Neutrino Flavor Conversion in Supernovae
Slow and Fast Conversions

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Introduction

Neutrinos from Supernova

- A core-collapse supernova (CCSN) is a star explosion that happens when a massive star ($M \gtrsim 8 M_\odot$) ends its nuclear fuel, collapsing into itself.
- In this process, a large number of neutrinos are emitted ($\sim 10^{53}$ erg) in a time window of about 10 seconds.
- However, **we still do not know the flavor conversion mechanism** for these neutrinos.

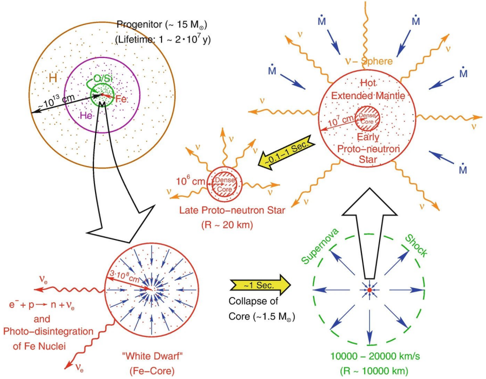

Figure: Stages of a core-collapse supernova. (Figure extracted from [2])

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[2] H. -Th. Janka. "Neutrino Emission from Supernovae". In: (Feb. 2017). DOI: 10.1007/978-3-319-21846-5_4. arXiv: 1702.08713 [astro-ph.HE]

Neutrinos Flavor Evolution

- The evolution for the (anti)neutrino density matrix ρ in a given phase space cell $d^3\vec{p}d^3\vec{x}$ [3]

$$\left(\frac{\partial}{\partial t}. + \vec{v}\cdot\vec{\nabla}_x\right)\rho(t,\vec{x},\vec{p}) = -i[H(t,\vec{x},\vec{p}),\rho(t,\vec{x},\vec{p})] \tag{1}$$

- Where the Hamiltonian is given by

$$H(t,\vec{x},\vec{p}) = H_{vac} + H_{mat} + H_{\nu\nu} \tag{2}$$

- In which the neutrino-neutrino potential is given by[4]

$$H_{\nu\nu}(\vec{p}) = \sqrt{2}G_F\int\frac{d^3\vec{p}'}{(2\pi)^3}(1-\vec{p}\cdot\vec{p}')[\rho(\vec{p}')-\bar{\rho}(\vec{p}')] \tag{3}$$

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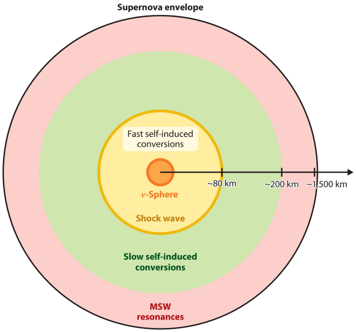
[3] Given that cell size is sufficiently large so that both $\vec{p}$ and $\vec{x}$ can be determined.

[4] G. Sigl and G. Raffelt. "General kinetic description of relativistic mixed neutrinos". In: *Nucl. Phys. B* 406 (1993), pp. 423–451. DOI: 10.1016/0550-3213(93)90175-0, Pedro Dedin Neto and Ernesto Kemp. "Neutrino–(anti)neutrino forward scattering potential for massive neutrinos at low energies". In: *Mod. Phys. Lett. A* 37.08 (2022), p. 2250048. DOI: 10.1142/S0217732322500481. arXiv: 2111.11480 [hep-ph].

Possible Regimes of Collective Conversion

- **Synchronized Oscillations (Slow):** happen when the total lepton number is large and all (anti)neutrino modes oscillate with the same synchronized frequency $\langle\omega\rangle$.
- **Bipolar Oscillations (Slow):** happen when the strength of the neutrino-neutrino interactions are more comparable to the vacuum, leading to pendulum-like oscillations with frequency $\sim \sqrt{\mu\omega}$.
- **Fast Conversions:** are the ones that happen even when $\omega = 0$ and then happen at time scales given by μ $(\mu \gg \omega)$.



