Towards a Nonperturbative Construction of the S-Matrix

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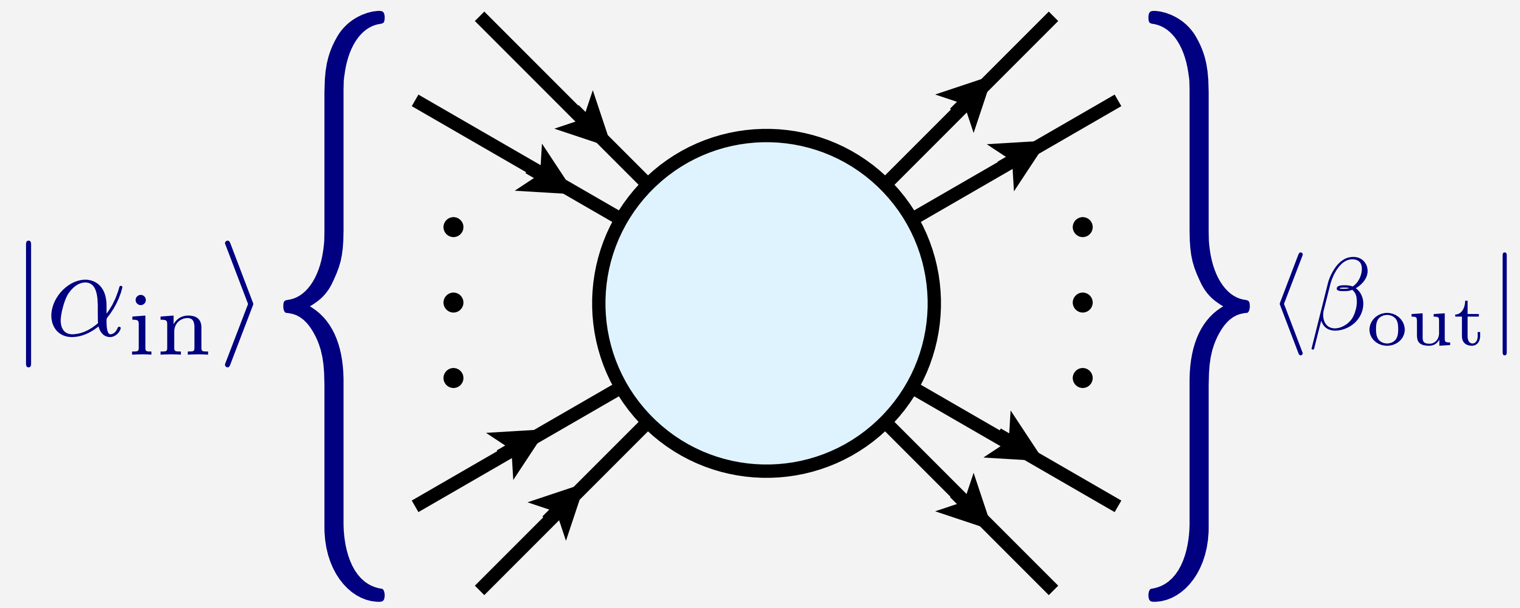
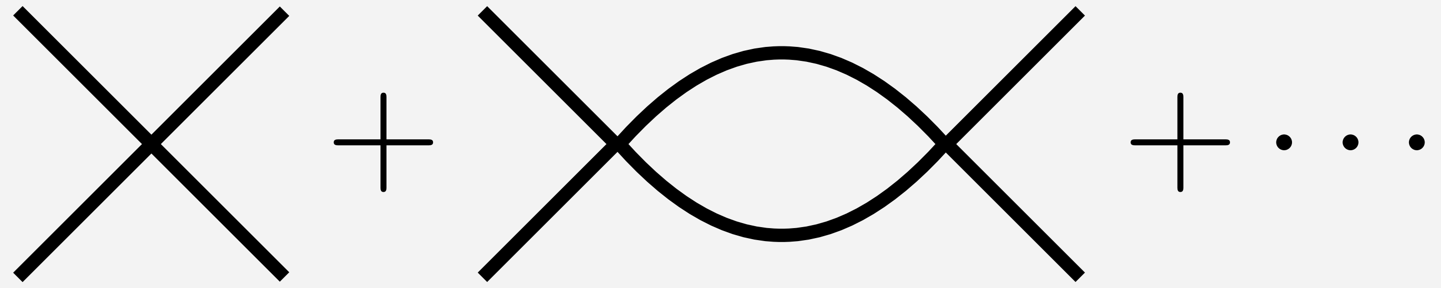
CERN Joint BSM/QCD Seminar, 28.10.22

Big Picture

S-Matrix: $\boxed{S_{\beta\alpha} \equiv \langle \beta_{\text{out}} | \alpha_{\text{in}} \rangle}$



Weak coupling: 

$n \sim \frac{1}{\lambda}$

How to compute S-matrix at **strong coupling** or **large particle number**?

Big Picture

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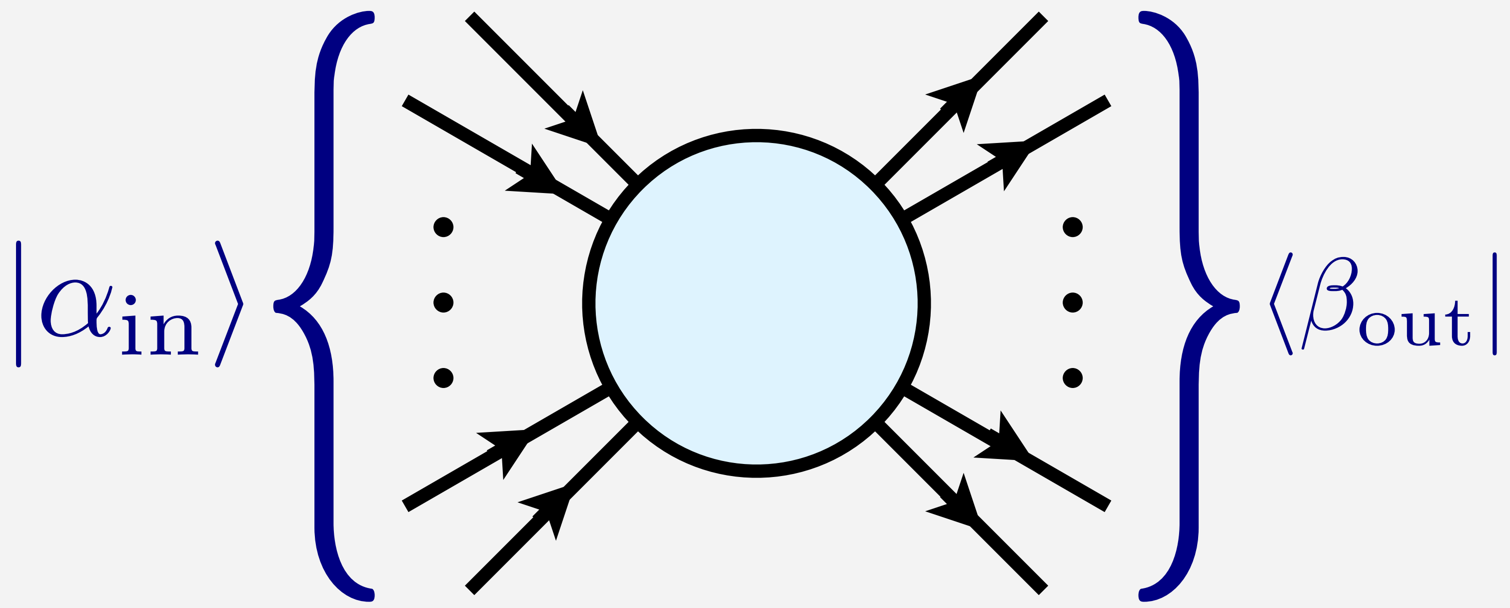

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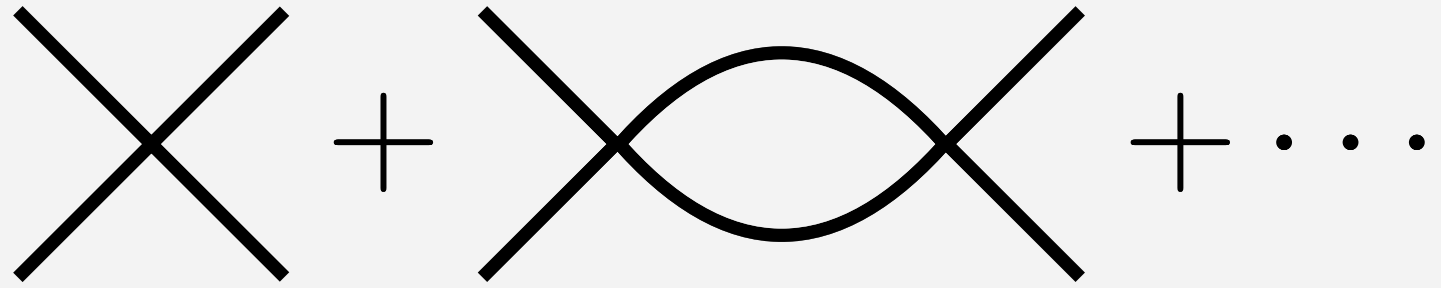

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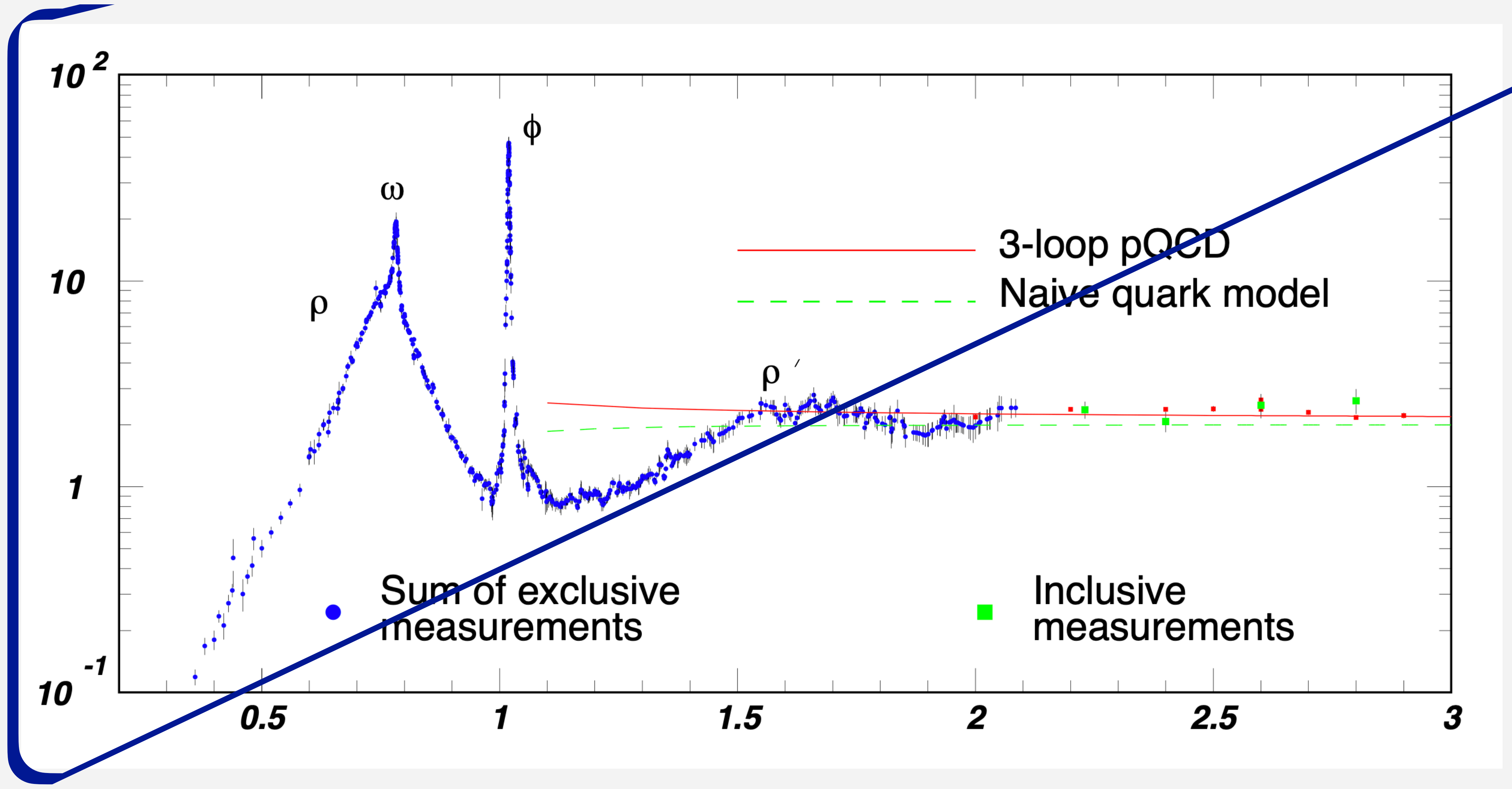
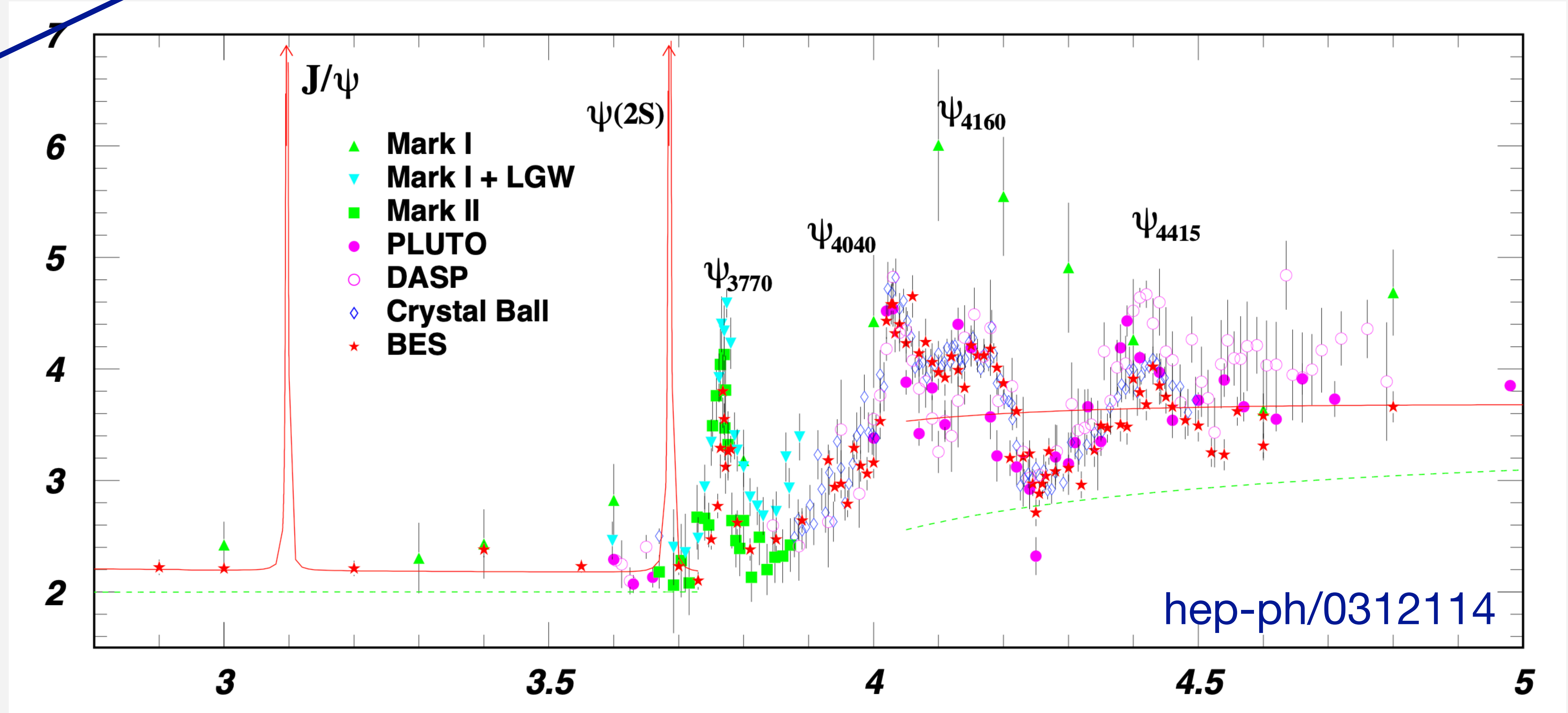
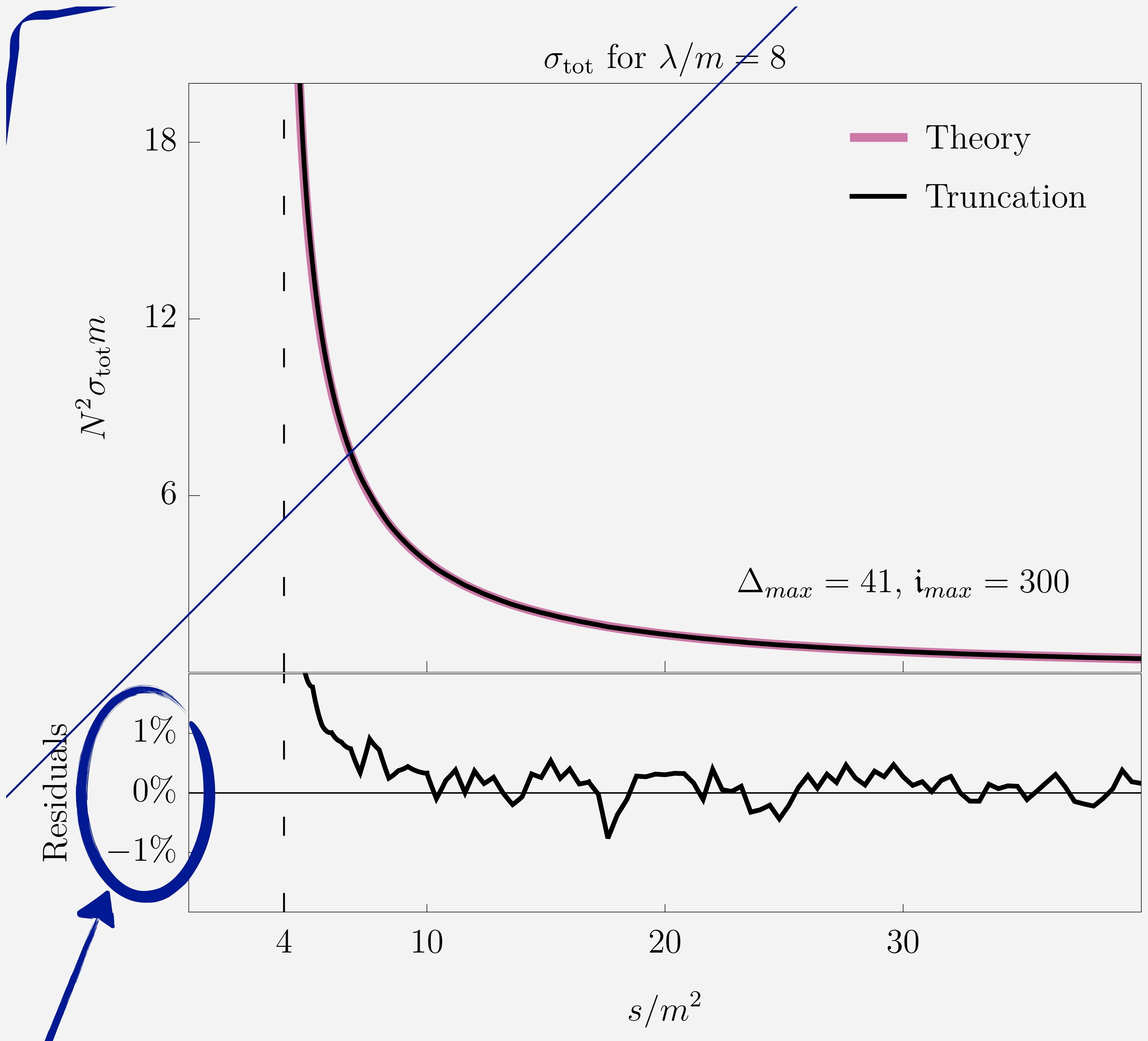




