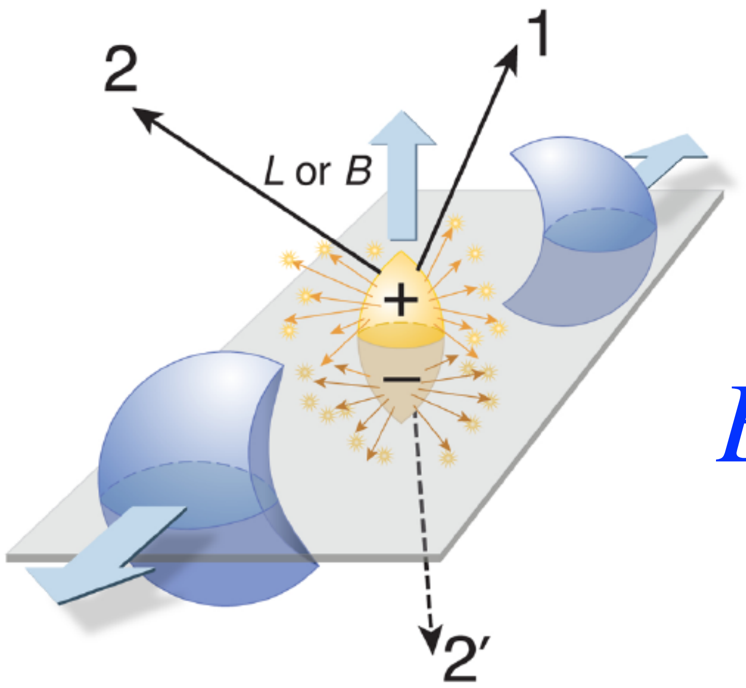
— Day 3 —

Some Implications from Anomalies

Probes to Topological Sector



Non-central collisions produce matter with net magnetic field and angular momentum (rotation).

$$B \sim 10^{15}\,\mathrm{T}$$

10^3 times larger than the surface magnetic field of the magnetar

$$J \sim 10^{7}\,\hbar$$

Largest spin states of nuclei (Yrast states) $< 100\,\hbar$

$U(1)_A$ Breaking by Anomaly

Triangle (ABJ) Anomalies

$$\boxed{\partial_\mu j_{\rm A}^\mu = \partial_\mu K^\mu}$$

Very interesting form!

QED:

$$\partial_\mu j_{\rm A}^\mu = -\frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

$$K^\mu = -\frac{e^2}{4\pi^2}\epsilon^{\mu\nu\alpha\beta}A_\nu\partial_\alpha A_\beta = -\frac{e^2}{8\pi^2}\epsilon^{\mu\nu\alpha\beta}A_\nu F_{\alpha\beta}$$

QCD:

$$\partial_\mu j_{\rm A}^{\mu\,a} = -\frac{g^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}\,F_{\mu\nu}F_{\alpha\beta}\,\mathrm{tr}t^a$$

Usually only the flavor-singlet is anomalous.

$$K^\mu = -\frac{e^2 N_f}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}A_\nu\big(F_{\alpha\beta} - \frac{2}{3}A_\alpha A_\beta\big)$$

$U(1)_A$ Breaking by Anomaly

Performing the space-time integration of

$$\partial_\mu j^\mu_{\rm A} = \partial_\mu K^\mu$$

Left-hand side is:

$$\int dt d^3x\, \partial_\mu j^\mu_A = \int dt\, \partial_t N_5 = N_5(t=\infty) - N_5(t=-\infty)$$

$$N_5 = \int d^3x\, j^0_A = \int d^3x\, \langle \bar\psi \gamma^0 \gamma_5 \psi \rangle$$

Right-hand side is:

$$\int dt d^3x\, \partial_\mu K^\mu = \int dt\, \partial_t \nu = \nu(t=\infty) - \nu(t=-\infty)$$

Winding Number Q_W

$$\nu = \int d^3x\, K^0 \quad \text{(Chern-Simons charge)}$$

$U(1)_A$ Breaking by Anomaly

Simple example: $$\nu = \int d^{d-1}x\, K^0$$

In the case of the U(1) electromagnetic theory:

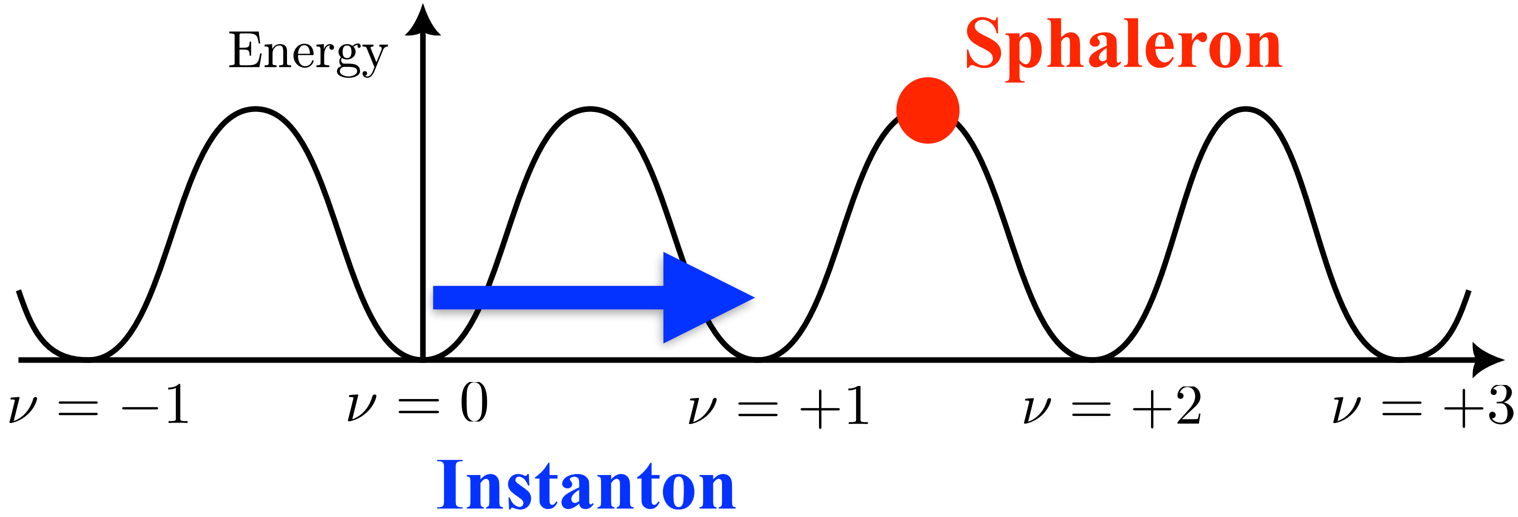
$$K^0 = -\frac{e^2}{8\pi^2}\epsilon^{ijk}A_i F_{jk} = \frac{e^2}{4\pi^2}\boldsymbol{A}\cdot\boldsymbol{B}$$

This itself is not gauge invariant, but the integral is!

Magnetic Helicity $$\nu = \frac{e^2}{4\pi^2}\int d^3x\, \boldsymbol{A}\cdot\boldsymbol{B}$$

Chiral anomaly dictates a new conservation law of the chiral charge and the magnetic helicity!

Theta Vacuum in QCD



True vacuum should be the "bloch state" $|\theta\rangle = \sum_\nu e^{i\theta\nu}|\nu\rangle$

Strong *θ*-term emerges from

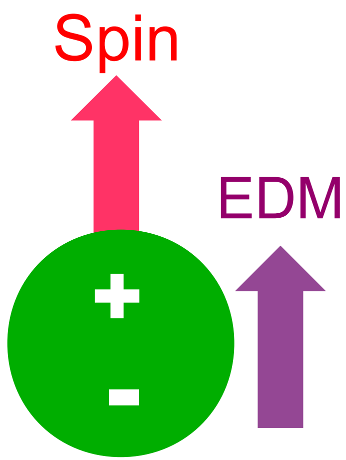
$$\langle\theta|\theta\rangle = \sum_{\nu_\infty,\nu_{-\infty}} \langle\nu_\infty|\nu_{-\infty}\rangle\, e^{-i\theta(\underbrace{\nu_\infty-\nu_{-\infty}}_{Q_W})}$$

Theta Vacuum in QCD

Genuine QCD Lagrangian should be

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2g^2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \theta\frac{1}{16\pi^2}\mathrm{tr}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

θ is one of the physical constants to be determined by the experiment ← Violation of P and CP (or T)



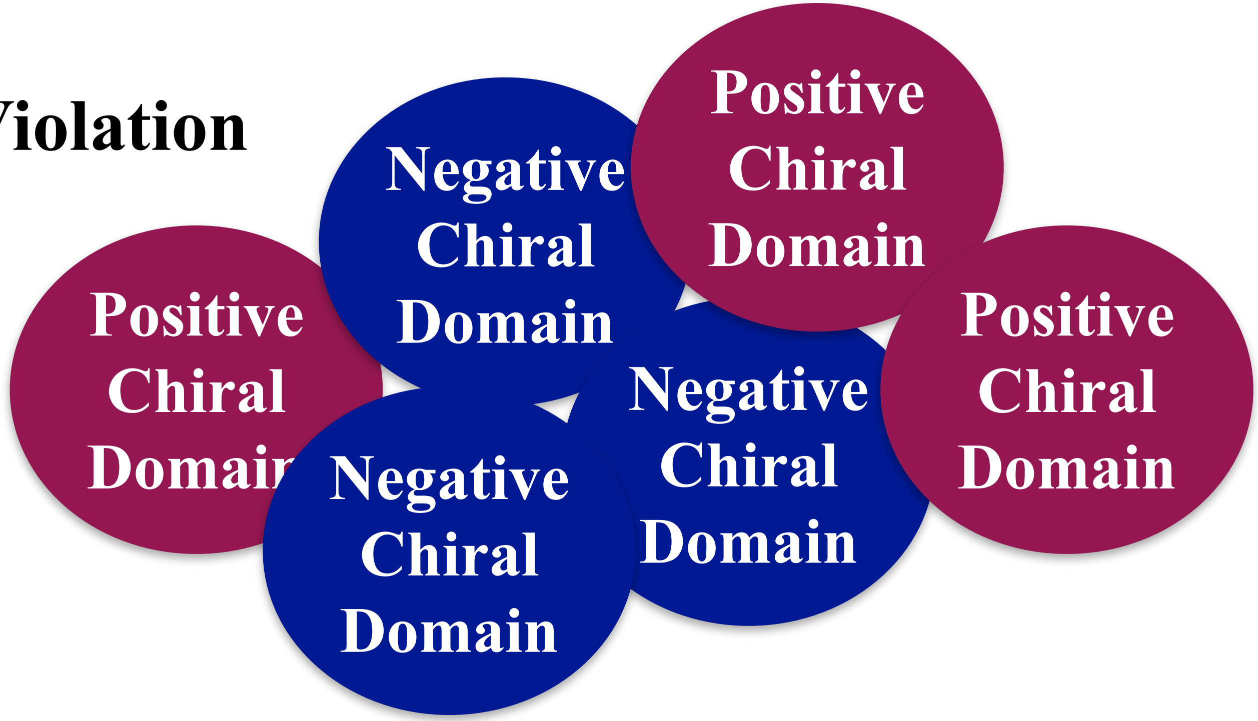
$\theta < 10^{-11}$ (or even smaller)

