
E = mc2
F − E + V = 2
∂2u/∂t2 = c2 ∂2u/∂x2
df/dt = lim h→0

Understanding quantum computing is not scary, it's just linear algebra

Mathematical aside 1 – Complex numbers and Dirac notation

$$c = a + ib,$$
$$c^* = a - ib,$$
$$a, b \in \mathbb{R}$$

$*$ = Complex conjugate
$\dagger$ = Hermitian conjugate

Assuming you know what a vector is
Let $\mathrm{d}, e \in \mathbb{C}^2$

ket is a column vector:

$$|d\rangle = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

bra is a row vector:

$$\langle e| = |e\rangle^\dagger = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}^\dagger = (e_1^* \quad e_2^*)$$

bra-ket is an inner product:

$$\langle e|d\rangle = (e_1^* \quad e_2^*)\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = d_1 e_1^* + d_2 e_2^*$$
$$= e_1^* d_1 + e_2^* d_2 = \langle d|e\rangle^*$$

ket-bra is a matrix multiplication:

$$|d\rangle\langle e| = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}(e_1^* \quad e_2^*)$$
$$= \begin{pmatrix} d_1 e_1^* & d_1 e_2^* \\ d_2 e_1^* & d_2 e_2^* \end{pmatrix}$$

Properties of Quantum Systems

1. Superposition
2. Entanglement
3. Interference
4. Decoherence and noise

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Quantum basis states

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

"." notation is used to represent multiplying

- **Orthogonal** states, i.e. $\langle 0|1\rangle = 1.0 + 0.1 = 0$ and $\langle 1|0\rangle = 0$ → $\langle \psi|\psi'\rangle = 0$
- **Normalized** states, i.e. $\langle 0|0\rangle = 1$ and $\langle 1|1\rangle = 1$ → $\langle \psi|\psi\rangle = 1$

<u>Example (1 qubit):</u>

$$|\psi\rangle = a|0\rangle + b|1\rangle,$$

where a, b are **amplitudes** (corresponding to probabilities)

$$a^2 + b^2 = 1,$$

ψ is called a **statevector.**

Quantum basis states - continued

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$ → z-basis states

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$ → x-basis states

$$|+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle), \qquad |-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$ → y-basis states

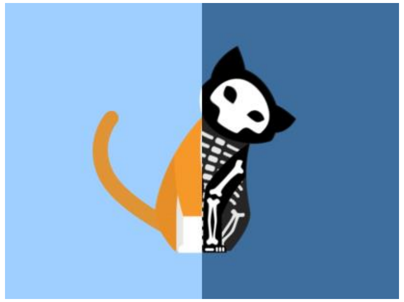
N.B. $|+i\rangle$ and $|-i\rangle$ are sometimes called $|R\rangle$ and $|L\rangle$ respectively

Will come back to these again when we discuss Bloch spheres

Superposition

$$|\psi\rangle = a|0\rangle + b|1\rangle,$$

- Statevector c (1 qubit) is not entirely $|0\rangle$ or entirely $|1\rangle$
- Linear combination of the two. Simultaneously both.
- Like flipping a coin and the coin is always rotating in the air
- Only when you do a **measurement**, you settle on **one** of the possible states
- **Probability** of measuring state $|0\rangle$ is a^2, and for $|1\rangle$ it's b^2
- A qubit packs more information than a classical bit
- Possibilities scale as $\mathbf{2^n}$, where n is the number of qubits



Born rule

- Postulate of quantum mechanics → probability that a measurement of a quantum system will yield a given result

- Suppose you have a state $|\psi\rangle$

- A measurement is performed in the orthonormal basis $(|x_1\rangle, |x_2\rangle)$

- $P(x_1) = |\langle x_1|\psi\rangle|^2, \; P(x_2) = |\langle x_2|\psi\rangle|^2$

- $\sum_i P(x_i) = 1$

Born rule - continued

- $P(x_1) = |\langle x_1|\psi\rangle|^2, \;\; P(x_2) = |\langle x_2|\psi\rangle|^2$
- $\sum_i P(x_i) = 1$