Punchline

- **Nonperturbative** recipe for directly computing S-matrix
- Uses **discrete**, **approximate** energy eigenstates (e.g. Hamiltonian truncation)
- Obtains **analytic** structure in complex plane



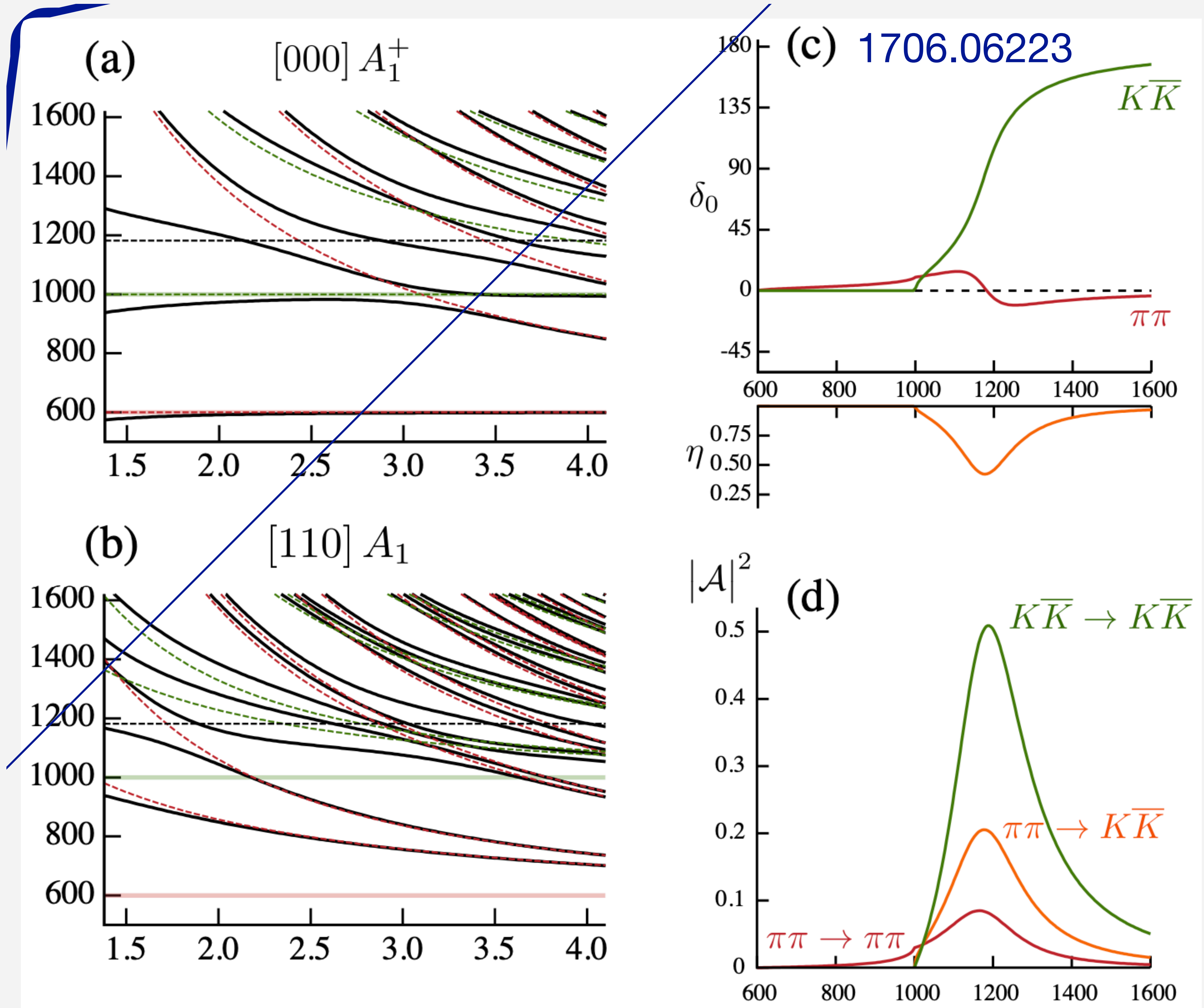
Percent-level error on laptop!

Menu

- Motivation and review
- LSZ with discrete spectrum
- Example: 2+1d $O(N)$ model at large N
- Future directions

Lattice Monte Carlo Methods

- **Euclidean** path integral based approach
- **Elastic** scattering obtained from **finite-volume** spectrum (Lüscher method)
- Numerically **ill-posed inverse problem** for obtaining amplitude from correlators
Bulava, Hansen ’19
- Limited to QFTs with **lattice formulation**



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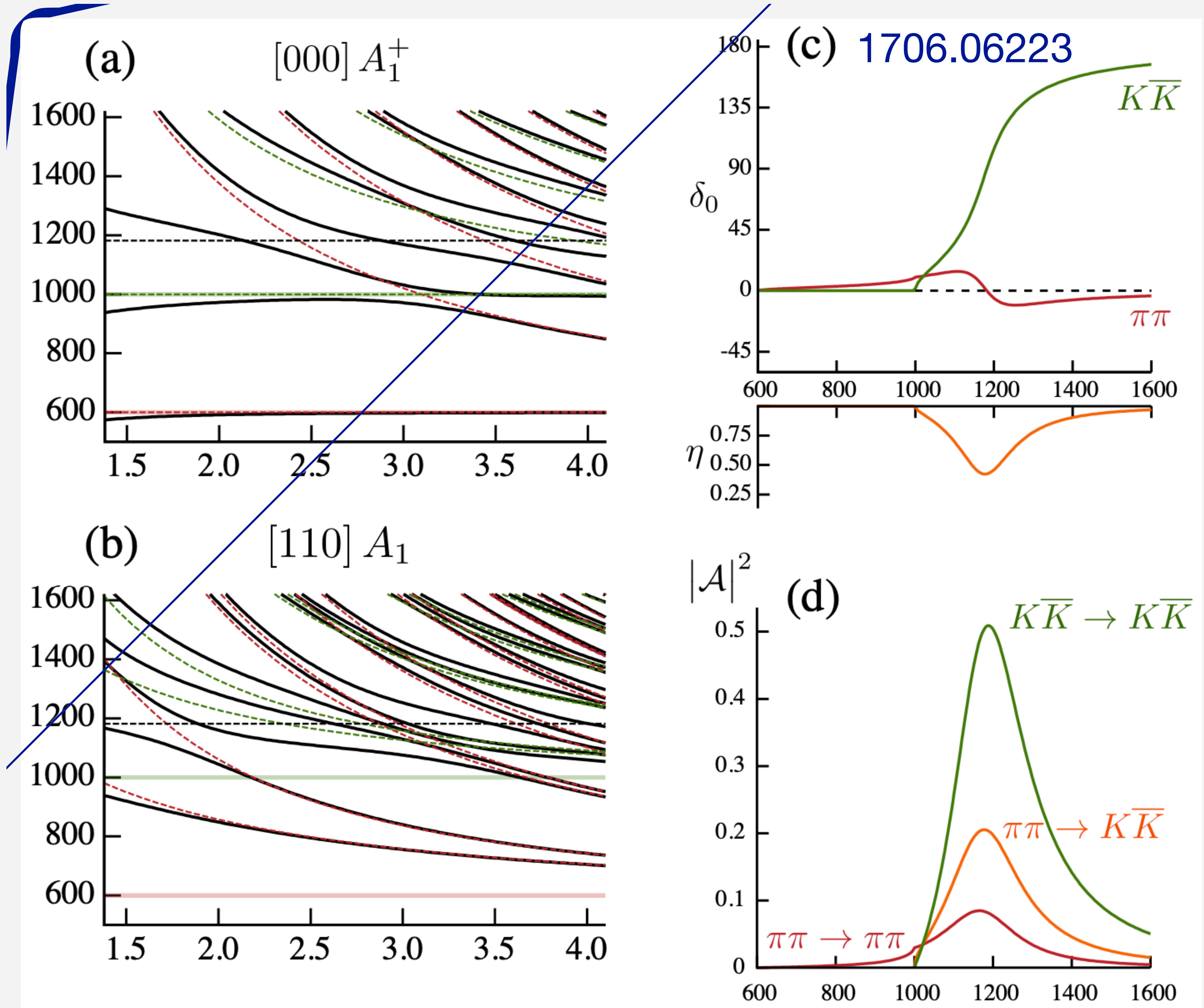


