

Gravitational Wave Astronomy

"Matter tells spacetime how to curve and spacetime tells matter how to move." - John Wheeler

- fundamental principle of general relativity (GR)

2nd key principle of relativity: No instant action at a distance

↳ information can never travel faster than C

What does this mean in the context of gravity?

Consider familiar system:



As the Moon orbits the Earth, causes tides
But the changing location of the Moon is
not known instantaneously, there is
a (very small!) delay $\sim 1.33\text{ s}$

light travel time
Earth-Moon

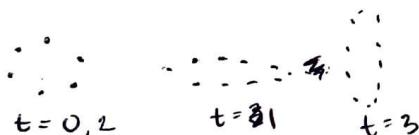
GR predicts that information about a changing grav. field is propagated via gravitational waves

- ↓
- ripples in spacetime; transverse waves (i.e., effect on matter is \perp to direction of prop.)
- predicted to travel at C (now confirmed, via GW170817)

Come in two polarizations,
named according to impact
on a ring of hypothetical
test particles:

↓

E/M + GW signals originated from
130 Mly away, arrived 1.7 s apart
 \therefore speeds can differ by no more
than $1.7\text{ s} / (130 \times 10^6\text{ ly}) \leq 10^{-15}$



Plus or "+" polarized



Cross or "X" polarized (45° rot. of + wave)

G-2

We describe the impact of GWs via a small perturbation to the flat Minkowski metric:

$$\underbrace{g_{\mu\nu}_{\text{metric}}}_{\text{flat Spacetime}} = \underbrace{\eta_{\mu\nu}}_{\text{flat Spacetime}} + \underbrace{h_{\mu\nu}}_{\text{perturbation}}, \quad h_{\mu\nu} \ll 1$$

How do we generate such perturbations?

$$h_{ij}(t) = \frac{2G}{c^4 D} \frac{\partial^2}{\partial t^2} Q_{ij}(t - r/c)$$

↑ ↑
note amplitude:
 $\frac{G}{c^4}$ extremely small
GWs full off w/ $1/d$ distance

↑
quadrupole moment tensor

Quadrupole formula

So, we need an accelerating quadrupole moment.

Possible source:

- asymmetric, rotating star - e.g. "mountain" on a NS

↳ produces continuous GWs; current LIGO data places limits on ellipticity only (& no detection)
- non-spherical gravitation collapse
(also not yet detected)
- binaries



⇒ slides (Hulse-Taylor PSR; binary merger animation)

Physical dependences of the GW strain

To evaluate the quad. formula, let's consider a simple Newtonian approx.
For a circular, equal-mass binary, can use Kepler's 3rd Law $\left(\frac{GM}{(2R)^3} = (2\pi f_{GW})^2 \right)$
to derive Q_{ij} :

$$Q_{xx} = 2MR^2 \cos^2 \phi$$

$$Q_{yy} = 2MR^2 \sin^2 \phi$$

$$Q_{yx} = Q_{xy} = 2MR^2 \sin \phi \cos \phi$$

$$Q_{iz} = 0$$

$$\phi = \phi_0 + \omega t \quad \text{is the time-dep. phase}$$

G-3

Plug into quadrupole formula to get

$$h_0 = \frac{G}{c^2} \frac{M_c}{D} \left(\frac{G}{c^3} \pi f_{GW} M_c \right)^{2/3}$$

↑
note: no longer
assuming equal masses

$$h(t) = h_0 \cdot \cos \phi(t), \text{ where}$$

$$\text{where } M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

is the "chirp mass"

and $f_{GW} = 2 \text{ forb}$
is the GW freq.
($\text{forb} = 2\omega_{\text{orb}}/2\pi$)

For characteristic values:

$$h_0 \approx 1.6 \times 10^{-22} \left(\frac{f_{GW}}{250 \text{ Hz}} \right)^{2/3} \left(\frac{M_c}{1.2 M_\odot} \right)^{5/3} \left(\frac{D}{40 \text{ Mpc}} \right)^{-1}$$

