Magnetic Resonance Imaging

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Gordon
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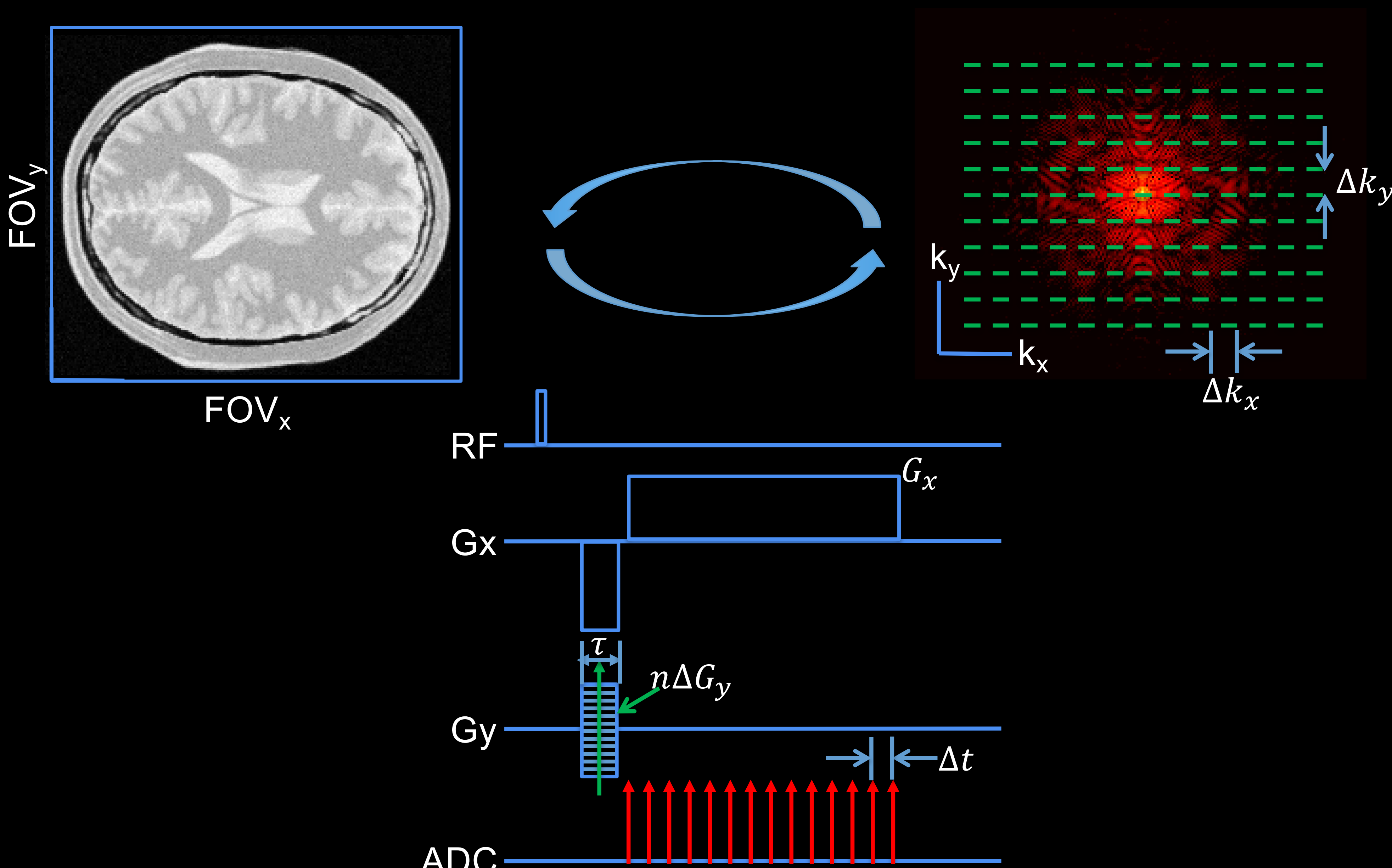
Medical
Imaging

Outline



FOV$_y$
FOV$_x$
Δk_y
k$_y$
k$_x$
Δk_x
RF
G_x
Gx
τ
$n\Delta G_y$
Gy
Δt
ADC

Outline

- Introduction
- MR physics
  - Spin
  - Bloch equation
  - Signal excitation and reception
  - Relaxation
- MR imaging
  - Projection-based MR imaging
  - Fourier transform-based MR imaging
  - k-space sampling
- Summary

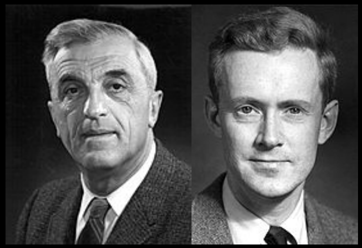
Magnetic Resonance Imaging

Introduction

Gordon Center for Medical Imaging

History

1944: Rabi
Physics (Measured magnetic moment of nucleus)



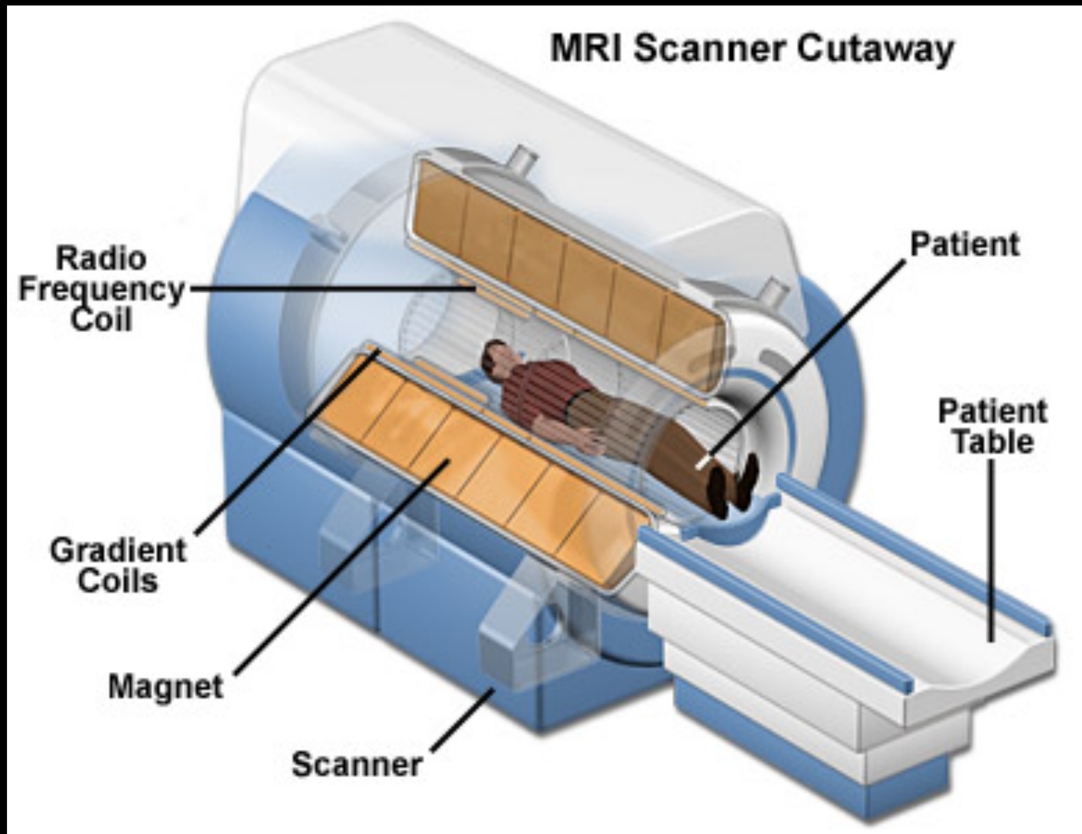
1952: Felix Bloch and Edward Mills Purcell
Physics (Basic science of NMR phenomena)

1991: Richard Ernst
Chemistry (High-resolution Pulsed FT-NMR)

2002: Kurt Wüthrich
Chemistry (3D molecular structure in solution by NMR)


2003: Paul Lauterbur & Peter Mansfield
Physiology or Medicine (MRI technology)

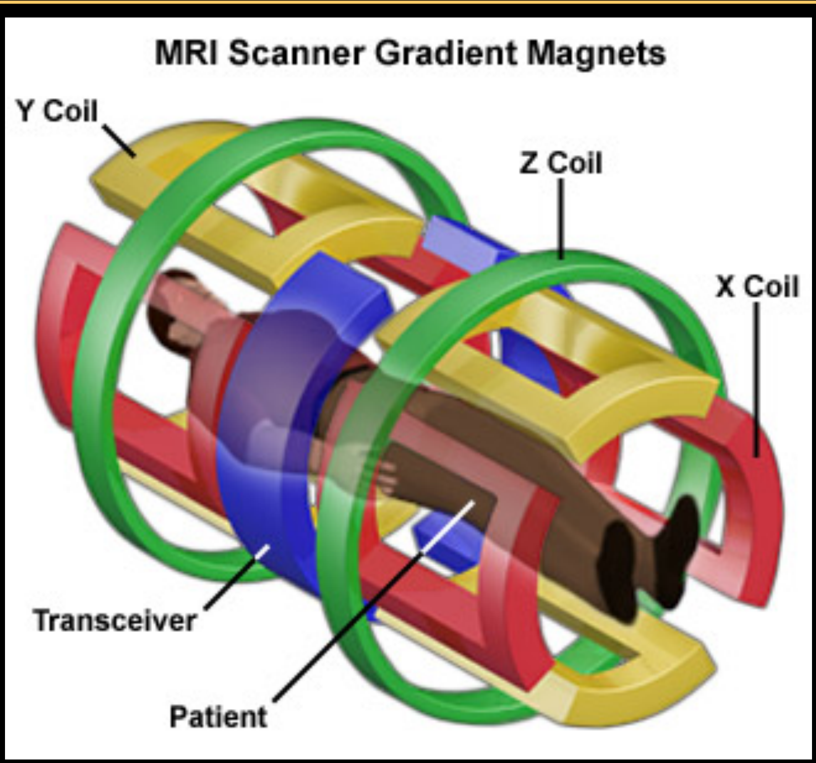
MRI scanner



Superconducting magnet

- Strong, static, and homogeneous magnetic field (1.5 ~ 10 T)
- A 3 T magnetic field is 60,000 times stronger than the Earth's magnetic field.
- < 1 ppm in a 25-cm diameter sphere
- Polarizer

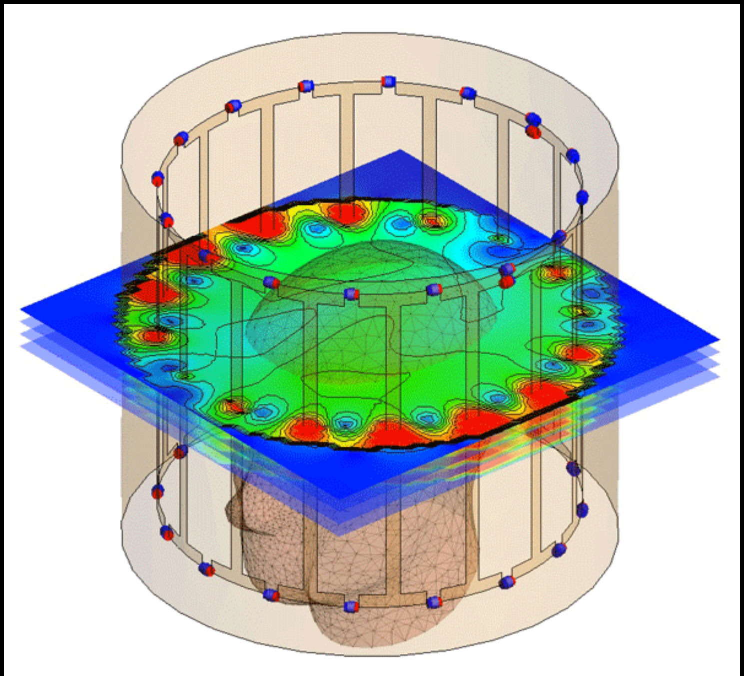
MRI scanner



Gradient coils

- An inhomogeneous magnetic field whose z-component varies linearly along a specific direction called the gradient direction.
- Typical specification: 40 mT/m, 200 T/m/s slew rate
- $B_0 \approx 3$ T, $G_x \approx 10$ mT/m or $\max(xG_x) \approx \frac{1}{1000}B_0$
- Spatial encoding

MRI scanner



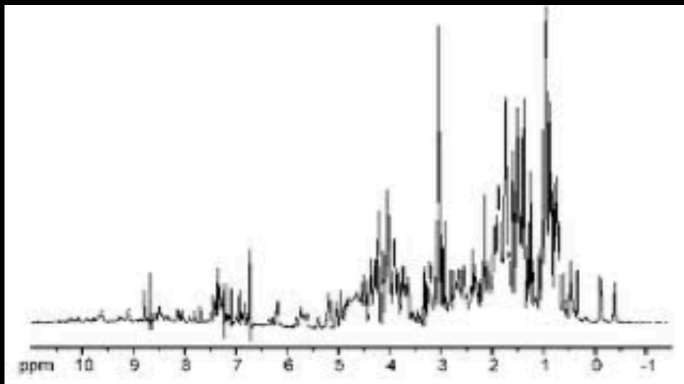
Radiofrequency (RF) coils

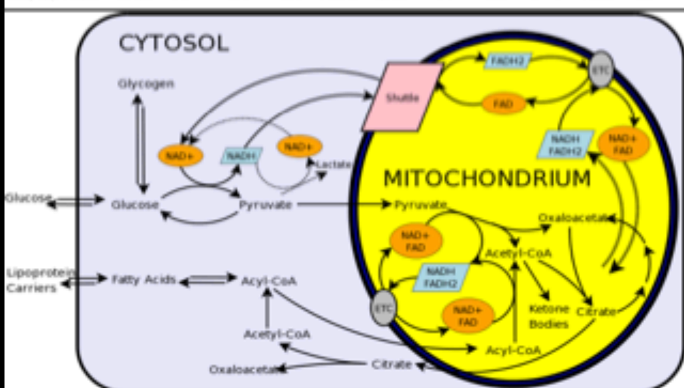
- Time-varying magnetic field at NMR resonance frequency (e.g., 60 ~300 MHz)
- Typical strength 0.1 Gauss
- Signal excitation and reception

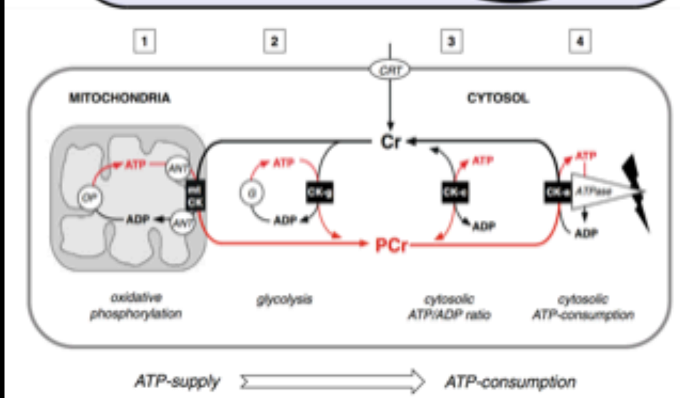
Courtesy of P. Futter, computed with FEKO

Magic of Spins

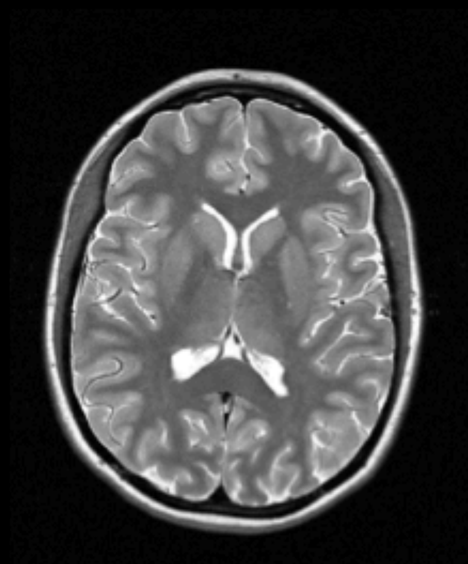
Spectroscopy

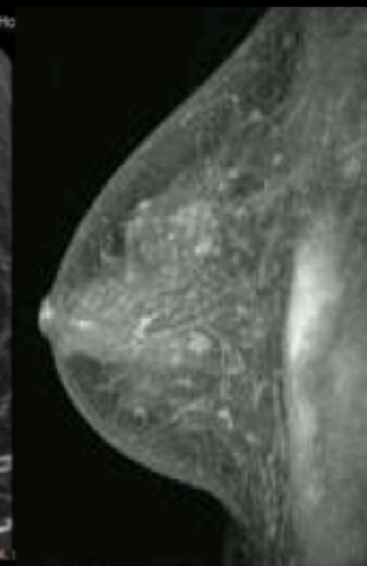


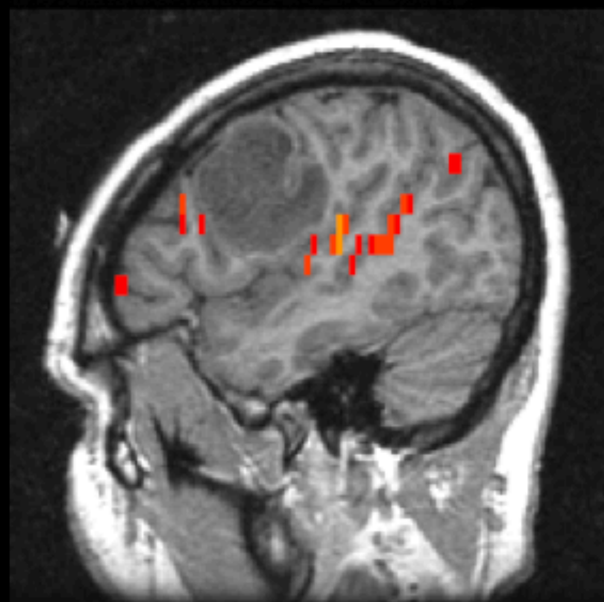


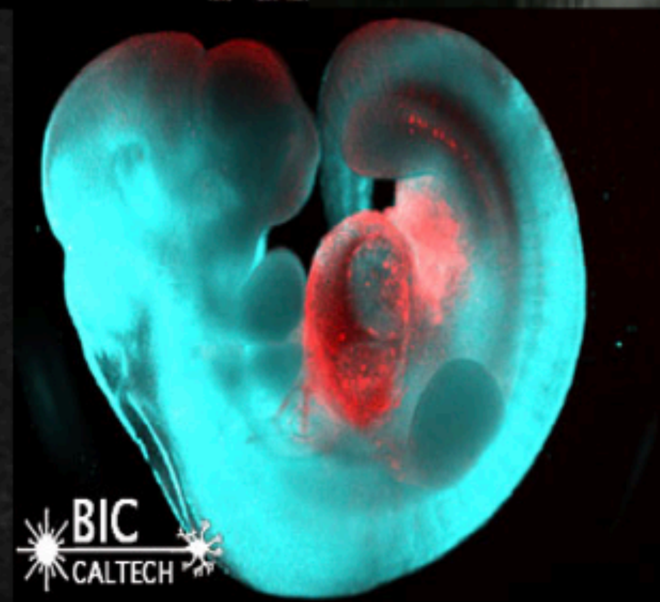


Imaging









Magnetic Resonance Imaging

MR Physics

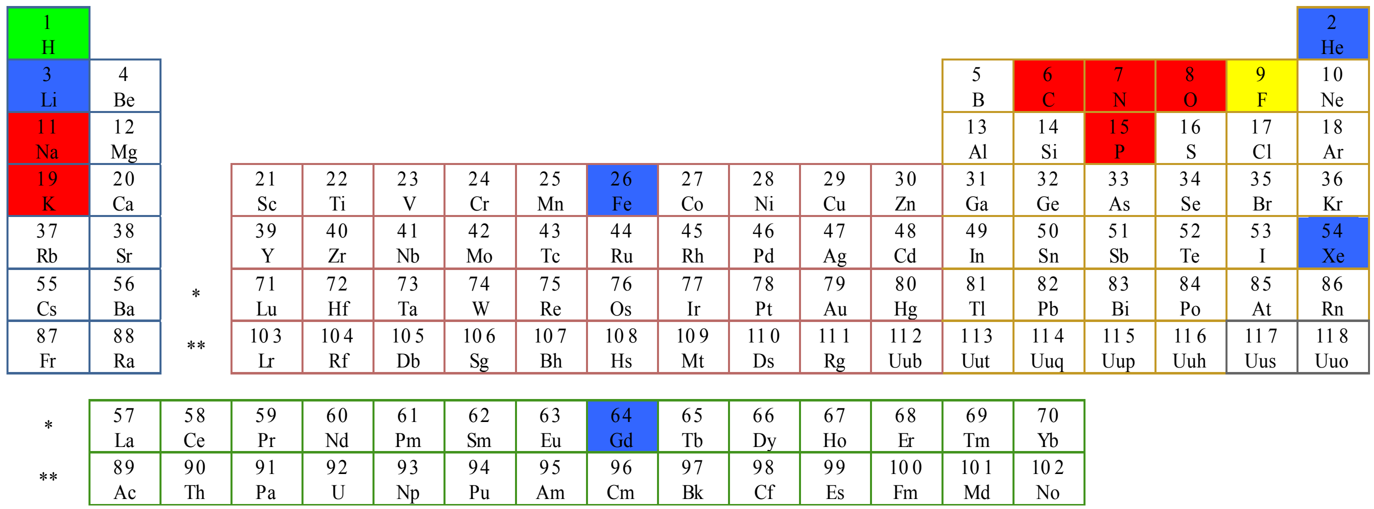
Spin

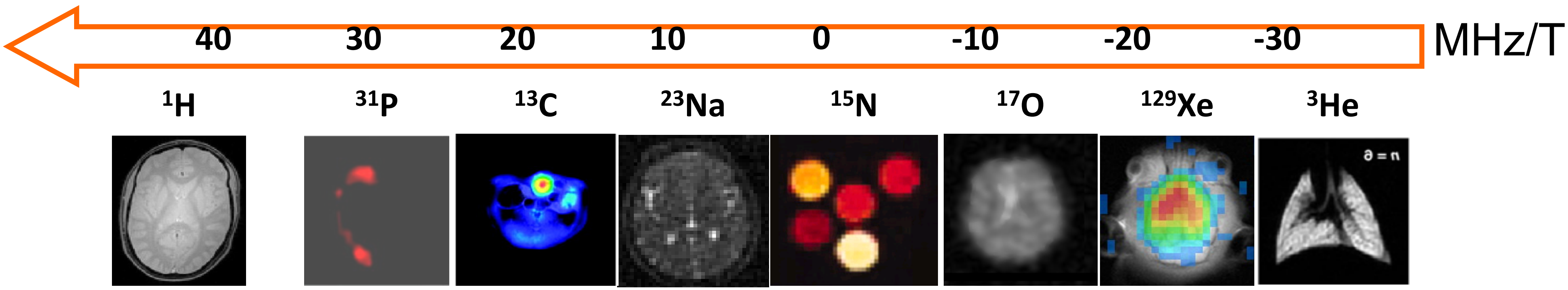
- Spin is an *intrinsic property* of particles just like mass, charge, etc.
- Nuclear magnetic moment

  - $\vec{\mu} = \gamma \vec{J}$, where $\vec{J}$ is the spin angular momentum
  - $\vec{\mu} = \gamma \hbar \sqrt{I(I+1)}$, where I is the nuclear spin quantum number.
  - $I = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$
    - Nuclei with an odd mass number have half-integral spin.
    - Nuclei with an even mass number and an even charge number have zero spin.
    - Nuclei with an even mass number but an odd charge number have integral spin.

