Jet substructure at the LHC with energy correlators

CERN QCD Seminar

Bianka Meçaj - Yale University

Based on 2205.03414 and 2210.09311

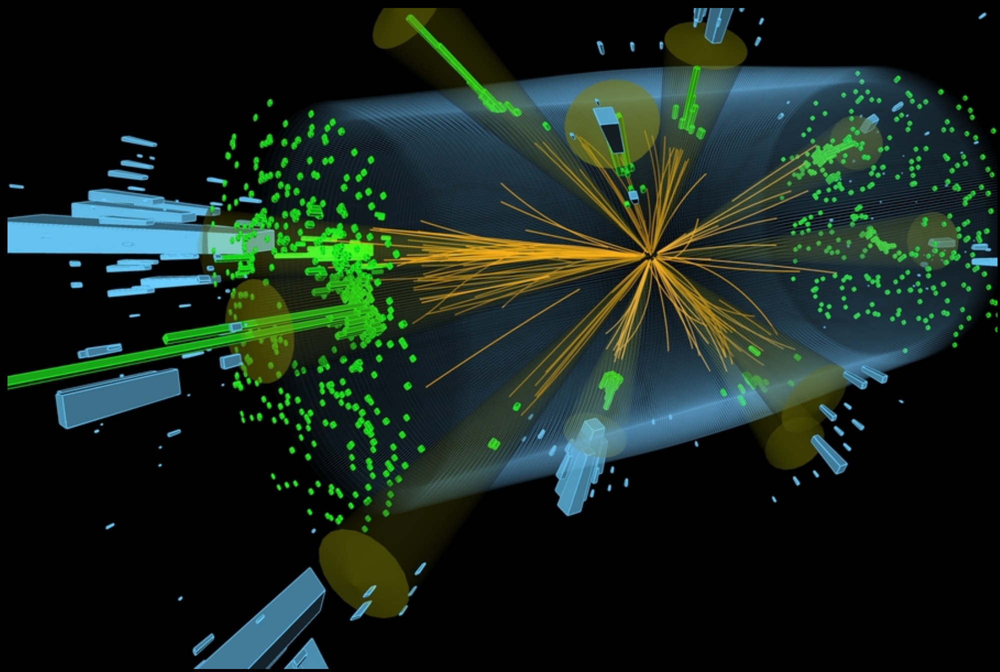
Evan Craft

Kyle Lee

Ian Moult

QCD at Hadron Colliders

Almost every LHC event contains jets



Jets are reconstructed using jet algorithms (anti-k_T)

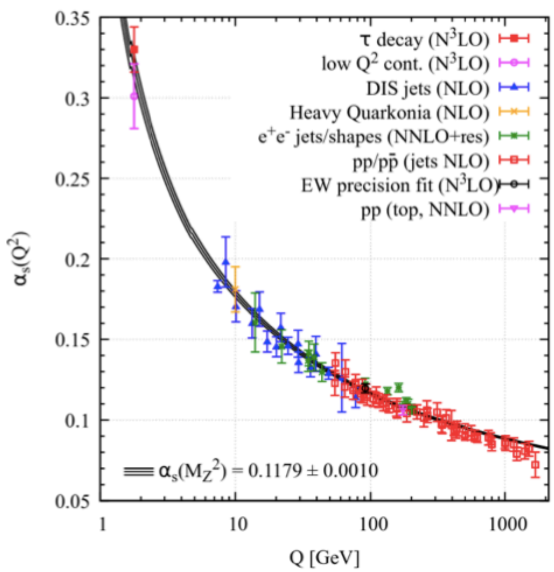
Cacciari, Salam 2006
Salam, Soyez 2007

How can we learn the most about underlying physics from the reconstructed jets?

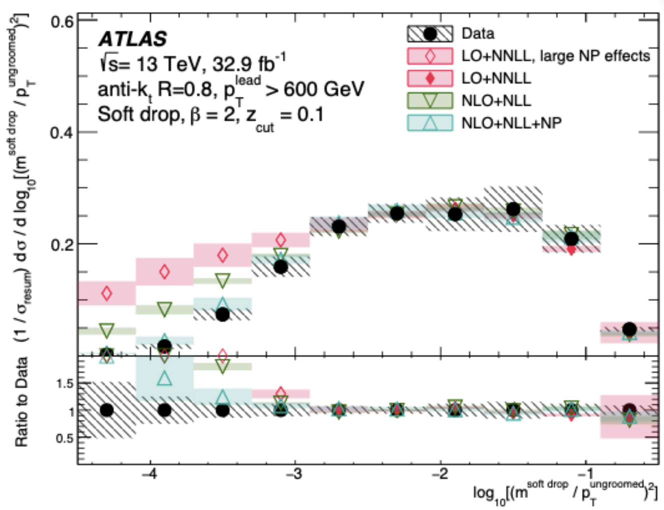
Jets at the LHC

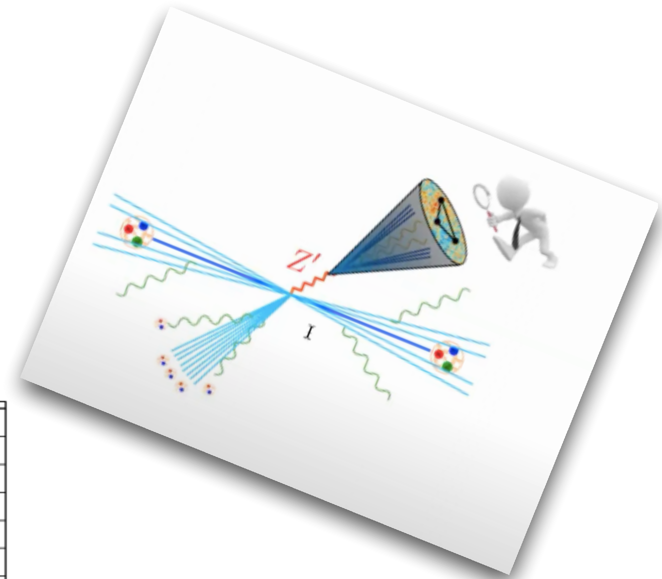
Jet substructure

QCD precision tests



New Physics

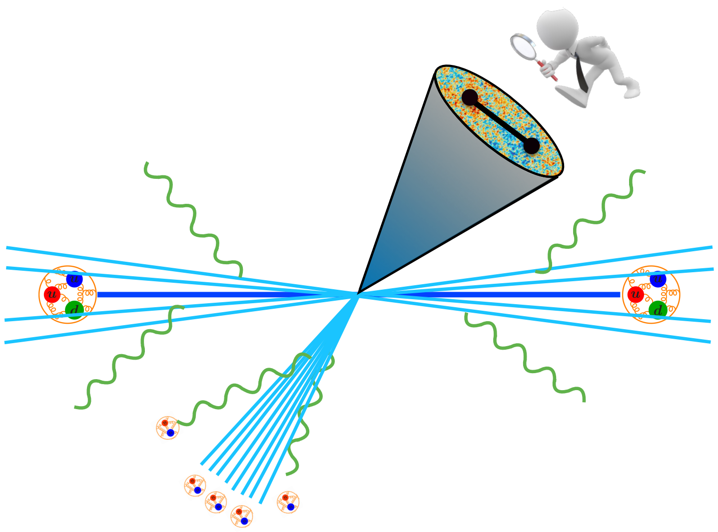




Both precision measurements and New Physics searches require precise description of jet cross sections.

Jet substructure

Study the internal structure of a jet



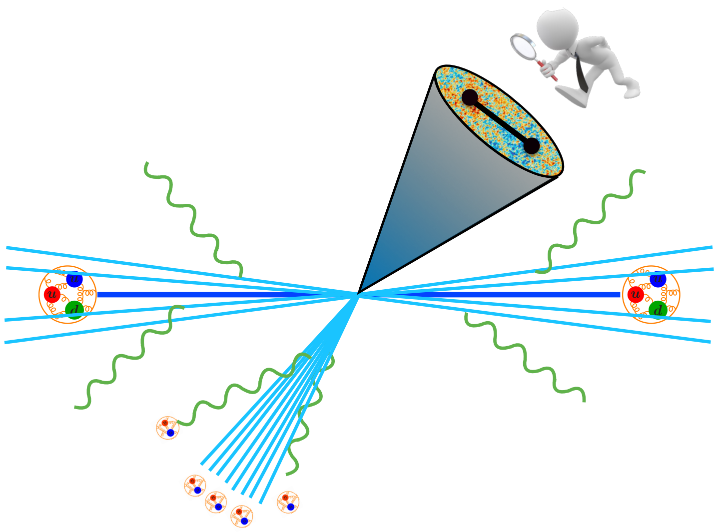
Any physics dynamics will be imprinted in the energy distributions inside the jet.

Asymptotic energy flux

- Study the statistical properties of energy flux within a jet.
- Particles within the jet are detected at infinity.
- This requires new theoretical tools to study jet substructure.

Jet substructure

Study the internal structure of a jet



Any physics dynamics will be imprinted in the energy distributions inside the jet.

Well-defined in QFT!

