



PhD School on QCD in Extreme Conditions

Lattice Field Theory for Extreme QCD - Part 2

Maria Paola Lombardo

INFN Firenze lombardo@fi.infn.it



Lattice Field Theory for Extreme QCD - Part 2

Symmetry and pattern of breaking from low to high temperature

Yang Mills

Massless QCD - light quarks

Scaling window

Interplay of chiral symmetry and confinement?

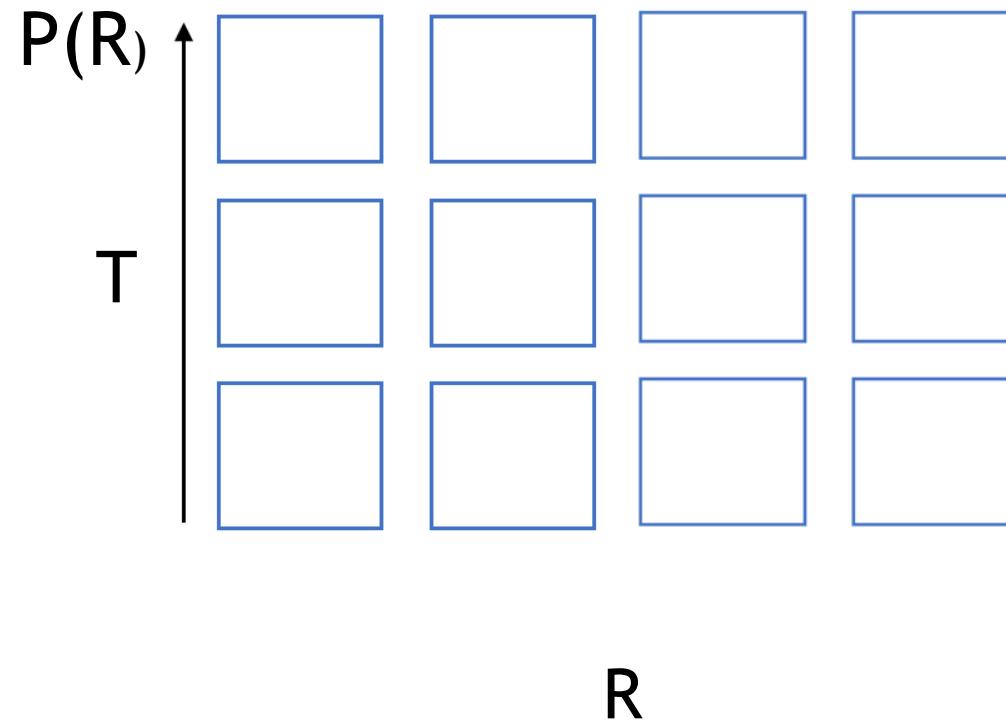
Aspects of the high temperature phase

Extended symmetry above TC?

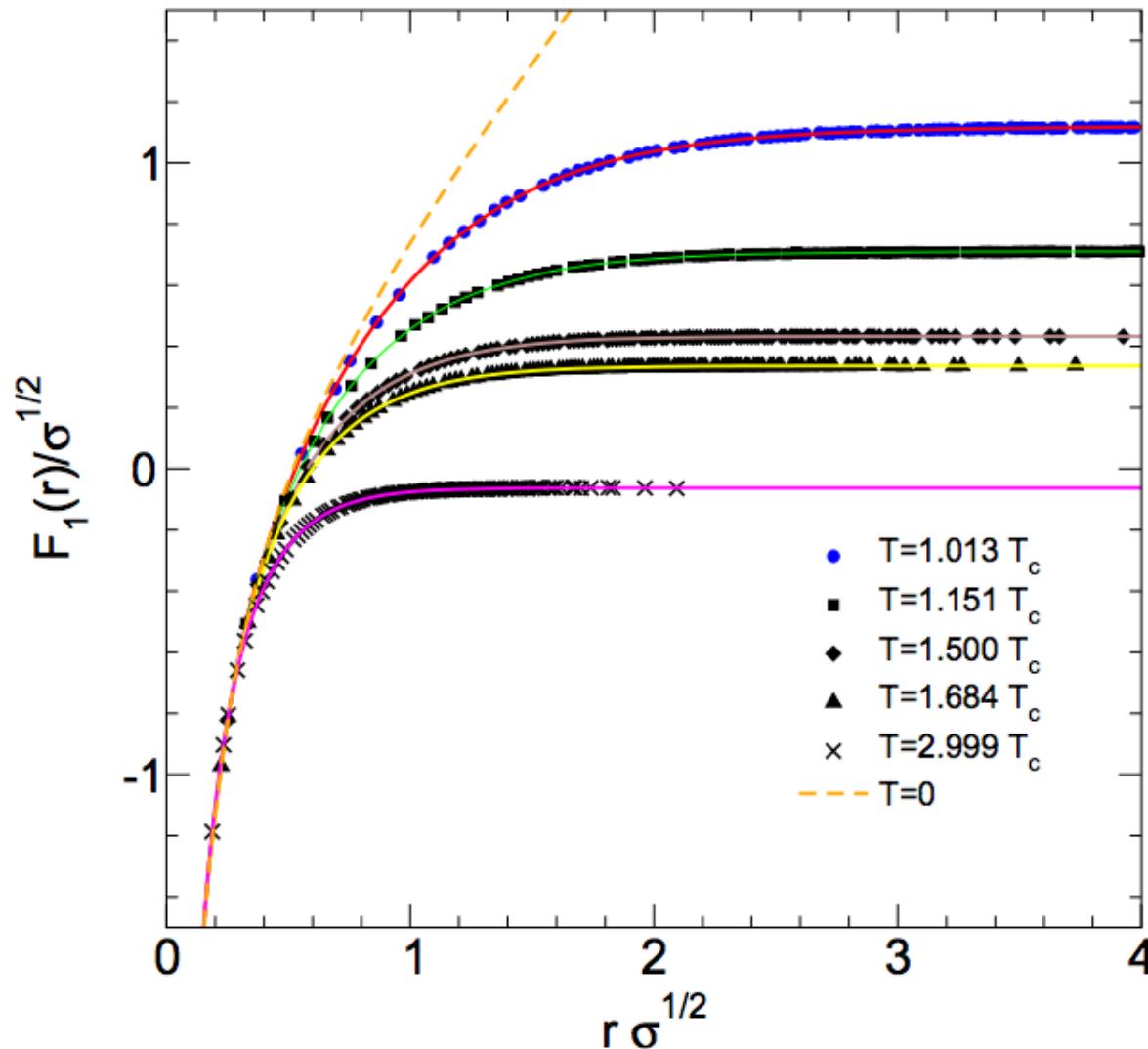
Heavy quarkonium → covered at the beginning of Part 3

Two point functions of Polyakov loop: alternative extraction of the potential

$$e^{-V(R,T)/T} \propto \langle P(\vec{0})P^\dagger(\vec{R}) \rangle \xrightarrow[R \rightarrow \infty]{} \propto e^{-\sigma R}$$

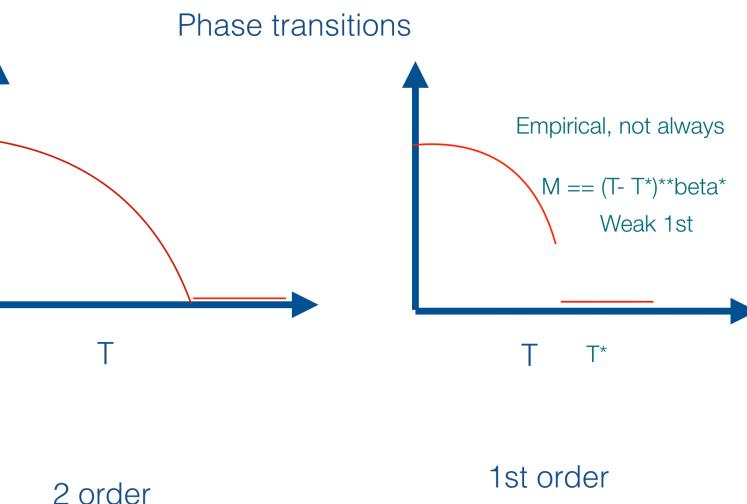


Temperature effects : no asymptotic string tension at high T



$T \rightarrow T_c : \sigma \rightarrow 0 ?$
 $\xi \rightarrow \infty ?$

or, rather, first order?

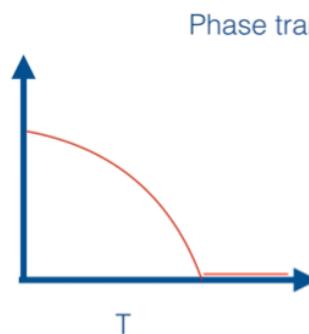


Polyakov loop as an order parameter for confinement in Yang-Mills

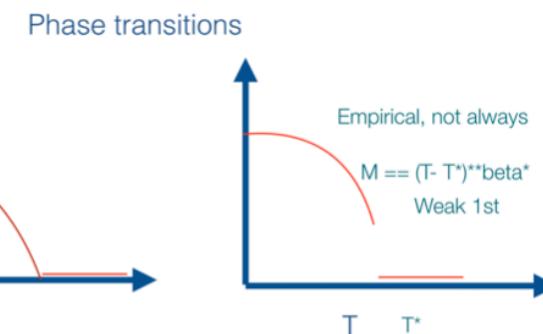
$$|P| = e^{-F/T}$$

$$|P| = 0 \rightarrow \text{no free quarks}$$

Same question as before:



2 order



1st order

Symmetry aspects of the confinement transition

For gauge transformations, periodicity up to an element of the center suffices : center symmetry

But: Polyakov loop is not invariant under center transformation

→ center transformation have physical content

Polyakov loop order parameter for center symmetry

Polyakov loop order parameter for confinement in Yang-Mills

Parametro d'ordine per $Z(N_c)$.

→ Confinement as a symmetry

For two colours: $Z(2)$: Ising model!!

→ Universality

'Build' Polyakov loop system: it is a cube of spins!!

→ dimensional reduction at T_c

Comment:

This reasoning concerns confinement of degrees of freedom (the quarks) that do not really exist in this theory!

Scaling and Universality : general aspects of 2nd order PTs

Magnetic
Equation of State

$$\frac{h}{M^\delta} = f\left(\frac{T - T_c}{M^{1/\beta}}\right)$$

Contains
Standard definition
Of critical exponents

For $h=0$ $M = A\left(\frac{T - T_c}{T_c}\right)^\beta$

For $T = T_c$ $M = h^{1/\delta}$

Universality

Equation of State,
hence
Critical Exponents

are determined by
global symmetries and dimensionality

Rationale:
diverging correlation length

In general:

	Magnets	Bosons	Fermions	Pol. Spins
External Field	h (magnetic field)	m (bare mass)	m (bare mass)	
Response Function	M (magnetization)	$\langle \sigma \rangle (m)$	$\langle \bar{\psi} \psi \rangle (m)$	
Order Parameter	$M_0 = M(h=0)$	$\langle \sigma \rangle (m=0)$	$\langle \bar{\psi} \psi \rangle (m=0)$	

$$\frac{h}{M^\delta} = f\left(\frac{T - T_c}{M^{1/\beta}}\right)$$

Symmetry of Yang-Mills

3-dimensional system of Polyakov loops

$SU(3)$: Z_3 center symmetry breaking at $T_c \rightarrow$ 1st order 3 state Potts model

$SU(2)$: Z_2 center symmetry breaking at $T_c \rightarrow$ 2nd order 2 state Potts model (Ising)

(a) Potts Model :



$Q=2$ (Ising model)