Electric Dipole Moment is sensitive to P and CP

Theta Vacuum in QCD

Even the heavy-ion collision would not change the physical constant, but…

Local Parity Violation (LPV)



Parity-violating condensate ~ background θ

Theta Vacuum in QCD

In the heavy-ion collision time-dependent (and space too) θ could be induced by the change in the QCD vacuum.

Such backgrounds would also induce a θ term in the electromagnetic sector → Chern-Simons-Maxwell theory

$$\mathcal{L}_{\rm CSM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \theta\frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

The last term is a surface term (in other words, topological invariant if integrated), and no contribution to dynamics as long as θ is constant, but…

Anomaly Induced Terms

Equations of Motion

Differentiating Chern-Simons Maxwell theory

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = \rho + \frac{e^2}{2\pi^2}\boldsymbol{\nabla}\theta\cdot\boldsymbol{B}$$

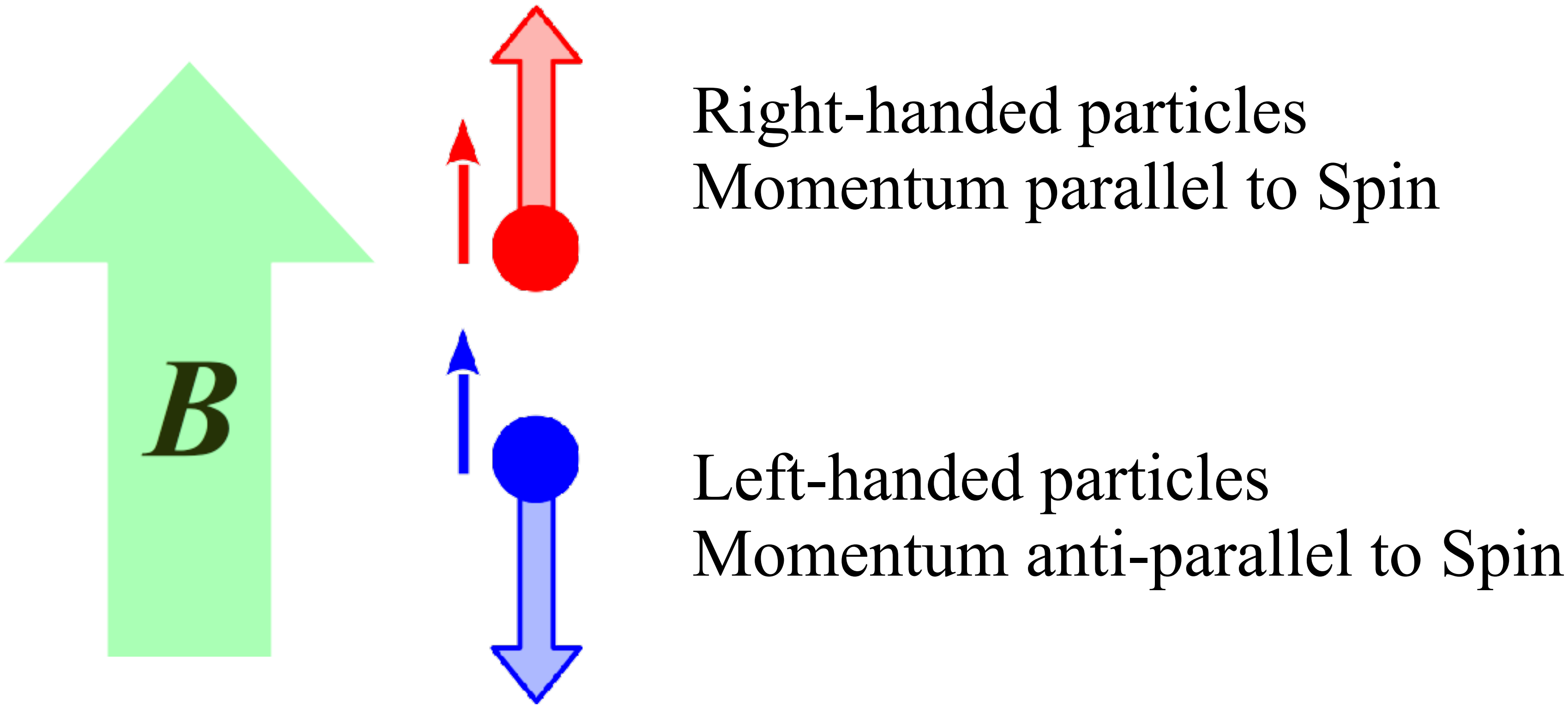
$$\boldsymbol{\nabla}\times\boldsymbol{B} - \frac{\partial\boldsymbol{E}}{\partial t} = \boldsymbol{j} + \frac{e^2}{2\pi^2}\left[\underline{\partial_0\theta\boldsymbol{B}} - \underline{\boldsymbol{\nabla}\theta\times\boldsymbol{E}}\right]$$

Chiral Magnetic Effect **AQHE**

Electric current along the magnetic direction!?

Interpretations

Chiral Magnetic Effect



$\partial_0\theta$ induces chirality imbalance → Nonzero current

often denoted as μ_5 (chiral chemical potential)

Similar Expected Effects

Various Chiral Effects

CME $\boldsymbol{j} \sim \mu_5 \boldsymbol{B}$

CSE $\boldsymbol{j}_5 \sim \mu \boldsymbol{B}$

Spin Polarization — **Material under *B***

Rotation

→ $\boldsymbol{B} \sim \mu \boldsymbol{\omega}$ → $\boldsymbol{j}_5 \sim \mu^2 \boldsymbol{\omega}$

Angular Velocity

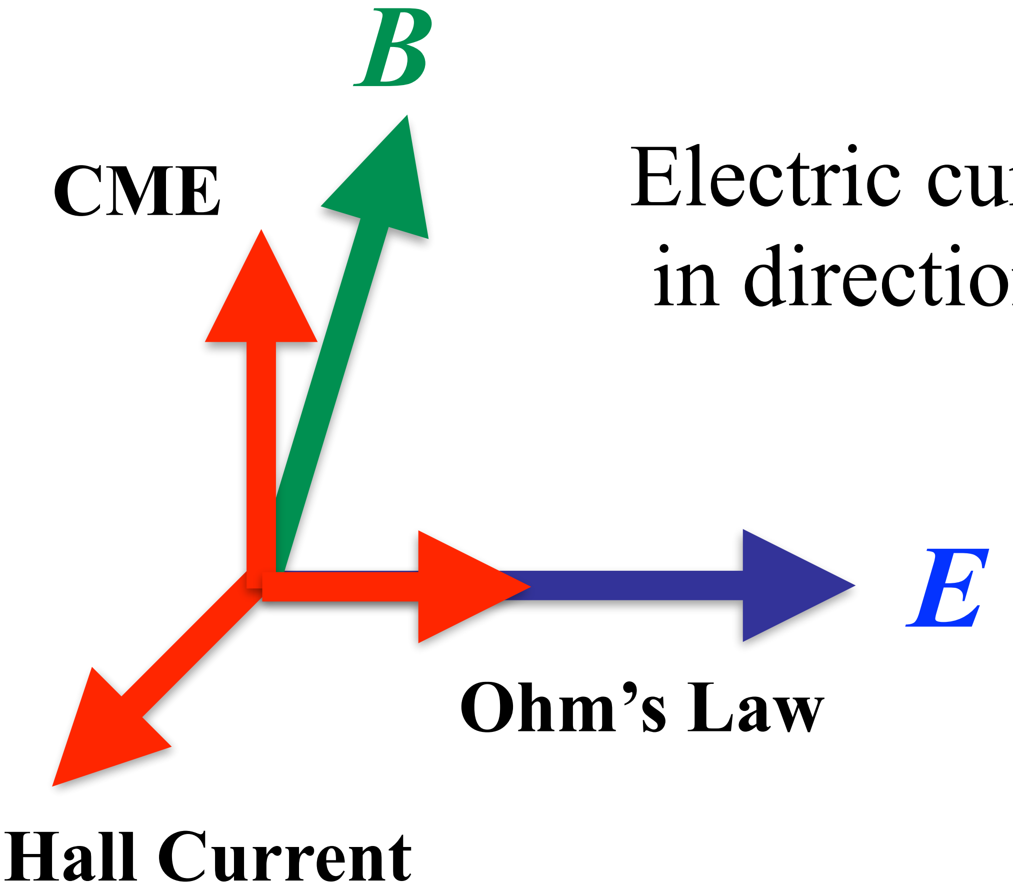
Experimental Setup

In the heavy-ion collision, it is very difficult to control the vacuum properties, and so far, there is no positive evidence for the chiral magnetic effect.

In condensed-matter physics situations, there is no way to control the time- and space-dependent θ angle (there are several proposals, though).

