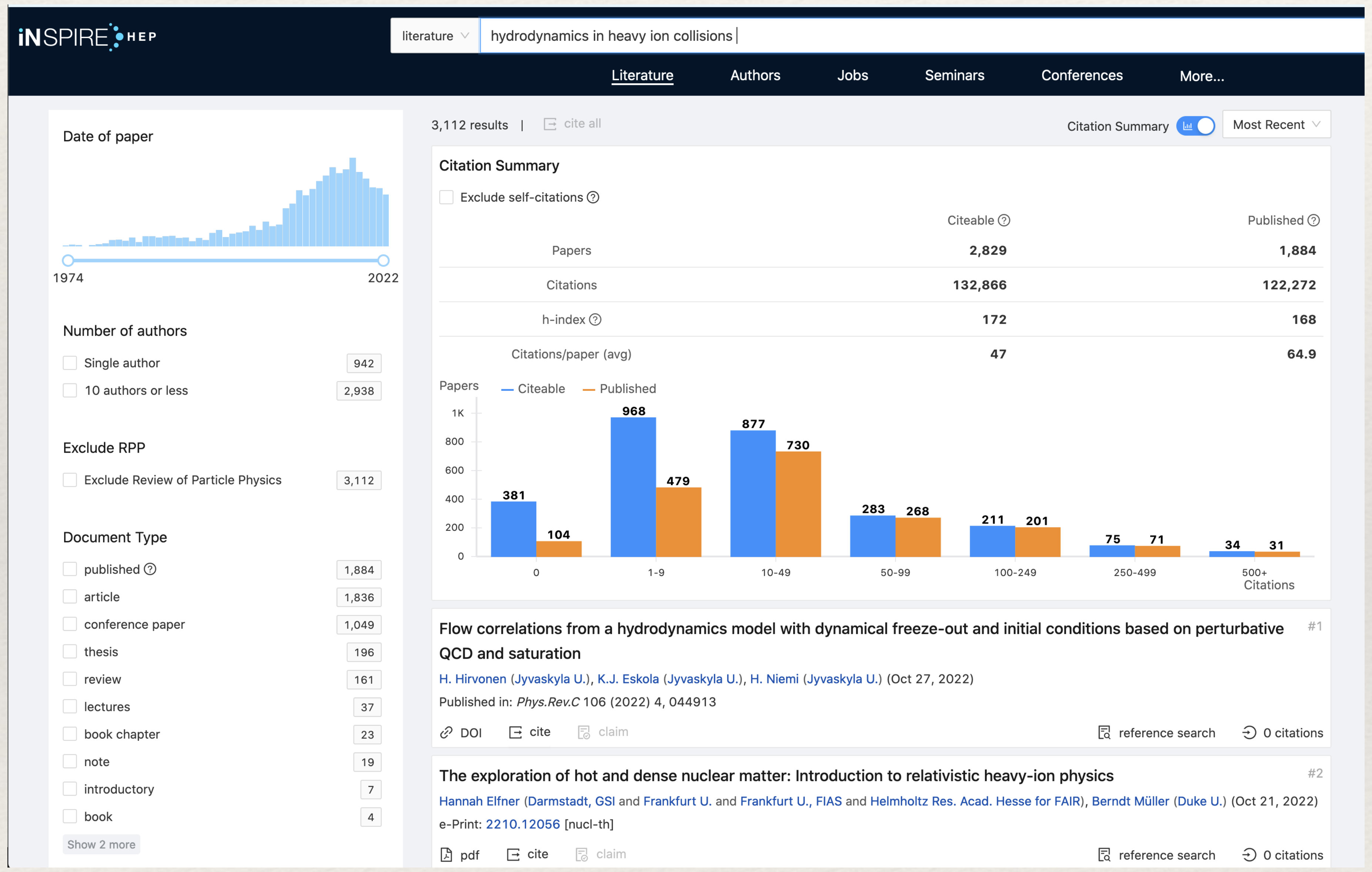
Hydrodynamics in heavy-ion collisions

ALICE-STAR Collaboration meeting 2022, IOP, Bhubaneswar.

- Victor Roy, NISER

Prelude

You may love hydro or you may hate hydro, but you can’t ignore hydro !



hydrodynamics in heavy ion collisions

3,112 results

Date of paper

1974 – 2022

Number of authors

☐ Single author	942
☐ 10 authors or less	2,938

Exclude RPP

☐ Exclude Review of Particle Physics	3,112

Document Type

☐ published	1,884
☐ article	1,836
☐ conference paper	1,049
☐ thesis	196
☐ review	161
☐ lectures	37
☐ book chapter	23
☐ note	19
☐ introductory	7
☐ book	4

Citation Summary

☐ Exclude self-citations

	Citeable	Published
Papers	2,829	1,884
Citations	132,866	122,272
h-index	172	168
Citations/paper (avg)	47	64.9

Flow correlations from a hydrodynamics model with dynamical freeze-out and initial conditions based on perturbative QCD and saturation #1

H. Hirvonen (Jyvaskyla U.), K.J. Eskola (Jyvaskyla U.), H. Niemi (Jyvaskyla U.) (Oct 27, 2022)

Published in: *Phys.Rev.C* 106 (2022) 4, 044913

0 citations

The exploration of hot and dense nuclear matter: Introduction to relativistic heavy-ion physics #2

Hannah Elfner (Darmstadt, GSI and Frankfurt U. and Frankfurt U., FIAS and Helmholtz Res. Acad. Hesse for FAIR), Berndt Müller (Duke U.) (Oct 21, 2022)

e-Print: 2210.12056 [nucl-th]

0 citations

Experimental and theoretical challenges in the search for the quark gluon plasma: The STAR Collaboration's critical assessment of the evidence from RHIC collisions #1

STAR Collaboration • John Adams (Birmingham U.) et al. (Jan, 2005)

Published in: *Nucl.Phys.A* 757 (2005) 102-183 • e-Print: nucl-ex/0501009 [nucl-ex]

3,622 citations

Formation of dense partonic matter in relativistic nucleus-nucleus collisions at RHIC: Experimental evaluation by the PHENIX collaboration #2

PHENIX Collaboration • K. Adcox (Vanderbilt U.) et al. (Oct, 2004)

Published in: *Nucl.Phys.A* 757 (2005) 184-283 • e-Print: nucl-ex/0410003 [nucl-ex]

3,313 citations

Anisotropy as a signature of transverse collective flow #3

Jean-Yves Ollitrault (Saclay) (Mar 20, 1992)

Published in: *Phys.Rev.D* 46 (1992) 229-245

1,393 citations

Electron Ion Collider: The Next QCD Frontier : Understanding the glue that binds us all #4

A. Accardi (Jefferson Lab and Hampton U.), J.L. Albacete (Orsay, IPN), M. Anselmino (INFN, Turin and Turin U.), N. Armesto (Santiago de Compostela U.), E.C. Aschenauer (Brookhaven) et al. (Dec, 2012)

Published in: *Eur.Phys.J.A* 52 (2016) 9, 268 • e-Print: 1212.1701 [nucl-ex]

1,295 citations

Collective flow and viscosity in relativistic heavy-ion collisions #5

Ulrich Heinz (Ohio State U.), Raimond Snellings (Utrecht U.) (Jan, 2013)

Published in: *Ann.Rev.Nucl.Part.Sci.* 63 (2013) 123-151 • e-Print: 1301.2826 [nucl-th]

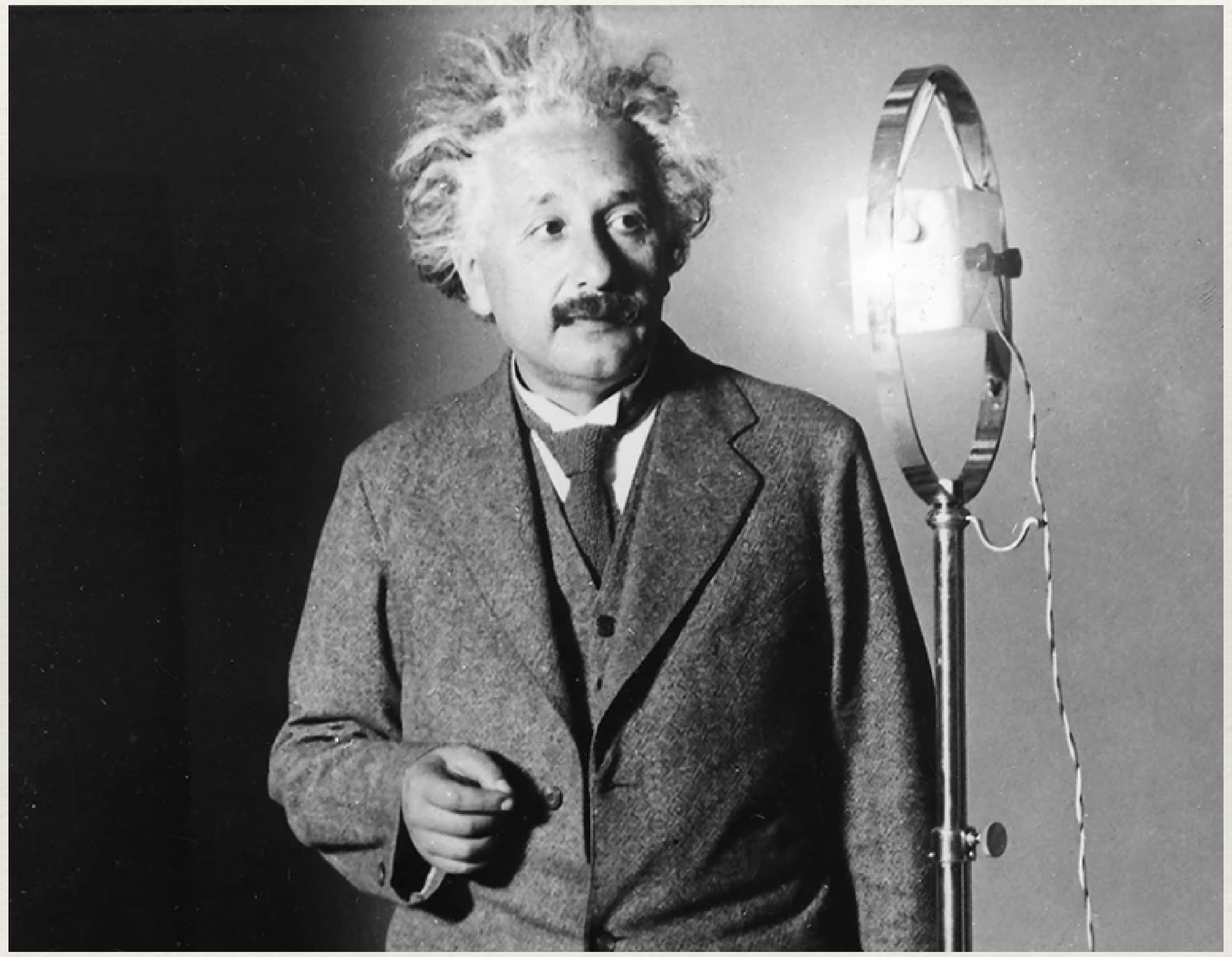
1,052 citations

Why hydro is so popular/works and how we use it in heavy ion collisions ?

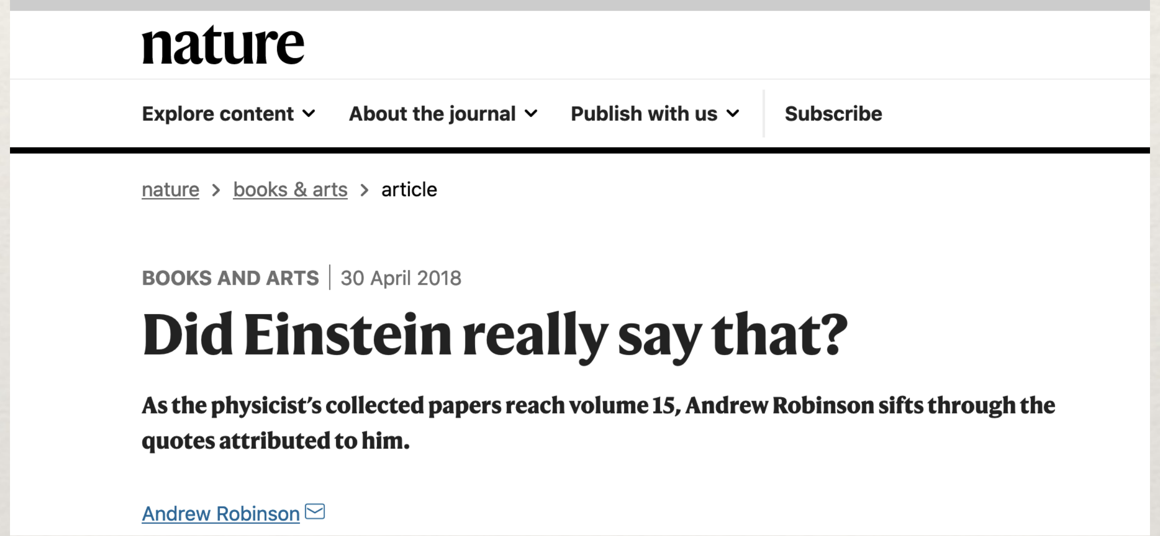
Why hydro is so popular?

One of the reason : it is a simple theory based on the fundamental conservation laws, worked excellently.

“Everything should be made as simple as possible, but no simpler”



https://www.nature.com/articles/d41586-018-05004-4



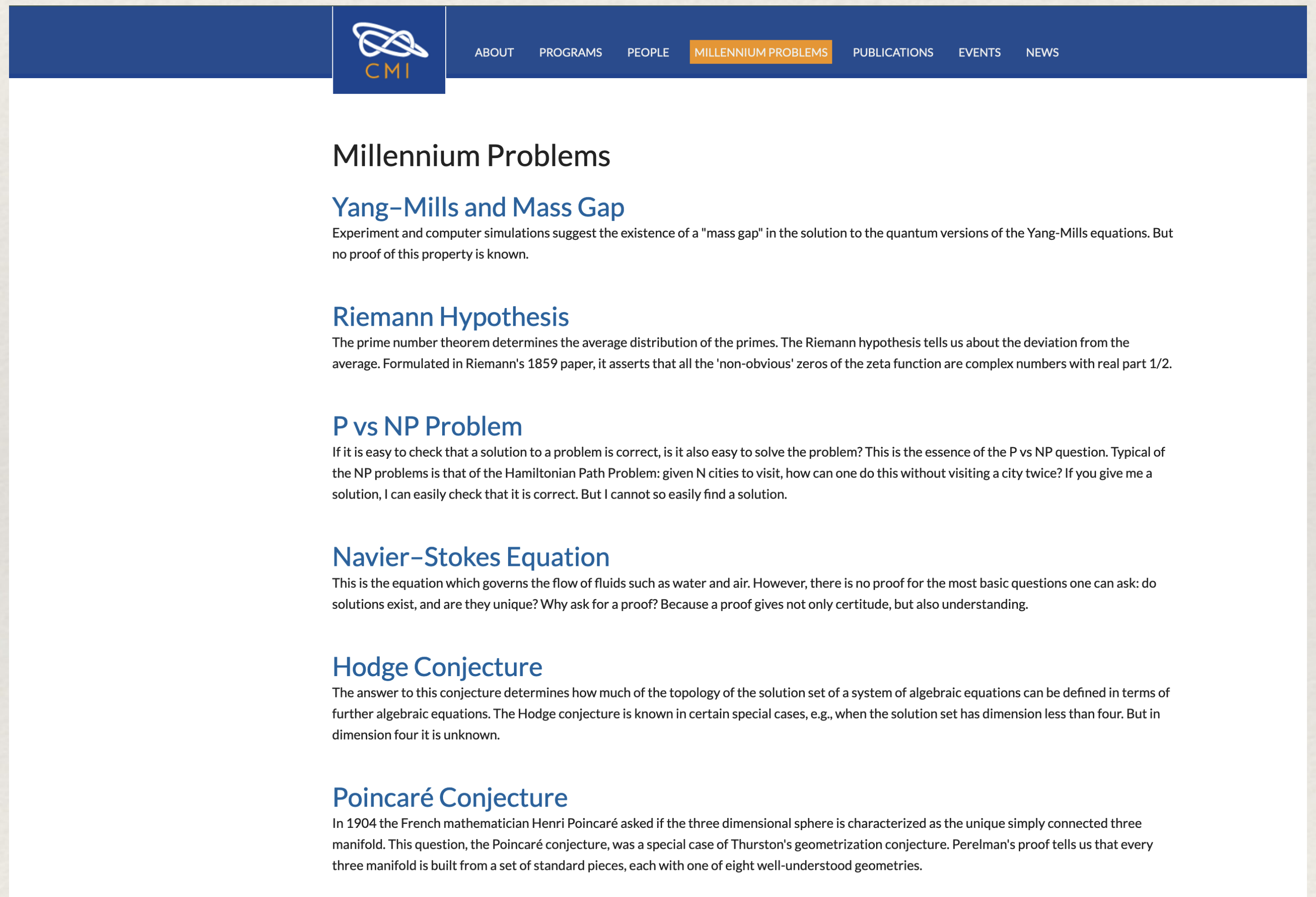
nature

BOOKS AND ARTS | 30 April 2018

Did Einstein really say that?

As the physicist’s collected papers reach volume 15, Andrew Robinson sifts through the quotes attributed to him.

Andrew Robinson



Millennium Problems

Yang–Mills and Mass Gap

Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

Riemann Hypothesis

The prime number theorem determines the average distribution of the primes. The Riemann hypothesis tells us about the deviation from the average. Formulated in Riemann's 1859 paper, it asserts that all the 'non-obvious' zeros of the zeta function are complex numbers with real part 1/2.

P vs NP Problem

If it is easy to check that a solution to a problem is correct, is it also easy to solve the problem? This is the essence of the P vs NP question. Typical of the NP problems is that of the Hamiltonian Path Problem: given N cities to visit, how can one do this without visiting a city twice? If you give me a solution, I can easily check that it is correct. But I cannot so easily find a solution.

Navier–Stokes Equation

This is the equation which governs the flow of fluids such as water and air. However, there is no proof for the most basic questions one can ask: do solutions exist, and are they unique? Why ask for a proof? Because a proof gives not only certitude, but also understanding.

Hodge Conjecture

The answer to this conjecture determines how much of the topology of the solution set of a system of algebraic equations can be defined in terms of further algebraic equations. The Hodge conjecture is known in certain special cases, e.g., when the solution set has dimension less than four. But in dimension four it is unknown.

Poincaré Conjecture

In 1904 the French mathematician Henri Poincaré asked if the three dimensional sphere is characterized as the unique simply connected three manifold. This question, the Poincaré conjecture, was a special case of Thurston's geometrization conjecture. Perelman's proof tells us that every three manifold is built from a set of standard pieces, each with one of eight well-understood geometries.

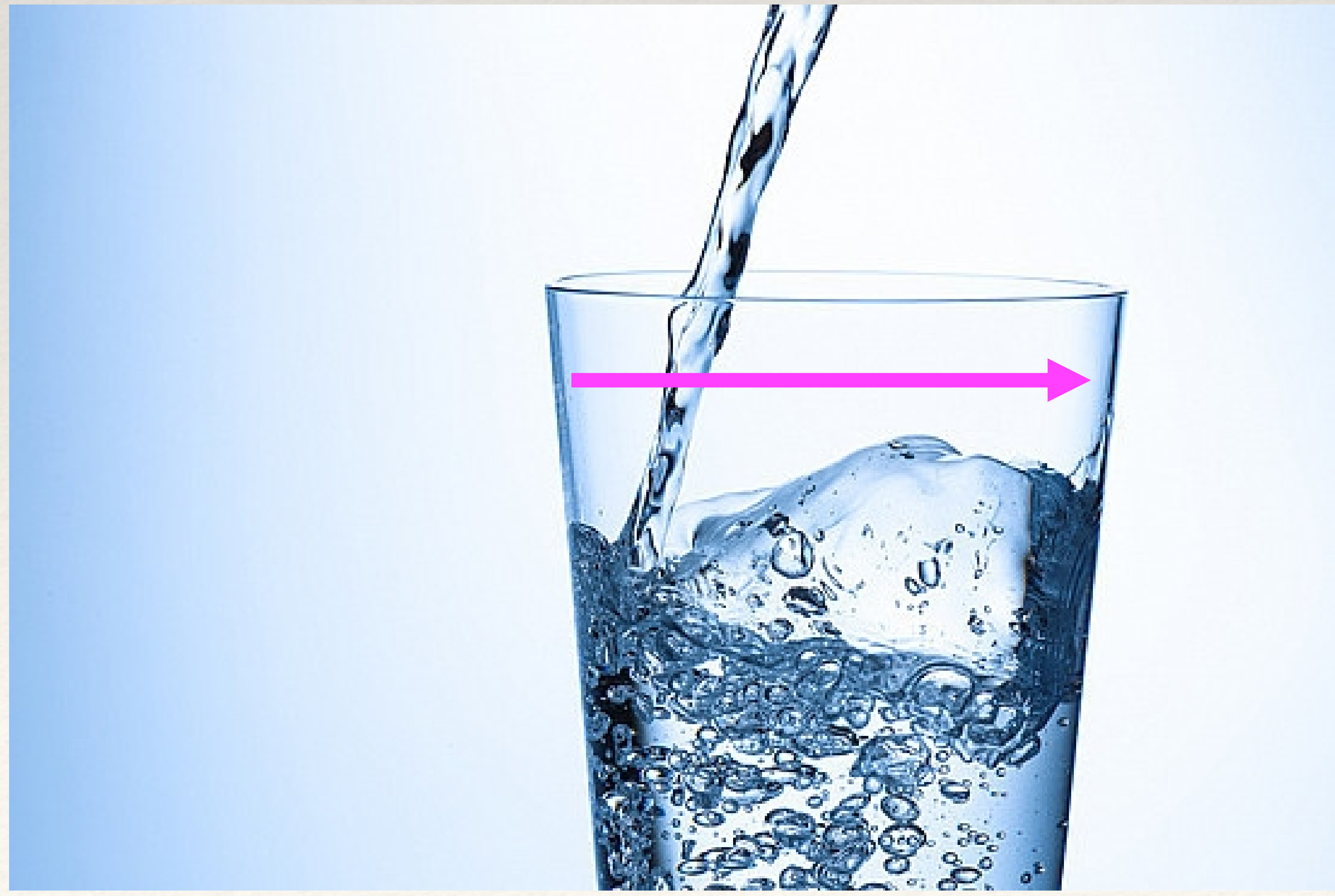
“The hardest thing in the world to understand is the income tax.”

The unusual effectiveness of hydro:1

One of the reason : it is a simple theory based on the fundamental conservation laws.