Nuclei used in MRI

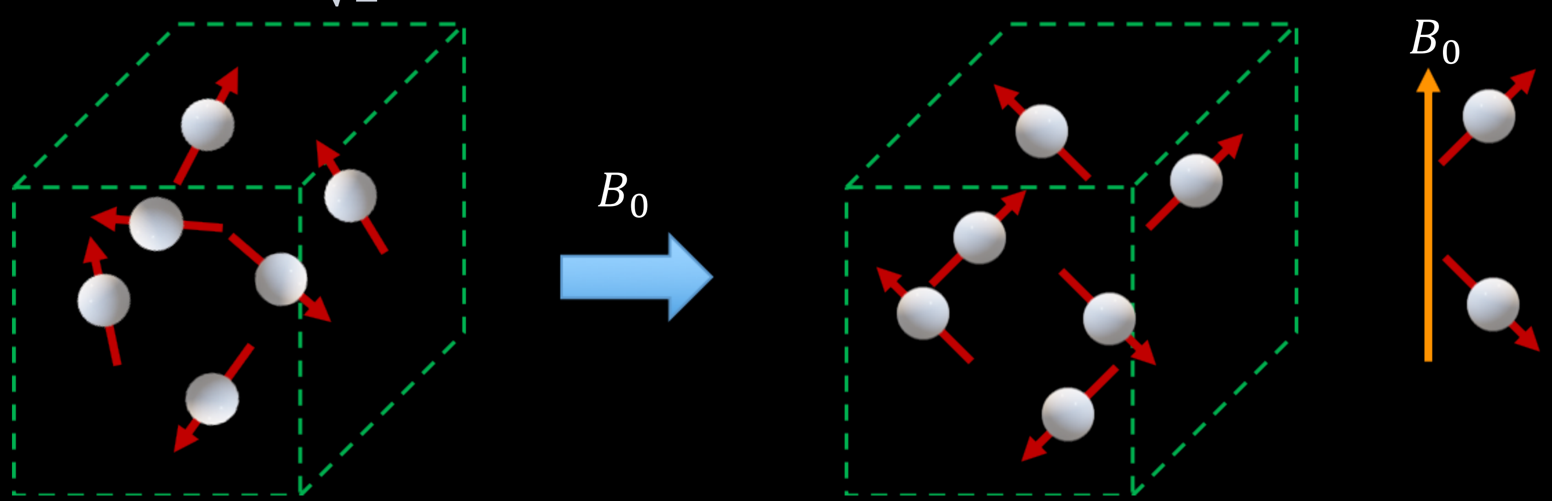
Nucleus	$\gamma / 2\pi$ (MHz/T)
^{1}H	42.576
^{2}H	6.53566
^{3}He	-32.434
^{7}Li	16.546
^{13}C	10.705
^{14}N	3.0766
^{15}N	-4.3156
^{17}O	-5.7716
^{23}Na	11.262
^{31}P	17.235
^{129}Xe	-11.777





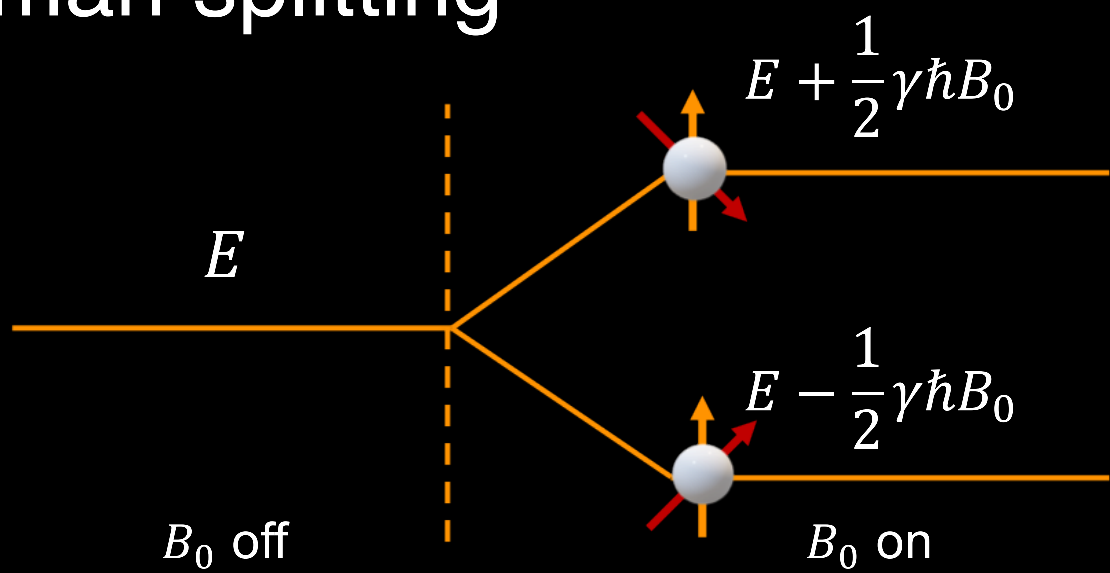
Magnetized nuclear spin system

- In the presence of an external magnetic field
  - $\mu_z = \gamma m_I \hbar$
  - $m_I = -I, -I+1, \dots, I$, is the magnetic quantum number
  - For spin-1/2 systems
    - $\mu_z = \pm \frac{\gamma\hbar}{2}$
    - $\mu_{xy} = \frac{\gamma\hbar}{\sqrt{2}} e^{i\phi}$, where ϕ is randomly distributed.



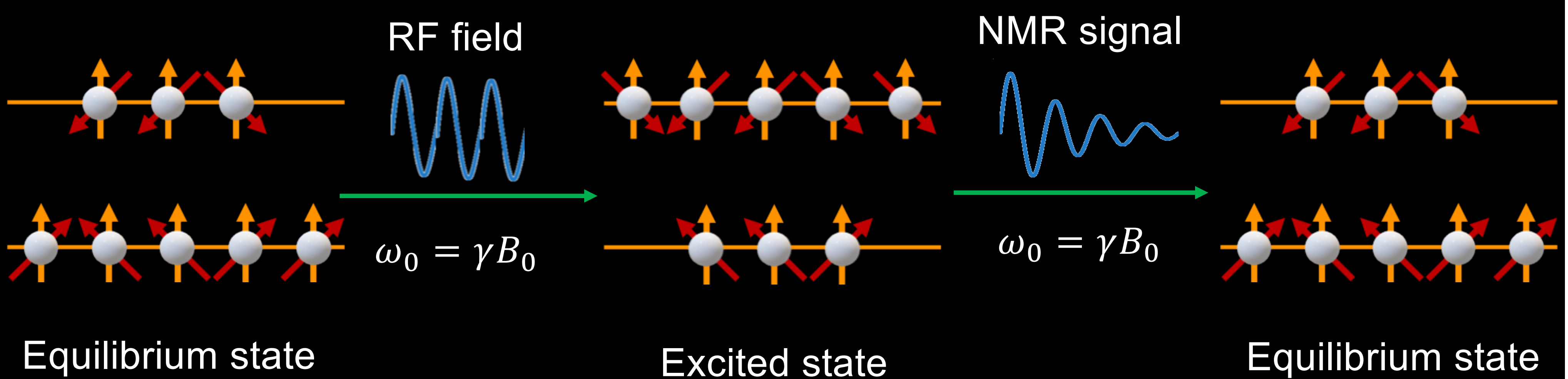
Magnetized nuclear spin system

- Zeeman splitting



$$\Delta E = \gamma\hbar B_0$$

- Nuclear magnetic resonance



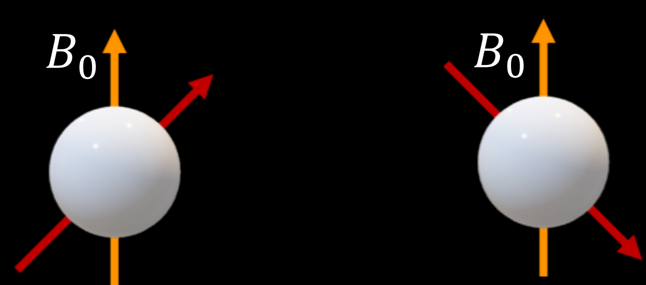
Bulk magnetization

- To describe the ensemble behavior of a spin system

$$\vec{M} = \sum_{n=1}^{N_s} \vec{\mu}_n$$

  – Spin-1/2 system

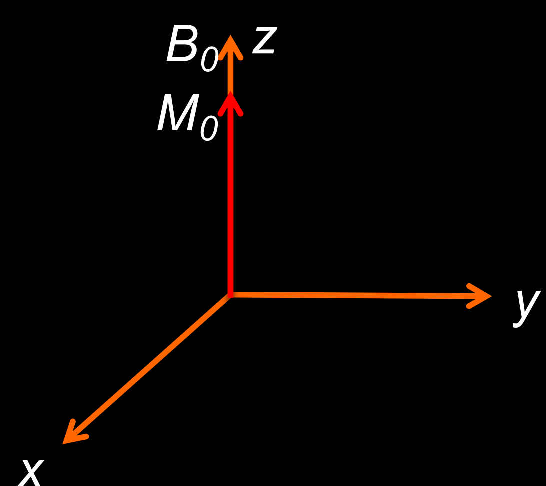
$$\vec{M} = \left(\sum_{n=1}^{N_\uparrow} \frac{1}{2}\gamma\hbar\hat{z} - \sum_{n=1}^{N_\downarrow} \frac{1}{2}\gamma\hbar\hat{z} \right) = (N_\uparrow - N_\downarrow)\,\frac{1}{2}\gamma\hbar\hat{z}$$



Bulk magnetization

- Bulk magnetization (spin-1/2)
  - $\frac{N_\uparrow - N_\downarrow}{N_s} \approx \frac{\gamma \hbar B_0}{2KT_s}$, N_s: total number of spins
  - At 1 Tesla, $\frac{N_\uparrow - N_\downarrow}{N_s} \approx 3\times10^{-6}$ (three in a million!)
  - $\vec{M}$ is parallel to the external magnetic field at equilibrium
  - M_0 is proportional to B_0

$$\vec{M} = \frac{\gamma^2 \hbar^2 B_0 N_s}{4KT_s}\ \hat{z}$$
$$= M_0\ \hat{z}$$

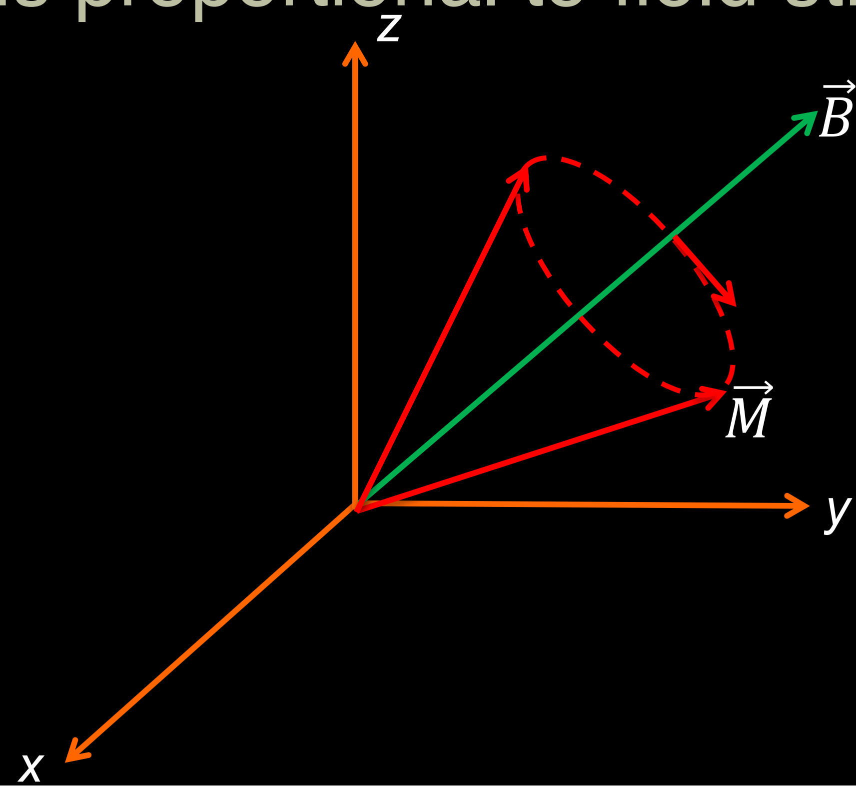


Bloch equation

- The governing equation of the dynamics of bulk magnetization
  - Ignoring relaxation, $\vec{M}$ rotates clockwise about the external magnetic field $\vec{B}$
  - Larmor frequency ω is proportional to field strength

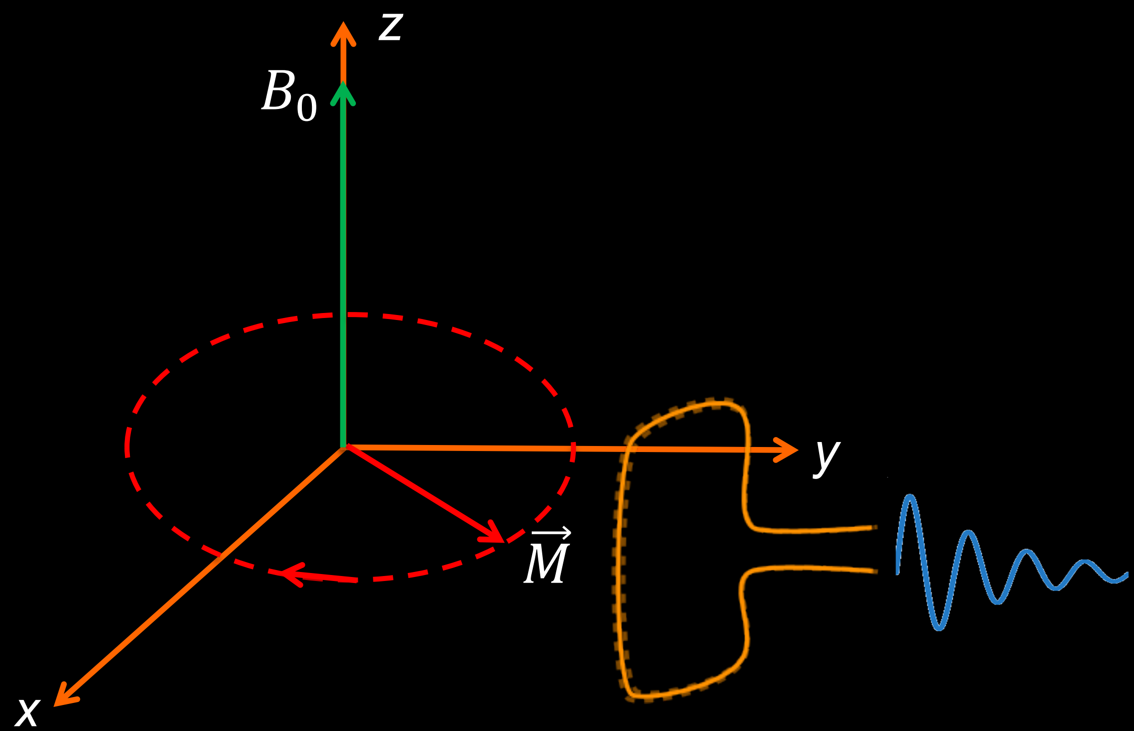
$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

$$\omega = \gamma \left| \vec{B} \right|$$



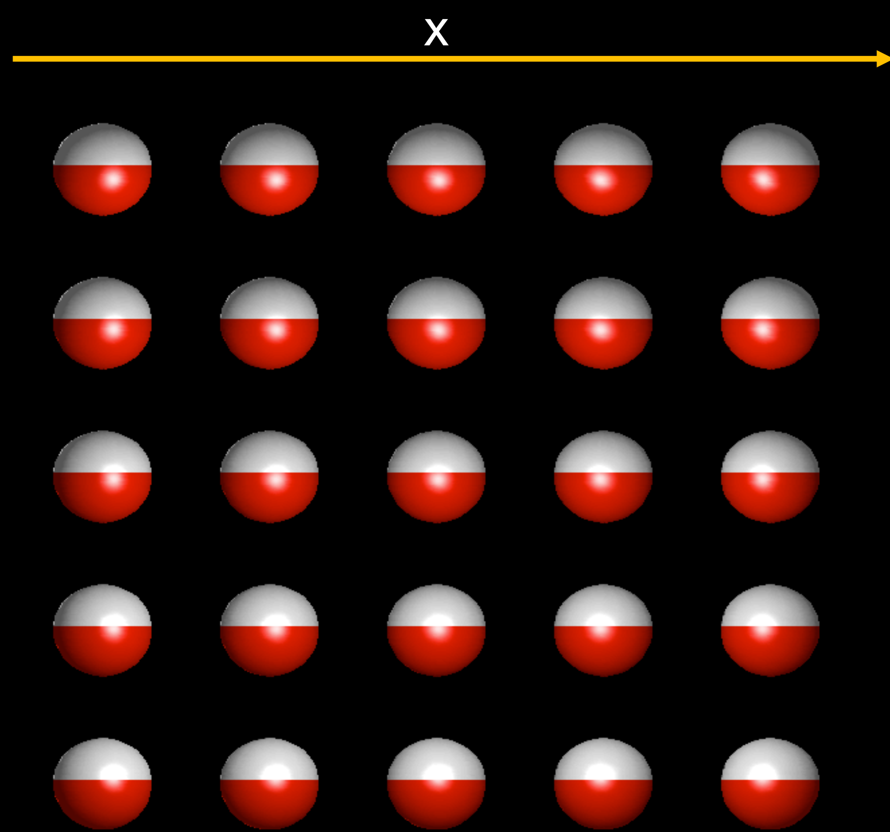
Example 1: Effects of main field $\vec{B}_0$

- Precession frequency is $\omega_0 = \gamma B_0$
  - γ is called the gyromagnetic ratio
  - $\gamma/2\pi = 42.58\ \mathrm{MHz/T}$ for ^{1}H
  - At $B_0 = 1.5$ T, $f_0 = \omega_0/2\pi \approx 64$ MHz (TV channel 3 in US or channel 2 in Australia)



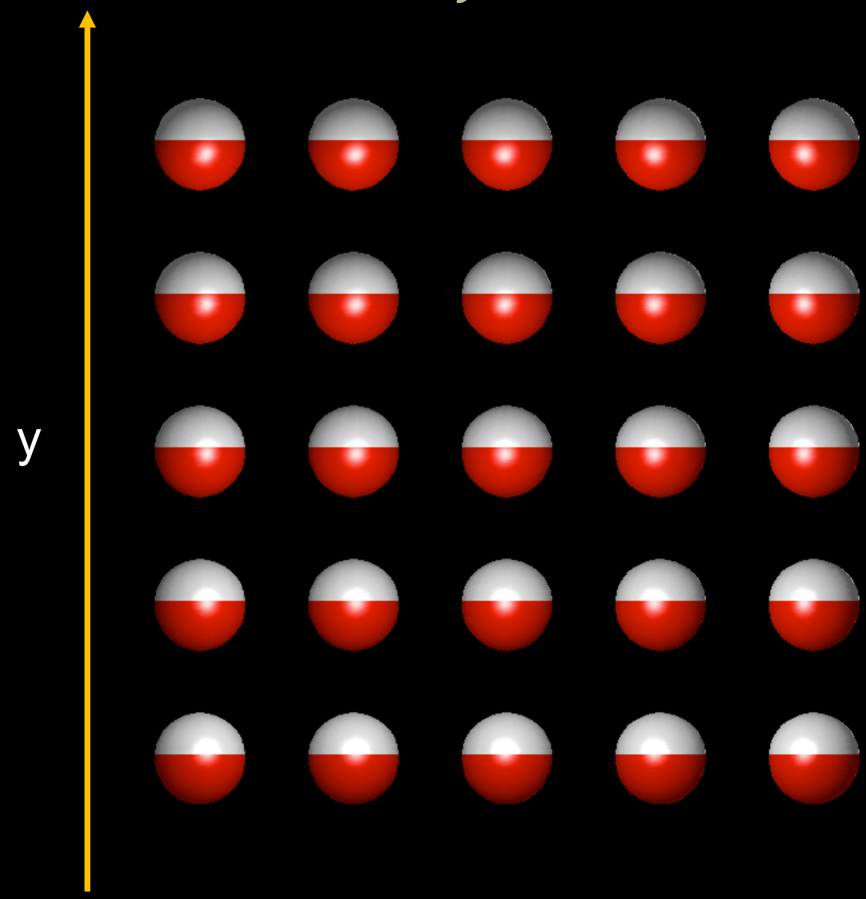
Example 2: Effects of Gradient field $\vec{G}$

- Gradient field: An *inhomogeneous* magnetic field whose *z-component* varies linearly along a specific direction called the gradient direction.
  - x-gradient: $\vec{B} = (B_0 + xG_x)\hat{z}$ or $\omega = (\omega_0 + \gamma x G_x)\hat{z}$



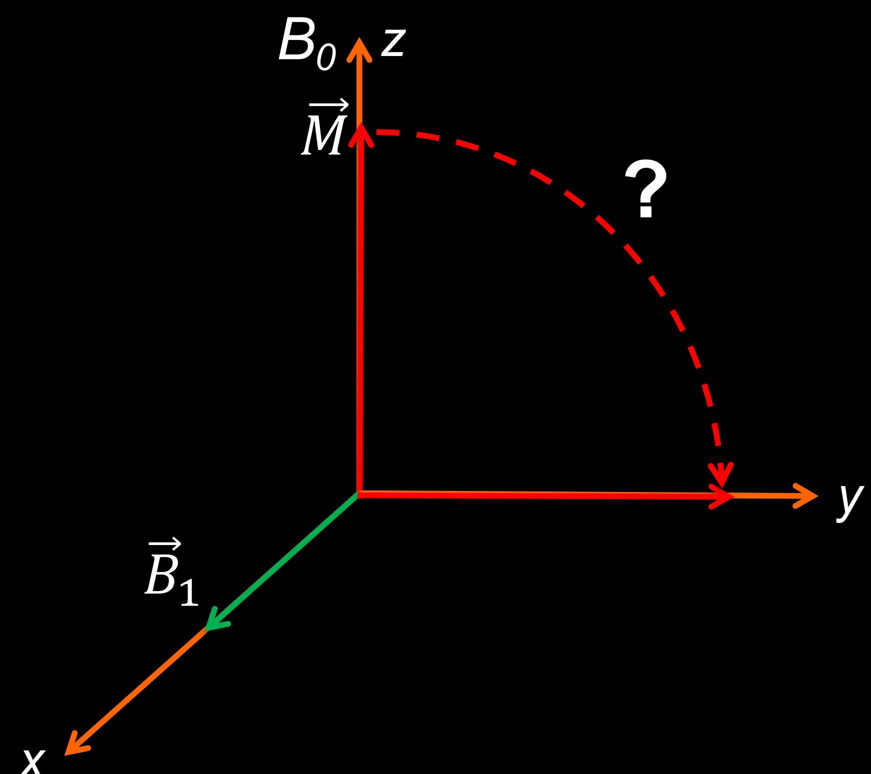
Example 2: Effects of Gradient field $\vec{G}$

- Gradient field: An *inhomogeneous* magnetic field whose *z-component* varies linearly along a specific direction called the gradient direction.
  - y-gradient: $\vec{B} = (B_0 + yG_y)\hat{z}$ or $\omega = (\omega_0 + \gamma y G_y)\hat{z}$



Non-selective signal excitation

- Excitation: Rotate $\vec{M}$ from the equilibrium state to the transversal plane.



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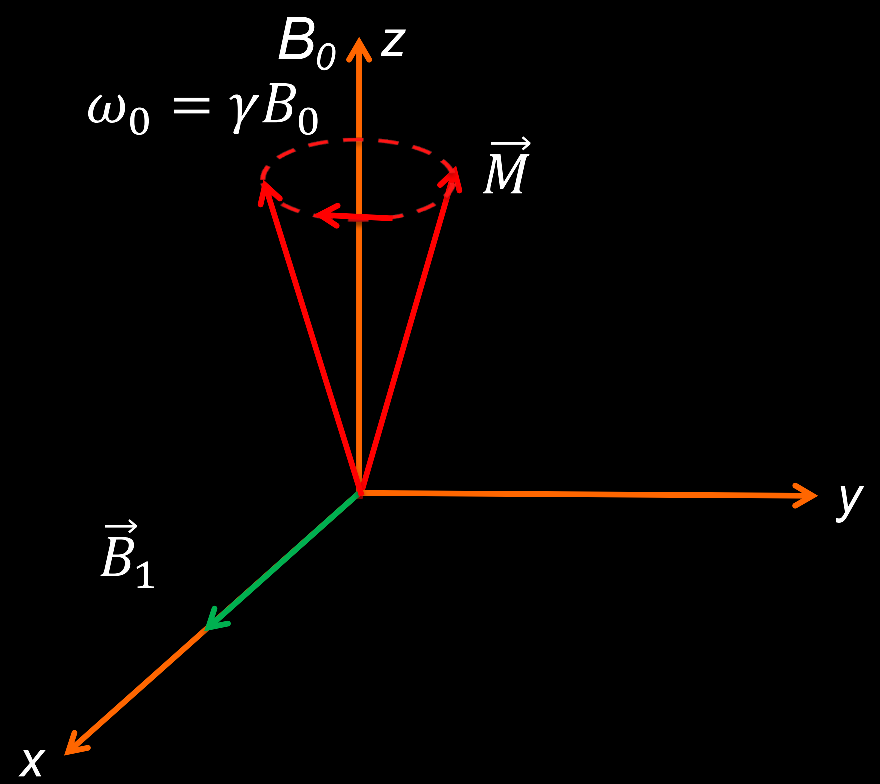
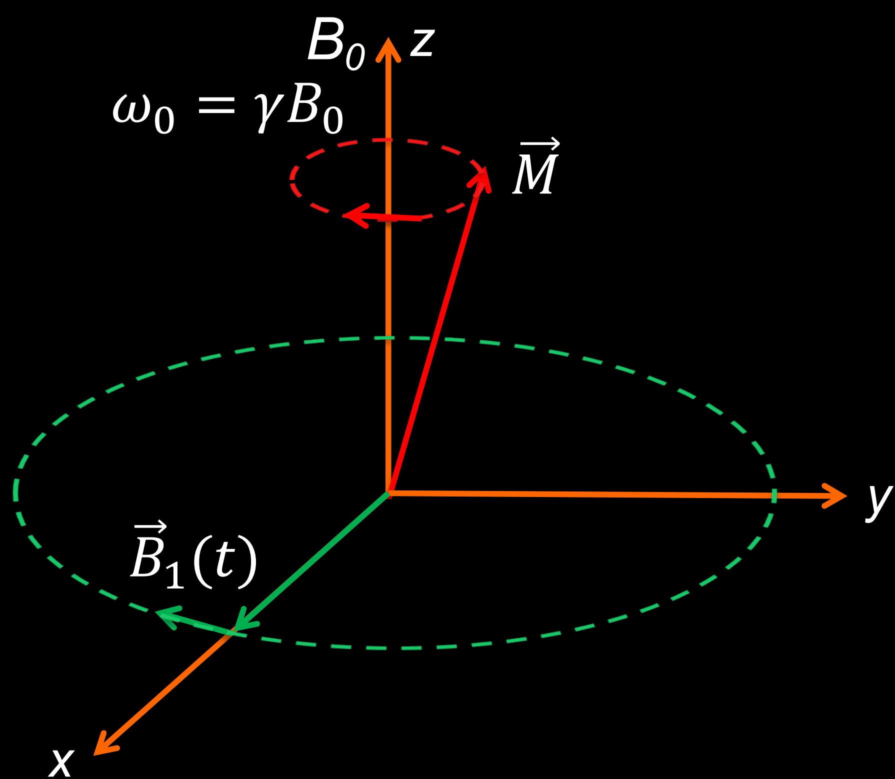


Non-selective signal excitation

- Excitation: Rotate $\vec{M}$ from the equilibrium state to the transversal plane.

