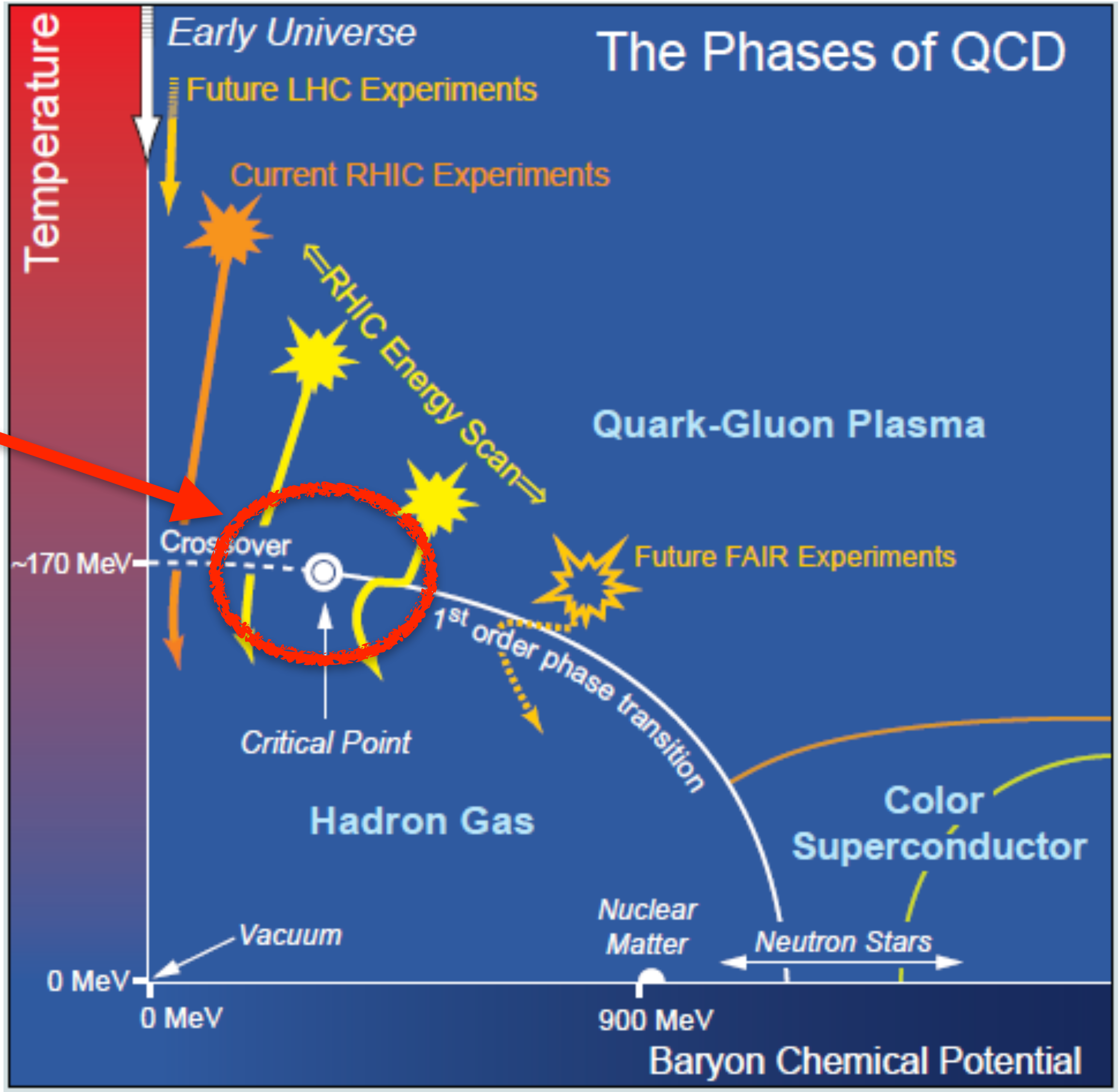
— Day 2 —

Theoretical Knowns and Many Unknowns at Low *T* and High Baryon Density

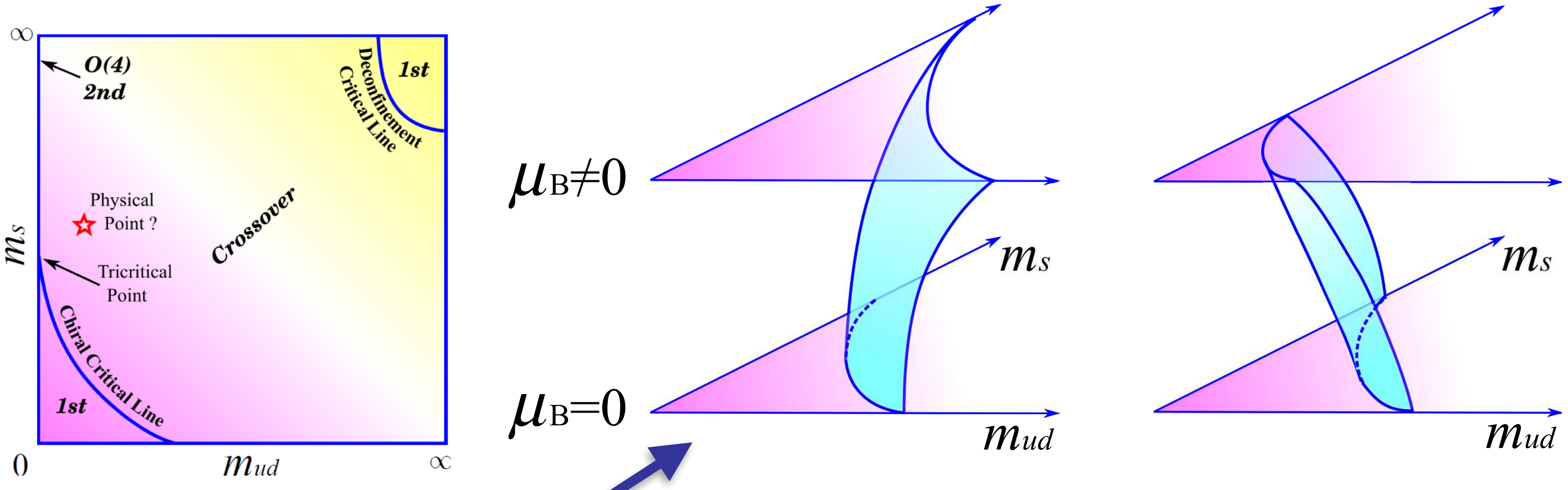
QCD Critical Point

QCD CP

End-point of
1st order phase
transition



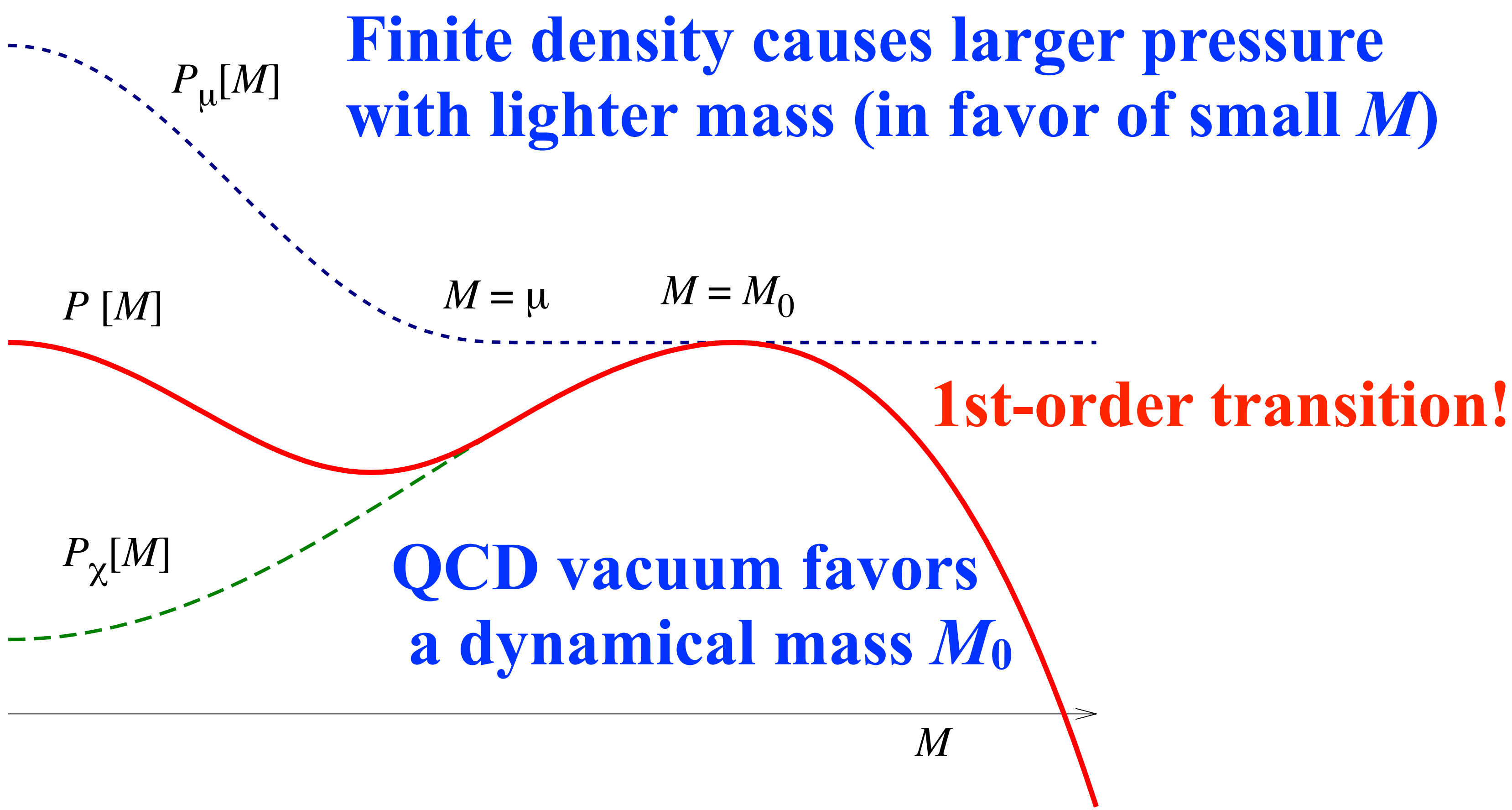
QCD Critical Point



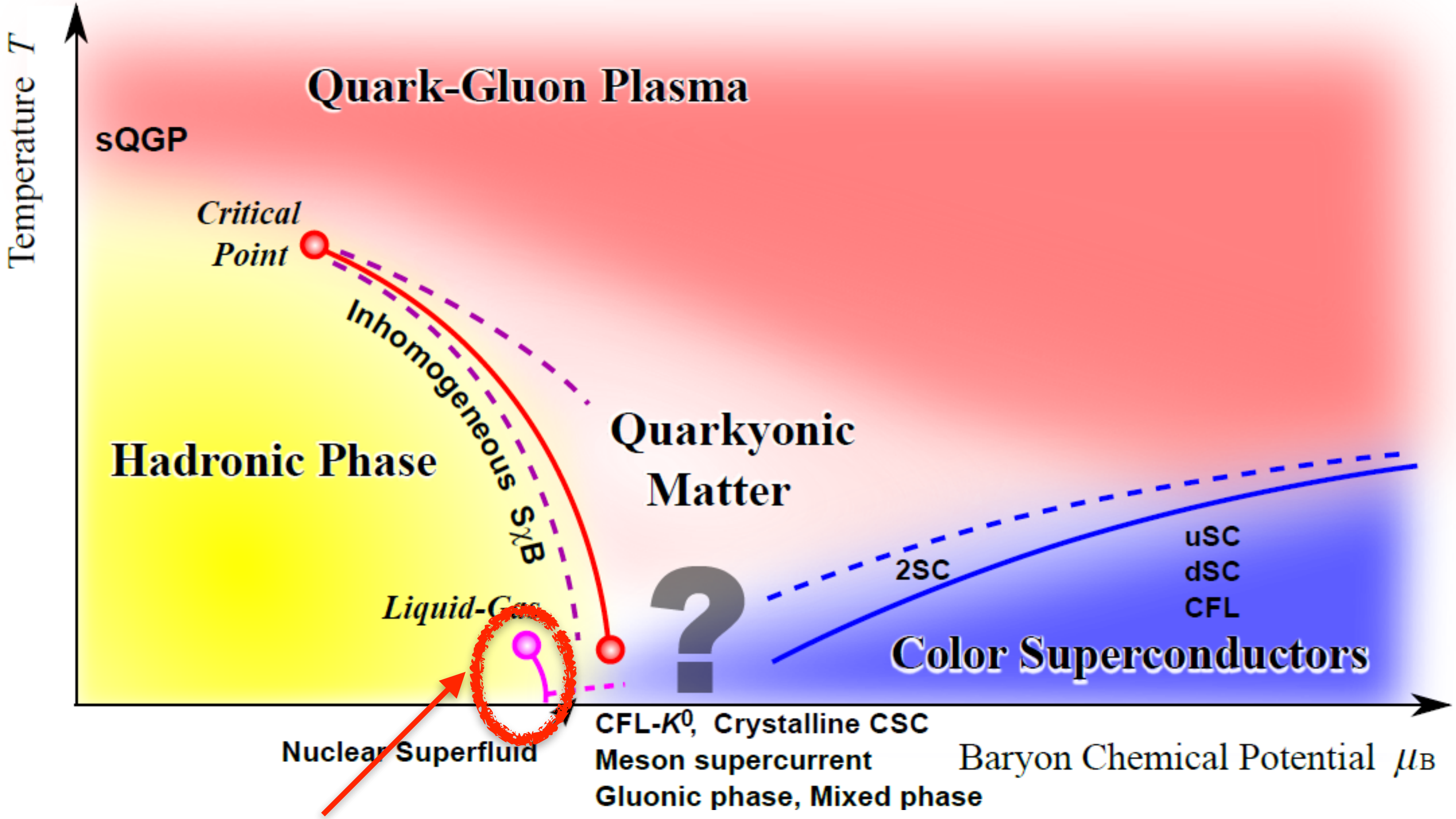
IF this is the case in QCD:
IF the curvature is large enough: → **QCD CP exists**

QCD CP is a hypothesis (needs exp. confirmation)

QCD Critical Point



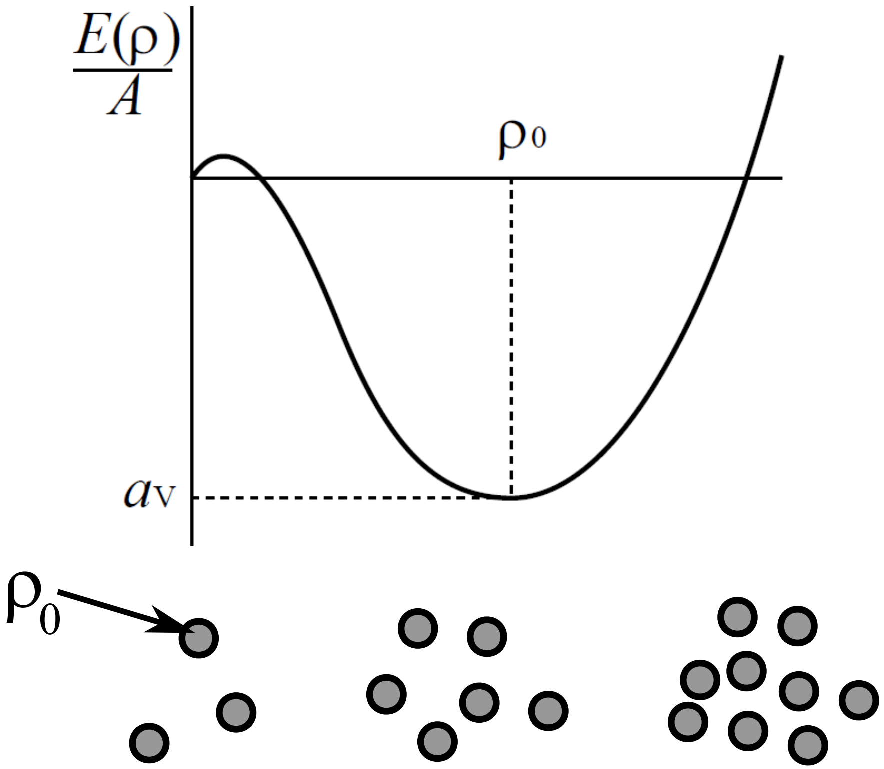
Another Critical Point



Liquid-gas phase transition

(Nuclear matter is a self-bound fermionic system)

