

Impact of nuclear matrix element calculations for future neutrinoless double beta decay searches

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$0\nu\beta\beta$

Hypothetical $(A, Z) \longrightarrow (A, Z + 2) + 2e^-$

- Forbidden in the Standard Model : $\Delta L = 2$
- The only known feasible way to prove the Majorana nature of ν

$0\nu\beta\beta$

Assuming the exchange of light Majorana neutrinos

$$(T_{1/2}^{-1})_\alpha = \widetilde{\Gamma}_\alpha(m_{\beta\beta}, M_{\alpha i}) = \frac{\Gamma_\alpha(m_{\beta\beta}, M_{\alpha i})}{\ln 2} = G_{0\nu} \times (g_A^2 |M_{0\nu}|)^2 \times m_{\beta\beta}^2$$

$M_{0\nu}$ Phase Space Factor (PSF) (kinematic) <small>arXiv:2009.10119</small>	$M_{0\nu}$ Nuclear Matrix Element (NME) $g_A = q g_A^{\text{bare}}$	Effective Majorana mass $\left \sum_j U_{ej}^2 m_j \right $
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- Forbidden in the Standard Model : $\Delta L = 2$
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Nuclear Models and Nuclear Matrix Elements

$$M_{0\nu} = M_{0\nu}^{\text{long}}$$

Long-range contribution to the decay rate induced by the exchange of light Majorana ν

[arXiv:2202.01787](https://arxiv.org/abs/2202.01787)

- Calculations performed by different groups by assuming $g_A^{\text{bare}} = 1.27$
- Data not available for all the isotopes
- Variation in $M_{0\nu}^{\text{long}}$ of a factor ~ 3

			^{76}Ge	^{82}Se	^{100}Mo	^{130}Te	^{136}Xe
Nuclear Shell Model	N1	2.89	2.73	-	2.76	2.28	
	N2	3.07	2.90	-	2.96	2.45	
	N3	3.37	3.19	-	1.79	1.63	
	N4	3.57	3.39	-	1.93	1.76	
	N5	2.66	2.72	-	3.16	2.39	
Quasiparticle Random Phase Approximation	Q1	5.09	-	-	1.37	1.55	
	Q2	5.26	3.73	3.90	4.00	2.91	
	Q3	4.85	4.61	5.87	4.67	2.72	
	Q4	3.12	2.86	-	2.90	1.11	
	Q5	3.40	3.13	-	3.22	1.18	
	Q6	-	-	-	4.05	3.38	
Energy-Density Functional theory	E1	4.60	4.22	5.08	5.13	4.20	
	E2	5.55	4.67	6.59	6.41	4.77	
	E3	6.04	5.30	6.48	4.89	4.24	
Interacting Boson Model	I1	5.14	4.19	3.84	3.96	3.25	
	I2	6.34	5.21	5.08	4.15	3.40	

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Large theoretical uncertainties!

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Quasiparticle Random Phase Approximation	Q1	5.29	-	-	1.37	1.55
	Q2	5.26	3.73	3.90	4.00	2.91
	Q3	4.85	4.61	5.87	4.67	2.72
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Short-range contribution

Phys. Rev. Lett. 120, 202001

To renormalize the $0\nu\beta\beta$ amplitude due to light Majorana ν exchange

$$M_{\alpha i} = M_{\alpha i}^{\text{long}} + M_{\alpha i}^{\text{short}}$$

Short-range contribution

Phys. Rev. Lett. 120, 202001

To renormalize the $0\nu\beta\beta$ amplitude due to light Majorana ν exchange

$$M_{\alpha i} = M_{\alpha i}^{\text{long}} + M_{\alpha i}^{\text{short}} = M_{\alpha i}^{\text{long}}(1 + n_{\alpha i})$$



$$n_{\alpha i} = \frac{M_{\alpha i}^{\text{short}}}{M_{\alpha i}^{\text{long}}}$$

Short-range contribution

Phys. Rev. Lett. 120, 202001

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Unknown value and sign
leading either to an
enhancement or suppression
of the expected decay rate

$$n_{\alpha i} = \frac{M_{\alpha i}^{\text{short}}}{M_{\alpha i}^{\text{long}}} \quad |n_{\alpha i}| \in$$

Nuclear Shell Model %	Quasiparticle Random Phase Approximation %
^{76}Ge	15 ÷ 42
^{82}Se	15 ÷ 42
^{100}Mo	-
^{130}Te	17 ÷ 47
^{136}Xe	17 ÷ 47

arXiv:2107.13354

Short-range contribution

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$M_{\alpha i} = M_{\alpha i}^{\text{long}} + M_{\alpha i}^{\text{short}} = M_{\alpha i}^{\text{long}}(1 + n_{\alpha i})$

What is the effect induced by this new short-range contribution?

Unknown value and sign
leading either to an enhancement or suppression of the expected decay rate

$$n_{\alpha i} = \frac{M_{\alpha i}^{\text{short}}}{M_{\alpha i}^{\text{long}}} \quad |n_{\alpha i}| \in$$

	Quasiparticle Shell Model %	Random Phase Approximation %
^{76}Ge	15 ÷ 42	32 ÷ 73
^{82}Se	15 ÷ 42	30 ÷ 70
^{100}Mo	-	49 ÷ 108
^{130}Te	17 ÷ 47	34 ÷ 77
^{136}Xe	17 ÷ 47	30 ÷ 70

arXiv:2107.13354

Future prospect

[arXiv:2202.01787](#)

^{76}Ge	LEGEND-1000 arXiv:2107.11462
^{136}Xe	nEXO arXiv:2106.16243
^{100}Mo	CUPID arXiv:2202.06279
^{130}Te	SNO+II arXiv:2104.11687
^{82}Se	SuperNEMO arXiv:0807.2188

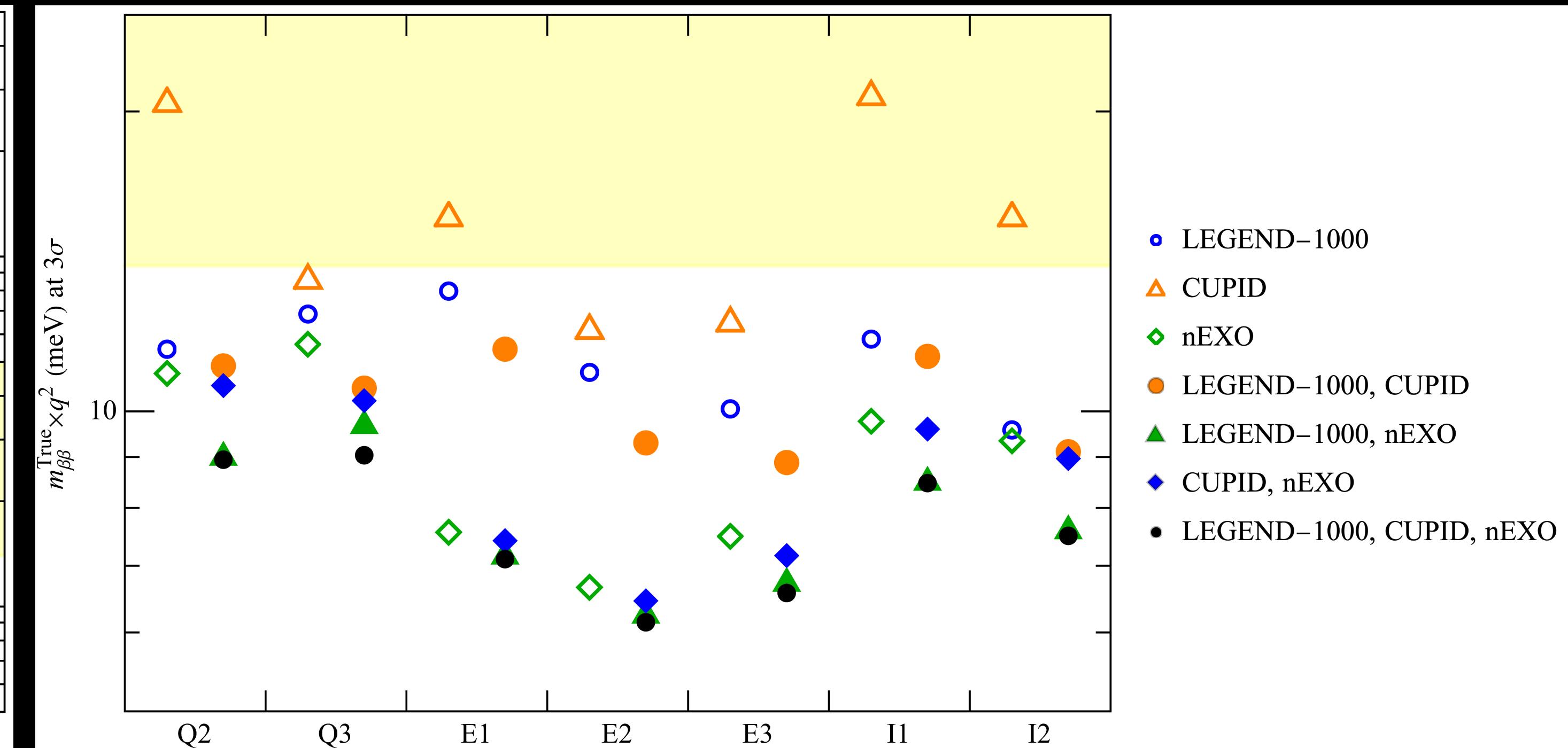
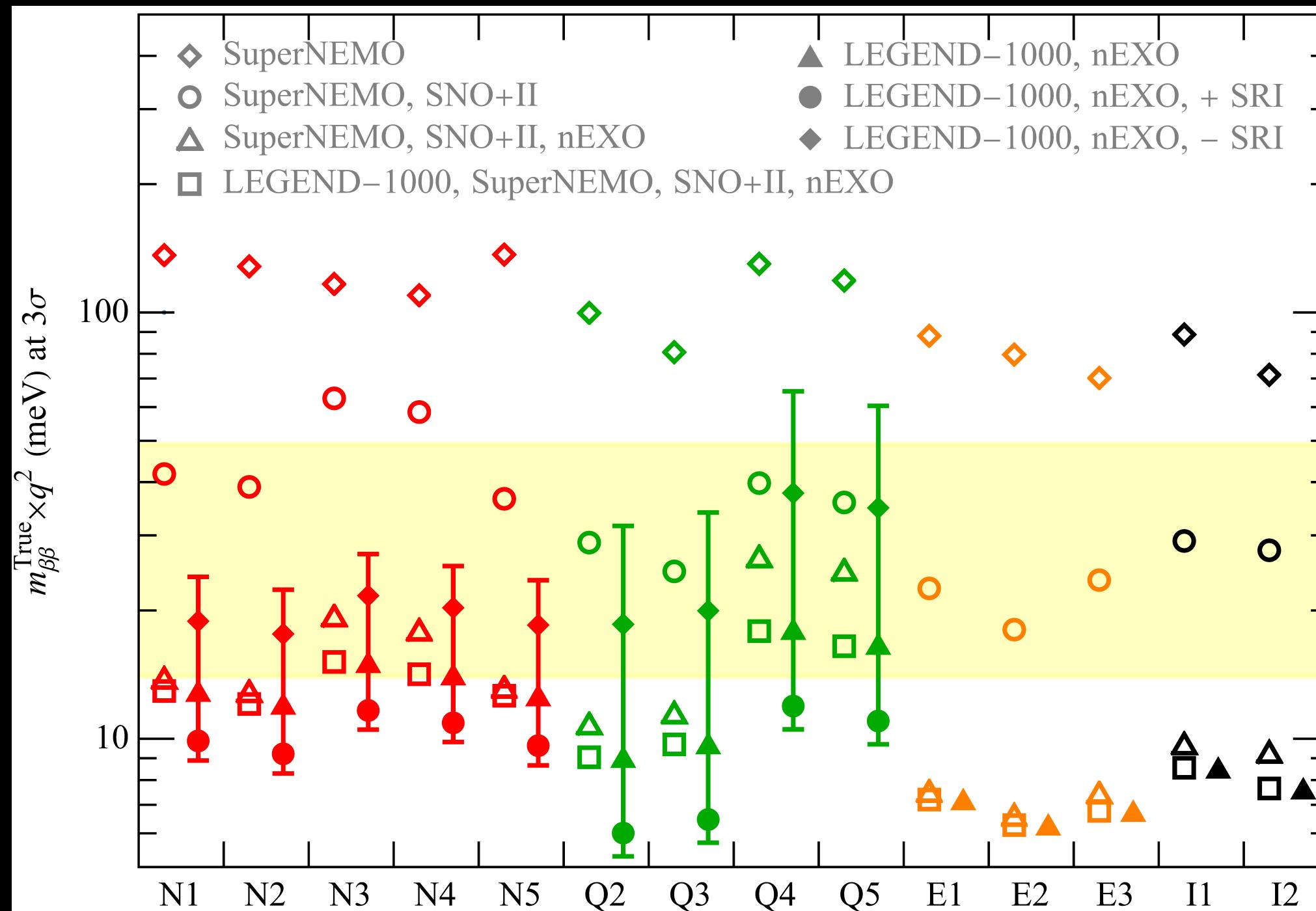
$$\left. \begin{aligned} S_{\alpha i}(m_{\beta\beta}, M_{\alpha i}) &= \ln 2 \cdot N_A \cdot \varepsilon_\alpha \cdot \left(\frac{T}{1 \text{ yr}} \right) \cdot \widetilde{\Gamma}_\alpha(m_{\beta\beta}, M_{\alpha i}) \\ B_\alpha &= b_\alpha \cdot \varepsilon_\alpha \cdot \left(\frac{T}{1 \text{ yr}} \right) \\ [\varepsilon] &= \text{mol} \cdot \text{yr} \quad [b] = \frac{\text{events}}{\text{mol} \cdot \text{yr}} \\ T &= 10 \text{ yr} \text{ in the following analysis} \end{aligned} \right] N_{\alpha i} = S_{\alpha i} + B_\alpha$$

$$\Delta\chi^2_{ij}(m_{\beta\beta}, M_{\alpha j}; m_{\beta\beta}^{\text{True}}, M_{\alpha i}^{\text{True}}) = 2 \sum_{\alpha} \left(N_{\alpha j} - N_{\alpha i}^{\text{True}} + N_{\alpha i}^{\text{True}} \ln \frac{N_{\alpha i}^{\text{True}}}{N_{\alpha j}} \right)$$

Future prospect

arXiv:2202.01787

@ 3σ ($\Delta\chi^2_{\text{tot}} = 9$)



- Big impact of the short-range term
- Uncertainties on both the size and sign of $|n_{\alpha i}|$
- LEGEND-1000 (^{76}Ge) + nEXO (^{136}Xe)

Nuclear model discrimination

Assuming that future $0\nu\beta\beta$ experiments detect a positive signal, will it be possible, via the combination of several experiments using different isotopes, to discriminate among the various NME models?