Non-selective signal excitation

- Excitation: Rotate $\vec{M}$ from the equilibrium state to the transversal plane.
  - Circular polarized radiofrequency (RF) field
    $$\vec{B}_1(t) = B_1(t)(\hat{x}\cos\omega_0 t - \hat{y}\sin\omega_0 t)$$
  - $B_1 \approx 0.1$ Gauss, $B_1(t)$ lasts several milliseconds

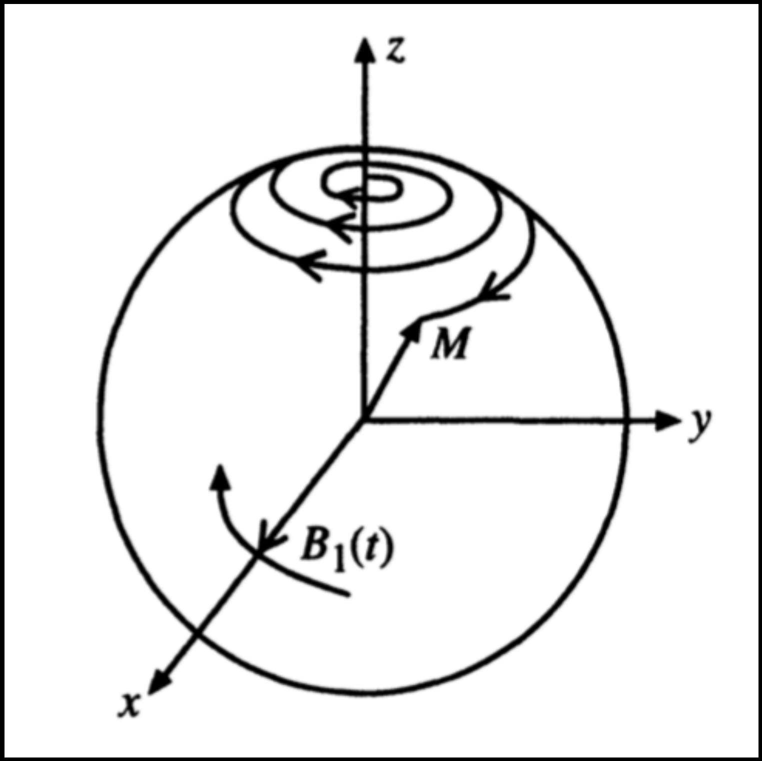


Non-selective signal excitation

- Excitation: Rotate $\vec{M}$ from the equilibrium state to the transversal plane.
  - Circular polarized radiofrequency (RF) field

$$\vec{B}_1(t) = B_1(t)(\hat{x}\cos\omega_0 t - \hat{y}\sin\omega_0 t)$$

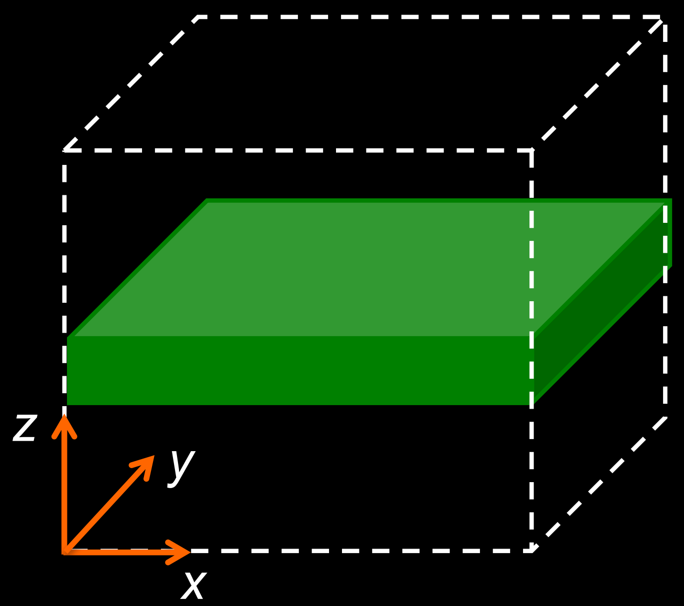
  - $B_1(t)$ typically lasts several milliseconds

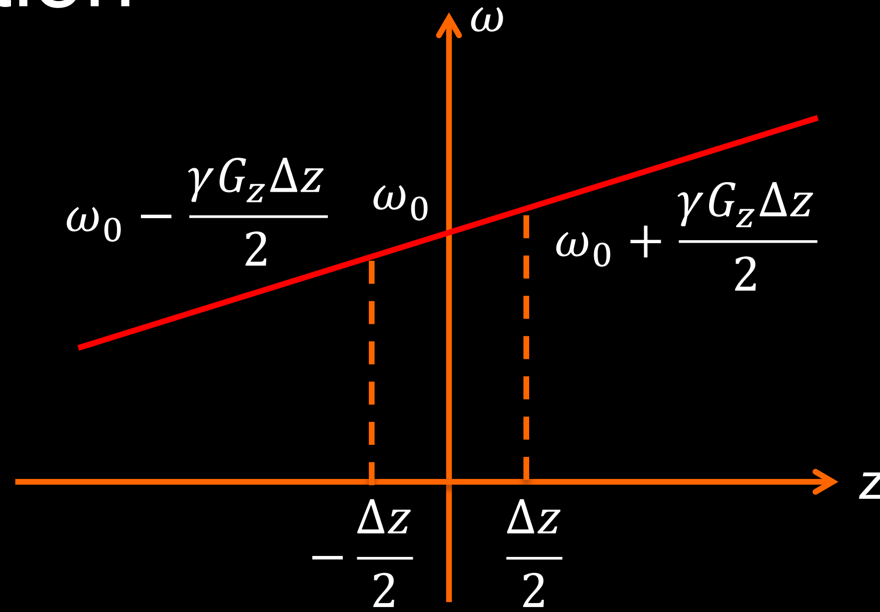


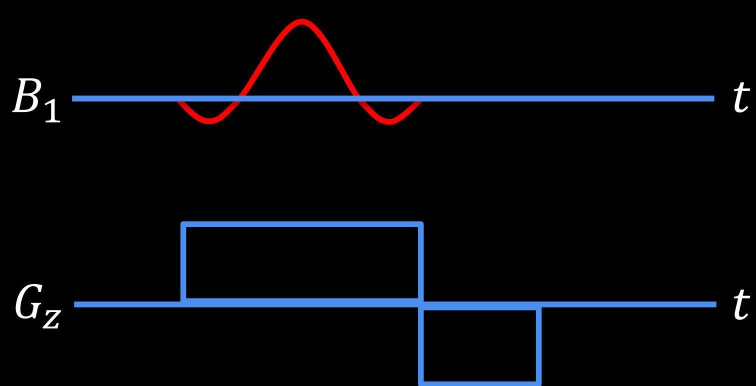
Courtesy of Z.-P. Liang

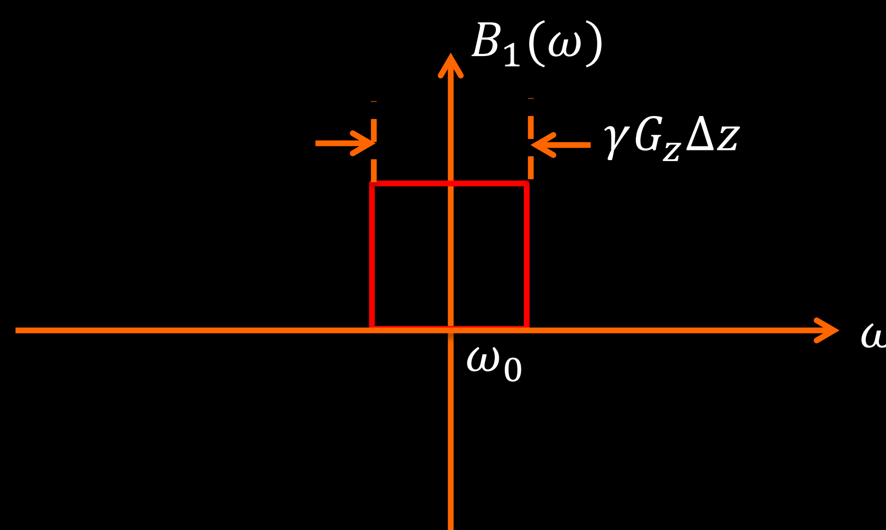
Selective signal excitation

- Slice-selective excitation



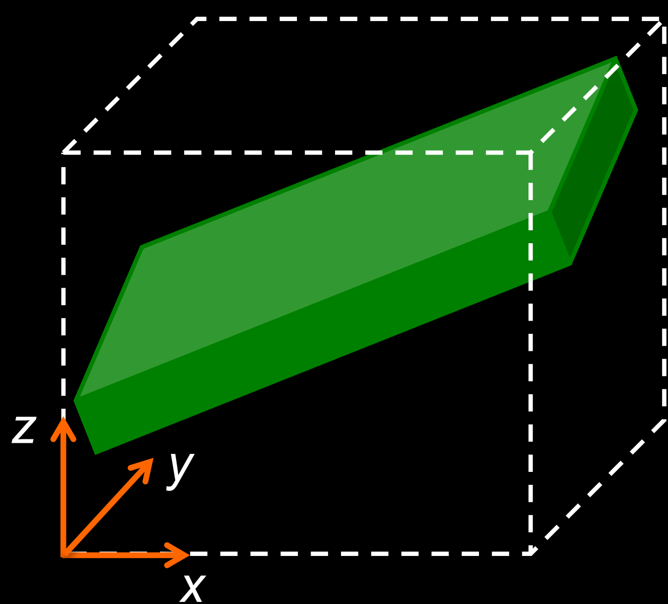


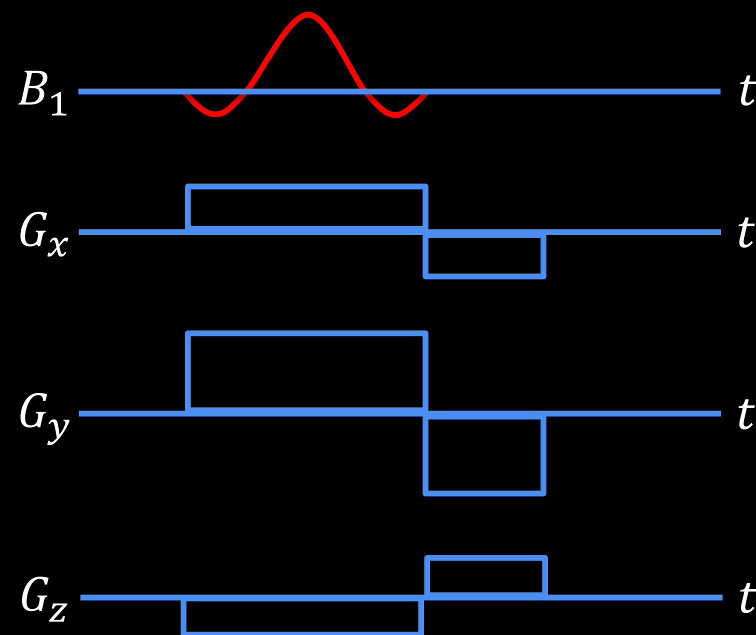




Selective signal excitation

- Slice-selective excitation
  - 2D/3D imaging in an oblique plane/volume

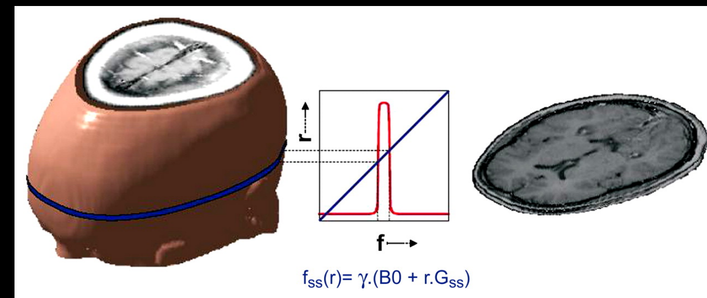




Selective signal excitation

One-dimensional RF pulse

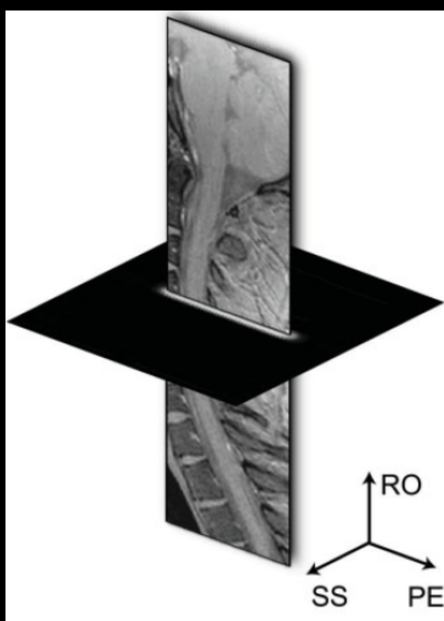
- Slice selection (Garroway 1974; Lauterbur, 1975; Mansfield, 1976; Hoult, 1977)



Multidimensional RF pulse: more flexible excitation pattern

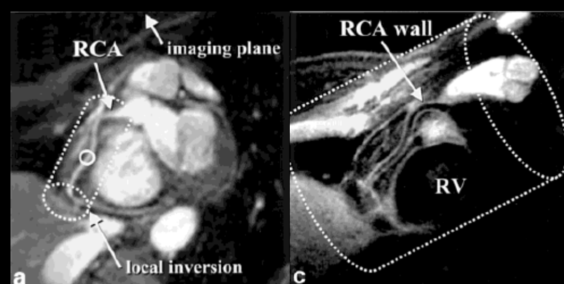
Reduced FOV imaging

Saritas et al., 2008.



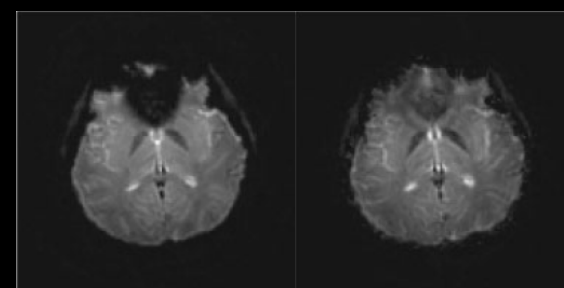
Local excitation

Botnar et al., 2001.



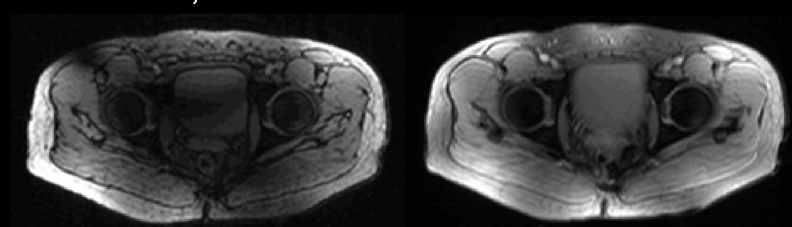
Signal recovery

Yip et al., 2006.



B_1 inhomogeneity correction

Kerr et al., 2008.



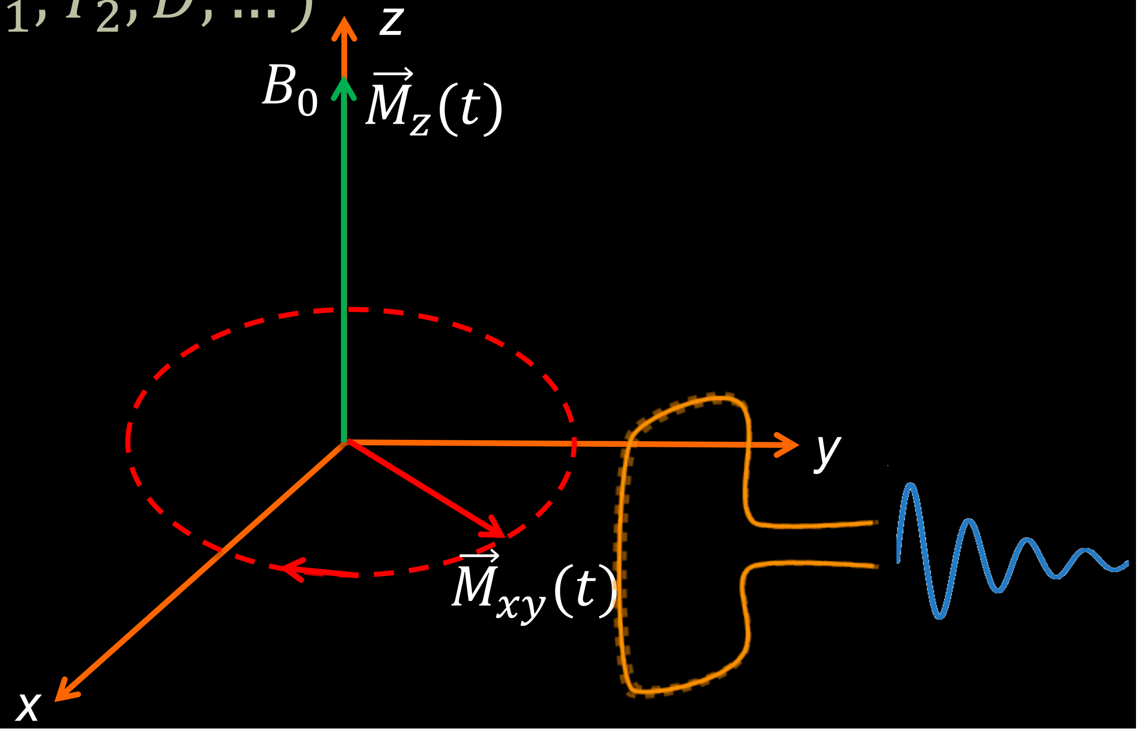
Motion tracking

Signal reception

- According to the principle of reciprocity:

$$V(t) = \omega_0 \int_{\text{Object}} \left|\vec{M}_{xy}(\vec{r})\right| \cdot \vec{B}_1^-(\vec{r}) \cos(-\omega_0 t + \phi(\vec{r}, t))\, d\vec{r}$$

  - $\vec{B}_1^-(\vec{r})$: B_1 field generated by a hypothetical unit current flowing in a receive coil.
  - $\left|\vec{M}_{xy}(\vec{r})\right| = M_0 \sin\theta\, f(T_1, T_2, D, \ldots)$
  - $\phi(\vec{r}, t)$
    - Gradient
    - B_0 inhomogeneity
    - Susceptibility
    - Chemical shift
    - Velocity
    - …

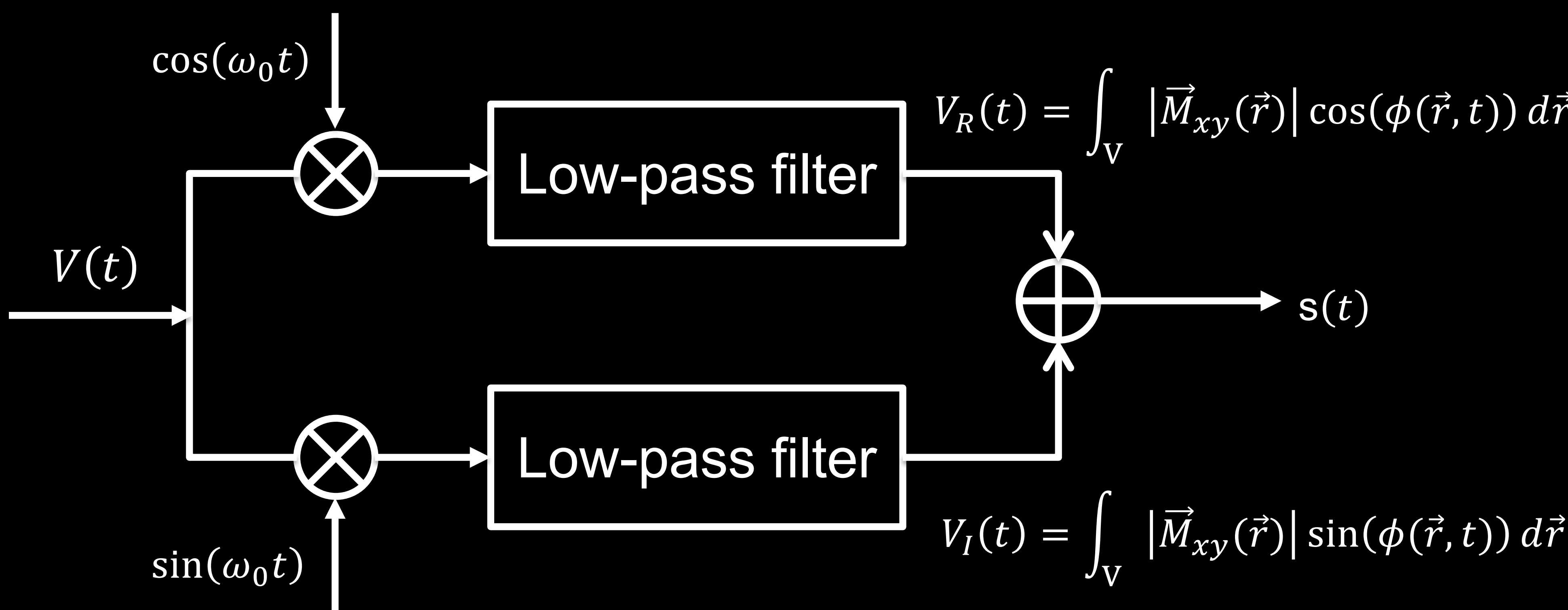


Signal reception

- Phase-sensitive detection

$$V(t) = \int_V \left|\vec{M}_{xy}(\vec{r})\right| \cos(-\omega_0 t + \phi(\vec{r},t))\, d\vec{r}$$

$$s(t) = V_R(t) + iV_I(t) = \int_V \left|\vec{M}_{xy}(\vec{r})\right| e^{i\phi(\vec{r},t)t} d\vec{r}$$



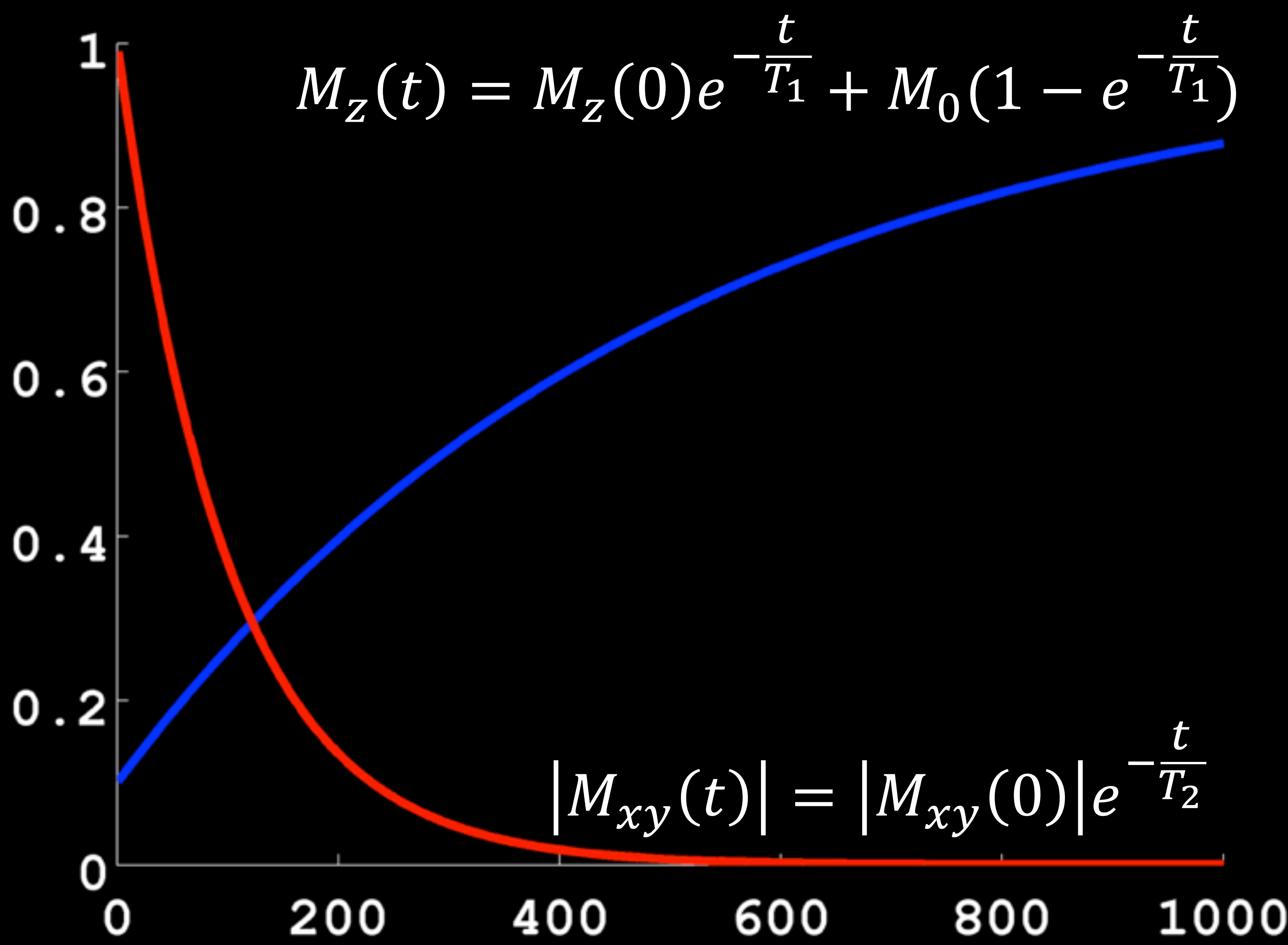
Relaxation

- Bloch equation

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} - \frac{M_x \hat{x} + M_y \hat{y}}{T_2} - \frac{(M_z - M_0)\hat{z}}{T_1}$$

