

# GEOGEBRA SUPPORTING MATERIALS FOR INTRODUCTORY UNIVERSITY COURSES

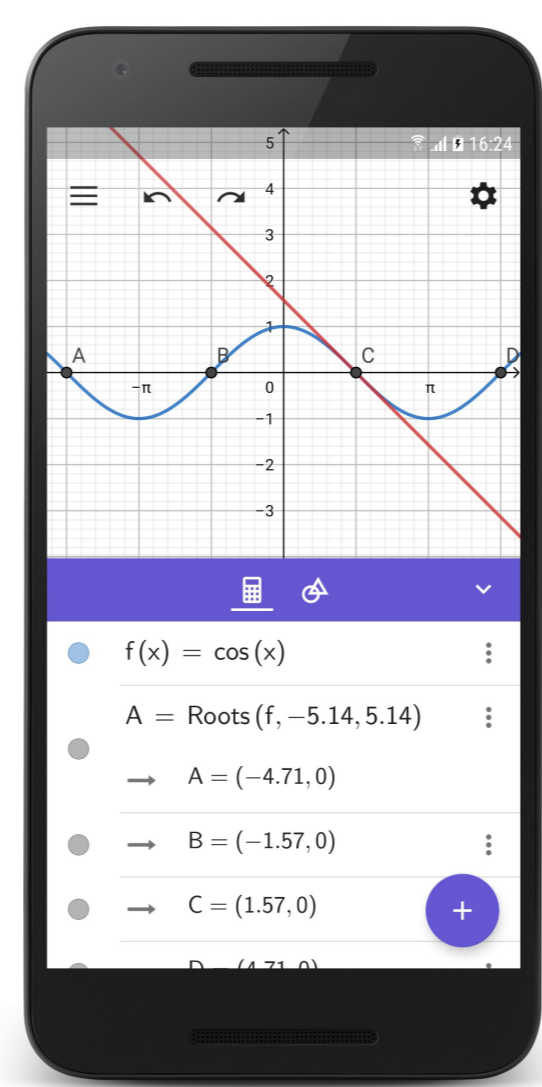
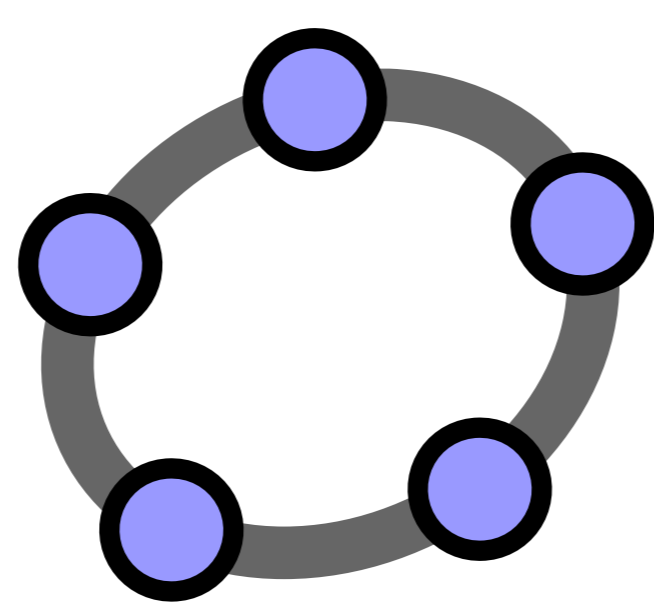