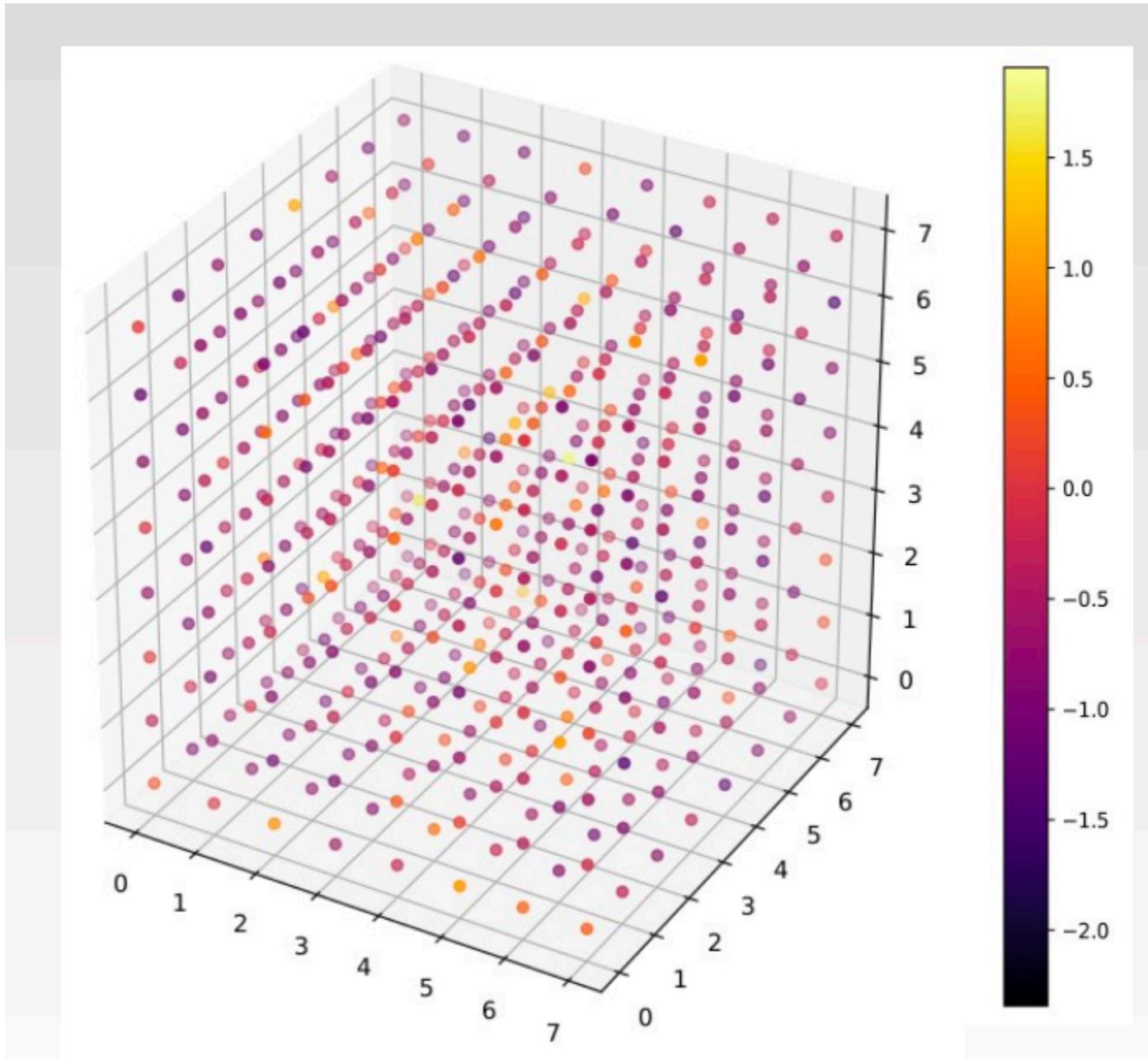


$Q=3$



$Q=4$

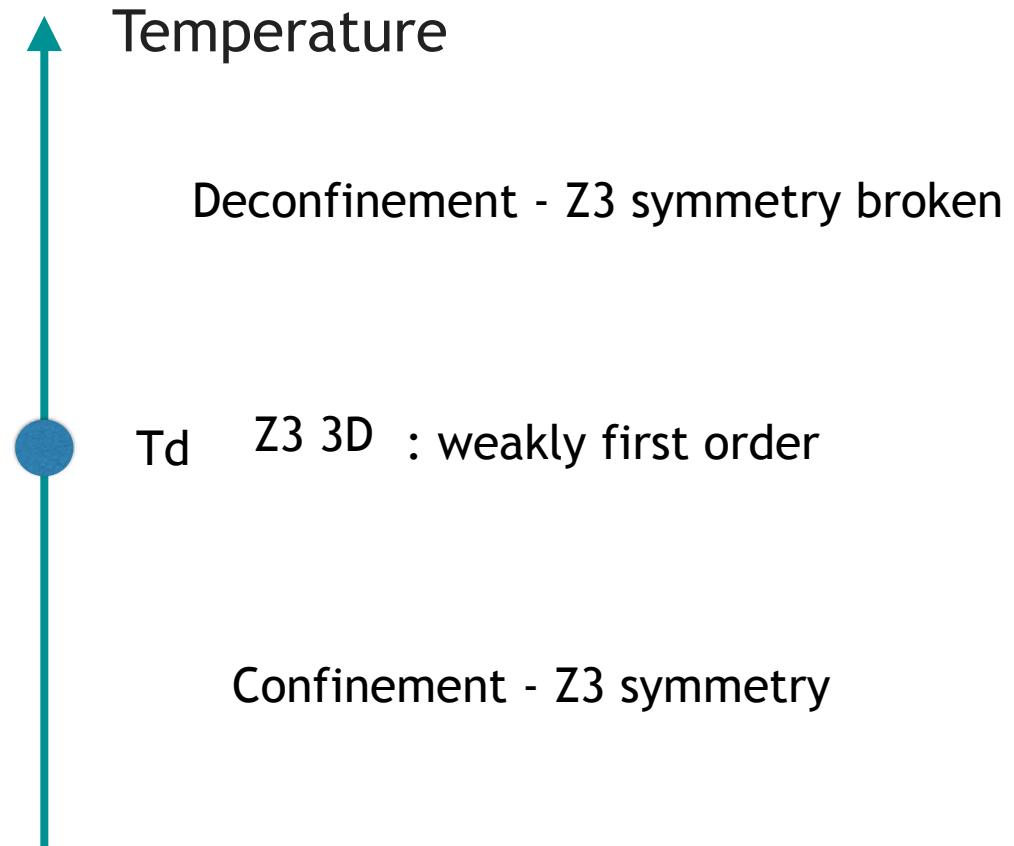
$|P|$ in a small
Lattice



Yang-Mills Theory $N_c = 2$ – Ising 3D universality class

Source		$SU(2)$	Ising
$\langle L \rangle$ $D\langle L \rangle$	β/ν	0.525(8)	0.518(7)
	$(1 - \beta)/\nu$	1.085(14)	1.072(7)
	$1/\nu$	1.610(16)	1.590(2)
	ν	0.621(6)	0.6289(8)
	β	0.326(8)	0.3258(44)
χ_ν $D\chi_\nu$	γ/ν	1.944(13)	1.970(11)
	$(1 + \gamma)/\nu$	3.555(15)	3.560(11)
	$1/\nu$	1.611(20)	1.590(2)
	ν	0.621(8)	0.6289(8)
	γ	1.207(24)	1.239(7)
	$\gamma/\nu + 2\beta/\nu$	2.994(21)	3.006(18)
g_r Dg_r $(\omega = 1)$	$-g_r^\infty$	1.403(16)	1.41
	$1/\nu$	1.587(27)	1.590(2)
	ν	0.630(11)	0.6289(8)

Phases of Yang-Mills SU(3)



Symmetries of QCD

$$\mathcal{L} = \sum_{a=1}^n \bar{q}_{La} \not{\partial} q_{La} + \bar{q}_{Ra} \not{\partial} q_{Ra} - m(\bar{q}_{La} q_{La} + \bar{q}_{Ra} q_{Ra}) + \theta \frac{g^2}{32\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} + \mathcal{L}_{gauge}$$

With $m = 0$, invariant under

$q_L \rightarrow V_L q_L q_R \rightarrow V_R q_R$, with $V \in U(n)$

Global symmetry:

$$U(n)_L \times U(n)_R \cong \underbrace{SU(n) \times SU(n)}_{\cdot} \times U(1)_V \times U(1)_A$$



baryon
number



Spontaneously Broken

Explicitly broken

Experimental evidence

$(\frac{n}{2} - 1)$ pseudoGB

Heavy η'

Explicit breaking of axial symmetry: solution of the UA(1) puzzle

Pseudoscalar spectrum

Particle name	Particle symbol	Antiparticle symbol	Quark content	Rest mass (MeV/c ²)
Pion ^[6]	π^+	π^-	$u\bar{d}$	$139.570\ 18 \pm 0.000\ 35$
Pion ^[7]	π^0	Self	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$ [a]	134.9766 ± 0.0006
Eta meson ^[8]	η	Self	$\frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}}$ [a]	547.862 ± 0.018
Eta prime meson ^[9]	$\eta'(958)$	Self	$\frac{u\bar{u}+d\bar{d}+s\bar{s}}{\sqrt{3}}$ [a]	957.78 ± 0.06
Kaon ^[12]	K^+	K^-	$u\bar{s}$	493.677 ± 0.016
Kaon ^[13]	K^0	\bar{K}^0	$d\bar{s}$	497.614 ± 0.024

UA(1) puzzle



Universality class of the high T transition:

theoretical (*lack of*) guidance

Parisen Toldin, Pelissetto, Vicari 2003

N_f	U(1) _A broken	U(1) _A restored
1	crossover or 1 st ord	$O(2) \rightarrow \mathbb{Z}_2$ or 1 st ord
2	$O(4) \rightarrow O(3)$ or 1 st ord Pisarski, Wilczek 1984	$U(2)_L \otimes U(2)_R \rightarrow U(2)_V$ or 1 st ord
≥ 3	1 st ord ?	1 st ord

Challenged by Cuteri, Philipsen, Sciarra 2020-2022

$N_f = ?$

T=0, no difference, just different #Goldstones

$$m_{u,d} = 0$$

$$N_f = 3$$

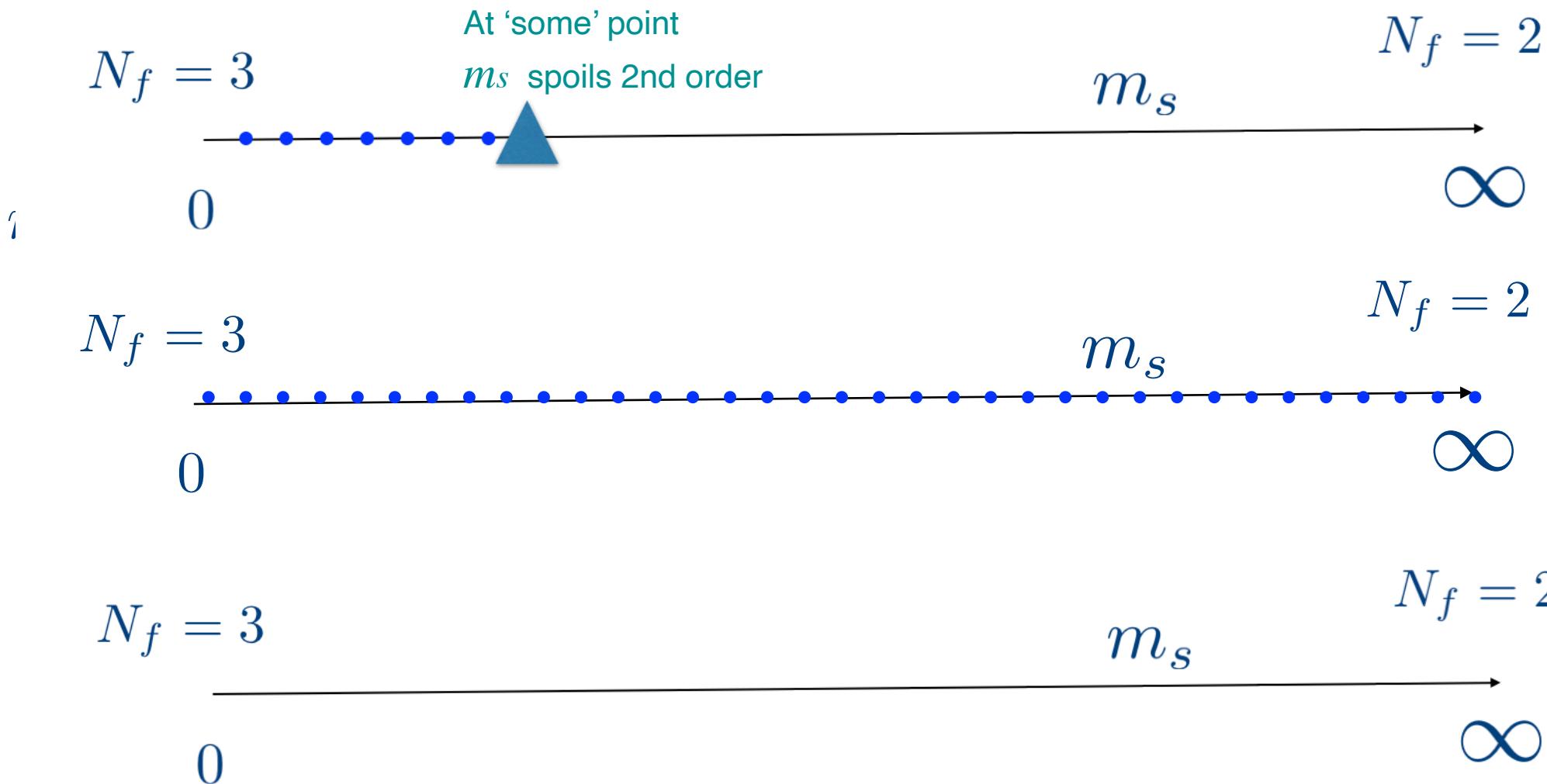
$$0$$

$$m_s$$

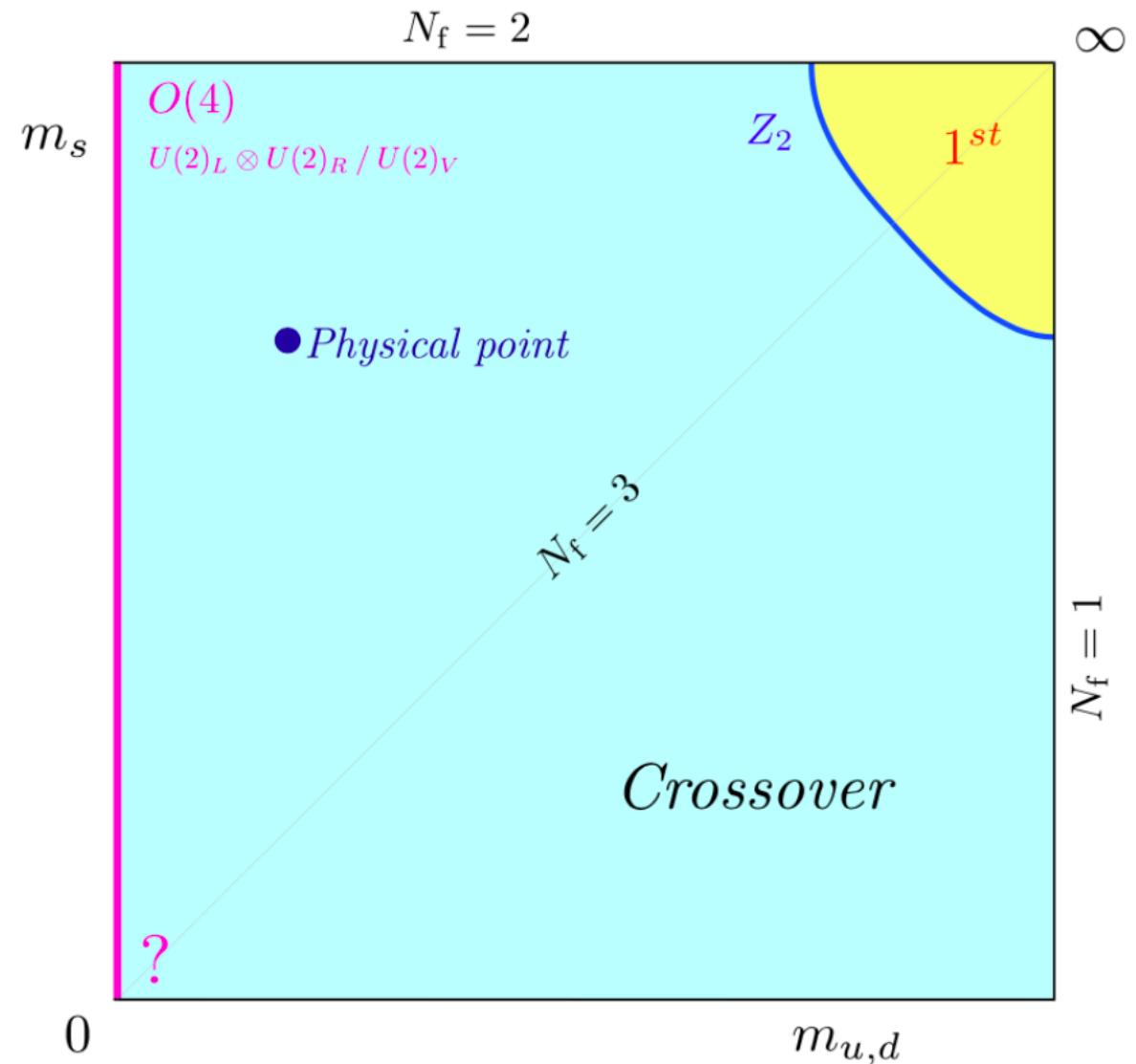
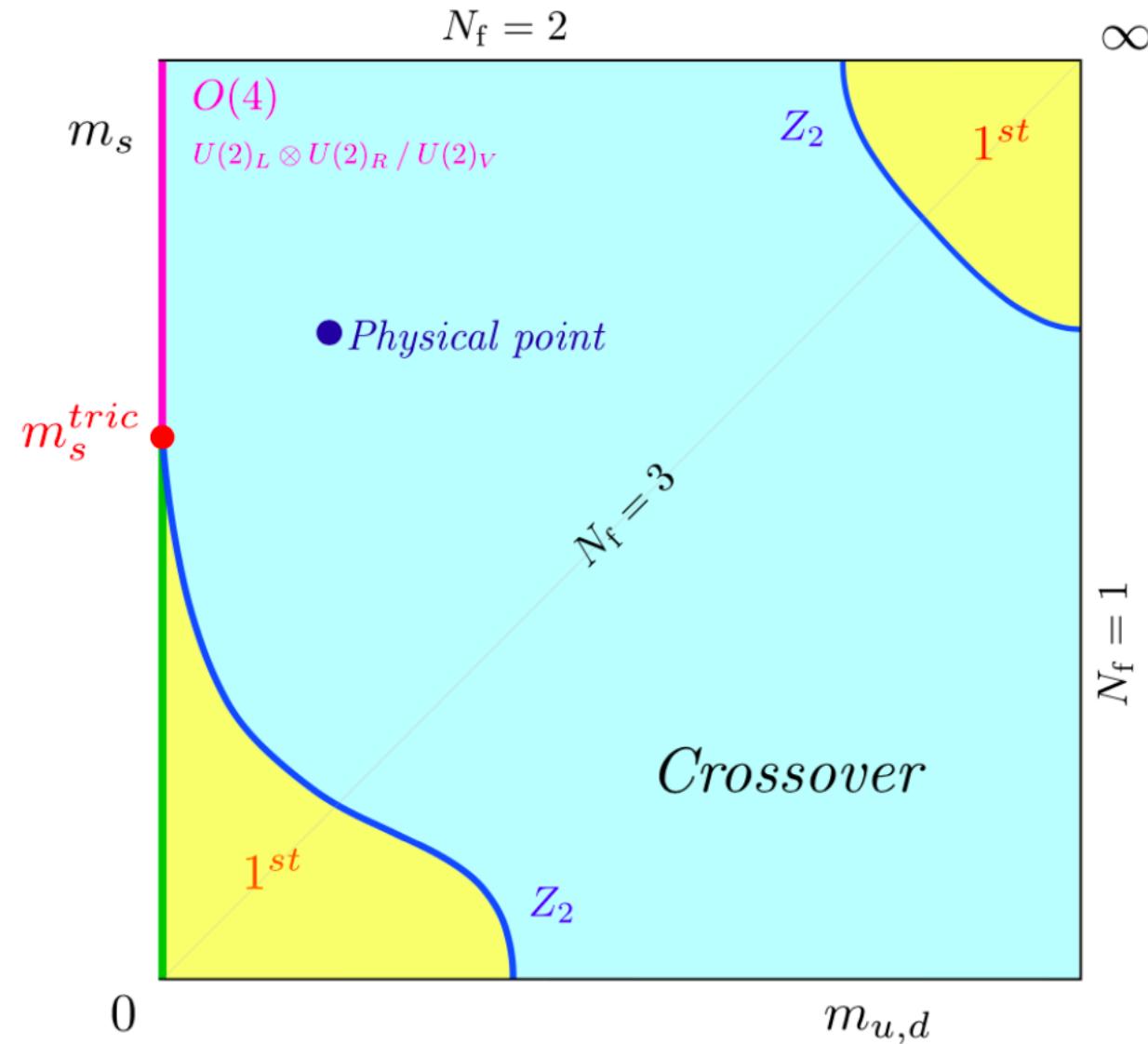
$$N_f = 2$$

