

Diffraction phases and losses in large-momentum-transfer atom interferometers

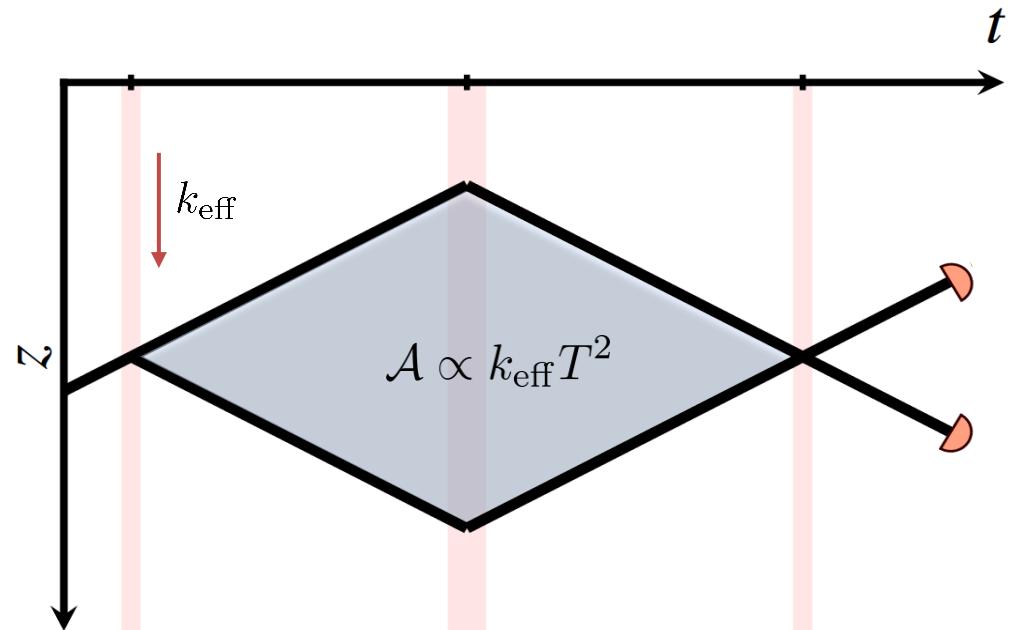
Klemens Hammerer

Naceur Gaaloul, Ernst Rasel, Christian Schubert

Jan-Niclas Siemß, Florian Fitzek

Leibniz Universität Hannover

Large-Momentum-Transfer & Very-Long-Baseline Atom Interferometry



atomic recoil & fine structure constant
gravimetry, gravity cartography, inertial navigation
gravitational constant
tests of the equivalence principle
gravitational waves
quantum clocks
dark matter...

Interferometers with metrological gain from LMT

Fine structure constant

R.H. Parker et al. Science 360 191 (2018)
L. Morel et al., Nature 588 61(2020)

Space-time Curvature across atomic Wave Function

P. Asenbaum et al. PRL 118 183602 (2017)

Gravitational Aharonov-Bohm effect

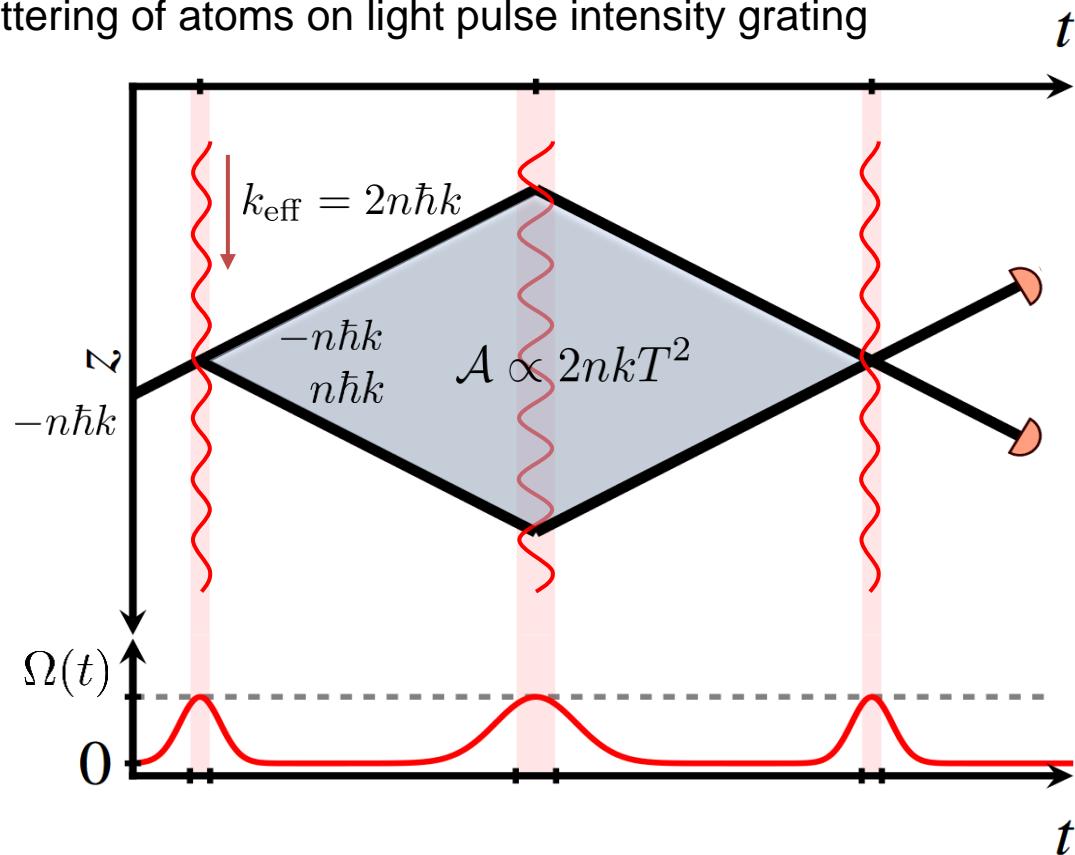
C. Overstreet et al. Science, 375 226 (2022)

Are all based on elastic scattering of atoms on light pulses in Bragg diffraction / Bloch oscillations

Large-Momentum-Transfer from elastic scattering: Bragg diffraction & Bloch oscillations

Bragg diffraction

= scattering of atoms on light pulse intensity grating



Kasevich group: PRL 127 100401 (2021), PRL 107 130403 (2011), PRA 86 011606 (2012)

Rasel group: PRL 116 173601 (2016), Nat. Comm. 12 2544 (2021)

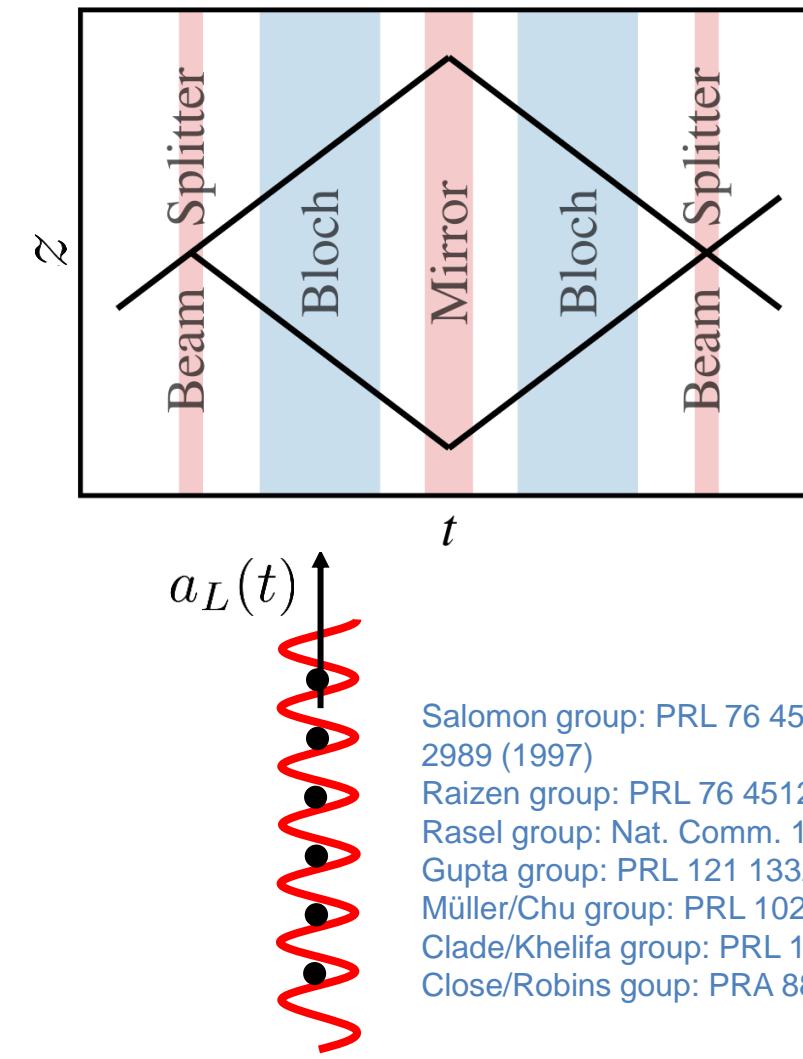
Gupta group: PRL 121 133201 (2018)

Müller/Chu group: PRL 100 180405 (2008)

Birkl group: arXiv:1603.08826

Bloch oscillations

= momentum transfer in accelerated optical lattice potential



Salomon group: PRL 76 4508 (1996), PRA 55 2989 (1997)

Raizen group: PRL 76 4512 (1996)

Rasel group: Nat. Comm. 12 2544 (2021)

Gupta group: PRL 121 133201 (2018)

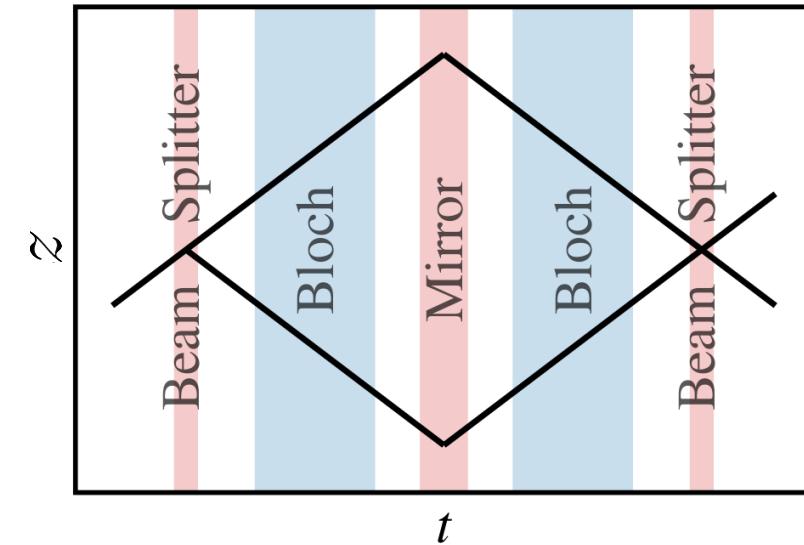
Müller/Chu group: PRL 102 240403 (2009)

Clade/Khelifa group: PRL 102 240402 (2009)

Close/Robins group: PRA 88 053620 (2013)

Large-Momentum-Transfer from elastic scattering: Bragg diffraction & Bloch oscillations

- ❖ What are the fundamental losses and phases involved?
- ❖ Favorable working points?
- ❖ Mitigation strategies for avoiding losses & suppressing phases?
→ design & technology choices for current & future detectors



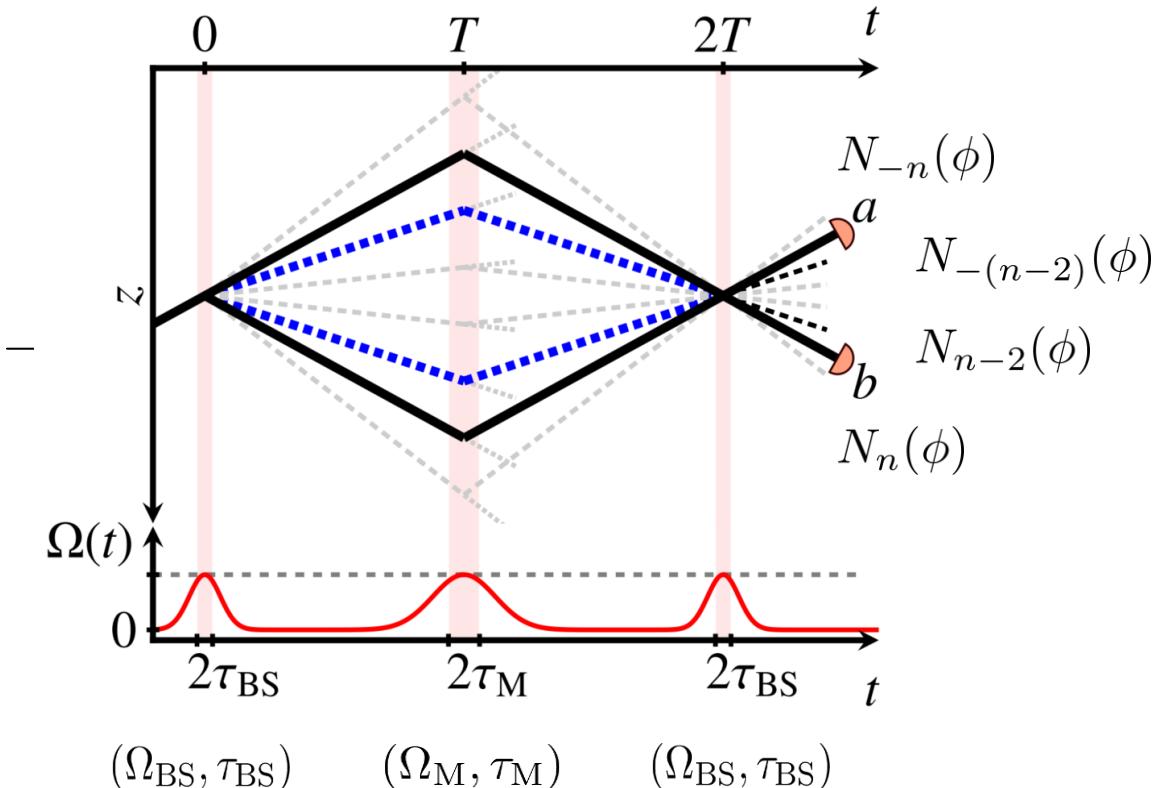
for symmetric double Bragg diffraction: E. Giese, A. Roura, G. Tackmann, E. M. Rasel, and W. P. Schleich PRA 88, 053608 2013