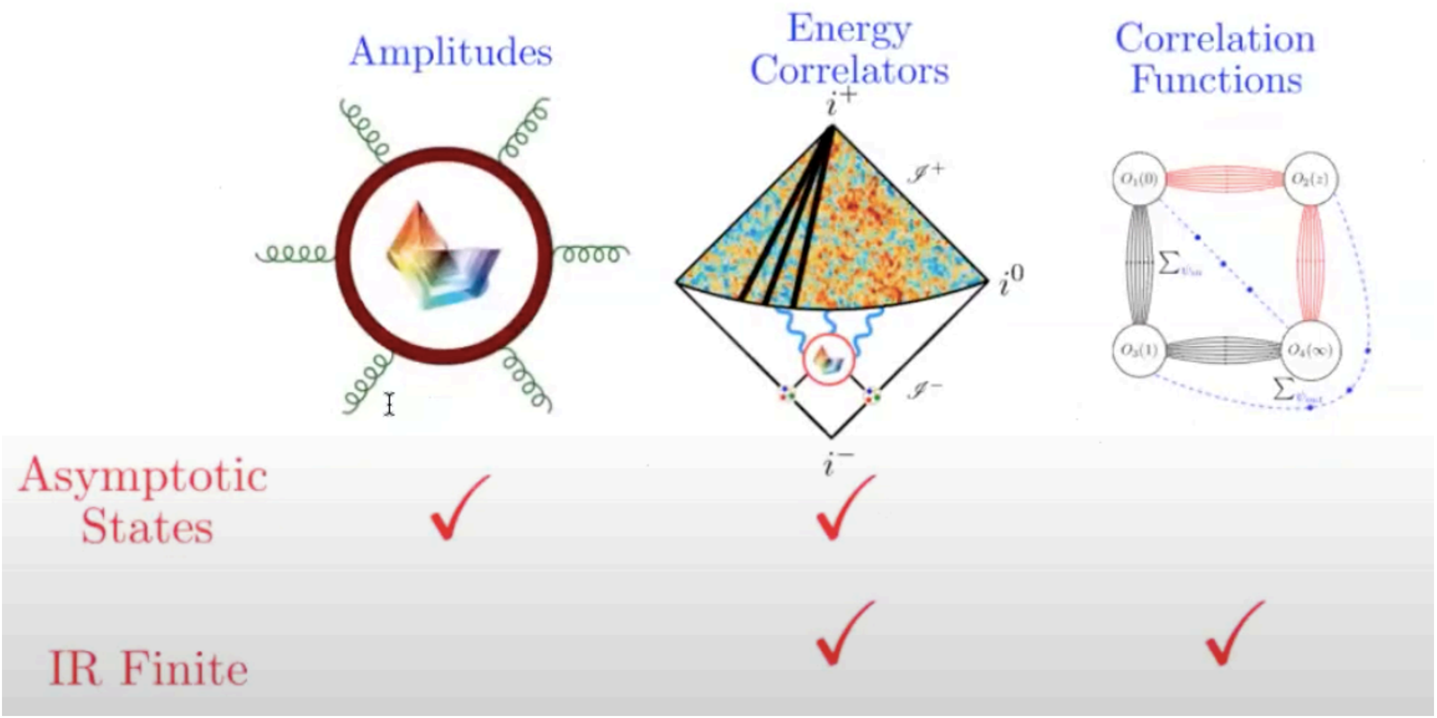
- Distribution of energy inside the jet is described by correlation functions of the energy flow operators ⇒energy correlators.

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2)\ldots\varepsilon(\vec{n}_n) \mid \Psi \rangle$$

[Basham, Brown, Ellis, Love]

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\, r^2 n^i T_{0i}(t, r\vec{n})$$

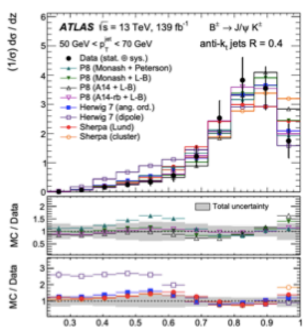
Energy Correlators



Heavy quarks at the LHC

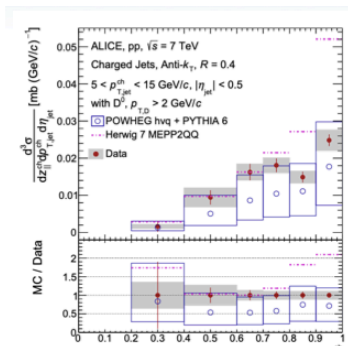
Effort to better understand their hadronization

- Many interesting processes include heavy quark effects: $h \to b\bar{b}, h \to c\bar{c}$



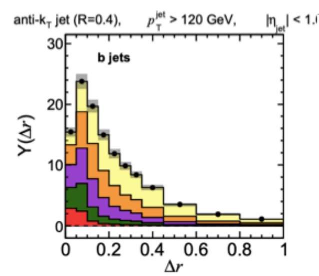
ATLAS Collaboration `21

ATLAS



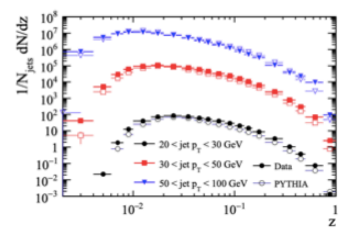
ALICE Collaboration `19

ALICE



CMS Collaboration `00

CMS

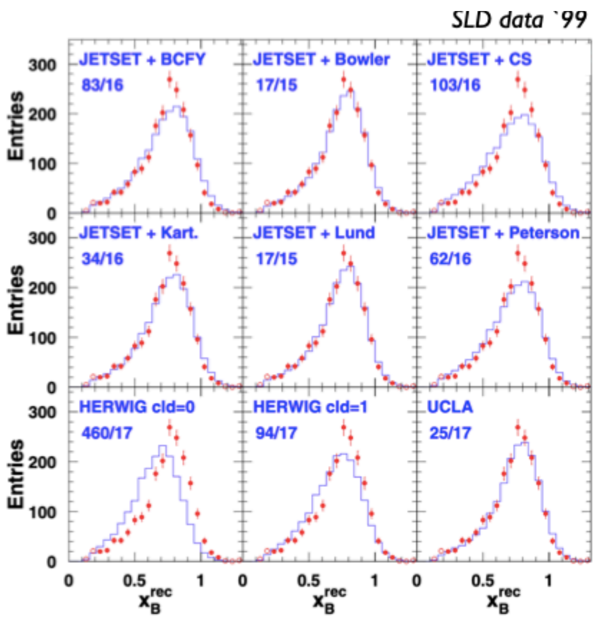


Kara Mattioli's Thesis `22

LHCb

Heavy Quarks in Parton Shower

Heavy quark effects



Parton Showers + different hadronization models compared to SLD data (`99)

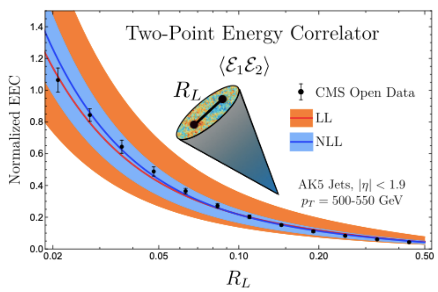
- **Monte Carlo work mostly well for light quarks.**
- Heavy quarks such as b-(beauty) and c-(charm) quarks are less understood how they develop in the shower.
- Their mass is non-negligible and this introduces an extra scale in the problem!

Energy Correlators

Progress on understanding the light-ray operators allows for the calculation and measurement of jet substructure properties from energy correlators.

$\Rightarrow$

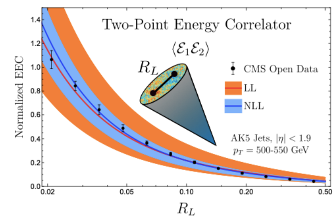
- **Scaling behaviour in the IR and UV region**
- **Intrinsic scale effects**

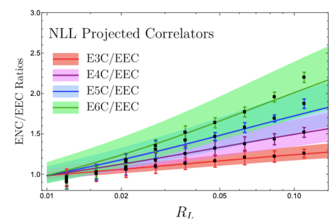


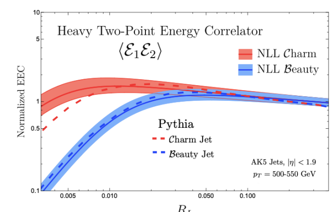
Energy correlators for jet substructure at LHC

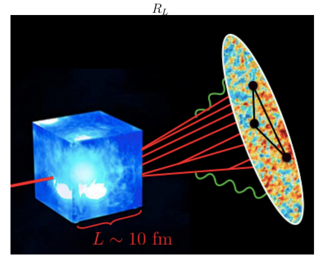
Outline

- **Scaling behavior**
- **Spectrum of the jet**
- **Intrinsic mass effects**
- **Applications**







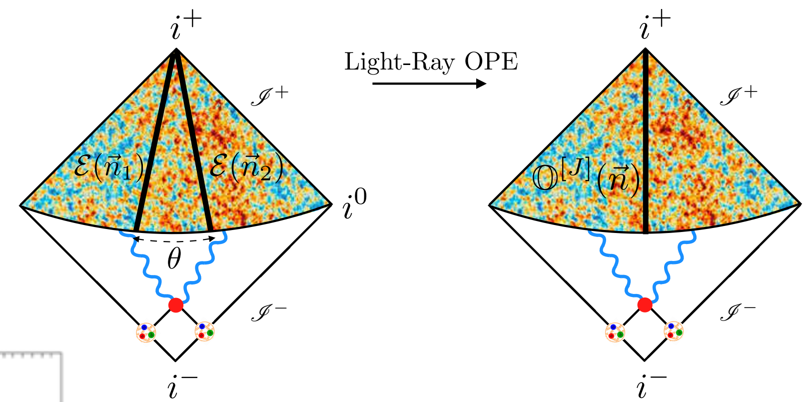


Scaling behavior

We will study energy correlators inside high energy jets at the LHC: small angle behavior

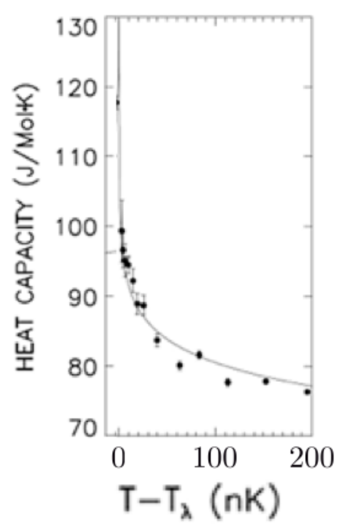
- Energy correlators admit an OPE

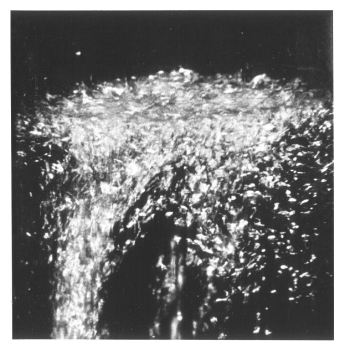
$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle \sim \sum \theta^{\gamma_i}\mathcal{O}_i(\vec{n}_1)$$