However, what we need is not necessarily the θ angle, but the parity-odd backgrounds should be introduced.

Experimental Setup

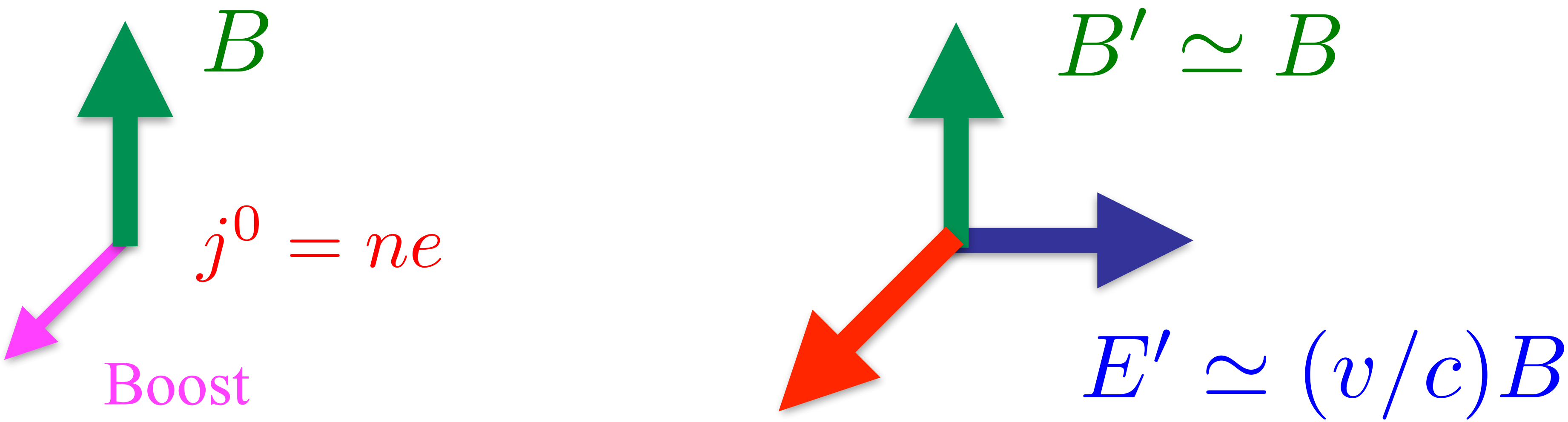


Electric currents flow in directions perpendicular to ***E***

Chiral imbalance provided by $\boldsymbol{E}\cdot\boldsymbol{B}$ not by μ_5

Experimental Setup

Textbook argument for "classical" Hall effect



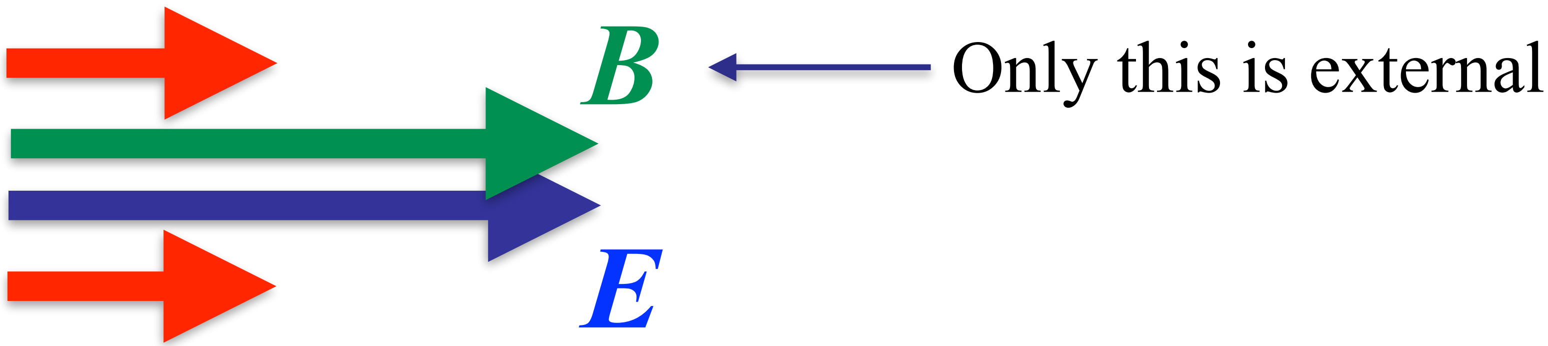
$$j_{\text{Hall}} = j' \simeq v \cdot ne = \frac{nec}{B} E$$

Classical Hall Conductivity

Experimental Setup

Generated Chirality

$$j_{\mathrm{CME}} = (\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{B} \;\propto\; B^2$$



$$j_{\mathrm{Ohm}} = \sigma\boldsymbol{E}$$

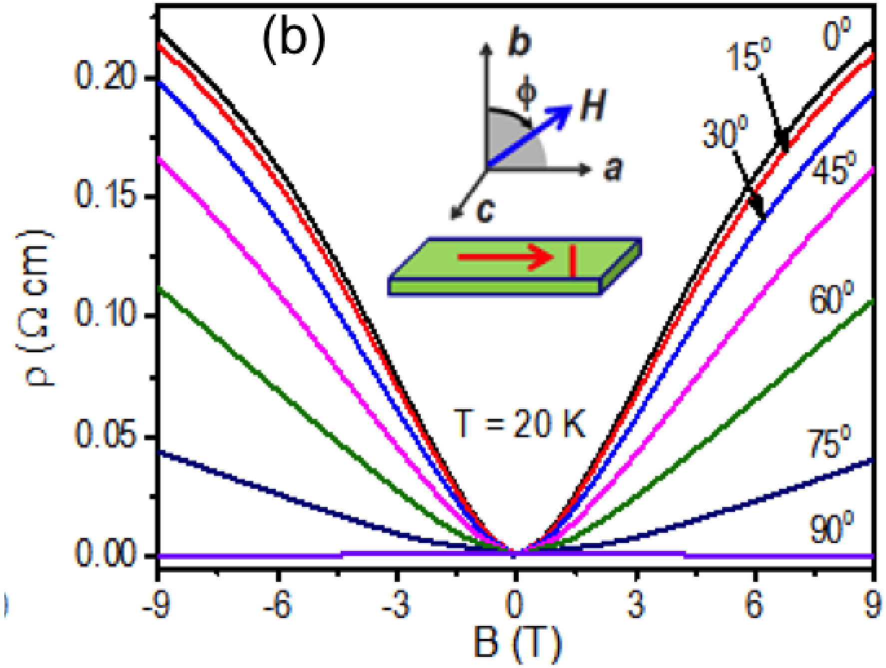
Signature for Anomaly

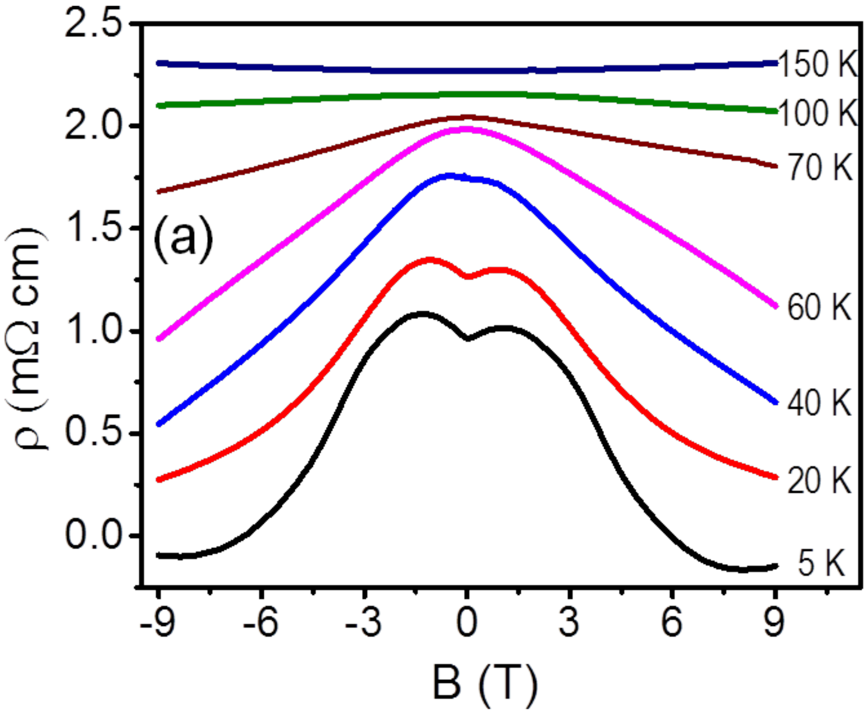
$$j = (\sigma_{\mathrm{Ohm}} + \sigma_{\mathrm{CME}})E \qquad \sigma_{\mathrm{CME}} \propto B^2$$

Son-Spivak (2012)

Experimental Setup

Lorentz force = "Classical" MR
Perpendicular *E* and *B* are Lorentz force free





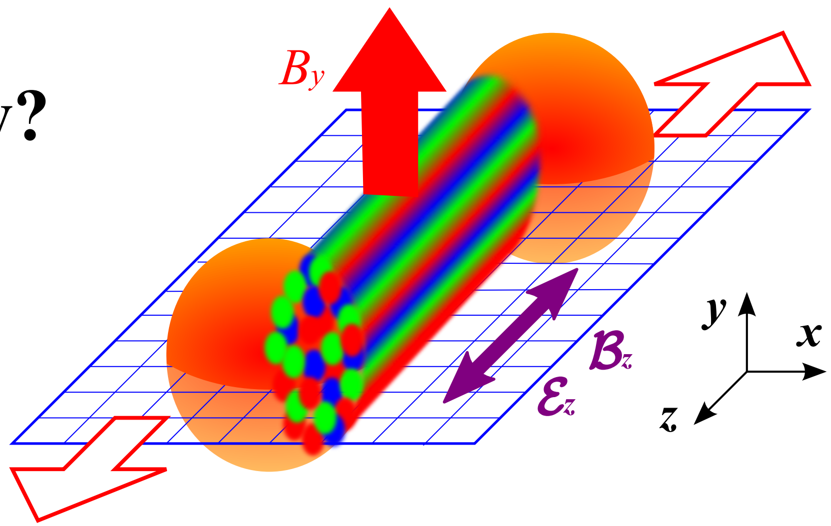
Li, Kharzeev, et al. (2016)

Negative "magnetoresistance"

Remarks

The community has invested a lot to find the topological effect in the heavy-ion collision. → Isobar Experiment

So far, no evidence… why?