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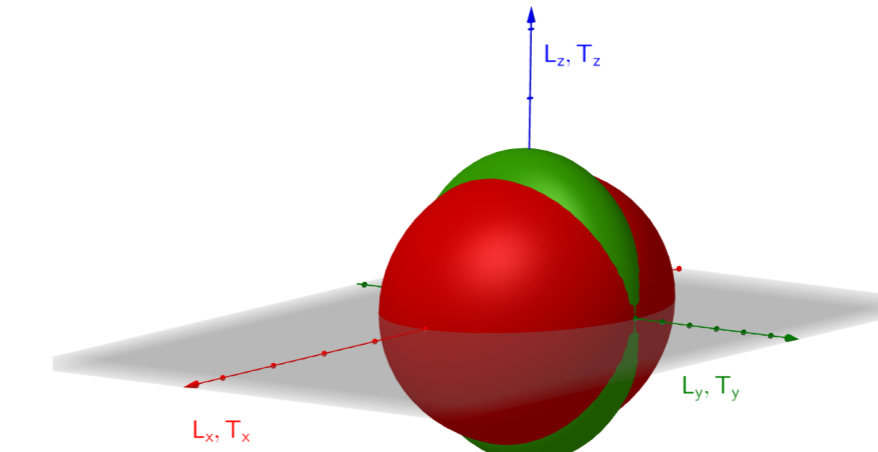
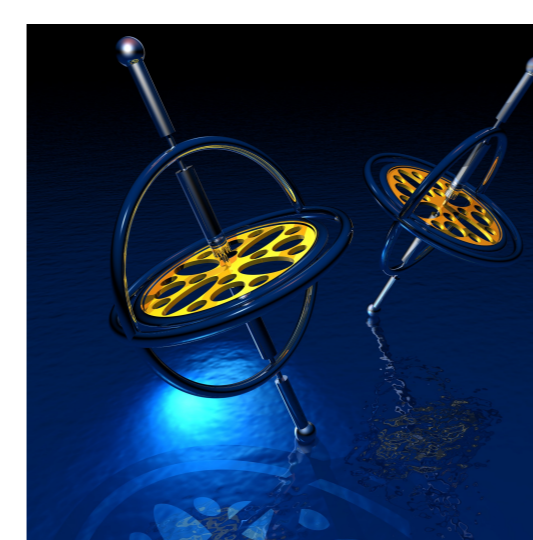
## Motivation: why GeoGebra?

- **GeoGebra** (a portmanteau of geometry and algebra) is an interactive geometry, algebra, statistics and calculus application, intended for learning and teaching mathematics and science from primary school to university level.
- **GeoGebra** is **freely** available on multiple platforms, with apps for desktops (Windows, macOS and Linux), tablets (Android, iPad and Windows) and web.
- Website: <https://www.geogebra.org>.
- **Easy sharing** of the models on-line.
- **Easy integration** into LMS materials (e.g. Moodle).
- Wide international community  $\implies$  rich reservoir of shared materials (e. g. [3–4, 6–8]).
- Provides numerical solving of algebraic equations, ODE's, simple animations.
- Leads to the more geometrical interpretation of some problems.



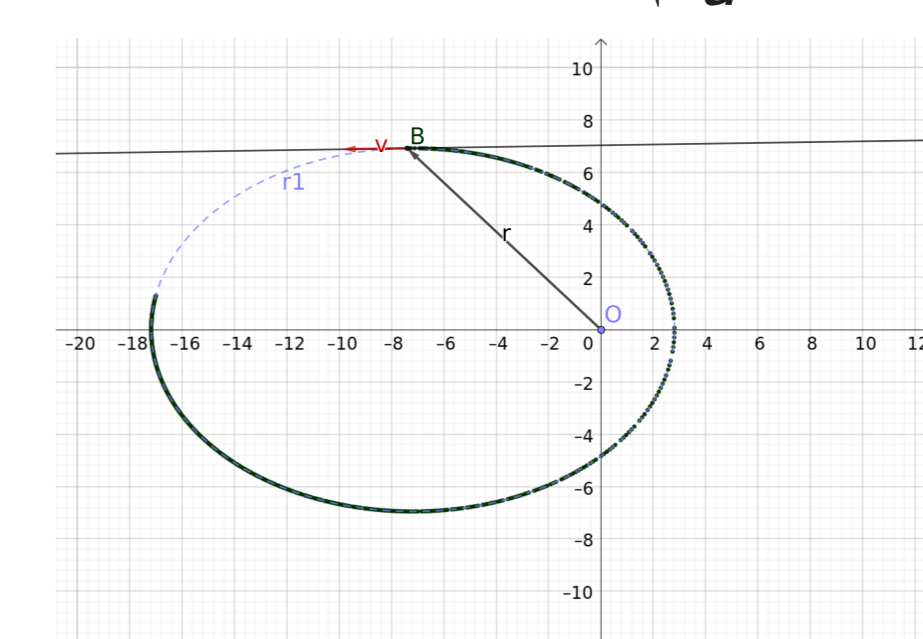
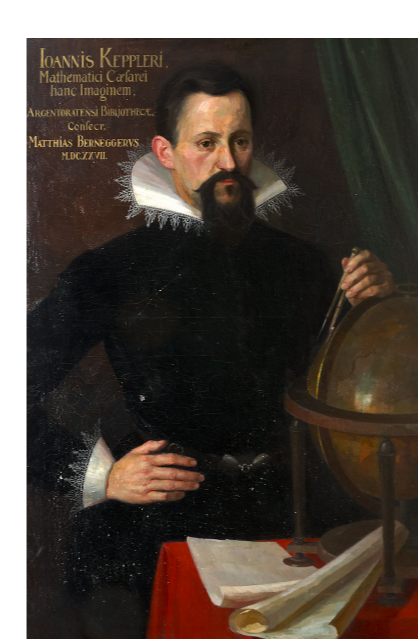
## Classical mechanics

