QGP - THEORY

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V ALICE-India School On Quark-Gluon Plasma,
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Content

1. Static properties: QCD Thermodynamics:
    i. Lattice QCD approach (LQCD),
    ii. Non interacting gas of quarks and gluons
    iii. Non interacting gas of hadrons (HRG)
    iv. QCD Phase Diagram
2. Static properties: Contact with experiments via *phase space integrated* observables:
    i. Mean hadron yields: Establish freezeout (HRG),
    ii. Fluctuations of conserved charges: Establish freeze out (HRG, LQCD)
3. Transport properties, Initial conditions: Kubo relations; Contact with experiments via phase space differential observables

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1. QCD Thermodynamics:

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1. QCD Thermodynamics

QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + i\bar{\psi}\gamma^\mu\left(\partial_\mu - ig\frac{\lambda_a}{2}A^a_\mu\right)\psi - m\bar{\psi}\psi$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf_{abc}A^b_\mu A^c_\mu$$

For conditions accessible in heavy ion collisions (HICs), the QCD coupling is large, standard perturbative techniques fail.

Lattice QCD approach

Direct determination of equilibrium properties of QCD medium by numerical evaluation of the QCD path integrals

Starting point, the vacuum-to-vacuum transition amplitude in the Feynman path integral formulation:

$$Z = \int \mathcal{D}A_\mu^a(x)\,\mathcal{D}\bar{\psi}(x)\,\mathcal{D}\psi(x)\,e^{i\int d^4x\,\mathcal{L}[A_\mu^a,\bar{\psi},\psi]}$$

The partition function of a grand canonical ensemble of quarks, antiquarks and gluons in thermal equilibrium is obtained by $t \rightarrow i\tau$ with range of τ: 0 to $\beta = \dfrac{1}{T}$ where T is the temperature of the system. Further, for consistency, the classical gluon fields need to obey periodicity while the fermion fields anti-periodicity in τ

Lattice QCD approach

Path integral for the thermal equilibrium ensemble average $\langle \hat{O} \rangle$ of an observable $\hat{O}$

$$\langle \hat{O} \rangle = \frac{\int \mathcal{D}A_\mu^a(\boldsymbol{x},\tau)\, \mathcal{D}\bar{\psi}(\boldsymbol{x},\tau)\, \mathcal{D}\psi(\boldsymbol{x},\tau)\, O[A,\bar{\psi},\psi]\, e^{-\int_0^\beta d\tau \int d^3x\, \mathcal{L}_\mathrm{E}[A,\bar{\psi},\psi]}}{\int \mathcal{D}A_\mu(\boldsymbol{x},\tau)\, \mathcal{D}\bar{\psi}(\boldsymbol{x},\tau)\, \mathcal{D}\psi(\boldsymbol{x},\tau)\, e^{-\int_0^\beta d\tau \int d^3x\, \mathcal{L}_\mathrm{E}[A,\bar{\psi},\psi]}}$$

The QCD Lagrangian being bilinear in the quark fields allow for an analytical evaluation:

$$\langle \hat{O} \rangle = \frac{\int \mathcal{D}A_\mu(\boldsymbol{x},\tau)\, \tilde{O}[A]\, \det\left[i\gamma^\mu \left(\partial_\mu - ig\frac{\lambda_a}{2}A_\mu^a\right) - m\right]\, e^{-\int_0^\beta d\tau \int d^3x\, \mathcal{L}_\mathrm{E}[A]}}{\int \mathcal{D}A_\mu(\boldsymbol{x},\tau)\, \det\left[i\gamma^\mu \left(\partial_\mu - ig\frac{\lambda_a}{2}A_\mu^a\right) - m\right]\, e^{-\int_0^\beta d\tau \int d^3x\, \mathcal{L}_\mathrm{E}[A]}}$$

In LQCD, this path integral is evaluated numerically by discretising space and time into $N_s^3 N_\tau$ lattice points. This method works well as long as the fermionic determinant is positive which is the case for zero baryon chemical potential, μ_B. At non-zero μ_B, we have the sign problem and only recently there has been significant progress in this direction.

Lattice QCD approach

Basic thermodynamic quantities from the partition function:

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T,V,\vec{\mu}) \ ,$$

$$\frac{\epsilon}{T^4} = -\frac{1}{VT^4} \left. \frac{\partial \ln Z(T,V,\vec{\mu})}{\partial 1/T} \right|_{\vec{\mu}/T \ \text{fixed}}$$

$$\frac{n_f}{T^3} = \frac{1}{VT^3} \frac{\partial \ln Z(T,V,\vec{\mu})}{\partial \hat{\mu}_f} \ ,$$

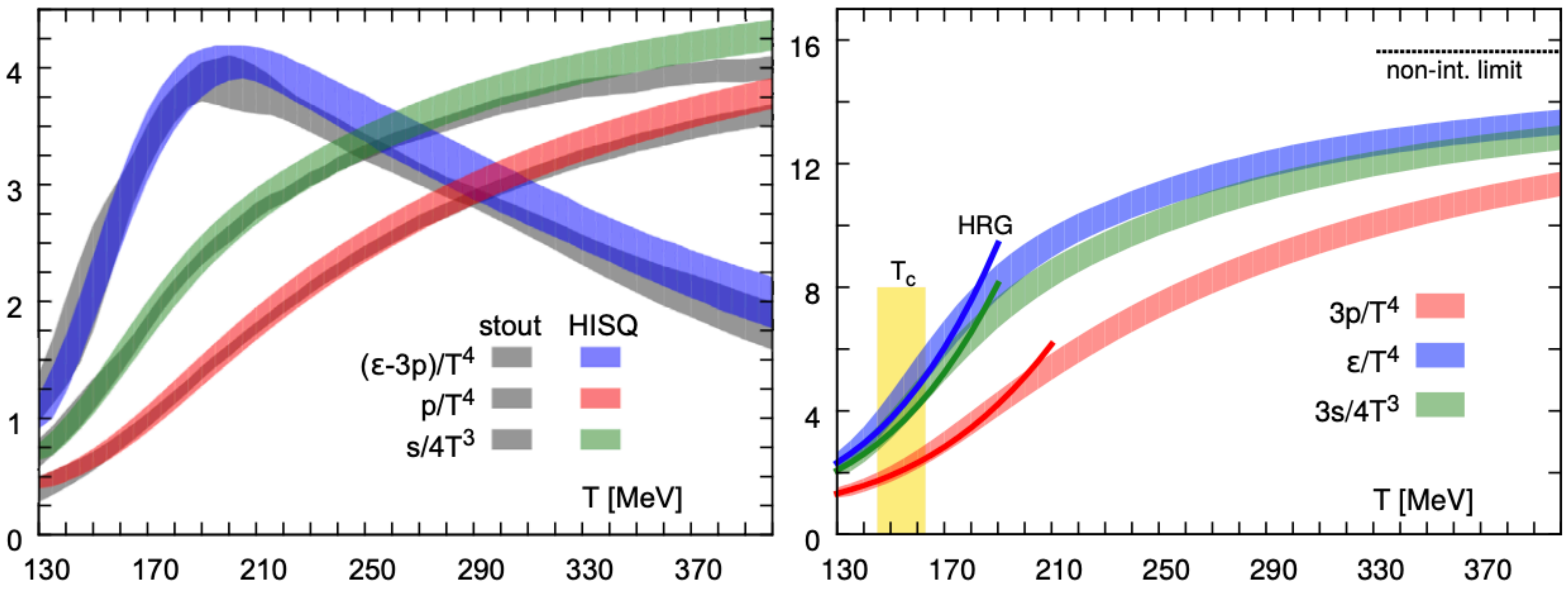
One approach to circumvent the sign problem is by incorporating Taylor expansion, for example, in 2+1 flavor:

$$\frac{P}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds}(T) \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k$$

$$\chi_{ijk}^{uds}(T) = \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_u^i \partial \hat{\mu}_d^j \partial \hat{\mu}_s^k} \right|_{\vec{\mu}=0}$$

Lattice QCD approach

Basic thermodynamic quantities from the partition function:



Ref: 1504.05274

High temperature limit

At high enough temperatures, m / T $\rightarrow 0$, where m is the quark mass. Massless QCD becomes good approximation. Further, asymptotic freedom implies quarks and gluons become progressively non-interacting. For the conditions accessible to HICs, massless limit of QCD with 3 flavours is relevant. The pressure in this limit:

$$\left(\frac{P}{T^4}\right)_{\text{ideal}} = \frac{8\pi^2}{45} + \sum_{f=u,d,s}\left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_f}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_f}{T}\right)^4\right]$$

Q.1: Starting from BE (gluons) and FD (quarks) distribution (see next slide), obtain the above pressure (count dof correctly)

Q.2: Find similar expressions for other thermodynamic quantities: energy density, entropy density, number density, speed of sound.

Low temperature limit

At low temperature, color confinement takes over and quarks and gluons are confined in colorless hadrons. Thus, baryons and mesons become relevant degrees of freedom in this region and a non-interacting gas of hadrons and all resonances- the hadron resonance gas (HRG) model provides a good approximation:

$$\ln \mathcal{Z}_{HRG}(T,V,\vec{\mu}) = \sum_{i \in\, mesons} \ln \mathcal{Z}^{M}_{m_i}(T,V,\vec{\mu}) + \sum_{i \in\, baryons} \ln \mathcal{Z}^{B}_{m_i}(T,V,\vec{\mu})$$

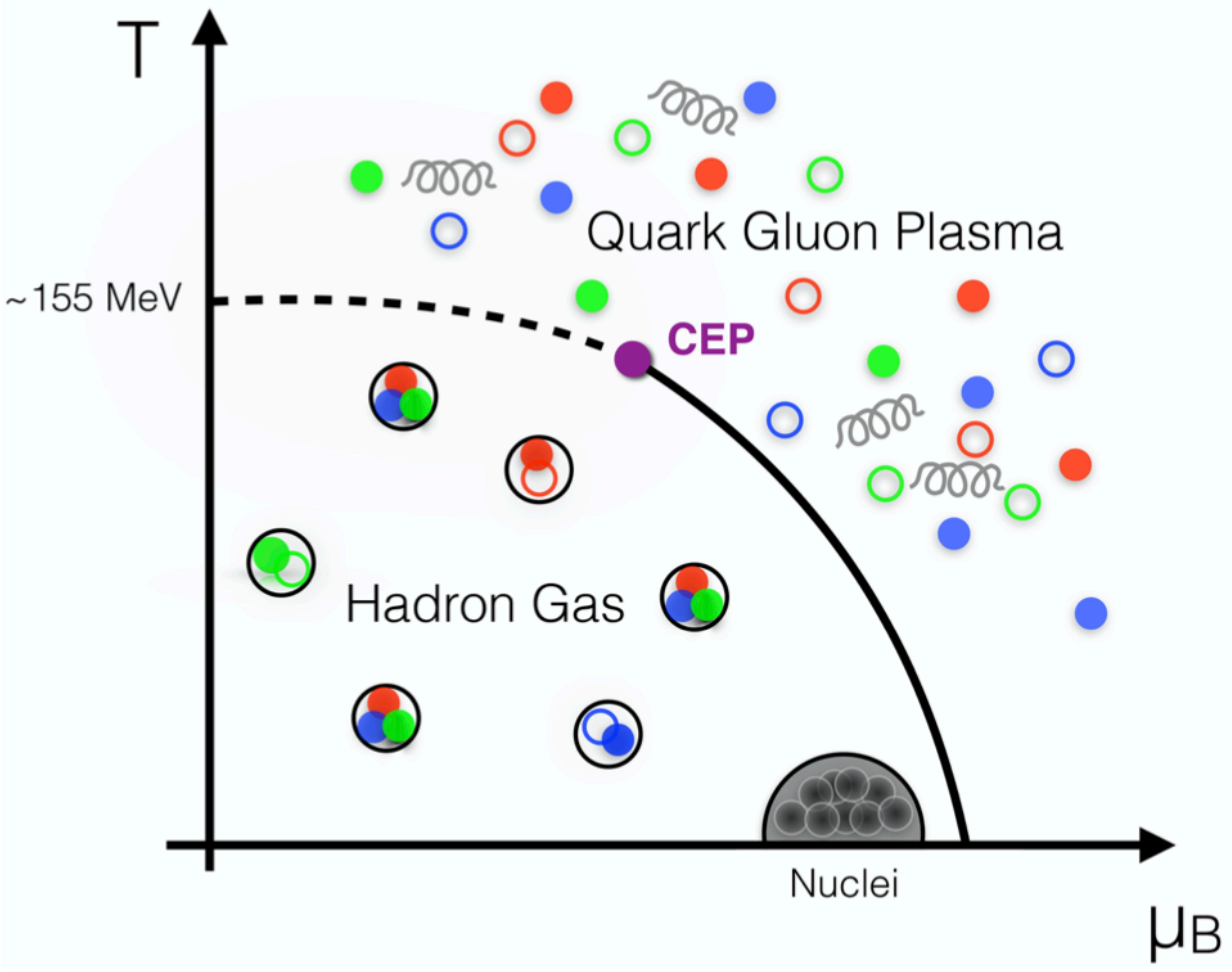
$$\ln \mathcal{Z}^{M/B}_{m_i}(T,V,\vec{\mu}) = \mp \frac{V}{2\pi^2} \int_0^\infty dk\, k^2 \, \ln(1 \mp z_i e^{-\varepsilon_i/T})$$

$$\varepsilon_i^2 = k^2 + m_i^2$$

$$z_i = \exp\left((B_i\mu_B + Q_i\mu_Q + S_i\mu_S)/T\right)$$

Q.3: Find expressions for the thermodynamic quantities: pressure, energy density, entropy density, number density, specific heat capacity.