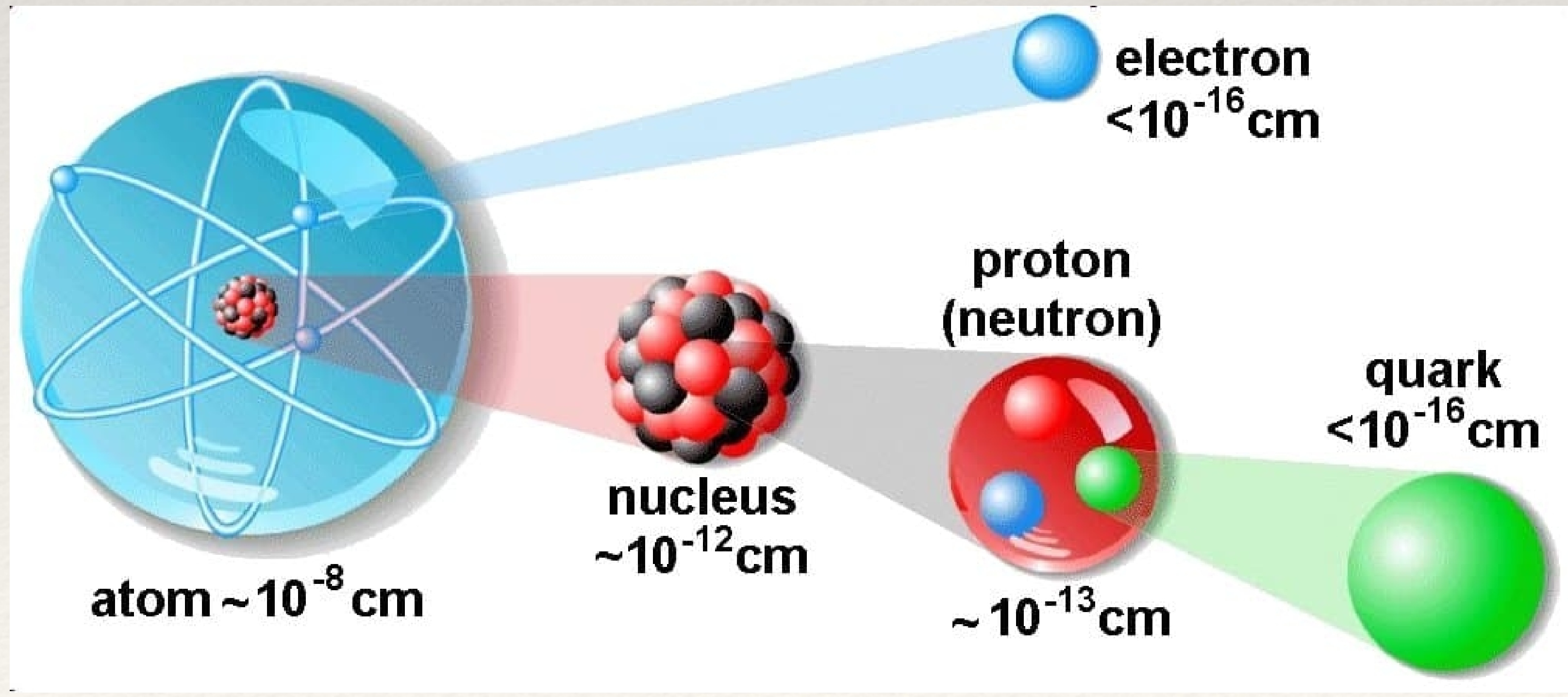
The other reason: the applicability of the frame work is based on the ratio of the two length scale and not the underlying interaction explicitly.

Ordinary fluid



Hydro should work when

Heavy Ion collisions



L ~ 1 m

L ~

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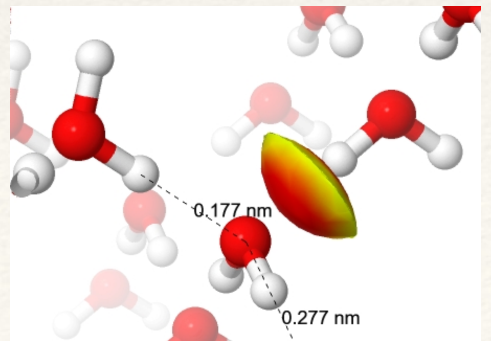
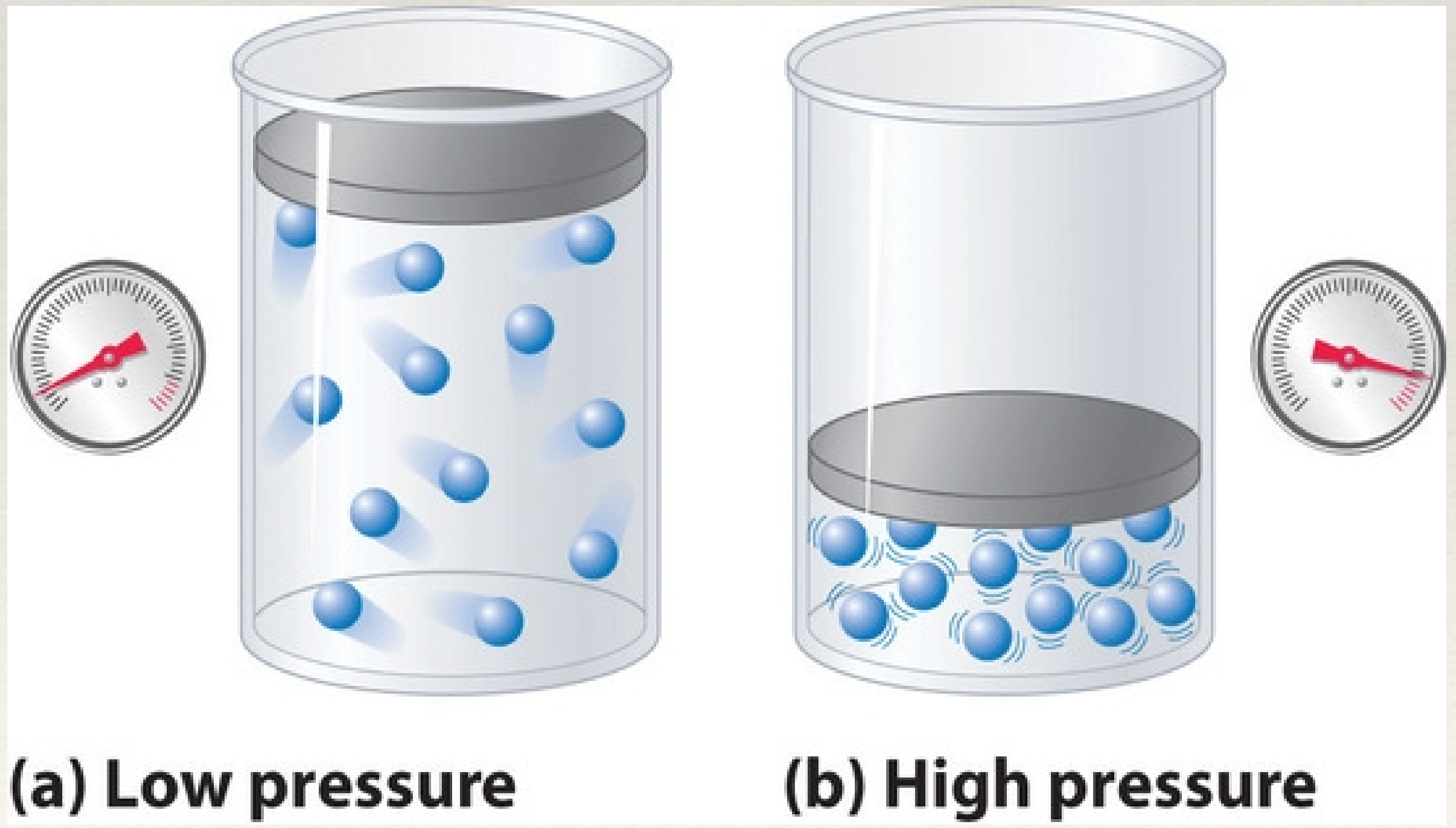


The unusual effectiveness of hydro:1

One of the reason : it is a simple theory based on the fundamental conservation laws.

The other reason: the applicability of the frame work is based on the ratio of the two length scale and not the underlying interaction explicitly.

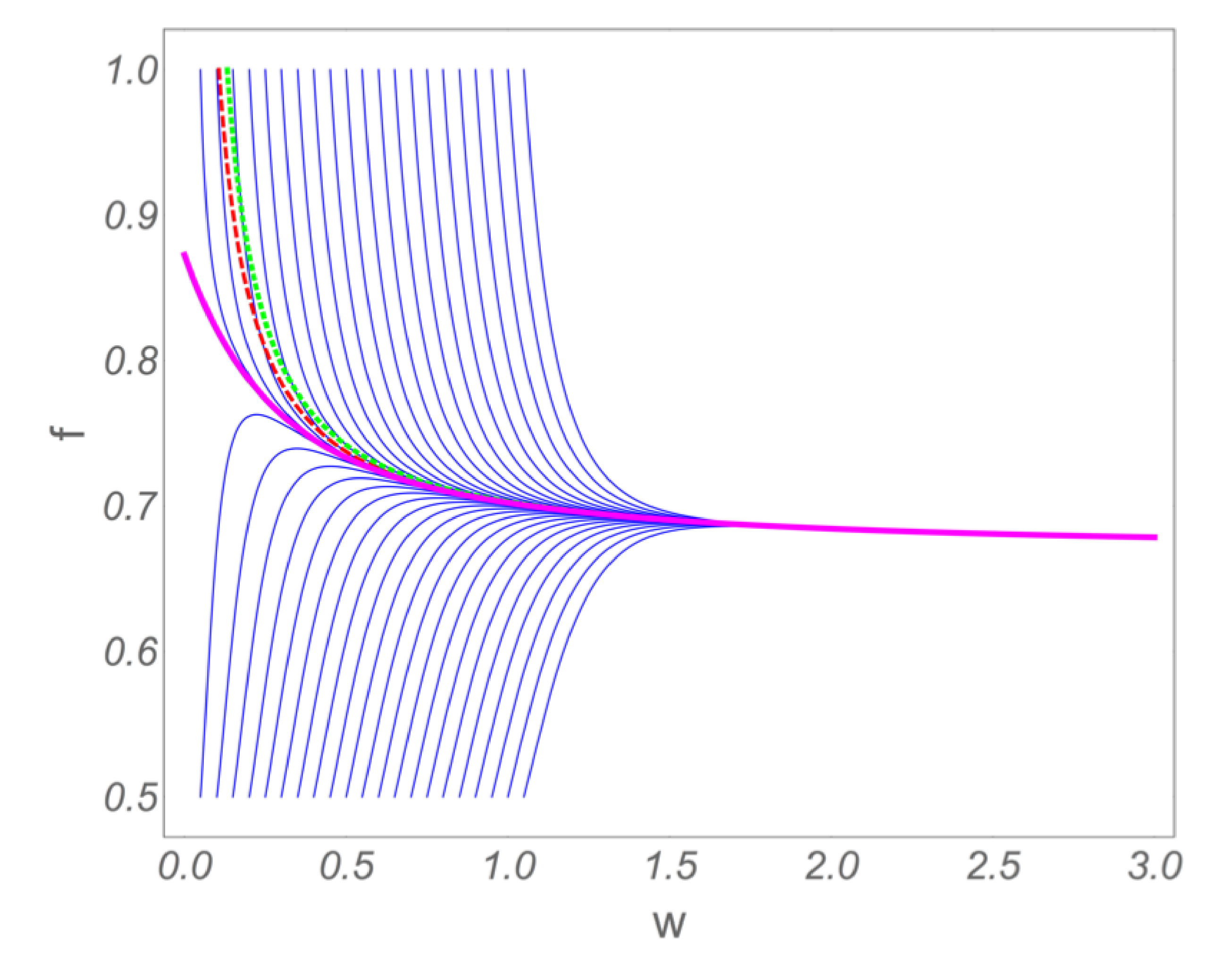
High density: in high energy heavy ion collisions the entropy density is quite high.



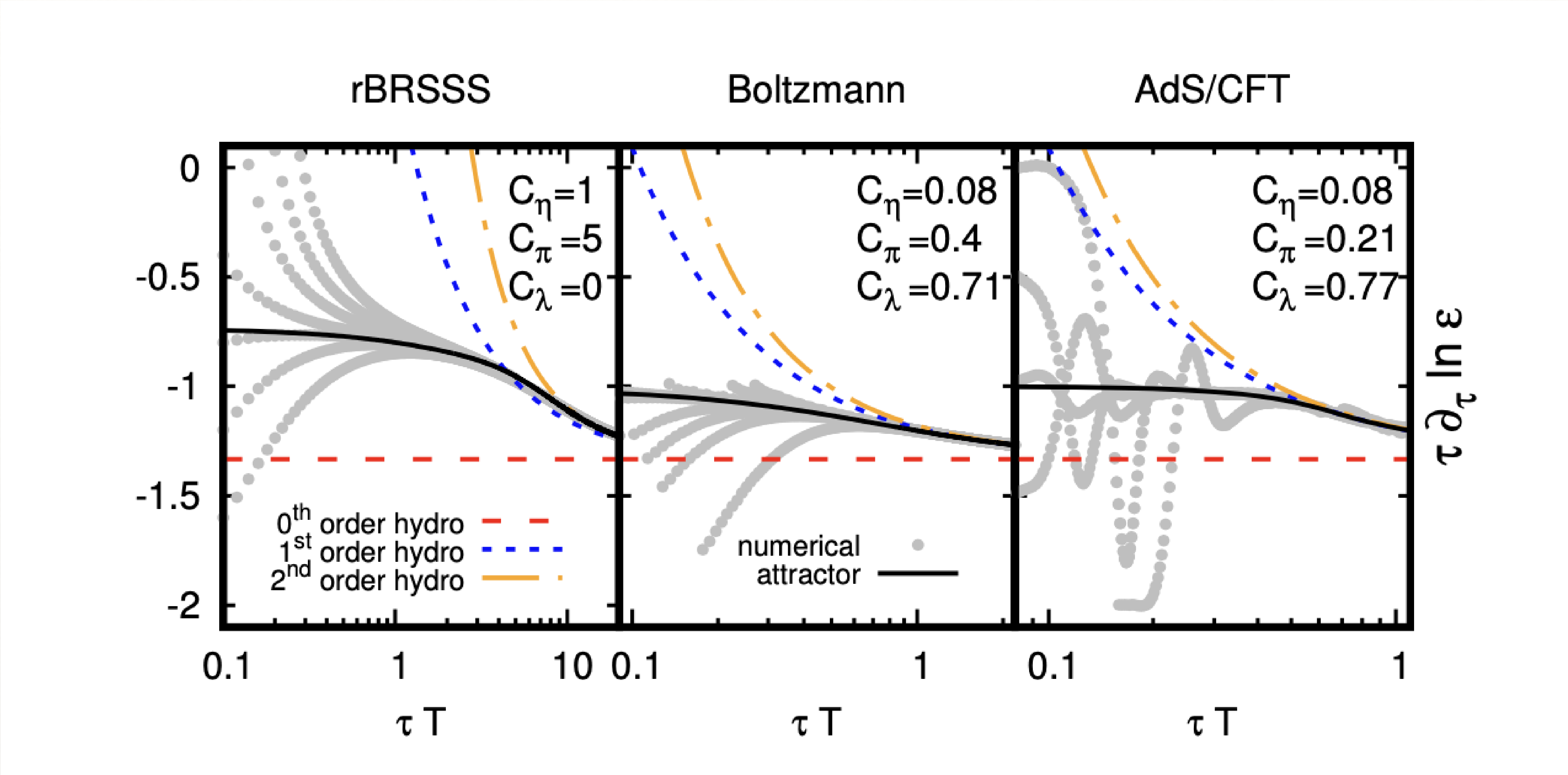
Deconfinement and long range force: both color and electromagnetic forces are at work.

The unusual effectiveness of hydro:2

Hydrodynamics attractor in 0+1 D Bjorken expansion



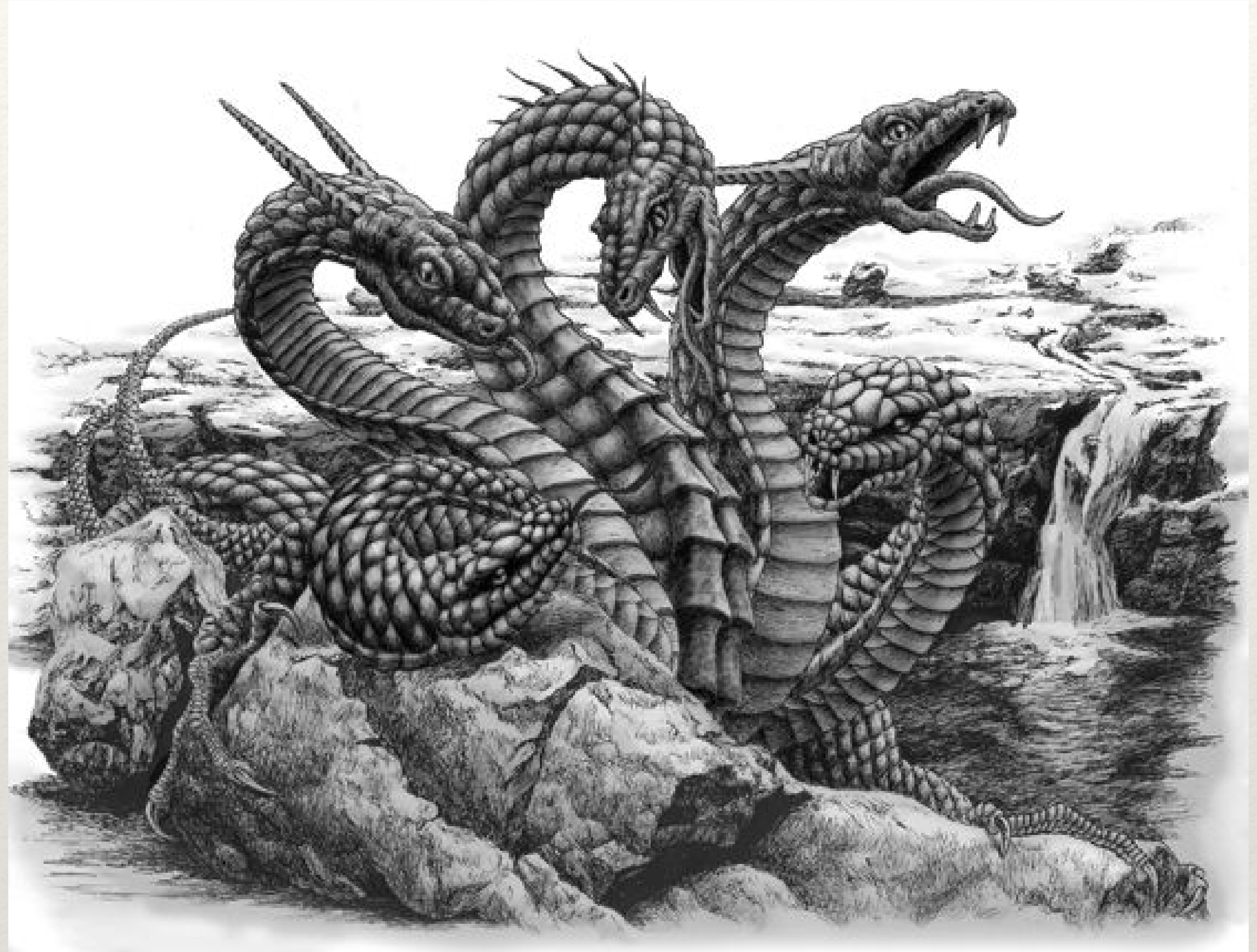
Hydrodynamics attractor in more general cases



Paul Romatschke Phys. Rev. Lett. 120, 012301

Many facets of hydrodynamics

- Non relativistic hydrodynamics
- Relativistic hydrodynamics
- Radiation hydrodynamics
- Magneto hydrodynamics
- Chiral hydrodynamics
- Spin hydrodynamics
- Superfluid hydrodynamics



1+1 D hydro - highly symmetric system.

1+2 D hydro - transverse expansion with azimuthal symmetry.

1+3 D hydro - no symmetry assumed.

Very few analytical solution exists.

No straight-forward way to deal with the causality in the relativistic version.

Greek Mythology: gigantic water-snake-like monster nine head (the number varies) one of which was immortal.

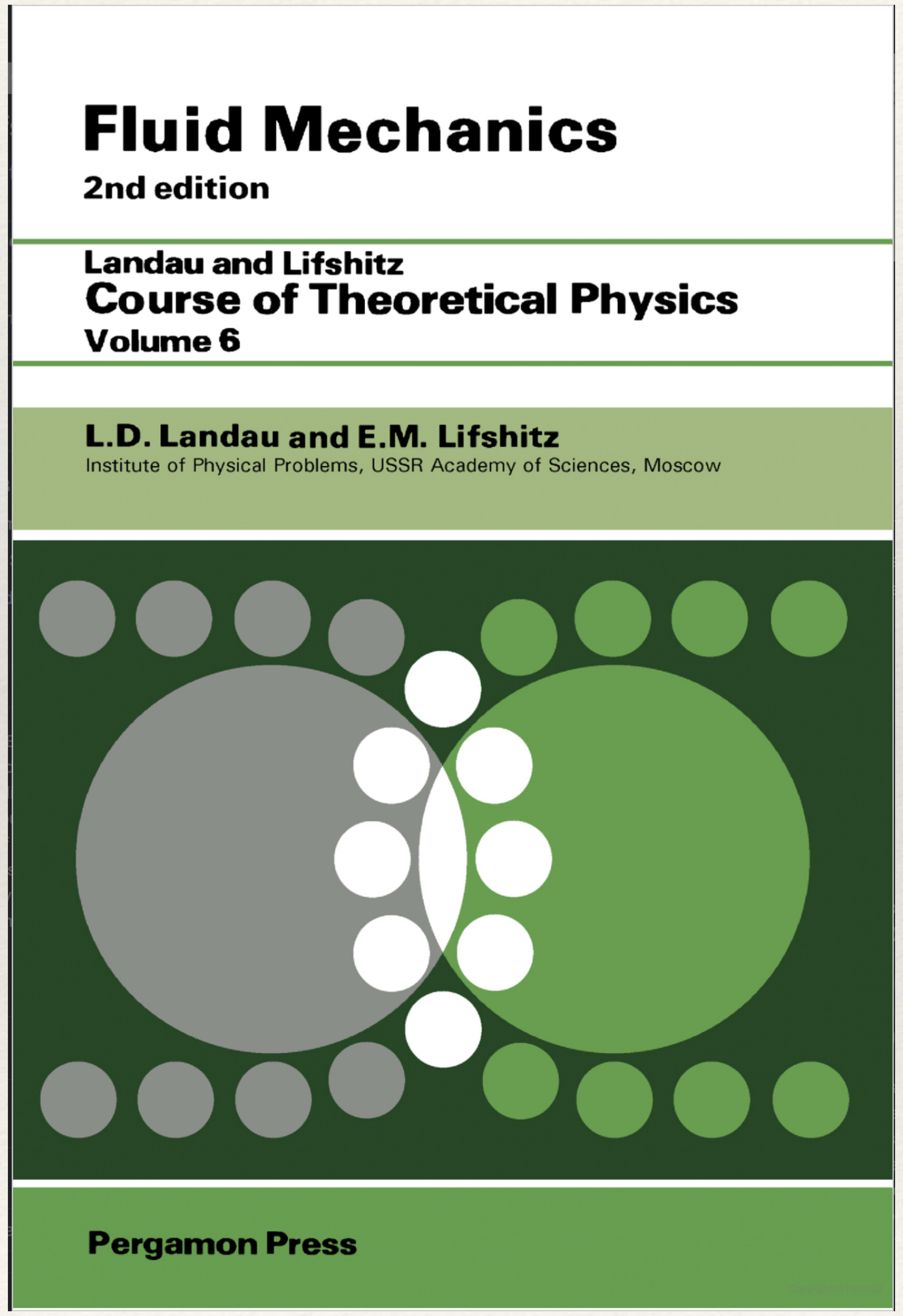
Plan of the lecture

- Non-relativistic hydrodynamics
  - -Ideal
  - -Viscous
- Short introduction to the relativistic hydrodynamics
- Hydrodynamics in heavy ion collisions
  - -numerical hydrodynamics
  - -connection to the experimental observables
  - -details of the hydro simulation (how to)