Topic #1: Bragg atom interferometers

Losses and diffraction phases in Bragg interferometers

P. Altin et al. New J. Phys. 15 023009 (2013).
 B. Estey et al. Phys. Rev. Lett. 115, 083002 (2015),
 R. H. Parker, et al. Phys. Rev. A 94, 053618 (2016)

Bragg diffraction is, fundamentally, a multi-port operation:



Relative atom number:

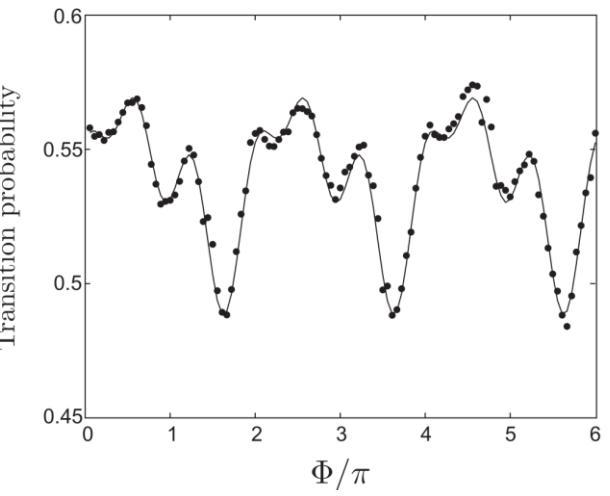
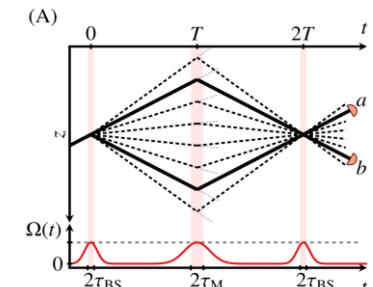
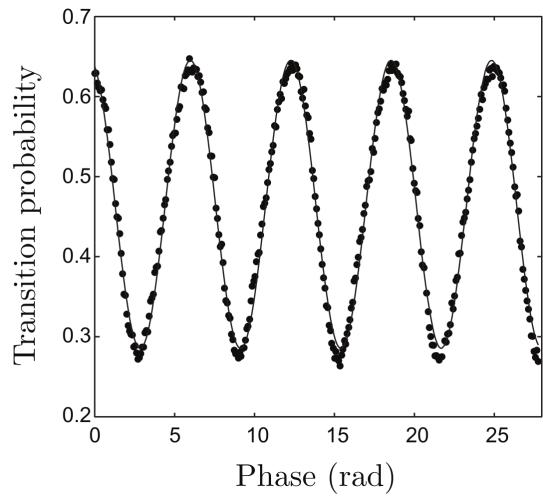
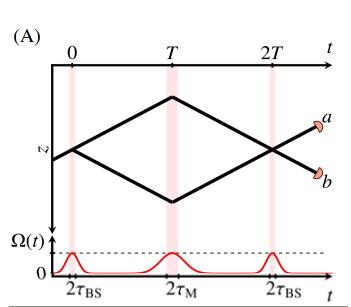
$$\begin{aligned}
 P_a(\phi) &= \frac{N_{-n}(\phi)}{N_{-n}(\phi) + N_n(\phi)} \\
 &= \frac{N_{-n}(\phi)}{N_{\text{tot}} - N_{-(n-2)}(\phi) + N_{n-2}(\phi)} \\
 &= P_0 + A_1 \cos(\phi + \varphi_1) + A_2 \cos(2\phi + \varphi_2) + \dots
 \end{aligned}$$

$A_i(\Omega_{BS}, \tau_{BS}, \Omega_M, \tau_M, T)$ $\varphi_i(\Omega_{BS}, \tau_{BS}, \Omega_M, \tau_M, T)$

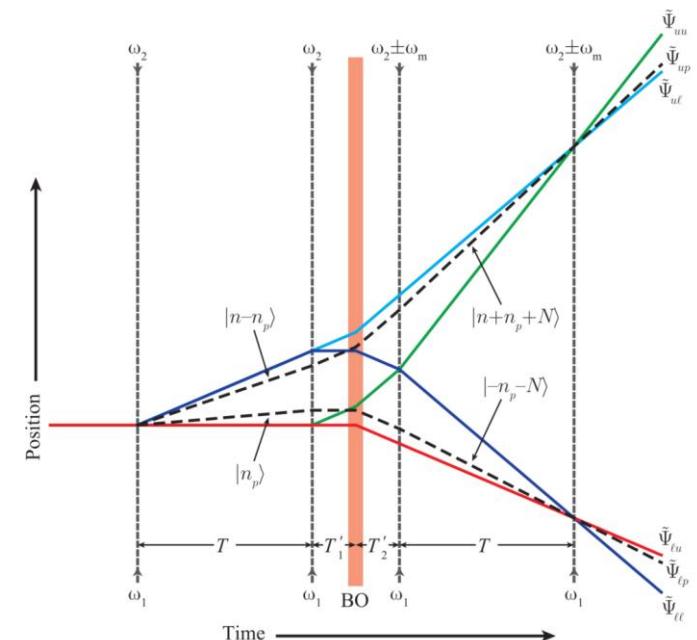
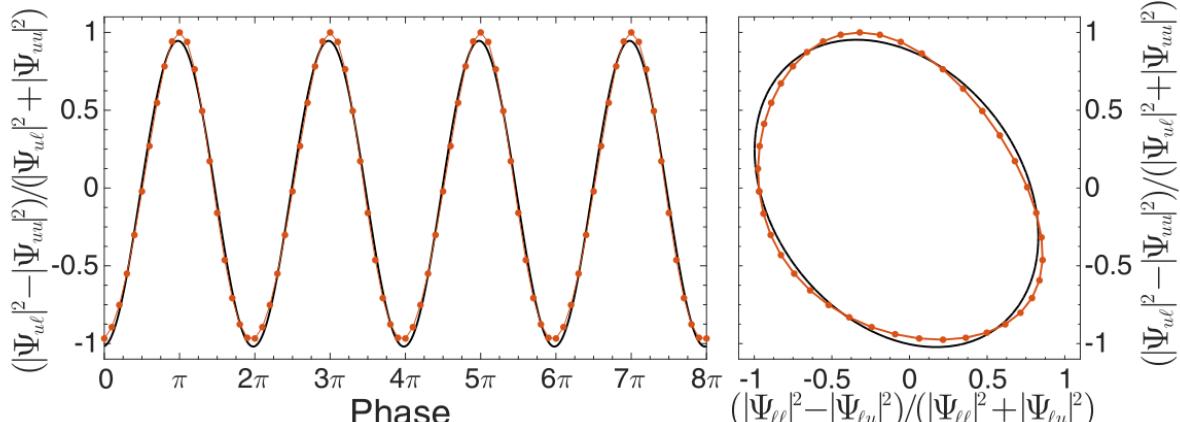
Diffraction phases in Bragg interferometers

cf. talk by A. Gauget

Close: P. Altin et al. New J. Phys. 15 023009 (2013)



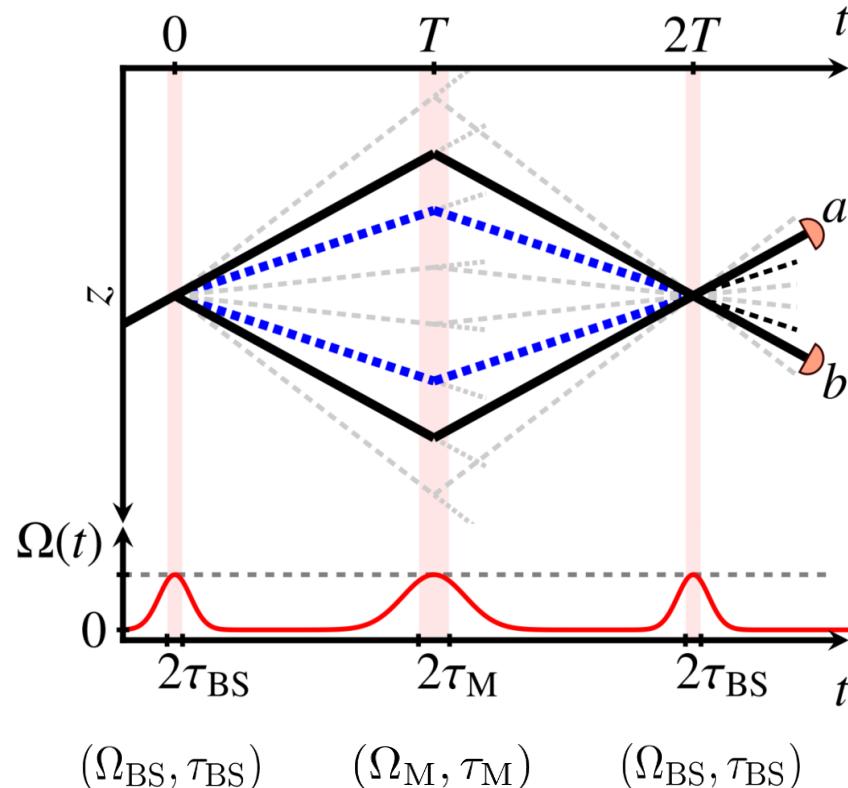
Müller: B. R. H. Parker, et al. Phys. Rev. A 94, 053618 (2016) [Theory, Monte Carlo Simulations]



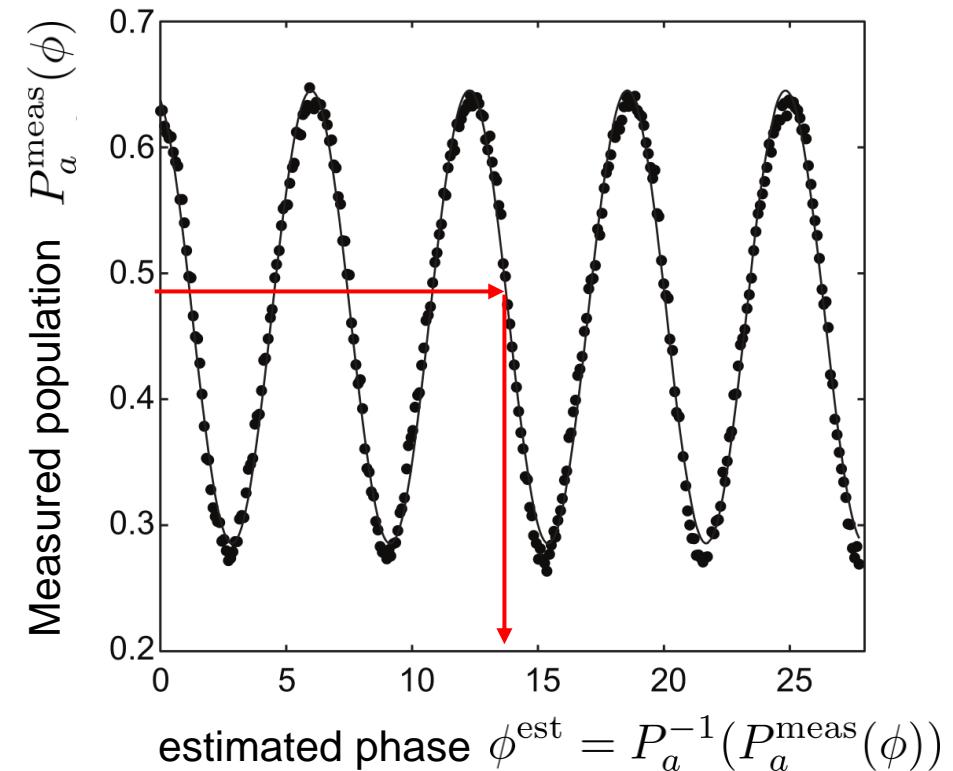
Diffraction phases in Bragg interferometers

P. Altin et al. New J. Phys. 15 023009 (2013).
 B. Estey et al. Phys. Rev. Lett. 115, 083002 (2015),
 R. H. Parker, et al. Phys. Rev. A 94, 053618 (2016)

Two-mode model: $P_a(\phi) = P_0 + A \cos(\phi)$



Reality: $P_a^{\text{meas}}(\phi) = P_0 + A_1 \cos(\phi + \varphi_1) + A_2 \cos(2\phi + \varphi_2) + \dots$

