

# Decoherence in Neutrino Oscillations

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@ CERN neutrino platform week 2023

Based on: 2204.10696

with Manfred Lindner and Werner Rodejohann



**IMPRS**  
for Precision Tests of Fundamental Symmetries  
INTERNATIONAL MAX PLANCK RESEARCH SCHOOL



# Outline

## Introduction

(Neutrino) decoherence

## Theoretic structure

The layer structure: QFT+open quantum system, micro. to macro.

## Phenomenology

Sensitivity, New methods

# Outline

## Introduction

(Neutrino) decoherence — concept & formalisms on the market

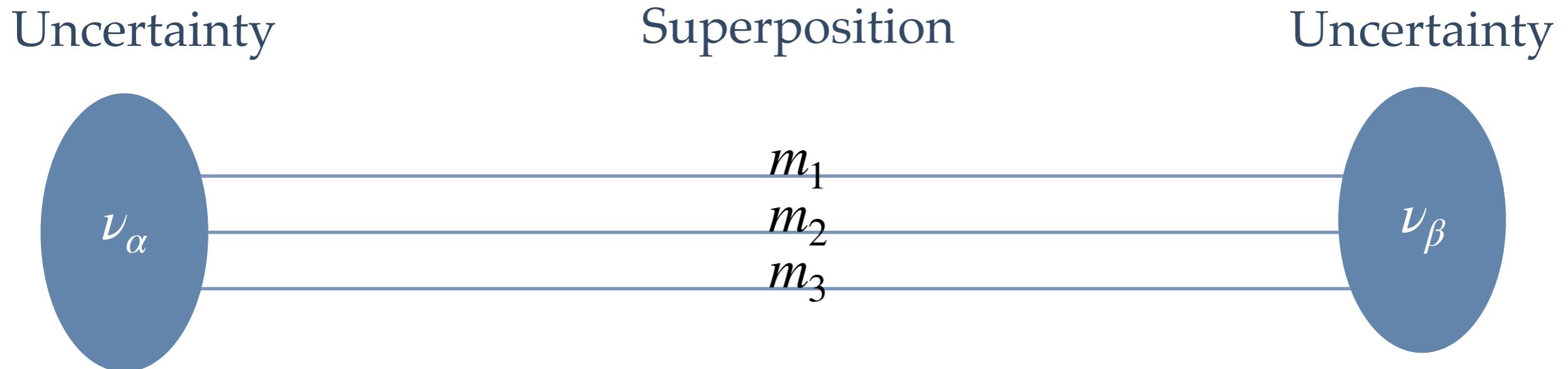
## Theoretic structure

The layer structure: QFT+open quantum system, micro. to macro.

## Phenomenology

Sensitivity, New methods

# Neutrino Oscillation — Coherence



B. Kayser (1981), J. Rich (1993), E. Akhmedov, A. Smirnov (2009)

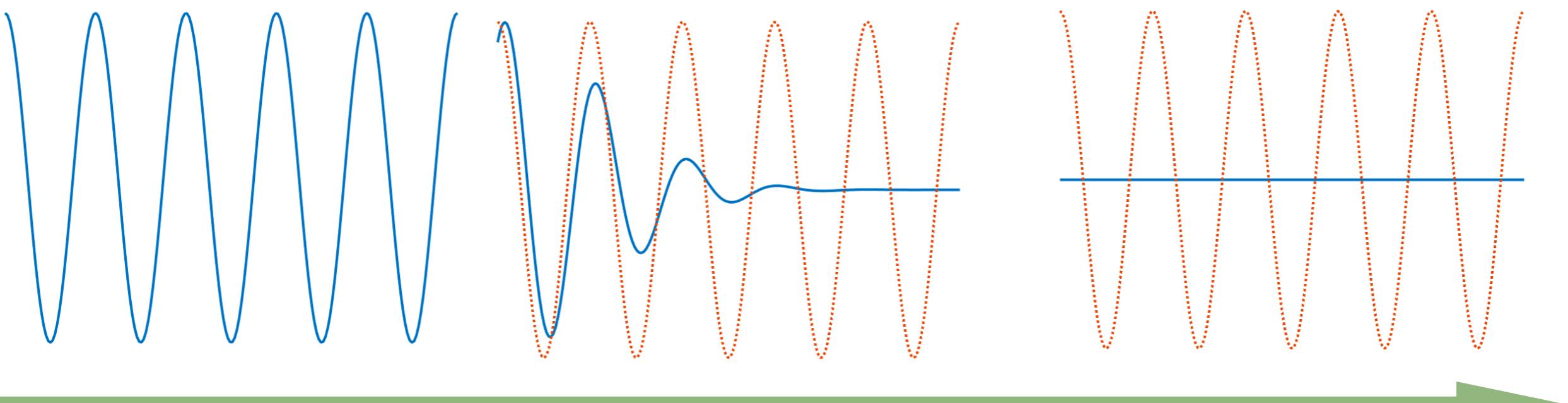
The uncertainties: give and take away coherence  
↑  
↓  
This talk

- ❖ Flavor transition probability:  $P_{\nu_\alpha \rightarrow \nu_\beta} = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* \exp\left(-i \frac{\Delta m_{jk}^2 L}{2E}\right)$
- ❖ Small mass splitting → Quantum effects at a macroscopic level
- ❖ Neutrinos interact weakly → Stable in a coherent state

# Decoherence

FTP:  $P_{3,\alpha \rightarrow \beta} = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* e^{-i \frac{\Delta m_{jk}^2 L_0}{2E_0}} \phi_{jk}(\vec{\sigma}; L_0, E_0) \quad 0 \leq |\phi_{jk}| \leq 1$

Coherent (Quantum)      Decoherent      Classical



Damping + Phase shift (if  $\phi_{jk}$  has an imaginary part)

- ❖ Open quantum system: lose information
- ❖ Wavepacket separation: lose mixing

# Decoherence - formalism of $\phi_{jk}$

Wavepackets:

$$\int [dp] e^{\frac{-(p-P)^2}{4\sigma^2}} e^{iE(p)t-ipx}$$

Directly for  $|\nu_\alpha\rangle$ : C. Giunti, C. W. Kim (1998), ...  
 for external particles: M. Beuthe (2001), ...  
 comparison (QM vs QFT): E. Akhmedov, J. Kopp (2010)

$$\phi_{jk}(L, E) \propto \exp \left[ - \left( \frac{\Delta m_{jk}^2 L \sigma}{2\sqrt{2} E^2} \right)^2 - \left( \frac{\Delta m_{jk}^2}{2\sqrt{2} E \sigma} \right)^2 \right]$$

Open quantum system:

$$\rho_S = \text{Tr}_E [\rho(t)]$$

Lindblad Equation:

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] - \mathcal{D}[\rho]$$

With some assumptions  
and conditions

*9 × 9 entries*

$$\phi_{jk}(L, E) \propto \exp \left[ - \left( \frac{\alpha m_{jk}^2 L}{E^n} \right)^2 \right]$$

$n = 0, \pm 1, \pm 2$  : quantum gravity,  
 e.g. matter effect fluctuation, neutrino absorption, ... [JUNO (2022) for a review]

