(Un)Knowns about QCD phases and prospects about dense QCD matter

Kenji Fukushima

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— PhD School on QCD in Extreme Conditions —

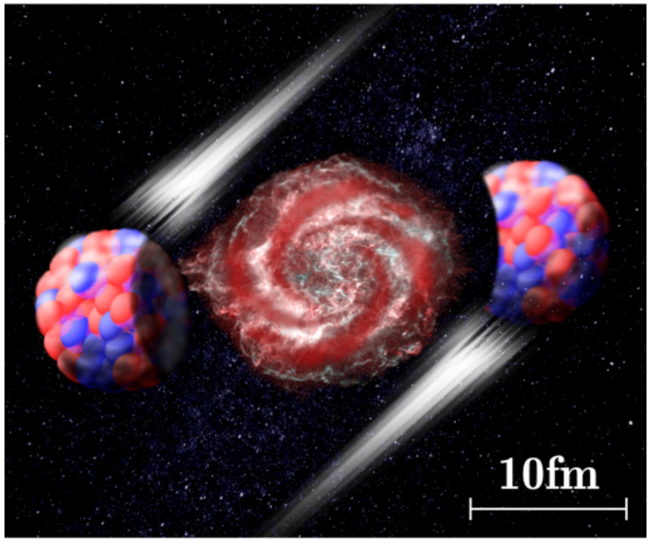
Useful References

*** 50 Years of Quantum Chroodynamics, Chap.7**
2212.11107 [hep-ph]

*** Nuclear Matter at High Density and Equation of State Chap.5**
Not yet readable on arXiv… sorry…

*** Little-Bang and Femto-Nova in Nucleus-Nucleus Collisions**
2009.03006 [hep-ph]

High Temp., High Density,
Strong *B*, Large Spin, …



Talk Plans

Knowns for QCD Matter at High *T* and Low Baryon Density

Theoretical Knowns and Many Unknowns at Low *T* and High Baryon Density

Some Implications from Anomalies

— Day 1 —

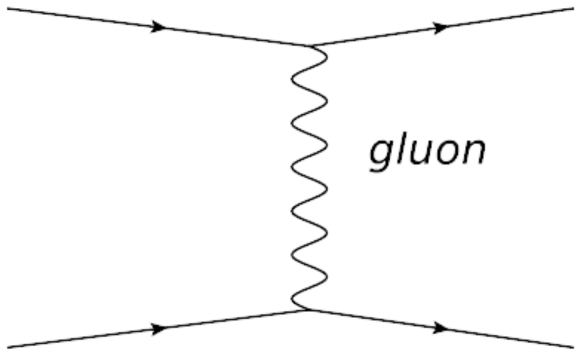
Knowns for QCD Matter at High *T* and Low Baryon Density

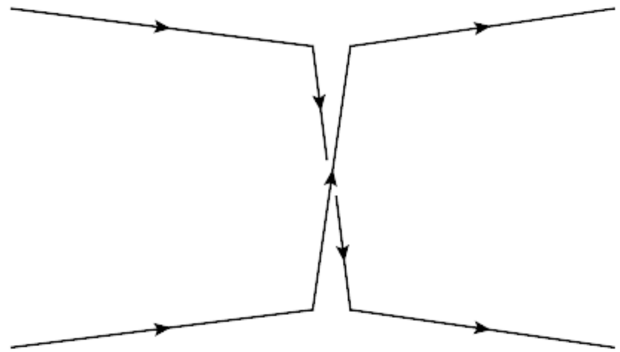
Quarks and Gluons

Quarks **spin-1/2 (fermions)** **6 flavors** **3 colors**

(transform in the SU(3) fundamental rep.)

quark **red** / **green** / **blue**





Gluons **spin-1 (bosons)** **8 colors** (in the adjoint rep.)

=3×3-1 (singlet)

$r\bar{r}$ $r\bar{g}$ $r\bar{b}$ $g\bar{r}$ $g\bar{g}$ $g\bar{b}$ $b\bar{r}$ $b\bar{g}$ $b\bar{b}$ $- (r\bar{r}+g\bar{g}+b\bar{b})$

Origin of the Mass

Quark Model

Phenomenological Mass Formula

$$M_{\text{hadron}} = \sum_i m_i + \Delta M$$

$$\Delta M = \sum \frac{4\pi\alpha_s}{9} \frac{\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j}{m_i m_j} |\psi(0)|^2$$

“Constituent Quark” $\underline{m_{u,d} \approx 360 \text{ MeV}}$

Origin of the Mass

Magnetic Moment of Spin-1/2 Particles

$$\mu = \frac{q\hbar}{2m}$$

Spin effect is more suppressed by larger mass

Origin of the Mass

Quark Model

Wave-function →

$$\mu_p = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$$

$$\mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u$$

"Constituent Quark"

$$\mu_u = \frac{q_u}{2m_q} = -2\mu_d \;\rightarrow\; \underline{m_q \approx 340\ \text{MeV}}$$

Origin of the Mass

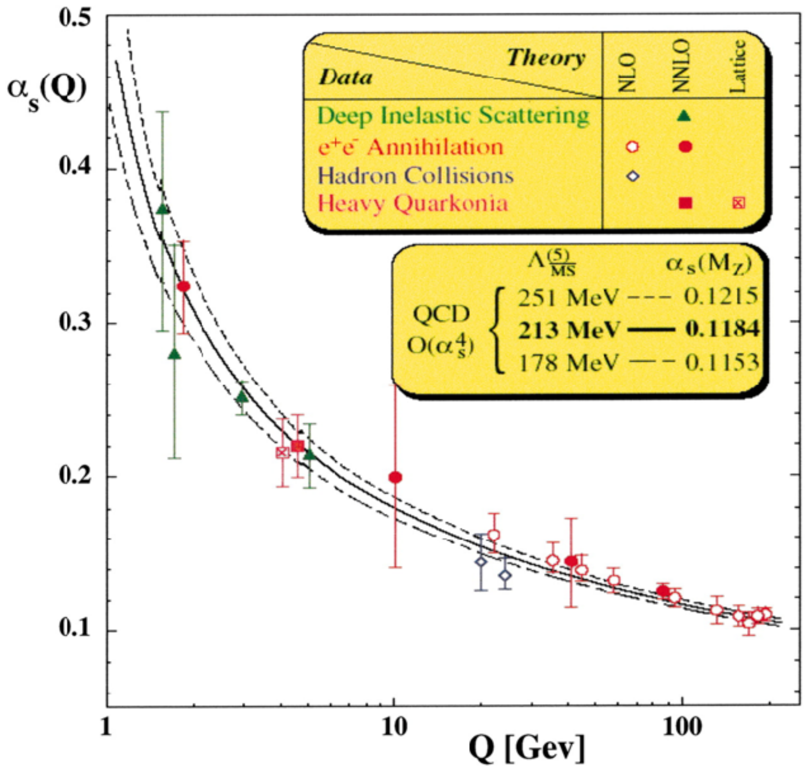
But, *up*- and *down*-quarks are almost massless!?

Flavor	Charge	Mass
u-quark	**(2/3)*e***	**~ 3MeV**
d-quark	**-(1/3)*e***	**~ 5MeV**
s-quark	**-(1/3)*e***	**~ 100MeV**
c-quark	**(2/3)*e***	**~ 1.3GeV**
b-quark	**-(1/3)*e***	**~ 4.2GeV**
t-quark	**(2/3)*e***	**~ 170GeV**

Origin of the Mass

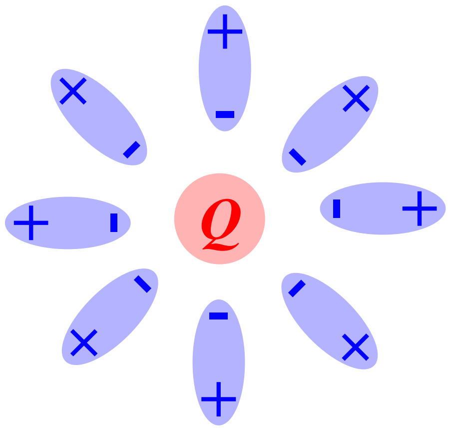
QCD Energy Scale

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2_{\rm QCD})}$$



$$\boxed{\Lambda_{\rm QCD} \sim 200\,\text{MeV}}$$

Coupling getting weaker at short distances (anti-screening)



Hadron size is fixed by the screening and the mass should be comparable to the QCD scale.

Chiral Symmetry

Massless QCD has global symmetry:

$$\mathrm{SU}(N_f)_{\mathrm{L}} \times \mathrm{SU}(N_f)_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{A}} \;\to\; \mathrm{SU}(N_f)_{\mathrm{V}}$$

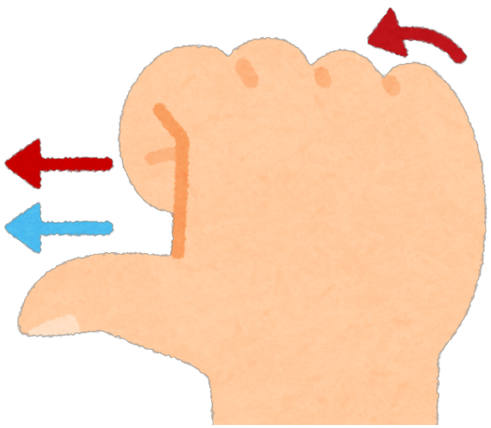
Massless (Chiral) Dirac Fermion

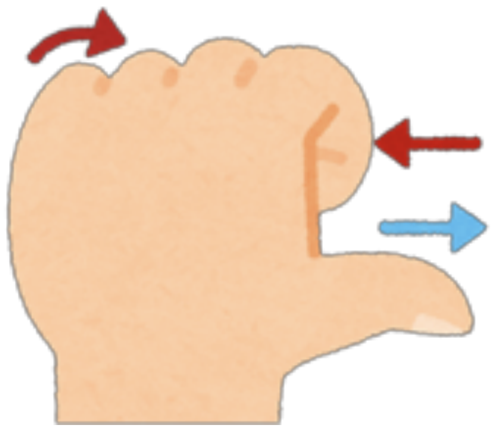
Right-handed Weyl Fermion

Left-handed Weyl Fermion

Spin

Momentum





Chiral Symmetry

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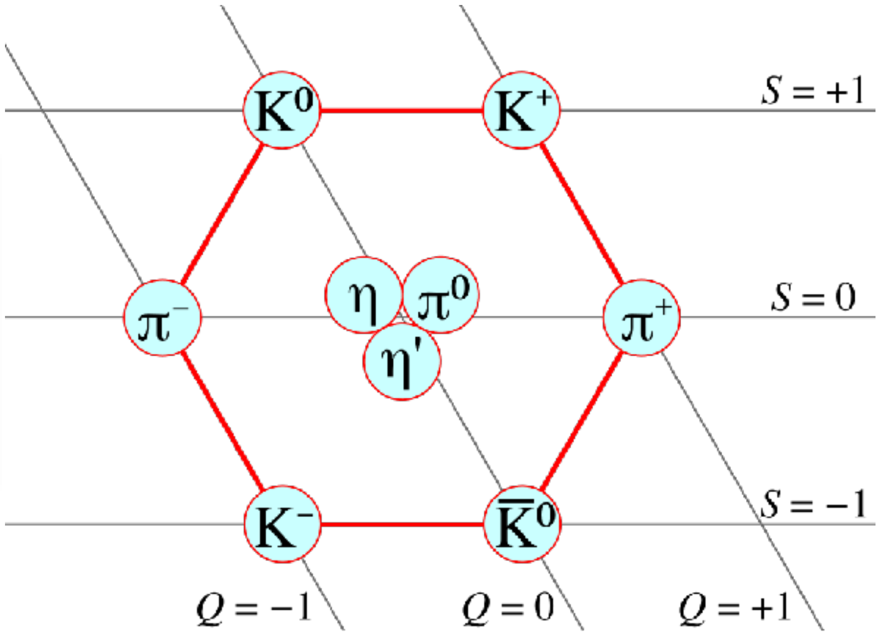
Anomalously Broken

Mass term: $m\bar{q}q$

induces *m* if

$$\boxed{\text{Chiral Condensate} \quad \langle \bar{q}q \rangle}$$

is not zero.



Spontaneous Breaking

Zero-point Oscillation Energy [Peskin-Schroeder]

$$= \int \frac{d^3p}{(2\pi)^3}\, \omega_{\mathbf{p}} \Big(a_{\mathbf{p}}^\dagger a_{\mathbf{p}} + \tfrac{1}{2}\big[a_{\mathbf{p}}, a_{\mathbf{p}}^\dagger\big] \Big). \qquad (2.31)$$

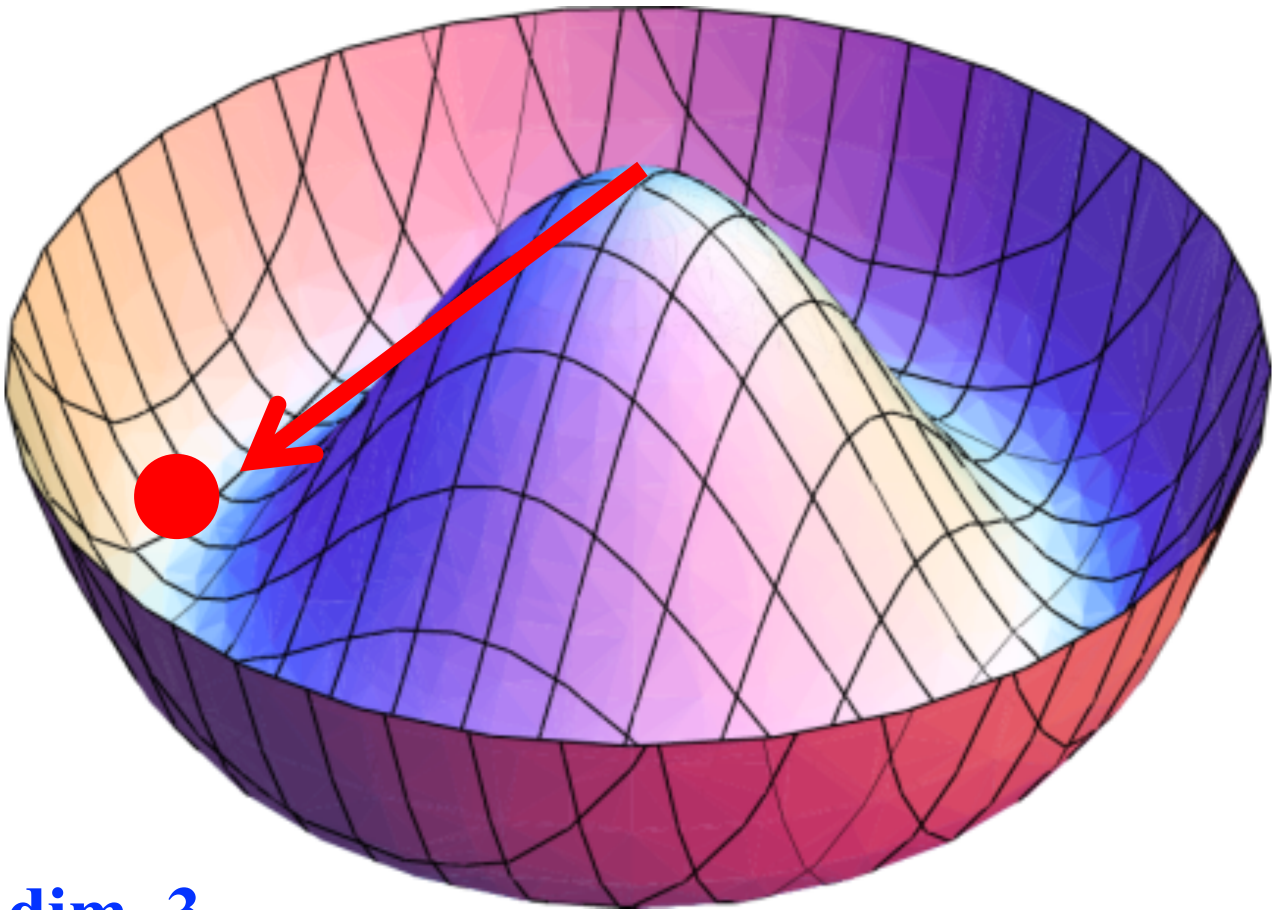
The second term is proportional to $\delta(0)$, an infinite c-number. It is simply the sum over all modes of the zero-point energies $\omega_{\mathbf{p}}/2$, so its presence is completely expected, if somewhat disturbing. Fortunately, this infinite energy shift cannot be detected experimentally, since experiments measure only energy *differences* from the ground state of H. We will therefore ignore this infinite constant term in all of our calculations. It is possible that this energy shift of the ground state could create a problem at a deeper level in the theory; we will discuss this matter in the Epilogue.