Another Critical Point



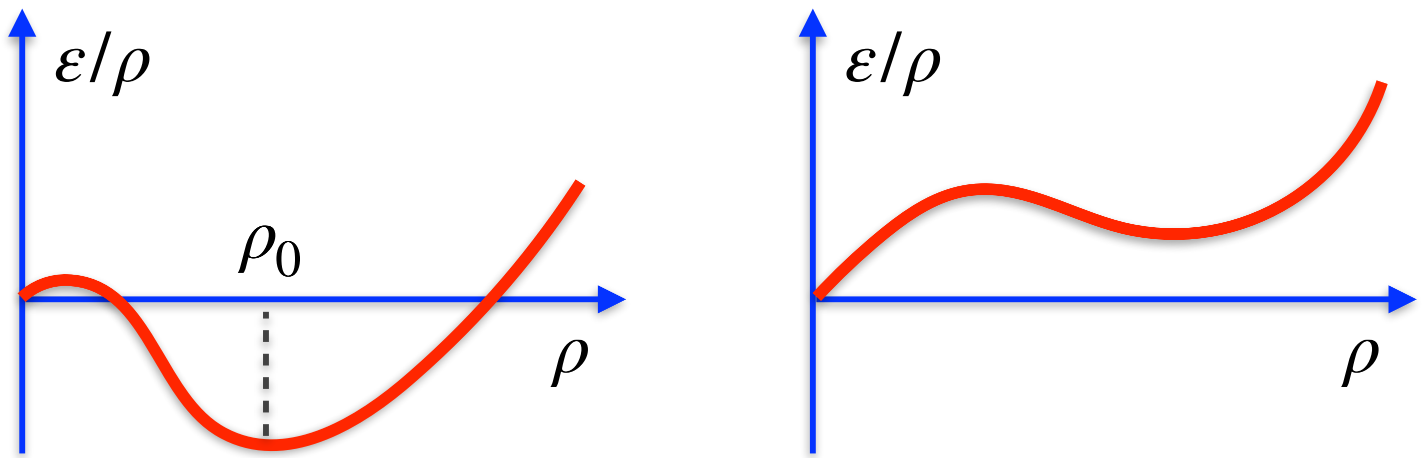
Self-bound fermionic systems have a preferred density.
Diluteness is realized as a "mixed phase" of nuclei.

No argument about whether quarks are self-bound?
Quark EoS is constrained by neutron stars > $2M_\odot$

Another Critical Point

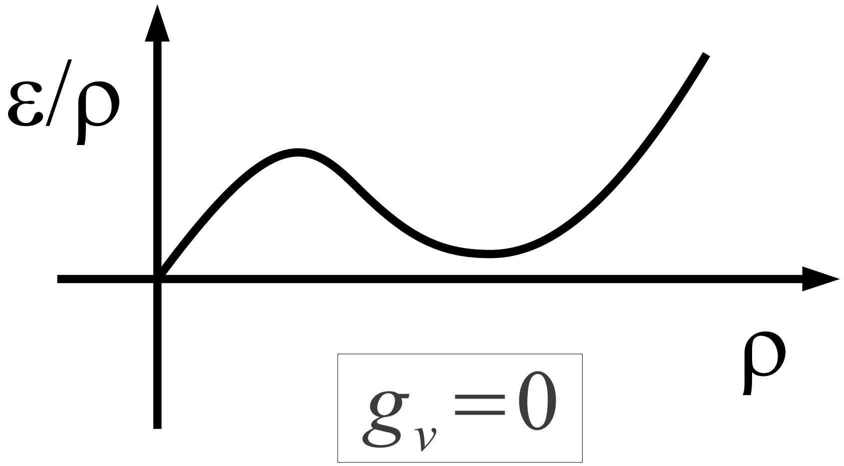
$$\frac{d}{d\rho}\left(\frac{\varepsilon}{\rho}\bigg|_{\text{gas}} - \frac{\varepsilon}{\rho}\bigg|_{\text{liquid}}\right) = \frac{p_{\text{gas}} - p_{\text{liquid}}}{\rho^2} = 0$$

Metastability **1st-order PT**

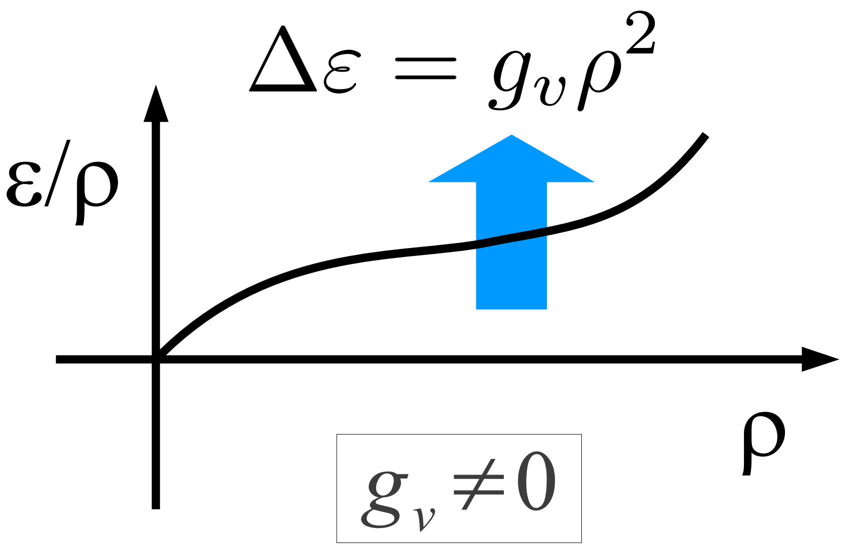


Nuclear Matter **Chiral Quark Models**

Another Critical Point



Meta-stable quark matter can have a 1st-order



Vector interaction easily washes out the QCD CP, but the spiral phase is robust.

Significant vector interaction is suggested from the neutron star observations…

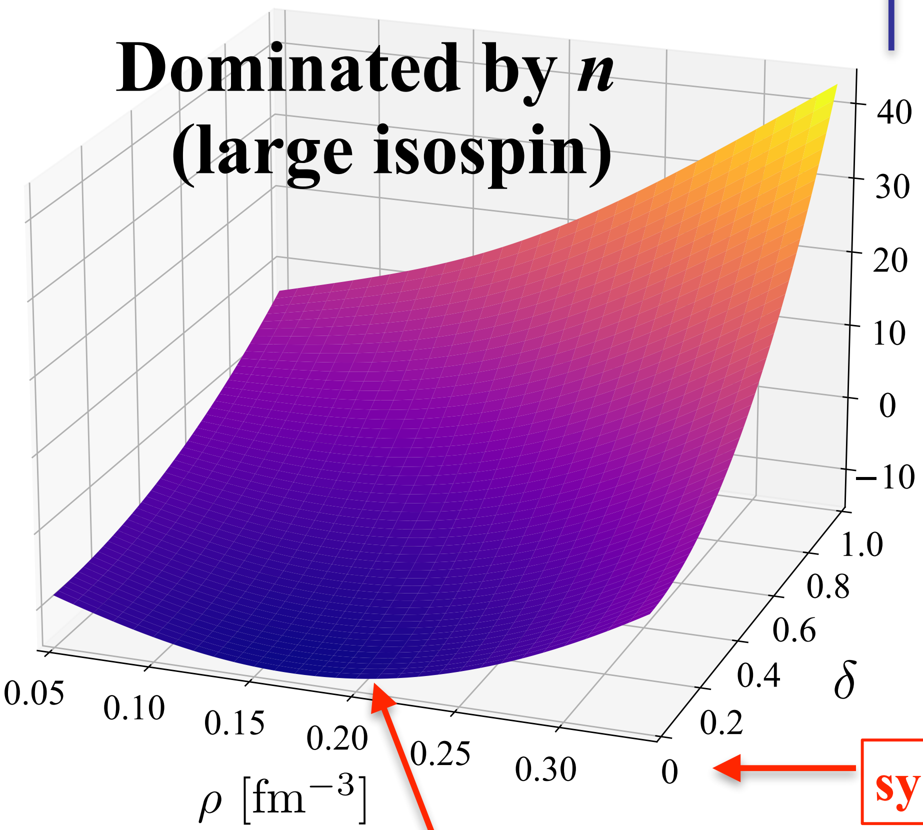
Another Critical Point

Neutron Star

Heavy-Ion Collision

Dominated by *n*
(large isospin)

Isospin fixed by *p/n* ratio
(small isospin)



$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

$$\rho = \rho_n + \rho_p$$

neutron matter

symmetric nuclear matter (HIC)

saturation density
(1st-order transition)

Phase structures may be very different.

Another Critical Point

Neutron Star

β equilibrium

$$\mu_s = 0$$

High baryon density should involve hyperons (Λ, Σ, etc)

EoS too soft? / Cooling too fast?

Hyperon Puzzle

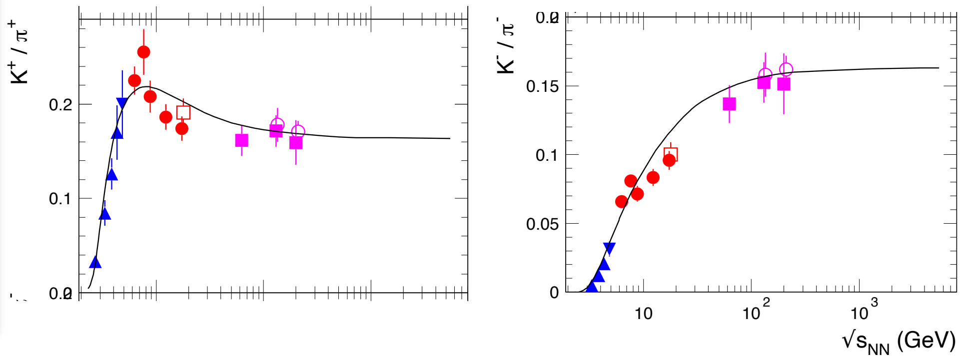
* Interactions may suppress hyperons (3-body forces YNN)
* Interactions may make EoS stiff (repulsive forces at high density)

Heavy-Ion Collision

Zero net strangeness

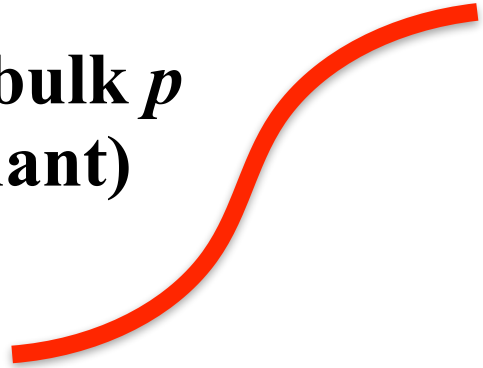
$$n_s = 0 \qquad (\mu_s \sim \frac{1}{3}\mu_B)$$

Hyperon strangeness is canceled by strange mesons ($\bar{s}$ in mesons sensitive to μ_B)

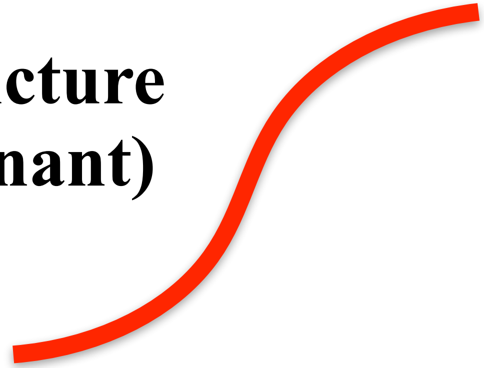


Observables for Criticality

Smooth bulk *p*
(dominant)



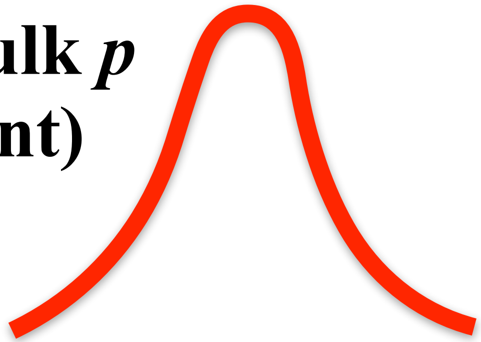
+ Fine structure
(sub-dominant)



Q : How to extract the difference ?

Observables for Criticality

Smooth bulk *p*
(dominant)



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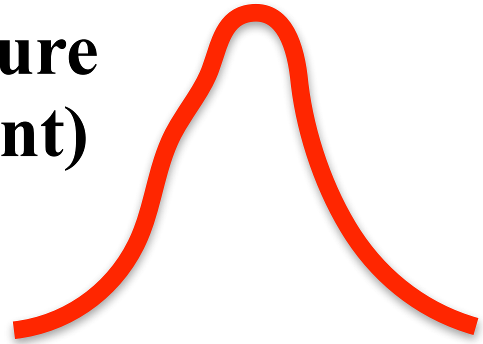


Q : How to extract the difference ?