Nuclear model discrimination

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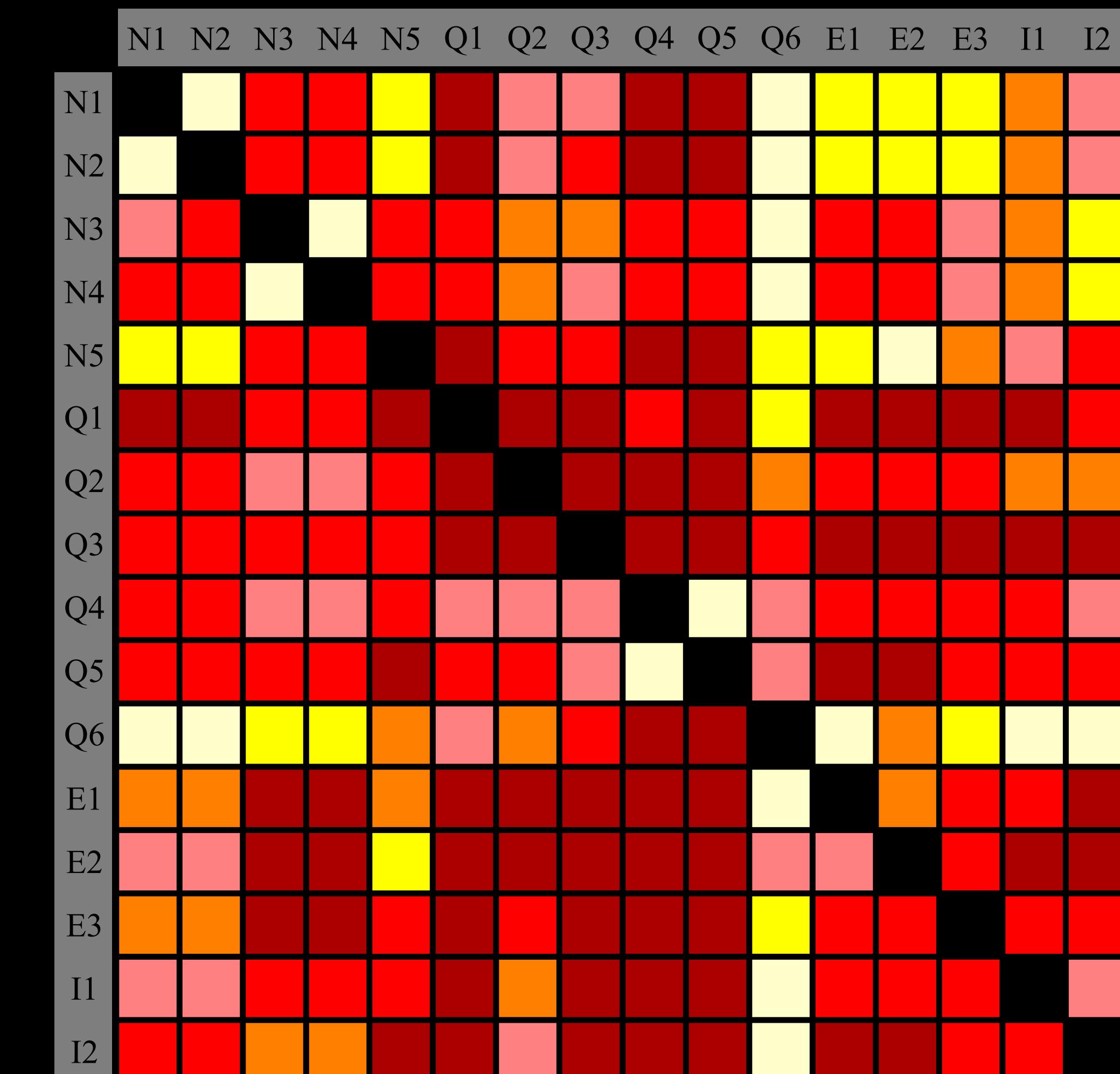
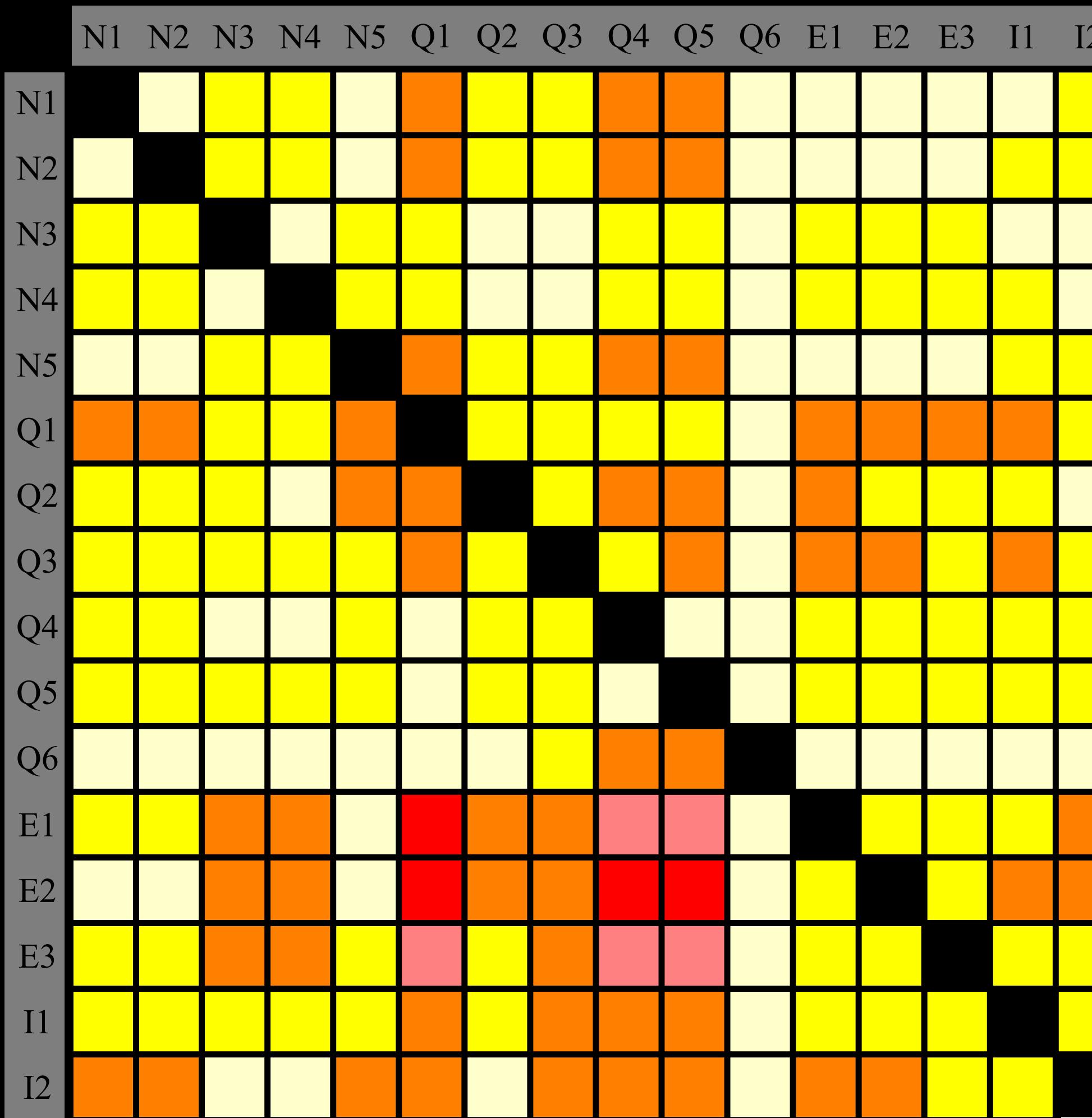
$$M_{\alpha i} = M_{\alpha i}^{\text{long}}$$

$$(\Delta\chi^2_{ij})_{\min} = \min_{m_{\beta\beta}} \Delta\chi^2_{ij}(m_{\beta\beta}, M_{\alpha j}; m_{\beta\beta}^{\text{True}}, M_{\alpha i}^{\text{True}}) \longrightarrow (\Delta\chi^2_{ij})_{\min} \neq 0 \implies \text{Nuclear model discrimination!}$$

$$M_{\alpha i} = M_{\alpha i}^{\text{long}}$$

$$m_{\beta\beta}^{\text{True}} = 10 \text{ meV}$$

$$m_{\beta\beta}^{\text{True}} = 40 \text{ meV}$$

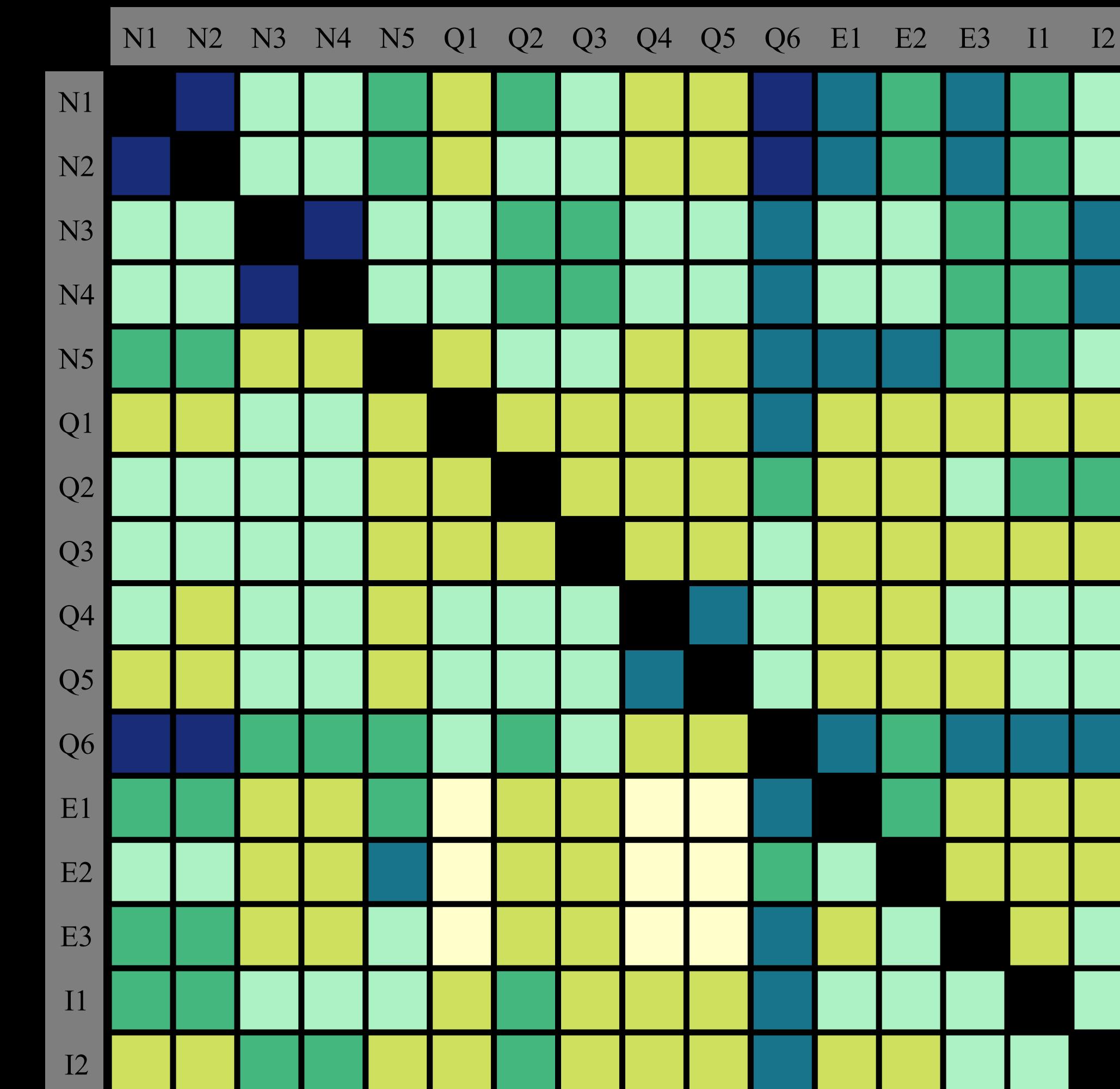


(Δχ²ᵢⱼ)ₘᵢₙ < 0.1
0.1 ≤ (Δχ²ᵢⱼ)ₘᵢₙ ≤ 1
1 < (Δχ²ᵢⱼ)ₘᵢₙ ≤ 4
4 < (Δχ²ᵢⱼ)ₘᵢₙ ≤ 9
9 < (Δχ²ᵢⱼ)ₘᵢₙ ≤ 25
(Δχ²ᵢⱼ)ₘᵢₙ > 25

$$M_{\alpha i} = M_{\alpha i}^{\text{long}}$$

@ 3σ ($\Delta\chi^2_{\text{tot}} = 9$)

- A large set of (i, j) model combinations allows a 3σ model discrimination in the Inverted Mass Ordering
- Nuclear model discrimination assuming $m_{\beta\beta}^{\text{True}} \leq 14$ meV is only possible when $i = \{\text{E1}, \text{E2}, \text{E3}\}$, $j = \{\text{Q1}, \text{Q4}, \text{Q5}\}$



■ $8 < m_{\beta\beta}^{\text{True}} \leq 14$ meV ■ $14 < m_{\beta\beta}^{\text{True}} \leq 30$ meV ■ $30 < m_{\beta\beta}^{\text{True}} \leq 49$ meV ■ $49 < m_{\beta\beta}^{\text{True}} \leq 100$ meV ■ $100 < m_{\beta\beta}^{\text{True}} \leq 500$ meV ■ $m_{\beta\beta}^{\text{True}} > 500$ meV