$$\infty$$

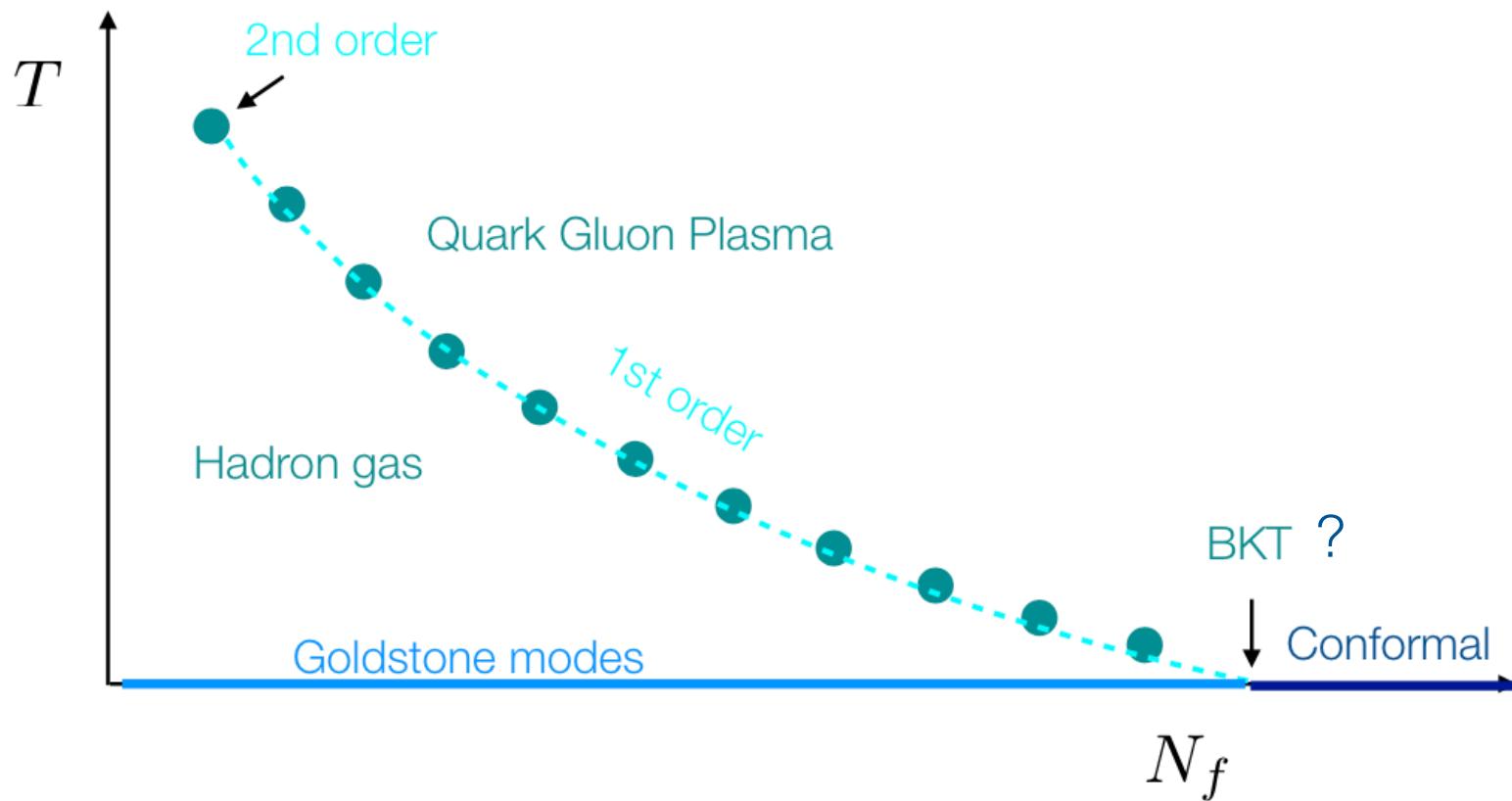
Strange mass as interpolator between $N_f=3$ and $N_f=2$ at high T



