

M-1.

Neutron Star Masses

The most reliable method for measuring masses of astro. objects is dynamically, using Kepler's 3rd Law



Limits # of systems for which we can get $M!$

> 3,300 known pulsars in the galaxy; only ~ 250 in binaries
(and fewer w/ correct geometry, companion, etc...)

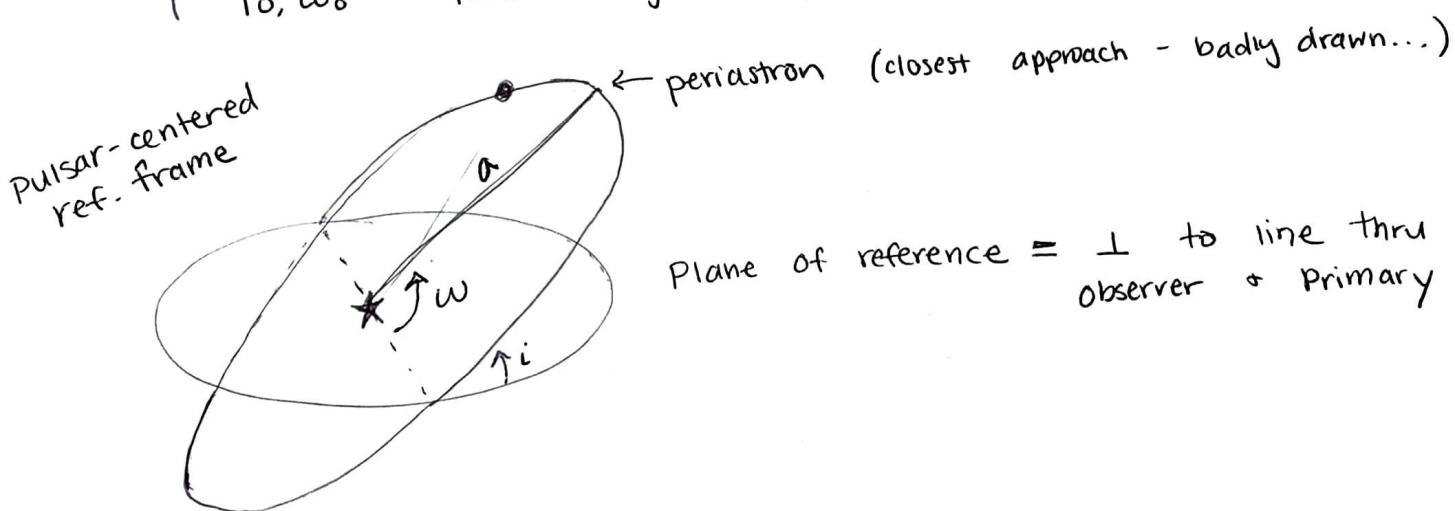
In Newtonian gravity, orbital motion is described by 5 Keplerian Parameters:

P_b - binary orbital period

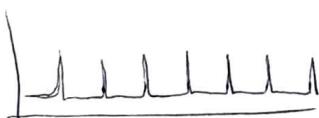
e - orbital eccentricity

$x = a \sin i$, projection of semi-major axis onto the line of sight, i = angle btwn orbital angular momentum vector & L.O.S.

T_0, ω_0 - time & angle of periastron



Orbit is measured via pulsar times of arrival (T.O.A.'s)

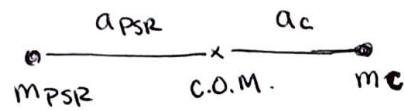


- Pulsar timing model fits TOAs to extract Keplerian parameters
- regularity of TOAs \rightarrow very precise params
- e.g., 1 ms precision in TOA \rightarrow determine PSR relative to center of orbit w/ 300 m accuracy

(c.f. most compact XRB has 38 min. orbit; 300,000 km separation)

M-2. How do we go from Kep. params to masses?

Re-draw diagram w/ center-of-mass frame:



A few definitions:

$$(i.) \quad a = a_{\text{PSR}} + a_C$$

$$(ii.) \quad M_{\text{PSR}} a_{\text{PSR}} = M_C a_C$$

Note:
assuming
circular
orbit

And let (iii) $v_{\text{PSR}} = \frac{2\pi}{P_b} \cdot a_{\text{PSR}} \sin i = \frac{2\pi}{P_b} \times v_{\text{PSR}}$

Kepler's 3 Law tells us:

$$\frac{G(M_{\text{PSR}} + M_C)}{a^3} = \left(\frac{2\pi}{P_b}\right)^2$$

use definitions i-iii. to rewrite this as:

$$f \equiv \frac{\frac{M_C \sin i}{(M_C + M_{\text{PSR}})^2}}{= \frac{4\pi^2 \times v_{\text{PSR}}^3}{T_O P_b^2}},$$

{ intrinsic source masses } { observable Kep. params.
(from TOAs) }

$$\text{where } T_O = \frac{GM_0}{C^2} \\ = 4.925 \text{ ms}$$

But, to break the degeneracy between $M_C, M_{\text{PSR}}, \sin i$, additional information is needed

Method 1: Measure mass ratio $q \equiv \frac{M_{\text{PSR}}}{M_C} = \frac{x_C}{x_{\text{PSR}}} \quad (\text{via eq. ii.})$

- we already have x_{PSR} from timing
- if companion is also a PSR, can time it to get x_C (rare!!!)
- if companion is optically bright, can use phase-resolved spectroscopy to track spectral lines $\rightarrow x_C$

↪ possible only for a small # of systems; still only gives limits on masses

M-3.