Various backgrounds:

Fluctuation contributions from flows etc. etc.

Not only *B* but also shell structures and nuclear shapes are slightly but significantly different!

Topological Effect at High Density

Physics in the large-Nc limit

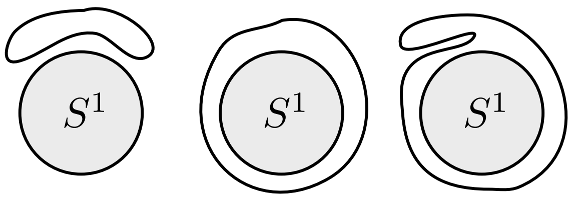
Only mesons (pions) and baryons are too heavy.

How to retrieve baryons?

Topological winding of pions!

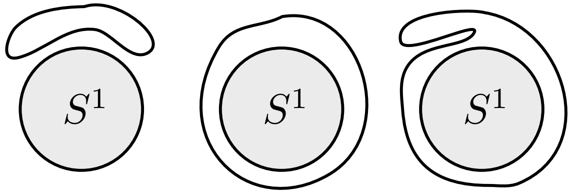
What is the topological winding?

The simplest example: fundamental homotopy group



$$\pi_1(S^1) = \mathbb{Z}$$

Topological Effect at High Density



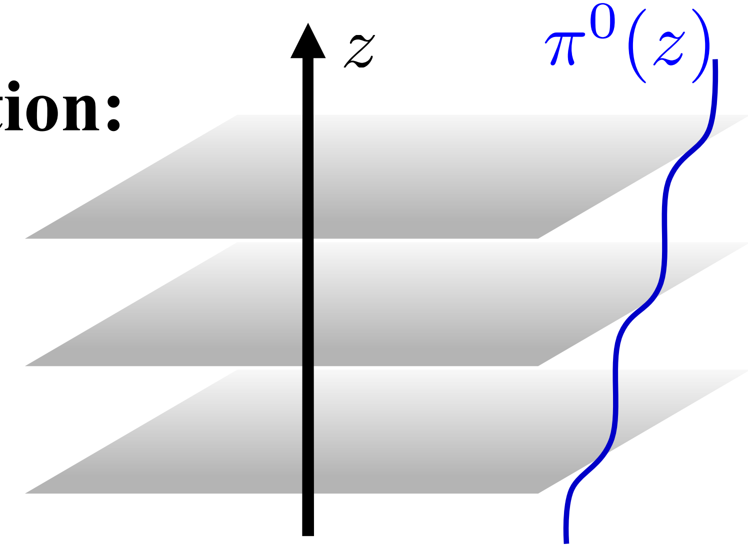
$$\pi_1(S^1) = \mathbb{Z}$$

Suppose a configuration, $\Sigma = e^{i\alpha(z)}$, with z periodic.

Going through one period in z, $\alpha(z)$ gets $2\pi n$ (which gives the same Σ). → *n* = (winding number)

One-dimensional pion configuration:

$$\Sigma = e^{i\pi^0(z)\tau^3/f_\pi}$$



Topological Effect at High Density

Three pions are "angular variables" on the sphere.

S^3 **Surface of 4-D sphare**

Or… the infinity in spatial 3-D *contracted* to a point (easier to make a picture? Monopole charge)

$$\Sigma = e^{i(\pi^1(\boldsymbol{x})\tau^1 + \pi^2(\boldsymbol{x})\tau^2 + \pi^3(\boldsymbol{x})\tau^3)/f_\pi}$$

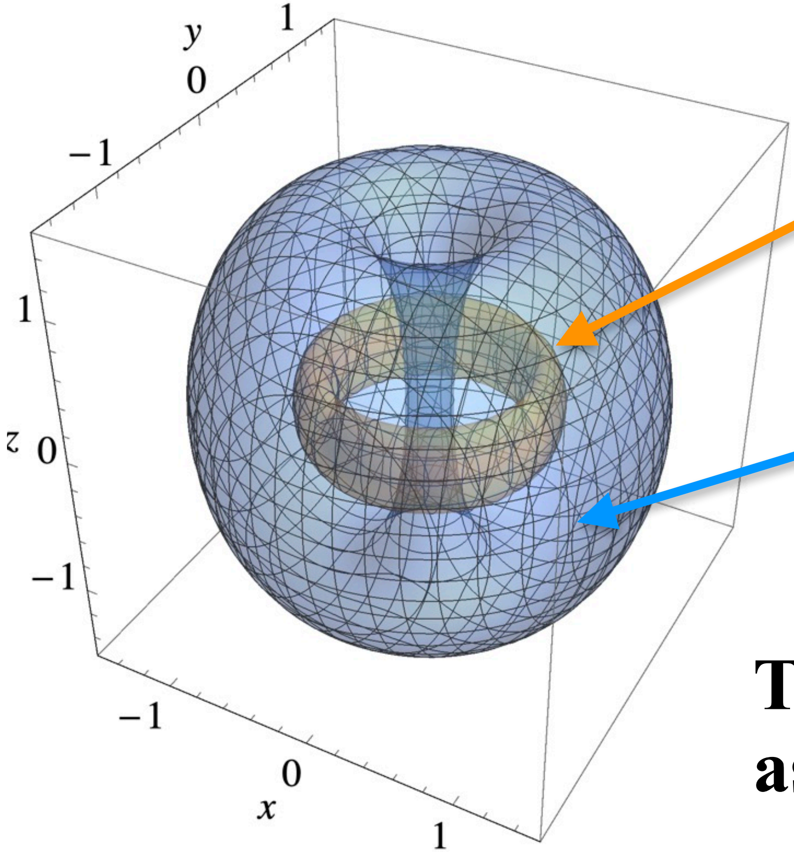
describes $\pi_3(S^3)$

Physics symbol for particles

Math symbol for S^3 in target space

Topological Effect at High Density

It is possible to make a winding configuration of neutral and charged pions. → Skyrmion



Dominated by charged pions

$$(\pi^+)^2 + (\pi^-)^2 = (\text{const.})$$

Dominated by neutral pions

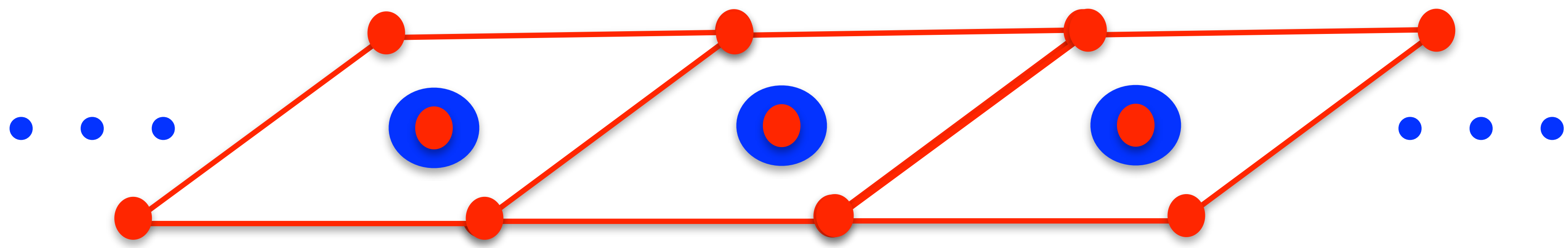
$$(\pi^3)^2 + (\sigma)^2 = (\text{const.})$$

This winding number is identified as the Baryon Number.

Topological Effect at High Density

In the large-*N*c limit, nuclear matter is given by multi-Skyrmions.

(Klebanov 1985, Goldhaber-Manton 1987)



If Skyrmions are solved with periodic bounadry condition (with some factor in isospin space), it is "matter" and by construction, this state is *inhomogeneous*.

Skyrme Crystals

Topological Effect at High Density

Couling to the magnetic field — 1D case

$$\Sigma = e^{i\pi^0(z)\tau^3/f_\pi}$$

A new term arises from anomaly!

$$\mathcal{H} = \frac{1}{2}(\boldsymbol{\nabla}\pi^0)^2 + \underline{m_\pi^2 f_\pi^2\left[1-\cos(\pi^0/f_\pi)\right]} - \underline{\frac{\mu}{4\pi^2 f_\pi}\boldsymbol{B}\cdot\boldsymbol{\nabla}\pi^0}$$