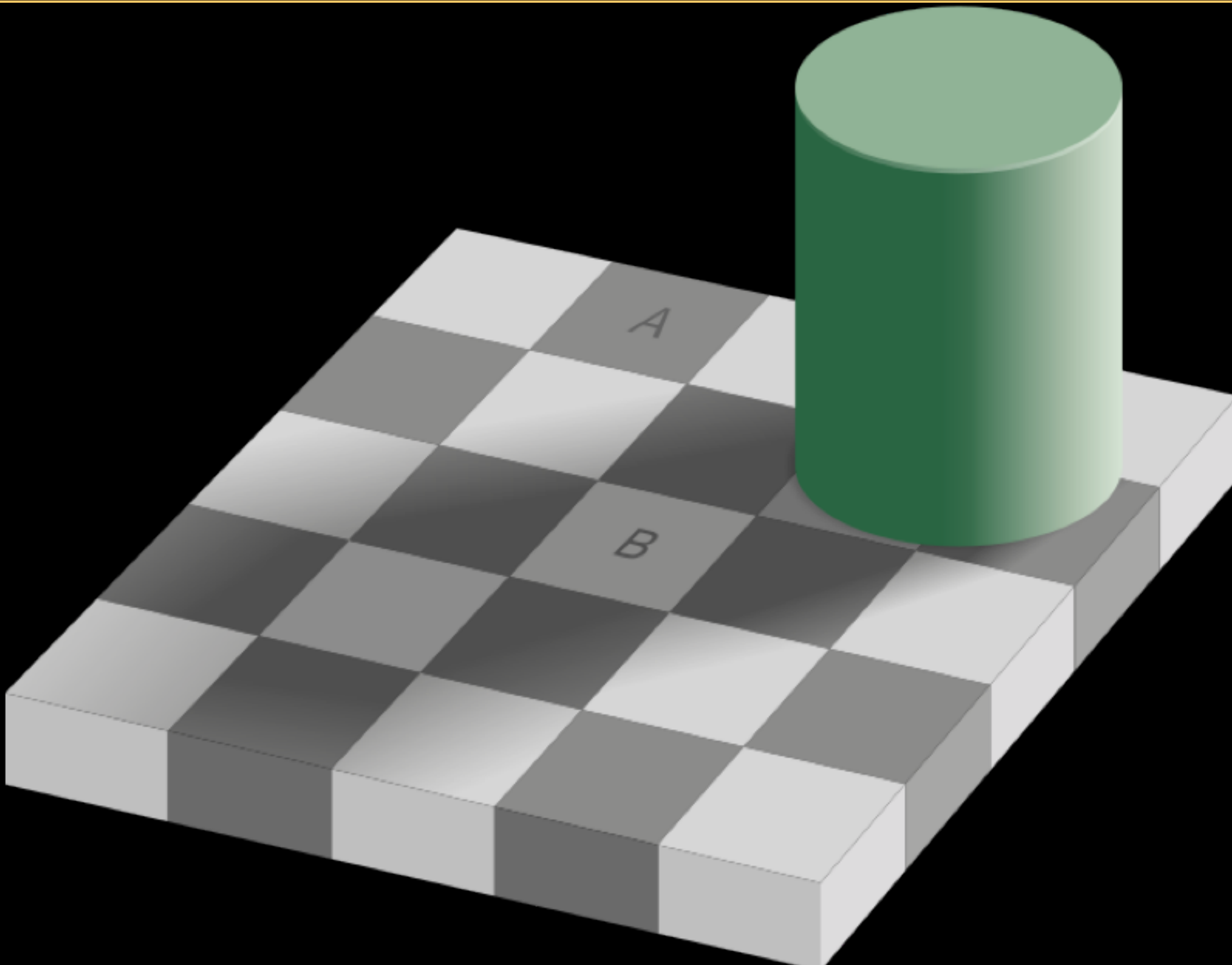
  - T_1 : Longitudinal relaxation time
  - T_2 : Transverse relaxation time
  - $M_0 \hat{z}$: Equilibrium state of the bulk magnetization

Relaxation

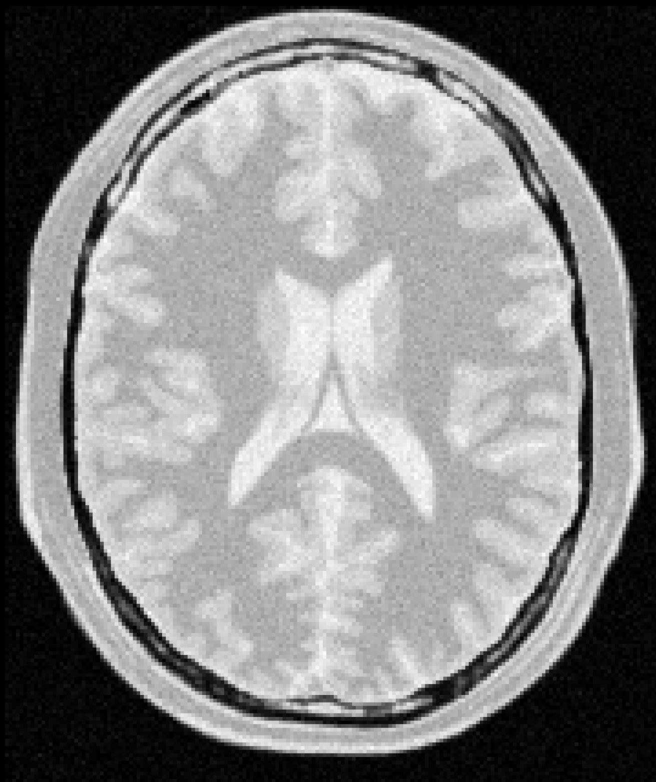


$M_z(t) = M_z(0)e^{-\frac{t}{T_1}} + M_0(1 - e^{-\frac{t}{T_1}})$
$|M_{xy}(t)| = |M_{xy}(0)|e^{-\frac{t}{T_2}}$
1
0.8
0.6
0.4
0.2
0
0
200
400
600
800
1000

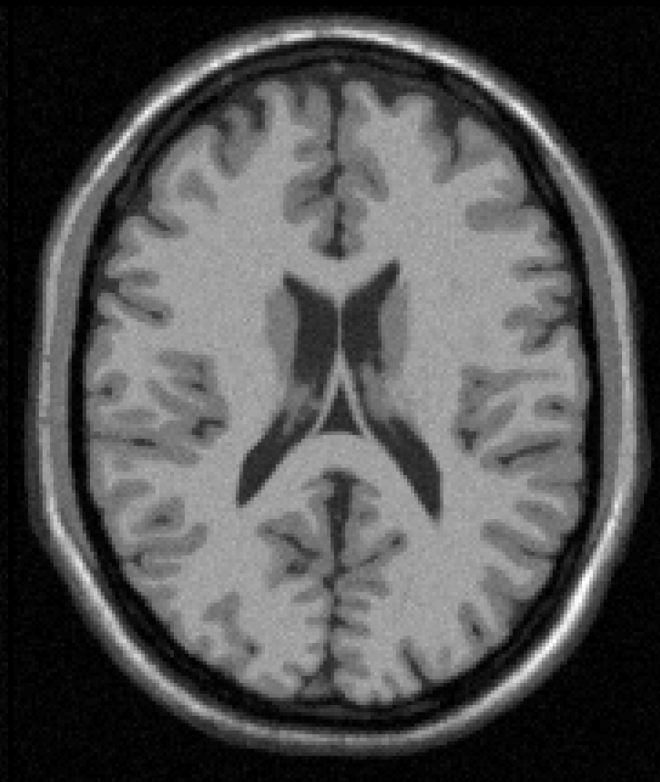
Contrast



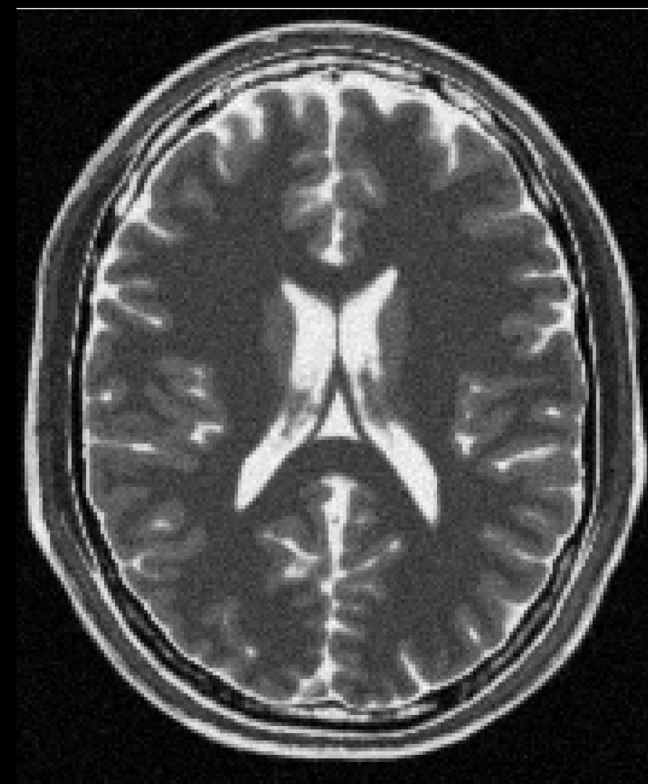
Contrast



Proton density



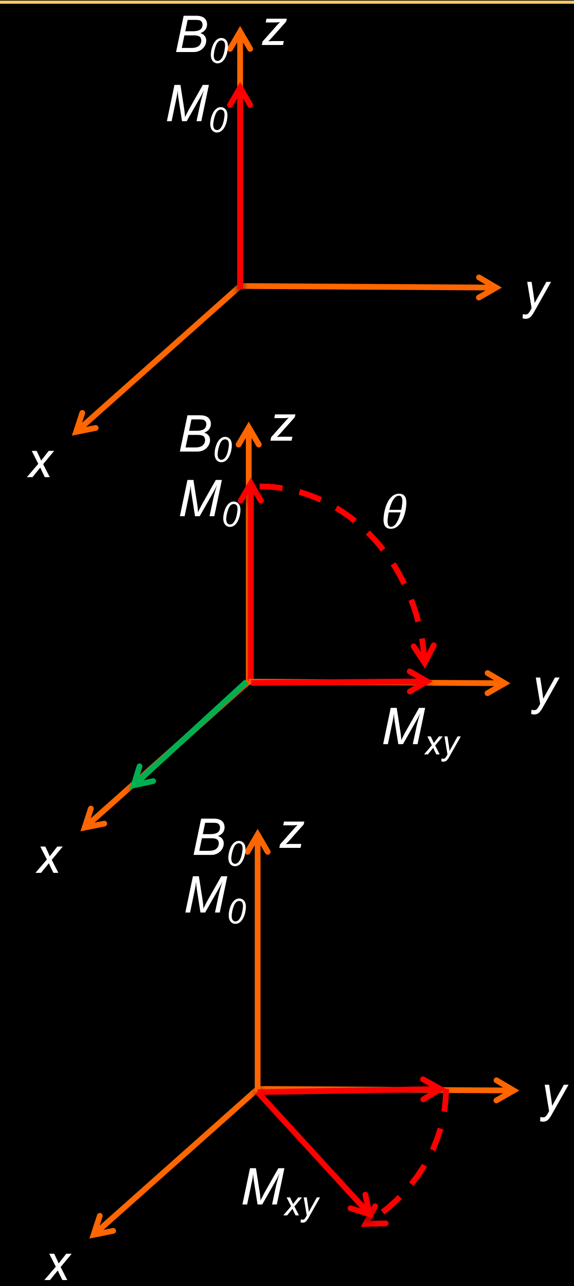
T1-weighted



T2-weighted

Review: MRI physics

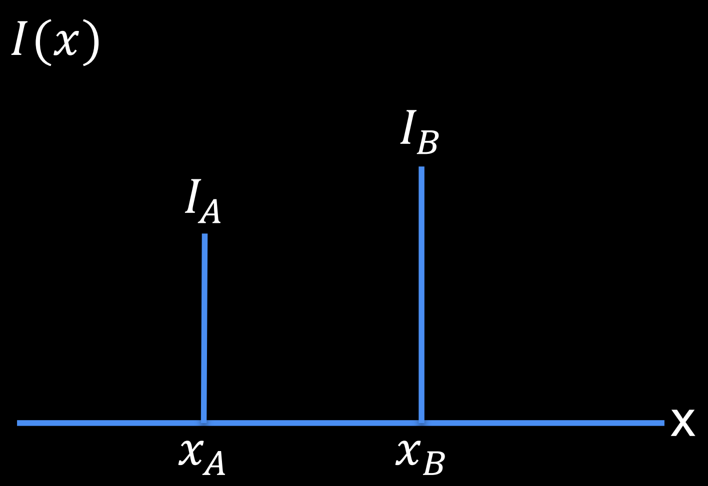
- Bulk magnetization
  - $M_0 = \sum_n^{N_s} \mu_{z,n}$
  - $M_0 = \frac{\gamma^2 h^2 B_0 N_s}{16\pi^2 K T_s}$ (1/2-spin system)
- Signal excitation
  - $|M_{xy}| = M_0 \sin\theta$
- Signal detection
  - $s(t) = M_0 \sin\theta \, e^{-i\omega_0 t}$
  - $\omega_0 = \gamma B_0$



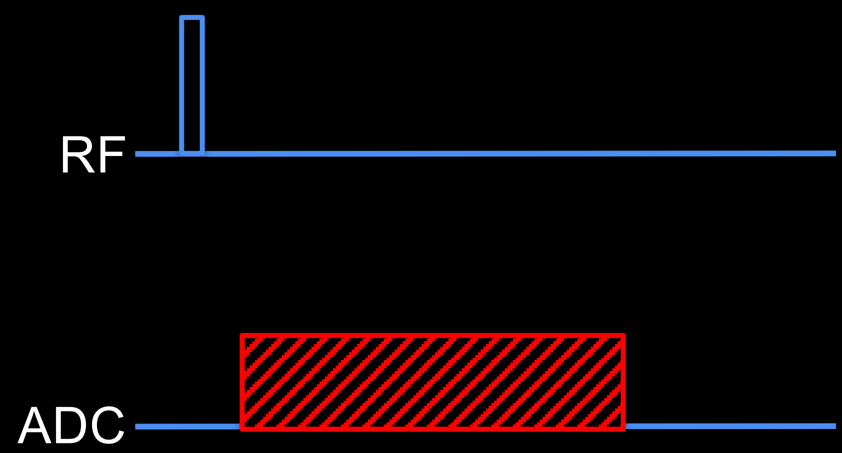
Magnetic Resonance Imaging

MR Imaging

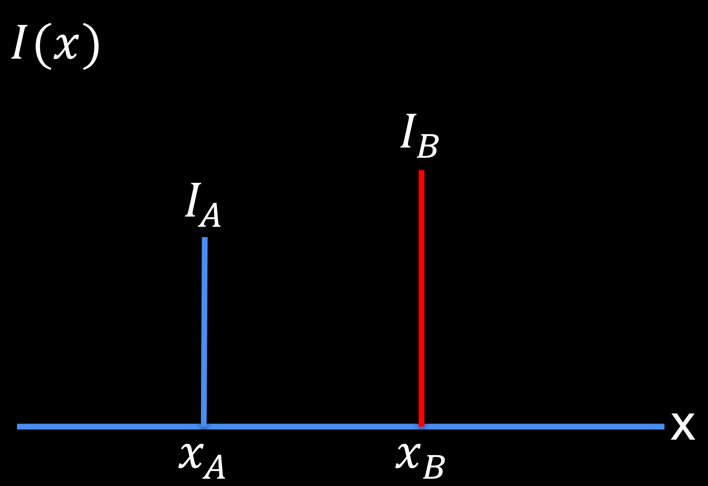
1D Imaging



$$s(t) = I_A e^{-i\omega_0 t} + I_B e^{-i\omega_0 t}$$

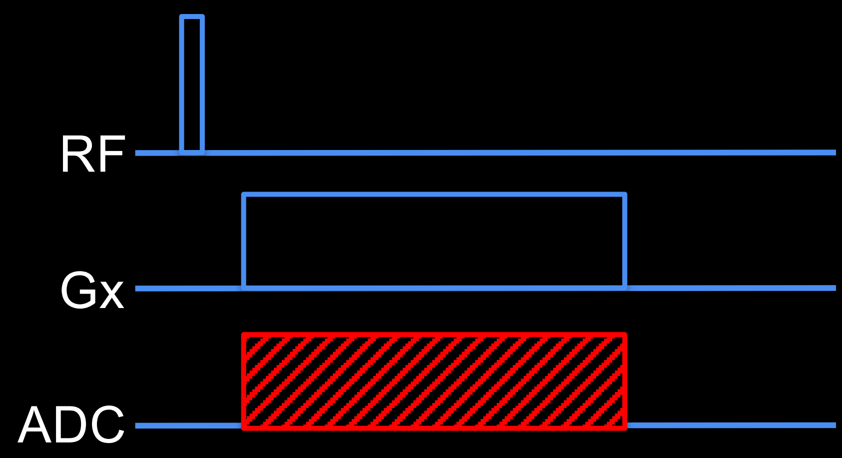


1D Imaging



$$s(t) = \left(I_A e^{-i\gamma G_x x_A t} + I_B e^{-i\gamma G_x x_B t}\right) e^{-i\omega_0 t}$$

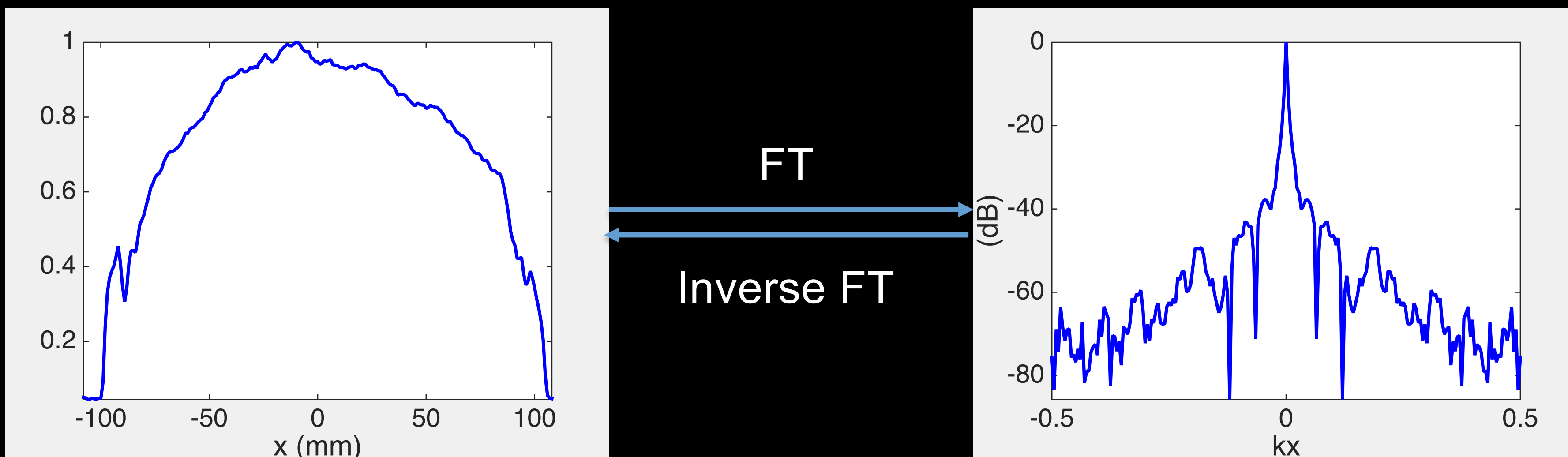
$$s(t) = I_A e^{-i\gamma G_x x_A t} + I_B e^{-i\gamma G_x x_B t}$$



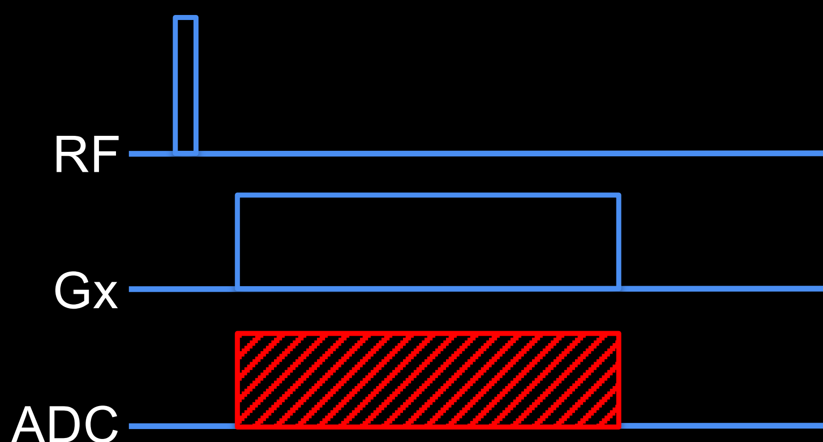
1D Imaging

$$s(t) = \int_x I(x)\, e^{-i\gamma G_x x t} dx$$

$$s(k_x) = \int_x I(x)\, e^{-i2\pi x \cdot k_x} dx,\, k_x = \frac{\gamma G_x t}{2\pi}$$