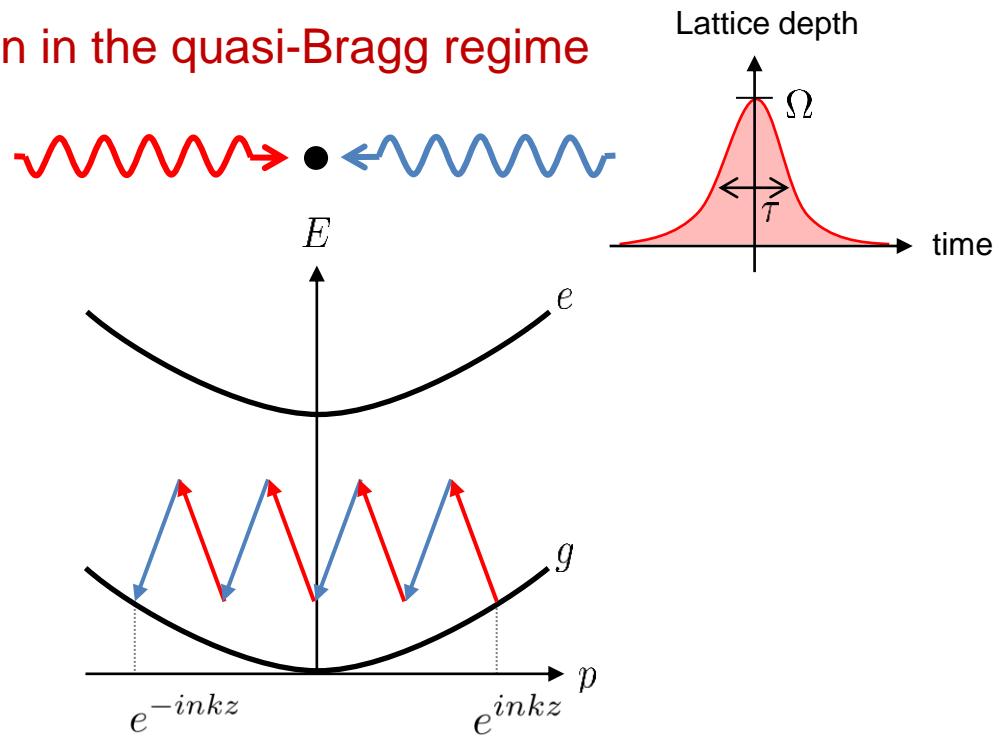


Phase estimation error: $\delta\phi(\Omega_{BS}, \tau_{BS}; \Omega_M, \tau_M, T) = P_a^{-1}(P_a^{\text{meas}}(\phi)) - \phi$

$$\underbrace{A_i(\Omega_{BS}, \tau_{BS}, \Omega_M, \tau_M, T)}_{\text{functional dependence?}} \quad \varphi_i(\Omega_{BS}, \tau_{BS}, \Omega_M, \tau_M, T)$$



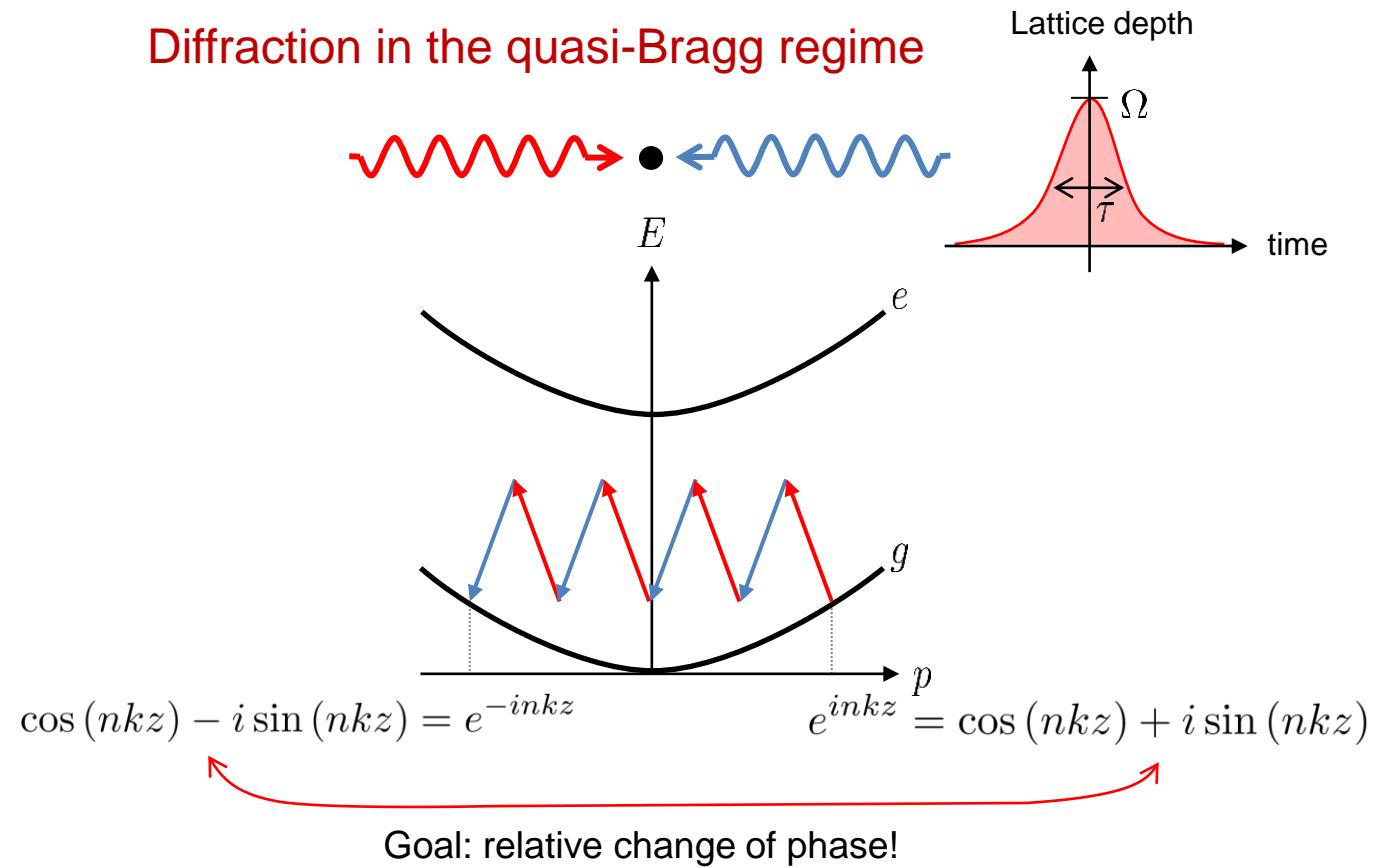
Diffraction in the quasi-Bragg regime



- Effective Hamiltonian approach (in analogy to deep-Bragg regime)

H. Müller et al. PRA 77, 023609 (2008)

Diffraction in the quasi-Bragg regime



- Effective Hamiltonian approach (in analogy to deep-Bragg regime)

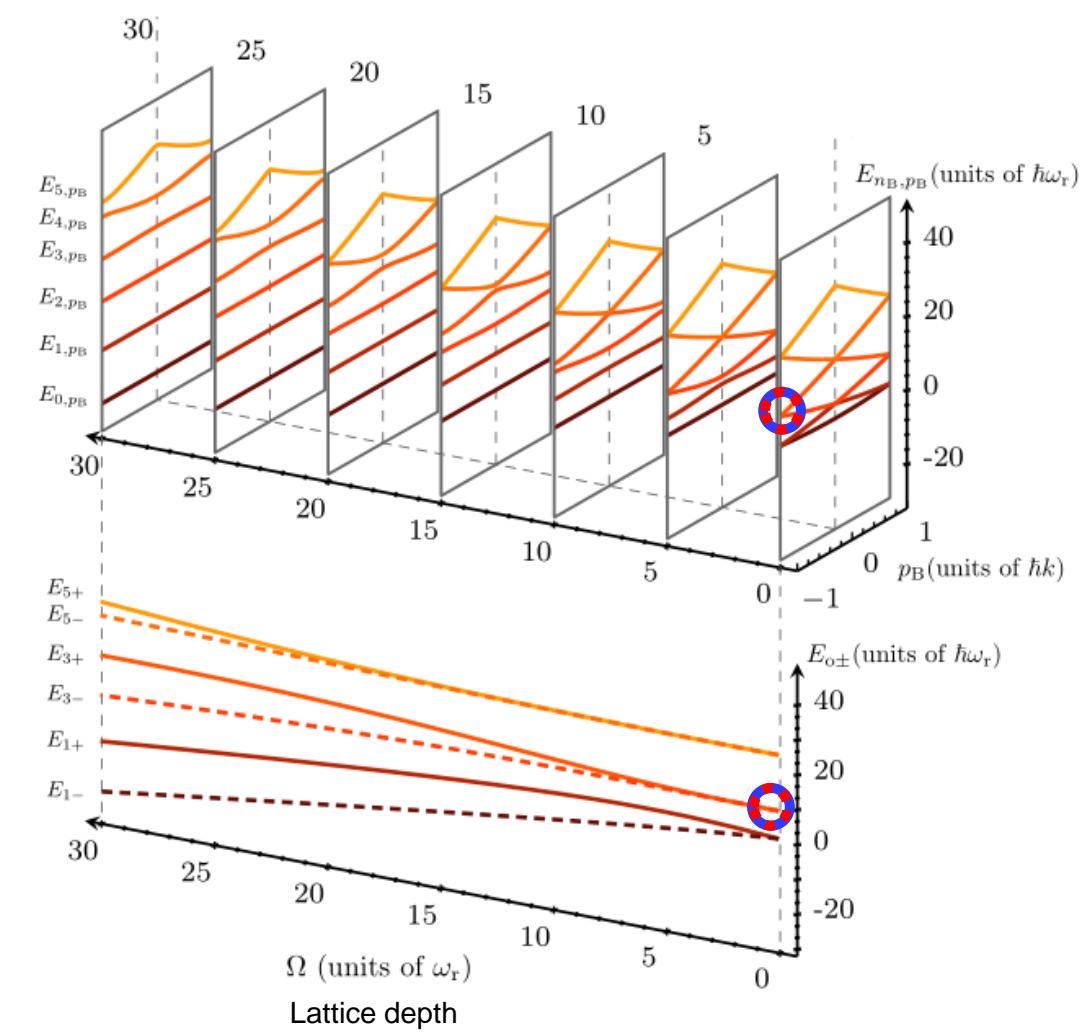
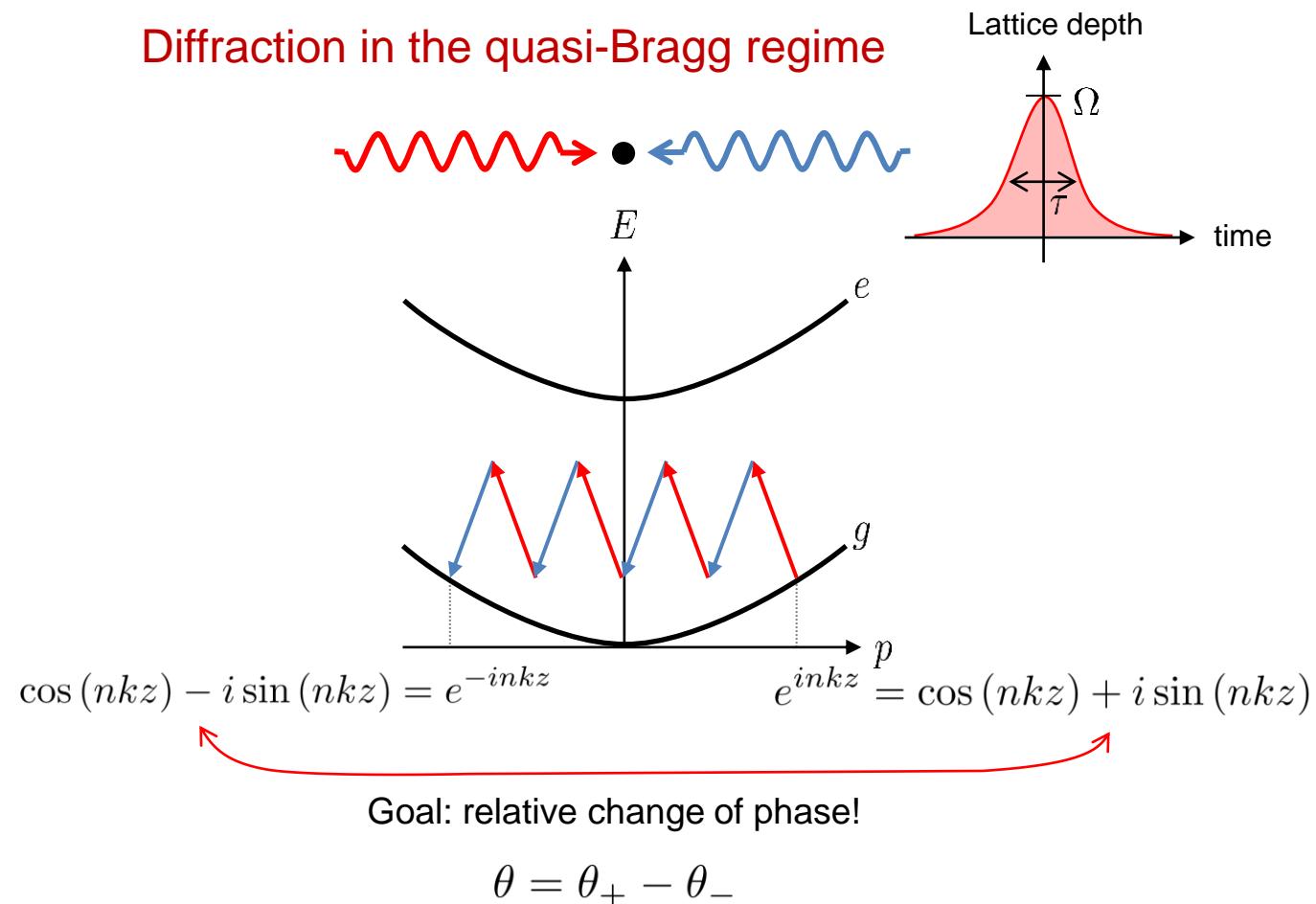
H. Müller et al. PRA 77, 023609 (2008)

- Understanding as an adiabatic dynamics in Bloch states

Gupta group: D. Gochnauer, et al., PRA 100, 043611 (2019)

J.-N. Siemß et al, PRA102, 033709 (2020)