Tamborra I, Shalgar S. 2021
Annu. Rev. Nucl. Part. Sci. 71:165–88

Figure: Expected regimes of flavor conversion inside a supernova. (Extracted from [6])

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[6] Irene Tamborra and Shashank Shalgar. "New Developments in Flavor Evolution of a Dense Neutrino Gas". In: *Ann. Rev. Nucl. Part. Sci.* 71 (2021), pp. 165–188. DOI: 10.1146/annurev-nucl-102920-050505. arXiv: 2011.01948 [astro-ph.HE]

Outline of the talk

- I want to talk about similarities and differences between slow and fast oscillations by considering 3 simple systems:
  1. Isotropic and Uniform Neutrino Gas: Dominated by Bipolar Slow Oscillations
  2. Non-isotropic gas with electron lepton number (ELN) angular crossing and $\omega = 0$: Dominated by Fast Flavor Conversions (FFC).
  3. Mixed system with ELN angular crossing and $\omega \neq 0$: Where we can see the influence of the vacuum component in the FFC.
- I'll conclude with a summary of some questions and problems that are still open in this field.
- Finally, I'll talk about the public repository been developed to host the codes in an open-source format so that anyone in the community can use it[7].

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[7] Pedro Dedin Neto. "Open-Source Numerical Solver for Neutrino Collective Effects – I: Isotropic Neutrino Gas". In: (Oct. 2022). arXiv: `2210.15770 [hep-ph]`.

Slow Conversions

Polarization Vector Formalism

- In a 2-families approximation, we can decompose complex matrices into Pauli matrices $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ so that the coefficients of this decomposition work as components of a 3-dimensional vector:

$$H_{vac} = -\omega \frac{1}{2} \vec{\sigma} \cdot \vec{B}, \quad H_{matt} = -\lambda \frac{1}{2} \vec{\sigma} \cdot \vec{L}, \quad \rho = \frac{1}{2} \mathbf{1} + \frac{1}{2} \vec{\sigma} \cdot \vec{P}, \tag{4a}$$

$$\omega \equiv \frac{\Delta m^2}{2E}, \quad \lambda \equiv \sqrt{2} G_F n_e, \quad \mu \equiv \sqrt{2} G_F n_\nu. \tag{4b}$$

- In this formalism, the evolution equation becomes a precession-like one.

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}_x \right) \vec{P}_{\nu,\vec{p}} = \vec{P}_{\nu,\vec{p}} \times \left[\omega \vec{B} + \lambda \vec{L} + \mu \int \frac{d^3 \vec{q}}{(2\pi)^3} (1 - \vec{p} \cdot \vec{q}) (\vec{P}_{\nu,\vec{q}} - \vec{P}_{\bar{\nu},\vec{q}}) \right] \tag{5}$$

- A similar equation holds for antineutrinos by changing the sign of the vacuum frequency $\omega \to -\omega$.

Uniform, Isotropic, and Mono-energetic Neutrino Gas

- For a uniform, Isotropic, and Mono-energetic Neutrino Gas

$$\dot{\vec{P}}_\nu = \vec{P}_\nu \times \left[\omega\vec{B} + \mu(\vec{P}_\nu - \vec{P}_{\overline{\nu}})\right], \qquad \dot{\vec{P}}_{\overline{\nu}} = \vec{P}_{\overline{\nu}} \times \left[-\omega\vec{B} + \mu(\vec{P}_\nu - \vec{P}_{\overline{\nu}})\right] \tag{6}$$

- If we define the difference $\vec{D} = \vec{P}_\nu - \vec{P}_{\overline{\nu}}$ and sum $\vec{S} = \vec{P}_\nu + \vec{P}_{\overline{\nu}}$ vectors, in addition to $\vec{Q} = \vec{S} - (\omega/\mu)\vec{B}$, we can write