- Example 1: state $|\psi\rangle = \frac{1}{\sqrt{3}}\left(|0\rangle + \sqrt{2}|1\rangle\right)$ is measured in the orthonormal basis $(|0\rangle, |1\rangle)$ [z-basis]
- $P(0) = |\langle 0|\psi\rangle|^2 = \left|\left\langle 0\right|\frac{1}{\sqrt{3}}\left(|0\rangle + \sqrt{2}|1\rangle\right)\right\rangle\Big|^2 = \left|\frac{1}{\sqrt{3}}\overbrace{\langle 0|0\rangle}^{1} + \frac{2}{\sqrt{3}}\cancelto{0}{\langle 0|1\rangle}\right|^2 = \frac{1}{3}$
- Try to prove that $P(1) = \frac{2}{3}$ $[P(0) + P(1) = 1]$

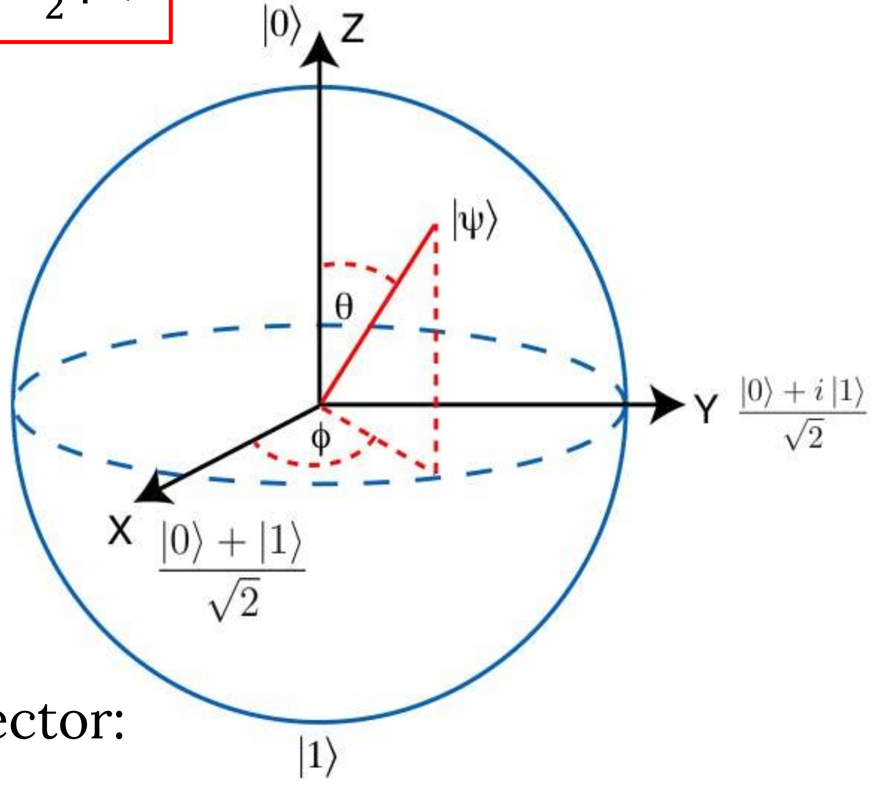
Born rule - continued

- Example 2: state $|\psi\rangle = \frac{1}{\sqrt{2}}\,(|0\rangle - |1\rangle)$ is measured in the orthonormal basis $(|+\rangle, |-\rangle)$ [x-basis]
- $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $\qquad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
- $P(+) = |\langle + | \psi \rangle|^2 = \left| \frac{1}{\sqrt{2}}(\langle 0| + \langle 1|) . \frac{1}{\sqrt{2}}\,(|0\rangle - |1\rangle) \right|^2 = \frac{1}{4} |\overbrace{\langle 0|0\rangle}^{1} - \cancelto{0}{\langle 0|1\rangle} + \cancelto{0}{\langle 1|0\rangle} - \overbrace{\langle 1|1\rangle}^{1}|^2 = 0$
- This is to be expected because $|\psi\rangle = \frac{1}{\sqrt{2}}\,(|0\rangle - |1\rangle) = |-\rangle$
- $\langle + | - \rangle = 0, \quad \langle - | - \rangle = 1$
- Try to prove that $P(-) = 1 \;\; [P(+) + P(-) = 1]$

