QED Corrections For Leptons & Nuclei In The MeV To GeV Regime

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CERN | NEUTRINO PHENO PROGRAM | MARCH 2023

Neutrino Theory Network

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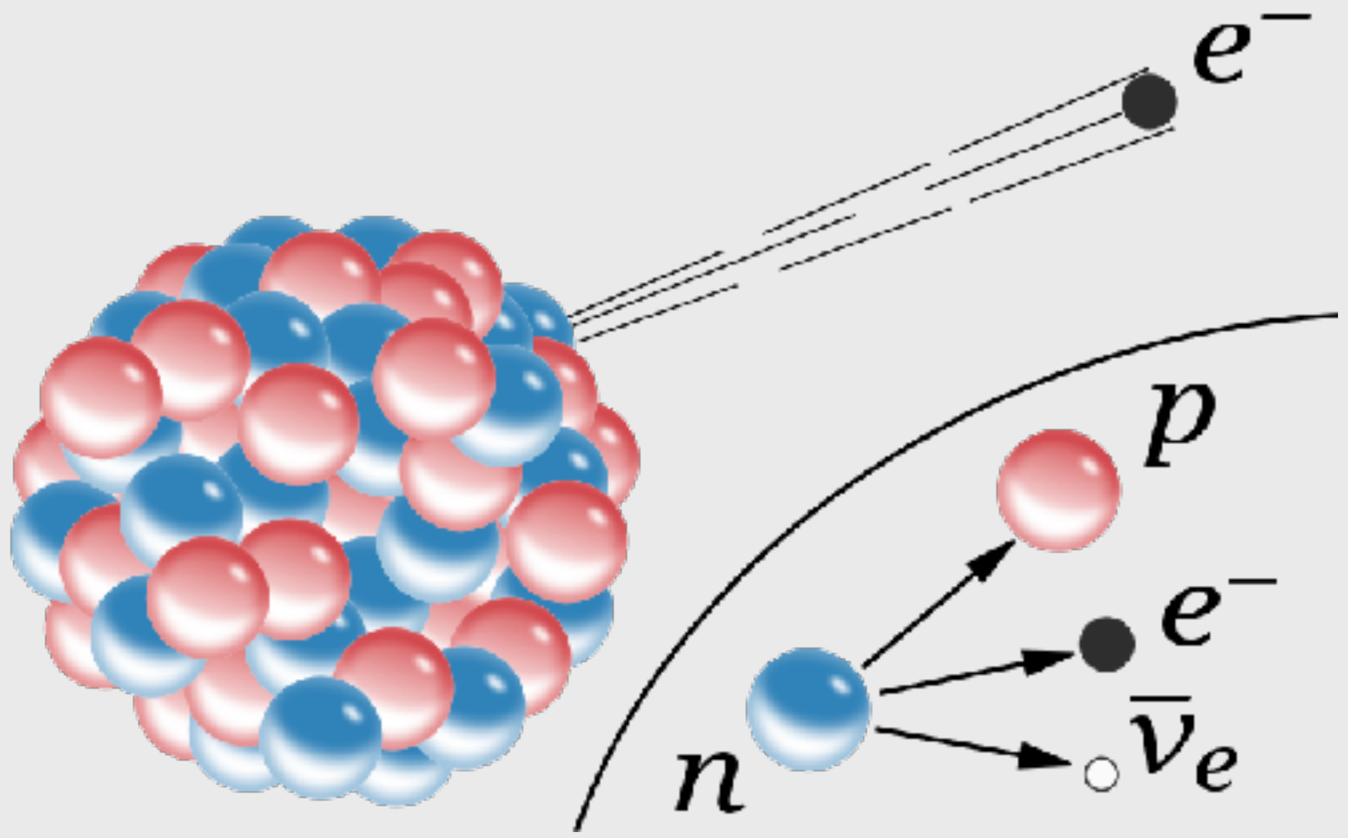
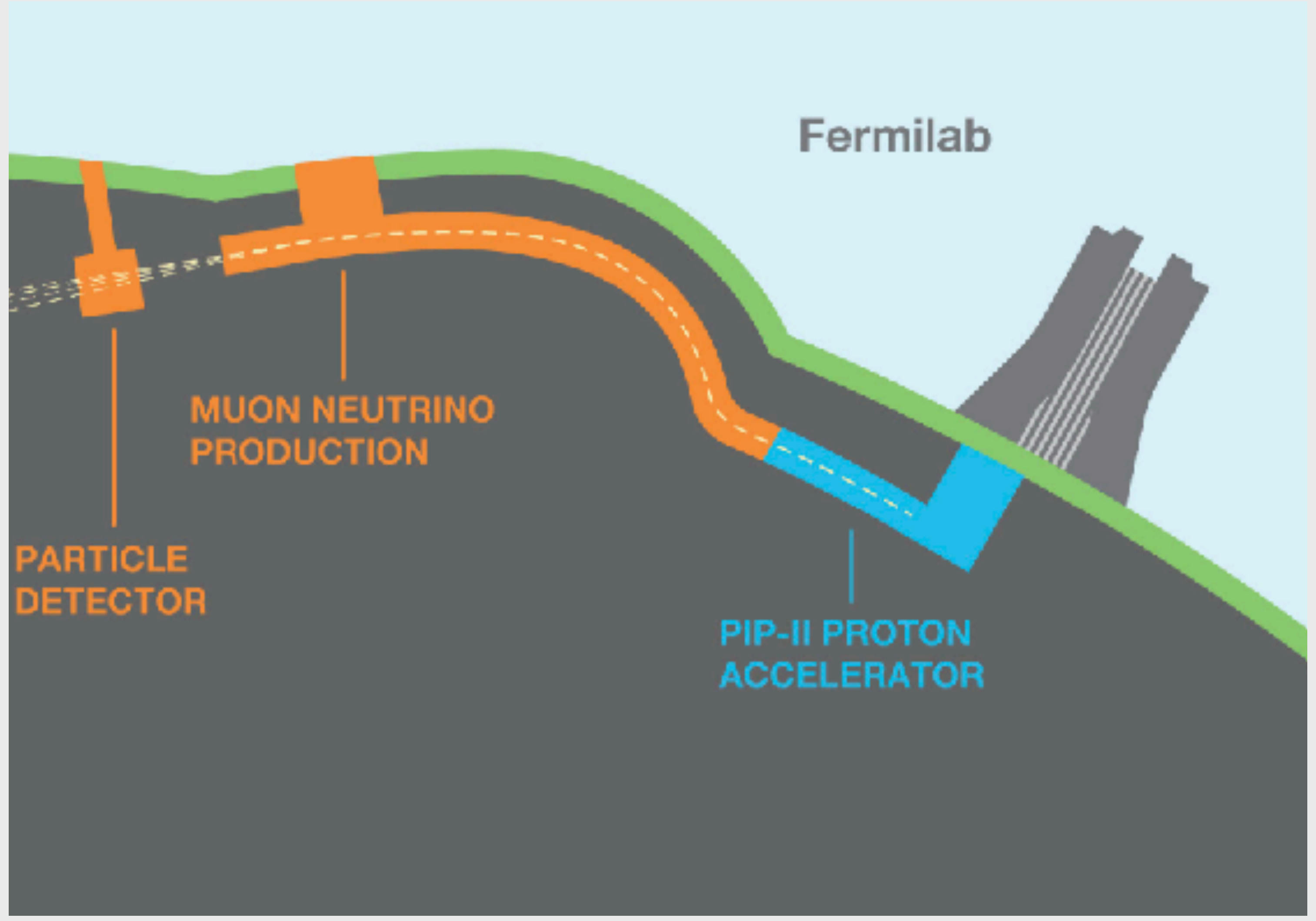
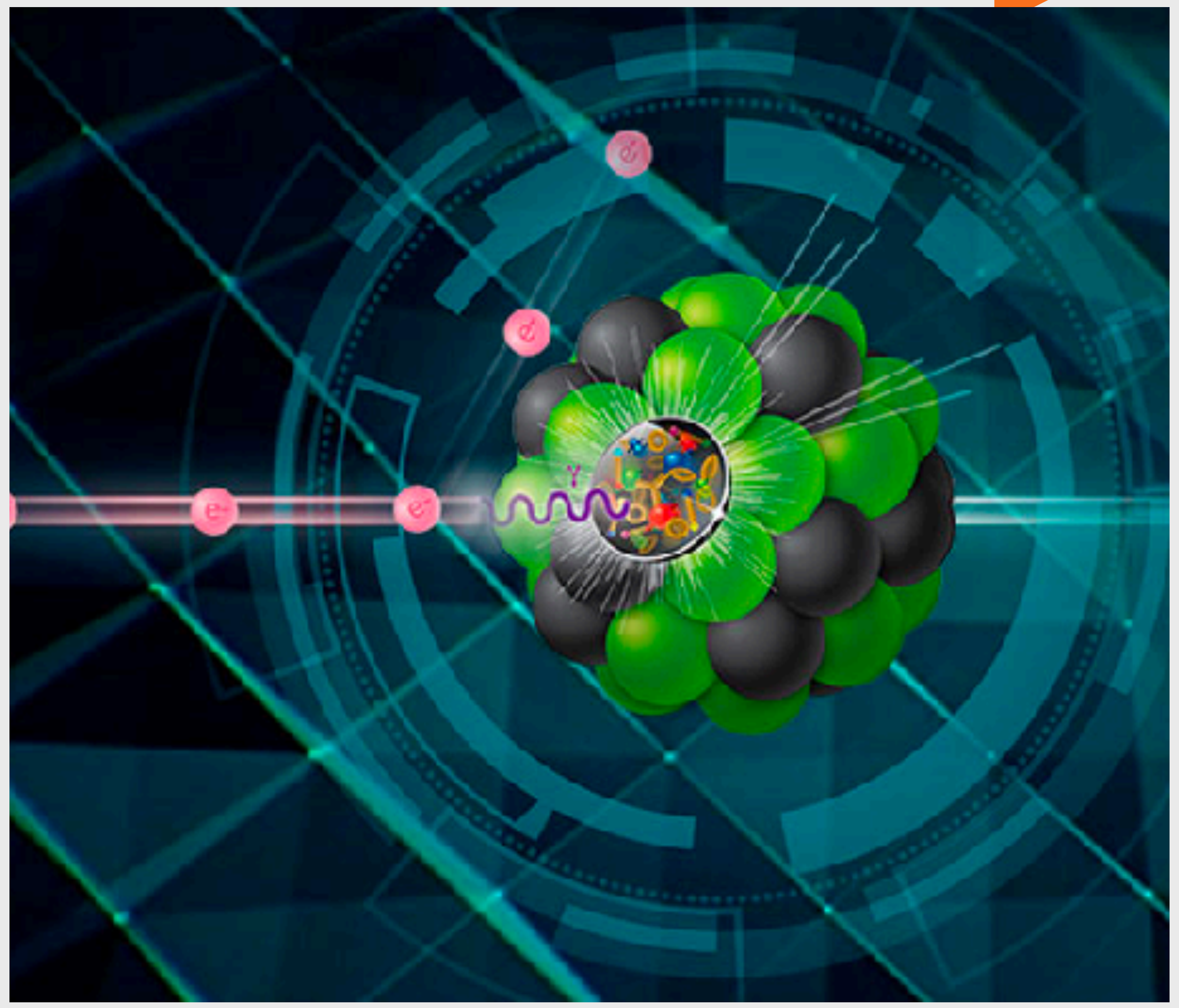
5 MeV   50 MeV   2 GeV   100 GeV







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Some Experiments Where This Matters

MISSING MOMENTUM SEARCHES FOR "DM"

LDMX NA64

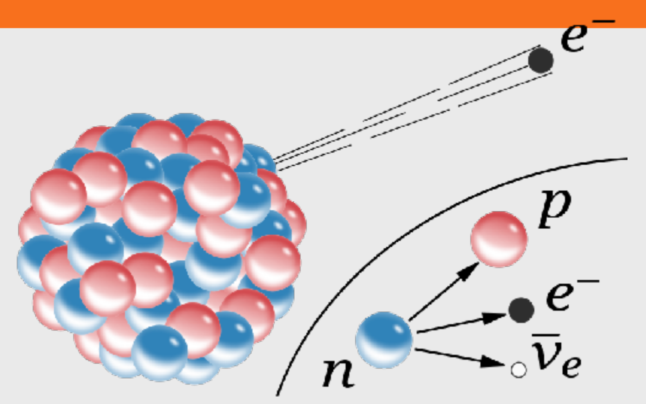
CHARGED LEPTON FLAVOUR VIOLATION

COMET Mu2e

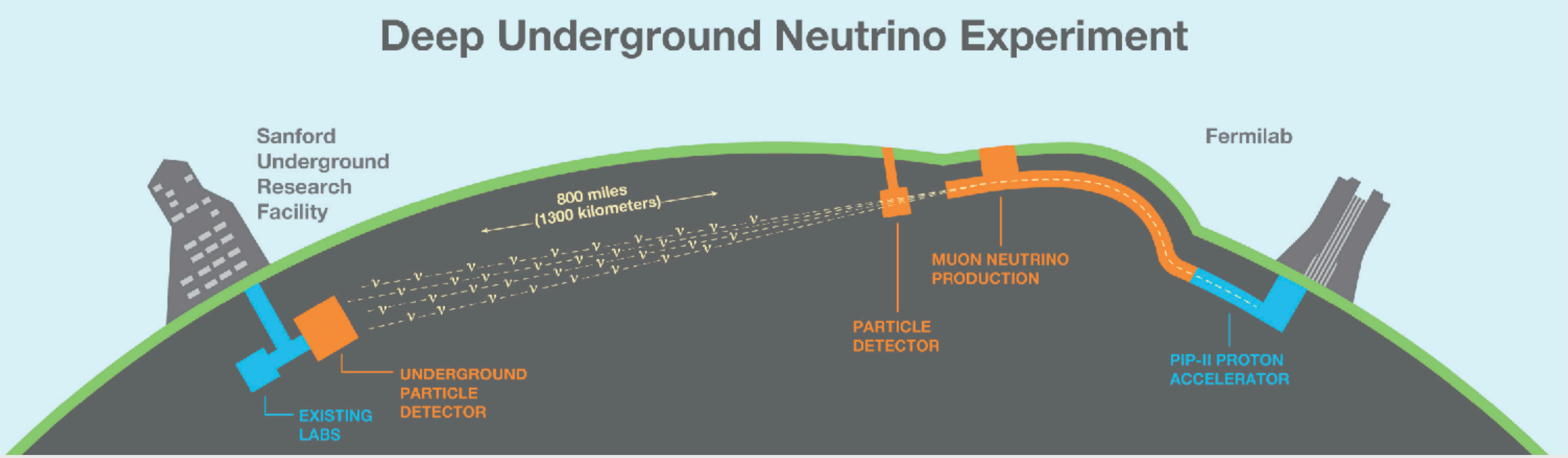
PRECISION NEUTRINO OSCILLATION EXPERIMENTS