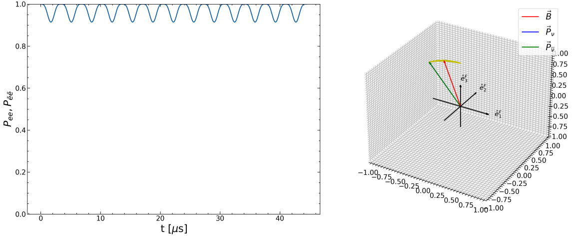
$$\dot{\vec{Q}} = \mu\vec{D} \times \vec{Q}, \qquad \dot{\vec{D}} = \omega\vec{Q} \times \vec{B} \tag{7}$$

Pendulum Analogy

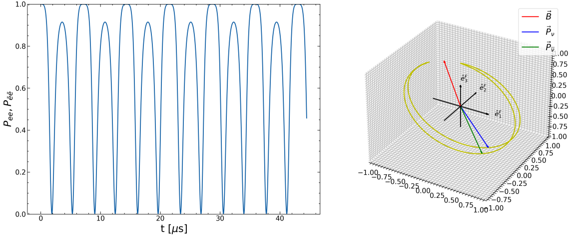
These equations are equivalent to a Pendulum attracted by a force field $\vec{F} = \omega\vec{B}$, with angular momentum $\vec{L} = \vec{D}$, position $\vec{R} = \vec{Q}$, and moment of inertia $I = \mu^{-1}$.

Survival Probability

- **Normal Hierarchy ($\Delta m^2 > 0$):** The system is attracted by $\vec{F} = |\omega|\vec{B}$

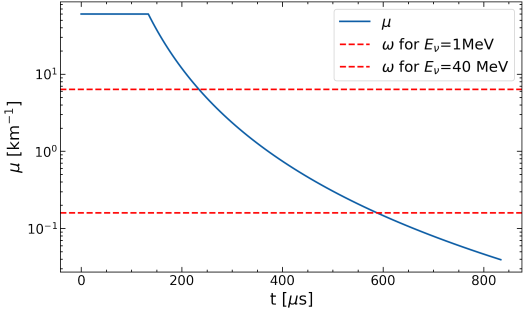


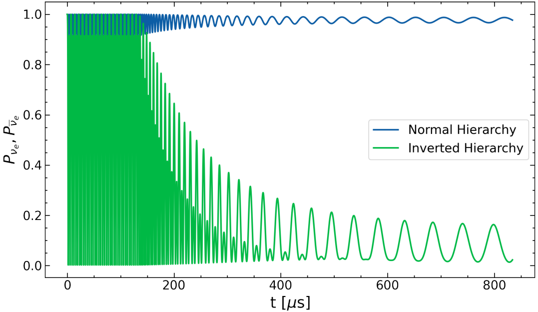
- **Inverted Hierarchy ($\Delta m^2 < 0$):** The system is attracted by $\vec{F} = -|\omega|\vec{B}$



Decreasing μ

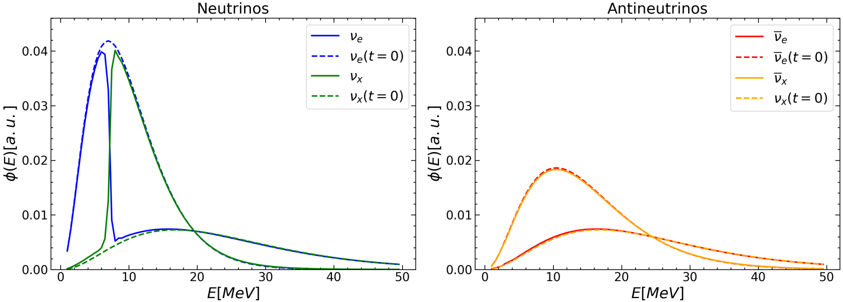
- If we have a decreasing μ along the neutrino propagation, such as in a supernova, we have an equivalent increase in the pendulum moment of inertia $I = \mu^{-1}$.
- Therefore, for an adiabatic decrease of μ, the oscillation is damped towards $\omega\vec{B}$, resulting in an almost complete conversion for the inverted mass hierarchy.