Nuclear model discrimination

Assuming that future $0\nu\beta\beta$ experiments detect a positive signal, will it be possible, via the combination of several experiments using different isotopes, to discriminate among the various NME models?

$$M_{\alpha i} = M_{\alpha i}^{\text{long}} + M_{\alpha i}^{\text{short}}$$

$$(\Delta\chi^2_{ij})_{\min} = \min_{m_{\beta\beta}, n_{\alpha j}} \Delta\chi^2_{ij}(m_{\beta\beta}, M_{\alpha j}; m_{\beta\beta}^{\text{True}}, M_{\alpha i}^{\text{True}}) \longrightarrow (\Delta\chi^2_{ij})_{\min} \neq 0 \implies \text{Nuclear model discrimination!}$$

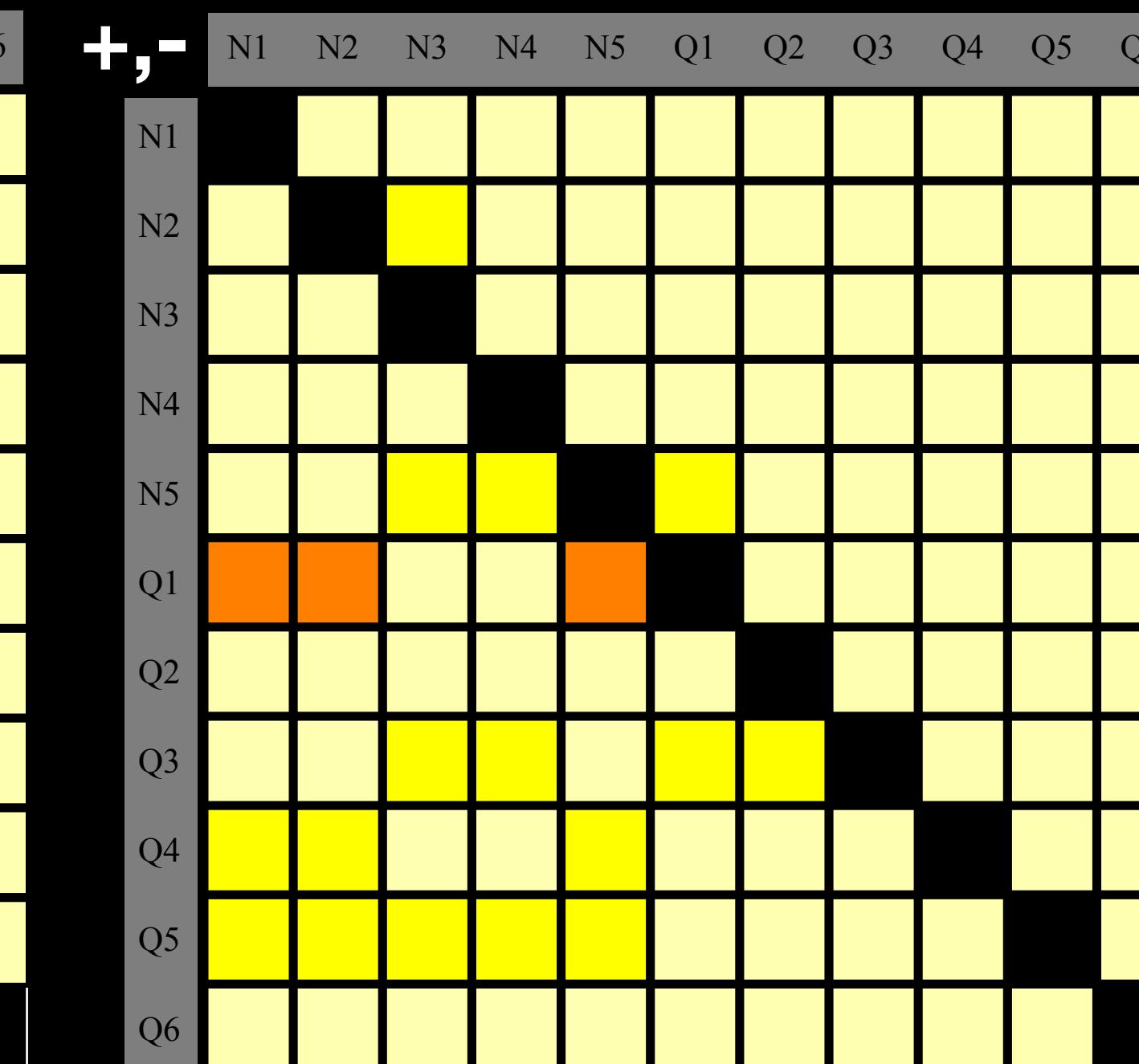
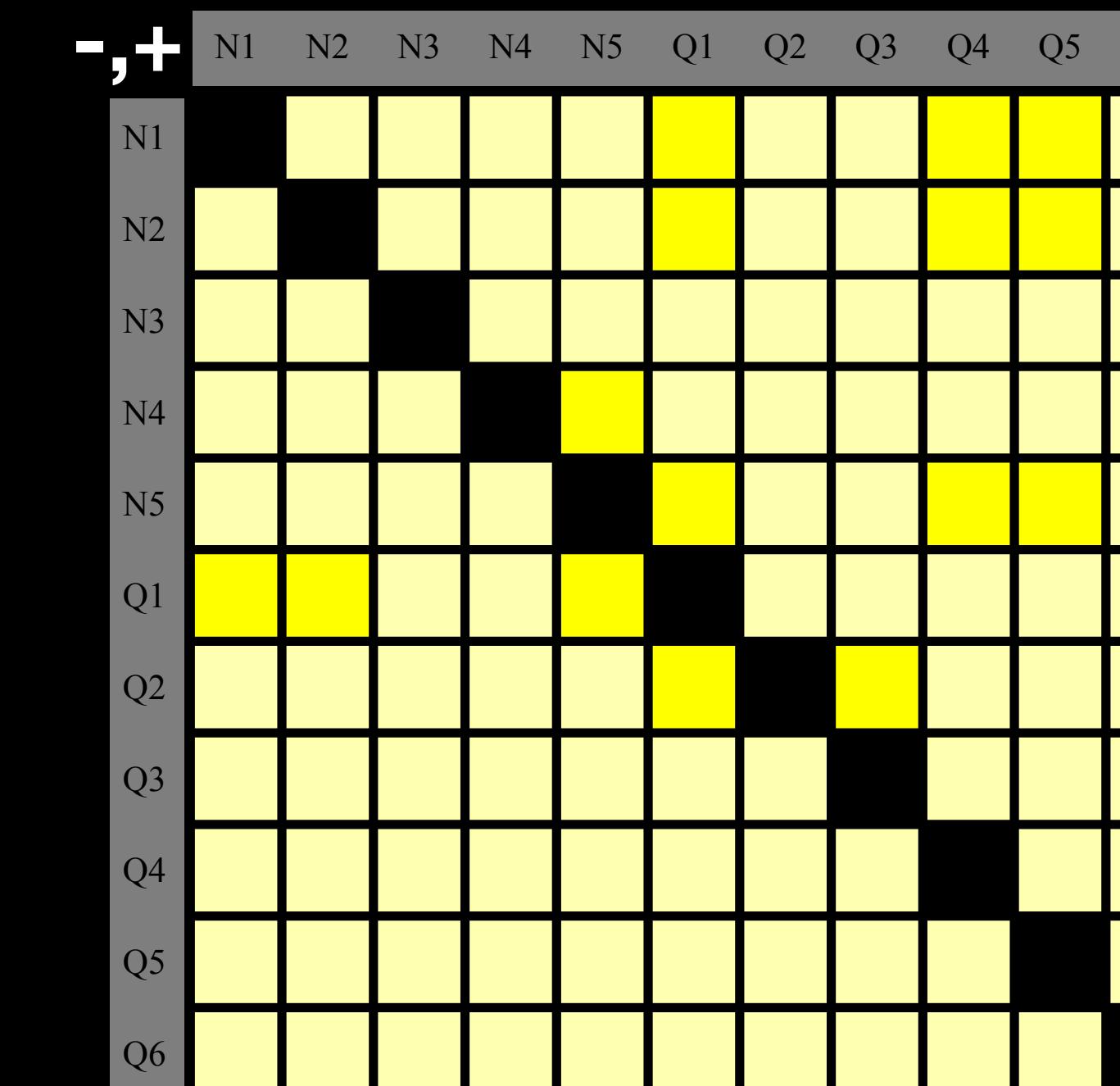
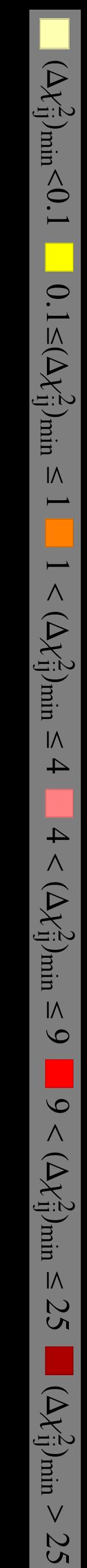
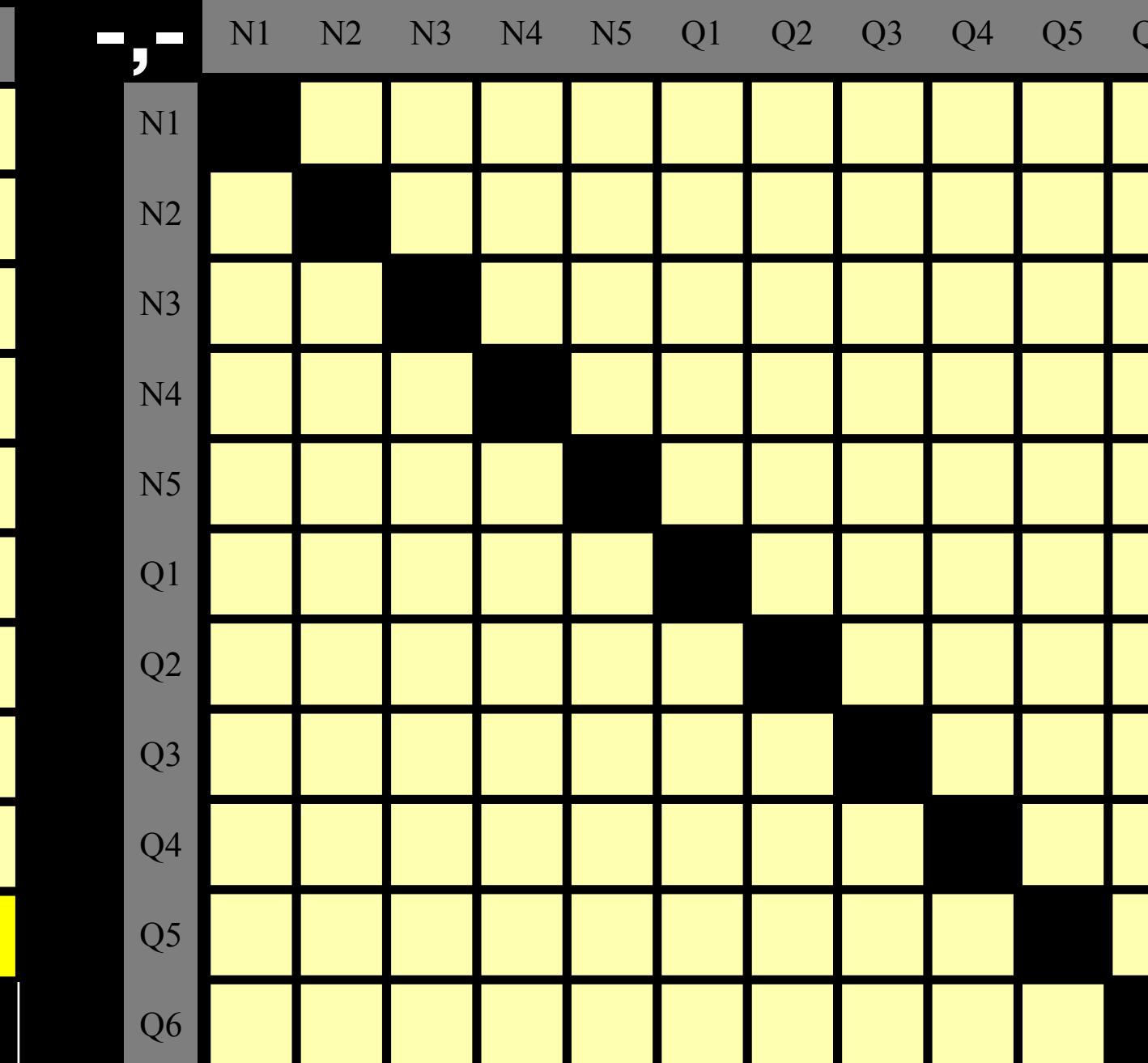
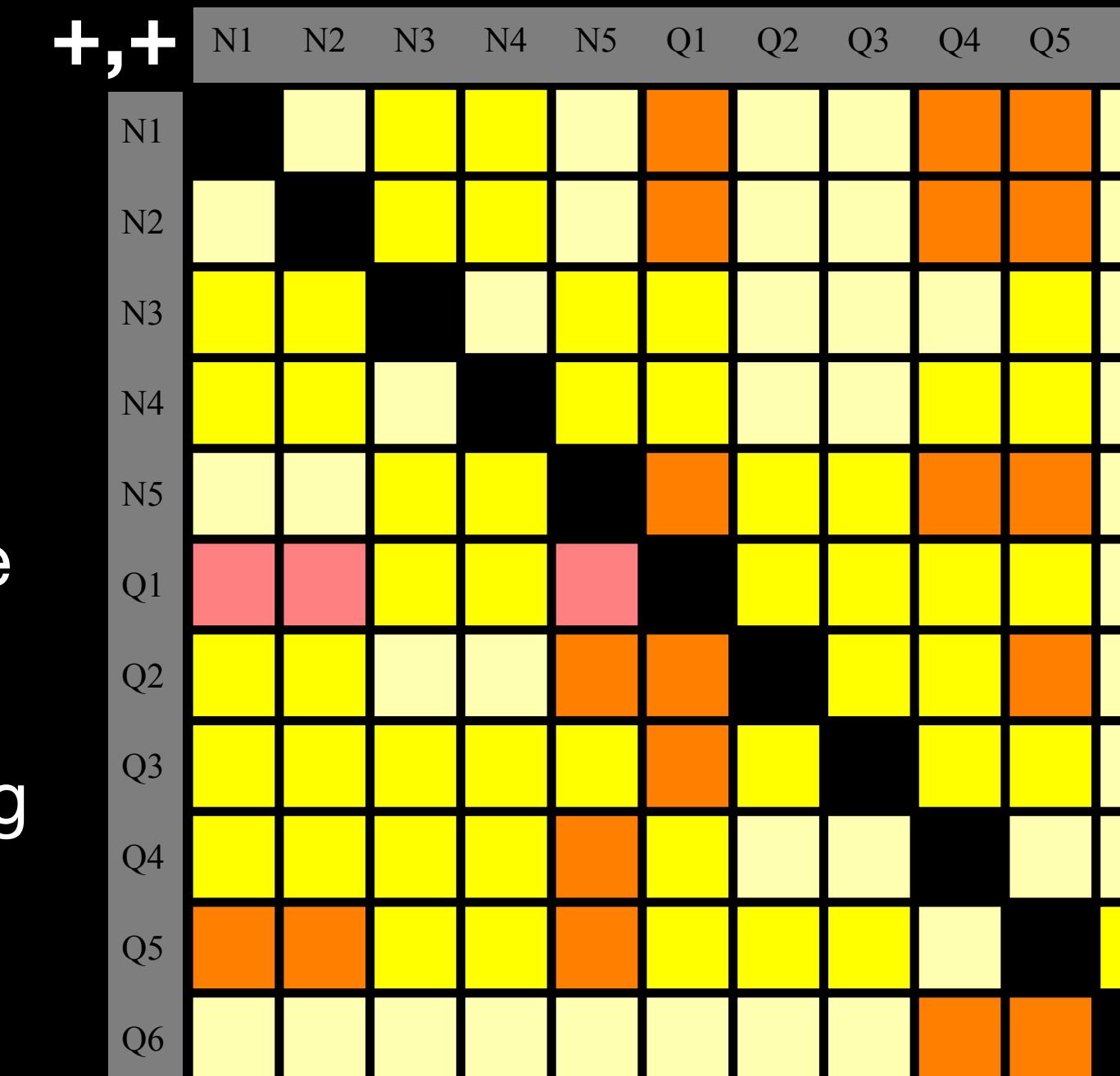
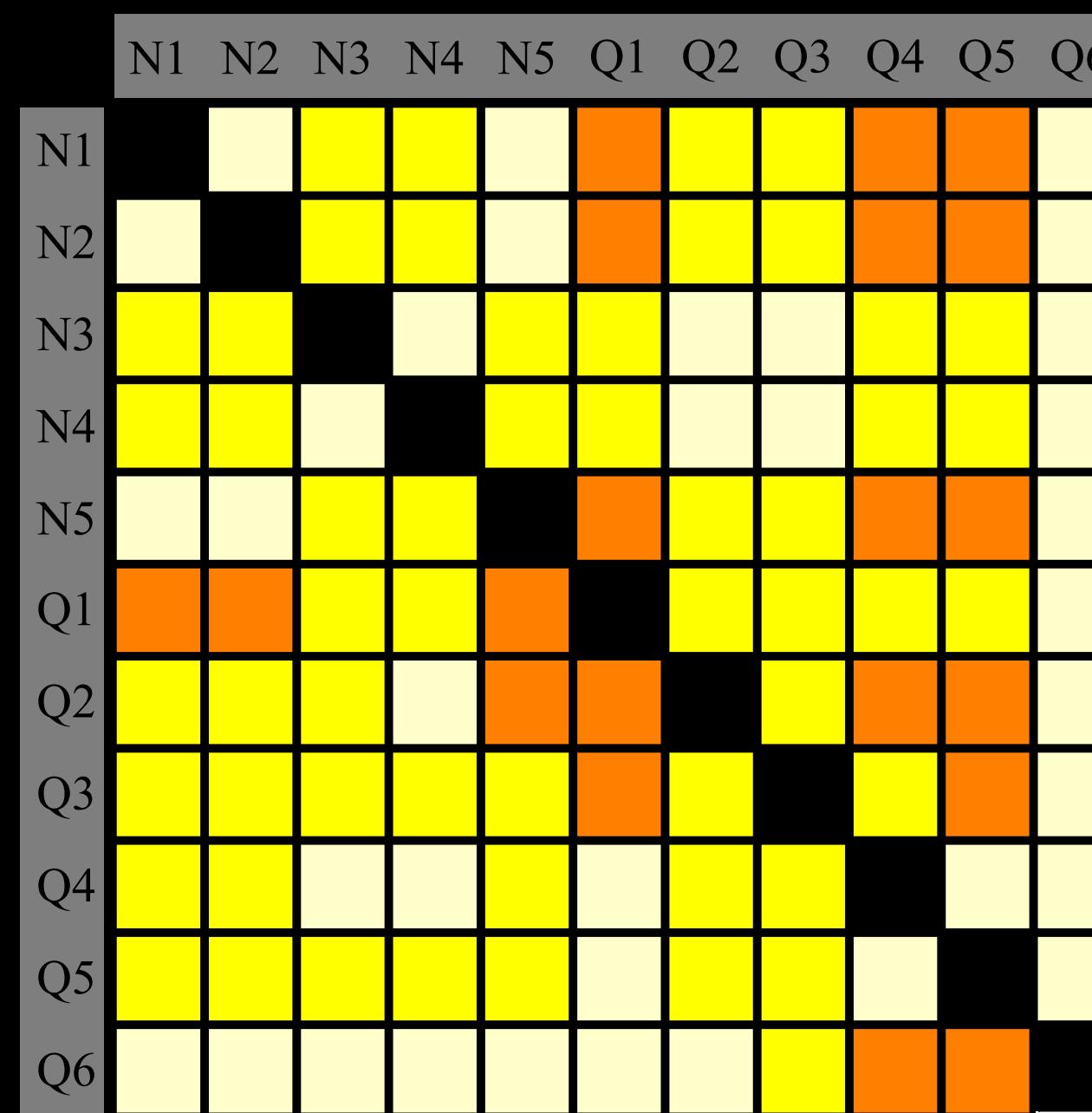
$$n_{\alpha j} = \frac{M_{\alpha j}^{\text{short}}}{M_{\alpha j}^{\text{long}}}$$