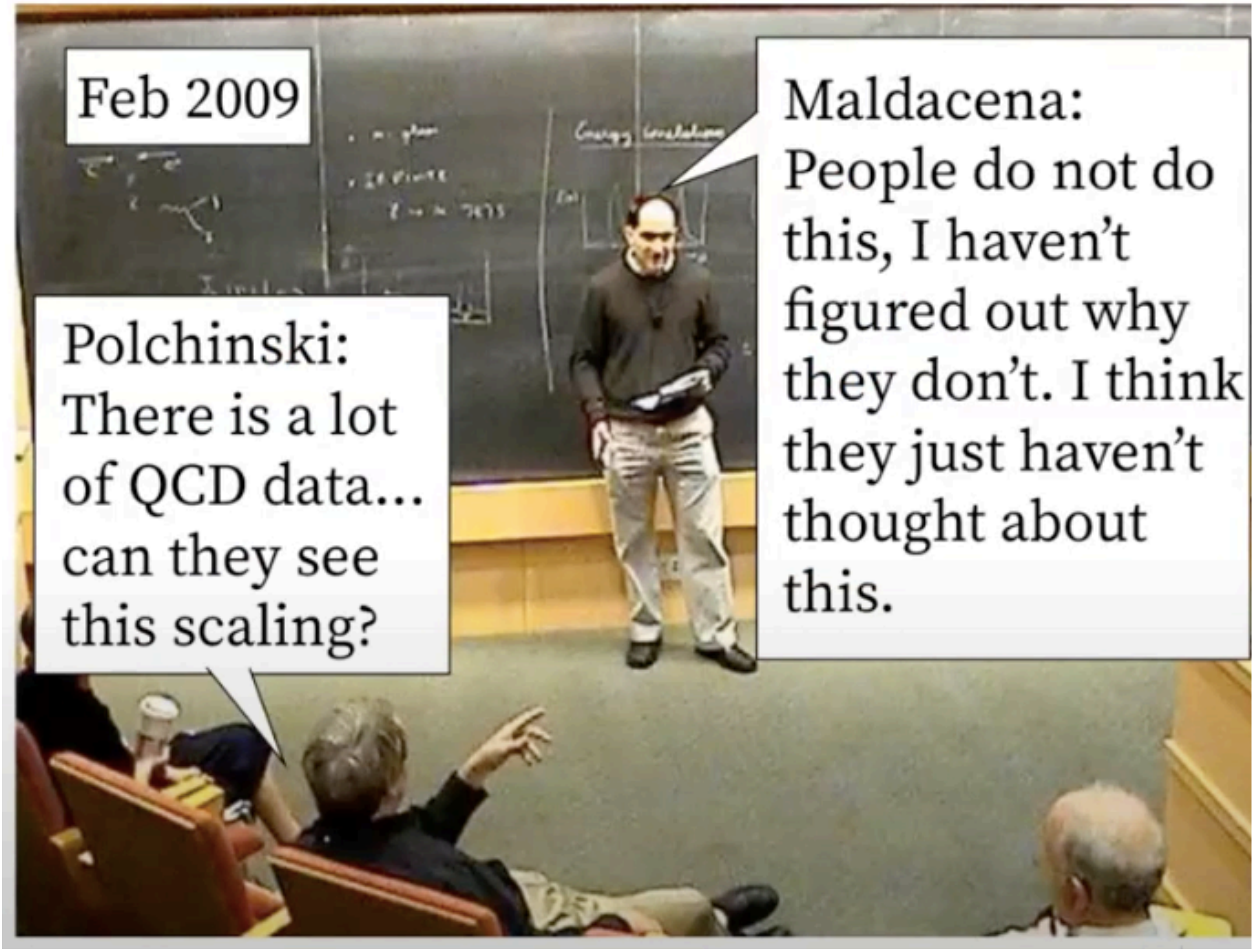


[Hofman, Maldacena]
[Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

- Universal scaling behavior in QFT as operators are brought together!



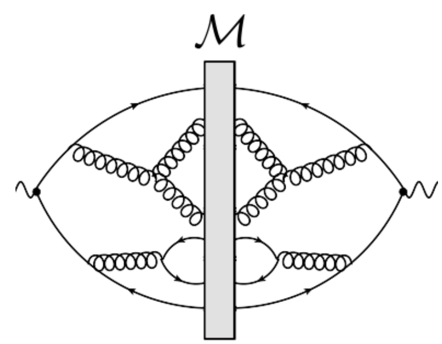



Feb 2009
Maldacena: People do not do this, I haven't figured out why they don't. I think they just haven't thought about this.
Polchinski: There is a lot of QCD data... can they see this scaling?

Event shape observables

Traditionally the way of thinking of jet substructure

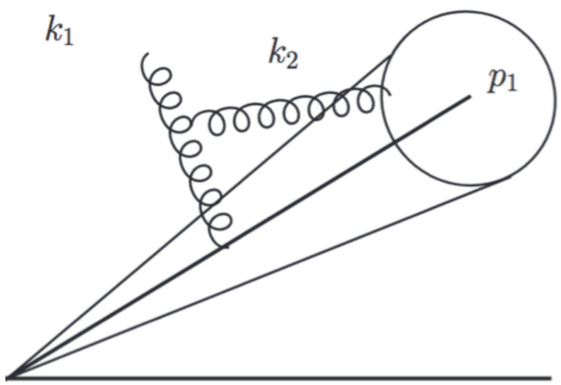
- Sometimes such observables can be complicated to compute at higher orders or for more complicated processes
- Underlying Event (UE) sensitivity
- Difficult to make generalized statements



$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

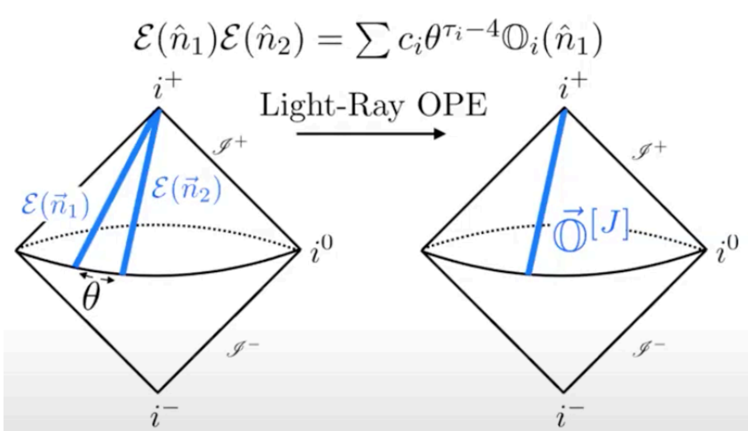
Free of non-global logarithms

- Jet substructure calculations usually suffer from non-global logs: soft and wide angle radiation.
- In jet physics such effects are removed with soft drop and grooming algorithms. ⇒ calculations depend on such algorithms, usually cumbersome
- For energy correlators such effects are suppressed (observable weighted by energy) ⇒ no need for jet grooming or soft drop!
- Energy Correlators are free of non-global logs



The light-ray OPE

- The leading scaling behaviour at the LHC is described by the leading terms in the OPE: twist two light-ray operators.
- Light-ray OPE is a rigorous and convergent expansion in CFT.

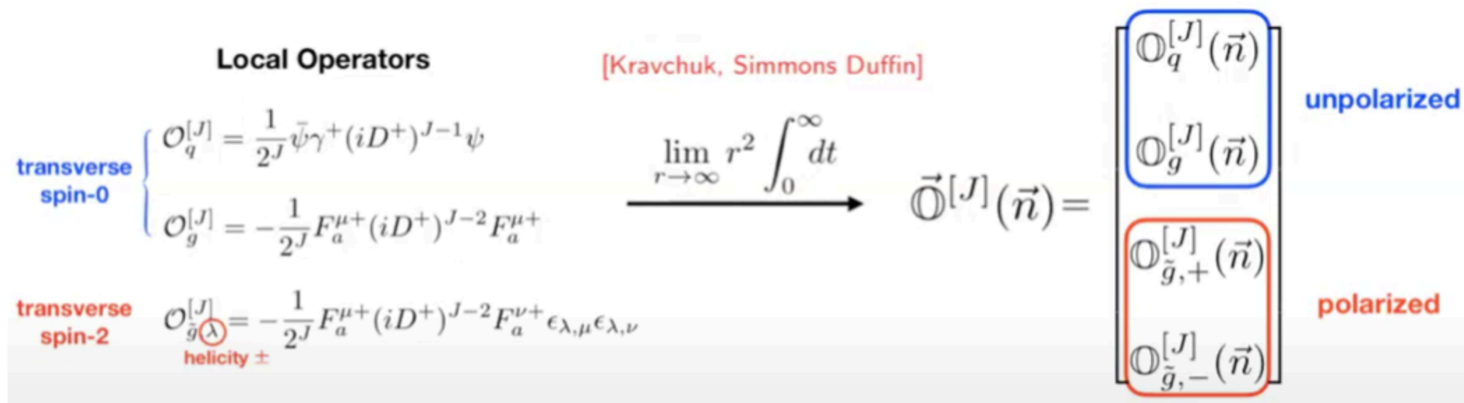


[Hofman, Maldacena]
[Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

Leading twist light-ray OPE

Control scaling at leading power

- Twist-2 operators in QCD are characterized by a spin J and transverse spin j=0,2.
- They can be transformed to a twist-2 light-ray operator vector parametrized by J



Leading twist light-ray OPE

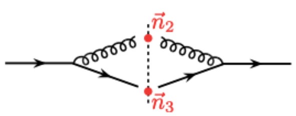
Explicit result in QCD

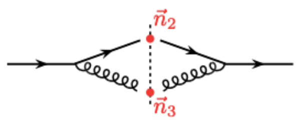
- Leading power expansion depends on coefficients that are analytic in J.

