Nuclear Medium Effects in Neutrino-Nucleus Deep Inelastic Scattering

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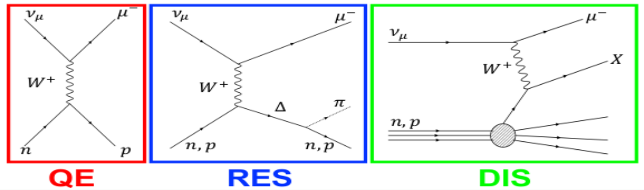
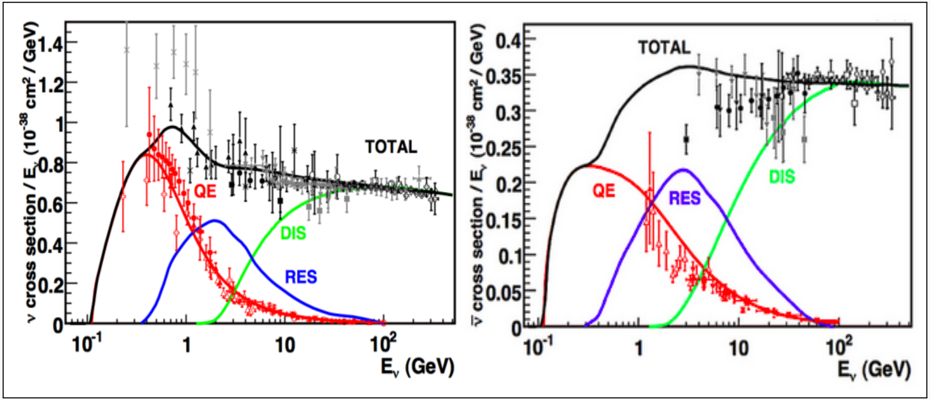
Outline

ν cross sections from different channels

Rev.Mod.Phys. 84 (2012) 1307-1341





General process for the deep inelastic scattering is
$l(k)+N(p)\longrightarrow l'(k')+X(p'),\quad l,l'=e^{\pm},\mu^{\pm},\nu_l,\bar{\nu}_l,\ N=n,p$

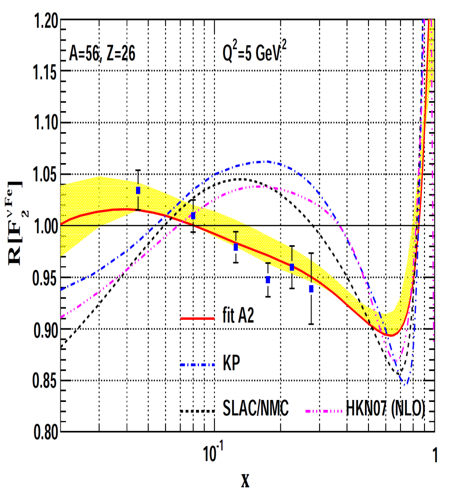
- $Q^2=-q^2=-(k-k')^2=4EE'\sin^2\frac{\theta}{2}$
- $M_N^2=p^2$
- $\nu=p.q=M_N(E-E')$
- $x=\frac{Q^2}{2M_N\nu}=\frac{Q^2}{2p.q}=\frac{Q^2}{2M_NEy}$
- $y=\frac{p.q}{p.k}=1-\frac{E'}{E}$
- $W^2=M_N^2+2p.q-Q^2$

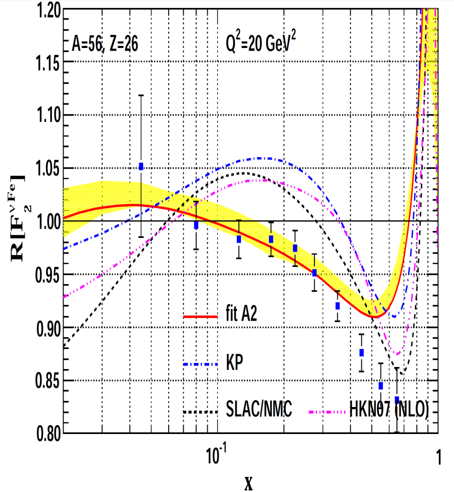
Phenomenological Efforts

Phenomenological group	data types used
EKS98	l+A DIS, p+A DY
HKM	l+A DIS
HKN04	l+A DIS, p+A DY
nDS	l+A DIS, p+A DY
EKPS	l+A DIS, p+A DY
HKN07	l+A DIS, p+A DY
EPS08	l+A DIS, p+A DY, $h^{\pm}, \pi^0, \pi^{\pm}$ in d+Au
EPS09	l+A DIS, p+A DY, π^0 in d+Au
nCTEQ	l+A DIS, p+A DY
nCTEQ	l+A and ν+A DIS, p+A DY
DSSZ	l+A and ν+A DIS, p+A DY, $\pi^0, \pi^{\pm}$ in d+Au

Paukkunen and Salgado:JHEP2010: “find no apparent disagreement with the nuclear effects in neutrino DIS and those in charged lepton DIS.”

CTEQ-Grenoble-Karlsruhe collaboration “observed that the nuclear corrections in ν-A DIS are indeed incompatible with the predictions derived from $l^{\pm}$-A DIS and DY data”





J G Morfin J. of Physics: Conf. Ser. 408 (2013) 012054; Kovarik et al. Phys.Rev.Lett. 106 (2011) 122301