Bloch Sphere

- Can express any normalized state as $|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\varphi}sin\frac{\theta}{2}|1\rangle$
- $\varphi \in [0, 2\pi)$ describes the relative phase (phase between different states – what interference relies on)
- $\theta \in [0, \pi]$ determines the probability to measure the state
- $P(0) = \cos^2\frac{\theta}{2}, \quad P(1) = \sin^2\frac{\theta}{2}, \quad P(0) + P(1) = 1$
- A state can be illustrated on the Bloch sphere
- The coordinates of a state are determined by the Bloch vector:

$$\bar{r} = \begin{pmatrix} sin\theta cos\theta \\ sin\theta sin\varphi \\ cos\theta \end{pmatrix}$$

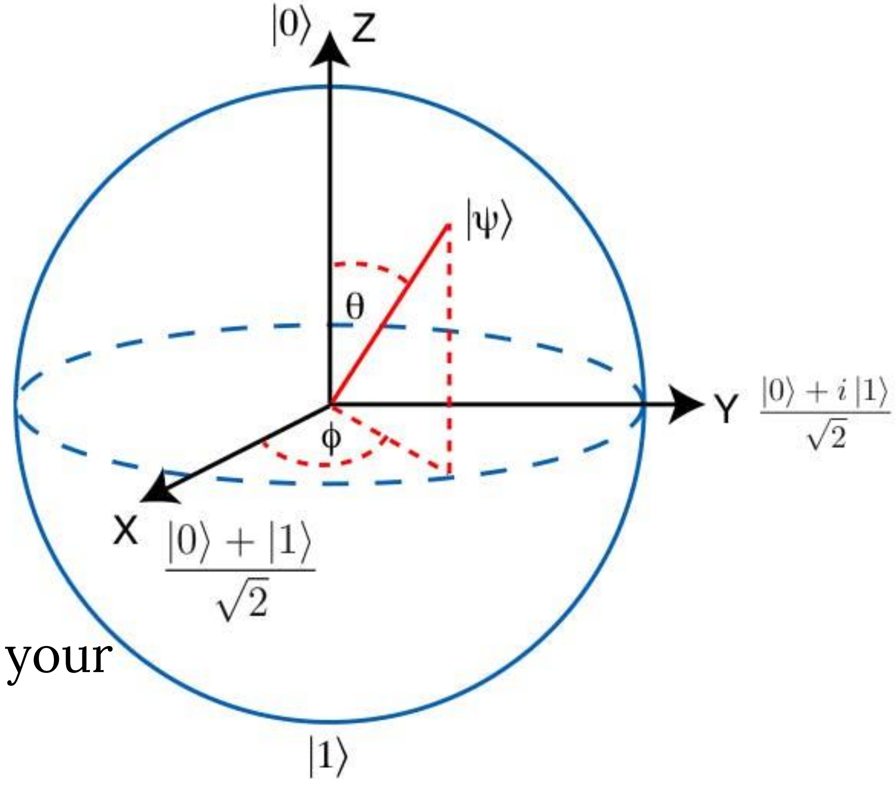


Bloch Sphere - continued

$$|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\varphi}sin\frac{\theta}{2}|1\rangle$$