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\vec{\mathcal{J}}\left[\widehat{C}_\phi(2) - \widehat{C}_\phi(3)\right]\vec{\mathbb{O}}^{[3]}(\hat{n}_1) + \cdots ,$$

$$\vec{\mathbb{O}}^{[J]}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\left[\widehat{C}_\phi(J) - \widehat{C}_\phi(J+1)\right]\vec{\mathbb{O}}^{[J+1]}(\hat{n}_1) + \cdots$$

[Chen, Moult, Zhu]



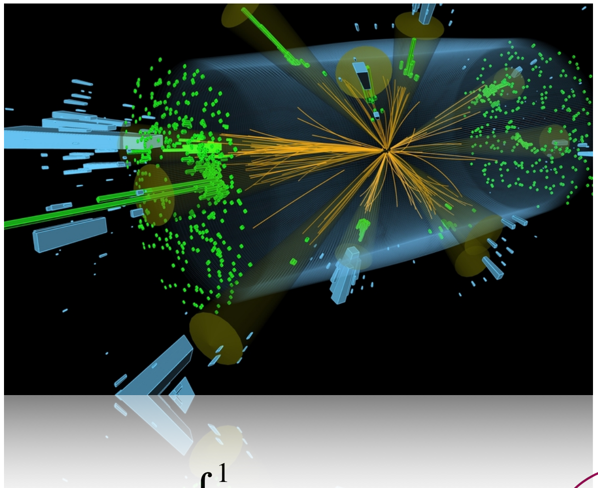


$$\widehat{C}_\phi(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 2n_f\gamma_{q\tilde{g}}(J)e^{-2i\phi}/2 & 2n_f\gamma_{q\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & \gamma_{g\tilde{g}}(J)e^{-2i\phi}/2 & \gamma_{g\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{\tilde{g}q}(J)e^{2i\phi} & \gamma_{\tilde{g}g}(J)e^{2i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{4i\phi} \\ \gamma_{\tilde{g}q}(J)e^{-2i\phi} & \gamma_{\tilde{g}g}(J)e^{-2i\phi} & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{-4i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) \end{pmatrix}$$

- OPE coefficients for polarized operators are power suppressed.

Energy Correlators at the LHC

Factorization Formula



$$\frac{\mathrm{d}\Sigma}{\mathrm{d}p_T \mathrm{d}\eta \mathrm{d}z} = \sum_i \mathscr{H}_i \left(p_T z, \eta, \mu\right) \otimes \int_0^1 dx\, x^N \mathscr{J}_{ij}(z, x, p_T R, \mu) J_j^{[N]}(z, x, \mu)$$

[Lee, BM, Moult]

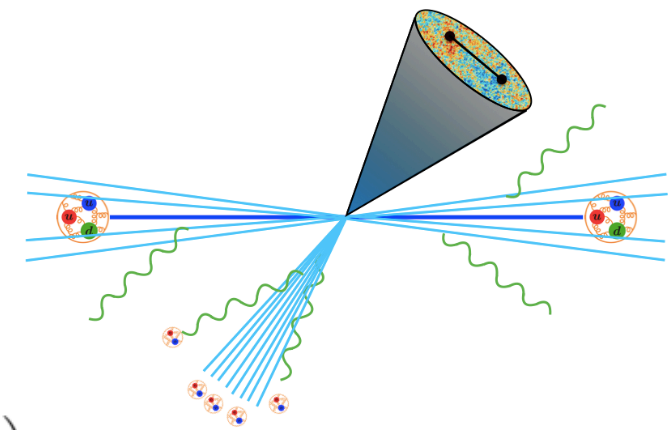
Hard function: includes pdfs

Matching coefficient, jet algorithm

Energy correlator jet function

Can calculate any higher point correlator at the LHC

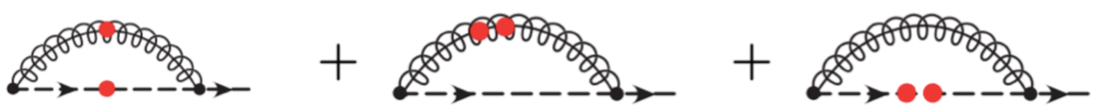
Energy correlator jet function



- EEC jet function definition:

$$J(z) = \sum_{i,j} \mathrm{Tr}\langle 0 \mid \frac{\bar{\not{n}}}{2}\xi_n(x)\delta(\omega - \bar{n}\cdot\mathcal{P})\bar{\xi}_n(0) \mid 0\rangle \frac{E_i E_j}{(Q/2)^2}\delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

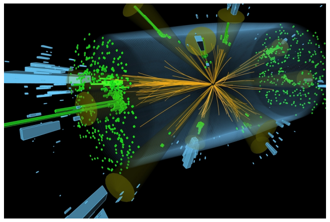
- There are three type of diagrams that contribute at one-loop



$\ni x(1-x)\delta(z - \frac{1-\cos\chi_{12}}{2})$ $\ni (1-x)^2\delta(z)$ $\ni x^2\delta(z)$

Two-point energy correlator

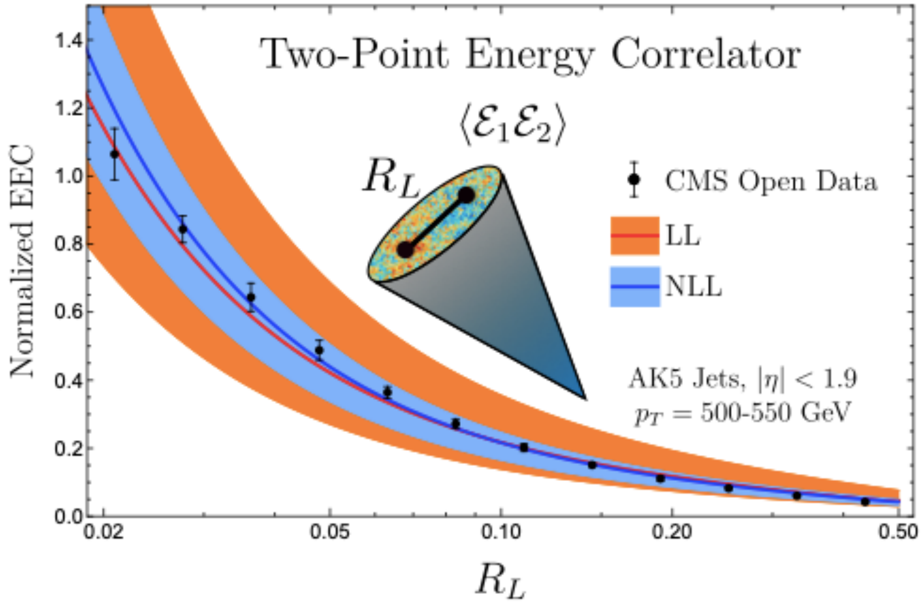
The simplest jet substructure observable



- The complicated LHC environment is described by a simple observable!
- Probe the OPE structure of $\langle \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2)\rangle$

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle \sim \sum \theta^{\gamma_i} \mathcal{O}_i(\vec{n}_1)$$

- A jet substructure observable that can test quantum scaling behavior of operators.



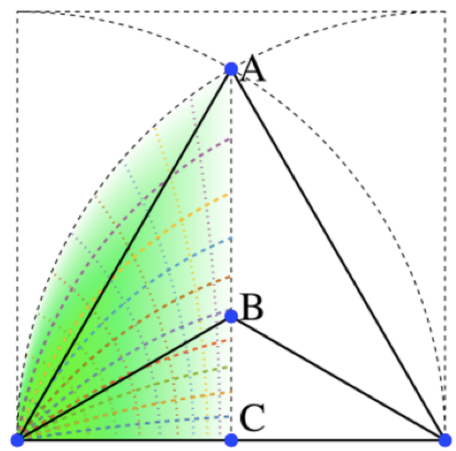
[Lee, BM, Moult]

Higher point correlators

Scaling behavior

- The high energies at the LHC are a suitable environment to test higher point correlation functions of the energy flow operator.
- For higher points one measures the "projected" correlators to the two-point correlator.
- Integrate over all shapes with fixed, largest angle R_L

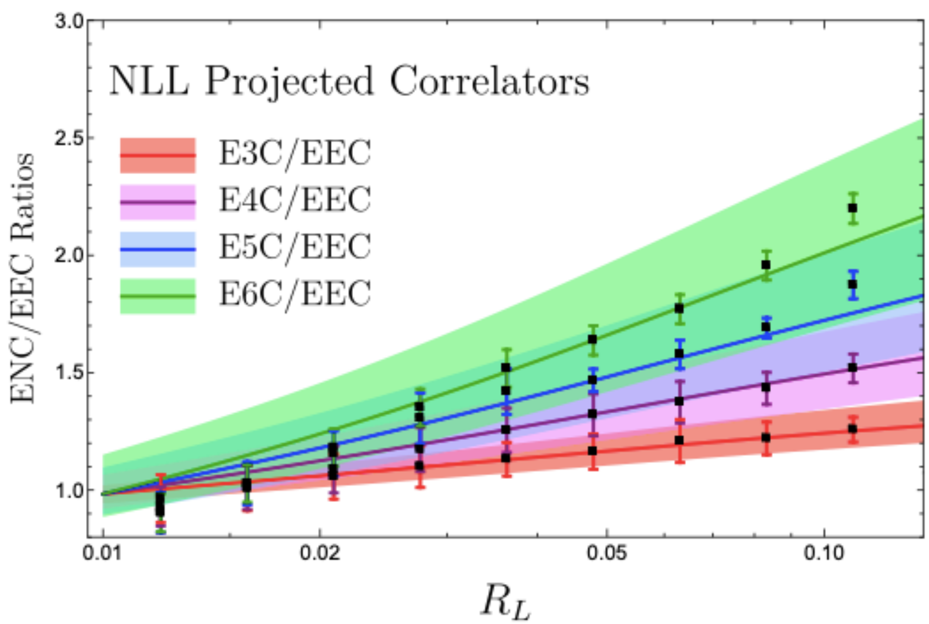
$$J_{\mathrm{EEC}}^{N-\mathrm{proj}}(R_L, x, \mu) = \int d\{\zeta\}\ \delta\left(R_L - \max[\{\zeta\}]\right)\ J_{\mathrm{EEC}}^{N}(\{\zeta\}, x, \mu)$$



Space of 3-point correlator

The jet spectrum

Higher-point correlators



[Lee, BM, Moult]

[Chen, Moult, Zhang, Zhu]