QCD Phase Diagram



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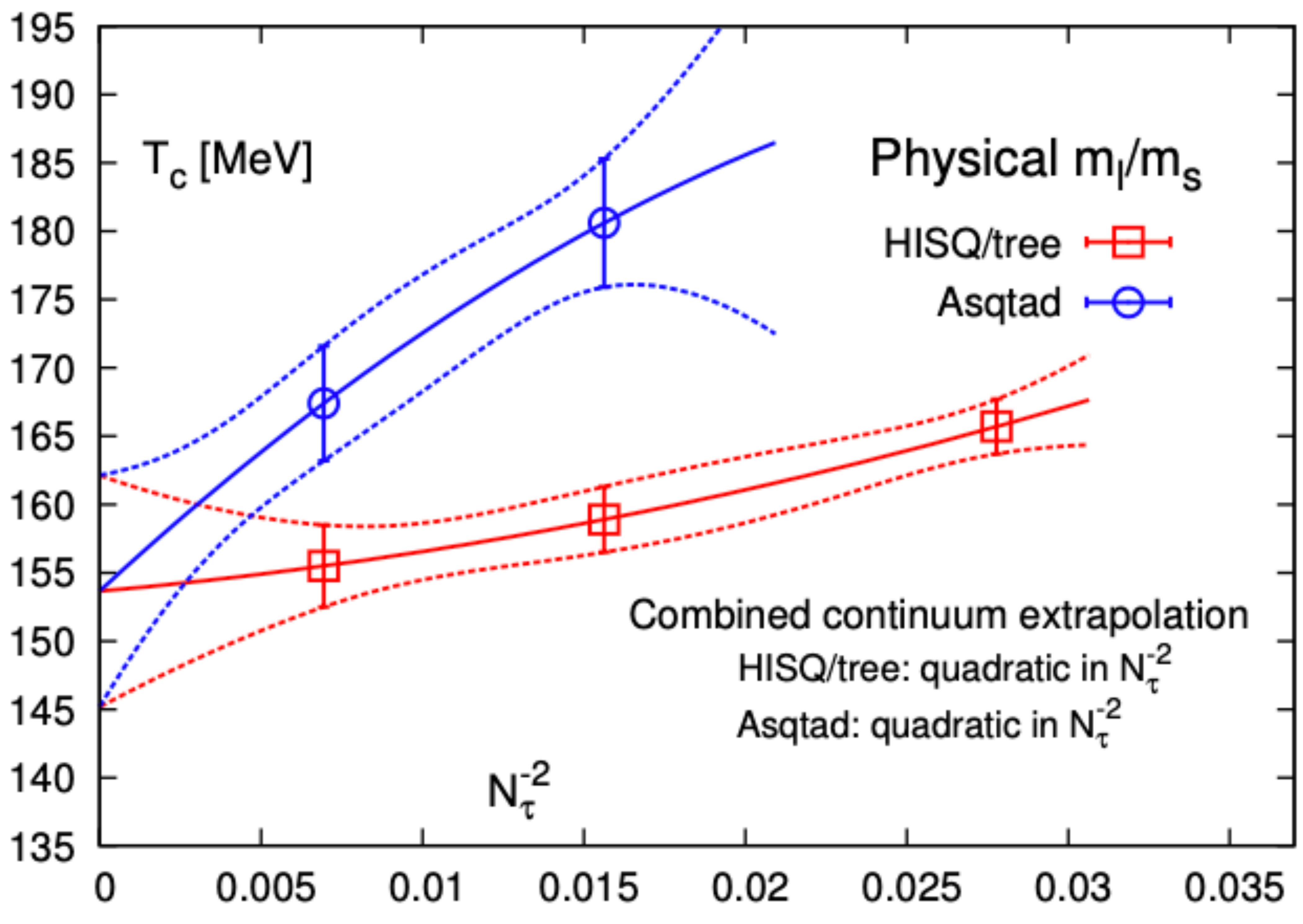
QCD has 3 conserved charges, namely baryon, electric and strangeness charges apart from energy. In the Grand-Canonical ensemble there are four corresponding Lagrange undetermined multipliers: namely T and the 3 chemical potentials: μ_B, μ_Q, μ_S

Hence most general QCD phase diagram is 4 dimensional with T, μ_B, μ_Q, μ_S as the axes

In HICs, the initial conditions of the incoming nuclei are such:
Net Strangeness = 0
Net Baryon / Net Electric = 2.5

This results in T-μ_B as the free parameters (assuming complete mixing)

QCD Phase Diagram

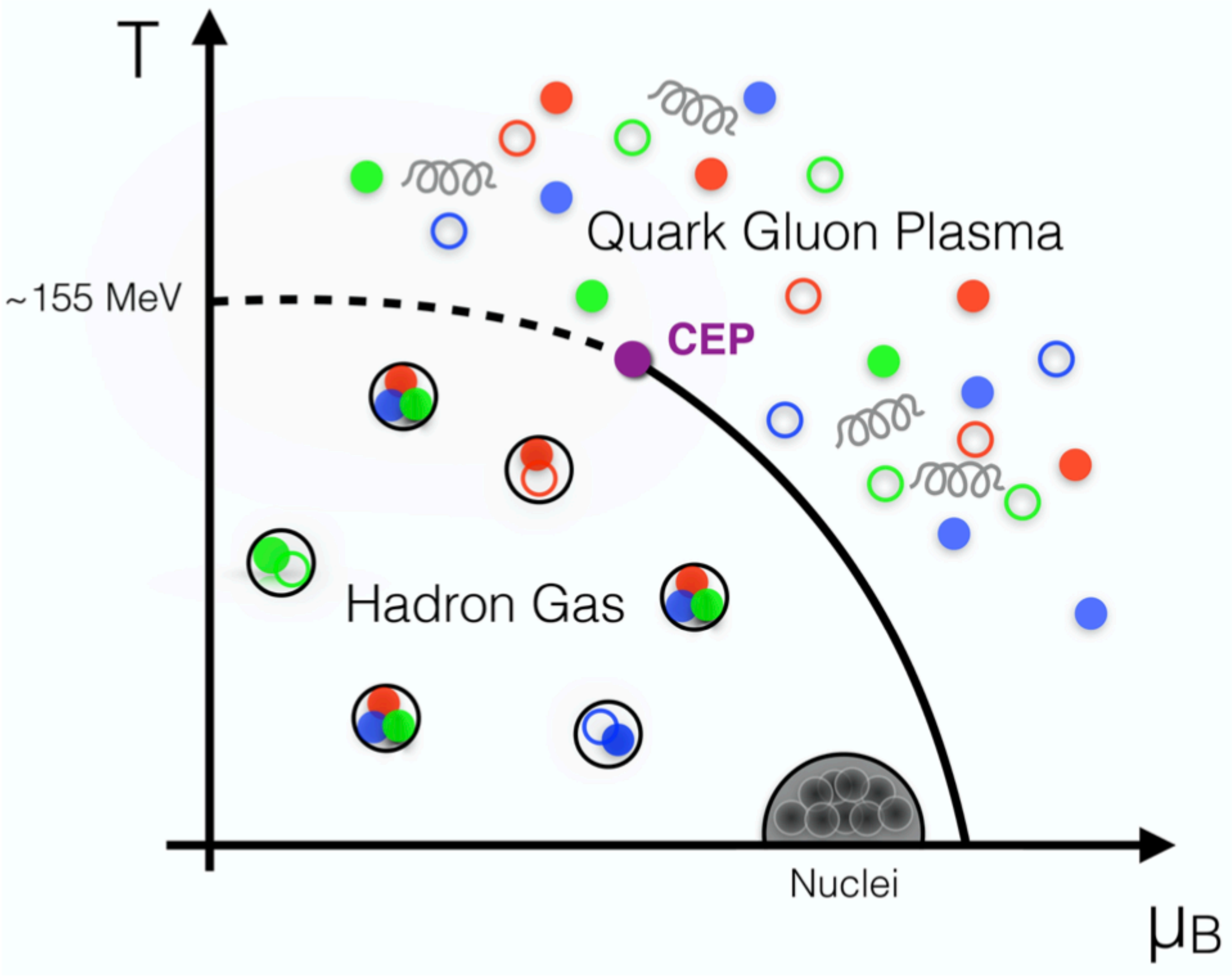


Chiral crossover temperature in (2+1) flavor physical QCD:

$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

Ref: 1504.05274

QCD Phase Diagram



Chiral crossover temperature in (2+1) flavor physical QCD:

$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

Curvature of the Chiral crossover temperature in (2+1) flavor physical QCD:

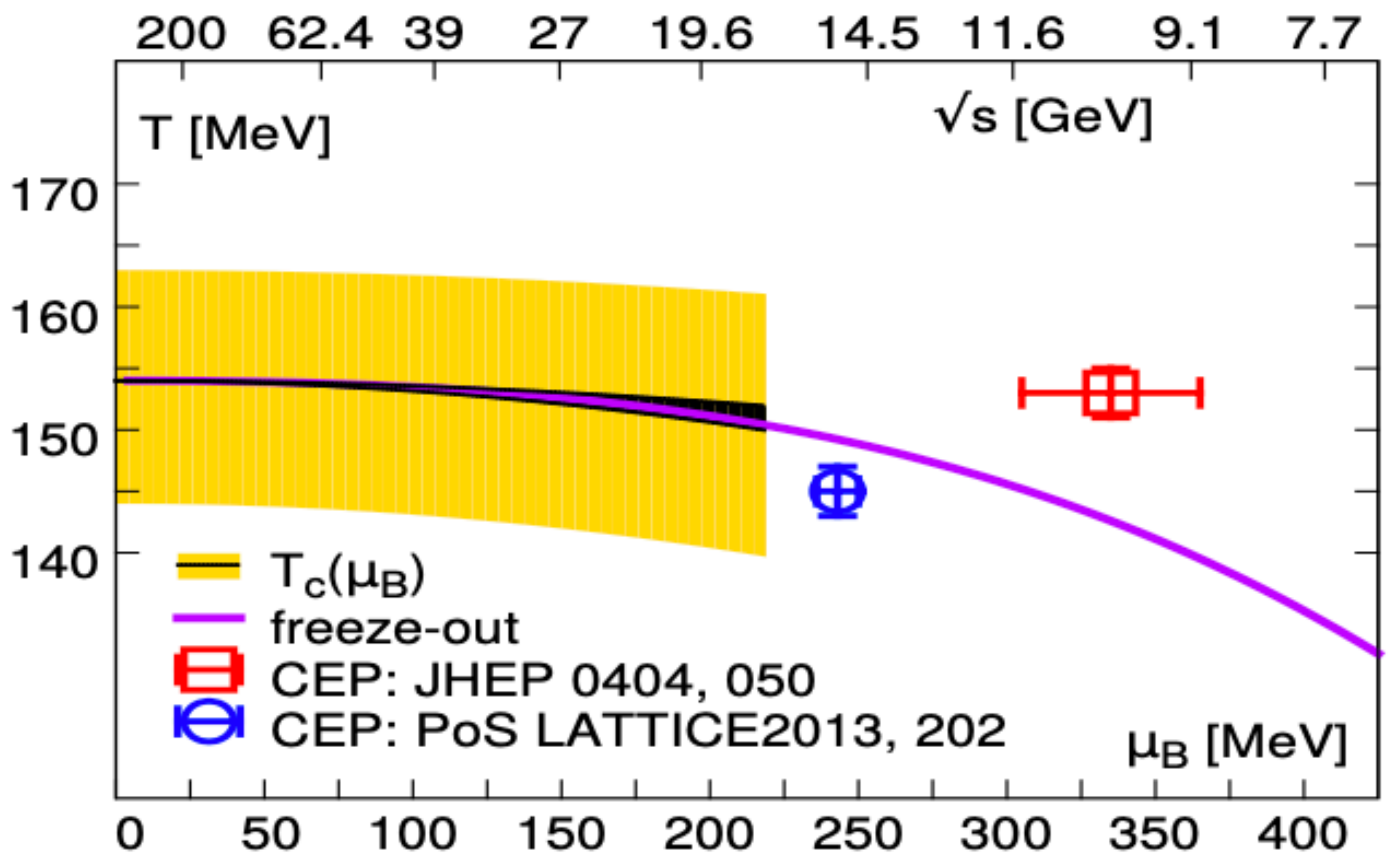
$$T_c^0(\mu_B) = T_c^0 \left[1 - 0.0066(7) \left(\frac{\mu_B}{T_c^0} \right)^2 + \mathcal{O}\left[\left(\frac{\mu_B}{T_c^0} \right)^4 \right] \right]$$