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Inputs: Diagrams, distributions



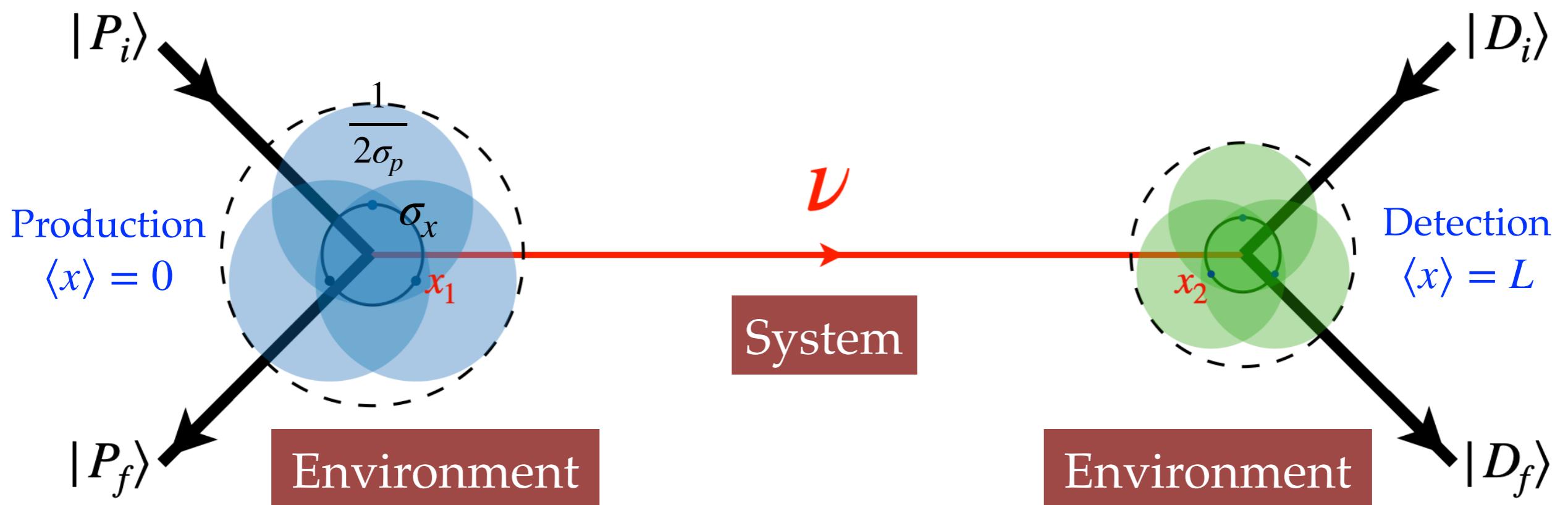
Output: Distribution of  $\phi_{jk}$  on the observational phase space (L, E here)

## Phenomenology

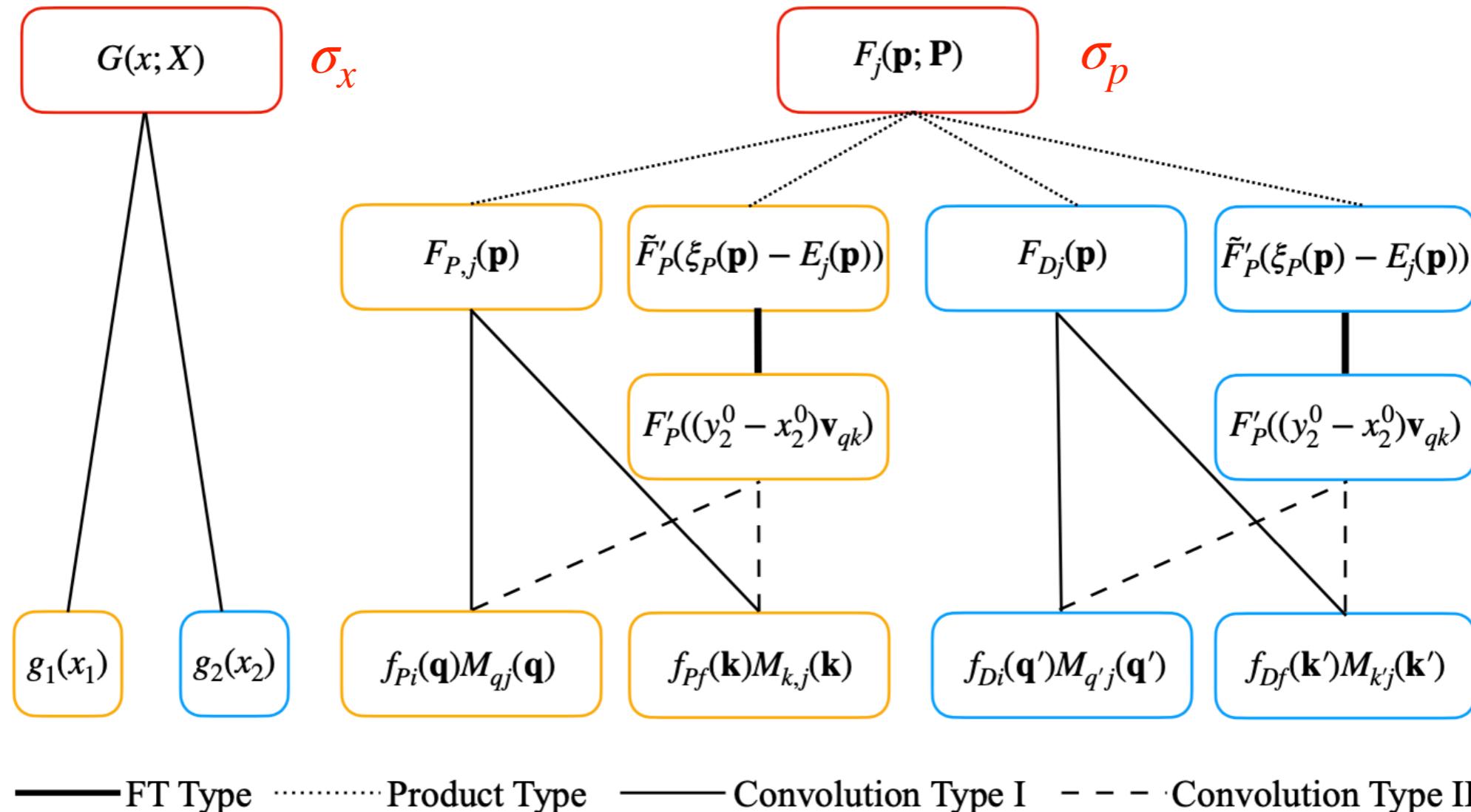
# QFT + Open Quantum System

- ⌘ Intrinsic uncertainties: unavoidable, can only be squeezed
- ⌘ Quantum superposition: sum over amplitude
- ⌘  $\sigma_x$ : (off-shell) neutrino interactions (related to the form factor)
- ⌘  $\sigma_p$ : (on-shell) external particle's WP size (mean free path, life time, ...)

(Just a parameterisation not the total position/ momentum uncertainty !)