symmetry breaking **WZW (anomaly)**

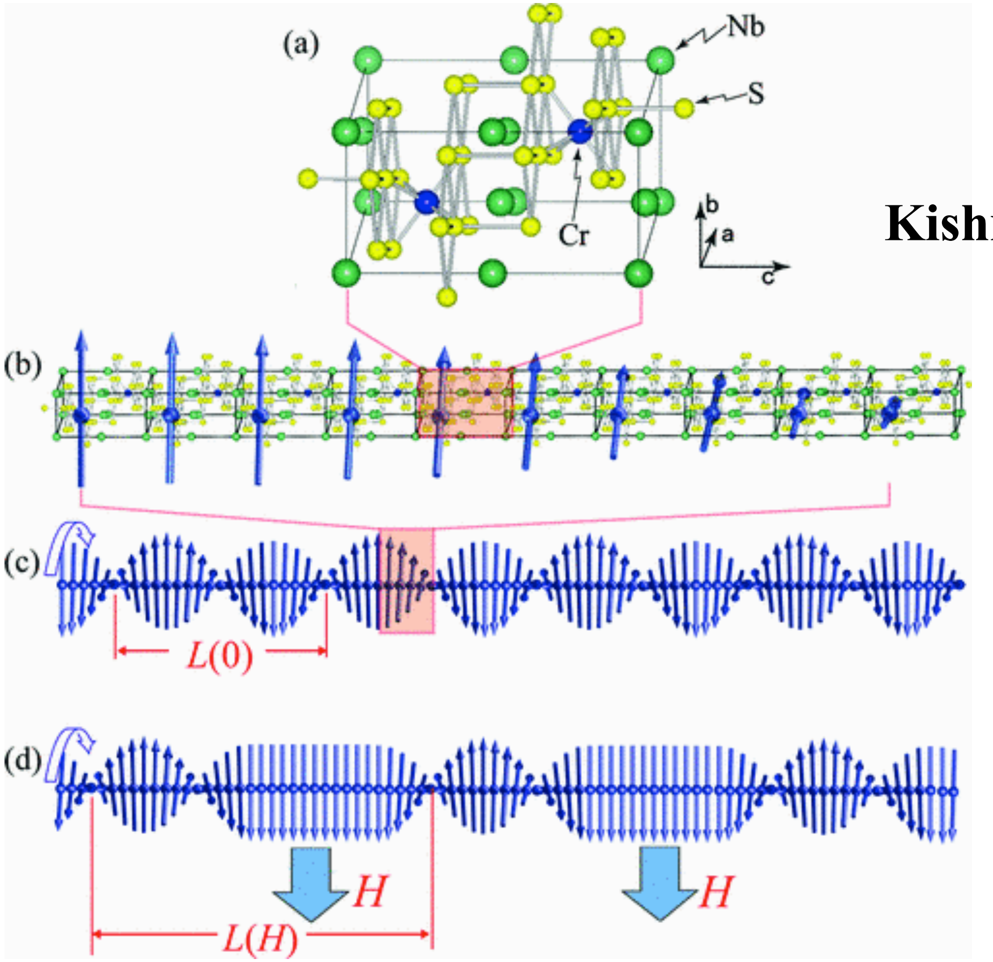
* Exactly the same Hamiltonian as the "Chiral Soliton Lattice" for a chiral magnet.

* The mass terms favors trivial $\pi^0 = 0,\ 2\pi$, etc.

* Finite baryon density requires inhomogeneous $\pi^0(z)$.

Topological Effect at High Density

Chiral Soliton Lattice



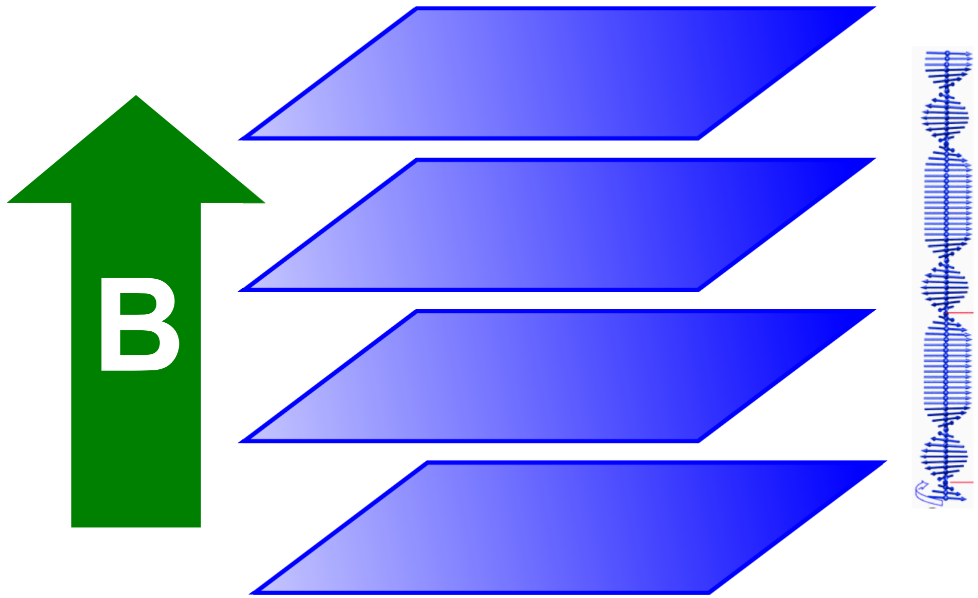
Kishine *et al.*, PRL108, 107202 (2012)

No symmetry breaking

Explicit breaking

$H \sim m$ (mass) in QCD

Topological Effect at High Density



Brauner-Yamamoto, JHEP (2016)

Baryon density localized on periodic domain walls

$$n_B = \frac{\partial \langle \mathcal{H} \rangle}{\partial \mu} = \frac{B_z}{4\pi^2 f_\pi} \partial_z \pi^0$$

This can be understood as Skyrmions under *B*:

$$\pi_3(\mathrm{SU}(2)) = \mathbb{Z} \longrightarrow \pi_1(\mathrm{U}(1)) = \mathbb{Z}$$

$$B = 0 \qquad\qquad B \to \infty$$

Another Anomaly — Trace

Trace anomaly tells us a lot about matter properties!

Conformality must be quantified by the trace of the (symmetrized) energy-momentum tensor.

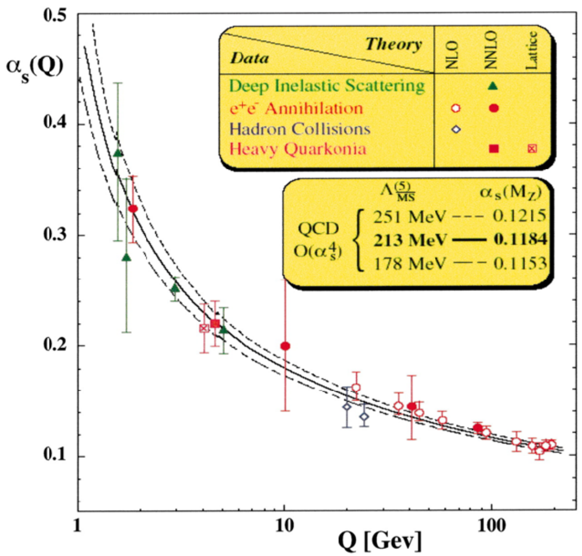
$$\Theta^{\mu}{}_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)}\partial_{\nu}\phi - \delta^{\mu}{}_{\nu}\mathcal{L}$$

QCD Lagrangian has almost no energy scale (other than small quark masses).

Is the trace vanishing???

Another Anomaly — Trace

Remember the running coupling constant!



$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2_{\rm QCD})}$$

The trace involved a parameter in the running coupling constant.

$$\Theta = \frac{\beta}{2g} F^a_{\mu\nu} F^{\mu\nu}_a + (1+\gamma_m) \sum_f m_f \bar{q}_f q_f$$

This is also an anomaly (breaking of conserved current).

Another Anomaly — Trace

Relation between the dilatation current and the energy-momentum tensor

Obviously, the massless scalar is scale invariant:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4!}\phi^4$$

However,

$$\Theta^\mu_{\ \mu} = \partial^\mu\phi\partial_\mu\phi - 4\mathcal{L} = -\partial^\mu\phi\partial_\mu\phi + \frac{\lambda}{3!}\phi^4 = -\partial^\mu\phi\partial_\mu\phi - \phi\Box\phi$$

This is not zero???

Another Anomaly — Trace

One can get the dilatation current from

$$x^\mu \to x'^\mu \simeq (1+\epsilon)x^\mu\,, \qquad \phi(x) \to \phi'(x') \simeq (1-\epsilon)\phi(x)$$

The corresponding Noether current is

$$j^\mu_{\rm dil} = (\partial^\mu \phi)(\phi + x^\nu \partial_\nu \phi) - x^\mu \mathcal{L} = \Theta^{\mu\nu} x_\nu + \frac{1}{2}\partial^\mu \phi^2$$

It is easy to confirm $\partial_\mu j^\mu_{\rm dil} = m^2 \phi^2$

The definition of conserved currents is not unique!

It is always possible to add "trivially" conserved parts.

Another Anomaly — Trace

The following energy-momentum tensor is also Okay!

$$\tilde{\Theta}_{\mu\nu} = \Theta_{\mu\nu} - \frac{1}{6}(\partial_\mu \partial_\nu - g_{\mu\nu}\Box)\phi^2$$

The following dilatation current is also Okay!

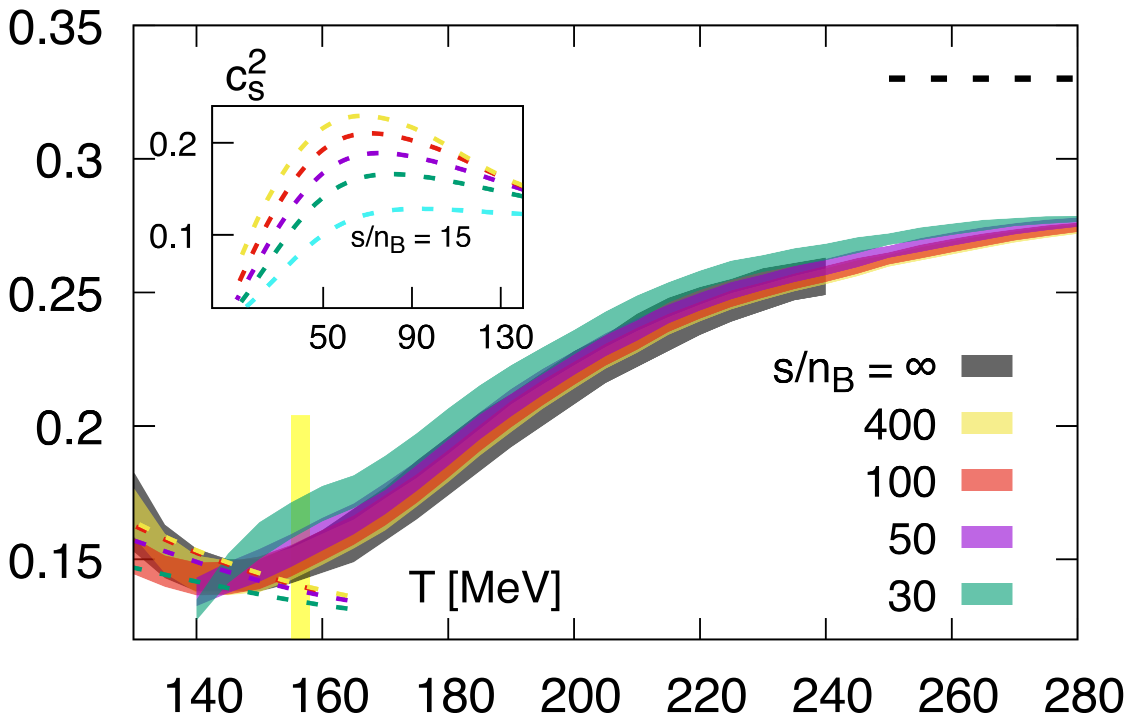
$$\tilde{j}^\mu_{\text{dil}} = j^\mu_{\text{dil}} + \frac{1}{6}\partial_\nu\left[(x^\mu \partial^\nu - x^\nu \partial^\mu)\phi^2\right]$$