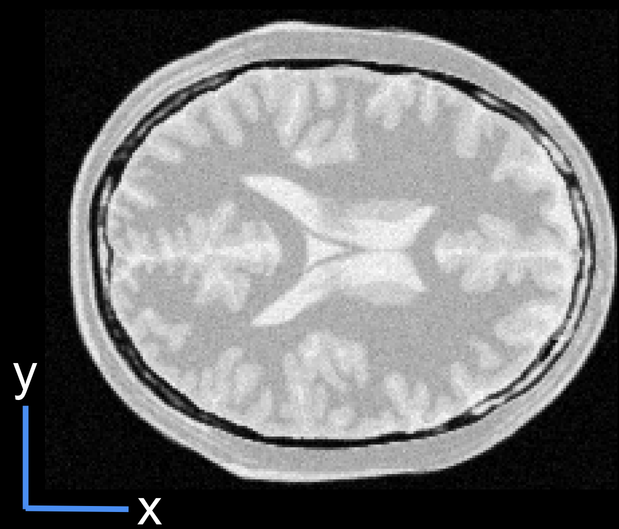


$I(x)$ $s(k_x)$

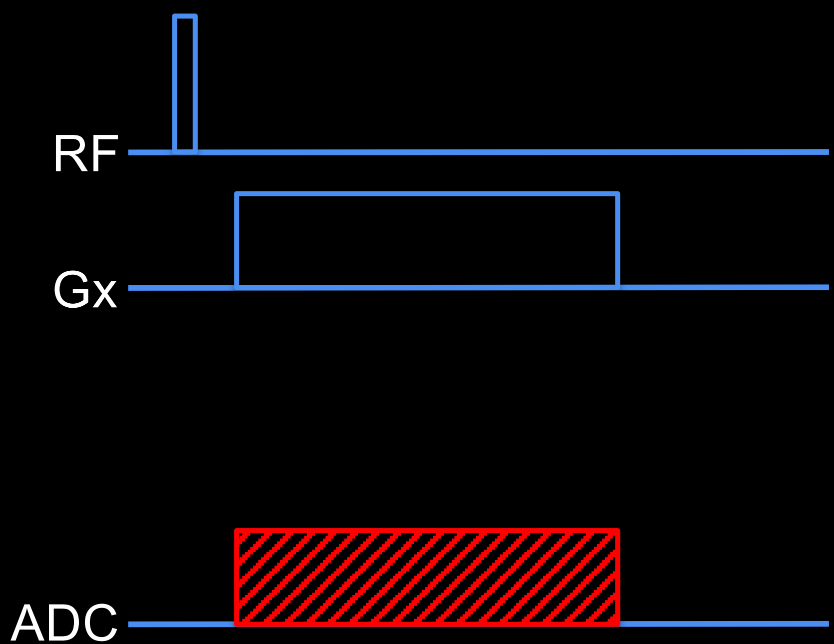


2D imaging

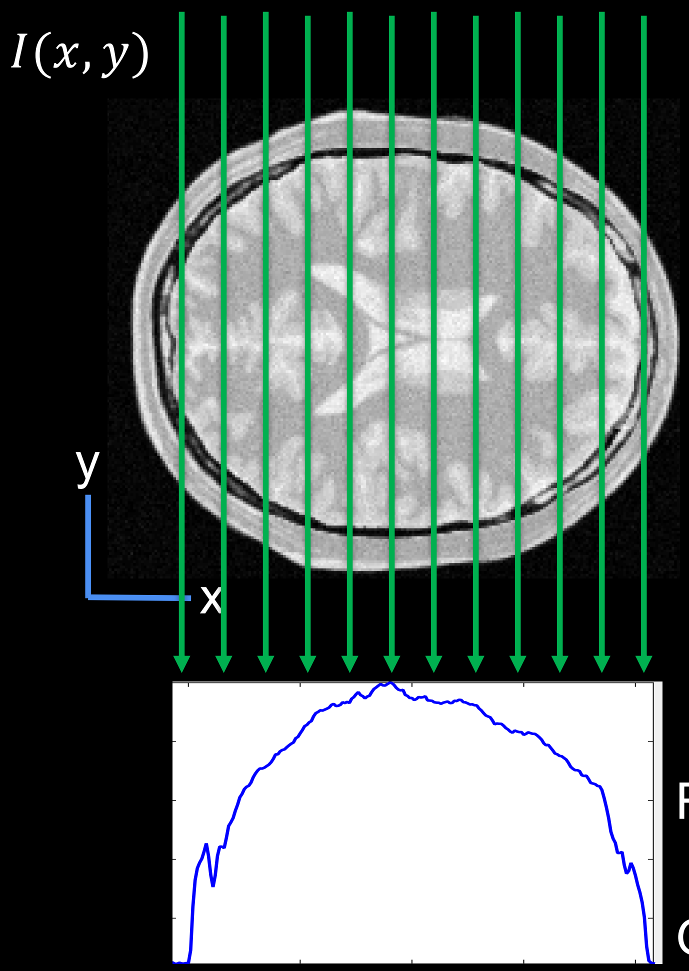
$I(x,y)$



$$s(t) = \iint_{x,y} I(x,y)\, e^{-i2\pi x\cdot k_x}\, dxdy\,,\; k_x = \frac{\gamma G_x t}{2\pi}$$



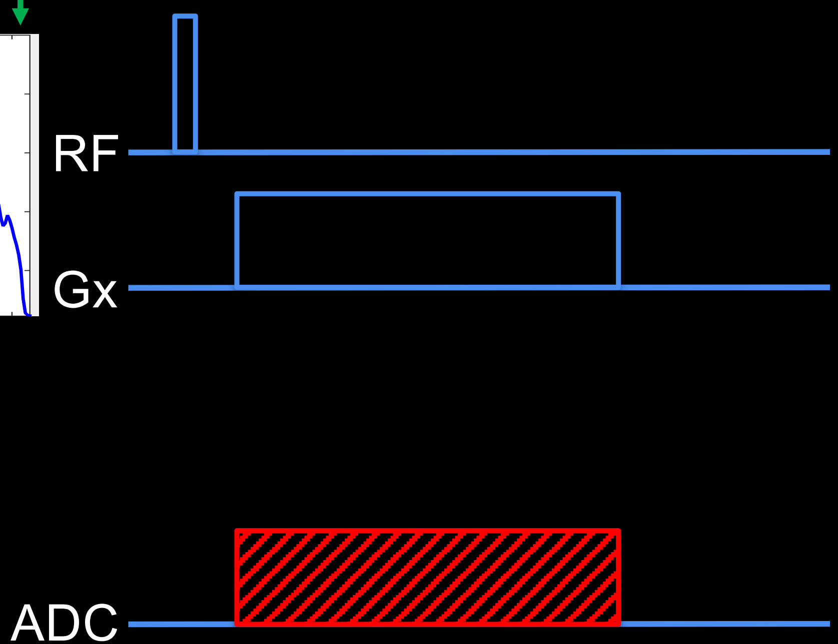
2D imaging



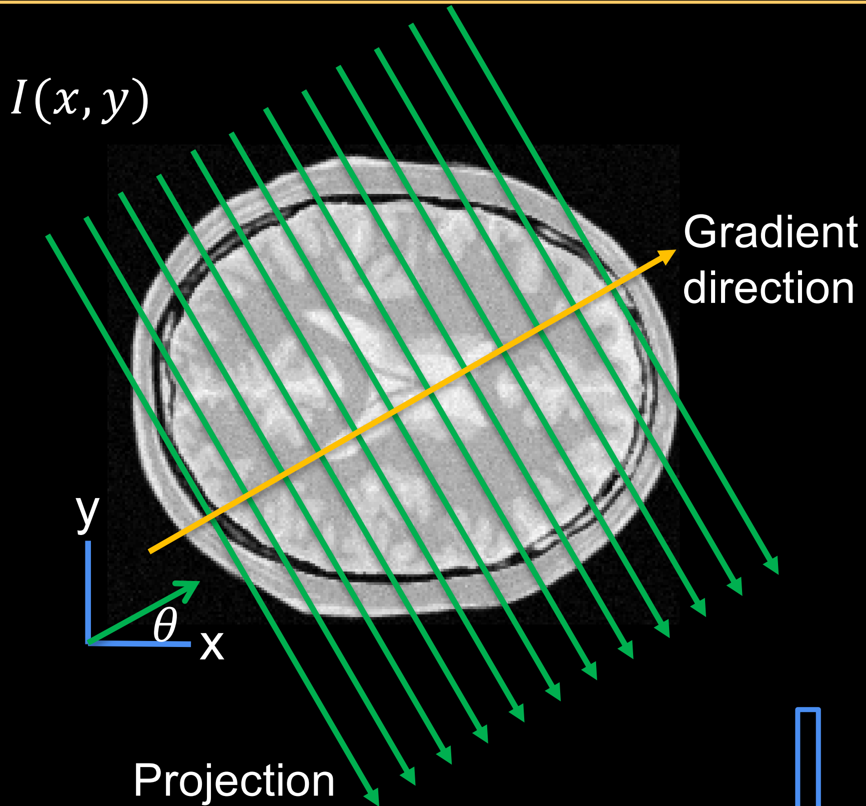
$$s(t) = \iint_{x,y} I(x,y) e^{-i2\pi x \cdot k_x} dxdy\,,\, k_x = \frac{\gamma G_x t}{2\pi}$$

$$s(k_x) = \int_x \left(\int_y I(x,y) dy \right) e^{-i2\pi x \cdot k_x} dx$$

Projection along y!



2D imaging

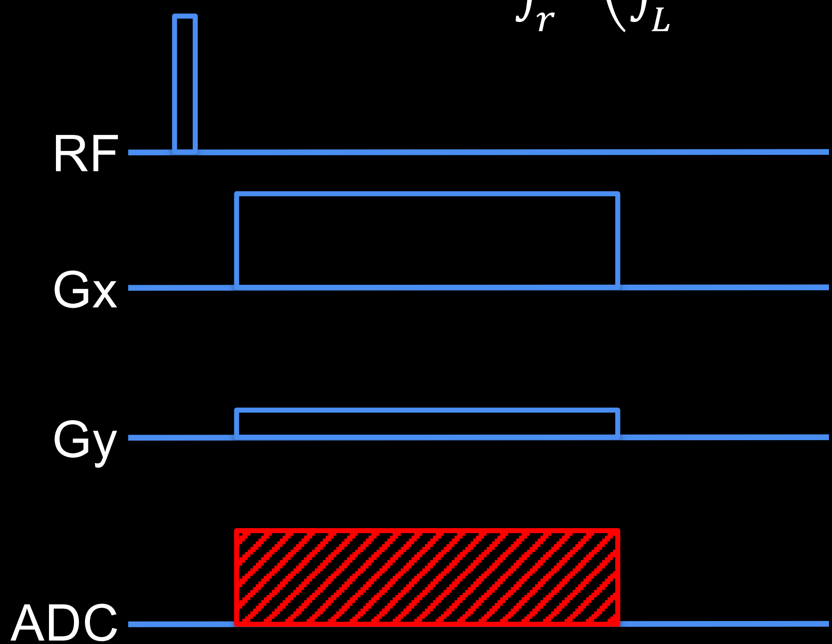


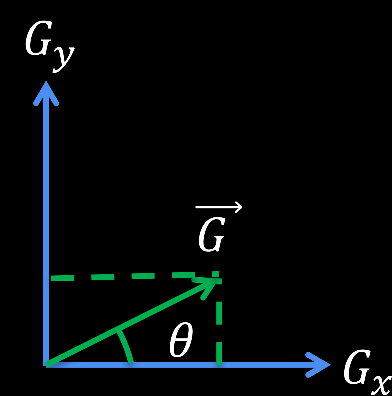
$$s(t,\theta) = \iint_{x,y} I(x,y)e^{-i\gamma|\vec{G}|(x\cos\theta + y\sin\theta)t}\,dxdy$$

$$= \int_r \underbrace{\left(\int_L I(x,y)\,dl\right)} e^{-i\gamma|\vec{G}|rt}\,dr$$

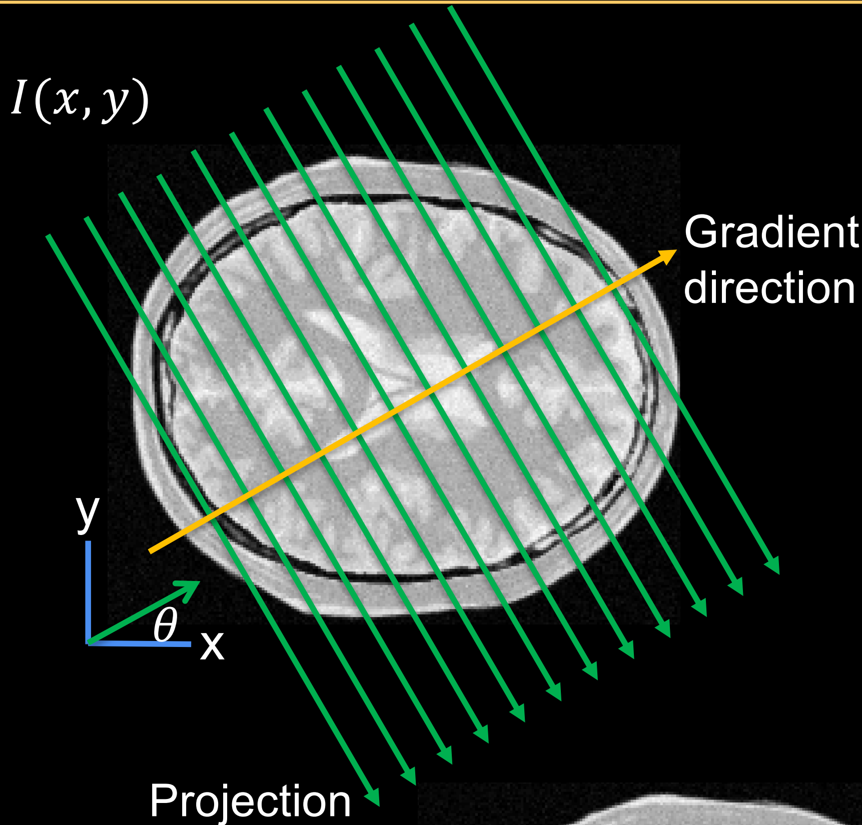
Projection along:
$x\cos\theta + y\sin\theta = r$

$$s(k,\theta) = \int_r \left(\int_L I(x,y)\,dl\right) e^{-i2\pi kr}\,dr\,,\ k = \frac{\gamma|\vec{G}|t}{2\pi}$$





2D imaging

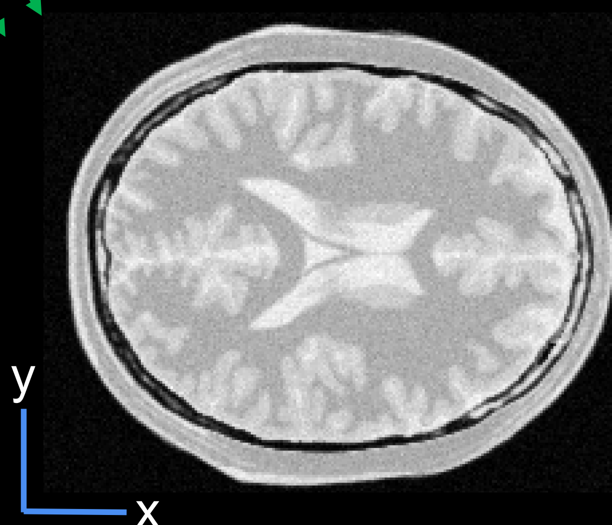


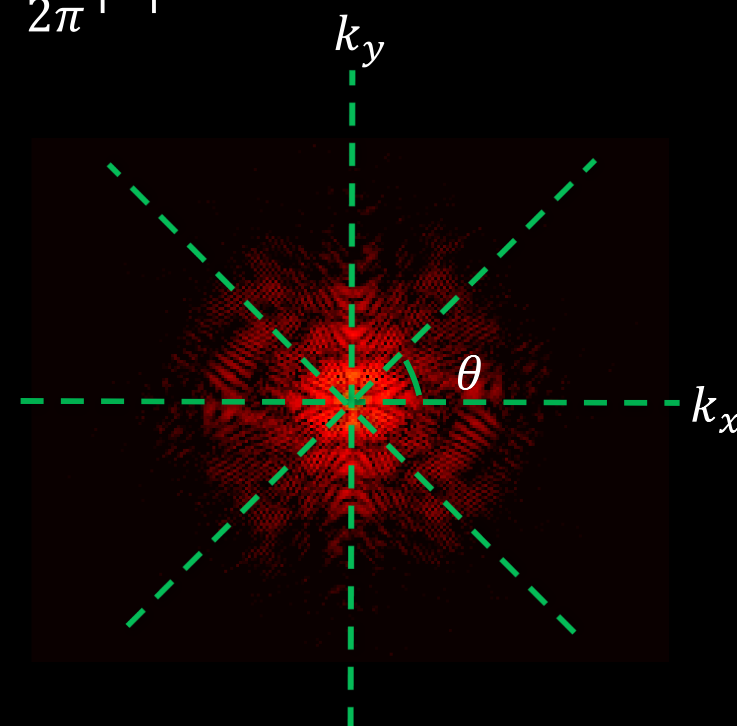
$$s(t,\theta) = \iint_{x,y} I(x,y)e^{-i\gamma|\vec{G}|(x\cos\theta + y\sin\theta)t}\,dxdy$$

$$s(k_x,k_y) = \iint_{x,y} I(x,y)e^{-i2\pi(xk_x + yk_y)}\,dxdy$$

$$k_y = \frac{\gamma}{2\pi}|\vec{G}|t\sin\theta$$

$$k_x = \frac{\gamma}{2\pi}|\vec{G}|t\cos\theta$$

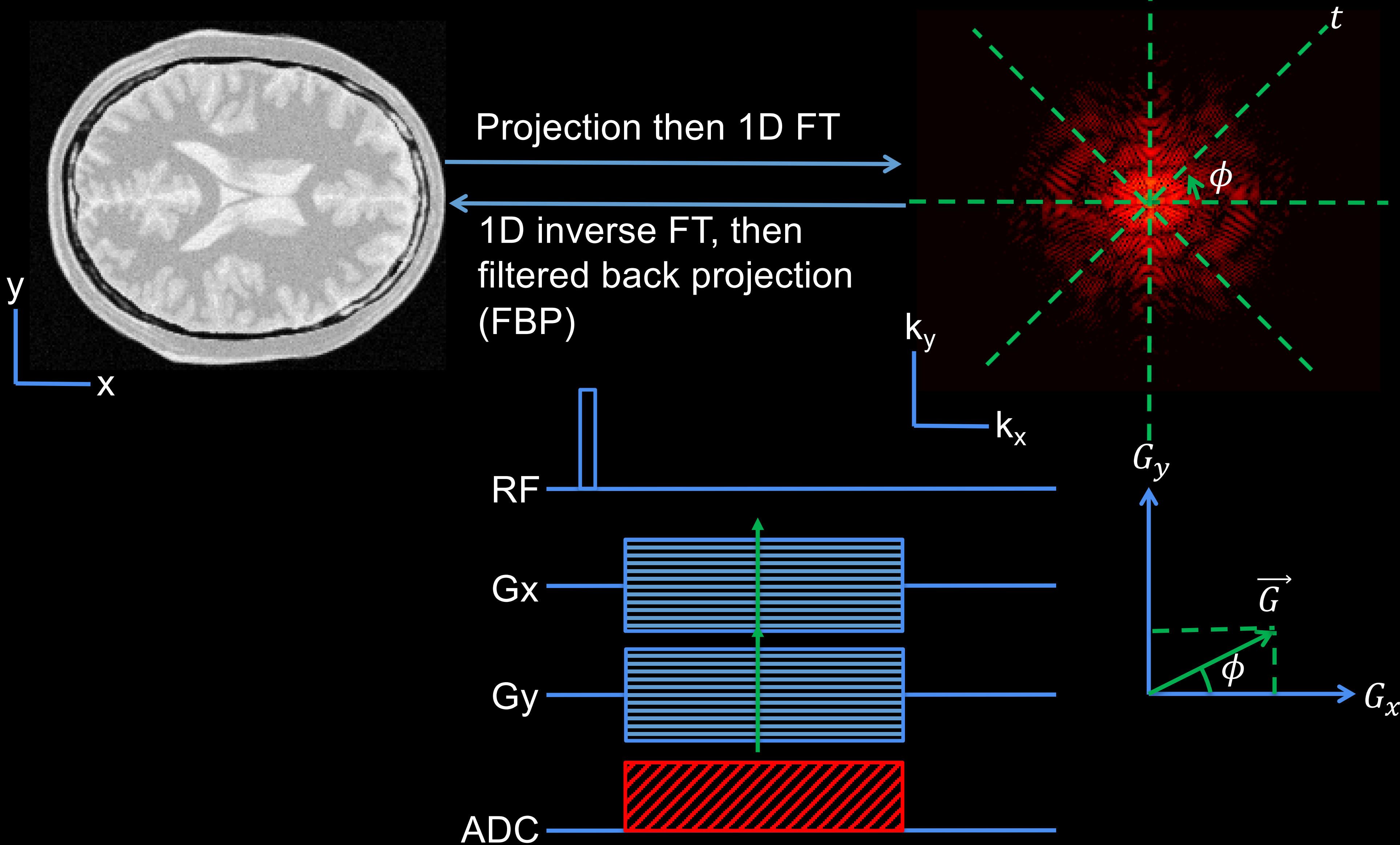




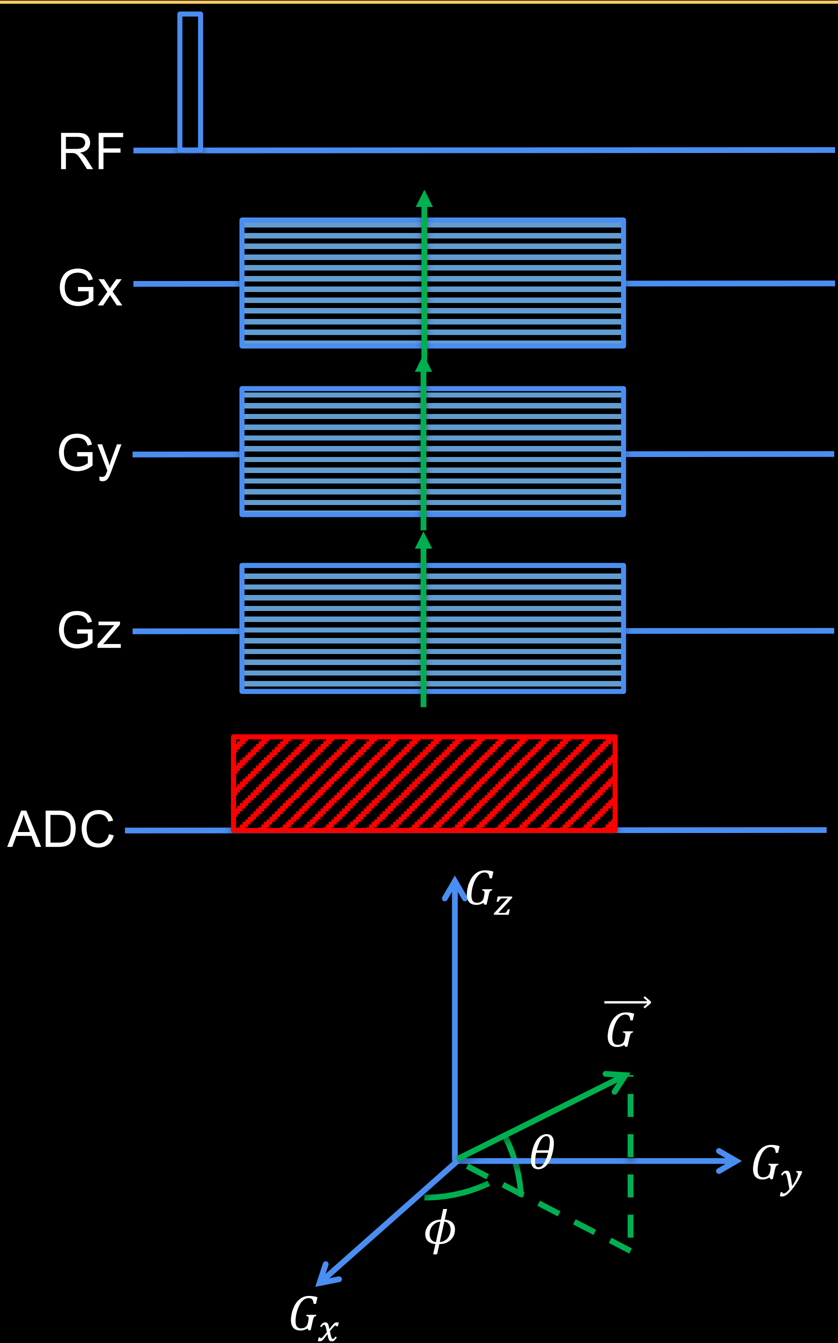
2D imaging: Radial sampling

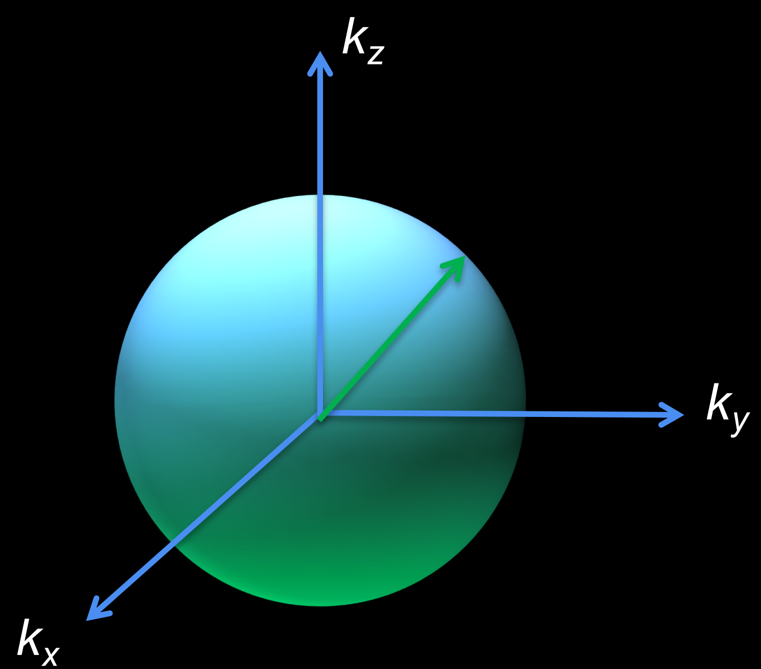
$I(x,y)$

$$s(t,\phi) = \iint_{x,y} I(x,y) e^{-i2\pi(x\cdot k_x + y\cdot k_y)} dxdy$$

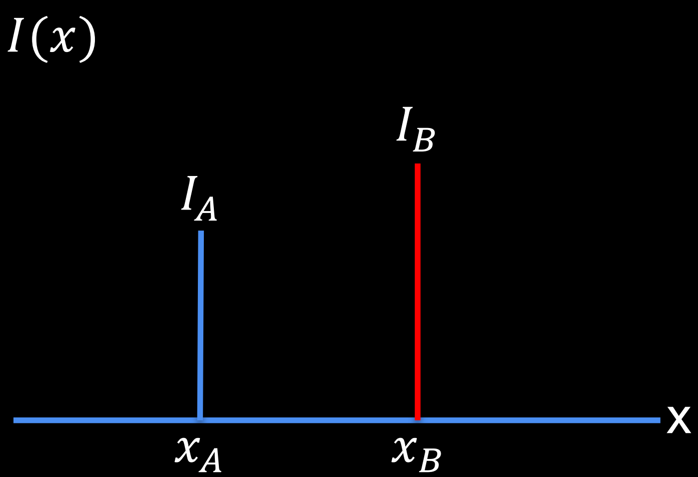


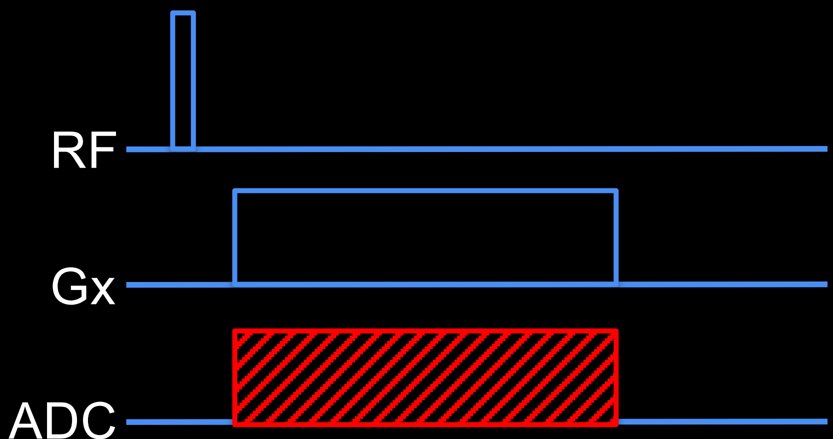
3D Imaging: Radial sampling





1D Imaging: Revisit

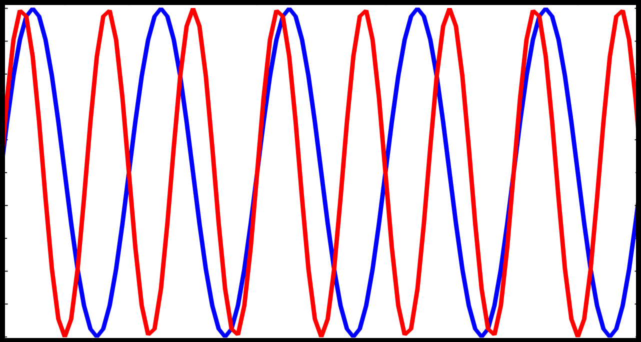




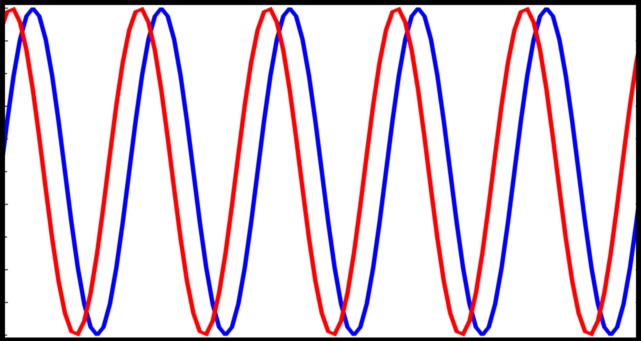
$$s(t) = I_A e^{-i\gamma G_x x_A t} + I_B e^{-i\gamma G_x x_B t}$$

Is it the only way to do it?