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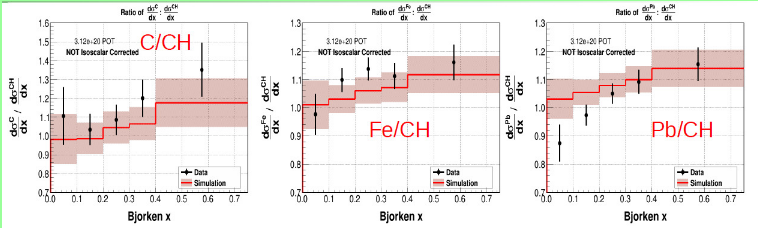
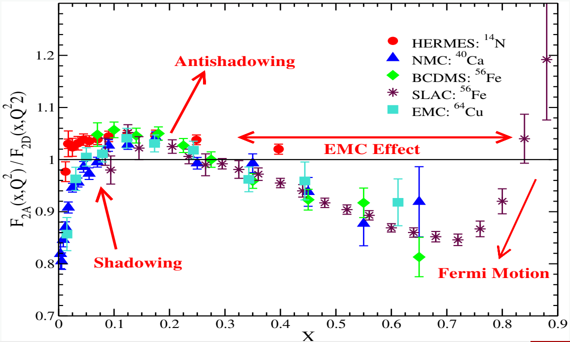


M. Sajjad Athar and S. K. Singh, The Physics of Neutrino Interactions (CUP, 2020)



It is important to understand nucleon dynamics and reduce the cross section uncertainty($\sim$20-25%) which is contributing to the systematic errors.

Motivation to study tau (anti)neutrinos

- DONuT, OPERA, SuperK and IceCube have observed tau (anti)neutrino events but with very limited statistics.
- SHiP, DUNE, HyperK, FASERν and DsTau are among the experiments which plan to observe $\nu_\tau/\bar{\nu}_\tau$ interactions on the different nuclear targets.
- We have performed calculations to theoretically study the $\nu_\tau/\bar{\nu}_\tau$—N induced DIS cross sections by taking into account, various perturbative (PDF evolution at NLO) and nonperturbative (kinematical and dynamical HT) effects.
- This is the first calculation, where ν_τ-A interaction cross section in the DIS region has been studied by taking into account, the nuclear medium effects like the Fermi motion, binding energy, nucleon correlation effects, mesonic contributions and the (anti)shadowing effects.

Formalism: $\nu_l/\bar{\nu}_l - N$ *scattering*

The basic reaction for the (anti)neutrino induced charged current deep inelastic scattering process on a free nucleon target is given by

$$\nu_l(k)/\bar{\nu}_l(k) + N(p) \quad \rightarrow \quad l^-(k')/l^+(k') + X(p'), \qquad (l = e, \nu, \tau)$$

The general expression for the double differential scattering cross section (DCX):

$$\frac{d^2\sigma}{dxdy} = \frac{yM_N}{\pi} \frac{E}{E'} \frac{|\mathbf{k}'|}{|\mathbf{k}|} \frac{G_F^2}{2} \left(\frac{M_W^2}{Q^2 + M_W^2} \right)^2 L_{\mu\nu} \, W_N^{\mu\nu} \,,$$

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Leptonic tensor: $L_{\mu\nu} = 8(k_\mu k'_\nu + k_\nu k'_\mu - k.k' g_{\mu\nu} \pm i\epsilon_{\mu\nu\rho\sigma}k^\rho k'^\sigma)$

Hadronic tensor:

$$W_N^{\mu\nu} = -g^{\mu\nu}\,W_{1N}(\nu,Q^2) + W_{2N}(\nu,Q^2)\frac{p^\mu p^\nu}{M_N^2} - \frac{i}{M_N^2}\epsilon^{\mu\nu\rho\sigma}p_\rho q_\sigma W_{3N}(\nu,Q^2) + \frac{W_{4N}(\nu,Q^2)}{M_N^2}q^\mu q^\nu$$

$$+\frac{W_{5N}(\nu,Q^2)}{M_N^2}(p^\mu q^\nu + q^\mu p^\nu) + \frac{i}{M_N^2}(p^\mu q^\nu - q^\mu p^\nu)W_{6N}(\nu,Q^2)\,.$$

$W_{iN}(\nu,Q^2)$ $(i = 1-6)$ are the weak nucleon structure functions

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In the limit of $Q^2 \rightarrow \infty$, $\nu \rightarrow \infty$, $x \rightarrow$ finite and $W_{iN}(\nu,Q^2)$ $(i = 1-5)$ are written in terms of the dimensionless nucleon structure functions as:

$$F_{1N}(x) = W_{1N}(\nu,Q^2) \qquad F_{2N}(x) = \frac{Q^2}{2xM_N^2}W_{2N}(\nu,Q^2) \qquad F_{3N}(x) = \frac{Q^2}{xM_N^2}W_{3N}(\nu,Q^2)$$

$$F_{4N}(x) = \frac{Q^2}{2M_N^2}W_{4N}(\nu,Q^2) \qquad F_{5N}(x) = \frac{Q^2}{2xM_N^2}W_{5N}(\nu,Q^2)$$

Formalism: $\nu_l/\bar{\nu}_l - N$ scattering

The differential scattering cross section is given by

$$\frac{d^2\sigma}{dxdy} = \frac{G_F^2 M_N E_\nu}{\pi(1+\frac{Q^2}{M_W^2})^2}\Big\{\Big[y^2x+\frac{m_l^2 y}{2E_\nu M_N}\Big]F_{1N}(x,Q^2)+\Big[\Big(1-\frac{m_l^2}{4E_\nu^2}\Big)-\Big(1+\frac{M_N x}{2E_\nu}\Big)y\Big]F_{2N}(x,Q^2)$$
$$\pm\Big[xy\Big(1-\frac{y}{2}\Big)-\frac{m_l^2 y}{4E_\nu M_N}\Big]F_{3N}(x,Q^2)+\frac{m_l^2(m_l^2+Q^2)}{4E_\nu^2 M_N^2 x}F_{4N}(x,Q^2)-\frac{m_l^2}{E_\nu M_N}F_{5N}(x,Q^2)\Big\}.$$

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For $\nu(\bar{\nu})$***-proton scattering***

$$\begin{aligned}
F_{2p}^{\nu}(x) &= 2x[d(x)+s(x)+\bar{u}(x)+\bar{c}(x)]\\
F_{2p}^{\bar{\nu}}(x) &= 2x[u(x)+c(x)+\bar{d}(x)+\bar{s}(x)]\\
xF_{3p}^{\nu}(x) &= 2x[d(x)+s(x)-\bar{u}(x)-\bar{c}(x)]\\
xF_{3p}^{\bar{\nu}}(x) &= 2x[u(x)+c(x)-\bar{d}(x)-\bar{s}(x)]
\end{aligned}$$