- Free non-symmetric gyroscope as an example of conservation of kinetic energy and angular momentum (a problem from [1])



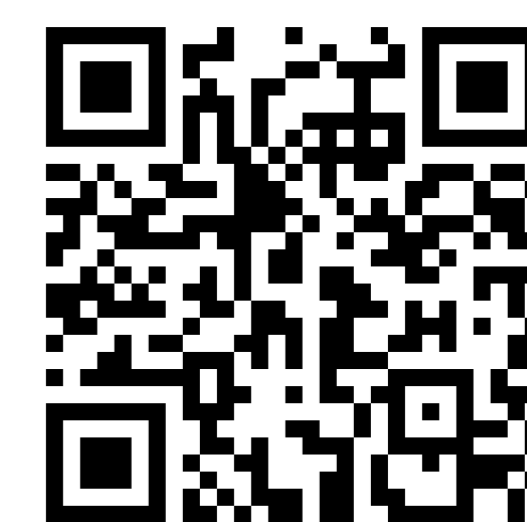
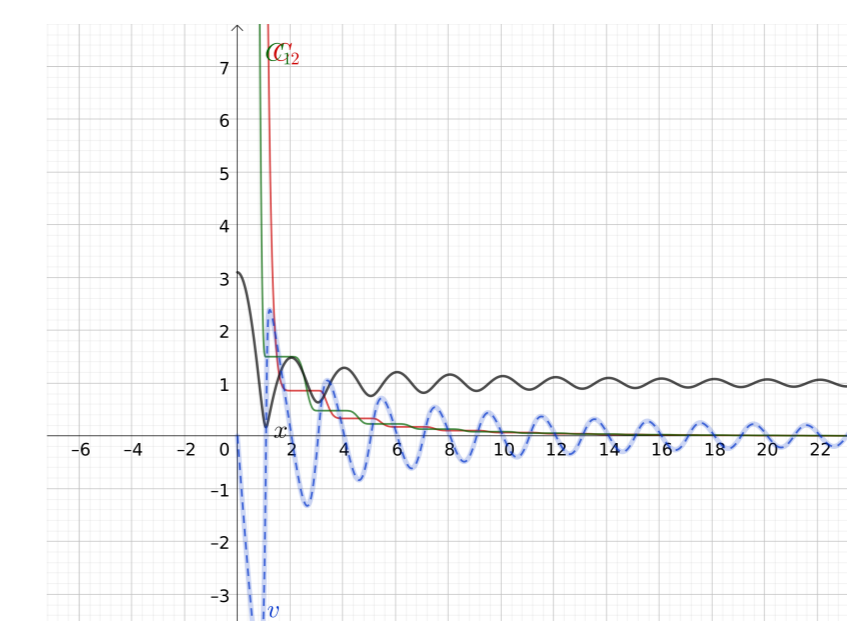
- Kepler's equation with numerical solution:

$$\Psi - \varepsilon \sin \Psi - \sqrt{\frac{GM}{a^3}} t = 0$$



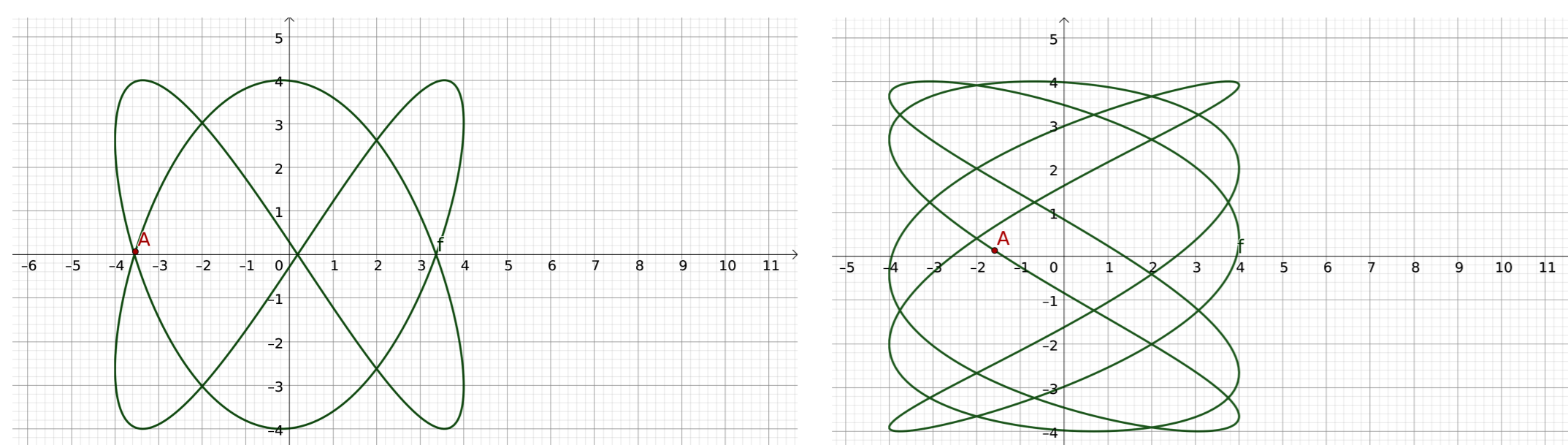
- Buquoy's variable mass problem formulated by J. F. A. Buquoy (1781–1851):

$$\ddot{y} = g \left( \frac{y_c}{y} - 1 \right) - \frac{1 + \text{sign}(\dot{y})}{2} \frac{\dot{y}^2}{y}, \quad y_c = \frac{F}{\eta g}, \quad g = 9.81 \text{ m s}^{-2}$$