NOTE: this result is for real μ_B; approaches with imaginary μ_B get a factor of 2 larger curvature

QCD CEP: ?

Ref: 1504.05274

QCD Phase Diagram



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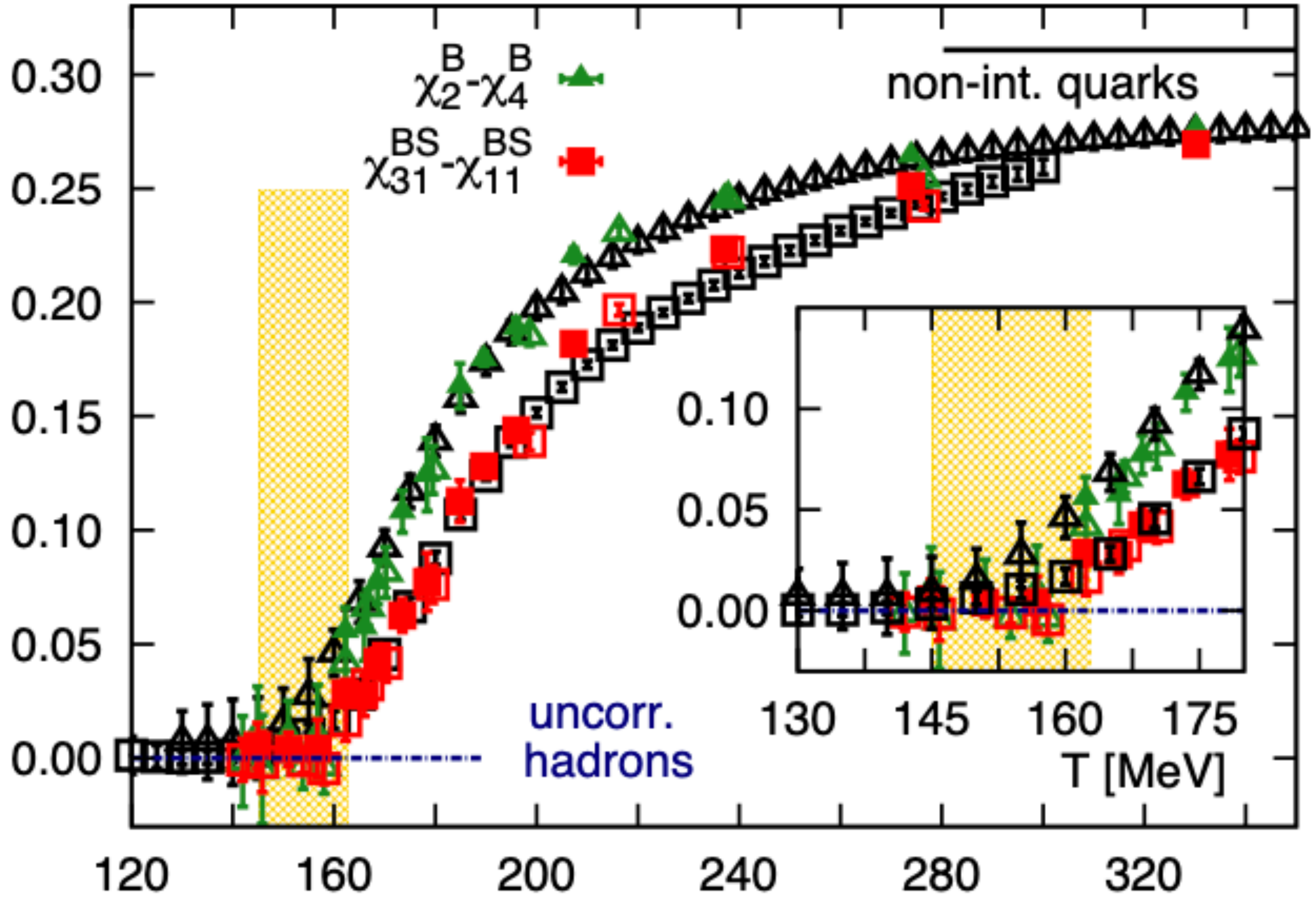
Color liberation



The observation of simultaneous liberation of color degrees of freedom as well as chiral transition is non-trivial and not well understood! Historically, this has been a long standing issue.

Q.4: Show that in HRG, the following are zero: $\chi_2^B - \chi_4^B$ and $\chi_{31}^{BS} - \chi_{11}^{BS}$

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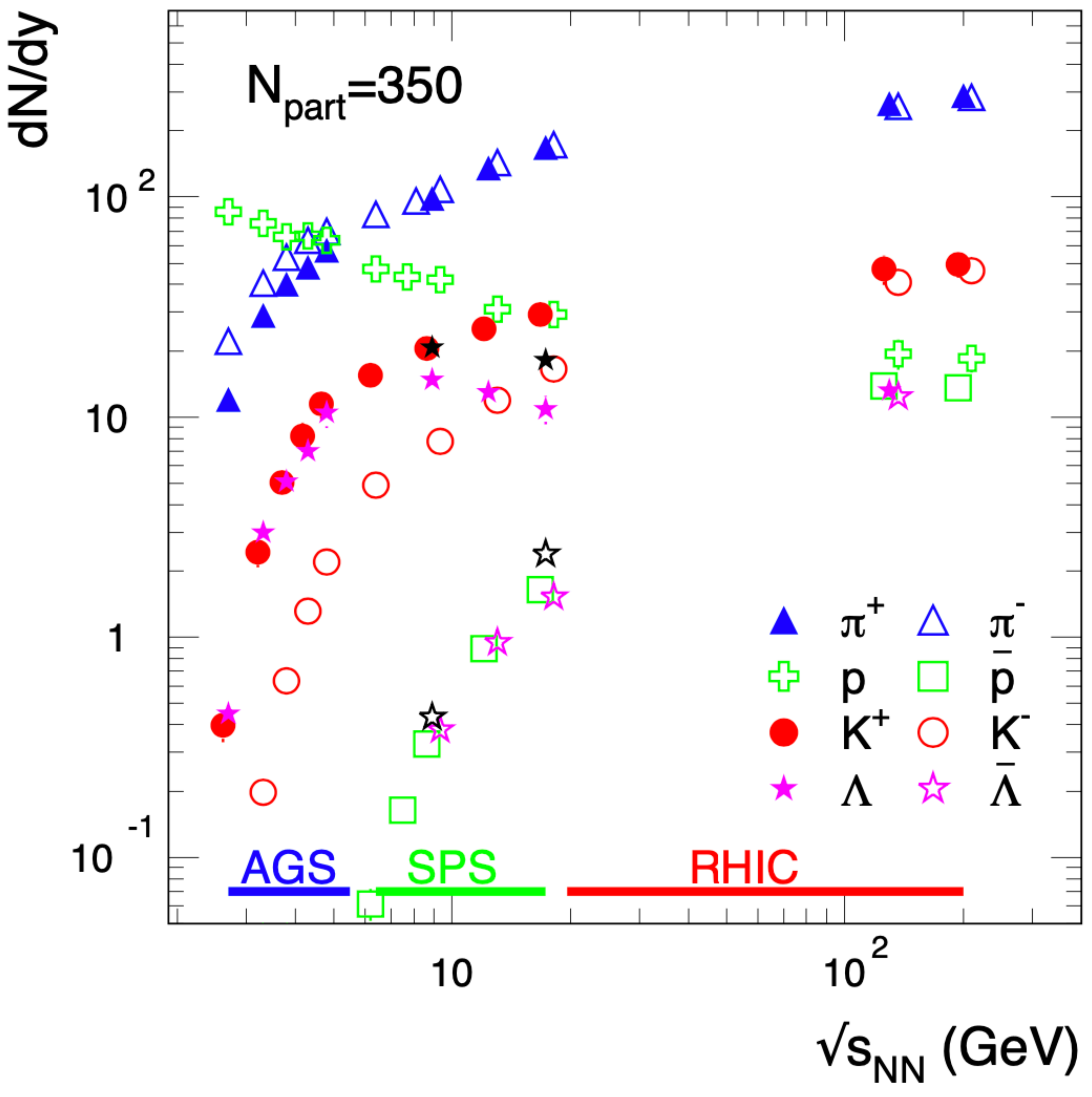
2. Contact with experiments via phase space integrated observables: I. Mean hadron yields

Mean hadron yields: Establish freeze out



Ref: arXiv: 0511071

NOTE:

1. The yields rise rapidly for small $\sqrt{s_{NN}}$, beyond $\sqrt{s_{NN}}$ > 20 GeV the mesons rise gently while anti-baryons continue to have a strong rise,
2. Yield of p and Λ decrease with increase in $\sqrt{s_{NN}}$,
3. The phase space integrated hadron yield can provide information about the thermodynamic conditions of the surface of last scattering. Good agreement between LQCD and HRG motivates fitting the yields with HRG