Need a (cheap) alternative to the lattice

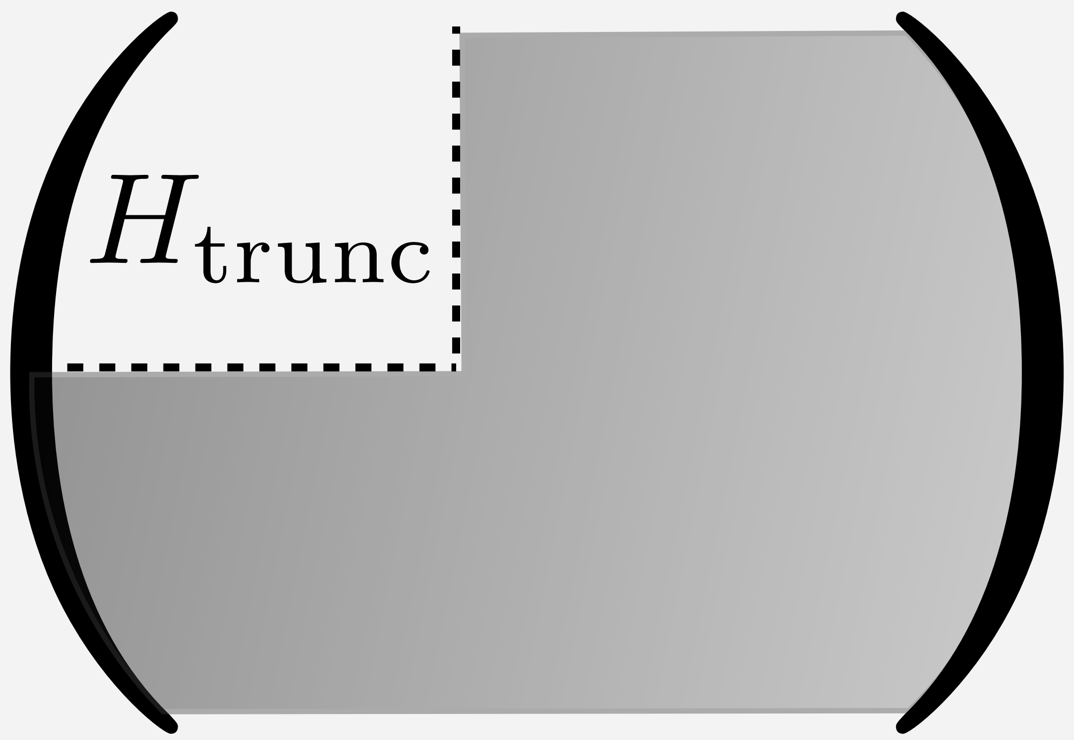
Hamiltonian Truncation

- **Lorentzian** method for studying strongly-coupled QFT **dynamics**
- Basic steps:

1) **Discretize** QFT Hilbert space
2) **Truncate** to finite-dimensional subspace
3) **Diagonalize** truncated Hamiltonian

- Approximation of **low-energy** eigenstates of **full QFT**

$$H = H_0 + V =$$

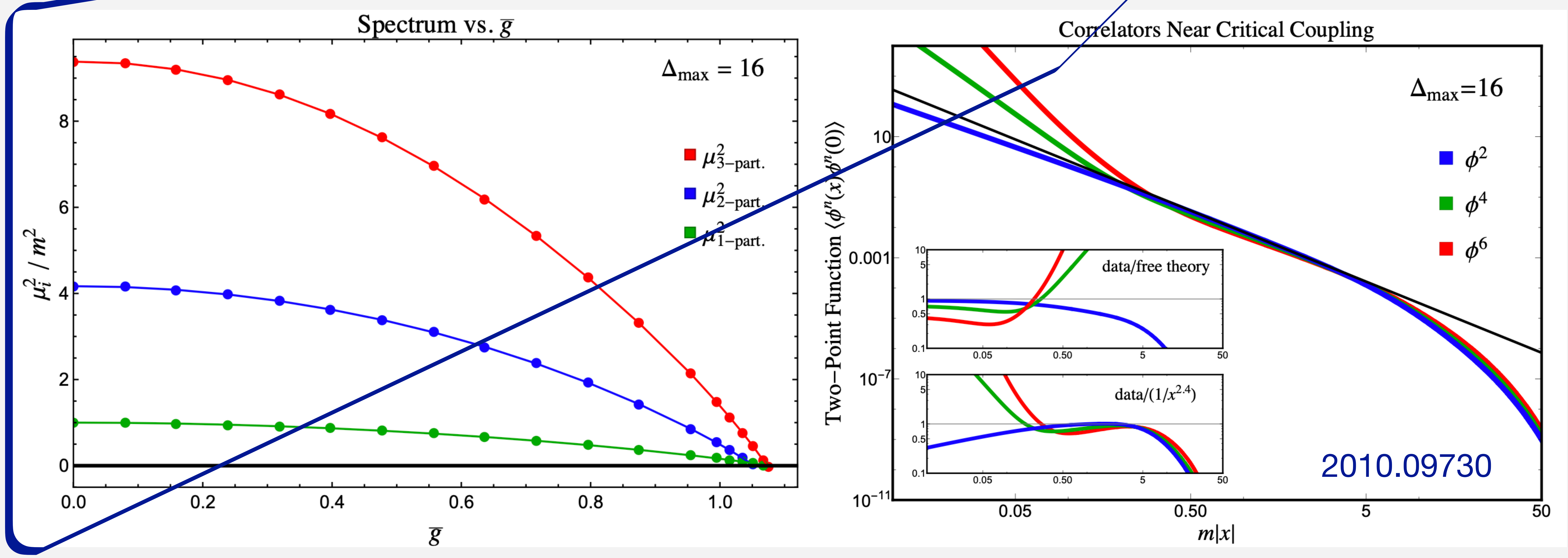


Solvable theory (e.g. free or CFT)

NOT small perturbation!

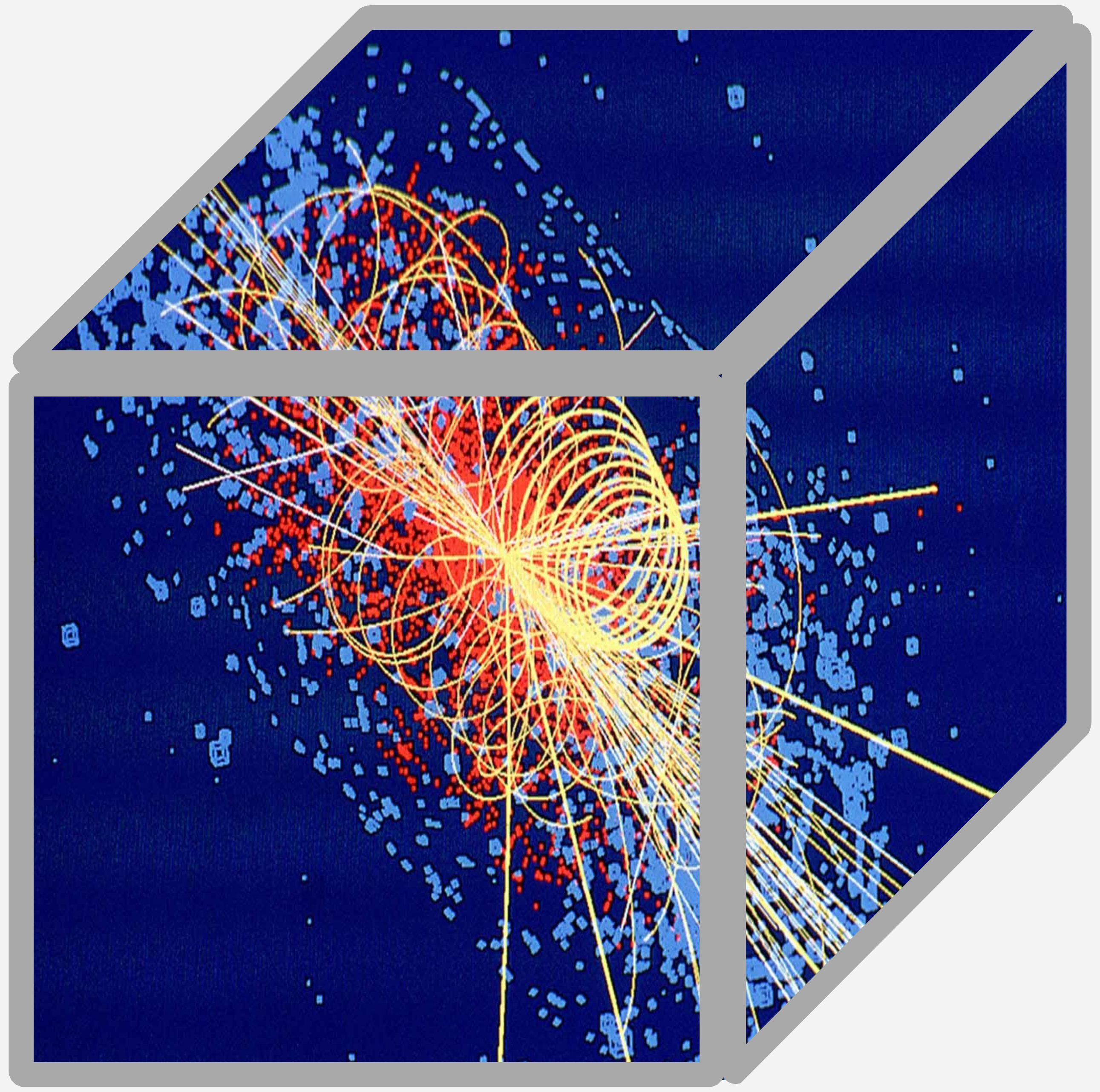
Hamiltonian Truncation

- Obtain **discrete** spectrum of **approximate** energy eigenstates
- Can use eigenstates to compute **dynamical** observables (correlation functions, form factors, etc)
- How do we obtain the S**-matrix** from the **energy eigenstates**?



Why is This Hard?

- S-matrix is overlap of **asymptotic states** $\langle \beta_{\mathrm{out}} | \alpha_{\mathrm{in}} \rangle$
- Numerical methods → **discrete** spectrum → IR cutoff → finite "box"
- **Prevents** identification of asymptotic states



What About LSZ?

- LSZ reduction connects S-matrix to **correlation functions**:

$$\langle \mathbf{p}_3, \mathbf{p}_4; \mathrm{out}|\mathbf{p}_1, \mathbf{p}_2; \mathrm{in}\rangle = \left(\frac{i}{\sqrt{Z}}\right)^2 \int \mathrm{d}x_2\, \mathrm{d}x_3\, e^{i(p_3 x_3 - p_2 x_2)} (\Box_3 + m^2)(\Box_2 + m^2) \langle \mathbf{p}_4 | T\{\phi(x_3)\phi(x_2)\}|\mathbf{p}_1\rangle$$

$$\sum_\alpha |M_\alpha^2\rangle\langle M_\alpha^2|$$

- Insert intermediate energy eigenstates $|M_\alpha^2\rangle$
- Connects asymptotic states with discrete spectrum in finite “box”
- **Problem:** $(\Box + m^2)$ gives **exact zeroes** on-shell, but $\sum_\alpha |M_\alpha^2\rangle\langle M_\alpha^2|$ only creates **approximate poles** in correlator → expression vanishes

LSZ for Discrete Spectrum

- Can use **equations of motion** to enforce cancellation exactly

$$\boxed{\frac{\delta S}{\delta \phi(x)} = -(\Box + m^2)\phi(x) + J(x)}$$

e.g. $J = -\frac{\lambda}{3!}\phi^3$ **for** $V = \frac{\lambda}{4!}\phi^4$

- **Nonperturbative** relation between correlators of $(\Box + m^2)\phi$ and J via **Dyson-Schwinger** equations:

$$\langle \mathbf{p}_4 | T\{\frac{\delta S}{\delta\phi(x_3)}\frac{\delta S}{\delta\phi(x_2)}\}|\mathbf{p}_1\rangle = i\,\langle \mathbf{p}_4|\frac{\delta^2 S}{\delta\phi(x_3)\delta\phi(x_2)}|\mathbf{p}_1\rangle$$

$$(\Box_3 + m^2)(\Box_2 + m^2)\langle \mathbf{p}_4|T\{\phi(x_3)\phi(x_2)\}|\mathbf{p}_1\rangle + \ldots$$

LSZ for Discrete Spectrum

- Rewrite Dyson-Schwinger equation as relation

$$(\Box_3 + m^2)(\Box_2 + m^2)\langle \mathbf{p}_4 | T\{\phi(x_3)\phi(x_2)\} | \mathbf{p}_1 \rangle = \langle \mathbf{p}_4 | T\{J(x_3)J(x_2)\} | \mathbf{p}_1 \rangle - i\delta^d(x_3 - x_2)\langle \mathbf{p}_4 | J'(x_2) | \mathbf{p}_1 \rangle$$

e.g. $J' = -\frac{\lambda}{2}\phi^2$ for $V = \frac{\lambda}{4!}\phi^4$

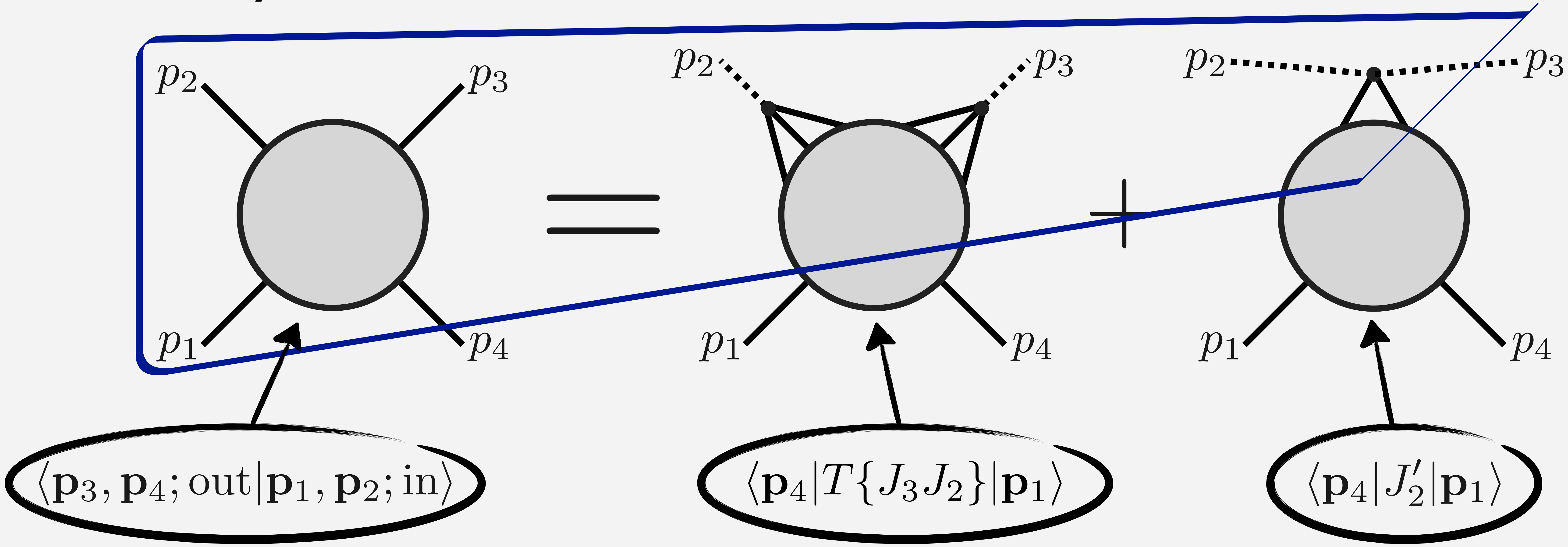
- Obtain alternative LSZ formula which is **manifestly smooth** on-shell:

$$\langle \mathbf{p}_3, \mathbf{p}_4; \mathrm{out} | \mathbf{p}_1, \mathbf{p}_2; \mathrm{in} \rangle = -\frac{1}{Z}\left[\int \mathrm{d}x_2\, \mathrm{d}x_3\, e^{i(p_3 x_3 - p_2 x_2)} \langle \mathbf{p}_4 | T\{J(x_3)J(x_2)\} | \mathbf{p}_1 \rangle - i \int \mathrm{d}x_2\, e^{i(p_3 - p_2)x_2} \langle \mathbf{p}_4 | J'(x_2) | \mathbf{p}_1 \rangle \right]$$