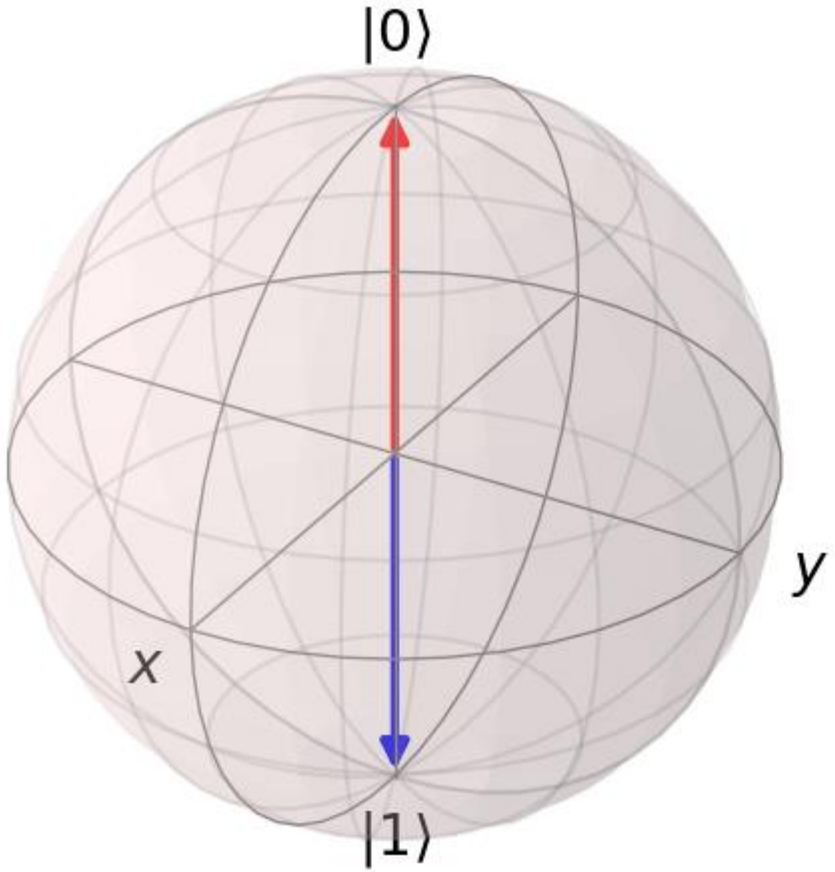
$$\bar{r} = \begin{pmatrix} sin\theta cos\theta \\ sin\theta sin\varphi \\ cos\theta \end{pmatrix}$$

- Example 1: $|0\rangle$: $\theta = 0$, $\bar{r} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$
- Example 2: $|1\rangle$: $\theta = \pi$ (180°), $\bar{r} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$
- Example 3: $|+\rangle$: $\theta = \frac{\pi}{2}$, $\varphi = 0$, $\bar{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$



- Note that angles on Bloch sphere are double what you get in your normal (Hilbert) space
- Z-measurement corresponds to a projection onto the z-axis
- Similarly for x and y-measurements

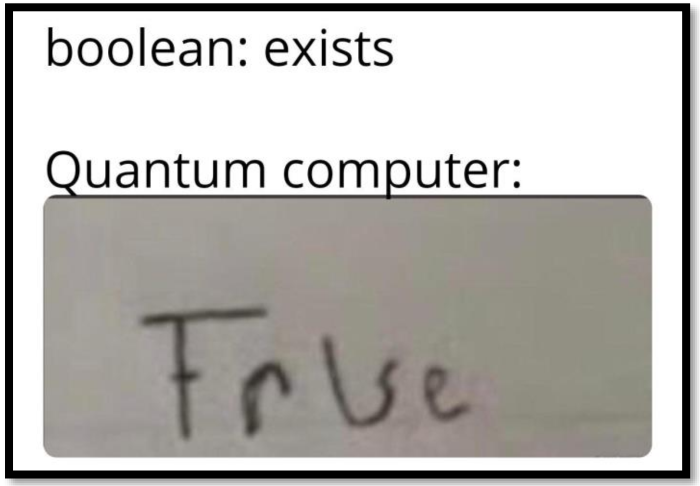
Bloch Sphere - continued



https://javafxpert.github.io/grok-bloch/

Superposition – takeaway points

- A classical bit can only be in the state corresponding to 0 or the state corresponding to 1
- Superposition is an ambiguous state in which a particle can be both a 0 and a 1
- The probability of measuring 0 or 1 for a qubit is in general neither 0% nor 100%
- The statevector collapses into one of the states when observed/an interaction with the environment occurs (like stopping a coin from flipping in the air)
- A Bloch sphere can be used to illustrate a quantum state