$$M_{\alpha i} = M_{\alpha i}^{\text{long}} + M_{\alpha i}^{\text{short}}$$

$$m_{\beta\beta}^{\text{True}} = 10 \text{ meV}$$

$|n_{\alpha i}|$ taken as the central value of the allowed range

$|n_{\alpha j}|$ free to vary in the corresponding range

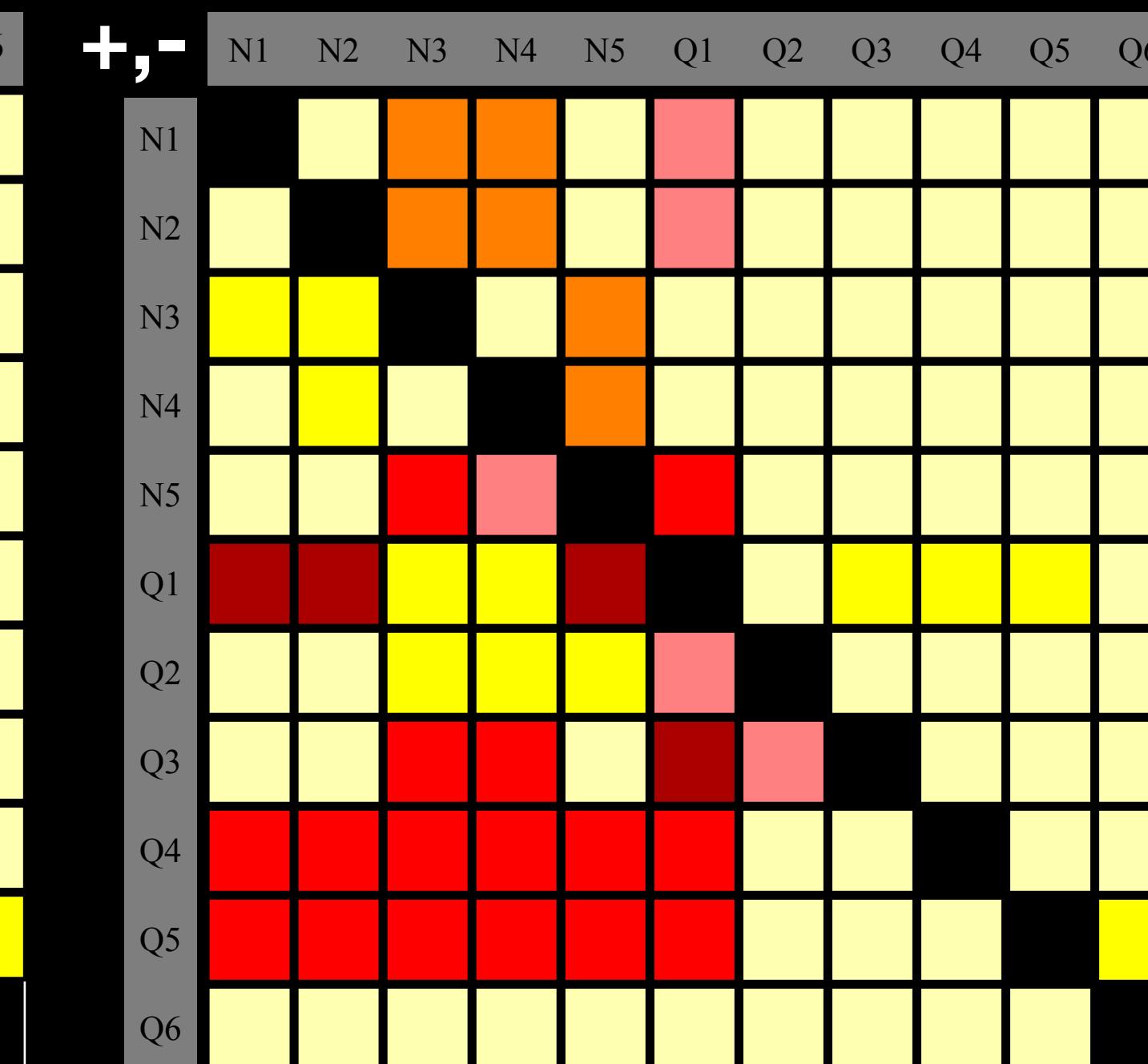
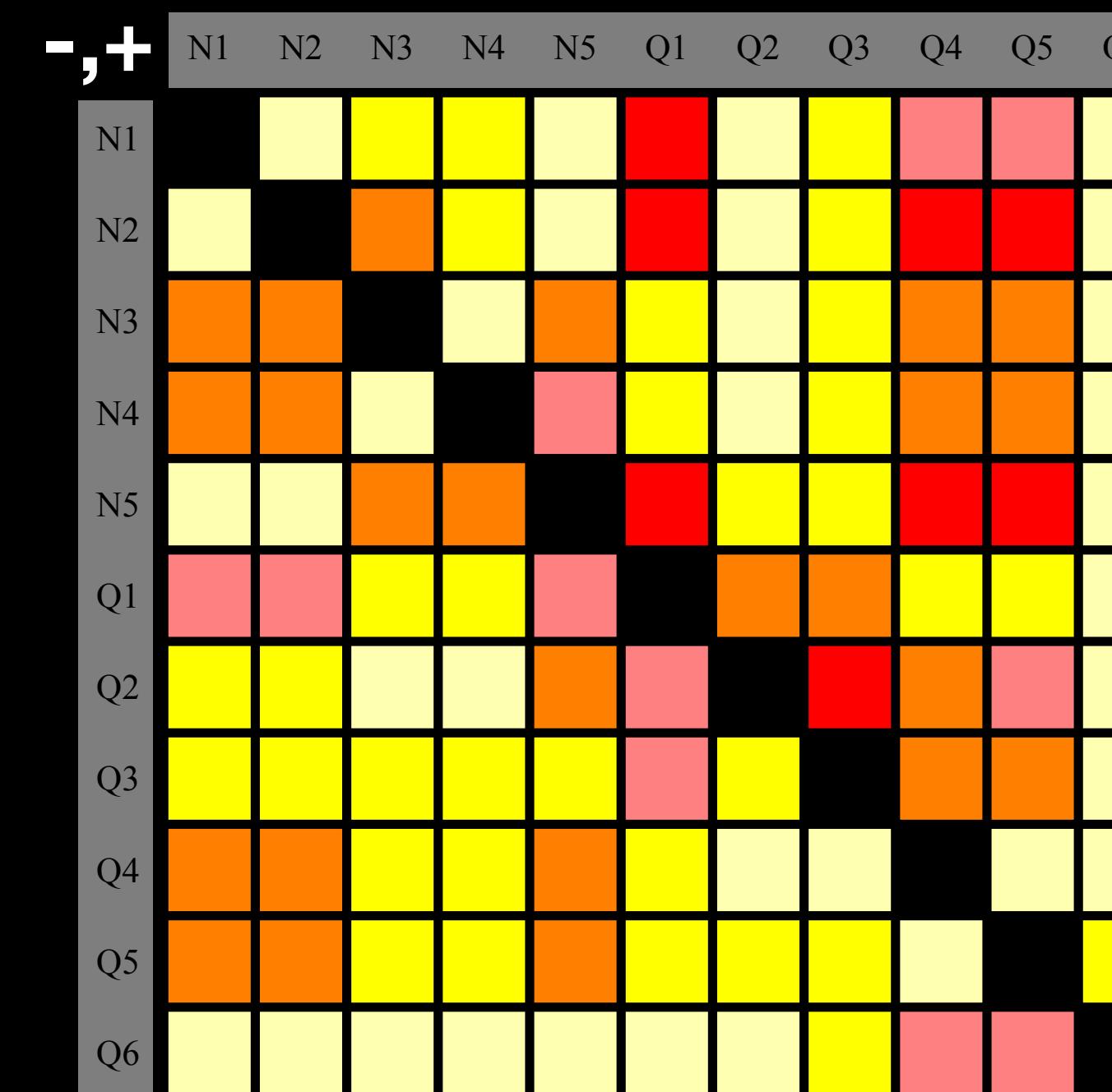
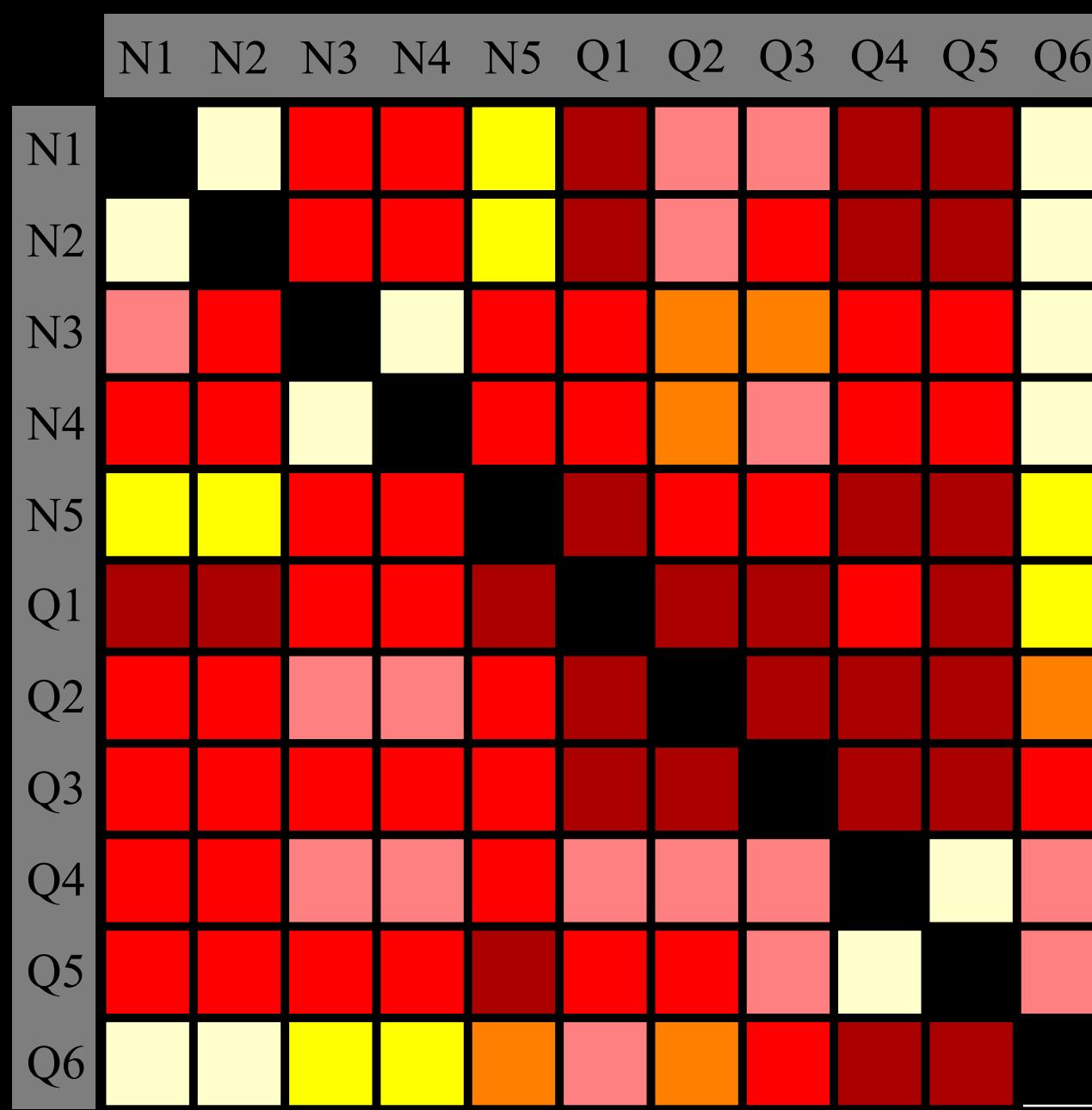
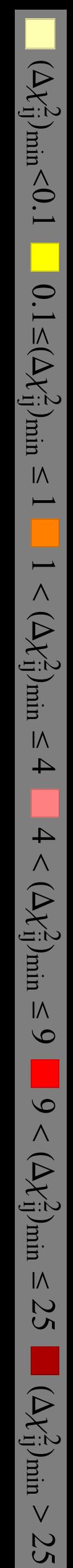
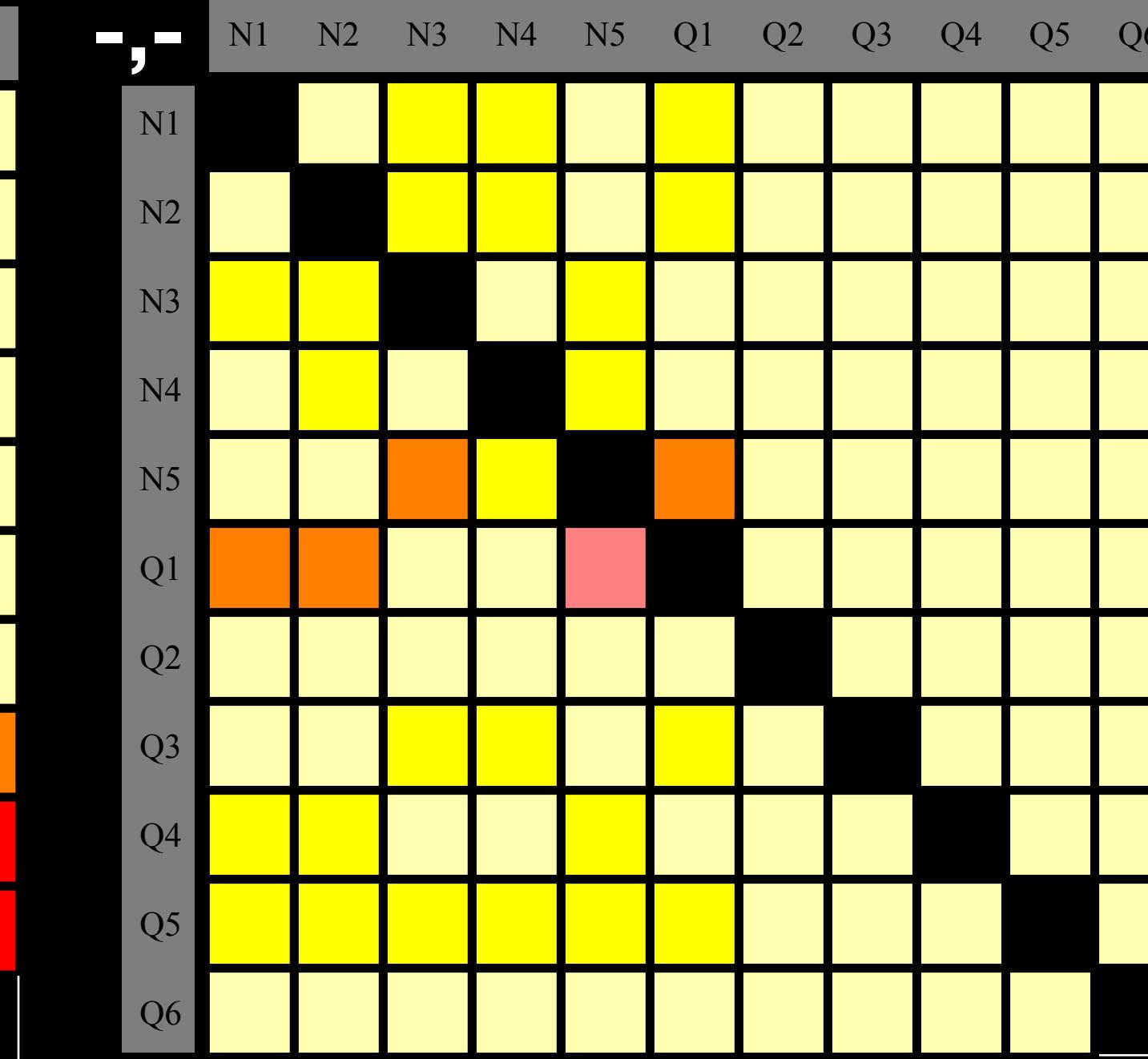
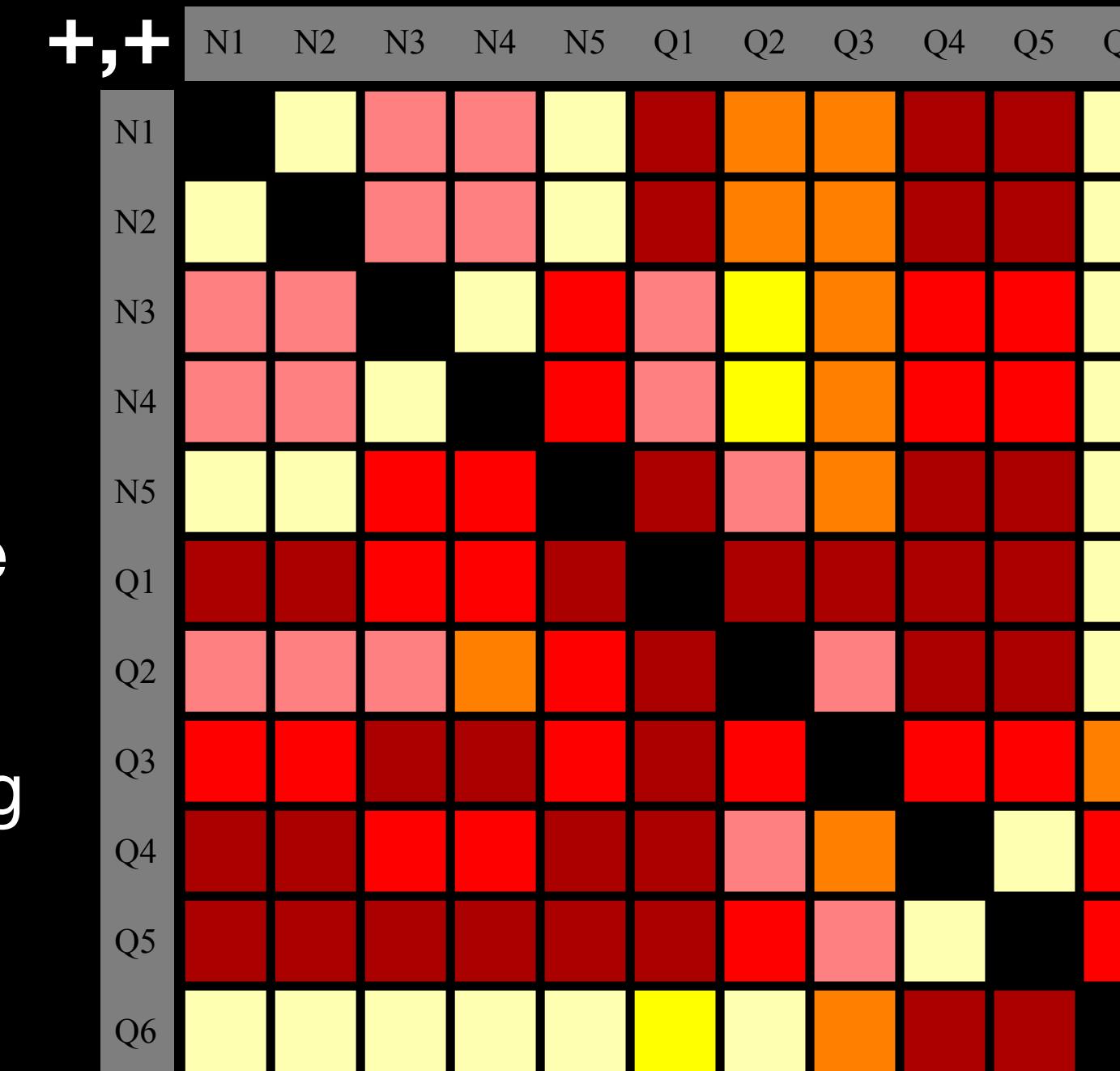