Multi-energetic Scenario - Spectral Split

- When considering a multi-energetic scenario, in a regime where $\mu \gg \omega$, each mode will evolve individually as a mono-energetic case[8].
- In a scenario of decreasing μ with inverted hierarchy and a positive total lepton number, only a fraction of the neutrinos will be allowed to change their flavor, giving rise to a **spectral split** in the converted spectrum.



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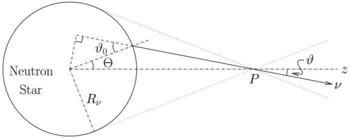
[8] Steen Hannestad et al. "Self-induced conversion in dense neutrino gases: Pendulum in flavour space". In: *Phys. Rev. D* 74 (2006). [Erratum: Phys.Rev.D 76, 029901 (2007)], p. 105010. DOI: `10.1103/PhysRevD.74.105010`. arXiv: `astro-ph/0608695`.

Connection to Supernovae - Bulb Model

- One way to connect this system to a supernova scenario is by using the spherically symmetric Bulb Model[9].
- In the so-called Single-Angle approximation, the neutrino-neutrino interactions have the same form as in the isotropic case, with a strength decreasing with the radius:

$$H_{\nu\nu} = \sqrt{2} G_F 2\pi D(r) \sum_{\alpha} \int [\rho_\alpha(p') - \overline{\rho}_\alpha(p')] dp' \qquad (8)$$



- Beyond the Single-Angle approximation, different angular modes ϑ_i, ϑ_j will experience different potential strength μ_{ij}, which can lead to different modes of oscillations resulting in a partial or total kinematic decoherence[10].

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[9] Huaiyu Duan et al. "Simulation of Coherent Non-Linear Neutrino Flavor Transformation in the Supernova Environment. 1. Correlated Neutrino Trajectories". In: *Phys. Rev. D* 74 (2006), p. 105014. DOI: `10.1103/PhysRevD.74.105014`. arXiv: `astro-ph/0606616`.

[10] Steen Hannestad et al. "Self-induced conversion in dense neutrino gases: Pendulum in flavour space". In: *Phys. Rev. D* 74 (2006). [Erratum: Phys.Rev.D 76, 029901 (2007)], p. 105010. DOI: `10.1103/PhysRevD.74.105010`. arXiv: `astro-ph/0608695`.

Fast Flavor Conversions (FFC)

- For a long time, it was thought that the final answer to the flavor conversion of supernova neutrinos would be single or multiple spectral splits with possible suppression of this effect due to kinematic decoherence.[11]
- However, it was noticed that if some symmetries are relaxed, the existence of **ELN crossing in the angular distribution** of neutrinos could lead to **conversions at the order of** μ, even with no vacuum mixing ($\omega = 0$).[12]
- This kind of phenomenon started to be called **fast flavor conversion (FFC)**, given that $\mu \gg \omega$ in the environments are relevant to collective oscillations.

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[11] Alessandro Mirizzi et al. "Supernova Neutrinos: Production, Oscillations and Detection". In: *Riv. Nuovo Cim.* 39.1-2 (2016), pp. 1–112. DOI: `10.1393/ncr/i2016-10120-8`. arXiv: `1508.00785 [astro-ph.HE]`.

[12] R. F. Sawyer. "Speed-up of neutrino transformations in a supernova environment". In: *Phys. Rev. D* 72 (2005), p. 045003. DOI: `10.1103/PhysRevD.72.045003`. arXiv: `hep-ph/0503013`.