Method 2: Measure post-Keplerian parameters

- relativistic timing effects; assuming GR is the correct theory of gravity then these can be related to the Kep. Params (P_b, χ, e) and masses at leading post-Newtonian order

- $\begin{cases} 1 \text{ pk params (+ Keplerian params)} \Rightarrow M_{\text{tot}} \text{ constrained} \\ \text{and } \therefore \text{can place limits on } M_1, M_2 \end{cases}$

$\begin{cases} 2 \text{ (or more) pk params } " " " \Rightarrow \text{constrain individual masses} \end{cases}$

→ Show slides

↳ pk params, double pulsar, NS mass tables + distribution

Method 3: if companion is an optically bright ~~bright~~ white dwarf to measure Balmer lines ($n_3 \rightarrow 2$) then can use phase-resolved spectroscopy to get V_{WD} . Combine: $q = \frac{M_{\text{PSR}}}{M_{\text{WD}}} = \frac{V_{\text{WD}}}{V_{\text{PSR}}} \leftarrow = 2\pi * \text{PSR} / P_b$

Similar to method 1, But the width of the Balmer line also gives the WD surface gravity. Can use mass-radius relation and surf. grav. $g = GM/R^2$ to isolate M_{WD} and get M_{PSR} .

↳ important result: J0348+0432 (Antoniadis + 2013)

• PSR+WD binary, $M_{\text{PSR}} = 2.01(4) M_\odot$

• this was the 2nd $\sim 2 M_\odot$ PSR, but w/ independent method

Comment on spider pulsars (w/ $M > 2 M_\odot$) :

- PSR timing gives Kep. params ($\therefore f$) but all other info comes from modeling the optical light-curves + spectra of the ablated companion
↳ heavily irradiated; short timescale variability; unevenly heated surface
↳ systematics are challenging

SKA: will increase # of NS masses by $\sim 10x$ by finding more binary PSRs + more precise PK params

R-1.

Neutron Star Radius Measurements

Two main techniques - both of which use the thermal emission from the NS surface

(1) Measure the angular size of the NS

(2) Detect the effects of the NS spacetime on the surface emission

Method 1 - Spectroscopic measurements of NS radii

Goal is to measure flux from NS surface + effective temperature (via spectrum, assuming a modified blackbody) to infer R

2 relations: $f_{bol} = \frac{L}{4\pi D^2}$, $L = f_c 4\pi R_{obs}^2 \sigma T_{eff}^4$

\uparrow
color-correction factor

$$\hookrightarrow R_{obs} = \left(\frac{f_{bol}}{\sigma T^4} \right)^{1/2} \cdot D$$

Generic technique for meas. size.

NS have extreme enough compactness to grav. lens their own surface emission s.t.

$$R_{obs} = \left(1 - \frac{2GM}{RC^2} \right)^{-1/2} R$$

always < 1

$\therefore R_{obs} > R$

$$\hookrightarrow R = \left(\frac{f_{bol}}{\sigma T^4} \right)^{1/2} \cdot D \cdot \sqrt{1 - \frac{2GM}{RC^2}}$$

↓

radius measurement is partially degenerate w/ mass

- need known distance - can be challenging
- sources in globular clusters are good for this

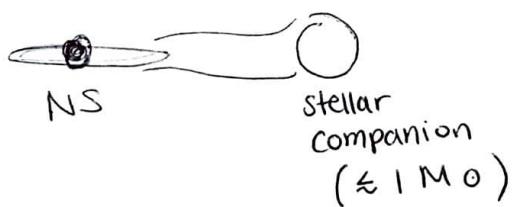
R-2.

Source selection

Want to observe emission "from the NS surface, the whole NS surface, and nothing but the NS surface" (F. Özel)

- { - little/no emission from magnetosphere ; accretion disk
 ↓
 so no radio pulsars!
- low magnetic fields (strong B-fields can induce non-uniform T distr.)
- Known distance

Low-mass X-ray binaries



Periodic accretion from stellar companion heats NS ; this heat is stored deep in the crust and later re-radiated via thermal surface emission

- low-B fields ($< 10^9$ G)
- expected to have uniform surface emission (unlike PSRs!) from (typically) H atmosphere, donated by H-rich companion
 - ↓
 - in some systems, He is indicated
 - heavier elements will settle very quickly due to strong g (timescale ~ minutes)