# Quantum Uncertainty Decomposition

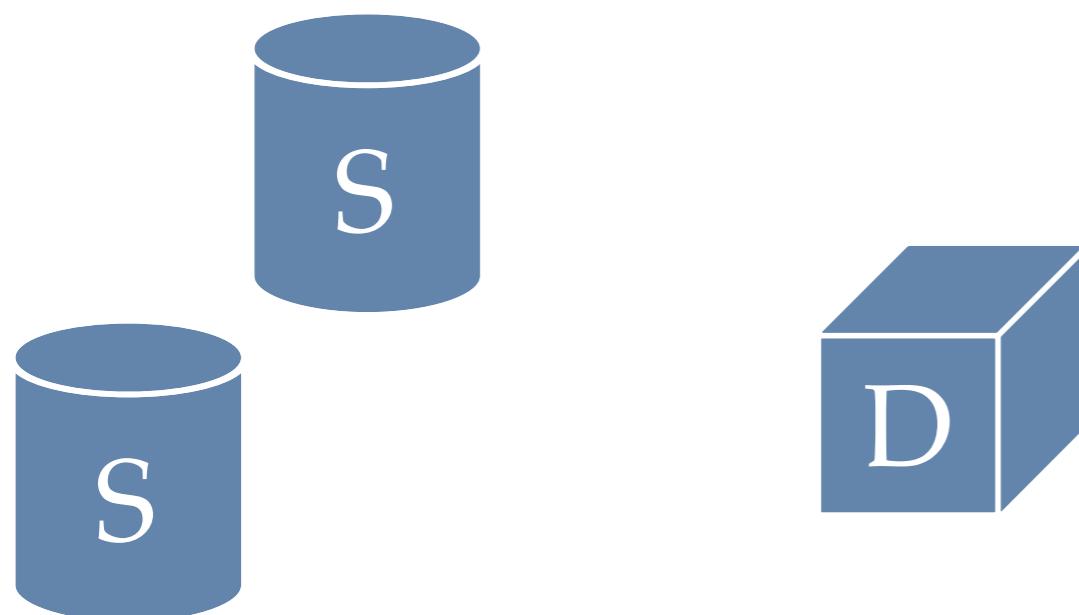
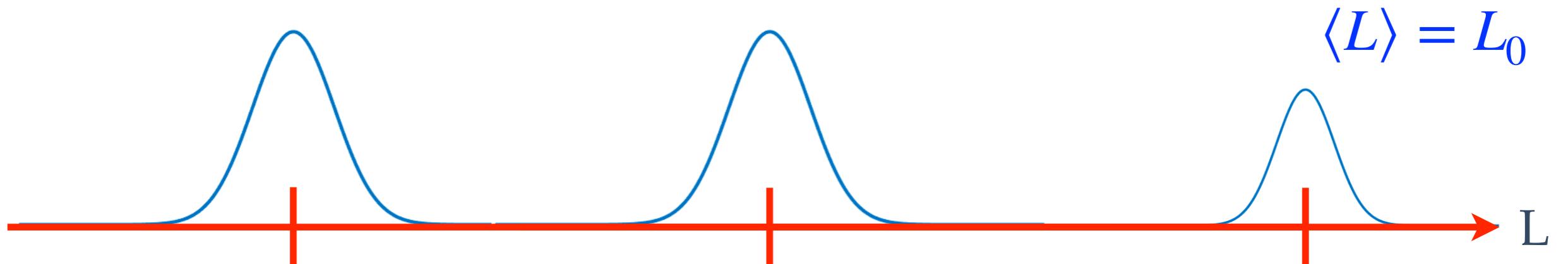


Type (notation for $\sigma_h$ )	Function relation	Width relation	Gaussian case
FT type ( $\tilde{\sigma}_f$ )	$h = \mathcal{FT}(f)$	NC	$\sigma_h = \frac{1}{2\sigma_f}$
Product type ( $\sigma_{fg}$ )	$h = f \times g$	PC, $\sigma_h < \{\sigma_f, \sigma_g\}$	$\frac{1}{\sigma_h^2} = \frac{1}{\sigma_f^2} + \frac{1}{\sigma_g^2}$
Convolution type I ( $\sigma_{f*g}$ )	$h = f * g$	PC, $\sigma_h > \{\sigma_f, \sigma_g\}$	$\sigma_h^2 = \sigma_f^2 + \sigma_g^2$

Function (width notation), Type	Width relation	Gaussian case
$H(y)$ ( $\sigma_H$ ), Convolution type I	PC, $\sigma_H > \{\sigma_f, \sigma_g\}$	$\sigma_H = \sigma_{f*g}$
$I(p)$ ( $\sigma_I$ ), Convolution type II	NC	$\sigma_I = 1/\sigma_{fg}$