A : Take the (higher) derivative !

$$\chi^{(n)}_{B,S} \equiv \frac{\partial^n}{\partial(\mu_{B,S}/T)^n} \frac{p}{T^4}$$

enhanced near QCD CP

Observables for Criticality

$$\frac{\sigma^2}{M} \equiv \frac{\chi_B^{(2)}}{\chi_B^{(1)}}\,, \qquad S\sigma \equiv \frac{\chi_B^{(3)}}{\chi_B^{(2)}}\,, \qquad \kappa\sigma^2 \equiv \frac{\chi_B^{(4)}}{\chi_B^{(2)}}$$

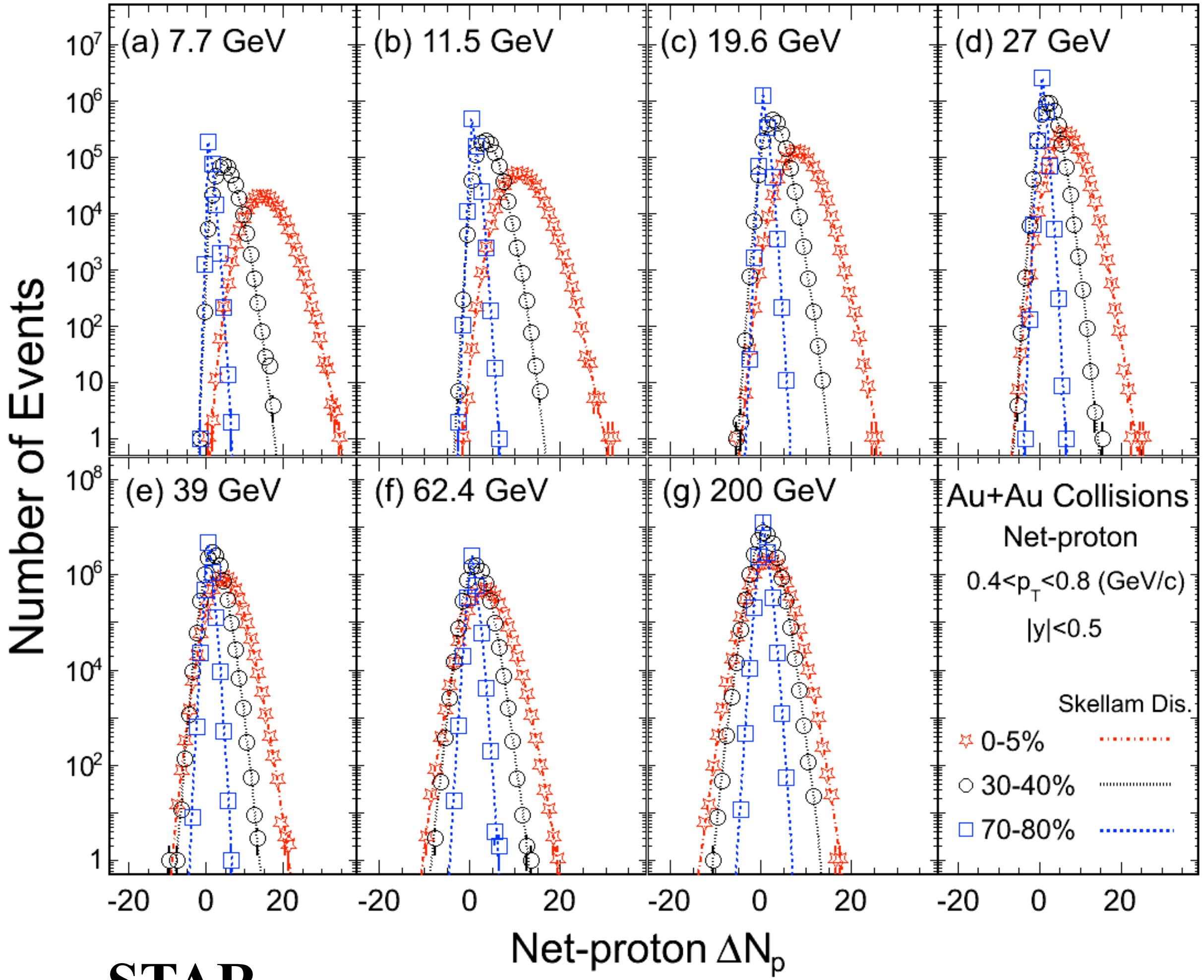
Skewness **Kurtosis**

HRG (non-interacting hadrons) + Boltzmann approx.

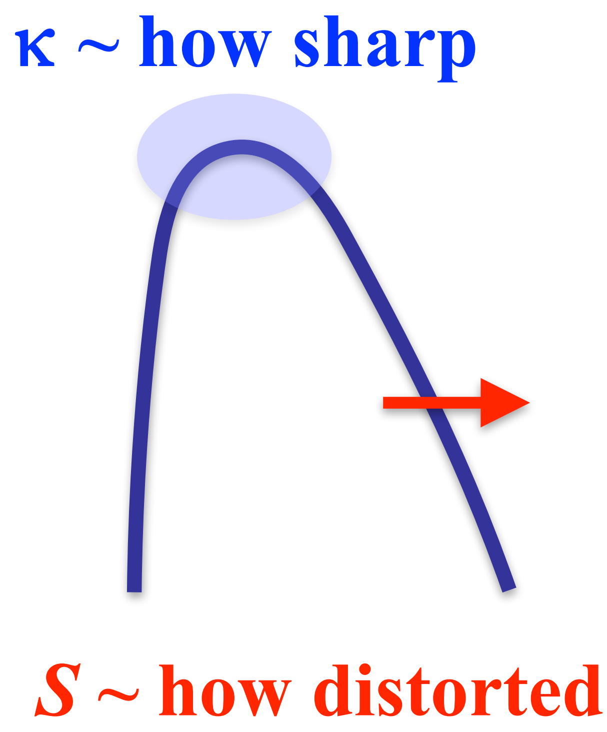
$$S\sigma = \tanh(\mu_{\mathrm{B}}/T)\,, \qquad \kappa\sigma^2 = 1$$

Karsch-Redlich (2011)

Observables for Criticality

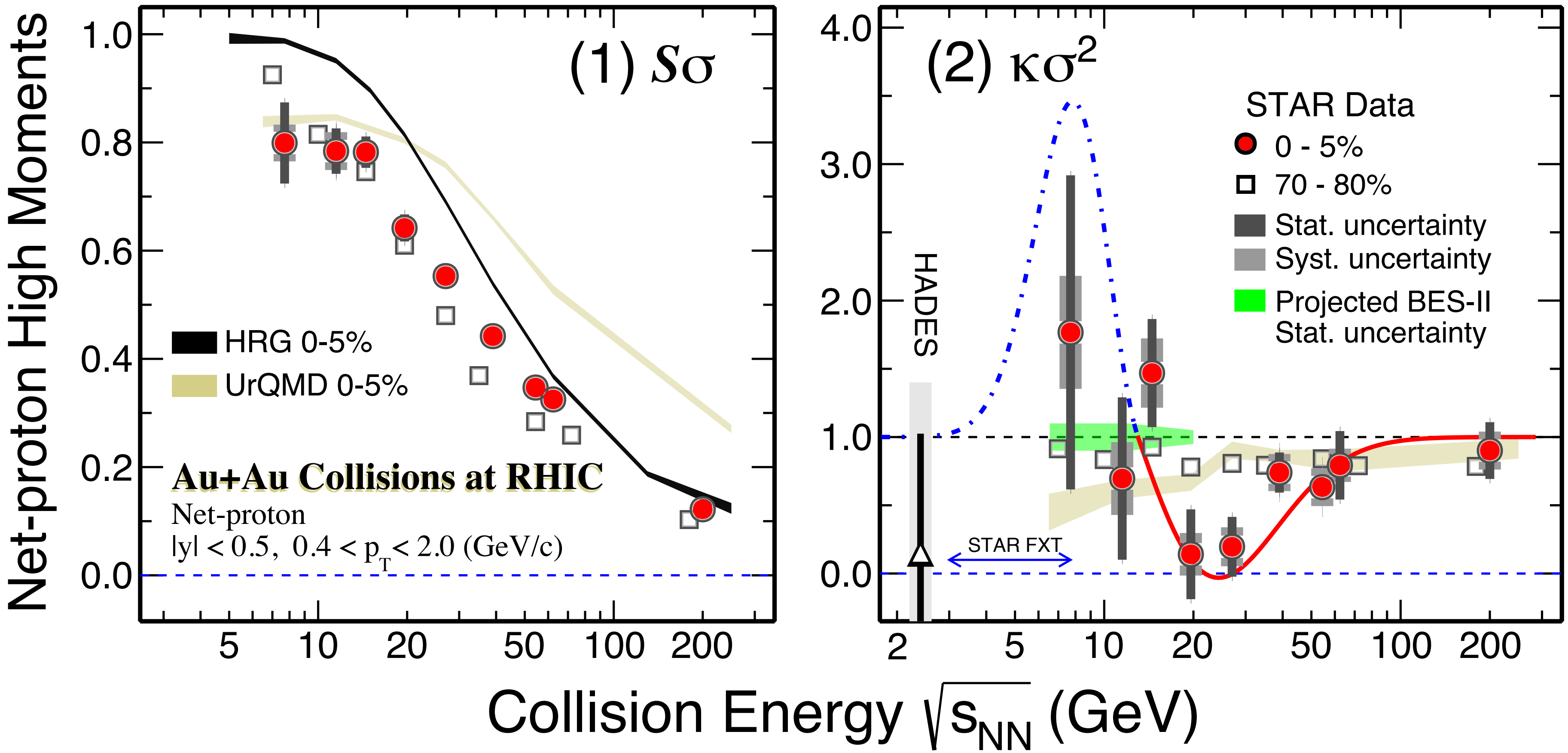


STAR

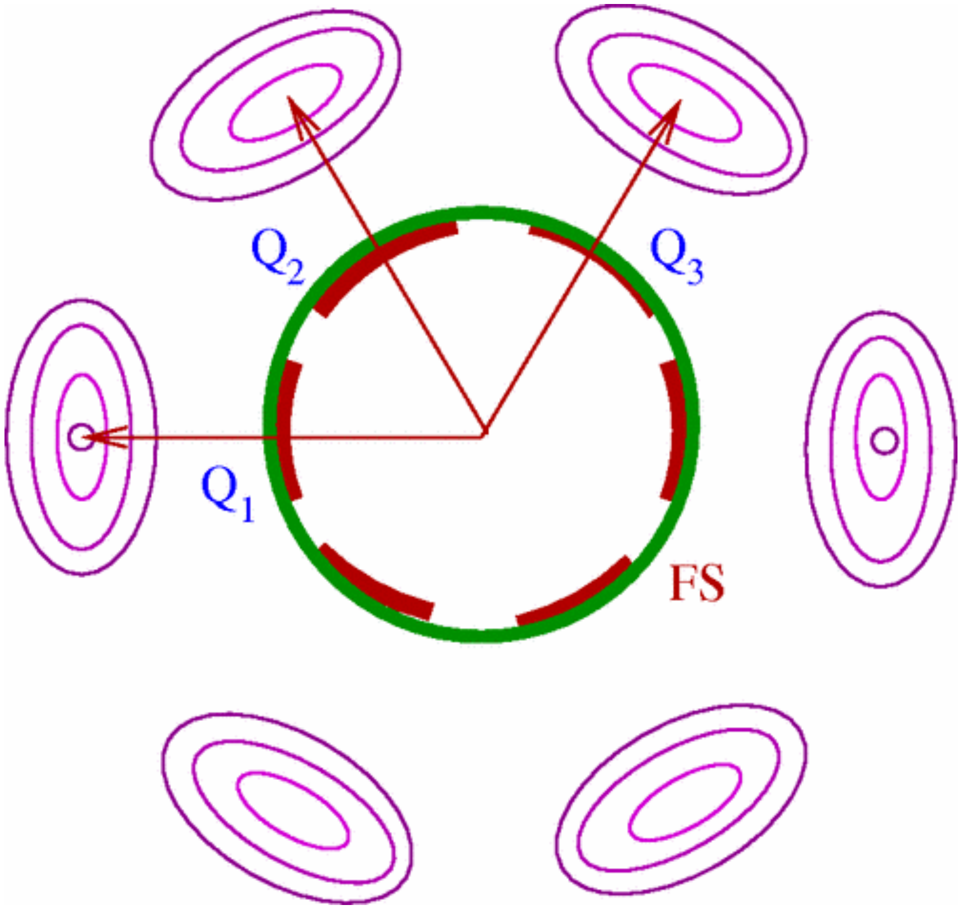


Observables for Criticality

QCD Critical Point discovered???



Inhomogeneity



High Density
(Large Fermi Sphere)

↓

Pseudo 1 Dimensional

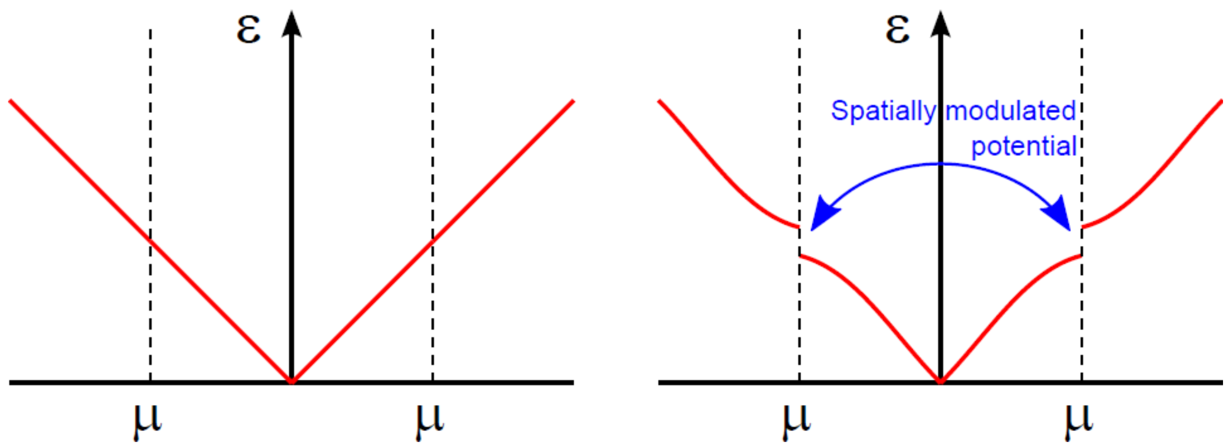
↓

Exotic Phases

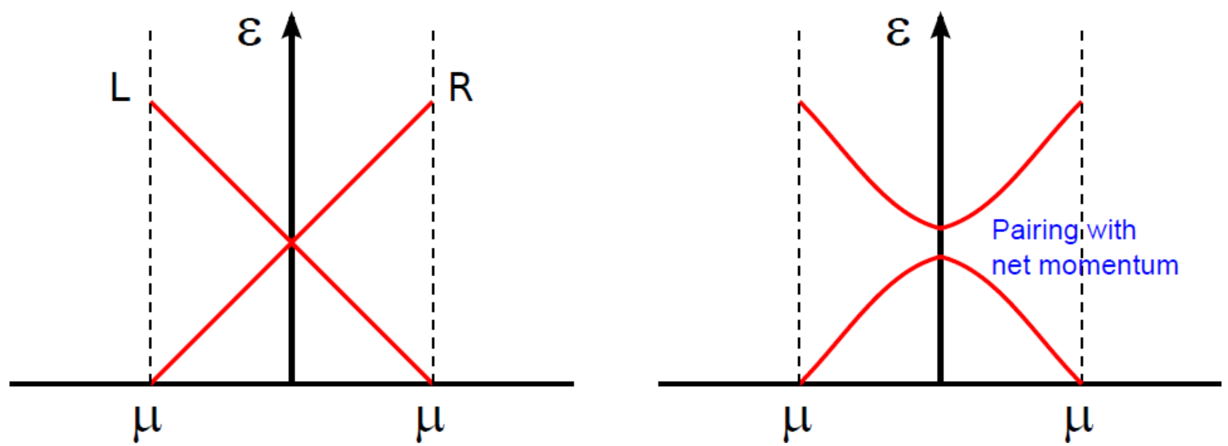
Kojo et al.