$$n_i = N_i/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T]\pm 1}$$

Mean hadron yields: Establish freeze out

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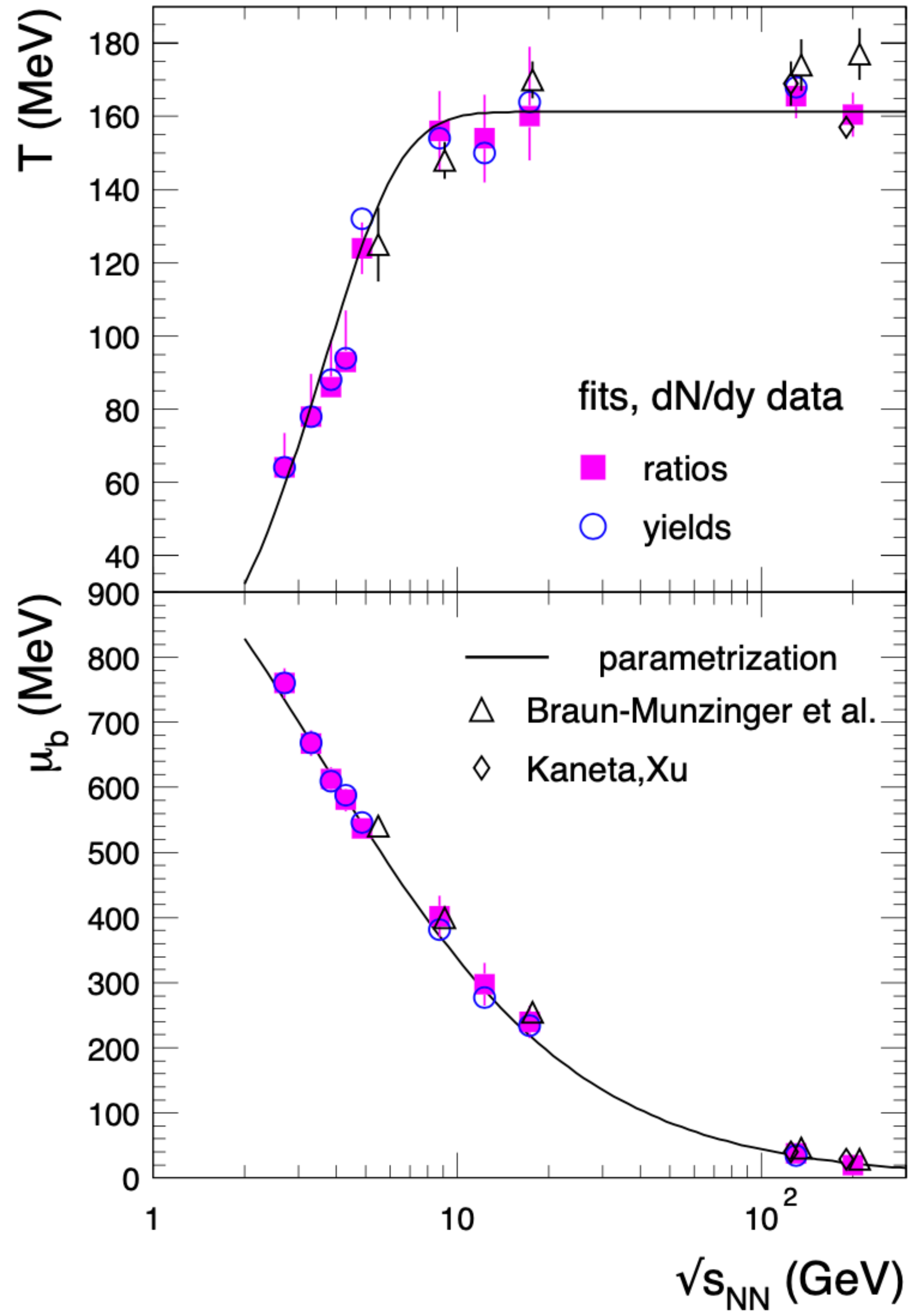
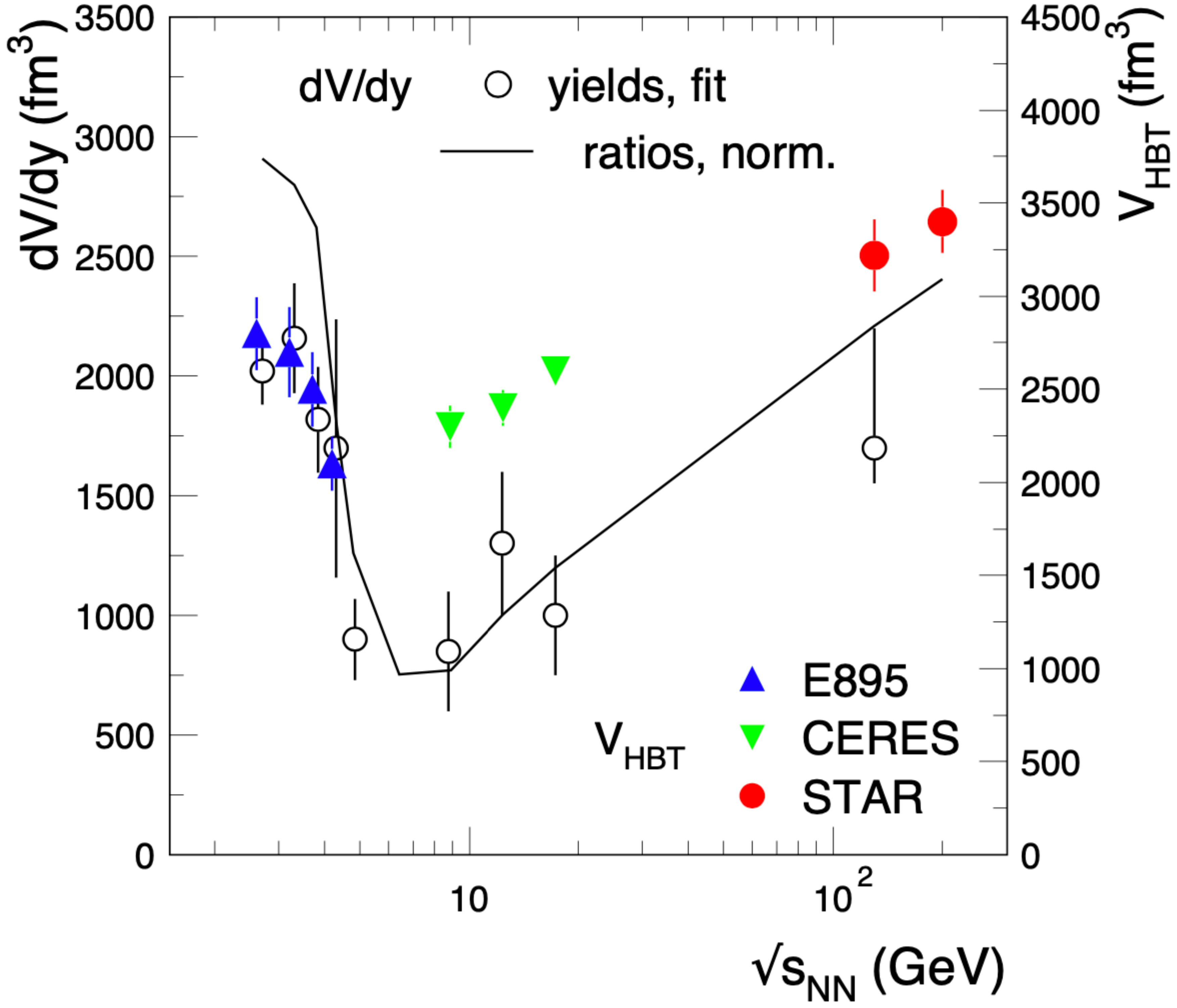
Mean hadron yields: Establish freeze out





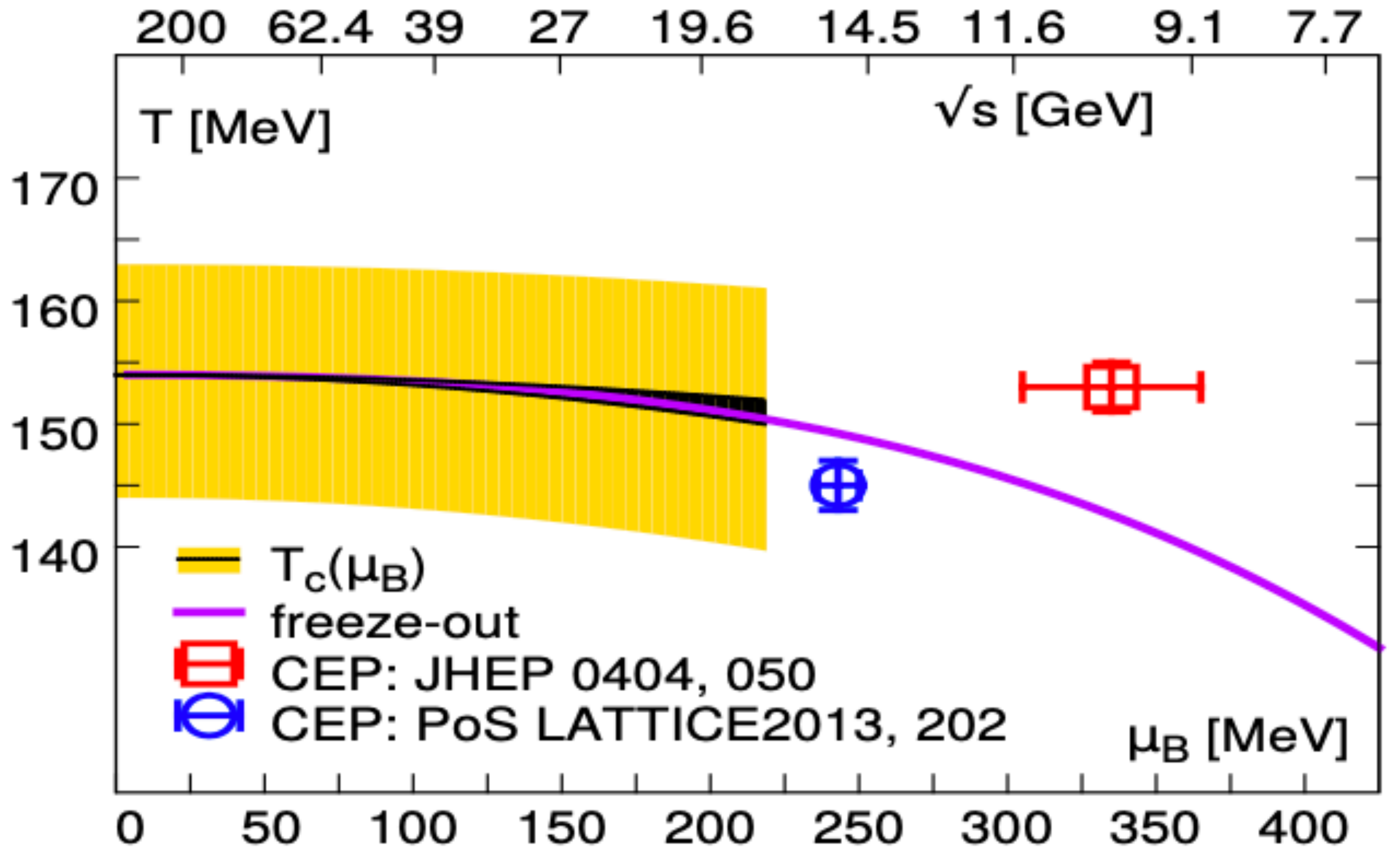
$$T[\mathrm{MeV}] = T_{lim}\left(1 - \frac{1}{0.7 + (\exp(\sqrt{s_{NN}}(\mathrm{GeV})) - 2.9)/1.5}\right)$$

$$\mu_b[\mathrm{MeV}] = \frac{a}{1 + b\sqrt{s_{NN}}(\mathrm{GeV})}$$

$a = 1303 \pm 120$ MeV and $b = 0.286 \pm 0.049$ GeV^{-1}

Ref: arXiv: 0511071

QCD Phase Diagram



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NOTE: this result is for real μ_B; approaches with imaginary μ_B get a factor of 2 larger curvature

QCD CEP: ?