Not true for QCD! $\omega_{\boldsymbol{p}} = \sqrt{\boldsymbol{p}^2 + \boxed{m^2}}$

Dynamical Quantity

Spontaneous Breaking

$$\boxed{N_f = 2}$$



$$\sigma \;\sim\; \langle\bar{\psi}\psi\rangle \neq 0$$

$$M \sim \lambda\langle\bar{\psi}\psi\rangle$$

Mass dim. 3

Mass dim. -2

$$\pi_i \;\sim\; \langle\bar{\psi}i\gamma_5\tau_i\psi\rangle = 0$$

Spontaneous Breaking

Zero-Point Oscillation Energy

$$-2\int^{\Lambda}\frac{d^3p}{(2\pi)^3}\sqrt{p^2+M^2}$$

Look at the curvature at the symmetric point.

$$\simeq -\frac{\Lambda^4}{8\pi^2}\left[2+\boxed{\xi^2}+\mathcal{O}(\xi^4)\right] \qquad \xi = M/\Lambda$$

negative

Interaction Effect

$$\frac{M^2}{2\lambda_\Lambda} = \frac{\Lambda^4}{2\hat{\lambda}_\Lambda}\boxed{\xi^2}$$

positive

Dynamical mass generated for $\hat{\lambda}_\Lambda > 2\pi^2$

Nambu—Jona-Lasinio 1961

Phases in Extreme Conditions

Origin of the Mass = QCD Vacuum

RHIC: From dreams to beams in two decades

Gordon Baym
Department of Physics, University of Illinois at Urbana-Champaign
Urbana, IL 61801, U.S.A.

This talk traces the history of RHIC over the last two decades, reviewing the scientific motivations underlying its design, and the challenges and opportunities the machine presents.

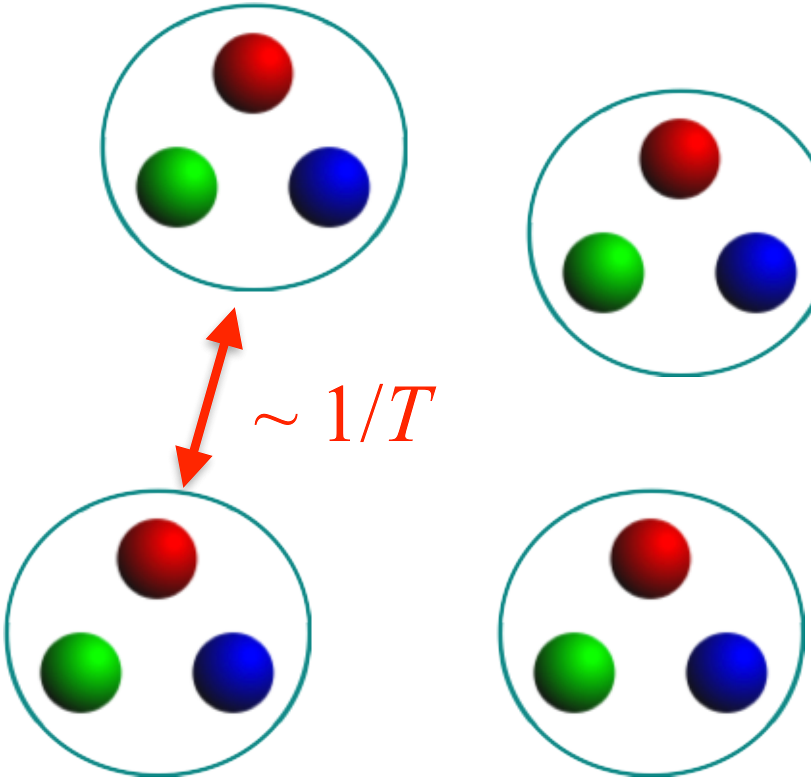
1. THE VERY EARLY DAYS

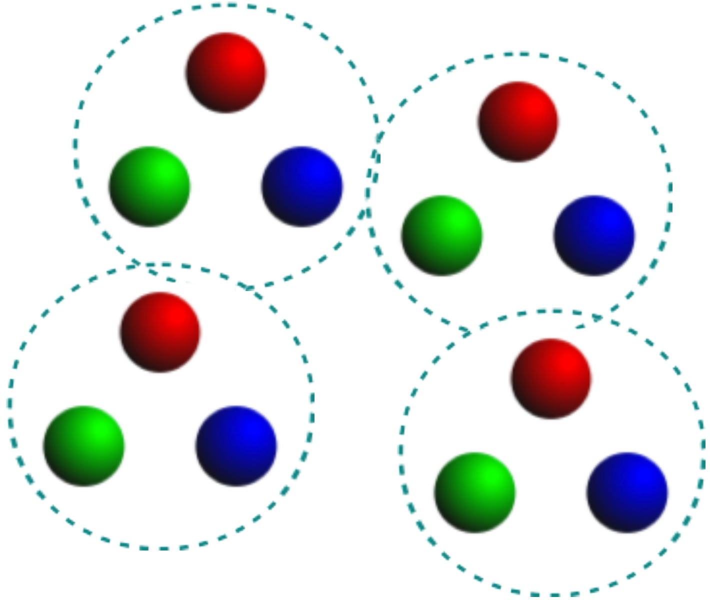
The opening of RHIC culminates a long history of fascination of nuclear and high energy physicists with discovering new physics by colliding heavy nuclei at high energy. As far back as the late 1960's the possibility of accelerating uranium ions in the CERN ISR for this purpose was contemplated [1]. The subject received "subtle stimulation" by the workshop on "Bev/nucleon collisions of heavy ions" at Bear Mountain, New York, organized by Arthur Kerman, Leon Lederman, Mal Ruderman, Joe Weneser and T.D. Lee in the fall of 1974 [1]. In retrospect, the Bear Mountain meeting was a turning point in bringing heavy ion physics to the forefront as a research tool. The driving question at the meeting was, as Lee emphasized, whether the vacuum is a medium whose properties one could change; "we should investigate," he pointed out, "... phenomena by distributing high energy or high nucleon density over a relatively large volume." If in this way one could restore broken symmetries of the vacuum, then it might be possible to create abnormal dense states of nuclear matter, as Lee and Gian-Carlo Wick speculated [2].

Vacuum
~ Medium?
~ Changeable??

Quark mass
changeable?
— Yes!

Phases in Extreme Conditions





Quarks are no longer confined
"Deconfinement"

Low *T*

High *T* ~ 200MeV

Phases in Extreme Conditions

~ 350MeV **High *T*** ~ a few MeV

Masses "melt" around the temperature ~ 200MeV

Almost massless fermions appear there!

Hot and Dense QCD Matter = Chiral Matter

QCD is a highly nontrivial theory with topological phenomena.

Quantum anomaly has been a central subject over half a century.

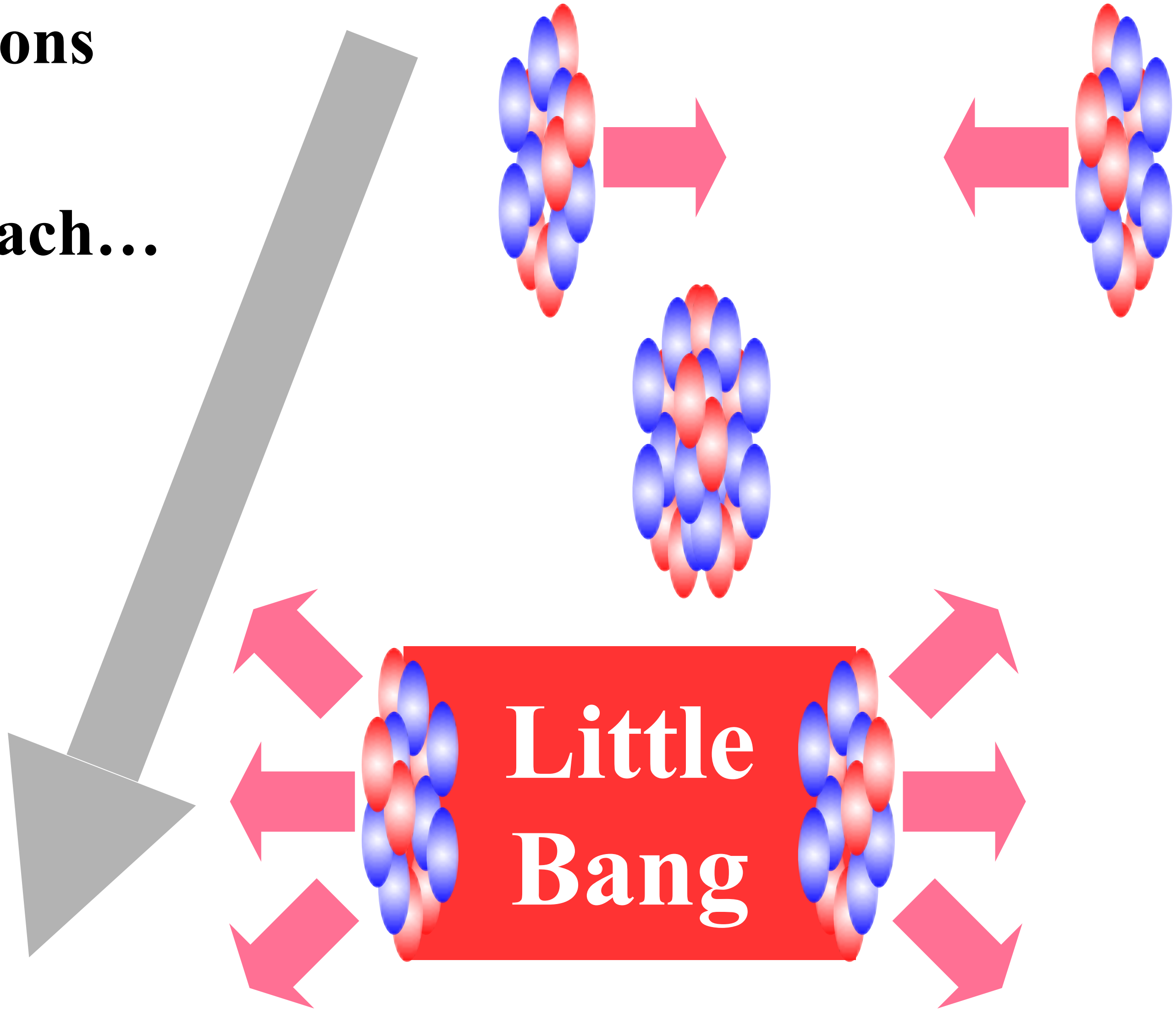
Phases in Extreme Conditions

Heavy-Ion Collisions

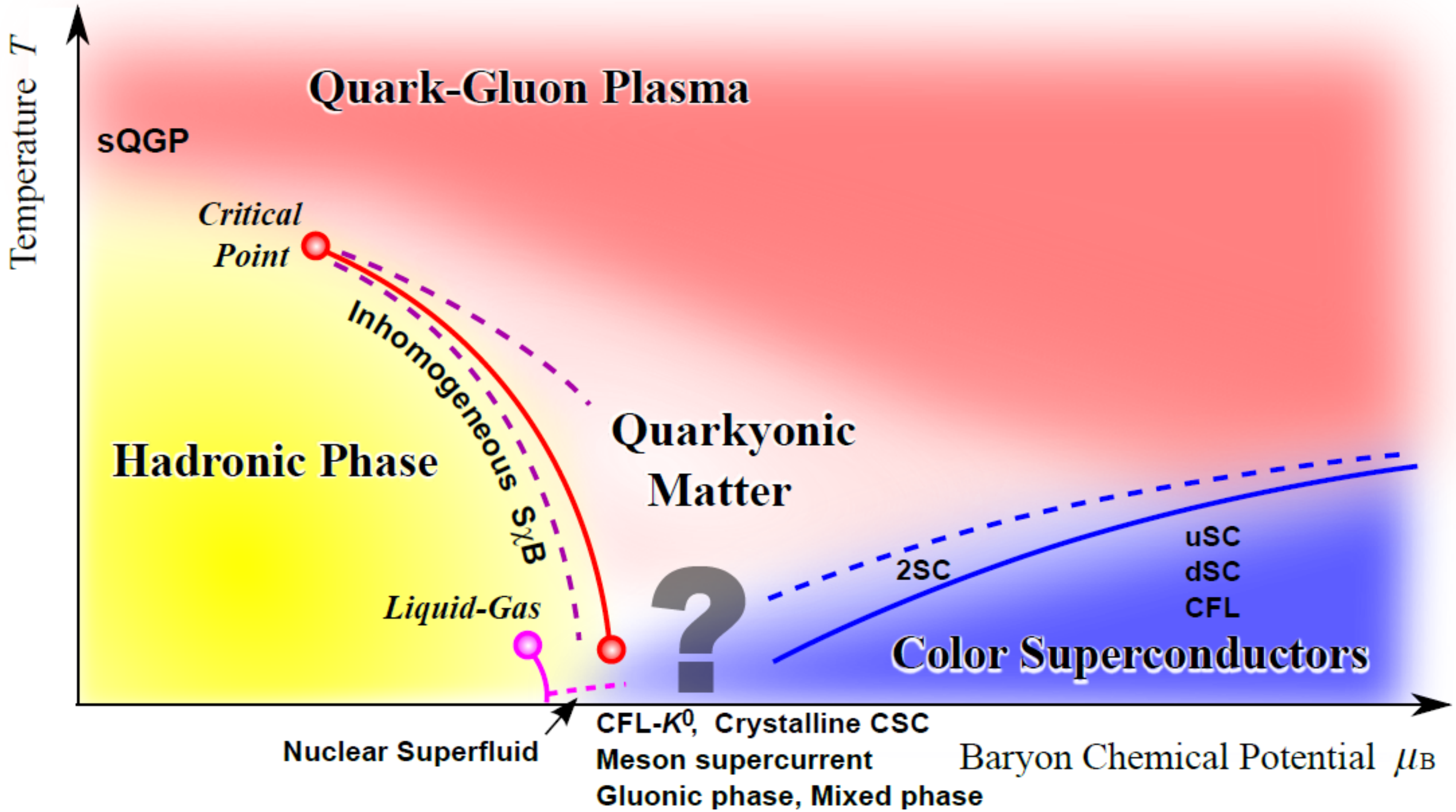
Two nuclei approach…

… and collide…

… and a quark gluon plasma (QGP) is made.