Inhomogeneity

Peierls Instability (Gross-Neveu model)



Overhauser Instability (Chiral Gross-Neveu model)



Inhomogeneity

Dirac Lagrangian in (1+1) D

$$\mathcal{L} = \bar{\psi}\big[(\partial_4+\mu)\gamma^4+\partial_3\gamma^3\big]\psi \qquad \psi = e^{-\mu\gamma^3\gamma^4 x_3}\psi'$$

$$= \bar{\psi}'\big(\partial_4\gamma^4+\partial_3\gamma^3\big)\psi' \qquad \bar{\psi} = \bar{\psi}' e^{-\mu\gamma^3\gamma^4 x_3}$$

Finite-density 1D theory = Zero-density 1D theory

IF $\langle\bar{\psi}'\psi'\rangle \neq 0$ **homogeneously, then….**

$$\langle\bar{\psi}\psi\rangle = \langle\bar{\psi}'\psi'\rangle\cos(2\mu x_3)$$

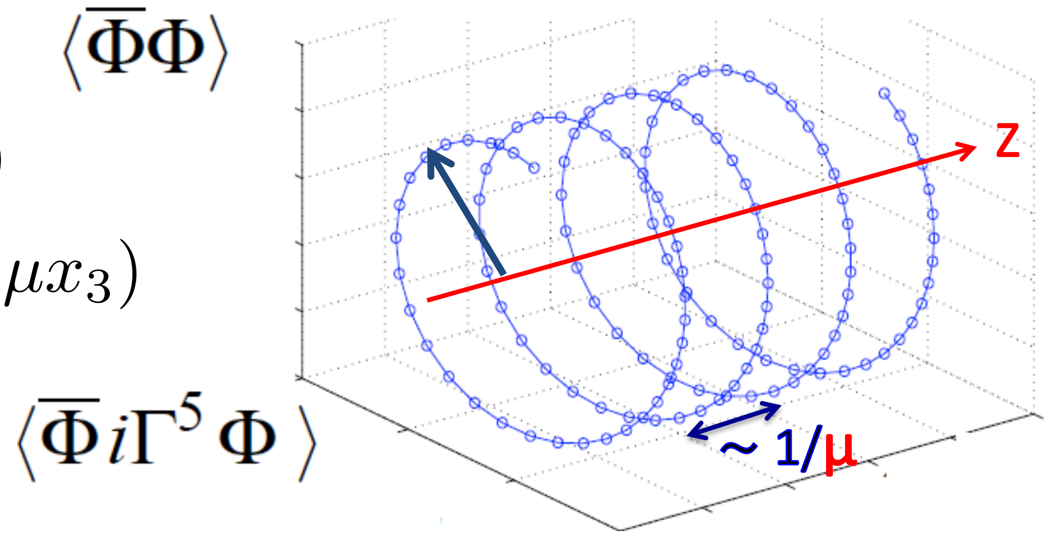
$$\langle\bar{\psi}\gamma^3\gamma^4\psi\rangle = \langle\bar{\psi}'\psi'\rangle\sin(2\mu x_3)$$

original condensates are helically inhomogeneous.

Inhomogeneity

Dirac Lagrangian in (1+1) D

$$\langle\bar{\psi}\psi\rangle = \langle\bar{\psi}'\psi'\rangle\cos(2\mu x_3)$$
$$\langle\bar{\psi}\gamma^3\gamma^4\psi\rangle = \langle\bar{\psi}'\psi'\rangle\sin(2\mu x_3)$$



This structure is called the Chiral Spirals.

There are two puzzles… however…

From where the density comes? Is this stable??

Inhomogeneity

Puzzle #1

$$\mathcal{L} = \bar{\psi}\big[(\partial_4 + \mu)\gamma^4 + \partial_3\gamma^3\big]\psi$$
$$= \bar{\psi}'\big(\partial_4\gamma^4 + \partial_3\gamma^3\big)\psi'$$

No μ any more

If μ dependence is completely gone, the density is ALWAYS zero??

$$\psi = e^{-\mu\gamma^3\gamma^4 x_3}\psi'$$
$$\bar{\psi} = \bar{\psi}' e^{-\mu\gamma^3\gamma^4 x_3}$$

This is a phase translation, shifting a momentum depending on the chirality!

Suppose that the theory has a UV cutoff… then…

Inhomogeneity

Integration Analytically Done

$$\Omega/V = -\int_{-\Lambda+\mu}^{\Lambda-\mu} \frac{dp}{2\pi}\frac{|\varepsilon(p)|}{2} - \int_{-\Lambda-\mu}^{\Lambda+\mu} \frac{dp}{2\pi}\frac{|\varepsilon(p)|}{2}$$

Right-handed Dispersion **Left-handed Dispersion**

$$= \Omega(\mu=0)/V - \frac{\mu^2}{2\pi}$$

No mass dependence?

$$n = -\frac{\partial}{\partial\mu}\frac{\Omega}{V} = \frac{\mu}{\pi}$$

Strangely, density is mass blind?

Inhomogeneity

Puzzle #2

What if there is no chiral spiral at all…

$$\Omega/V = \Omega(\mu=0)/V + \left(-\frac{p_F\mu}{2\pi} + \frac{M^2}{2\pi}\ln\left|\frac{p_F+\mu}{M}\right|\right)\underline{\theta(\mu-M)}$$

$$\delta\Omega/V = -\int_0^\mu d\mu\, n(\mu)$$

$$n = \frac{p_F}{\pi}\theta(\mu-M)$$

The spiral phase and the non-spiral phase, which is favored?

Inhomogeneity

Energy (or Density) Comparison

Density is larger in the spiral phase and the energy is lower.

Chiral Spiral **Homogeneous Phase**

$$n = \frac{\mu}{\pi} \quad \text{vs.} \quad n = \frac{p_F}{\pi}\theta(\mu - M)$$

Chiral spiral always wins!

Why is the density mass independent?

Inhomogeneity

Axial Anomaly in (1+1)D Theory

$$\partial_\mu j_A^\mu = -\frac{e}{2\pi} F_{01}$$

In (1+1) D the electric field *E* is the topological charge.

Dirac matrices satisfy: $\gamma^\mu \gamma^5 = -\epsilon^{\mu\nu}\gamma_\nu$

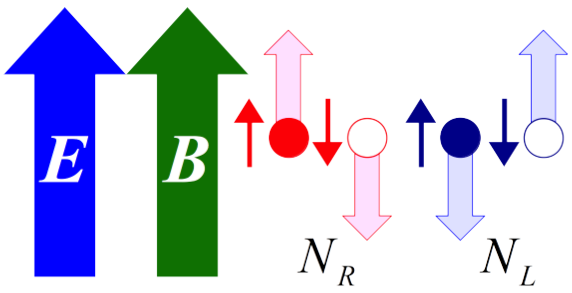
$$n = j_V^0 = j_A^1 = -\frac{e}{2\pi}\int dx\, F_{01} = \frac{e}{\pi}A^0 = \frac{\mu}{\pi}$$

Assuming that the mass is only dynamical.

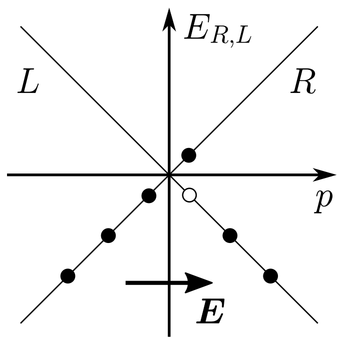
$U(1)_A$ Breaking by Anomaly

Chiral Anomaly in QED

Nielsen-Ninomiya (1983)



$$p_F^{(R)} = +qEt \quad p_F^{(L)} = -qEt$$



Chirality changing rate

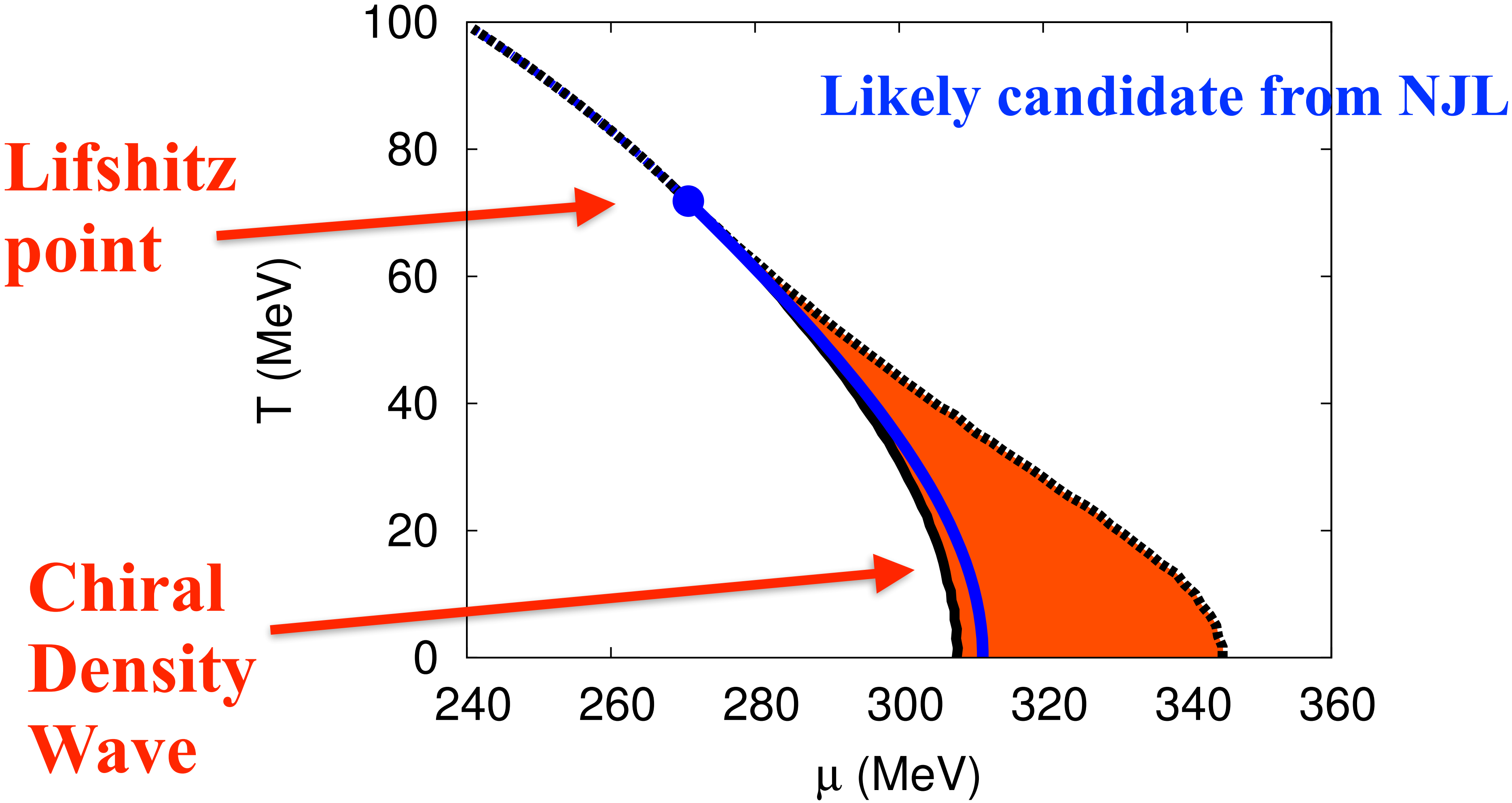
$$\frac{dN_{R,L}}{dz\,d^2x} = \frac{p_F^{(R,L)}}{2\pi} \cdot \frac{qB}{2\pi}$$

$$\frac{dN_5}{dtd^3x} = \frac{q^2}{2\pi^2}\boldsymbol{E}\cdot\boldsymbol{B} \quad \Rightarrow \quad \partial_\mu j_A^\mu = -\frac{q^2}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Inhomogeneity vs. Criticality

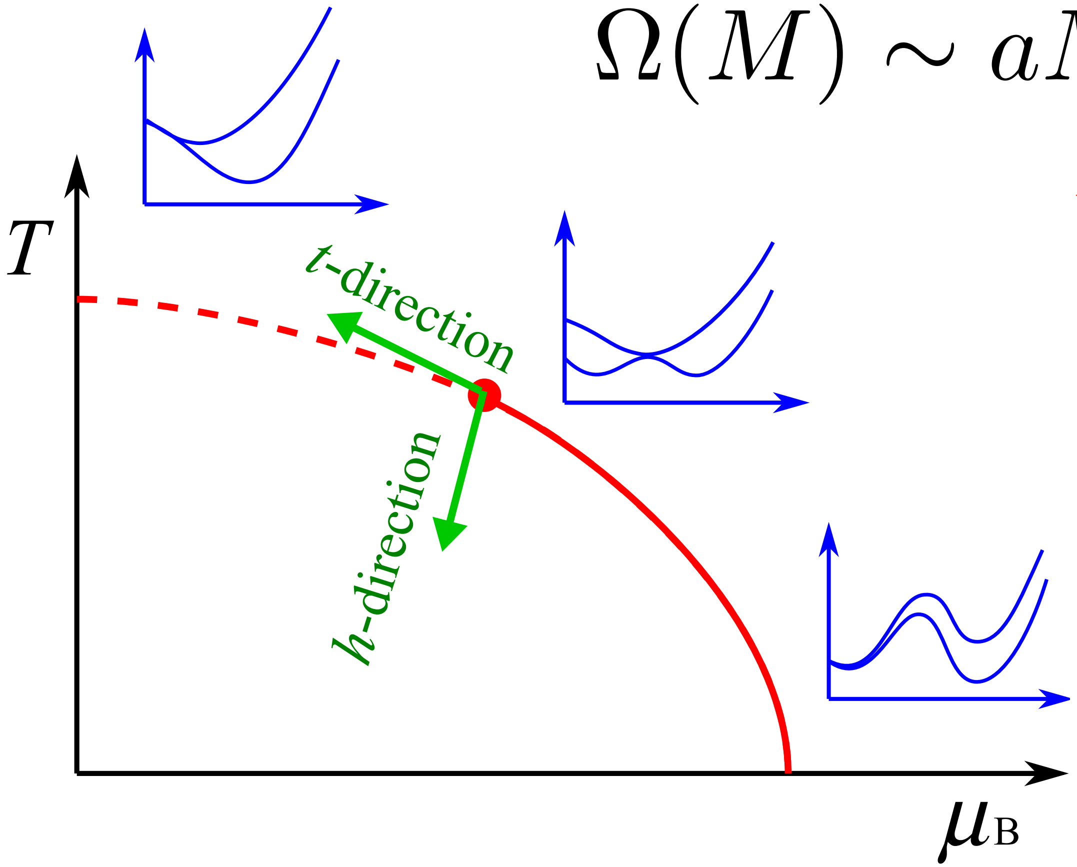
(3+1) D Theory?

Review: Buballa-Carignano (2014)



Inhomogeneity vs. Criticality

Possible relation to the QCD Critical Point



$$\Omega(M) \sim aM^2 + bM^4 + \underline{cM^6}$$

Assumed to be positive for stability

$a = 0$: 2-nd order

$a = b = 0$: Tricritical

Inhomogeneity vs. Criticality

In the massless NJL model: **Nickel (2008)**

$$\Omega(M) \sim aM^2 + bM^4 + cM^6$$

Inhomogeneous condensates induce $\partial M \neq 0$

$$\Omega(M, q) \to aM^2 + bM^4 + cM^6 + dq^2 M^2 + \cdots$$

Spatial inhomogeneity occurs for *d* < 0 (Lifshitz point)

It happens to result in $b \propto d$ **!**

→ **Lifshitz point and QCD CP coincide!**

Inhomogeneity vs. Criticality

Fluctuation effects

It is known by now that phonon fluctuations wash out the inhomogeneous condensates but a remnant remains = Quasi Long-Range Order

Hidaka-Kamikado-Kanazawa-Noumi (2015)

Chiral condensate vanishes with IR divergence at finite *T*, but the power-law correlation persists, indicating that higher-order condensates survive…

$$\langle M(x) \rangle = 0 \qquad \langle M^2(x) \rangle \neq 0 \qquad (\mathbb{Z}_2)_R \times (\mathbb{Z}_2)_L$$

This is called "Stern Phase"

Further Higher Density

Stern Phases

$$\langle\bar{\psi}\psi\rangle = \langle\bar{\psi}_{\mathrm{R}}\psi_{\mathrm{L}} + \bar{\psi}_{\mathrm{L}}\psi_{\mathrm{R}}\rangle$$

Is this a unique way to break chiral symmetry?