For $\nu(\bar{\nu})$***-neutron scattering***

$$\begin{aligned}
F_{2n}^{\nu}(x) &= 2x[u(x)+s(x)+\bar{d}(x)+\bar{c}(x)]\\
F_{2n}^{\bar{\nu}}(x) &= 2x[d(x)+c(x)+\bar{u}(x)+\bar{s}(x)]\\
xF_{3n}^{\nu}(x) &= 2x[u(x)+s(x)-\bar{d}(x)-\bar{c}(x)]\\
xF_{3n}^{\bar{\nu}}(x) &= 2x[d(x)+c(x)-\bar{u}(x)-\bar{s}(x)].
\end{aligned}$$

At the leading order

Callan-Gross relation:

$$F_2(x)=2xF_1(x)$$

Albright-Jarlskog relations:

$$F_4(x)=0 \qquad F_2(x)=2xF_5(x)$$

In this work MMHT PDFs parameterization (Harland-Lang *et al.*, Eur. Phys. J. C **75**, no. 5, 204 (2015)) has been used.

Charm quark is considered to be a massive object and in four flavor scheme we consider:

$$F_{iN}(x,Q^2)=F_{iN}^{n_f=4}(x,Q^2)=\underbrace{F_{iN}^{n_f=3}(x,Q^2)}_{\text{for massless}(u,d,s)\text{ quarks}}+\underbrace{F_{iN}^{n_f=1}(x,Q^2)}_{\text{for massive charm quark}}$$

Details in ref. Ansari *et al.*, Phys. Rev. D **102**, 113007 (2020)

Perturbation and nonperturbative effects at nucleon level

In the kinematic region of low and moderate Q^2, both the higher order perturbative and the nonperturbative ($\propto \frac{1}{Q^2}$) QCD effects come into play.

- Perturbative effects like the QCD corrections at the next-to-next-to-leading order (NNLO) in the strong coupling constant α_s.
- In the present work we have evaluated the structure functions at NLO, following the works of Kretzer and Reno (Phys. Rev. D **66**, 113007 (2002); ibid **69**, 034002 (2004)) and Jeong and Reno (Phys. Rev. D **82**, 033010 (2010).)
- The nonperturbative effect like:
  - The target mass correction (TMC) is associated with the finite mass of the target nucleon, and is incorporated using Ref. Kretzer and Reno, Phys. Rev.D **69**, 034002 (2004).
  - HT originates due to the interactions of struck quarks with the other quarks via gluon exchange and is incorporated following the works of Dasgupta *et al.*, Phys. Lett. B **382**, 273 (1996).
- The effects of tau lepton mass, charm quark mass and threshold effect have been taken into account

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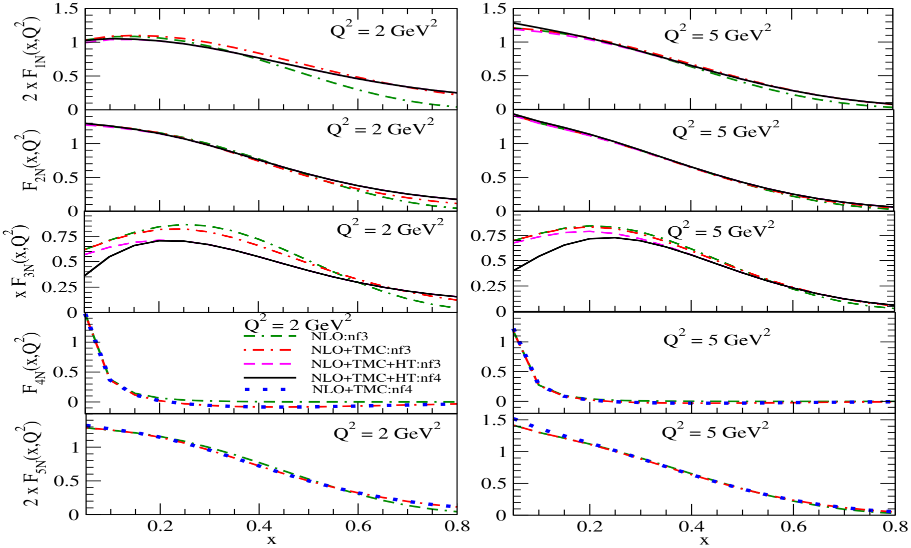
Free nucleon structure functions: $F_{iN}^{WI}(x,Q^2)$ *vs* x $(i=1-5)$



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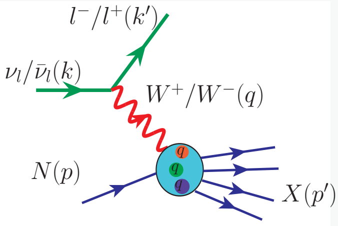
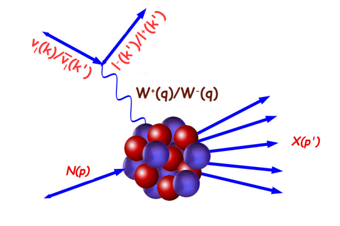
$\nu_l/\bar{\nu}_l - A$ scattering





- Fermi motion, binding energy and nucleon correlations through spectral function and is calculated using Lehmann's representation for the relativistic nucleon propagator.
- Nuclear many body theory is used to calculate it for an interacting Fermi sea in nuclear matter. A local density approximation is then applied to translate these results to finite nuclei.
- There are virtual mesons associated with the nucleon bound inside the nucleus. These meson clouds get strengthened by the strong attractive nature of nucleon-nucleon interactions.
- This leads to an increase in the interaction probability of virtual mediating quanta with the meson cloud. The effect of meson cloud is more pronounced in heavier nuclear targets and dominate in the intermediate region of $x(0.2<x<0.6)$.
- The shadowing suppression at small x occurs due to coherent multiple scattering of quark-anti quark pair coming from the virtual boson with destructive interference of the amplitudes and is incorporated following the works of Kulagin and Petti. Phys. Rev. D **76**, 094033(2007).