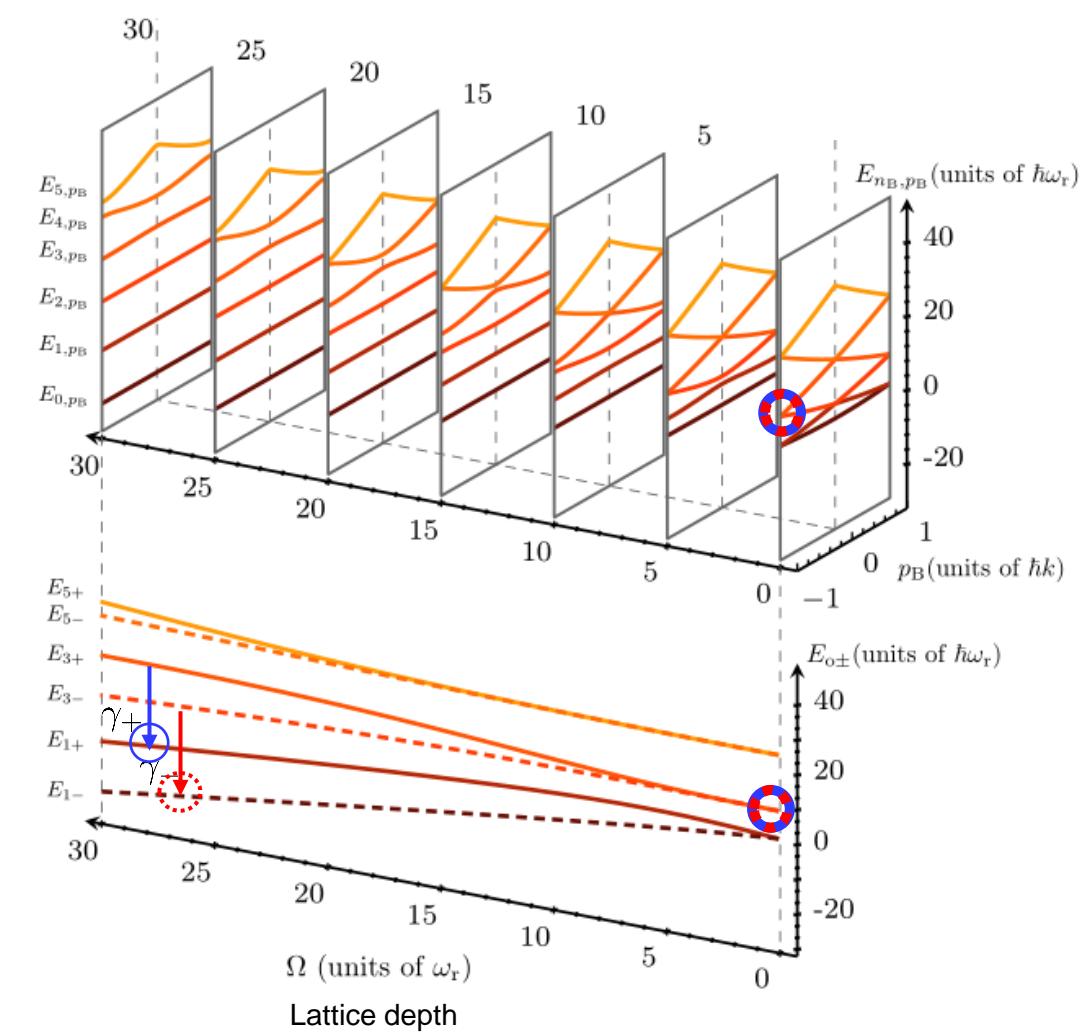
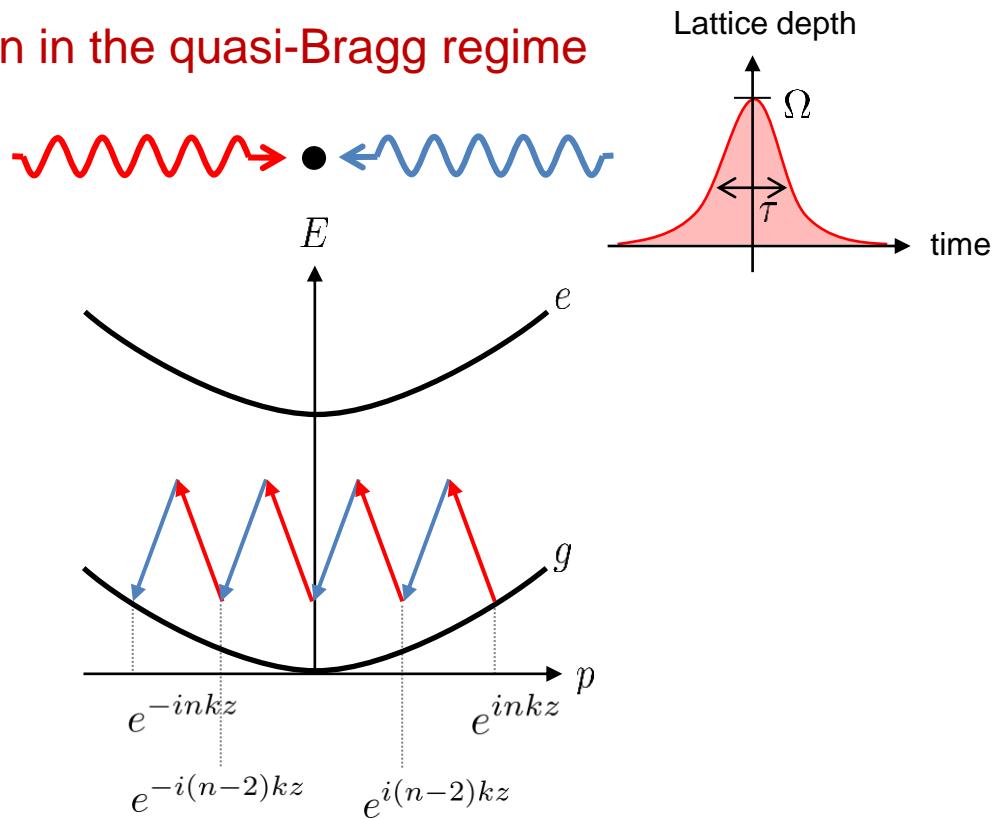
Diffraction in the quasi-Bragg regime



$$e^{i\theta_+} \cos(nkz) + i e^{i\theta_-} \sin(nkz)$$

$$\theta_{\pm}(\Omega, \tau) = \int dt E_{\pm}(t; \Omega, \tau) / \hbar$$

Diffraction in the quasi-Bragg regime



$$\cos(nkz) + i \sin(nkz)$$

Bragg scattering matrix

$$S = e^{-i\frac{\Phi-i\Gamma}{2}} \begin{pmatrix} e^{-inkz} & e^{inkz} \\ \cos\left(\frac{\Theta-i\gamma}{2}\right) & -i\sin\left(\frac{\Theta-i\gamma}{2}\right) \\ -i\sin\left(\frac{\Theta-i\gamma}{2}\right) & \cos\left(\frac{\Theta-i\gamma}{2}\right) \end{pmatrix} \begin{pmatrix} e^{-inkz} \\ e^{inkz} \end{pmatrix}$$

adiabatic phases $\Phi = \theta_+ + \theta_-$ $\Theta = \theta_+ - \theta_-$

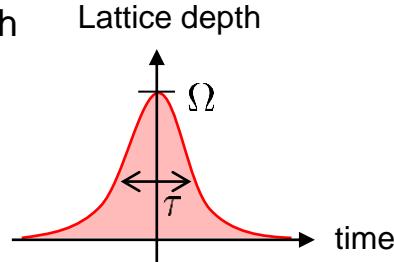
$$\theta_{\pm}(\Omega, \tau) = \int dt E_{\pm}(t; \Omega, \tau) / \hbar$$

Landau-Zener losses $\Gamma = \gamma_+ + \gamma_-$ $\gamma = \gamma_+ - \gamma_-$

$$\gamma_{\pm}(\Omega, \tau)$$

+ perturbation due to Doppler-detuning & momentum width

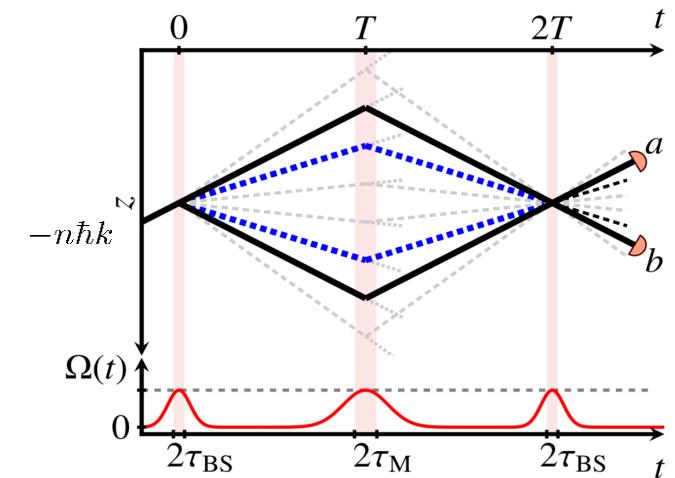
J.-N. Siemß et al, PRA102, 033709 (2020)



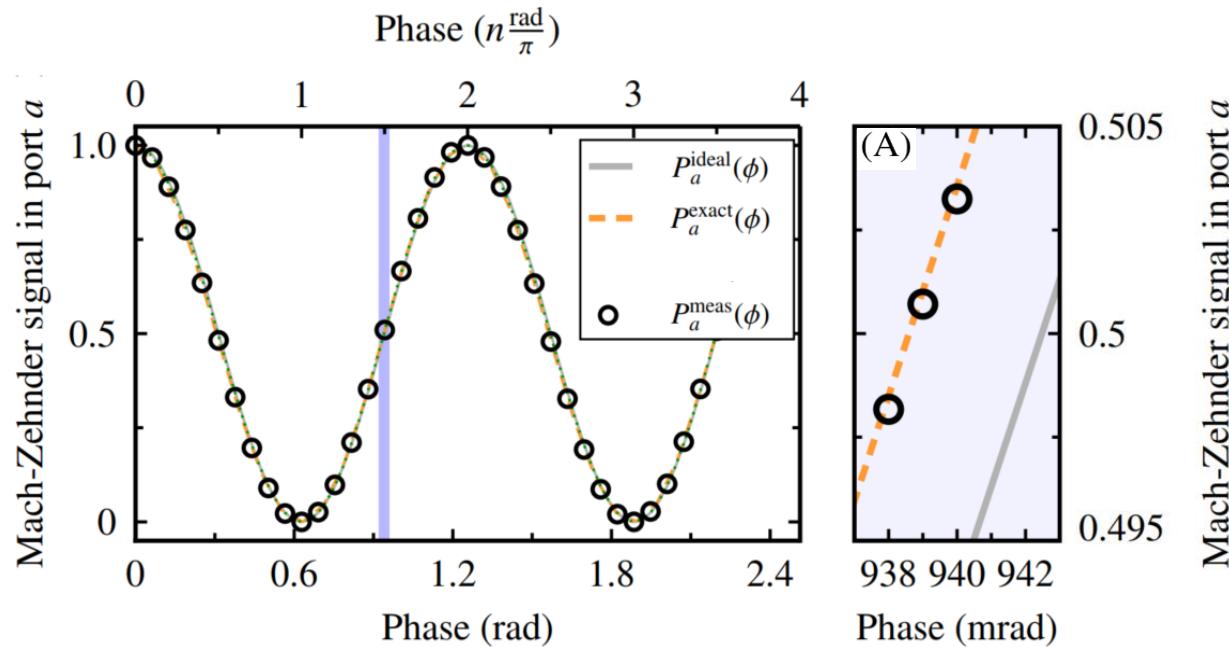
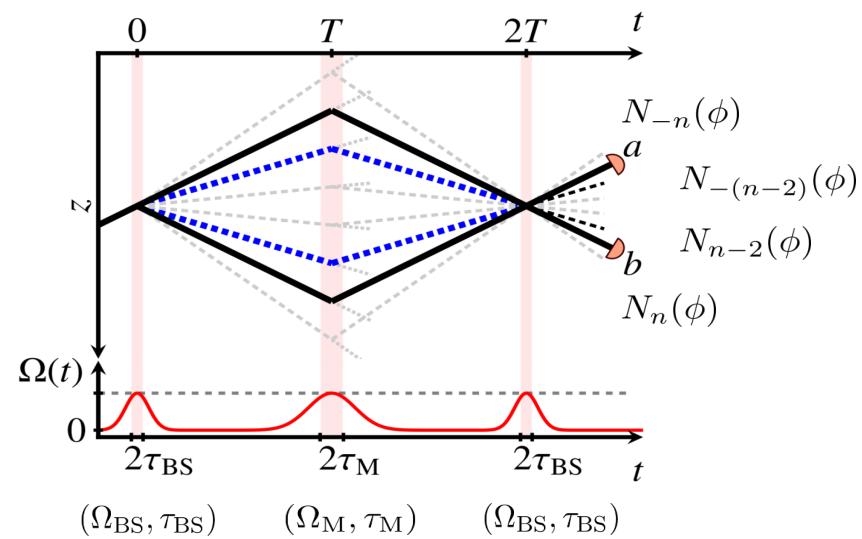
Extension to dominant parasitic states:

$$S = \begin{pmatrix} e^{-inkz} & e^{-i(n-2)kz} & e^{i(n-2)kz} & e^{inkz} \\ S_{-n,-n} & S_{-n,-(n-2)} & S_{-n,+(n-2)} & S_{-n,+n} \\ S_{-(n-2),-n} & S_{-(n-2),-(n-2)} & S_{-(n-2),+(n-2)} & S_{-(n-2),+n} \\ S_{+(n-2),-n} & S_{+(n-2),-(n-2)} & S_{+(n-2),+(n-2)} & S_{+(n-2),+n} \\ S_{+n,-n} & S_{+n,-(n-2)} & S_{+n,+(n-2)} & S_{+n,+n} \end{pmatrix} \begin{pmatrix} e^{-inkz} \\ e^{-i(n-2)kz} \\ e^{i(n-2)kz} \\ e^{inkz} \end{pmatrix}$$

Interferometers with multiple ports and paths



Phase model of complete interferometer



UATIS: Florian Fitzek, et al. Sci Rep 10, 22120 (2020)

Relative atom numbers:

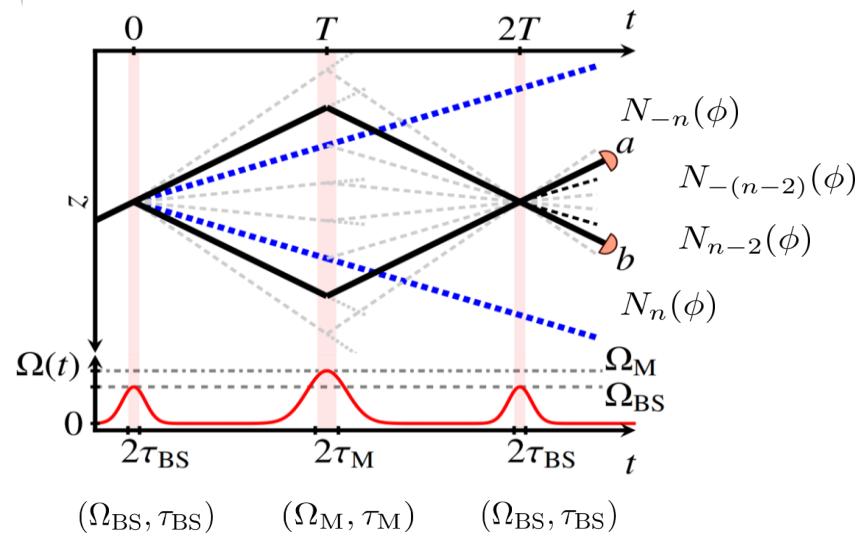
$$\begin{aligned} P_a^{\text{exact}}(\phi) &= \frac{N_{-n}(\phi)}{N_{\text{tot}} - N_{-(n-2)}(\phi) + N_{n-2}(\phi)} \\ &= P_0 + \sum_{i=1}^{\infty} A_i \cos(j \phi + \varphi_i) \end{aligned}$$

- Accurate model
- Signal template not practical due to number of fit parameters

$$S(\Omega_{BS}, \tau_{BS}, \Omega_M, \tau_M, T) \rightarrow A_i(\Omega_{BS}, \tau_{BS}, \Omega_M, \tau_M, T)$$

$$S(\Omega_{BS}, \tau_{BS}, \Omega_M, \tau_M, T) \rightarrow \varphi_i(\Omega_{BS}, \tau_{BS}, \Omega_M, \tau_M, T)$$

Suppression of parasitic paths by tailored mirror pulses

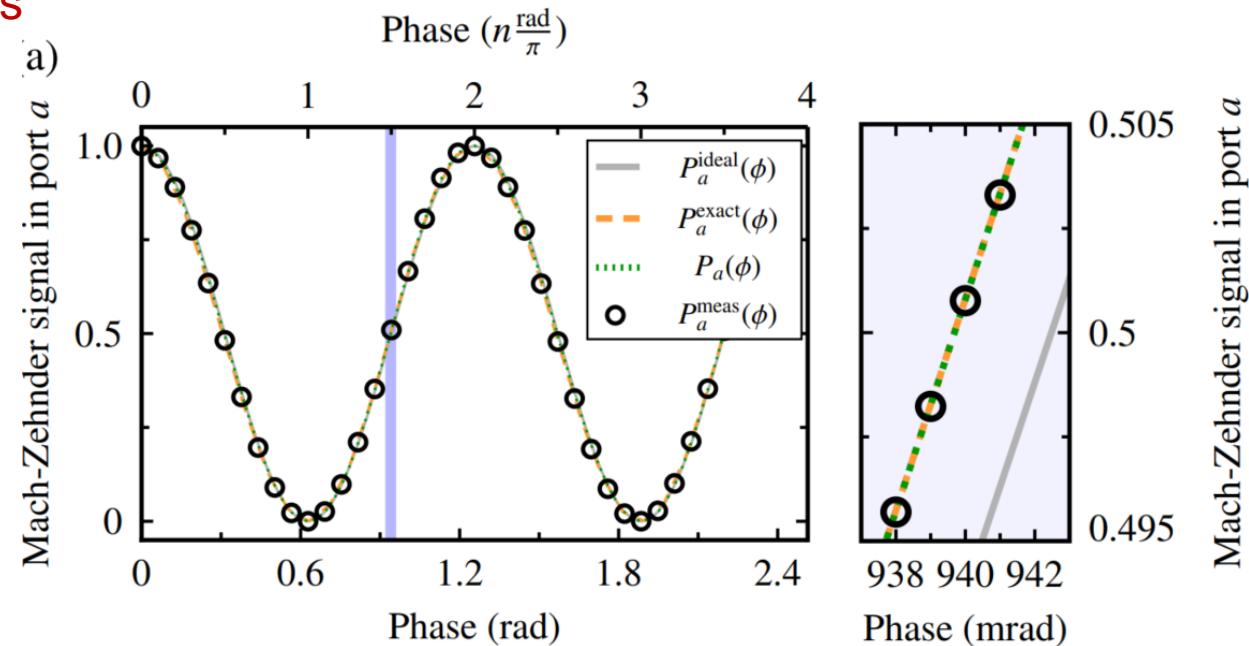


Simplified signal template:

$$\begin{aligned} P_a(\phi) &= \frac{N_{-n}(\phi)}{N_{\text{tot}} - N_{-(n-2)}(\phi) + N_{n-2}(\phi)} \\ &= P_0 + \sum_{j=1}^3 A_j \sin \left(j \cdot \left(n\phi + \gamma + \frac{\pi}{2} \right) \right) + \mathcal{O}[\gamma^3] \end{aligned}$$

$$S(\Omega_{BS}, \tau_{BS}, \Omega_M, \tau_M, T) \rightarrow A_i(\Omega_{BS}, \tau_{BS}, \Omega_M, \tau_M, T)$$

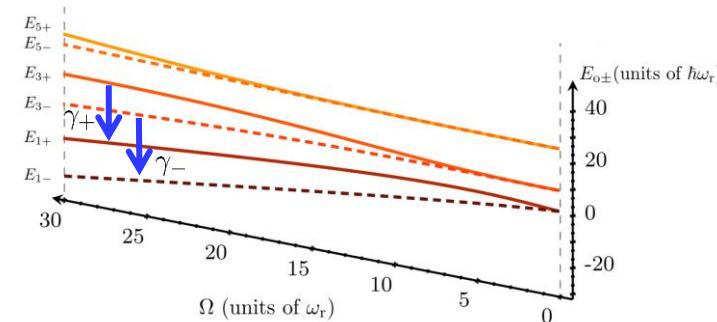
$$S(\Omega_{BS}, \tau_{BS}, \Omega_M, \tau_M, T) \rightarrow \varphi_i(\Omega_{BS}, \tau_{BS}, \Omega_M, \tau_M, T)$$



UATIS: Florian Fitzek, et al. Sci Rep 10, 22120 (2020)

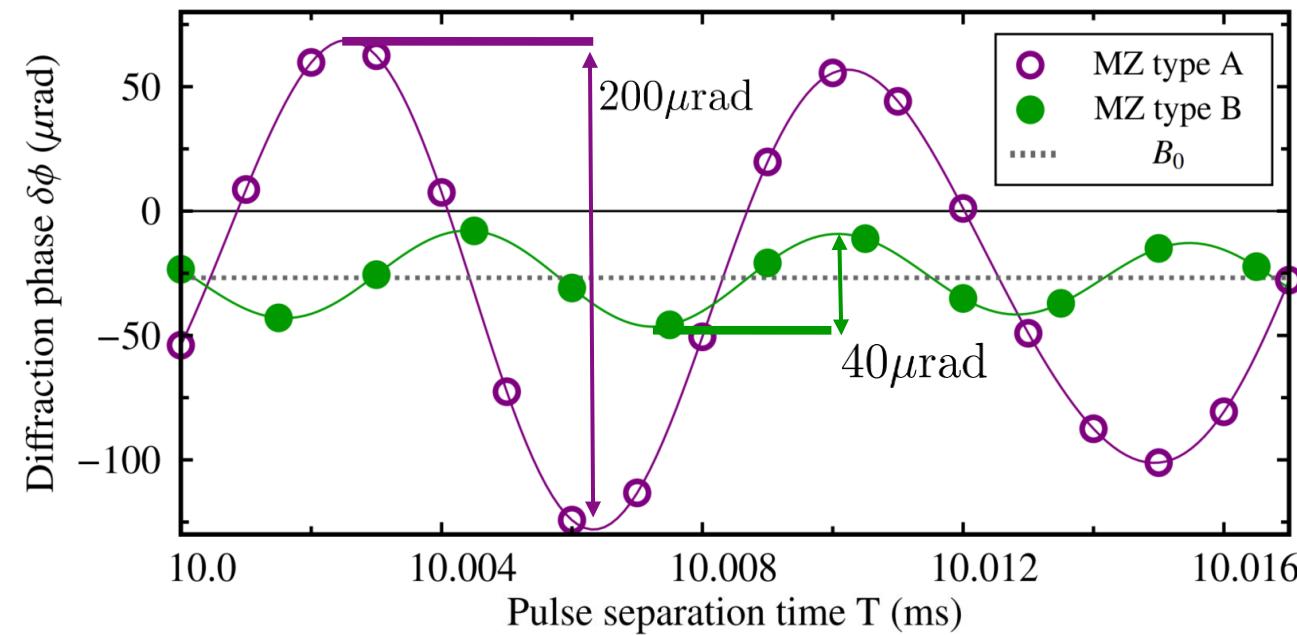
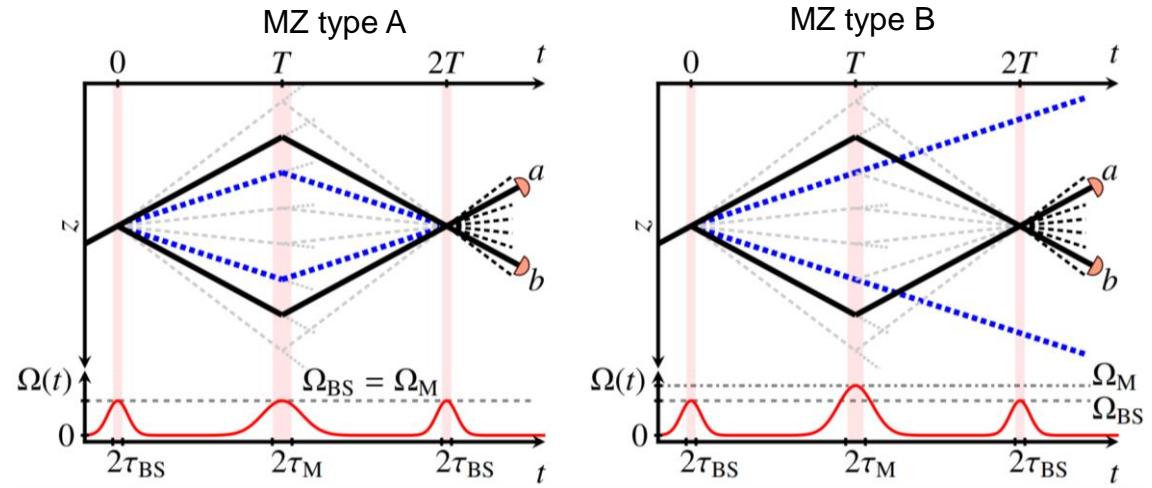
remaining diffraction phase
= LZ loss parameter!

$$\gamma = \gamma_+ - \gamma_-$$



Diffraction phase for MZ interferometer ($n = 5$)

$$\begin{aligned}\delta\phi &= \phi^{\text{est}} - \phi = P_a^{-1}(P_a^{\text{meas}}(\phi)) - \phi \\ &= P_a^{-1}(P_a^{\text{meas}}(\phi))|_{\gamma=0} - \frac{\gamma}{n} - \phi\end{aligned}$$



Fit to diffraction phase:

$$f(T) = B_0 + B_1 \cos(4\omega_r T + \eta_1) + B_2 \cos(6\omega_r T + \eta_2)$$

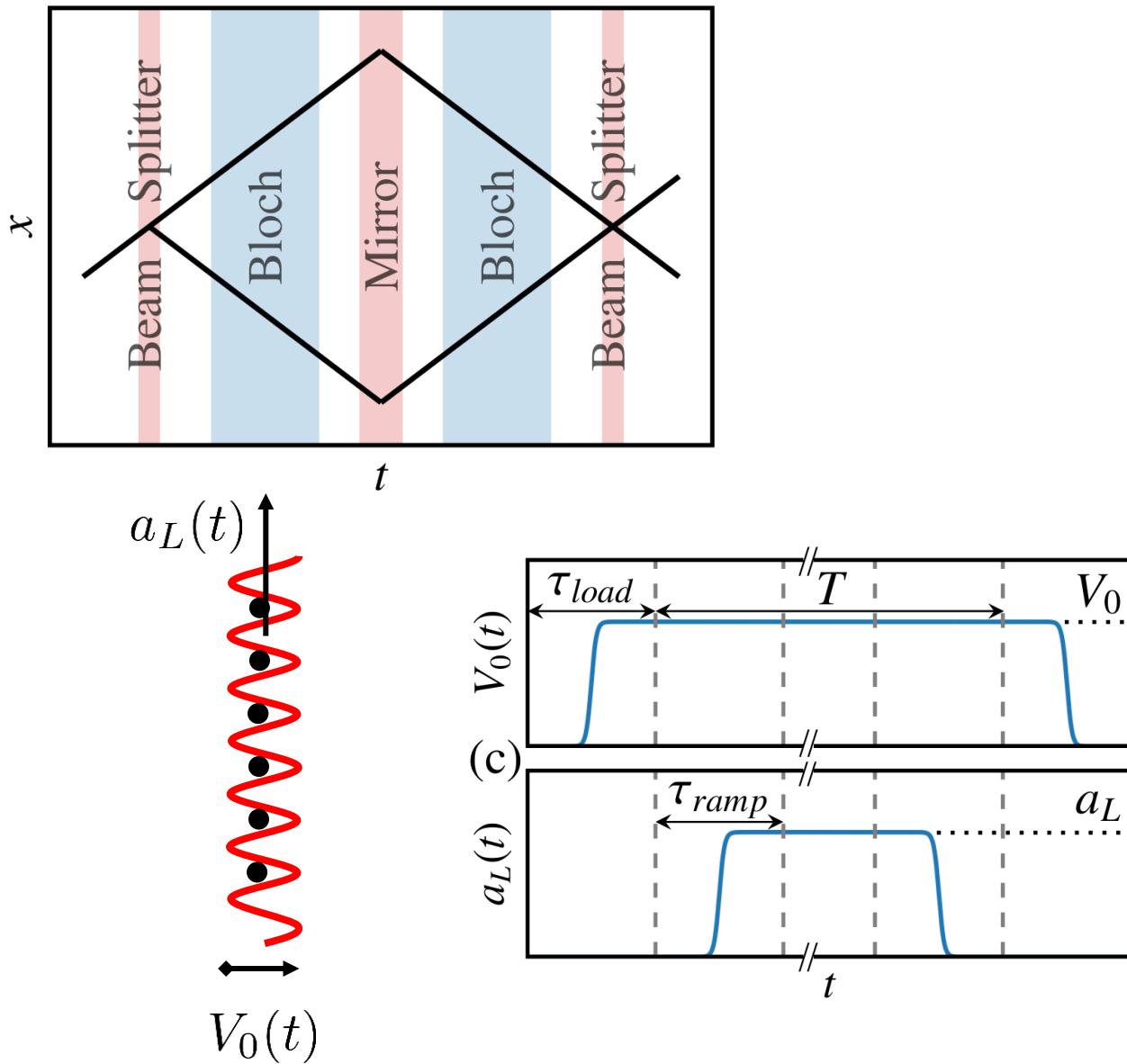
Reduction of Peak-to-peak (PP) values by a factor of 5