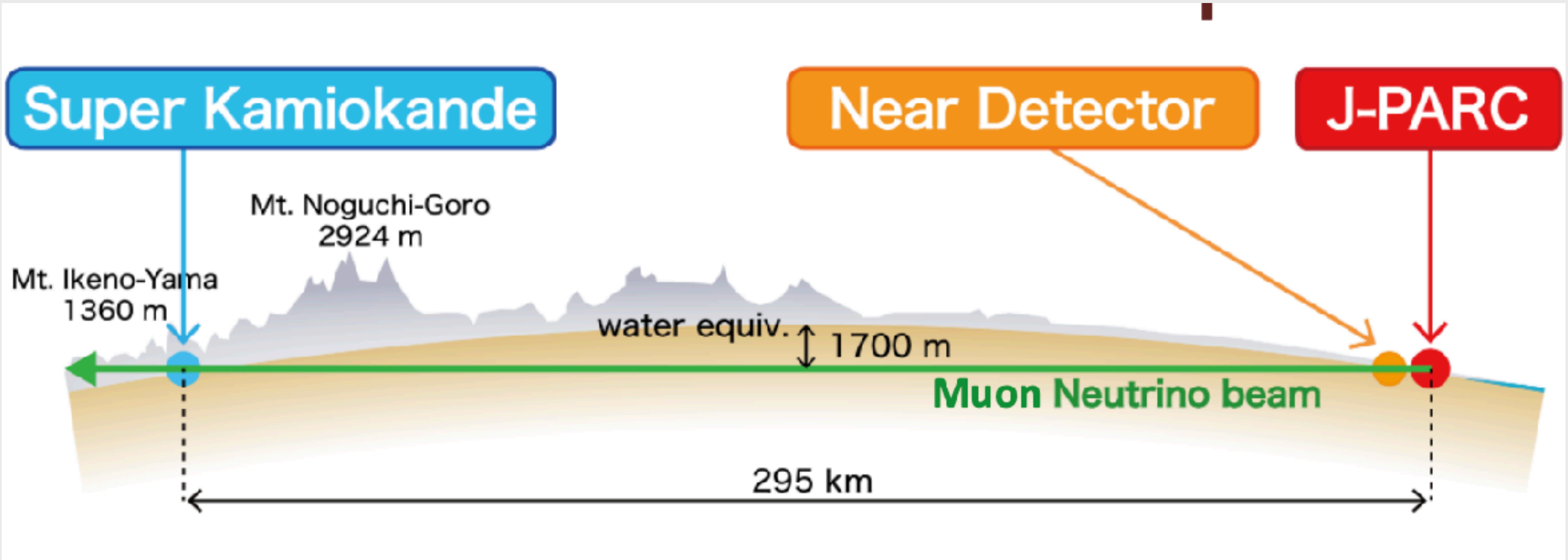
DUNE NOvA T2K

CKM & β-DECAY



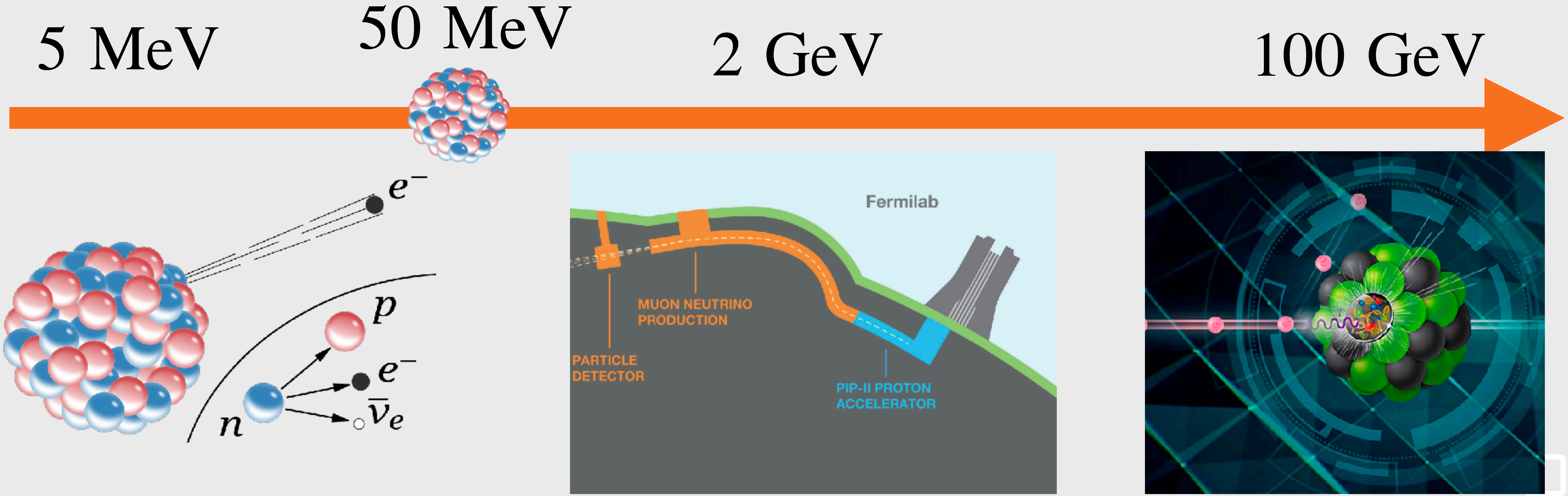
When Do QED Effects Matter?





A Multiscale Problem

- Need an approach which straddles nuclear scales and is flexible for high- and low-energy probes.



PART 1

BIG PICTURE

- Why is this important **for you?**
- Why is this a hard & interesting problem?
- Why is effective field theory the natural tool?

PART 2

WHAT'S NEW

- All order Coulomb results for a **charged current.**
- Structure dependence and nuclear coherence (big Z).
- Bridge between QFT and treatment in nuclear codes.

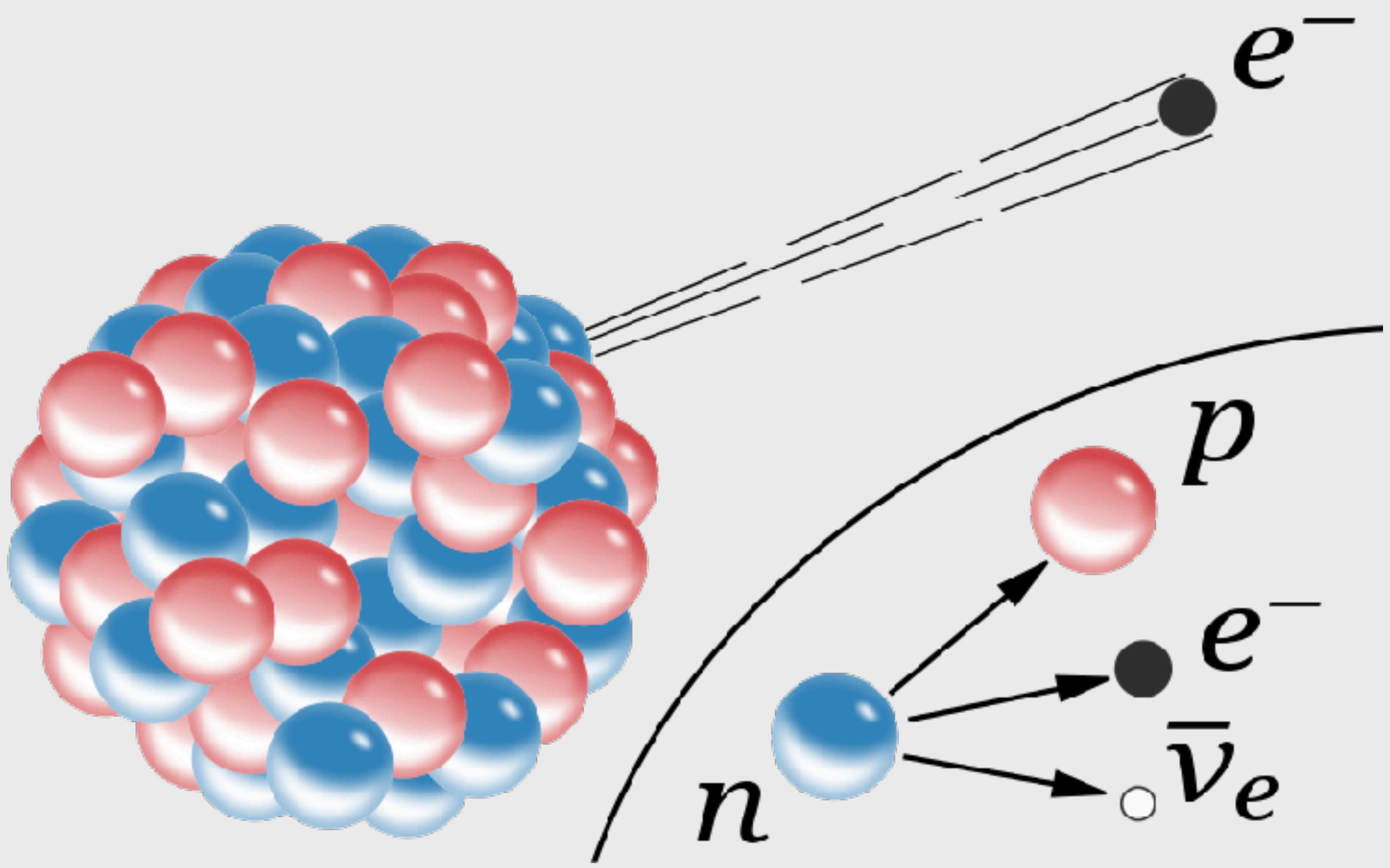
PART 3

APPLICATIONS

- Factorization theorems
- Superallowed beta decay (nucleus, low energy lepton)
- Neutrino nucleon scattering (high energy, no nucleons)

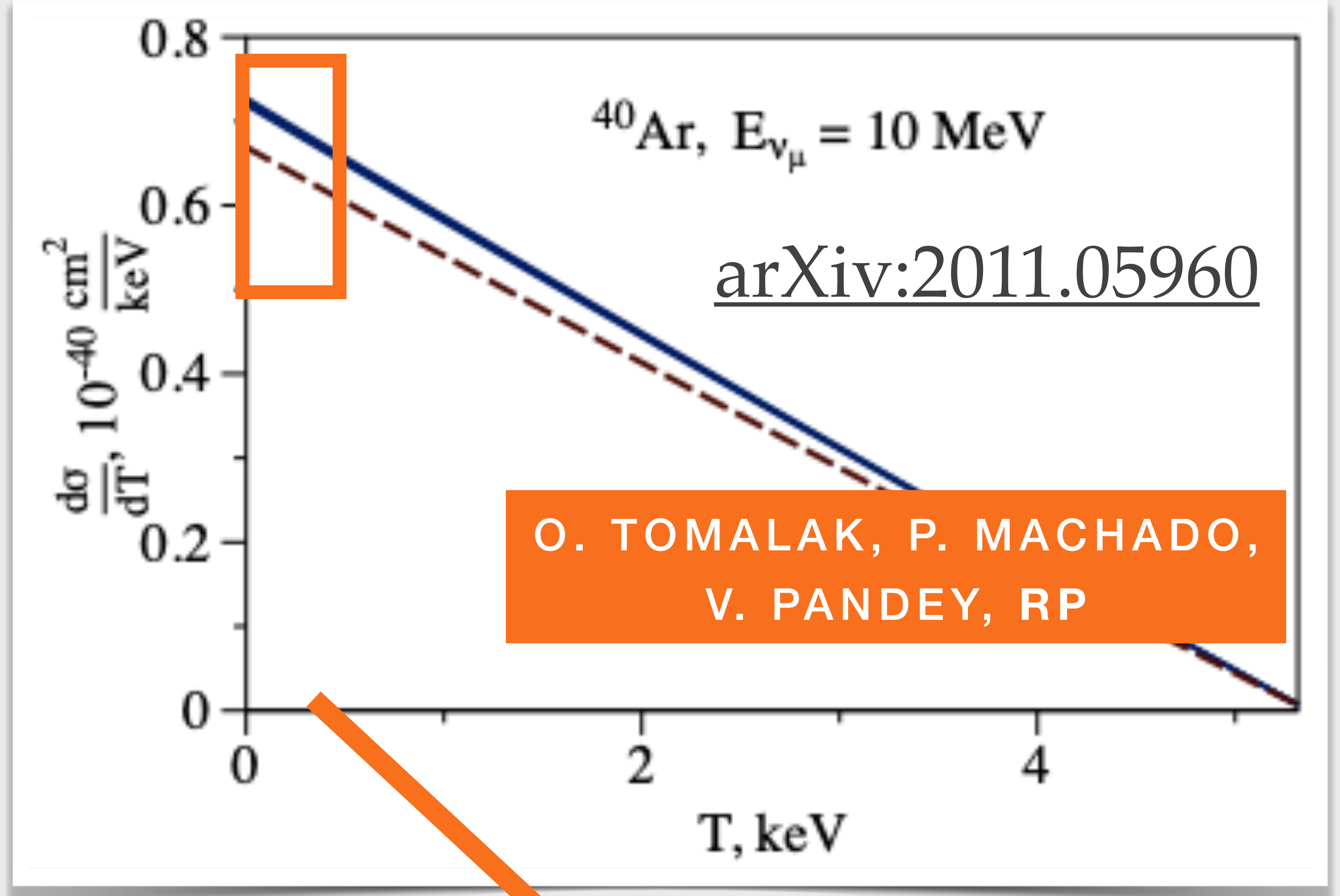
When Do QED Effects Matter?

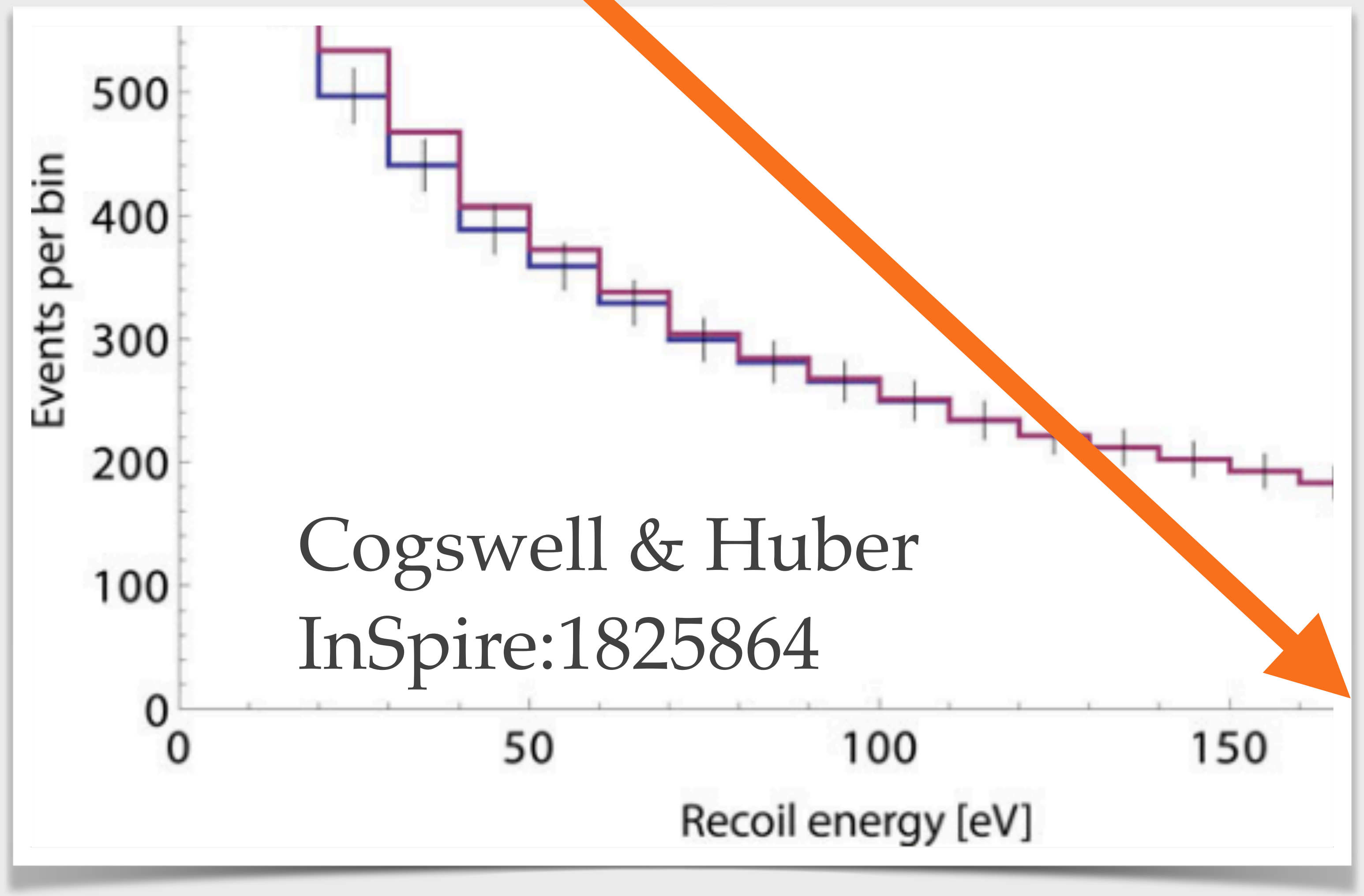
- Extractions of V_{ud} are at 10^{-4}
- Rely on $0^+ \to 0^+$ beta decays.
- Require radiative corrections up to (at least) $O(Z^2\alpha^3)$.