No exact zeroes

LSZ for Discrete Spectrum

- **Example:** for ϕ^4 theory, have schematic representation:



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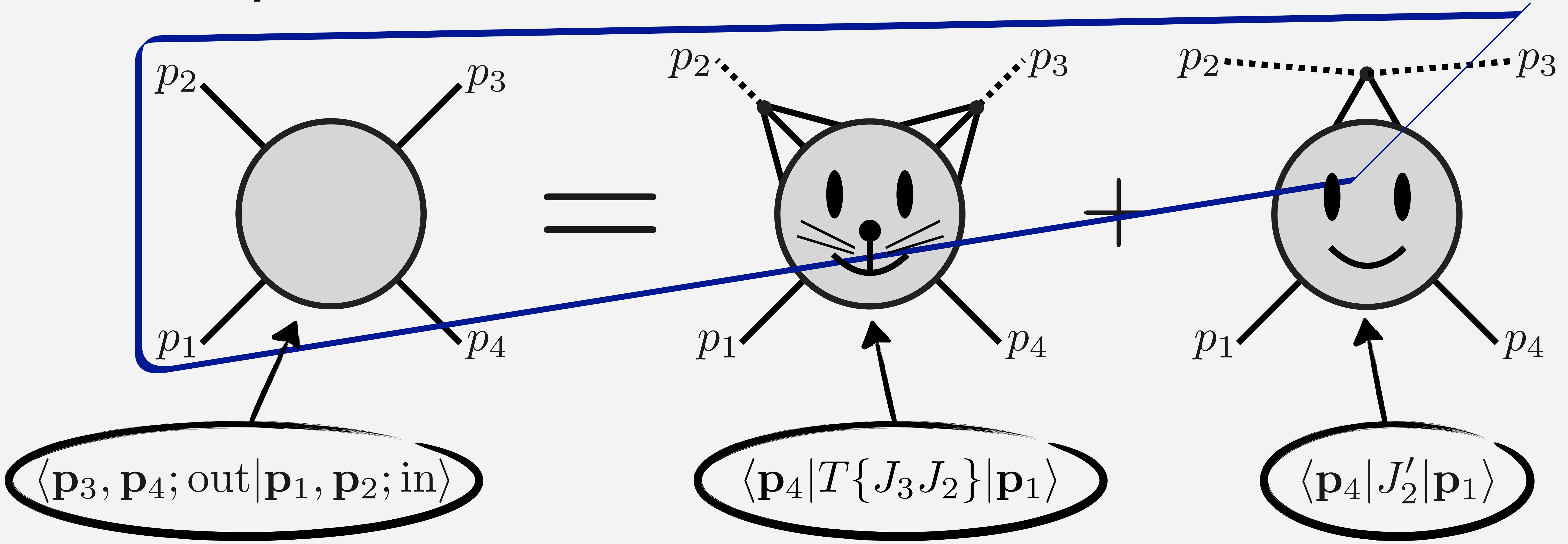

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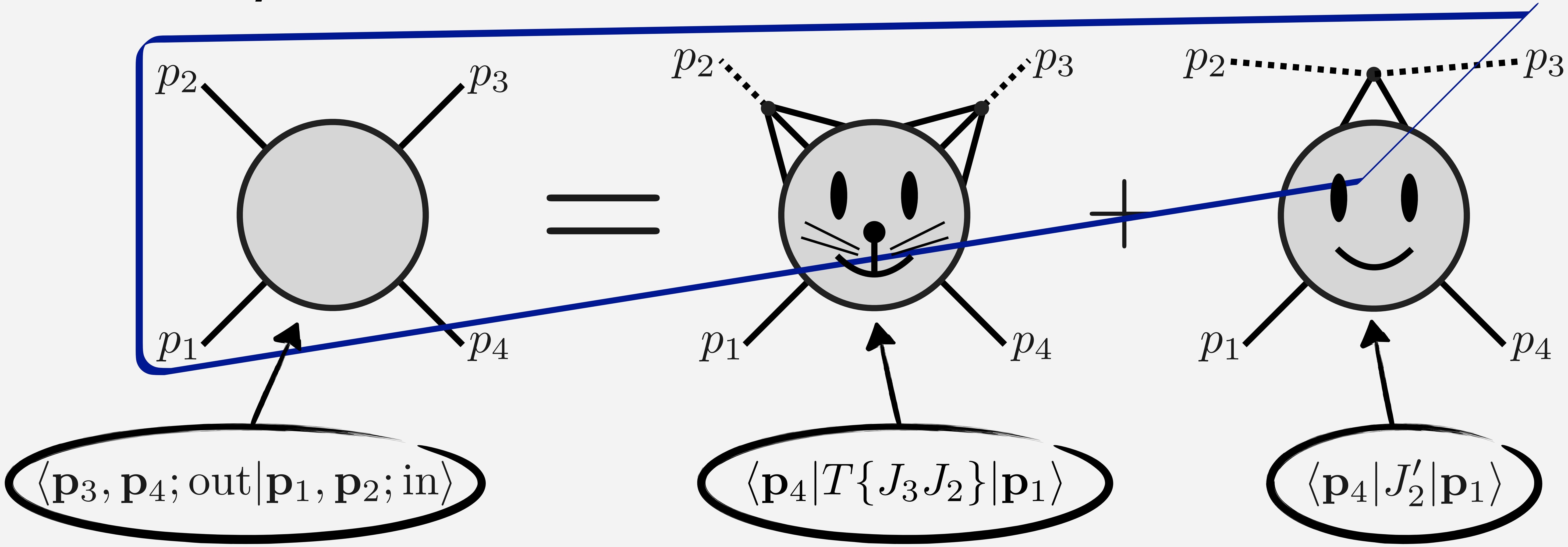

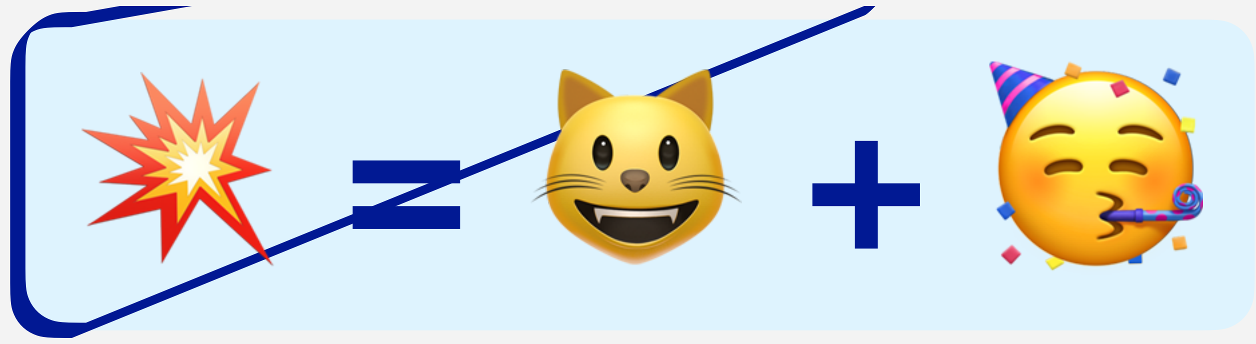

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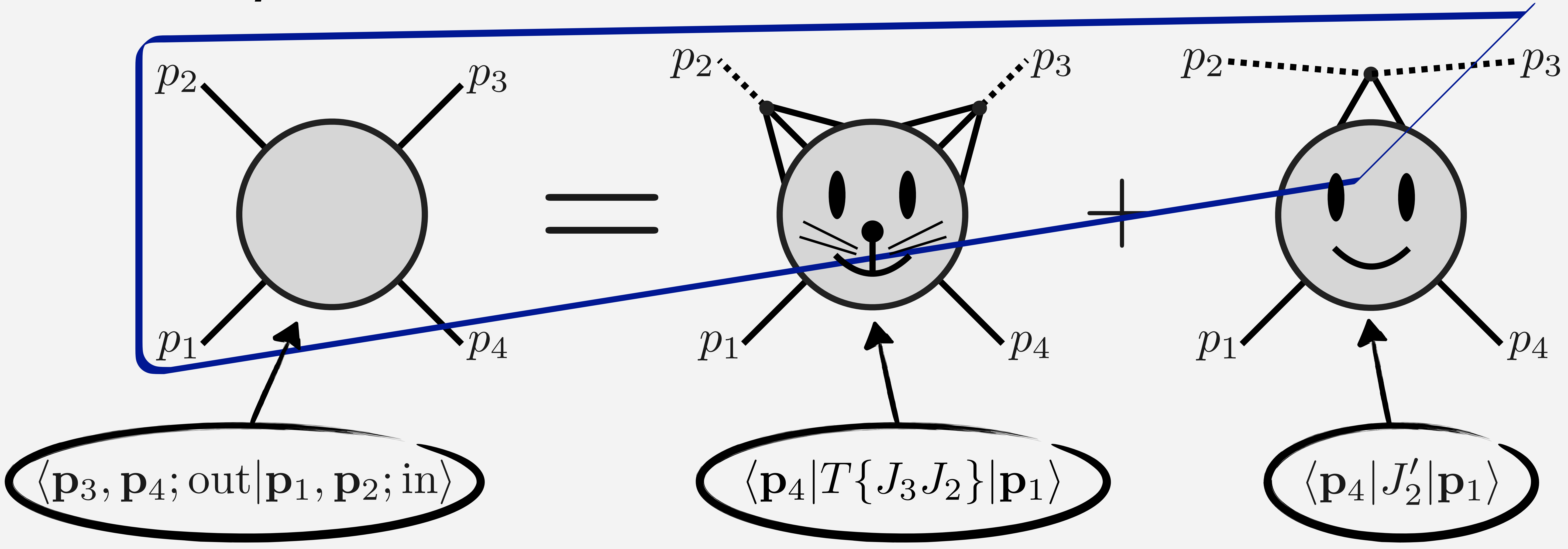
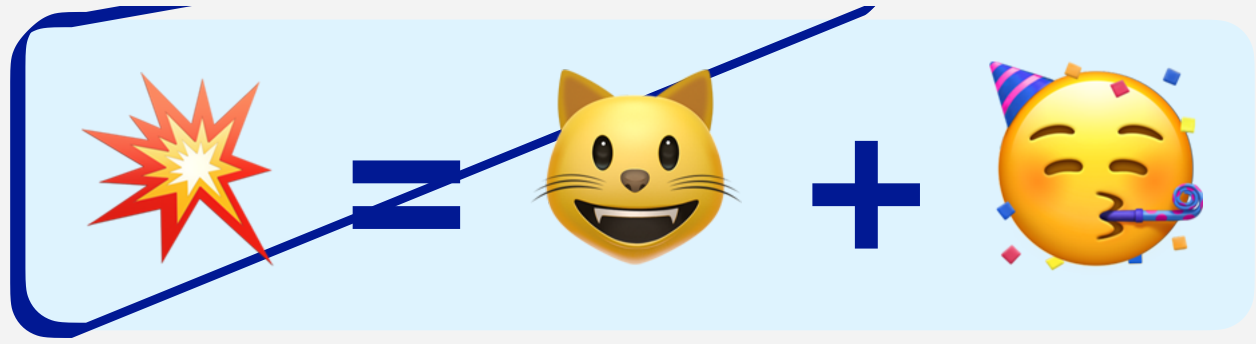


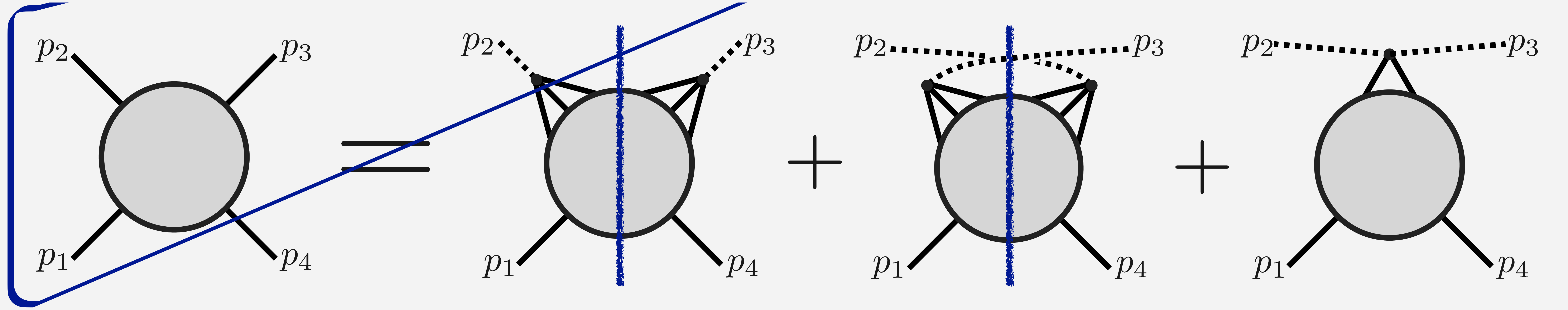


- **Note:** physical mass m^2 in LSZ formula does not generically match bare mass m_0^2 in e.o.m. $\to J \supset (m^2 - m_0^2)\phi$

Sum over Intermediate States

- Inserting sum over energy eigenstates:

$$\mathcal{M}(s,t) = \frac{1}{Z}\Bigg[\sum_\alpha \Bigg(\frac{\langle \mathbf{p}_4|J(0)|M_\alpha^2;\mathbf{p}_1+\mathbf{p}_2\rangle\langle M_\alpha^2;\mathbf{p}_1+\mathbf{p}_2|J(0)|\mathbf{p}_1\rangle}{M_\alpha^2 - s - i\epsilon} + \frac{\langle \mathbf{p}_4|J(0)|M_\alpha^2;\mathbf{p}_1-\mathbf{p}_3\rangle\langle M_\alpha^2;\mathbf{p}_1-\mathbf{p}_3|J(0)|\mathbf{p}_1\rangle}{M_\alpha^2 - t - i\epsilon}\Bigg) + \langle \mathbf{p}_4|J'(0)|\mathbf{p}_1\rangle\Bigg]$$