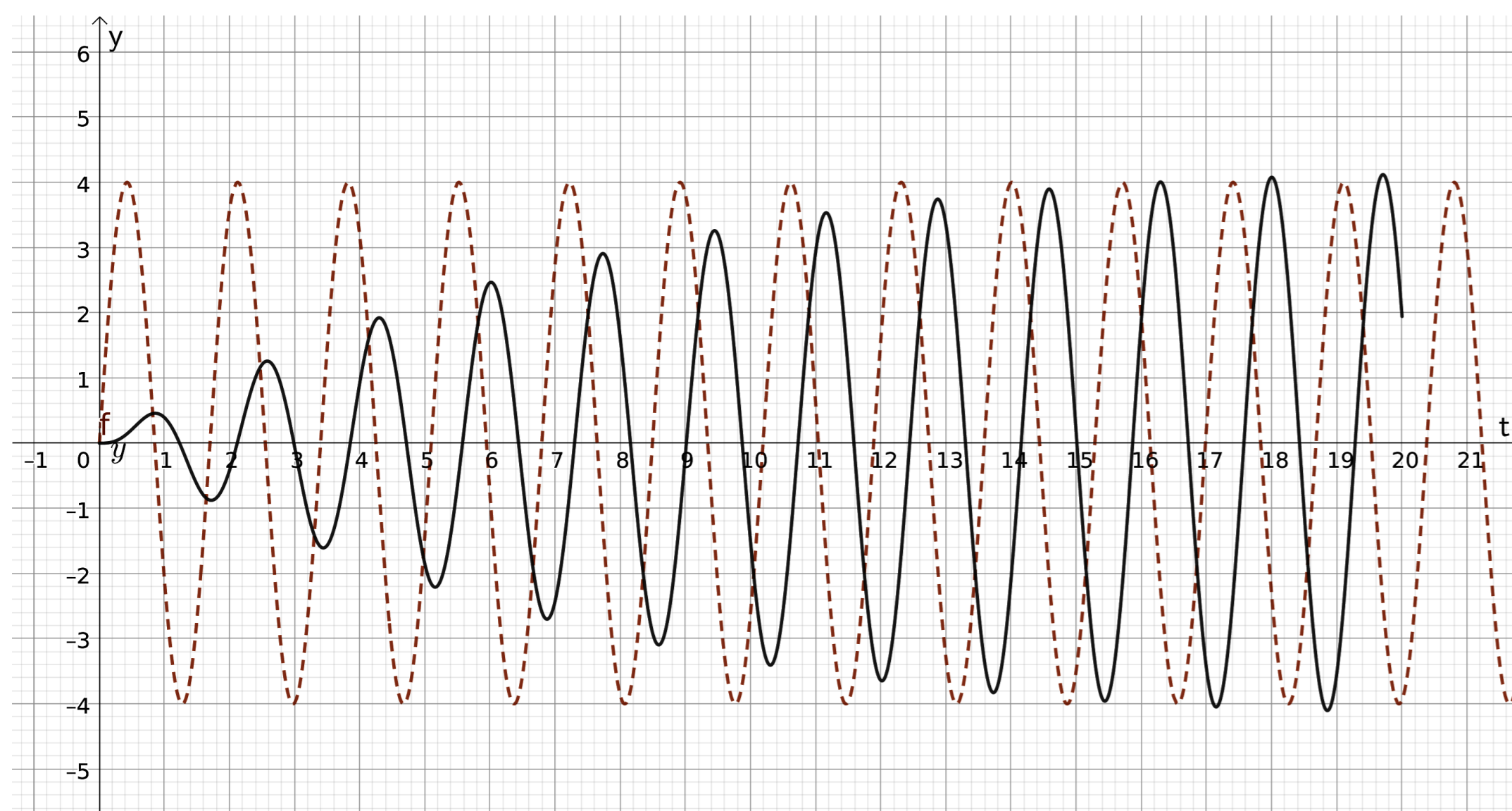


## Physics courses for future chemists

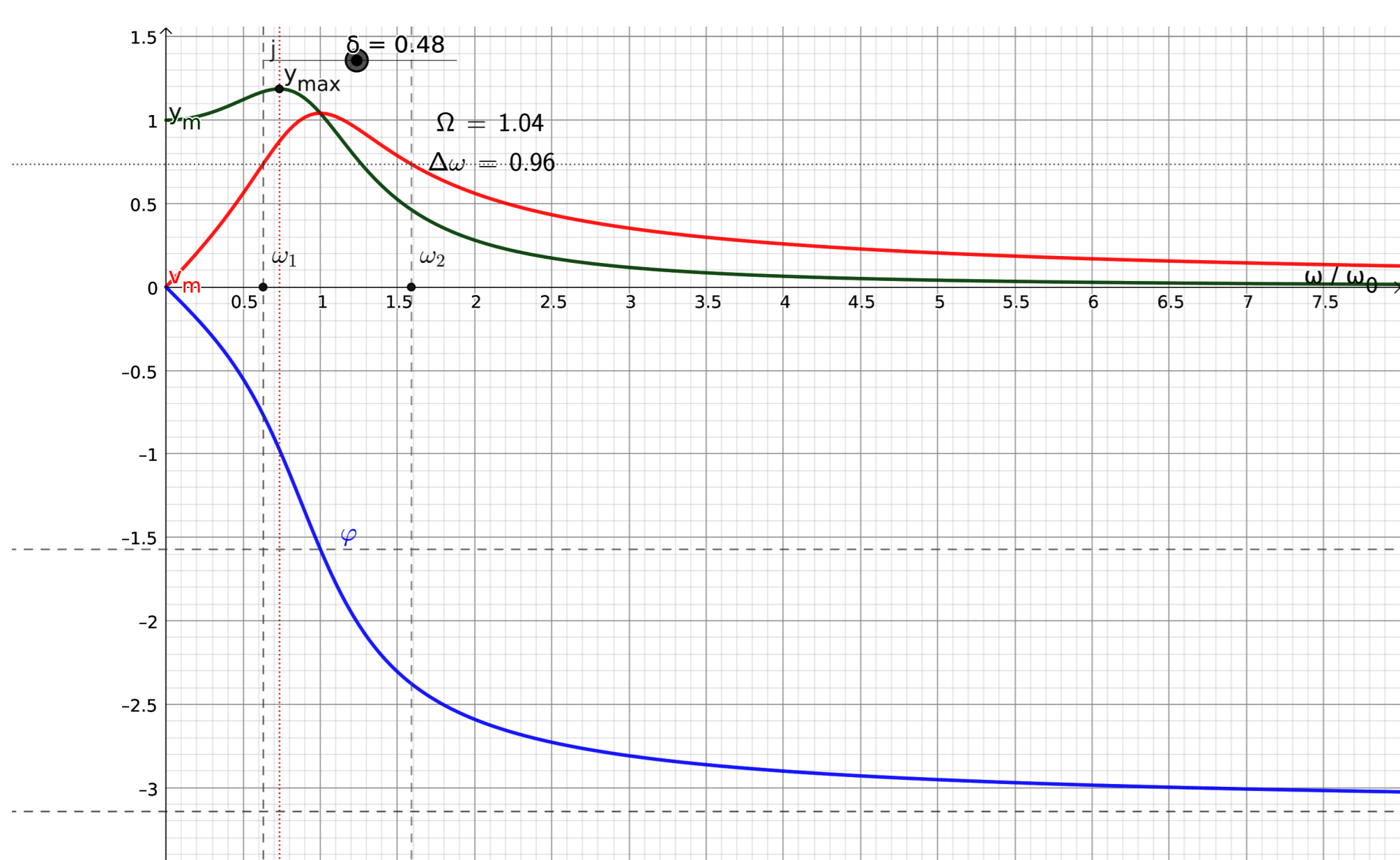
- Lissajous curves



- Forced oscillations



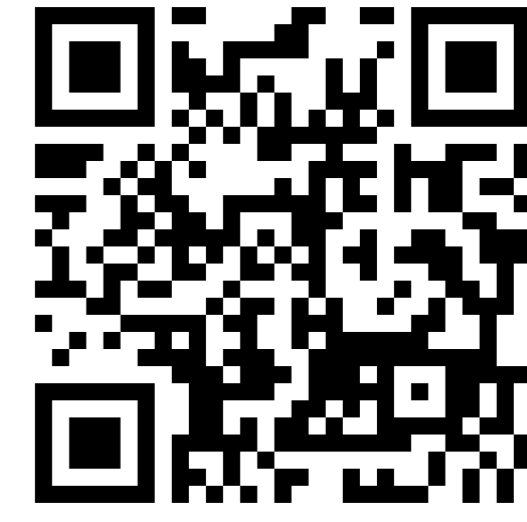
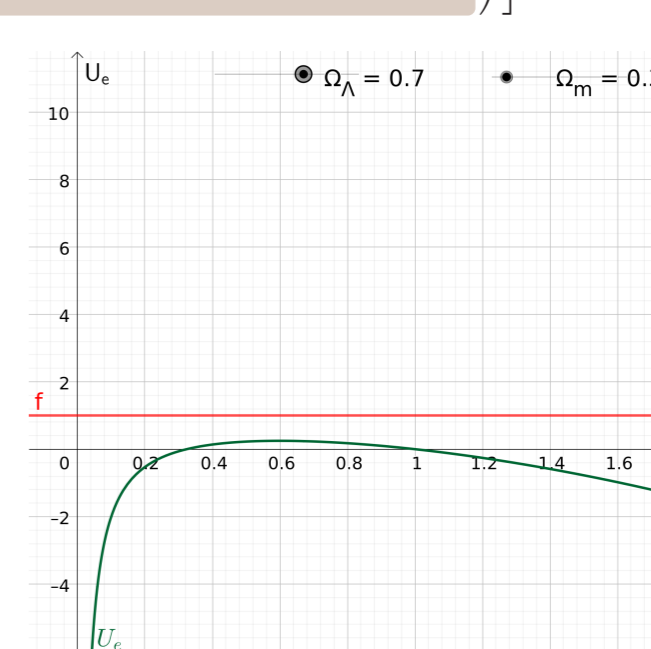
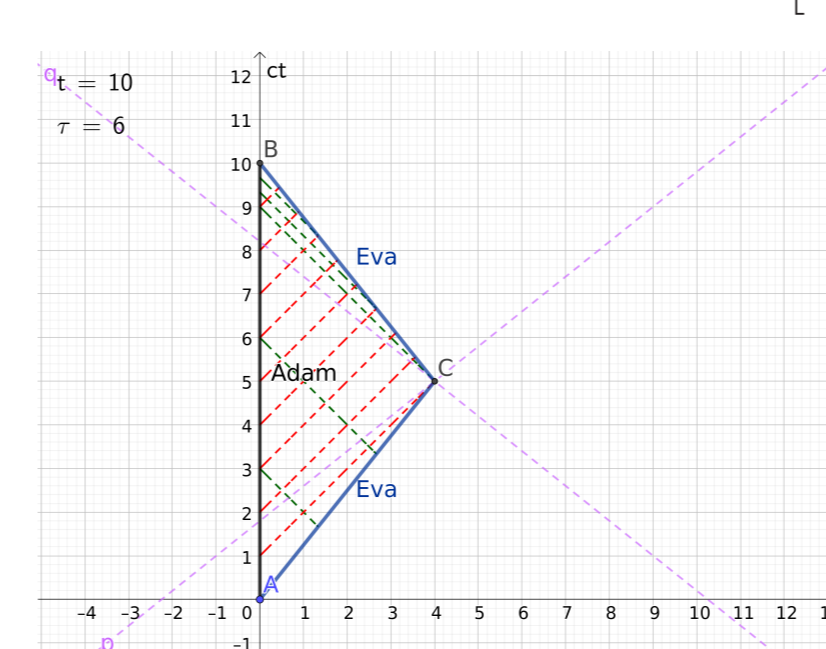
- Resonance curve



## Special relativity and intro to cosmology

- Special relativity is a rather abstract topic  $\implies$  useful illustrate relativistic effects visually [2,5]
- Twin paradox and effective potential for Friedmann universe expansion equation

$$\dot{y}^2 = H_0^2 \left[ 1 - \left( \Omega_{m0} + \Omega_{\Lambda0} - \frac{\Omega_{m0}}{y} - \Omega_{\Lambda0} y^2 \right) \right], \quad y = \frac{a}{a_0}$$