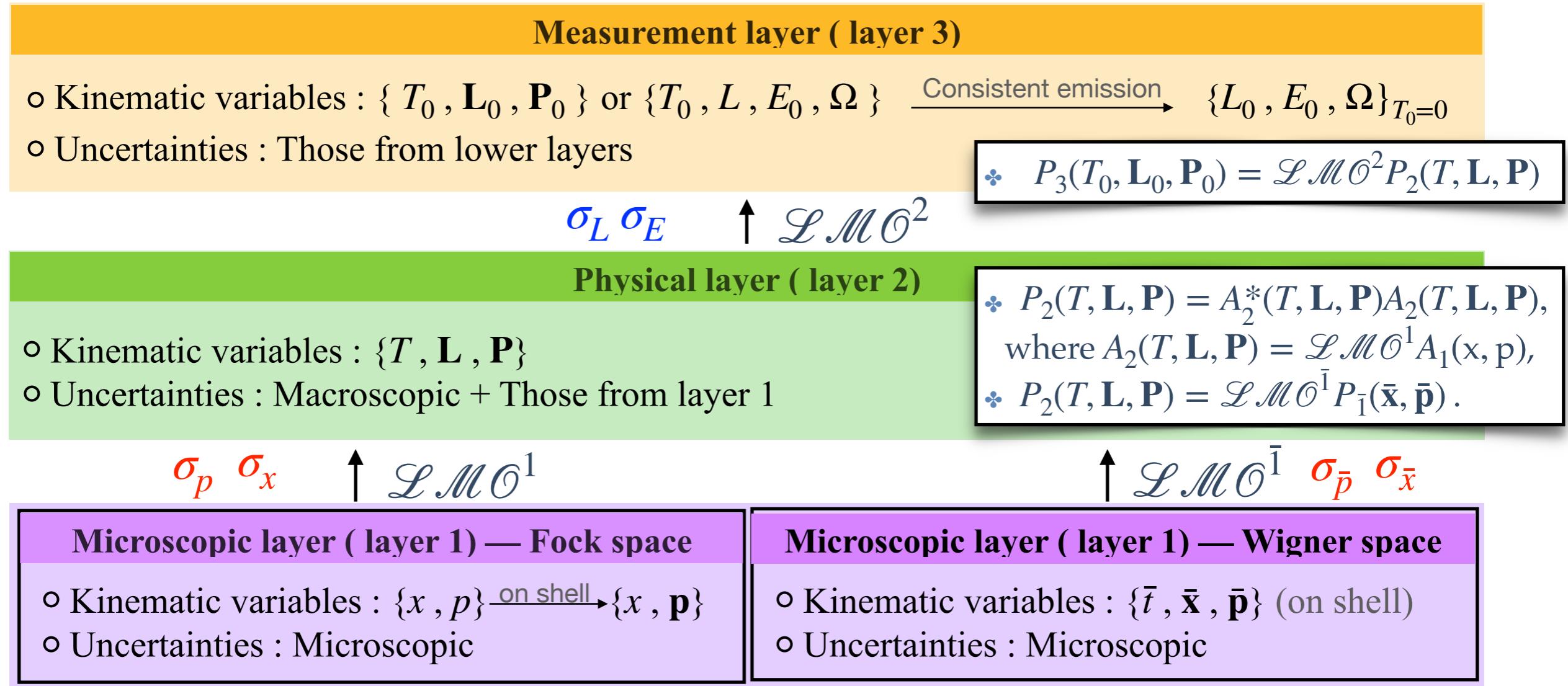
# Classical Uncertainties

- ❖ Statistical averaging — sum over the probability (amplitude<sup>2</sup>)
- ❖ Reflects our ignorance — avoidable
- ❖ What do we get in the end? Expectation value of the FTP



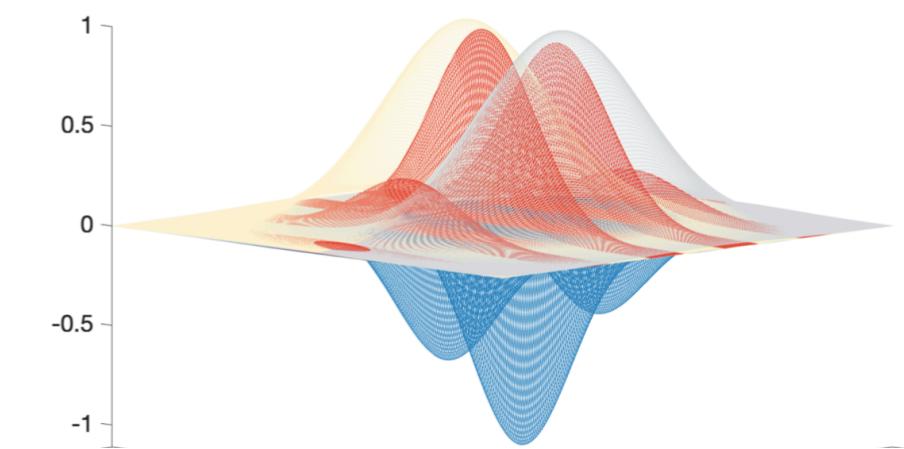
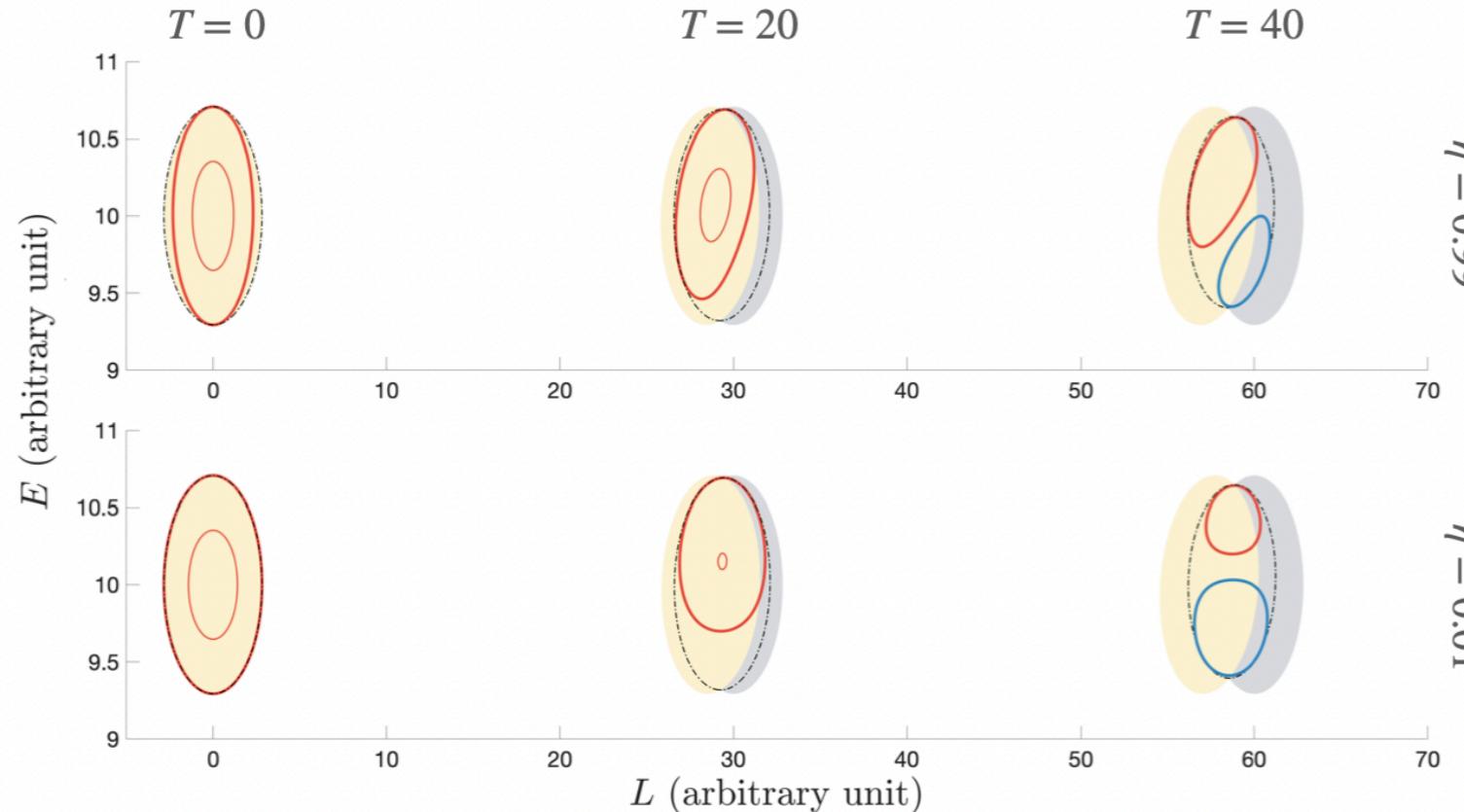
- ❖  $\sigma_L$  : production profile, space time (gravitation) fluctuation ...
- ❖  $\sigma_E$  : energy reconstruction model, energy resolution ...

# The (Phase Space) Layer Structure



*The layer moving operators ( $\mathcal{LMO}$ ) contain the uncertainty parameters*

# Decoherence on the Physical Layer



$$P_{3,jk} = \frac{\text{State decoherence term}}{\text{Phase decoherence term}} \times e^{i\psi_{jk}}$$

State decoherence term: Two overlapping circles, one blue and one red, with a dashed line connecting them.