Parameters $(\Omega_{\text{BS}}, \tau_{\text{BS}})$ minimize parasitic couplings to 0.18%: $\frac{\gamma}{5} \approx 280 \mu\text{rad}$

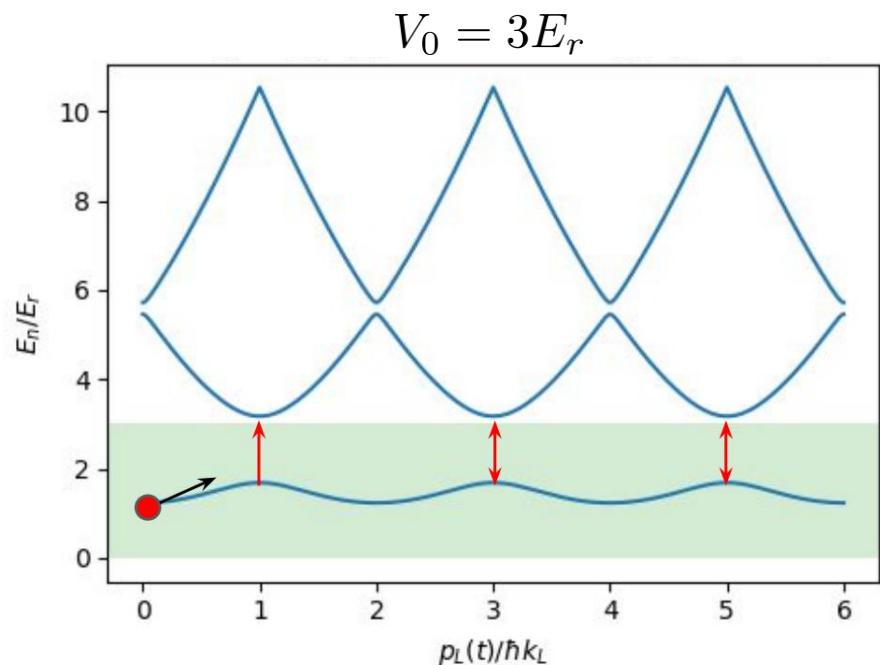
R. H. Parker, et al. Phys. Rev. A 94, 053618 (2016)

Topic #2: Bloch oscillations

Bloch oscillations



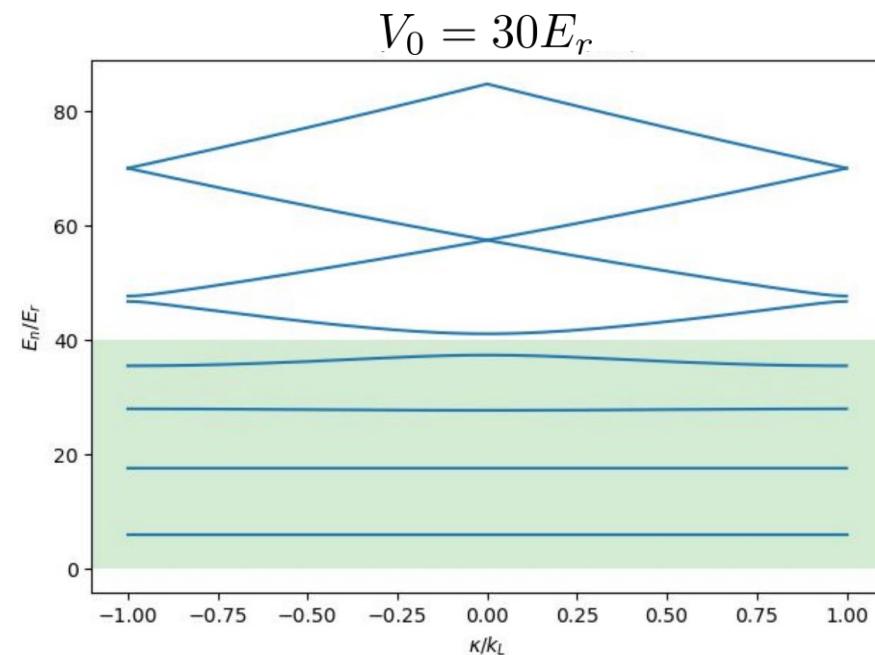
Shallow vs. deep lattices



adiabatic theorem using instantaneous Bloch states
Landau-Zener physics at avoided crossings

$$P_{\text{LZ}} = e^{-F_c/F}, \quad F_c = \frac{\pi}{32} \frac{V_0^2}{E_r} k_L$$

Peik et al., PRA 55 (1997)



deep lattices required for strong accelerations $a \gtrsim 100m/s^2$

- no avoided crossings
- LZ loss formula does not apply

Hamiltonian in lattice frame

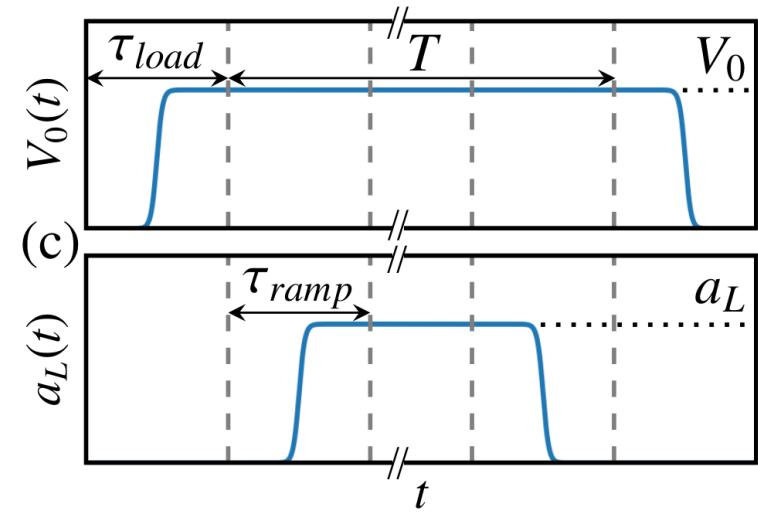
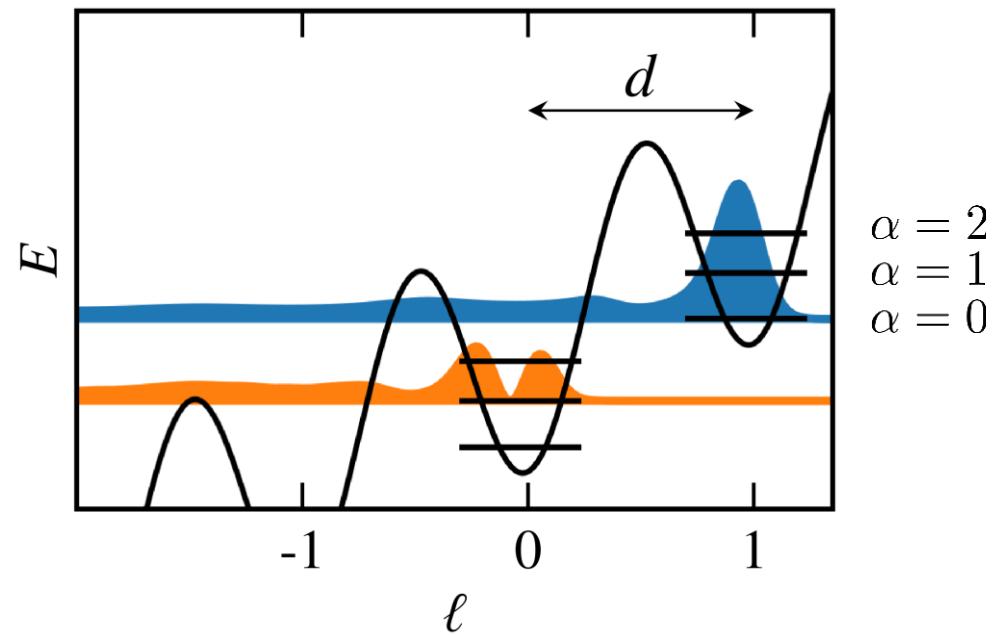
$$H(t) = \frac{p^2}{2m} + 2\hbar\Omega \cos^2(k_L x) + m a_L(t) x$$

instantaneous eigenstates = Wannier-Stark states

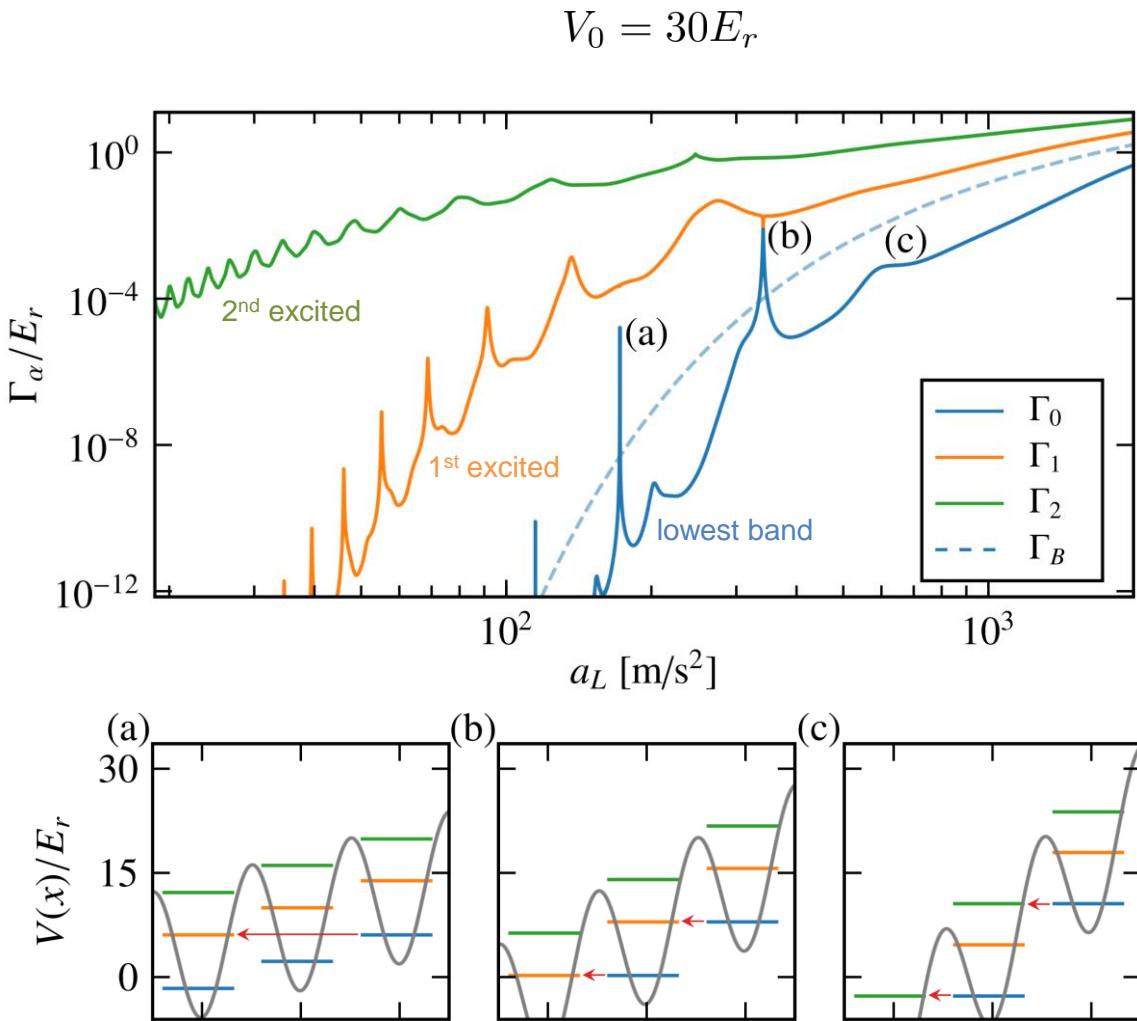
$$H(t) |\Psi_{\alpha,\ell}(t)\rangle = \left(E_{\alpha,0}(t) + d\ell m a_L(t) - i \frac{\Gamma_\alpha(t)}{2} \right) |\Psi_{\alpha,\ell}(t)\rangle$$