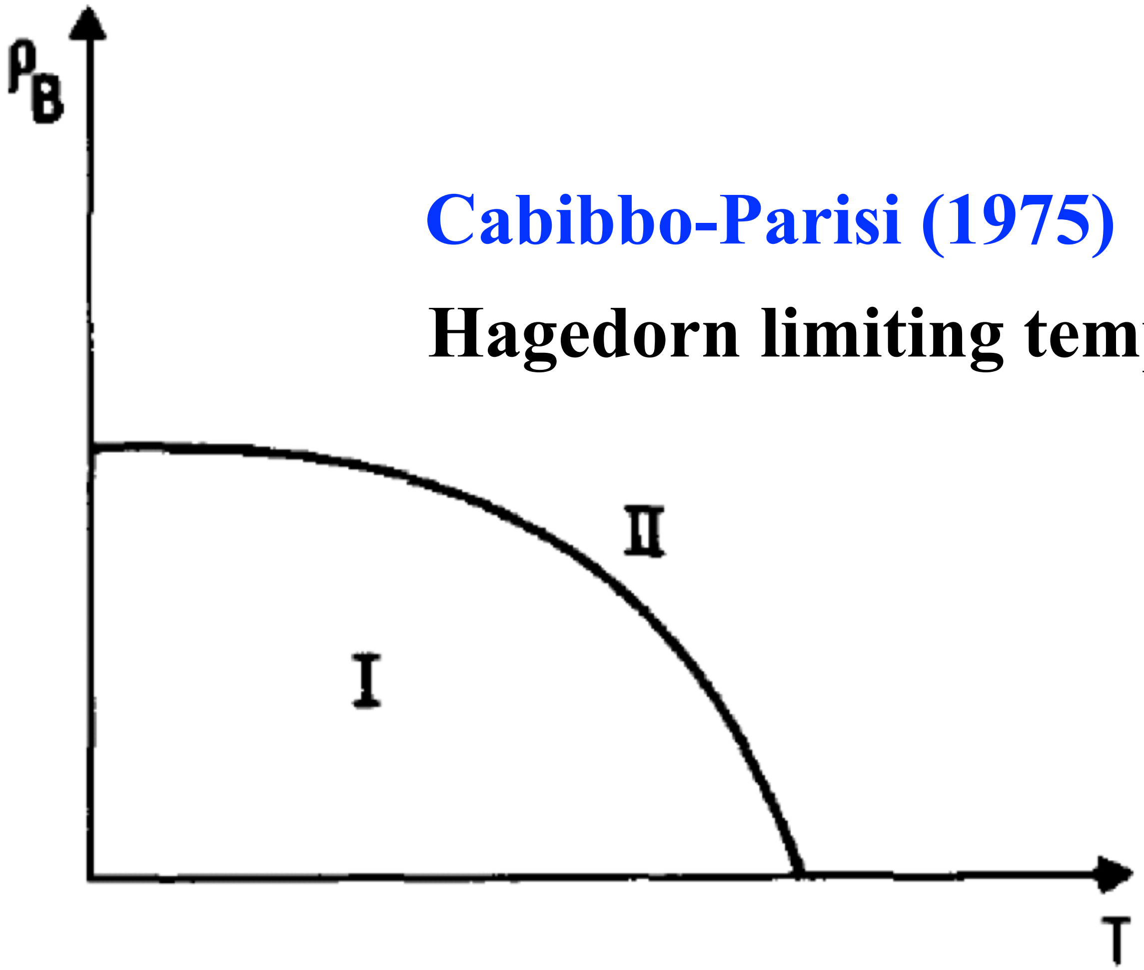
Probing the QCD Phase Diagram



Some inhomogeneous phase is suggested!?

Probing the QCD Phase Diagram

The very first phase diagram of QCD matter!

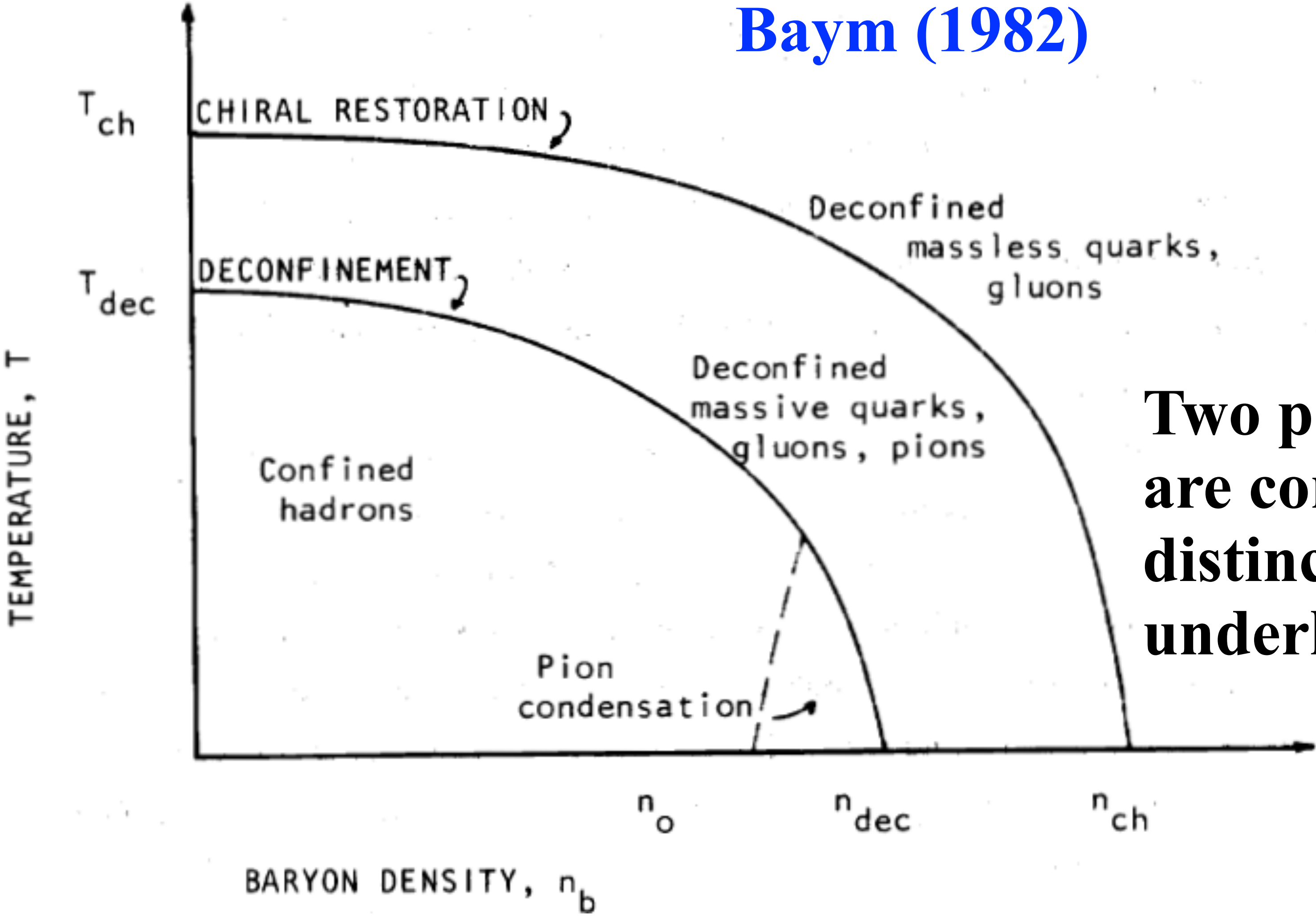


Cabibbo-Parisi (1975)

Hagedorn limiting temperature

Probing the QCD Phase Diagram

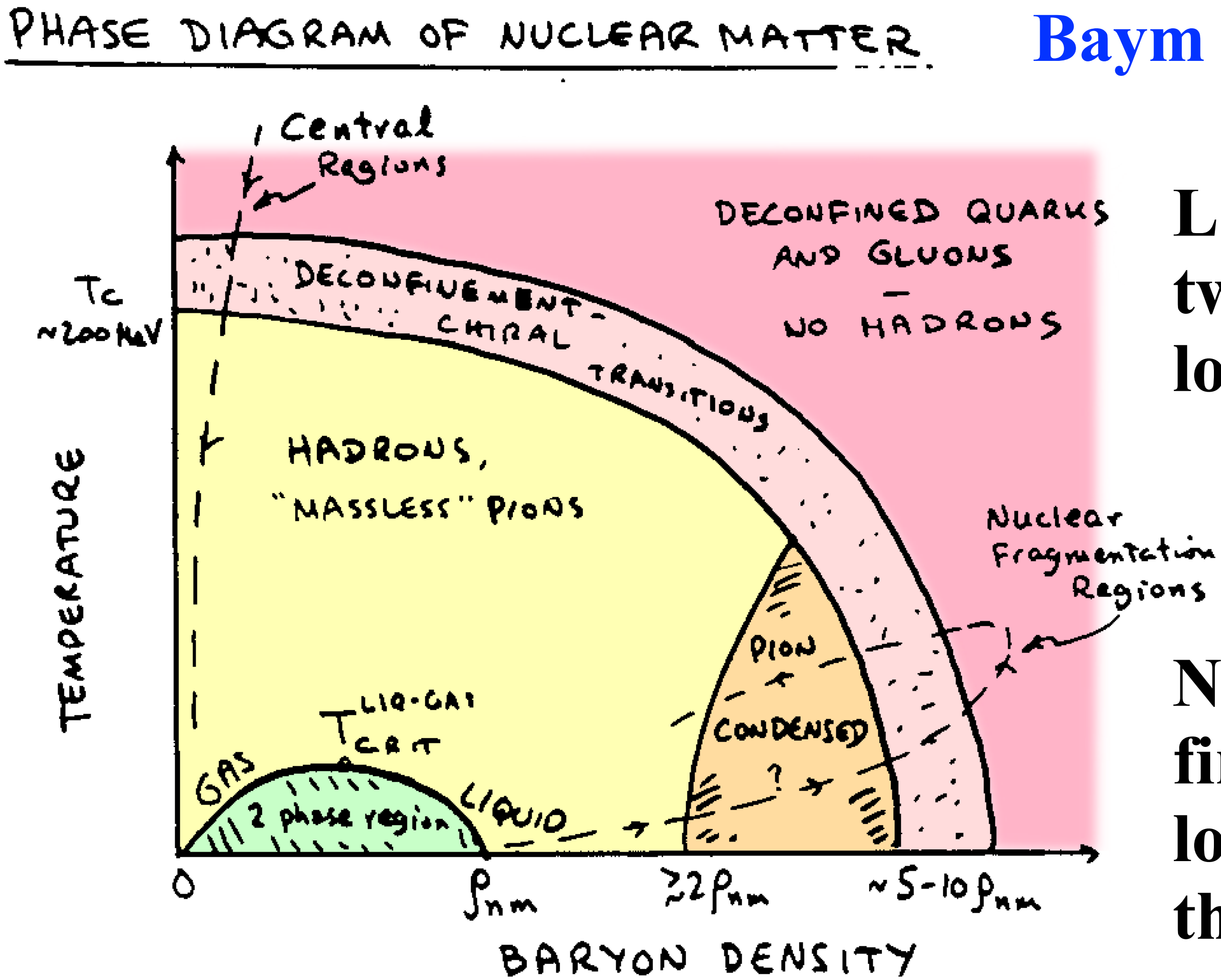
Baym (1982)



Two phase transitions are considered to be distinct… different underlying physics.

Probing the QCD Phase Diagram

Baym (1986)



Lattice-QCD implies two transitions are locked together!?

No lattice data at finite density — locking there is a theoretical conjecture.