Text book definition of hydrodynamics

Non-relativistic classical description

Fluid Mechanics- Landau-Lifshitz

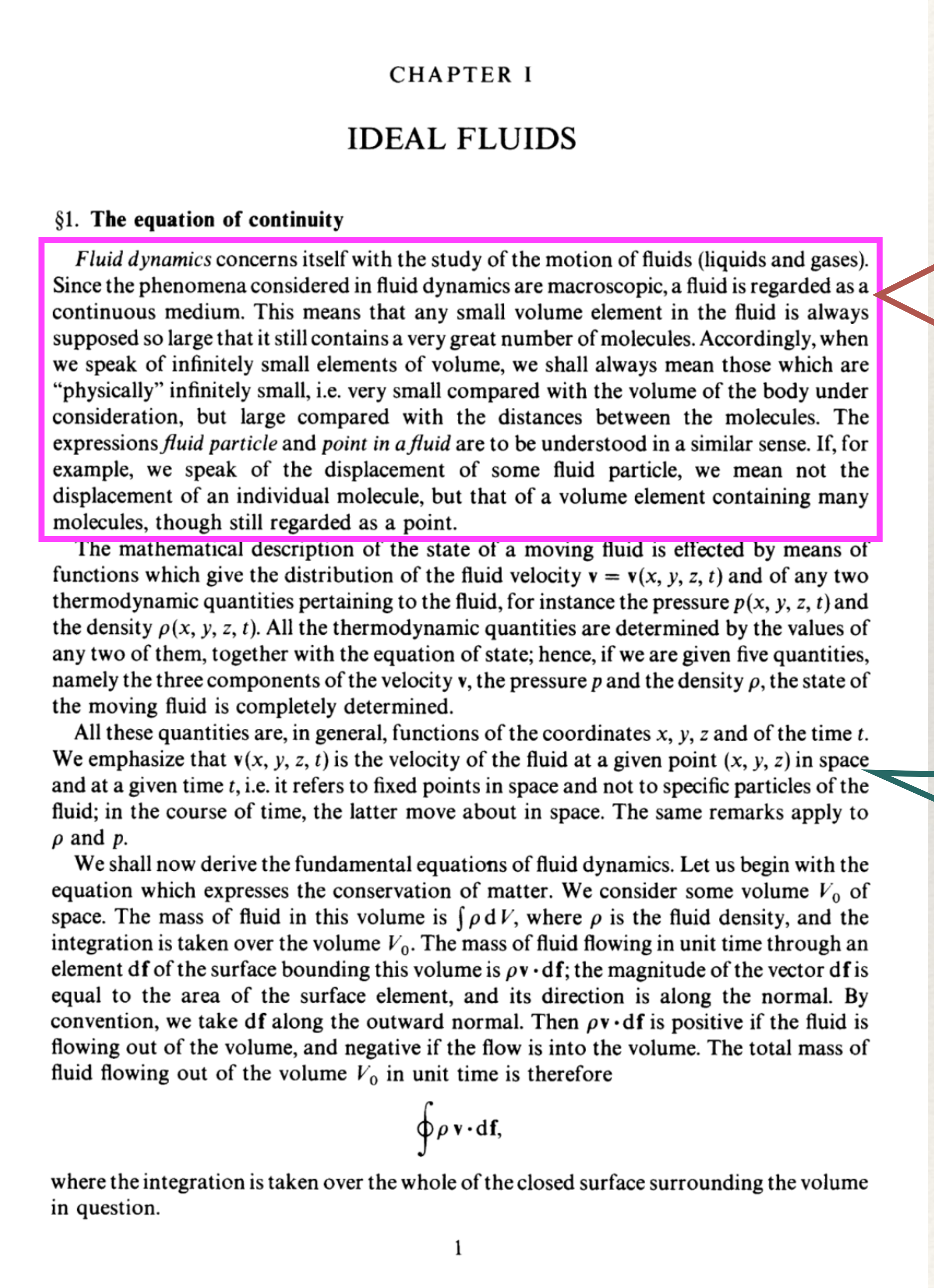


Fluid Mechanics
2nd edition

Landau and Lifshitz
Course of Theoretical Physics
Volume 6

L.D. Landau and E.M. Lifshitz
Institute of Physical Problems, USSR Academy of Sciences, Moscow

Pergamon Press



CHAPTER I

IDEAL FLUIDS

§1. The equation of continuity

Fluid dynamics concerns itself with the study of the motion of fluids (liquids and gases). Since the phenomena considered in fluid dynamics are macroscopic, a fluid is regarded as a continuous medium. This means that any small volume element in the fluid is always supposed so large that it still contains a very great number of molecules. Accordingly, when we speak of infinitely small elements of volume, we shall always mean those which are "physically" infinitely small, i.e. very small compared with the volume of the body under consideration, but large compared with the distances between the molecules. The expressions *fluid particle* and *point in a fluid* are to be understood in a similar sense. If, for example, we speak of the displacement of some fluid particle, we mean not the displacement of an individual molecule, but that of a volume element containing many molecules, though still regarded as a point.

The mathematical description of the state of a moving fluid is effected by means of functions which give the distribution of the fluid velocity $\mathbf{v} = \mathbf{v}(x, y, z, t)$ and of any two thermodynamic quantities pertaining to the fluid, for instance the pressure $p(x, y, z, t)$ and the density $\rho(x, y, z, t)$. All the thermodynamic quantities are determined by the values of any two of them, together with the equation of state; hence, if we are given five quantities, namely the three components of the velocity $\mathbf{v}$, the pressure p and the density ρ, the state of the moving fluid is completely determined.

All these quantities are, in general, functions of the coordinates x, y, z and of the time t. We emphasize that $\mathbf{v}(x, y, z, t)$ is the velocity of the fluid at a given point (x, y, z) in space and at a given time t, i.e. it refers to fixed points in space and not to specific particles of the fluid; in the course of time, the latter move about in space. The same remarks apply to ρ and p.

We shall now derive the fundamental equations of fluid dynamics. Let us begin with the equation which expresses the conservation of matter. We consider some volume V_0 of space. The mass of fluid in this volume is $\int \rho \, \mathrm{d}V$, where ρ is the fluid density, and the integration is taken over the volume V_0. The mass of fluid flowing in unit time through an element $\mathrm{d}\mathbf{f}$ of the surface bounding this volume is $\rho \mathbf{v} \cdot \mathrm{d}\mathbf{f}$; the magnitude of the vector $\mathrm{d}\mathbf{f}$ is equal to the area of the surface element, and its direction is along the normal. By convention, we take $\mathrm{d}\mathbf{f}$ along the outward normal. Then $\rho \mathbf{v} \cdot \mathrm{d}\mathbf{f}$ is positive if the fluid is flowing out of the volume, and negative if the flow is into the volume. The total mass of fluid flowing out of the volume V_0 in unit time is therefore

$$\oint \rho \mathbf{v} \cdot \mathrm{d}\mathbf{f},$$

where the integration is taken over the whole of the closed surface surrounding the volume in question.

- Fluid dynamics concerns itself with the study of the motion of the fluid (liquid & gas).
- Any small volume of fluid contains large number of molecules (Knudsen number).
- When we speak of the displacement of some fluid element (or particle) it does not mean the displacement of individual molecule.

- State of a fluid is given by dynamic and thermodynamic variables

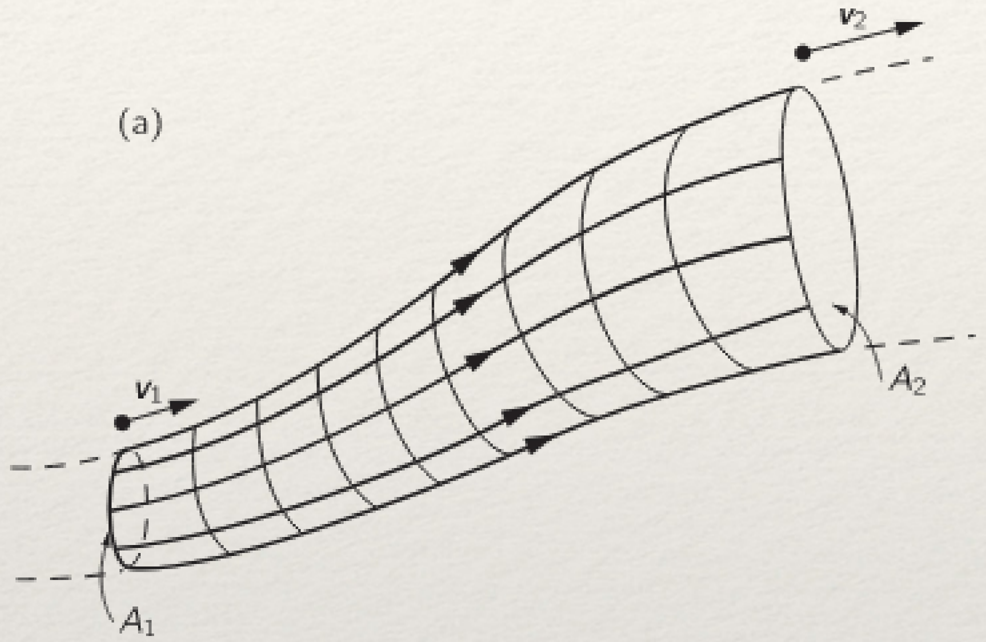
  Velocity field:

  Pressure field:

  Mass density:

Non-relativistic formulations: co-moving derivative

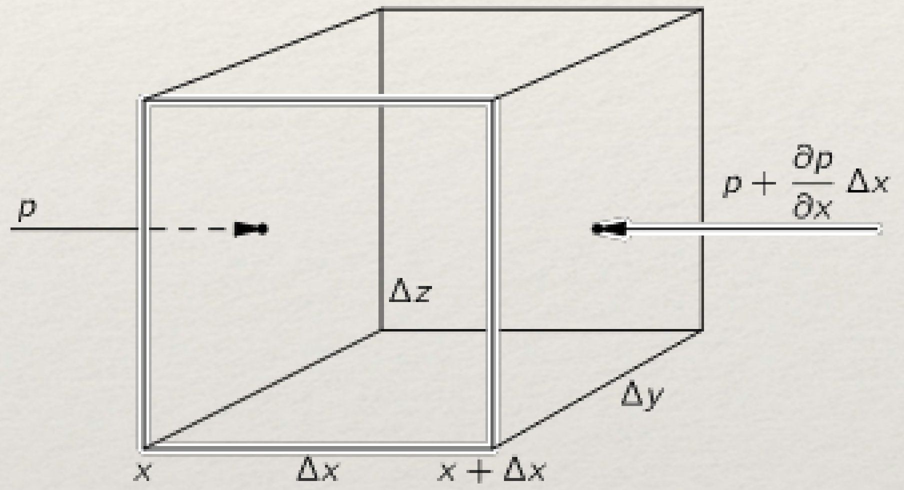
The change to field picture introduces new concepts of co-moving derivative



Material derivative/co-moving derivative:

Non-relativistic formulations: Euler equation

The Newton's law for fluid element



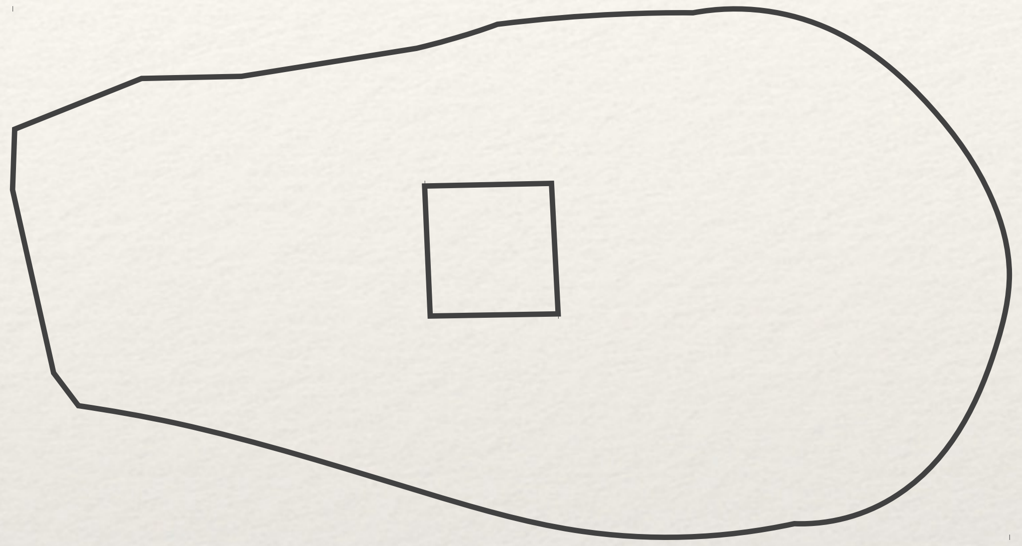
Force per unit volume =Force/(area*length) =pressure/length

Euler equation for ideal fluid

Source of non-linearity

Non-relativistic formulations: Mass conservation

What about the density?



Change in mass in the elemental volume = flow of mass through the surface

With source :

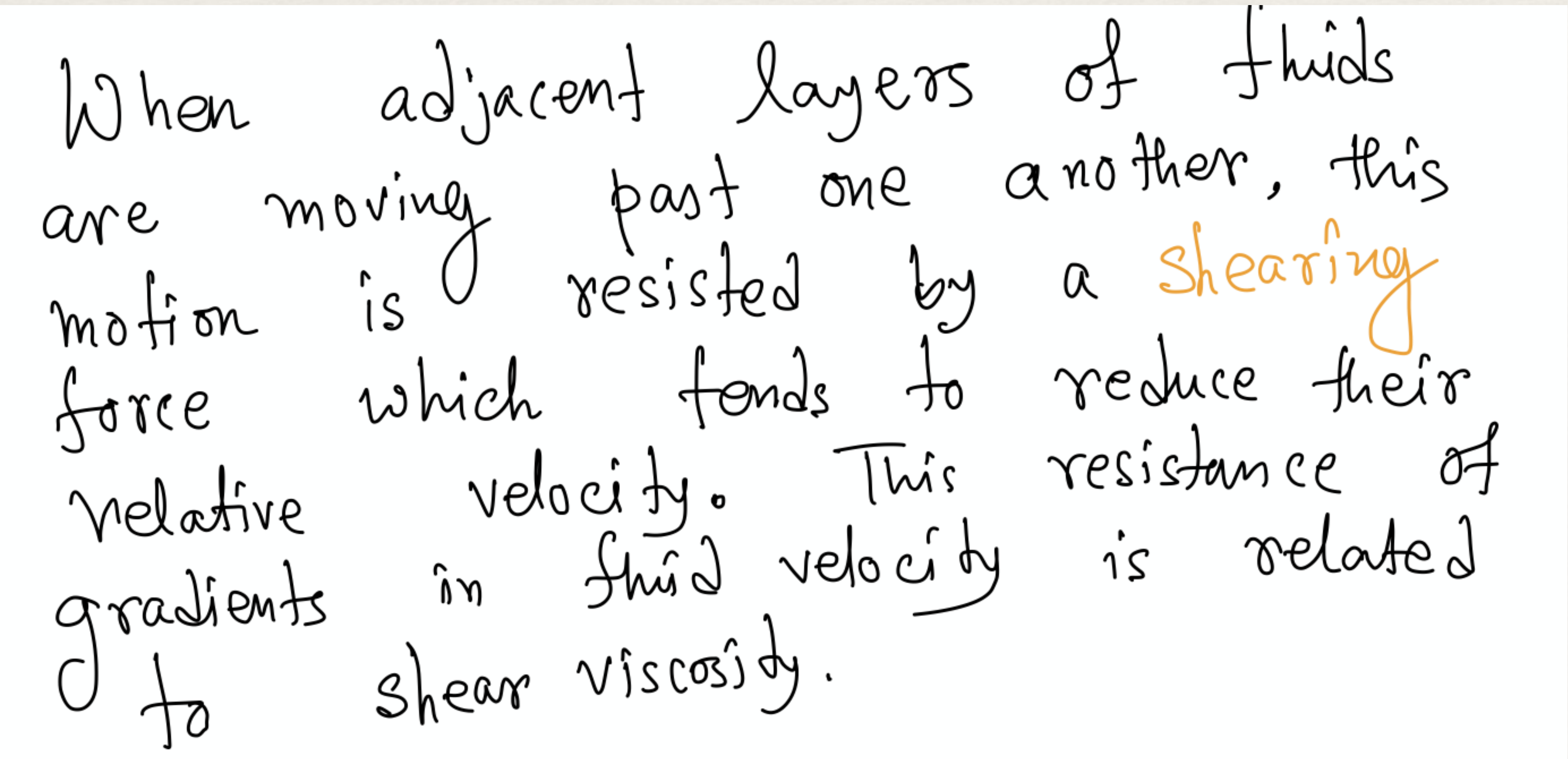
Change in mass in the elemental volume = flow of mass through the surface + contribution from the source

Non-relativistic formulations:Viscosity 1

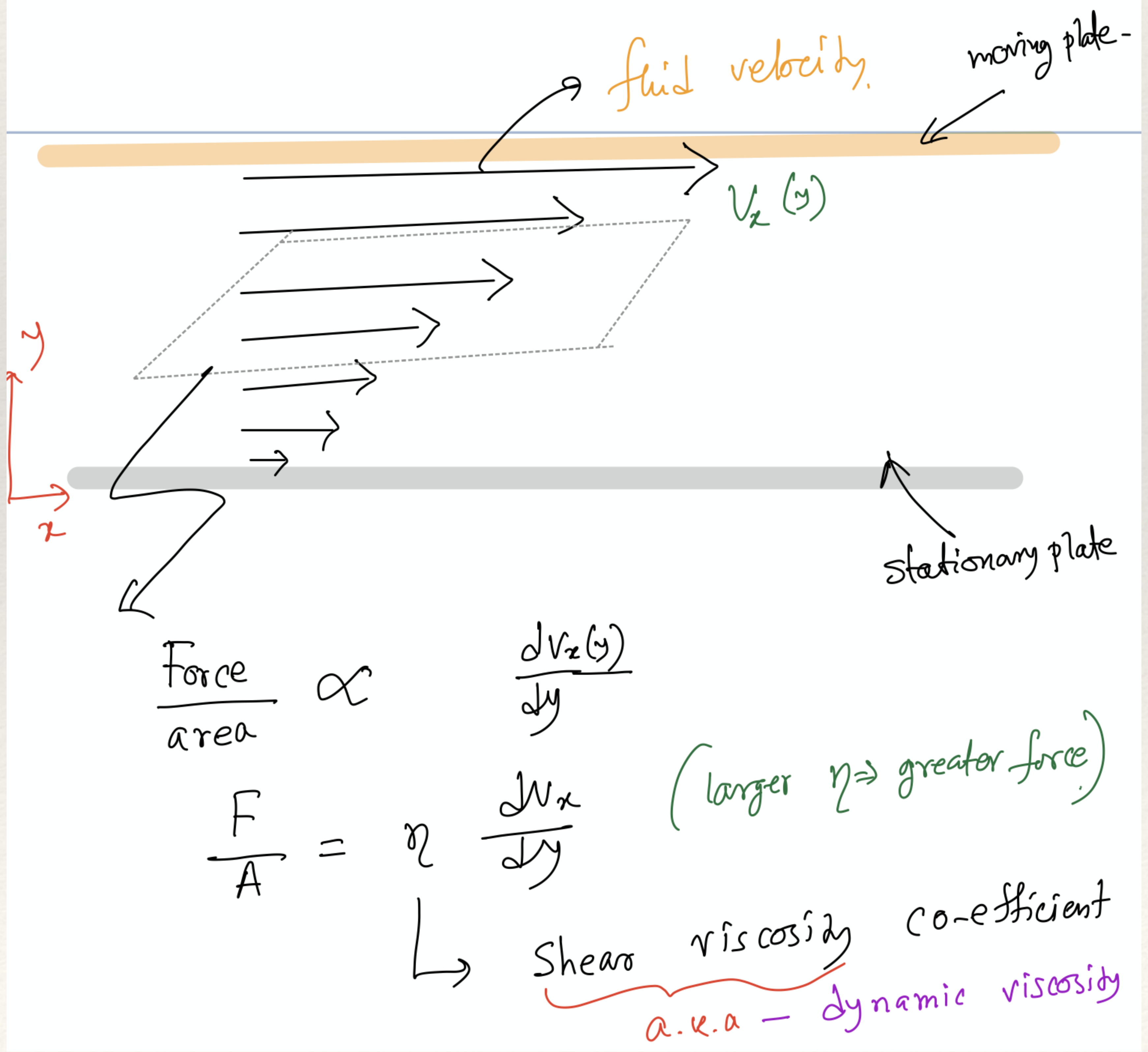
At finite temperature almost all real fluid resist any relative motion: analogous to friction in solids

More specifically non-equilibrium processes inside fluid is associated to thermodynamic irreversibility

Irreversibility results in energy dissipation in fluid. viscosity (shear & bulk) , thermal conduction.



When adjacent layers of fluids are moving past one another, this motion is resisted by a shearing force which tends to reduce their relative velocity. This resistance of gradients in fluid velocity is related to shear viscosity.



$$\frac{\text{Force}}{\text{area}} \propto \frac{dv_z(y)}{dy}$$

$$\frac{F}{A} = \eta \frac{dV_x}{dy}$$

(larger $\eta \Rightarrow$ greater force)

↳ Shear viscosity co-efficient

a.k.a — dynamic viscosity

is usually a function of temperature

Non-relativistic formulations:Viscosity 2

How does the dissipative terms contribute to the fluid equation fo motion?