Fast Conversions

FFC - Stability and ELN Crossing

- The necessary conditions for these types of conversions to happen are:
  1. A small initial mixing, encoded by the off-diagonal element of the density matrix ρ_{ex} (two-families approximation);
  2. A crossing in the electron lepton number (ELN) in the angular distribution[13];
- One can access the necessary conditions for flavor conversion by Linear Stability Analysis around the initial conditions where $|\rho_{ex}| \ll |\rho_{ee}|, |\rho_{xx}|$.

$$i\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}\right)\rho_{ex} = (H_{ee} - H_{xx})\rho_{ex} + (\rho^0_{xx} - \rho^0_{ee})\mu \int d\vec{p}'(1 - \hat{p}\cdot\hat{p}')(\rho_{ex} - \bar{\rho}_{ex}) \quad (9a)$$

$$\rho_{ex}(\vec{x}, t, \vec{p}) = Q(\vec{p})e^{-i(\Omega t - \vec{k}\cdot\vec{x})}, \qquad \bar{\rho}_{ex}(\vec{x}, t, \vec{p}) = \bar{Q}(\vec{p})e^{-i(\Omega t - \vec{k}\cdot\vec{x})} \quad (9b)$$

- So that fast growth is achieved if $\mathrm{Im}\{\Omega\} \neq 0$ (for temporal instabilities) or $\mathrm{Im}\{k_i\} \neq 0$ (for spatial instabilities).

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[13] Basudeb Dasgupta. "Collective Neutrino Flavor Instability Requires a Crossing". In: *Phys. Rev. Lett.* 128.8 (2022), p. 081102. DOI: `10.1103/PhysRevLett.128.081102`. arXiv: `2110.00192 [hep-ph]`.

Uniform and Mono-energetic Neutrino Gas with ELN crossing

- One simple system that shows **fast oscillations** ($\omega = 0$) is an uniform and mono-energetic neutrino gas with axial symmetry ($v = \cos\theta_{\vec{p}}$) and a crossing in the ELN angular distribution.

$$\dot{\vec{P}}_{\nu,v} = \vec{P}_{\nu,v} \times \mu \int du(1-u.v)(\vec{P}_{\nu,u} - \vec{P}_{\bar{\nu},u}), \quad (10a)$$

$$\dot{\vec{P}}_{\bar{\nu},v} = \vec{P}_{\bar{\nu},v} \times \mu \int du(1-u.v)(\vec{P}_{\nu,u} - \vec{P}_{\bar{\nu},u}). \quad (10b)$$

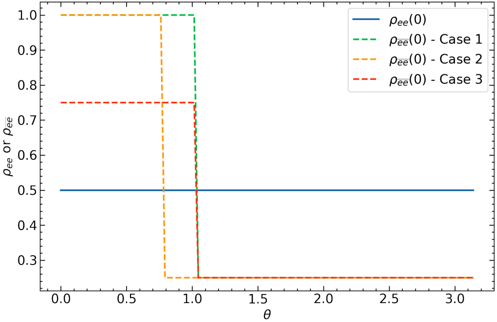

Figure: (Anti)neutrino angular distribution (Same as in [15])

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[15] Irene Tamborra and Shashank Shalgar. "New Developments in Flavor Evolution of a Dense Neutrino Gas". In: *Ann. Rev. Nucl. Part. Sci.* 71 (2021), pp. 165–188. DOI: 10.1146/annurev-nucl-102920-050505. arXiv: 2011.01948 [astro-ph.HE]

Gyroscopic Pendulum

- Defining the **angular moments** $\vec{M}_n$, and going to a frame co-rotating around $\vec{M}_0$, we can write the EoM as follows

$$\vec{M}_n = \int_{-1}^{+1} dv v^n (\vec{P}_{\nu,v} - \vec{P}_{\overline{\nu},v}), \qquad \dot{\vec{M}}_n = \vec{M}_{n+1} \times \vec{M}_1 \tag{11}$$