Physics of (De)Confinement

A very simple "bag model" picture

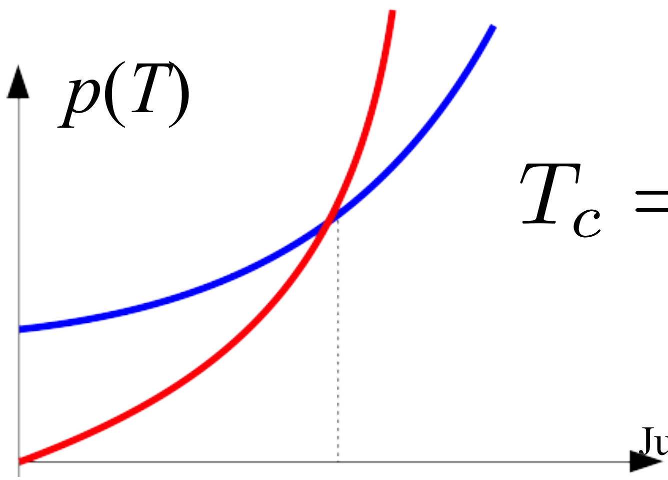
Confined Phase

$$p_{\text{hadron}}(T) = \frac{3\pi^2}{90}T^4 + B$$

Deconfined Phase

$$p_{\text{pert}}(T) = \frac{(16+21)\pi^2}{90}T^4$$

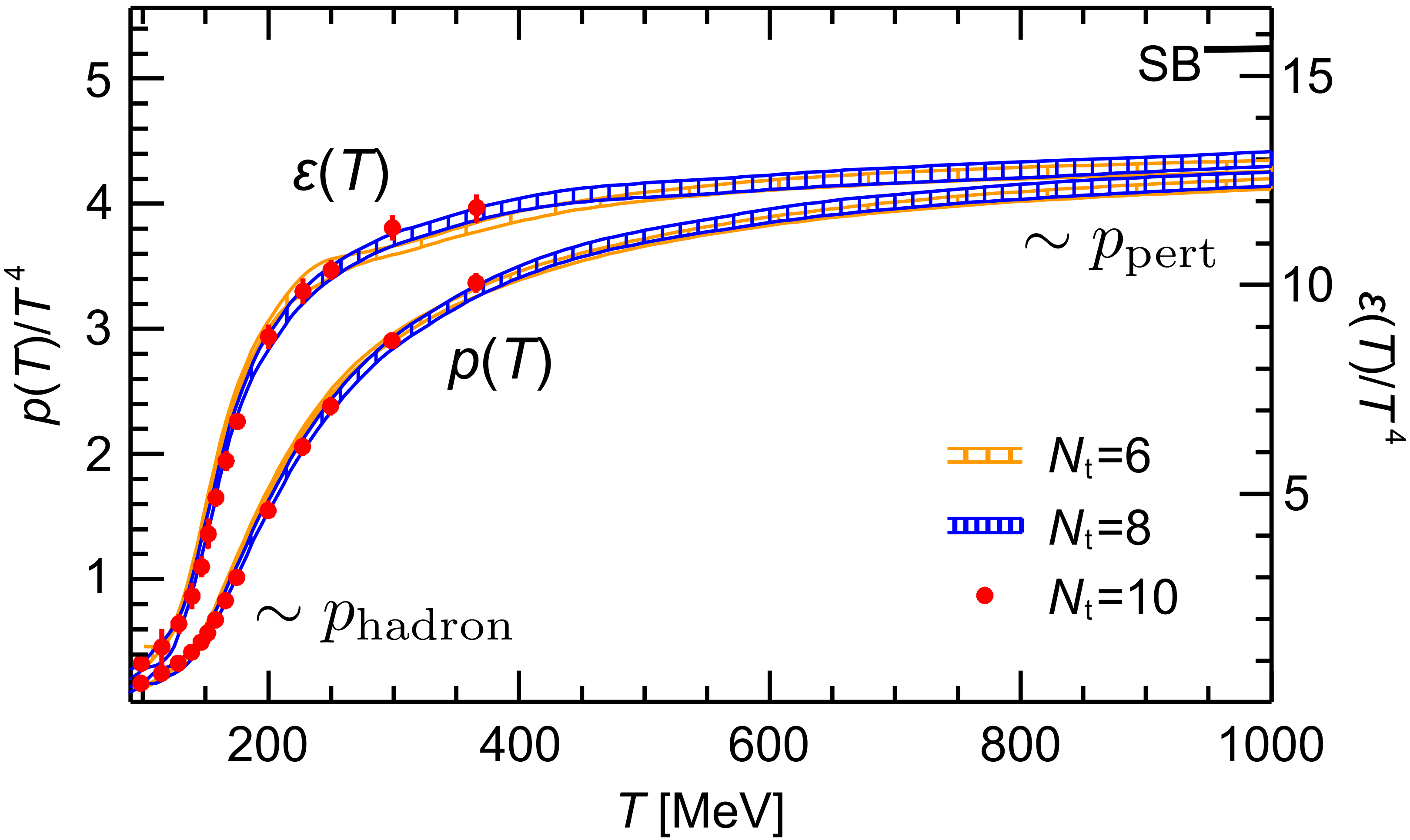
$B^{1/4} \sim \Lambda_{\text{QCD}}$ **confining pressure in the QCD vacuum**



$$T_c = \left[\frac{90}{(37-3)\pi^2}B\right]^{1/4} \sim 160\ \text{MeV}$$

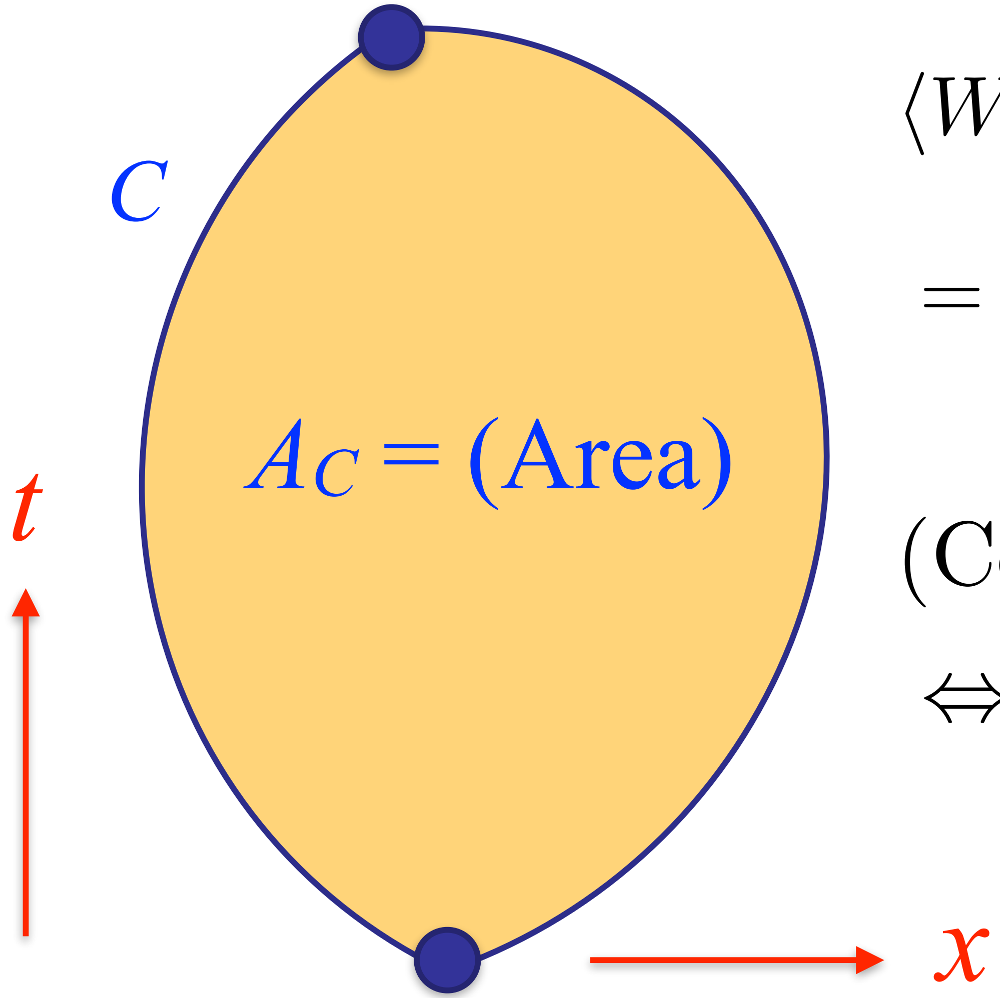
Physics of (De)Confinement

Wuppertal-Budapest (2010)



Physics of (De)Confinement

Confinement of Quarks (Wilson 1974)



$$\langle W(C) \rangle$$

$$= \left\langle \mathrm{tr} P \exp\left[ig \oint_C dx^\mu A_\nu \right] \right\rangle$$

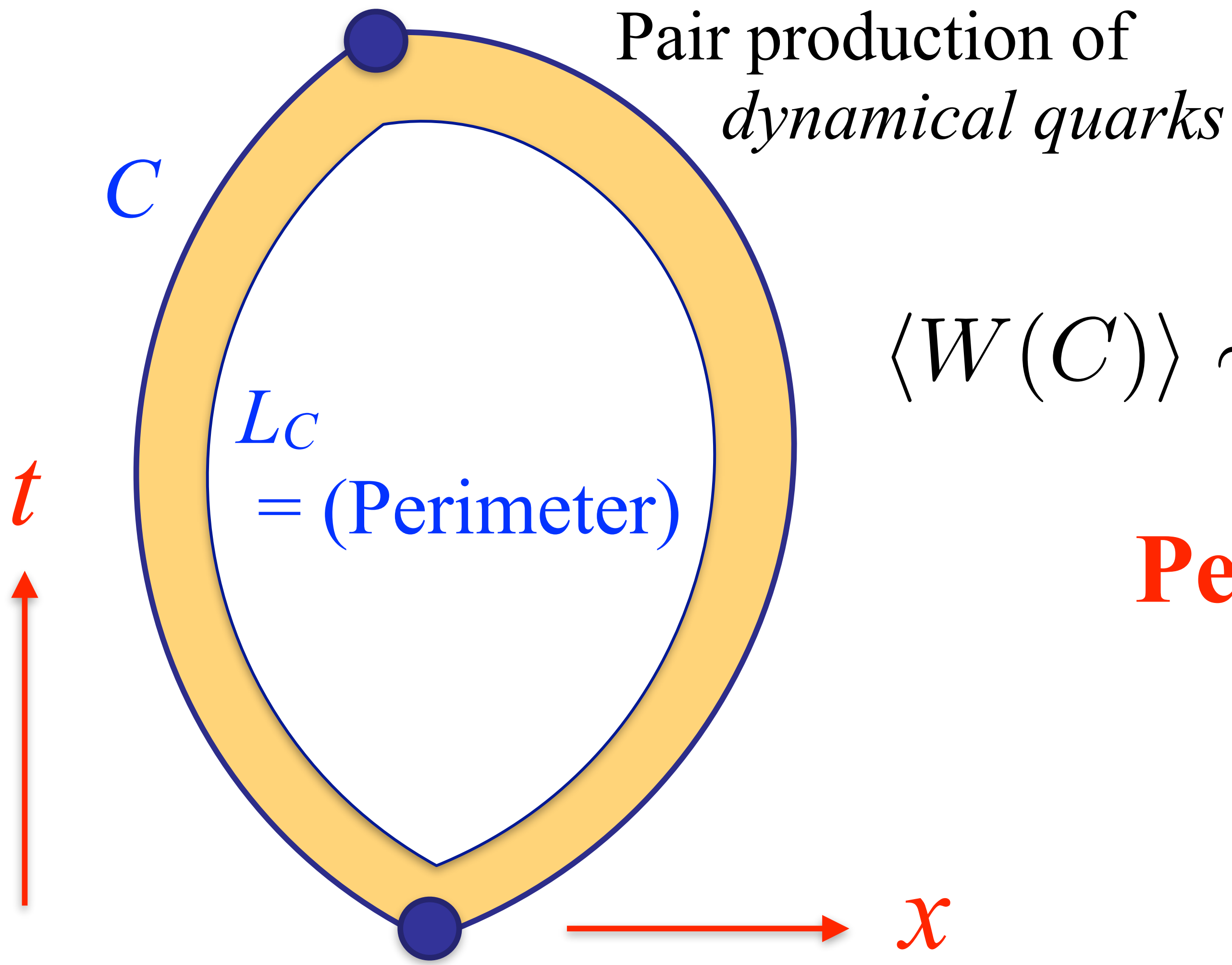
$$(\text{Confinement})$$

$$\Leftrightarrow \langle W(C) \rangle \sim \exp[-\# A_C]$$

Area Law

Physics of (De)Confinement

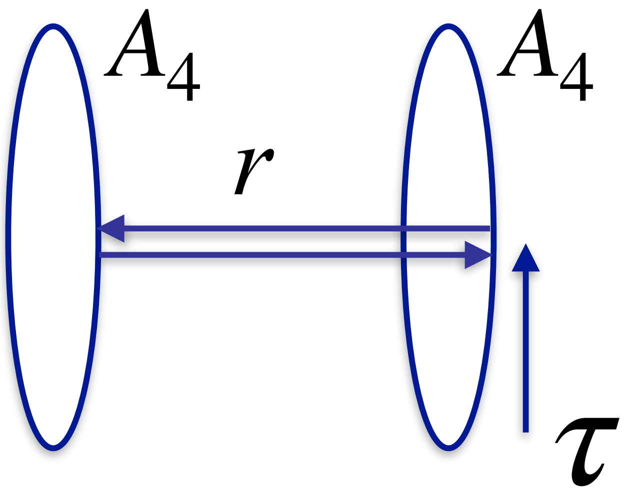
Confinement of Quarks (Wilson 1974)



$$\langle W(C)\rangle \sim \exp[-\# L_C]$$

Perimeter Law

Physics of (De)Confinement



$$W(C) = \mathrm{tr}L(0)\,\mathrm{tr}L^\dagger(r)$$

$$\langle W(C)\rangle = \langle \mathrm{tr}L(0)\mathrm{tr}L^\dagger(r)\rangle$$

$$= \exp[-f_{q\bar{q}}(r)/T]$$

$$\boxed{\begin{aligned} &\to |\langle \mathrm{tr}L\rangle|^2 \quad (r\to\infty) \\ &= \exp[-2f_q/T] \end{aligned}}$$

$$Z = \mathrm{tr}\, e^{-\hat{H}/T}$$

$$it \;\Leftrightarrow\; \tau$$

$$A_\mu(\tau+\beta) = A_\mu(\tau)$$

$$\psi(\tau+\beta) = -\psi(\tau)$$

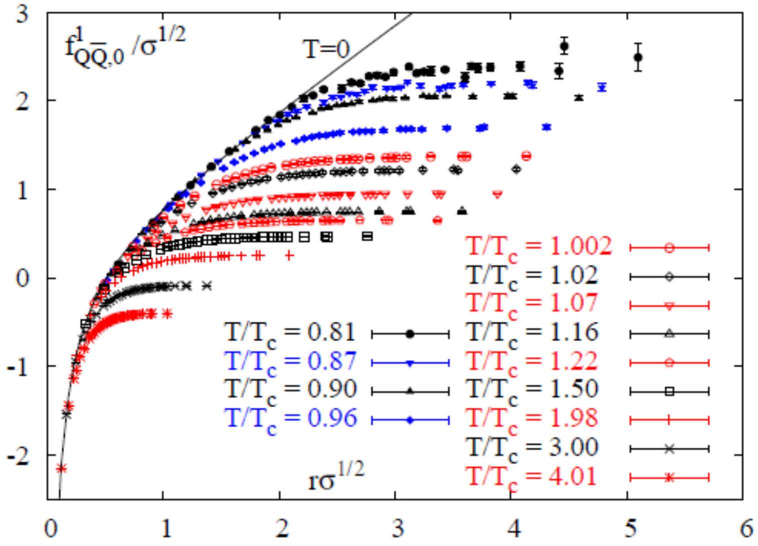
Polyakov Loop $\Phi = \dfrac{1}{3}\langle \mathrm{tr}L\rangle \sim e^{-f_q/T}$