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Freezeout curve

2. Contact with experiments via phase space integrated observables: II. Fluctuations of conserved charges

Fluctuations of conserved charges

From theory

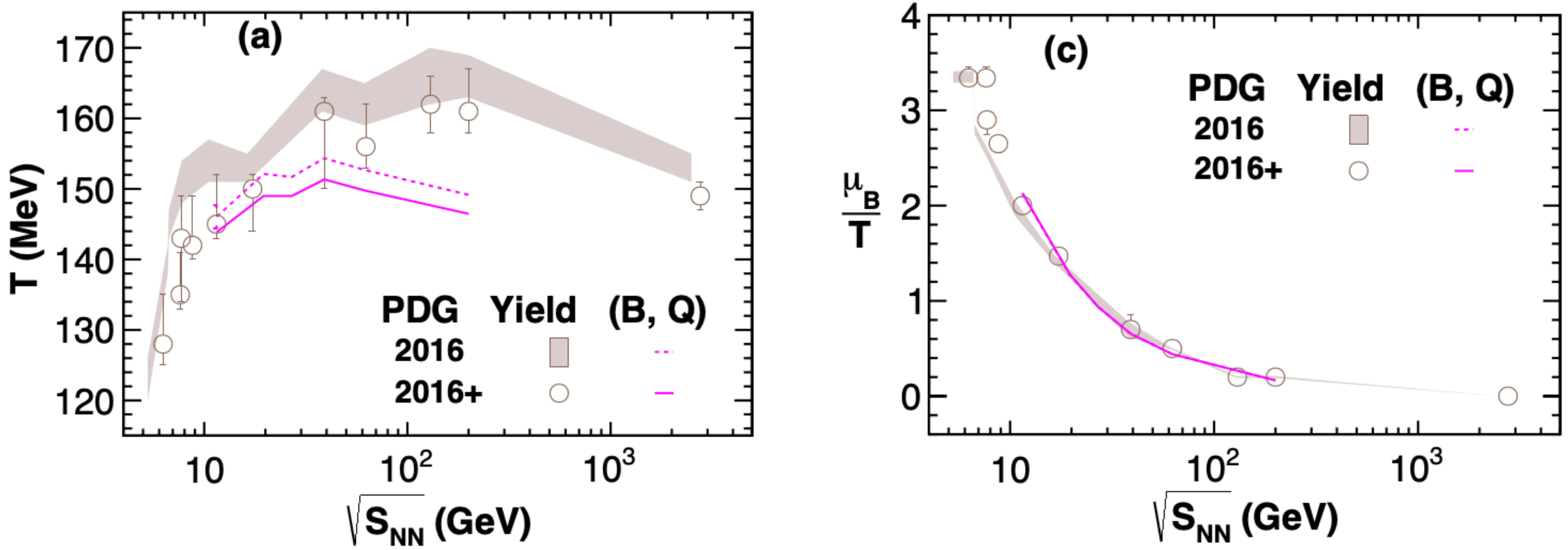
$$\chi_1^Q\left(T,\mu_B\right)=\frac{1}{VT^3}\left\langle N_Q\right\rangle\ ,$$
$$\chi_2^Q\left(T,\mu_B\right)=\frac{1}{VT^3}\left\langle\left(\delta N_Q\right)^2\right\rangle\ ,$$
$$\chi_3^Q\left(T,\mu_B\right)=\frac{1}{VT^3}\left\langle\left(\delta N_Q\right)^3\right\rangle\ ,$$
$$\chi_4^Q\left(T,\mu_B\right)=\frac{1}{VT^3}\left[\left\langle\left(\delta N_Q\right)^4\right\rangle-3\left\langle\left(\delta N_Q\right)^2\right\rangle^2\right]$$

$$\delta N_Q=N_Q-\left\langle N_Q\right\rangle$$

From experiments

$$M_Q\left(\sqrt{s}\right)=\left\langle N_Q\right\rangle$$
$$S_Q\left(\sqrt{s}\right)=\frac{\left\langle\left(\delta N_Q\right)^3\right\rangle}{\sigma_Q^3}$$
$$\sigma_Q^2\left(\sqrt{s}\right)=\left\langle\left(\delta N_Q\right)^2\right\rangle\ ,$$
$$\kappa_Q\left(\sqrt{s}\right)=\frac{\left\langle\left(\delta N_Q\right)^4\right\rangle}{\sigma_Q^4}-3$$

Fluctuations of conserved charges: HRG



Ref: 1708.08152

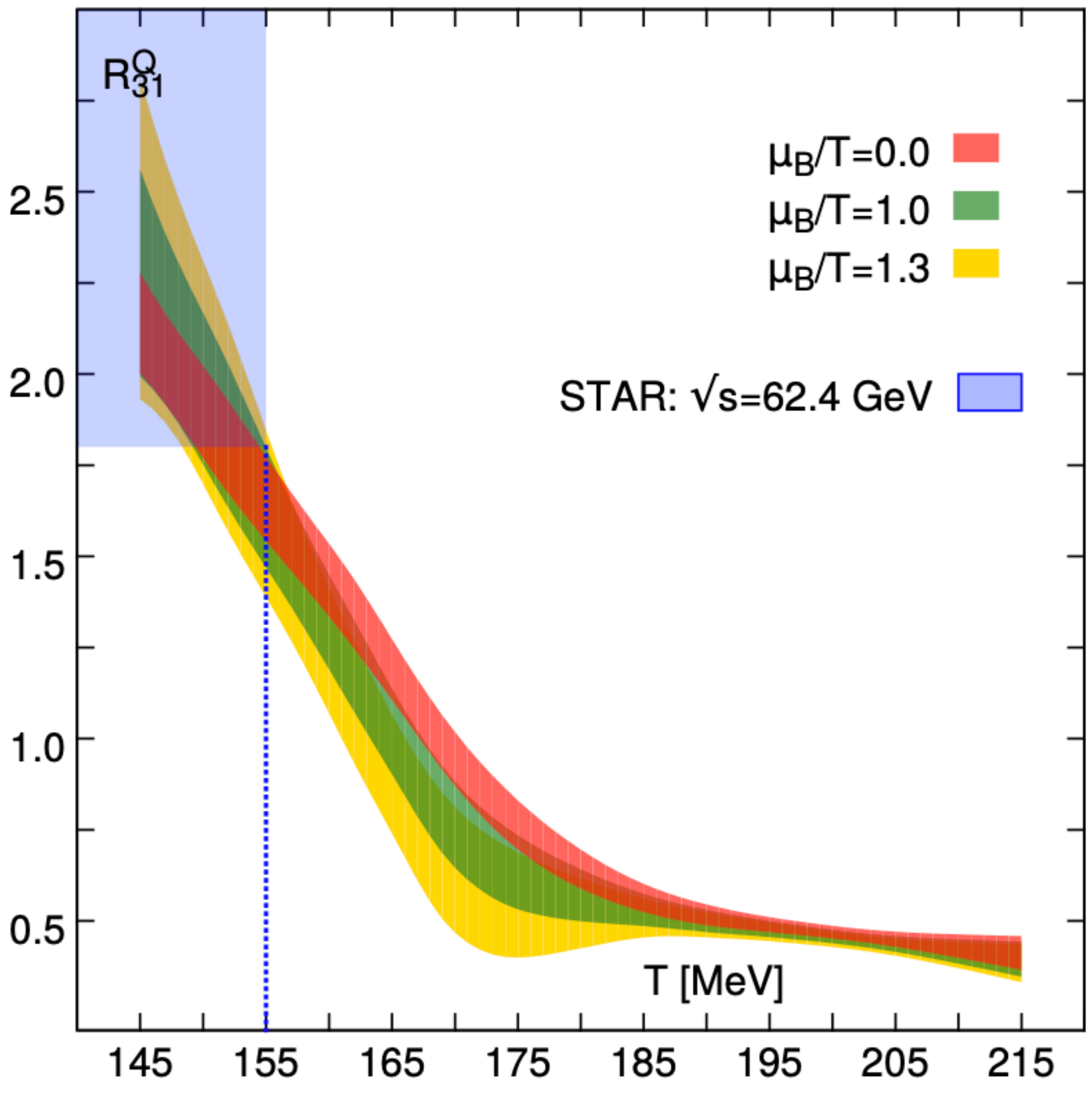
Fluctuations of conserved charges

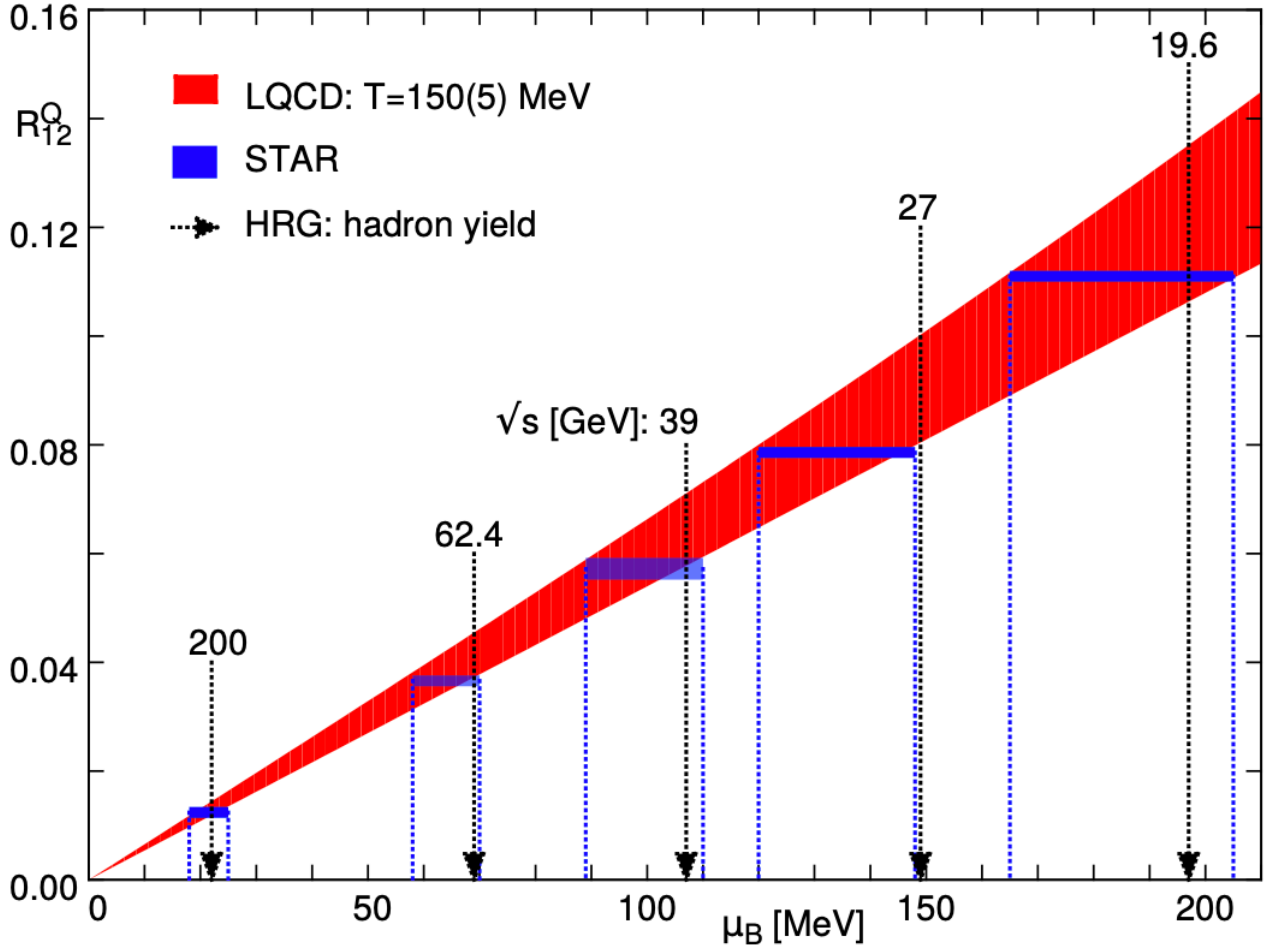
LQCD meets experiments: construct suitable volume independent ratios

$$\frac{M_Q\left(\sqrt{s}\right)}{\sigma_Q^2\left(\sqrt{s}\right)} = \frac{\chi_1^Q\left(T,\mu_B\right)}{\chi_2^Q\left(T,\mu_B\right)} \equiv R_{12}^Q$$

$$\frac{S_Q\left(\sqrt{s}\right)\sigma_Q^3\left(\sqrt{s}\right)}{M_Q\left(\sqrt{s}\right)} = \frac{\chi_3^Q\left(T,\mu_B\right)}{\chi_1^Q\left(T,\mu_B\right)} \equiv R_{31}^Q$$