[Force/volume] the viscous force per unit area F/A=

To match the dimension we can take the following term F/V=

Almost correct

If we consider a rigid body like rotation of the fluid —> there should be no viscous forces between the adjacent layers

Simple application of hydro equation

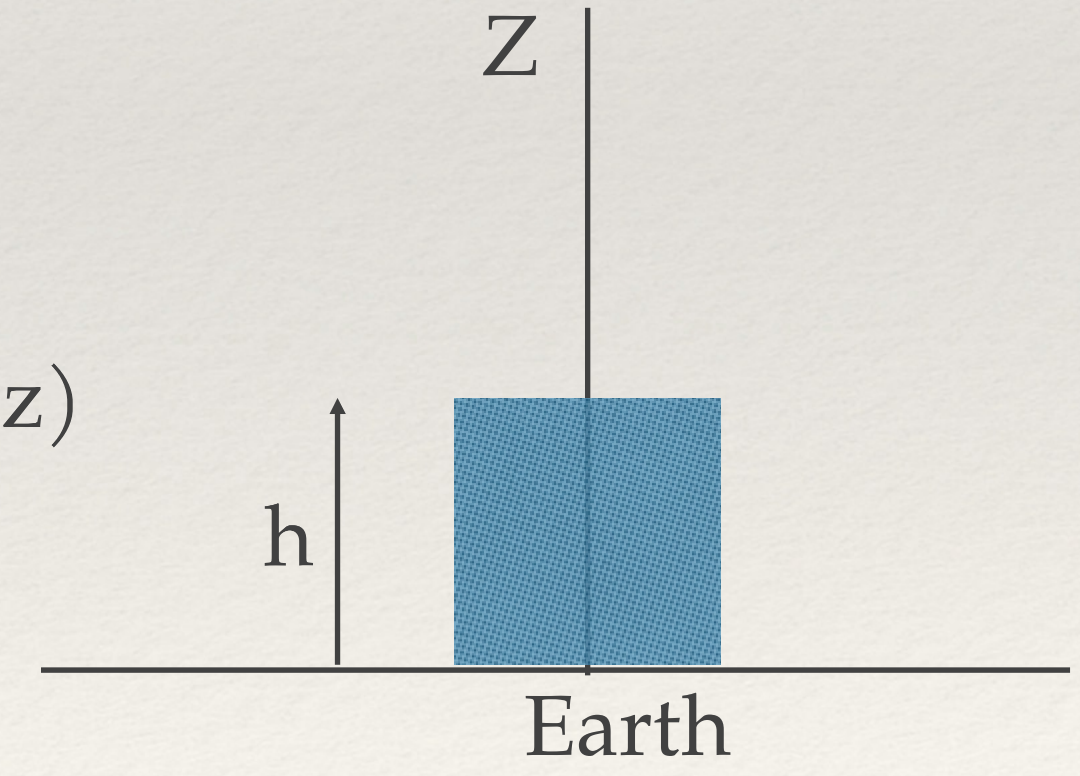
Consider the following simple situation

Fluid is in an uniform gravitational field and at rest —> v(x,y,z,t)=0

We also consider a non-viscous fluid

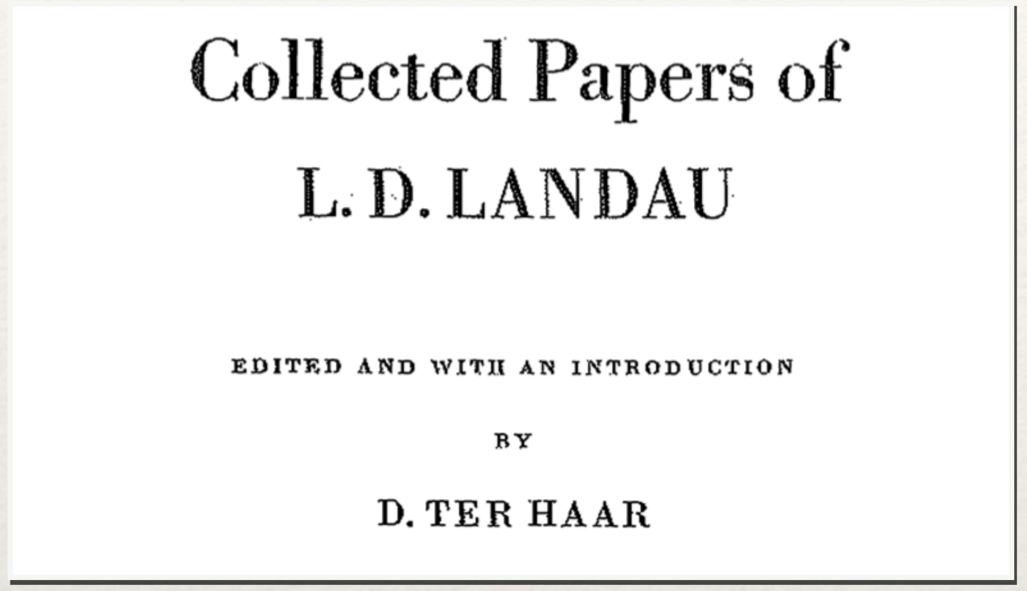
Euler equation

(-ve sign corresponds to the fact ‘g’ pointed towards -z)



When the fluid is in motion and viscosity is non-zero, the problem becomes much more harder !

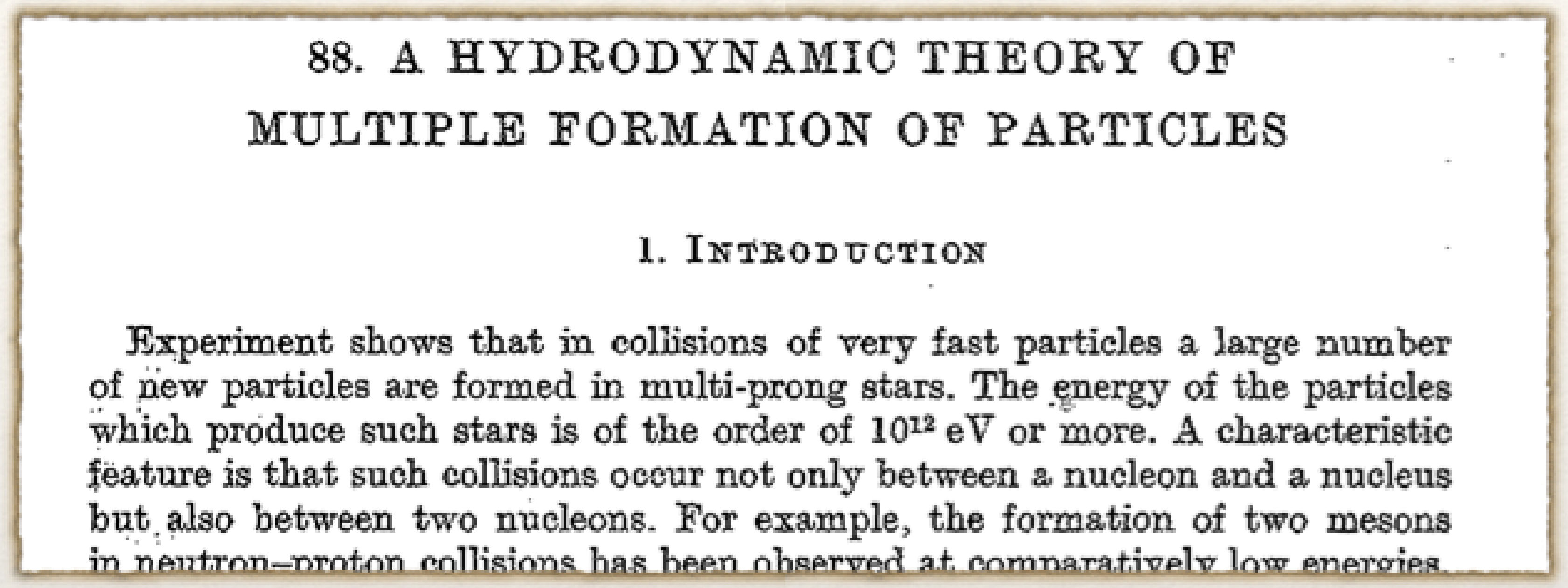
What is the connection to heavy-ion collisions



Collected Papers of
L. D. LANDAU

EDITED AND WITH AN INTRODUCTION
BY
D. TER HAAR

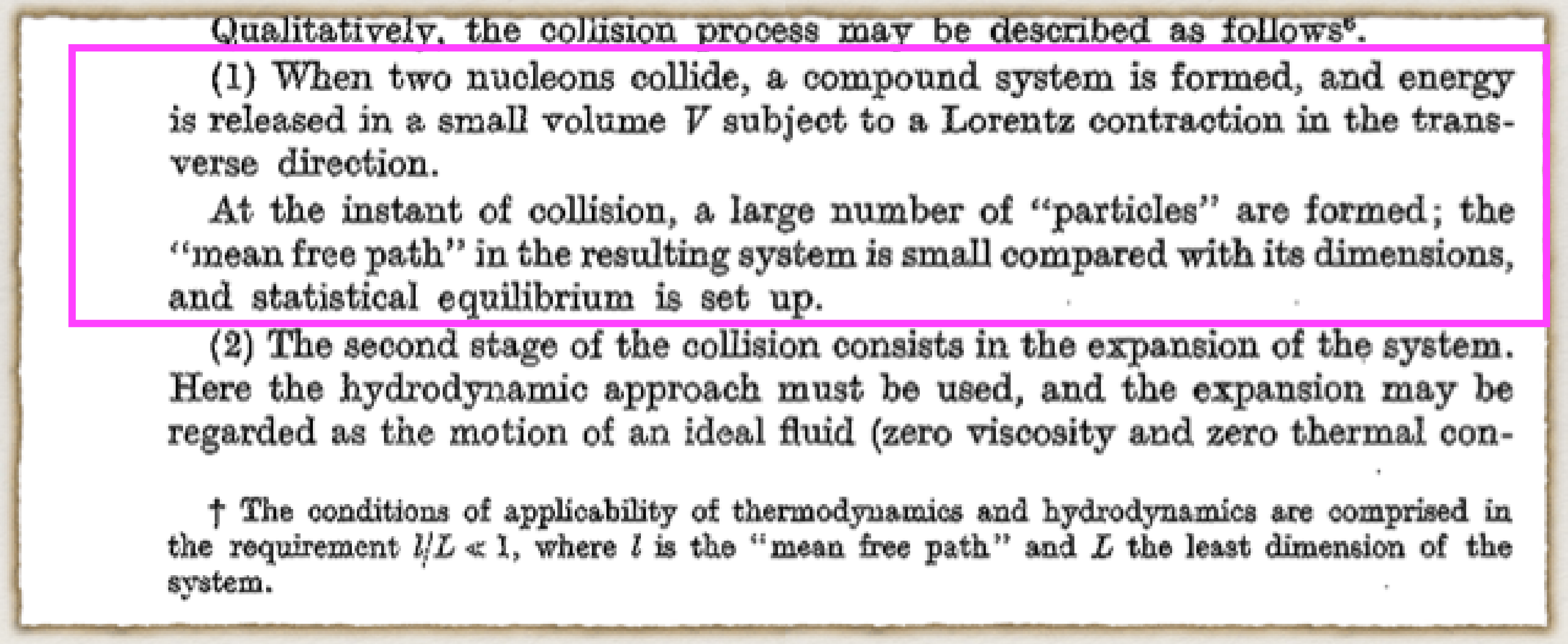
Lev Landau



88. A HYDRODYNAMIC THEORY OF MULTIPLE FORMATION OF PARTICLES

1. INTRODUCTION

Experiment shows that in collisions of very fast particles a large number of new particles are formed in multi-prong stars. The energy of the particles which produce such stars is of the order of 10^{12} eV or more. A characteristic feature is that such collisions occur not only between a nucleon and a nucleus but also between two nucleons. For example, the formation of two mesons in neutron–proton collisions has been observed at comparatively low energies.



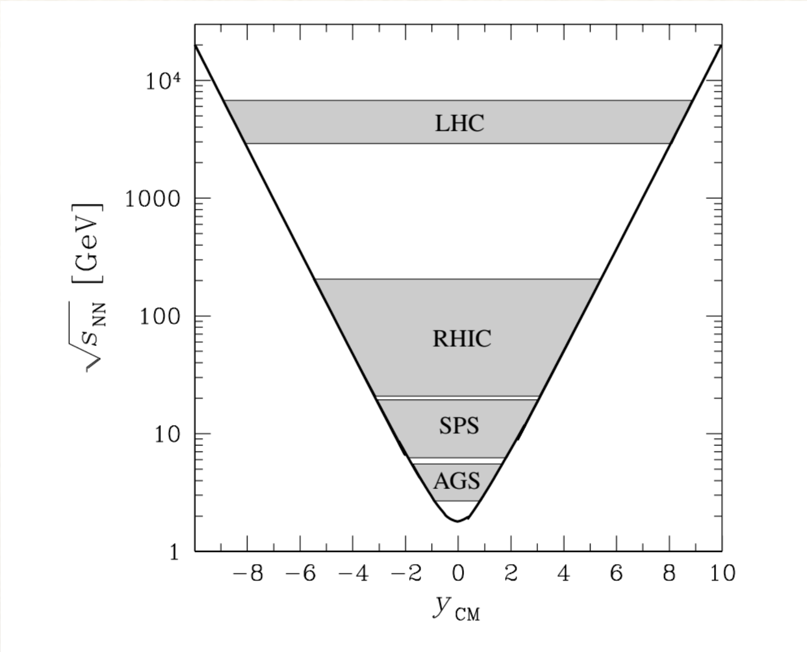
Qualitatively, the collision process may be described as follows[6].

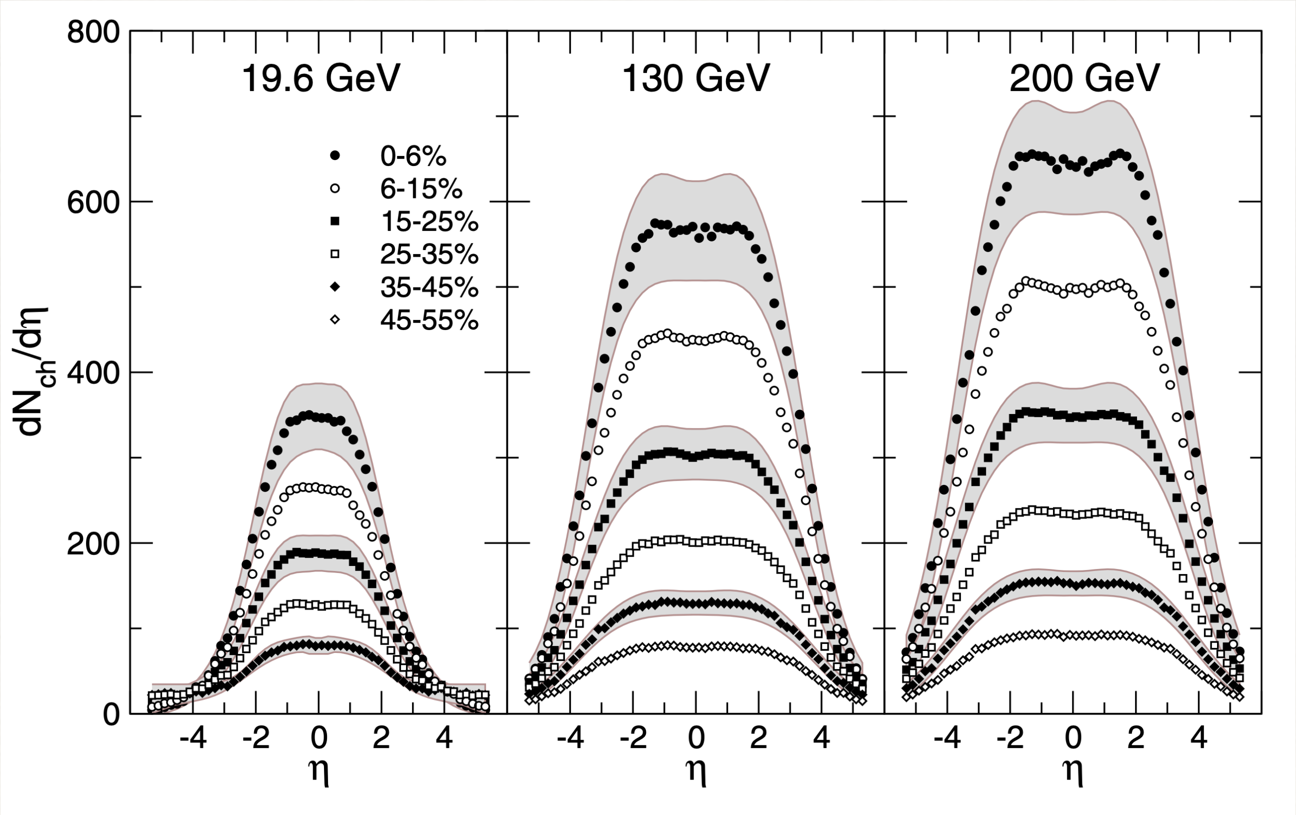
(1) When two nucleons collide, a compound system is formed, and energy is released in a small volume V subject to a Lorentz contraction in the transverse direction.

At the instant of collision, a large number of "particles" are formed; the "mean free path" in the resulting system is small compared with its dimensions, and statistical equilibrium is set up.

(2) The second stage of the collision consists in the expansion of the system. Here the hydrodynamic approach must be used, and the expansion may be regarded as the motion of an ideal fluid (zero viscosity and zero thermal con-

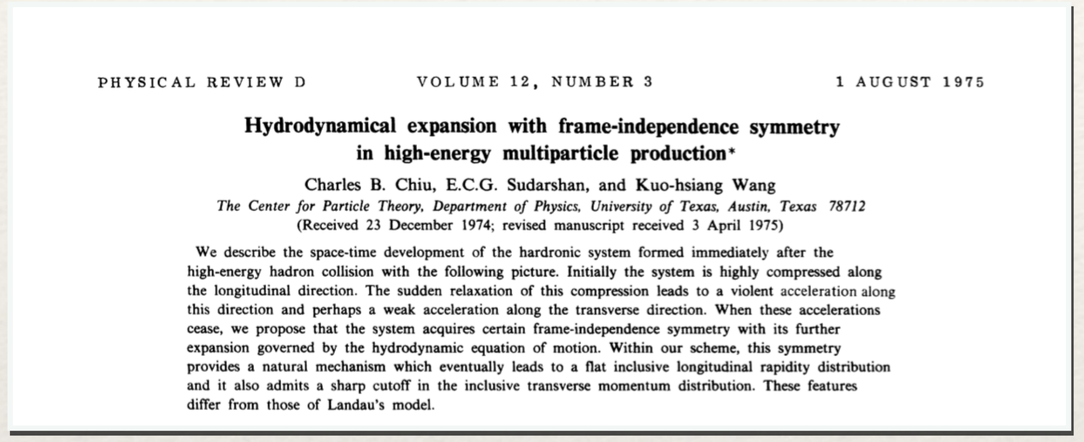
† The conditions of applicability of thermodynamics and hydrodynamics are comprised in the requirement $l/L \ll 1$, where l is the "mean free path" and L the least dimension of the system.





What is the connection to heavy-ion collisions

Initial entropy density is related to the final multiplicity



PHYSICAL REVIEW D VOLUME 12, NUMBER 3 1 AUGUST 1975

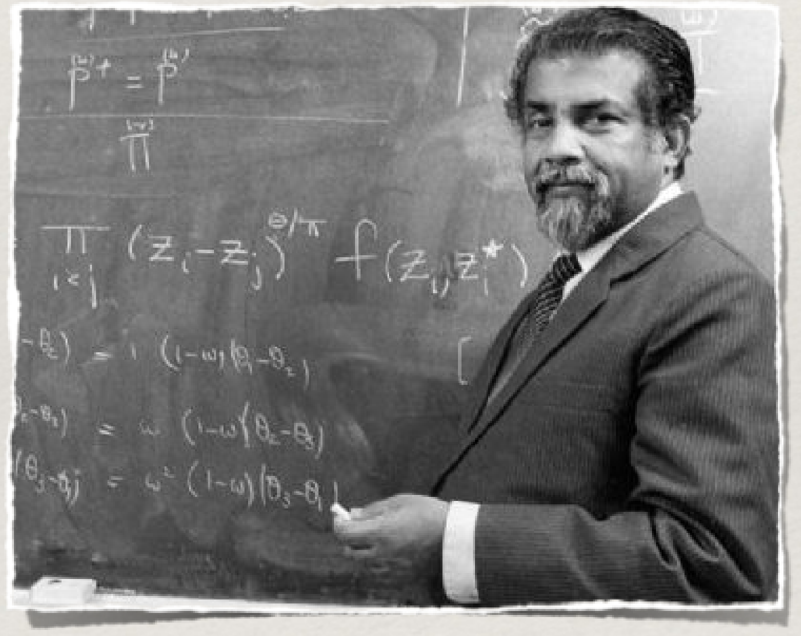
Hydrodynamical expansion with frame-independence symmetry in high-energy multiparticle production*

Charles B. Chiu, E.C.G. Sudarshan, and Kuo-hsiang Wang

The Center for Particle Theory, Department of Physics, University of Texas, Austin, Texas 78712

(Received 23 December 1974; revised manuscript received 3 April 1975)

We describe the space-time development of the hardronic system formed immediately after the high-energy hadron collision with the following picture. Initially the system is highly compressed along the longitudinal direction. The sudden relaxation of this compression leads to a violent acceleration along this direction and perhaps a weak acceleration along the transverse direction. When these accelerations cease, we propose that the system acquires certain frame-independence symmetry with its further expansion governed by the hydrodynamic equation of motion. Within our scheme, this symmetry provides a natural mechanism which eventually leads to a flat inclusive longitudinal rapidity distribution and it also admits a sharp cutoff in the inclusive transverse momentum distribution. These features differ from those of Landau's model.



ECG Sudarshan

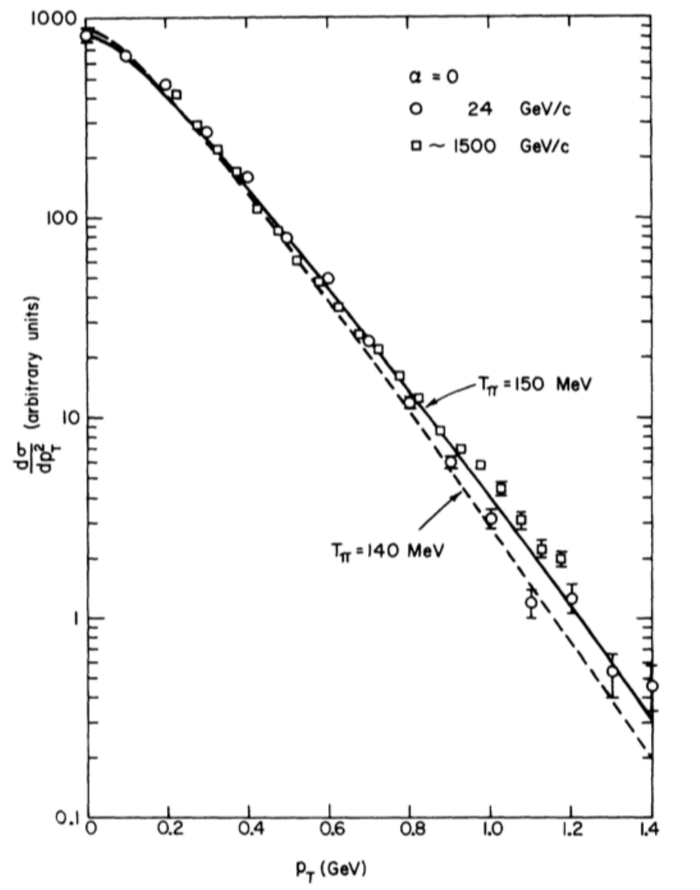
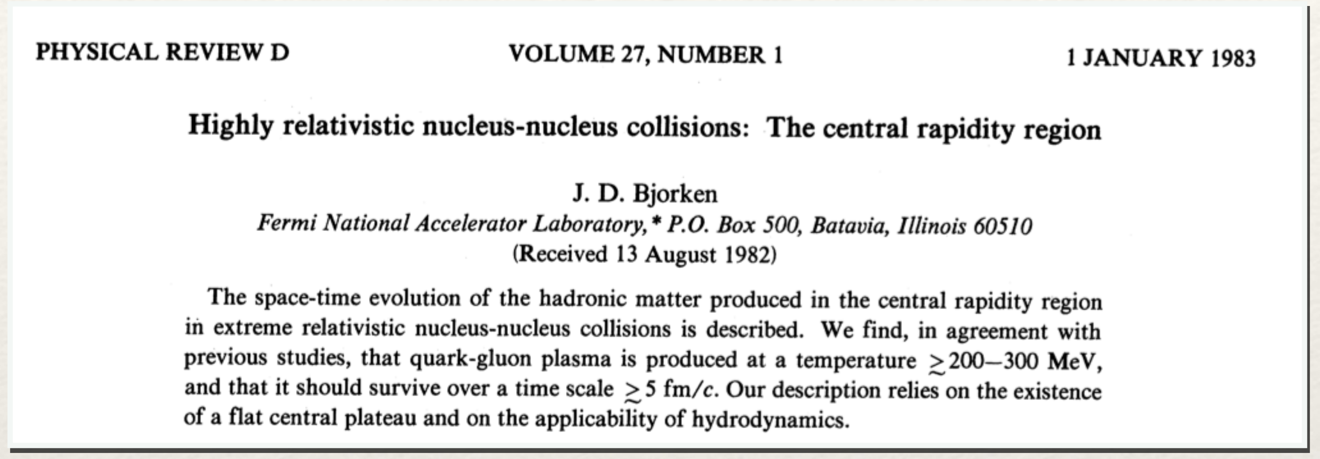


FIG. 1. Pion inclusive transverse momentum distribution at $\alpha = 0$. The theoretical curves are predictions of Eq. (26), with $T_\pi = 140$ and 150 MeV. For the data see Ref. 21.