- By assuming that there are only **3 linear independent angular moments** , which is the case if we have a **single ELN angular crossing**[16], the EoM of motion can be written as

$$\dot{\vec{M}}_1 = \mu \vec{M}'_2 \times \vec{M}_1, \qquad \dot{\vec{M}}'_2 = \gamma \vec{M}_0 \times \vec{M}_1 \tag{12}$$

Gyroscopic Pendulum Analogy

These equations are equivalent to a Gyroscopic Pendulum attracted by a force field $\vec{F} = \gamma \vec{M}_0$ ($\gamma = \mu v_1 v_2 v_3$), with total angular momentum $\vec{J} = \vec{M}'_2$ ($\vec{M}'_2 = \vec{M}_2 - (v_1 + v_2 + v_3)\vec{M}_1$), position $\vec{R} = \vec{M}_1$, and moment of inertia $I = \mu^{-1}$.

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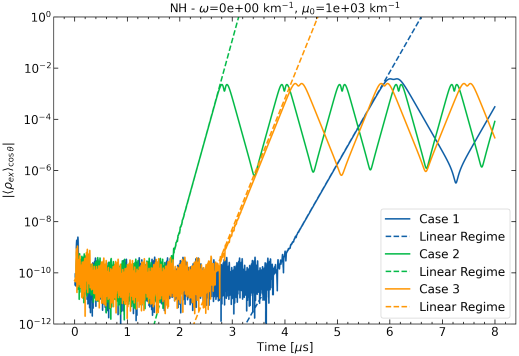
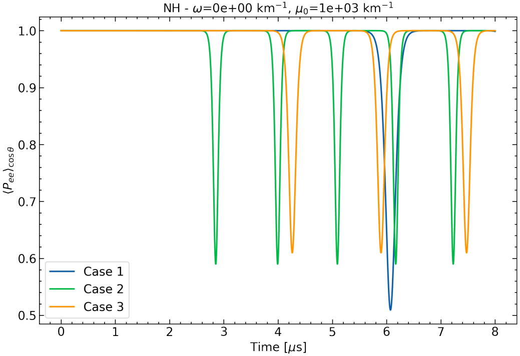
[16] Ian Padilla-Gay, Irene Tamborra, and Georg G. Raffelt. "Neutrino Flavor Pendulum Reloaded: The Case of Fast Pairwise Conversion". In: *Phys. Rev. Lett.* 128.12 (2022), p. 121102. DOI: 10.1103/PhysRevLett.128.121102. arXiv: 2109.14627 [astro-ph.HE].

Numerical Solution

- As we have seen, the survival probability $\langle P_{ee} \rangle_\theta$ has an oscillatory behavior, as expected from the gyroscopic pendulum equations.
- Also, on the left, we can see the fast increase in flavor conversion, encoded by ρ_{ex}, in agreement with the linear regime analysis.





Connection to Supernovae - Decoupling Reagion

- Neutrinos ν_e, $\bar{\nu}_e$, and ν_x (ν_μ, ν_τ) decouple from different regions ($R_{\nu_e} > R_{\bar{\nu}_e} > R_{\nu_x}$) due to the different cross sections.
- This makes the $\bar{\nu}_e$ distribution more forward peaked than ν_e near the decoupling region.
- So we should expect FFC to happen near the decoupling region, which could influence a lot the revival of the shock wave as proposed by models with neutrino-driven explosions.

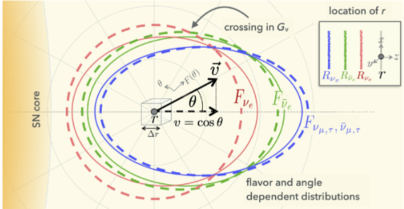


Figure: (Anti)neutrinos angular distribution near the decoupling region (Extracted from[17])

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[17] Soumya Bhattacharyya and Basudeb Dasgupta. "Fast Flavor Depolarization of Supernova Neutrinos". In: *Phys. Rev. Lett.* 126.6 (2021), p. 061302. DOI: 10.1103/PhysRevLett.126.061302. arXiv: 2009.03337 [hep-ph].