- Potentially relevant for reactor neutrino fluxes
  (*cough* Patrick Huber *cough*)

When Do QED Effects Matter?

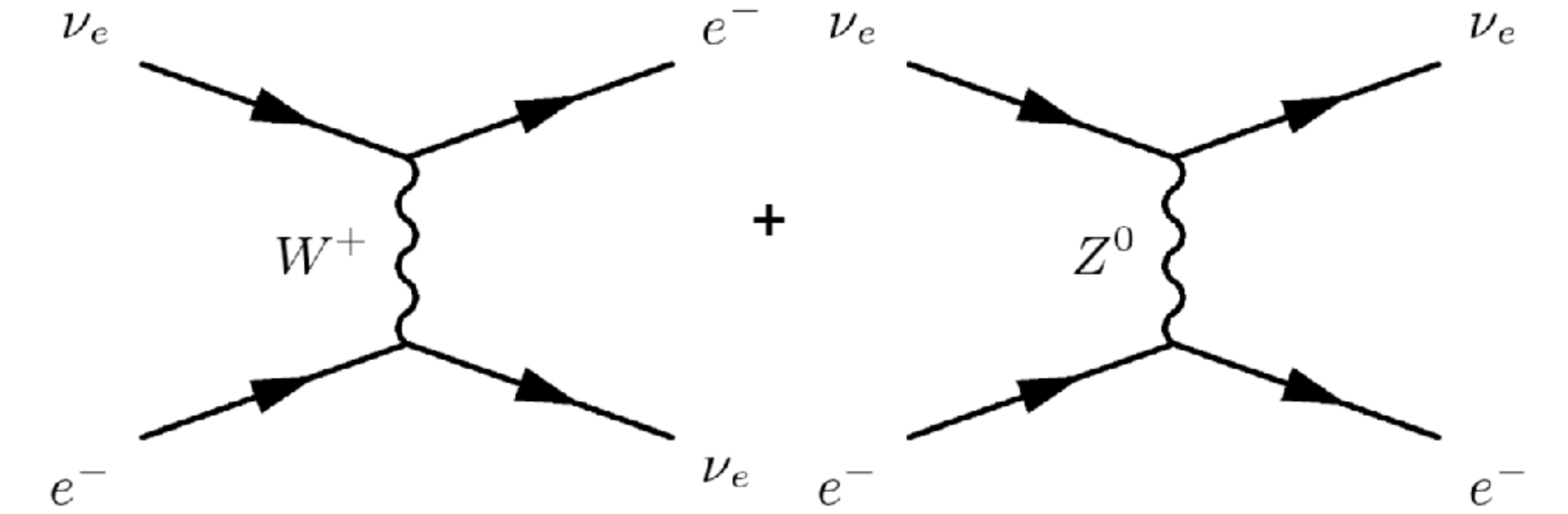


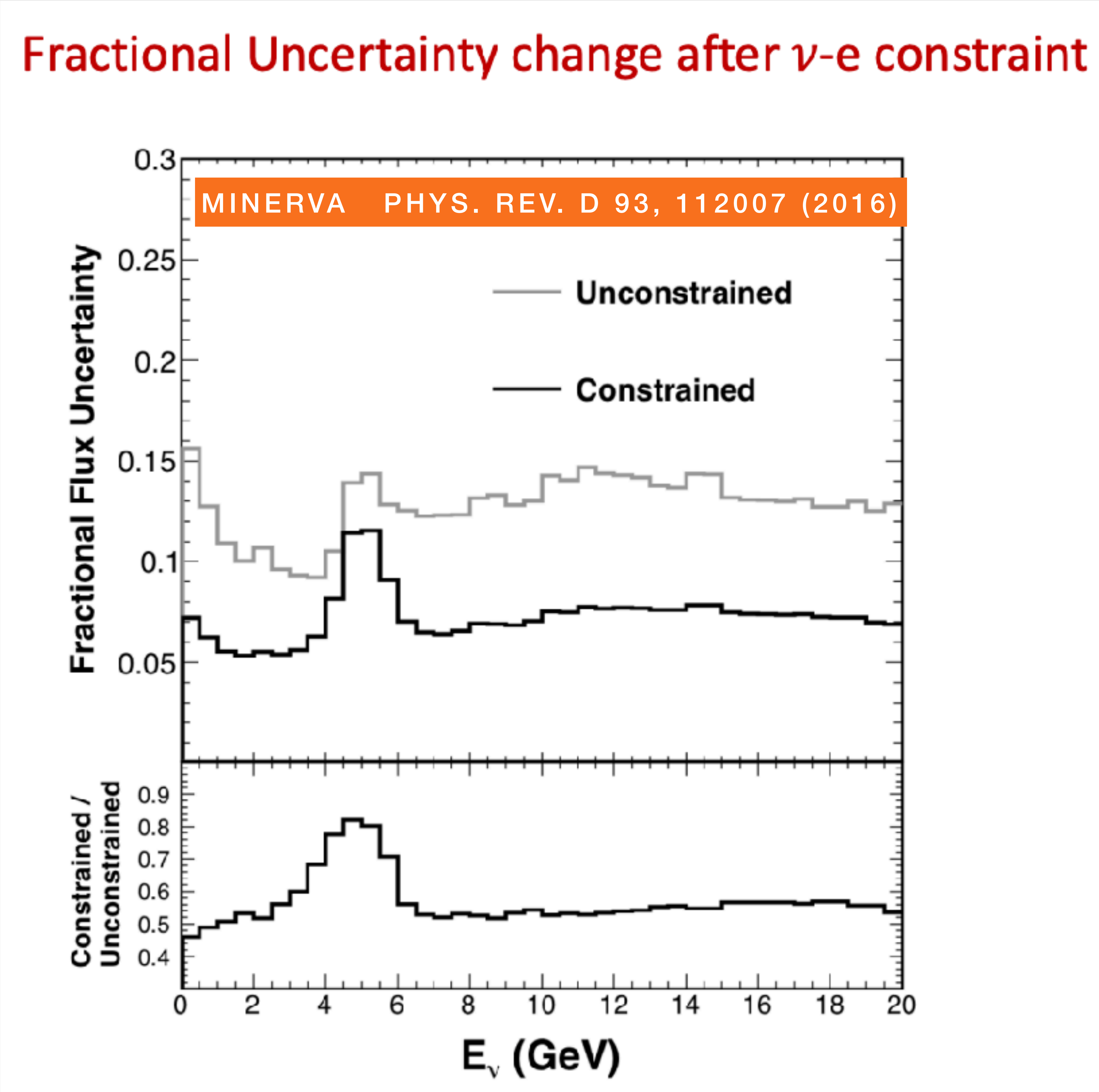


O. TOMALAK,

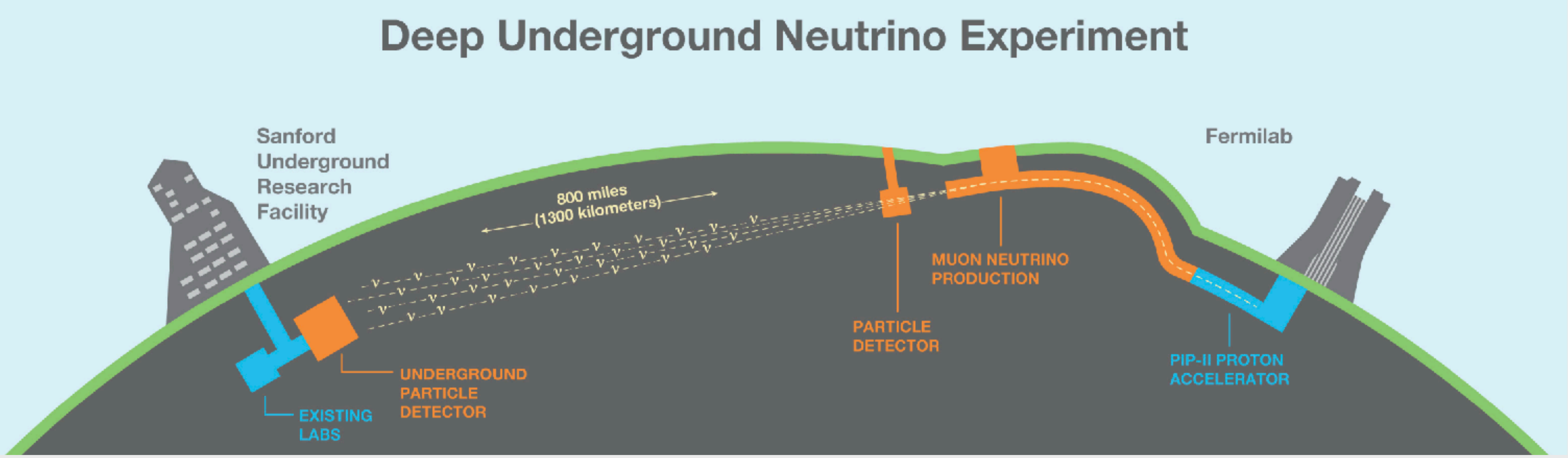
When Do QED Effects Matter?

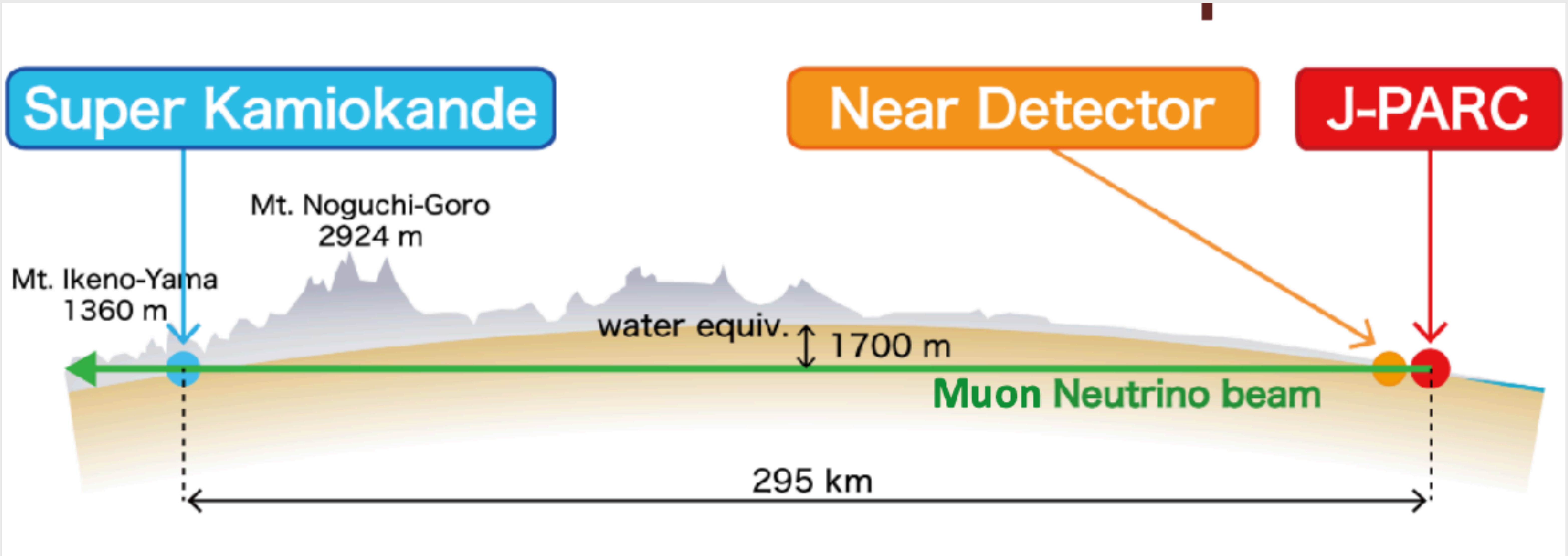
- Neutrino electron scattering used for beam-normalization.
- Requires 1-loop QED (see Tomalak & Hill 2019).