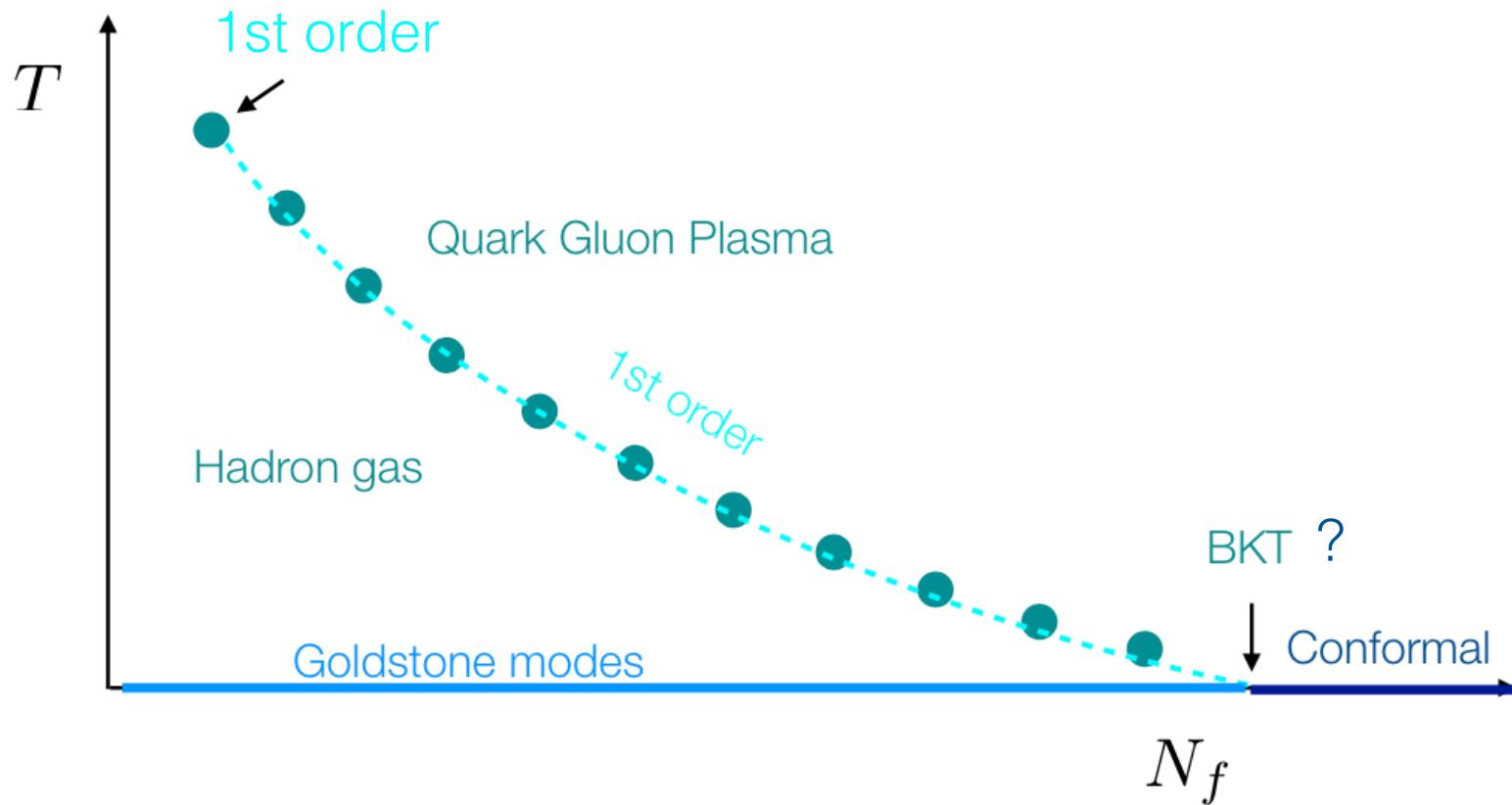
Old and New scenario



Beyond Nf=3 - Scenario 1

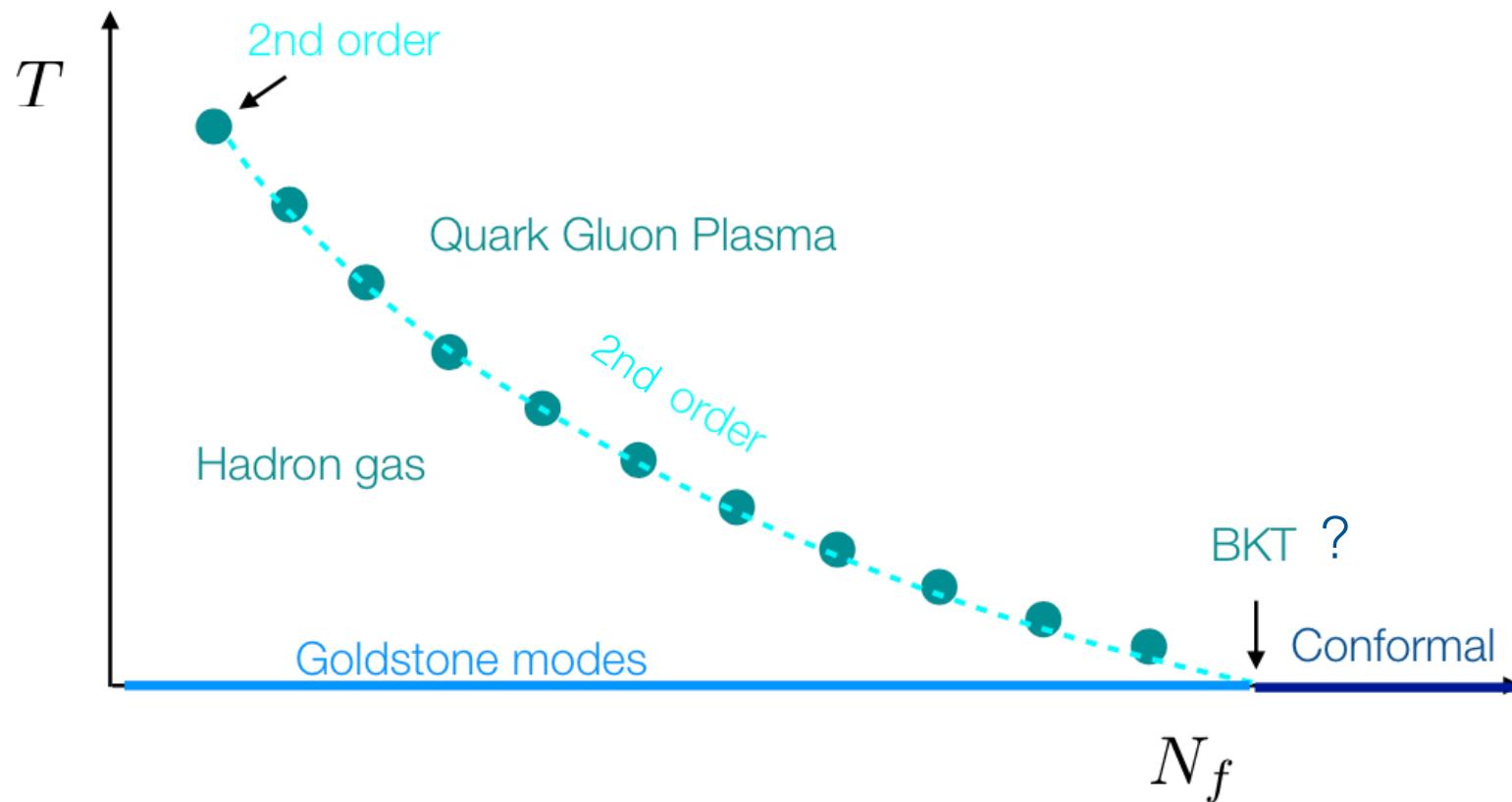


Beyond Nf=3 - Scenario 2



Beyond Nf=3 - Scenario 3

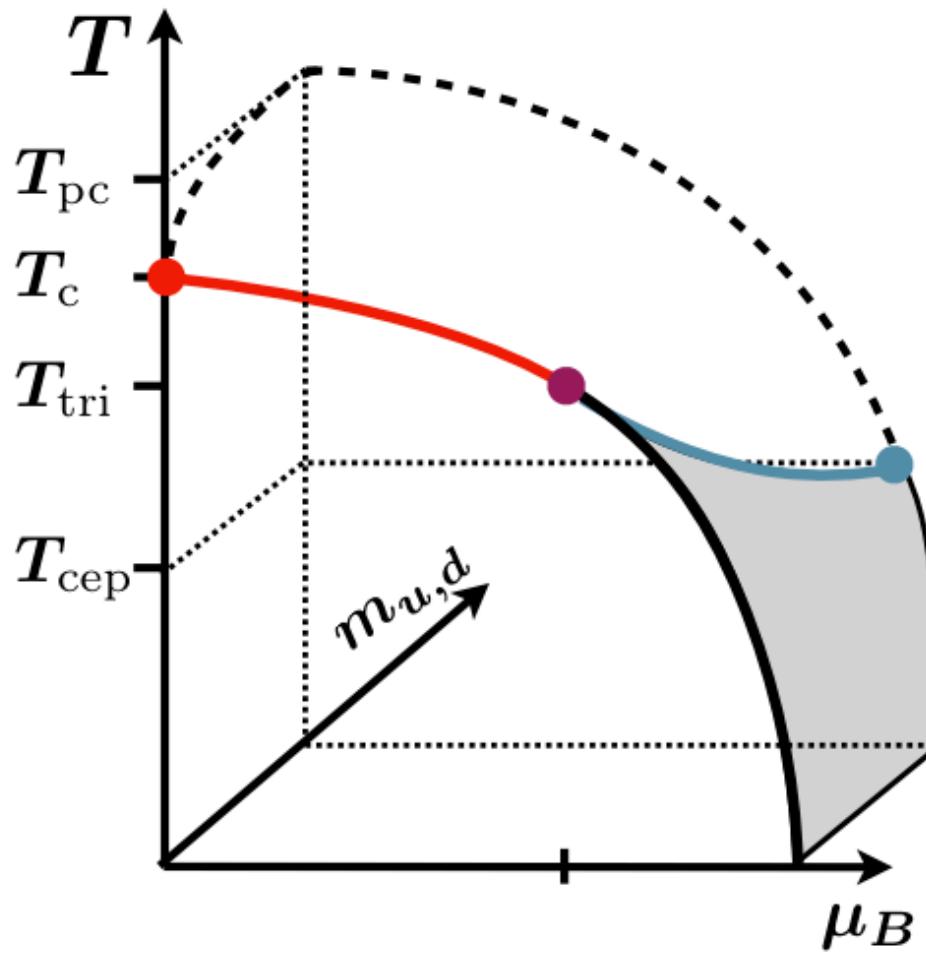
Cuteri, Philipsen, Sciarra



*How “far”
in mass
and temperature
does T_c influence the
QGP?*

Possible answer:
within the scaling window
of the theory

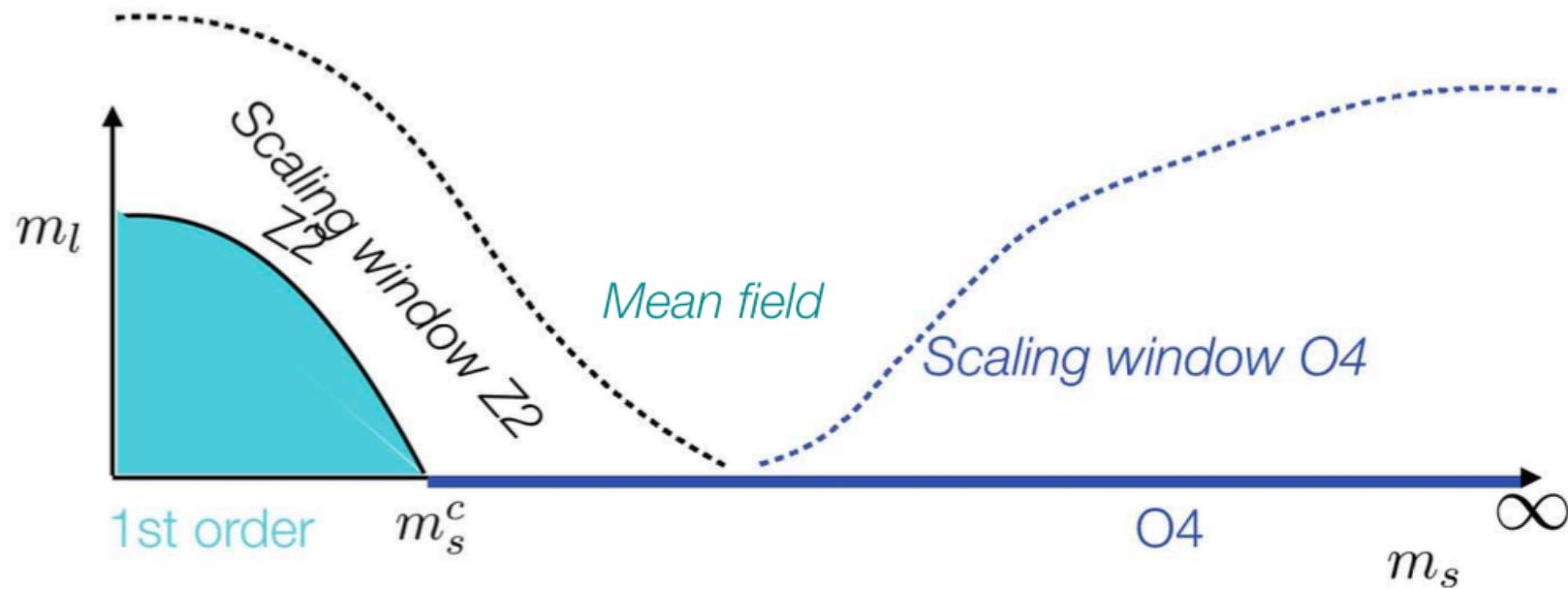
$$T_c \simeq 132 \text{ MeV}$$



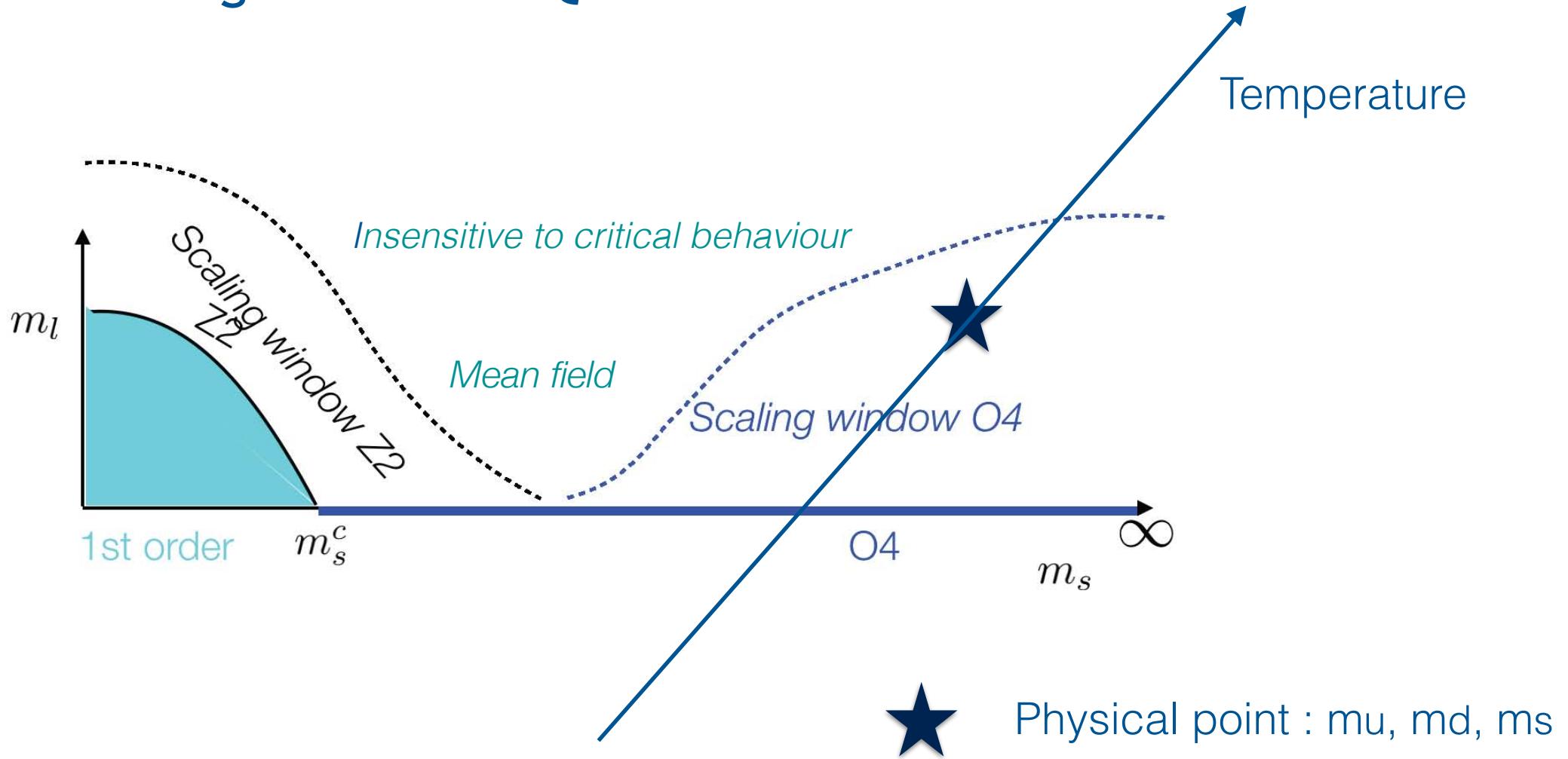
Karsch 2019

Byproducts of the study of the scaling window:
Universality class at T_c . Value of T_c , upper bound to T_{cep}

Switching on the light mass: a possible Scenario 1



Where is the scaling window in QCD in mass and T?



The magnetic equation of State:

$$h = M^\delta f(t/M^{1/\beta}).$$

$M \equiv \bar{\psi}\psi$, $h \equiv m_q$, $t \equiv T - T_c$, m_q is the quark mass and T_c is the critical temperature

Three strategies to identify the scaling behaviour:

- direct comparison with the Equation of State
- the study of the dependence of the pseudo-critical temperatures on the breaking field, also known as scaling of pseudo-critical temperatures
- definition of RG invariant quantities, which do not depend on the breaking field at the critical point.

Byproduct: critical temperature in the chiral limit

Significant source of scaling violations:

additive linear mass corrections to $\bar{\psi}\psi$

Variation on the EoS

also mentioned in the PhD thesis by Wolfgang Unger

'Beating' the regular terms/additive renormalization
for more stringent universality checks

$$\Delta_3 \equiv (\bar{\psi}\psi - m\chi_L) \equiv (\bar{\psi}\psi - m \frac{\partial \bar{\psi}\psi}{\partial m}) \equiv m(\chi_T - \chi_L)$$

Kotov, MpL, Trunin

Transverse and longitudinal susceptibilities

$$\chi_T = \frac{\bar{\psi}\psi}{m}$$

$$R_\pi \equiv \chi_T^{-1}/\chi_L^{-1}$$

$$\frac{1}{R_\pi(t, m)} = \delta - \frac{x}{\beta} \frac{f'(x)}{f(x)},$$

$$\chi_L = \frac{\partial \bar{\psi}\psi}{\partial m}.$$

$$R_\pi(0, m) = \frac{1}{\delta}$$

Kocic, Kogut, MpL;
Karsch, Laermann

Equation of State for Δ_3

- linear terms in m drop in

$$\Delta_3 \equiv (\bar{\psi}\psi - m\chi_L) \equiv (\bar{\psi}\psi - m \frac{\partial \bar{\psi}\psi}{\partial m})$$

Use: $M = h^{1/\delta} f_G(t/h^{1/\beta\delta})$ (parametrization in:

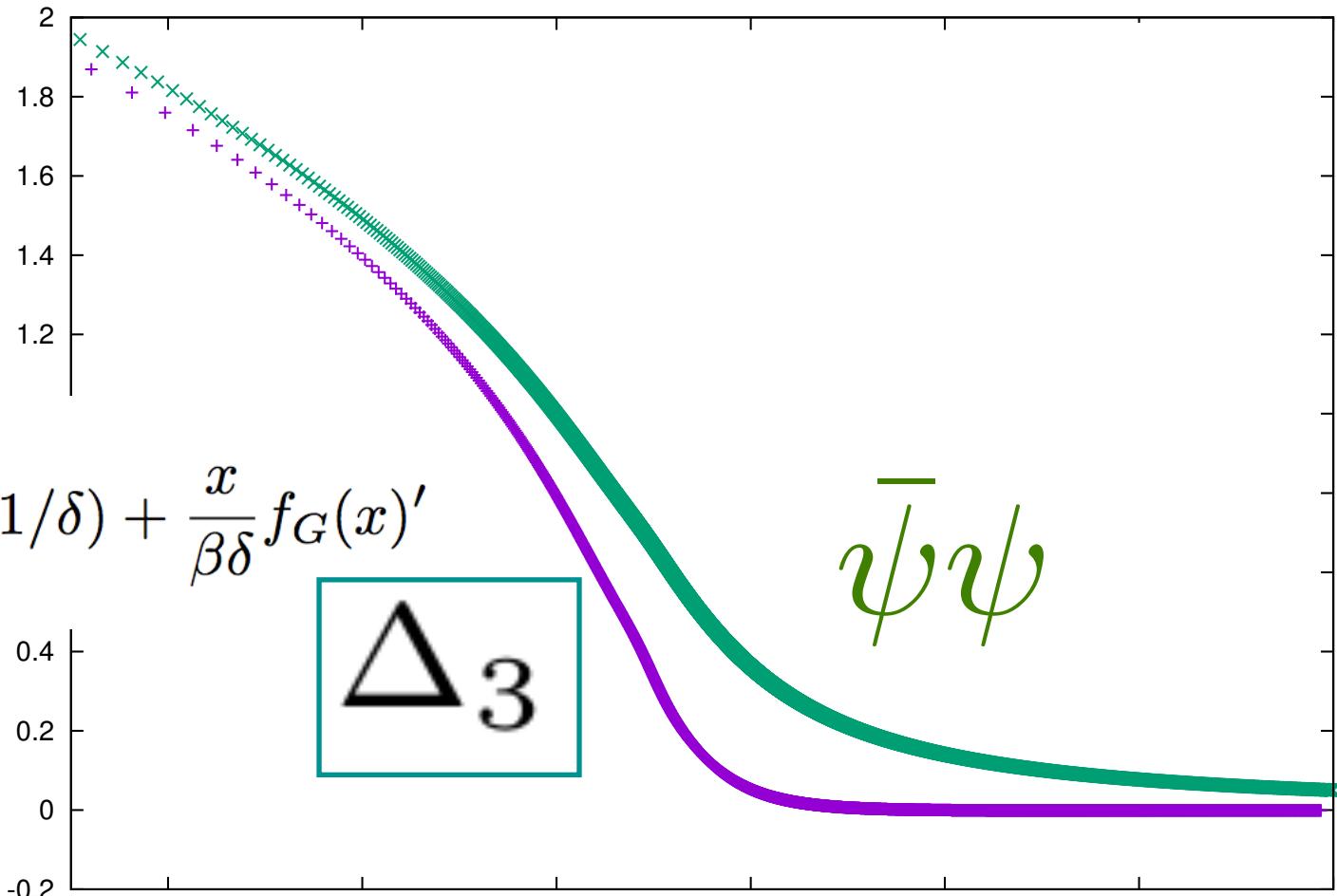
J.Engels and F.Karsch, Phys. Rev. D 85, (2012)

To get EoS for Δ_3

$$\Delta_3 = m^{1/\delta-1} f_G(t/m^{1/\beta\delta}) - 1/\delta m^{1/\delta-1} f_G(t/m^{1/\beta\delta}) + m^{1/\beta\delta+1} f'_G((t/m^{1/\beta\delta})$$

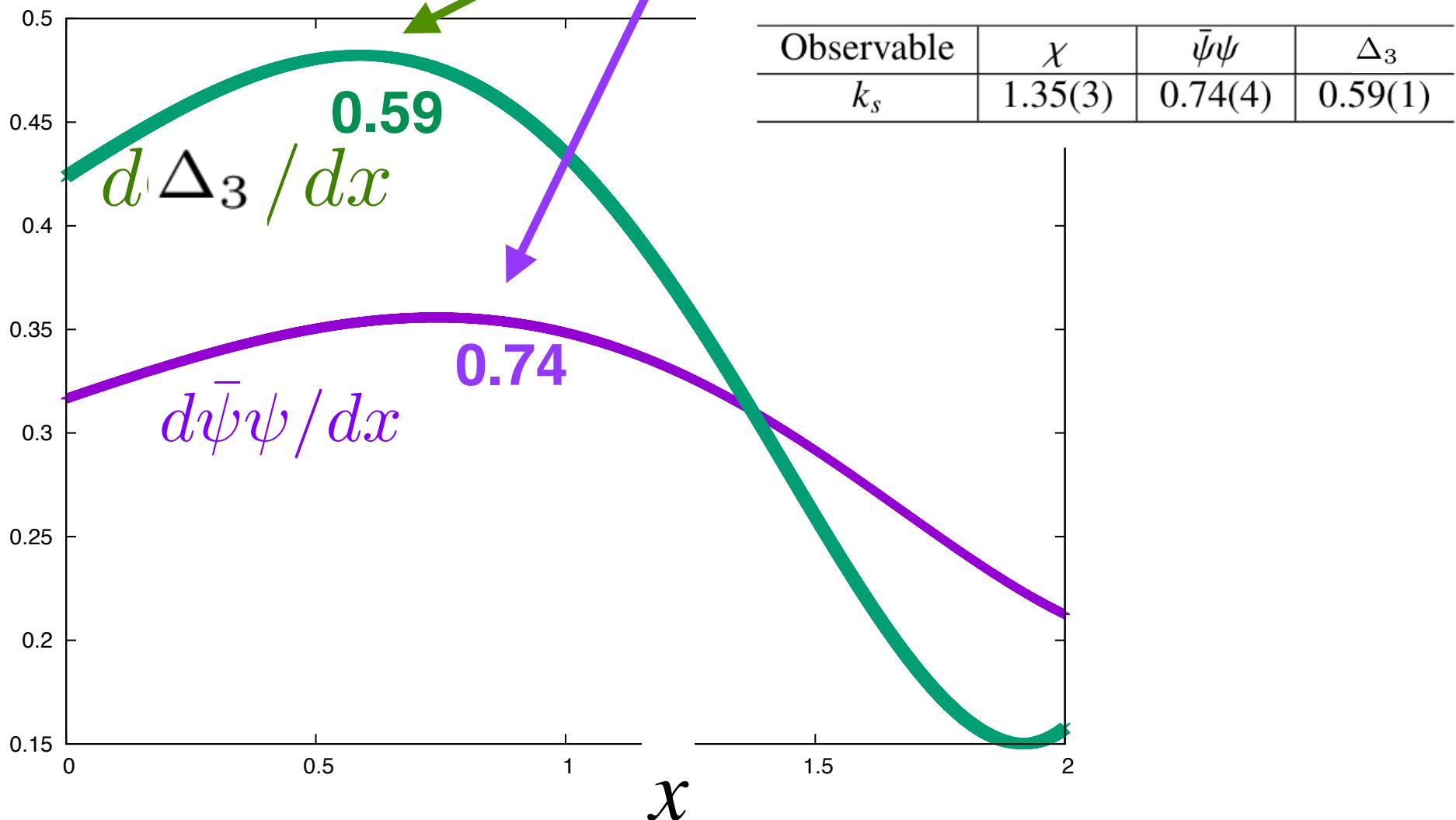
$$\frac{\Delta_3}{m^{1/\delta}} = f_G(x)(1 - 1/\delta) + \frac{x}{\beta\delta} f'_G(x)$$

$$\frac{\Delta_3}{m^{1/\delta}} = f_G(x)(1 - 1/\delta) + \frac{x}{\beta\delta} f_G(x)' \quad x = t/h^{1/\beta\delta}$$