Mixing Fast and Slow

FFC dependency on ω

- When considering both vacuum oscillations ω and non-isotropic angular distributions (with ELN crossing), the equations for a uniform gas become [18]

$$\dot{\vec{P}}_{\nu,\vec{p}} = \vec{P}_{\nu,\vec{p}} \times \left[\omega_{\vec{p}}\vec{B} + \mu \int \frac{d\vec{q}}{(2\pi)^3}(1 - \hat{p}\cdot\hat{q})(\vec{P}_{\nu,\vec{q}} - \vec{P}_{\bar{\nu},\vec{q}})\right] \tag{13a}$$

$$\dot{\vec{P}}_{\bar{\nu},\vec{p}} = \vec{P}_{\bar{\nu},\vec{p}} \times \left[-\omega_{\vec{p}}\vec{B} + \mu \int \frac{d\vec{q}}{(2\pi)^3}(1 - \hat{p}\cdot\hat{q})(\vec{P}_{\nu,\vec{q}} - \vec{P}_{\bar{\nu},\vec{q}})\right] \tag{13b}$$

- These equations cannot be translated to a pendulum-like equation, and, as we will see, it loses the oscillatory behavior asymptotically.

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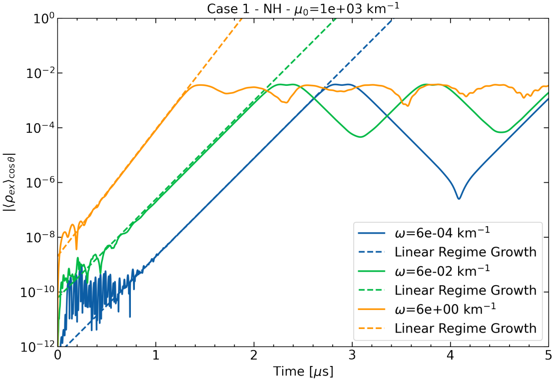
[18] The presence of a matter potential, $\lambda \neq= 0$ will only change the mixing parameters $\Delta m^2, \theta_V \rightarrow \Delta m^2_M, \theta_M$.

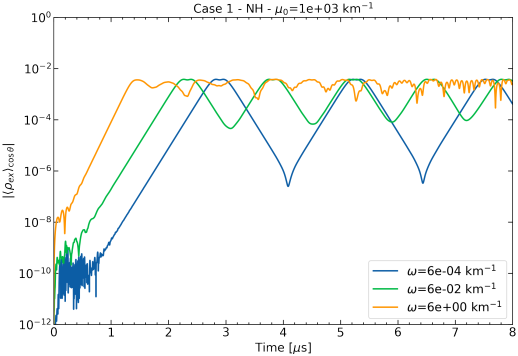
FFC dependency on ω - Linear Regime

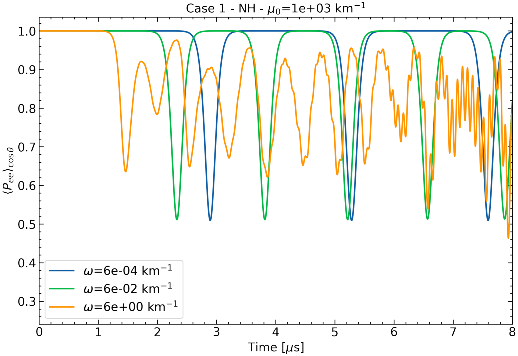
- ω does not change the fast growth of flavor conversion ($\propto \mu$) too much nor the existence of instabilities that lead to flavor conversion.
- However, ω rules the time when fast conversion growth begins, as it is responsible for giving the initial small flavor mixture seed of the linear regime.



FFC dependency on ω - Non-linear Regime

- When ω starts to become comparable to μ ($\omega \lesssim \mu$), it affects the non-linear regime, leading to a loss of the periodic conversion pattern, as already discussed in[19].





