Cross diffusion coefficients of hot and dense hadronic matter

Hiranmaya Mishra

School of Physical Science, NISER, Bhubaneswar, India

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Outline

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- Boltzmann equation and transport coefficients
- Landau-Lifstiz condition of fit and positivity of and transport coefficient
- Hadron resoance gas model (HRG/EVHRG))
  - Thermodynamics
  - Relaxation time estimation
- Results
- Summary and Outlook

Introduction

- Transport properties of hot/dense matter are important for heavy ion collision (HIC), cosmology and important for near equillibrium evolution of any thermodynamic system
- The most studied transport coefficient is perhaps shear viscosity η. In HIC spatial anisotrpy of colliding nuclei gets converted to momentum anisotropy throgh a hydro evoln. The equllibriation is decided by η. ($\frac{\eta}{s} \sim \frac{1}{4\pi}$, the KSS bound)
- The bulk viscosity ζ - thought earlier to be not important for HIC hydro evolution. Argument: $\zeta \sim (\epsilon - 3p)/T^4$ that vanishes for ideal gas. However, lattice simulation $\Rightarrow$ large $(\epsilon - 3p)/T^4$ near T_c. This,in turn, can give rise to different physical effects (Cavitation).

Introduction -Contd. · · ·

- Otherthan viscosites, the electrical conductivies have been studied on lattice, pQCD and effective models of strong interaction physics. This coefficient has been important in magneto hydrodynamical simulations. Time evolution of magnetic field depend crucially on σ_{el}.
- The temperature and chemical potential dependence of transport coefficients may reveal the location of phase transition
- In principle, one can estimate transport coefficients using Kubo formulation but, QCD is strongly coupled for the energies accessible at HIC. Lattice QCD simulation is numerically challanging and also has problems in doing simulations at finite baryon densities.
- We shall approach the problem here within Boltzmann kinetic equation within relaxation time approximation
- Diffusion of conserved charges is usually neglected.

Diffusion coeff.$\cdots$

Diffusion is a dissipative process which occurs as soon as inhomogenity arise in a conserved quantity.

Ficks Law: $\mathbf{j}_q = -\kappa_q \nabla n_q(x)$ (non-relativistic)

- High energy heavy ion collisions, with almost vanishing ρ_B, effects of diffusion is expected to be small.
- It can be important for BES, FAIR, NICA physics.
- Fluctuations of conserved charges plays an important role to find the critical point.
- Diffusion plays an important role in the time evolution of conserved charges

Diffusion coeff.···

- Strongly interacting matter carry a multitude of conserved quantum numbers: B,S,Q.
- Diffusion currents of conserved charges must be coupled with each other. Gradients of a given charge density can generate diffusion current of any of the other charges.

Therefore, in the presence of multiple conserved charges, one has a generalized Fick's law,

$$\begin{pmatrix} \Delta J^i_B \\ \Delta J^i_Q \\ \Delta J^i_S \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \begin{pmatrix} D^i \alpha_B \\ D^i \alpha_Q \\ D^i \alpha_S \end{pmatrix}.$$

$\alpha_q = \mu_q / T$ with $q = B, S, Q$ and $D = u^\mu \partial_\mu$; $D^\mu = \partial^\mu - u^\mu D$ and $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is the projector orthogonal to fluid four velocity u^μ. $\kappa_{qq'}$ denotes the multicomponent diffusion matrix.We shall try to estimate the diffusion matricx element for hadronic matterusing Boltzman equation within relxation time approximation

Formalism: Relativistic Boltzmann Equation

In a mixture of multi component species, the single particle distribution function $f_a(x, p_a)$ ($a = 1, N$,:species index), in the absence of any external force, evolves in space time through a relativistic Boltzmann equation

$$p_a \cdot \partial f_a(x, p_a) = (u \cdot p_a) \sum_b \mathcal{C}_{ab}[f] \equiv \mathcal{C}_a$$

$$\mathcal{C}_{ab}[f] = \frac{1}{2} \sum_{c,d} \int dP_b dP_c{}' dP_d{}' \left[f_c f_d \tilde{f}_a \tilde{f}_b - f_a f_b \tilde{f}_c \tilde{f}_d \right] W_{ab \to cd}.$$

$\tilde{f}_a = (1 - \kappa f_a(x, p_a)/g_a)$;with $\kappa = \pm 1, 0$ for fermions and bosons, classical particles. The transition rate corresponding to binary scattering $W_{ab \to cd} = \frac{1}{16}(2\pi)^4 |M_{ab \to cd}|^2 \times \delta^4(p_a + p_b - p_c' - p_d')$.