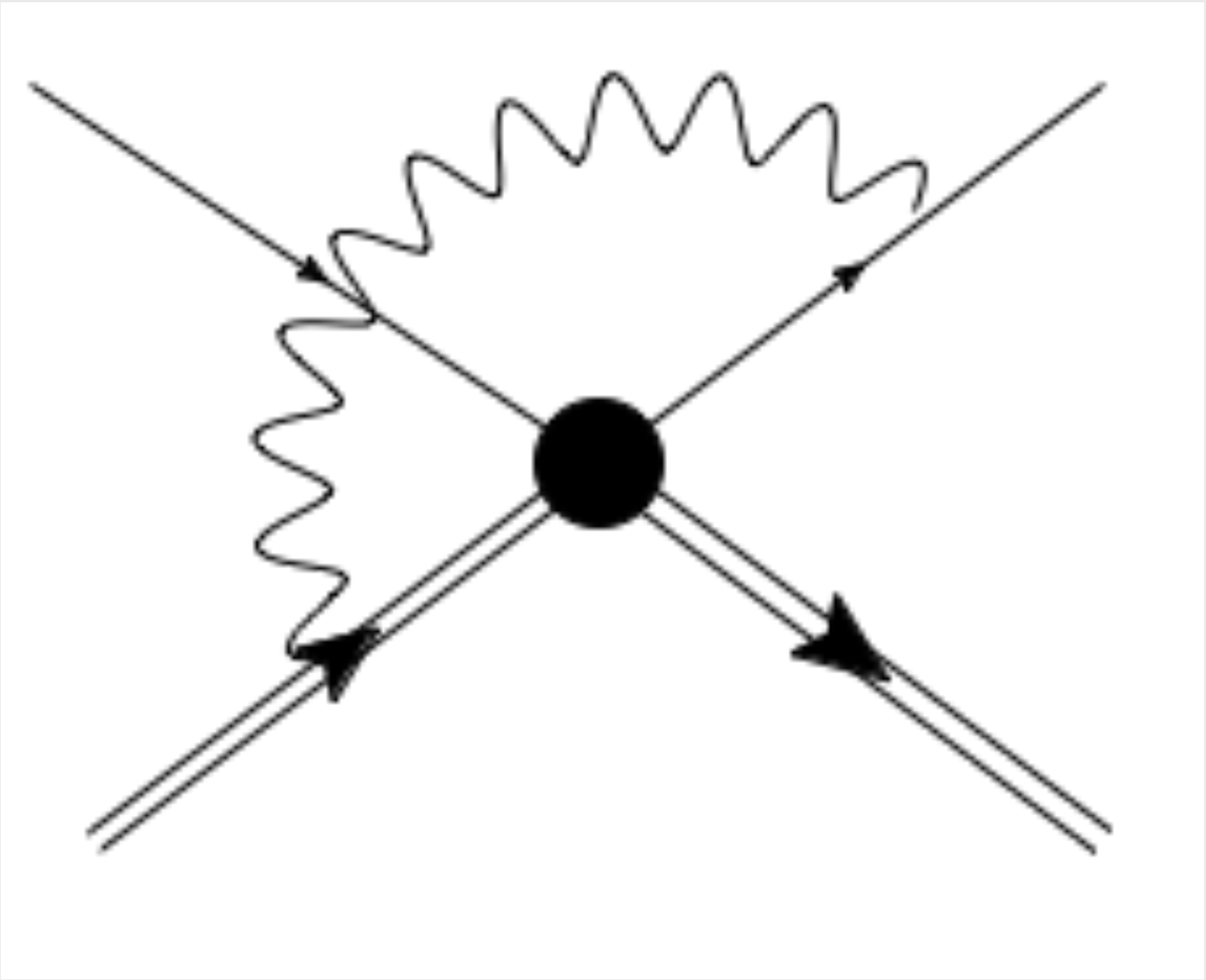
When Do QED Effects Matter?

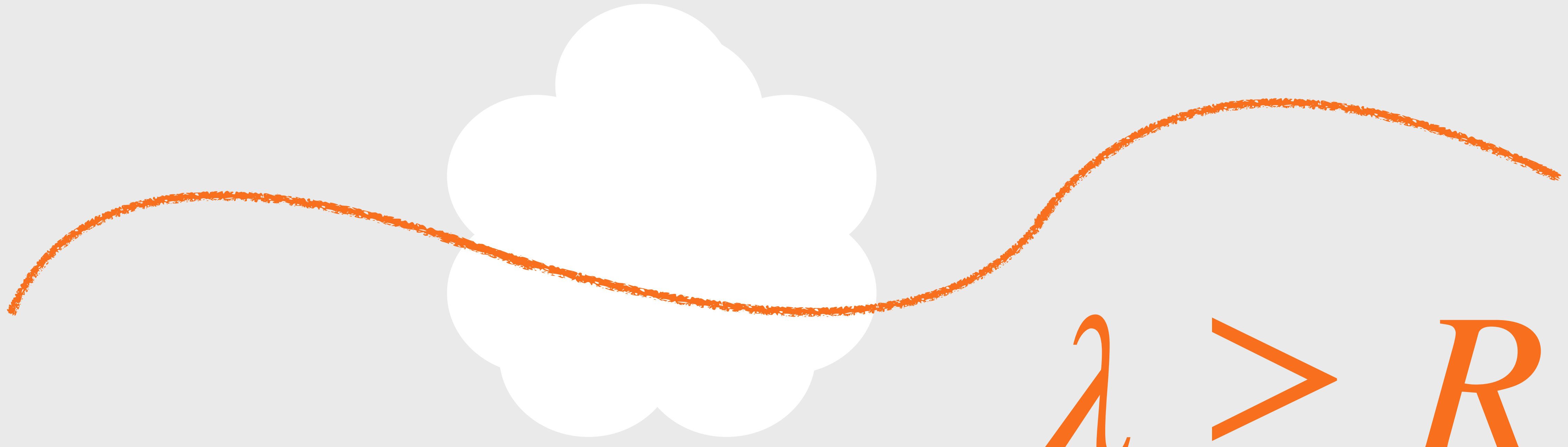




How Do We Account For Nuclear Structure Inside Loops?

- Loops connect high energy leptons to very heavy nuclei.
- At long wavelengths the nucleus can couple ***coherently*** to photons.
- Radiative correction are enhanced by the the ***charge of the nucleus.***





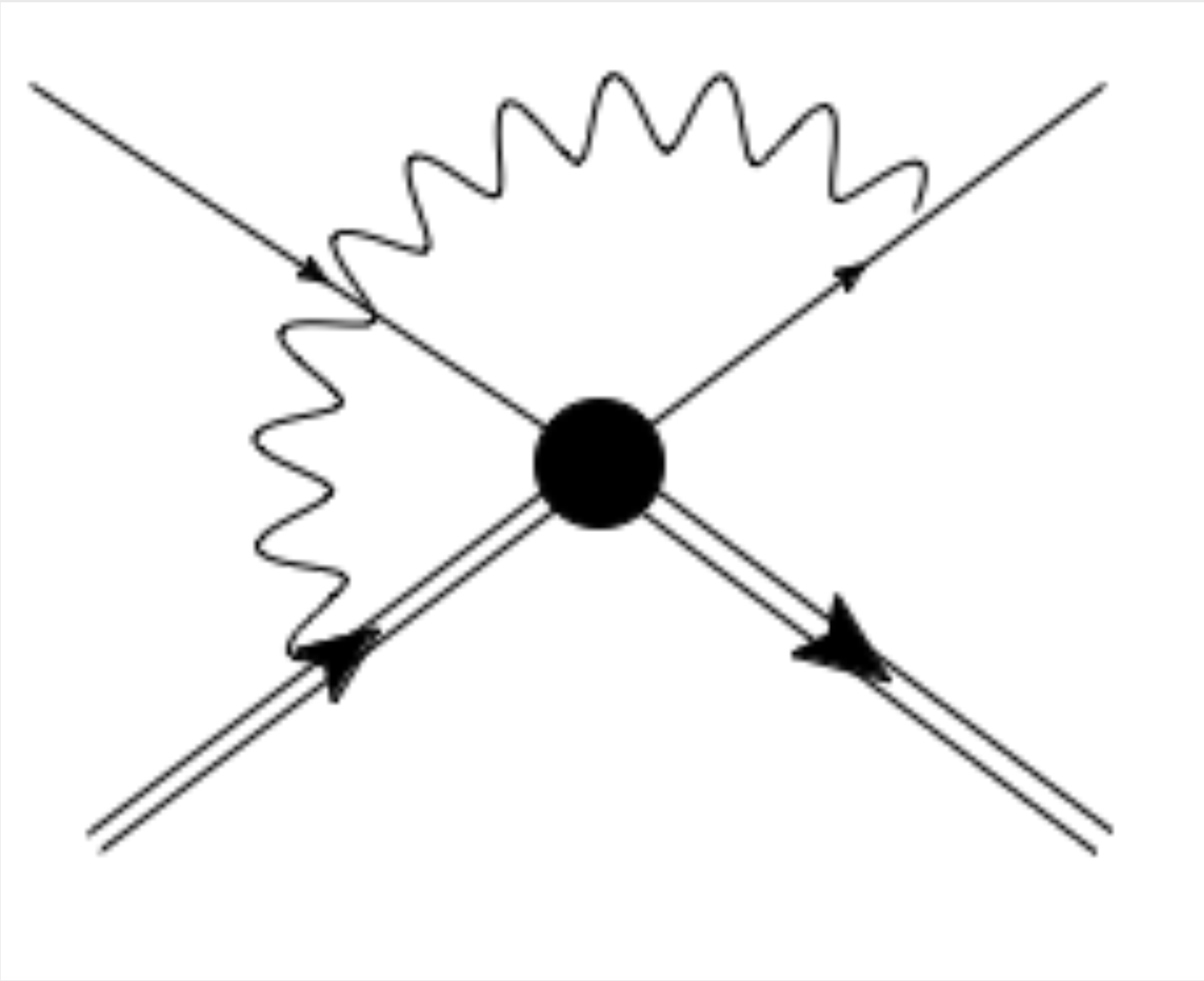
$$\alpha \to Z\alpha \quad \text{for} \quad \mathbf{L} \ll 1/R$$

LOOP MOMENTUM.

$$\lambda \gtrsim R$$

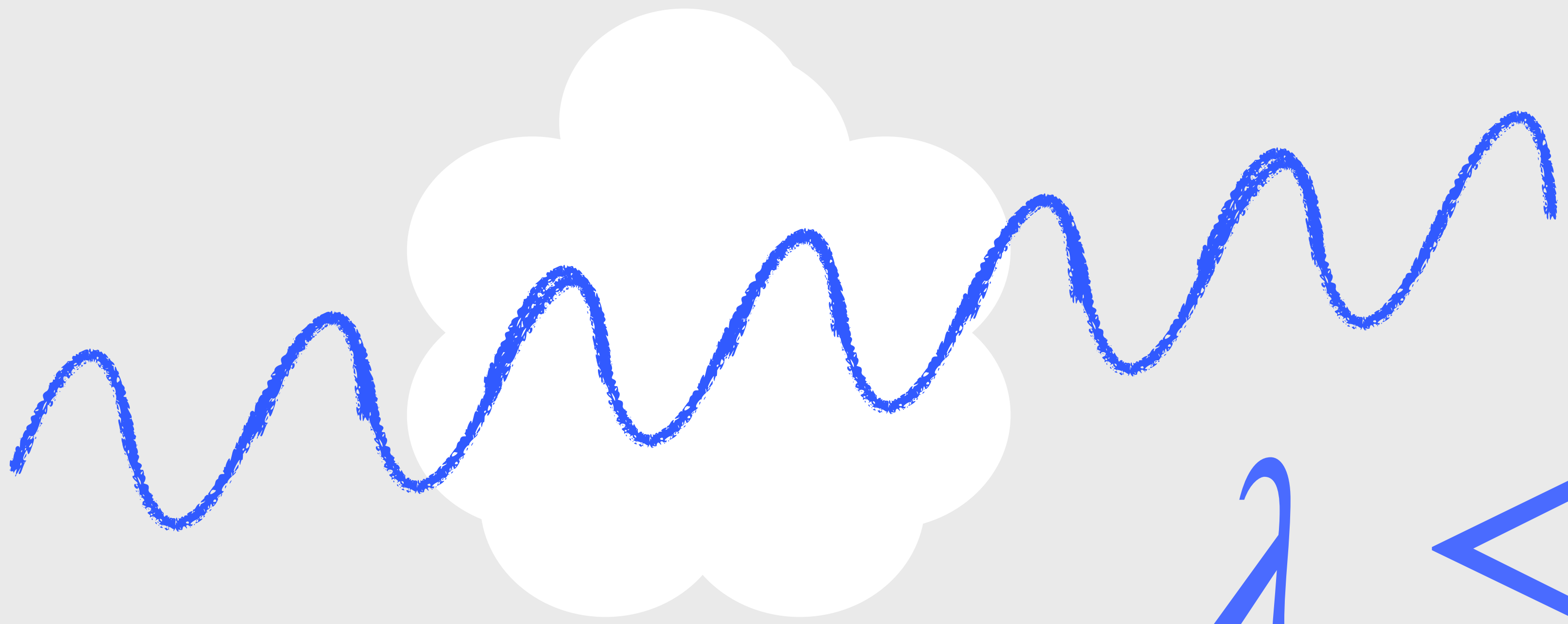
How Do We Account For Nuclear Structure Inside Loops?

- Loops connect high energy leptons to very heavy nuclei.
- At short wavelengths the photon couples ***to nucleons (or even quarks)***.
- Radiative corrections are smaller, but involve ``the brown muck''.



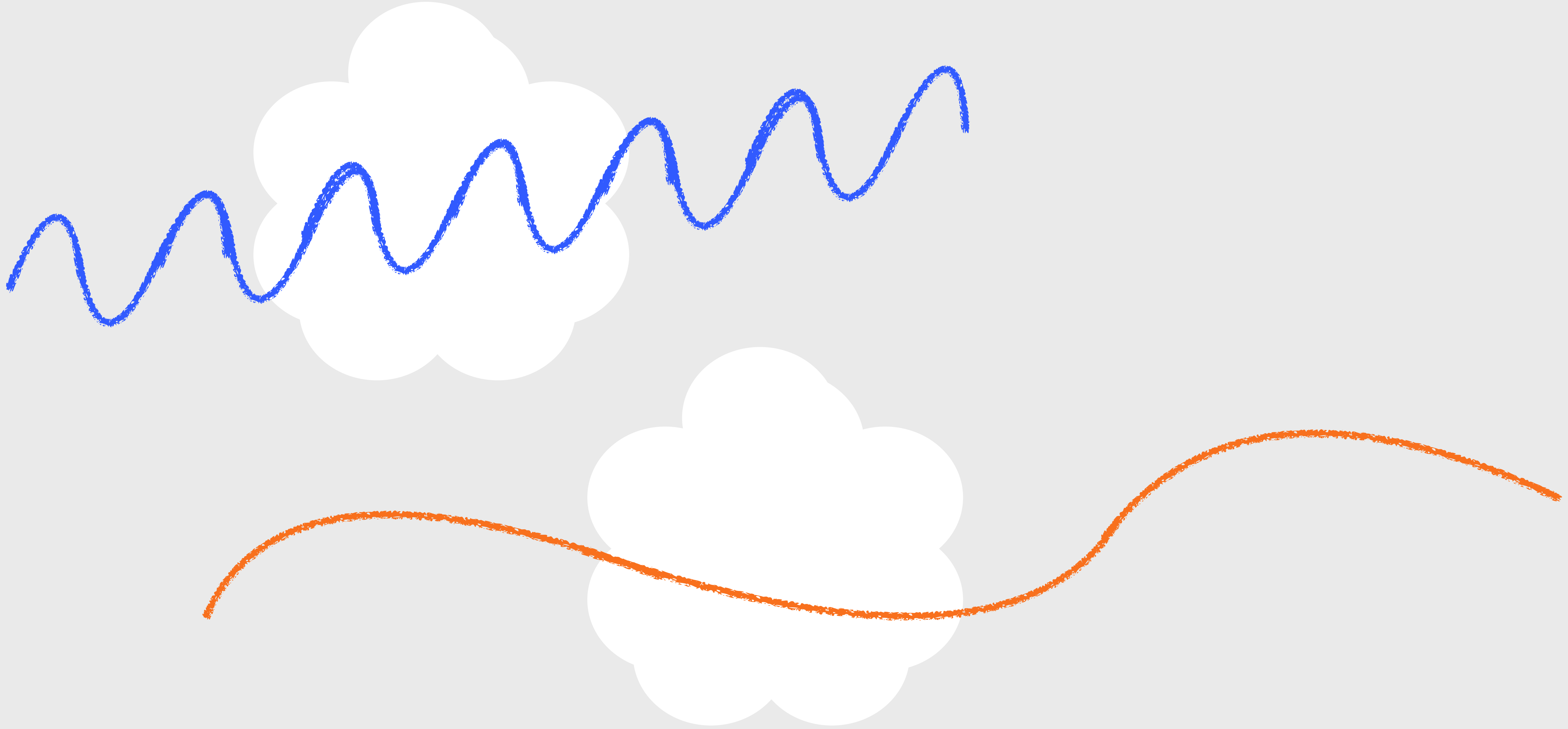
$$\alpha \quad \text{for} \quad \mathbf{L} \gg 1/R$$