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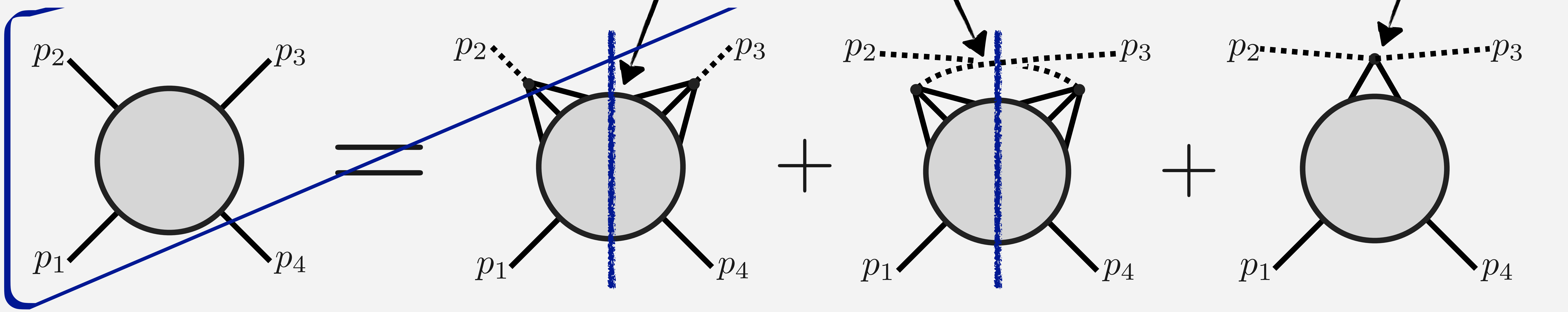
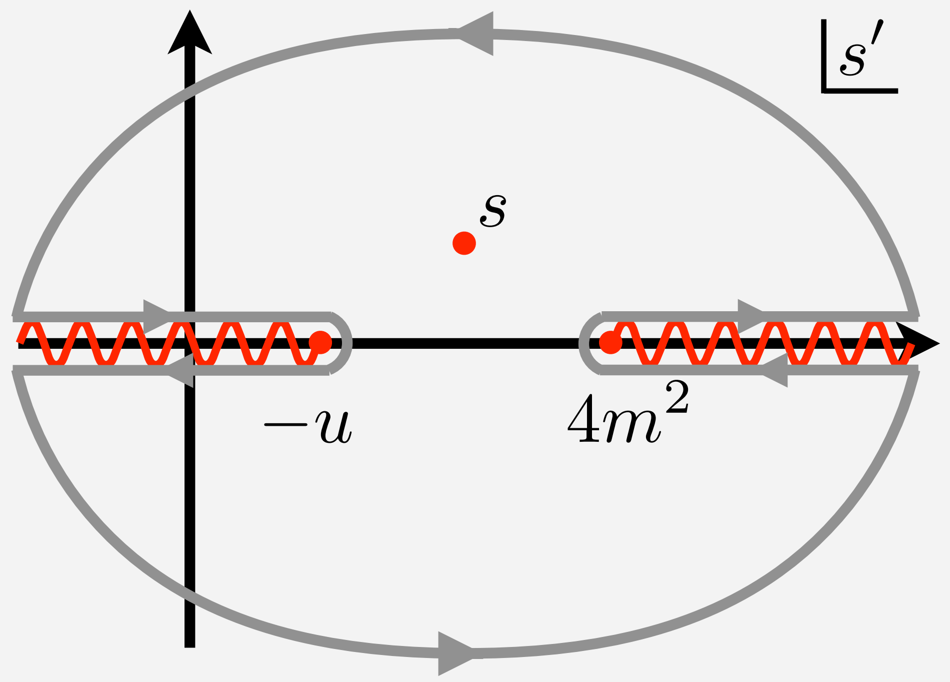


Dispersion Relation



- Reproduces fixed-u **dispersion relation:**

$$\mathcal{M}(s,t) = \frac{1}{\pi}\int ds' \frac{\mathrm{Im}[\mathcal{M}(s',t')]}{s'-s-i\epsilon} + \frac{1}{\pi}\int dt' \frac{\mathrm{Im}[\mathcal{M}(s',t')]}{t'-t-i\epsilon} + \text{subtraction terms}$$

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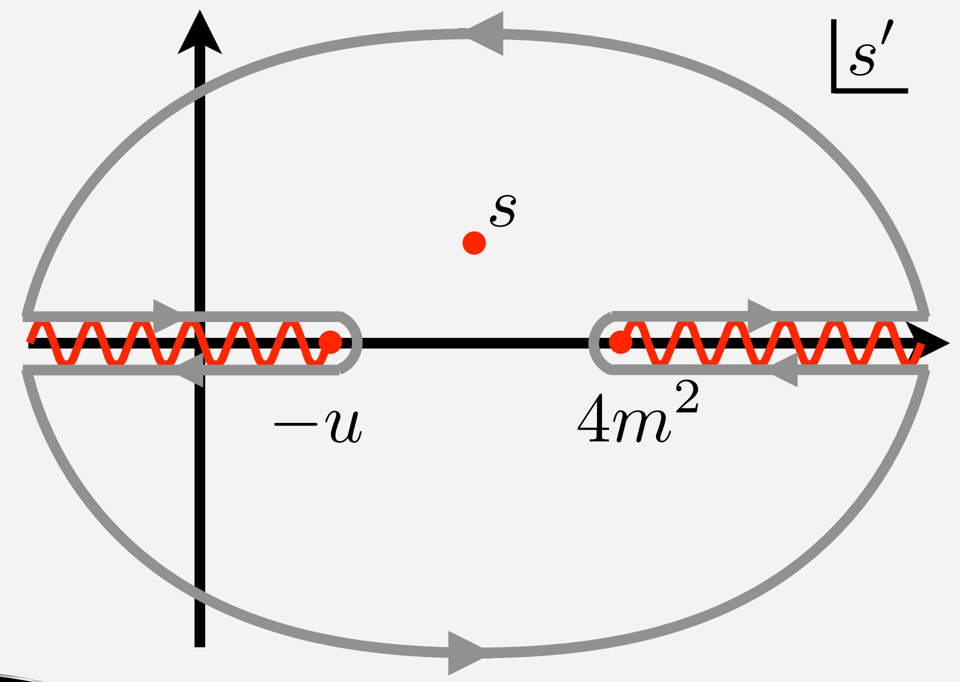

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Recap

$$\langle \mathbf{p}_3, \mathbf{p}_4; \mathrm{out}|\mathbf{p}_1, \mathbf{p}_2; \mathrm{in}\rangle \overset{\text{LSZ}}{\Longrightarrow} \langle \mathbf{p}_4|\phi_3\phi_2|\mathbf{p}_1\rangle \overset{\text{Schwinger-Dyson}}{\Longrightarrow} \langle \mathbf{p}_4|J_3J_2|\mathbf{p}_1\rangle \overset{\mathbb{1}\simeq\sum_\alpha |M_\alpha^2\rangle\langle M_\alpha^2|}{\Longrightarrow} \langle \mathbf{p}_4|J_3|M_\alpha^2\rangle\langle M_\alpha^2|J_2|\mathbf{p}_1\rangle$$

- Numerically stable
- Computable from discrete energy eigenstates

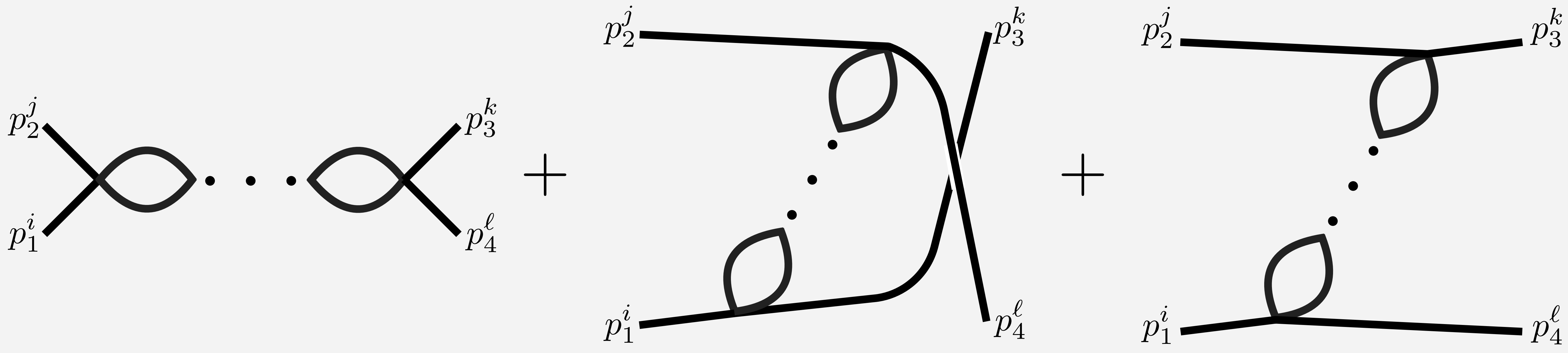
Example: O(N) Model at Large N

$$\boxed{\mathcal{L} = \frac{1}{2}\partial^\mu \phi^i \partial_\mu \phi^i - \frac{1}{2} m^2 \phi^i \phi^i - \frac{\lambda}{4N}(\phi^i\phi^i)(\phi^j\phi^j)}$$

$(d = 2+1)$

- **Solvable** at $N \to \infty$ for all values of m, λ
- Scattering amplitude computable from summing chains of loop diagrams:

$$\mathcal{M}^{ijk\ell}(s,t) = \frac{1}{N}\Big(\mathcal{M}(s)\delta^{ij}\delta^{k\ell} + \mathcal{M}(t)\delta^{ik}\delta^{j\ell} + \mathcal{M}(u)\delta^{i\ell}\delta^{jk}\Big)$$



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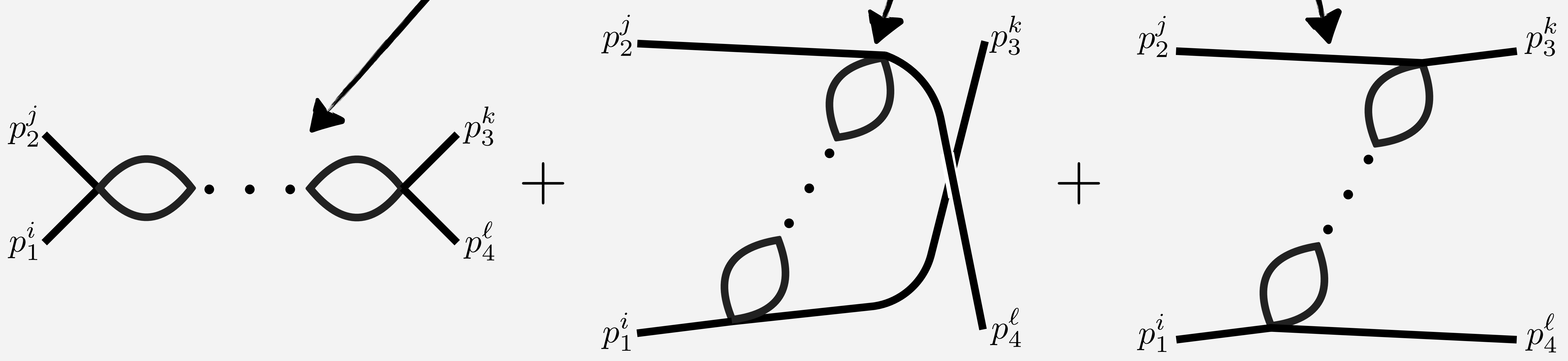


Lightcone Conformal Truncation

- Describe Hilbert space in terms of **UV CFT**: $H = H_0 + V$

$(\partial\vec{\phi})^2 \qquad m^2\vec{\phi}^2 + \frac{\lambda}{N}(\vec{\phi}^2)^2$

1) **Discretize:** use basis of CFT local operators

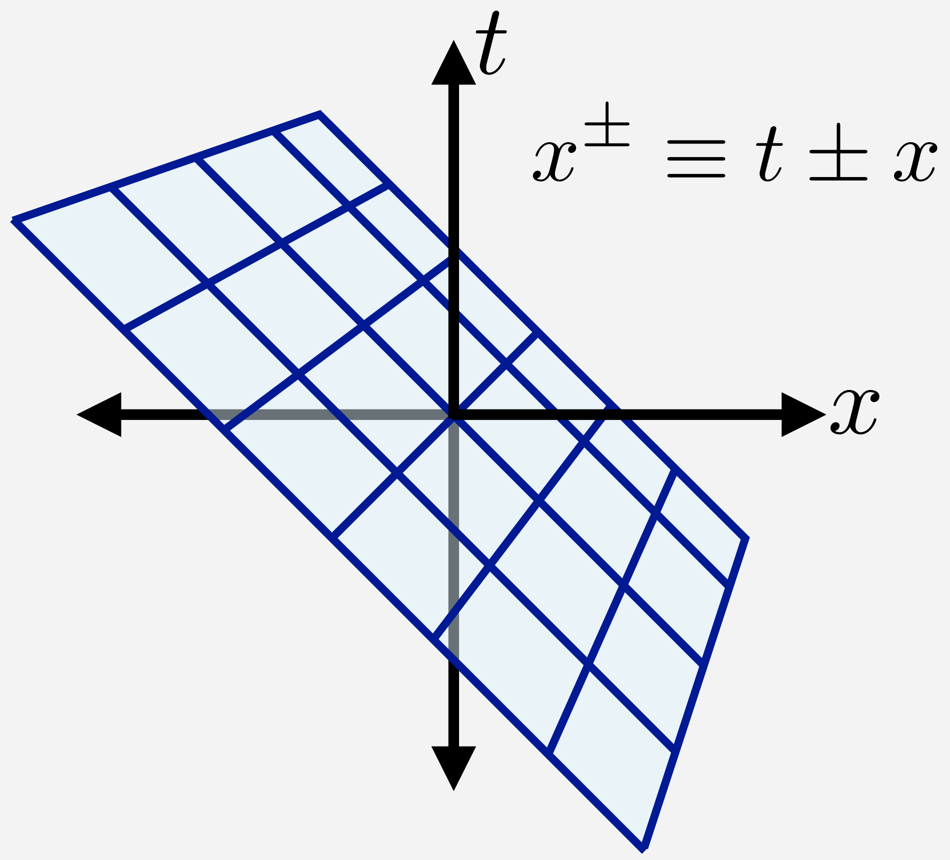
$$\boxed{|\mathcal{O};p_\mu\rangle \equiv \int \mathrm{d}x\, e^{-ipx}\mathcal{O}(x)|0\rangle}$$

with energy divided into discrete bins (in fixed $\vec{p}$ frame)

2) **Truncate:** restrict to operators with $\Delta \leq \Delta_{\max}$, $\mathfrak{i}_{\max}$ bins for each $\mathcal{O}$

3) **Diagonalize:** construct Hamiltonian from free theory data and diagonalize numerically

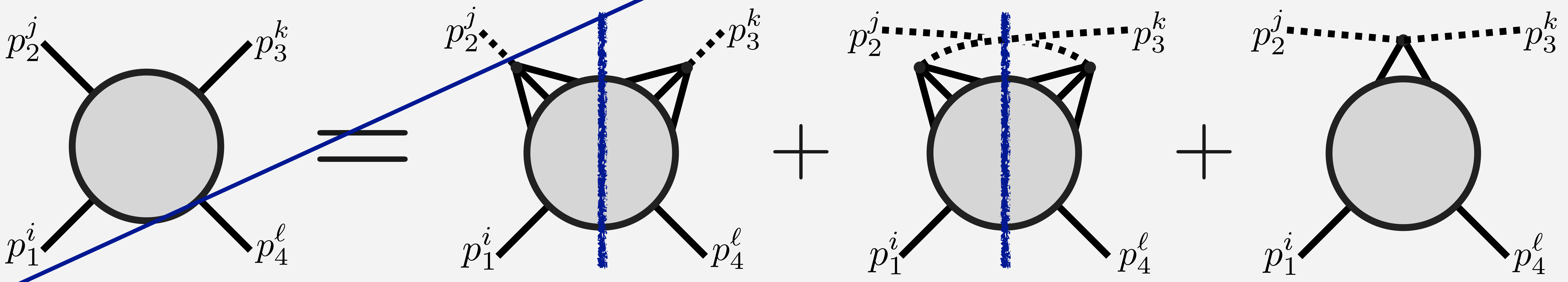
- States formulated in **lightcone quantization**