M. Glück et al. Physics Reports 366, 103 (2002)

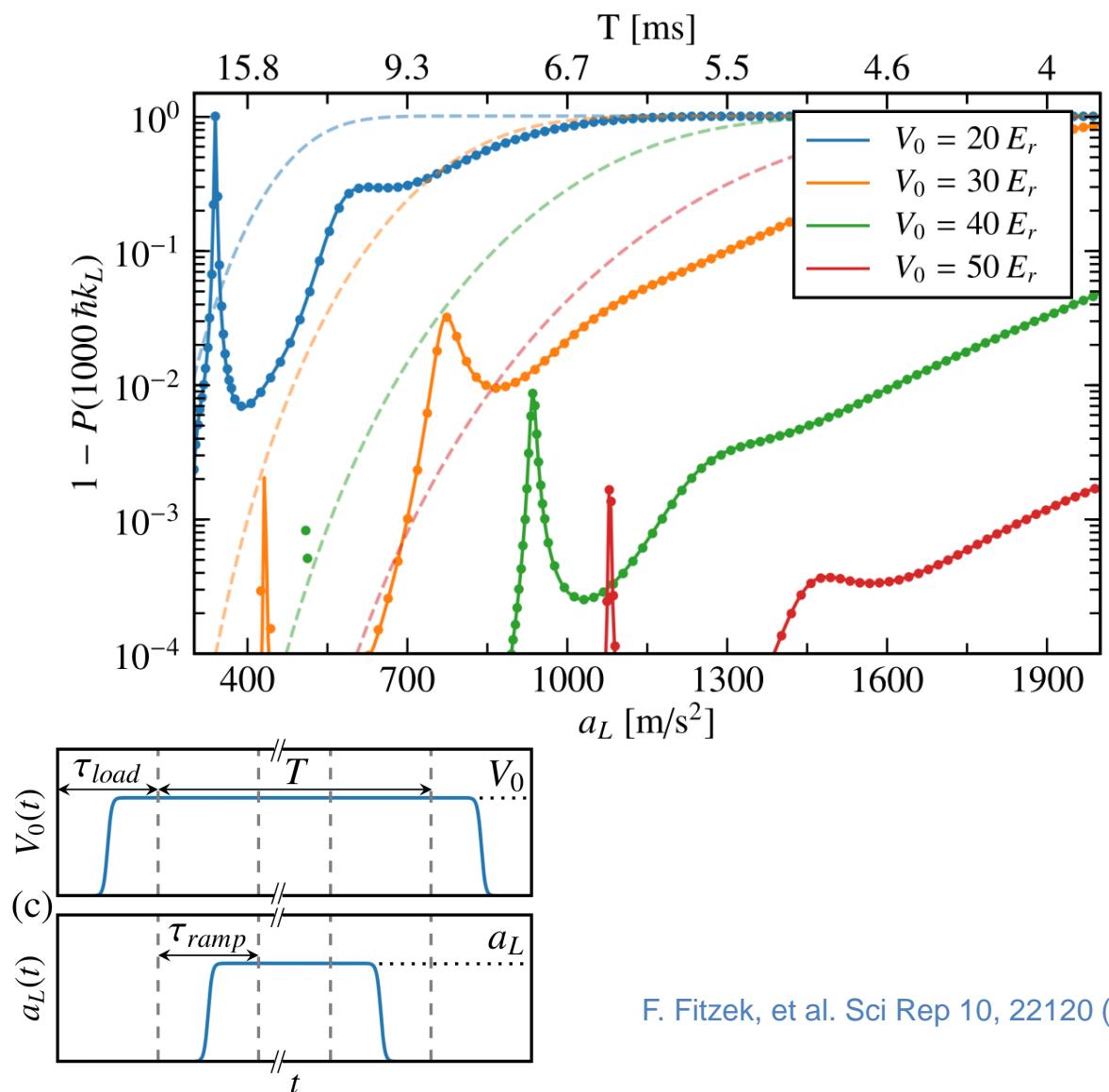
adiabatic dynamics



Losses to continuum in Bloch oscillations



probability to loose atoms after $1000\hbar k$ momentum transfer



Phase & phase uncertainty in Bloch oscillations

$$|\psi(t)\rangle = \sum_{\ell} c_{\ell} e^{-i \int_0^t dt' E_{0,0}(t')/\hbar} e^{-id\ell p_L(t)/\hbar} e^{-\int_0^t dt' \Gamma_0(t')/\hbar} |\Psi_{0,\ell}(t)\rangle$$

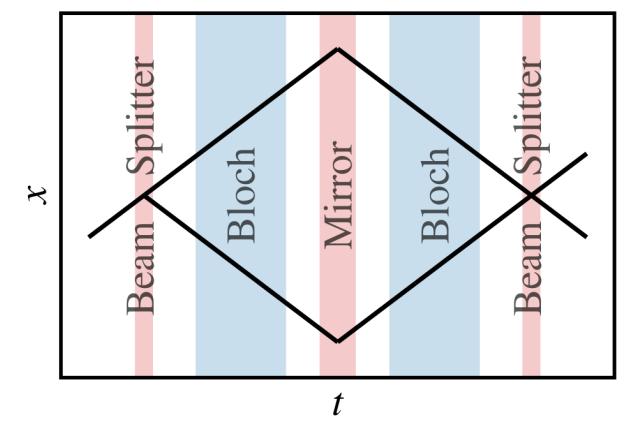
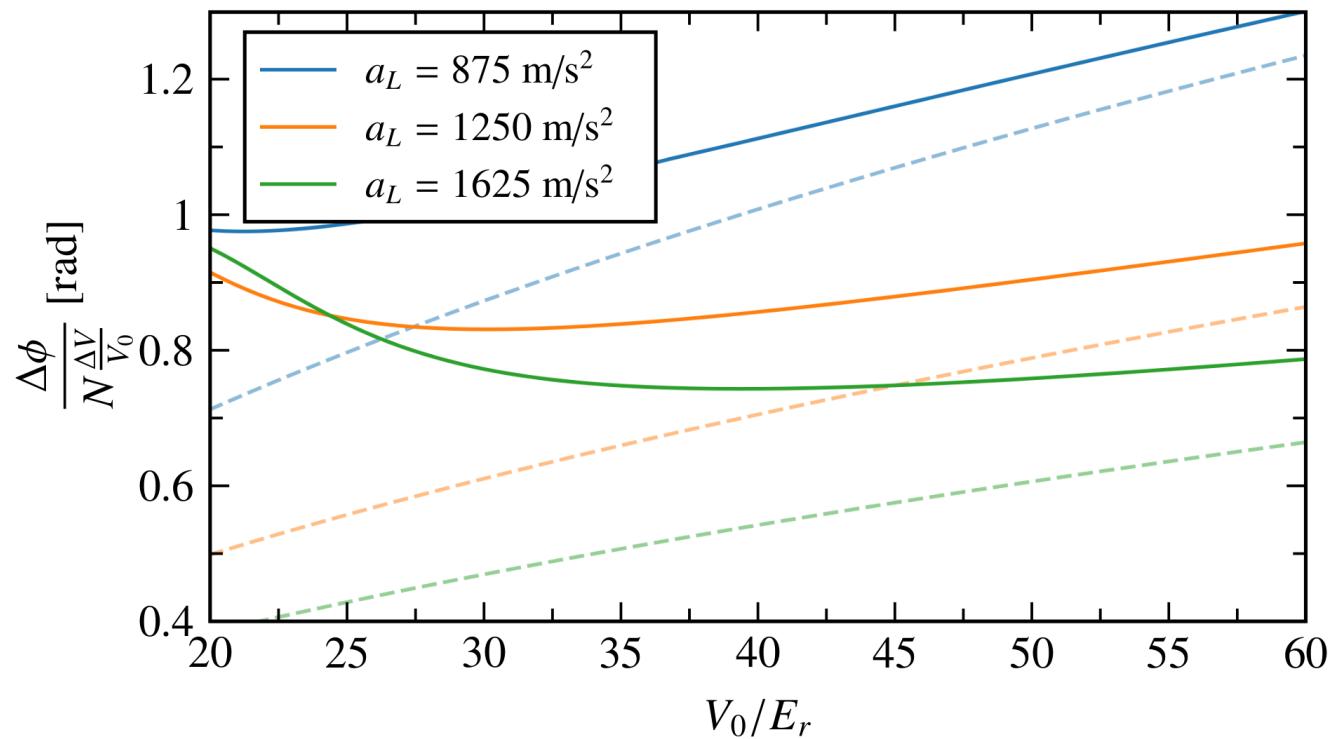
phases

phase per Bloch oscillation

$$\frac{\phi}{N} = E_{0,0} T_B / \hbar$$

phase deviation per Bloch oscillation & per relative deviation in lattice depth

$$\frac{\Delta\phi}{N \frac{\Delta V}{V_0}} = 2\pi \left| \frac{\partial E_{0,0}}{\partial V_0} \right| \frac{V_0}{d m a_L}$$



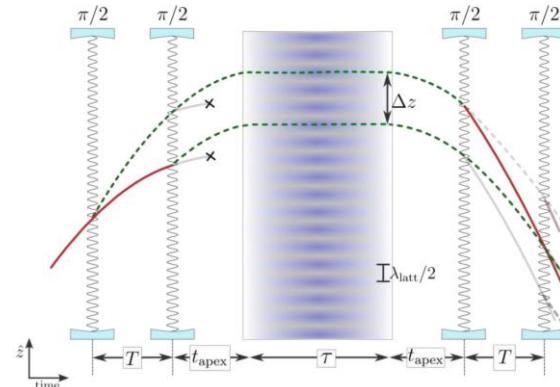
Phase & phase uncertainty in Bloch oscillations: case studies

Example 1: suspending atoms in cavity potential

Panda et al., arXiv:2210.07289 (2022)

Jitter in cavity axis → intensity variations between arms of $\frac{\Delta V}{V} \simeq 5 \times 10^{-7}$

Phase uncertainty at 1min suspension time $\Delta\phi \simeq 0.6$ rad

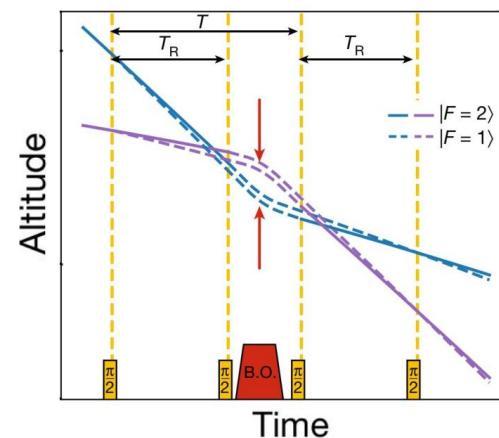


Example 2: fine structure measurement

Morel et al., Nature 588, 61-65 (2020)

Relative one-shot-uncertainty $\frac{\Delta(\frac{h}{m})}{\frac{h}{m}} \simeq 10^{-9}$ achieved at phase resolution of $\Delta\phi \simeq 1$ mrad

→ intensity variations between arms of $\frac{\Delta V}{V} \simeq 1.5 \times 10^{-6}$



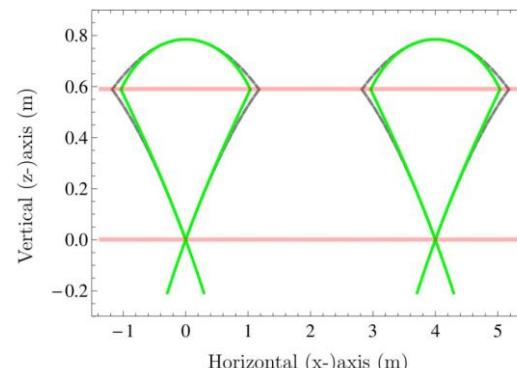
Example 3: ELGAR

Canuel et al., Clas. Quant. Gravity (2020)

LMT beam splitter based on Bragg diffraction & Bloch oscillations

Targeted phase resolution $\Delta\phi \simeq 1 \mu\text{rad}$ → intensity variations between arms of

$$\frac{\Delta V}{V} \simeq 10^{-9}$$



Summary

#1 Bragg diffraction

- Analytical theory for Bragg diffraction based on the adiabatic theorem [Siemss et al PRA 102, 033709 \(2020\)](#)
- Diffraction phases due to parasitic paths and open output ports
- Suppression of parasitic paths by adapted mirror pulse [Siemss et al arXiv:2208.06647](#)
- Control over diffraction phases with accuracy on μrad -level

#2 Bloch oscillations

- Analysis of Bloch oscillations in deep lattices & for large accelerations
- Adiabatic description in terms of Wannier-Stark states
- Finite lifetime due losses to continuum & resonant tunnelling
- Phase deviation due to intensity fluctuations

Jan-Niclas Siemß,
Florian Fitzek

Naceur Gaaloul,
Ernst Rasel,
Christian Schubert



Collaborative Research Centre
Designed Quantum States of Matter

Group

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Kasper Kusmierenk
Maja Scharnagl
Timm Kielinski
Julian Günther
Ivan Vybornyi
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Jakob Schweer