$$M_{\alpha i} = M_{\alpha i}^{\text{long}} + M_{\alpha i}^{\text{short}}$$

$$m_{\beta\beta}^{\text{True}} = 40 \text{ meV}$$

$|n_{\alpha i}|$ taken as the central value of the allowed range

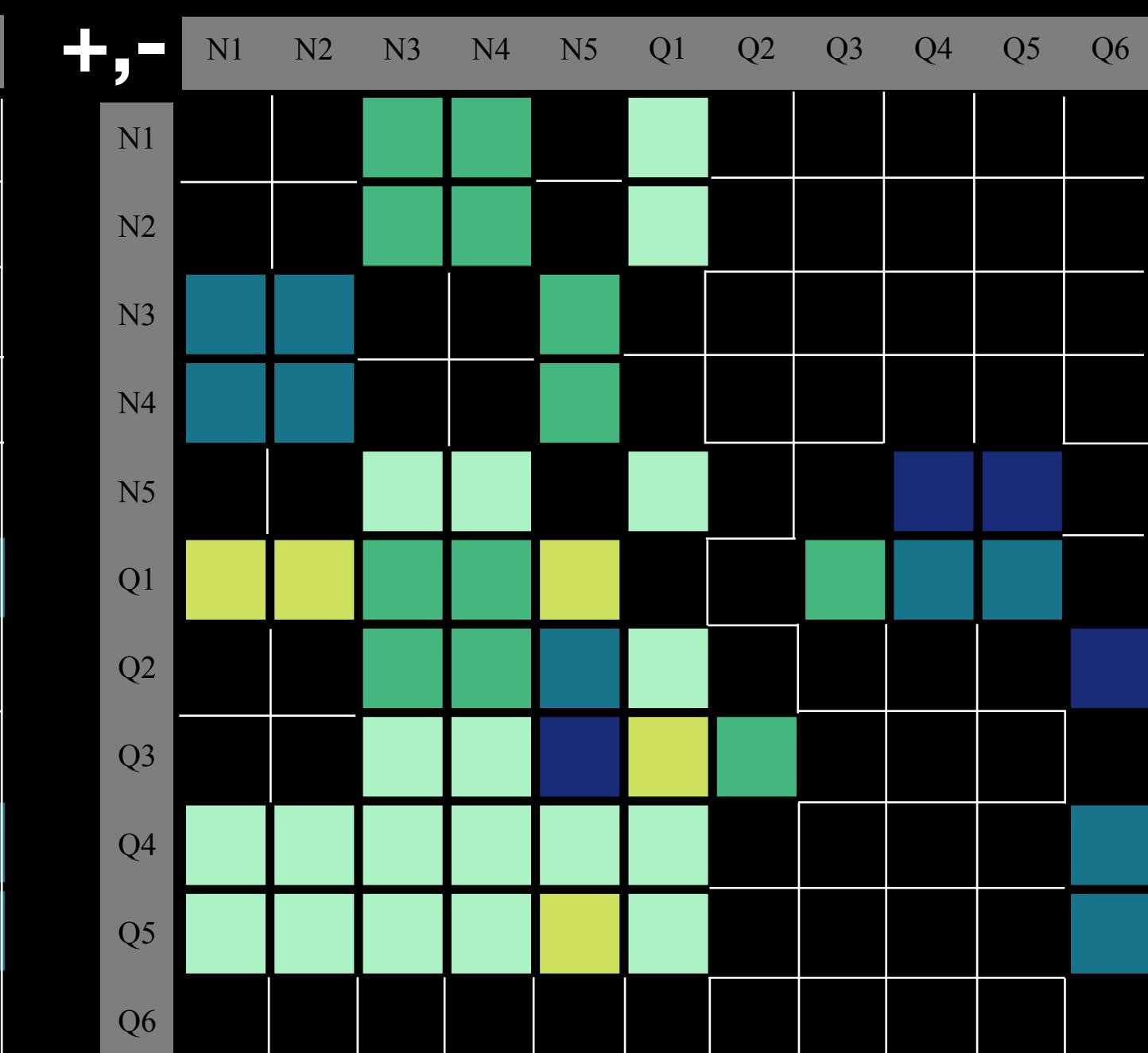
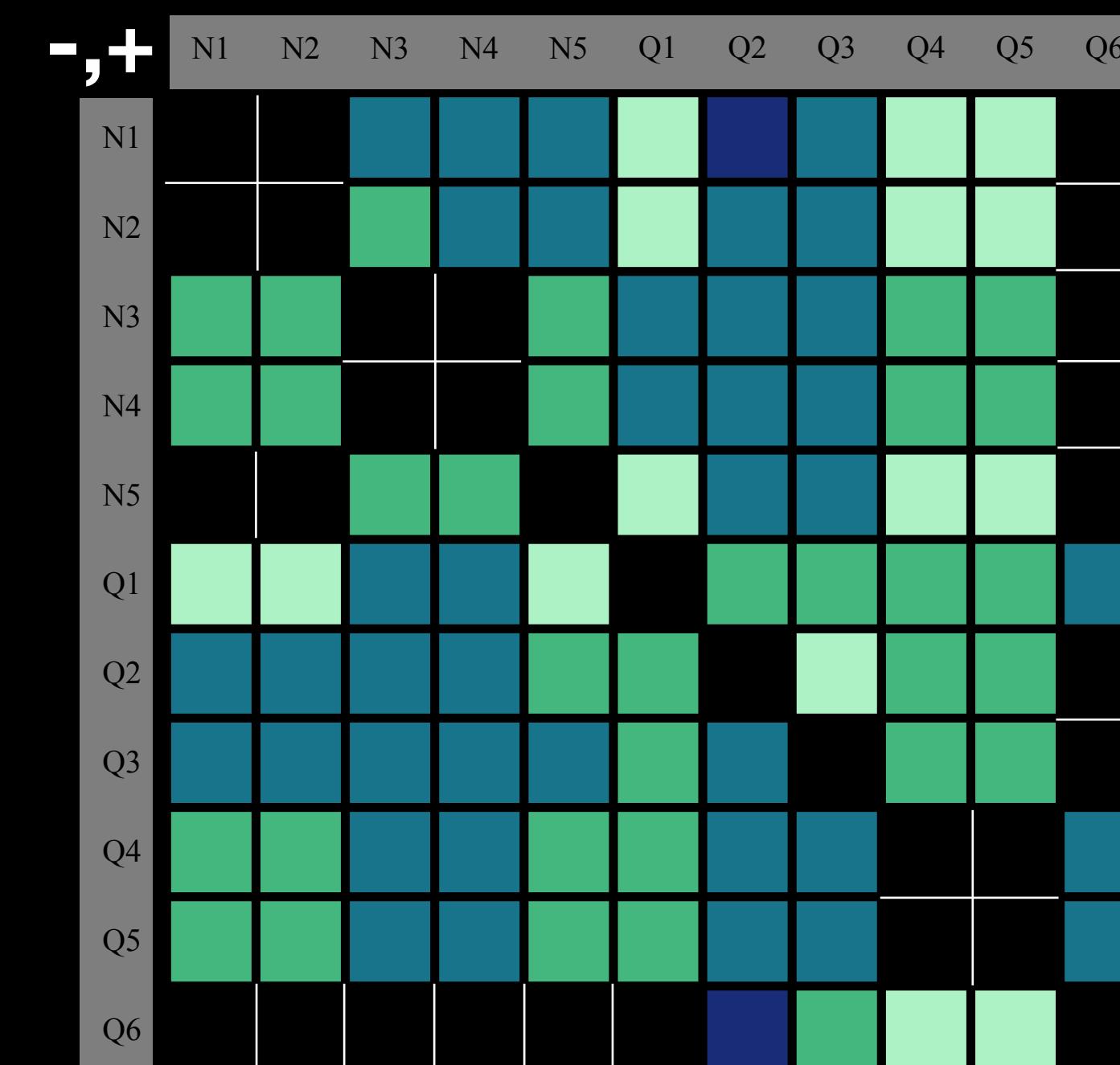
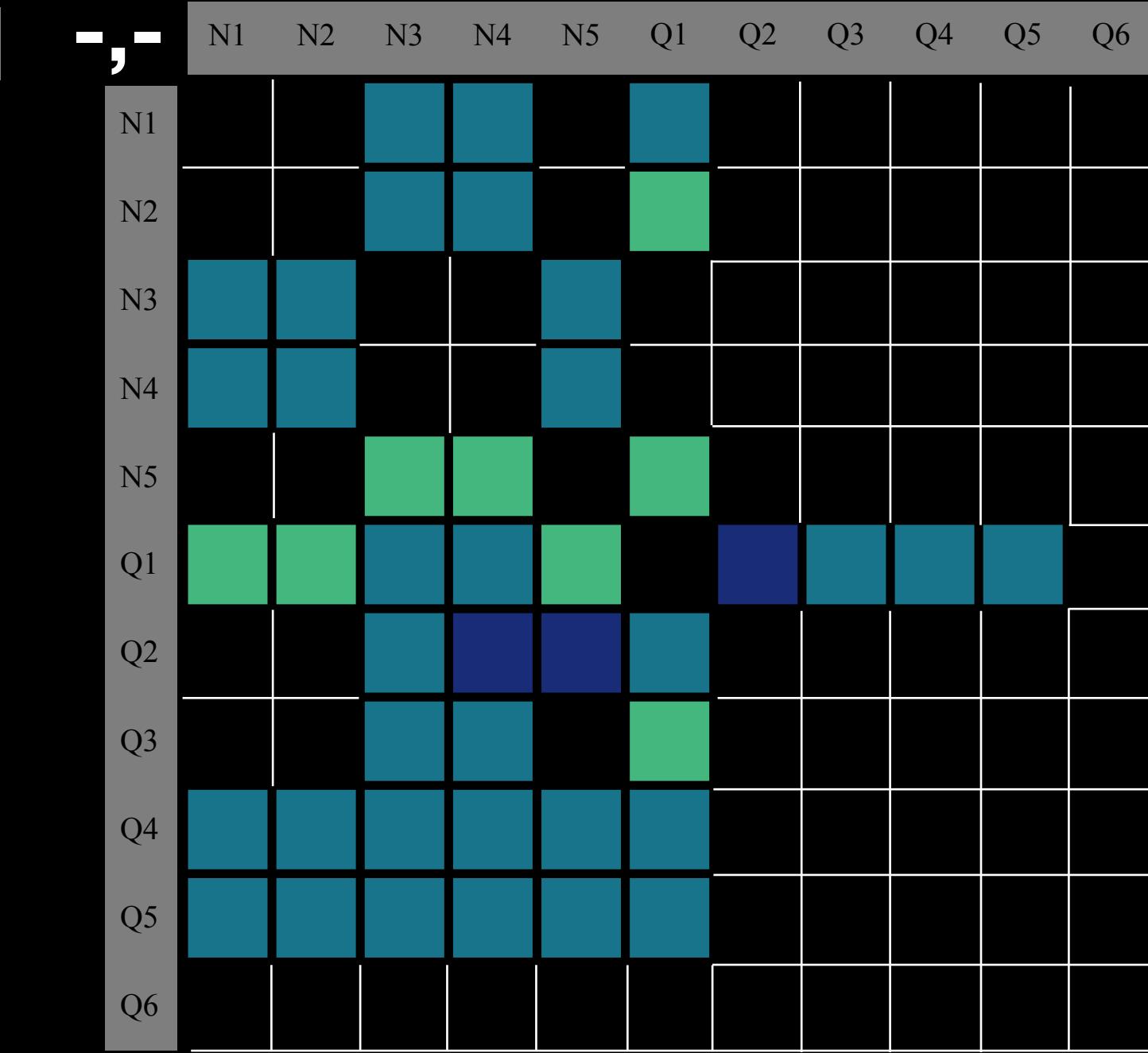
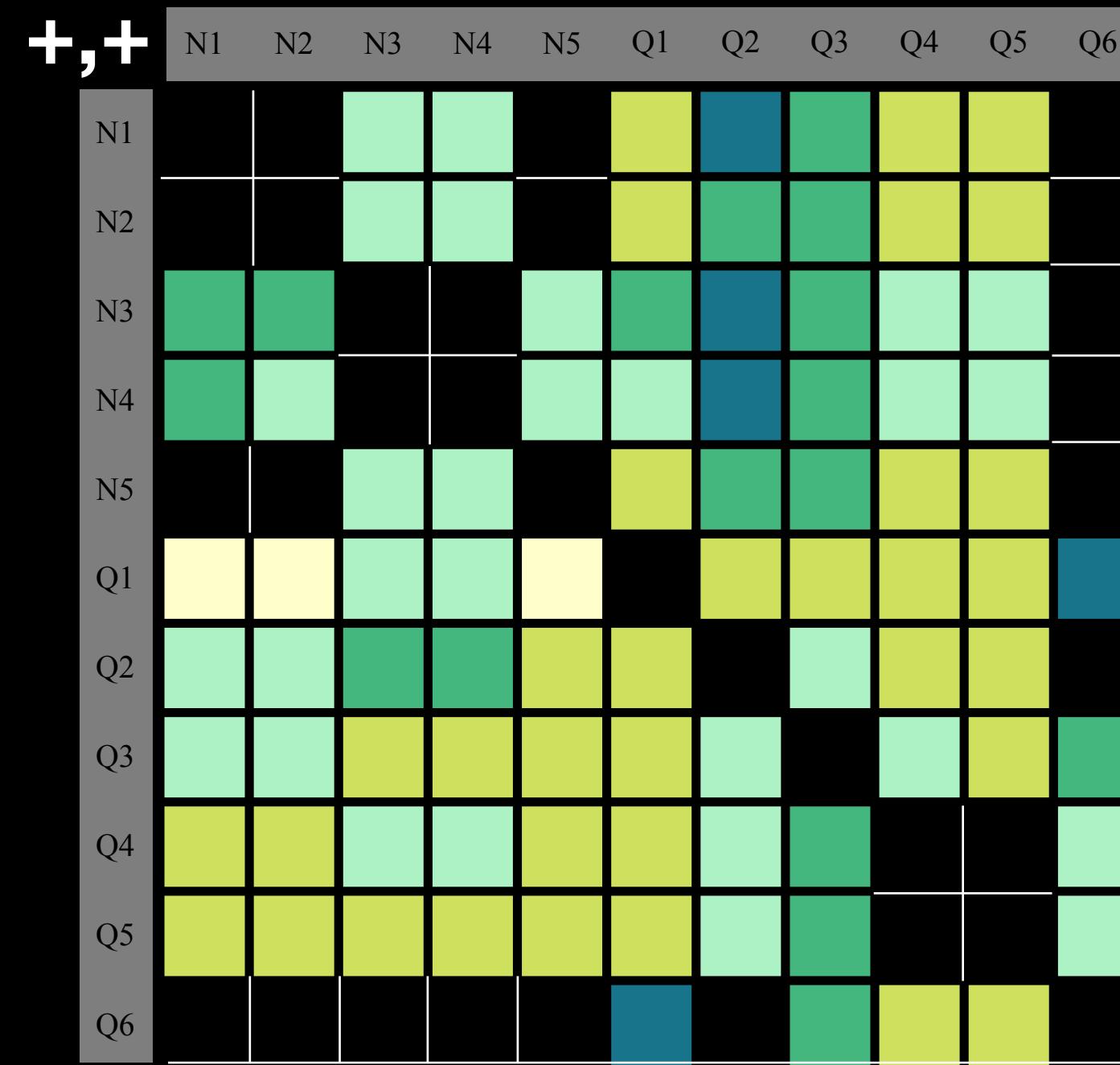
$|n_{\alpha j}|$ free to vary in the corresponding range



$$M_{\alpha i} = M_{\alpha i}^{\text{long}} + M_{\alpha i}^{\text{short}}$$

@ 3σ ($\Delta\chi^2_{\text{tot}} = 9$)

Black squares indicate the cases where discrimination at 3σ is not possible for realistic values of $m_{\beta\beta}^{\text{True}}$ and exposure times T



Legend: $8 < m_{\beta\beta}^{\text{True}} \leq 14 \text{ meV}$ (yellow), $14 \text{ meV} < m_{\beta\beta}^{\text{True}} \leq 30 \text{ meV}$ (light green), $30 \text{ meV} < m_{\beta\beta}^{\text{True}} \leq 49 \text{ meV}$ (medium green), $49 \text{ meV} < m_{\beta\beta}^{\text{True}} \leq 100 \text{ meV}$ (dark green), $100 \text{ meV} < m_{\beta\beta}^{\text{True}} \leq 500 \text{ meV}$ (blue), $m_{\beta\beta}^{\text{True}} > 500 \text{ meV}$ (dark blue).