Generalized LSZ Formula

- Generalizing formula to $O(N)$ model:

$$\mathcal{M}^{ijk\ell}(s,t) = \frac{1}{Z}\left[\sum_\alpha \left(\frac{\langle \mathbf{p}_4^\ell | J^k | M_\alpha^2; \mathbf{p}_1 + \mathbf{p}_2\rangle\langle M_\alpha^2; \mathbf{p}_1 + \mathbf{p}_2 | J^j | \mathbf{p}_1^i\rangle}{M_\alpha^2 - s - i\epsilon} + \frac{\langle \mathbf{p}_4^\ell | J^j | M_\alpha^2; \mathbf{p}_1 - \mathbf{p}_3\rangle\langle M_\alpha^2; \mathbf{p}_1 - \mathbf{p}_3 | J^k | \mathbf{p}_1^i\rangle}{M_\alpha^2 - t - i\epsilon}\right) + \langle \mathbf{p}_4^\ell | J'^{\,jk} | \mathbf{p}_1^i\rangle\right]$$



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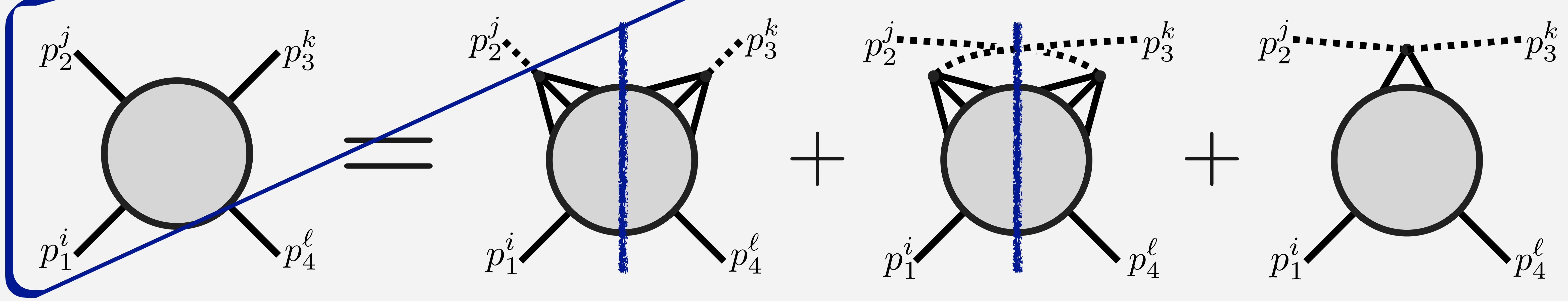
“s-kinematics” term

$$\mathcal{M}^{ijk\ell}(s,t) = \frac{1}{Z}\left[\sum_\alpha\left(\frac{\langle \mathbf{p}_4^\ell|J^k|M_\alpha^2;\mathbf{p}_1+\mathbf{p}_2\rangle\langle M_\alpha^2;\mathbf{p}_1+\mathbf{p}_2|J^j|\mathbf{p}_1^i\rangle}{M_\alpha^2-s-i\epsilon} + \frac{\langle \mathbf{p}_4^\ell|J^j|M_\alpha^2;\mathbf{p}_1-\mathbf{p}_3\rangle\langle M_\alpha^2;\mathbf{p}_1-\mathbf{p}_3|J^k|\mathbf{p}_1^i\rangle}{M_\alpha^2-t-i\epsilon}\right) + \langle \mathbf{p}_4^\ell|J'^{\,jk}|\mathbf{p}_1^i\rangle\right]$$

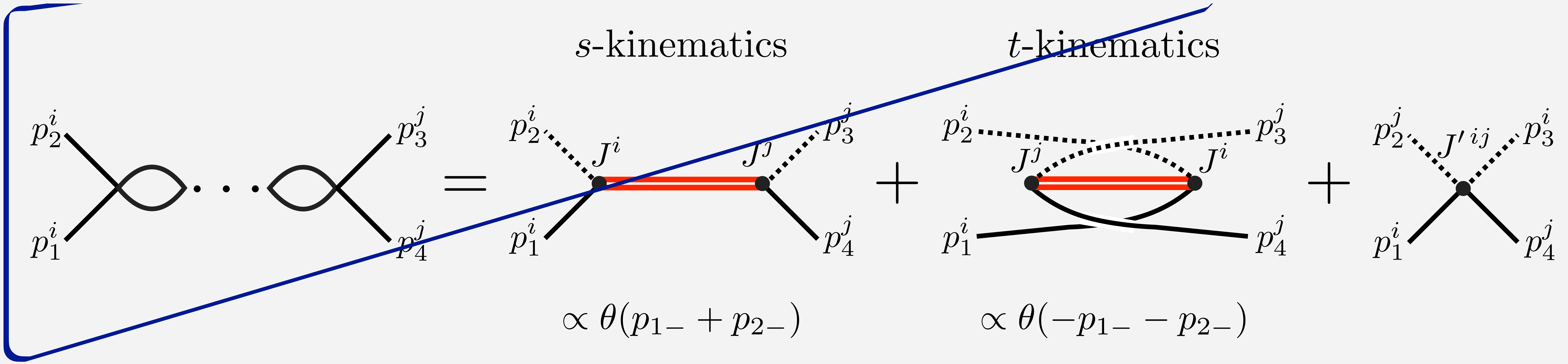
“t-kinematics” term



S-Flavor Amplitude

$$\boxed{\mathcal{M}^{ijk\ell}(s,t) = \frac{1}{N}\left(\mathcal{M}(s)\delta^{ij}\delta^{k\ell} + \mathcal{M}(t)\delta^{ik}\delta^{j\ell} + \mathcal{M}(u)\delta^{i\ell}\delta^{jk}\right)}$$

"s-flavor" amplitude



$$\mathcal{M}(s) = \frac{1}{N}\left[\sum_\alpha\left(\frac{\langle \mathbf{p}_4^j|J^j|M_\alpha^2;\mathbf{p}_1+\mathbf{p}_2\rangle\langle M_\alpha^2;\mathbf{p}_1+\mathbf{p}_2|J^i|\mathbf{p}_1^i\rangle}{M_\alpha^2 - s - i\epsilon} + \frac{\langle \mathbf{p}_4^j|J^i|M_\alpha^2;\mathbf{p}_1-\mathbf{p}_3\rangle\langle M_\alpha^2;\mathbf{p}_1-\mathbf{p}_3|J^j|\mathbf{p}_1^i\rangle}{M_\alpha^2 - t - i\epsilon}\right) + \langle \mathbf{p}_4^j|J'^{ij}|\mathbf{p}_1^i\rangle\right]$$

$\mathcal{S}$-Flavor Amplitude

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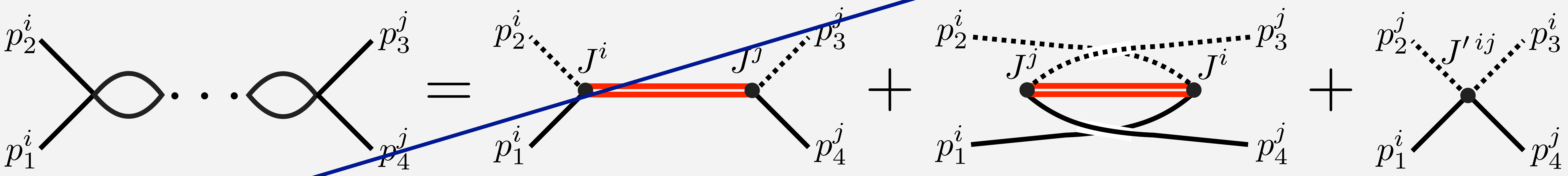
"s-flavor" amplitude

s-kinematics $\quad$ t-kinematics



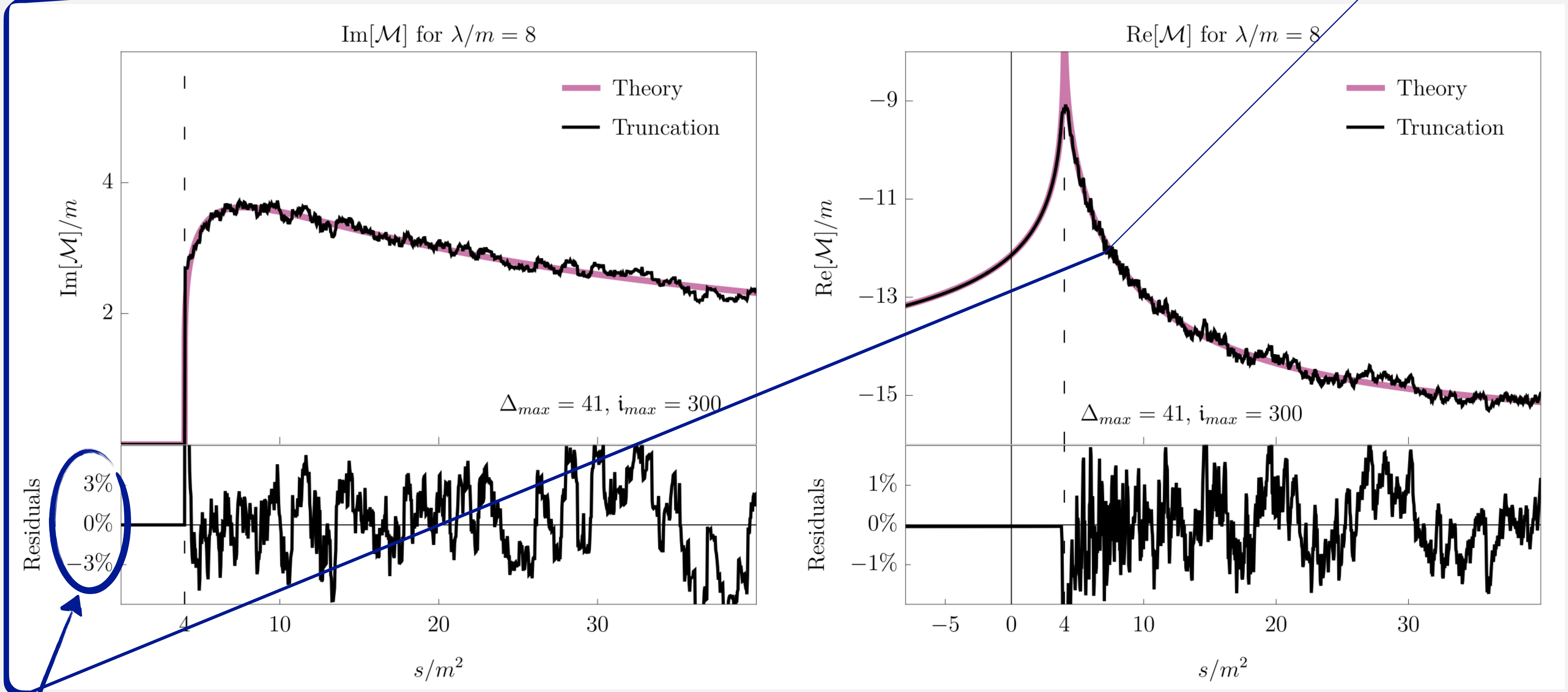
$\propto \theta(p_{1-} + p_{2-})$ $\quad$ $\propto \theta(-p_{1-} - p_{2-})$

$$\mathcal{M}(s) = \frac{1}{N}\left[\sum_\alpha \left(\frac{\langle \mathbf{p}_4^j|J^j|M_\alpha^2;\mathbf{p}_1+\mathbf{p}_2\rangle\langle M_\alpha^2;\mathbf{p}_1+\mathbf{p}_2|J^i|\mathbf{p}_1^i\rangle}{M_\alpha^2 - s - i\epsilon} + \frac{\langle \mathbf{p}_4^j|J^i|M_\alpha^2;\mathbf{p}_1-\mathbf{p}_3\rangle\langle M_\alpha^2;\mathbf{p}_1-\mathbf{p}_3|J^j|\mathbf{p}_1^i\rangle}{M_\alpha^2 - t - i\epsilon}\right) - \langle \mathbf{p}_4^j|J'^{ij}|\mathbf{p}_1^i\rangle\right]$$

$\mathrm{Im}[\mathcal{M}]$

-2λ

Results (s-flavor)



Percent-level error

$$\mathcal{M}^{ijk\ell}(s,t) = \frac{1}{N}\left(\mathcal{M}(s)\delta^{ij}\delta^{k\ell} + \mathcal{M}(t)\delta^{ik}\delta^{j\ell} + \mathcal{M}(u)\delta^{i\ell}\delta^{jk}\right)$$

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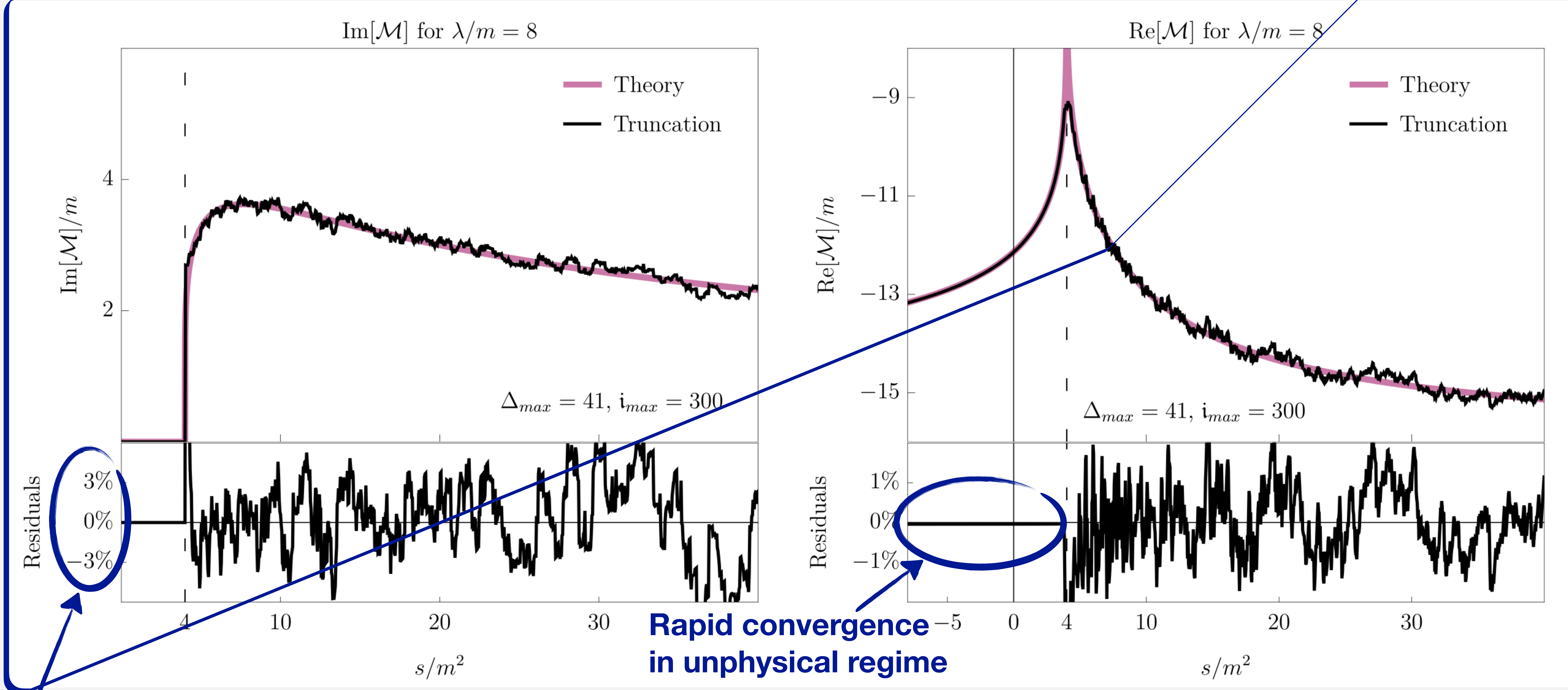