Physics of (De)Confinement

Screening Effect in the Confined Phase

$$f_{q\bar{q}}(r) \to 2(\text{hadron mass}) \quad (r \to \infty)$$

$$f_q \to (\text{hadron mass}) \quad (\text{in "conf." phase})$$



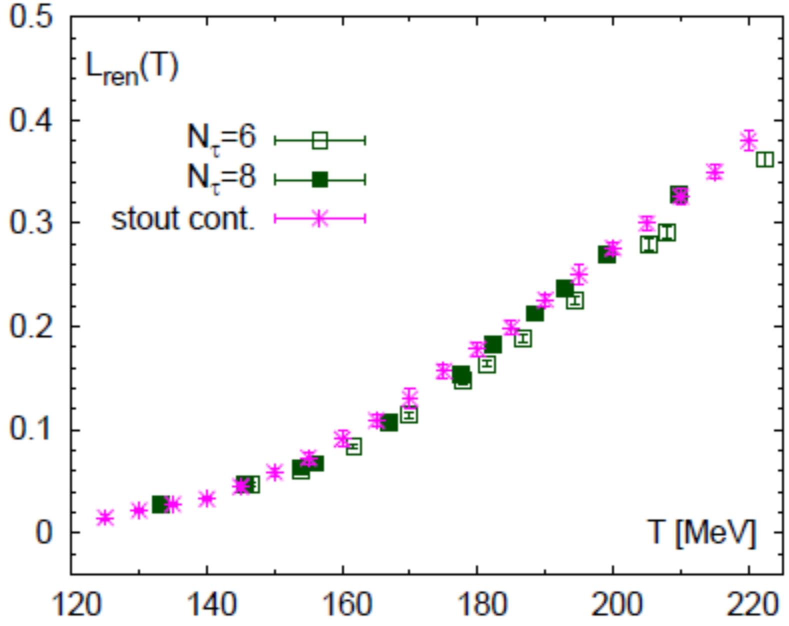
Linear potential is "screened" at large distances

No way to *define* confinement at finite *T*

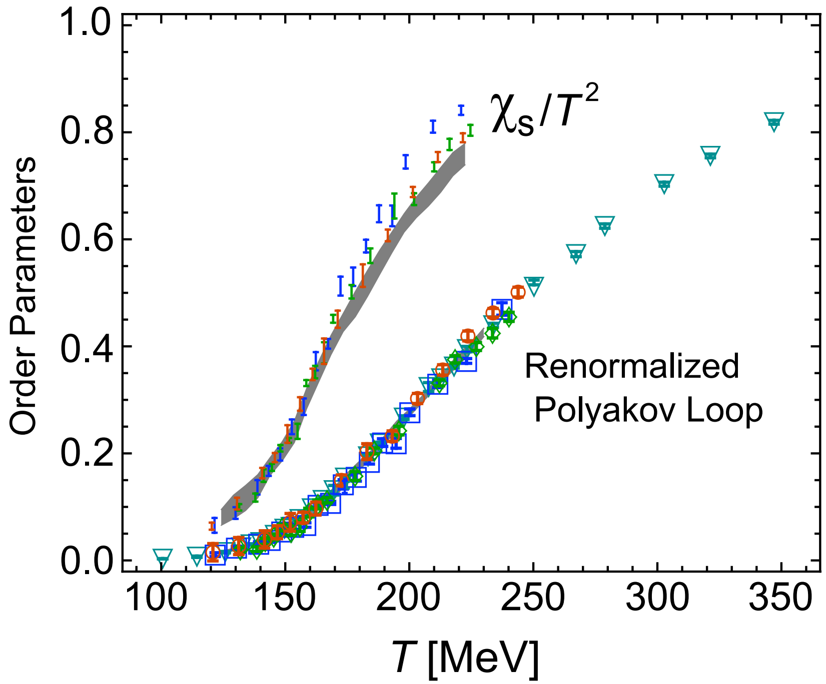
Physics of (De)Confinement

Polyakov loop increases very smoothly:

Lattice from BNL-Bielefeld (2010)



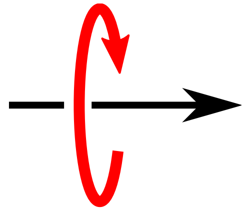
Lattice from Wuppertal-Budapest (2010)

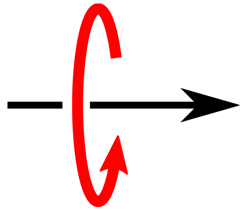


There is no clear-cut T_c for deconfinement.

Physics of Chiral Restoration

$$\mathrm{SU}(N_f)_{\mathrm{L}} \times \mathrm{SU}(N_f)_{\mathrm{R}} \times \cancel{\mathrm{U}(1)_{\mathrm{A}}} \;\to\; \mathrm{SU}(N_f)_{\mathrm{V}}$$





Shall be explained on Day 3.

For $N_{\mathrm{f}} = 2$

2nd-order "expected" from the universality

$$\mathrm{SU}(2) \times \mathrm{SU}(2) \;\to\; \mathrm{SU}(2)$$

$$\mathrm{SO}(4) \;\to\; \mathrm{SO}(3)$$

Massless π^0, π^+, π^-

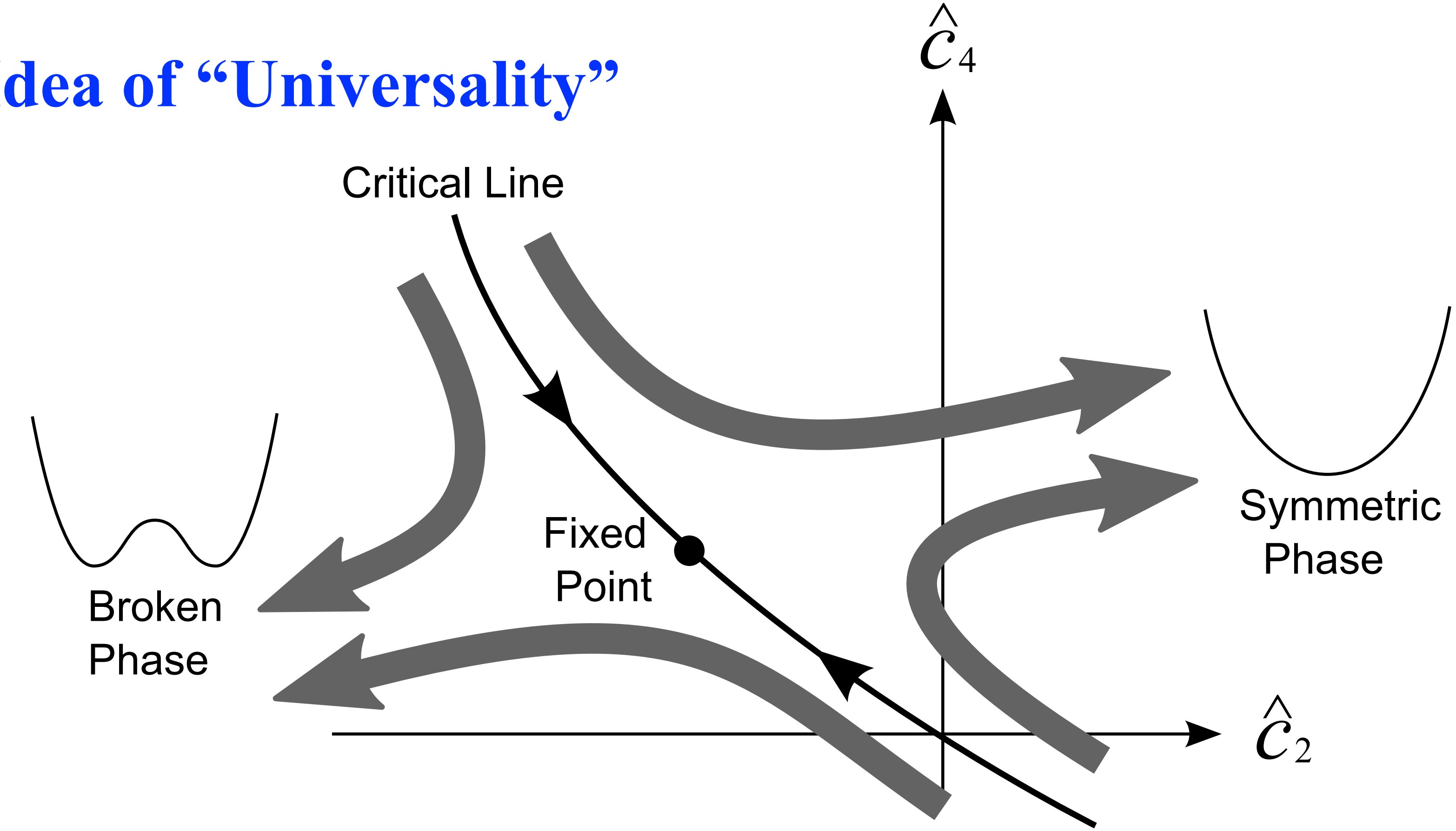
Massive σ

Degenerate $\sigma, \pi^0, \pi^+, \pi^-$

Physics of Chiral Restoration

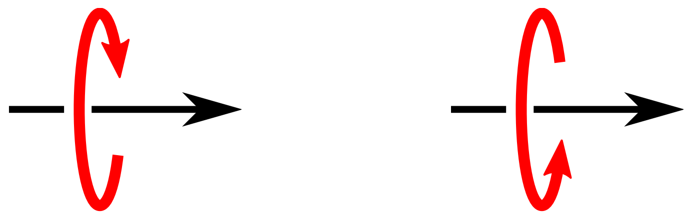
$$\Gamma = \int d^d x \left[(\partial\phi)^2 + c_2|\phi|^2 + c_4|\phi|^4 + \cdots \right]$$

Idea of "Universality"



Physics of Chiral Restoration

$$\mathrm{SU}(N_f)_\mathrm{L} \times \mathrm{SU}(N_f)_\mathrm{R} \times \cancel{\mathrm{U}(1)_\mathrm{A}} \;\to\; \mathrm{SU}(N_f)_\mathrm{V}$$



Shall be explained on Day 3.

U(1)$_\mathrm{A}$ Breaking Interaction

$$\begin{aligned}\det[\bar\psi_i(1+\gamma_5)\psi_j] \;&\to\; \det[R_{jm}\bar\psi_n(1+\gamma_5)\psi_m]L^\dagger_{ni}\,] \\ &= \det[R]\,\det[L^\dagger]\,\det[\bar\psi_i(1+\gamma_5)\psi_j]\end{aligned}$$

$$\langle\bar u u\rangle\langle\bar d d\rangle\langle\bar s s\rangle \sim \langle\bar q q\rangle^3$$

1st-order transition is strongly favored.

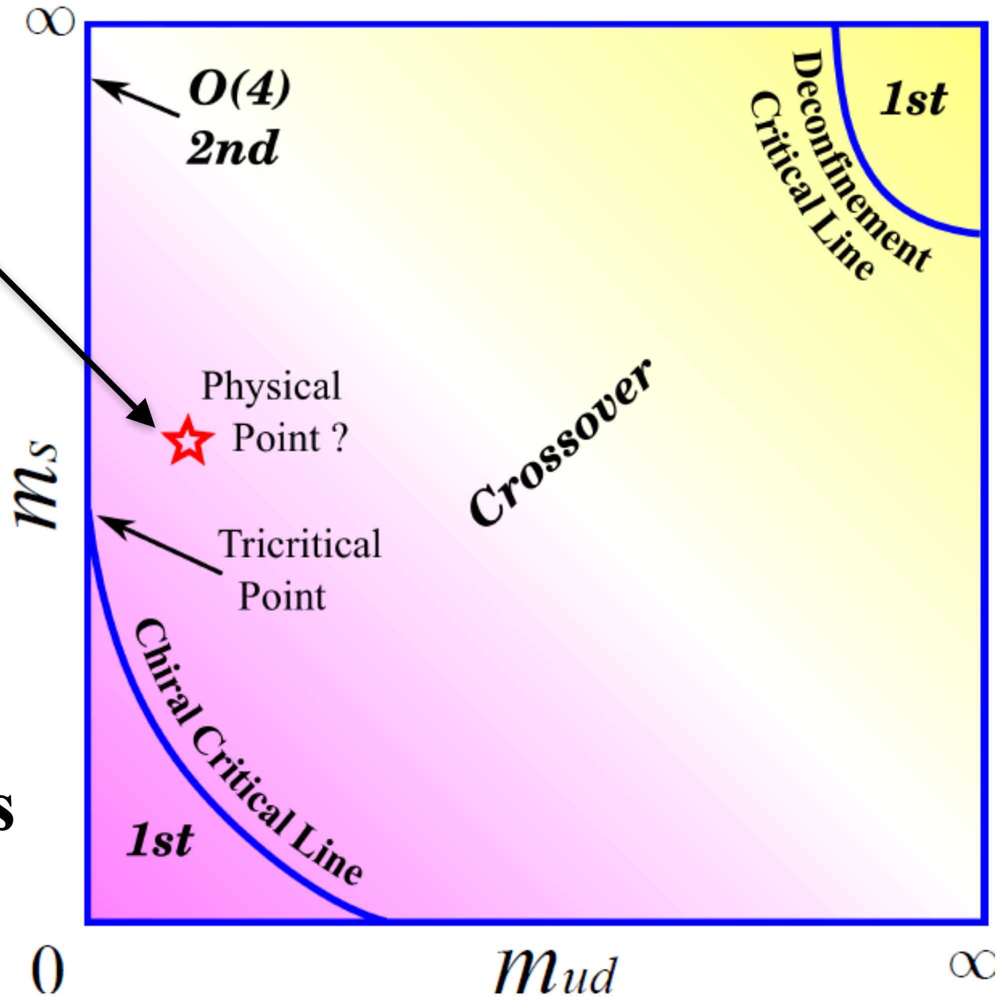
Physics of Chiral Restoration

Located in the crossover region

Close to the O(4) 2nd?