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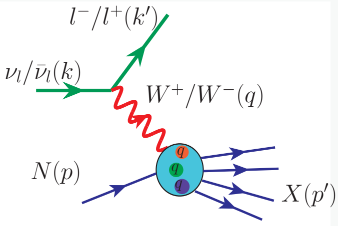

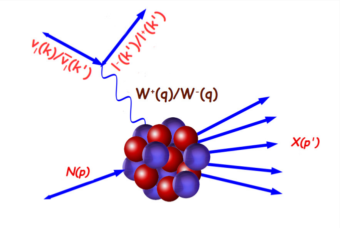

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Nuclear hadronic tensor:

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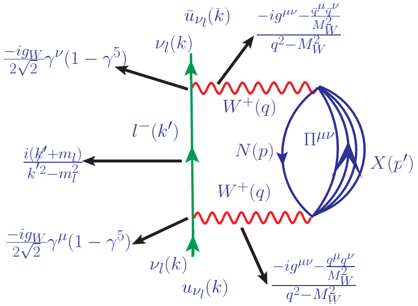
- To calculate the scattering cross section for a neutrino interacting with a target nucleon in the nuclear medium, we write

$$d\sigma_A = -2\frac{m_\nu}{|\mathbf{k}|} Im\Sigma(k) d^3r.$$



- The neutrino self energy $\Sigma(k)$:

$$\Sigma(k) = \frac{iG_F}{\sqrt{2}}\int\frac{d^4q}{(2\pi)^4}\frac{4L^{WI}_{\mu\nu}}{m_l}\frac{1}{(k'^2-m_l^2+i\varepsilon)}\left(\frac{M_W}{q^2-M_W^2}\right)^2\Pi^{\mu\nu}(q),$$

  - $\Pi^{\mu\nu}(q)$ written in terms of the nucleon propagator (G_l) and meson propagator (D_j):

$$\Pi^{\mu\nu}(q) = \left(\frac{G_F M_W^2}{\sqrt{2}}\right)\times\int\frac{d^4p}{(2\pi)^4}G(p)\sum_X\sum_{s_p,s_l}\prod_{i=1}^{N}\int\frac{d^4p'_i}{(2\pi)^4}\prod_l G_l(p'_l)\prod_j D_j(p'_j)$$
$$<X|J^\mu|N><X|J^\nu|N>^*(2\pi)^4\,\delta^4\left(k+p-k'-\sum_{i=1}^{N}p'_i\right),$$

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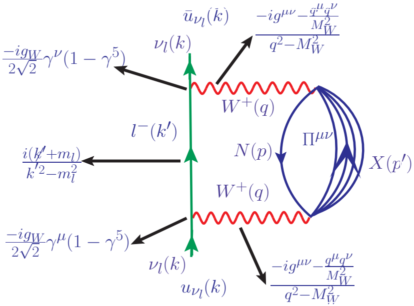


- $\Pi^{\mu\nu}(q)$ written in terms of the nucleon propagator (G_l) and meson propagator (D_j):

$$\Pi^{\mu\nu}(q) \quad = \quad \left(\frac{G_F M_W^2}{\sqrt{2}}\right) \times \int \frac{d^4p}{(2\pi)^4} G(p) \sum_X \sum_{s_p, s_l} \prod_{i=1}^{N} \int \frac{d^4p'_i}{(2\pi)^4} \prod_l G_l(p'_l) \prod_j D_j(p'_j)$$

$$< X|J^\mu|N > < X|J^\nu|N >^* (2\pi)^4 \delta^4\left(k + p - k' - \sum_{i=1}^{N} p'_i\right),$$

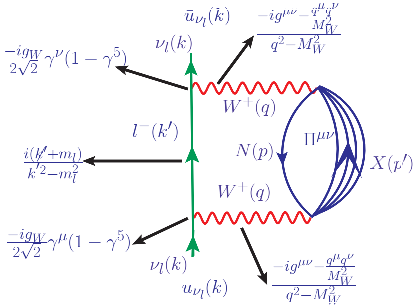
Formalism: $\nu_l/\bar{\nu}_l - A$ scattering

- To calculate the scattering cross section for a neutrino interacting with a target nucleon in the nuclear medium, we write

$$d\sigma_A = -2\frac{m_\nu}{|\mathbf{k}|} Im\Sigma(k) d^3r.$$

- The neutrino self energy $\Sigma(k)$:

$$\Sigma(k) = \frac{iG_F}{\sqrt{2}}\int \frac{d^4q}{(2\pi)^4}\frac{4L^{WI}_{\mu\nu}}{m_l}\frac{1}{(k'^2-m_l^2+i\epsilon)}\left(\frac{M_W}{q^2-M_W^2}\right)^2 \Pi^{\mu\nu}(q),$$



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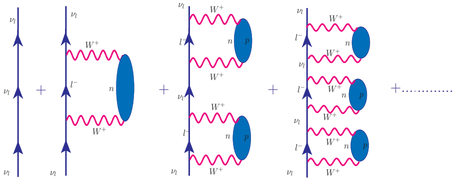
$$G(p) = \frac{M_N}{E(\mathbf{p})}\sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p})\left[\int_{-\infty}^{\mu} d\omega \frac{S_h(\omega,\mathbf{p})}{p^0-\omega-i\eta} + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega,\mathbf{p})}{p^0-\omega+i\eta}\right],$$



The spectral function $F_{iA,N}(x_A,Q^2)$ $(i=1-5)$ are obtained as:

$$F_{iA,N}(x_A,Q^2) = 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M_N}{E_N(\mathbf{p})}\int_{-\infty}^{\mu} dp^0\, S_h(p^0,\mathbf{p},\rho(r)) \times f_{iN}(x,Q^2),$$

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Zaidi *et al.* Phys. Rev. D **101** (2020), 033001, Phys. Rev. D **99** (2019), 093011