Rapid convergence in unphysical regime

Percent-level error

$$\mathcal{M}^{ijk\ell}(s,t) = \frac{1}{N}\left(\mathcal{M}(s)\delta^{ij}\delta^{k\ell} + \mathcal{M}(t)\delta^{ik}\delta^{j\ell} + \mathcal{M}(u)\delta^{i\ell}\delta^{jk}\right)$$

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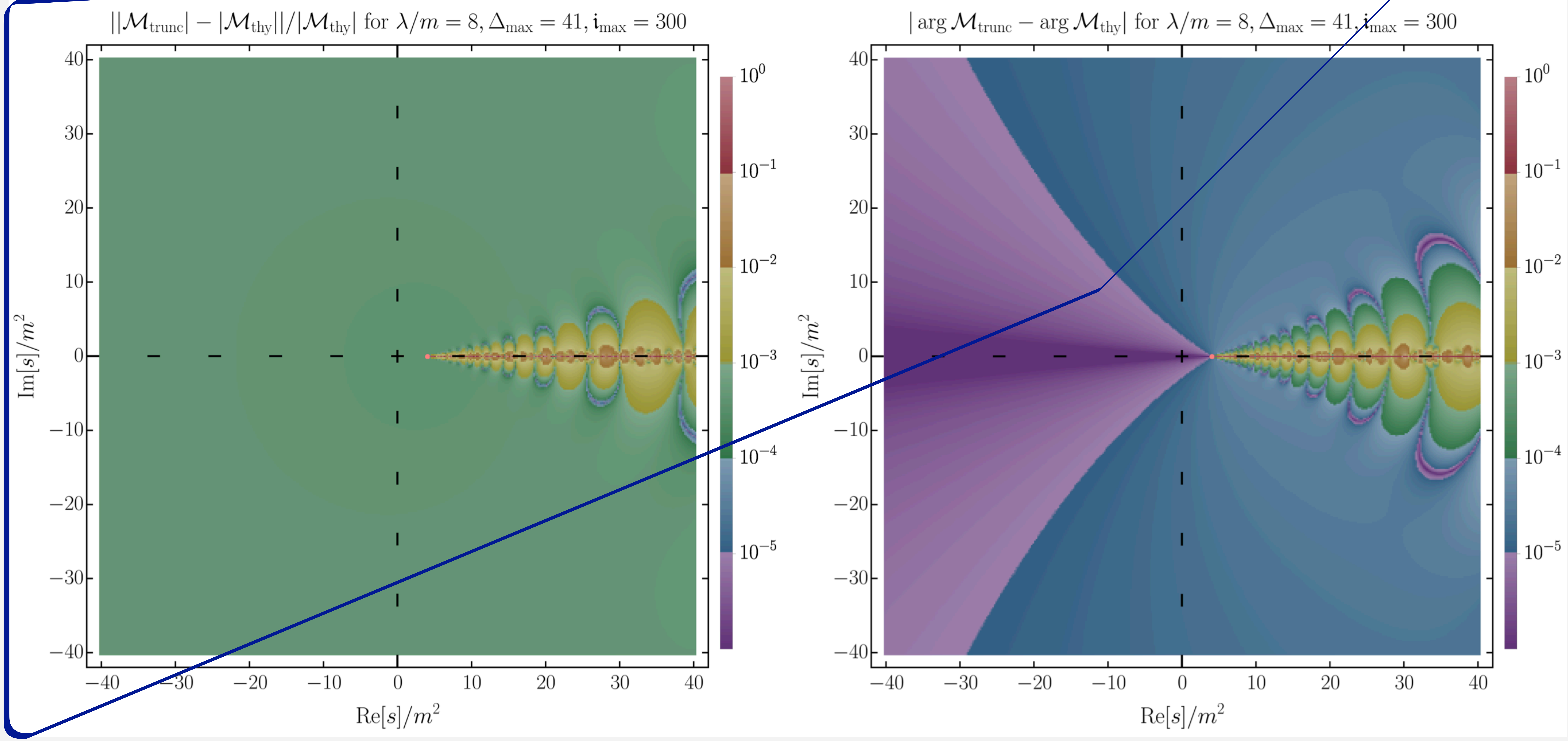

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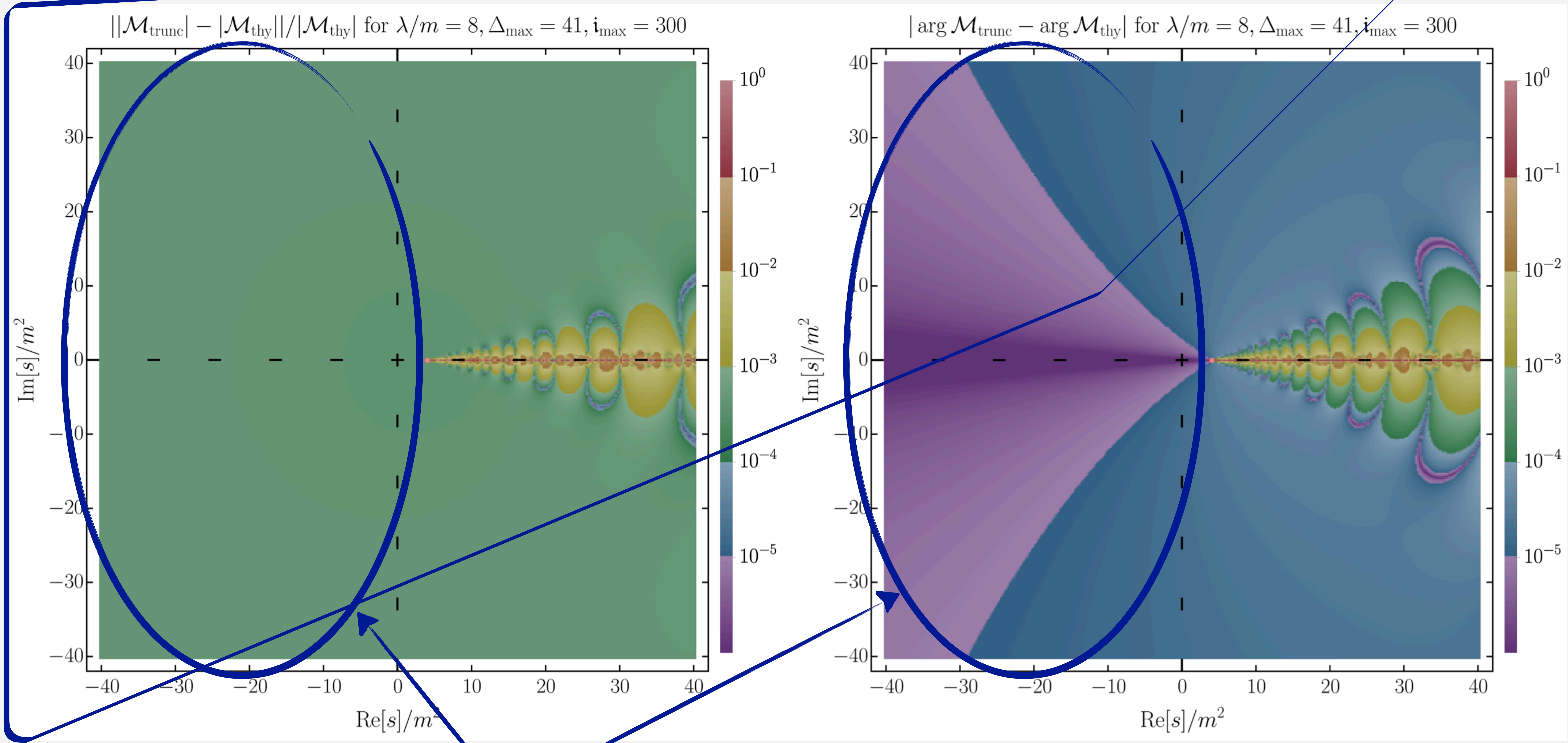


Rapid convergence throughout complex plane

$$\mathcal{M}^{ijk\ell}(s,t) = \frac{1}{N}\left(\mathcal{M}(s)\delta^{ij}\delta^{k\ell} + \mathcal{M}(t)\delta^{ik}\delta^{j\ell} + \mathcal{M}(u)\delta^{i\ell}\delta^{jk}\right)$$

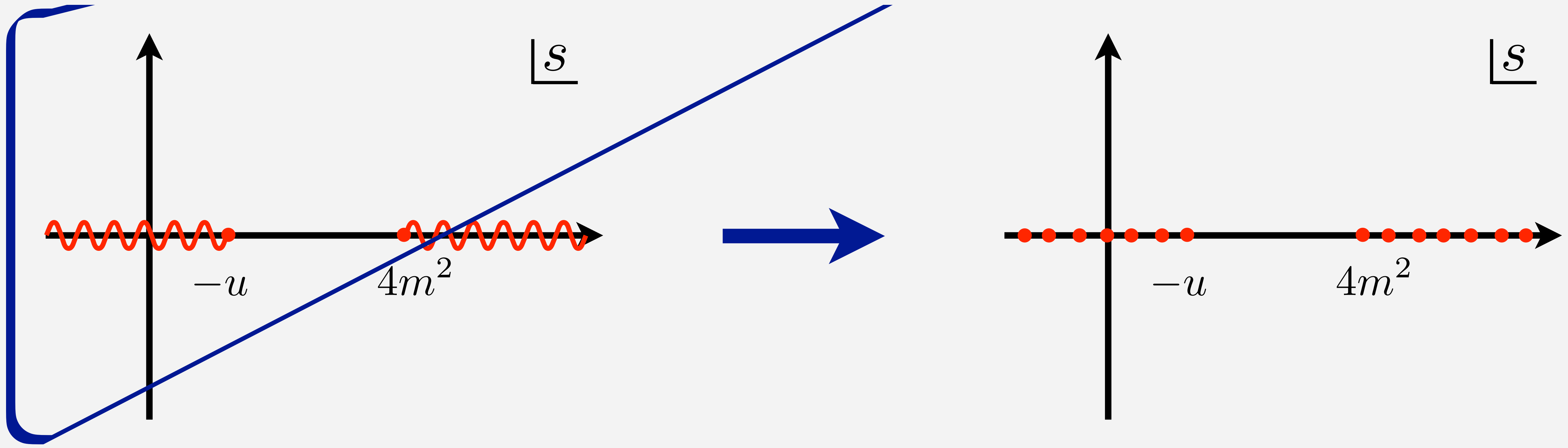
Future Directions

Full Analytic Structure

- $\langle \mathbf{p}_4 \,|\, T\{\phi_3\phi_2\} \,|\, \mathbf{p}_1 \rangle$ studies arbitrary $s \in \mathbb{C}$ for fixed $u \leq 0$
- To study arbitrary s, t, and u:

$$(\Box_4 + m^2)\cdots(\Box_1 + m^2)\langle T\{\phi_4\phi_3\phi_2\phi_1\}\rangle = \langle T\{J_4 J_3 J_2 J_1\}\rangle + \text{contact terms}$$

- Can we access full multi-sheeted structure?



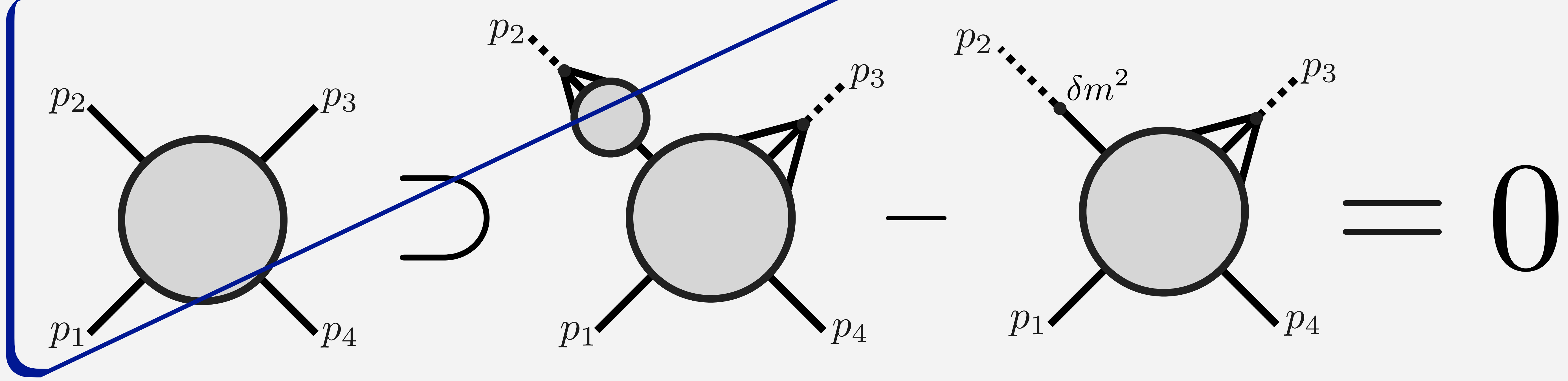
Future Directions

Finite N Theories

- **Same approach** as large N, include general particle number states
- Two minor complications:

  1) Field strength renormalization $Z \equiv \langle \Omega \,|\, \phi(0) \,|\, \mathbf{p} \rangle \neq 1$

  2) Mass shift $\delta m^2 \equiv m^2 - m_0^2 \neq 0 \rightarrow J \supset \delta m^2 \phi$



Future Directions

Bound States

- LSZ can be used for **any operator** $\mathcal{O}$ with same quantum numbers as one-particle state $|\mathbf{p}\rangle$
- Crucial for computing scattering of **bound states** ($\pi \sim \bar{q}\gamma_5 q$)
- Can we use e.o.m. for **constituent** fields or **Ward identities** for stress-energy tensor or conserved currents?
- More generally, use OPE for $[V, \mathcal{O}]$?