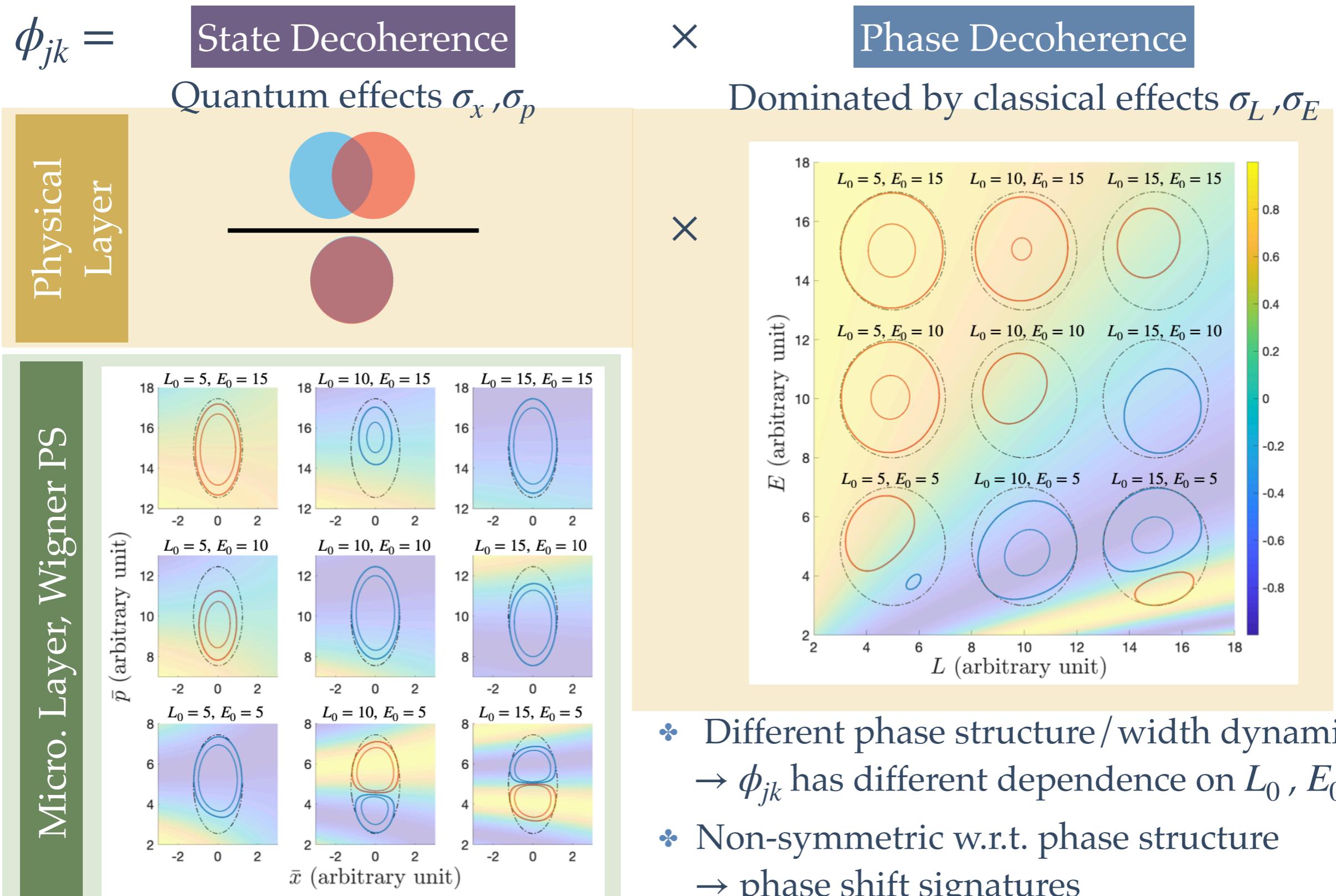
Phase decoherence term: Two overlapping circles, one blue and one red, with diagonal stripes of alternating blue and red colors.

**PWO effect:**

$$\frac{\int dx \Gamma(x; L)}{\int dx |\Gamma(x; L)|} = e^{i(\eta(x)|_{x=L} - \beta)} \Phi(L)$$

for  $\Gamma(x; L) \equiv |\Gamma(x; L)| e^{i\eta(x)}$

# Classification of Decoherence



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## Phenomenology

Sensitivity: What to expect? Where to look? How far are we?

New method: Direct measurement of the oscillation phase (shift)

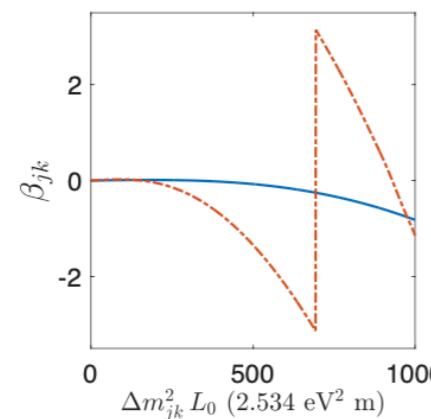
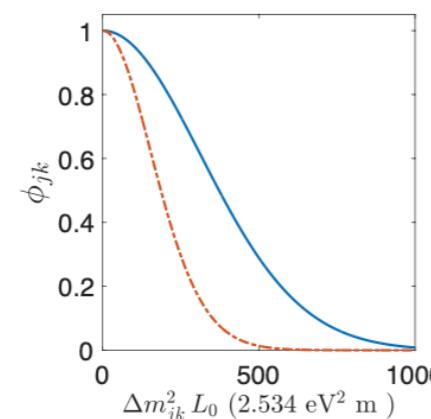
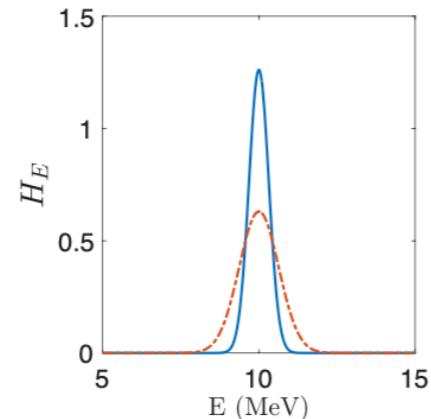
# Decoherence Signatures — Damping & Phase Shift

Weighting distribution:  
(Uncertainty)

Damping term:

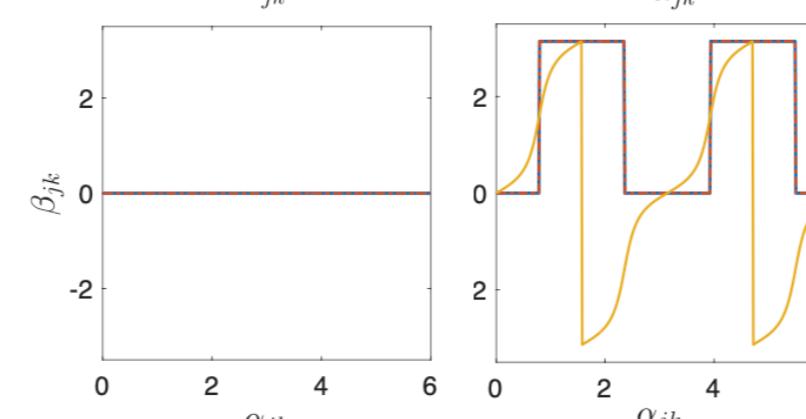
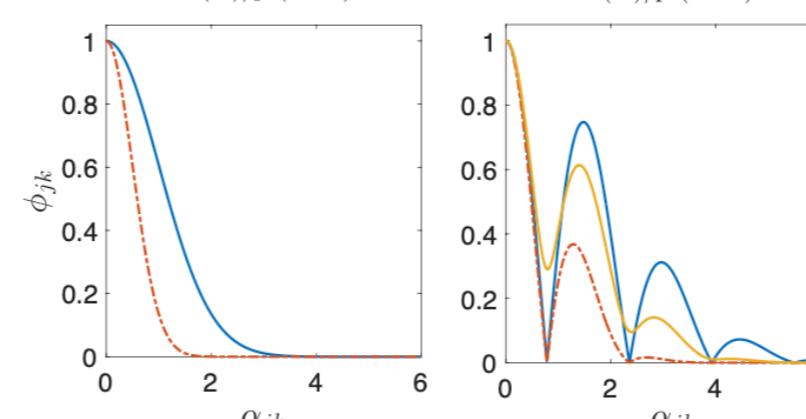
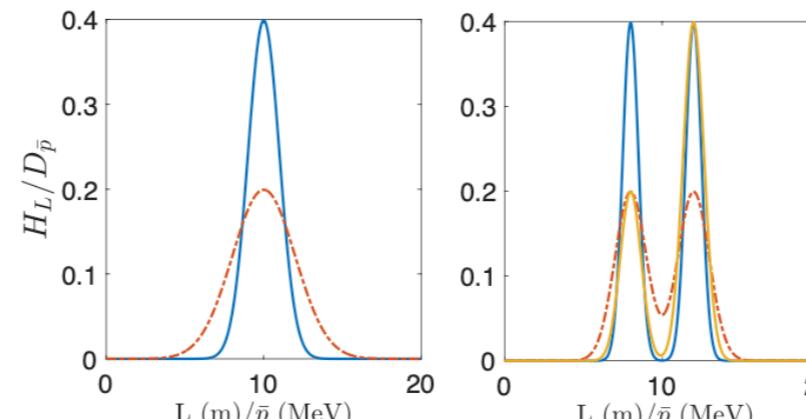
Phase shift term:

Classical E



Energy resolution:  
 $\sigma_E = \sigma_E^o \sqrt{E_0}$

Classical L / quantum ( $x, p \rightarrow \bar{p}$ )



$$\sigma_{\bar{p}} = \frac{\sigma_p}{1 + 4\sigma_x^2 \sigma_p^2}$$

$$\text{For } H_L: \alpha_{jk} = \frac{\Delta m_{jk}^2}{2E_0}$$

$$\text{For } D_{\bar{p}}: \alpha_{jk} = \frac{\Delta m_{jk}^2 L_0}{2E_0^2}$$

# Decoherence Signatures — Damping & Phase Shift

Weighting distribution:  
(Uncertainty)

Damping term:

Phase shift term:

$$\text{Energy resolution: } \sigma_E = \sigma_E^o \sqrt{E_0}$$

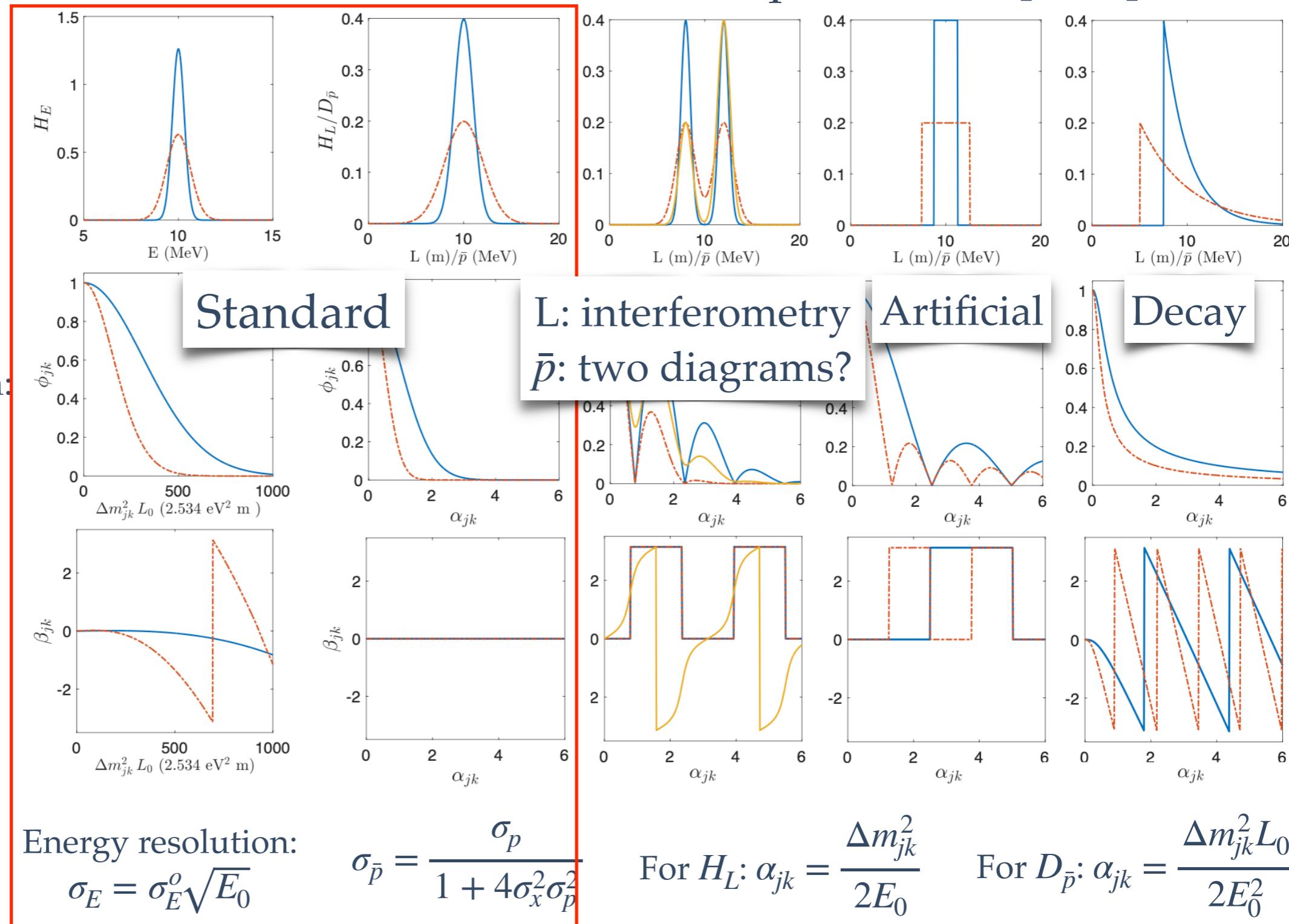
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Classical E

Classical L / quantum ( $x, p \rightarrow \bar{p}$ )



# Sensitivity for Reactor Neutrinos

Lower energy → more PWO effect

Where to look? & How far are we?