LOOP MOMENTUM.



$$\lambda \lesssim R$$

Scales In The Problem



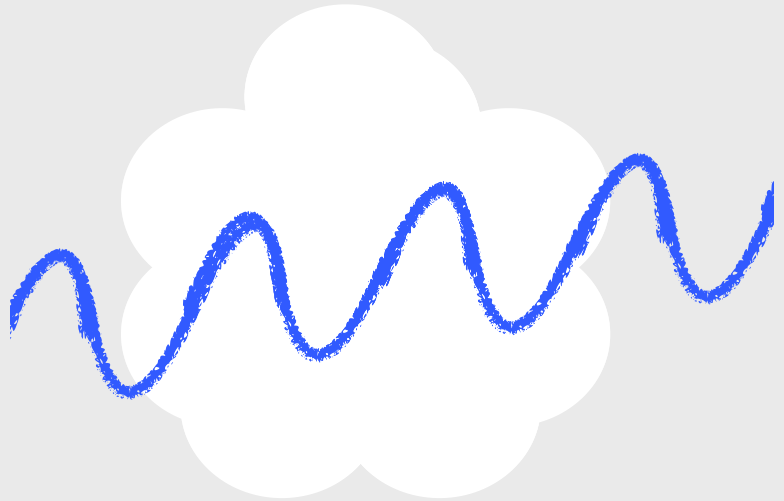
Nuclear Mass	40 GeV	α
QCD scale	1 GeV	
Lepton Energy	600 MeV	
Nuclear Radius	50 MeV	$Z\alpha$

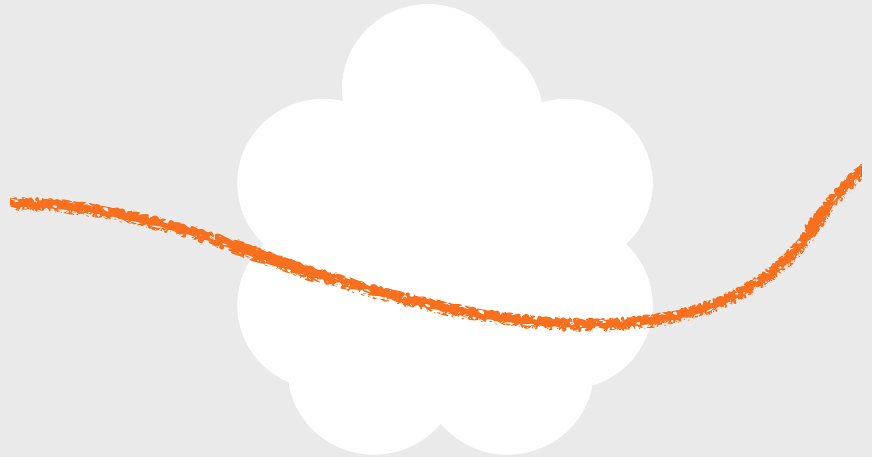
$$\int \frac{\mathrm{d}^4 L}{(2\pi)^4}$$

- Loop integrals have all these scales!

Effective Field Theory

- Separate scales in loop.
- Two EFTs ↔ Two regions.





$$\int_{\mathrm{EFT_I}} [\mathrm{d}L] \quad + \quad \int_{\mathrm{EFT_{II}}} [\mathrm{d}L]$$

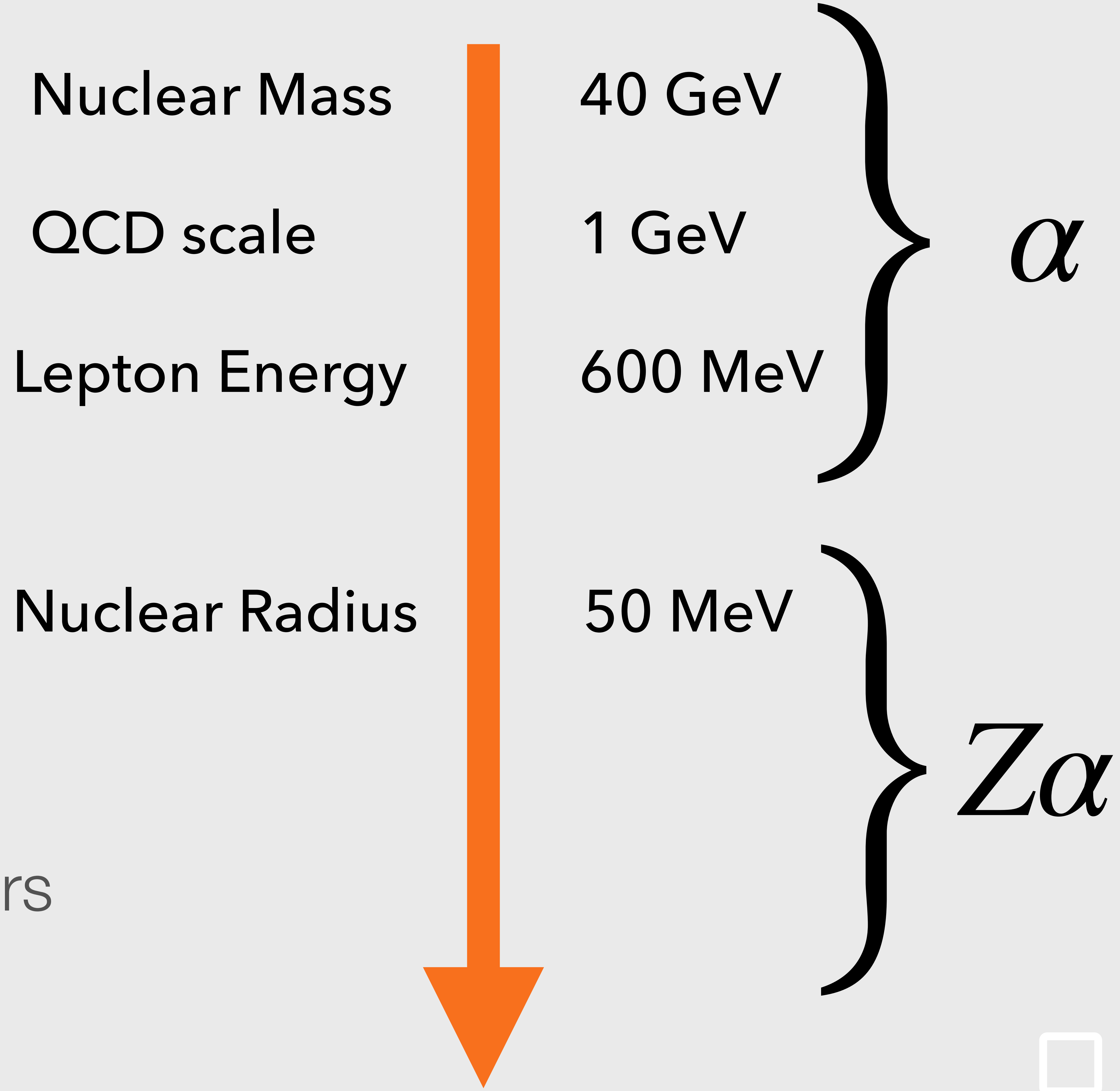
Nuclear Mass	40 GeV	$\mathrm{EFT_I}$
Lepton Energy	600 MeV	$\mathrm{EFT_I}$
Nuclear Radius	50 MeV	$\mathrm{EFT_{II}}$

Why EFT?

$$\mathscr{M}(\{\text{all the scales}\})$$

$$= \mathscr{M}_1(E_1)\mathscr{M}_2(E_2)\mathscr{M}_3(E_3)\ldots$$

- Problem factorizes
- Each matrix element has only 1 scale.
- Nuclear scales can be separated from others (e.g. Coulomb, collinear radiation etc.)



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- Why is this a hard & interesting problem?
- Why is effective field theory the natural tool?

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WHAT'S NEW

- All order Coulomb results for a **charged current.**
- Structure dependence and nuclear coherence (big Z).
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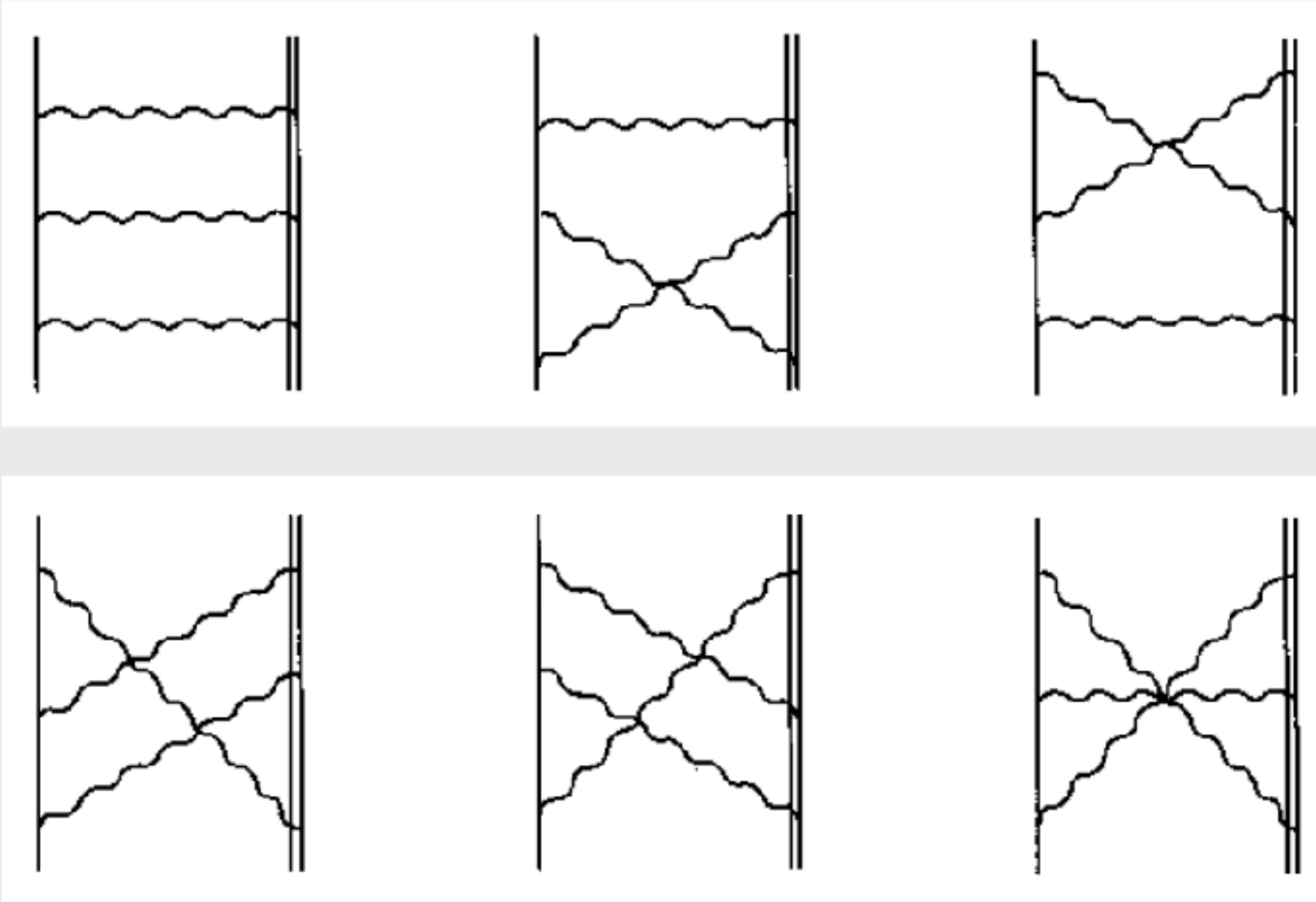
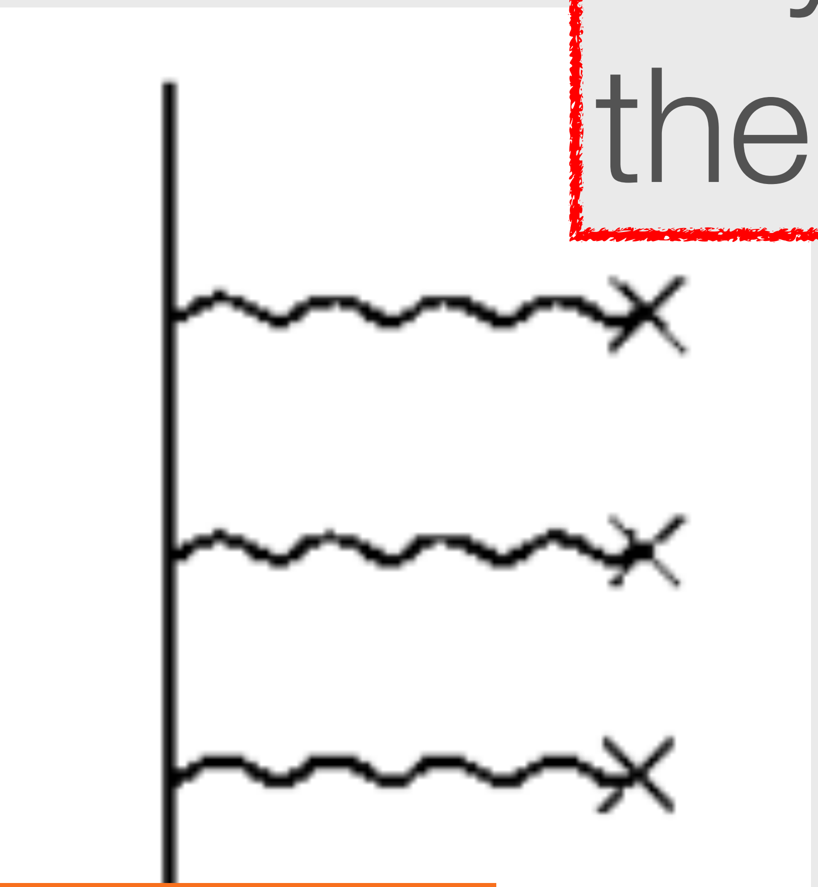
- Superallowed beta decay (nucleus, low energy lepton)
- Neutrino nucleon scattering (high energy, no nucleons)

Coulomb Physics Review: Crossed

- It is “well known” (but obscure) that the sum of all crossed ladder diagrams reduces to the Dirac equation with Coulomb field.
- Assumes $|q|R \ll 1$, $|q|/M \ll 1$ and $Z_A = Z_B$.