Future Directions

Gauge Theories

- **Two** possible approaches:

Free Quarks, Gluons

$$+gA^\mu J_\mu + gA^3 + g^2 A^4$$

Banks-Zaks (e.g. $SU(\cancel{3}), N_f = 16$)

$$+m\bar{q}q \quad (N_f - 3)$$

QCD

- **Marginal ($d = 3 + 1$)**
- **Not Gauge-Invariant**

- **Relevant**
- **Gauge-Invariant**

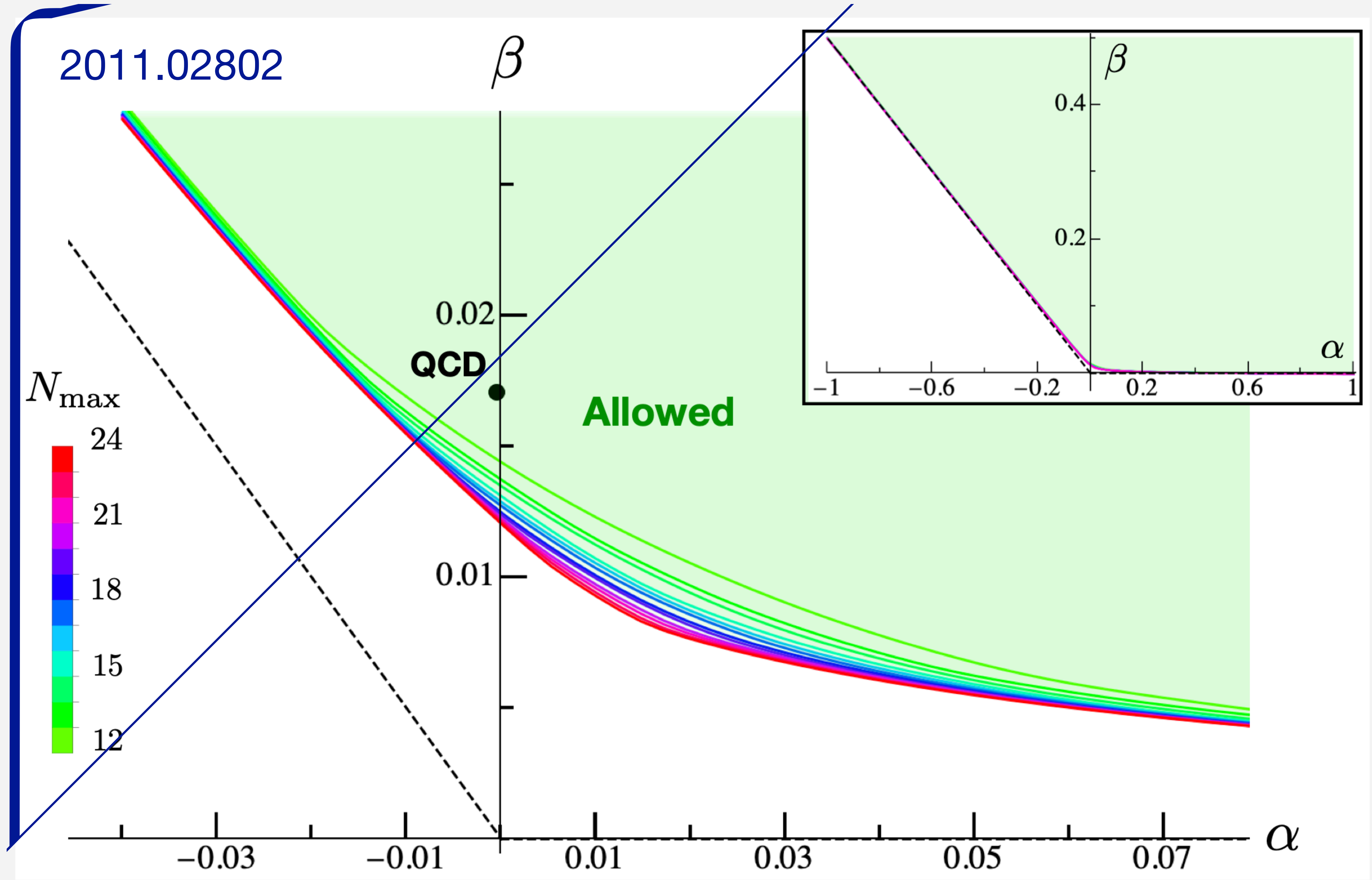
Let's Do This!

- **Same approach** can be implemented in **many numerical frameworks**
- Wide variety of exciting **open questions** (both conceptual & technical)
- Would **greatly benefit** from insight on QCD, BSM, experiment, & more
- If you're interested in **any aspect** of QFT (Standard Model, model-building, EFT, amplitudes, etc) there's an application for **you**!

BACKUP SLIDES

S-Matrix Bootstrap

- Nonperturbative constraints on space of S-matrices consistent with **analyticity**, **unitarity**
- Can input information on **low-energy** EFT behavior
- Difficult to isolate **individual** QFTs



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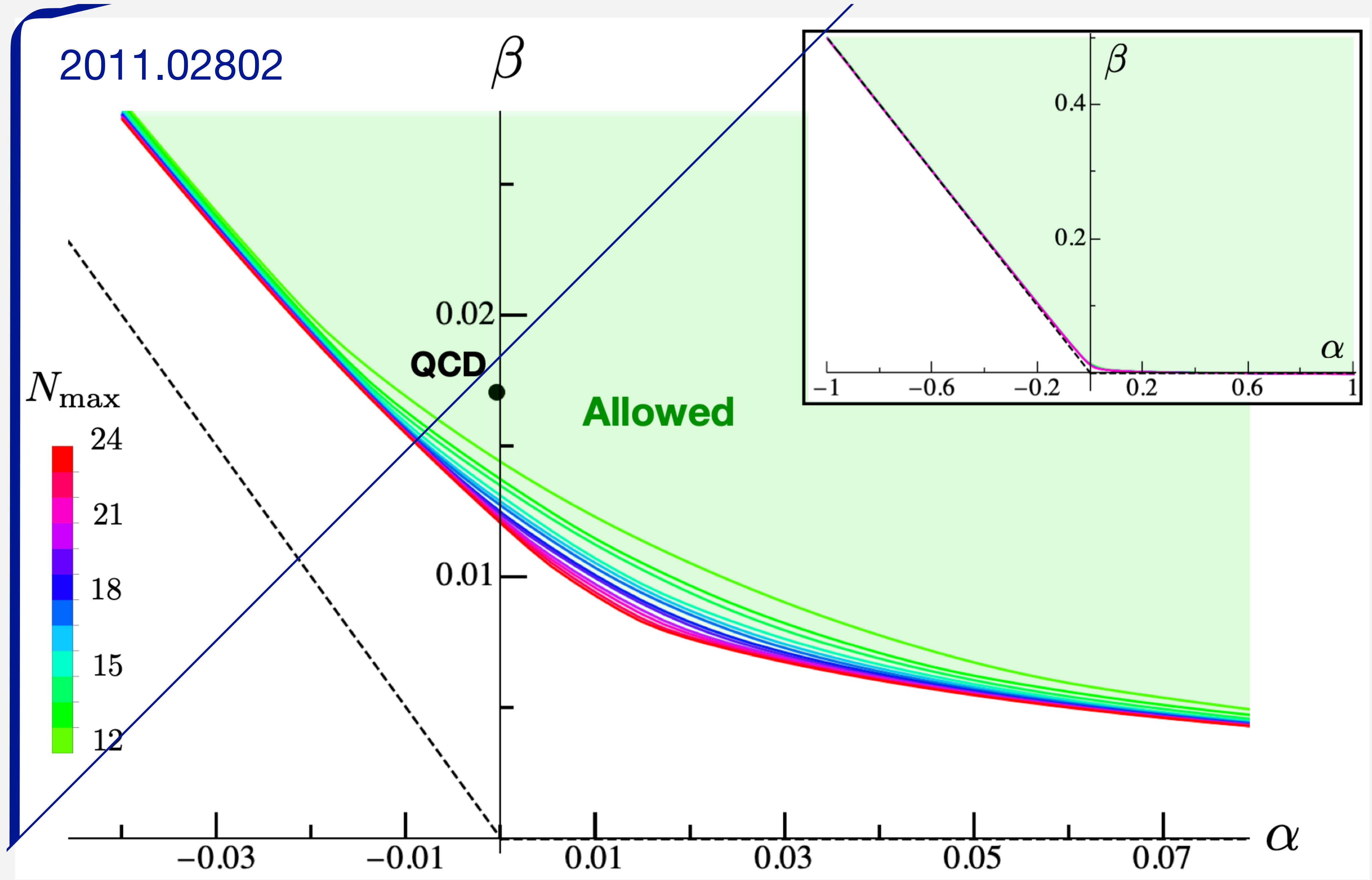
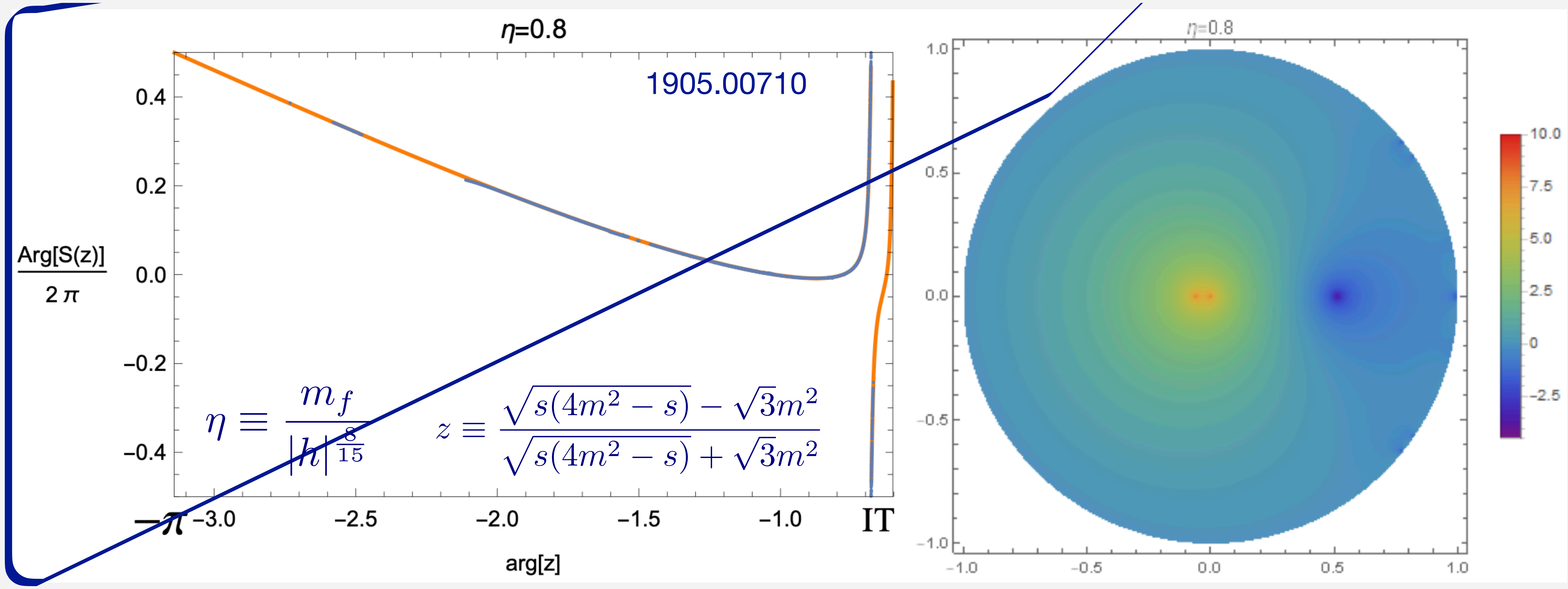


Need a way to study specific S-matrices

Truncation and the S-Matrix

1+1d Ising Field Theory (Gabai, Yin '19)

- Compute **elastic** scattering from eigenvalue spectrum (via Lüscher)
- **Analytically continue** to complex plane using unitarity, analyticity

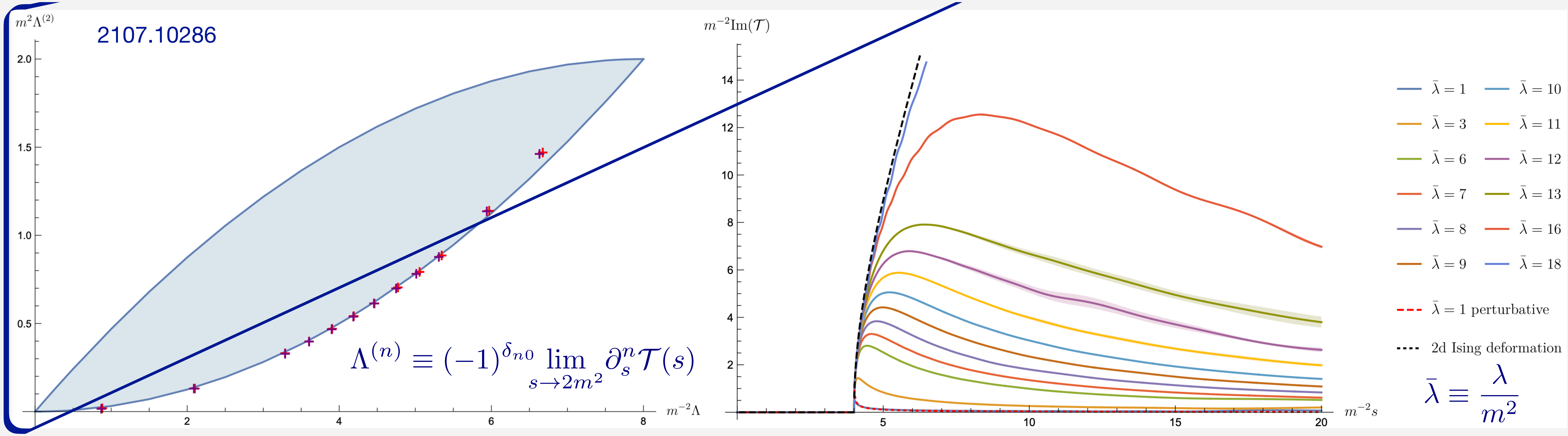


Truncation and the S-Matrix

1+1d ϕ^4 Theory (Chen, Fitzpatrick, Karateev '21)

- Directly compute **form factors** $\langle p \,|\, \mathcal{O} \,|\, p' \rangle$
- Use combined **form factor + S-matrix bootstrap** to derive **theory-specific** bounds on S-matrix

Karateev, Kuhn, Penedones '19



T- and U-Flavor Amplitudes