Stern proposed the following:

$$\left\langle \bar{\psi}\frac{\lambda^a}{2}(1-\gamma_5)\psi \cdot \bar{\psi}\frac{\lambda^a}{2}(1+\gamma_5)\psi \right\rangle = \left\langle \bar{\psi}_{\mathrm{R}}\lambda^a\psi_{\mathrm{L}} \cdot \bar{\psi}_{\mathrm{L}}\lambda^a\psi_{\mathrm{R}} \right\rangle$$

However, this possibility was immediately falsified from the QCD inequality by Kogan et al.
(pseudo-scalar susceptibility should be the largest)

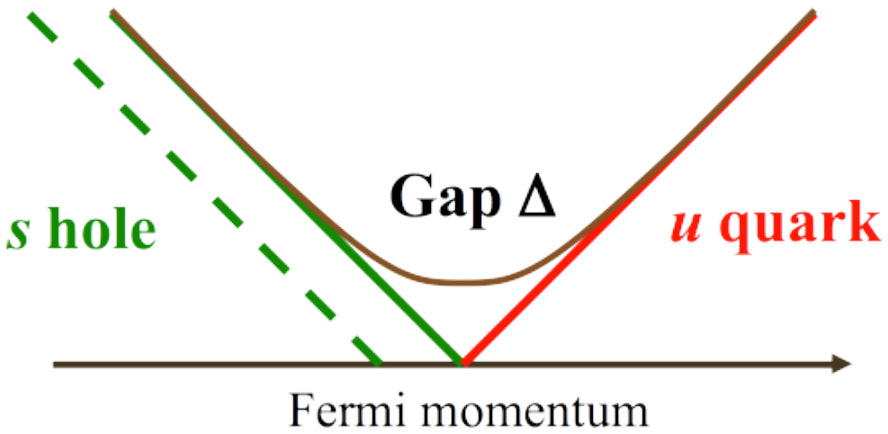
Further Higher Density

QCD inequality breaks down at finite density

→ Color Super Conductivity

Fermi Surface $\mu_q \sim 500\ \mathrm{MeV} \ \to \ \rho \sim 10\rho_0$

Attractive Force $\mathbf{3} \times \mathbf{3} \to \bar{\mathbf{3}}$



$$\sqrt{p_F^2 + m_s^2} = \mu_q$$

$$\to \ p_F \simeq \mu_q - \frac{m_s^2}{2\mu_q}$$

Gap and Fermi surface mismatch are of the same order

Further Higher Density

Color Interaction

$$(t^a)_{ij}(t^a)_{kl} = -\frac{N_c+1}{4N_c}(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}) + \frac{N_c-1}{4N_c}(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})$$

Color Triplet (antisymmetric)
Attractive

Color Sextet (symmetric)
Repulsive

Dominant

Only this channel considered
(**flavor**) (**spin**) (~~**orbital**~~)
should be symmetric

Always mixed with triplet
No new physics brought in
Harmlessly neglected

Further Higher Density

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \otimes \mathbf{6}$$

Quantum numbers and operators			
J^P	Color	Flavor	Operator
0^+	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\bar{\psi}_C \gamma_5 \psi$, $\bar{\psi}_C \gamma_0 \gamma_5 \psi$
1^+	$\bar{\mathbf{3}}$	$\mathbf{6}$	$\bar{\psi}_C \gamma_i \psi$, $\bar{\psi}_C \sigma_{0i} \psi$
0^-	$\bar{\mathbf{3}}$	$\mathbf{6}$	$\bar{\psi}_C \psi$, $\bar{\psi}_C \gamma_0 \psi$
1^-	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\bar{\psi}_C \gamma_i \gamma_5 \psi$, $\bar{\psi}_C \sigma_{ij} \psi$

Further Higher Density

Spin-dependent Part **Breit Interaction**

$$H_{\text{color-spin}} = \alpha_s \sum_{i \neq j} M_{ij} \underbrace{(\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j)}_{\text{color}} \underbrace{(\boldsymbol{s}_i \cdot \boldsymbol{s}_j)}_{\text{spin}}$$

> spin-singlet (antisymmetric) + flavor triplet (antisymmetric)

$$(\boldsymbol{s}_i \cdot \boldsymbol{s}_j)|\mathbf{0}\rangle = -(3/4)|\mathbf{0}\rangle$$

Good Diquark

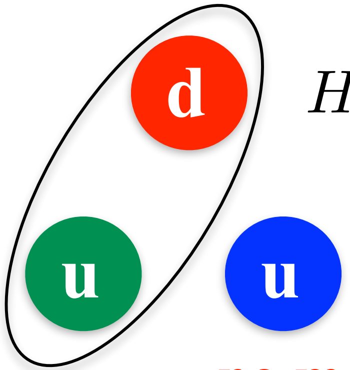
> spin-triplet (symmetric) + flavor sextet (symmetric)

$$(\boldsymbol{s}_i \cdot \boldsymbol{s}_j)|\mathbf{1}\rangle = +(1/4)|\mathbf{1}\rangle$$

Bad Diquark

Further Higher Density

$\boxed{N : S = 1/2}$



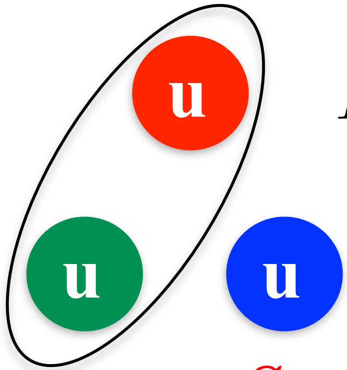
$$H_{\text{color-spin}} = -\frac{3}{4}C$$

$S = 0$

no more s-s int.

$$H_{\text{color-spin}} = -\frac{3}{4}C$$

$\boxed{\Delta : S = 3/2}$



$$H_{\text{color-spin}} = +\frac{1}{4}C$$

$S = 1$

$\boldsymbol{S} \cdot \boldsymbol{s} = 1/2$

$$H_{\text{color-spin}} = +\frac{3}{4}C$$

$$m_{\text{bad}} - m_{\text{good}} \approx \frac{2}{3}(M_\Delta - M_N)$$ confirmed in lattice QCD

Further Higher Density

Diquark Condensate (NOT GAUGE INV!)

$$\Delta_{\alpha i} \;\propto\; \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk}\langle\bar{\psi}_{\beta j} i\gamma^5 C \bar{\psi}^T_{\gamma k}\rangle$$

Color-Flavor Locking Ansatz

Δ_{ud}	Δ_{ds}	Δ_{su}
up-down	**down-strange**	**strange-up**
up-down	**down-strange**	**strange-up**

Further Higher Density

Gauge Invariant Characterization

$$(\varphi_{\mathrm{L}})_{\alpha i} \sim \epsilon_{\alpha\beta\gamma}\,\epsilon_{ijk}(\psi_{\mathrm{L}})^T_{\beta j}C(\psi_{\mathrm{L}})_{\gamma k}$$

$$(\varphi_{\mathrm{R}})_{\alpha i} \sim \epsilon_{\alpha\beta\gamma}\,\epsilon_{ijk}(\psi_{\mathrm{R}})^T_{\beta j}C(\psi_{\mathrm{R}})_{\gamma k}$$

Stern phase order parameter $\langle\varphi_{\mathrm{R}}^\dagger\varphi_{\mathrm{L}}\rangle + \langle\varphi_{\mathrm{L}}^\dagger\varphi_{\mathrm{R}}\rangle$

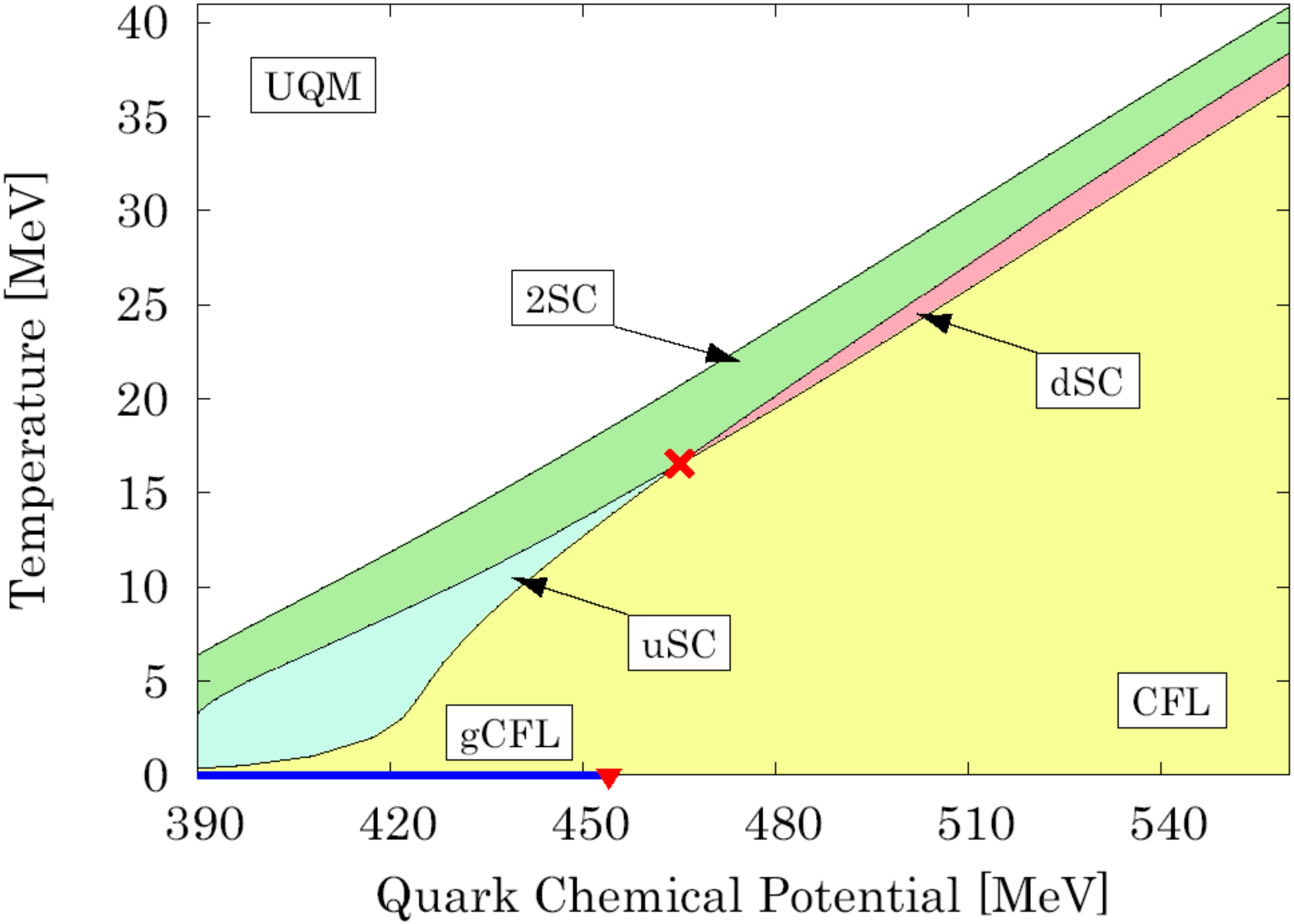
$$(\mathbb{Z}_2)_R \times (\mathbb{Z}_2)_L$$

cf. Color superconductor is not topological unlike QED because the Cooper pair is (anti) triplet charged. Color sextet condensates would change the story…

Further Higher Density

$\Delta_{ud}, \Delta_{ds}, \Delta_{su} \neq 0$	CFL Phase
$\Delta_{ds} = 0, \Delta_{su}, \Delta_{ud} \neq 0$	uSC Phase
$\Delta_{su} = 0, \Delta_{ds}, \Delta_{ud} \neq 0$	dSC Phase
$\Delta_{ud} = 0, \Delta_{ds}, \Delta_{su} \neq 0$	sSC Phase
$\Delta_{ds} = \Delta_{su} = 0, \Delta_{ud} \neq 0$	2SC Phase
$\Delta_{su} = \Delta_{ud} = 0, \Delta_{ds} \neq 0$	2SCds Phase
$\Delta_{ud} = \Delta_{ds} = 0, \Delta_{su} \neq 0$	2SCsu Phase

Further Higher Density

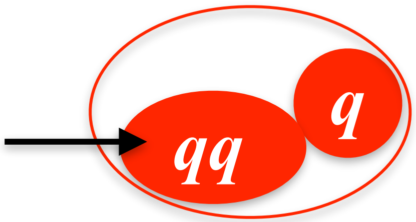


Further Higher Density

Matching of Symmetry Breaking Patterns

Baryons: 8+1 (low-lying) **Quarks: 3color × 3flavor = 9**

Flavor Triplet



Condensate

Excitation

$\langle ud \rangle \ \langle ds \rangle \ \langle su \rangle$ **Diquark condensates break chiral symmetry in the same way as the hadronic phase.**

Diquarks realize duality between baryons and quarks!

Dense QCD may have more stringent duality than crossover at high *T*…

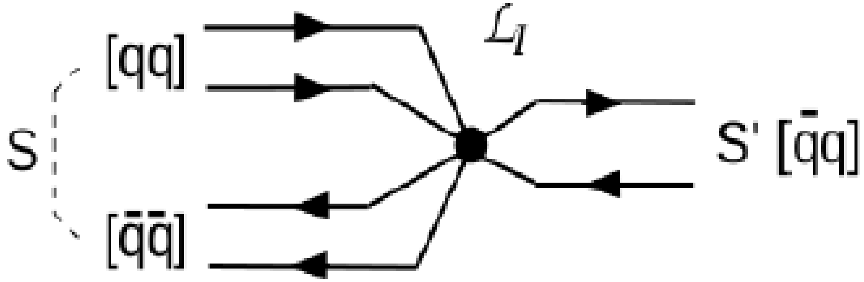
Further Higher Density

U(1)$_A$ breaking interaction

$$\det \bar{\psi}_{Lj}\psi_{Ri} + \det \bar{\psi}_{Rj}\psi_{Li}$$

$$\to \det R_{im}\bar{\psi}_{Ln}\psi_{Rm}L^{\dagger}_{nj} + \det L_{im}\bar{\psi}_{Rn}\psi_{Lm}R^{\dagger}_{nj}$$

For N_f =3, this is a six point interaction:

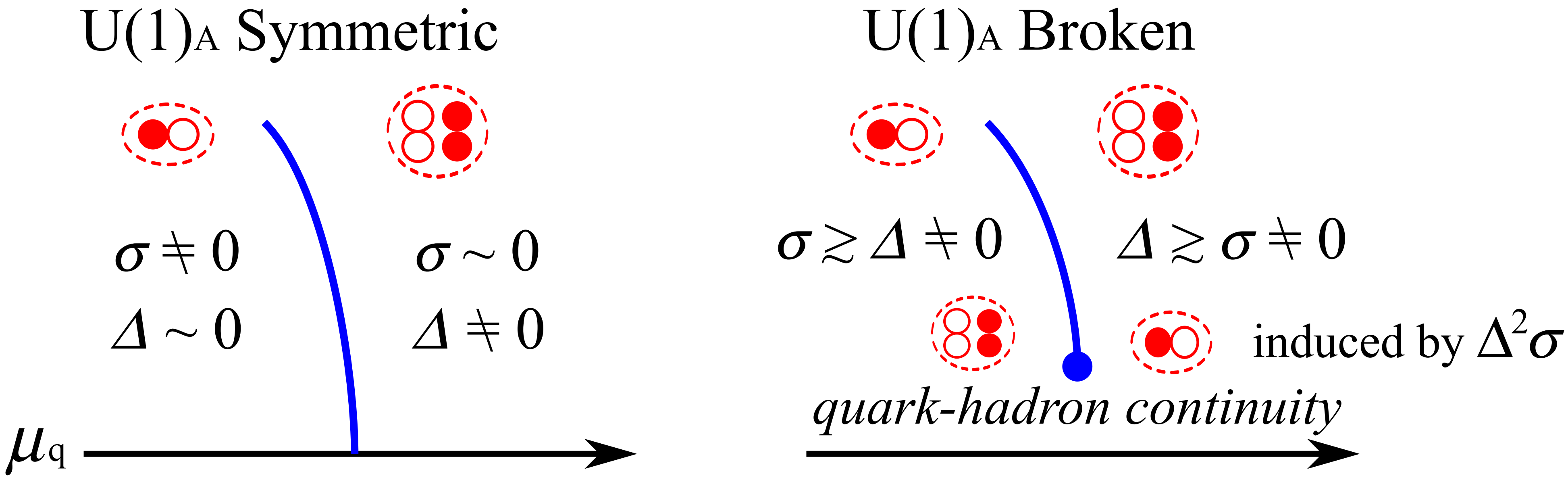


’t Hooft-Isidori-Maiani-
-Polosa-Riquer (2008)

$$\sim \langle\psi\psi\rangle\langle\bar{\psi}\bar{\psi}\rangle\langle\bar{\psi}\psi\rangle$$

Anomaly induces a mixing between mesons and diquarks

Further Higher Density



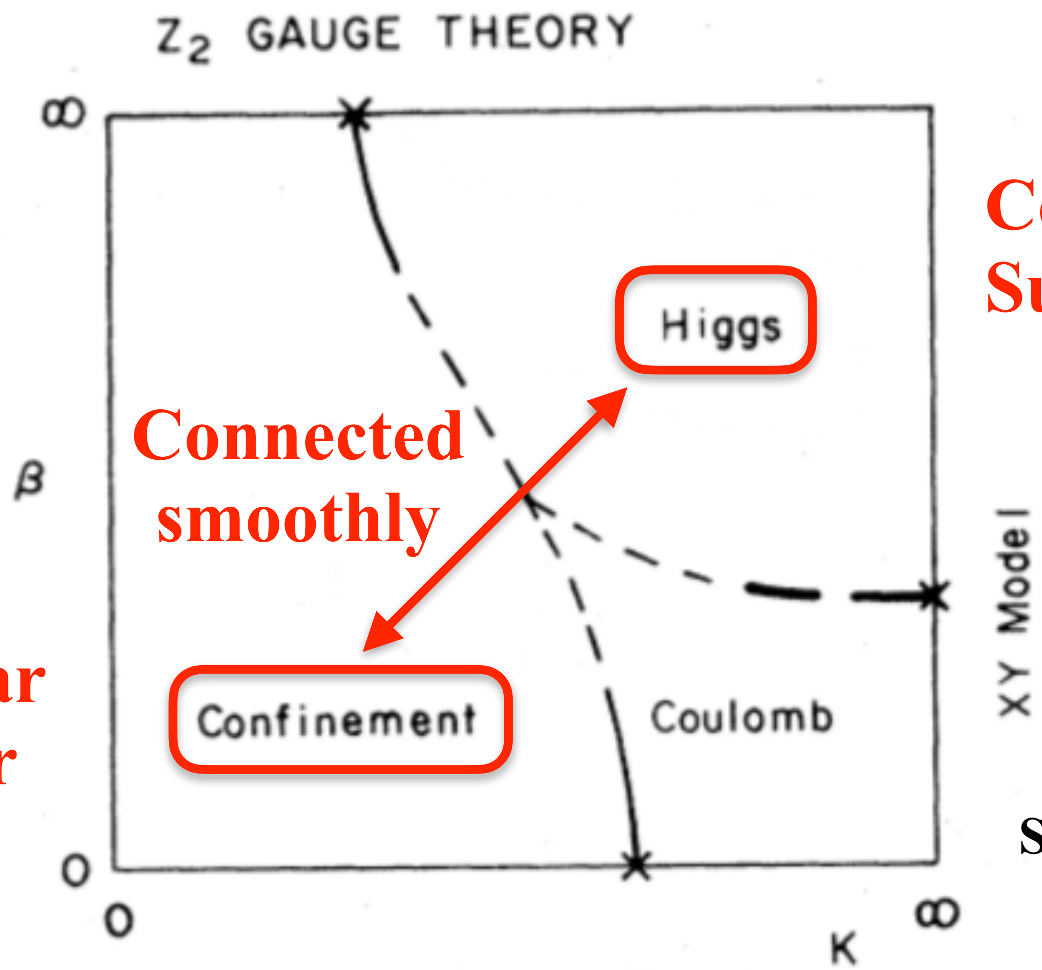
No phase transition because $\sim \Delta\Delta^* M$

Hatsuda-Tachibana-
-Yamamoto-Baym (2006)

$U(1)_A$ breaking interaction

Further Higher Density

Fradkin-Shenker (1979)



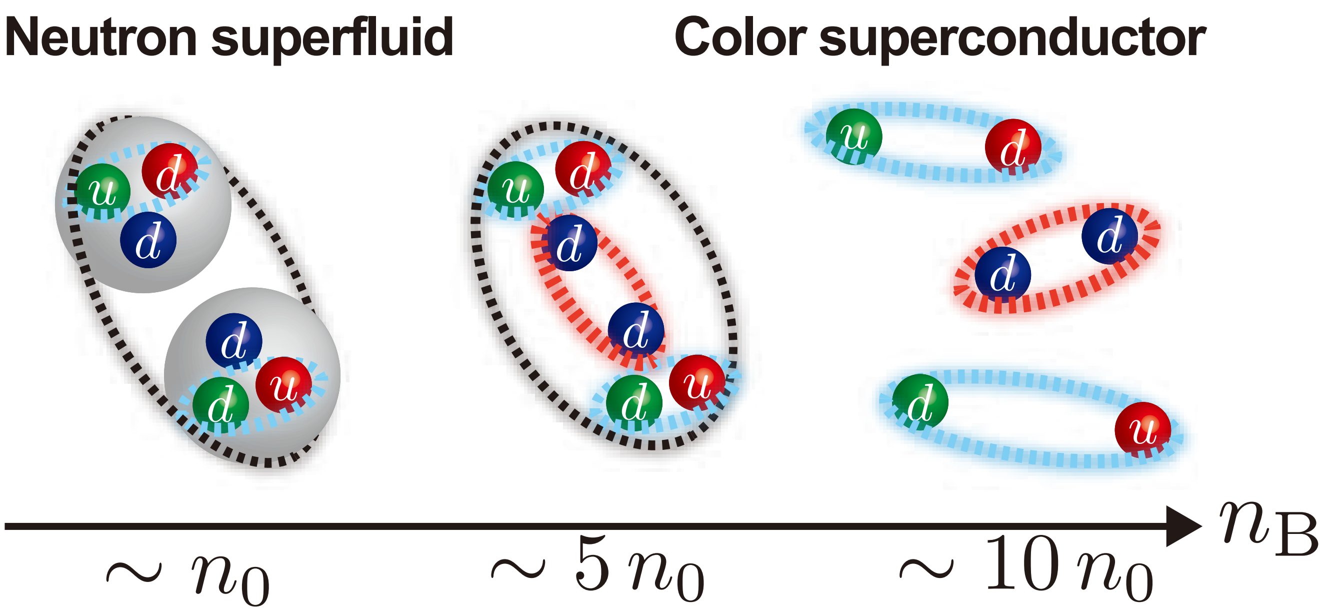
Color Superconductor

Nuclear Matter

Schaefer-Wilczek, PRL (1999)

Further Higher Density

Fujimoto-Fukushima-Weise (2020)



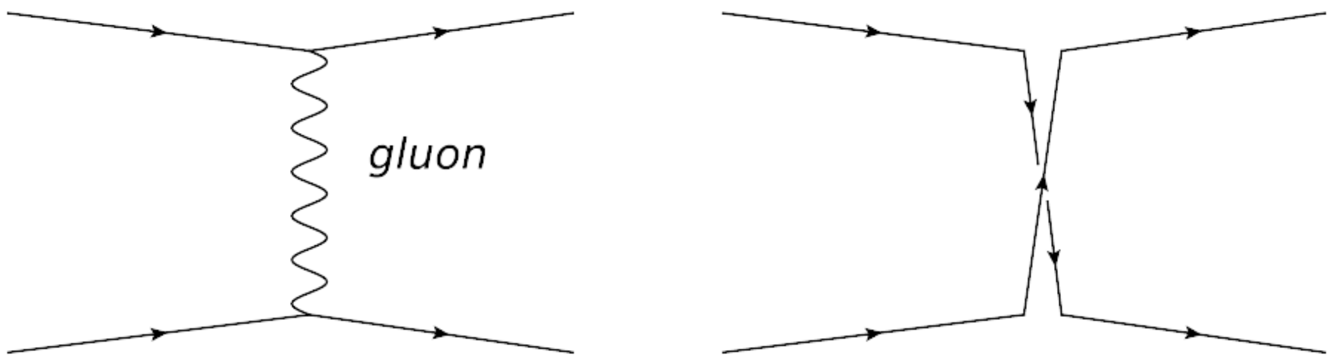
No change in global symmetry
No need to have a phase transition

Three is nearly Infinity

Quarks **spin-1/2 (fermions)** **6 flavors** N_c **colors**

(transform in the SU(N_c) fundamental rep.)

quark **red / green / blue**

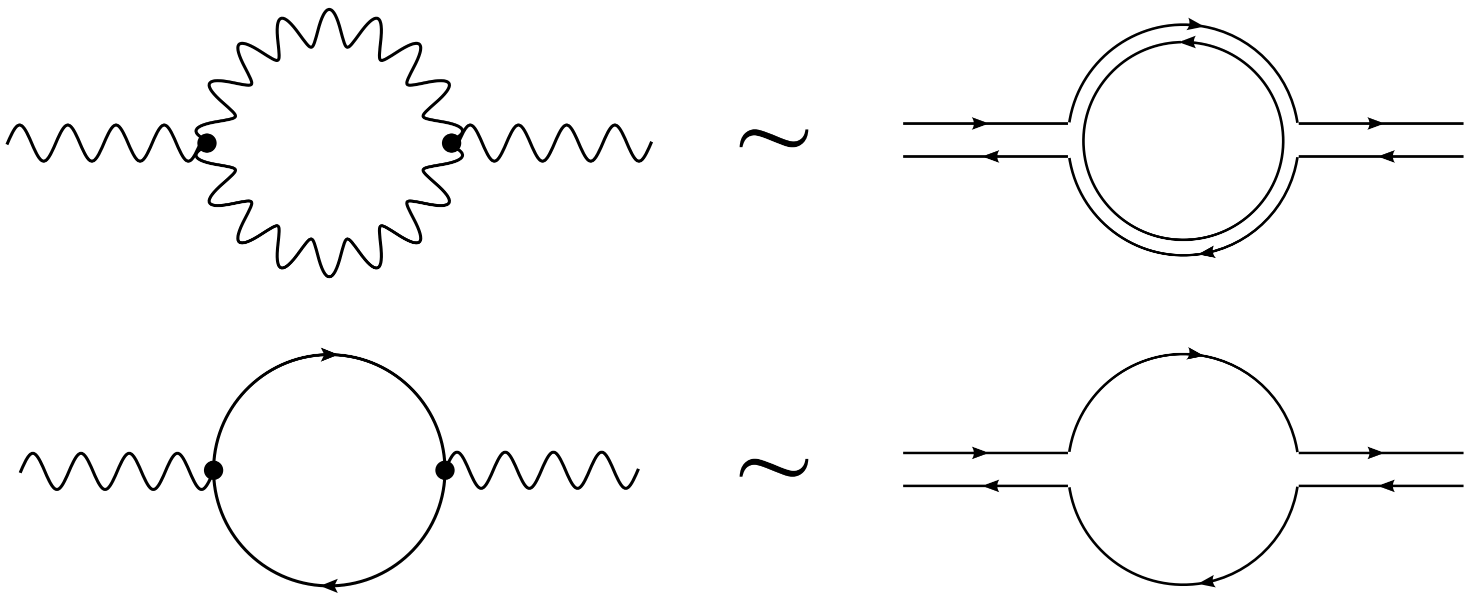


Gluons **spin-1 (bosons)** $N_c^2 - 1$ **colors**

$r\bar{r}$ $r\bar{g}$ $r\bar{b}$ $g\bar{r}$ $g\bar{g}$ $g\bar{b}$ $b\bar{r}$ $b\bar{g}$ $b\bar{b}$ - $(r\bar{r}+g\bar{g}+b\bar{b})$

Three is nearly Infinity

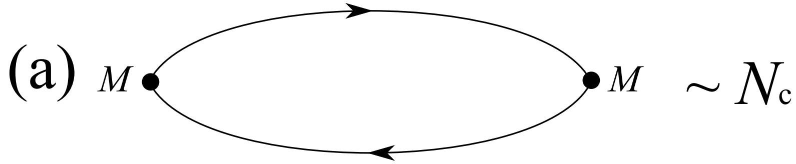
Theoretical Preparation: Large-Nc Counting