$|M_{ab \to cd}(\sqrt{s}, \Omega)|^2 = 64\pi^2 s \frac{p_{ab}}{p_{cd}} \frac{d\sigma_{ab \to cd}}{d\Omega}$

Boltzmann Equation contd. $\cdots$

Consider small perturbation from equllibrium

$$f_a(x,p) = f_a^{(0)}(p)\,(1+\phi^a(x,p))$$

The equilibrium distribution function in the classical limit is given by,

$$f_a^{(0)} = g_a \exp(-\beta u\cdot p_a + \beta\sum_q q_a\mu_q) \equiv g_a \exp(-\beta u\cdot p_a + \alpha_a),$$

$\alpha^a = \sum_q q_a\alpha_q \equiv \beta\sum_q q_a\mu_q$, $\alpha_q = \beta\mu_q$ and μ_q corresponding to different chemical potentials (e.g. $q = B, S, Q$). To identify the transport cefficients, the energy momentum tensor and the conserved current

$$T^{\mu\nu} = -Pg^{\mu\nu} + \omega u^\mu u^\nu + \Delta T^{\mu\nu}, \qquad J_q^\mu = n_q u^\mu + \Delta J_q^\mu.$$

u^μ is the velocity of energy flow normalized as $u_\mu u^\mu = 1$ and $\omega = \varepsilon + P$ is the enthalpy.

$T^{\mu\nu}, J_q^\mu$ CONTD. $\cdots$

The dissipative correction to the energy-momentum tensor due to viscosity,

$$\Delta T^{\mu\nu} = \eta\left(D^\mu u^\nu + D^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\theta\right) + \zeta\Delta^{\mu\nu}\theta \equiv \eta\Sigma^{\mu\nu} + \zeta\Delta^{\mu\nu}\theta$$

and, the dissipative contribution to the conserved current is given as,

$$\Delta J_q^\mu = \sum_{q'} \kappa_{qq'} D^\mu \alpha_{q'},$$

which is a relativistic form of Fick's law generalised to different conserved charges q with $\kappa_{qq'}$ being the diffusion matrix coefficient The diffusion current is generated by the gradient in the thermal potential α_q. $\theta = \partial \cdot u$ is the expansion scalar; $D = u^\mu \partial_\mu$; $D^\mu = \partial^\mu - u^\mu D$ and $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is the projector orthogonal to u^μ.

$T^{\mu\nu}, J_q^\mu$ CONTD. $\cdots$

Interms of microscopic distribution functions

$$T^{\mu\nu} = \sum_a \int \frac{d^3p_a}{(2\pi)^3} \frac{p_a^\mu p_a^\nu}{E_a} f_a, \qquad J_q^\mu = \sum_a q_a \int \frac{d^3p_a}{(2\pi)^3} \frac{p_a^\mu}{E_a} f_a.$$

The non-equilibrium part ϕ_a of the distribution function f_a leads to the non-equilibrium contributions $\Delta T^{\mu\nu}$ and ΔJ_q^μ. This means ϕ_a should have the same tensor structure as $\Delta T^{\mu\nu}$ and ΔJ_q^μ.

$$\phi_a = -A_a\theta - \sum_q B_a^q p_a^\mu D_\mu \alpha_q + C_a p_a^\mu p_a^\nu \Sigma_{\mu\nu}$$

where, the functions A_a, B_a^q and C_a are functions of magnitude of momentum .