Pion transverse momentum spectra



PHYSICAL REVIEW D VOLUME 27, NUMBER 1 1 JANUARY 1983

Highly relativistic nucleus-nucleus collisions: The central rapidity region

J. D. Bjorken

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510*

(Received 13 August 1982)

The space-time evolution of the hadronic matter produced in the central rapidity region in extreme relativistic nucleus-nucleus collisions is described. We find, in agreement with previous studies, that quark-gluon plasma is produced at a temperature $\gtrsim 200$–300 MeV, and that it should survive over a time scale $\gtrsim 5$ fm/c. Our description relies on the existence of a flat central plateau and on the applicability of hydrodynamics.

J. D. Bjorken

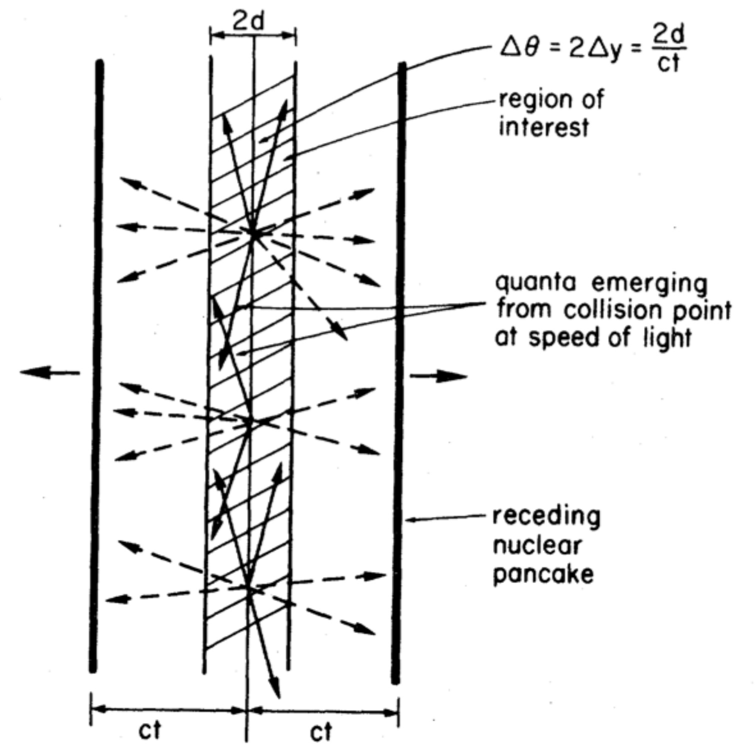
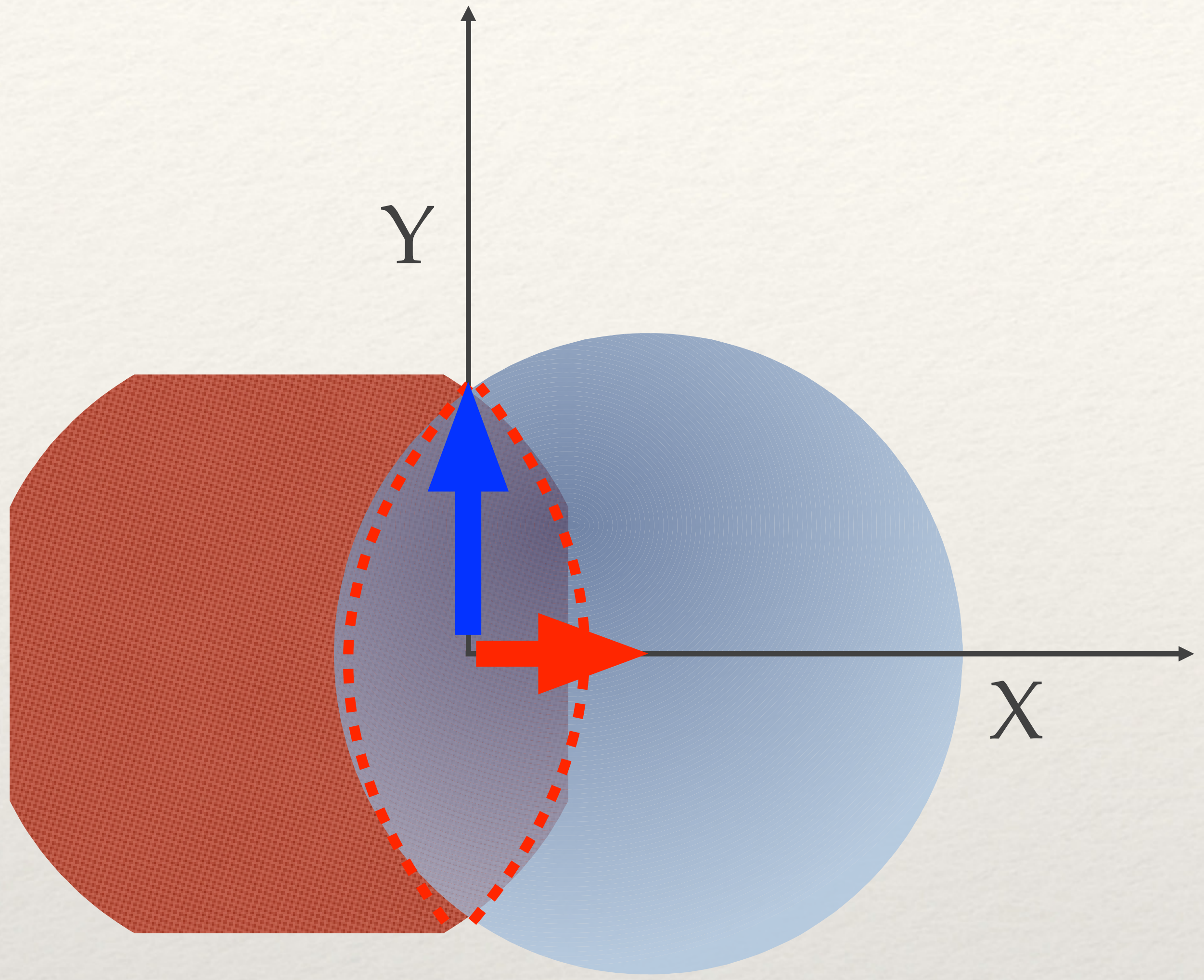
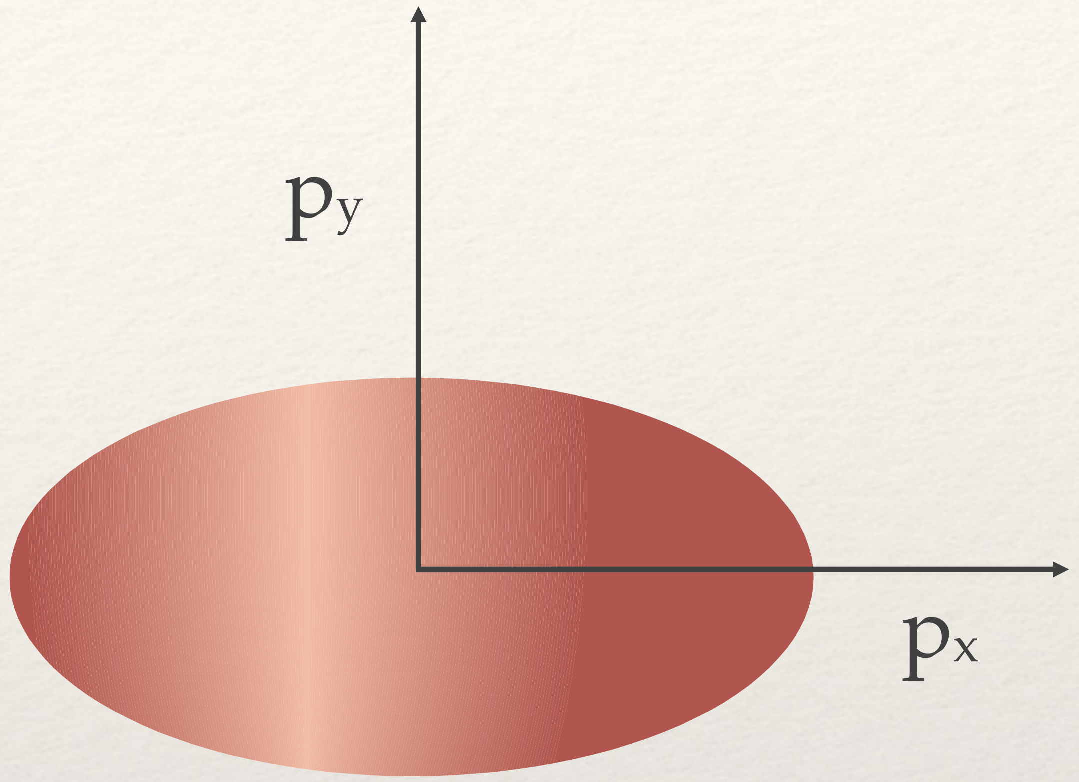


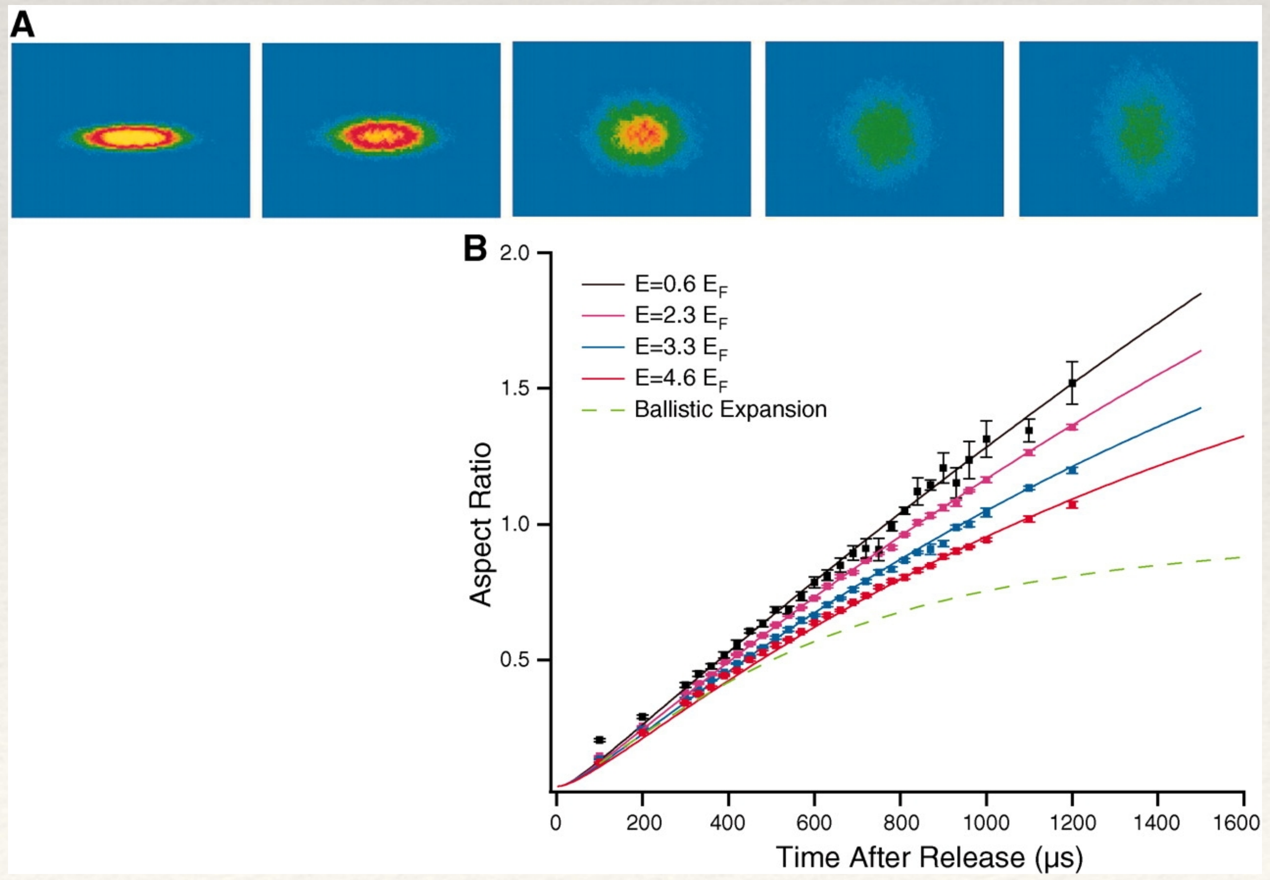
FIG. 2. Geometry for the initial state of centrally produced plasma in nucleus-nucleus collisions.

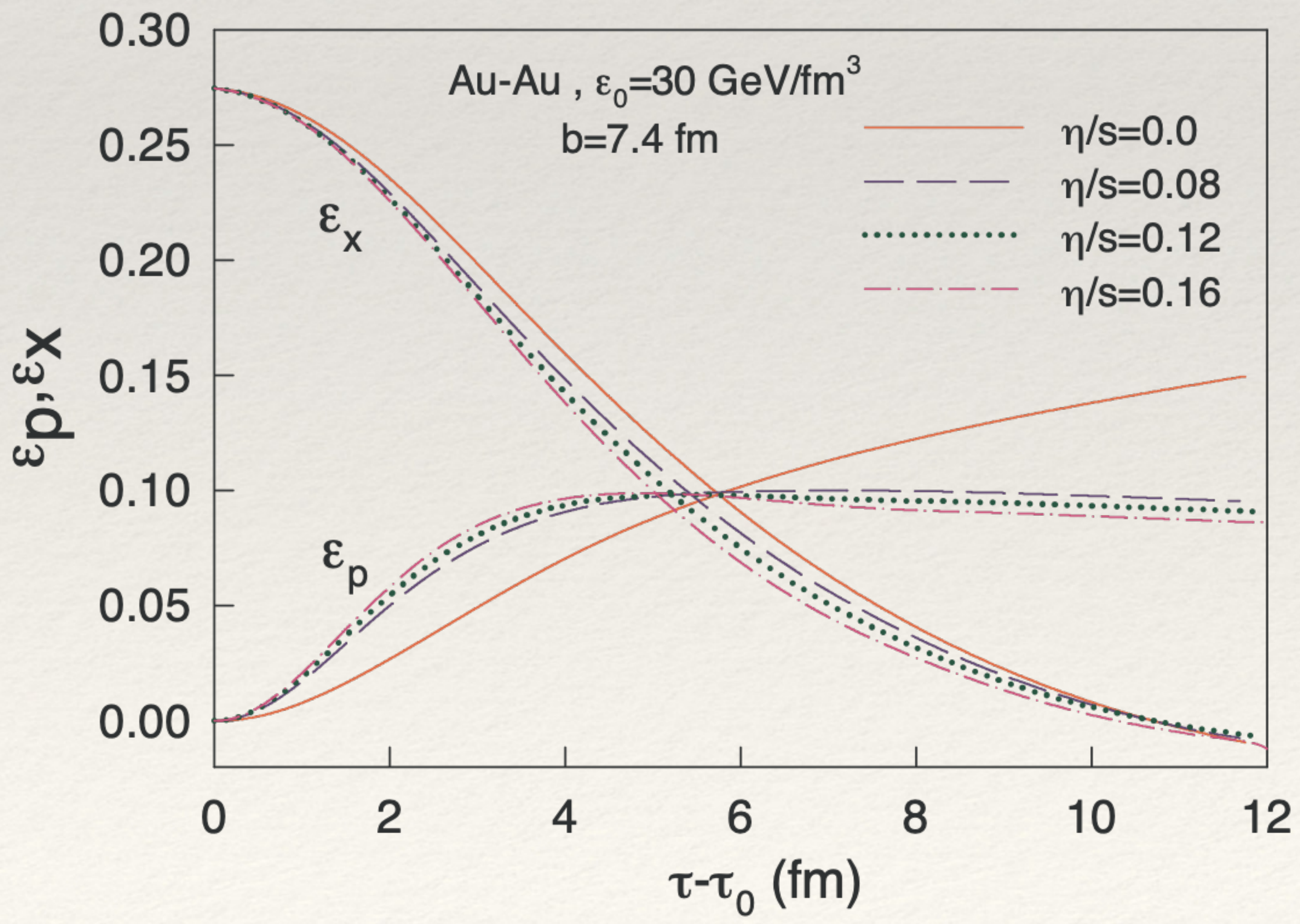
What is the connection to heavy-ion collisions

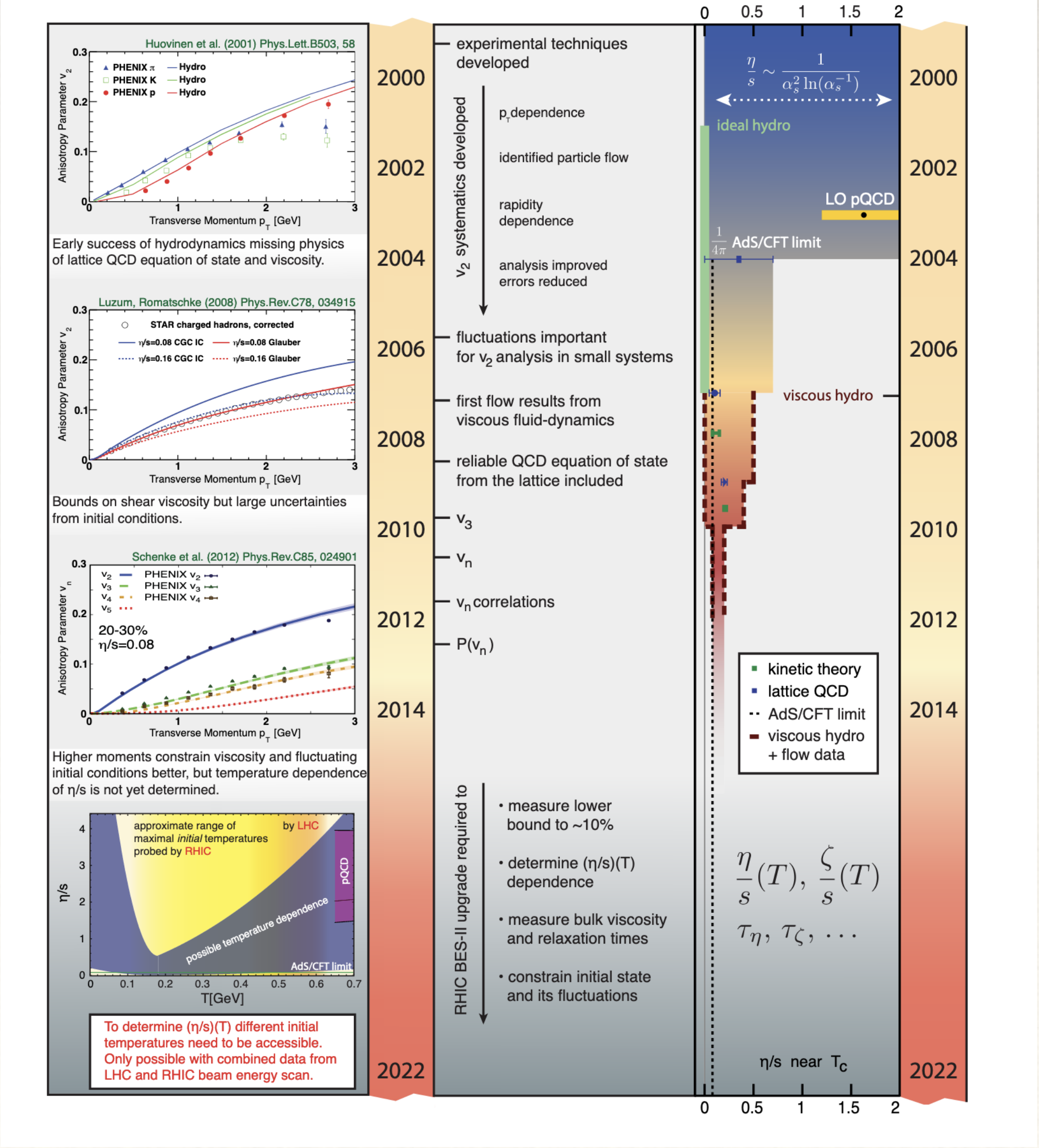
Flow harmonics











Early success of hydrodynamics missing physics of lattice QCD equation of state and viscosity.

Bounds on shear viscosity but large uncertainties from initial conditions.

Higher moments constrain viscosity and fluctuating initial conditions better, but temperature dependence of η/s is not yet determined.

To determine (η/s)(T) different initial temperatures need to be accessible. Only possible with combined data from LHC and RHIC beam energy scan.

Why relativistic hydrodynamics ?

Mass of the constituent ~ system temperature (average K.E)

Mass density is not a good degrees of freedom
since it does not take into account the K.E which may be ~ or greater than the rest mass of the particles

Energy density~

Energy per particle ~

Energy density~

Kinetic energy

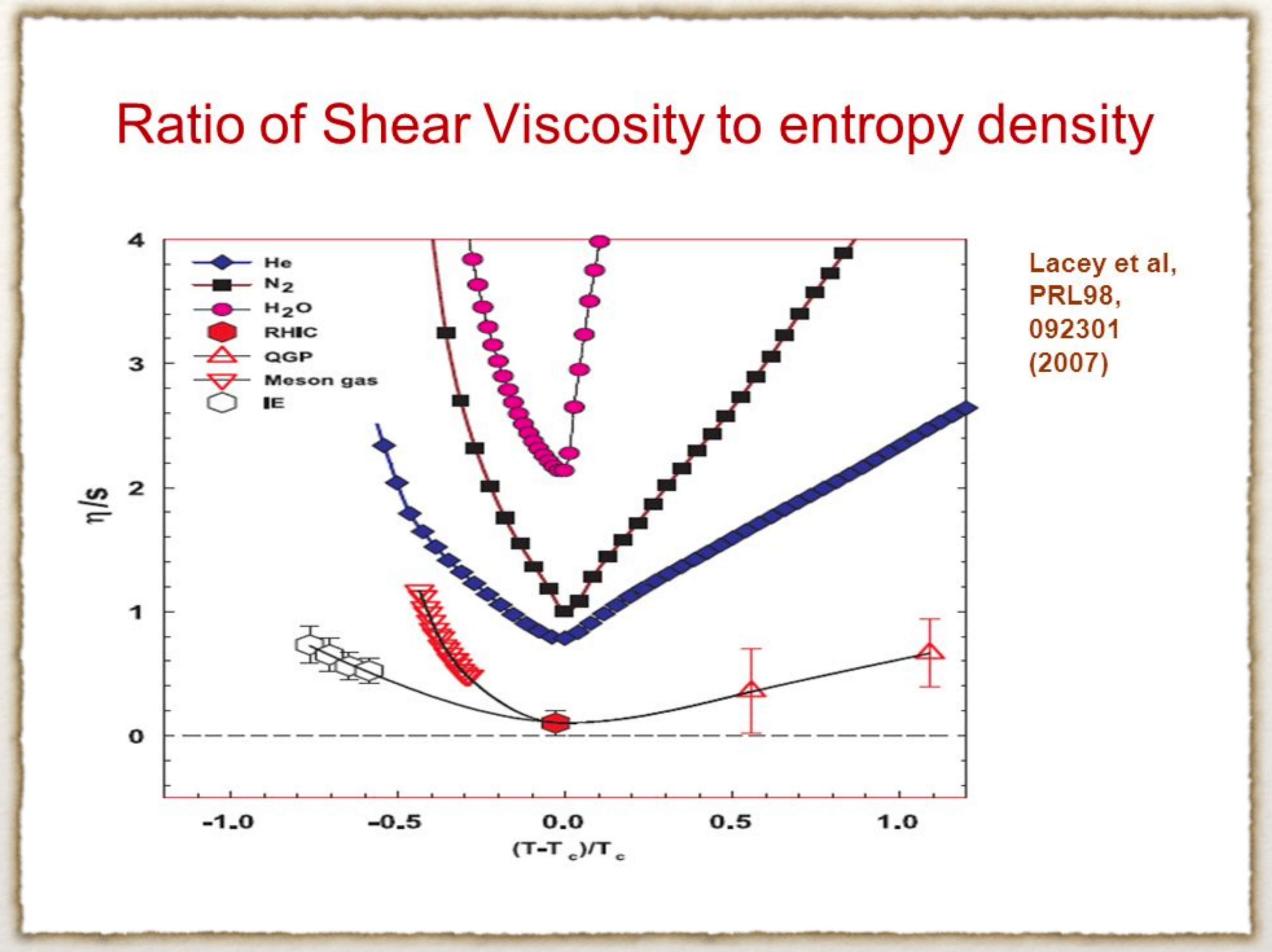
Fluid velocity reaches few percent of speed of light!