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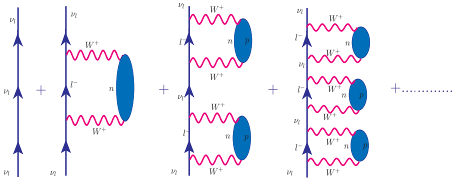

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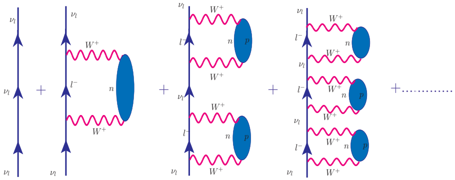

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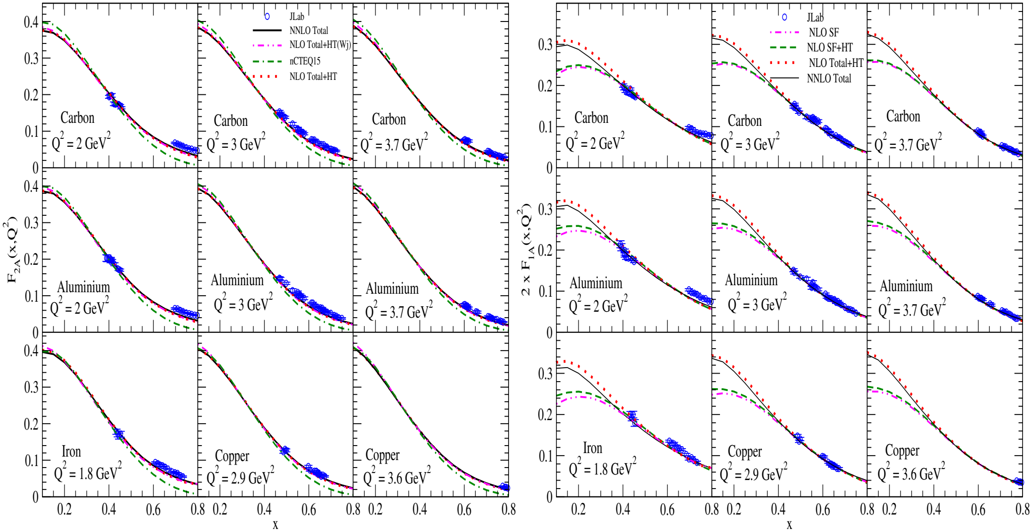
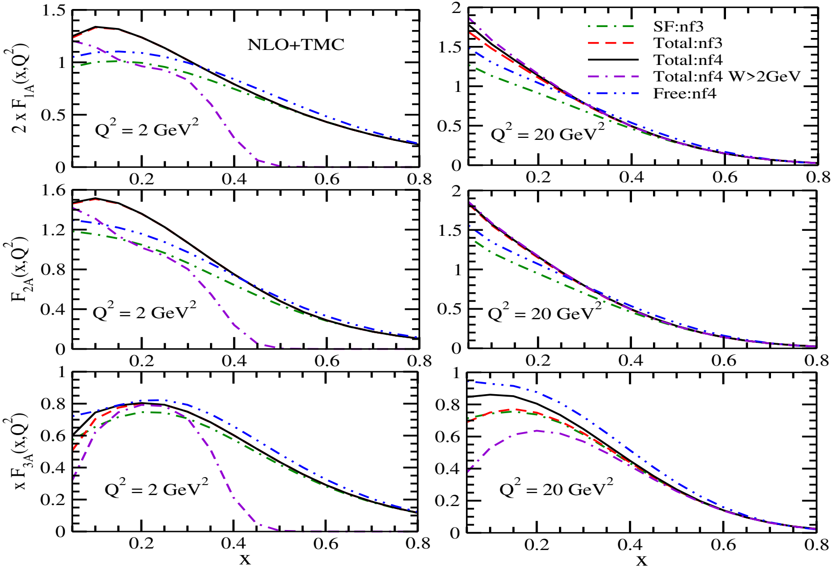
Zaidi *et al.* Phys. Rev. D **101** (2020), 033001, Phys. Rev. D **99** (2019), 093011

EM Nuclear Structure Functions

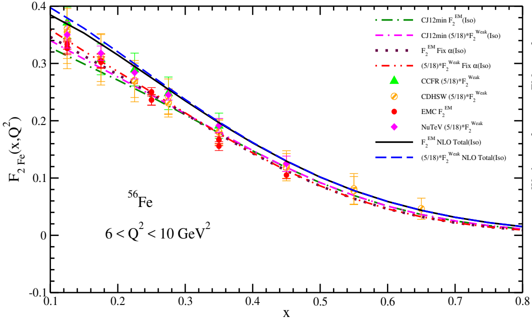


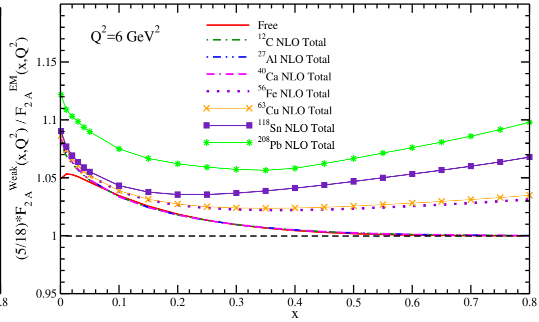
Zaidi *et al.*, Phys. Rev. D **99**, 093011 (2019).