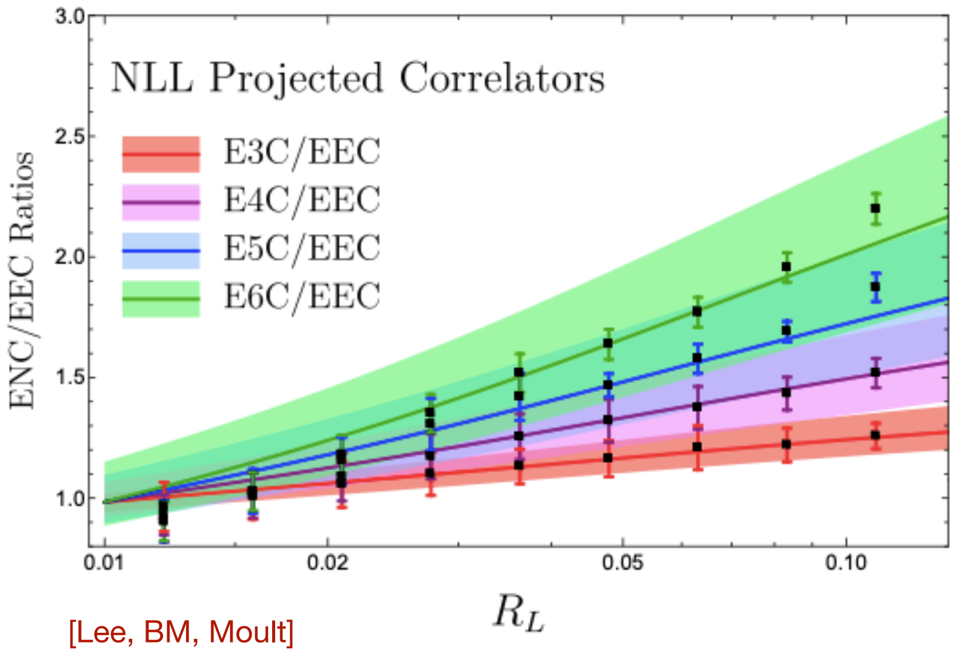
- Can be observed at the high energies at the LHC at high precision
- Ratio of the higher-point correlators with the two-point isolates anomalous scaling!
- The anomalous scaling behavior depends on N (slope increases with N)

⇓

- First hand probe of the anomalous dimensions of QCD operators.
- Non-perturbative effects cancel in the ratio

The jet spectrum

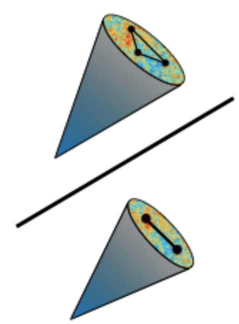
Higher-point correlators



[Lee, BM, Moult]

[Chen, Moult, Zhang, Zhu]

Asymptotic energy flux directly probes the spectrum of (twist-2) lightray operators at the quantum level!



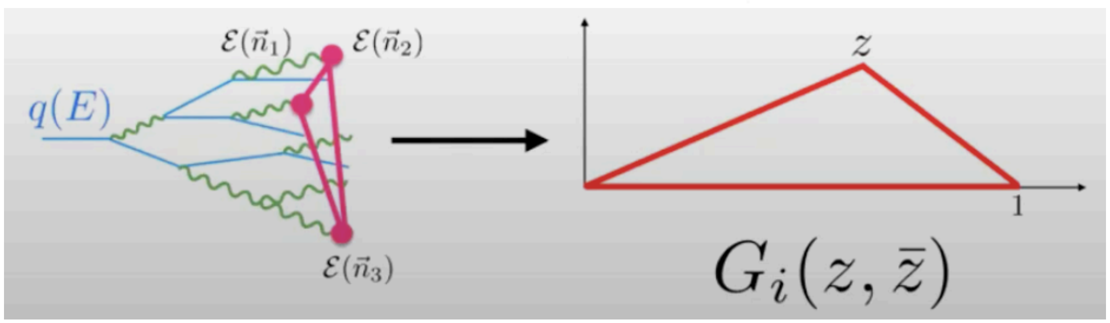
$$\frac{\langle \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_{J-1} \rangle}{\langle \mathcal{E}_1 \mathcal{E}_2 \rangle} \sim \frac{\langle \mathbb{O}^{[J]} \rangle}{\langle \mathbb{O}^{[3]} \rangle}$$

Higher point correlators at higher orders

Can probe the nature of interactions in more details

- Explicit results for $N > 2$ were presented by Hofman and Maldacena

$$\langle \mathcal{E}(\vec{n}_1)\cdots\mathcal{E}(\vec{n}_n)\rangle = \left(\frac{q}{4\pi}\right)^n \Bigg[1 + \sum_{i<j}\frac{6\pi^2}{\lambda}[(\vec{n}_i.\vec{n}_j)^2 - \frac{1}{3}] + \\ + \frac{\beta}{\lambda^{3/2}}[\sum_{i<j<k}(\vec{n}_i.\vec{n}_j)(\vec{n}_j.\vec{n}_k)(\vec{n}_i.\vec{n}_k) + \cdots] + o(\lambda^{-2})\Bigg]$$



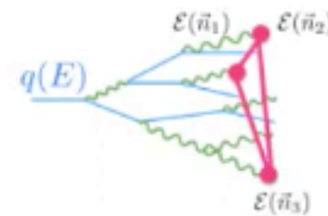
Should be able to observe this shape dependance!

Example: Three point correlator

1→3 splitting

To compute in perturbation theory, one integrates over the energy fraction of particles, with the angles of observed particles fixed.

At lowest order in perturbation theory, one has:



$$\int d\omega_1 d\omega_2 d\omega_3 \; \delta(1-\omega_1-\omega_2-\omega_3) \; (\omega_1\omega_2\omega_3) \; P_{1\to 3}$$

Consider for illustration a simple Mandelstam invariant in the splitting function $P_{1\to3} \supset \frac{1}{s_{123}}$.

One obtains integrals of the form:

Slide by I.Moult

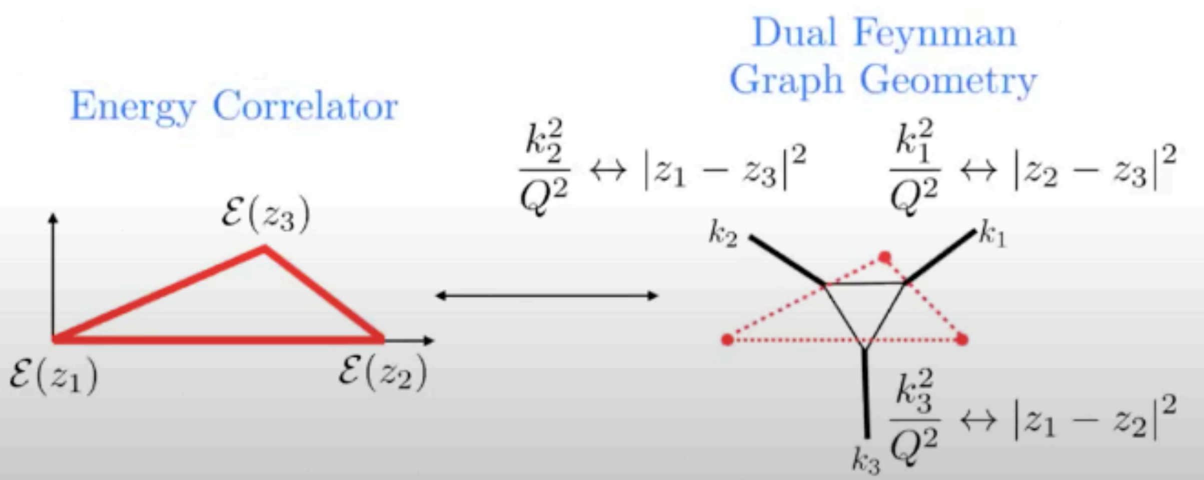
$$\int d\omega_1 d\omega_2 d\omega_3 \delta(1-\omega_1-\omega_2-\omega_3) \frac{\omega_1\omega_2\omega_3}{\omega_1\omega_2 z_{12}^2 + \omega_1\omega_3 z_{13}^2 + \omega_2\omega_3 z_{23}^2}$$

Example: Three point correlator

1→3 splitting

This is recognized as a dual Feynman loop integral, where the $|z_{ij}|^2$ are the dual coordinates:

$$x_i^\mu - x_{i+1}^\mu = p_i^\mu \, , x_{ij}^2 = (x_i - x_j)^2 = (p_i + \cdots p_{j-1})^2 \, ,$$
$$x_{ij}^2 \leftrightarrow |z_{ij}|^2$$



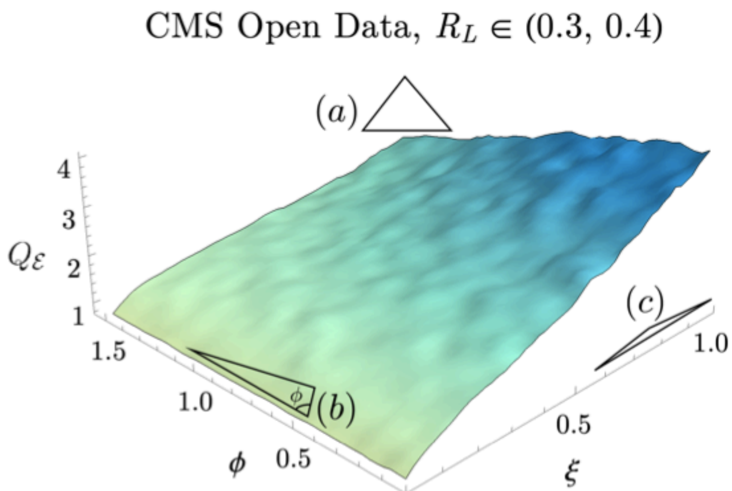
Structure of such integrals well understood.