Relativistic Ideal hydrodynamics

Non-relativistic	*Relativistic*
Conservation of mass:	
Euler equation:	
	Where

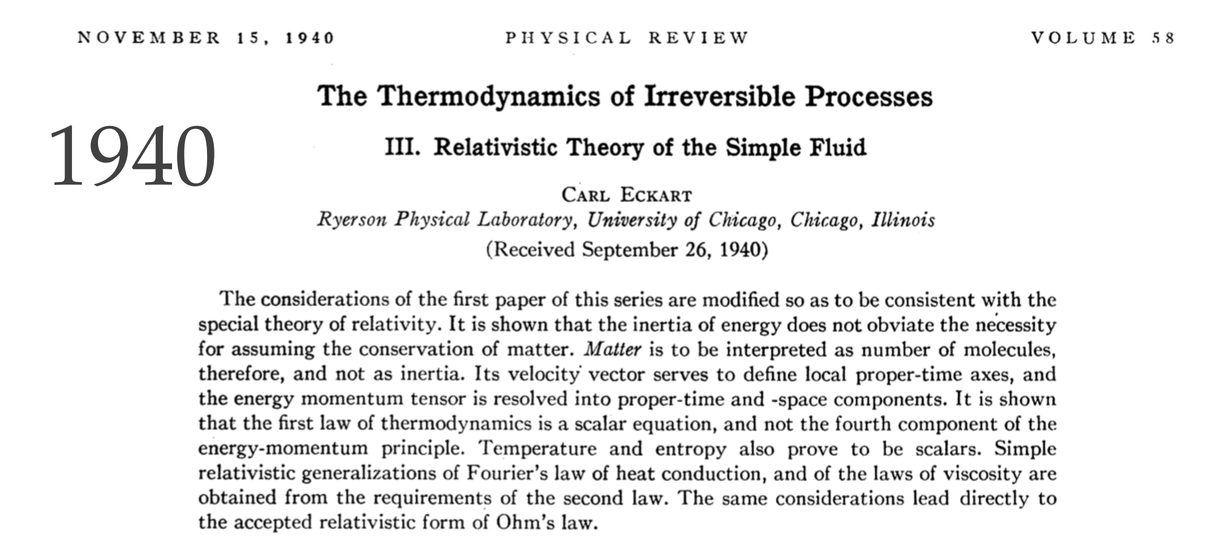
Relativistic viscous hydrodynamics

In nature ideal fluid does not exists !
Superfluid helium also has small but finite shear viscosity!


Ratio of Shear Viscosity to entropy density

Lacey et al, PRL98, 092301 (2007)

Carl Eckart


NOVEMBER 15, 1940 PHYSICAL REVIEW VOLUME 58

1940

The Thermodynamics of Irreversible Processes

III. Relativistic Theory of the Simple Fluid

Carl Eckart
Ryerson Physical Laboratory, University of Chicago, Chicago, Illinois
(Received September 26, 1940)

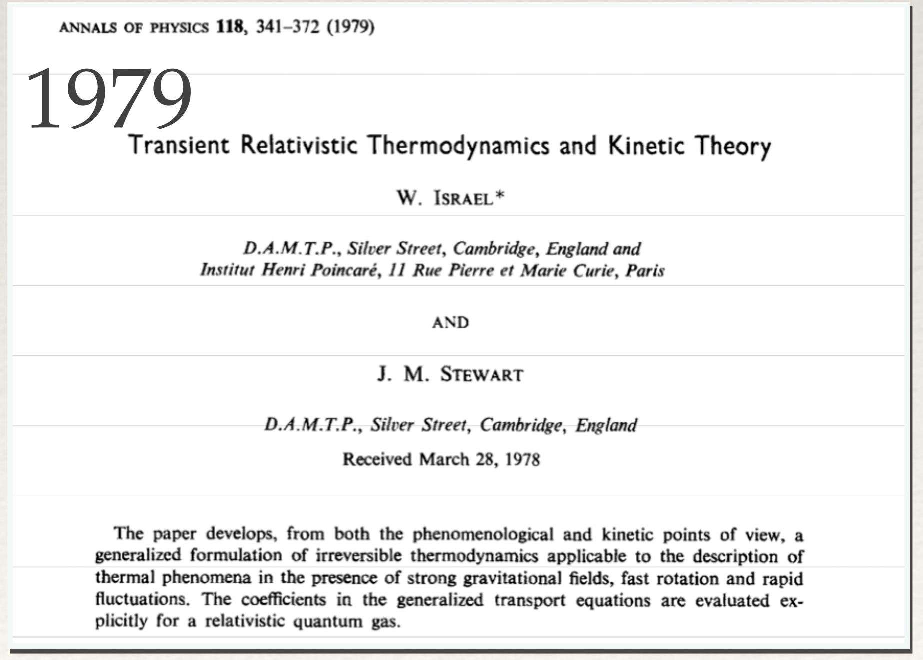
The considerations of the first paper of this series are modified so as to be consistent with the special theory of relativity. It is shown that the inertia of energy does not obviate the necessity for assuming the conservation of matter. *Matter* is to be interpreted as number of molecules, therefore, and not as inertia. Its velocity vector serves to define local proper-time axes, and the energy momentum tensor is resolved into proper-time and -space components. It is shown that the first law of thermodynamics is a scalar equation, and not the fourth component of the energy-momentum principle. Temperature and entropy also prove to be scalars. Simple relativistic generalizations of Fourier's law of heat conduction, and of the laws of viscosity are obtained from the requirements of the second law. The same considerations lead directly to the accepted relativistic form of Ohm's law.

Relativistic generalisation of
Navier-Stokes theorem

Relativistic N-S suffers from the
Problem of acausality(signal propagation faster than light)


W Israel


J M Stewart


ANNALS OF PHYSICS **118**, 341–372 (1979)

1979

Transient Relativistic Thermodynamics and Kinetic Theory

W. Israel*

D.A.M.T.P., Silver Street, Cambridge, England and
Institut Henri Poincaré, 11 Rue Pierre et Marie Curie, Paris

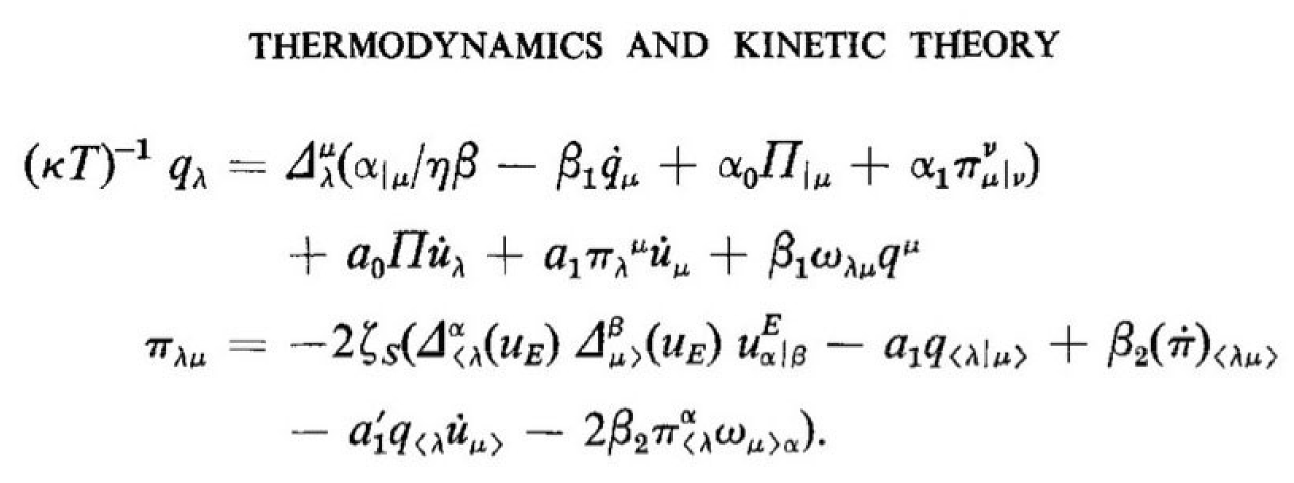
AND

J. M. Stewart

D.A.M.T.P., Silver Street, Cambridge, England

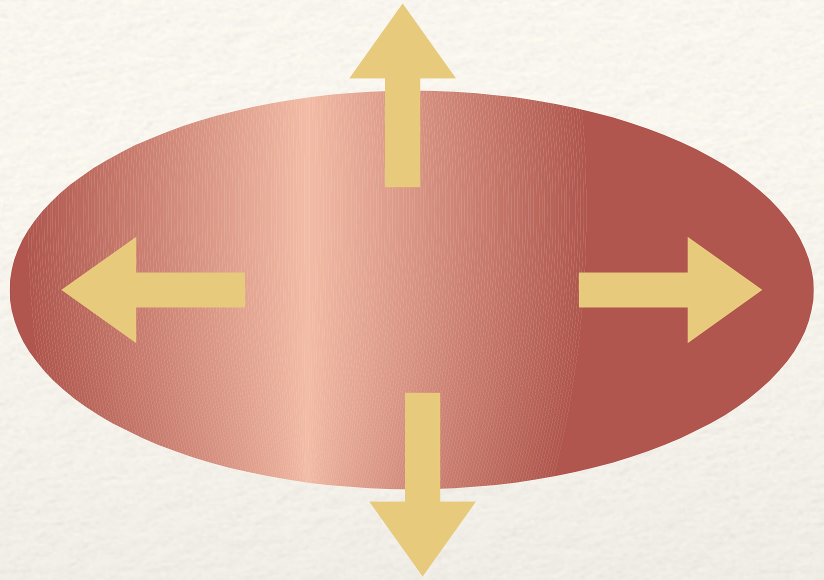
Received March 28, 1978

The paper develops, from both the phenomenological and kinetic points of view, a generalized formulation of irreversible thermodynamics applicable to the description of thermal phenomena in the presence of strong gravitational fields, fast rotation and rapid fluctuations. The coefficients in the generalized transport equations are evaluated explicitly for a relativistic quantum gas.


THERMODYNAMICS AND KINETIC THEORY

$$\begin{aligned}(\kappa T)^{-1}\, q_\lambda &= \Delta^{\mu}_{\lambda}(\alpha_{|\mu}/\eta\beta - \beta_1 \dot{q}_\mu + \alpha_0 \Pi_{|\mu} + \alpha_1 \pi^{\nu}_{\mu|\nu}) \\ &\quad + a_0 \Pi \dot{u}_\lambda + a_1 \pi_\lambda{}^{\mu} \dot{u}_\mu + \beta_1 \omega_{\lambda\mu} q^\mu \\ \pi_{\lambda\mu} &= -2\zeta_S(\Delta^{\alpha}_{\langle\lambda}(u_E)\, \Delta^{\beta}_{\mu\rangle}(u_E)\, u^{E}_{\alpha|\beta} - a_1 q_{\langle\lambda|\mu\rangle} + \beta_2(\dot{\pi})_{\langle\lambda\mu\rangle} \\ &\quad - a_1' q_{\langle\lambda} \dot{u}_{\mu\rangle} - 2\beta_2 \pi^{\alpha}_{\langle\lambda} \omega_{\mu\rangle\alpha}).\end{aligned}$$

The energy-momentum tensor



(Conservation of mass)

Fluid at rest has finite energy density and pressure

(energy, mass) density = (energy,mass)/volume

(Euler equation)

pressure=force/area=momentum/(time x area)

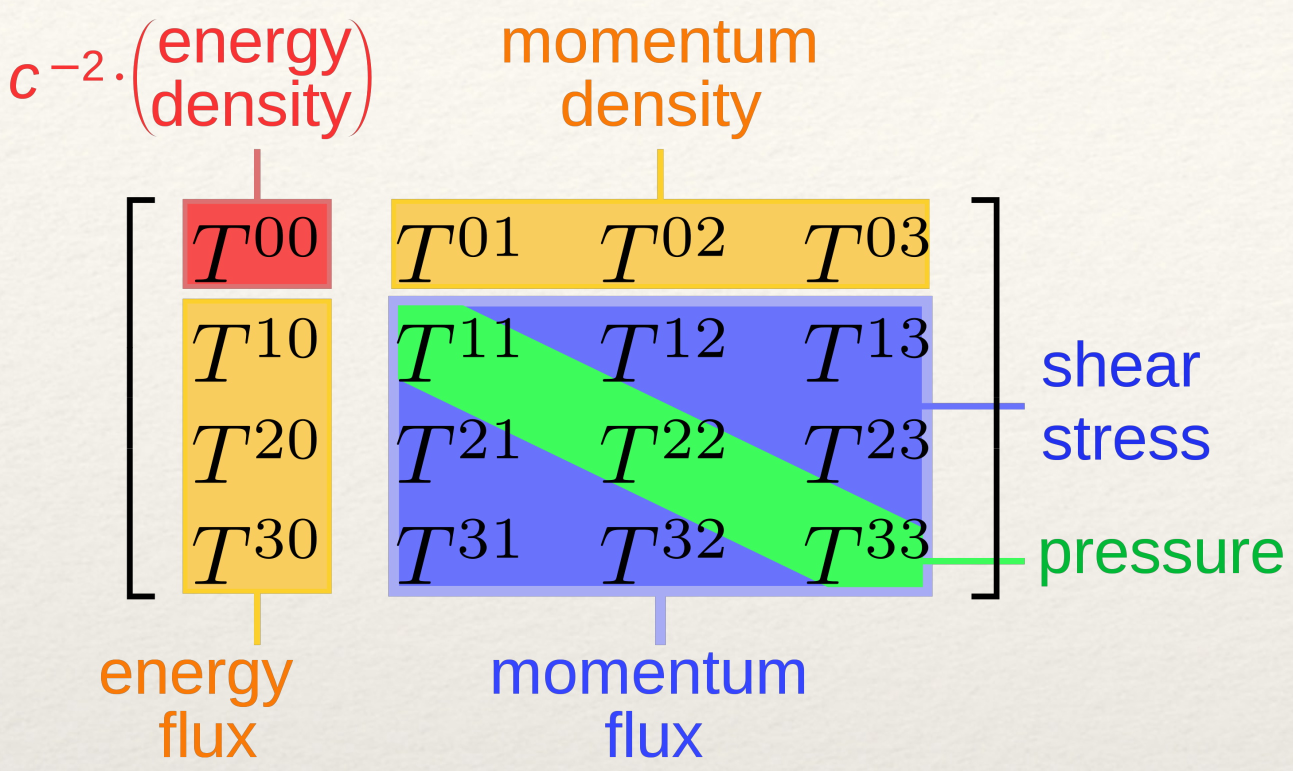
energy, momentum, and volumes change under Lorentz transformation (arbitrary reference frame)

Physical laws must be invariant under Lorentz transformation Tensors are the right quantity to work with

Lorentz transformation

Energy-momentum is conserved:

Connection between the microscopic world to the macroscopic world: EoS



Need for an Equation of State (EoS):

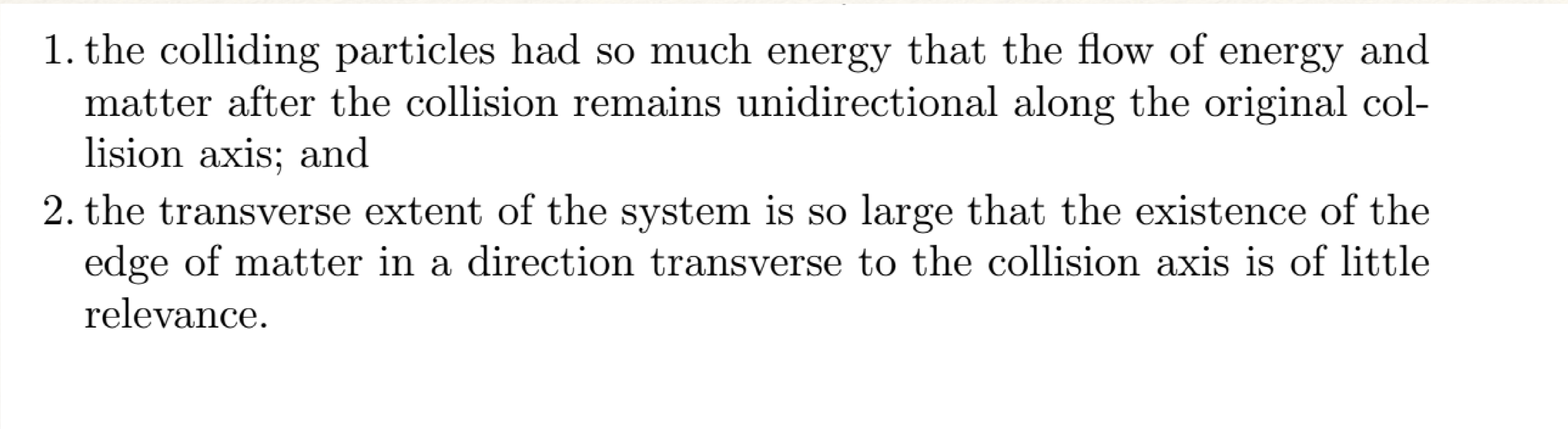
Four equations

Five unknowns

For a masses Boltzmann gas:

Part- 2

Relativistic hydrodynamics: simple applications in HIC: Bjorken flow

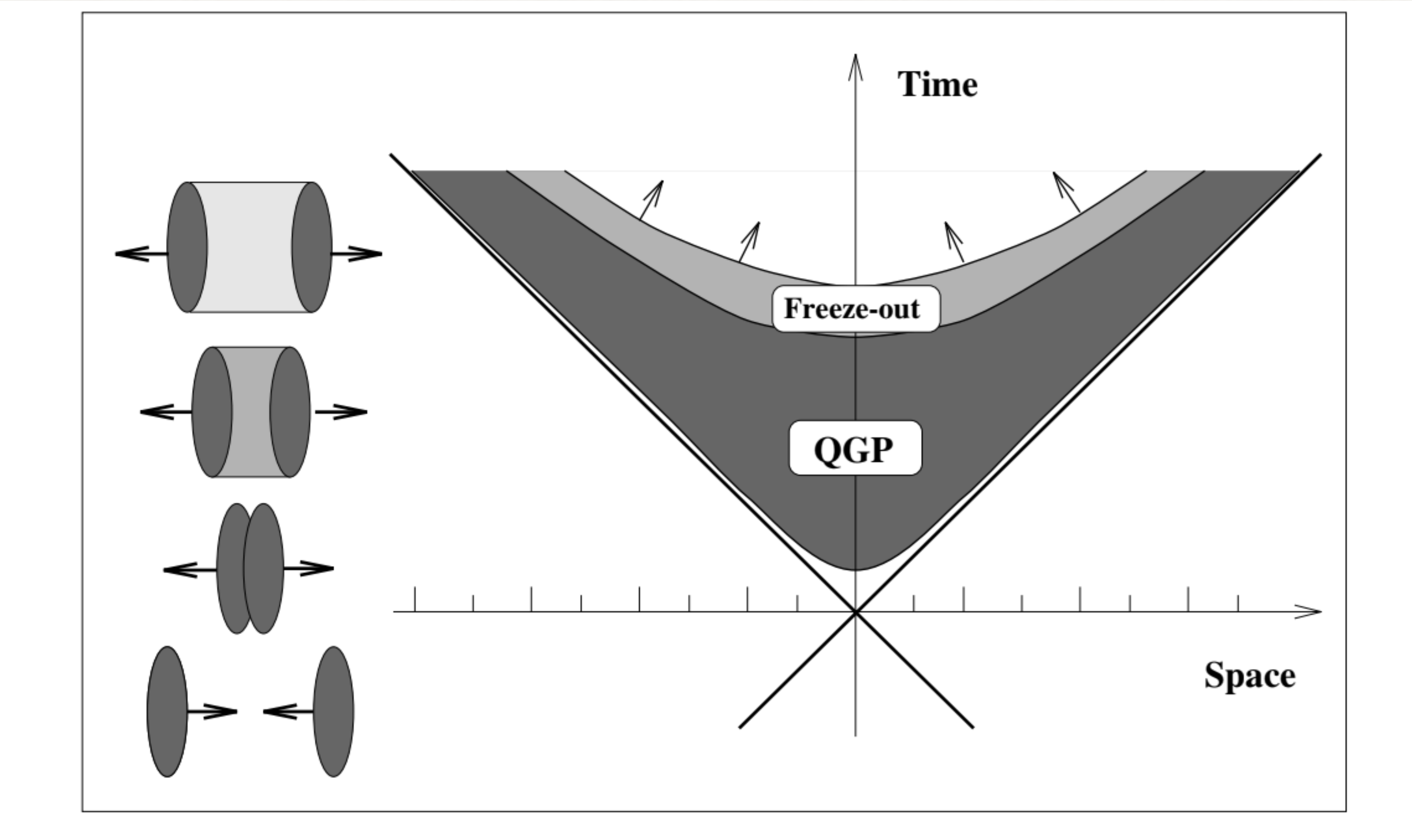


1. the colliding particles had so much energy that the flow of energy and matter after the collision remains unidirectional along the original collision axis; and
2. the transverse extent of the system is so large that the existence of the edge of matter in a direction transverse to the collision axis is of little relevance.