$$\Delta J_q^i == \sum_{a,q'} q_a \int \frac{d^3p_a}{(2\pi)^3} \frac{p_a^2}{3E_a} f_a^{(0)} B_a^{q'} D^i \alpha_{q'}$$

$$= \sum_{q'} \kappa_{qq'} D^i \alpha_{q'}, \qquad (1)$$

Boltzmann equation contd. $\cdots$

To obtain he departure from equllibrium functions A, B_a^q and C_a; use Boltzmann equation within Chapman-Enscog approximation. i.e. expand both sides of the equation upto first order in ϕ_a.Taking the derivative of the LHS of the Boltzmann equation upto first order leads to

$$p_a^\mu \partial_\mu f_a^{(0)} = -f_a^{(0)} \Big[E_a^2 D\beta - E_a D\alpha^a + \beta p_a^\mu p_a^\alpha \left(\frac{1}{2} \Sigma_{\mu\alpha} + \frac{1}{3} \Delta_{\mu\alpha} \theta \right) + p_a^\mu \sum_q \left(\frac{E_a n_q}{\omega} - q_a \right) D_\mu \alpha_q \Big].$$

while the collision term becomes

$$\mathcal{C}_a = \frac{1}{2} E_a f_a^{(0)} \sum_{b,c,d} \int \frac{dp_b dp_c' dp_d'}{(2\pi)^{3\times 3}} f_b^{(0)} W(a, b|c, d) \left(\phi_c + \phi_d - \phi_a - \phi_b \right).$$

Boltzmann equation contd. ···

For diffusive processes$\phi_a = \phi_a(p_a^\mu) \simeq -\sum_q B_a^q D_\mu \alpha_q$ Solve for the deviation function B_a^q in the **relaxation time approximation** i.e. all particles are in equllibrium except for the species a appearing in Boltzmann equation for f_a This leads to

$$B_{a-part}^q = \frac{\tau_a}{E_a}\left(q_a - \frac{E_a n_q}{\omega}\right).$$

with the energy dependent relaxation time

$$\tau_a^{-1}(E_a) = \sum_{b,c,d} \frac{1}{2} \int \frac{d^3p_b d^3p'_c d^3p'_d}{(2\pi)^3} f_b^{(0)} W(a,b|c,d).$$

$\Rightarrow$

$$\kappa_{qq'} = \sum_a \int \frac{d^3p_a}{(2\pi)^3} \frac{p_a^2}{3E_a^2} \left(q_a - \frac{n_q E_a}{\omega}\right) \tau_a \left(q'_a - \frac{n_{q'} E_a}{\omega}\right) f_a^{(0)}.$$

Diffusion coefficients in HRG model

We shall estimate diffusion matrix elements within HRG model. This needs thermodynamics for the hadrons and the relaxation time for them. Both of these we estimate in Hdaron resonance gas model.
(Ideal HRG)

$$\ln Z^{id} = \sum_a \ln Z_a^{id} = \sum_a \pm \frac{V g_a}{2\pi^2} \int_0^\infty dp\ p^2 \ln[1 \pm \exp(-\beta(E_a - \mu_a))].$$

Here V is the volume of the system, g_a is the degeneracy factor, E_a is the single-particle energy,
$\mu_a = \sum_q q_a \mu_q \equiv B_a \mu_B + S_a \mu_S + Q_a \mu_Q$
Excluded volume HRG

$$P^{ex}(T, \mu_1, \mu_2, ...) = \sum_a P_a^{id}(T, \tilde{\mu}_1, \tilde{\mu}_2, ..); \qquad \tilde{\mu}_a = \mu_a - V_a^{ex} P^{ex}(T, \mu_1,$$

Relaxation time

$$\tau_a^{-1} \equiv w_a(E_a) = \sum_b \int \frac{d^3 p_b}{(2\pi)^3} \sigma_{ab} v_{ab} f_b^{(0)}$$

Energy averaged relaxation time

$$\tau_a^{-1} = \frac{\int \frac{d^3 p_a}{(2\pi)^3} f_a^{(0)}(E_a) w_a(E_a)}{\int \frac{d^3 p_a}{(2\pi)^3} f_a^{(0)}(E_a)} \equiv \sum_b n_b \langle \sigma_{ab} v_{ab} \rangle$$

Thermal averaged cross section

$$\langle \sigma_{ab} v_{ab} \rangle = \frac{\int d^3 p_a d^3 p_b f_a^{(0)}(E_a) f_b^{(0)}(E_b) \sigma_{ab} v_{ab}}{\int d^3 p_a d^3 p_b f_a^{(0)}(E_a) f_b^{(0)}(E_b)}.$$