Fluctuations of conserved charges: LQCD





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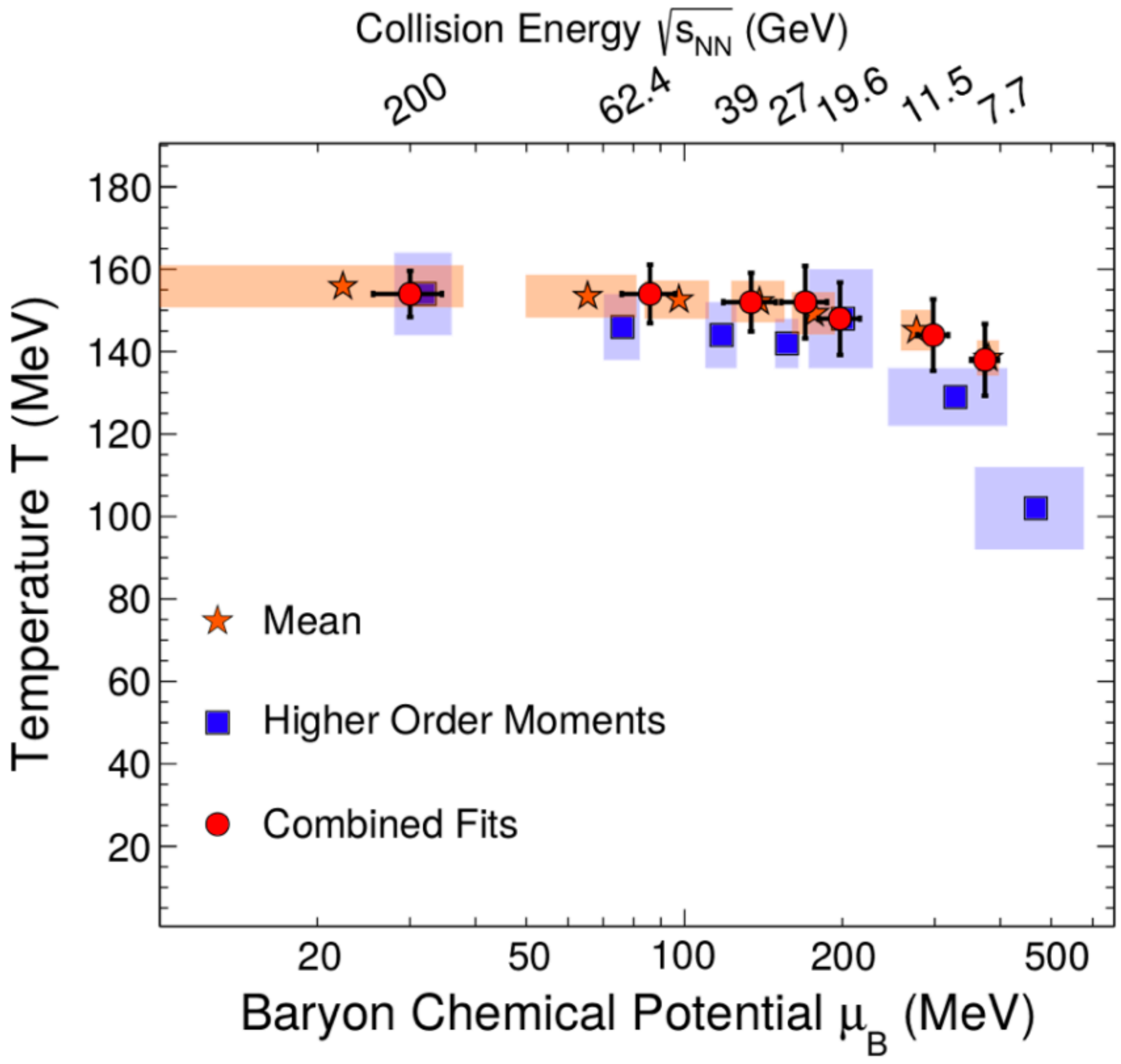
Freezeout: mean, higher moments



Ref:2103.13641

NOTE: higher order estimates of freeze out temperature are systematically lower,
Does it imply mean and higher moments do not freezeout together?

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3. Contact with experiments via phase space differential observables

Static vs expanding medium

The HRG/LQCD represents a static medium in complete thermal and chemical equilibrium at freezeout. For example, in HRG

$$n_i = N_i/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T] \pm 1}$$

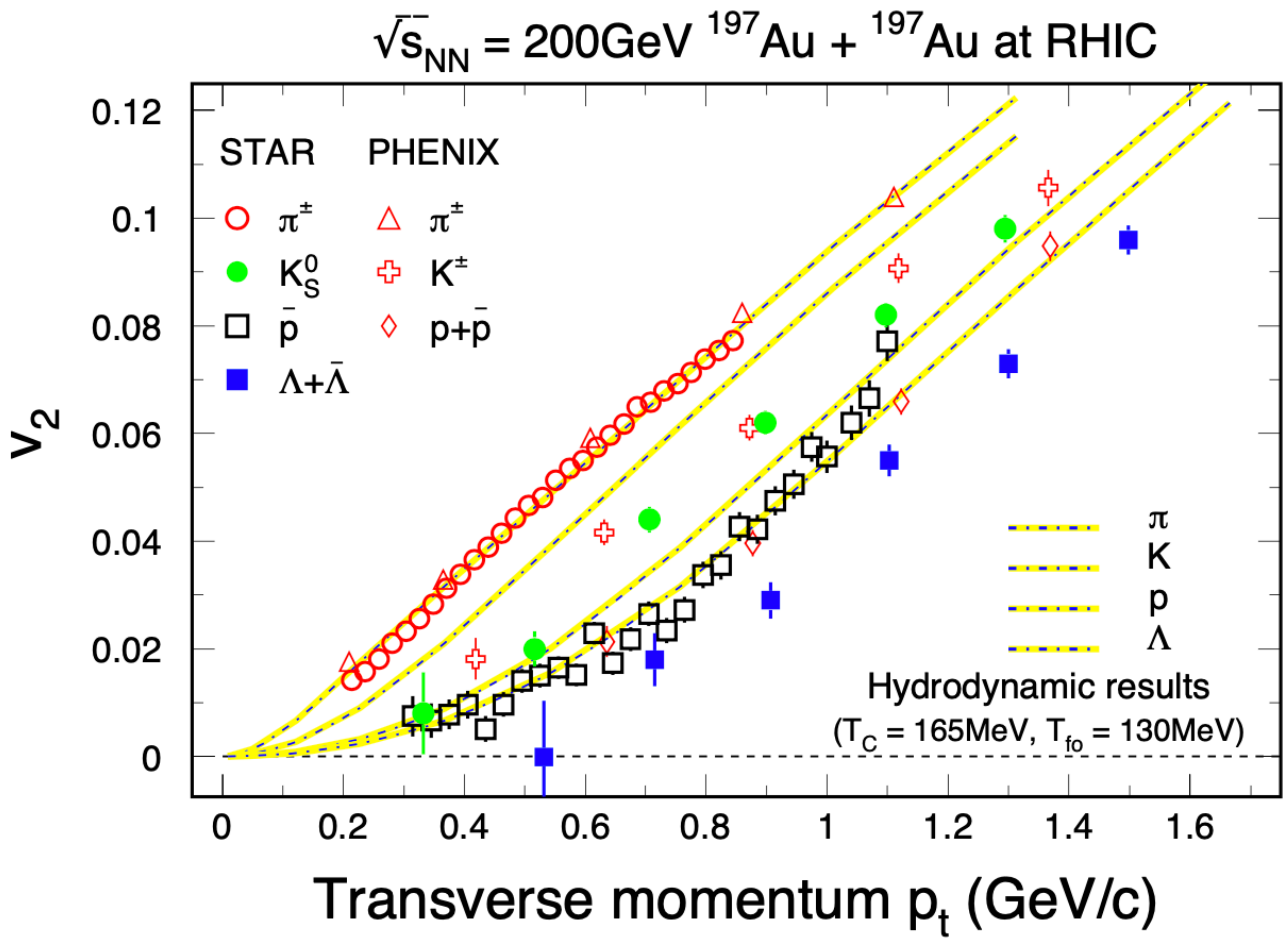
As a result, all directions are equivalent, at least the transverse directions at mid-rapidity

The momentum space occupation of the emerging hadrons from the fireball at the time of freezeout may be studied by suitable Fourier decomposition:

$$\frac{d^3N}{p_T dp_T dy d\phi} = \frac{d^2N}{p_T dp_T dy}\frac{1}{2\pi}\left[1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \Psi_n)\right]$$

$$v_n(p_T, y) = \langle \cos[n(\phi - \Phi_R)]\rangle = \frac{\int_0^{2\pi} d\phi \cos[n(\phi - \Phi_R)] \frac{d^3N}{p_T dp_T dy d\phi}}{\int_0^{2\pi} d\phi \frac{d^3N}{p_T dp_T dy d\phi}}.$$