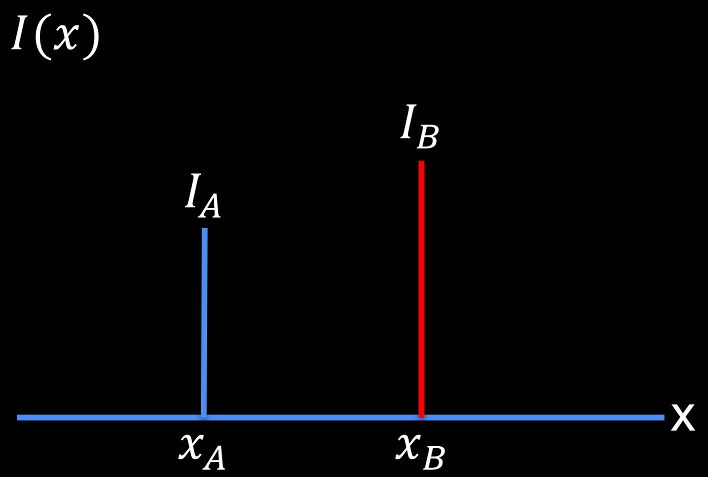
Frequency



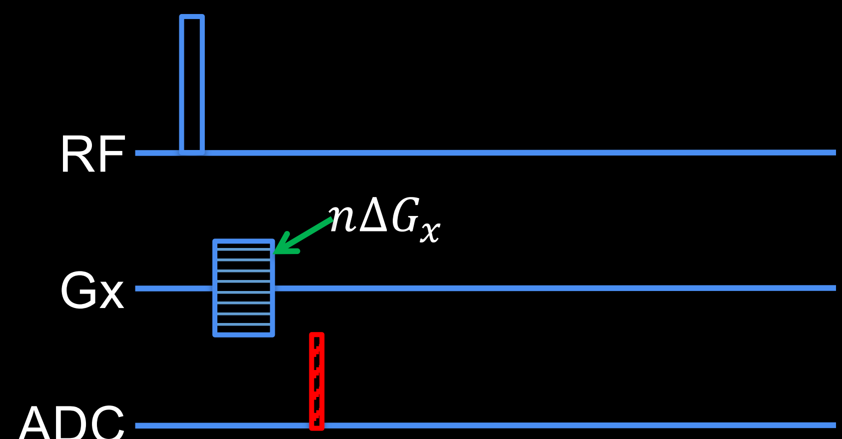
Phase?



1D Imaging: Revisit



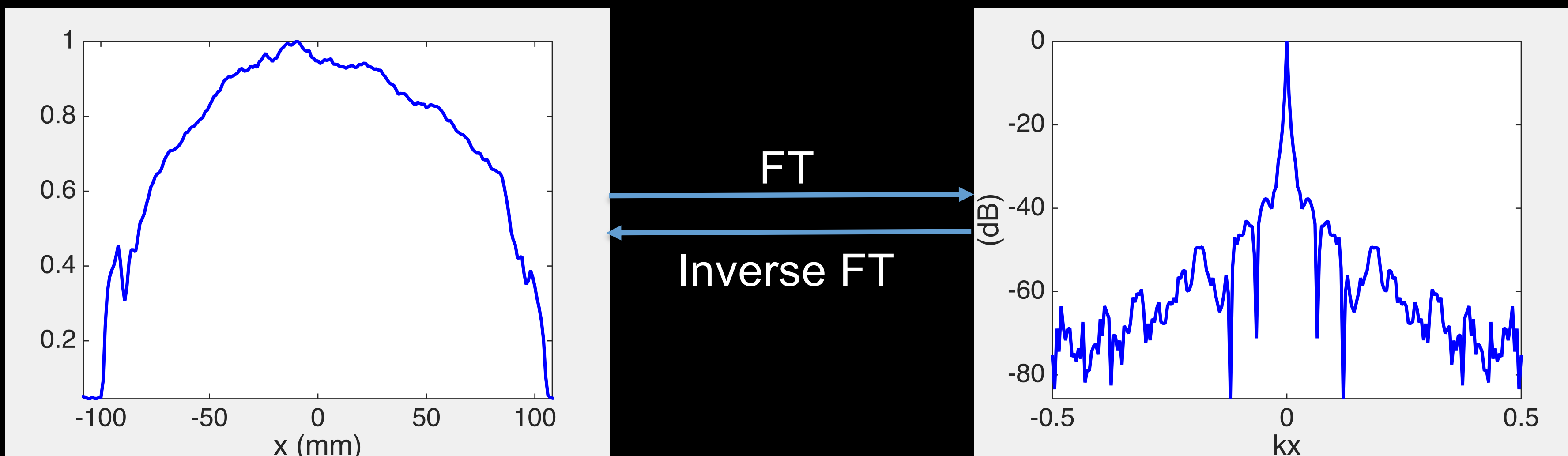
$$s(n) = I_A e^{-i\varphi_A(n)} + I_B e^{-i\varphi_B(n)}$$
$$\varphi_A(n) = \gamma n \Delta G_x \tau x_A$$
$$\varphi_B(n) = \gamma n \Delta G_x \tau x_B$$



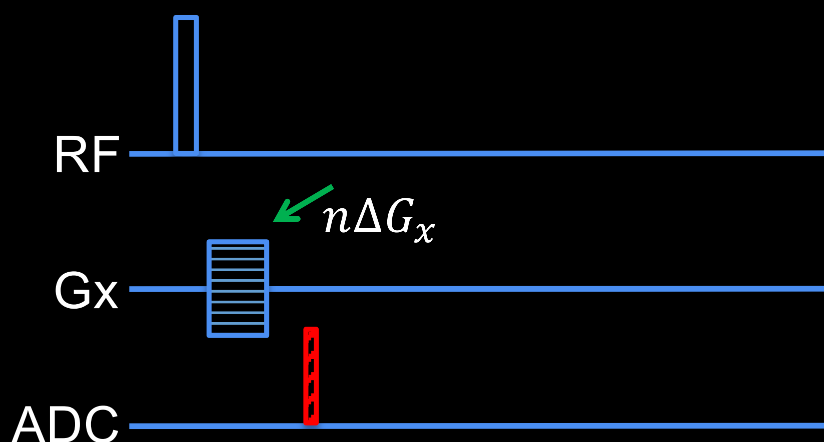
1D Imaging: Revisit

$$s(k_x) = \int_x I(x)\, e^{-i2\pi x\cdot k_x} dx,$$

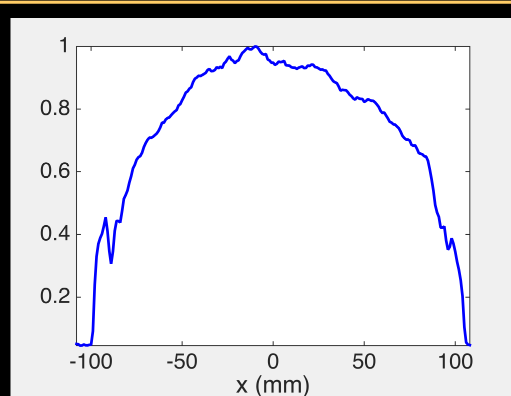
$$k_x = \frac{\gamma n \Delta G_x \tau}{2\pi}$$

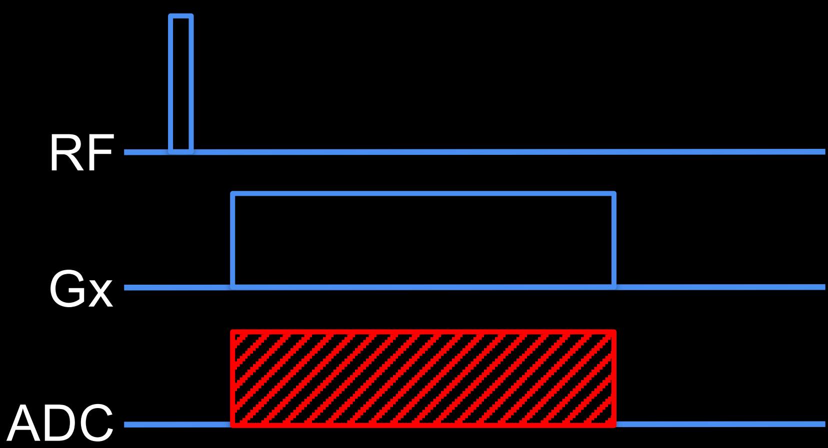


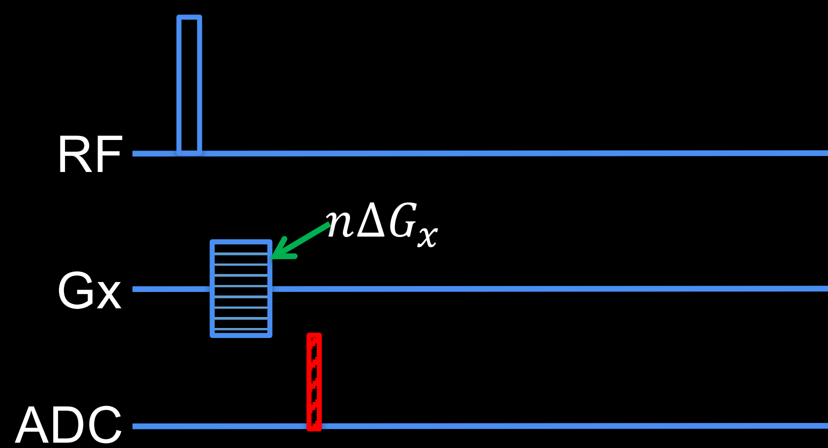
$I(x)$ $s(k_x)$



1D imaging: Frequency vs. Phase encoding







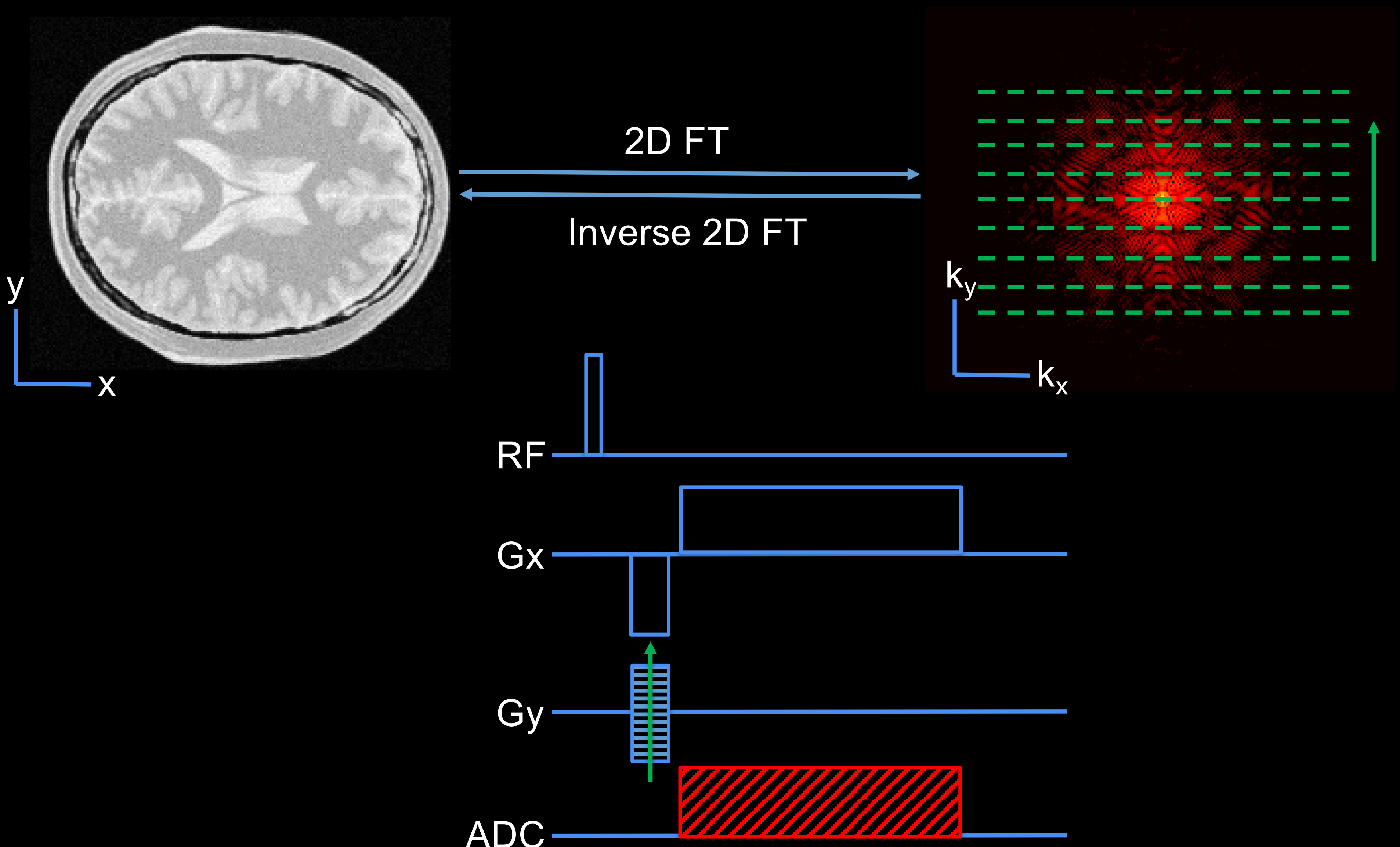
$$s(n) = \int_x I(x)\, e^{-i2\pi x\cdot k_x} dx, k_x = \frac{\gamma n G_x \Delta t}{2\pi}$$

$$s(n) = \int_x I(x)\, e^{-i2\pi x\cdot k_x} dx, k_x = \frac{\gamma n \Delta G_x \tau}{2\pi}$$

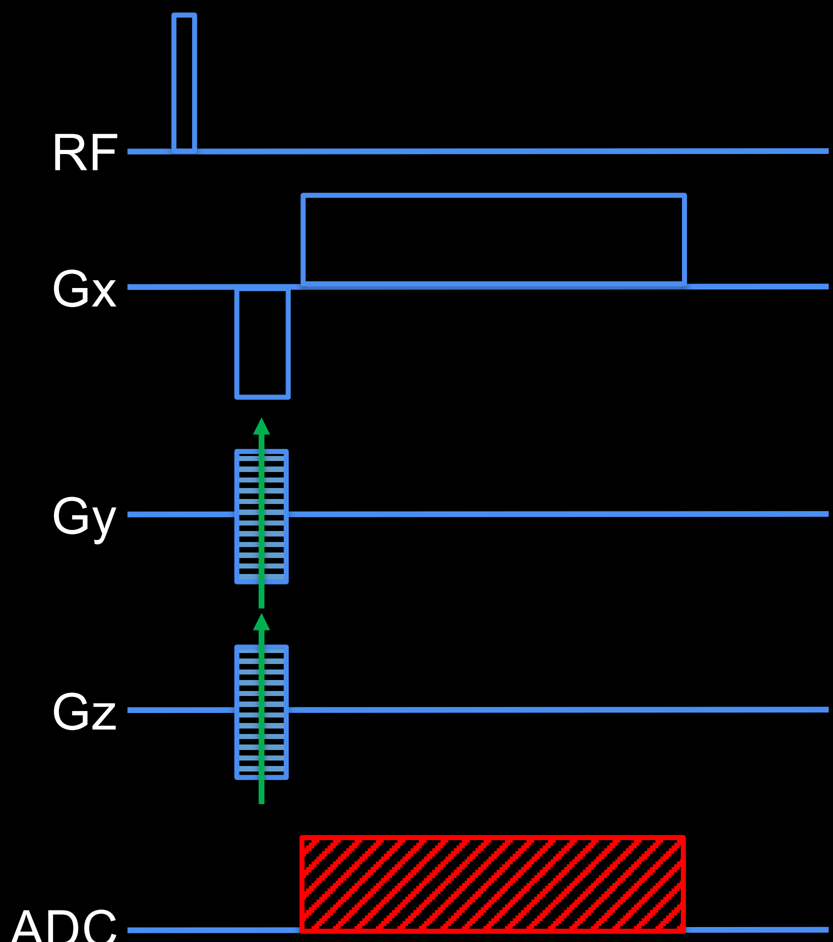
2D imaging: Cartesian sampling

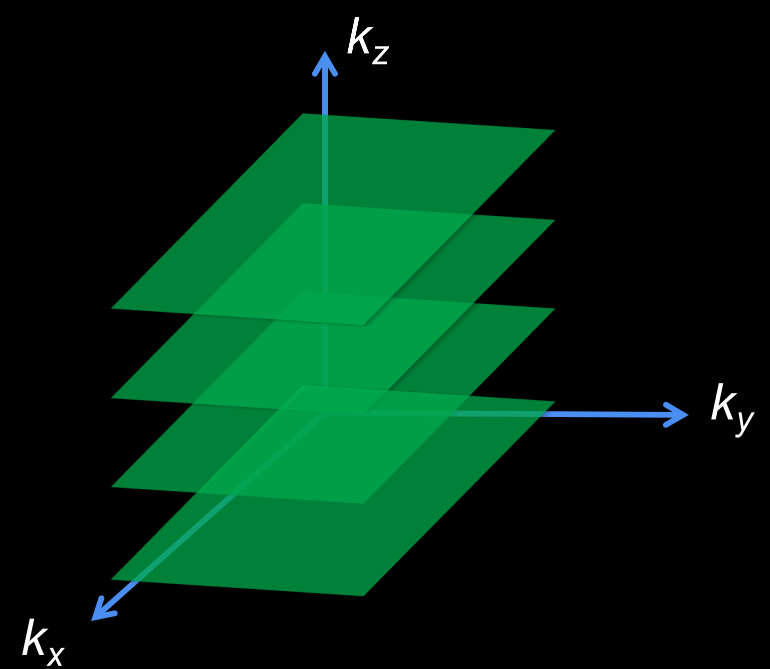
$I(x, y)$

$$s(k_x, k_y) = \iint_{x,y} I(x, y) e^{-i2\pi(x \cdot k_x + y \cdot k_y)} dx dy$$