↑
freq. when NS
are ~ 4 stellar
radii apart

↑
Chirp mass for
a $1.4 + 1.4 M_\odot$
binary

↑
distance of
1st BNS merger

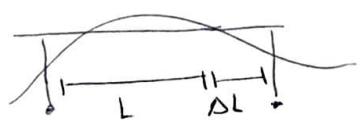
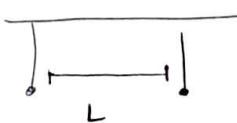
Scaling:

- as objects get closer, f_{GW} increases \rightarrow strain ampl. increases
- strong scaling w/ mass:
 $2 20 M_\odot \text{ BHs} \approx 84 \times$ louder than $2 1.4 M_\odot \text{ NSs}$
- GW strain falls off w/ $1/D$ (cf. $\text{EM} \sim 1/D^2$)

But amplitude is incredibly small!

Direct detection

- GWs introduce a tidal force that affects the relative trajectory of nearby objects
- Consider 2 suspended test particles, sep'd by L

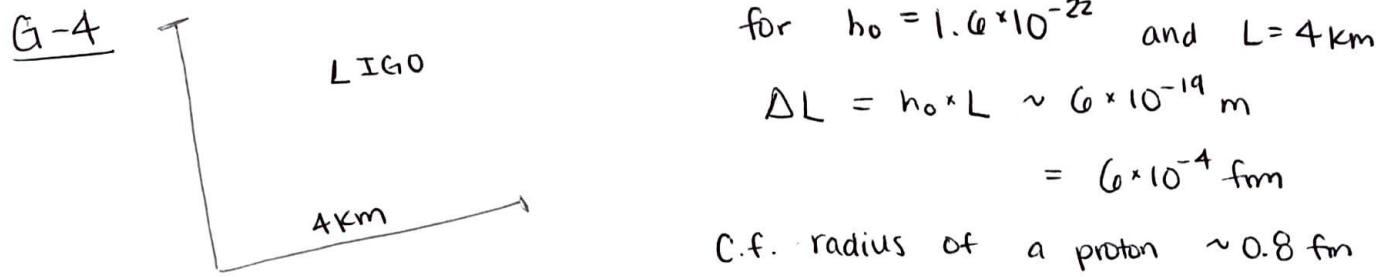


$$h \sim \frac{\Delta L}{L}$$

↑
magnitude
of astrophysical
signal

relative displacement;
determined by experimental
sensitivity

length of arm;
determined by
experimental set-up



Displacement is $\frac{1}{1000\text{th}}$ the radius of a proton

\Rightarrow LIGO slide

Can also use quadrupole formula to derive GW luminosity:

$$L_{\text{GW}} = \frac{\partial E_{\text{GW}}}{\partial t} = \frac{G}{c^5} \frac{1}{5} \left\langle \ddot{Q}_{ij} \ddot{Q}^{ij} \right\rangle \sim \frac{d}{dt} (h \cdot r)$$

$$\sim 10^{50} \left(\frac{f_{\text{GW}}}{250 \text{ Hz}} \right)^{10/3} \left(\frac{M_c}{1.2 M_\odot} \right)^{10/3} \text{ erg/s}$$

- for merging BHs, GWs can briefly "outshine" (in GW energy) the luminosity of all stars in the visible universe
- incredible amt. of power

Can use this to define a decay timescale:

$$t_{\text{decay}} \sim \frac{E_{\text{orb}}}{\dot{E}_{\text{orb}}} \quad , \quad \text{where } E_{\text{orb}} \text{ can be estimated via Keplerian motion}$$

$$\dot{E}_{\text{orb}} = -\dot{E}_{\text{GW}}$$

$$\sim 140 \left(\frac{M_c}{1.2 M_\odot} \right) \left(\frac{f}{30 \text{ Hz}} \right)^{-8/3} \text{ s}$$

\nwarrow ~lower limit LIGO is sensitive to
• orbital separation $\sim 350 \text{ km}$

Finally,

$$N_{\text{cycles}} = \int_{\text{flower}}^{\text{upper}} t_{\text{decay}} df$$

\checkmark very sensitive to freq.
low end of range

\hookrightarrow for GW170817, init. analysis from 30-2040 Hz $\rightarrow N \sim 2500$ cycles
Re-analysis from 23-2040 Hz $\rightarrow N \sim 4200$ cyc. ($\sim 60\%$ incr!)