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Entanglement

- A feature of quantum mechanics that is not present in classical mechanics (i.e. confusing)
- Describes correlations between particles (on a quantum level)
- A group of particles are generated, interact, or share spatial proximity in a certain way
- Grouped particles are so intertwined that one constituent cannot be fully described without considering the other(s)
- Quantum state of each particle of the group cannot be described independently of the state of the others, even if separated by huge distances
- Described by Einstein as “spooky action at a distance”
- Root of very deep and very fundamental disagreements on how we define quantum mechanics and how we interpret it

Entanglement - continued

- Subject of the 2022 Nobel Prize in Physics

- In March 2022, NASA announced it would be sending a quantum entanglement experiment to space

Example 1:

- You are hungry, so you order a burger and a hotdog from a local restaurant

- You receive your order in 2 identical, unlabeled boxes

- If you open one box, and discover it contains a burger, then, by definition, the other box contains a hotdog (assuming the restaurant didn't mess up your order)

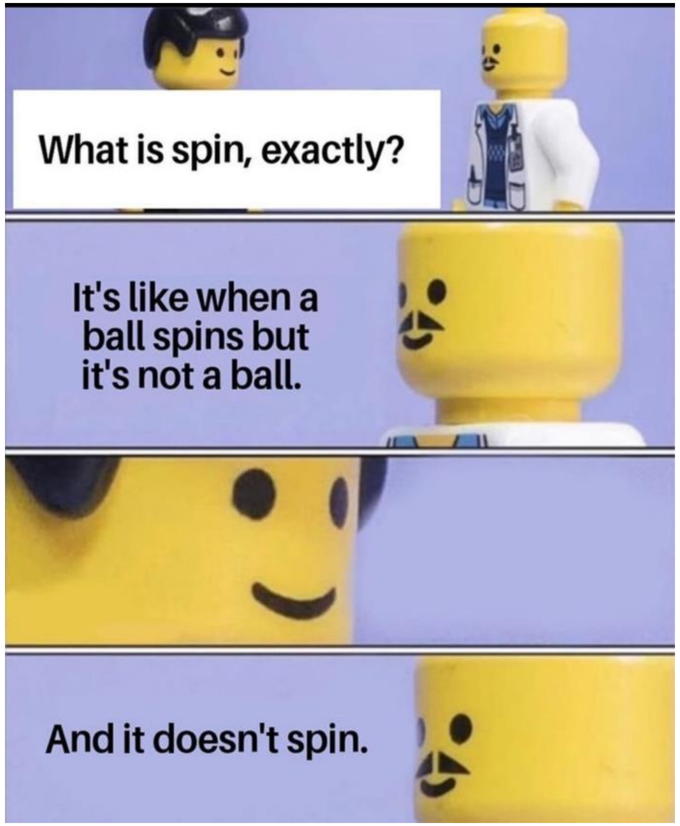
- This is also true even if you travel with one of the boxes and leave the other one behind

Entanglement - continued

- Measurements of physical properties such as position, momentum, spin, and polarization performed on entangled particles can, in some cases, be found to be perfectly correlated

<u>Example 2:</u>

- ➢ Have 2 entangled electrons such that their total spin is known to be zero (but both are in a state of superposition)
- ➢ If you measure one of them to be spin-up, then, by definition, the other one will have to be spin-down



<u>Example 3:</u>

- ➢ Have 2 entangled photons such that their total polarization is known to be zero (again, both are in a state of superposition)
- ➢ If you measure one of them to be polarised 90 degrees clockwise, then, by definition, the other one will have to be 90 degrees anticlockwise

Entanglement – takeaway points

- A special connection between two qubits
- A feature of quantum mechanics is that some particles are connected and correlated to each other
- Once you know a property of one of them, you can deduce the property of the other
- Each particle cannot be described independently
- This property does not change with distance