Relaxation time contd.···

Assuming hard sphere scatterring for the hadrons

$$\tau_a^{-1} == \sum_b n_b \langle \sigma_{ab} v_{ab} \rangle$$

$$\langle \sigma_{ab} v_{ab} \rangle = \frac{\sigma}{8 T m_a^2 m_b^2 K_2(m_a/T) K_2(m_b/T)} \times \int_{(m_a+m_b)^2}^{\infty} ds \times \frac{[s-(m_a-m_b)^2]}{\sqrt{s}} \times [s-(m_a+m_b)^2] K_1(\sqrt{s}/T),$$

where, the hard sphere scattering cross section can be expressed as $\sigma = 4\pi R^2$.

$$\kappa_{qq'} = \sum_a \int \frac{d^3 p_a}{(2\pi)^3} \frac{p_a^2}{3E_a^2} \left(q_a - \frac{n_q E_a}{\omega} \right) \tau_a \left(q'_a - \frac{n_{q'} E_a}{\omega} \right) f_a^{(0)}.$$

Results: κ_{BB}

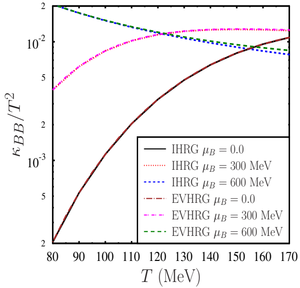

Fig. 1-a

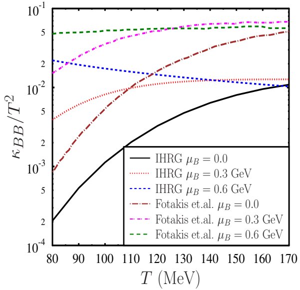

Fig. 1-b

Figure: **left :** variation of κ_{BB}/T^2 with T and μ_B. $\mu_Q = 0 = \mu_S$. Among all the hadrons baryonic contribution is dominant over mesonic contribution in κ_{BB}/T^2. **right** we show the qualitative and quantitative comparison among the results as obtained in thea IHRG model and the results obtained in Ref. Fotakis etal. In this case we obtained our results in the IHRG model considering $n_S = 0$ and $\mu_Q = 0$.

Results: κ_{SS}

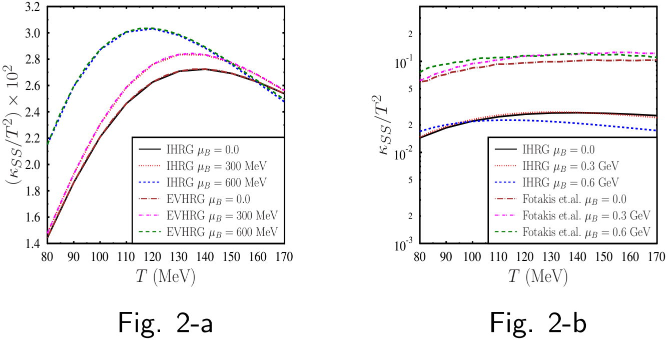

Figure: **left :** variation of κ_{SS}/T^2 with T and μ_B. $\mu_Q = 0 = \mu_S$. Among all the hadrons mesonic contribution is dominant over baryonic contribution in κ_{BB}/T^2. **right** we show the qualitative and quantitative comparison among the results as obtained in thea IHRG model and the results obtained in Ref. Fotakis etal. In this case we obtained our results in the IHRG model considering $n_S = 0$ and $\mu_Q = 0$.

Results: κ_{QQ}

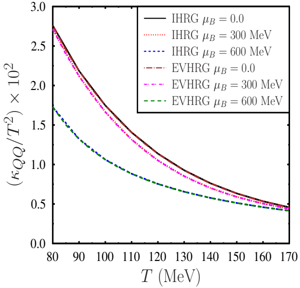

Fig. 3-a

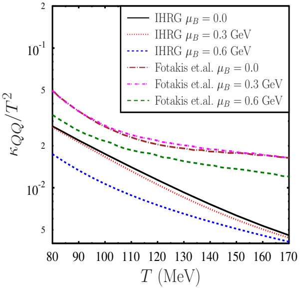

Fig. 3-b

Figure: **left :** variation of κ_{QQ}/T^2 with T and μ_B. $\mu_Q = 0 = \mu_S$. Among all the hadrons mesonic contribution is dominant over baryonic contribution in κ_{BB}/T^2. **right** we show the qualitative and quantitative comparison among the results as obtained in thea IHRG model and the results obtained in Ref. Fotakis etal. In this case we obtained our results in the IHRG model considering $n_S = 0$ and $\mu_Q = 0$.