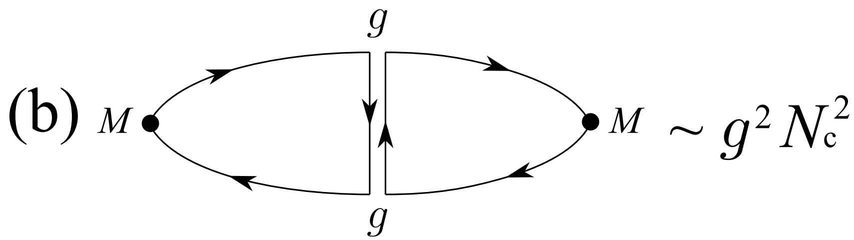


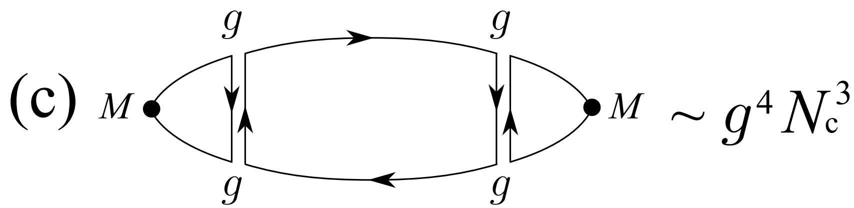
Three is nearly Infinity

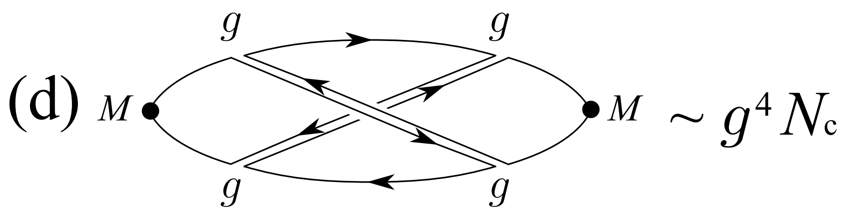
Theoretical Preparation: Large-Nc Counting

$$g^2 \sim 1/N_c$$







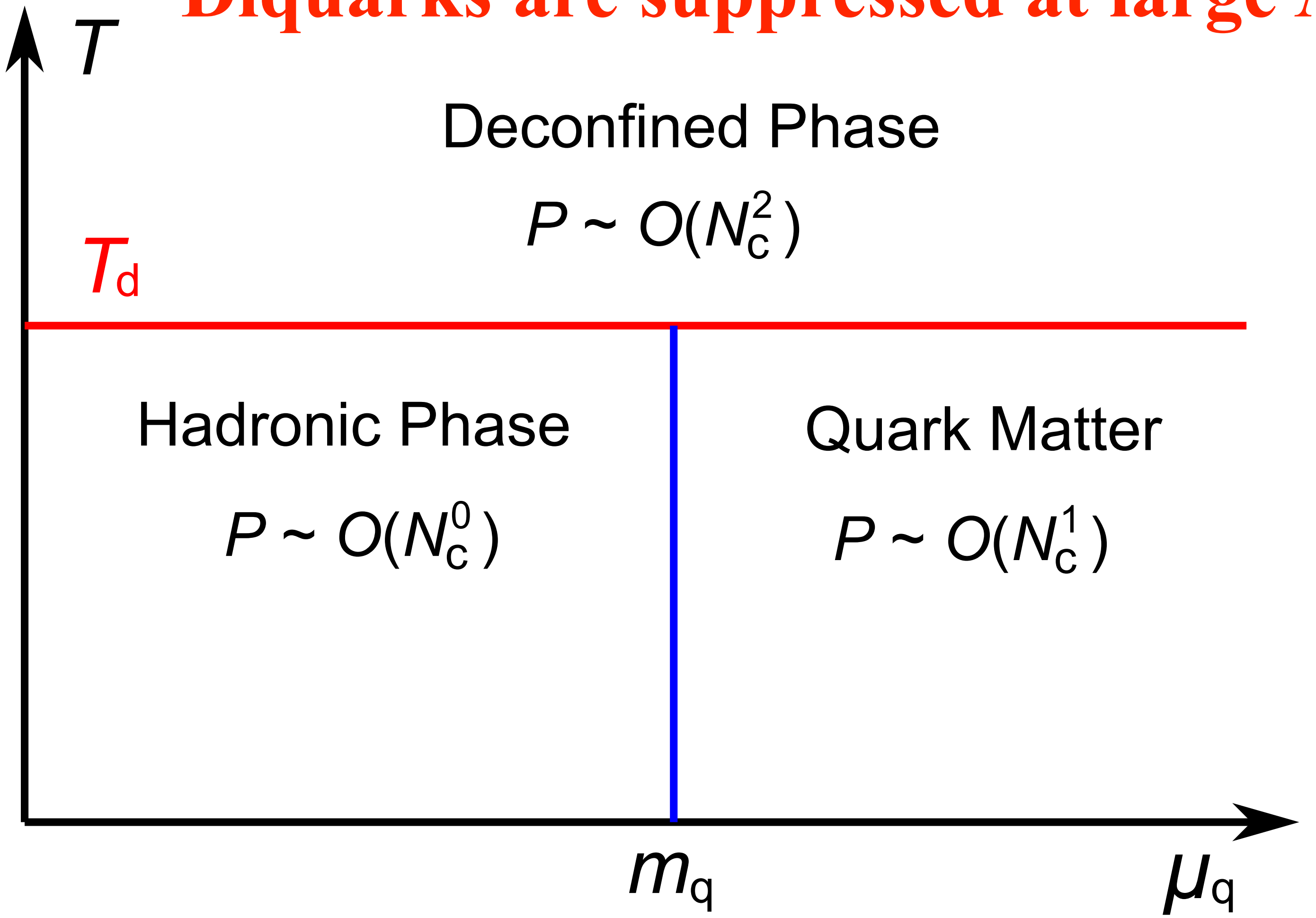


Non-planar diagrams and quark loops suppressed!

Three is nearly Infinity

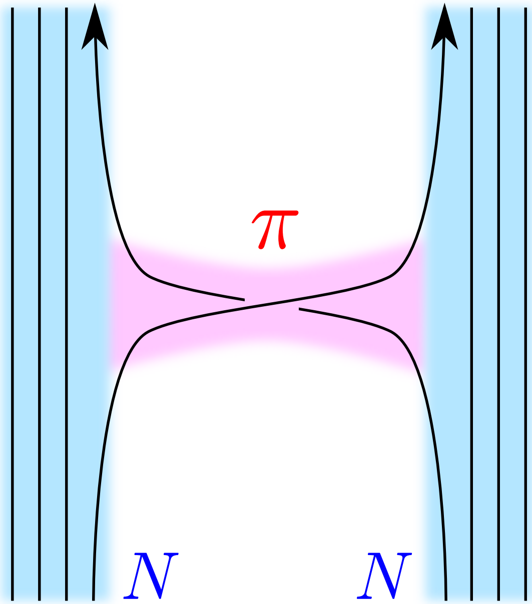
Strongly Interacting Baryons ~ Free Quarks

Diquarks are suppressed at large *N*c.



Three is nearly Infinity

This is NOT the end of the story!



If there are infinitely many quarks, mesons do not interact, but baryons do interact very strongly!

Pressure of Quark Matter

Kinetic Energy ~ O(N_c)

Pressure of Baryonic Matter

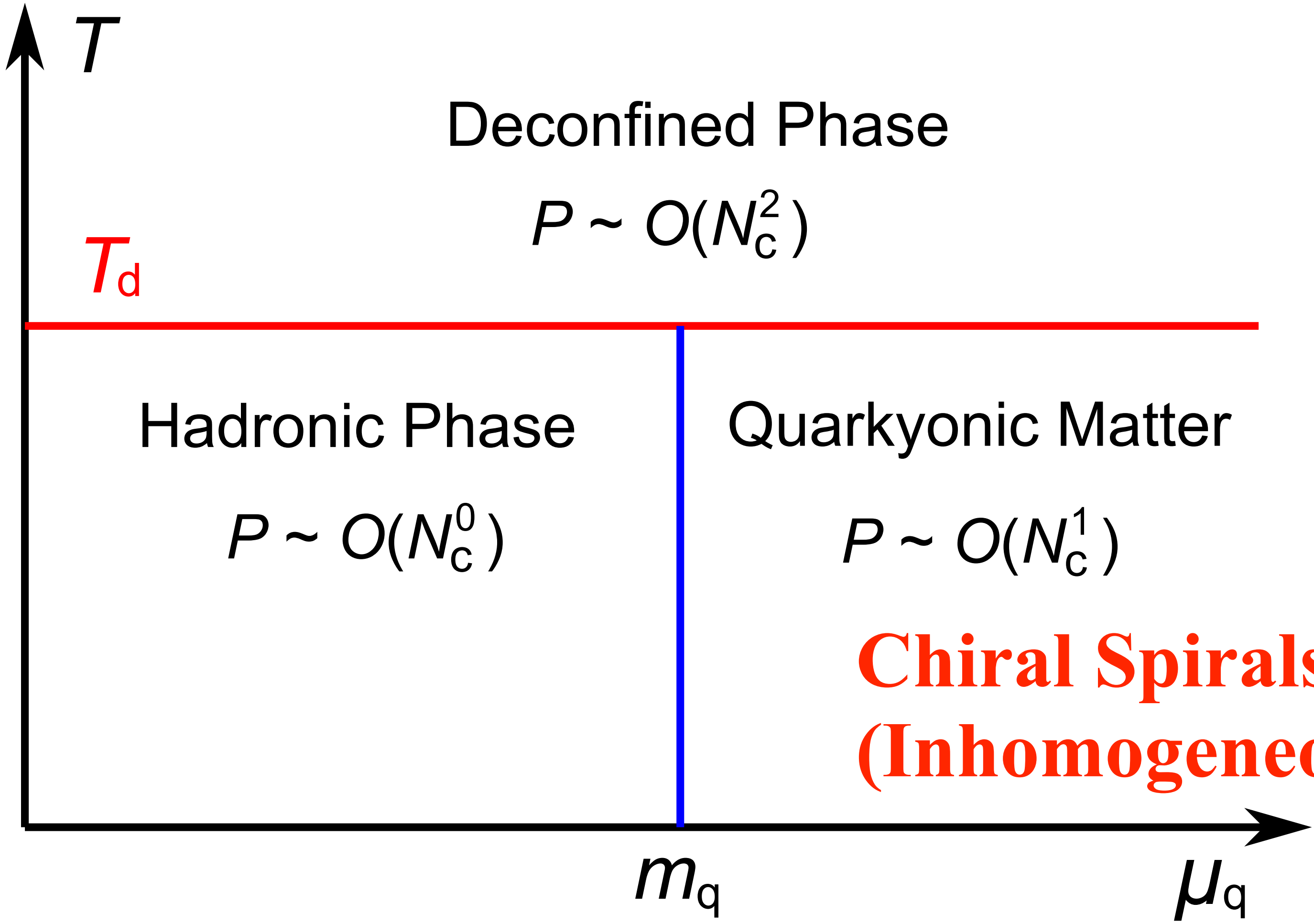
Interaction Energy ~ O(N_c)

Quarkyonic

McLerran-Pisarski (2008)

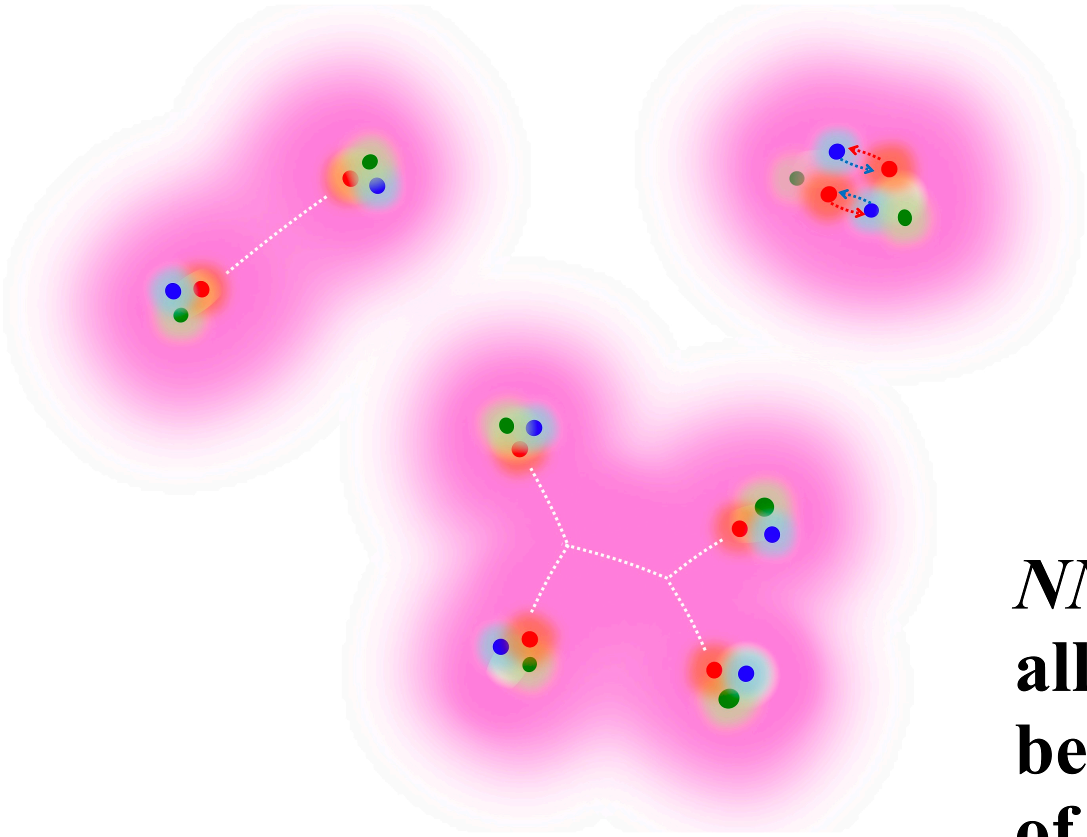
Three is nearly Infinity

Strongly Interacting Baryons ~ Free Quarks



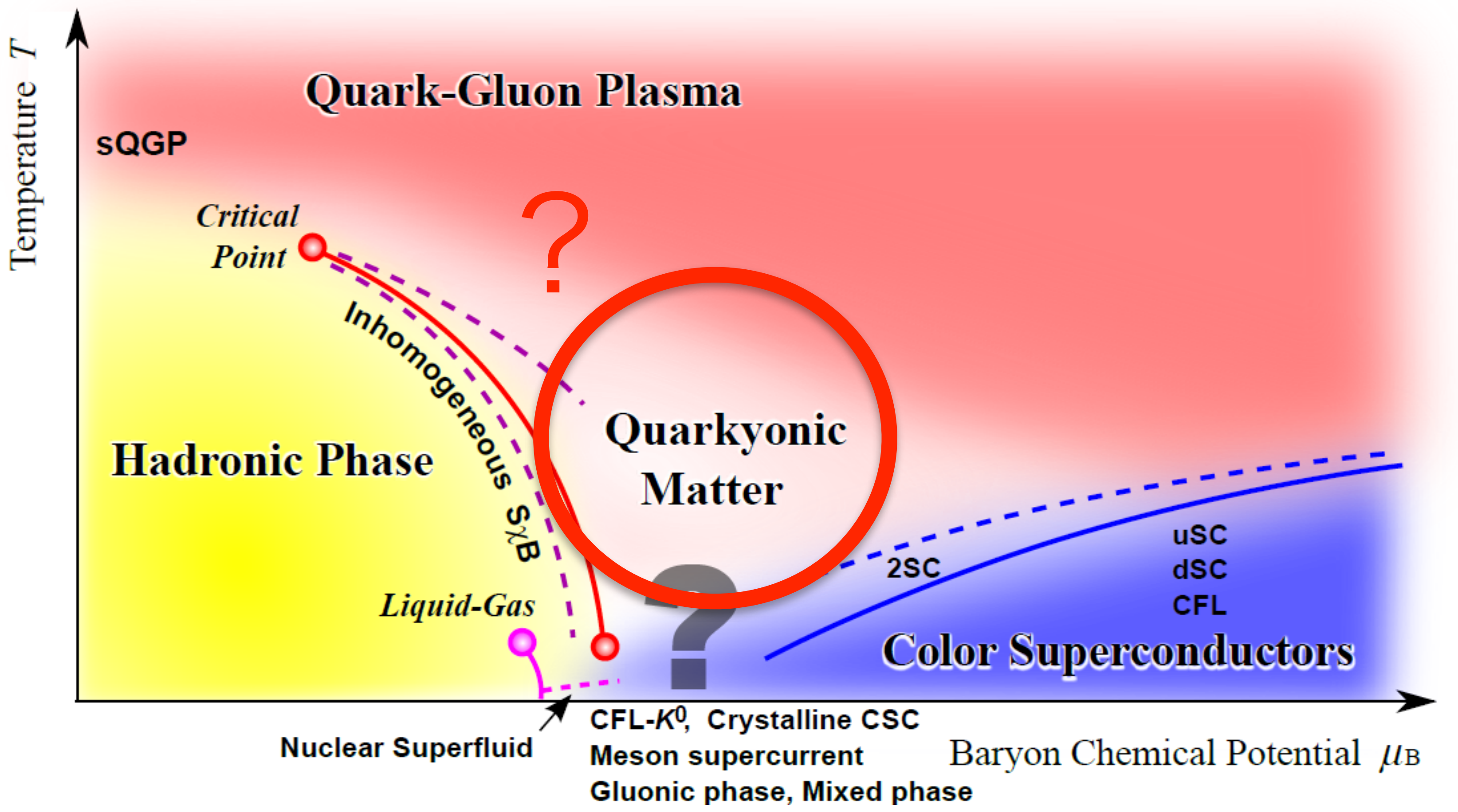
Chiral Spirals (Inhomogeneous!)