## Conclusion

- ✓ GeoGebra models are a suitable supplement to solutions to some problems within university courses, bringing a firmer understanding of discussed solutions.
- ✓ Some students often use this tool at secondary schools and use this tool for preparing graphs, even for bachelor's thesis.

## References

- [1] Chorlton F 1963 Textbook of Dynamics (London: D. van Nostrand Company Ltd.).
- [2] Kersting M and Blair D 2021 *Teaching Einsteinian Physics in Schools* (London New York: Routledge).
- [3] Rodríguez Y, Santana A and Mendoza L M 2013 Physics education through computational tools: the case of geometrical and physical optics *Phys. Educ.* **48** 621.
- [4] Solvang L and Haglund J 2021 How can GeoGebra support physics education in upper-secondary school—a review *Phys. Educ.* **56** 055011.
- [5] Takeuchi T 2010 *An Illustrated Guide to Relativity* (Cambridge University Press).
- [6] Teichrow A and Erb R 2020 How augmented reality enhances typical classroom experiments: examples from mechanics, electricity and optics *Phys. Educ.* **55** 065029.
- [7] Tomić M K, Aberšek B and Pesek I GeoGebra as a spatial skills training tool among science, technology engineering and mathematics students *Computer Applications in Engineering Education* **27** 1506–1517.
- [8] Walsh T 2017 Creating interactive physics simulations using the power of GeoGebra *The Physics Teacher* **55** 316–7.

<https://www.geogebra.org/u/lukasrichterek>