RESULTS: MIXED DIFFUSION COEFFICIENT κ_{BQ}

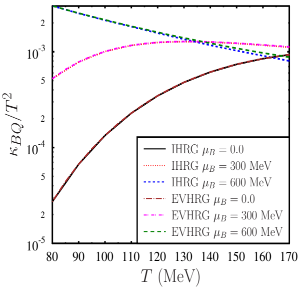

Fig. 4-a

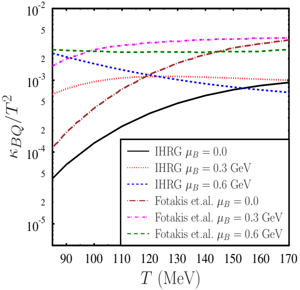

Fig. 4-b

Figure: **left :** variation of κ_{BQ}/T^2 with T and μ_B. $\mu_Q = 0 = \mu_S$. Among all the hadrons mesonic contribution is dominant over baryonic contribution in κ_{BB}/T^2. **right** we show the qualitative and quantitative comparison among the results as obtained in thea IHRG model and the results obtained in Ref. Fotakis etal. In this case we obtained our results in the IHRG model considering $n_S = 0$ and $\mu_Q = 0$.

Results: Mixed diffusion coefficient κ_{BS}

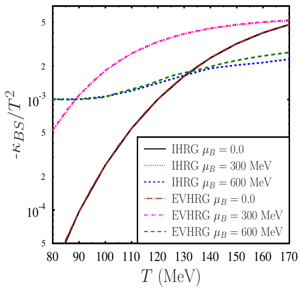

Fig. 5-a

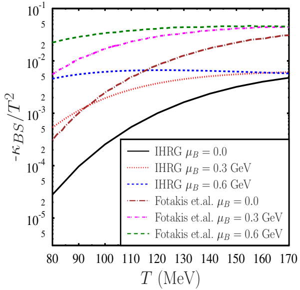

Fig. 5-b

Figure: **left :** variation of κ_{BS}/T^2 with T and μ_B. $\mu_Q = 0 = \mu_S$. Among all the hadrons mesonic contribution is dominant over baryonic contribution in κ_{BB}/T^2. **right** we show the qualitative and quantitative comparison among the results as obtained in thea IHRG model and the results obtained in Ref. Fotakis etal. In this case we obtained our results in the IHRG model considering $n_S = 0$ and $\mu_Q = 0$.

Results: Mixed diffusion coefficient κ_{SQ}

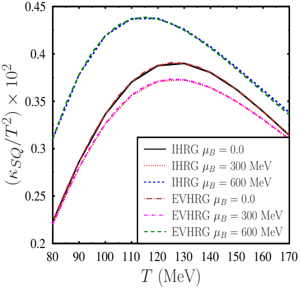

Fig. 5-a

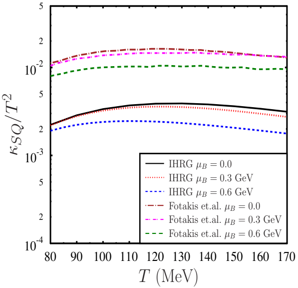

Fig. 5-b

Figure: **left :** variation of κ_{SQ}/T^2 with T and μ_B. $\mu_Q = 0 = \mu_S$. Among all the hadrons mesonic contribution is dominant over baryonic contribution in κ_{BB}/T^2. **right** we show the qualitative and quantitative comparison among the results as obtained in thea IHRG model and the results obtained in Ref. Fotakis etal. In this case we obtained our results in the IHRG model considering $n_S = 0$ and $\mu_Q = 0$.

Summary,Conclusion and Outlook

- We consiidered here the diffusuion matrix associated with various conserved charges.The diagonal components here are manifestly positive. The off diagonal components can be positive or negative.
- Using HRG model within the hard sphere scatterring approximation, we estimated various elements of $\kappa_{qq'}$.
- The off diagonal elements are of similar order as the diagonal elements. The cross conductivities are non negligible.
- Need to study hydrodynamics with multiple conserved charges.
- Fluctuation of conserved charges need to be explored with cross dissipation coefficients.
- $\cdots$
- $\cdots$