Derivatives:
give scaling
of pseudo
critical
temperature
 T_c
with mass

$$T_c = T_c(0) + k_s m_\pi^{2/\beta\delta}$$

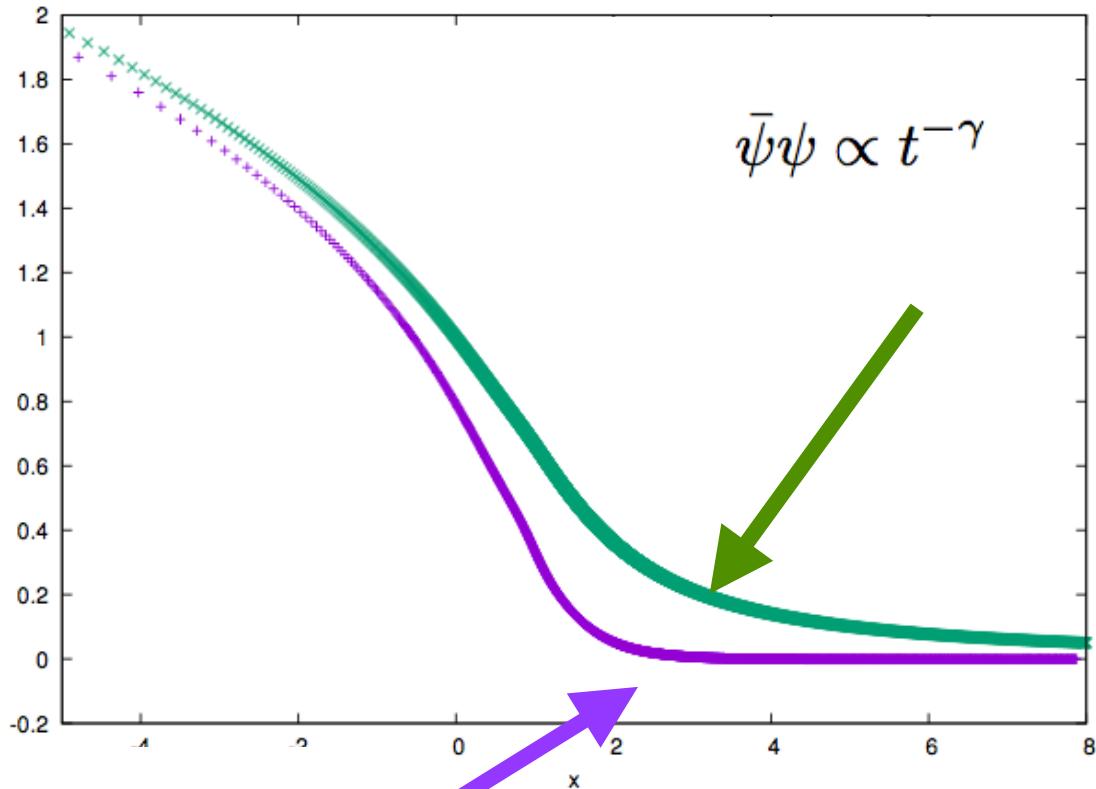


Asymptotic behavior - high T expansion

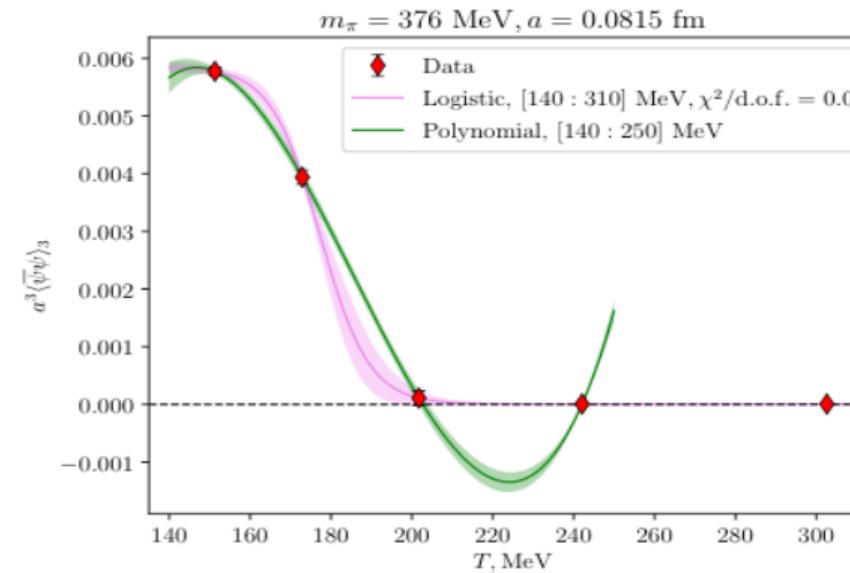
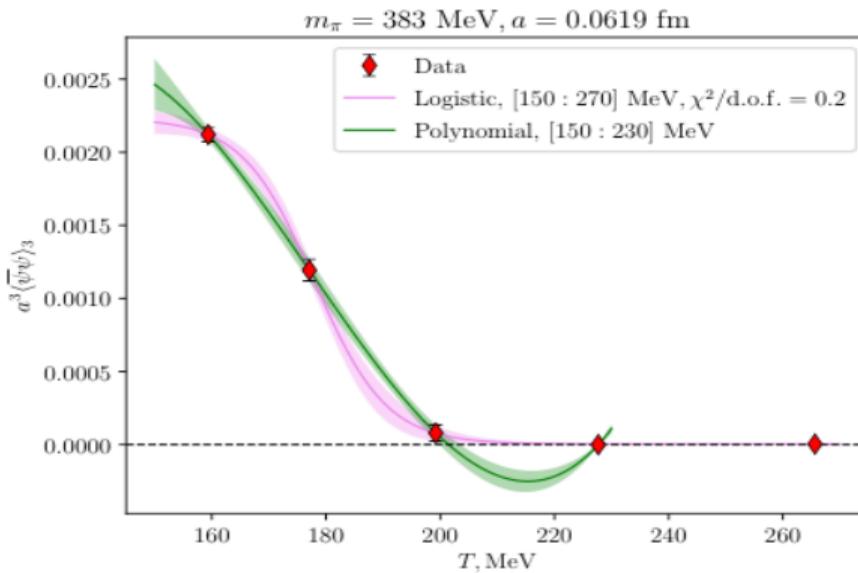
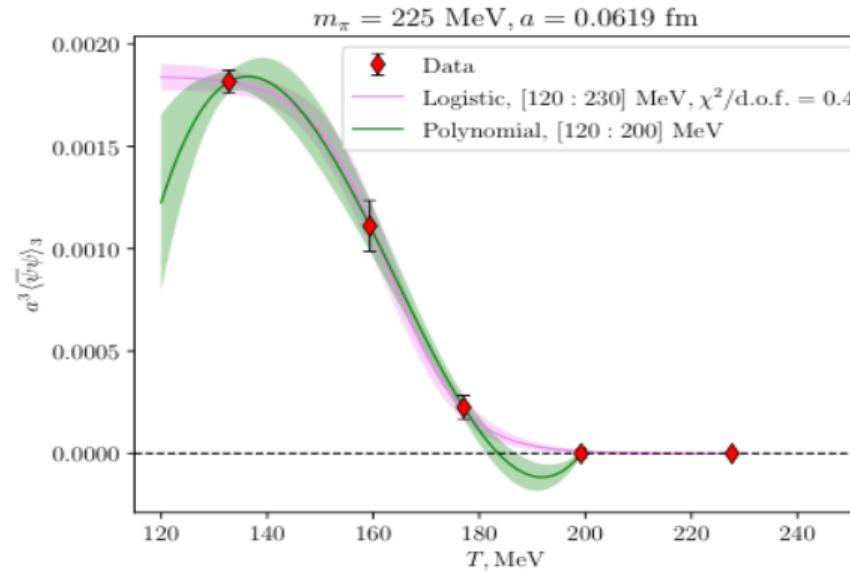
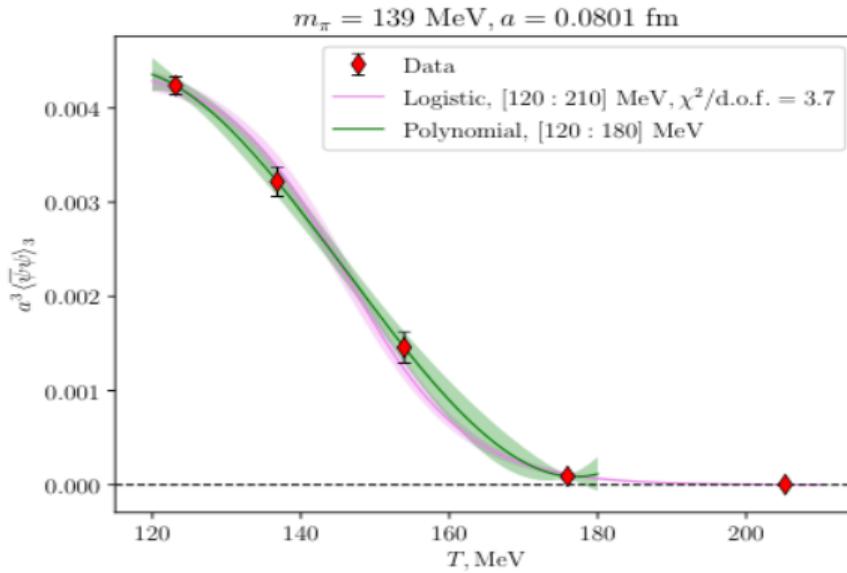
$$f_G(x) = x^{-\gamma} \sum_{n=0}^{\infty} d_n x^{-2n\Delta}$$

again, linear term
drops in Δ_3

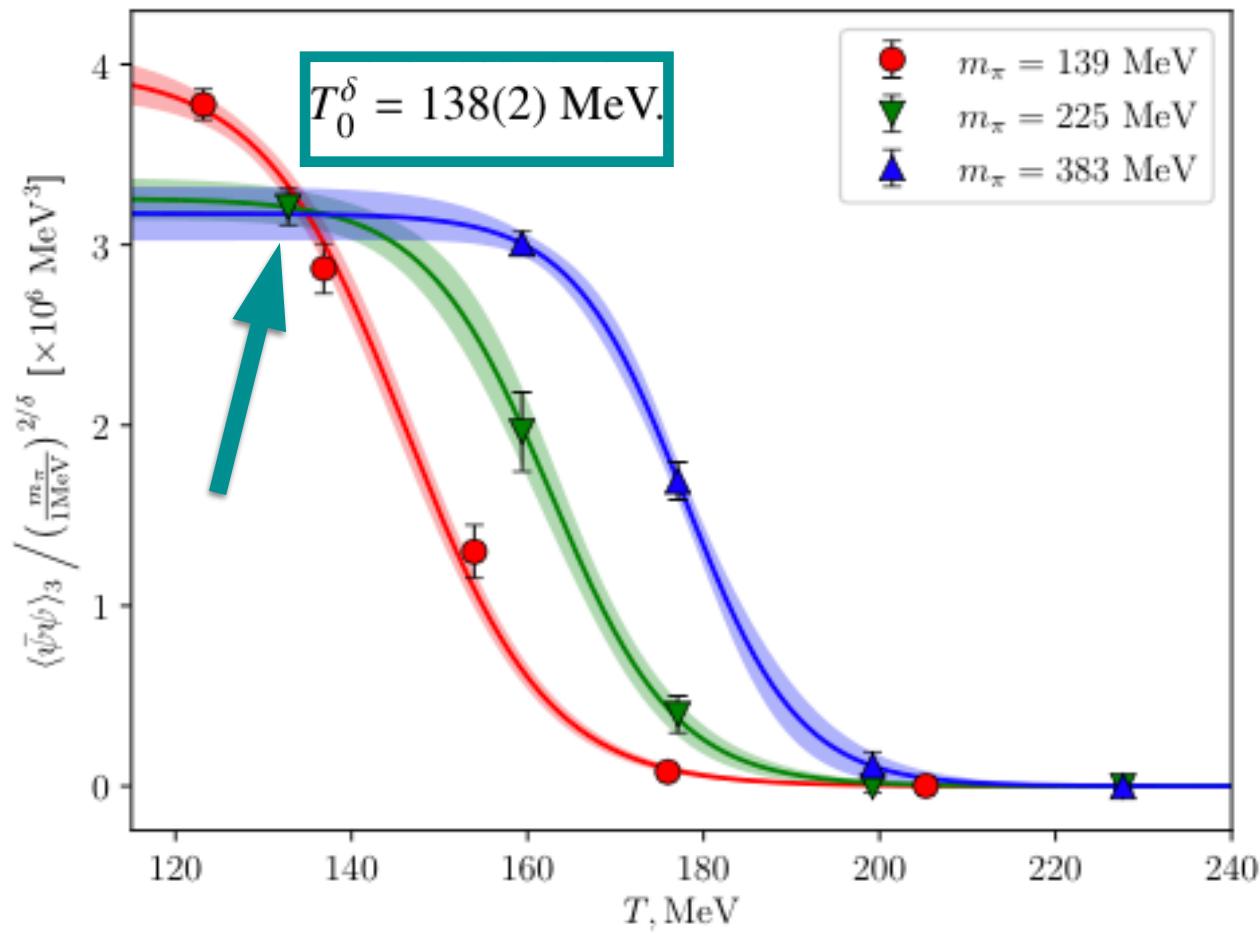
$$\Delta_3 \propto t^{-\gamma-2\beta\delta}$$



Bare Δ_3



Scaling at the critical point: searching for $\langle \bar{\psi}\psi \rangle_3 (T = T_0) = A m_\pi^{2/\delta}$

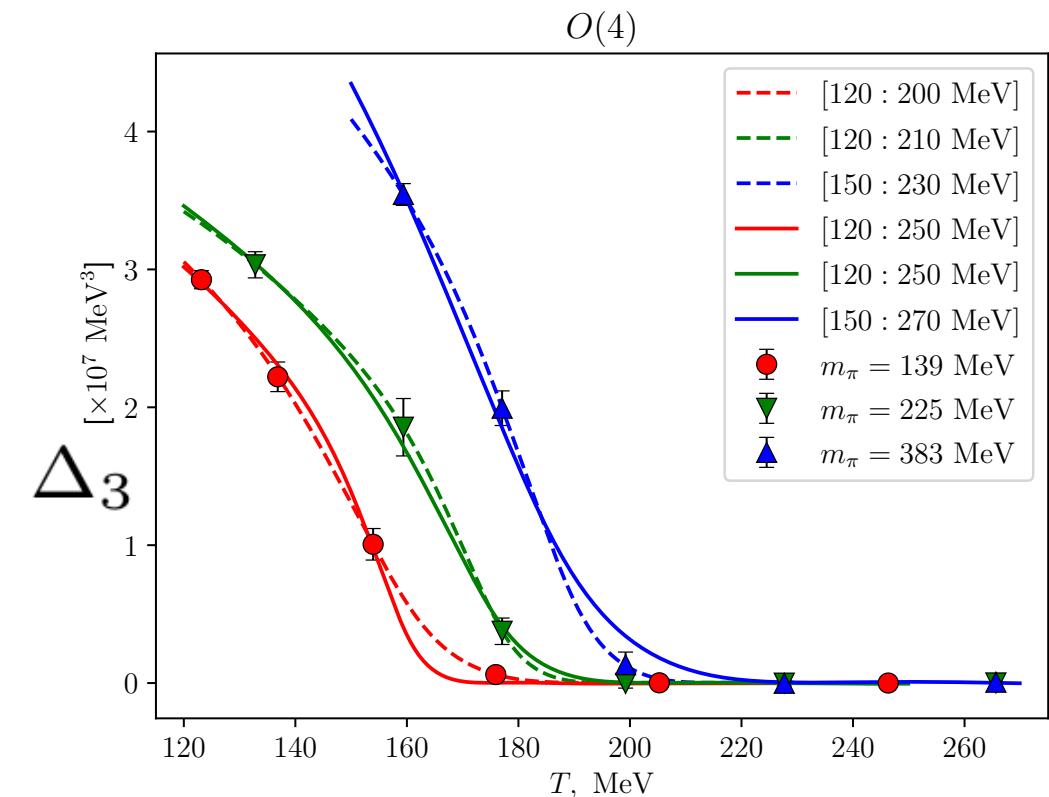
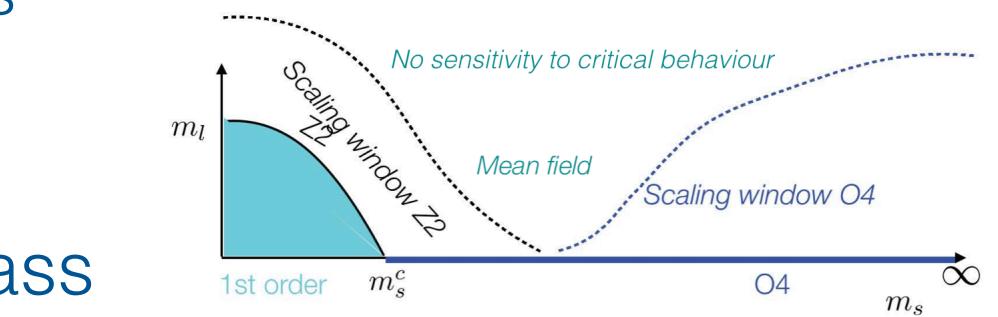
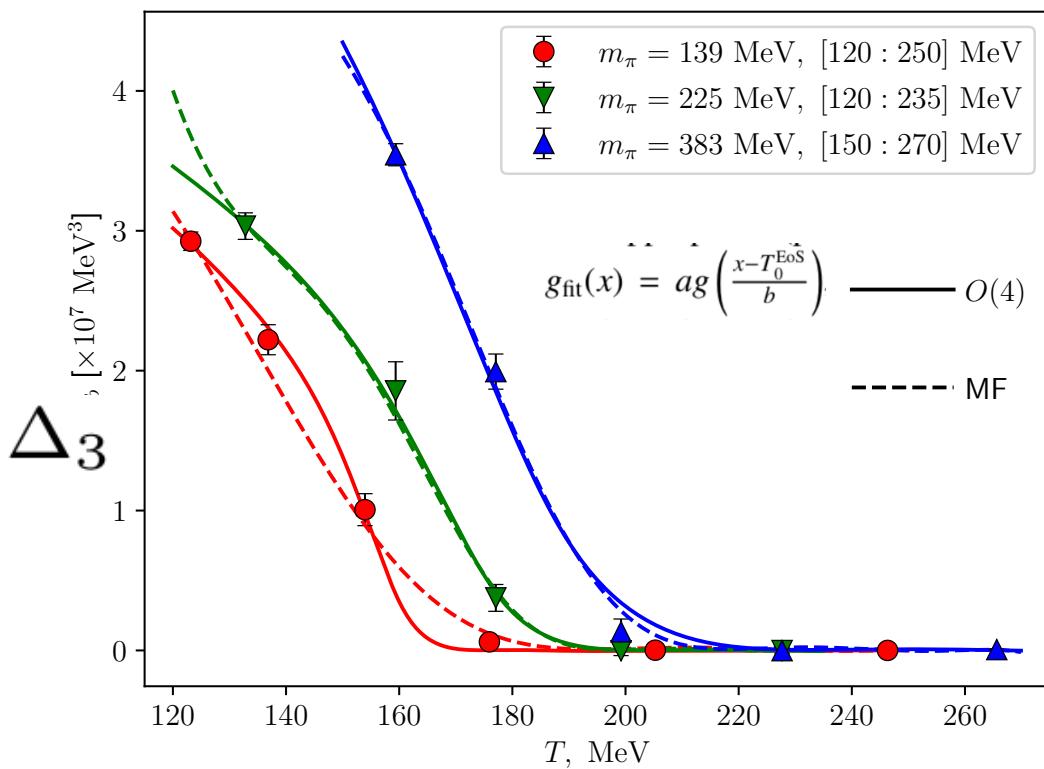