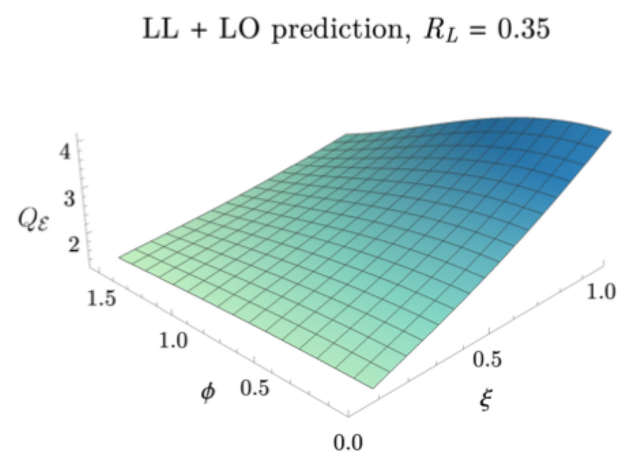
Slide by I.Moult

Non-Gaussianities inside high energy jets

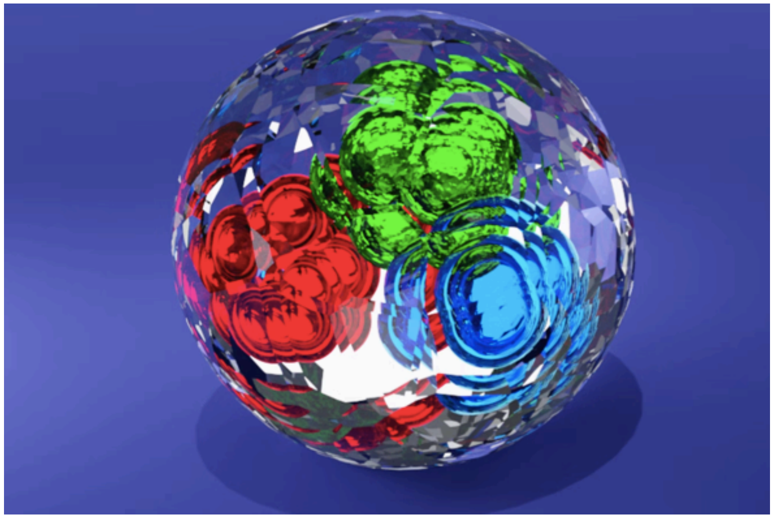
Theoretical control on high point functions

- **Can be measured on LHC data**

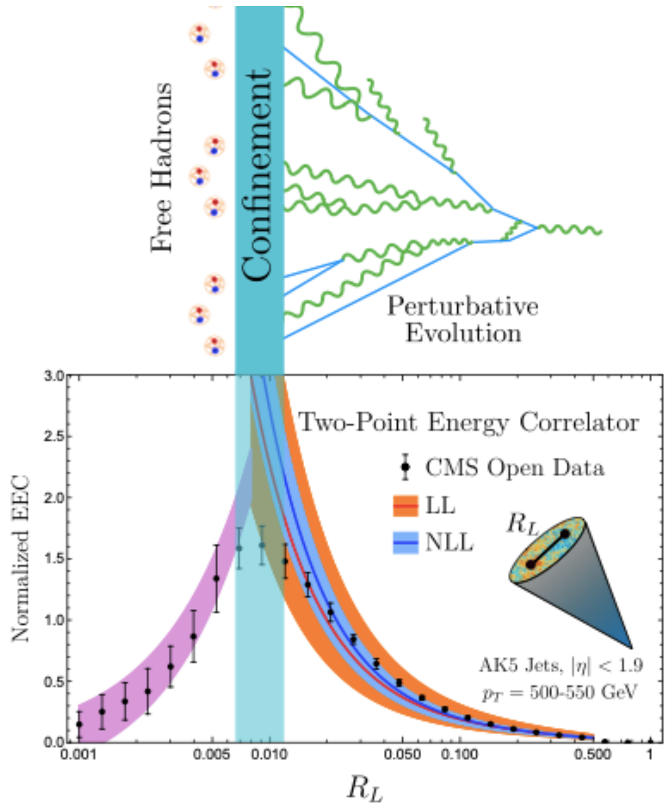




[Chen, Moult, Thaler, Zhu]



Confinement transition in jet substructure?

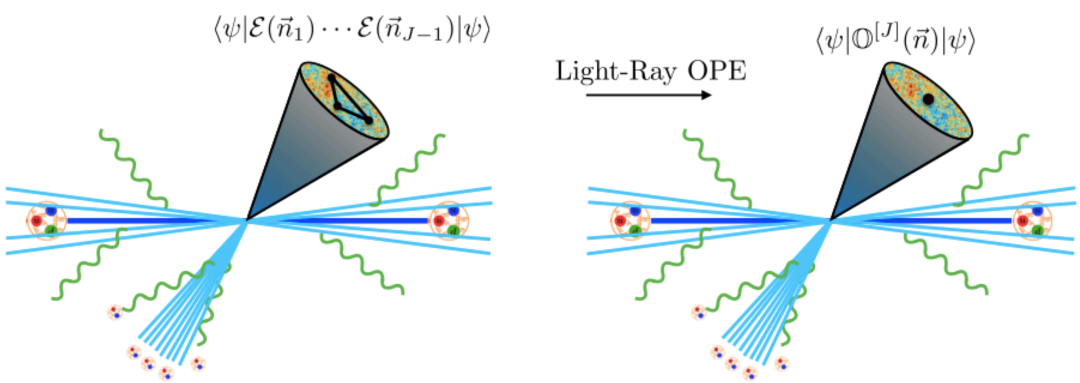


Any underlying dynamics will be imprinted in the energy correlators, including hadronization transition.

Jet substructure from first principles!

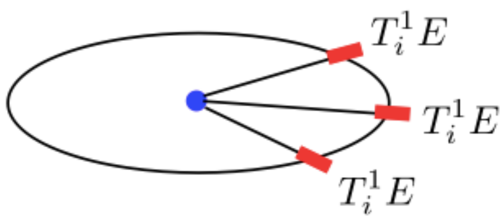
- Energy correlator is a jet substructure observable defined from first principles in QFT

⇒ No room for ambiguity what it's being measured in theory.



- Formalism we have presented can be applied for any conserved charge for LHC processes.
- No jet grooming or pruning is needed to extract the final results, pure QFT calculation!
- Not sensitive to soft and wide angle radiations.

Implementation on tracks



$$E_i \to \int dx_i \, x_i T_i(x_i) E_i = T_i^{(1)} E_i$$

Multiply by the first moment of the track function

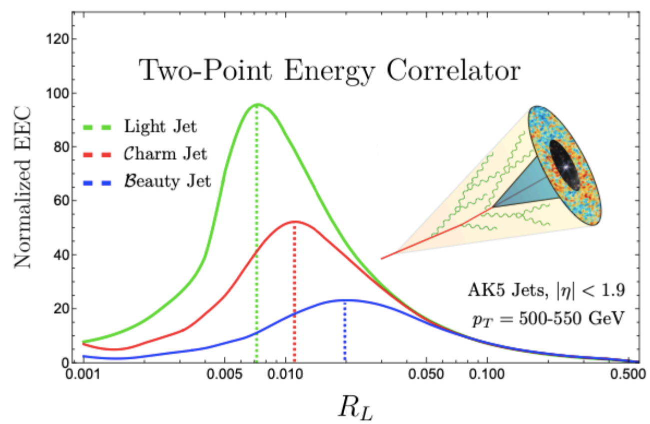
- Incorporate information not only from the calorimeter but also from the tracks.

[Li, Moult, van Velzen, Waalewijn, Zhu]

- Possible using track functions.
- Better precision

The anomalous dimension can also be measured from these first moments!

Beautiful and Charming Energy Correlators



Energy Correlators on heavy jets

Introduce an additional scale

- **At the LHC energies there is access to the transition phase from massless to massive behaviour $\Rightarrow$ more complexity**
- **Also very interesting!**
    - **Can probe intrinsic mass effects of quarks before confinement into hadrons**

Factorization theorem

Can compute any higher point correlators on massive quarks at LHC at NLL

Hard function (NNLO)

[Czakon, Generet, Mitov, Poncelet; 2021]

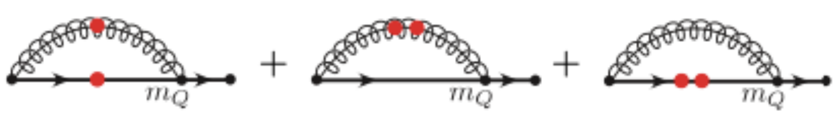
$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

$\mu_H \sim p_T$

$\mu_J \sim p_T R$

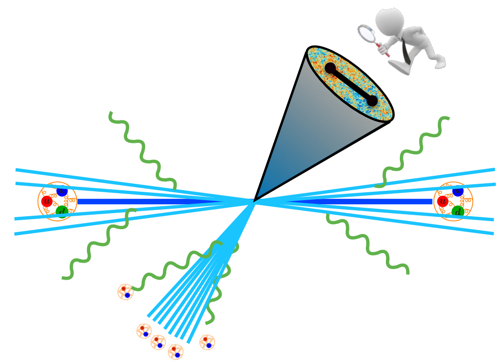
[Craft, Lee, BM, Moult]

Massive Energy Correlator Jet Function (NLO)



Heavy quark jet function

Result



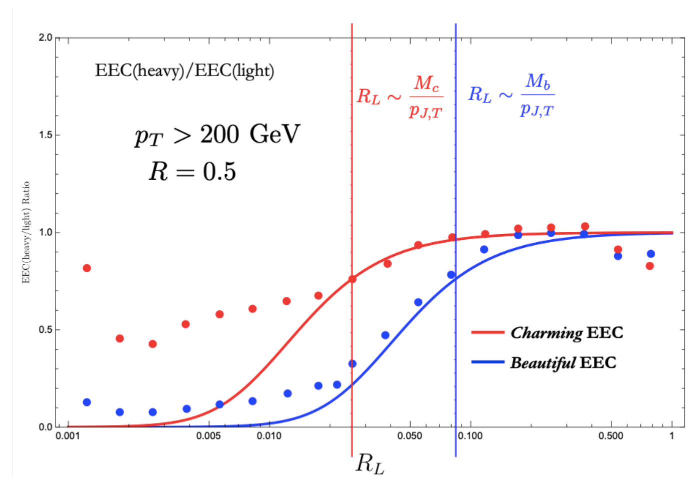
$$J_Q^{\text{bare}}(z,M,\mu) = \delta(z)\left(1+\frac{\alpha_s C_F}{4\pi}\left[-\left(\gamma_{qq}^{(0)}(3)+\gamma_{gq}^{(0)}(3)\right)\left(\frac{1}{\epsilon_{\text{UV}}}+\ln\frac{\mu^2}{M^2}\right)-\frac{19}{6}\right]\right)$$

$$+\frac{\alpha_s C_F}{\pi}\frac{1}{z}\left[\frac{3}{4}-\frac{5}{2}\delta^2-\frac{\delta^4}{1+\delta^2}+3\delta^3\arctan\left(\frac{1}{\delta}\right)+\frac{1}{2}\delta^2(1-\delta^2)\ln\frac{\delta^2}{1+\delta^2}\right]$$