Then, we can prove:

$$\tilde{j}^\mu_{\text{dil}} = \tilde{\Theta}^{\mu\nu} x_\nu \quad \Rightarrow \quad \tilde{\Theta}^\mu{}_\mu = \partial_\mu \tilde{j}^\mu_{\text{dil}} = 0$$

Speed of Sound at High T

Speed of sound with increasing *T* at small densities



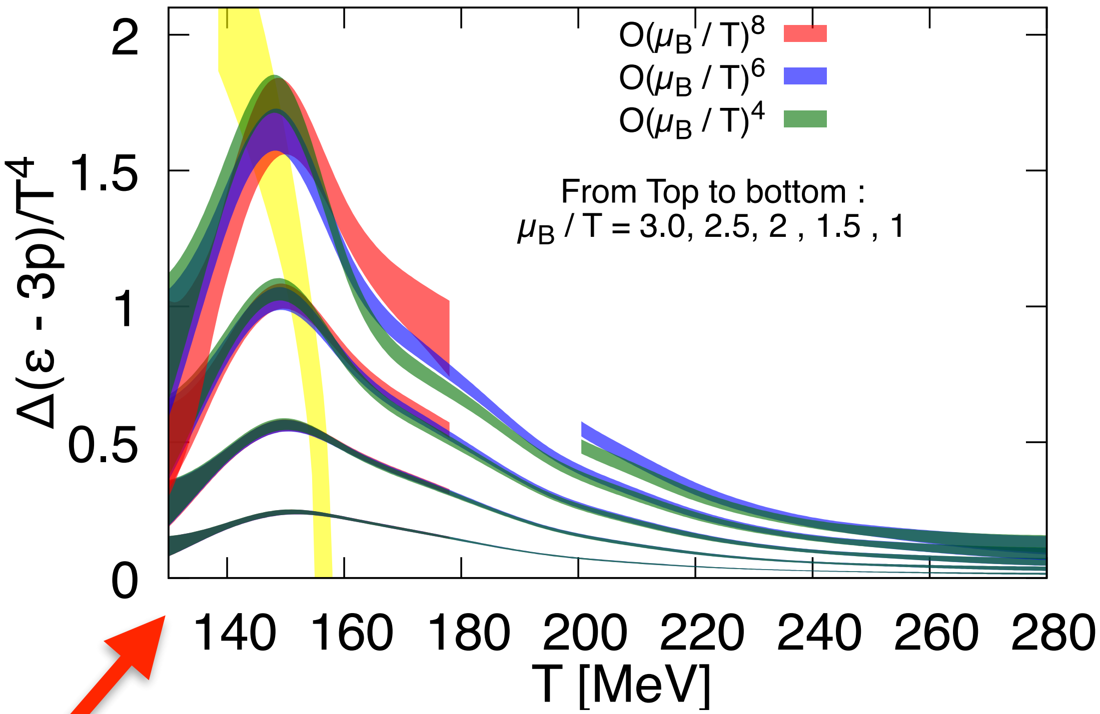
HotQCD Collab. (2212.09043)

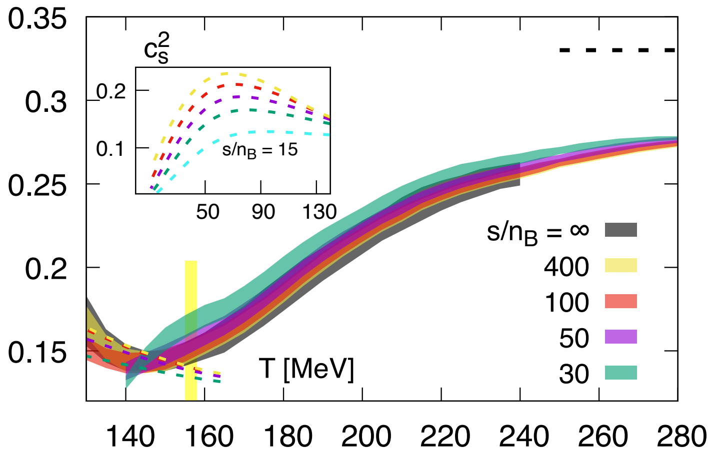
Suppressed around the phase transition (~1st-order PT) and approaching the conformal limit at high *T*

Trace Anomaly at High T

The energy-momentum tensor in hydro variables:

$$\Theta^{\mu}{}_{\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$





Any relation? Yes!!

Trace Anomaly at High T

Fujimoto-Fukushima-McLerran-Praszalowicz (2022)

Measure of conformality: $\Delta \equiv \frac{\langle \Theta \rangle_{T,\mu_{\rm B}}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$

$$v_s^2 = \frac{dP}{d\varepsilon} = v_{s,\,\rm deriv}^2 + v_{s,\,\rm non\text{-}deriv}^2$$

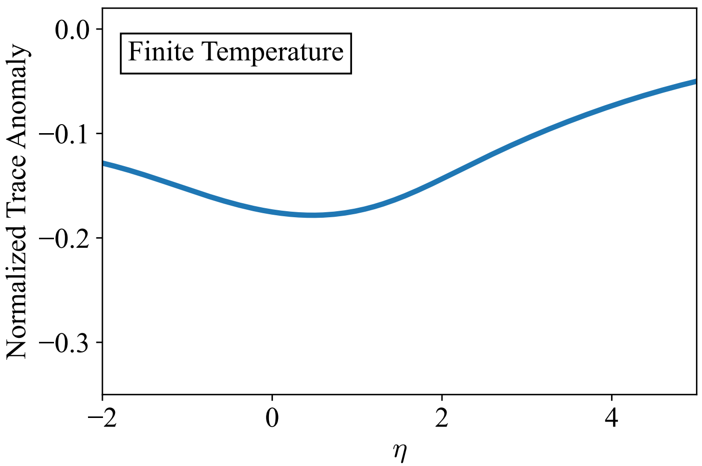
$$v_{s,\rm deriv}^2 \equiv -\varepsilon \frac{d\Delta}{d\varepsilon} \qquad v_{s,\,\rm non\text{-}deriv}^2 \equiv \frac{1}{3} - \Delta$$