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Interference

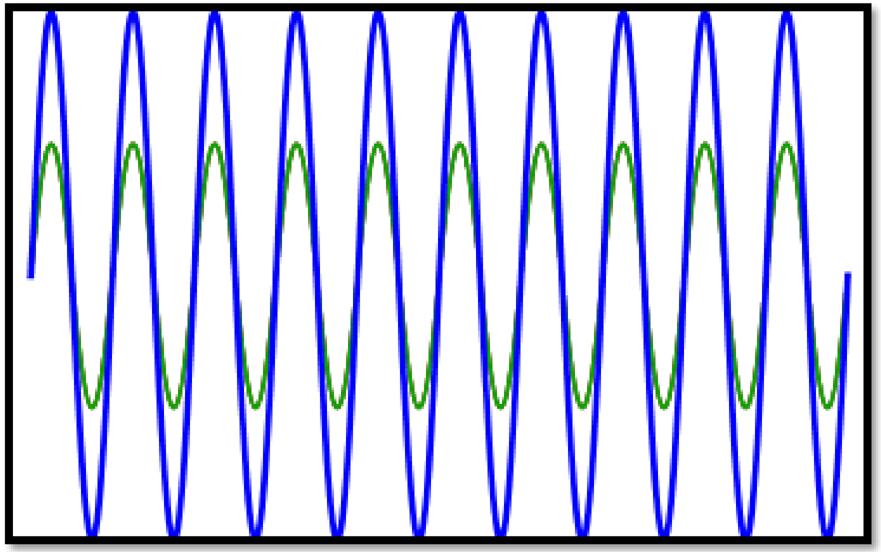
- Due to "**wave-particle duality**", at the subatomic (quantum) level, particles have wavelike properties
- This is why electron orbitals are depicted as clouds of probability instead of as orbits like planets around the sun
- In quantum computing, **interference** is used to affect probability amplitudes.
- Each qubit corresponds to a "wave" (described by a wave function)
- Each of the waves correspond to a specific path to solving a problem. There are correct paths to solving the problem and wrong paths as well
- All paths are available at the same time due to the principle of superposition which allows the qubits to "have" all the possible states simultaneously

Interference - continued

- One codes a “phase” onto each of the states of the qubits or “waves”.
- Qubits could then interact together and exhibit constructive and destructive interference.
- By applying certain quantum algorithms and “phases” to try to solve the problem, we should get a constructive interference at the best path to our solution (i.e. the correct solution)
- We should get a destructive interference for the other paths that we do not require
- This is how most optimization problems work with Quantum Computing
- We will get a more concrete example of an optimization problem and a potential solution with **Grover’s algorithm** (next lesson)

Interference – takeaway points

- Quantum systems can behave as waves
- Waves can interfere constructively or destructively
- We use constructive interference to increase the probability of finding the best solution
- We use destructive interference to decrease the probability of finding the wrong solution



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Decoherence and noise

- Quantum states are very sensitive and vulnerable to errors
- These errors arise from **decoherence**, a process in which the **environment** interacts with the qubits, uncontrollably changing their quantum states and causing information stored by the quantum computer to be lost
- Decoherence could come from many aspects of the environment:
  - ➢ changing magnetic and electric fields
  - ➢ radiation from warm objects nearby
  - ➢ cross talk between qubits
- This causes "noise" and uncertainty when it comes to measurements coming out of a quantum computer
- This "noise" needs to be mitigated

Quantum advantage (“supremacy”)

- “Quantum advantage” refers to the demonstrated and measured success to process a **real-world problem** faster on a quantum computer than on a classic computer
- Some people say that “quantum supremacy” means the same thing and both terms can be used interchangeably
- Some other people say “quantum supremacy” means that no classical computer can do the work of the quantum computer at all within a set of reasonable parameters
- The term “quantum advantage” is now the more preferred term
- Google claimed they achieved a “quantum advantage” back in 2019 with a paper in Nature using a quantum computer with 54 qubits, with IBM presenting arguments against this before the paper even released

Types of quantum computing qubits

1. Superconducting quantum computers

- Superconducting electrical circuits
- No resistance to the flow of direct electrical current
- Used by IBM, Google, Intel, Amazon
- Computations can be performed faster than on other qubits
- Take advantage of existing printable circuit processes that are efficient and available
- Most straightforward approach to creating a quantum computers
- Quickly experience decoherence
- Demand error correction techniques
- Only operate in very cold environments (below 100mK)
- Each superconducting qubit is different from others and therefore requires continuous calibration for operations