Contour: relative (to RENO) statistic enhancement required for the sensitivity

Current reactor bound (RENO+DYB+KL):  $\sigma_{\bar{p}} > 0.47 \text{ MeV}$

[A. de Gouvea, V. Romeri, C. Ternes (2021)]

JUNO sensitivity:  $\sigma_{\bar{p}} > 3 \times 10^{-3} \text{ MeV}$

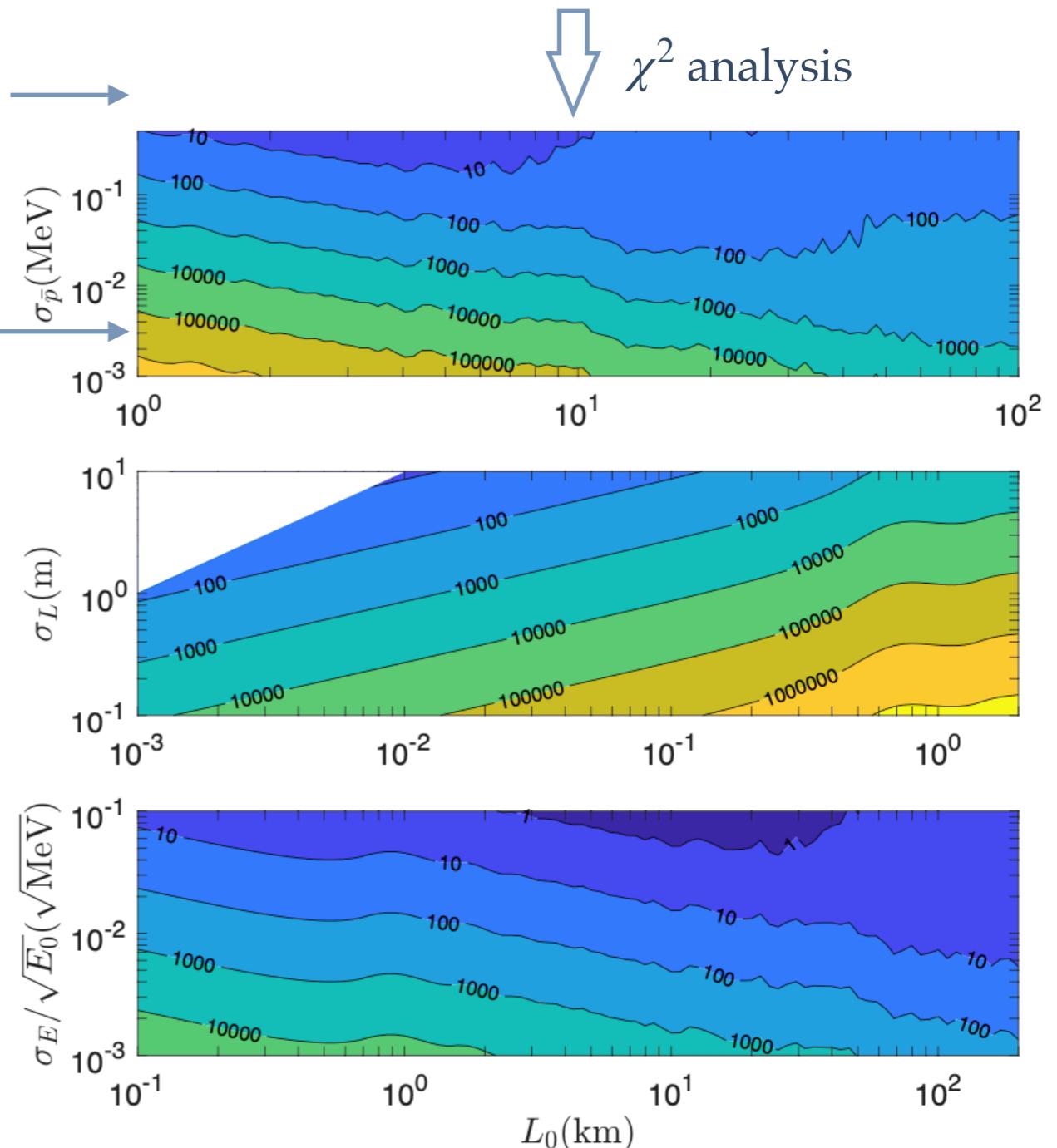
[JUNO collaboration (2022)]

- The usually neglected (quantum) localisation term, contributes manly from classical uncertainties

- $\phi_{jk}$  does not depend on  $L_0$

→ can go to near source for hight statistics

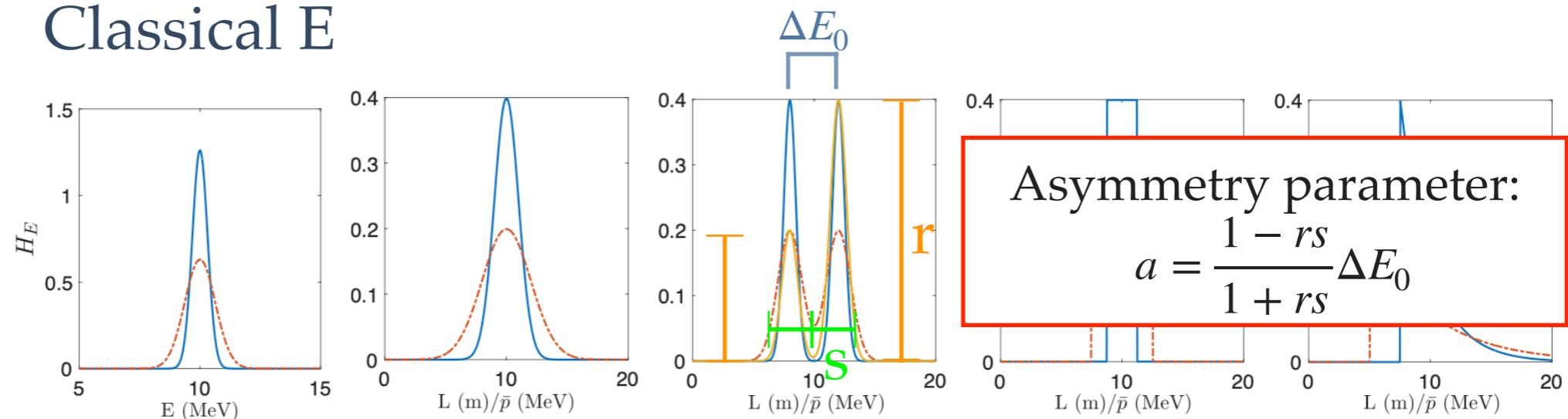
- Smearing from the energy resolution  
→ Difficult to discriminate from  $\sigma_{\bar{p}}$