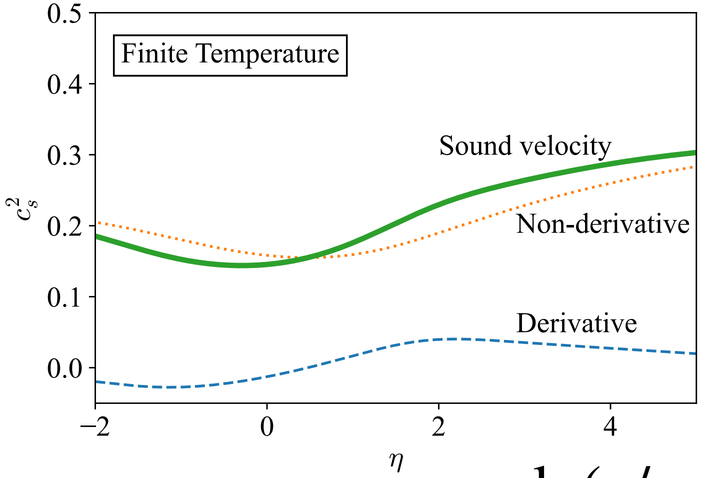
Derivative **Non-Derivative**

Trace Anomaly at High T

Finite Temperature — Non-Derivative Dominant

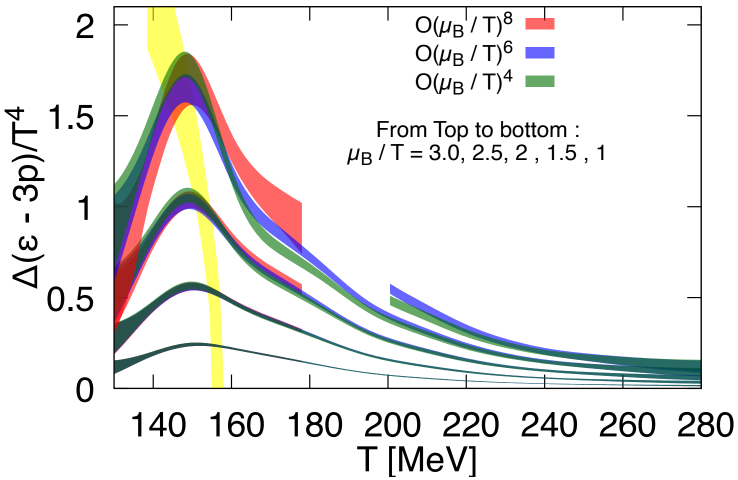
Sign Flipped $-\Delta$

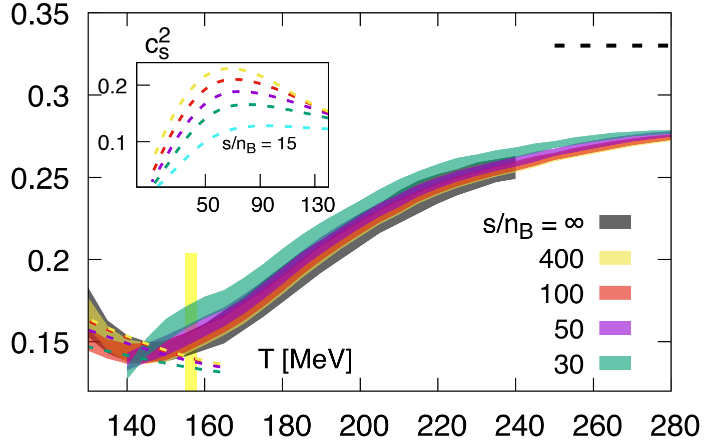




$$\eta = \ln(\varepsilon/\varepsilon_0)$$

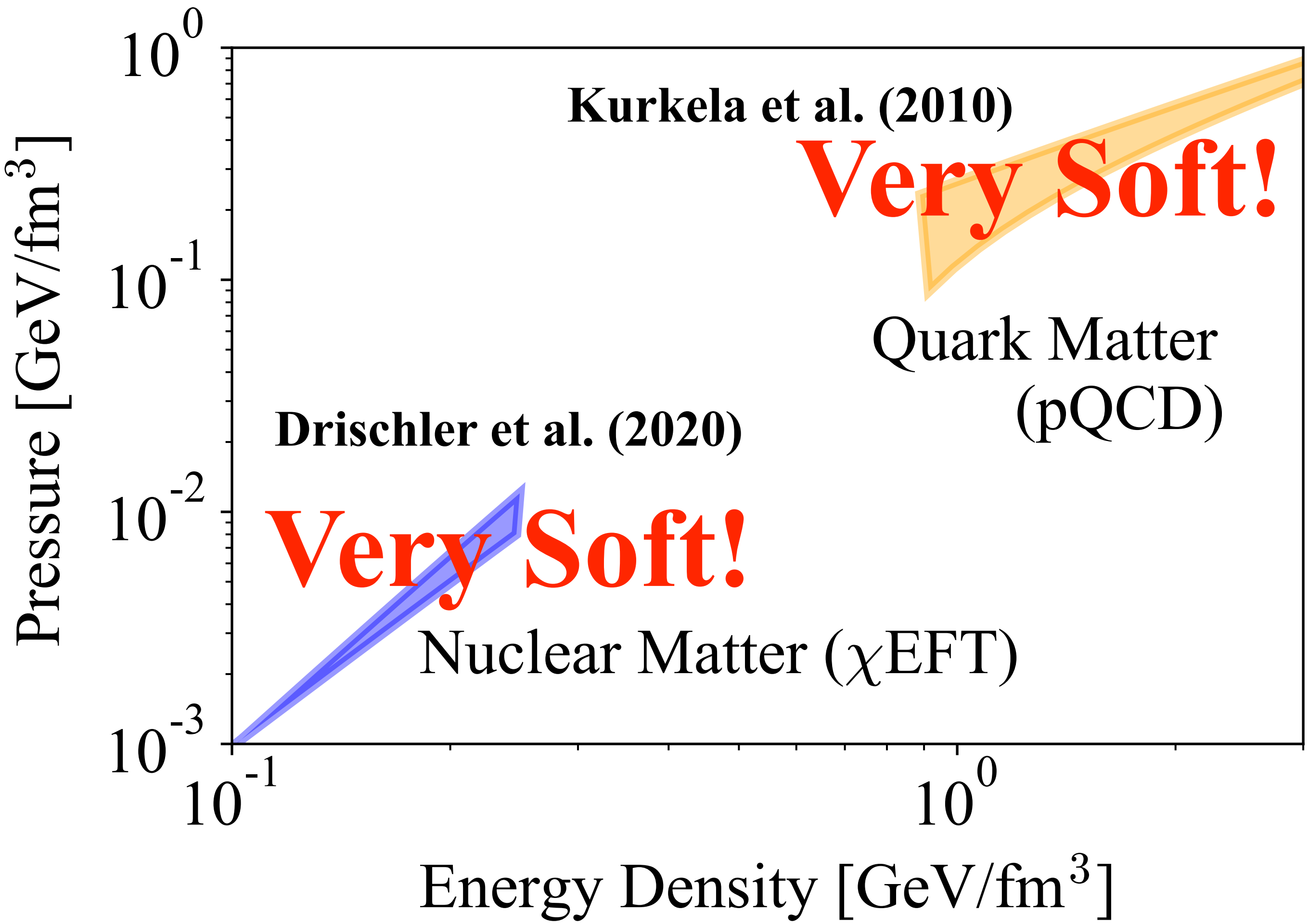
Speed of Sound





Speed of Sound at High Density

Ab Initio EoS in the Low and High Density Regions

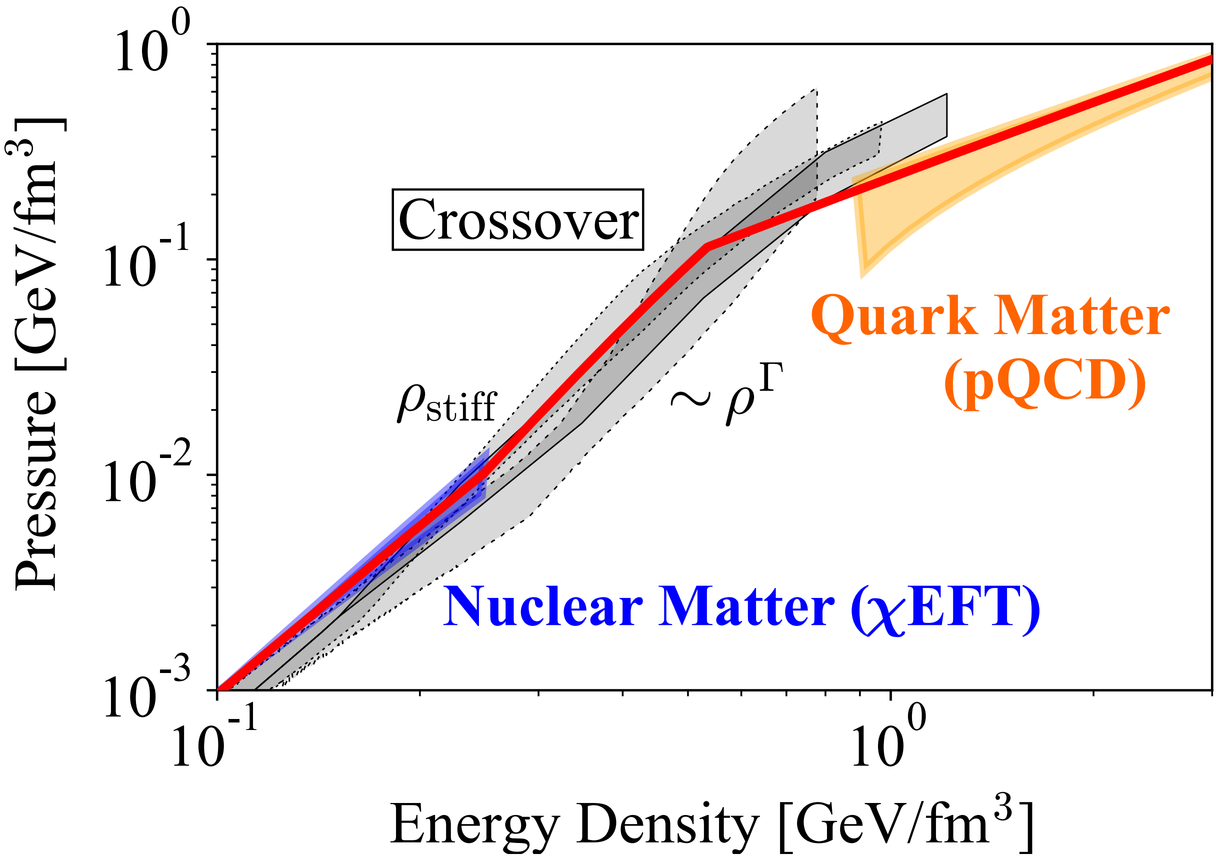


Stiffening in the intermediate region is necessary to support massive neutron stars.

A peak is just unavoidable!

Speed of Sound at High Density

Fujimoto-Fukushima-Hotokezaka-Kyutoku (2022)



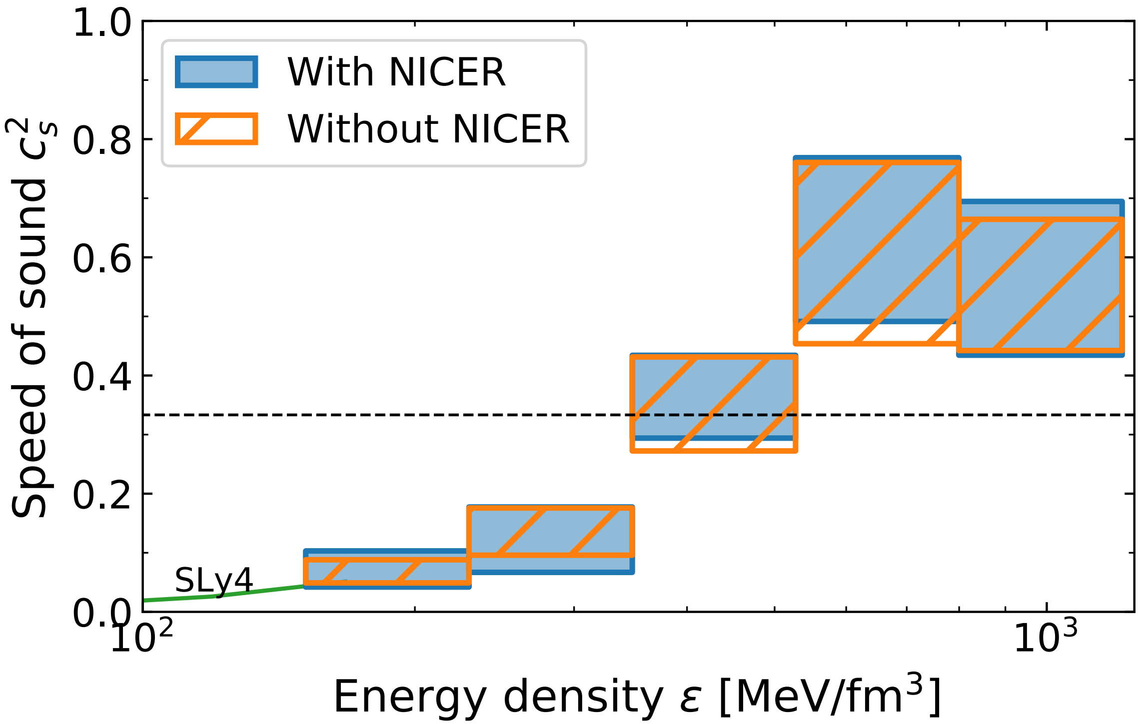
[Crossover EOS]
Quark matter onset
~ Softening point

Many other ways for interpolation not excluded (systematic studies needed further)

We have performed the gravitational wave simulation.