Searching for the scaling window in mass

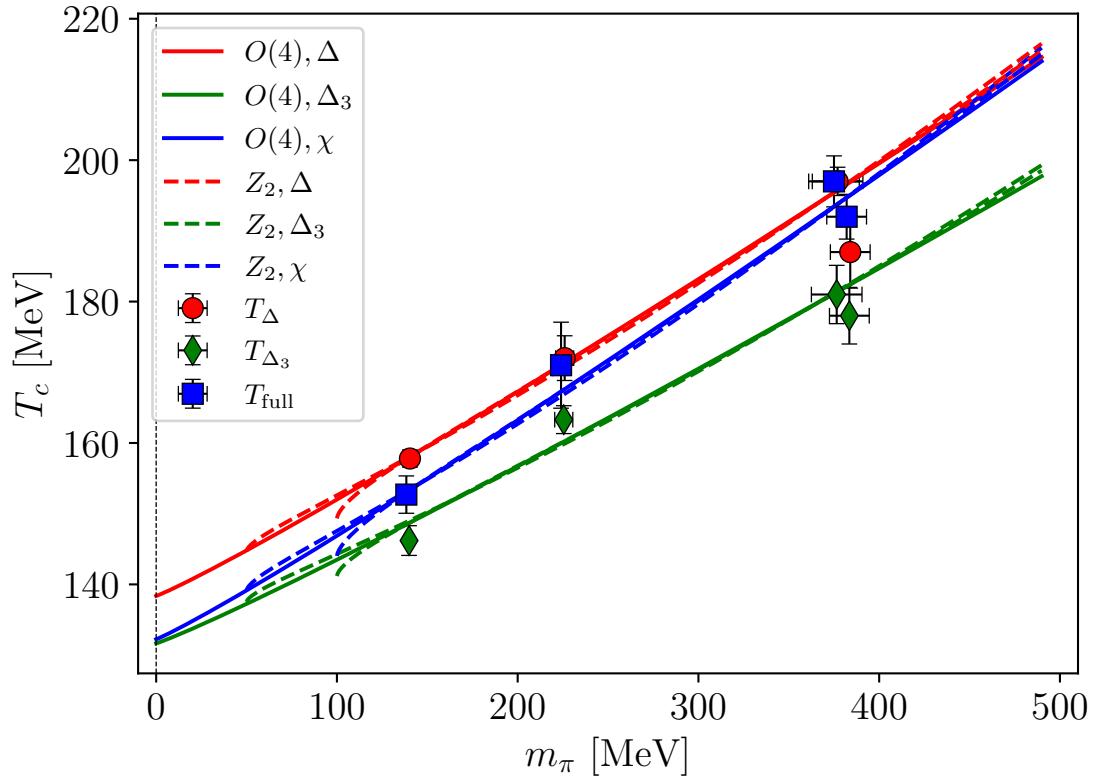
O(4) or mean field?

Unrealistic T_0 from O4 at high mass

$$T_{\text{EOS}} = 142(2), 159(3), 174(2) \text{ MeV}$$



Scaling of the pseudo critical temperatures



Consistent (not a proof) with O4

Robust extrapolation:

$$T_0 \equiv T_c(m_\pi \rightarrow 0) = 134^{+6}_{-4} \text{ MeV}$$

Check O4:

$$T_c(m_\pi) = T_0 + A z_p m_\pi^{2/\beta\delta}$$

Observable	T_0 [MeV]	$z_p/z_{\bar{\psi}\psi_3}$	$z_p/z_{\bar{\psi}\psi_3}$ O(4)	z_p O(4)
χ	132(4)	1.24(17)	2.45(4)	1.35(3)
$\langle\bar{\psi}\psi\rangle$	138(2)	1.15(24)	1.35(7)	0.74(4)
$\langle\bar{\psi}\psi\rangle_3$	132(3)	1	1	0.55(1)

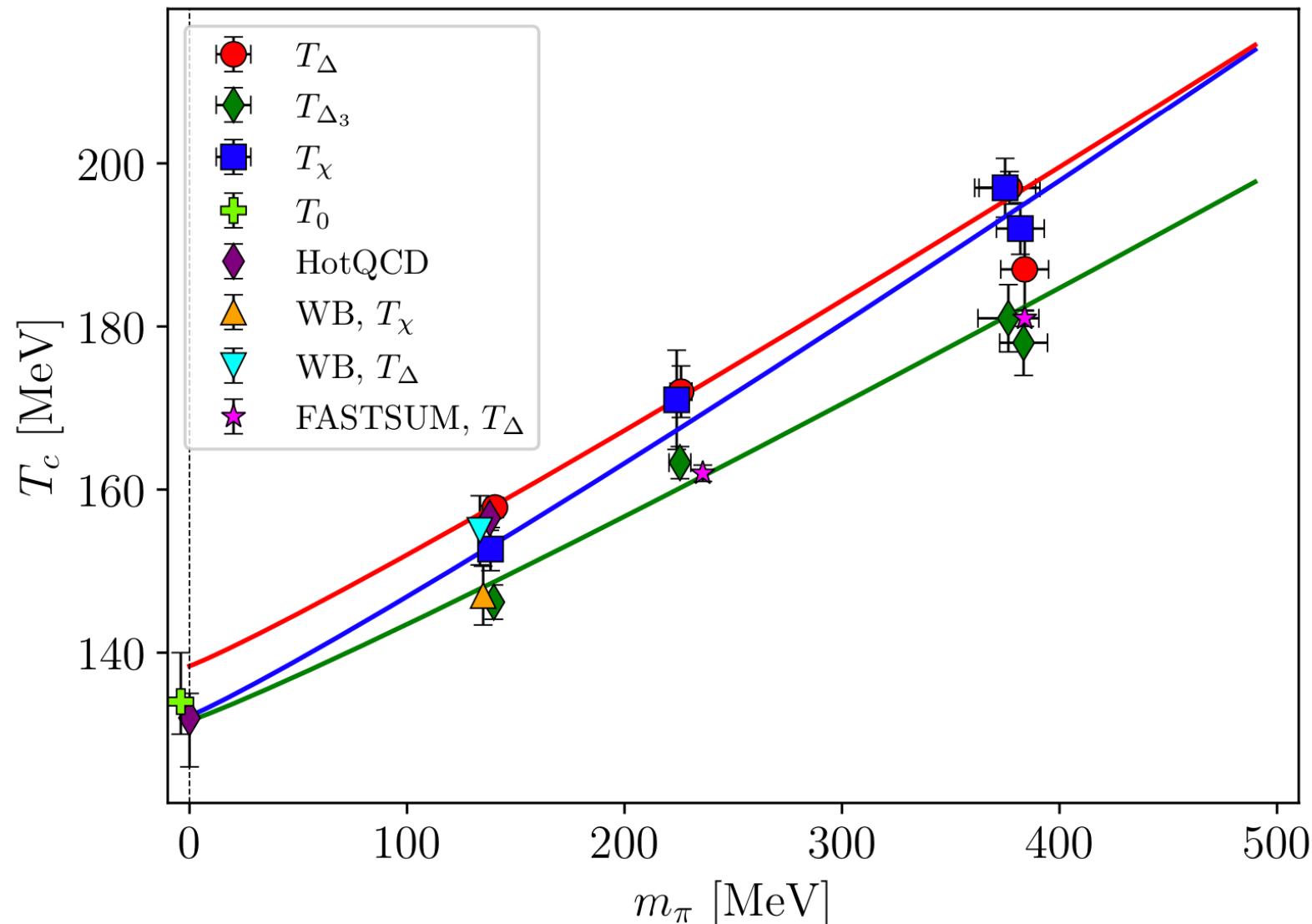
O4 vs Z₂

$$T_c(m_\pi) = T_0 + B(m_\pi^2 - m_c^2)^{1/\beta\delta}$$

$m_c = 100$ MeV still OK

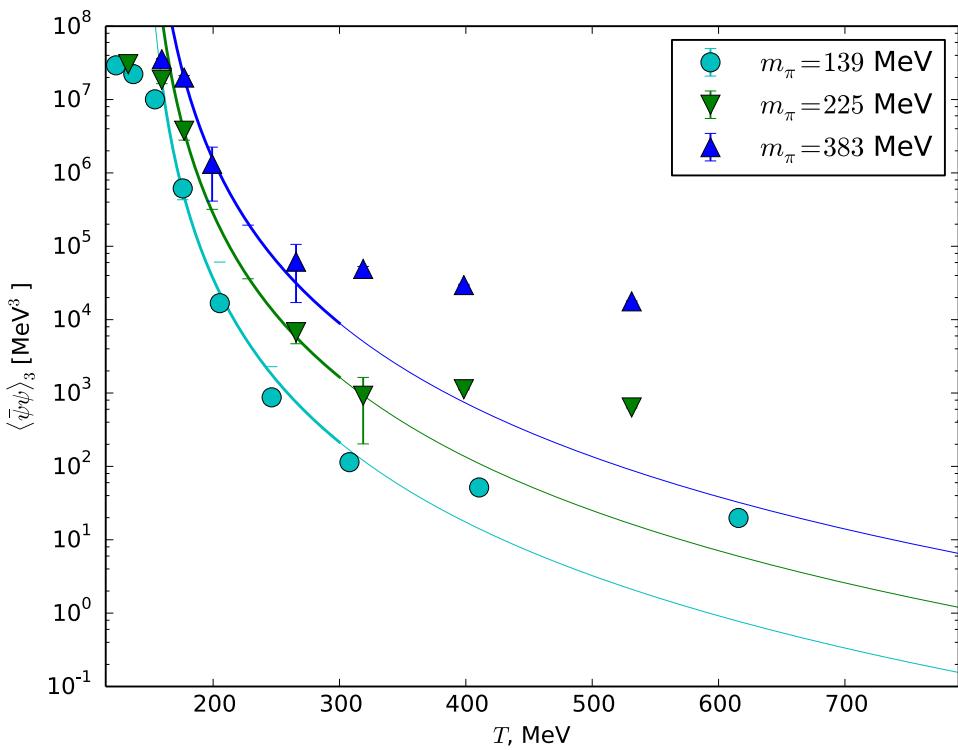
$m_c = 0$ still OK, indistinguishable from O4

Comparisons: pseudo critical temperatures, and chiral extrapolation



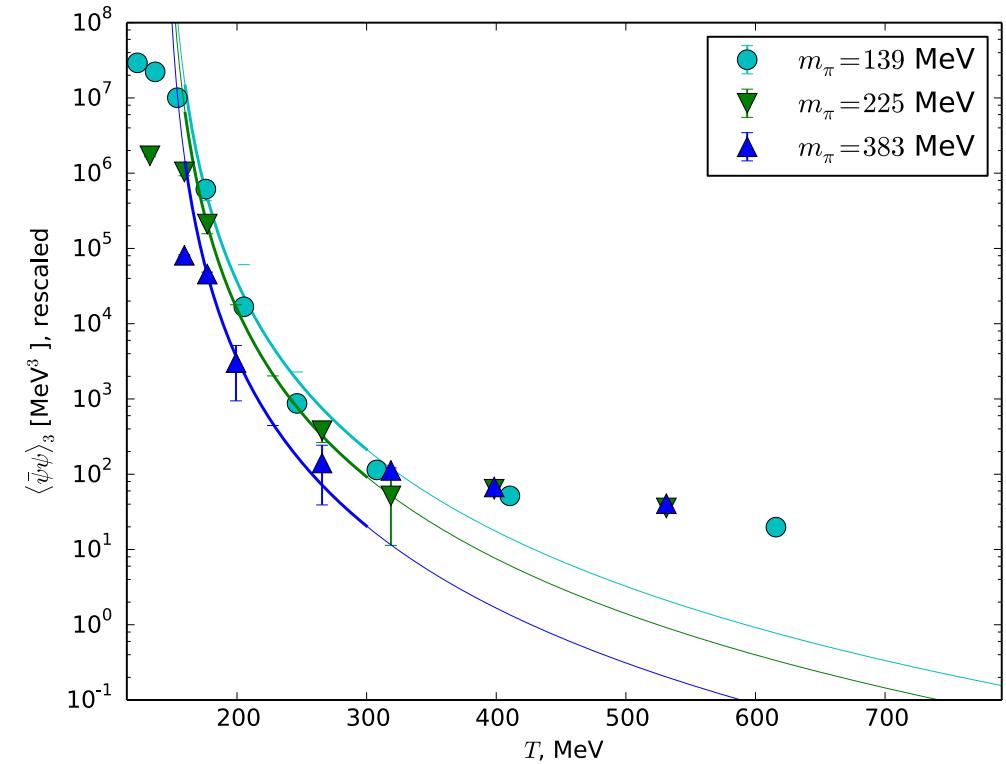
Searching for the scaling window in temperature

'Forgotten' microscopic dynamics



$$\Delta_3 \propto t^{-\gamma - 2\beta\delta} \quad T < 300 \text{ MeV}$$

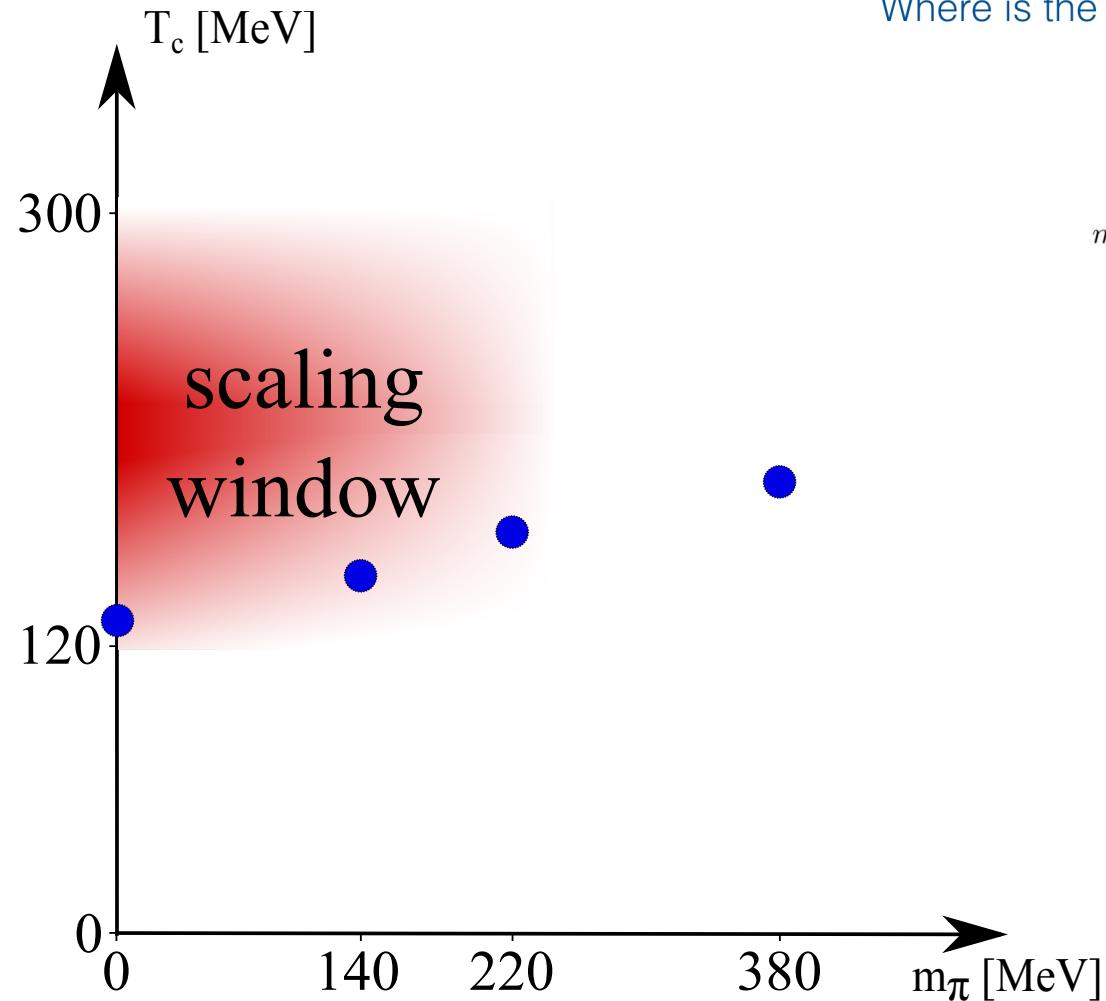
'Forgotten' critical behaviour..