Leibniz
Universität
Hannover

Theoretical
Quantum Optics

Quantum
Metrology

Quantum
Information



SQUEIS

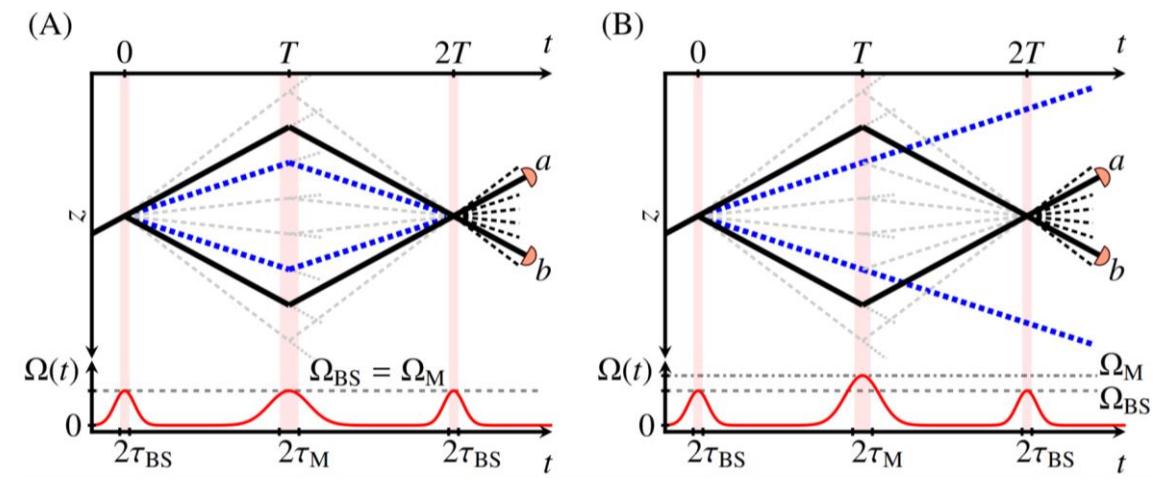
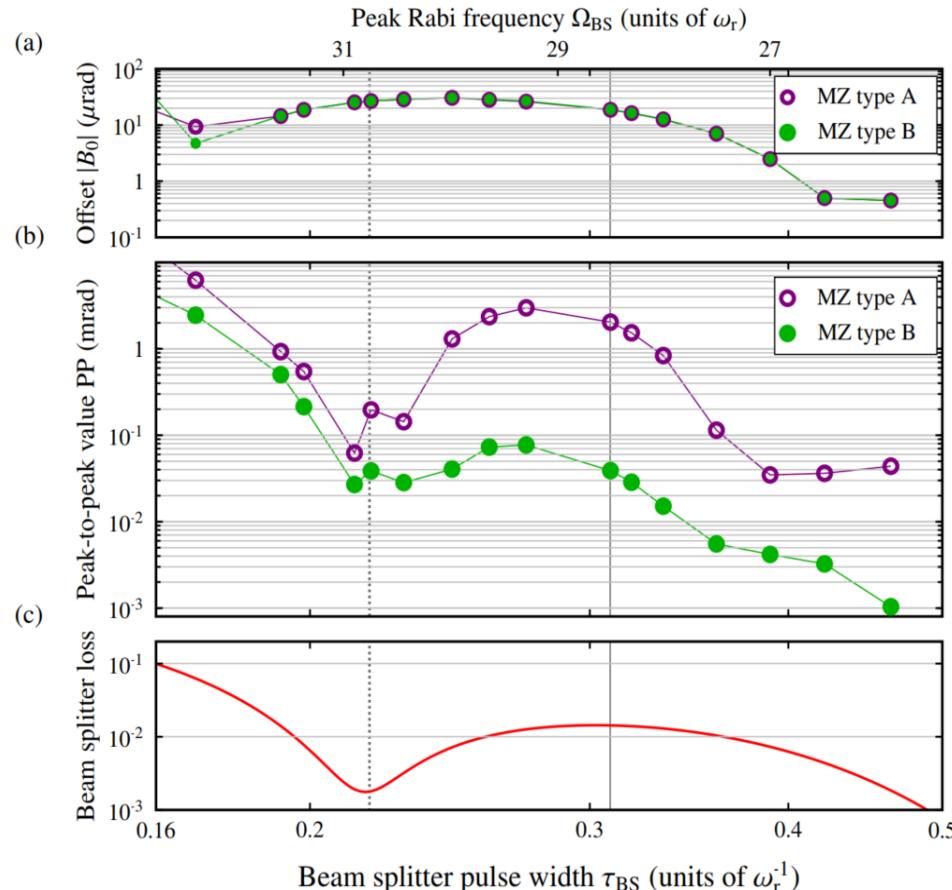


ATIQ

Diffraction phase for MZ interferometer ($n = 5$)

Systematic error for numerical simulations [1]

$$\begin{aligned}\delta\phi &= \phi^{\text{est}} - \phi = P_a^{-1}(P_a^{\text{meas}}(\phi)) - \phi \\ &= P_a^{-1}(P_a^{\text{meas}}(\phi))|_{\gamma=0} - \frac{\gamma}{n} - \phi\end{aligned}$$



Fit to systematic error:

$$f(T) = B_0 + B_1 \cos(4\omega_r T + \eta_1) + B_2 \cos(6\omega_r T + \eta_2)$$

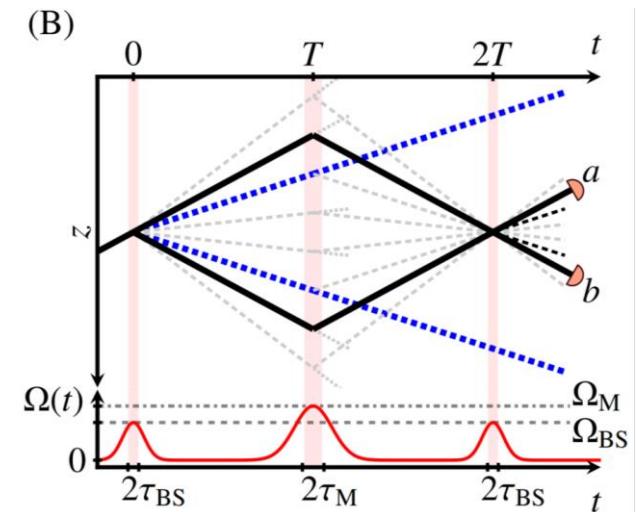
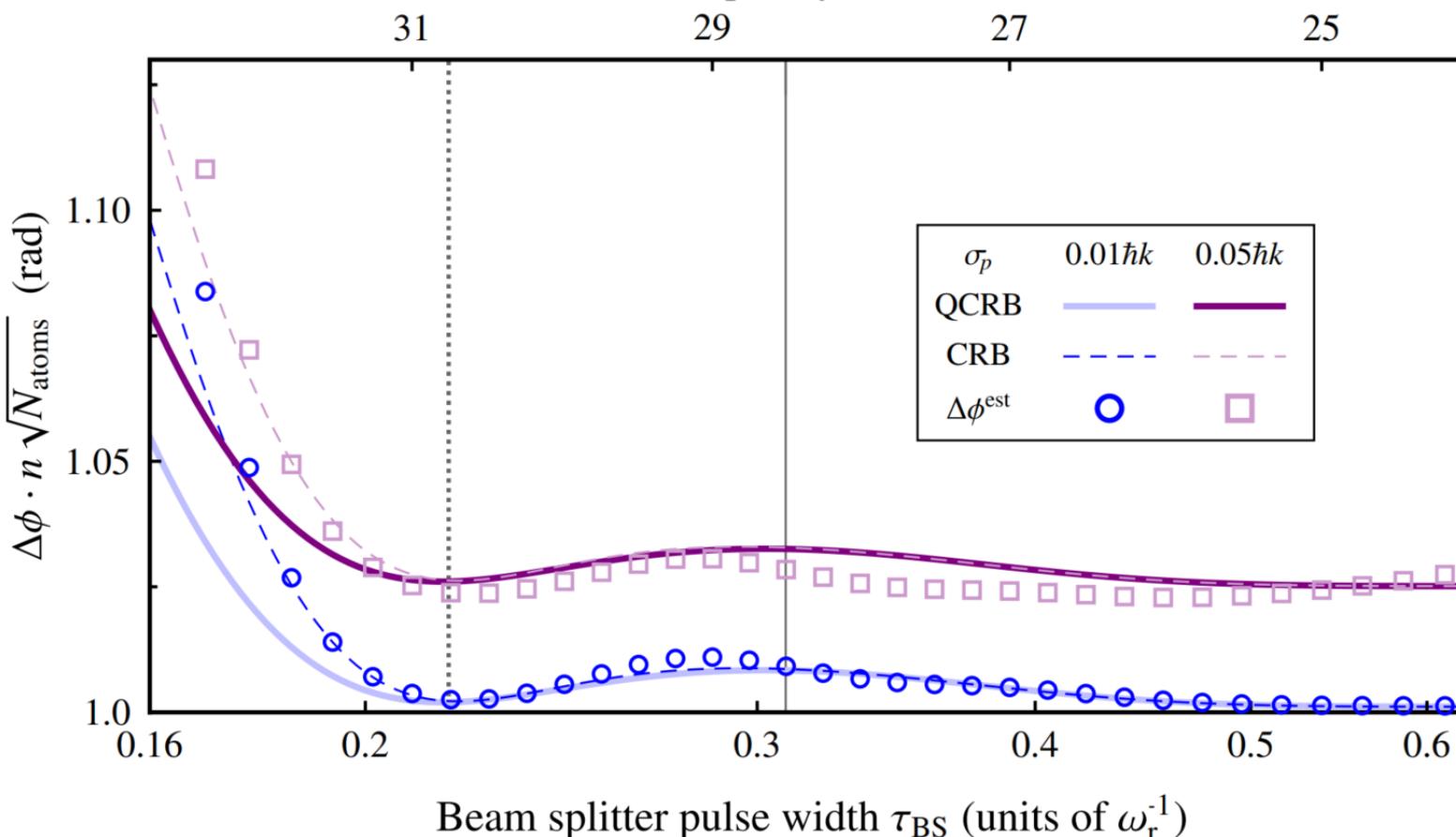
- Diffraction phases of few μrad for select pulse parameters
- Up to two orders of magnitude in suppression (including γ/n)

Projection noise limit for Bragg atom interferometers

Phase sensitivity for numerical simulations [1]

$$\Delta\phi^{\text{est}} \approx \sqrt{\frac{P_a(\phi)(1-P_a(\phi))}{N_a(\phi)+N_b(\phi)}} \frac{1}{|\partial_\phi P_a(\phi)|}$$

Peak Rabi frequency Ω_{BS} (units of ω_r)



$$|\psi_{\text{out}}\rangle = S |\psi_{\text{in}}\rangle$$

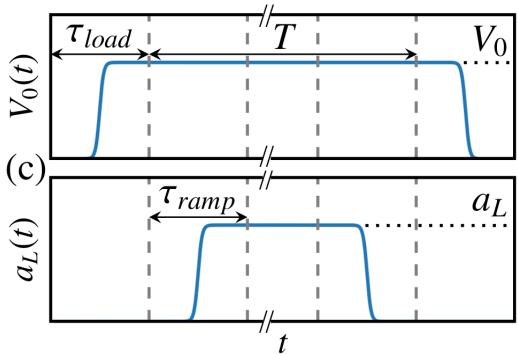
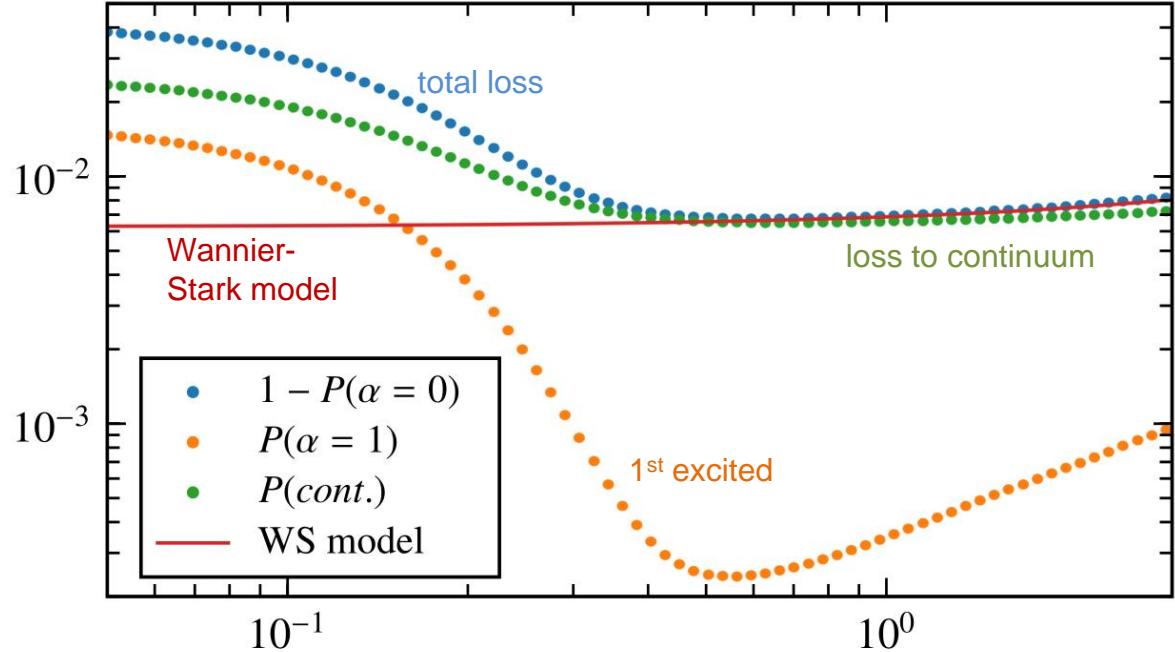
QCRB: Quantum Cramér-Rao bound

CRB: classical Cramér-Rao bound considering counting atoms in ports a and b

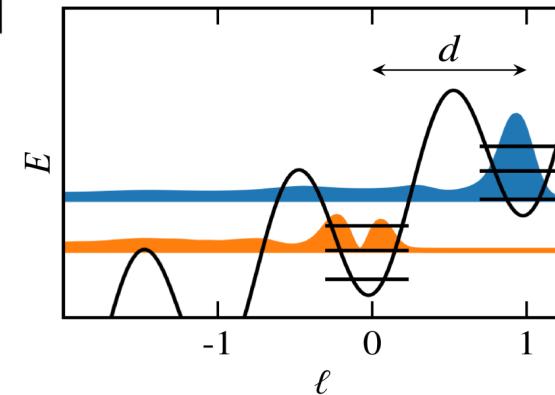
Losses to continuum vs. non-adiabatic losses

non-adiabatic losses due to fast acceleration ramp

Losses



τ_{ramp} [ms]



probability to loose atoms after $1000\hbar k$ momentum transfer