Nuclear structure functions: $2xF_{1A}(x,Q^2)$, $F_{2A}(x,Q^2)$ *and* $xF_{3A}(x,Q^2)$ *for* ^{40}Ar



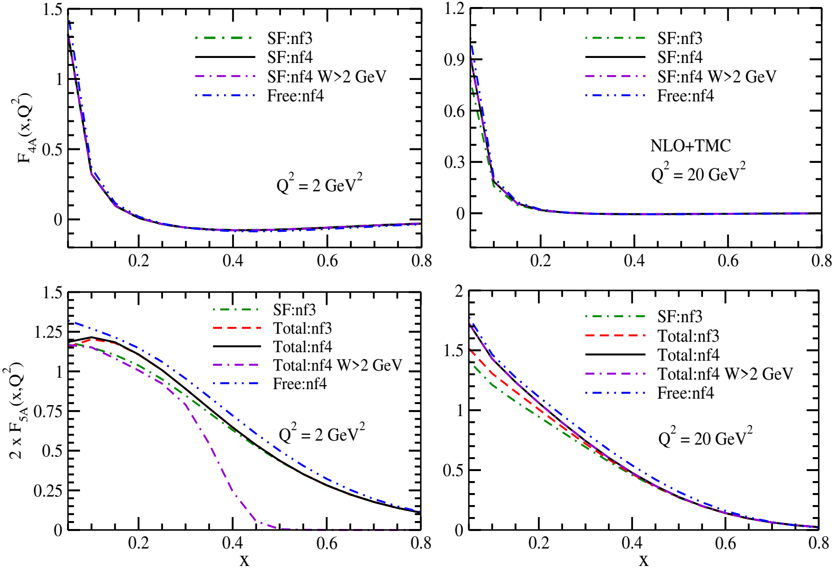
Main take away for Nuclear Medium Effects, Left: are different in EM and Weak interactions. Right: show A dependence



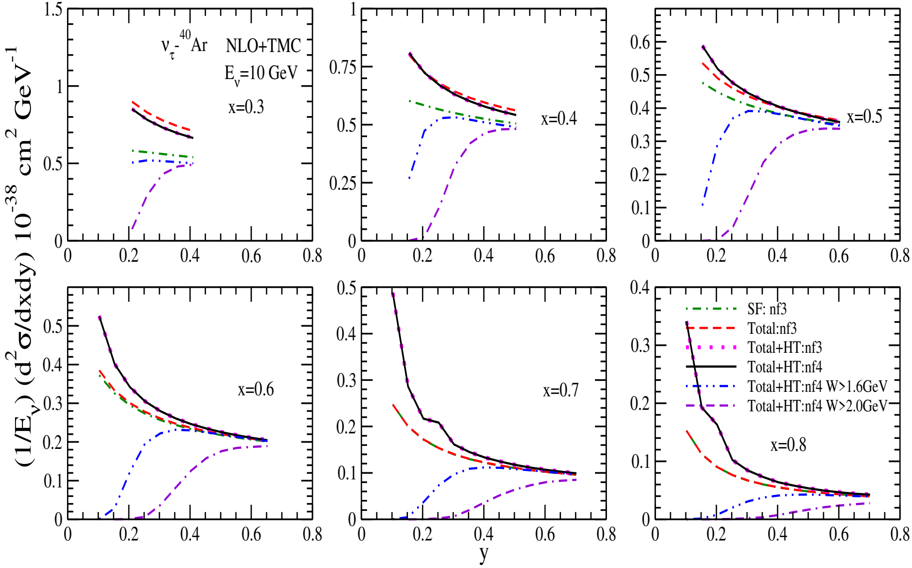


Haider *et al.*, Nuc Phys A, 955, 2016, 58

Nuclear structure functions: $F_{4A}(x,Q^2)$ *and* $2xF_{5A}(x,Q^2)$ *for* ^{40}Ar



$\frac{1}{E_\nu}\frac{d^2\sigma}{dxdy}$ vs y at $E_\nu = 10GeV$



Conclusions

- Perturbative and nonperturbative effects are quite important in the evaluation of nucleon structure functions as well as the differential cross section. These effects are important in the different regions of x and Q^2.
- We have studied nuclear medium effects in electromagnetic and weak nuclear structure functions.
- The theoretical results presented here show that the difference between $F_2^{EM}(x,Q^2)$ and $F_2^{Weak}(x,Q^2)$ is quite small at large $x(x > 0.3)$. Nuclear Medium Effects are A dependent.
- The additional structure functions $F_4(x,Q^2)$ and $F_5(x,Q^2)$ leads to a significant reduction in the $\nu_\tau/\bar{\nu}_\tau - N(A)$ cross section as compared to muon and electron neutrino cross sections.
- These theoretical results would be helpful for MINERνA, MicroBooNE, NOvA, and upcomig DUNE experiment.
- These results would also be helpful in atmospheric neutrino analysis and other experiments which are planning to observe the ν_τ events.

Conclusions

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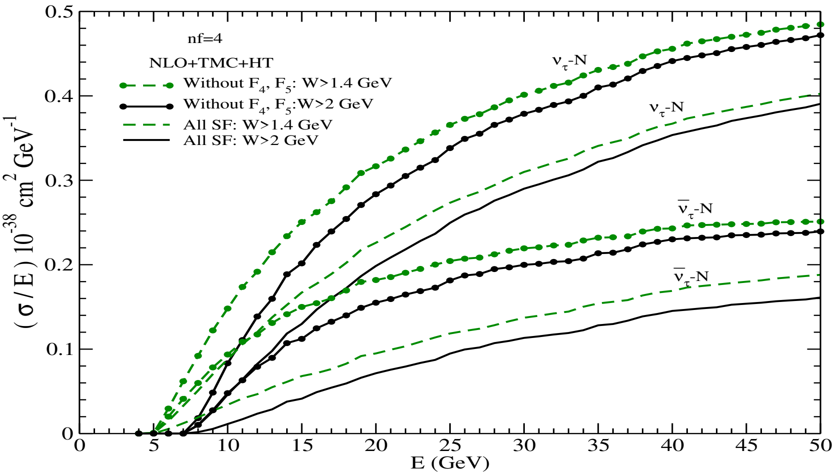
Thank You!