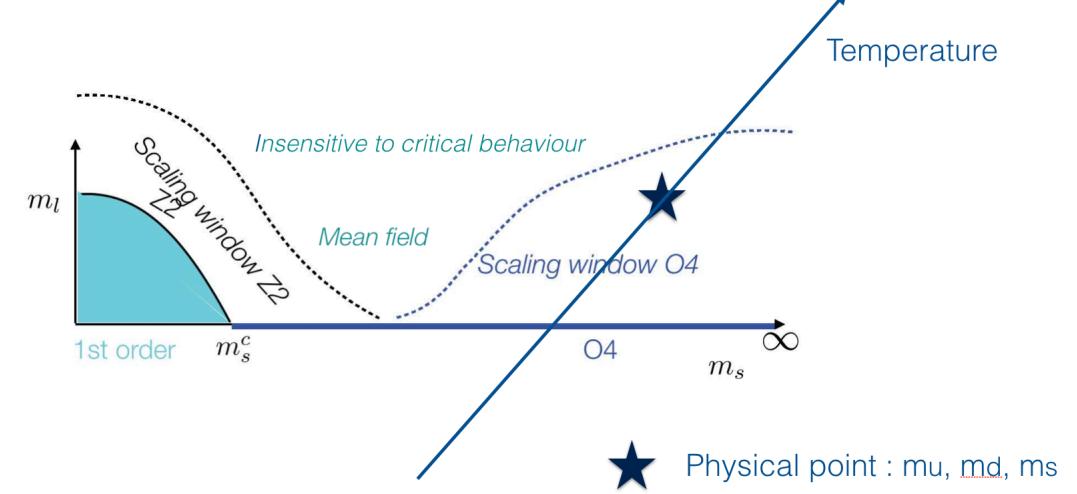
$$\Delta_3 \propto m_\pi^6 \quad T > 300 \text{ MeV}$$

Open issue 1:

Scaling window ???

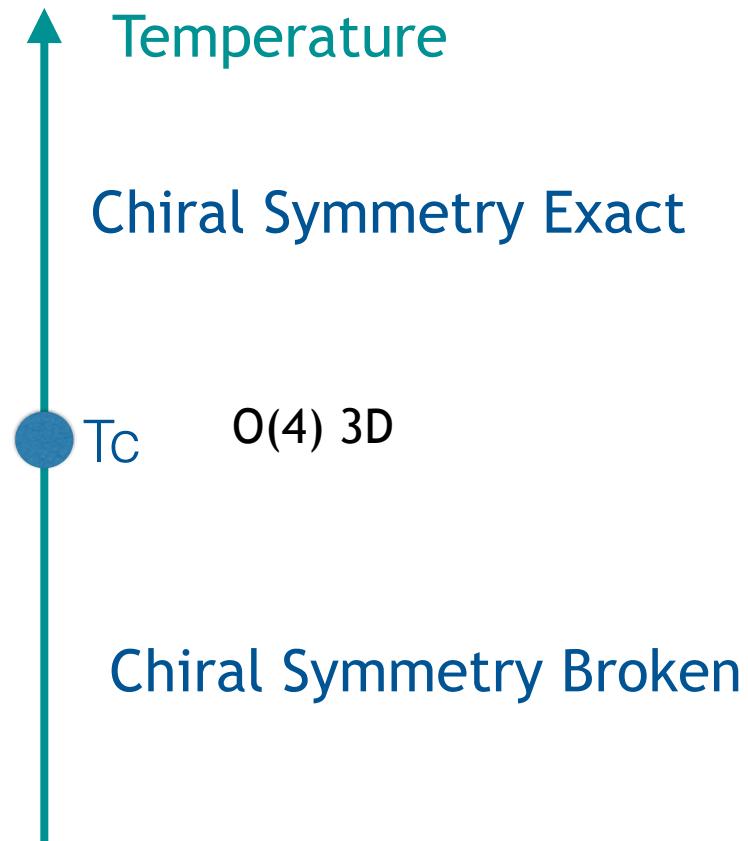


Where is the scaling window in QCD in mass and T ?



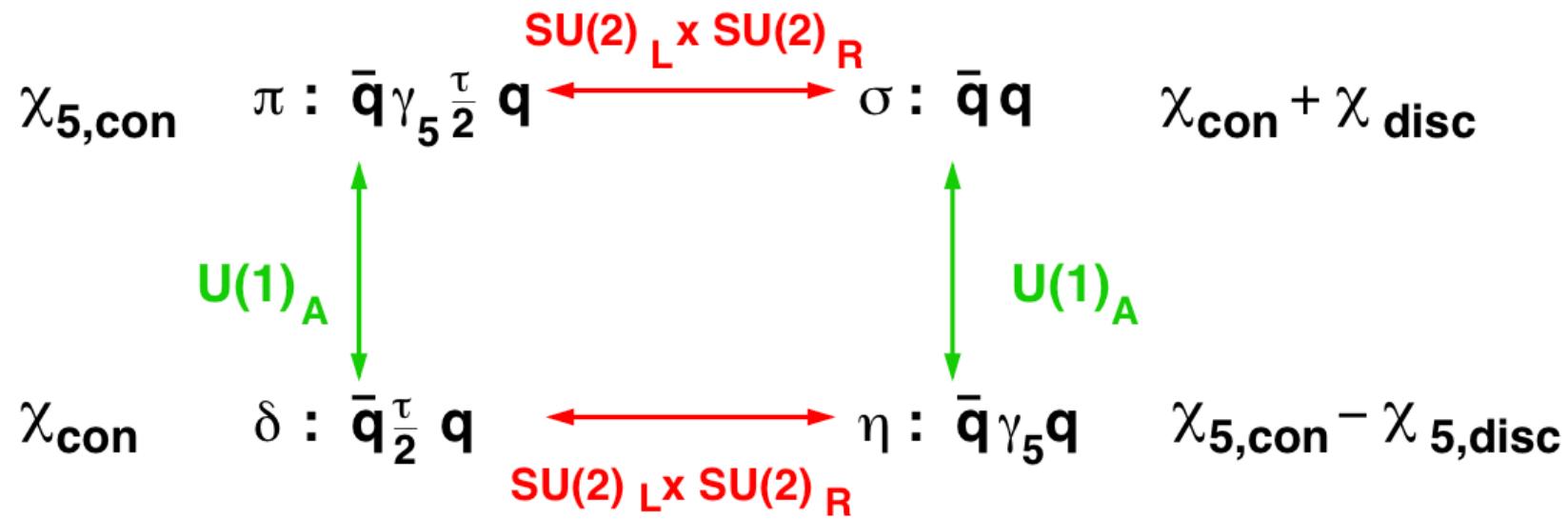
Comparisons with FRG approaches

Phases of massless QCD



Axial Symmetry - strategy

Two flavour:



Order parameter: $\bar{q} q$

Two point functions of composite fermions: meson spectrum

Basic :

$$\mathcal{O} = (\bar{\psi}\psi)_y(\bar{\psi}\psi)_x,$$

$$\int [dU][d\psi][d\bar{\psi}] \bar{\psi}_y^{u,a} \psi_y^{d,a} \bar{\psi}_x^{d,b} \psi_x^{u,b} e^{-S} = \int [dU] (M_{x,y}^{-1,u}[U])^{ab} (M_{y,x}^{-1,d}[U])^{ba} \det M e^{-S_g}.$$

$$\langle 0 | H^\dagger(T) H(0) | 0 \rangle = \sum_n \frac{A_{\text{src},n} A_{\text{snk},n}}{2E_n} e^{-E_n T}$$

Spectral decomposition
(generalizes asymptotic exponential decay)

Insert appropriate gamma matrices to create different quantum numbers

For composite spectrum
the task is to identify the asymptotic exponent \mathbf{m}

$$G(t)_{t \rightarrow \infty} \rightarrow e^{-\mathbf{m}t}$$

Pattern of symmetry revealed by correlators
And reflected by masses

Open issue 2:

Fate of $:U(1)_A:$?

To make a long story short, no decisive results

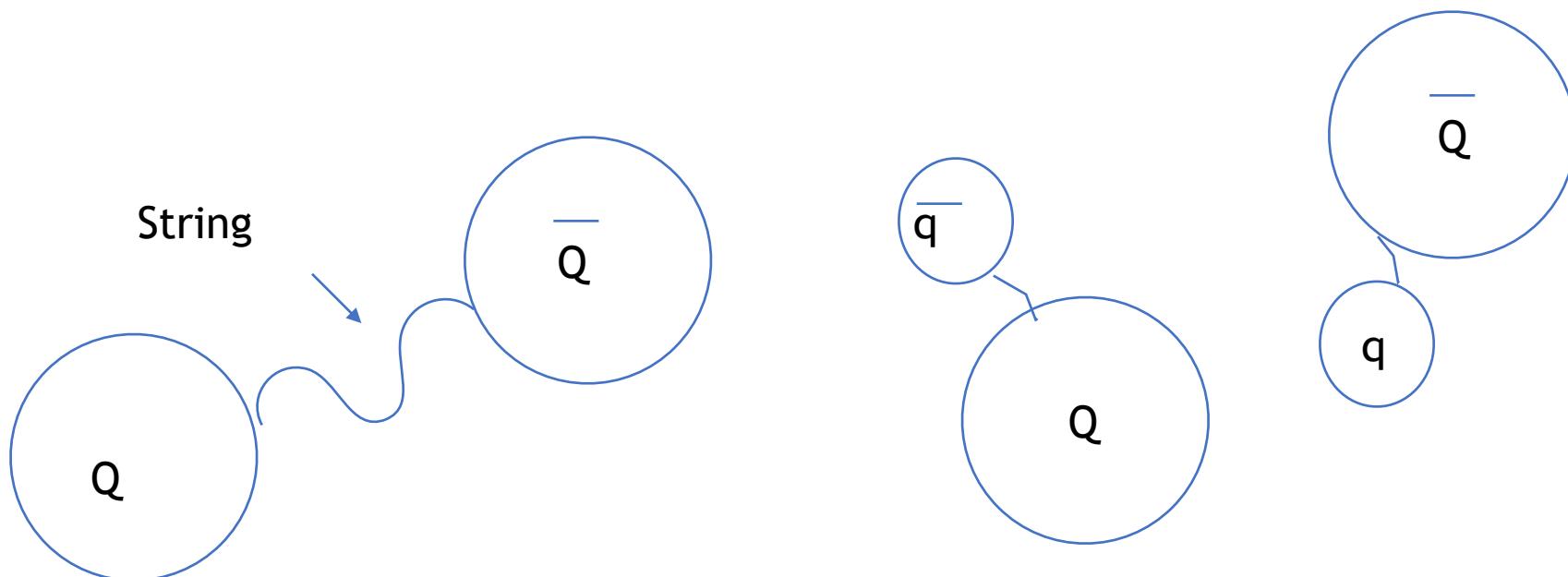
Probably still broken above T_c

The fate of the anomaly and its impact on the transition remains an important open problem

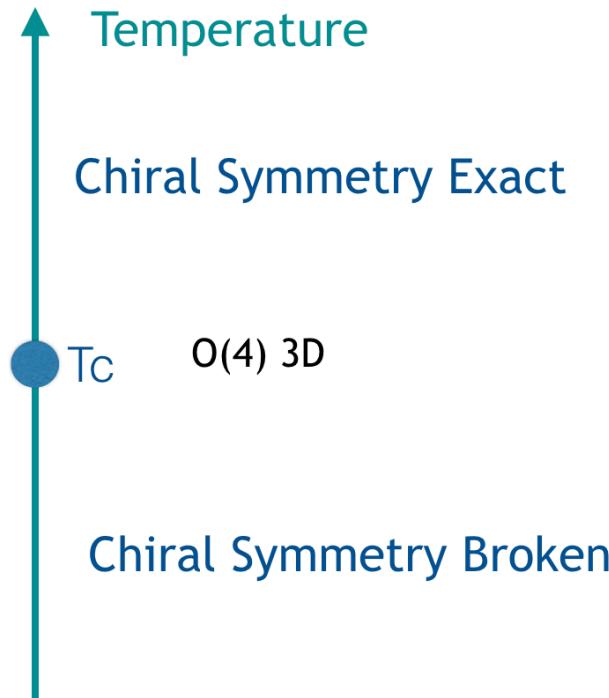
Polyakov loop again : YM vs QCD

Used as order parameter for the Yang-Mills transition

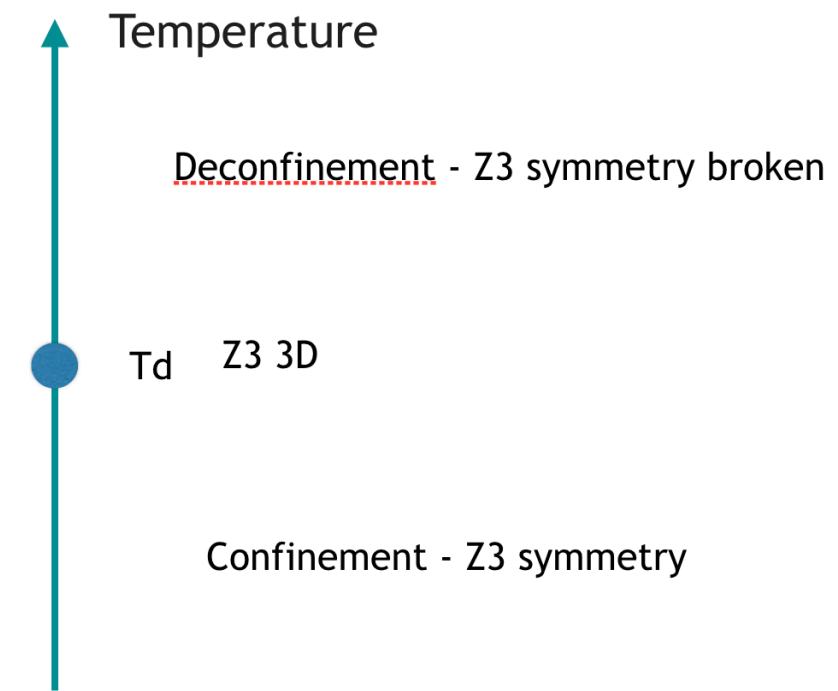
No longer an order parameter with matter fields: the string can break due to recombination with light quarks popping out of the vacuum