Static vs expanding medium

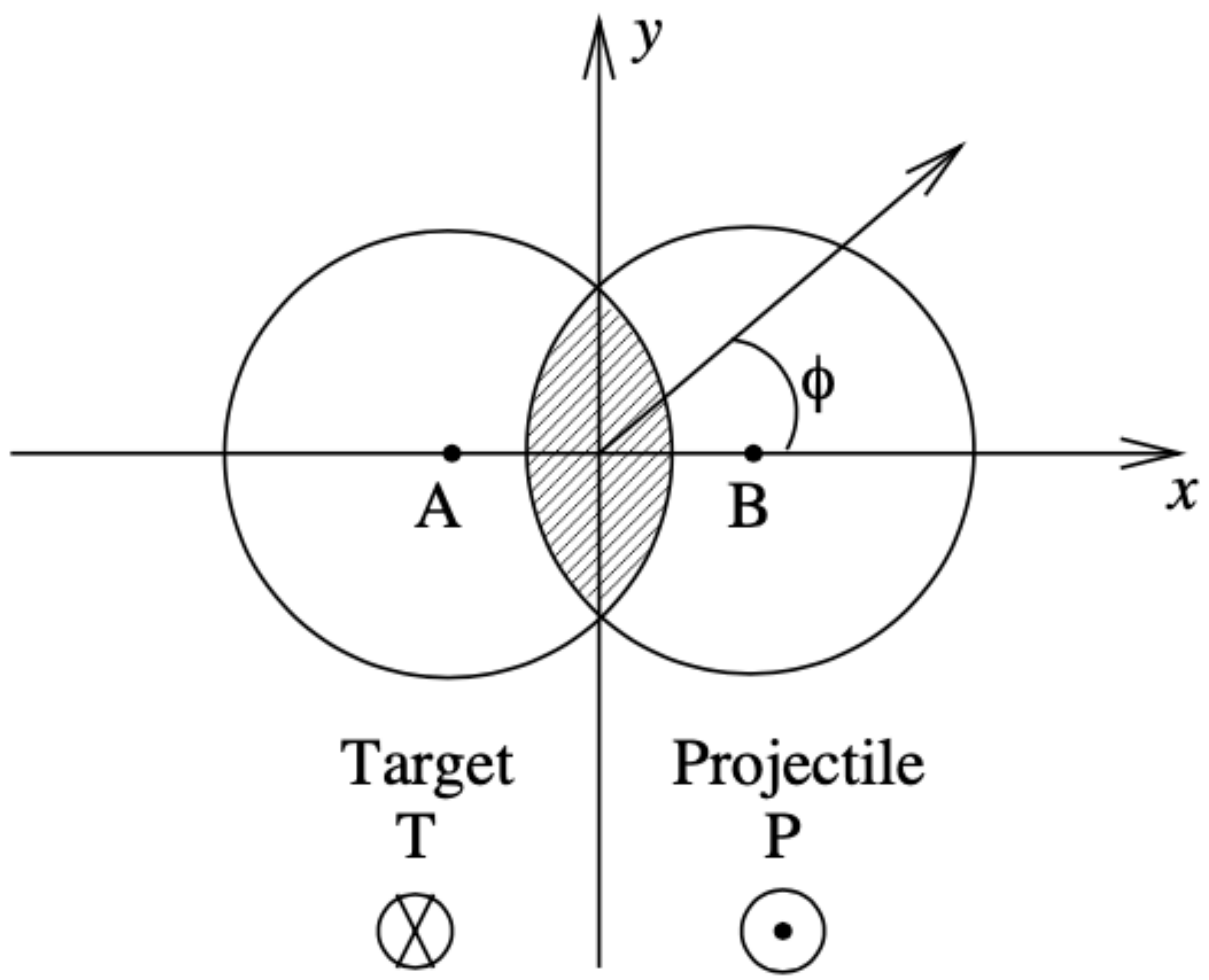


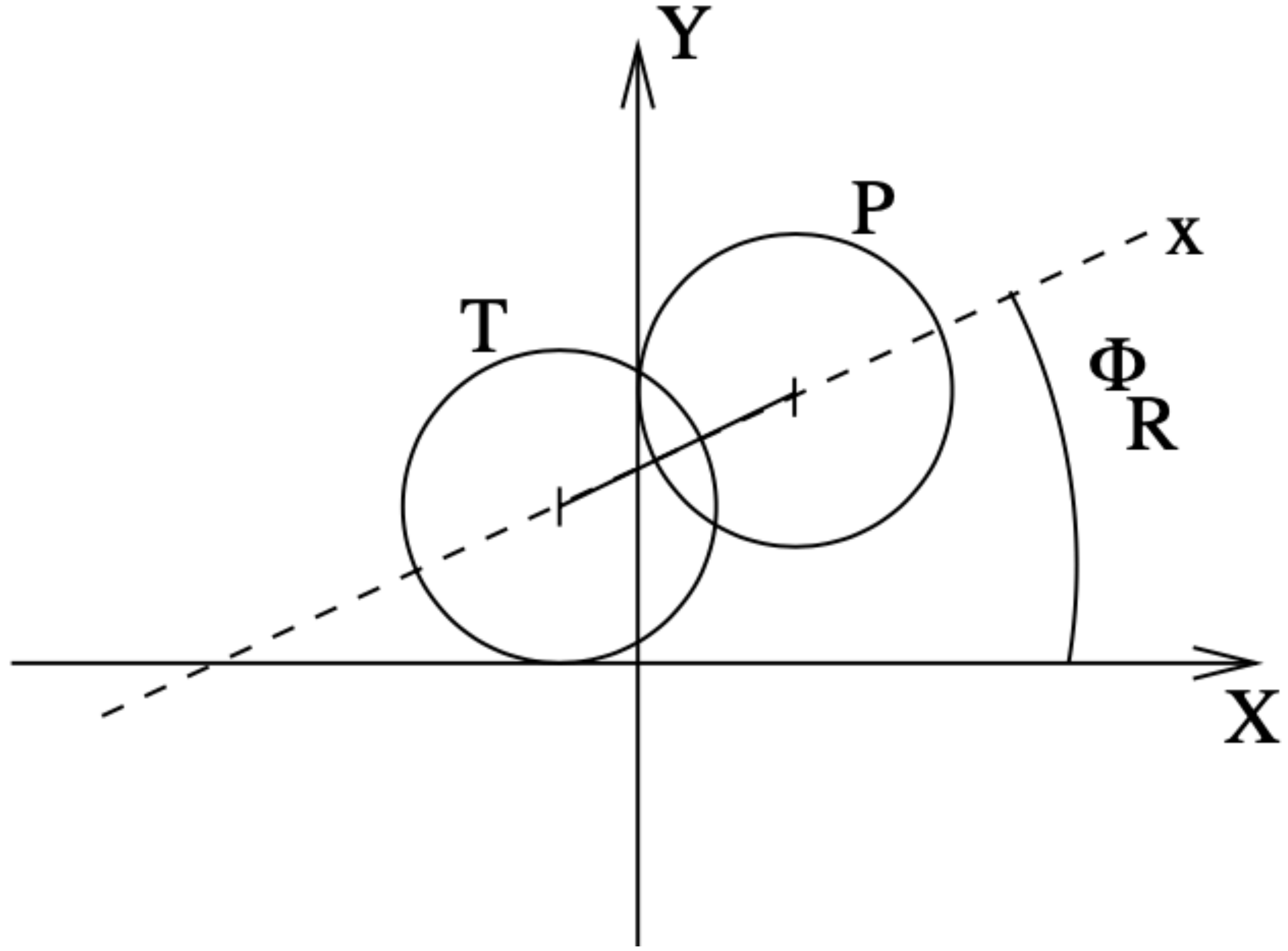
Non-zero second harmonic flow coefficient at freezeout !

The phase space differential observable like the second harmonic flow coefficient reveal the medium at freezeout has more features than just being in thermal and chemical equilibrium.

Non-zero flow coefficient reveal breaking of azimuthal symmetry in the transverse plane. Such breaking of azimuthal symmetry is only possible in the initial condition.

Initial Geometry: Nucleus

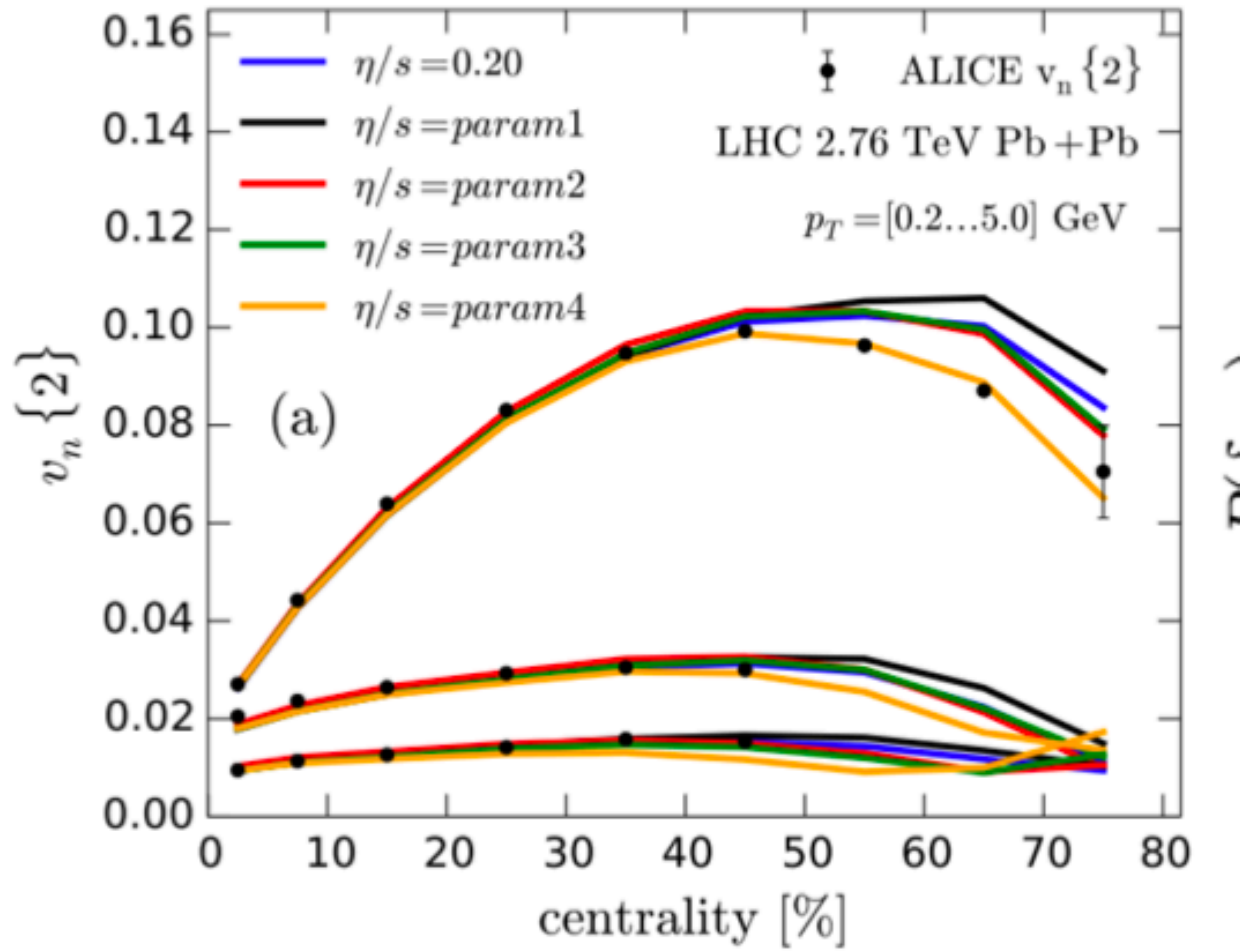




$$\frac{d^3N}{p_T dp_T dy d\phi} = \frac{d^2N}{p_T dp_T dy}\frac{1}{2\pi}\left[1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \Phi_R)\right]$$

NOTE: for A+A collision, there is symmetry of $\phi - \phi_R \rightarrow \phi - \phi_R + \pi$, thus only even harmonics contribute

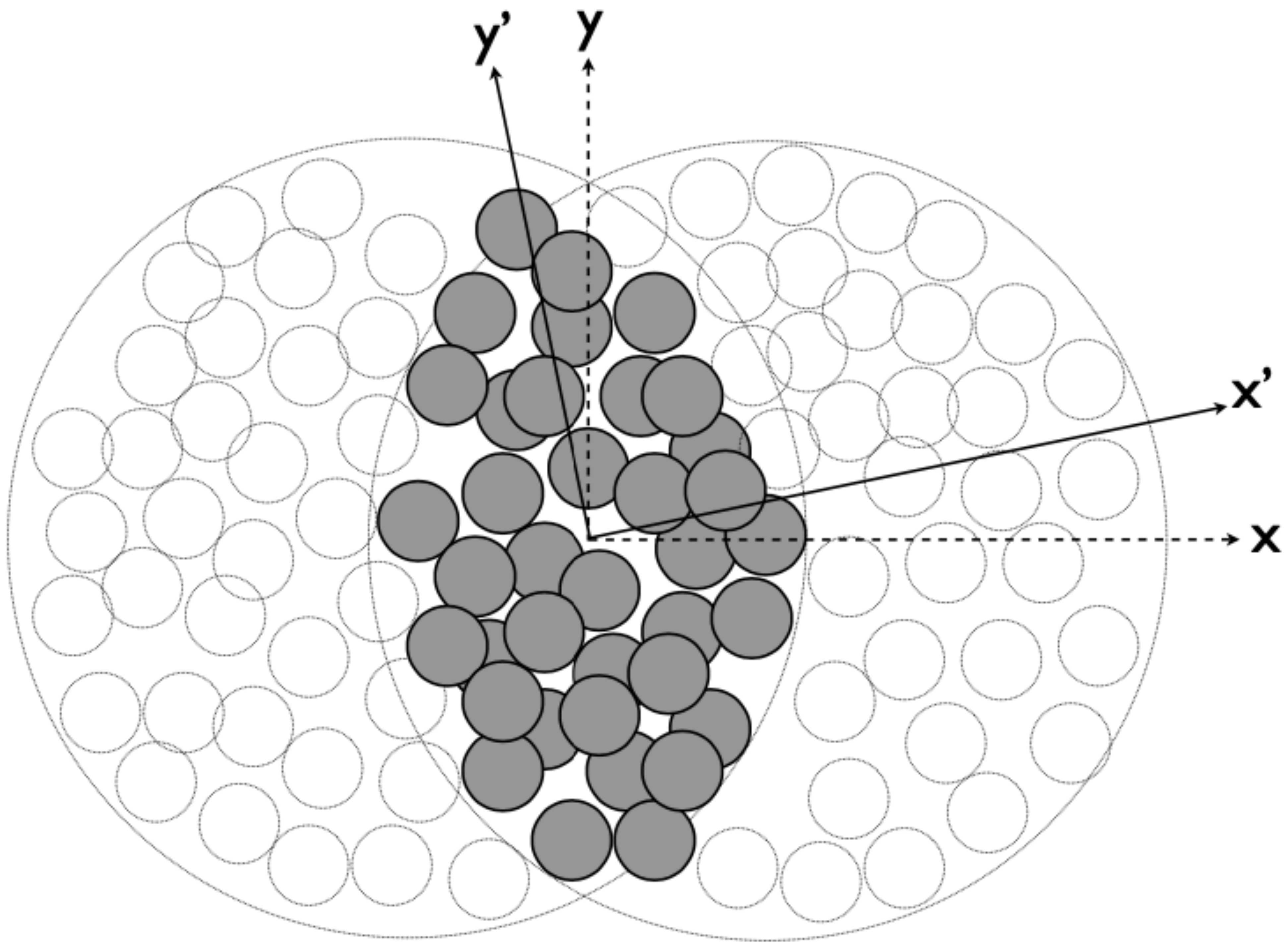
Initial Geometry



Sizable odd harmonic flow!