Crossover confirmed by finite-*V* analysis

Aoki et al. (2006)

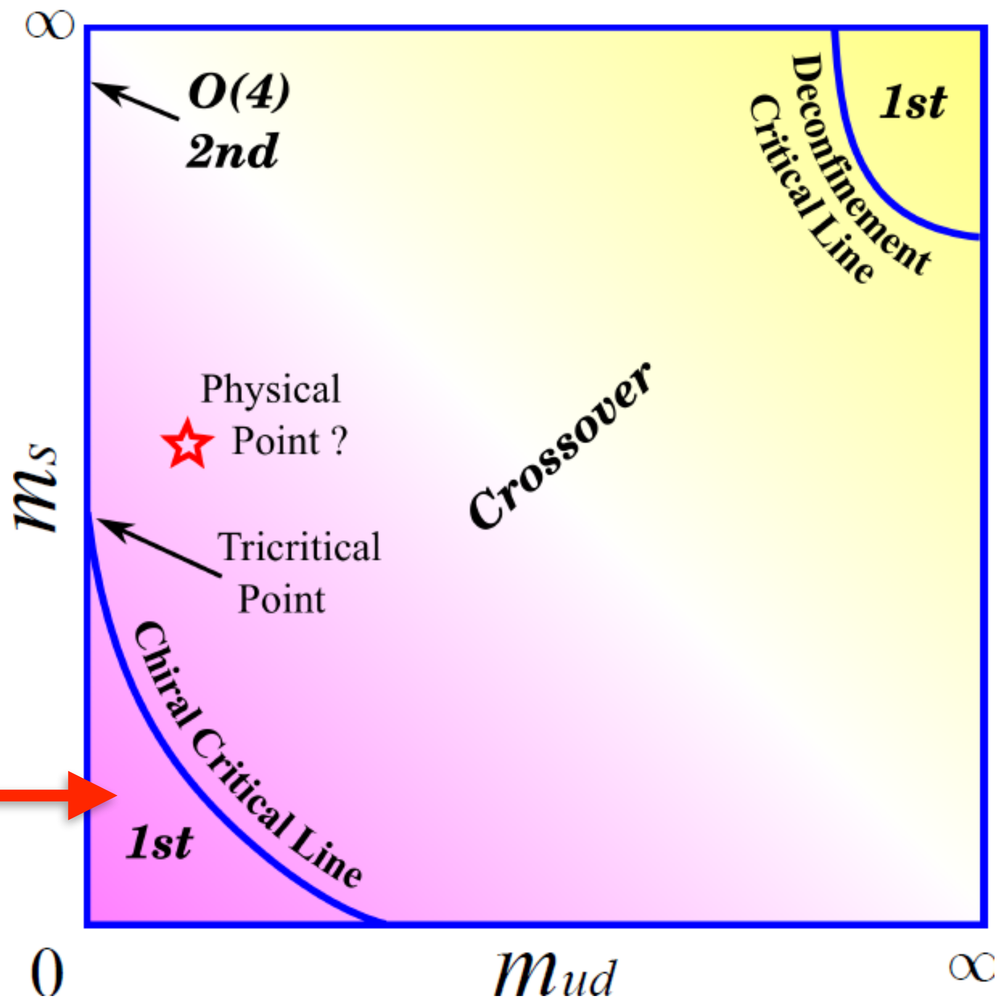


Quenched Limit

Physics of Chiral Restoration

Pisarski-Wilczek (1984)

If U(1)$_A$ is restored then symmetry is not O(4) but O(4) × O(1) and the leading order ε expansion cannot find a fixed point…
1st-order phase transition?



Recent lattice-QCD suggests the 1st-order region is tiny or even entirely vanishing!?

See; Philipsen (2022)

Relation Between Two Transitions

Gluon Sector

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \qquad (A_\mu \to A_B + \mathcal{A})$$

$$A_{\mathrm{B4}} = \frac{2\pi}{g\beta} \mathrm{diag}(q_1, q_2, \ldots, q_{N_\mathrm{c}}) = \frac{2\pi}{g\beta} \sum_{i=1}^{N_\mathrm{c}} q_i \delta_i \qquad \Big(\sum_i q_i = 0 \Big)$$

$$D_{\mathrm{B4}} \mathcal{A}_\mu = \partial_4 \mathcal{A}_\mu - ig[A_{\mathrm{B4}}, \mathcal{A}_\mu] = \partial_4^{(i,j)} \mathcal{A}_\mu^{(i,j)} t_{(i,j)}$$

$$\partial_4^{(i,j)} = \partial_4 - 2\pi i \delta_{\mu 4} q_{ij} \qquad q_{ij} = q_i - q_j$$

A_4 appears like an imaginary chemical potential

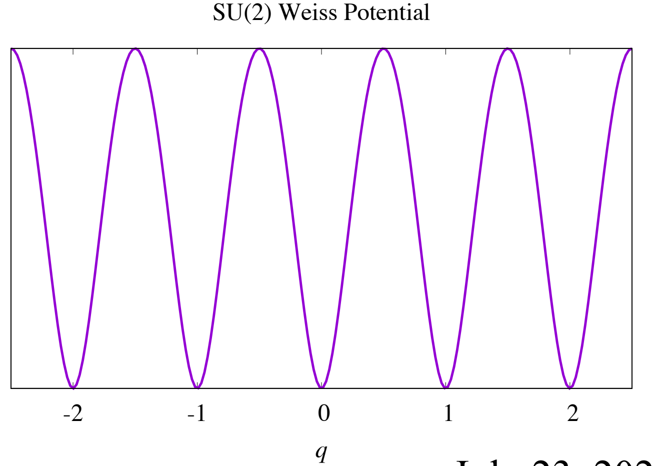
Relation Between Two Transitions

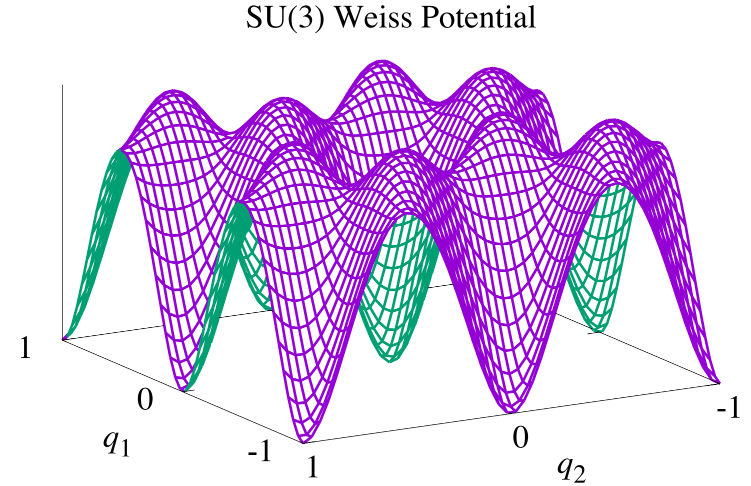
Gluon Sector $A_4 \sim$ **Colored imaginary chemical potential**

$$V_{\rm glue}[q] = 2V\int\frac{d^3p}{(2\pi)^3}\sum_{i>j}\Big[\ln\big(1-e^{-\beta|\boldsymbol{p}|+2\pi i q_{ij}}\big)+\ln\big(1-e^{-\beta|\boldsymbol{p}|-2\pi i q_{ij}}\big)\Big]$$

This momentum integration is analytically done:

$$V_{\rm glue}^{\rm Weiss}[q] = \frac{4\pi^2 V}{3\beta^3}\sum_{i>j}(q_{ij})^2_{\rm mod1}\big[(q_{ij})_{\rm mod1}-1\big]^2$$



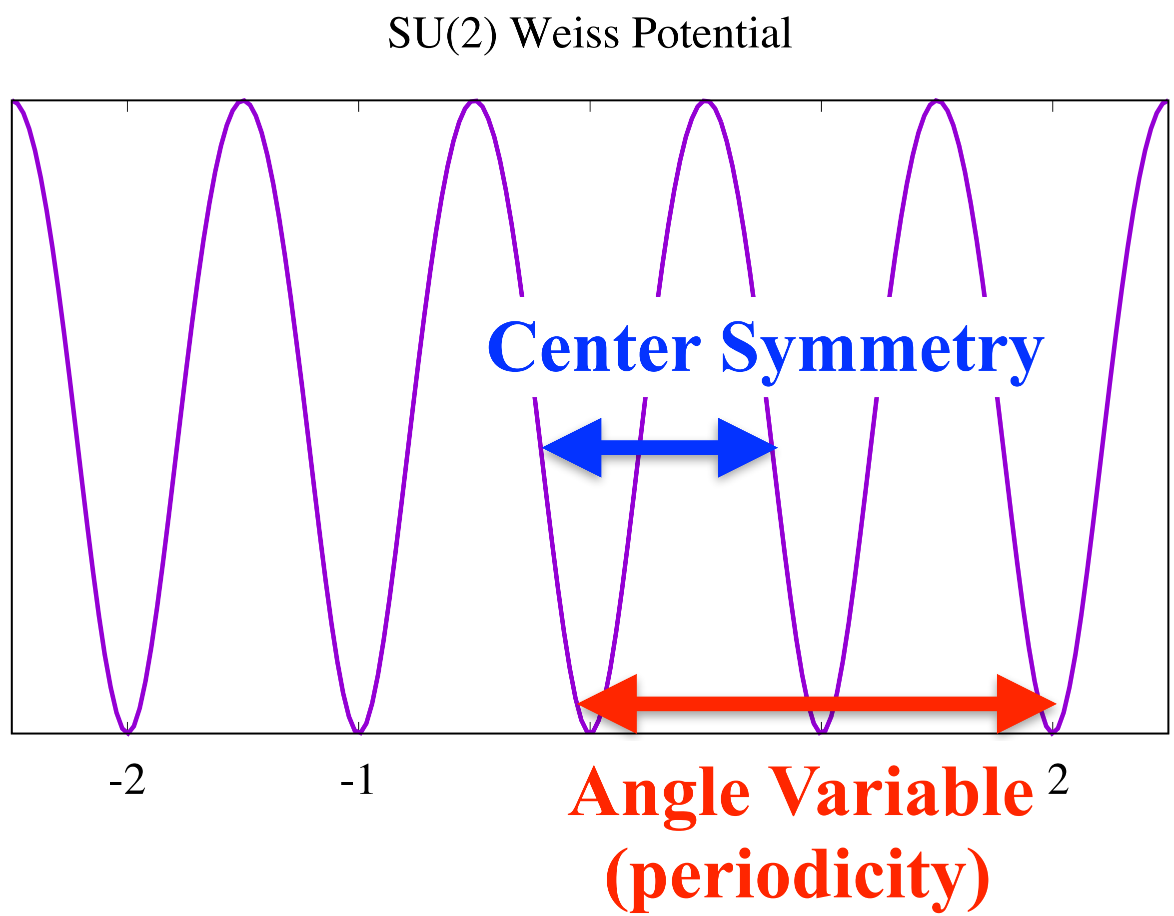


Relation Between Two Transitions

Gluon Sector

One loop potential has spontaneous symmeetry breaking and the perturbative vacuum is found in the "broken" phase.

Potential curvature is the Debye mass.



Relation Between Two Transitions

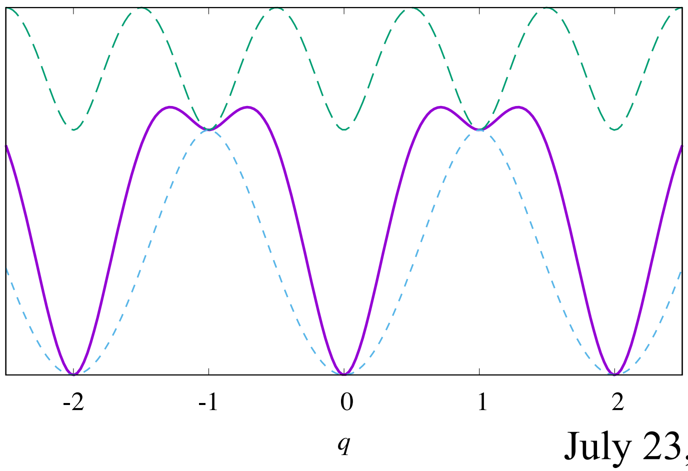
Quark Sector

$$V_{\text{quark}}[q] = -2N_{\text{f}}TV \int \frac{d^3p}{(2\pi)^3} \sum_{i=1}^{N_{\text{c}}} \Big[\ln\big(1 + e^{-\beta(|\boldsymbol{p}|-\mu)+2\pi i q_i}\big) + \ln\big(1 + e^{-\beta(|\boldsymbol{p}|+\mu)-2\pi i q_i}\big) \Big]$$

$$= -N_{\text{f}}V \frac{4\pi^2}{3\beta^4} \sum_{i=1}^{N_{\text{c}}} \left(q_i + \frac{1}{2} - i\frac{\beta\mu}{2\pi} \right)^2_{\text{mod}1} \left[\left(q_i + \frac{1}{2} - i\frac{\beta\mu}{2\pi} \right)_{\text{mod}1} - 1 \right]^2 .$$

Complex at finite μ → Sign Problem
No way to fix the optimal Polyakov loop…!?

SU(2) Full Weiss Potential (N_{f}=1)



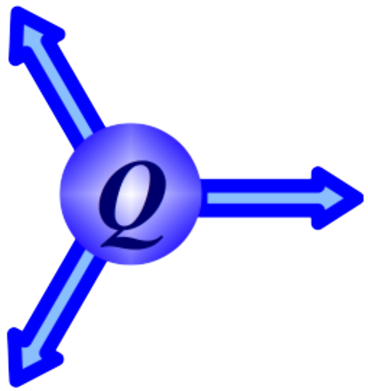
Periodicity from symmetry is broken by quarks.