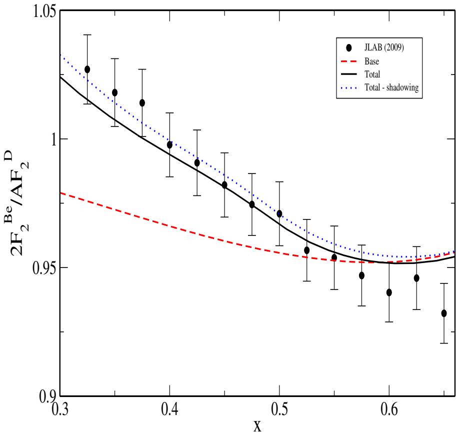
Backup

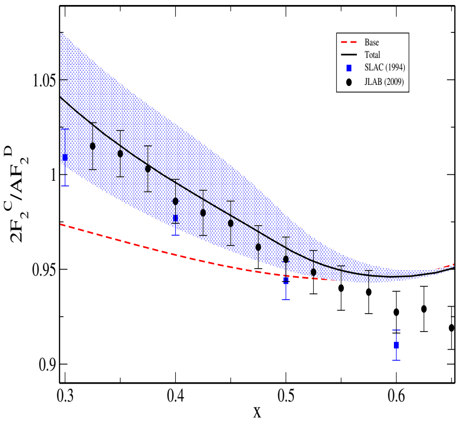
$\frac{\sigma}{E}$ vs E for $\nu_\tau/\bar{\nu}_\tau - N$ DIS



Total scattering cross section with center of mass energy cut of 1.4 GeV and 2 GeV for $\nu_\tau - N$ and $\bar{\nu}_\tau - N$ DIS.
Ansari *et al.*, Phys. Rev. D **102**, 113007 (2020)

EM nuclear structure function $\frac{2F_2^A}{AF_2^D}(A = Be,\ C)$ vs x





M. Sajjad Athar *et al.*, Nucl. Phys A **857**, 29(2011)

Nuclear structure functions

The spectral function $F_{iA,N}(x_A, Q^2)$ are obtained as:

$$F_{iA,N}(x_A, Q^2) = 4\int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M_N}{E_N(\mathbf{p})} \int_{-\infty}^{\mu} dp^0\, S_h(p^0, \mathbf{p}, \rho(r)) \times f_{iN}(x, Q^2),$$

where $i = 1-5$ and

$$f_{1N}(x,Q^2) = AM_N\left[\frac{F_{1N}(x_N,Q^2)}{M_N} + \left(\frac{p^x}{M_N}\right)^2 \frac{F_{2N}(x_N,Q^2)}{\nu_N}\right],$$

$$f_{2N}(x,Q^2) = \left[\frac{Q^2}{(q^z)^2}\left(\frac{|\mathbf{p}|^2 - (p^z)^2}{2M_N^2}\right) + \frac{(p^0 - p^z\gamma)^2}{M_N^2}\left(\frac{p^z Q^2}{(p^0 - p^z\gamma)q^0 q^z} + 1\right)^2\right] \times \left(\frac{M_N}{p^0 - p^z\gamma}\right) \times F_{2N}(x_N,Q^2),$$

$$f_{3N}(x,Q^2) = A\frac{q^0}{q^z} \times \left(\frac{p^0 q^z - p^z q^0}{p\cdot q}\right) F_{3N}(x_N,Q^2), \qquad f_{4N}(x,Q^2) = A\, F_{4N}(x_N,Q^2),$$

$$f_{5N}(x,Q^2) = A\; F_{5N}(x_N,Q^2) \times \frac{2x_N}{M_N\nu_N} \times (a_1 + a_2 + a_3),$$

$$a_1 = \frac{|\mathbf{q}|q_0}{q^2}\left(\frac{|\mathbf{q}| + q_0}{|\mathbf{q}| + 2q^0}\right) \times \left\{-p_x^2 + \frac{|\mathbf{q}|^2}{Q^2}\frac{M_N\nu_N}{q_0(p_N^0 - \gamma p_N^z)}\left(\frac{Q^2}{|\mathbf{q}|^2}\left[\frac{|\vec{p}_N|^2 - p_N^{z2}}{2}\right] + (p_N^0 - \gamma p_N^z)^2\left[1 + \frac{p_N^z Q^2}{q^0|\mathbf{q}|(p_N^0 - \gamma p_N^z)}\right]^2\right)\right\},$$

$$a_2 = \left(\frac{q_0}{|\mathbf{q}| + 2q^0}\right)\left(p_N^0 + \frac{q^0}{2x_N}\right)^2\left(1 + \frac{p_N^z + |\mathbf{q}|/2x_N}{p_N^0 + q^0/2x_N}\right), \qquad a_3 = \left(\frac{p_N^0 q^0}{2x_N}\right)\left(1 + \frac{p_N^z q^0}{p_N^0(|\mathbf{q}| + 2q^0)}\right).$$

Nuclear structure functions

The mesonic structure functions $F_{iA,a}(x_a,Q^2),\ \ (i=1,2,5;a=\pi,\rho)$ are obtained as:

$$F_{iA,a}(x_a,Q^2) \quad = \quad -6\kappa\int d^3r\int\frac{d^4p}{(2\pi)^4}\theta(p^0)\ \delta ImD_a(p)\ 2m_a\ \ f_{ia}(x_a),$$

where

$$f_{2a}(x_a) \quad = \quad \left(\frac{m_a}{p^0\ -\ p^z\ \gamma}\right)\left[\frac{Q^2}{(q^z)^2}\left(\frac{|\mathbf{p}|^2\ -\ (p^z)^2}{2m_a^2}\right)+\frac{(p^0\ -\ p^z\ \gamma)^2}{m_a^2}\times\left(\frac{p^z\ Q^2}{(p^0\ -\ p^z\ \gamma)q^0q^z}\ +\ 1\right)^2\right]F_{2a}(x_a)$$

$$f_{1a}(x_a) \quad = \quad AM_N\left[\frac{F_{1a}(x_a)}{m_a}\ +\ \frac{|\mathbf{p}|^2\ -\ (p^z)^2}{2(p^0\ q^0\ -\ p^z\ q^z)}\ \frac{F_{2a}(x_a)}{m_a}\right],\qquad f_{5a}(x_a)=\frac{2x_a}{m_a\nu}(a_1+a_2+a_3)\ F_{5a}(x_a),$$