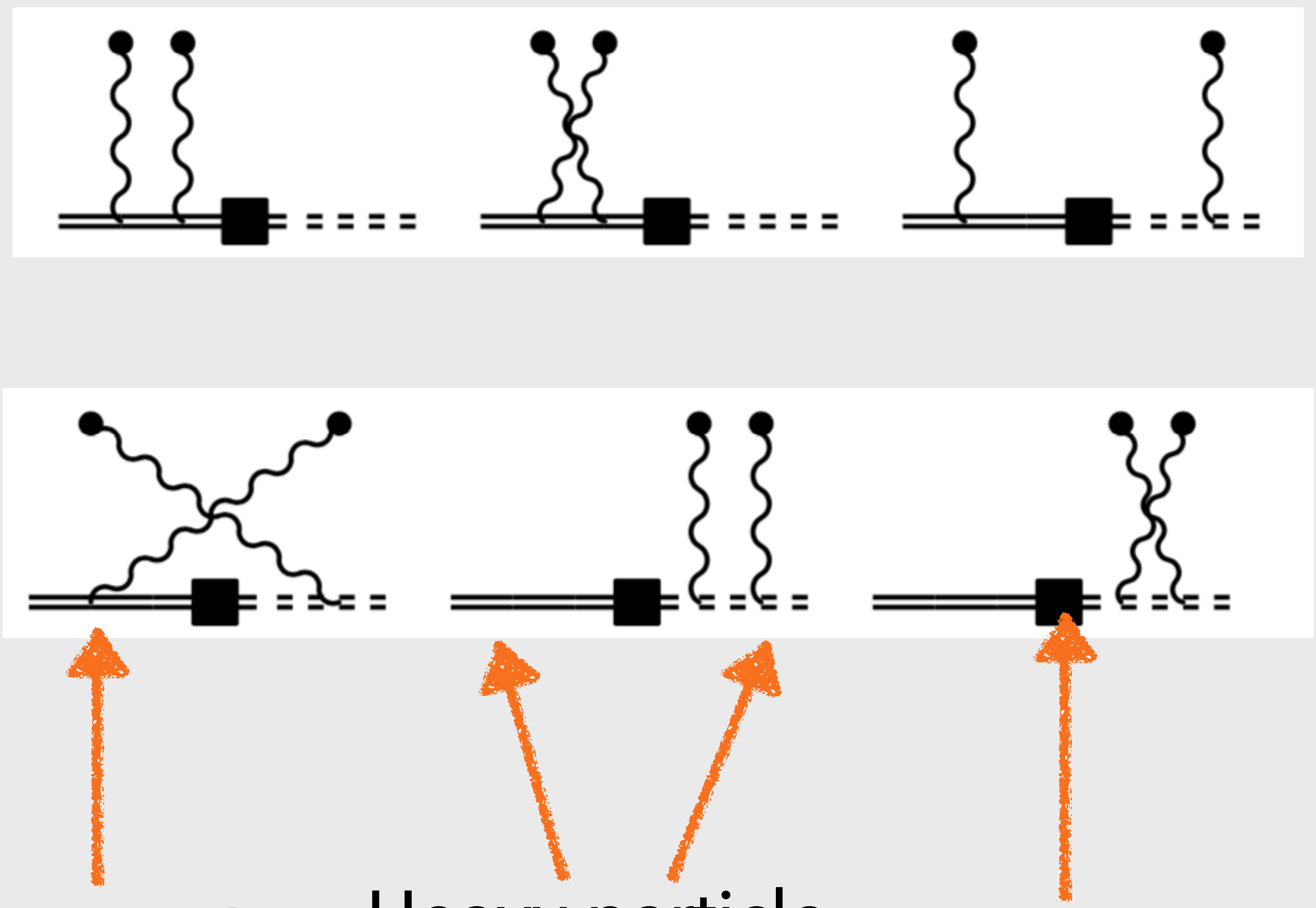


Key simplification is the large mass limit.

WEINBERG VOL 1, 13.6

SEE ALSO: BRODSKY (1969), YENNIE (1964), NEGHABIAN (1983)

Crossed Ladders: Same Idea New Slide



$iZe\ F(q^2)\ v_\mu$

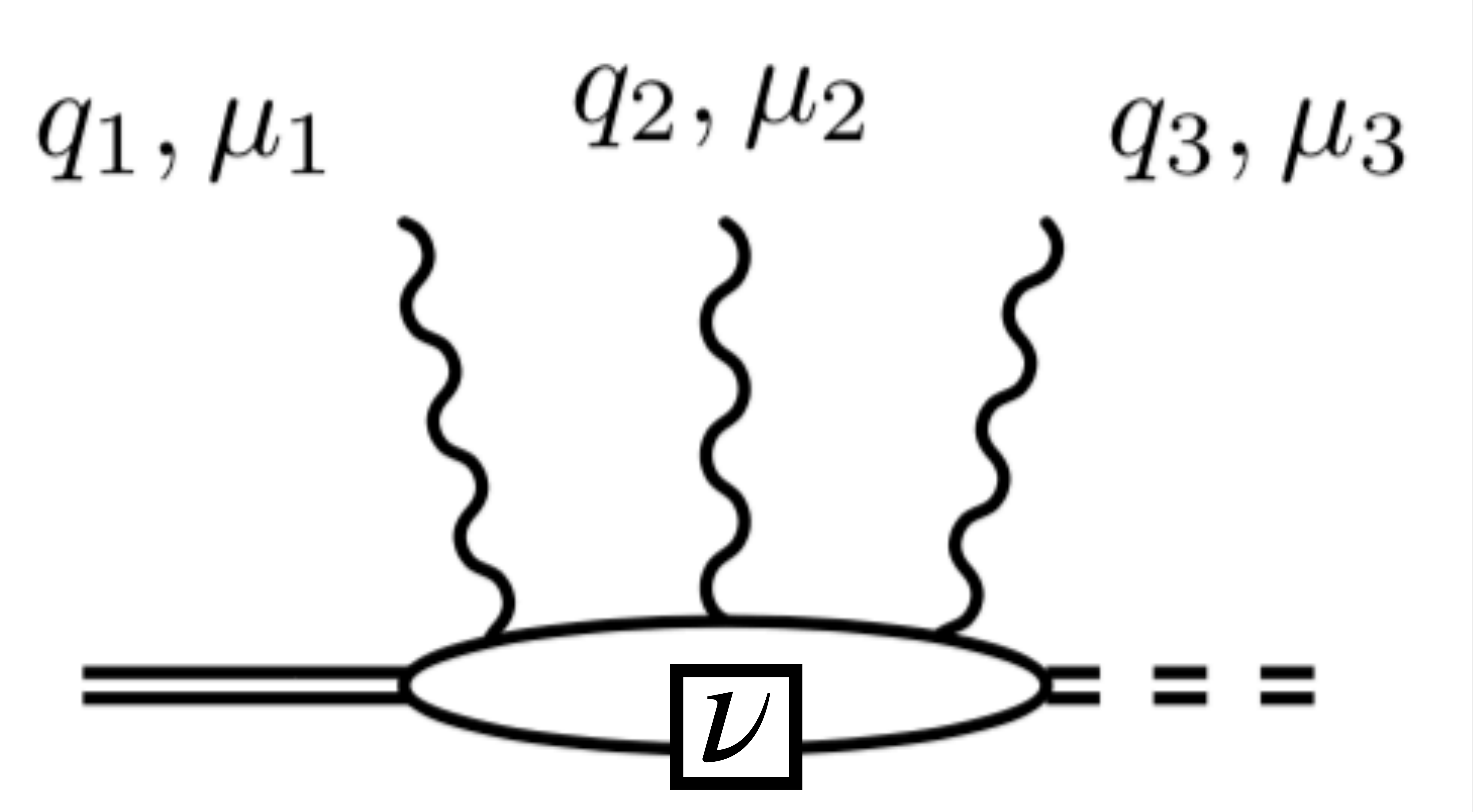
- Sum of all diagrams equivalent to external field sourced by charge distribution.

$$\frac{2m}{(p+q)^2 - m^2} \to \frac{1}{v \cdot q}$$

New Analysis With Charged Currents

$$G_{\mu_1\ldots\mu_N\,;\,\nu}(q_1,\ldots q_N) = \int [\mathrm{d}^4x]\, \mathrm{e}^{\mathrm{i}\sum_i q_i\cdot x_i} \langle B\,|\,T\{J_{\mu_1}(x_1)\ldots\mathcal{J}_\nu\ldots J_{\mu_N}(x_N)\}\,|A\rangle$$