Figure: IBM Q System One

Types of quantum computing qubits - continued

2. Optical quantum computers

- Use particles of light to carry and process information
- Photons are manipulated with mirrors, beam splitters, and phase shifters
- Used by Xanadu, PsiQuantum, UST of China
- Photonic qubits can operate at room temperature (and also low temperatures)
- Photons have long coherence time
- Photons are not highly affected by the environment
- Can be easily integrated into existing optical–based infrastructures
- Novel emerging technology (not mature)
- Bulky in size
- Due to the interaction of several signals, computation is a complex process

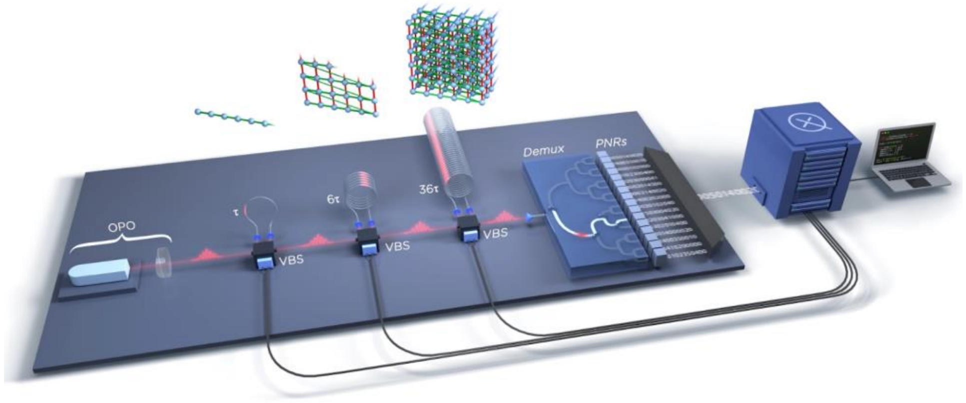

Figure: Xanadu Borealis

Types of quantum computing qubits - continued

3. <u>Trapped ion quantum computers</u>

- Charged atoms (ions) trapped in place by lasers and magnetic fields
- Outermost particle, an electron, orbiting the nucleus demonstrates superposition and can be used as a qubit
- Quantum information is encoded in the electronic energy levels of ions as they are suspended in vacuum
- Used by IonQ, Quantinuum, Oxford Ionics
- Significantly longer time to decoherence
- An ion trap can operate at room temperature
- Better performance if the ions are cooled to somewhere around 4K
- Currently slower than their superconducting qubits
- Vacuum required to hold, or trap, the ions
- Novel emerging technology (not mature)

Figure: [IonQ Harmony]

Types of quantum computing qubits - continued

Other types of qubits include:

- Annealing
- Quantum dot
- Color center
- Neutral atoms in optical tweezer array
- Topological quantum computers (Majorana zero mode)
- Electron-on-helium
- Cold atom
- Defect based
- Nuclear magnetic resonance

Shortcomings and difficulties (uncertain future)

- Qubits are very small and very sensitive
- Quantum computers are difficult to engineer and program. It becomes challenging to find skilled individuals to operate and maintain the necessary machinery
- Quantum computing works, but **will it scale**?
- Need something in the realm of **1,000,000 stable qubits** to be able to face any kind of useful real-life problem
- Decoherence and noise
- We only have a few ($O(10)$) useful quantum algorithms so far
- Many quantum computers need extremely low temperatures and/or complete isolation from external factors to operate well

Summary

- Quantum computers are an exciting prospect based on several quantum properties including superposition, entanglement, interference, and decoherence

- Understanding quantum computing is not scary, it's just linear algebra

- Quantum supremacy means a quantum computer doing one specific thing better than a classical computer

- There are many types of quantum computers, all with advantages and disadvantages

- It is still early days for quantum computing. The future is still uncertain

Congratulations, you are now familiar with the basics of quantum computing! 🥳

Blooket

https://play.blooket.com

Thank you for your attention!

https://forms.gle/YWiECGZXbiPEfTcA8

Reminders

- Please fill out original survey if you haven't already (only takes a couple of minutes)

- Please create accounts on IBM and Xanadu if you haven't already (need them for tomorrow's practice sessions)

Connect with me

https://ahmedabdelmotteleb.github.io/