Symmetry argument

- Approximate boost-invariance along the beam line near mid rapidity
- Translational invariance in the transverse plane
- Rotational invariance in the transverse plane

The line element in the Minkowski space (Milne co-ordinate):



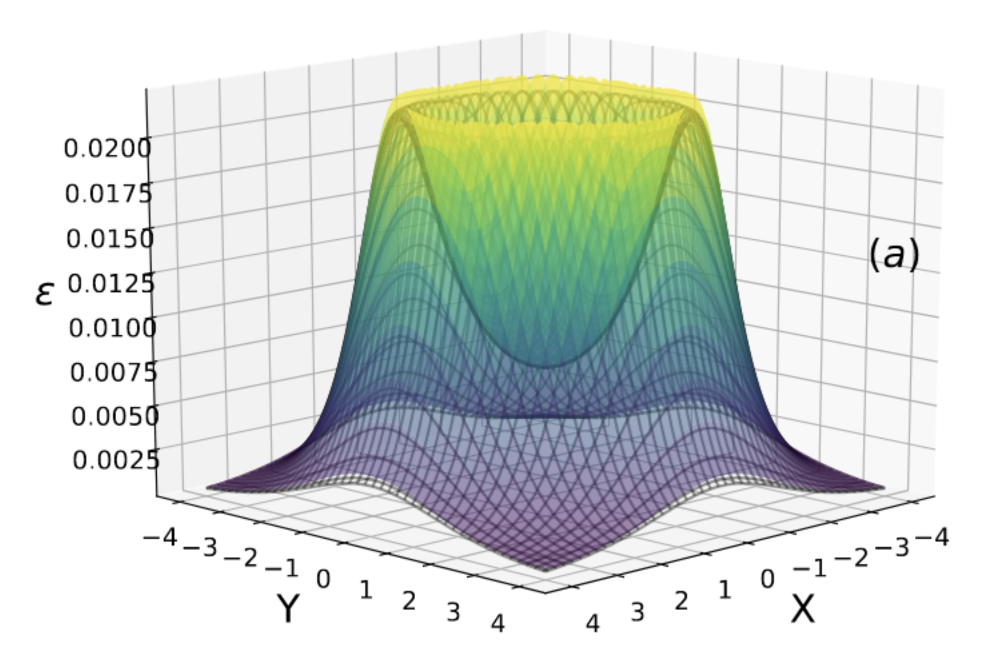
Milne co-ordinate system	*Cartesian co-ordinate*
Local four velocity	

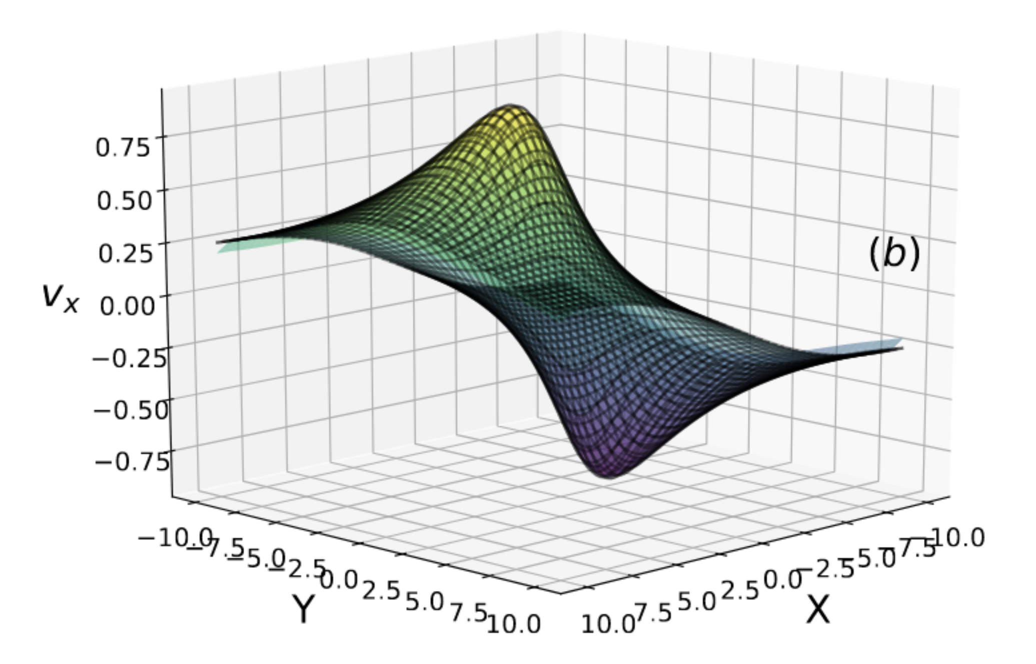
Fluid is at rest in Milne system, we only need to find energy density evolution !

Relativistic hydrodynamics: simple applications in HIC:Gubser flow

Transverse + longitudinal viscous expansion of fluid

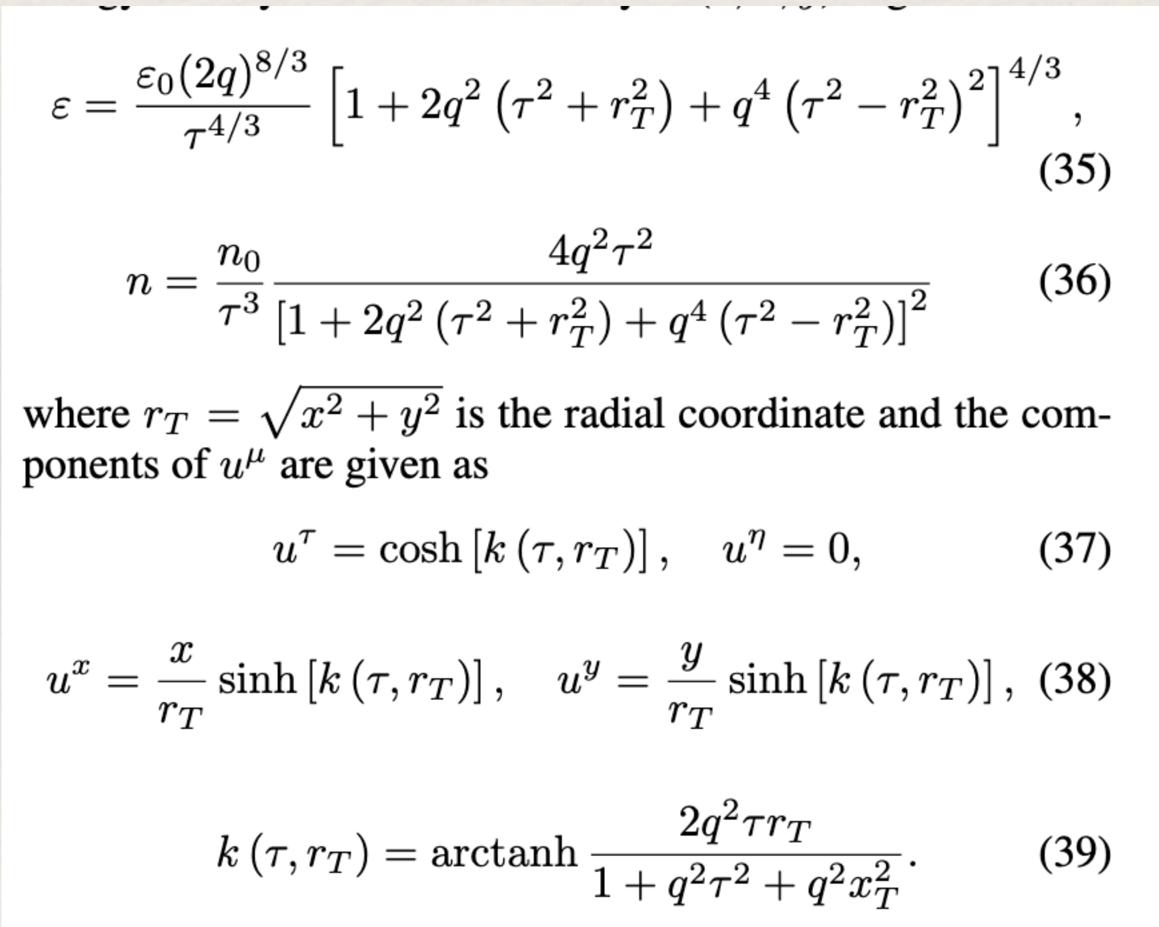
Steven S Gubser





Symmetry argument

- Approximate boost-invariance along the beam line near mid rapidity
- ~~Translational invariance in the transverse plane~~
- Rotational invariance in the transverse plane



$$\varepsilon = \frac{\varepsilon_0 (2q)^{8/3}}{\tau^{4/3}} \left[1 + 2q^2\left(\tau^2 + r_T^2\right) + q^4\left(\tau^2 - r_T^2\right)^2\right]^{4/3}, \tag{35}$$

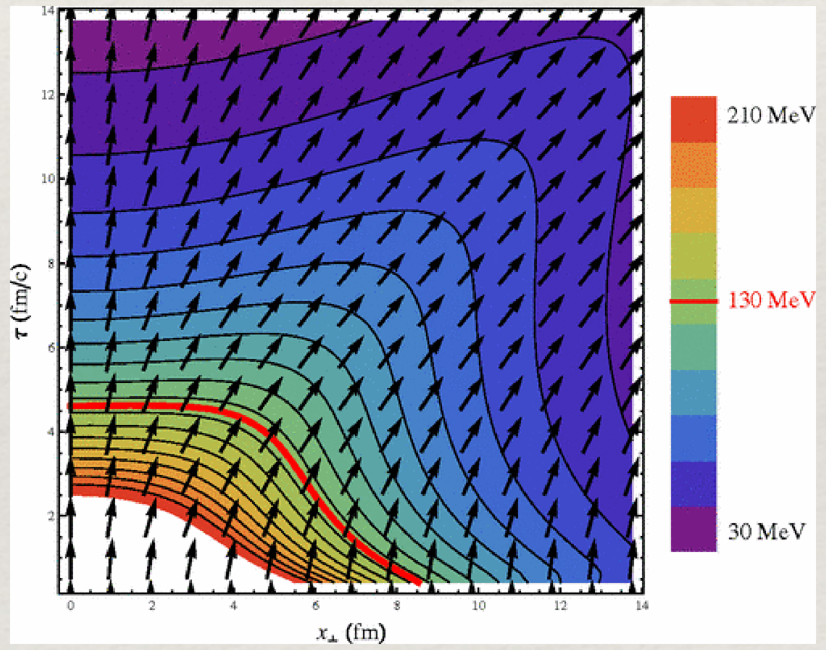
$$n = \frac{n_0}{\tau^3} \frac{4q^2\tau^2}{\left[1 + 2q^2\left(\tau^2 + r_T^2\right) + q^4\left(\tau^2 - r_T^2\right)\right]^2} \tag{36}$$

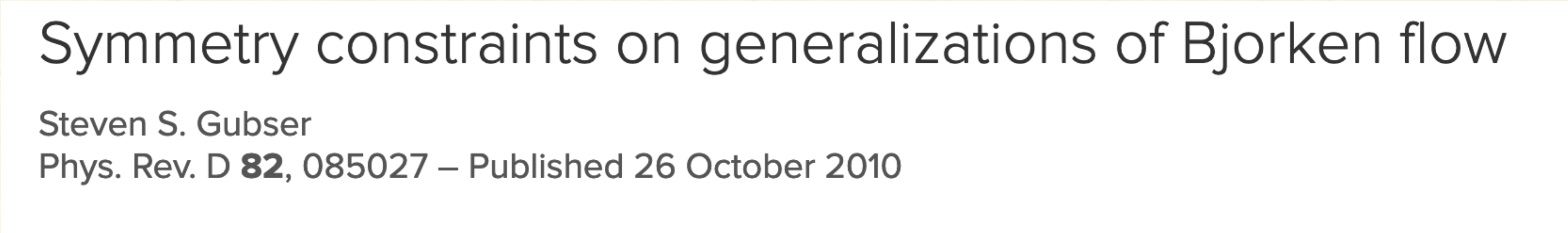
where $r_T = \sqrt{x^2 + y^2}$ is the radial coordinate and the components of u^μ are given as

$$u^\tau = \cosh\left[k\left(\tau, r_T\right)\right], \quad u^\eta = 0, \tag{37}$$

$$u^x = \frac{x}{r_T} \sinh\left[k\left(\tau, r_T\right)\right], \quad u^y = \frac{y}{r_T} \sinh\left[k\left(\tau, r_T\right)\right], \tag{38}$$

$$k\left(\tau, r_T\right) = \operatorname{arctanh} \frac{2q^2\tau r_T}{1 + q^2\tau^2 + q^2 x_T^2}. \tag{39}$$

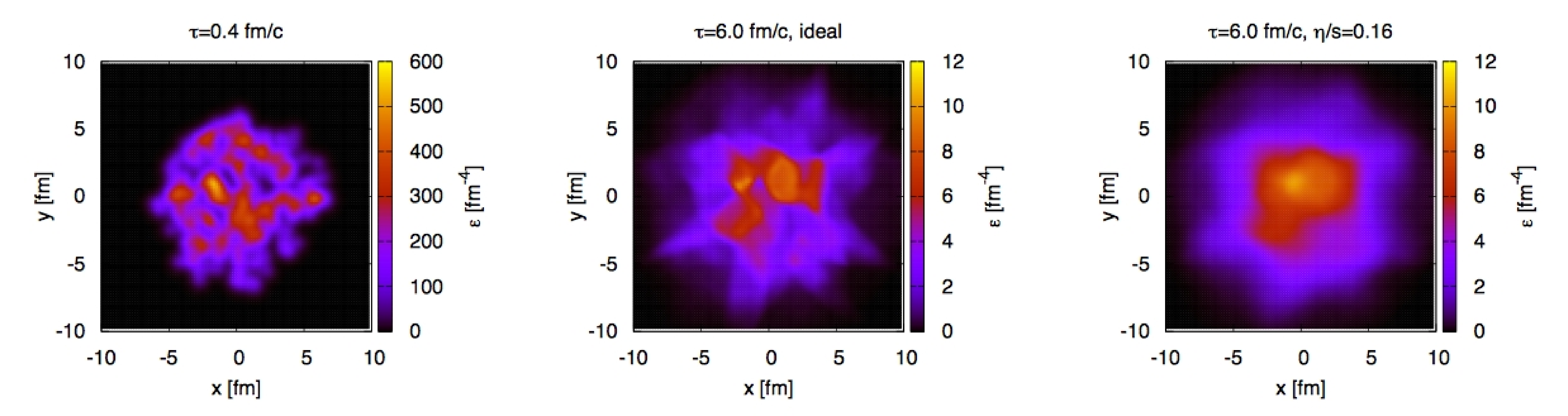




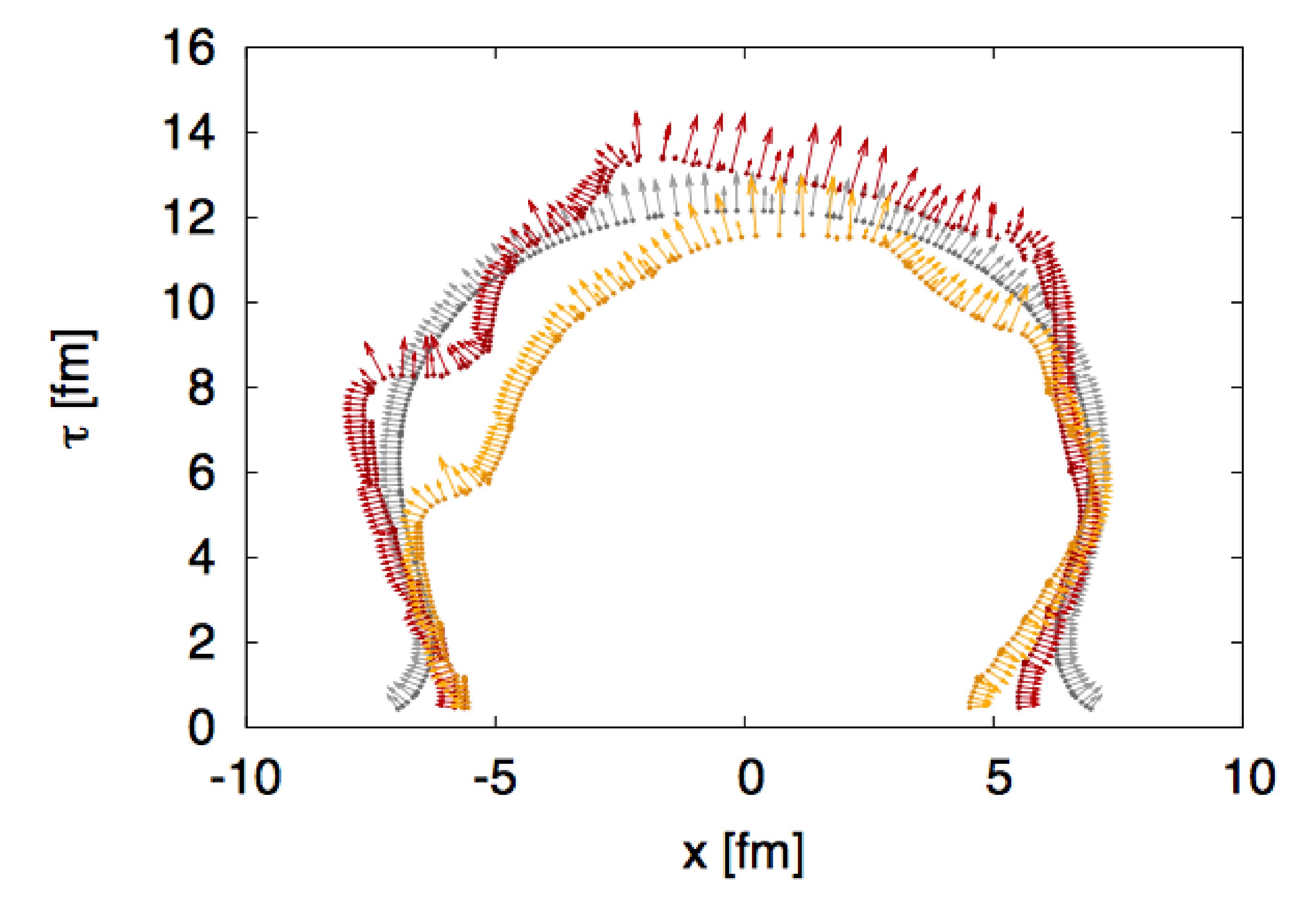
Symmetry constraints on generalizations of Bjorken flow

Steven S. Gubser
Phys. Rev. D **82**, 085027 – Published 26 October 2010

Relativistic hydrodynamics: Need for numerical hydro



Complex geometry

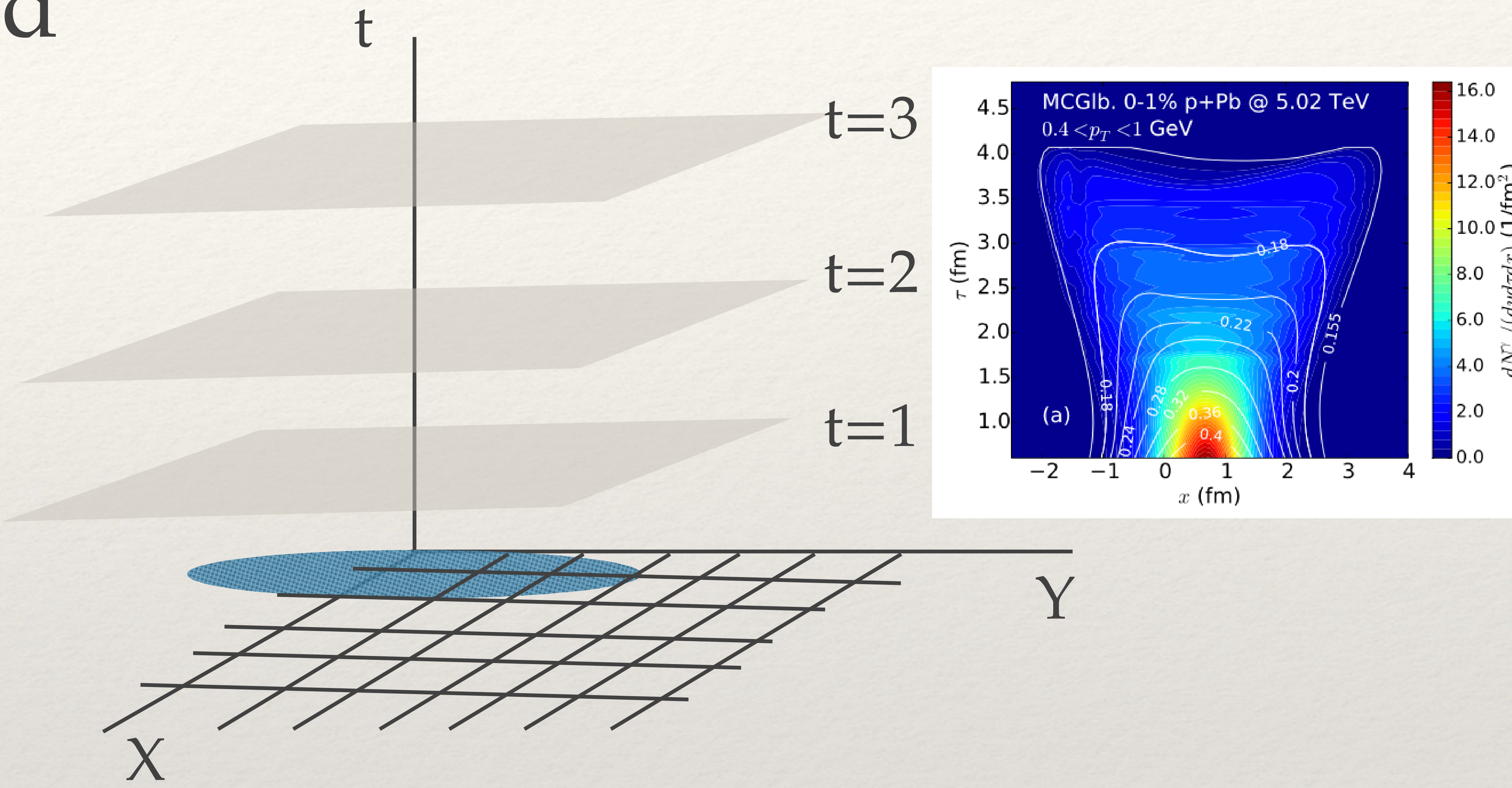


Complicated freezeout hyper surface

What is under the hood?

Numerical relativistic hydrodynamics

Relativistic ideal fluid



Four equations

Similarly for

When we consider no special symmetry one needs to solve
3(spatial) + 1(temporal) equations —> 3+1 -d hydro.

Let us consider that we have a translational symmetry
along 'z' direction and

2(spatial) + 1(temporal) equations —> 2+1 -d hydro.

Given energy-momentum tensor components on the spatial grid at initial time

We need to evaluate conserved quantities on each
space grid point at later times by solving the coupled partial diff eqn until freeze out.