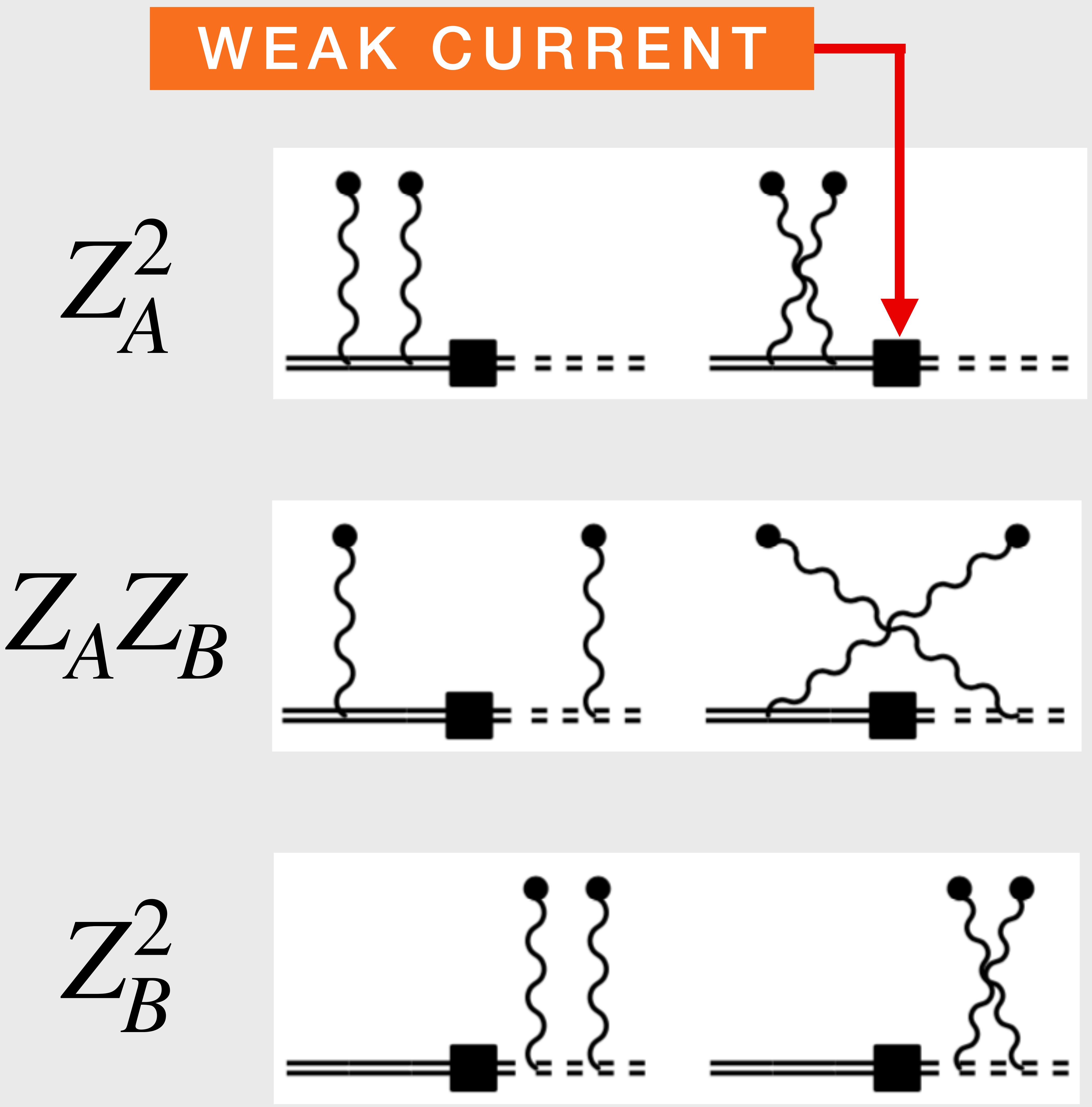
WEAK CURRENT

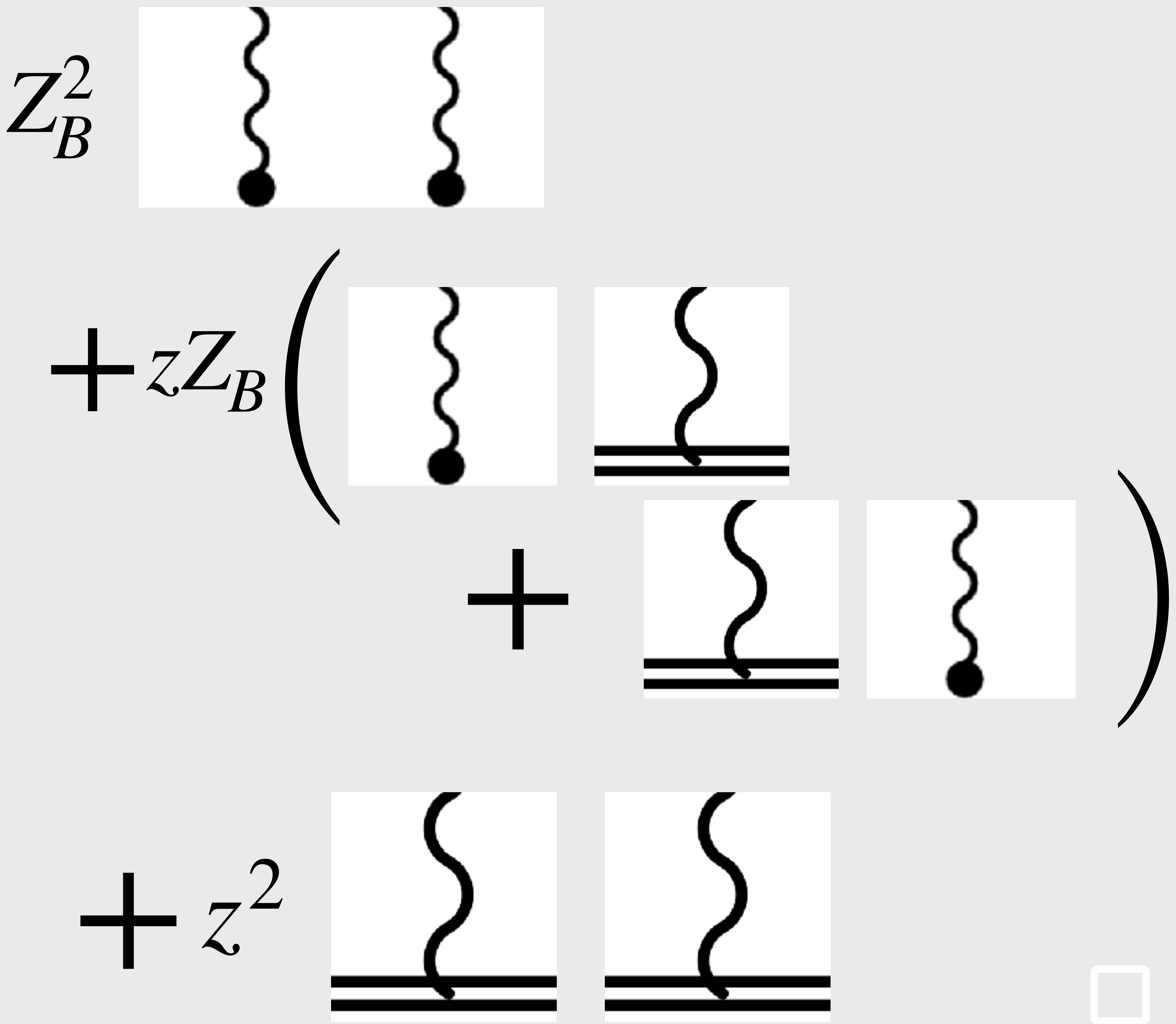


$$\langle B\,|\,\big(\mathrm{Big\ Blob}\big)\,|A\rangle$$

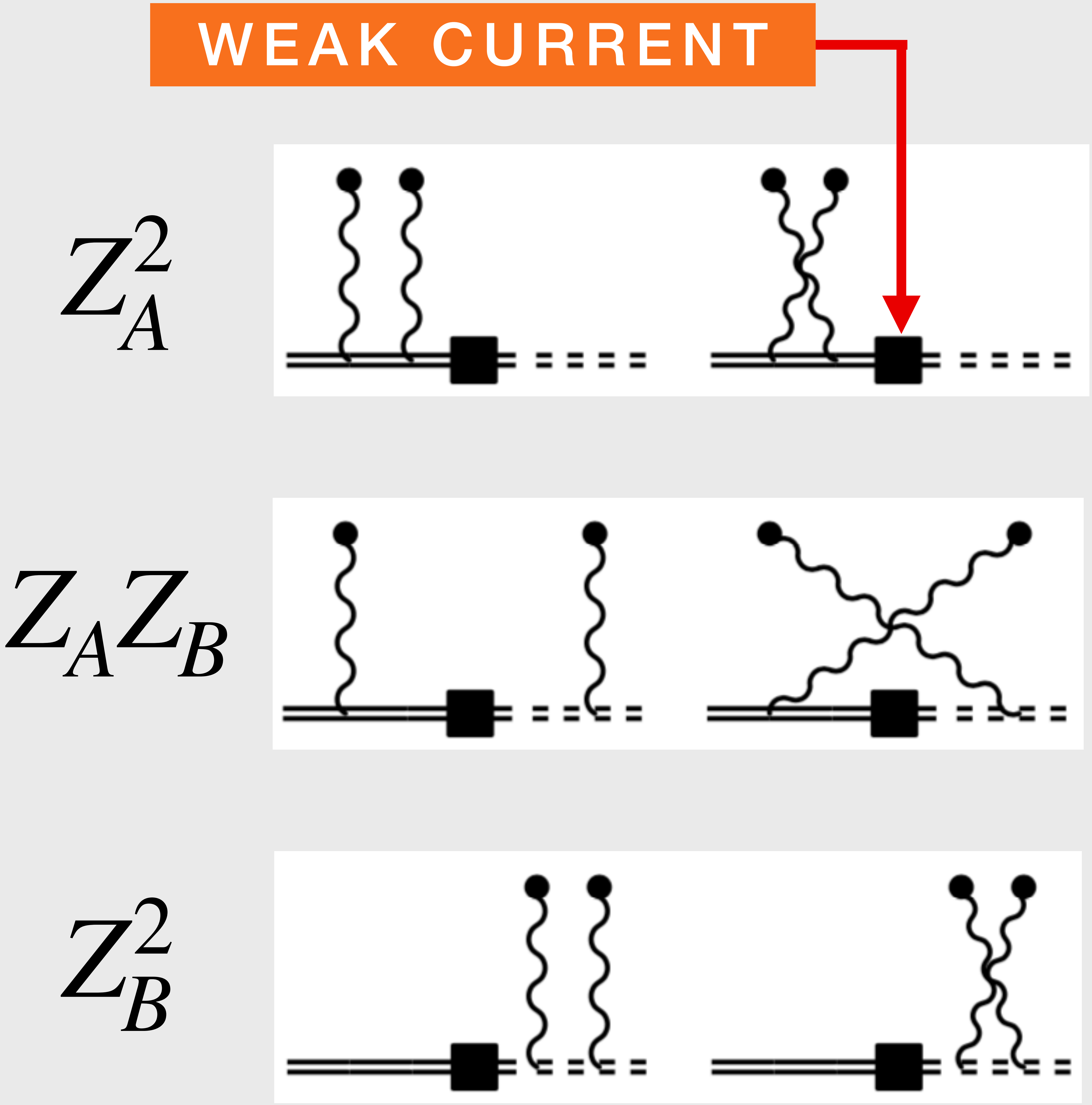
$$Z_A \neq Z_B$$

Charged Currents





Charged Currents



$$Z_B^2 \left(2 \times \text{ Coulomb} \right)$$

$$+ z Z_B \left(\text{Coulomb} + \text{Eikonal} \right)$$

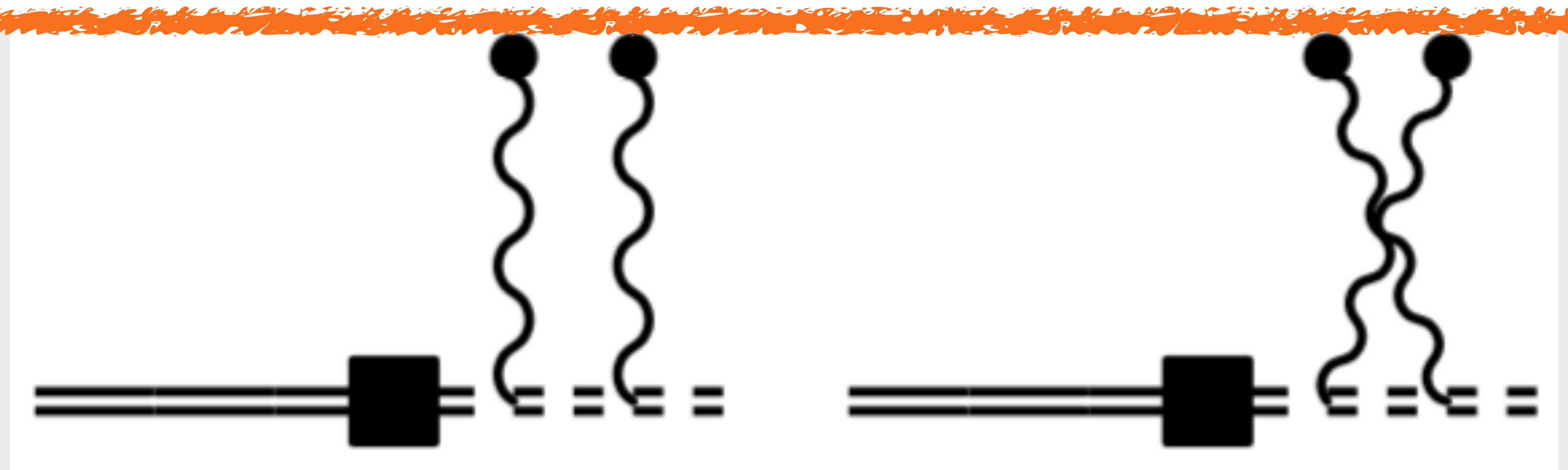
$$+ z^2 \left(2 \times \text{ Eikonal} \right)$$

Charged Currents

Main Idea

- We have successfully generalized the formalism to account for charged currents.
- Necessary step for charged current reactions on nuclei.

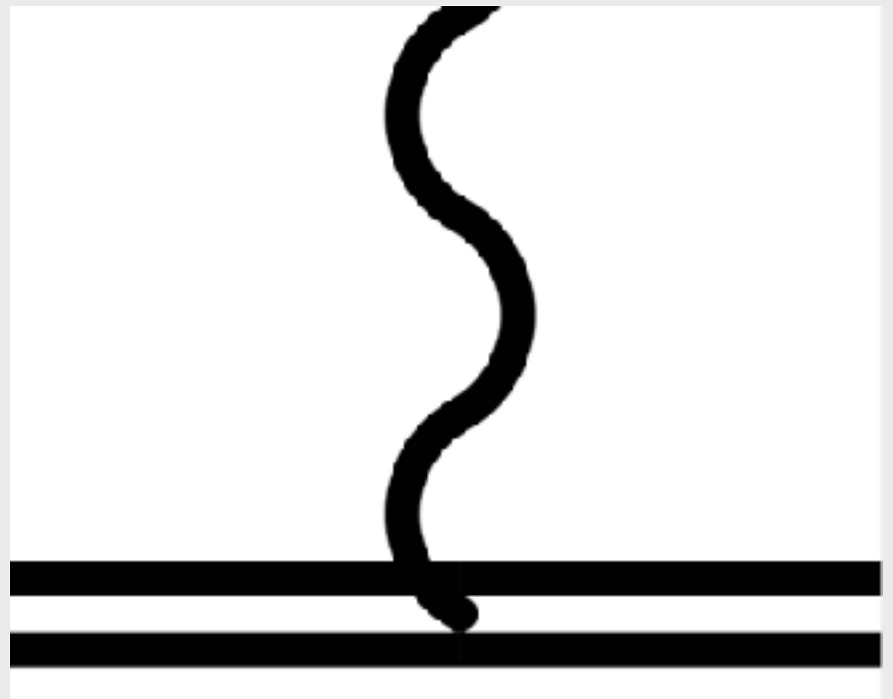
Z_B^2



Structure Dependence

- Nucleus is an extended object.
- How do we include this inside loops consistently?
- Answer is surprisingly simple. Related to EFT called NR-QED.

$$= \frac{i}{v \cdot q + i\varepsilon}$$



$$= iZe\, v_\mu + \left(F(q^2) - 1\right)\left(v^\mu - \frac{v \cdot q}{q^2} q^\mu\right)$$

$$qR \sim O(1)$$

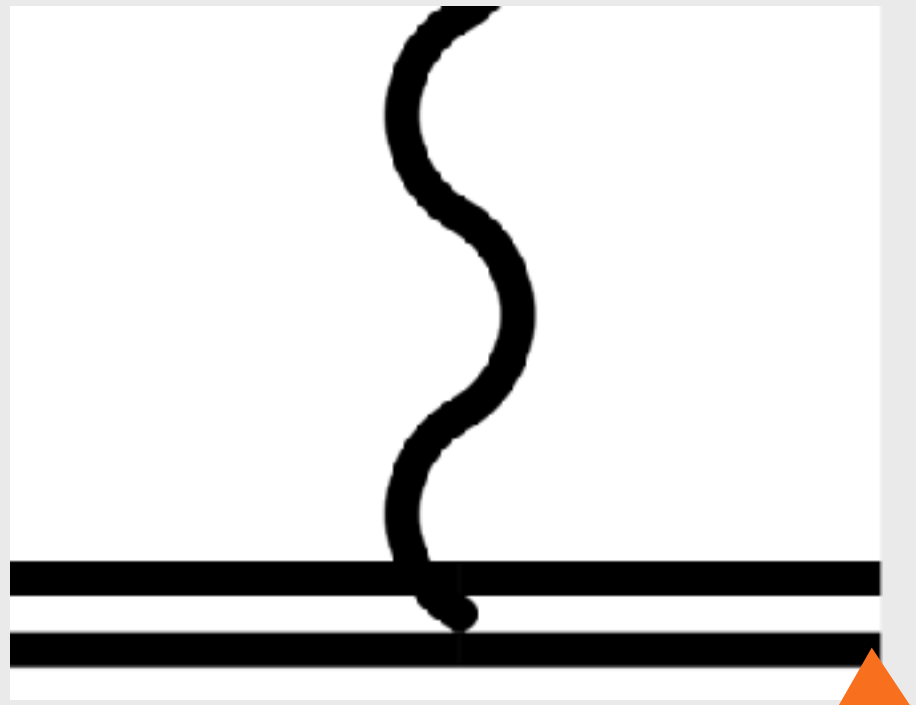
$$\int_{\mathrm{EFT_{II}}} [\mathrm{d}L]$$

Structure Dependence And Gauge Invariance

$$\left(v^\mu - \frac{v \cdot q}{q^2} q^\mu\right) = v^\mu \left(g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)$$

$$= \frac{\mathrm{i}}{v \cdot q + \mathrm{i}\varepsilon}$$

$$\nabla \cdot \mathbf{E} = v_\mu \partial_\nu F^{\mu\nu}$$



Minimal Coupling

$$= \mathrm{i}Ze\ v_\mu$$

$$+ \left(F(q^2) - 1\right) \left(v^\mu - \frac{v \cdot q}{q^2} q^\mu\right)$$

WHY THIS FUNNY FEYNMAN RULE?

E-Field Is Gauge Inv.

Structure Dependence

Main Idea

- Inclusion of nuclear structure allows us to capture the region where the nucleus looses coherence inside loops.
- Necessary for matching from ~10 MeV scale, up to the ~ 200 MeV scale.

$$\int_{\text{EFT}_{\text{II}}} \quad + \left(F(q^2) - 1\right)\left(v^\mu - \frac{v \cdot q}{q^2} q^\mu\right)$$

Wavefunctions And Feynman Diagrams

- Coulomb effects historically handled with ``distorted waves''

- What are the equivalent effects in Feynman diagrams?

Use Lippmann-Schwinger Equation!

$$|\psi_p^{(\pm)}\rangle = |\phi_p\rangle + \frac{1}{H - E_p \pm i\varepsilon} V |\phi_p\rangle + \frac{1}{H - E_p \pm i\varepsilon} V \frac{1}{H - E_p \pm i\varepsilon} V |\phi_p\rangle + \dots$$

Wavefunctions And Feynman Diagrams

- Coulomb effects historically handled with ``distorted waves''

- What are the equivalent effects in Feynman diagrams?

Loop With A Phase Factor!

$$\langle x | \psi_p^{(\pm)} \rangle = \mathrm{e}^{\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \left(1 + \int \frac{d^3Q}{(2\pi)^3} \frac{1}{2\mathbf{P}\cdot\mathbf{Q} + \mathbf{Q}^2 \pm \mathrm{i}\varepsilon} \frac{Z\alpha}{\mathbf{Q}^2} \mathrm{e}^{\mathrm{i}\mathbf{Q}\cdot\mathbf{x}} + \ldots \right)$$

Wavefunctions And Feynman Diagrams

Main Idea

- We now have a constructive understanding of how wavefunction methods and diagramatics wed together.
- Important for interfacing with existing nuclear codes, and to understand old literature and reproduce/validate against old literature (or find mistakes).

$$\langle x|\psi_p^{(-)}\rangle = e^{i\mathbf{p}\cdot\mathbf{x}}\left(1+\int \frac{}{(2\pi)^3}\,\frac{}{2\mathbf{P}\cdot\mathbf{Q}+\mathbf{Q}^2\pm i\varepsilon}\,\frac{}{\mathbf{Q}^2}\,e^{i\mathbf{Q}\cdot\mathbf{x}}+\dots\right)$$

Take Home Message

- A complete theory for QED interactions with heavy, extended, composite particles.
- Can systematize coherently enhanced radiative corrections.
- Understanding of wavefunctions vs diagramatic perturbation theory builds Rosetta stone.

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- Neutrino nucleon scattering (high energy, no nucleons)

Multi-Scale Problem

Full Theory

- Full theory has many scales.
- Want good perturbative control.

Nuclear Mass	40 GeV
Nucleon mass	1 GeV
Lepton Energy	600 MeV
Fermi Momentum	200 MeV
Nuclear Radius	50 MeV

$$\mathscr{M}(M_A, m_N, m_\ell, E_\ell, R, \Lambda_{QCD}, p_F)$$

Tower Of EFTs

- Factorization lets us treat one-scale at a time.
- No double counting!