G-5.

Impact of tides

- Presence of tides adds an additional quadrupole moment to the system \rightarrow acts to accelerate the inspiral
- Enters via the phase evolution of $h(t)$

$$h(t) = h_0 \boxed{\cos \phi(t)}$$

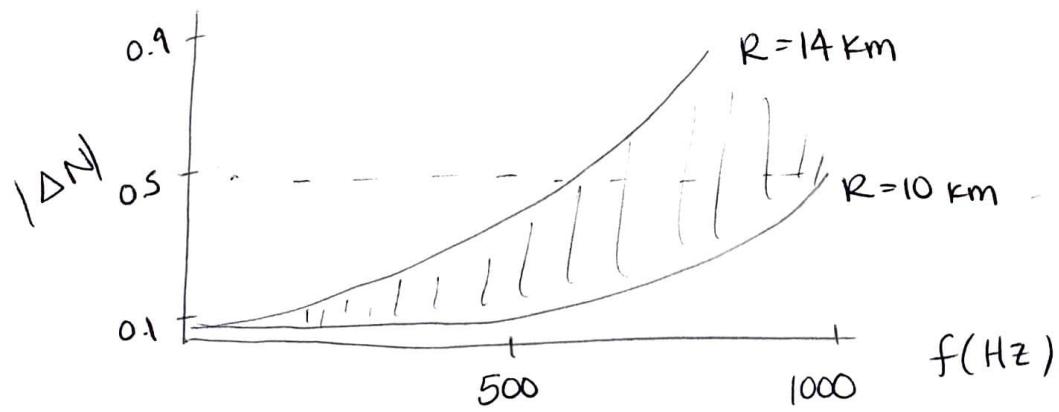
↑
~ point-particle + tidal correction

Change to # of orbital cycles, due to tides:

$$\Delta N \approx -0.01 \left(\frac{M}{1.4 M_{\odot}} \right) \left(\frac{K_2}{0.26} \right) \left(\frac{R}{10 \text{ km}} \right)^5 \left(\frac{f}{100 \text{ Hz}} \right)^{5/3}$$

↑
value for a
Newtonian, $n=1$ polytrope

cumulative phase shift:



- Larger stars produce a larger tidal effect, but still < 1 cycle!

What is K_2 ? Tidal Love number $\begin{cases} \text{Flanagan + Hinderer 2008} \\ \text{Hinderer 2010} \end{cases}$

- describes tidal polarizability of matter, assuming a static, linearized perturbation to the metric, due to an external tidal field
- depends on M , EOS ; can compute analogously to M-R relations, but for a perturb. metric

$K_2 \sim 0.05 - 0.15$
for Hadronic EOSs



G-6.

What enters the phase is not κ_2 alone, but rather combination of κ_2 , M, and R that we call the tidal deformability

$$\Lambda = \frac{2}{3} \kappa_2 \left(\frac{GM}{RC^2} \right)^{-5}$$

(enters at 5th Post-Newtonian order in the phase)

Compact NSs / small radii

\hookrightarrow small Λ , "hard" to deform

large radii NSs

\hookrightarrow large Λ , "easy" to deform

$$\phi_{\text{gw}}(t) = f(M_c, q, f_{\text{gw}}, t_c, \phi_c, \tilde{\Lambda})$$

↑
binary avg. of the component Λ 's

$$\tilde{\Lambda} = \frac{16}{13} \left[\frac{(12m_2 + m_1)m_1^4 \Lambda_1}{(m_1 + m_2)^5} + 1 \leftrightarrow 2 \right]$$

\Rightarrow slides