The mass should not affect the UV behavior of the jet function.
This can be seen from comparing the UV poles with the light quark jet function.

$$J_q^{\text{bare}}(z,\mu) = \delta(z)+\frac{\alpha_s C_F}{4\pi}\left[\delta(z)\left(-\frac{3}{\epsilon_{\text{UV}}}-\frac{37}{3}\right)+3\frac{Q^2}{\mu^2}\mathcal{L}_0\left(\frac{Q^2}{\mu^2}z\right)\right]$$

$$= \delta(z)+\frac{\alpha_s C_F}{4\pi}\left[\delta(z)\left(-\left(\gamma_{qq}^{(0)}(3)+\gamma_{gq}^{(0)}(3)\right)\frac{1}{\epsilon_{\text{UV}}}-\frac{37}{3}\right)+3\frac{Q^2}{\mu^2}\mathcal{L}_0\left(\frac{Q^2}{\mu^2}z\right)\right]$$



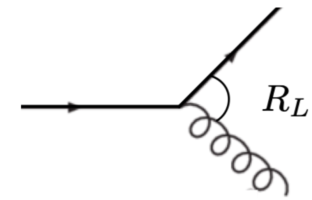
[Craft, Lee, BM, Moult]

Massive jets

Massive Energy Correlator Jet Function

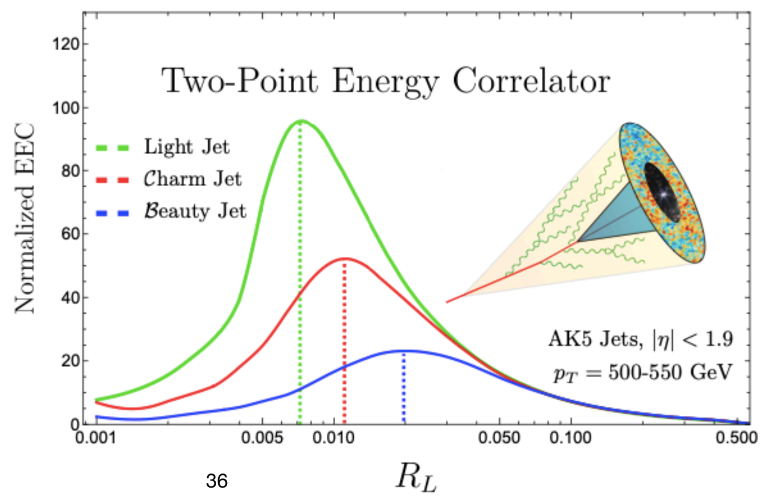
$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

Hard function



Virtuality $\sim p_T R_L + m_Q^2$

- Formation time changes with the mass of the quark.
- Can clearly see this from the two-point EEC.

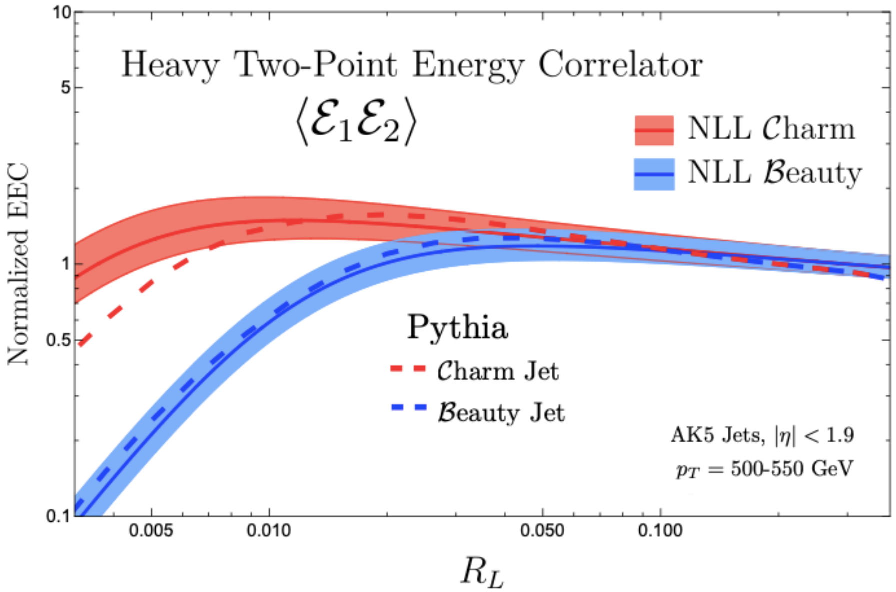


[Craft, Lee, BM, Moult]

Massive two point correlator

First massive jet substructure observable at NLL

- Scaling behaviour identical to massless case for larger scales.
- A turn-over for $R_L \to m_Q/p_T$
- The change in the slope is perturbative effect contrary to massless jets: $R_L \to \Lambda_{QCD}/p_T$
- The turn-over region is of interest for improving heavy quark description is parton shower.

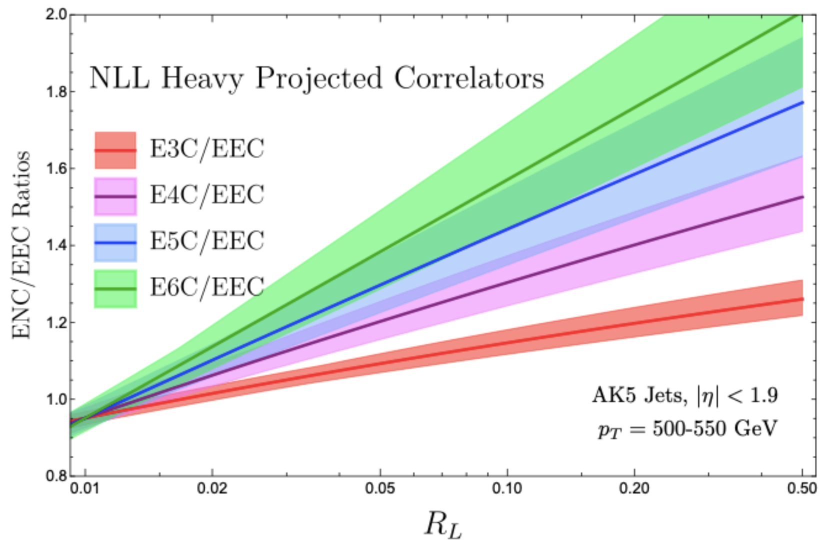


[Craft, Lee, BM, Moult]

Projected energy correlators

Resolve the UV scaling behaviour

- Ratios of higher point correlators with the two point EEC are independent of IR effects, including quark mass.
- The exact behaviour as the massless case.
- Non-trivial cross check of the factorization theorem!
- Anomalous dimensions should not be affected by the IR physics.



[Craft, Lee, BM, Moult]

Dead-cone effect in QCD

Fundamental phenomena

- Parton-shower pattern depends on the mass of the emitting parton.
- Angular suppression $\propto \frac{M}{E}$.

First observation for QCD by ALICE collab in [2106.05713]

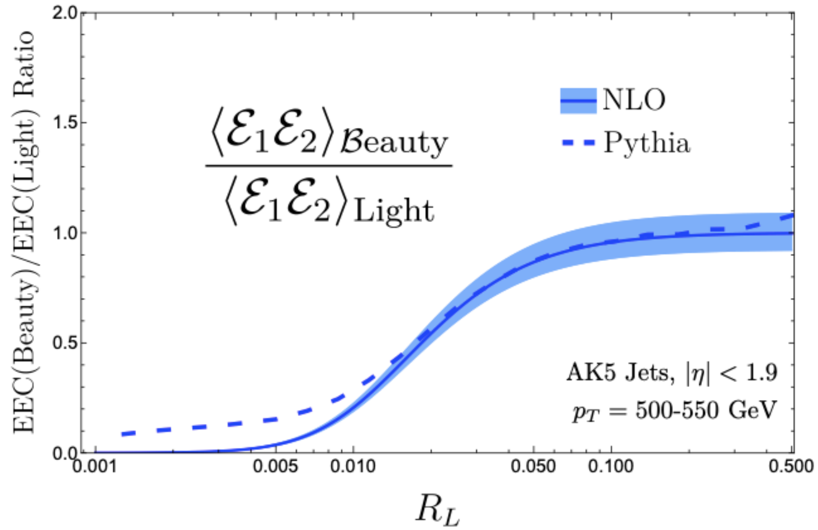
Observable used for the observation of the dead-cone effect in LHC data

$$R(\theta) = \frac{1}{N^{\mathrm{D^0\,jets}}} \frac{\mathrm{d}n^{\mathrm{D^0\,jets}}}{\mathrm{d}\ln(1/\theta)} \Big/ \frac{1}{N^{\mathrm{inclusive\,jets}}} \frac{\mathrm{d}n^{\mathrm{inclusive\,jets}}}{\mathrm{d}\ln(1/\theta)} \Bigg|_{k_{\mathrm{T}}, E_{\mathrm{Radiator}}}$$

- Not possible to calculate it from first principles in QFT.