Nuclear Mass	40 GeV
Nucleon mass	1 GeV
Lepton Energy	600 MeV
Fermi Momentum	200 MeV
Nuclear Radius	50 MeV

$\mathrm{EFT_I}$, $\mathrm{EFT_{II}}$, $\mathrm{EFT_{III}}$, $\mathrm{EFT_{IV}}$, $\mathrm{EFT_V}$

$$\mathcal{M}_1(M_A)\mathcal{M}_2(m_N)\mathcal{M}_3(E_\ell)\mathcal{M}_4(p_F)\mathcal{M}_5(R)$$

Nucleon-Level Factorization

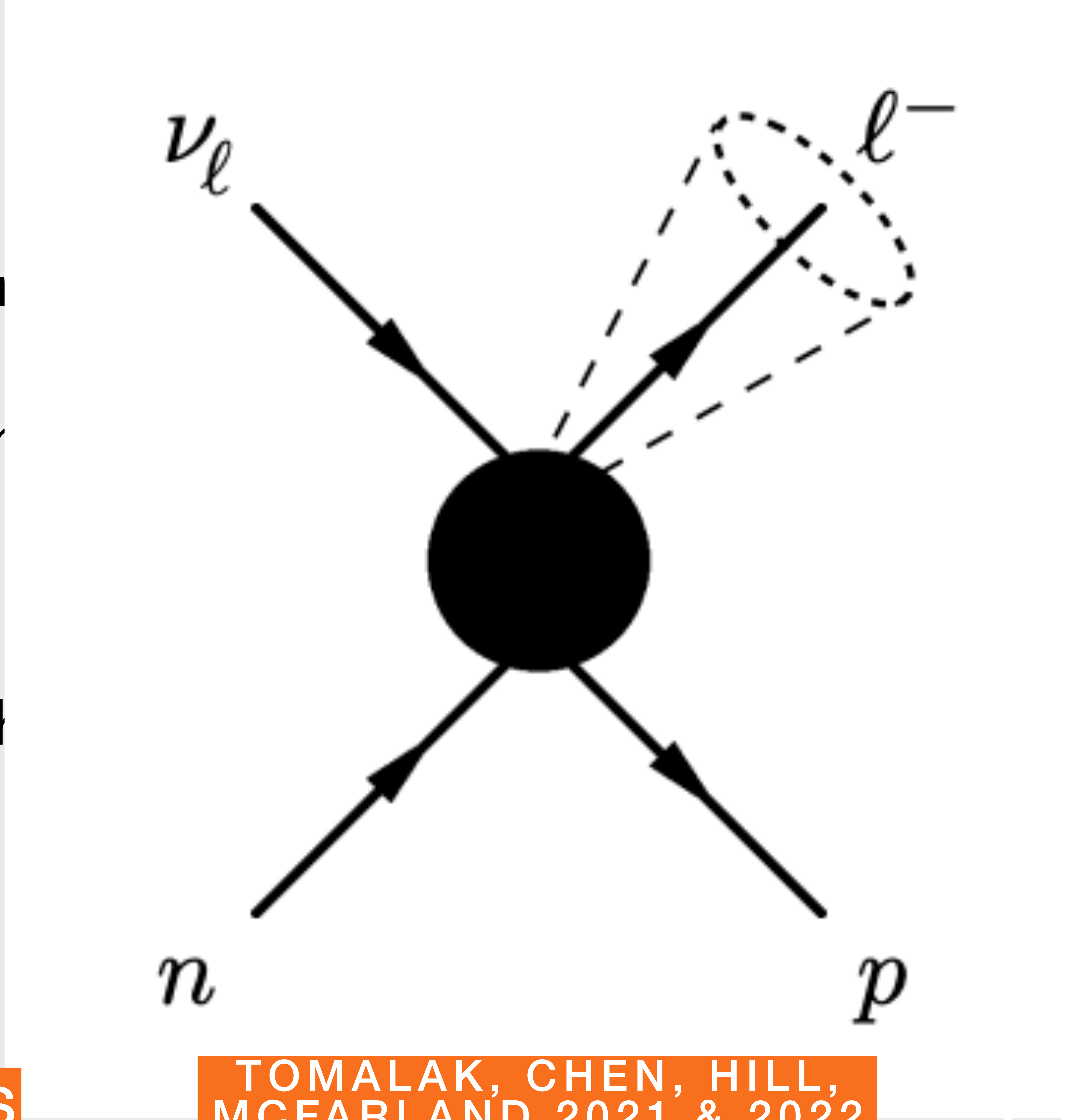
- Neutrino nucleon scattering.
- Soft-Hard-Collinear/Jet factorization.
- Tested explicitly at 1-loop

$$\mathscr{M} = \mathscr{M}_H \otimes S \otimes J$$

Nucleon

Lepton

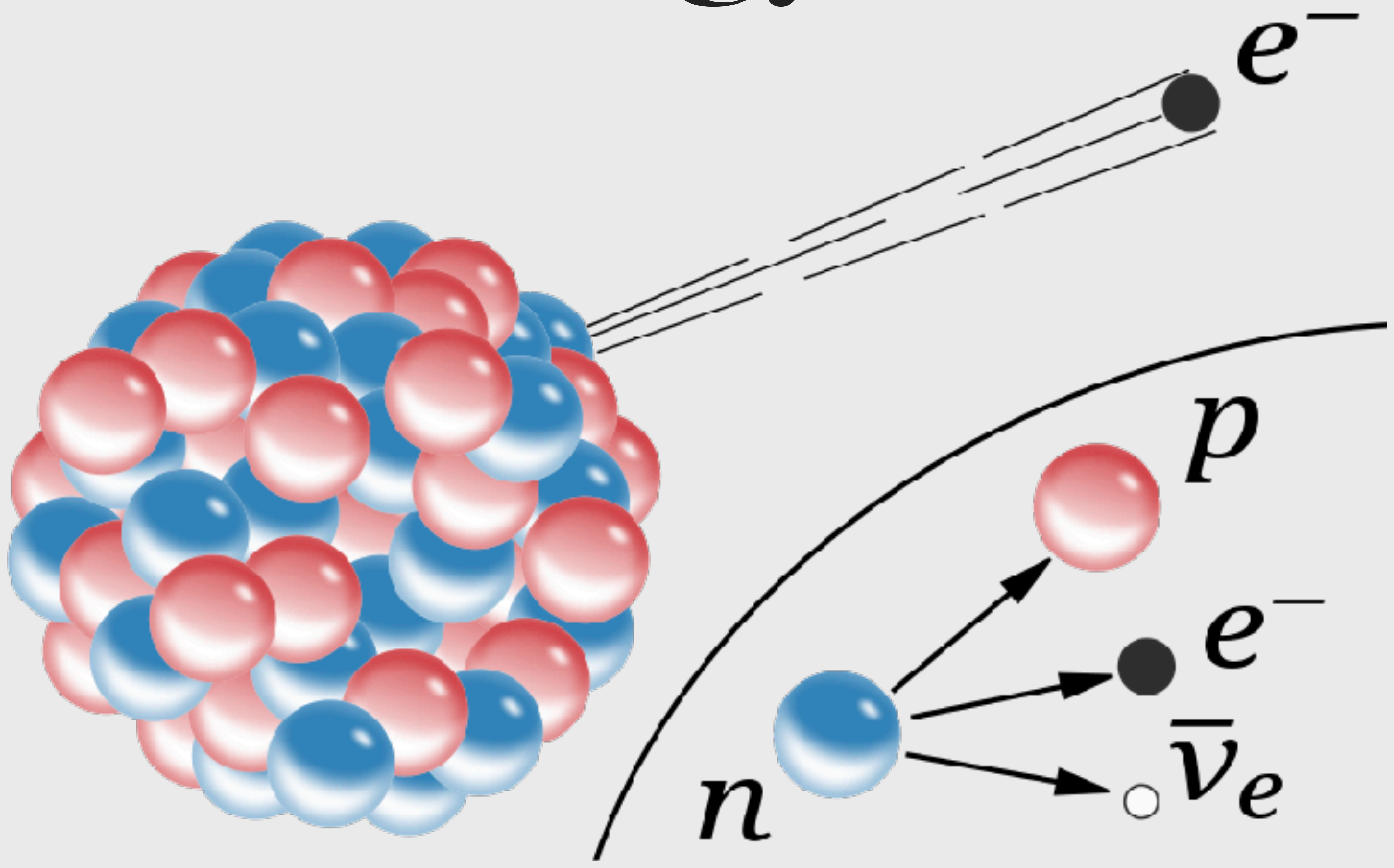
Soft ph



ANALOGOUS TO CSS IN COLLIDER PHYSICS

TOMALAK, CHEN, HILL, MCFARLAND 2021 & 2022

Low-Energy Neutrino Nucleus Factorization



Nuclear Scales	200 MeV
Nuclear radius	50 MeV
Lepton Energy	5 MeV
Electron mass	0.5 MeV
Atomic scales Soft photons	1 keV

- Beta decay.
- 30 MeV neutrino scattering.

Low-Energy Neutrino Nucleus Factorization

Nuclear Scales	200 MeV
Nuclear radius	50 MeV
Lepton Energy	5 MeV
Electron mass	0.5 MeV
Atomic scales Soft photons	1 keV

$$\mathscr{M} = \mathscr{M}_H \otimes S \otimes \mathscr{M}_{\mathrm{UV}}$$

LEPTON MOMENTUM ($\mathscr{M}_H$)

ATOMIC SCREENING (S)

NUCLEAR PHYSICS ($\mathscr{M}_{\mathrm{UV}}$)

First QED Factorization For Nuclei

SEE NTN TALK FROM LAST SUMMER

Low-Energy Neutrino Nucleus Factorization

Lepton Energy

Electron mass

$\}\mathrm{EFT}_{\mathrm{V}}$

INFINITELY HEAVY POINT-LIKE NUCLEI

RELATIVISTIC LEPTON

$$\mathscr{M}_H = \sum_{m,n} Z^m \alpha^n \mathscr{M}_H^{(m,n)}$$

SOME ADVERTISING FOR WORK TO APPEAR

- Full re-summation of m=n terms. Proof of factorization at all orders in $Z\alpha$.
- Rigorous QED corrections for e.g. beta decay.
- Anomalous dimension up to $O(Z^2\alpha^4)$ [in progress]

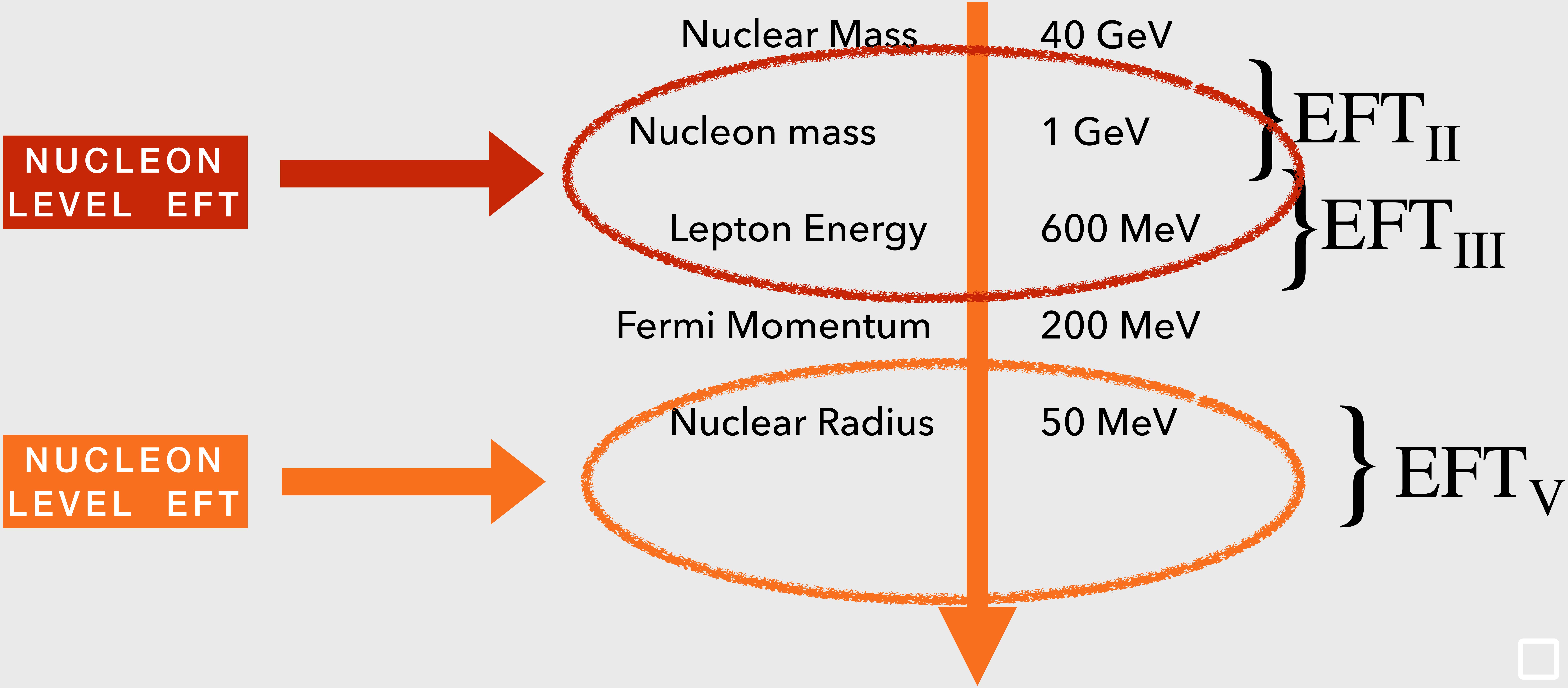
What We *Really* Want

FOR DUNE / T2K

- Lepton momentum in ~GeV regime.
- Moderate nuclear charge $Z \sim 10$.
- Observables are EM jets.

Nuclear Mass	40 GeV	$\mathrm{EFT_I}$
Nucleon mass	1 GeV	$\mathrm{EFT_{II}}$
Lepton Energy	600 MeV	$\mathrm{EFT_{III}}$
Fermi Momentum	200 MeV	$\mathrm{EFT_{IV}}$
Nuclear Radius	50 MeV	$\mathrm{EFT_V}$

What We Have Now



What We Have Now

Main Idea

- We understand how factorization works in two limiting cases.
- More work is required to ``wed'' the theories together.
- It is clear *a priori* that certain regions will overlap with scales relevant for nuclear matrix elements [Difficult problem!]

Conclusions & Outlook

Summary

- Generalization of old Coulomb physics results to charged currents and composite objects.
- Factorization theorems now exist for free nucleons for GeV energies, and point-like nuclei for MeV energies.

Take Home Message

- Precision neutrino physics means new challenges (and "new" physics).
- Radiative corrections ``entangle'' a hierarchy of scales.
- Effective field theory can help simplify analysis when scales separate.

Send Us Your Students!

- Deadline March 31st
- Please send your students!

14th

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07–19 August 2023

The International Neutrino Summer School provides training for the next generation of neutrino physicists in both theory and experiment. The school brings together graduate students and postdocs, along with some of the best teachers and researchers in neutrino physics. It provides an intense, two-week-long learning experience in an open and interactive environment that covers the full range of modern neutrino physics.

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