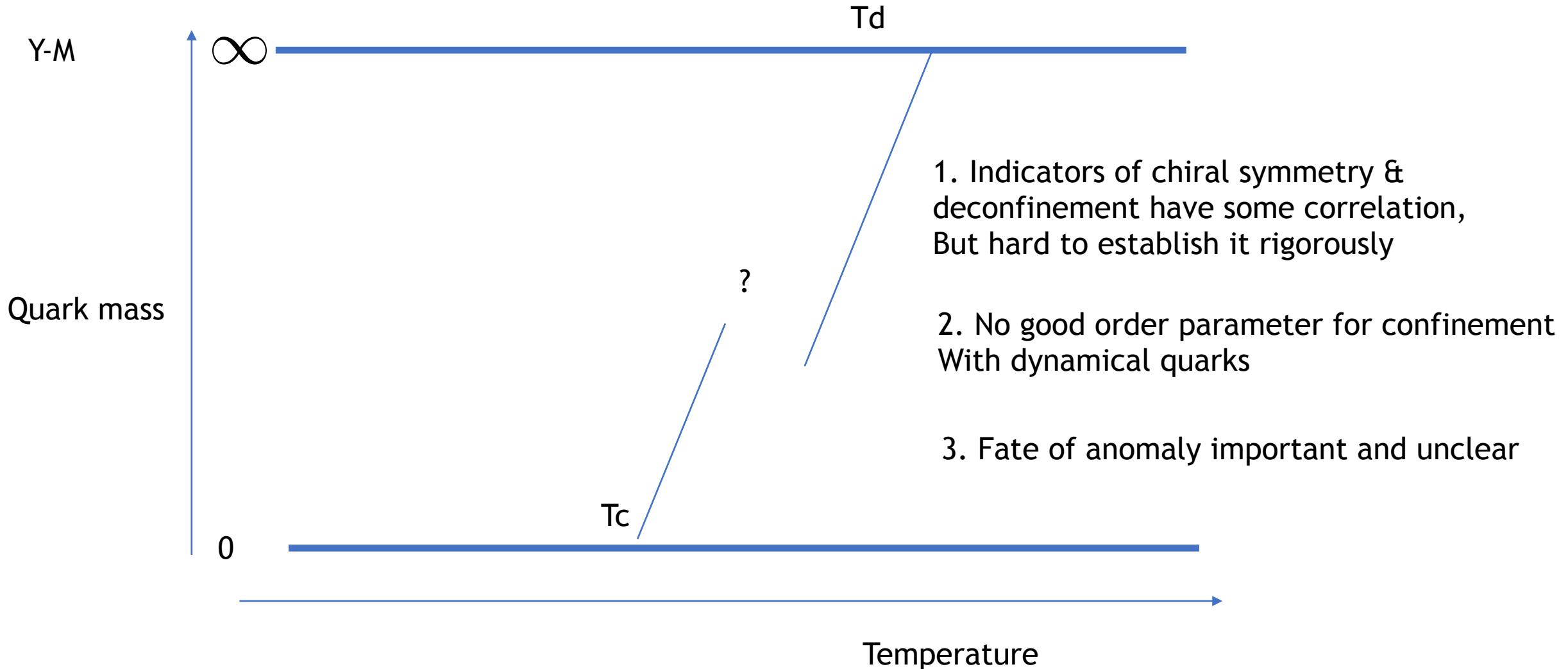
Phases of massless QCD



Phases of Yang Mills

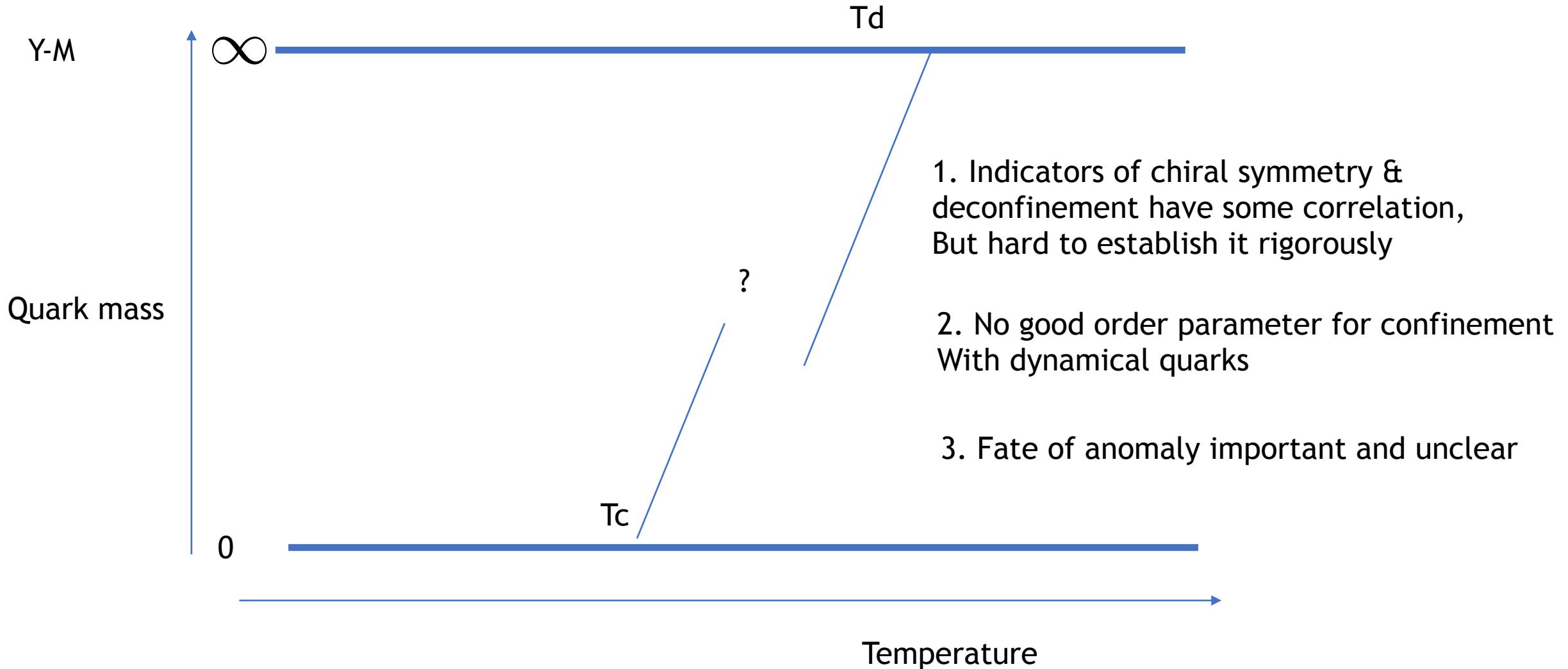


Phases of QCD in the temperature, mass plane



Open issue 3 - major question, interplay of chiral symmetry and confinement

Phases of QCD in the temperature, mass plane



Aspects of the high temperature phase

Extended symmetry above T_c ?

Heavy Quarks

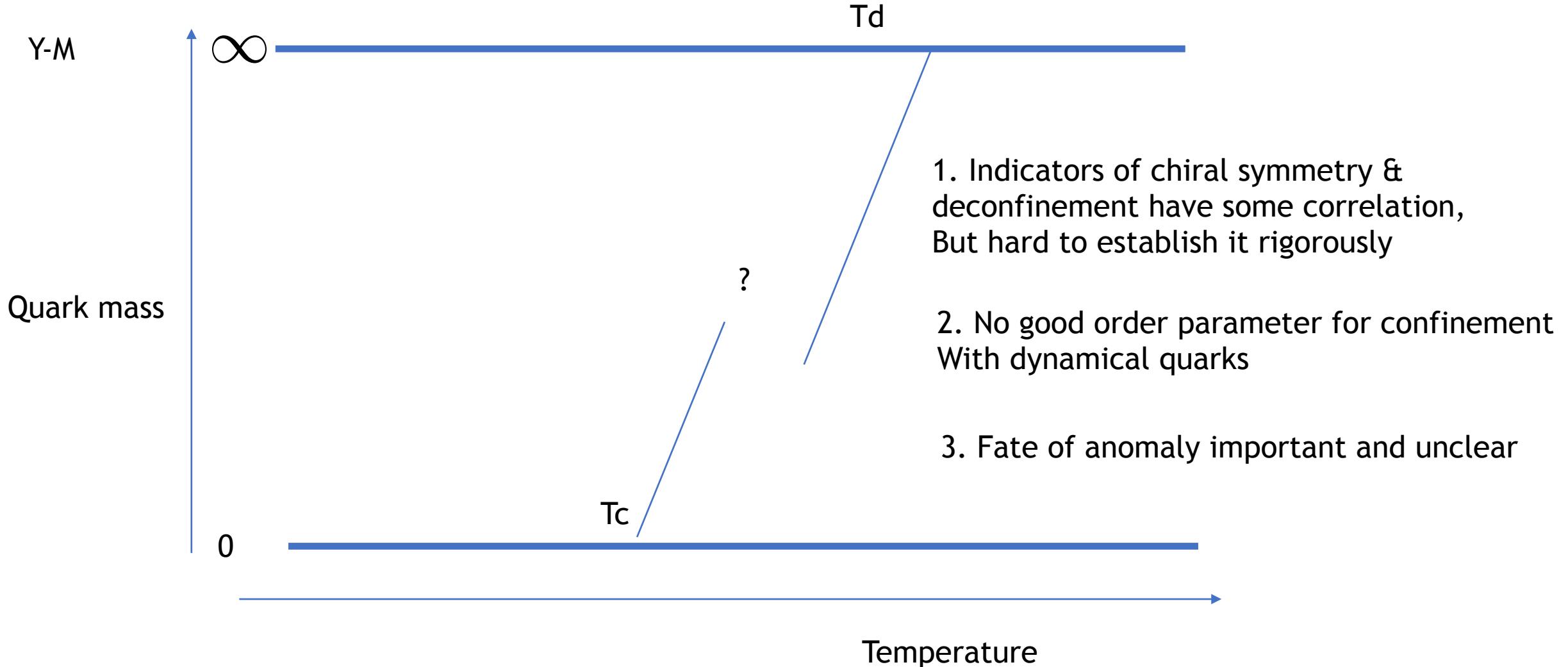
Quarkonia
Potential models
Temperature effects
NRQCD

Topology

UA(1) problem
Strong CP problem and the QCD axion
Topological susceptibility
Axion cosmology and lattice

Open issue 4 - nature of the strongly coupled phase above Tc

Phases of QCD in the temperature, mass plane

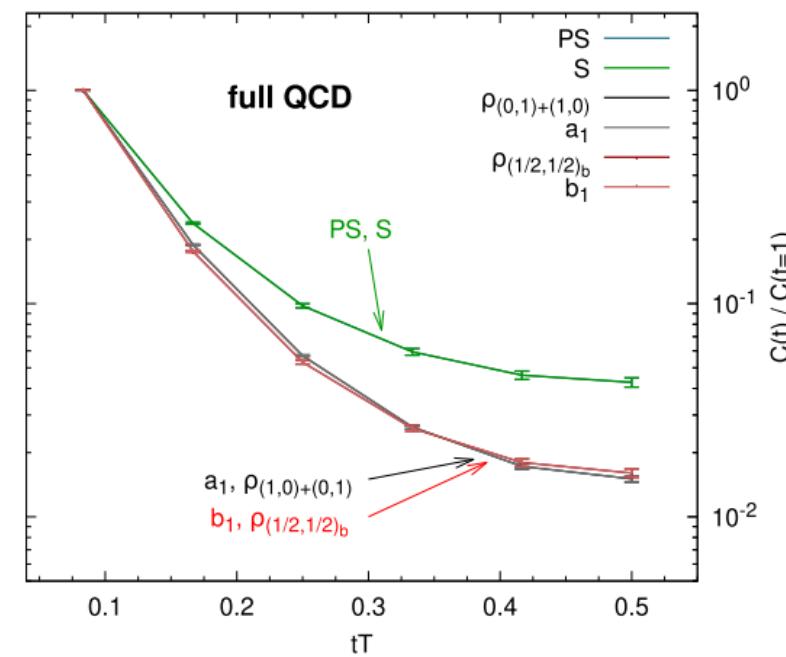
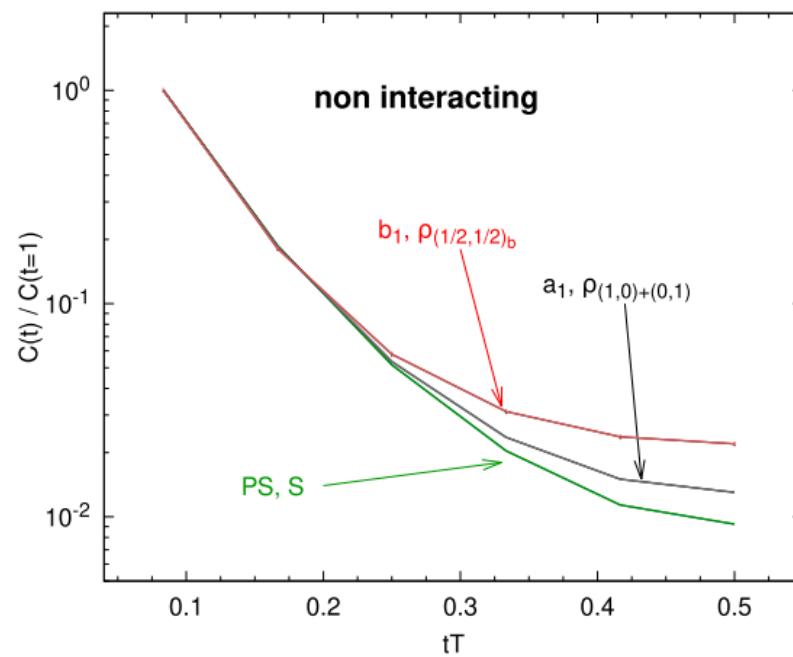


Speculation:

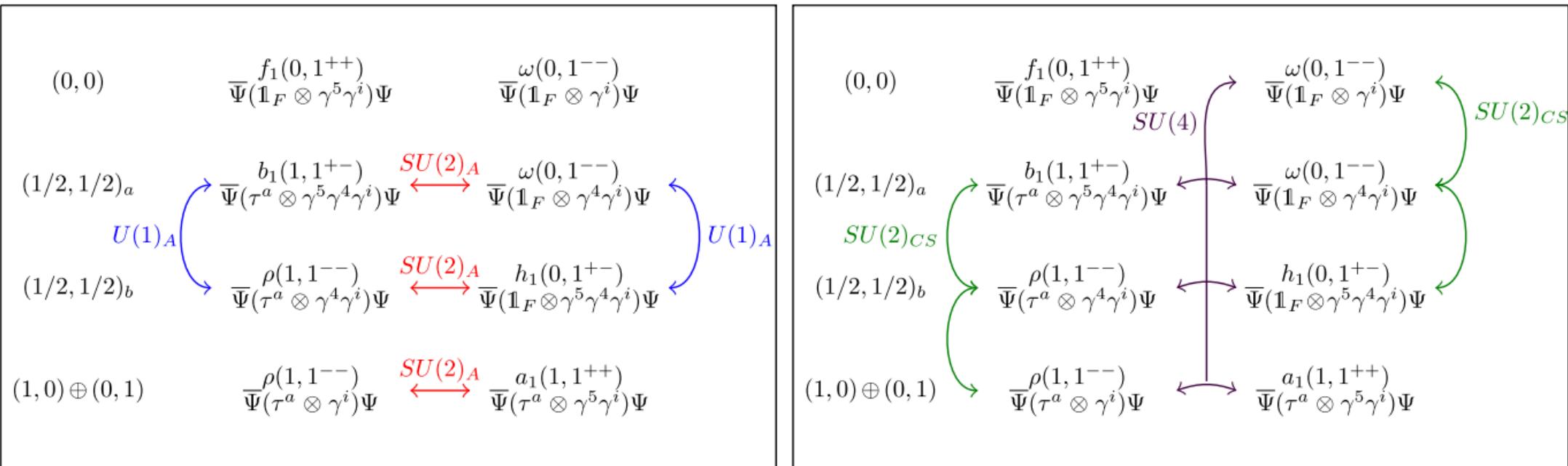
L. Glozman et al.

The stringy fluid

C. Rohrhofer et al. / Physics Letters B 802 (2020) 135245



Proposal: extended chiral-spin (CS) symmetry



An observation:

According to PDG: a1 (1260) and b1 (1235) are almost degenerate in ordinary conditions

This ‘near degeneracy’ may be understood in terms of simple potential models

A simple minded interpretation of the CS symmetry follows

Potential models analysis at high temperature may shed some further light

Some references:

On symmetry aspects of confinement:

Benjamin Svetitsky, Phys. Rept. 132 (1986), 1-53

Reviews of lattice finite temperature QCD, with introductory material:

Owe Philipsen, <https://arxiv.org/pdf/1912.04827.pdf>
Heng-Tong Ding <https://arxiv.org/pdf/2002.11957.pdf>