In conclusion

Assuming that future $0\nu\beta\beta$ experiments detect a positive signal, will it be possible, via the combination of several experiments using different isotopes, to discriminate among the various NME models?

This project has received funding and support from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 860881-HIDDeN

In conclusion

Assuming that future $0\nu\beta\beta$ experiments detect a positive signal, will it be possible, via the combination of several experiments using different isotopes, to discriminate among the various NME models?

YES (depending on the value of $m_{\beta\beta}^{\text{True}}$)!

The short-range term could affect considerably both the sensitivities and the nuclear model discrimination power of next-generation $0\nu\beta\beta$ experiments:

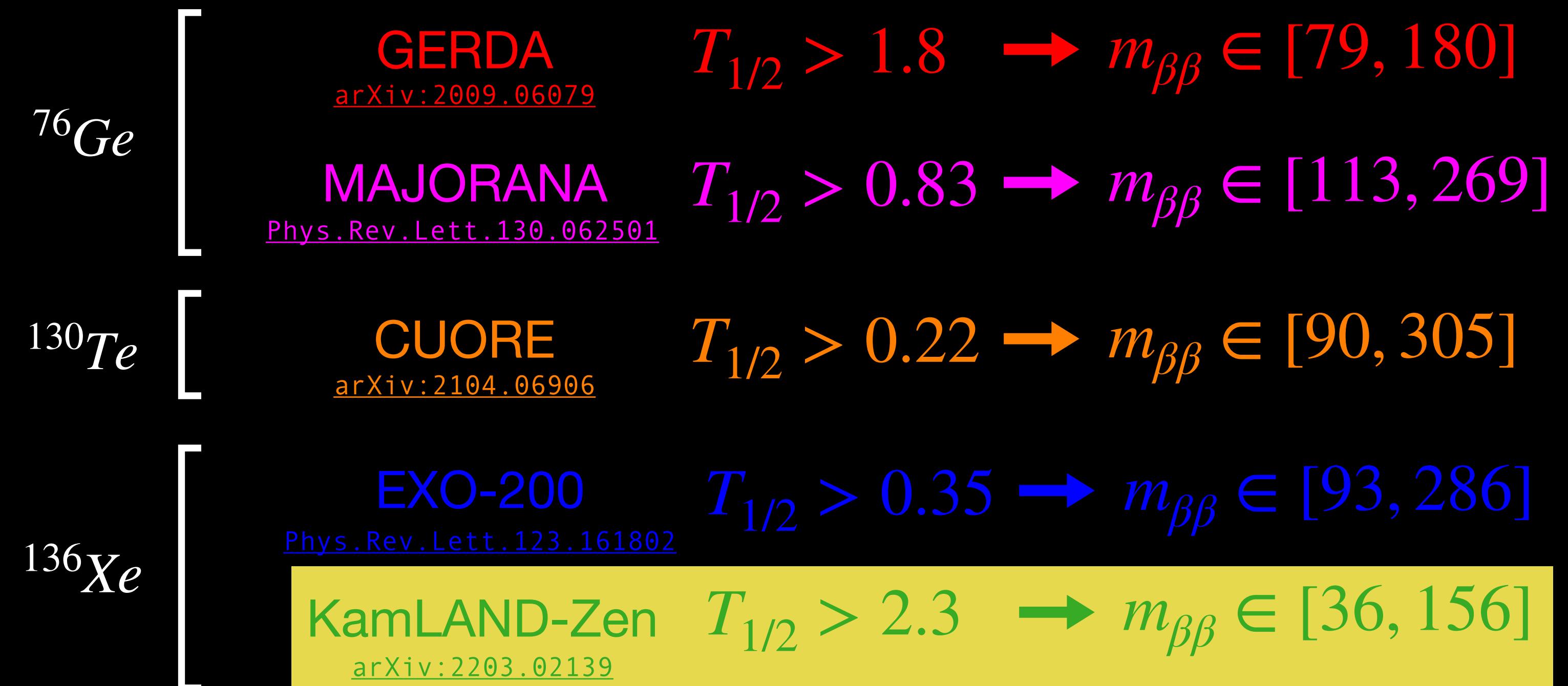
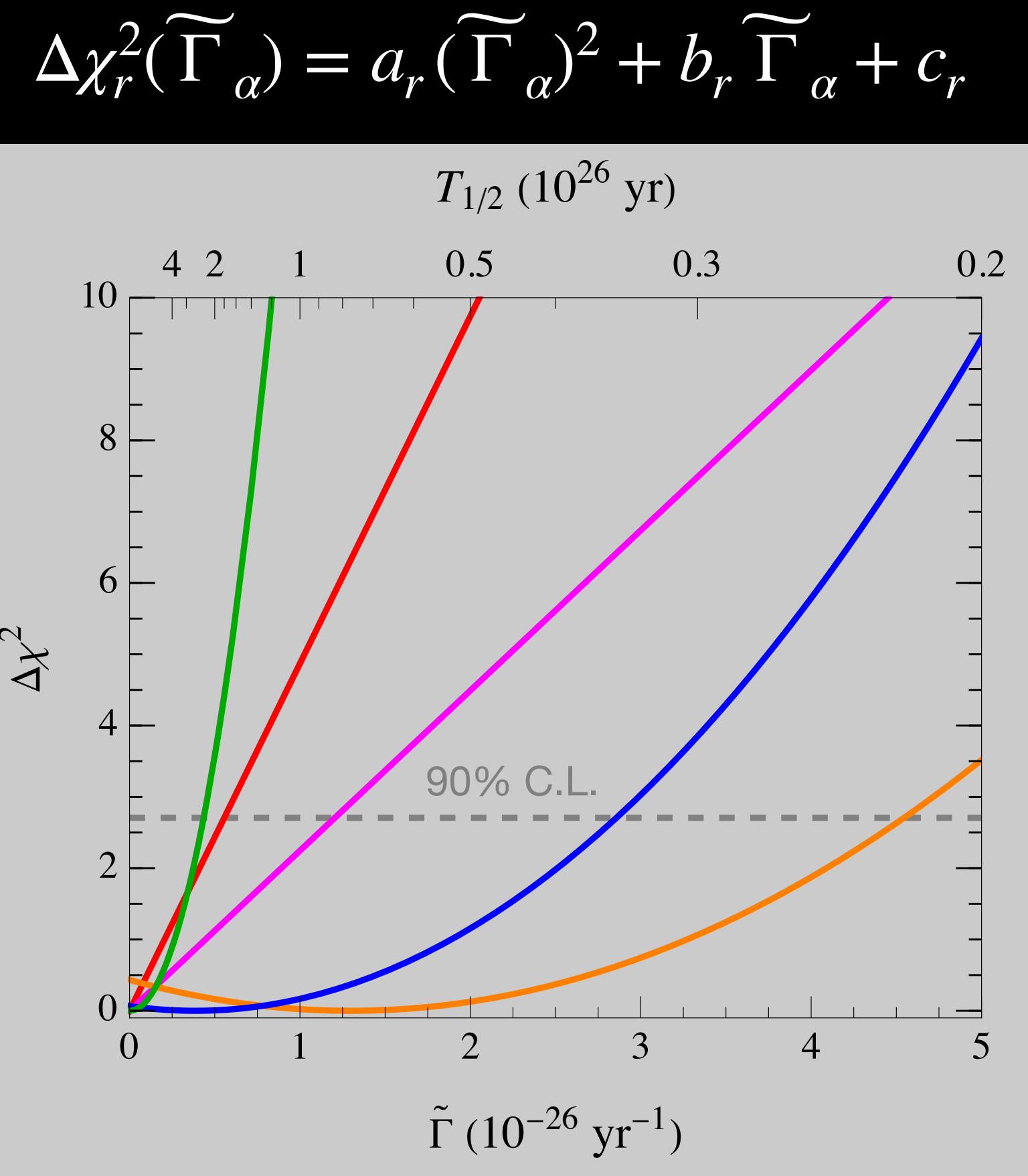
- The most sensitive projects are LEGEND-1000 and nEXO, whose sensitivity to $m_{\beta\beta}$ will cover most part of the inverted mass ordering region for many NME models. However, unfortunate short-range interaction interference might prevent these advanced setups to reach this region.
- Discriminating between different NME calculations will be possible for a broad range of NME models, even though the presence of the short-range contribution will essentially destroy this sensitivity, unless its sign is known to be positive.

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Backup

Current picture

[arXiv:2204.09569](#)



Inverted Mass Ordering : $m_{\beta\beta} \in [14, 49] \text{ meV}$

Backup

Current picture

[arXiv:2204.09569](https://arxiv.org/abs/2204.09569)

Sensitivity on $m_{\beta\beta}$:

$$\Delta\chi_r^2(\widetilde{\Gamma}_\alpha) = a_r (\widetilde{\Gamma}_\alpha)^2 + b_r \widetilde{\Gamma}_\alpha + c_r$$

$$\widetilde{\Gamma}_\alpha(m_{\beta\beta}, M_{\alpha i}) = G_{0\nu} (g_A^2 |M_{0\nu}|)^2 m_{\beta\beta}^2$$

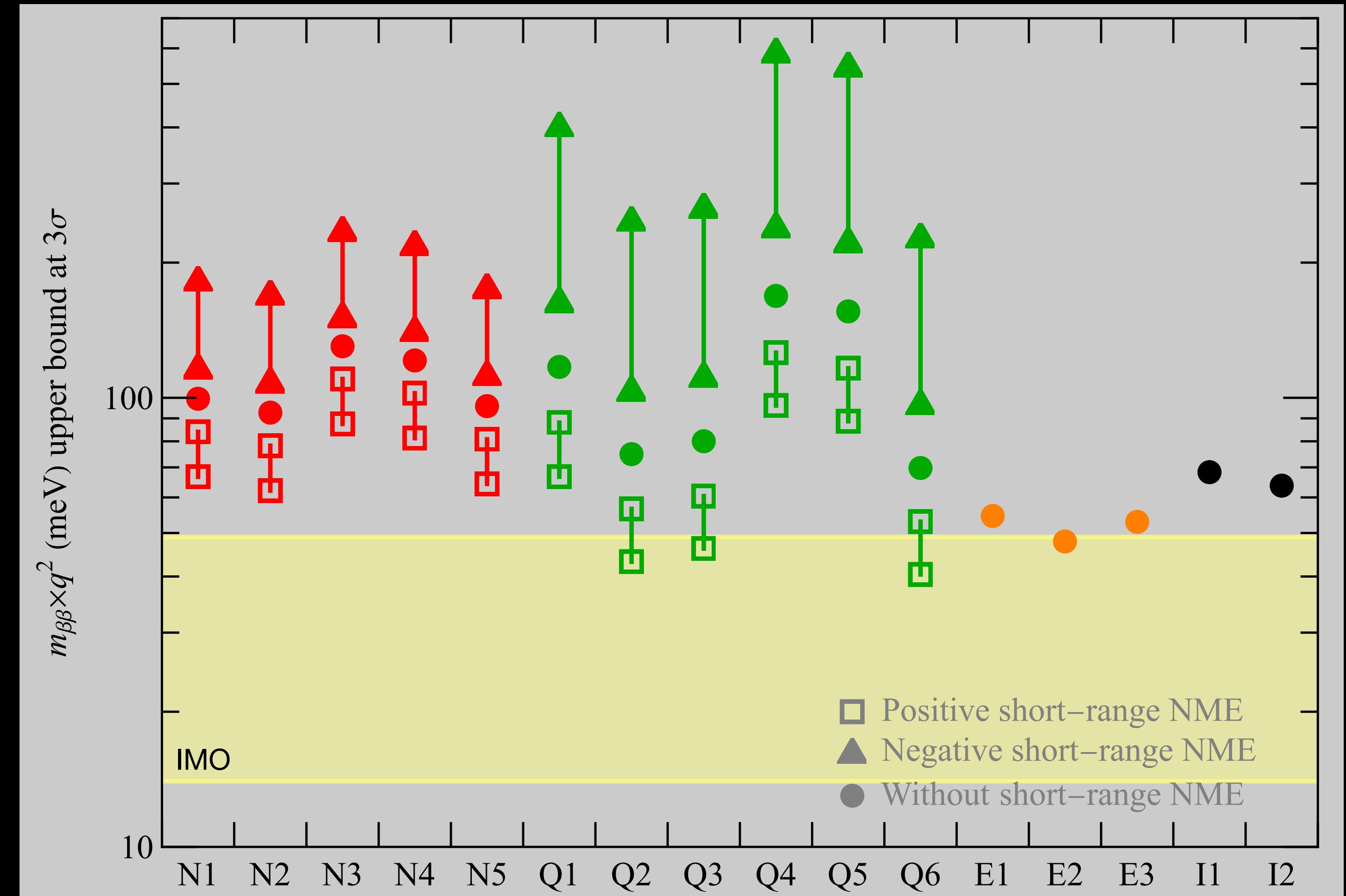


$$\chi_{\text{tot}}^2(m_{\beta\beta}) = \sum_r \Delta\chi_r^2(m_{\beta\beta})$$

$$\Delta\chi_{\text{tot}}^2(m_{\beta\beta}) = \chi_{\text{tot}}^2(m_{\beta\beta}) - \chi_{\text{tot,min}}^2(m_{\beta\beta})$$

- Impact of the short-range term
- Uncertainties on both the size and sign of $|n_{\alpha i}|$

@ 3σ ($\Delta\chi_{\text{tot}}^2 = 9$)



Backup

$$\Delta\chi^2_r(\widetilde{\Gamma}_\alpha) = a_r(\widetilde{\Gamma}_\alpha)^2 + b_r \widetilde{\Gamma}_\alpha + c_r$$

arXiv:2204.09569

Nuclide	Experiment	a_r	b_r	c_r	$T_{1/2}^{90}/10^{26}\text{yr}$
^{76}Ge	GERDA	0.000	4.871	0.000	1.8
	MAJORANA	0.000	2.246	0.000	0.83
^{130}Te	CUORE	0.257	-0.667	0.433	0.22
^{136}Xe	KamLAND-Zen	14.315	0.000	0.000	2.3
	EXO-200	0.443	-0.342	0.066	0.35

Updated with recent results

Backup

$$S_{\alpha i}(m_{\beta\beta}, M_{\alpha i}) = \ln 2 \cdot N_A \cdot \varepsilon_\alpha \cdot \left(\frac{T}{1 \text{ yr}} \right) \cdot \widetilde{\Gamma}_\alpha(m_{\beta\beta}, M_{\alpha i})$$

$$B_\alpha = b_\alpha \cdot \varepsilon_\alpha \cdot \left(\frac{T}{1 \text{ yr}} \right)$$

[arXiv:2202.01787](#)

Experiment	Isotope	ε [mol·yr]	b [events/(mol·y)]	PSF $[\text{yr}^{-1} \text{ eV}^{-2}]$
LEGEND-1000	^{76}Ge	8736	$4.9 \cdot 10^{-6}$	$2.36 \cdot 10^{-26}$
SuperNEMO	^{82}Se	185	$5.4 \cdot 10^{-3}$	$10.19 \cdot 10^{-26}$
CUPID	^{100}Mo	1717	$2.3 \cdot 10^{-4}$	$15.91 \cdot 10^{-26}$
SNO+II	^{130}Te	8521	$5.7 \cdot 10^{-3}$	$14.2 \cdot 10^{-26}$
nEXO	^{136}Xe	13700	$4.0 \cdot 10^{-5}$	$14.56 \cdot 10^{-26}$

Backup

What if sign unknown?