Relation Between Two Transitions

Equivalent but more useful expression

$$
\begin{aligned}
V_{\text{quark}}[q] &= -2N_{\mathrm{f}}TV\int\frac{d^3p}{(2\pi)^3}\mathrm{tr}\Big[\ln\big[1+L\,e^{-\beta(\varepsilon_p-\mu)}\big]+\ln\big[1+L^\dagger e^{-\beta(\varepsilon_p+\mu)}\big]\Big] \\
&= -2N_{\mathrm{f}}TV\int\frac{d^3p}{(2\pi)^3}\Big[\ln\big(1+3\ell\,e^{-\beta(\varepsilon_p-\mu)}+3\ell^* e^{-2\beta(\varepsilon_p-\mu)}+e^{-3\beta(\varepsilon_p-\mu)}\big) \\
&\qquad +\ln\big(1+3\ell^* e^{-\beta(\varepsilon_p+\mu)}+3\ell\,e^{-2\beta(\varepsilon_p+\mu)}+e^{-3\beta(\varepsilon_p+\mu)}\big)\Big]\,,
\end{aligned}
$$

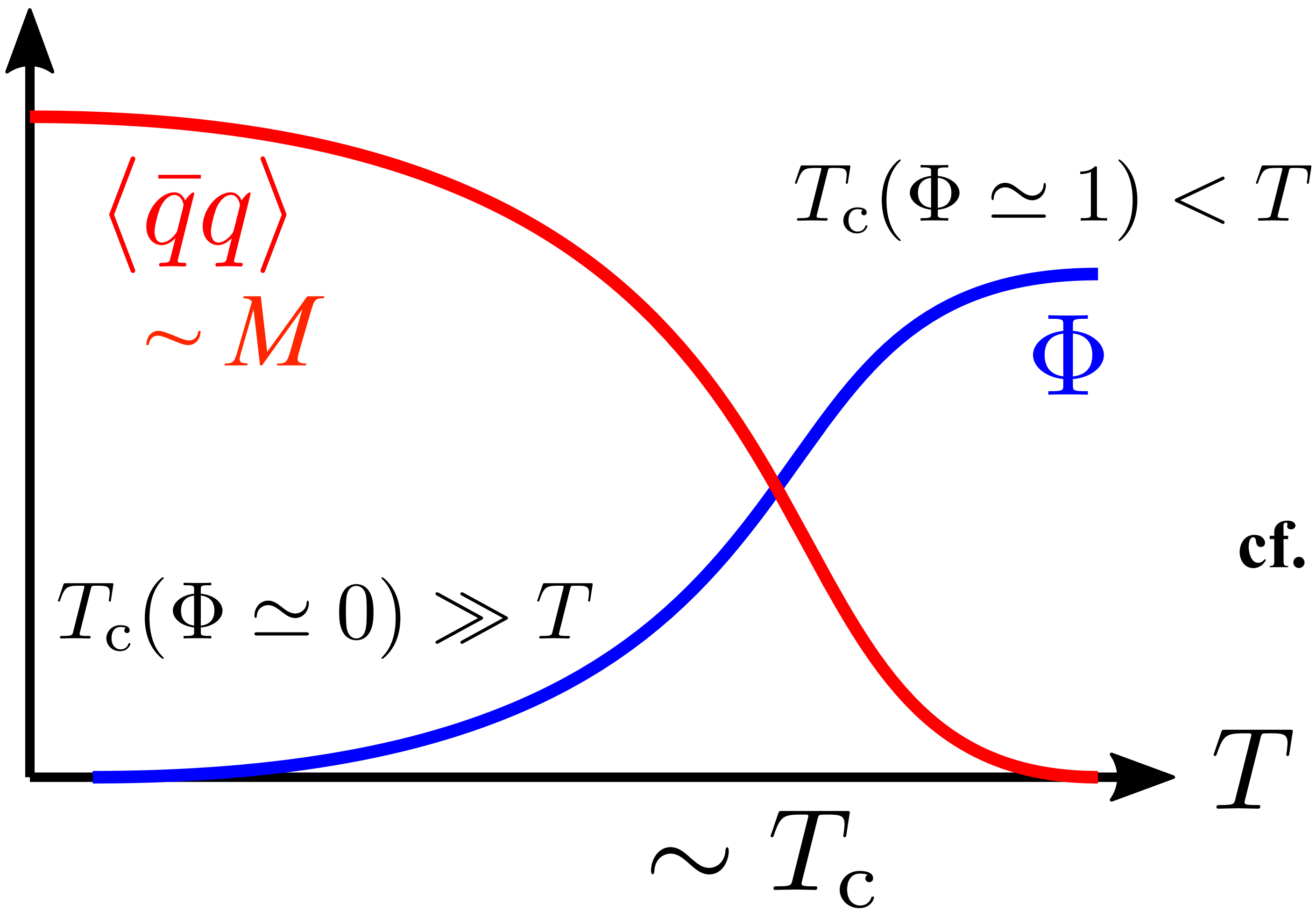
This gives a natural coupling betw'n $\langle\bar{q}q\rangle$ and Φ.



$$1+e^{i2\pi/3}+e^{-i2\pi/3}=0$$

Polyakov loop = medium screening

Relation Between Two Transitions

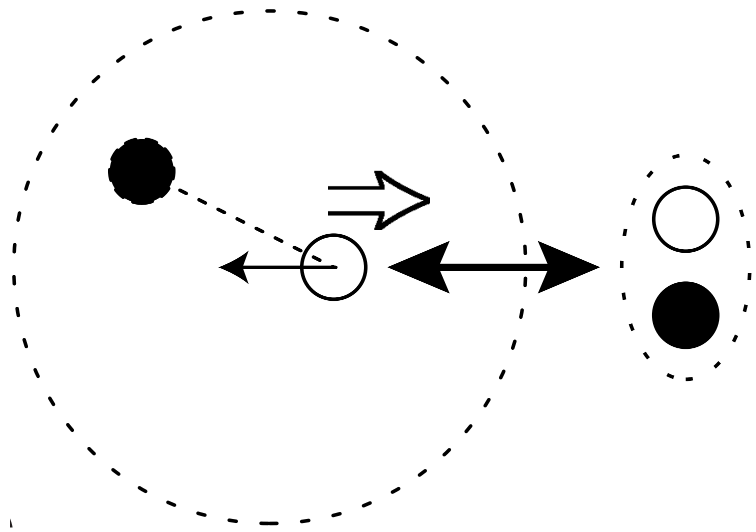


cf. PNJL

Very simple but robust idea to make them locked

Relation Between Two Transitions

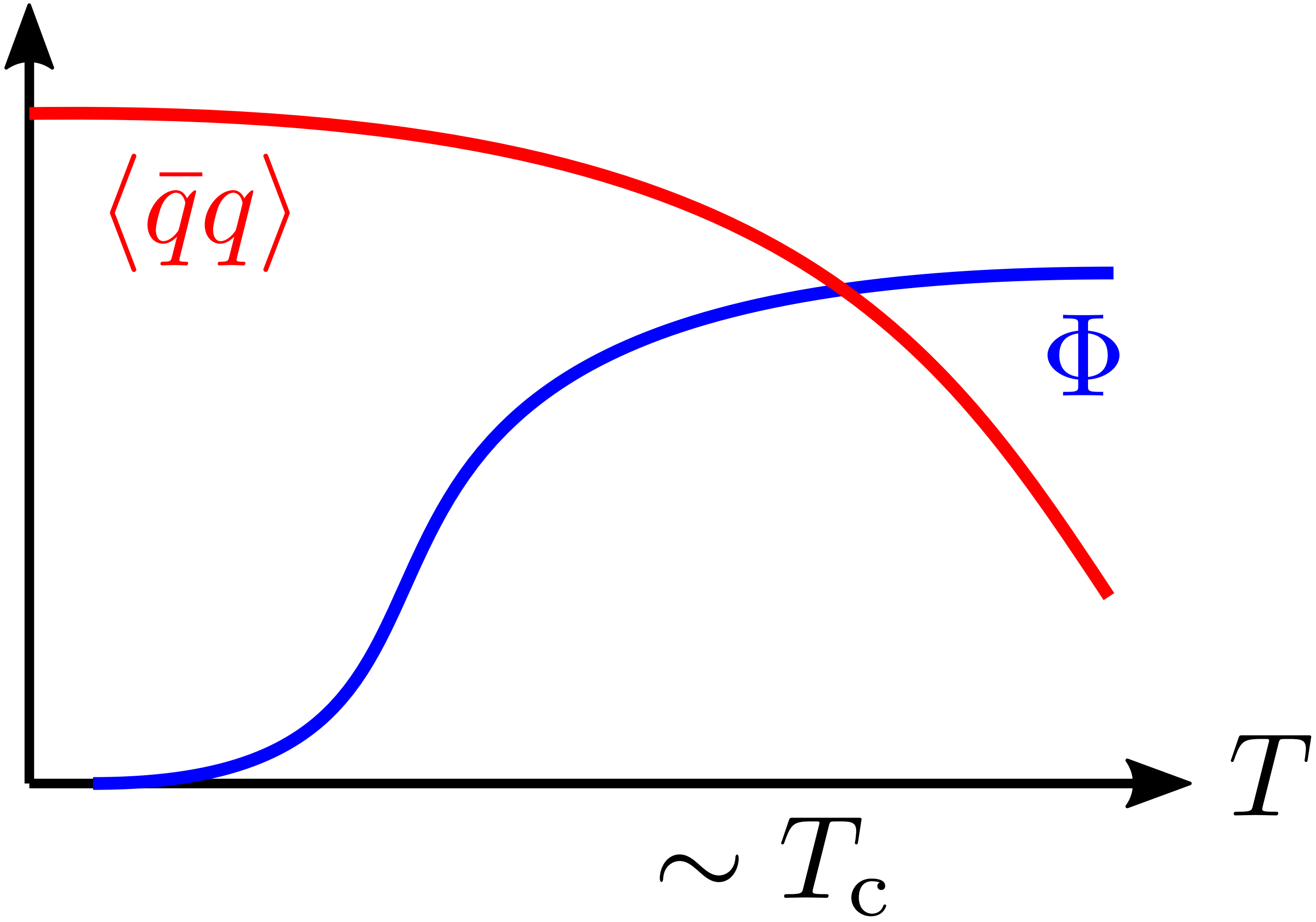
Intuitive arguments (Casher)



If quarks are confined by the spherical potential, how can quarks flip their chirality?

Confinement → Chiral Symmetry Breaking

Relation Between Two Transitions



Is this also possible? Yes, e.g. adjoint quarks

Relation Between Two Transitions

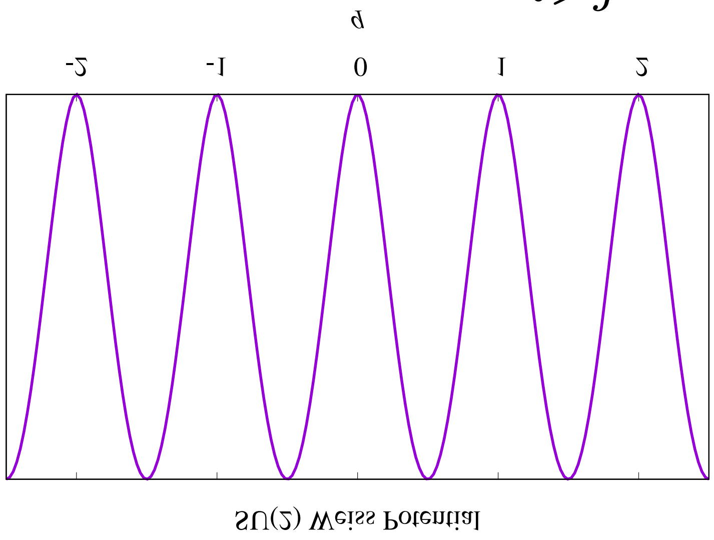
What if there are adjoint quarks?

With periodic boundary condition (in a box)

Gluon+Adjoint Quark

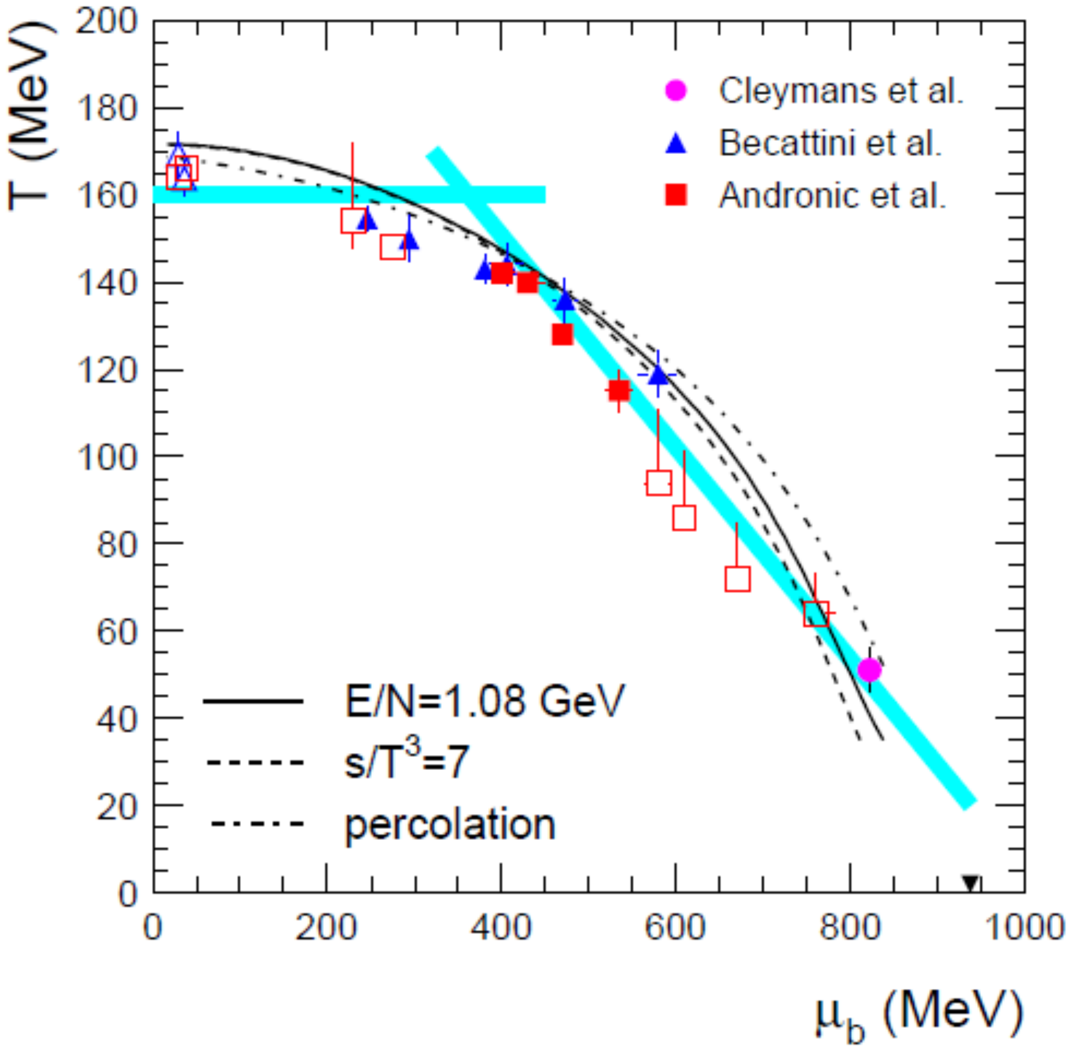
$$\left(\frac{1}{2}-N_f\right)\frac{8\pi^2 V}{3L^3}\sum_{i>j}(q_{ij})^2_{\mathrm{mod}1}\left[(q_{ij})_{\mathrm{mod}1}-1\right]^2$$

Upside down!



Confinement occurs almost trivially and perturbatively. Small box → Large box?