Intrinsic mass effects

Dead-cone effect



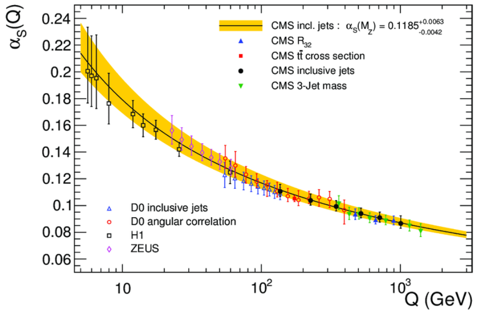
[Craft, Lee, BM, Moult]

- **Ratios of the massive and massless EEC isolate mass (IR) effects.**
- **A transition region related to the quark mass, which is perturbatively calculable.**
- **Excellent agreement with MC.**
- **Small angle suppression can be interpreted as a dead-cone effect.**

Applications of these results

- Precision measurements,

example: strong coupling, since the anomalous dimensions are proportional to α_s.



- Better jet modeling in MC simulations, especially for heavy quarks.

- Higher order NLL are important for better precision in parton showers:

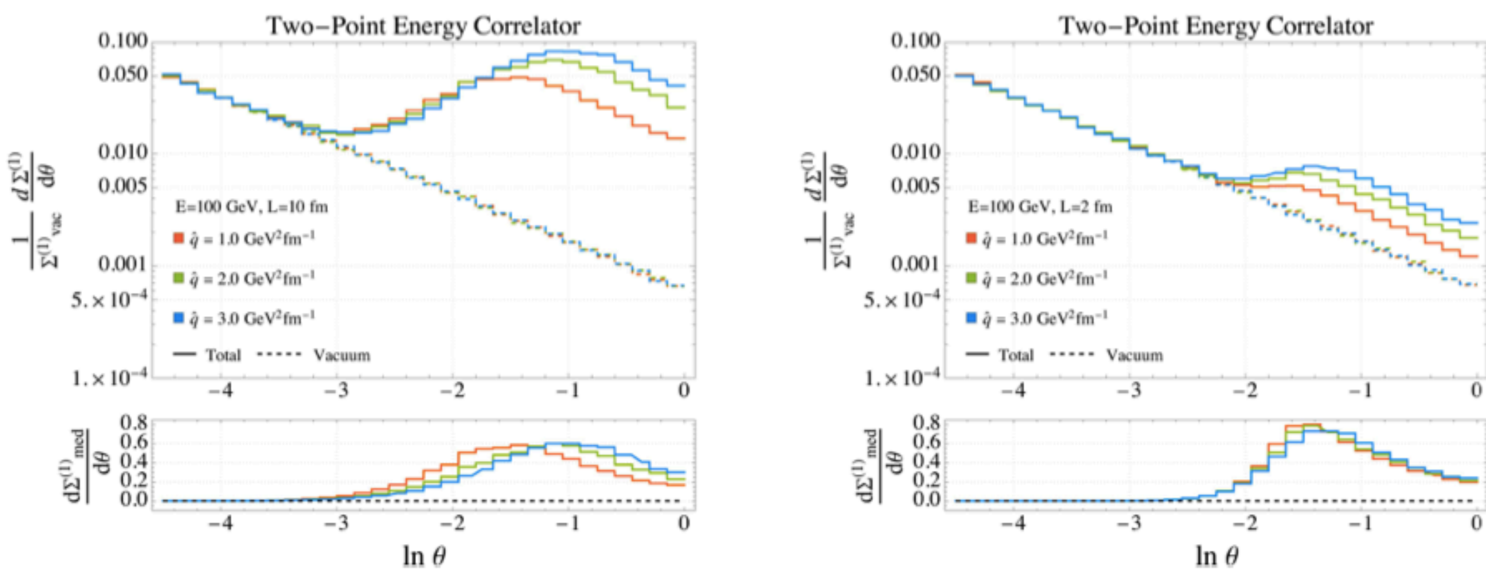
“reference resummation” for testing DGLAP finite moments.

Applications of these results

Heavy Ions

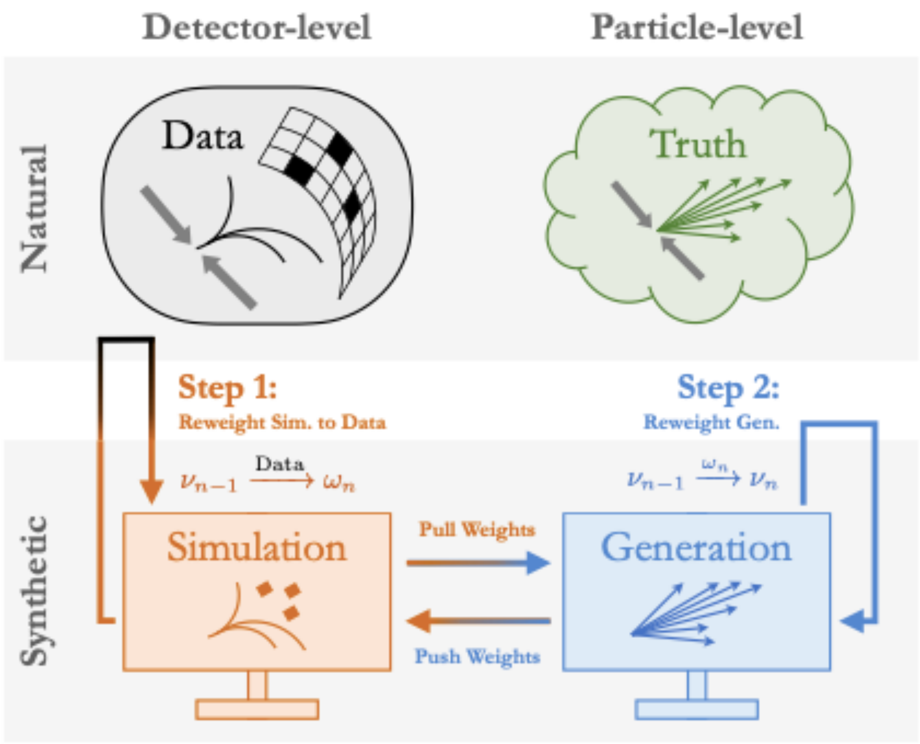
- **Might help understand properties of the QGP.**
- **Intrinsically that is a multi-scale problem too, global properties of plasma.**

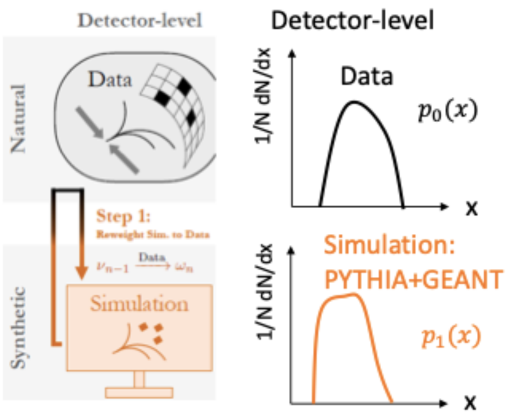
[Andres, Dominguez, Kunnawalkam Elayawalli, Holguin, Marquet, Moult]



Energy Correlators and Machine Learning

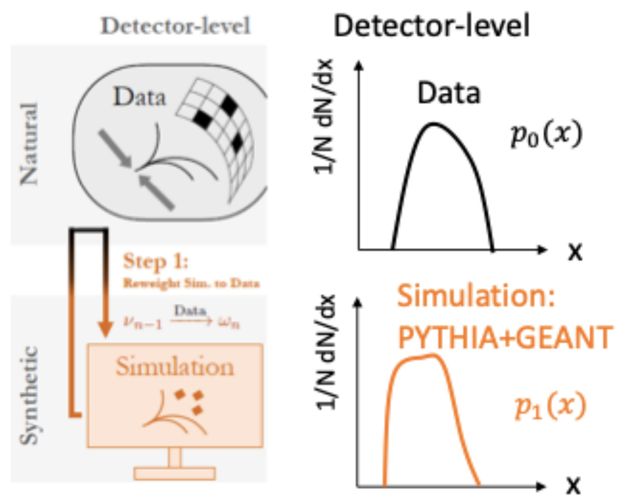
Unfolding for detector effects





Energy Correlators and Machine Learning
Omnifold

- Simultaneously unfold for multiple observables; suitable for energy correlators
- **Correlation information is preserved**
- Unbinned method

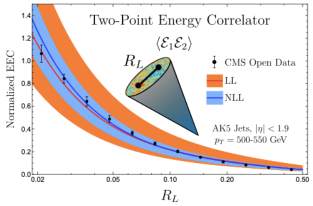


Conclusions

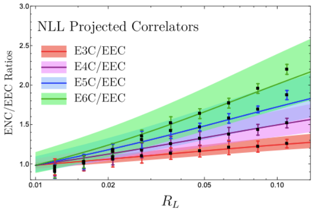
- Factorization formula for calculating energy correlators study jet substructure at the LHC.

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}p_T \mathrm{d}\eta \mathrm{d}z} = \sum_i \mathscr{H}_i\left(p_T z, \eta, \mu\right) \otimes \int_0^1 dx\, x^N \mathscr{J}_{ij}(z, x, p_T R, \mu)\, J_j^{[N]}(z, x, \mu)$$

- Can probe a universal scaling behavior of QFT in the complicated LHC environment.



- Higher-point correlators can be calculated for LHC and probe anomalous scaling dimension of operators.

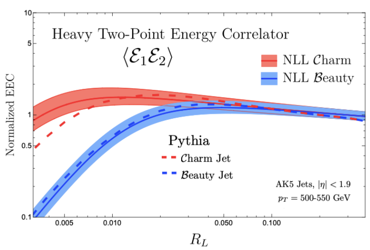


Conclusions

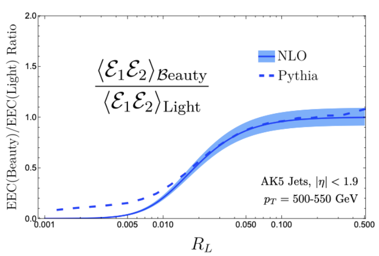
- A simple factorization formula for massive quark jets: massive EEC first jet substructure observable at NLL

$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

- Intrinsic mass effects of strongly interacting elementary particles.



- Energy Correlators can be used to observe the dead-cone effect.



- There is a myriad of future applications of such jet observables that can be applied to both QCD in the vacuum and heavy ions

Thank You!