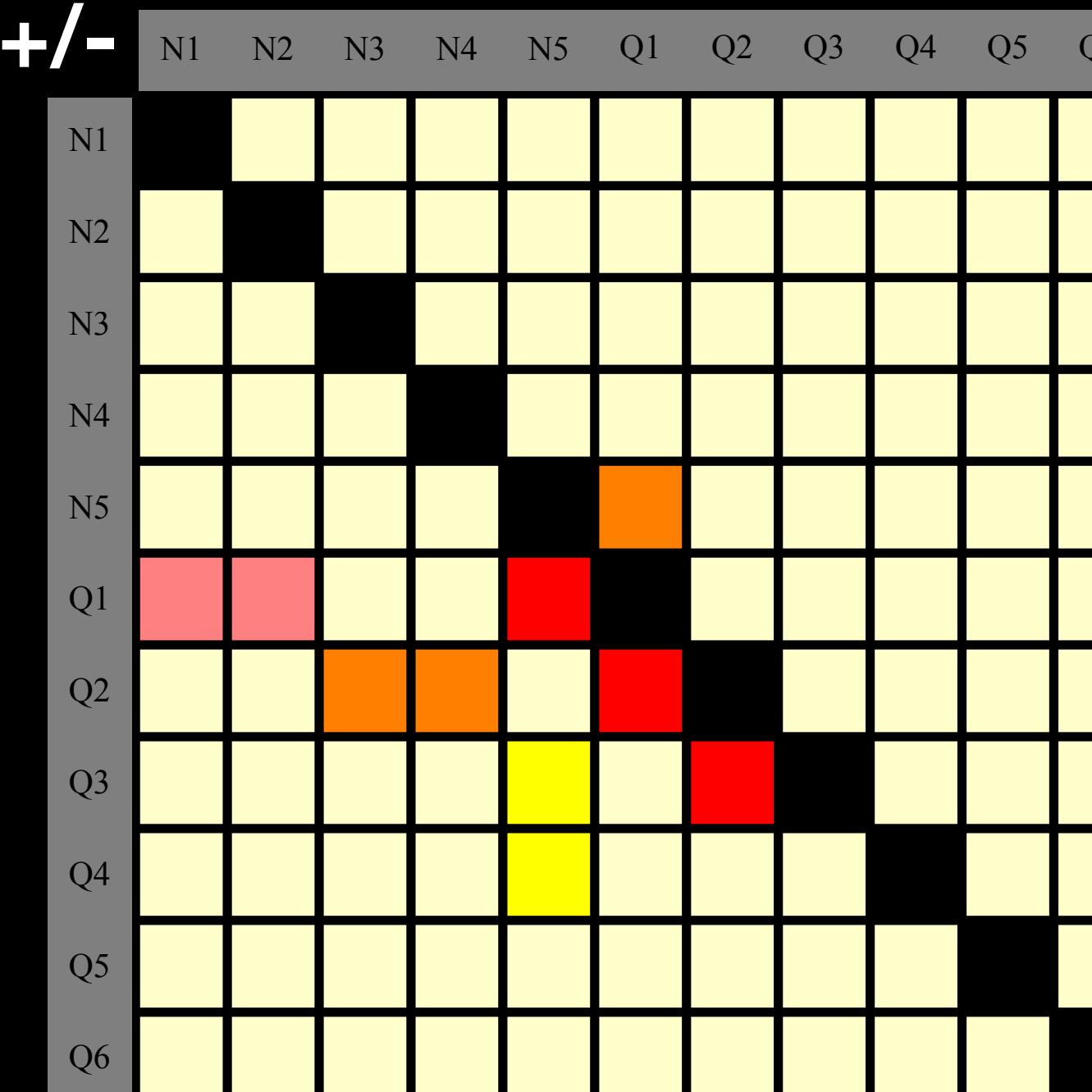
$$m_{\beta\beta}^{\text{True}} = 60 \text{ meV}$$

$|n_{\alpha i}|$ taken as the central value of the allowed range

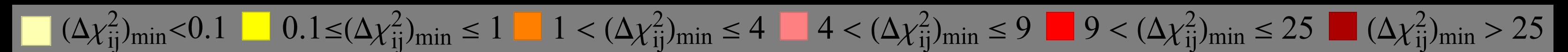
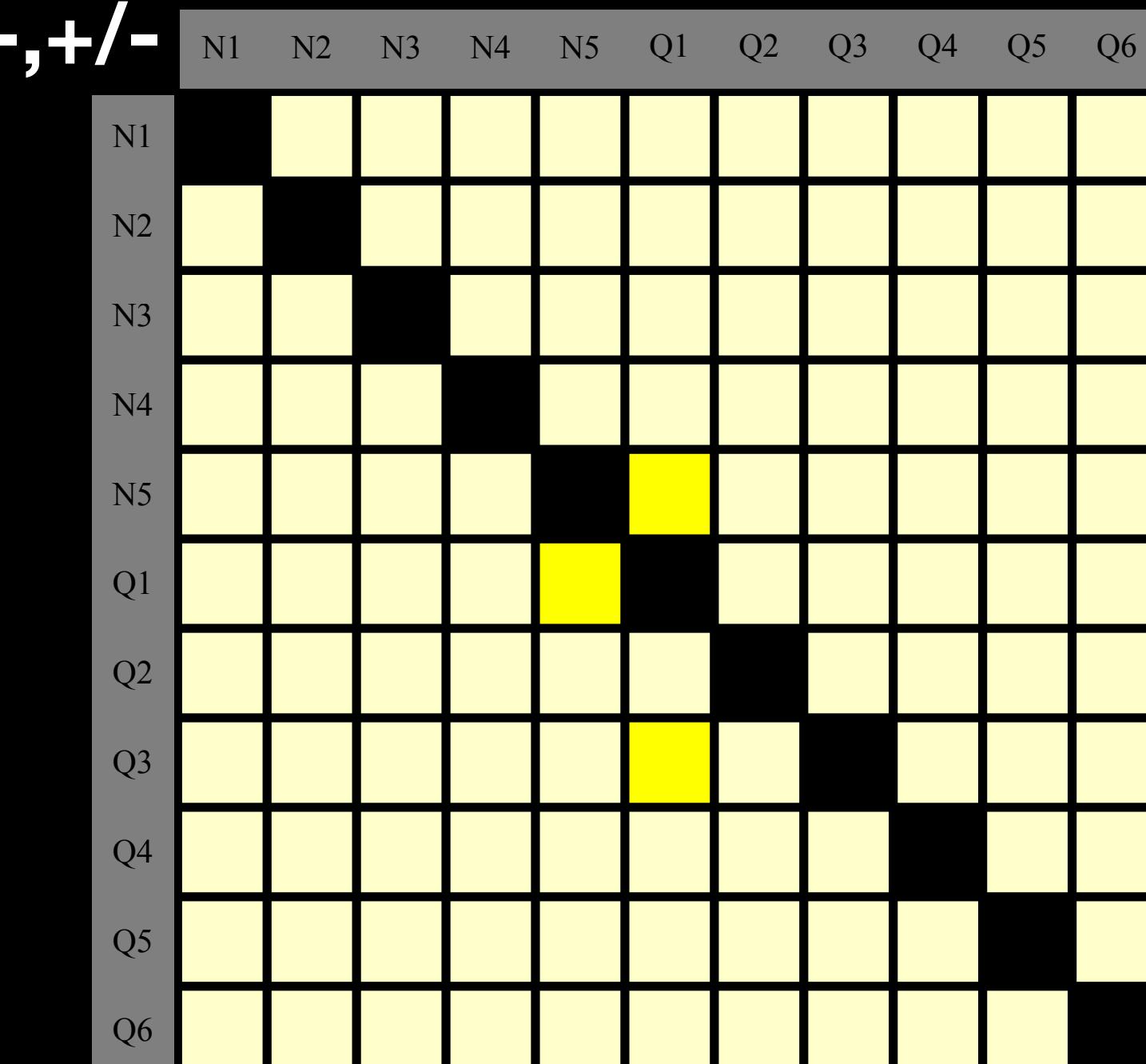
$n_{\alpha j}$ free to vary in the union of the corresponding positive and negative ranges

Discrimination power gets weaken for smaller $m_{\beta\beta}^{\text{True}}$

+,-/-



-,+/-



Not promising nuclear model discrimination!

Backup

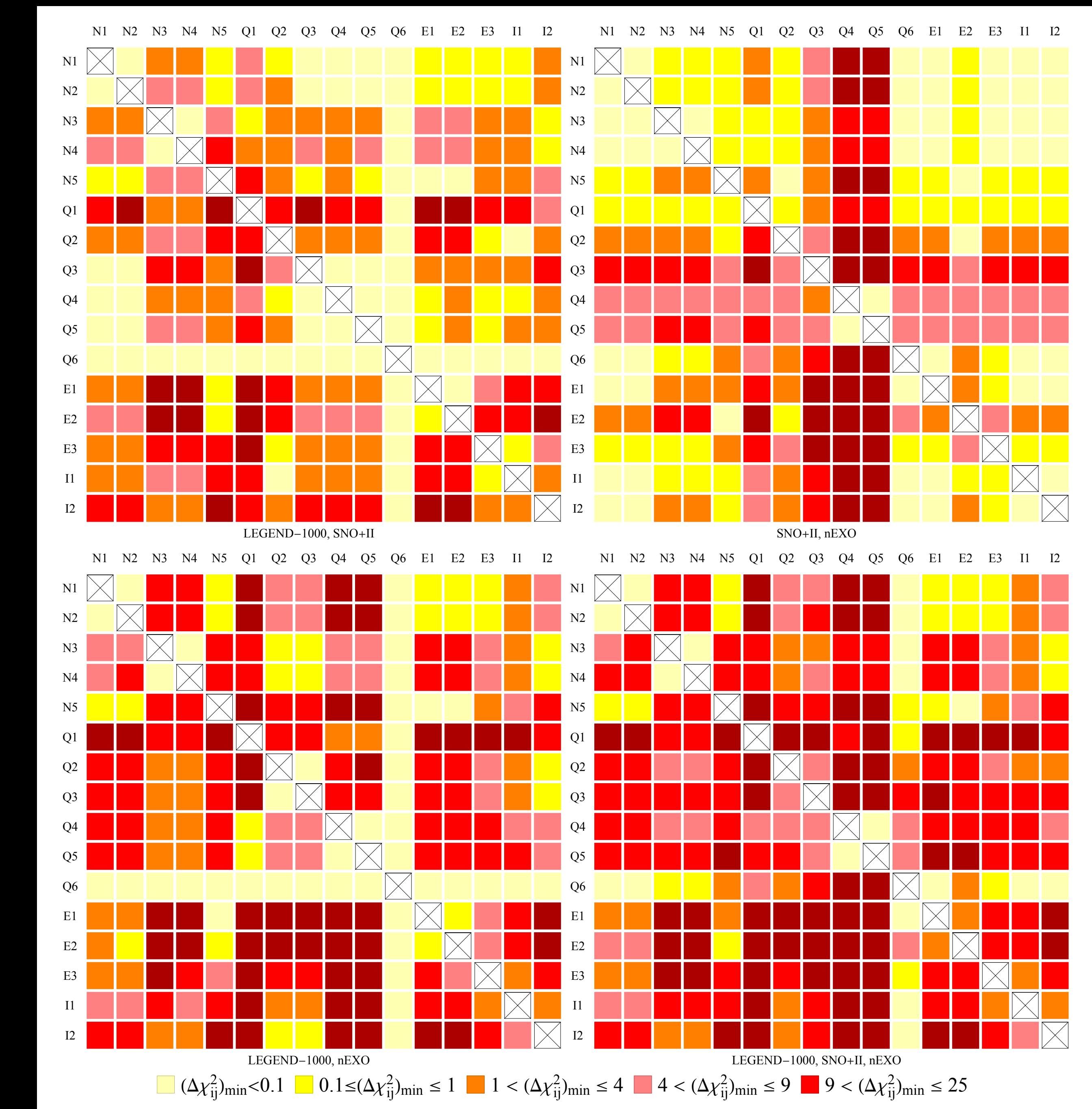
$$M_{\alpha i} = M_{\alpha i}^{\text{long}}$$

$$m_{\beta\beta}^{\text{True}} = 40 \text{ meV}$$

Nuclear model discrimination power for different combinations of future experiments.

All Nuclear models considered.

The involvement of SNO+II significantly improves the discrimination potential and increase the number of distinguishable models.



Backup

$$M_{\alpha i} = M_{\alpha i}^{\text{long}}$$

$$m_{\beta\beta}^{\text{True}} = 40 \text{ meV}$$

Nuclear model discrimination power for different combinations of future experiments.

The involvement of CUPID significantly improves the discrimination potential and increase the number of distinguishable models.