Phenomenology



Experimentally determined with a clear physics picture

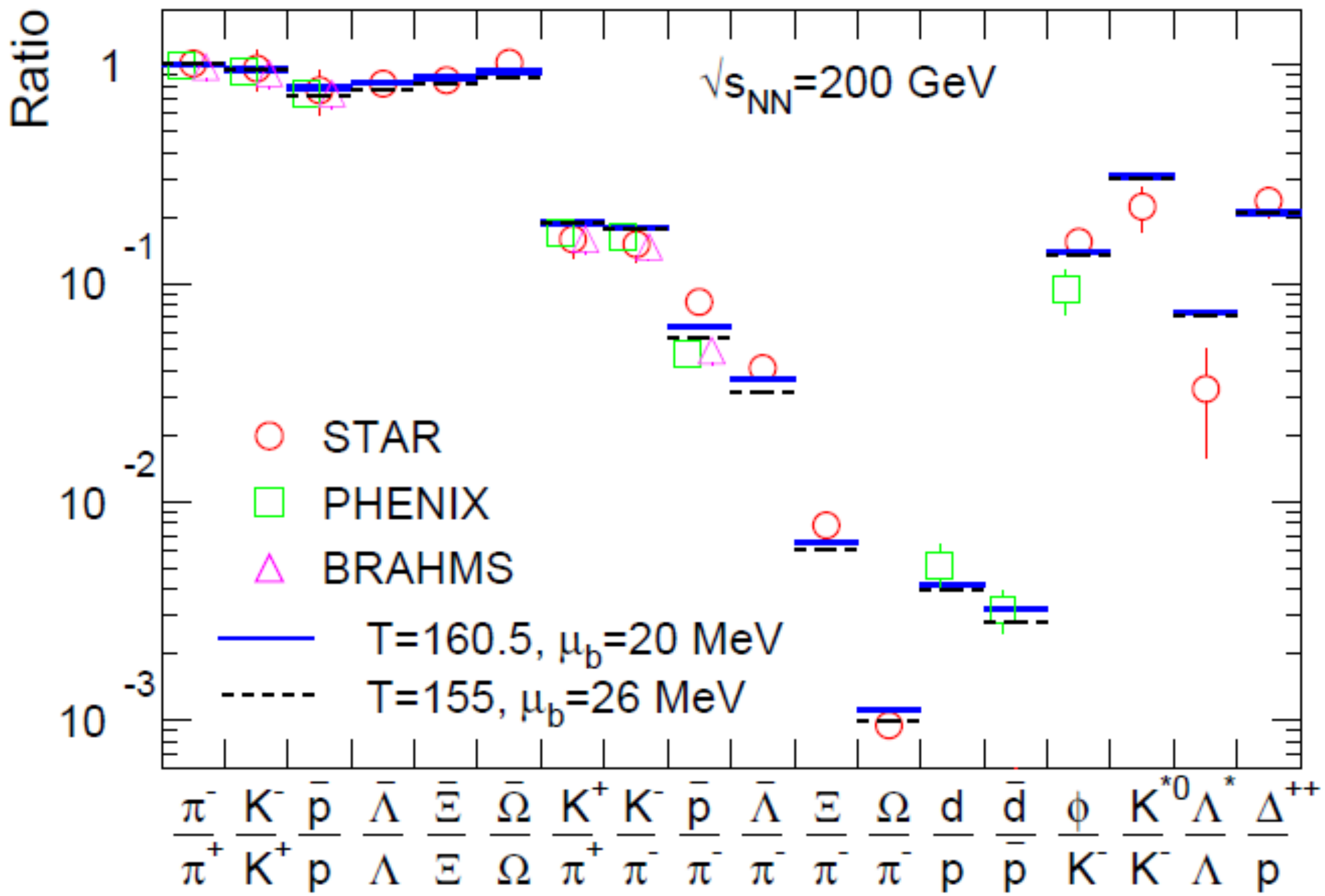
Deconfinement = Hagedorn?

This tells us a lot of insights on deconfinement physics

Andronic et al. (2010)

Phenomenology

How to determine *T* and *μ* "experimentally"

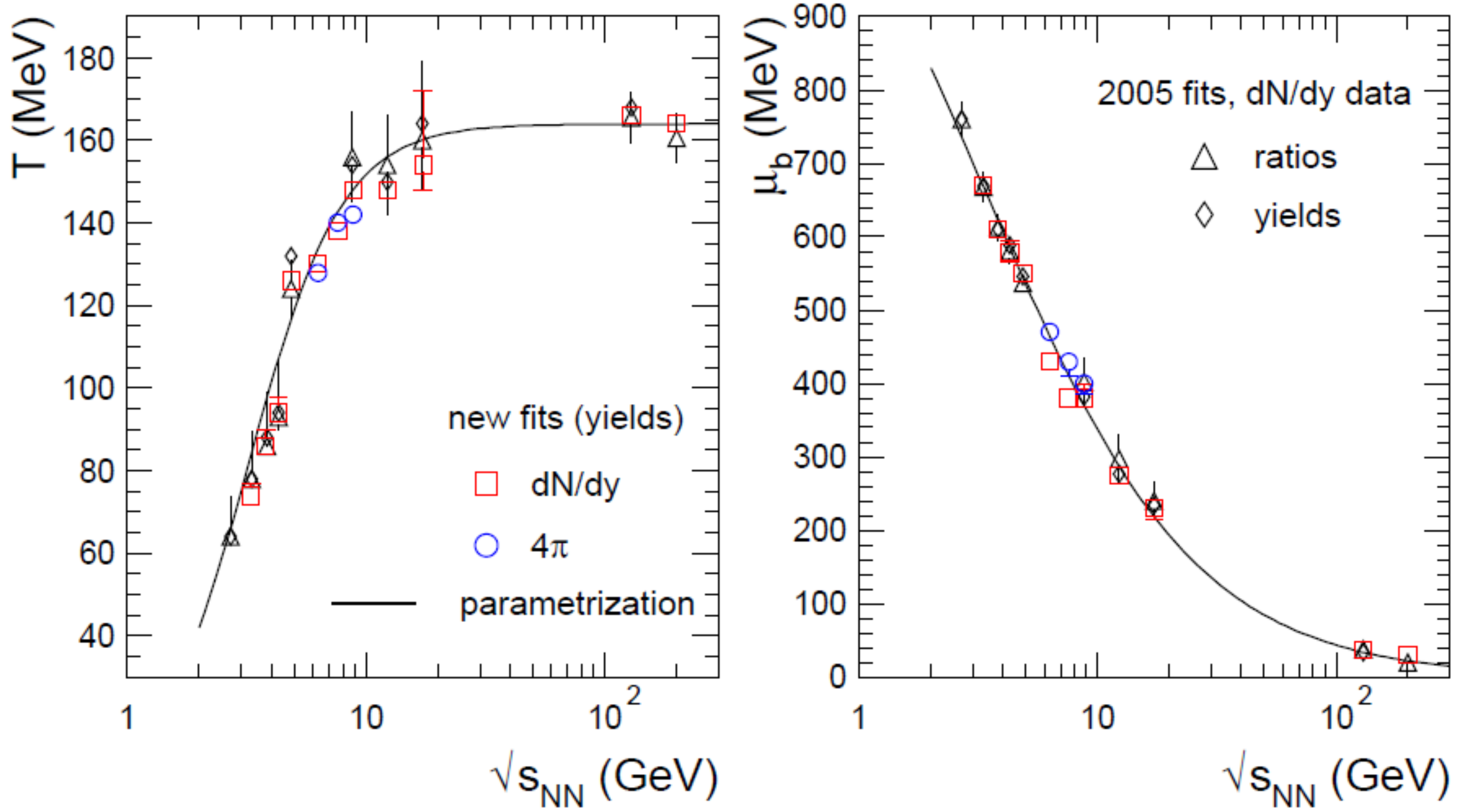


Thermal Model Fit works gooe!

Andronic Braun-Munzinger Stachel…

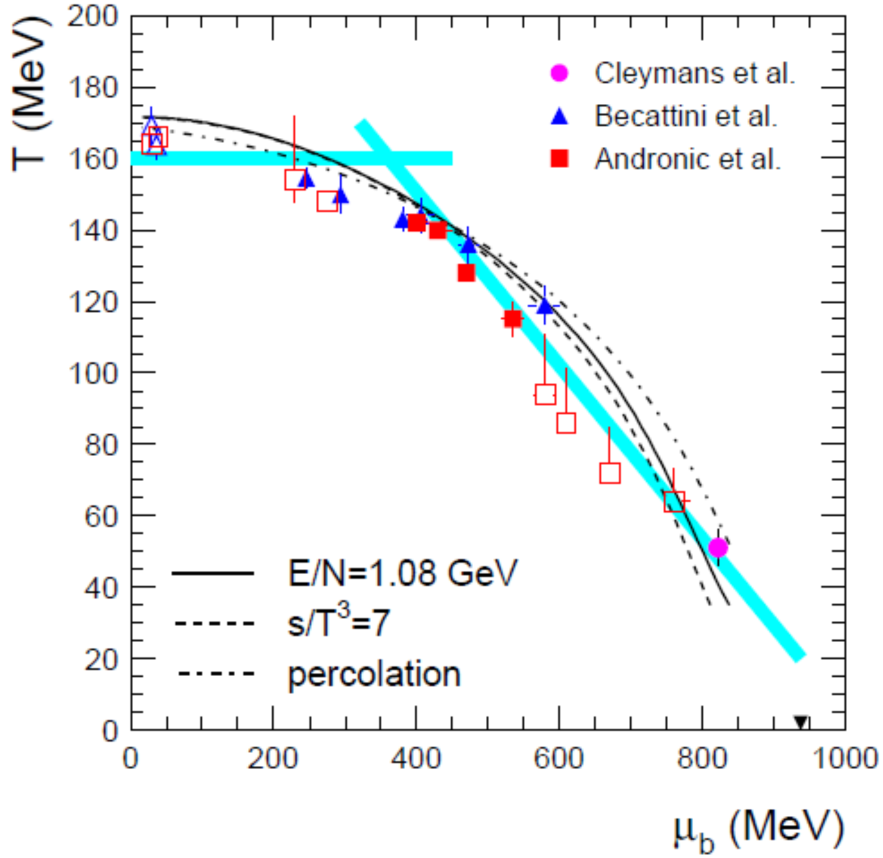
Phenomenology

$$(\text{Mapping})\ \sqrt{s_{NN}}\ \Leftrightarrow T,\ \mu_B$$



Phenomenology

Phase Diagram = Two Hagedorn Transition Lines



Mesonic Hagedorn Transition

$$Z = N \int dm\, \rho(m)\, e^{-m/T}$$

$$\rho(m) = e^{m/T_H}$$

$$T_c = T_H$$

Baryonic Hagedorn Transition

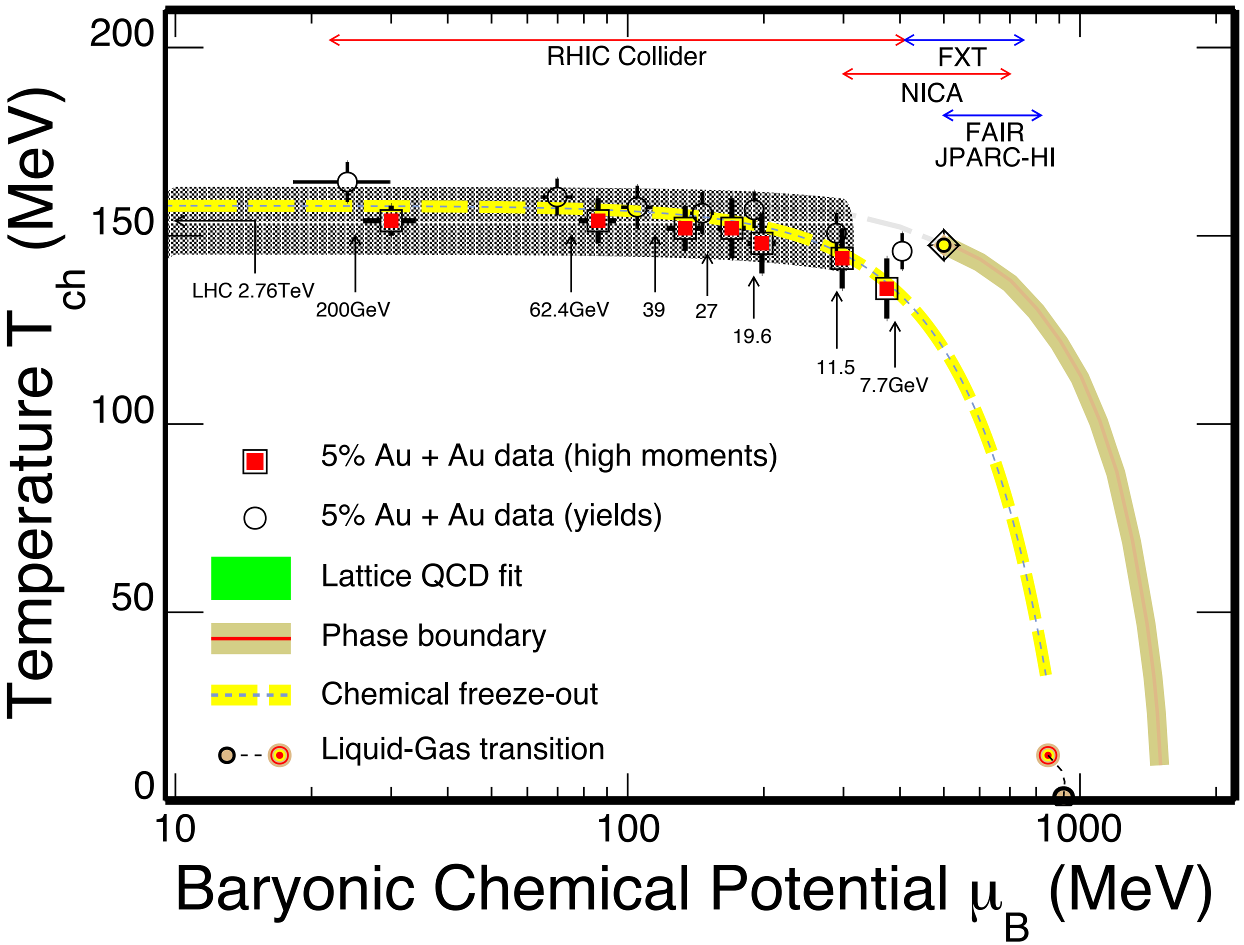
$$Z = N \int dm\, \rho_B(m)\, e^{-(m-\mu_B)/T}$$

$$\rho_B(m) = e^{m_B/T_B}$$

$$T_c = (1 - \mu_B/m_B) T_B$$

Phenomenology

"Experimentally Determined" Phase Diagram



For full information see; 2009.03006 [hep-ph]