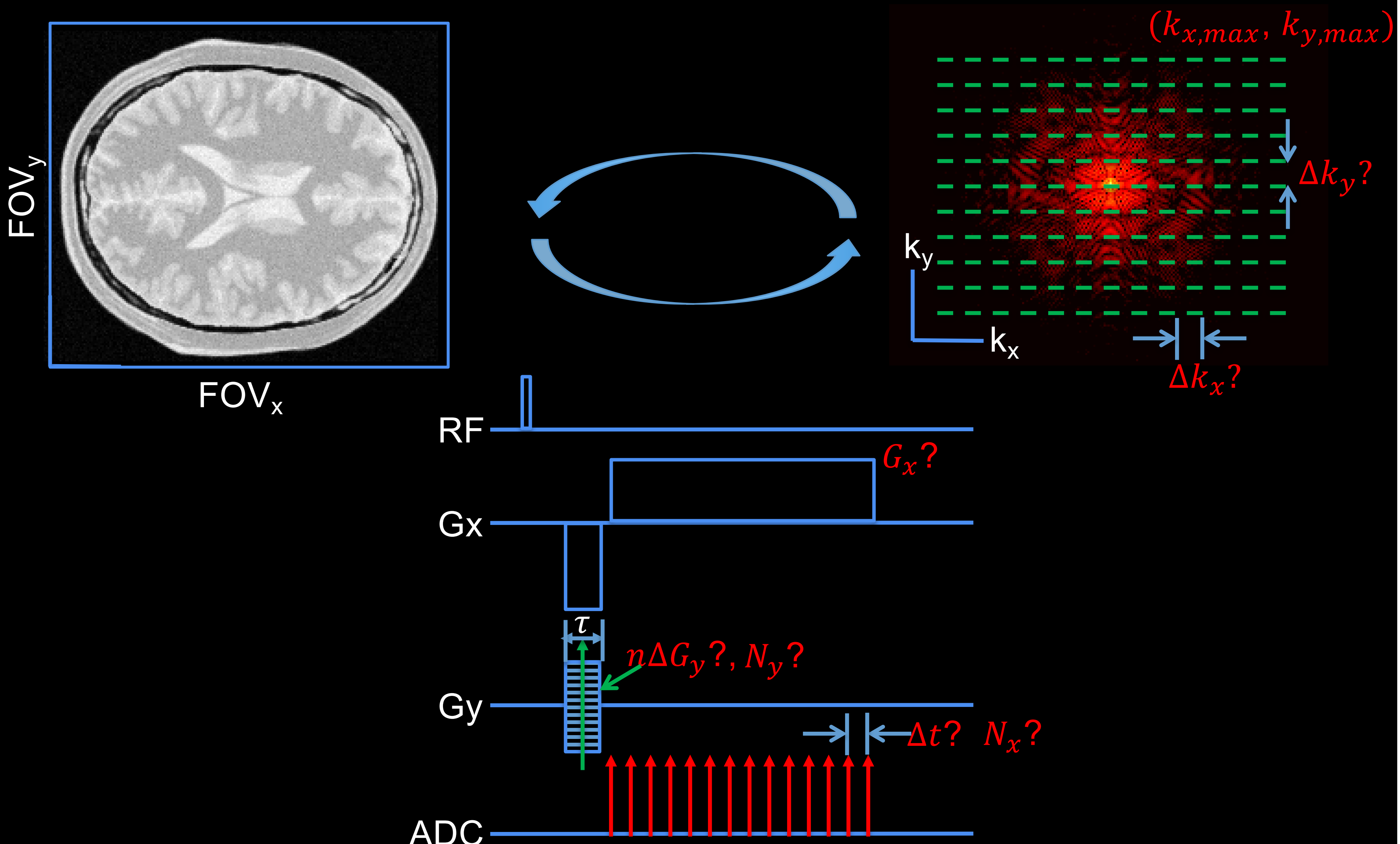


3D Imaging: Cartesian sampling

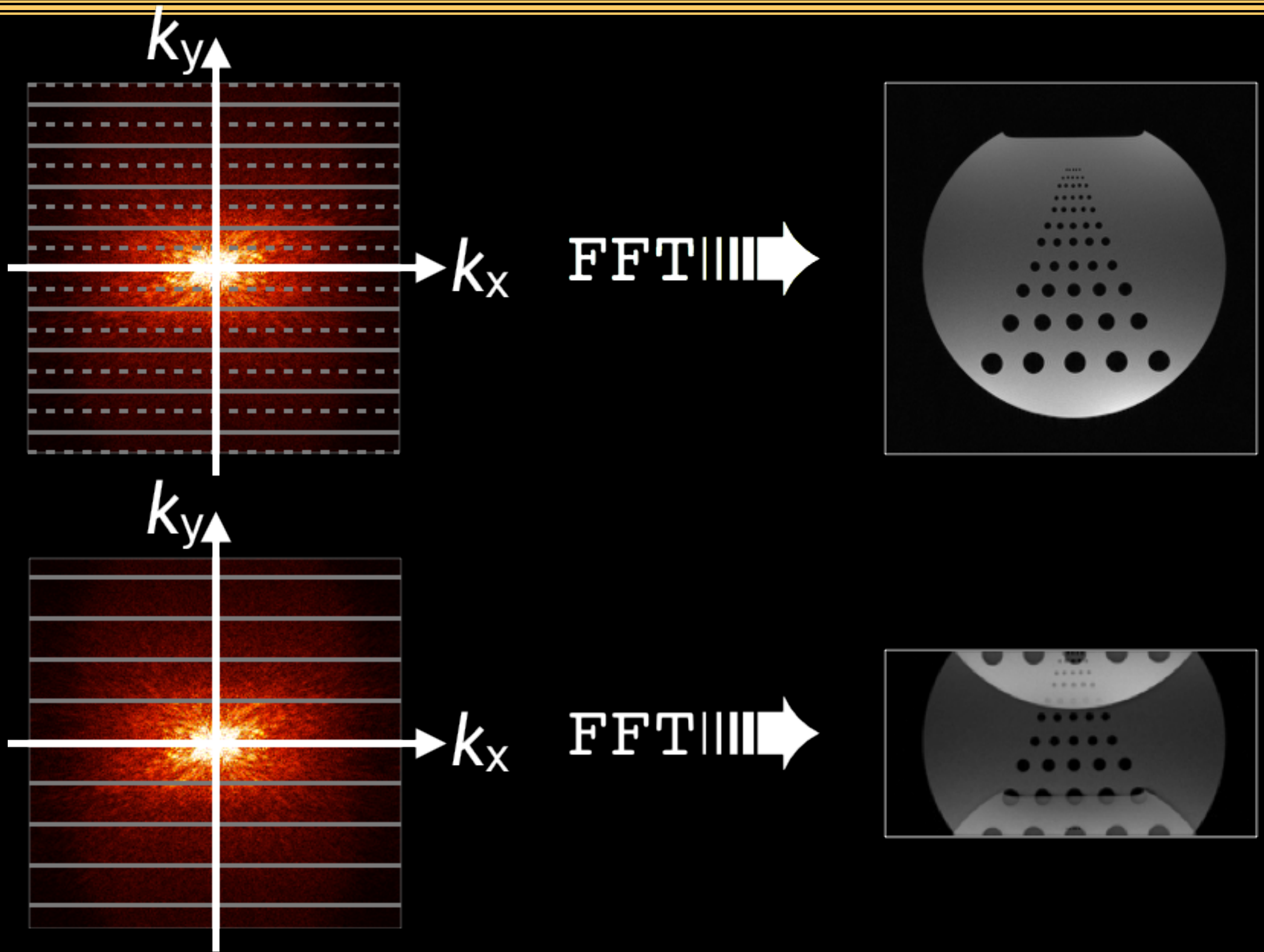




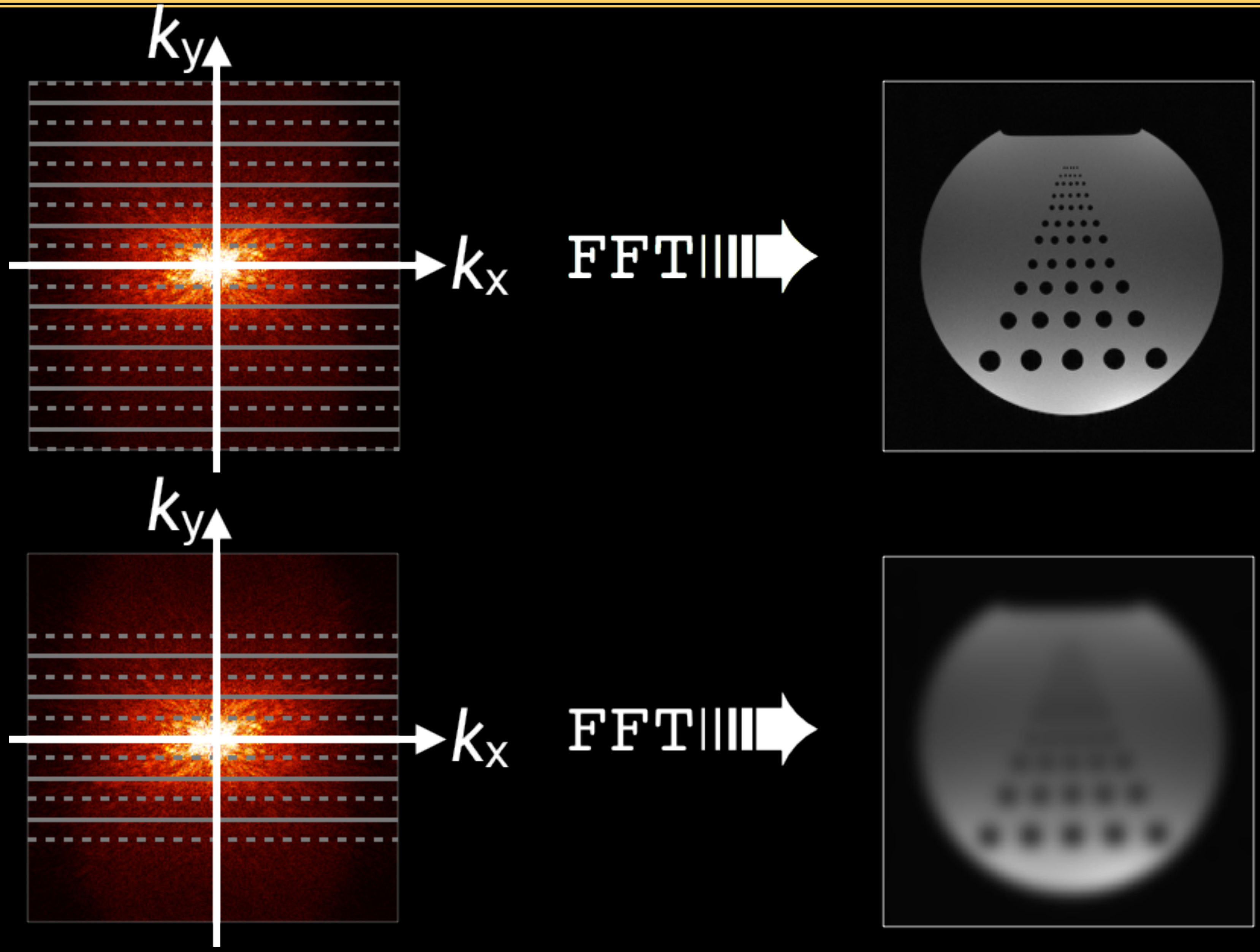
k-space sampling



k-space: Aliasing



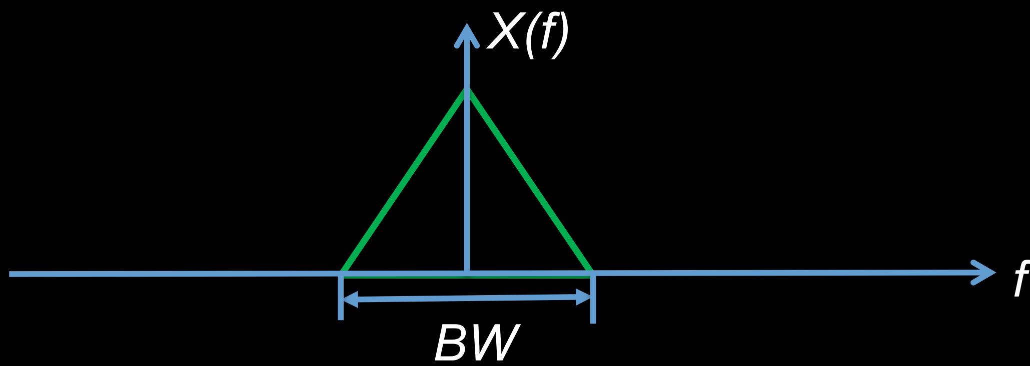
k-space: Resolution



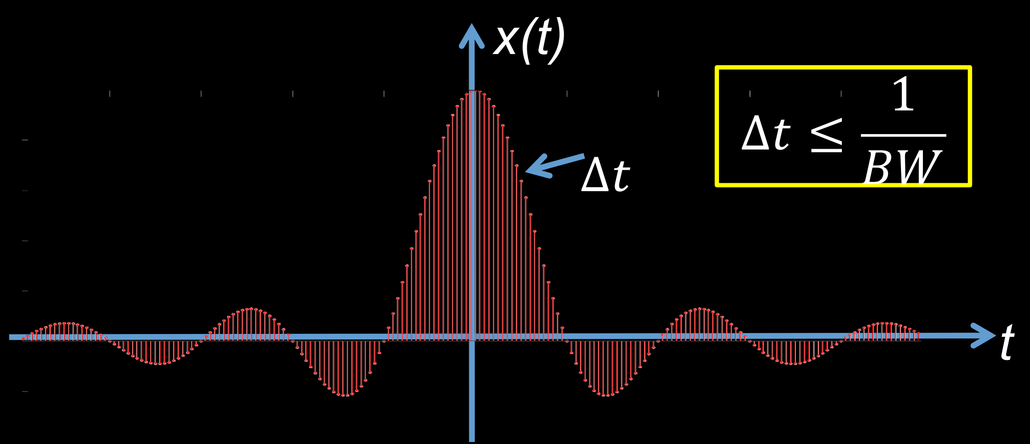
k-space sampling: Field of View

- Nyquist sampling theorem

Frequency domain:



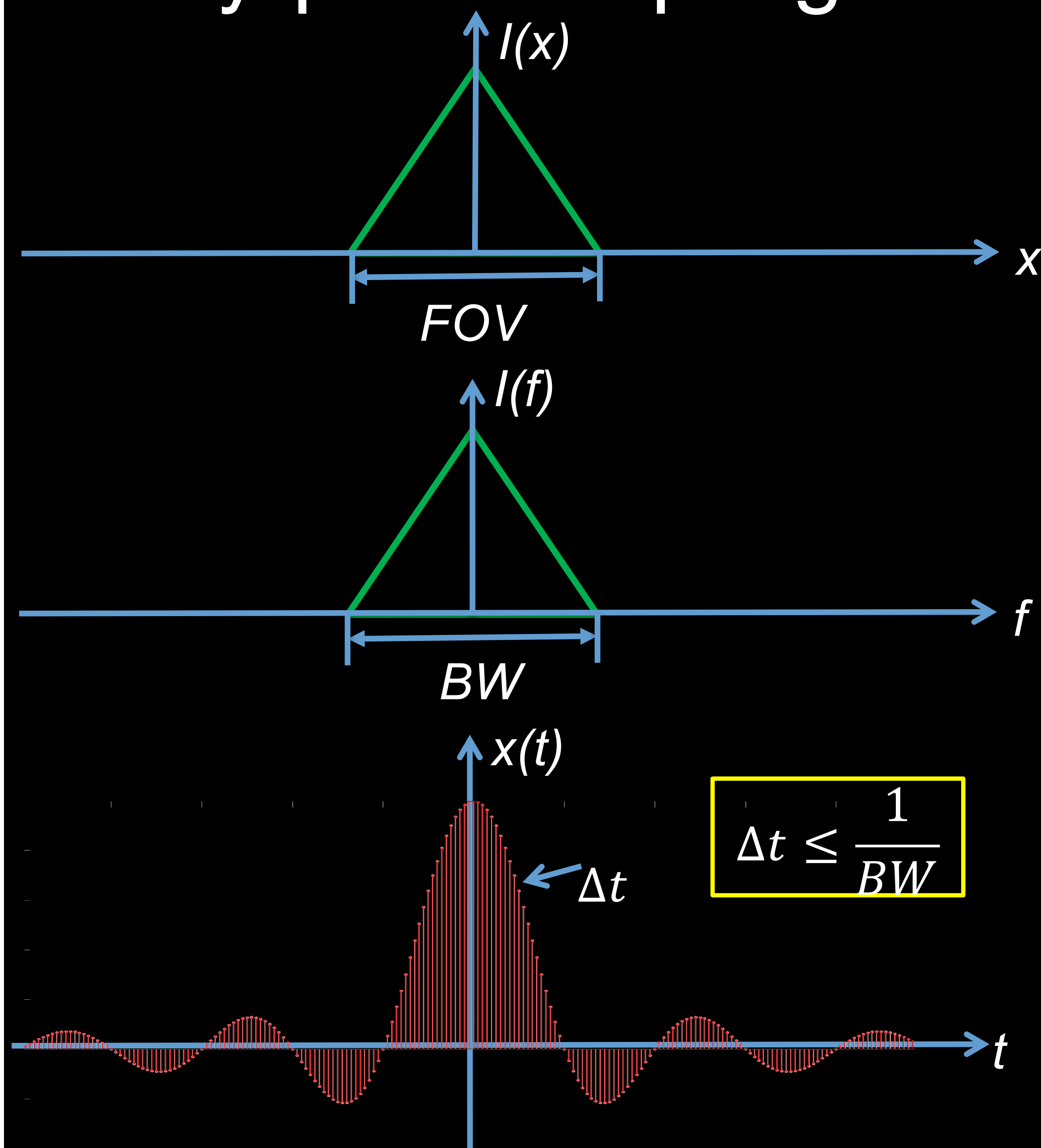
Time domain:



$$\Delta t \leq \frac{1}{BW}$$

k-space sampling: Field of View

- Nyquist sampling theorem



$$\Delta t \leq \frac{1}{BW}$$

$$I(x) = 0, if\ |x| \leq \frac{FOV}{2}$$

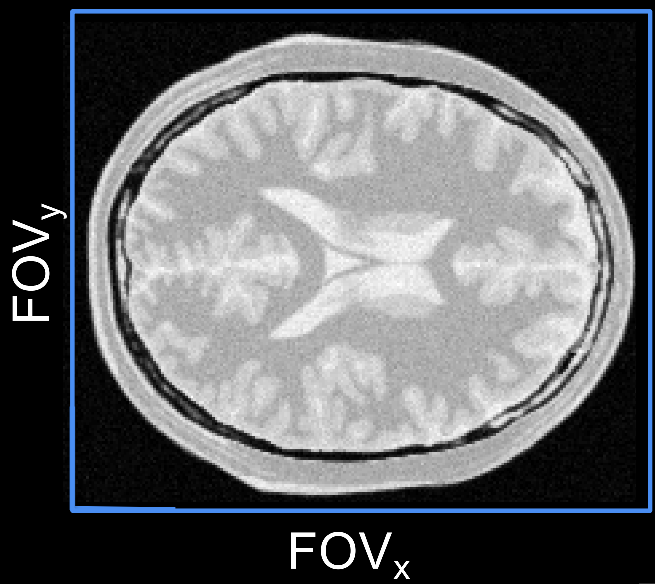
$$I(f) = 0, if\ |f| \leq \frac{BW}{2} = \frac{1}{2}\frac{\gamma G_x FOV}{2\pi}$$

$$\Delta t \leq \frac{1}{BW} = \frac{1}{\gamma G_x FOV/2\pi}$$

$$\Delta k_x = \frac{\gamma G_x \Delta t}{2\pi} \leq \frac{1}{FOV}$$

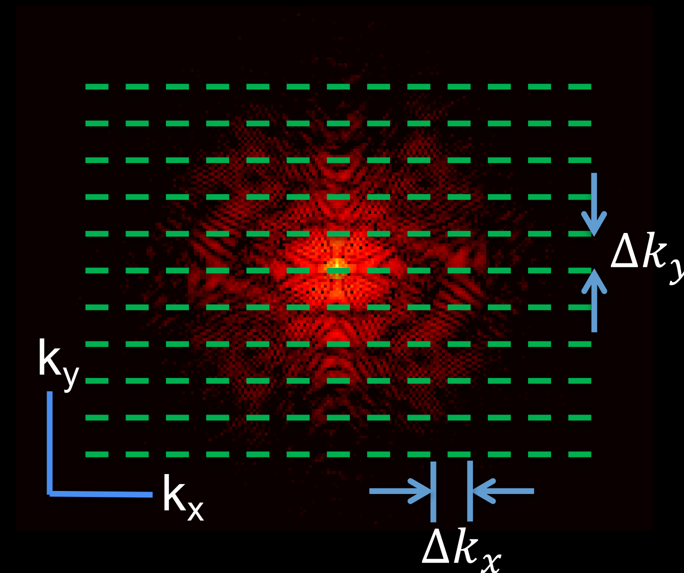
$$s(n\Delta k_x) = \int_x I(x)\, e^{-i2\pi x \cdot k_x} dx,\ k_x = \frac{\gamma n G_x \Delta t}{2\pi}$$

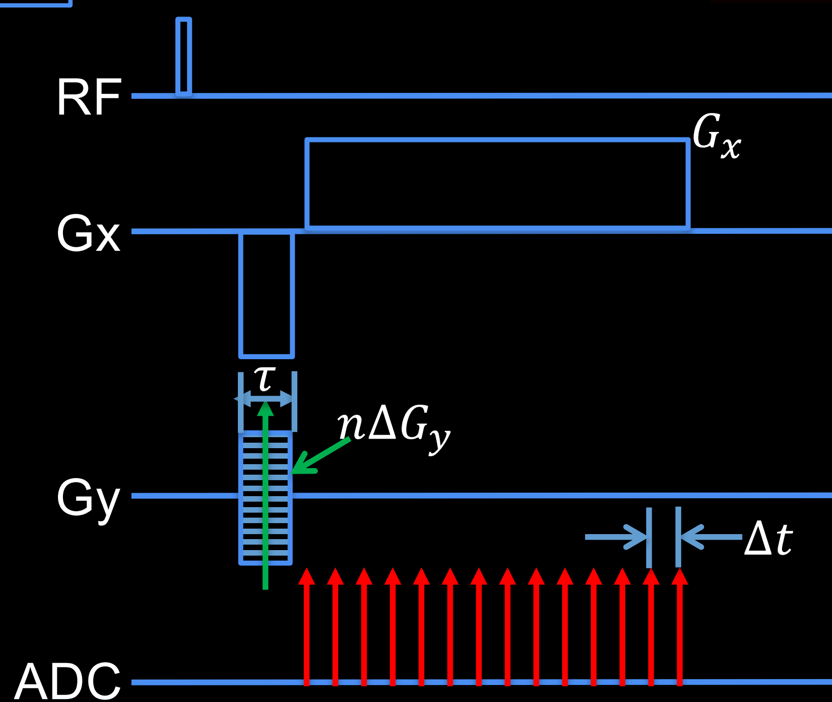
k-space sampling: Field of View



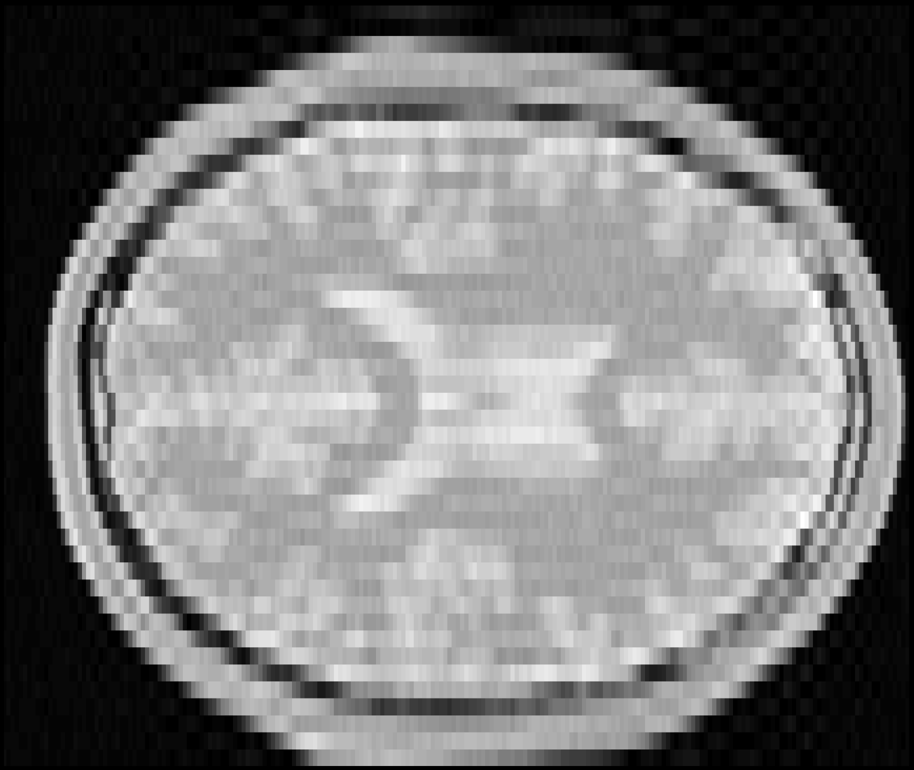
$$\Delta k_x = \frac{\gamma G_x \Delta t}{2\pi} \leq \frac{1}{FOV_x}$$

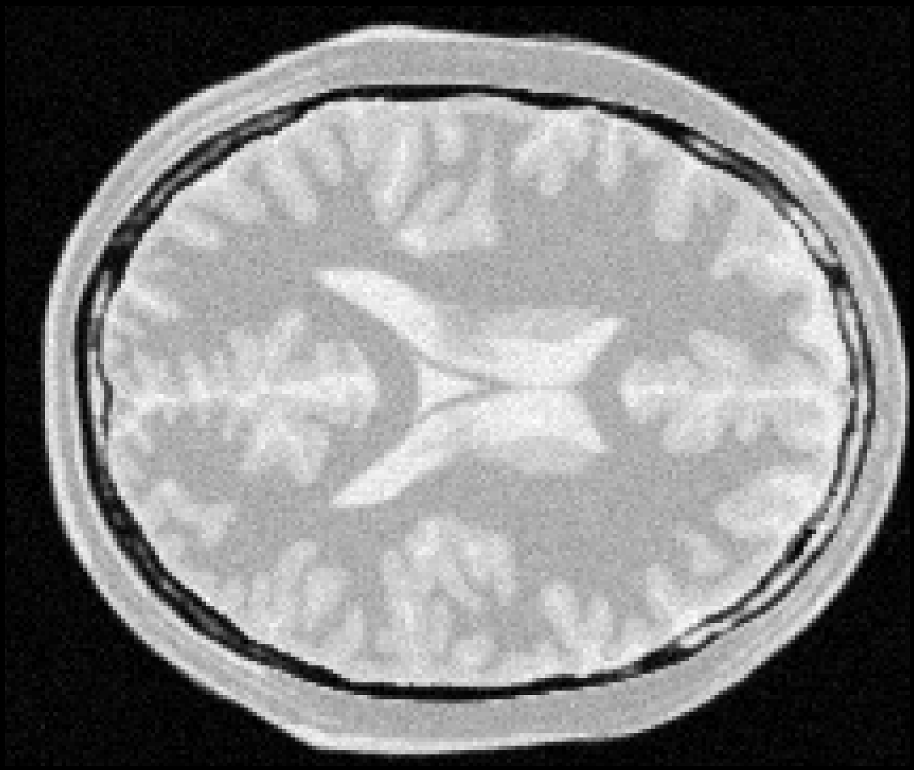
$$\Delta k_y = \frac{\gamma \Delta G_y \tau}{2\pi} \leq \frac{1}{FOV_y}$$