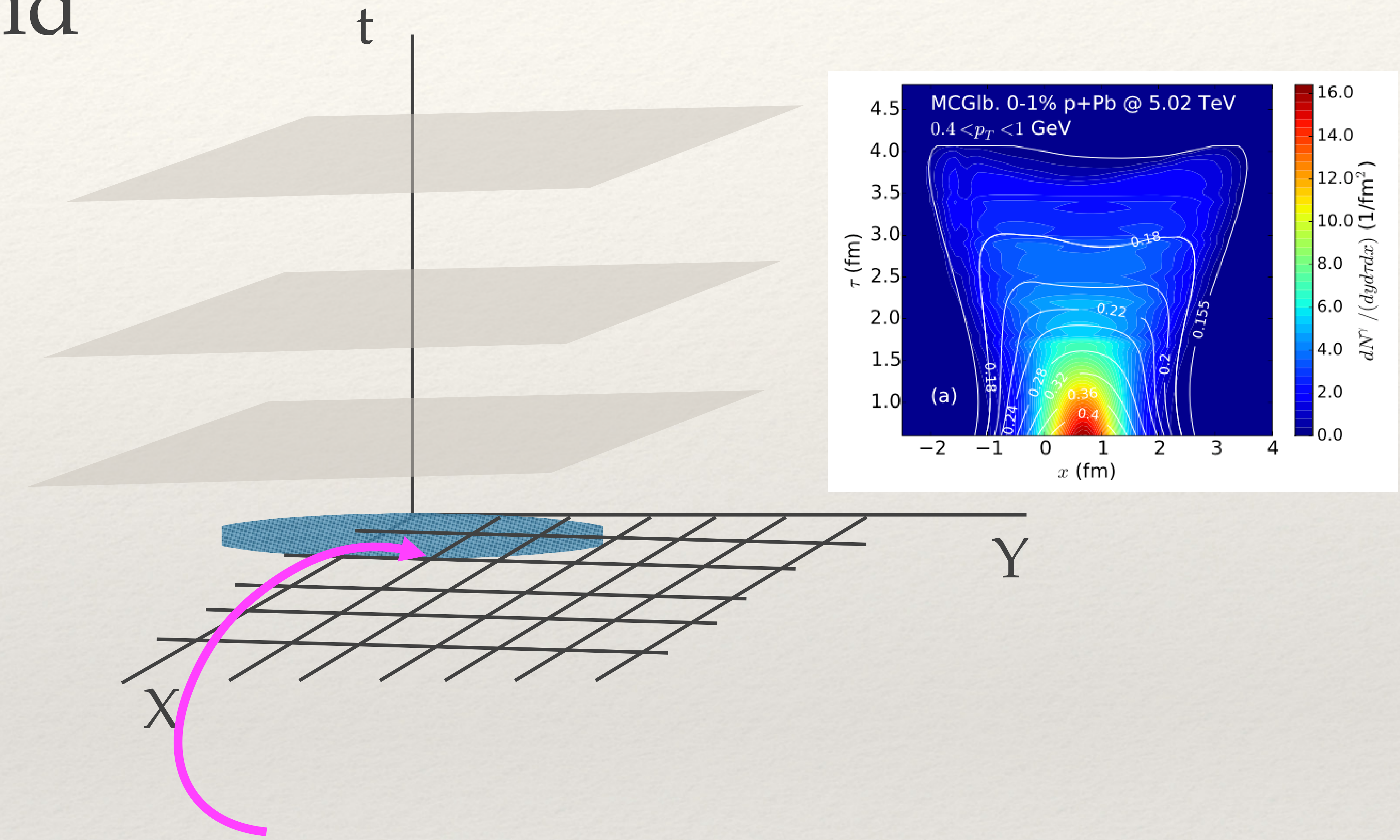
What is under the hood?

Numerical relativistic hydrodynamics

Relativistic ideal fluid

Four equations

In Milne coordinate with boost invariance



To solve these set of equations we need specialised algorithms : SHASTA, Kurganov-Tadmor, Riemann Solver etc.

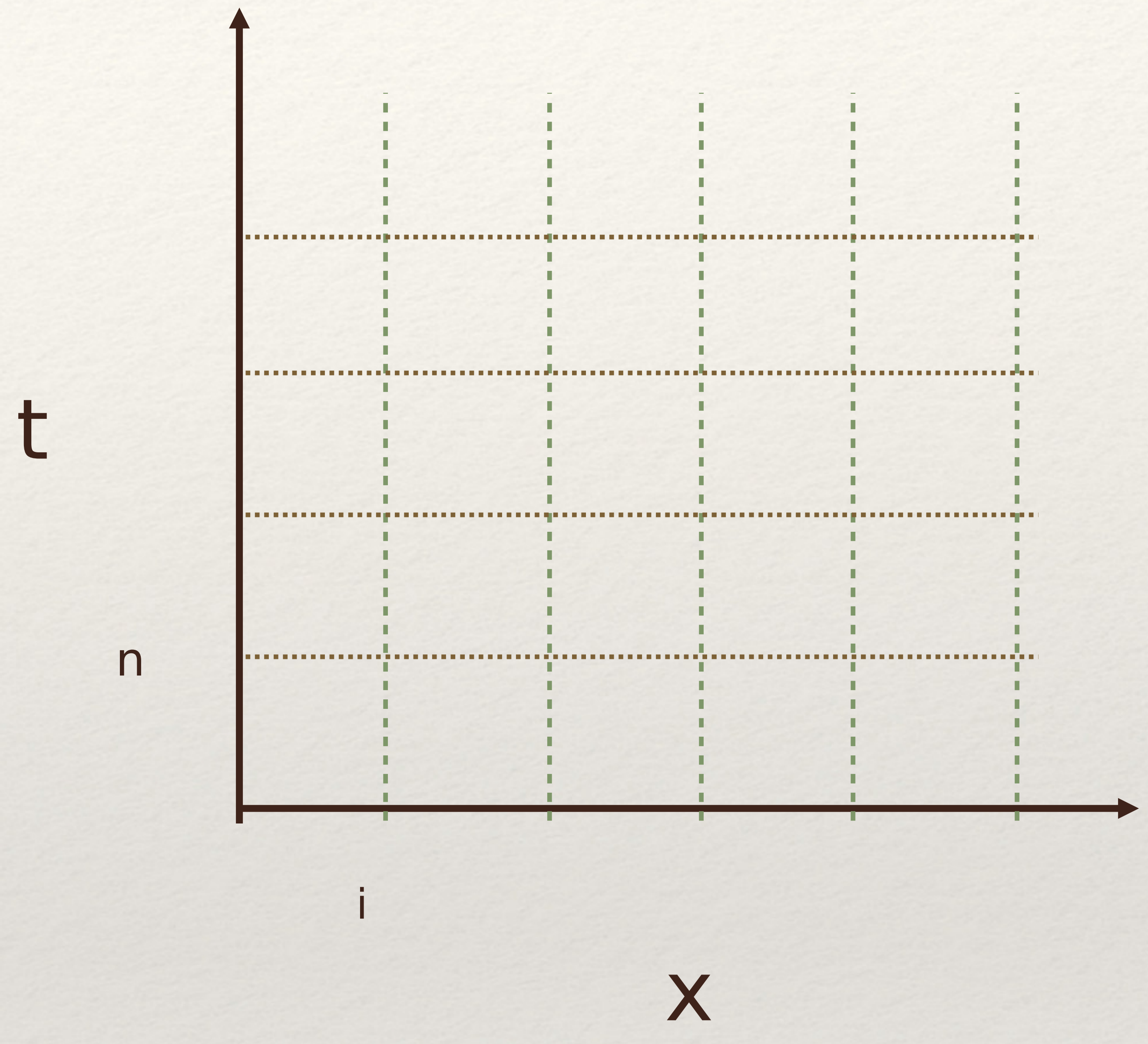
Numerical solution of differential equations

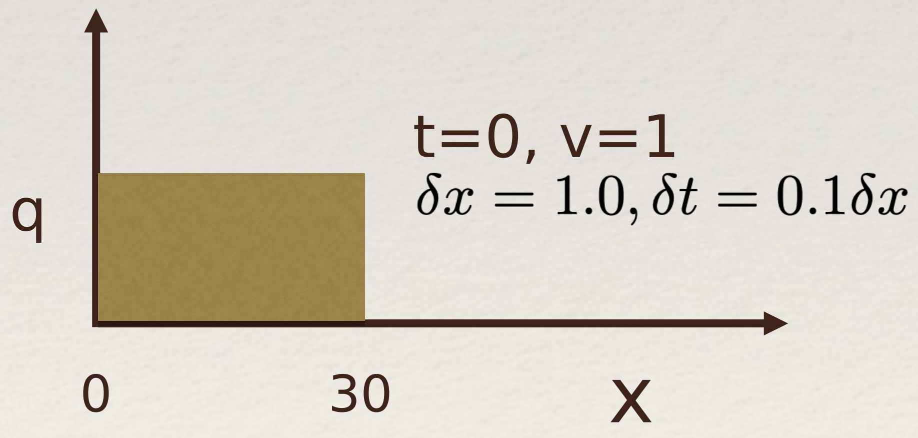
$$\frac{\partial q}{\partial t} + v_x \frac{\partial q}{\partial x} = 0$$ Advection equation

Simple cantered difference scheme gives

$$q_i^{n+1} = q_i^n - \frac{v\delta t}{2\delta x}(q_{i+1}^n - q_{i-1}^n)$$

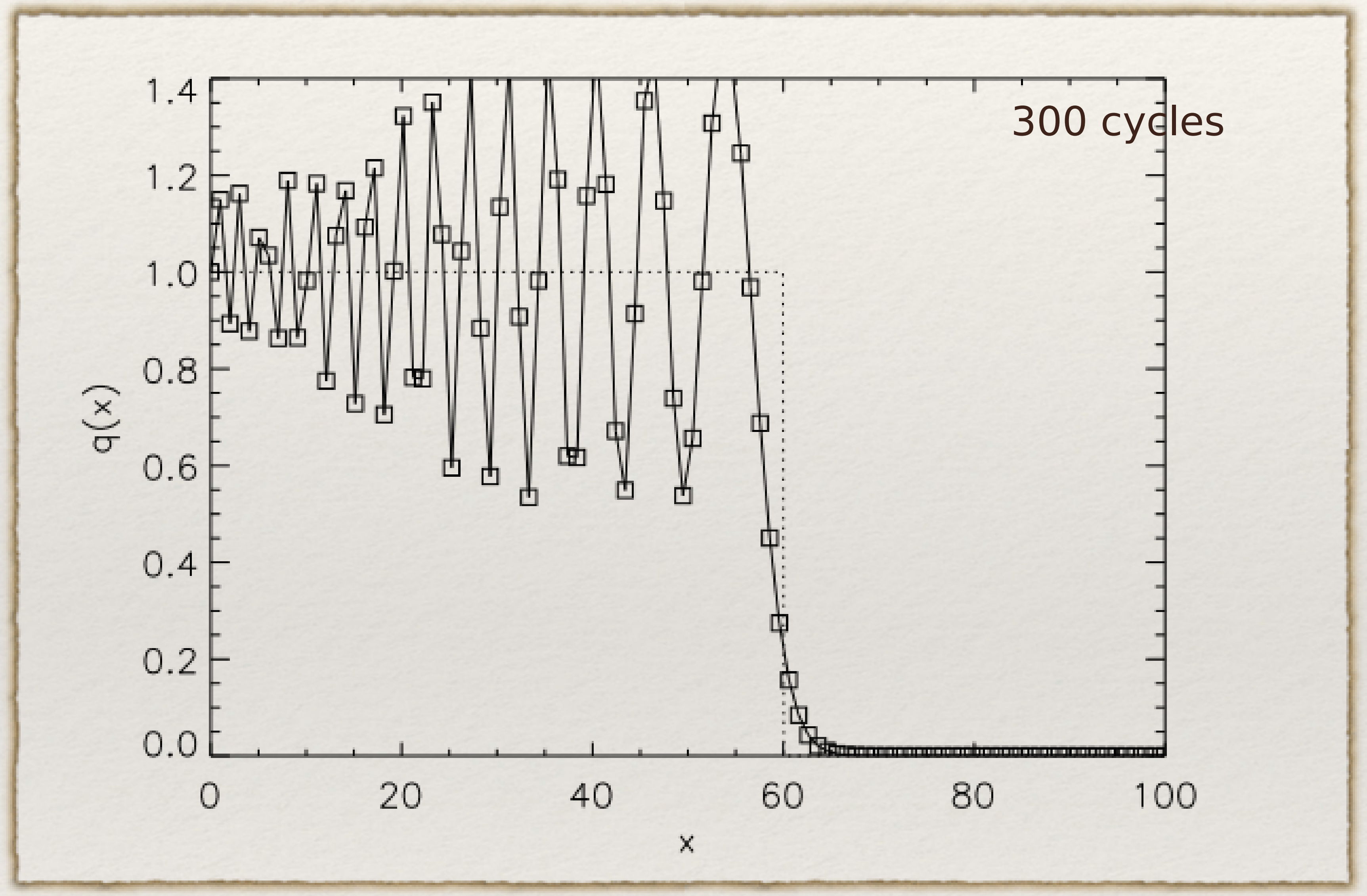
Starting from an initial time we keep on solving for later times





Handling error is very important !

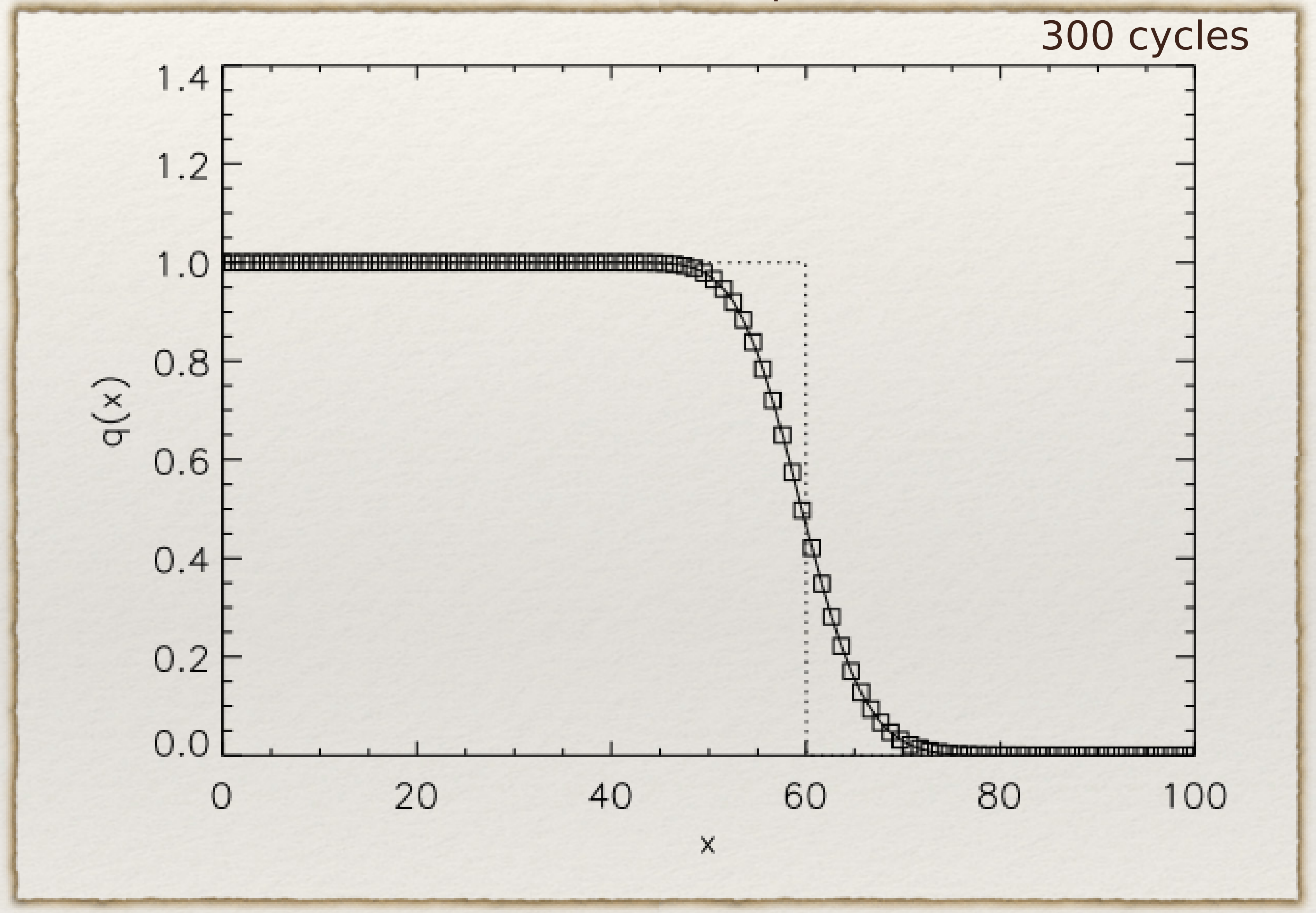
Simple finite difference scheme produces spurious results



2nd order accuracy in space derivative

Many stable algorithm->unphysical diffusion

upwind scheme



1st order accuracy in space derivative

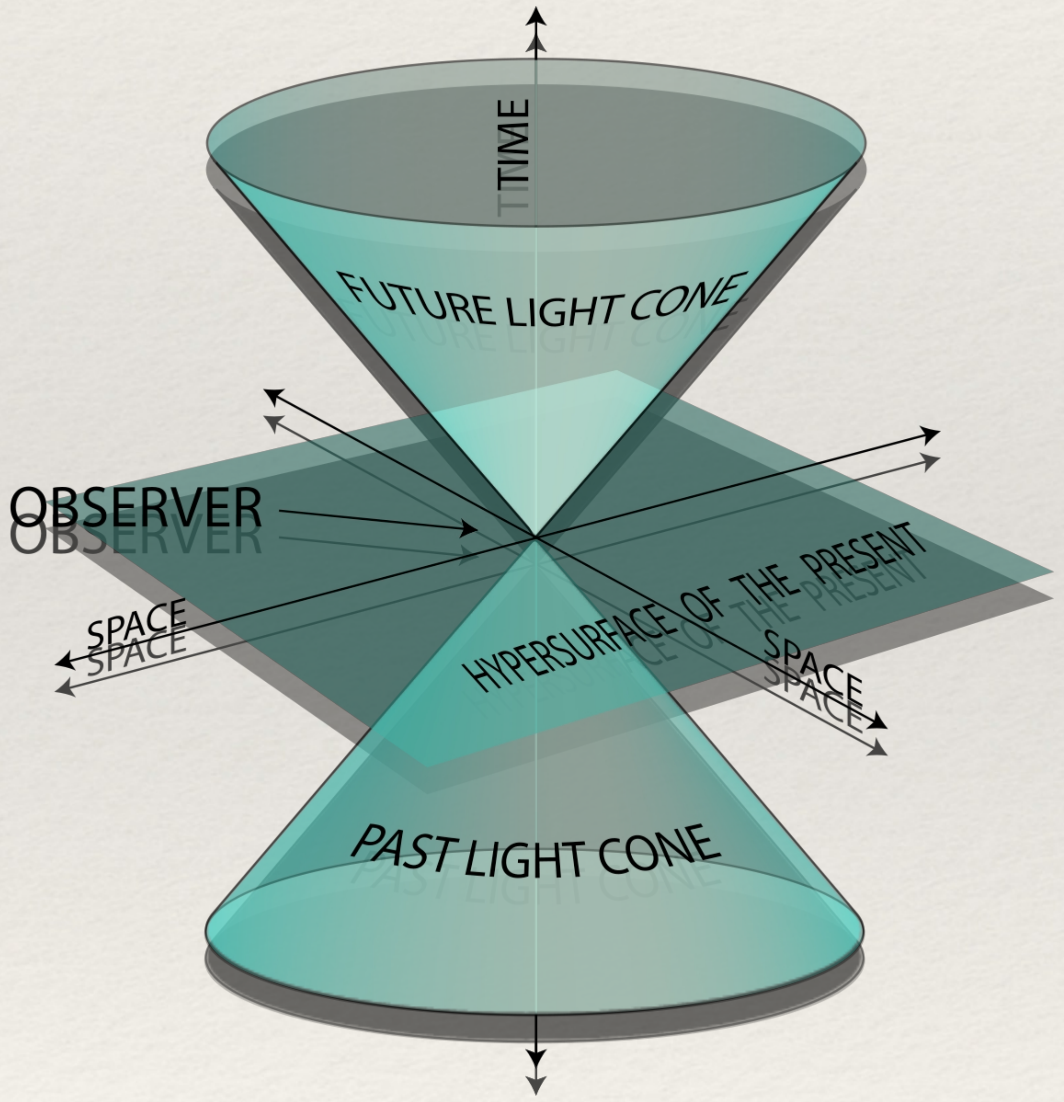
Heavy-ion collisions: how do you get particles from fluid ?

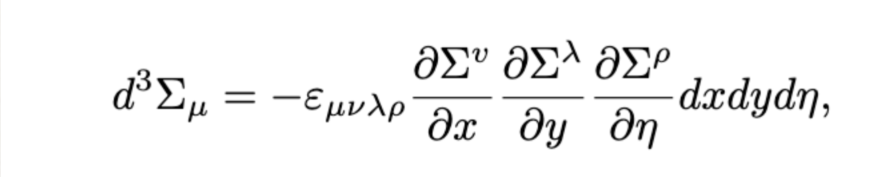
Cooper-Frye formulation: $$E\frac{dN}{dp^3} \equiv \underline{\int_\sigma d\sigma_\mu p^\mu f(x,p) \approx \sum_\sigma \Delta\sigma_\mu p^\mu f(x,p)}:$$

From hydrodynamics

3 dimensional space —> 2d surface (xy,yz,zx)

4 dimensional space-time —> 3d hyper-surface (xyz,xyt,yzt,zxt)





$$d^3\Sigma_\mu = -\varepsilon_{\mu\nu\lambda\rho}\frac{\partial\Sigma^\nu}{\partial x}\frac{\partial\Sigma^\lambda}{\partial y}\frac{\partial\Sigma^\rho}{\partial \eta}dxdyd\eta,$$

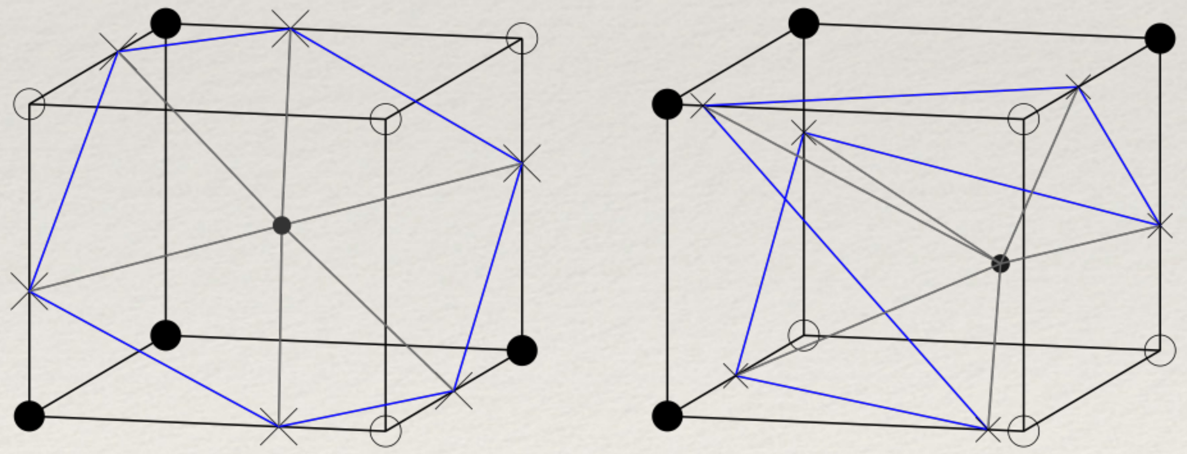


Fig. 5. Examples of triangularisation of the polygon in a simple and a complicated case.

Relativistic numerical hydrodynamics: flow chart

STEP -1 : *Initial conditions*

- Setting primitive variables: energy / entropy density obtained from some model (Glauber, IP-Galsma, CGC etc).
- Pressure is obtained from the energy/entropy density using EoS.
- Velocities are set either to zero or to some values from educated guess.

Conserved quantities:

- calculate components of from the given initial data.

STEP -2 : *Time evolve conserved quantities*

- Conserved quantities in the next time step is obtained by solving the conservation equation.
- Calculate the new velocities from the conserved quantities.
- Recover primitive variables from the conserved quantities.
- Check for the freeze out condition.

STEP -3 : *Store freeze out information*

- In each time step and run the simulation till all fluid elements are frozen out.

STEP -4 : *Calculate observables from freeze out data*

- invariant yields using stored freeze out information and Cooper-Frye formula .
- calculate other relevant observables such as flow harmonics, correlations.

Need more study

- Inclusion of critical slowing down due to CP

ncorporate the initial strong electromagnetic field : relativistic magnetohydrodynamics

- Hydrodynamization and attractor solution
- Particlization and alternative of Cooper-Frye prescription

Active participation from young students like you!