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[19] Shashank Shalgar and Irene Tamborra. "Dispelling a myth on dense neutrino media: fast pairwise conversions depend on energy". In: *JCAP* 01 (2021), p. 014. DOI: `10.1088/1475-7516/2021/01/014`. arXiv: `2007.07926 [astro-ph.HE]`.

More Complex Scenarios

- In more realistic scenarios, non-uniformity and collisions need to be considered, making it even more difficult to predict the full non-linear neutrino evolution.
- Different approaches have been taken in the literature, and the final answer to this problem is still something in dispute. As examples:
    1. In[20], it is shown that in a 1D model with periodic boundary conditions (as representative of a small patch of a supernova environment) depolarization (flavor equipartition) should be expected by averaging over a small patch in space.
    2. In[21], a model with neutrino flavor conversion, advection, and collisions is solved numerically, showing a non-trivial interplay between the slow component ω and flavor instabilities, where they find that flavor equipartition is not achieved in general.

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[20] Soumya Bhattacharyya and Basudeb Dasgupta. "Elaborating the ultimate fate of fast collective neutrino flavor oscillations". In: *Phys. Rev. D* 106.10 (2022), p. 103039. DOI: `10.1103/PhysRevD.106.103039`. arXiv: `2205.05129 [hep-ph]`.

[21] Shashank Shalgar and Irene Tamborra. "Neutrino Flavor Conversion, Advection, and Collisions: The Full Solution". In: (July 2022). arXiv: `2207.04058 [astro-ph.HE]`.

Conclusions

Conclusions

- The flavor conversion mechanism in supernovae is still an open question with a variety of possible phenomena emerging from different models;
- We saw that there are plenty of similarities between slow and fast bipolar oscillations in their simplest form, such as the oscillatory behavior with an analogy to a gyroscopic pendulum;[22]
- However, when considering scenarios with both ELN crossing and vacuum oscillations $\omega \neq 0$, the full nonlinear solution and its asymptotic flavor conversion are still something to be better understood;
- Moreover, when including non-uniformity and possible collisions near the decoupling region of the neutrinos, it is even more difficult to get a predictive answer, with flavor equipartition being discussed in the literature as the possible final outcome.

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[22] Damiano F. G. Fiorillo and Georg G. Raffelt. "Slow and fast collective neutrino oscillations: Invariants and reciprocity". In: *Phys. Rev. D* 107.4 (2023), p. 043024. DOI: `10.1103/PhysRevD.107.043024`. arXiv: `2301.09650 [hep-ph]`.

Open-source Code

- The numerical implementation of neutrinos systems presented here can be found in an open-source format in the public repository `https://github.com/pedrodedin/Neutrino-Collective-Effects.git`
- The **current version** only includes the scenarios with **slow conversions**. However, we intend to include fast conversions in the future.
- A more detailed description of the physics and code implementation can be found in the pre-print version of Pedro Dedin Neto. "Open-Source Numerical Solver for Neutrino Collective Effects – I: Isotropic Neutrino Gas". In: (Oct. 2022). arXiv: `2210.15770 [hep-ph]`

Thank you!

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[2] Soumya Bhattacharyya and Basudeb Dasgupta. "Fast Flavor Depolarization of Supernova Neutrinos". In: *Phys. Rev. Lett.* 126.6 (2021), p. 061302. DOI: `10.1103/PhysRevLett.126.061302`. arXiv: `2009.03337 [hep-ph]`.

[3] Basudeb Dasgupta. "Collective Neutrino Flavor Instability Requires a Crossing". In: *Phys. Rev. Lett.* 128.8 (2022), p. 081102. DOI: `10.1103/PhysRevLett.128.081102`. arXiv: `2110.00192 [hep-ph]`.

[4] Pedro Dedin Neto. "Open-Source Numerical Solver for Neutrino Collective Effects – I: Isotropic Neutrino Gas". In: (Oct. 2022). arXiv: `2210.15770 [hep-ph]`.

References II

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