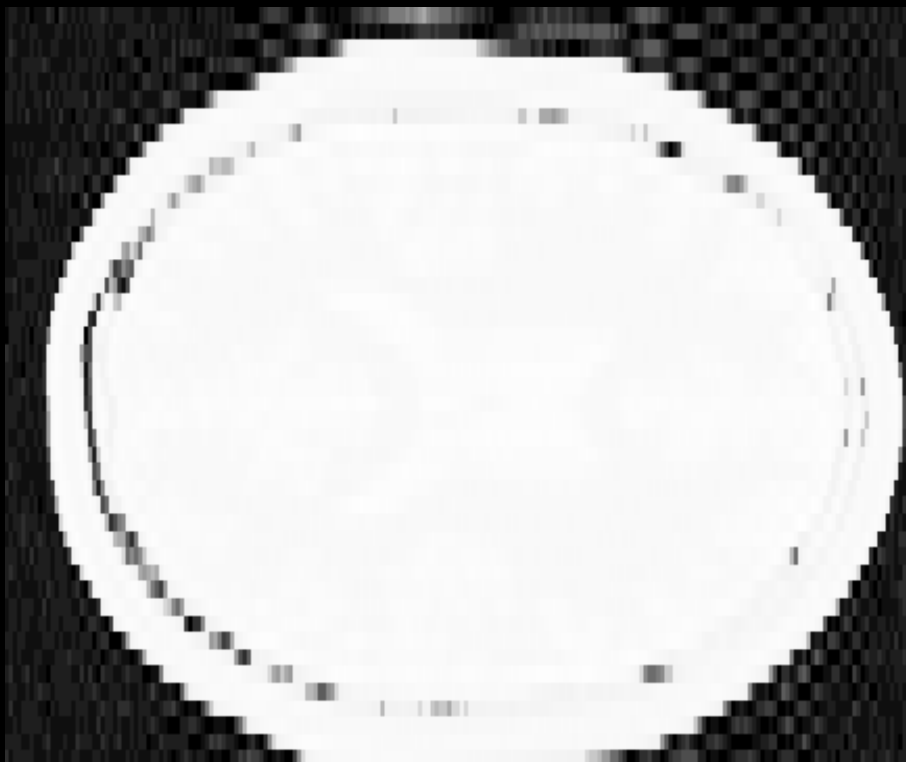


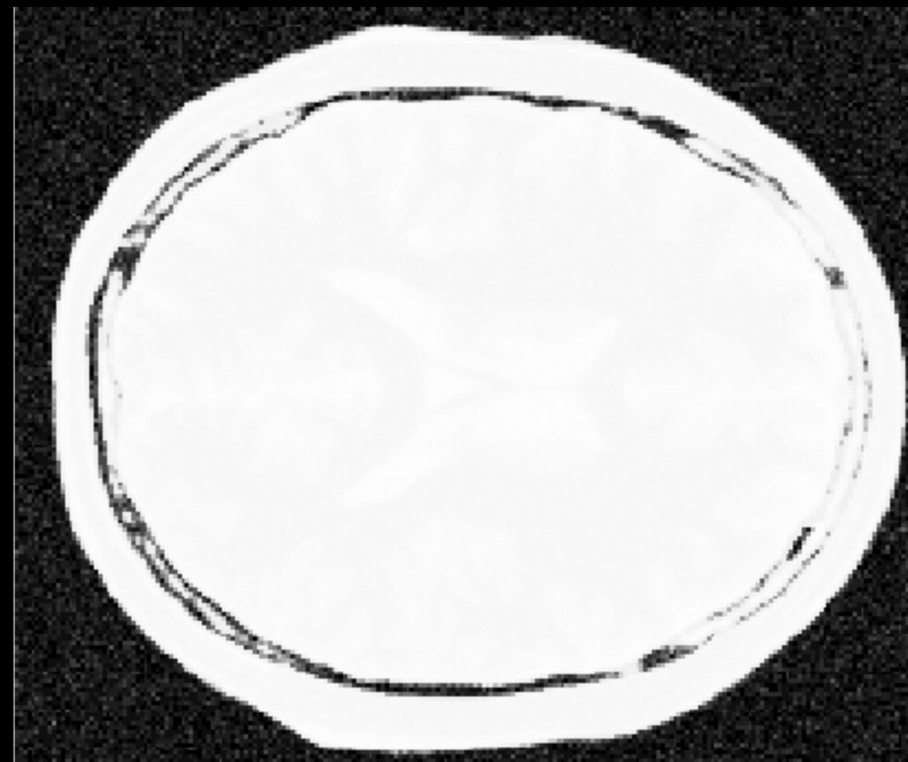
k-space sampling: Resolution





k-space sampling: Resolution

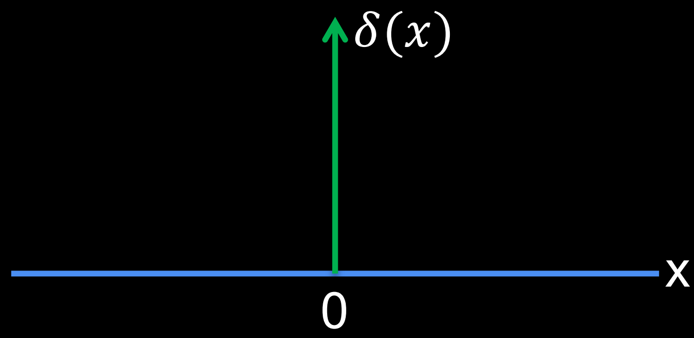




k-space sampling: Resolution

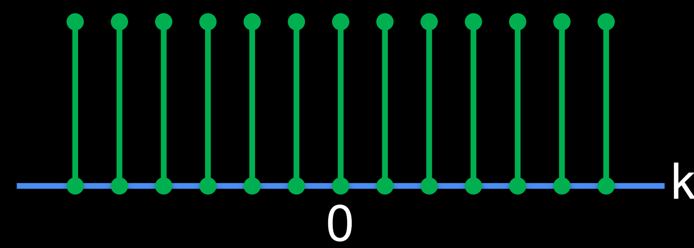
- Point-spread-function

A point source in the image domain



k-space

$$s(n\Delta k) = \int_{-\infty}^{\infty} \delta(x)\; e^{-i2\pi n\Delta kx} dx \equiv 1$$



Properties of Dirac delta function

$$\delta(x) = 0, for\ x \neq 0$$

$$\int_{-\infty}^{\infty} \delta(x) dx = 1$$

$$\int_{-\infty}^{\infty} \delta(x)\varphi(x) dx = \varphi(0)$$

Reconstruction

$$h(x) = \Delta k \sum_{n=-N/2}^{\frac{N}{2}-1} 1 e^{i2\pi n\Delta kx} = e^{-i\pi\Delta kx} \Delta k \frac{\sin(\pi N\Delta kx)}{\sin(\pi\Delta kx)}$$

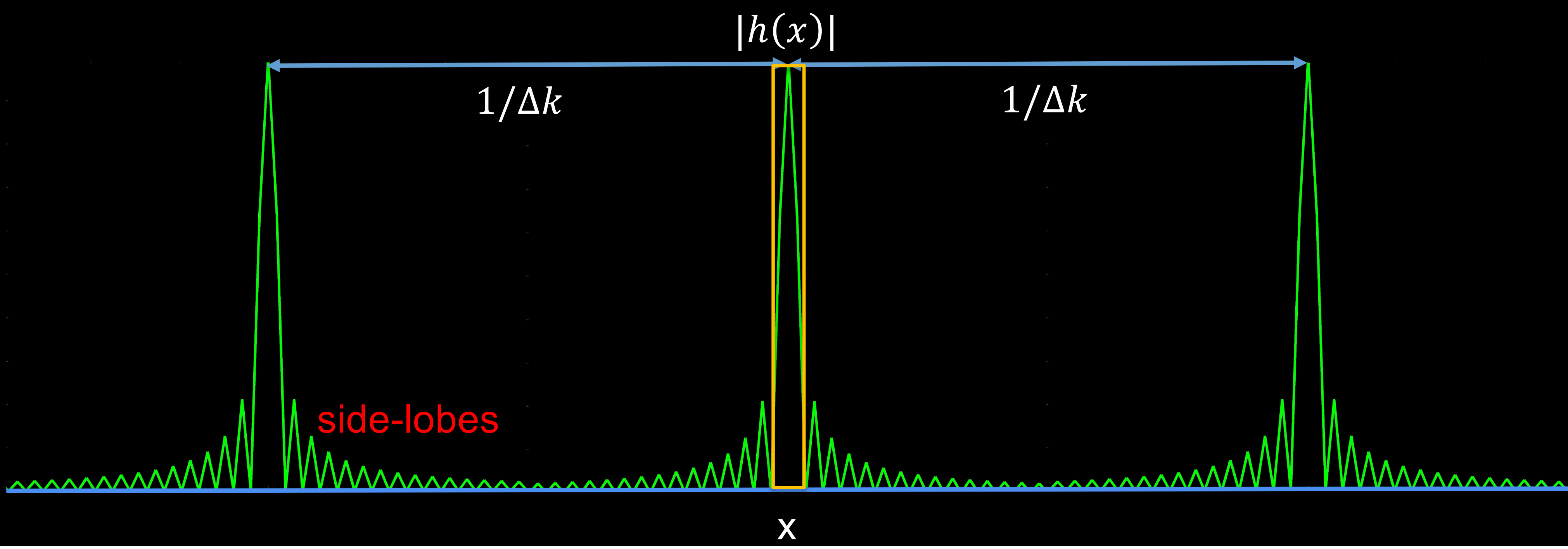
k-space sampling: Resolution

- Point-spread-function

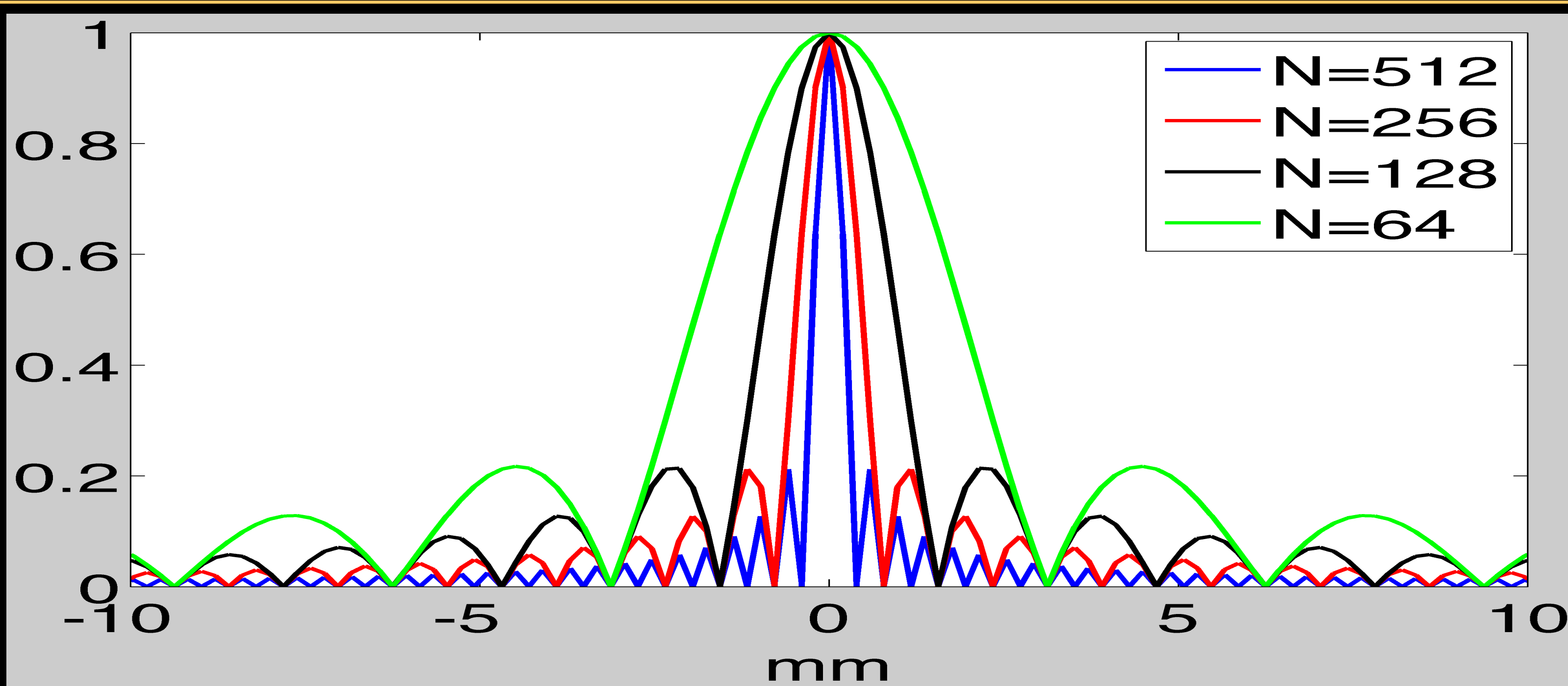
$$h(x) = e^{-i\pi\Delta kx}\Delta k\,\frac{\sin(\pi N\Delta kx)}{\sin(\pi\Delta kx)}$$

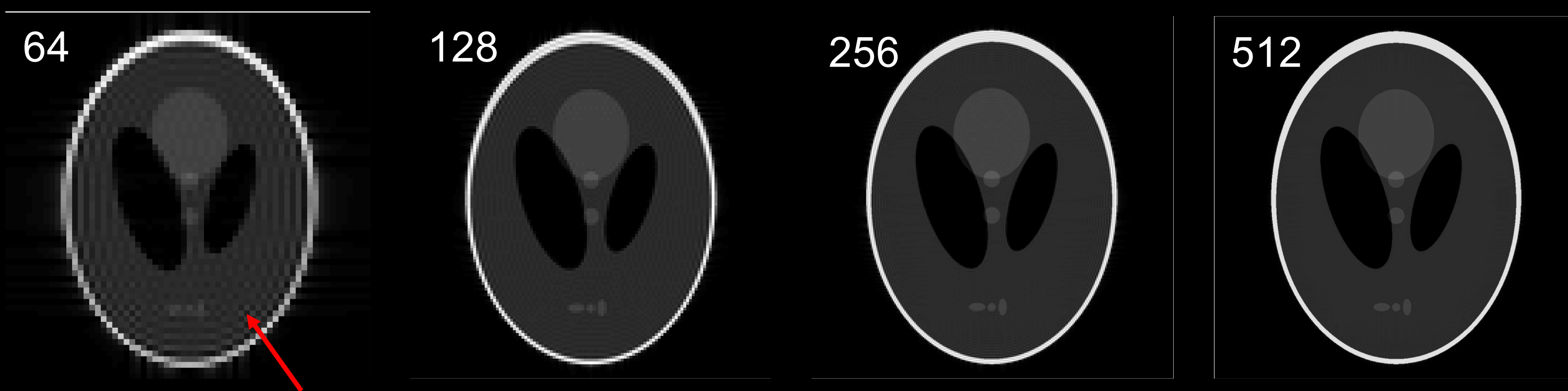
$$\hat{I}(x) = I(x) * h(x)$$

$$W_h = \frac{1}{h(0)}\int_{-FOV/2}^{FOV/2} h(x)dx = \frac{FOV}{N}$$

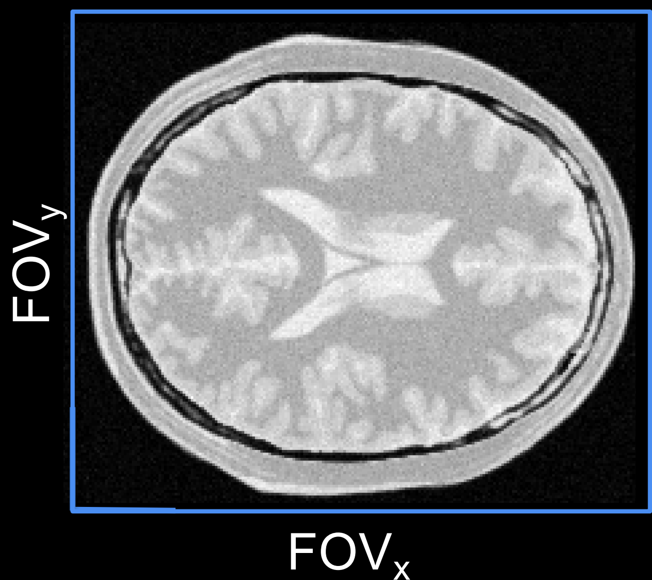


k-space sampling: Resolution





k-space sampling

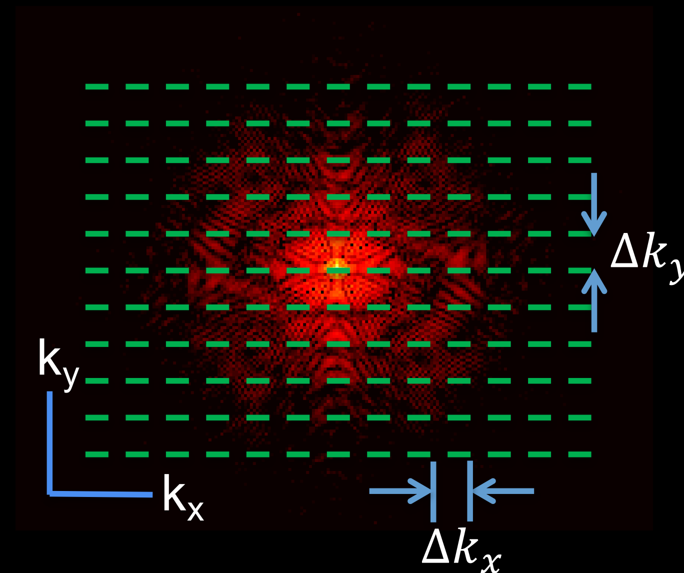


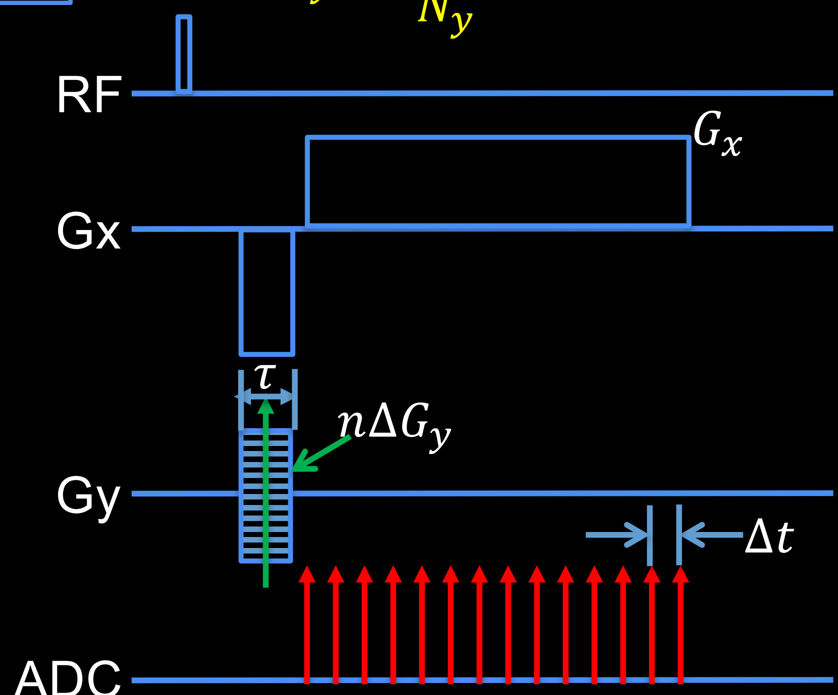
$$\Delta k_x = \frac{\gamma G_x \Delta t}{2\pi} \leq \frac{1}{FOV_x}$$

$$\Delta k_y = \frac{\gamma \Delta G_y \tau}{2\pi} \leq \frac{1}{FOV_y}$$

$$\Delta_x = \frac{FOV_x}{N_x}$$

$$\Delta_y = \frac{FOV_y}{N_y}$$





Summary

- NMR physics
  - Spins and Bloch equation
- MR signal excitation and reception
- MR signal encoding & decoding
  - Frequency encoding
  - Phase encoding
  - Projection-based MR imaging
  - Fourier transform-based MR imaging
- K-space sampling
  - Field of View
  - Resolution

Omitted topics

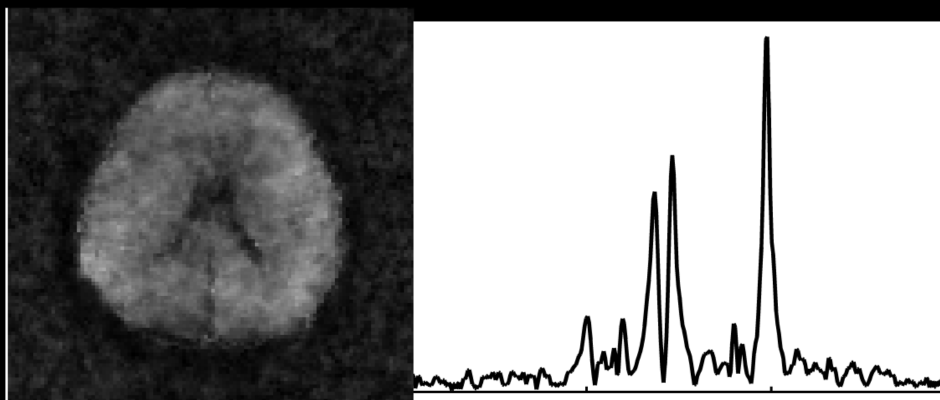
- MR physics
  - Noise and signal-to-noise ratio
  - RF pulse design
  - Fast imaging sequence
    - RARE, FLASH, bSSFP, BURST, …
  - Contrast
    - BOLD signal, diffusion, perfusion, susceptibility, spectroscopic imaging, flow imaging, …
- MR imaging
  - k-space sampling trajectories and non-Cartesian reconstruction
  - Constrained image reconstruction
    - Parallel imaging, compressed sensing, low-rank matrix/tensor based reconstruction, model based reconstruction, …
  - Parameter mapping
    - T1, T2, T2*, B0, B1, water/fat separation, MR fingerprinting, …
- MR hardware
  - Gradient coil design
  - RF coil design
  - Hardware imperfection and compensation
  - MRI using Earth's field
- Applications of MR

Future readings

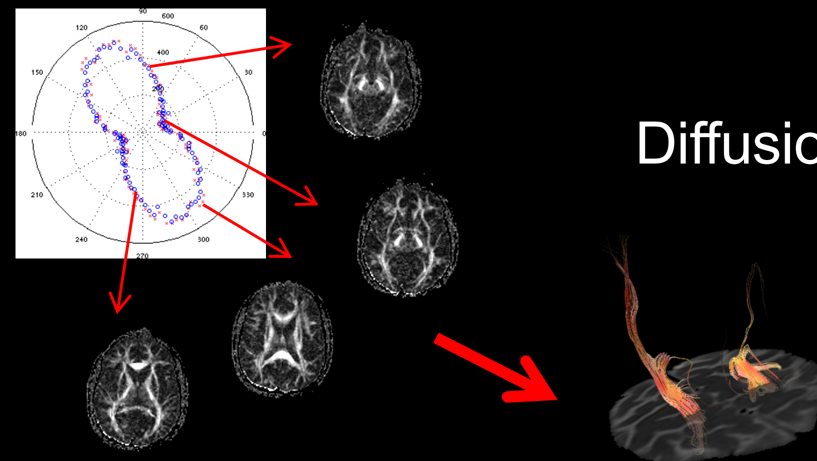
- Quantum mechanics
  - M. H. Levitt, *Spin Dynamics: Basics of Nuclear Magnetic Resonance* (2nd Edition), John Wiley & Sons, Ltd., 2018.
- EM field in MR
  - J. M. Jin, *Analysis and Design in Magnetic Resonance Imaging,* CRC Press, 1998.
- MRI
  - Z.-P. Liang & P. C. Lauterbur, *Principles of Magnetic Resonance Imaging: A Signal Processing Perspective*, IEEE Press/John Wiley, 1999.
  - R. W. Brown, et al., *Magnetic Resonance Imaging: Physical Principles and Sequence Design* (2nd Edition), Wiley-Blackwell, 2014
- MRI sequence programming
  - M. A. Bernstein, et al., *Handbook of MRI Pulse Sequences*, Academic Press, 2004.

There are much more!

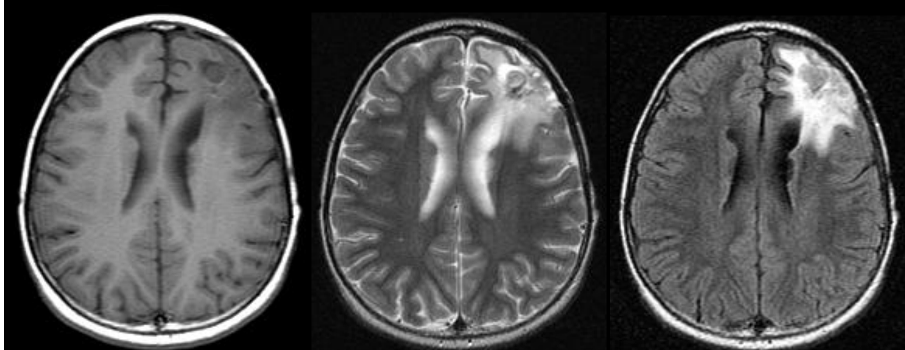
There is nothing that nuclear spins will not do for you, as long as you treat them as human beings. -- Erwin Hahn



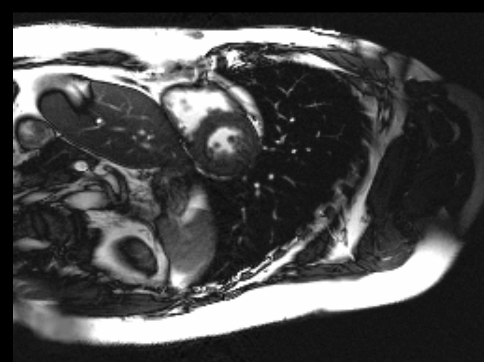
MRSI



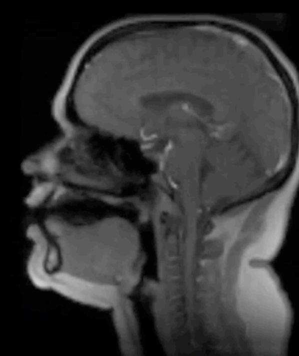
Diffusion



T1, T2 changes



Cardiac imaging



Speech imaging

Thank you for your attention.