Continuous Crossover



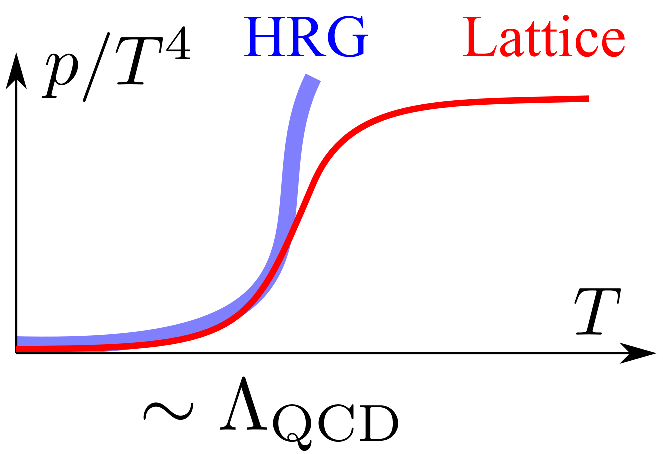
***NN*, *NNN*, *NNNN*, ….**
all many-body interactions
become the same order
of O(N_c) around ~ 2n_0

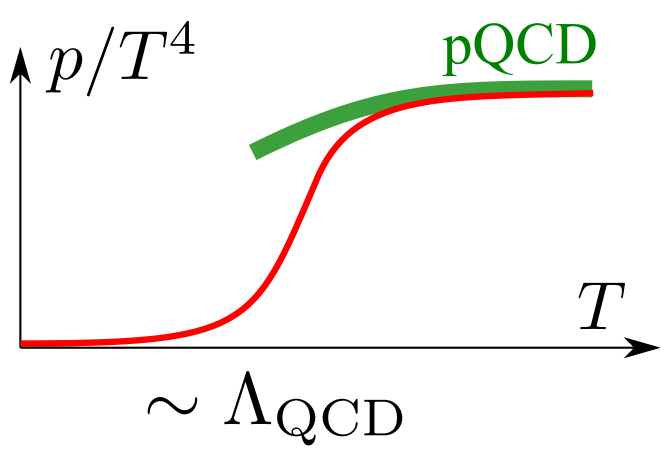
Continuous Crossover



Continuous Crossover

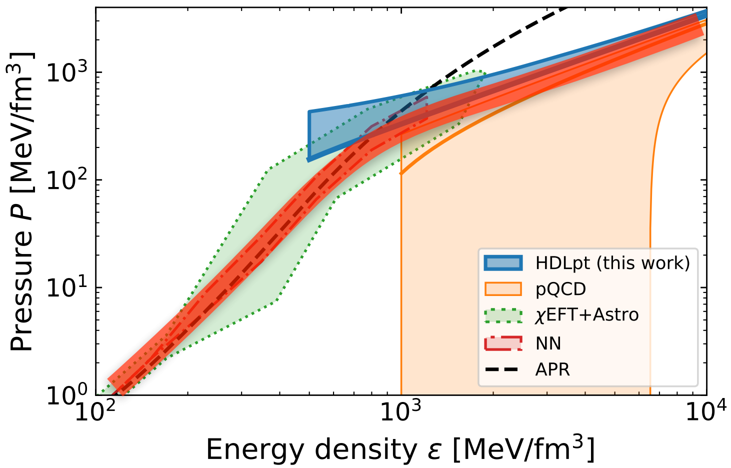
High-T has been understood by HRG + pQCD





High-Density

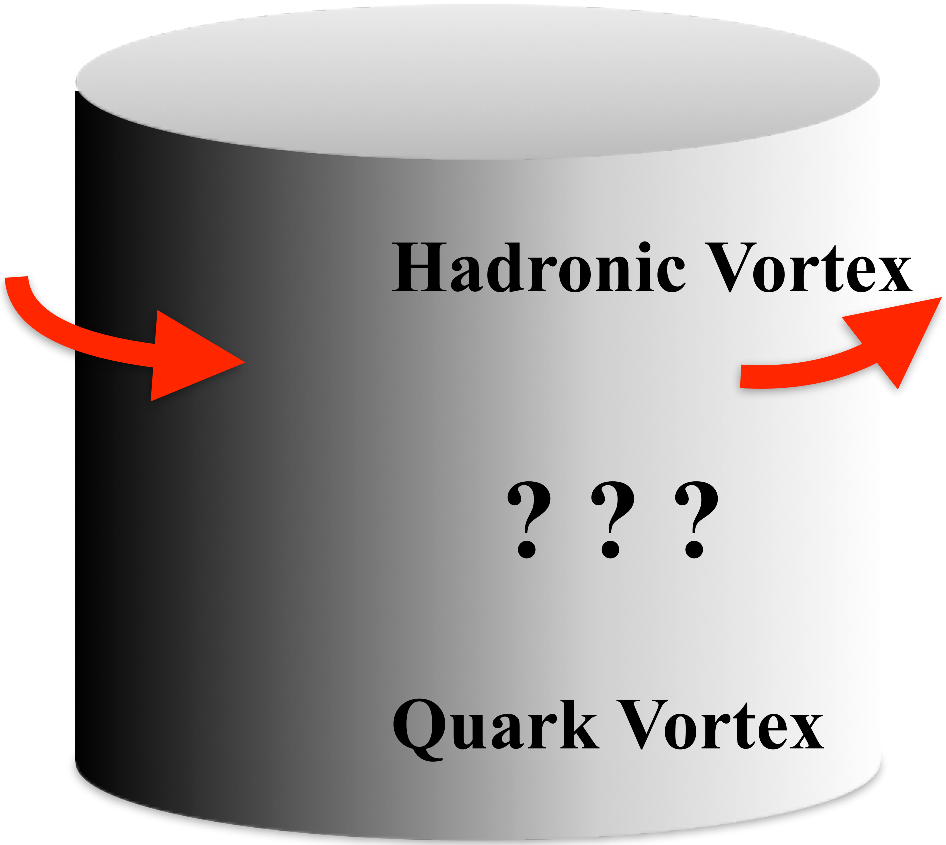
A duality region where the hadrons and quarks may coexist (quarkyonic).



pQCD

First-order Phase Tansition?

Controversy



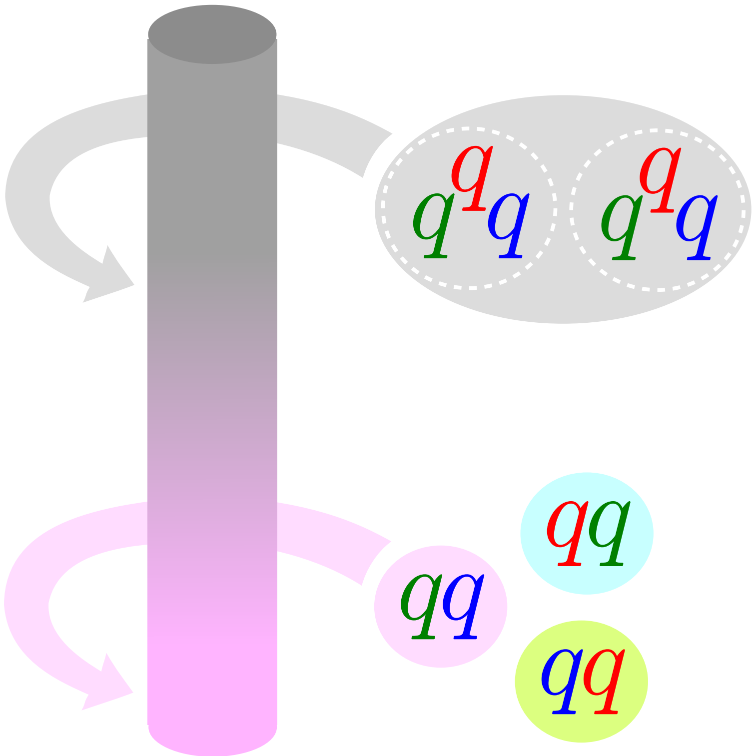
Rotate the bucket filled with quarks
Upper part : Hadronic Vortex
Lower part : Quark Vortex

How can they be connected?

First-order Phase Tansition?

Controversy

Alford-Baym-Fukushima-Hatsuda-Tachibana (2018)



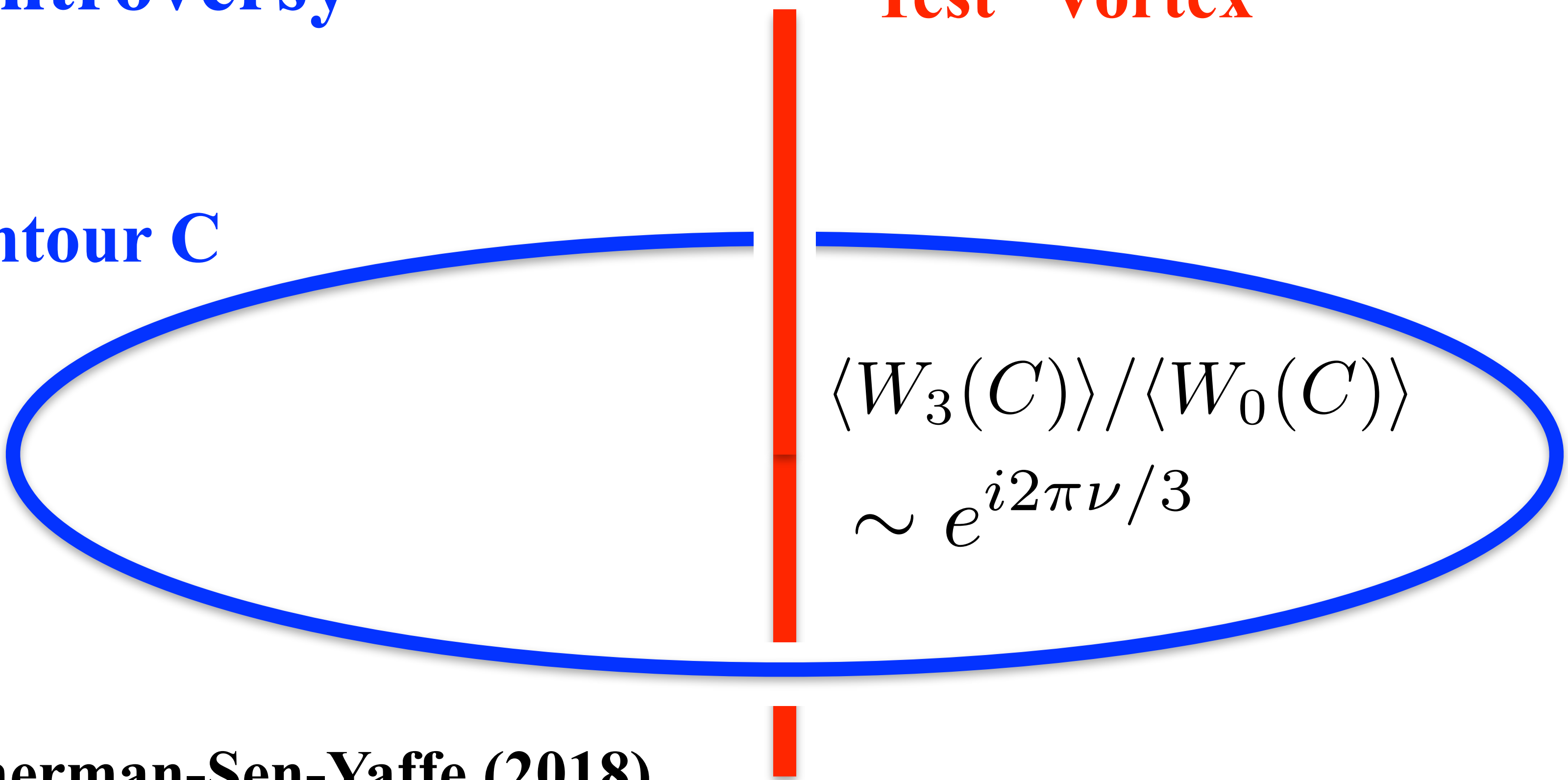
We proposed a scenario of the vortex continuity, but…

First-order Phase Tansition?

Controversy

"Test" Vortex

Contour C



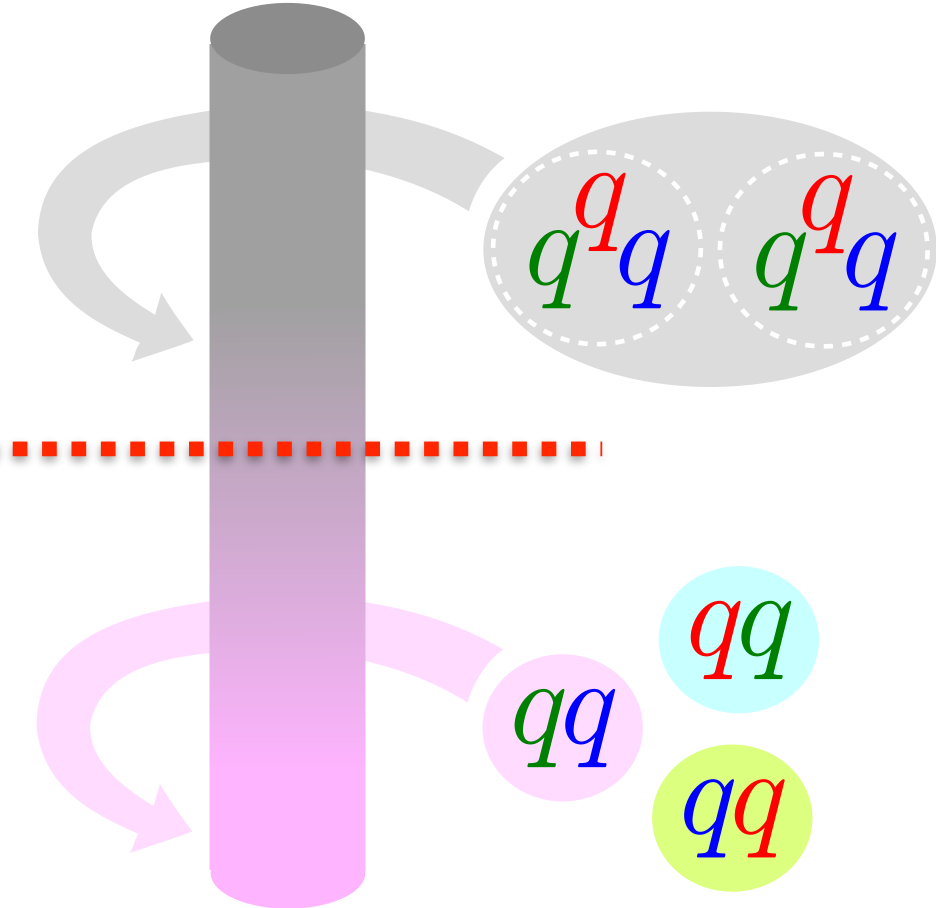
Cherman-Sen-Yaffe (2018)

Hadronic phase has no color flux and no phase…
Distinguishable?

First-order Phase Tansition?

Controversy

There must be a discontinuous interface?



Hirono-Tanizaki argued that a single (global) vortex is energetically not allowed, so the argument is not strict…?

There might be a first-order transition, not resolved yet…