Speed of Sound at High Density

Two non-trivial features



A peak in the speed of sound???

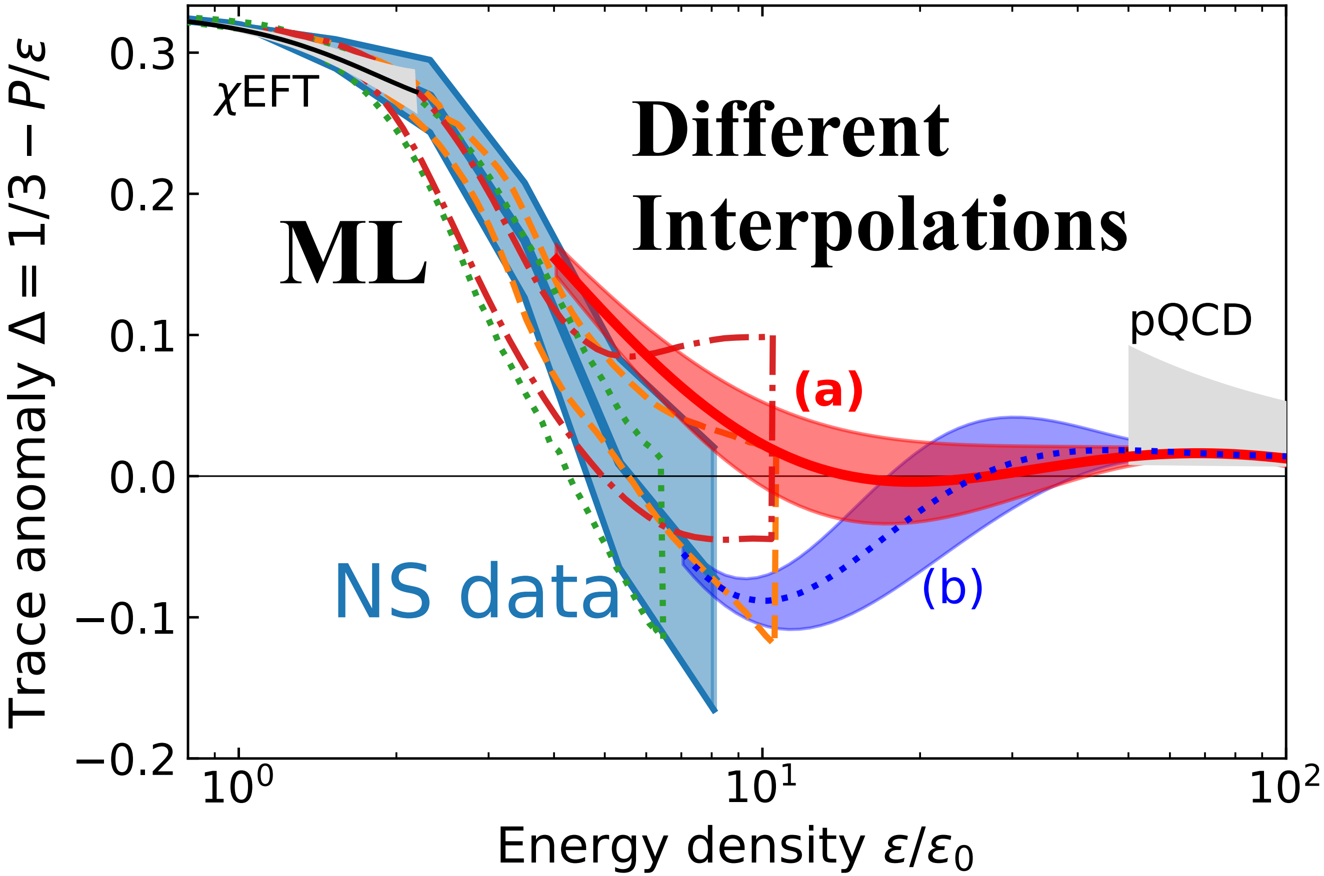
Exceeding the conformal limit???

Some hint to Quarkyonic?

(McLerran-Reddy)

Trace Anomaly at High Density

Fujimoto-Fukushima-McLerran-Praszalowicz (2022)

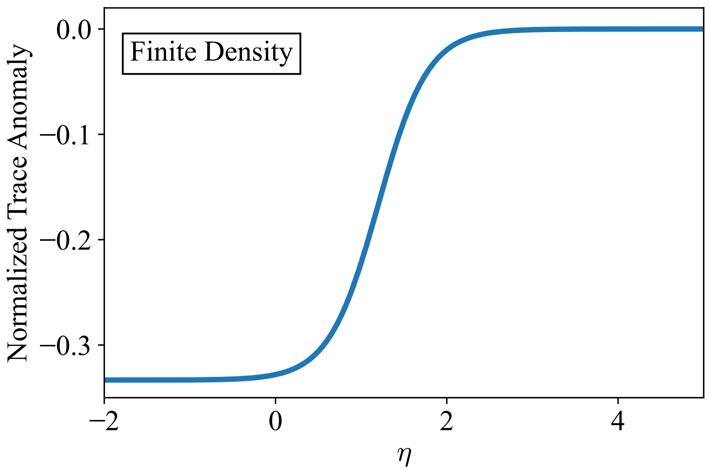


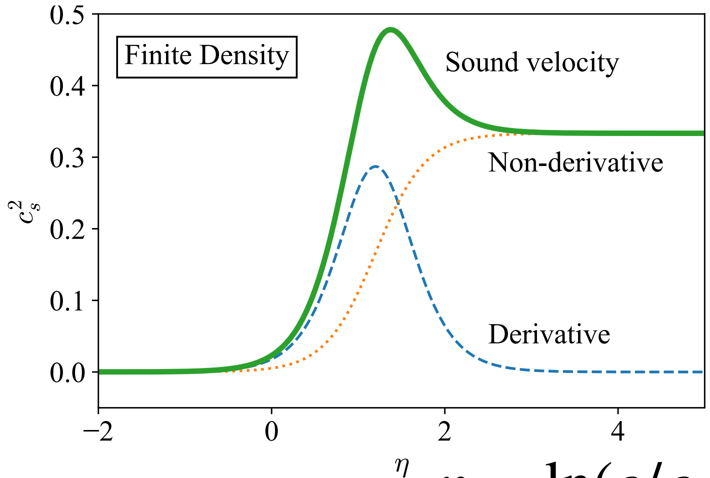
Very different from the finite temperature case!

Trace Anomaly at High Density

Finite Density — Derivative Dominant

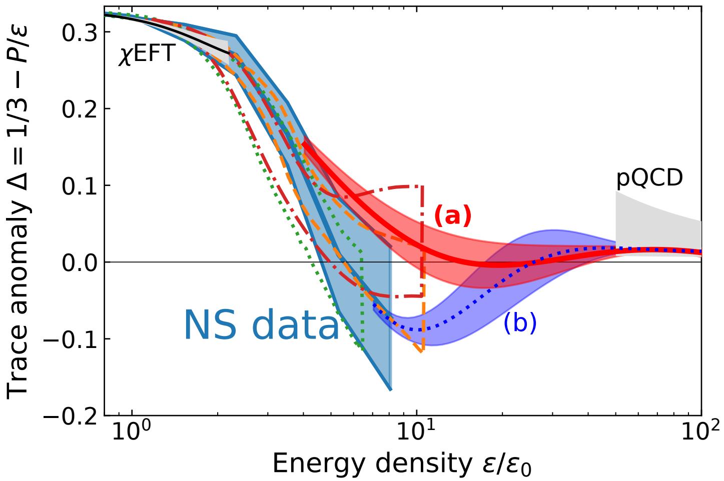
Sign Flipped $-\Delta$

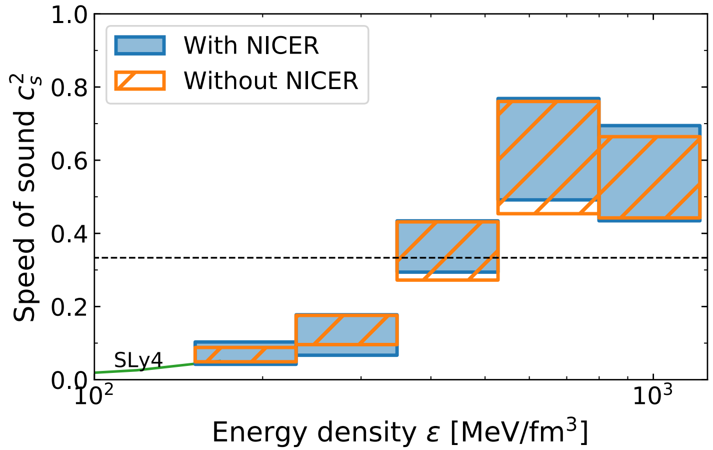




$\eta = \ln(\varepsilon/\varepsilon_0)$

Speed of Sound





Remarks

These results suggest that the peak in the speed of sound indicates nearly conformal metter (restoration of scale invariance) in dense matter in the neutron star.

In the community more and more authors have confirmed this conclusion with different methods.

Some fundamental explanation from Quantum Field Theory is necessary… any idea?

Any hint for exotic phases (like Quarkyonic, CSL, etc.) ?

Any questions welcome!