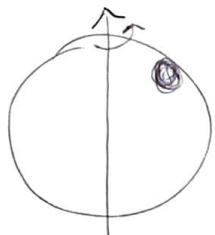
2 types of spectroscopic measurements :

1. system is in quiescence
2. bursting phase

⇒ Slides

R-3. Method 2 - Pulse Profile Modeling w/ NICER

Physical picture: rotation-powered MSPs w/ thermal hotspot offset from \hat{z}



- powered by return current from magnetic field lines at the polar cap(s)

- thermal emission is in soft X-ray (2-6 keV)
 - steady, so can be avgd. over many cycles
(important b/c sources are dim)

Sources / Primary targets — 2 in binaries:

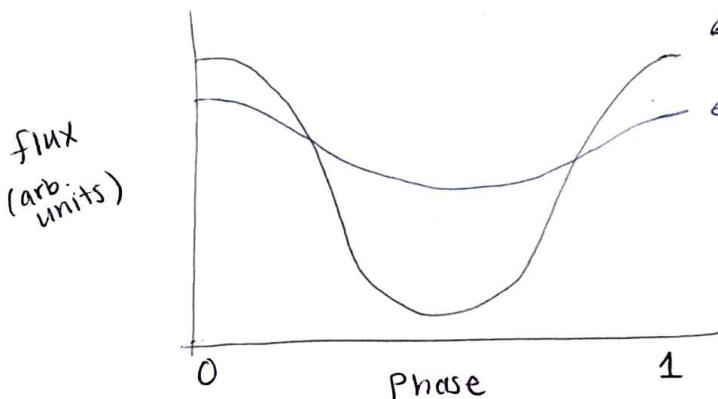
J0437-4751 - mass known to 5% ; $f = 173 \text{ Hz}$
J0740+6620 - massive PSR ~~f = 398~~
 $f = 340 \text{ Hz}$

— 2 isolated PSRs: J0030+0451 - $f = 205 \text{ Hz}$
J2124-3358 - $f = 203 \text{ Hz}$

Radius constraints for 2 sources, J0030 + J0740, have been published

Key refs: Miller et al. 2019, 2021
Riley et al. 2019, 2021

Theoretical waveforms and dependence on NS spacetime:



small M/R (non-compact)

large M/R (very compact)

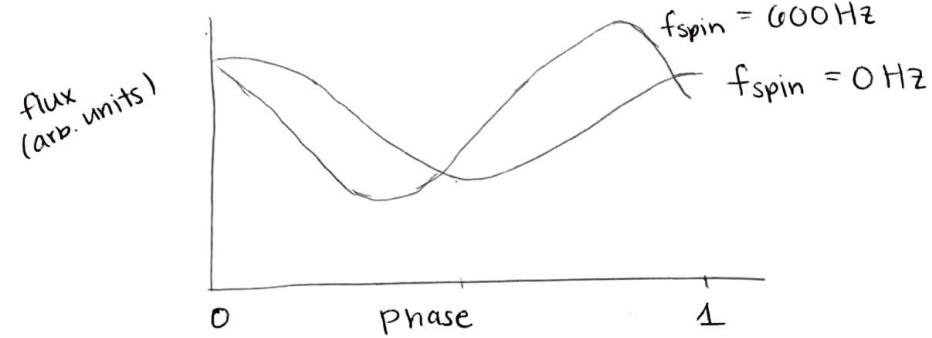
Recall:

$$\frac{GM}{rc^2} \left. \begin{array}{l} \text{sun} \sim 10^{-6} \\ \text{NS} \sim 0.1 \cdot 10^{-3} \end{array} \right\} (non-rot.) BH = 0.5$$

- as M/R increases, pulsed fraction decreases due to grav. lensing
- effect first predicted: Pechenik et al. 1983

R-4.

Special relativistic effects of doppler boosting + aberration



In absence of SR effects,
profile would be symmetric

- Doppler boosting makes the blueshifted side appear brighter, s.t. the max flux occurs at an earlier phase + the profiles becomes asymmetric.

- effect \sim L.O.S. velocity
- angular velocity is known from pulse freq.
 \hookrightarrow can be used to measure R , if inclination (i) and spot geometry (α) are known



- SR effects are indep. of the normalization, so errors in flux calib. are not important
- more rapidly spinning NS \rightarrow stronger effect

- * In principle, can combine these two (SR + GR) effects to measure M and R ; in practice, constraints are still very correlated w/o indep. mass

\downarrow energy-dep. shape of light-curve
of these can help resolve some degeneracies.

Challenges

- Both Doppler boosting + grav. lensing are degenerate w/ $\sin(i), \sin(\alpha)$
- Need to incl. higher-order spin corrections
 - oblateness $\sim 5\text{-}30\%$ effect at 600 Hz (Morsink 2007; Psaltis+ 2014)
 - quad. mom. $\sim 1\text{-}5\%$ effect (Psaltis + 2014)
- hot spot shape, size, T, T distribution, #, + beaming pattern all uncertain
- foreground from magnetosphere can be significant
 \downarrow
 these are all MSPs!