$$a_1 \quad = \quad \frac{|\mathbf{q}|q_0}{q^2}\left(\frac{|\mathbf{q}|+q_0}{|\mathbf{q}|+2q^0}\right)\times\left\{-p_x^2+\frac{|\mathbf{q}|^2}{Q^2}\frac{m_a\nu}{q_0(\gamma p_N^z-p_N^0)}\left(\frac{Q^2}{|\mathbf{q}|^2}\left[\frac{|\vec{p}_N|^2-p_N^{z2}}{2}\right]+(\gamma p_N^z-p_N^0)^2\left[1+\frac{p_N^z Q^2}{q^0|\mathbf{q}|(\gamma p_N^z-p_N^0)}\right]^2\right)\right\}$$

$$a_2 \quad = \quad \left(\frac{q_0}{|\mathbf{q}|+2q^0}\right)\left(p_N^0+\frac{q^0}{2x_a}\right)^2\left(1+\frac{p_N^z+|\mathbf{q}|/2x_a}{p_N^0+q^0/2x_a}\right);\quad a_3=\left(\frac{p_N^0q^0}{2x_a}\right)\left(1+\frac{p_N^zq^0}{p_N^0(|\mathbf{q}|+2q^0)}\right).$$

$D_a(p)$ is the meson(π or ρ) propagator in the nuclear medium and is written as

$$D_a(p)=[p_0^2-\mathbf{p}^2-m_a^2-\Pi_a(p_0,\mathbf{p})]^{-1},\qquad \text{with}\qquad \Pi_a(p_0,\mathbf{p})=\frac{f^2}{m_\pi^2}\frac{C_\rho\ F_a^2(p)\mathbf{p}^2\Pi^*}{1-\frac{f^2}{m_\pi^2}V_j'\Pi^*},$$

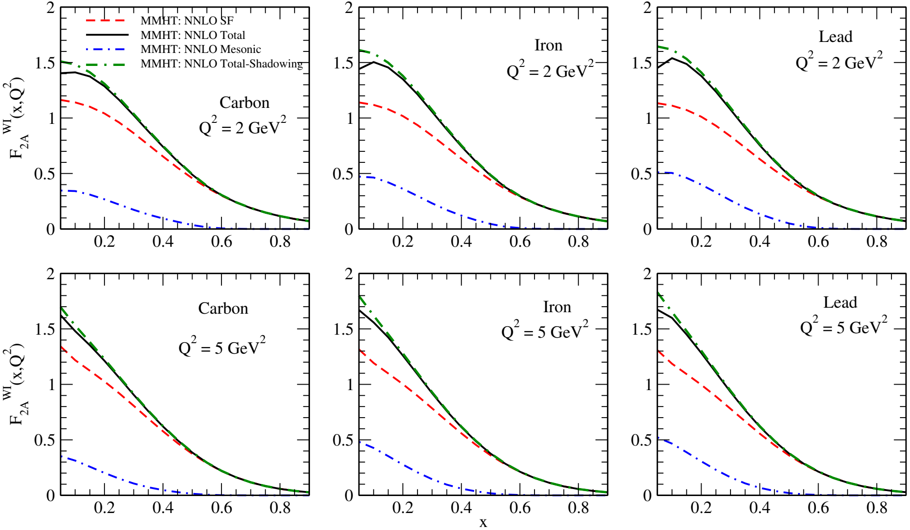
Zaidi *et al.* Phys.Rev. D**101** (2020) no.3, 033001, Phys.Rev. D**99** (2019) no.9, 093011

Nuclear structure functions

$\kappa = 1(2)$ for pion(rho meson), $\nu = \frac{q_0(\gamma p_N^z - p_N^0)}{m_a}$, $x_a = -\frac{Q^2}{2p\cdot q}$, m_a is the mass of the meson(π or ρ).

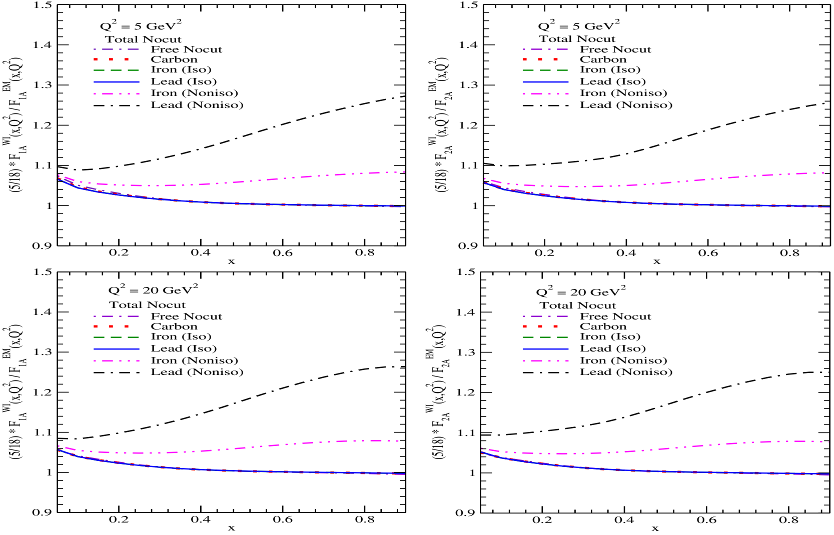
where, $C_\rho = 1(3.94)$ for pion(rho meson). $F_a(p) = \frac{(\Lambda_a^2 - m_a^2)}{(\Lambda_a^2 - p^2)}$ is the πNN or ρNN form factor, Λ_a=1 GeV (fixed by Aligarh-Valencia group) and $f = 1.01$. V_j' is the longitudinal(transverse) part of the spin-isospin interaction for pion(rho meson), and Π^* is the irreducible meson self energy that contains the contribution of particle-hole and delta-hole excitations.

Weak Nuclear Structure Functions



Athar and Morfin, Jour. Phys. G 48, 034001 (2021).

NME in Weak & EM interactions



Zaidi *et al.* Phys Rev. D **101**, 033001 (2020)