Need to break the $\phi - \phi_R \rightarrow \phi - \phi_R + \pi$ symmetry in the initial geometry

Initial Geometry: Nucleus+nucleons



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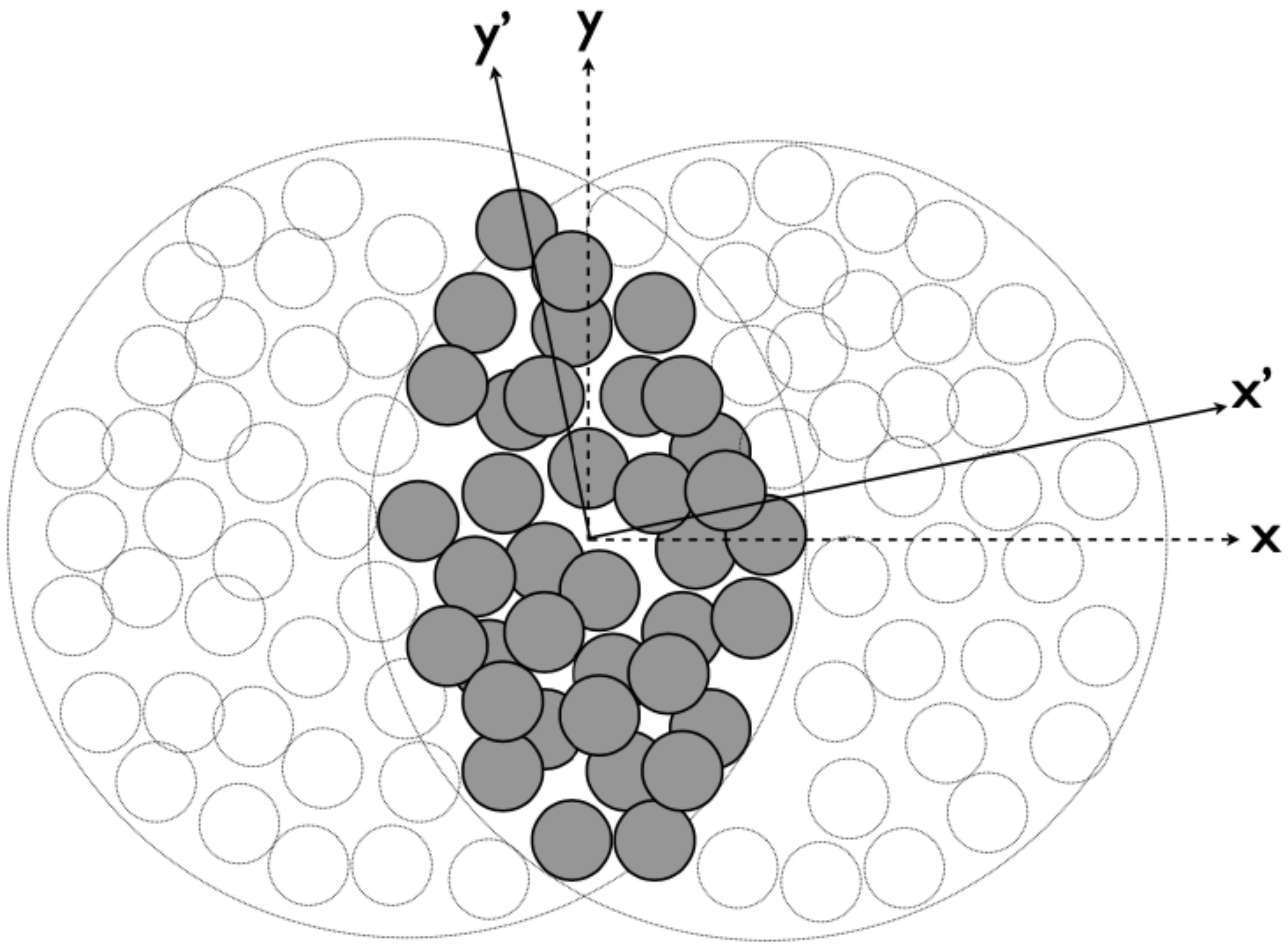
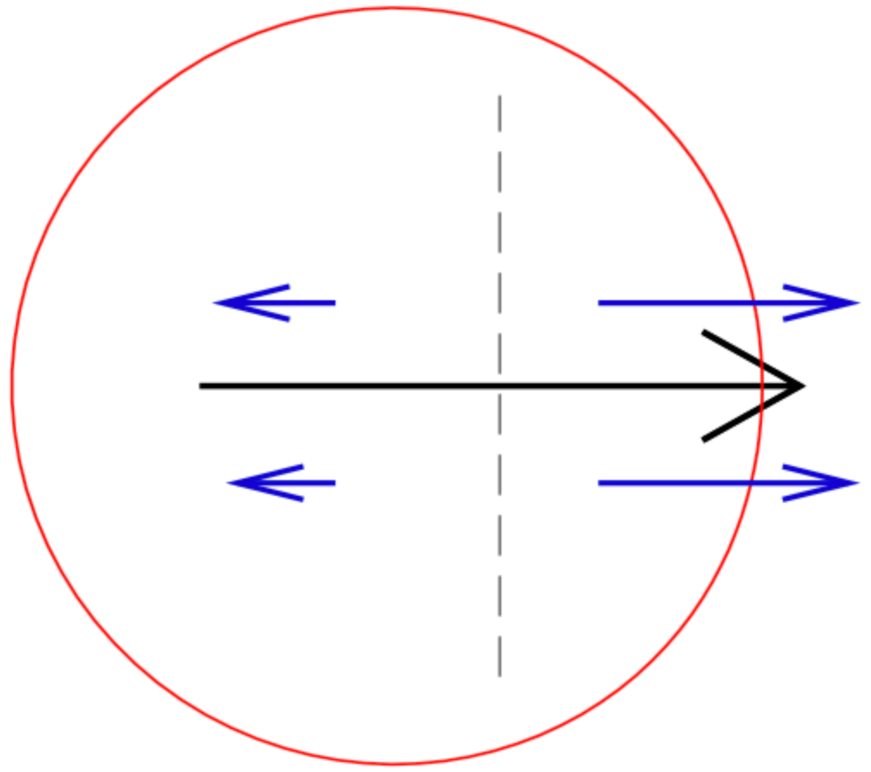
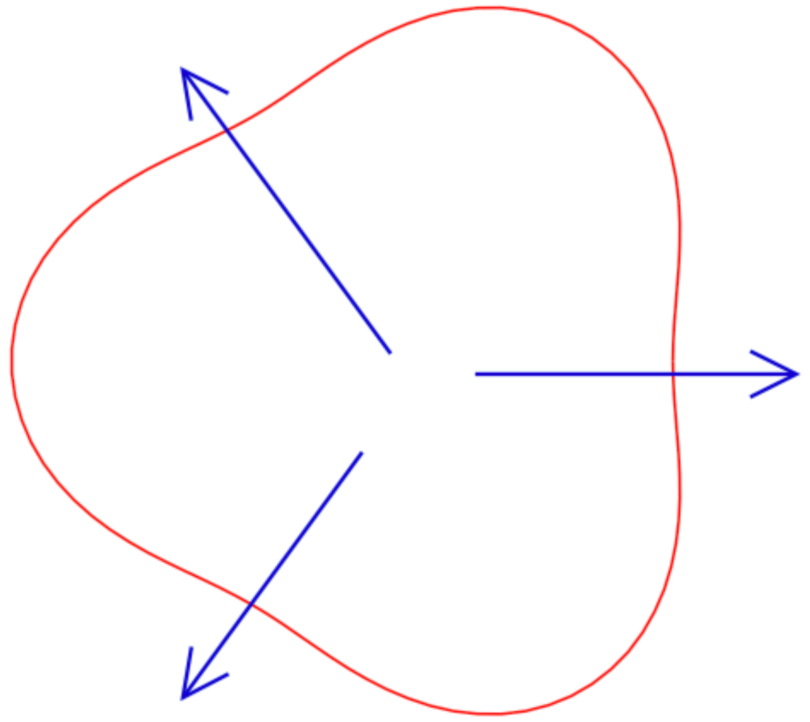
$$\frac{d^3N}{p_T dp_T dy d\phi} = \frac{d^2N}{p_T dp_T dy} \frac{1}{2\pi} \left[1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \Psi_n) \right]$$

Initial Geometry: Nucleus+nucleons



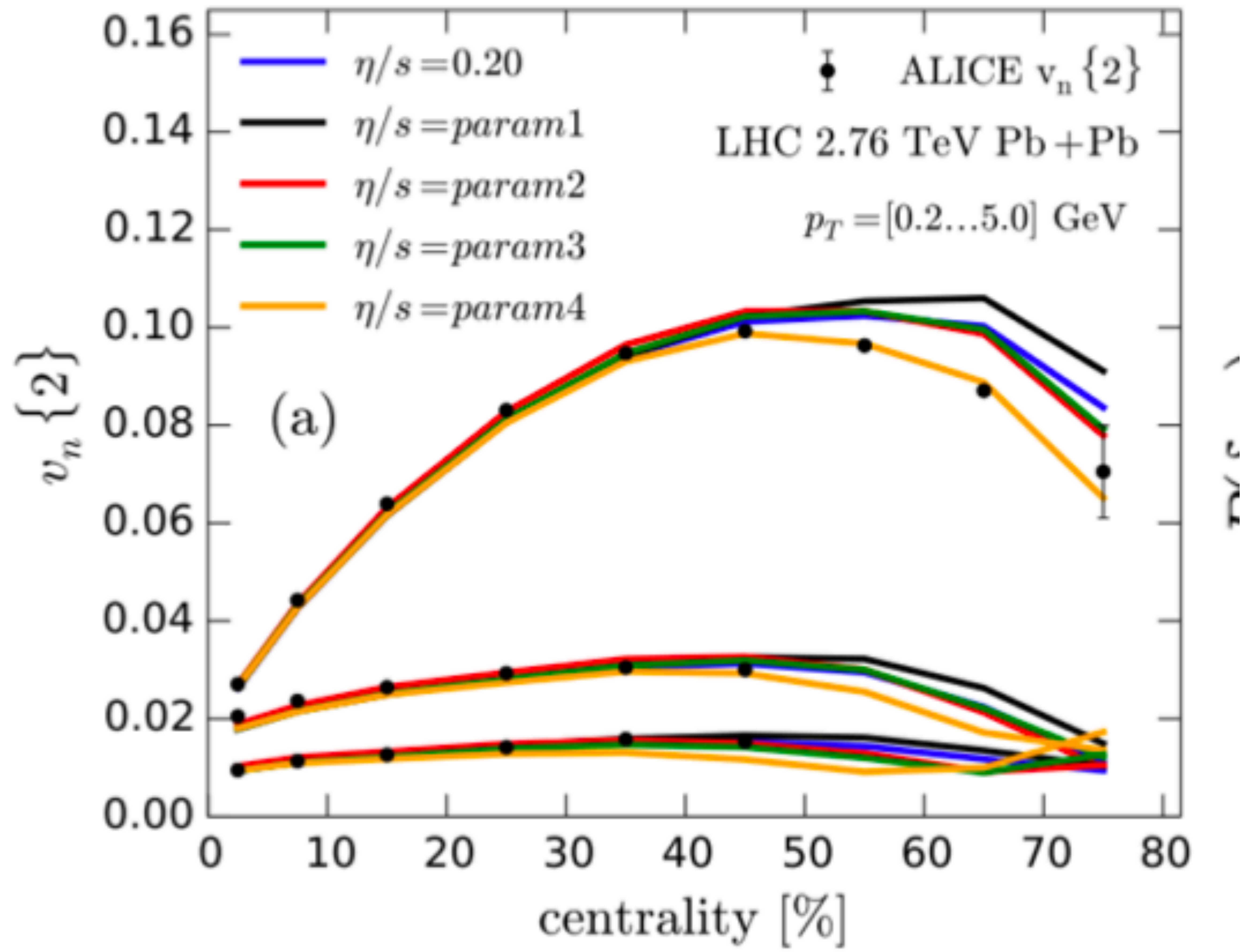


Dipole asymmetry gives rise to v_1



Triangular asymmetry gives rise to v_3

Initial Geometry+medium transport properties



To explain the phase space differential observables like flow at freezeout one needs to model the evolution of the fireball with appropriate initial conditions (where the symmetries are broken) and then appropriate medium transport properties like shear, bulk viscosities etc that control the magnitude of these flow coefficients

Thus one learns about initial condition as well as medium properties by phase space differential observables

More on this to be covered by Victor