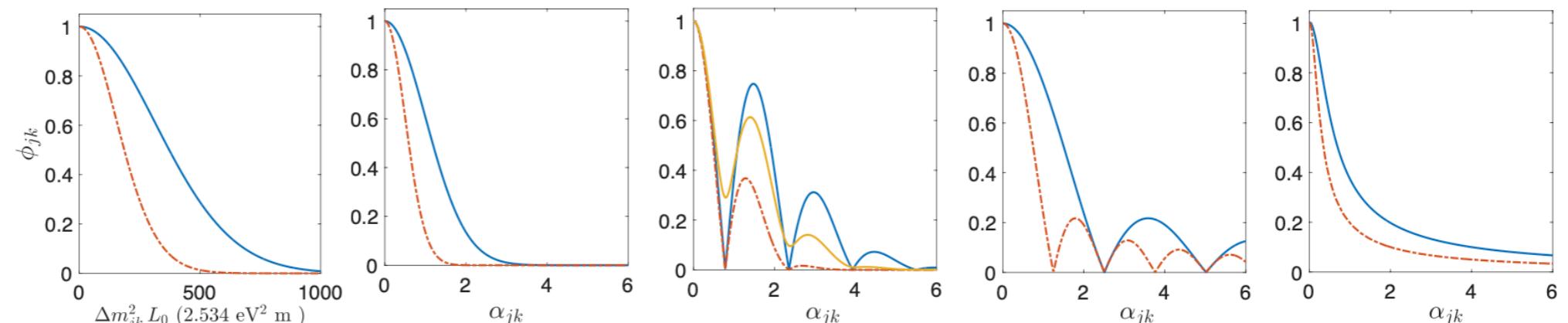
# Decoherence Signatures — Damping & Phase Shift

## Classical E

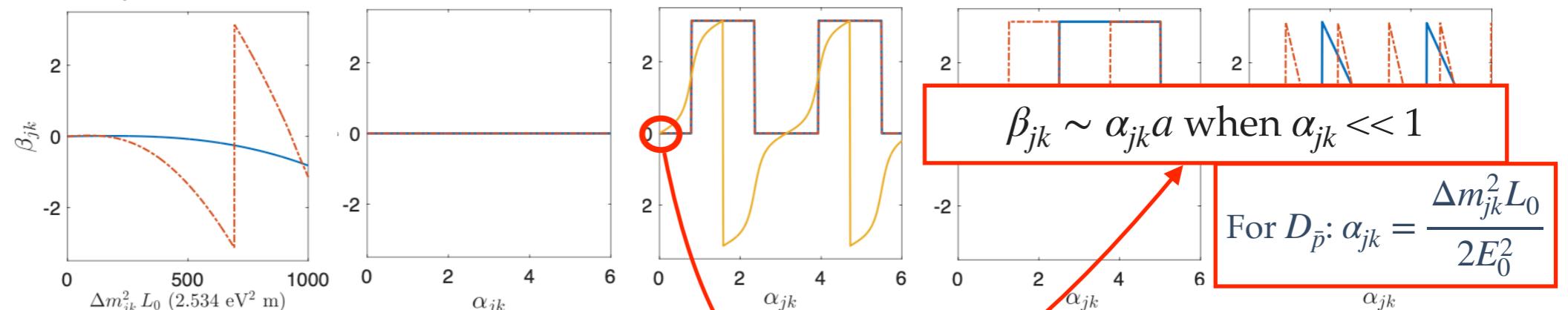
Weighting distribu  
(Uncertainty)



Damping term



Phase shift ter



$$\sigma_{\bar{p}} = \frac{\sigma_p}{1 + 4\sigma_x^2 \sigma_p^2}$$

CERN, 14/03/2023

For  $H_L$ :  $\alpha_{jk} = \frac{\Delta m_{jk}^2}{2E_0}$

# New Method: Direct Phase Measurement

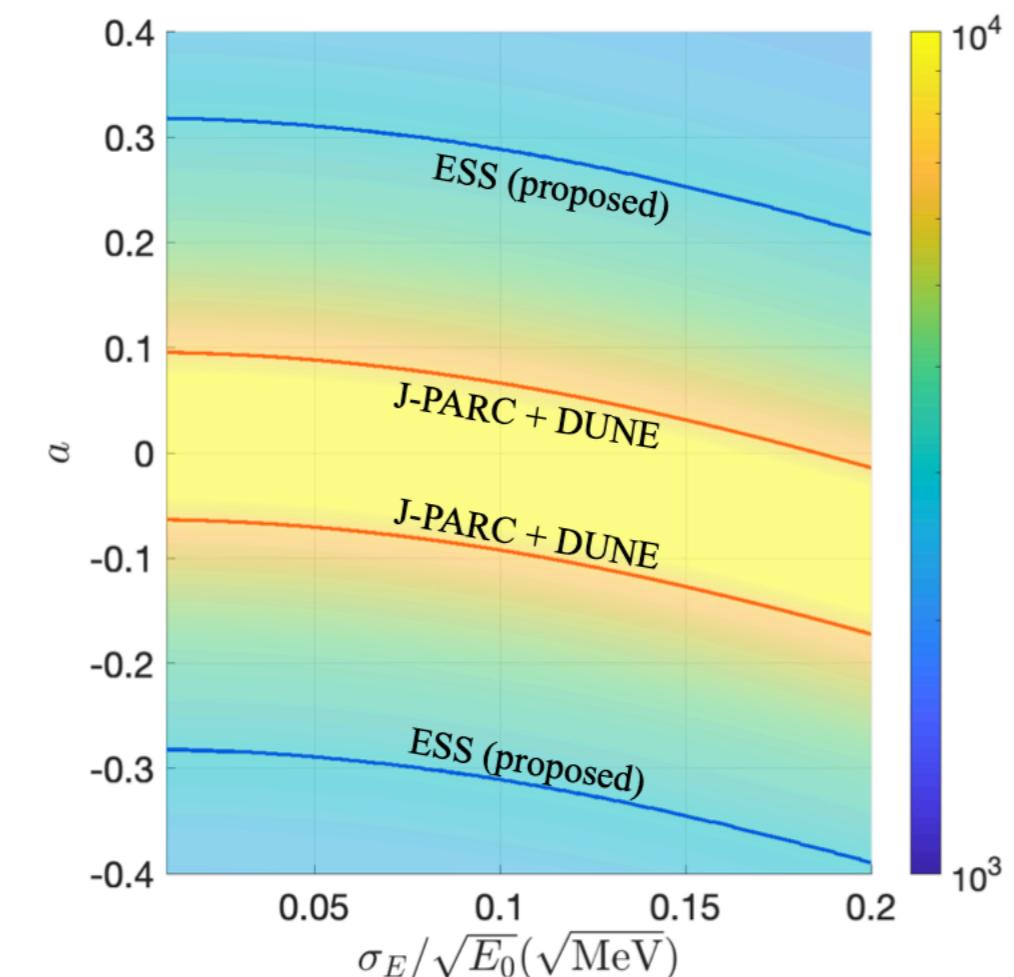
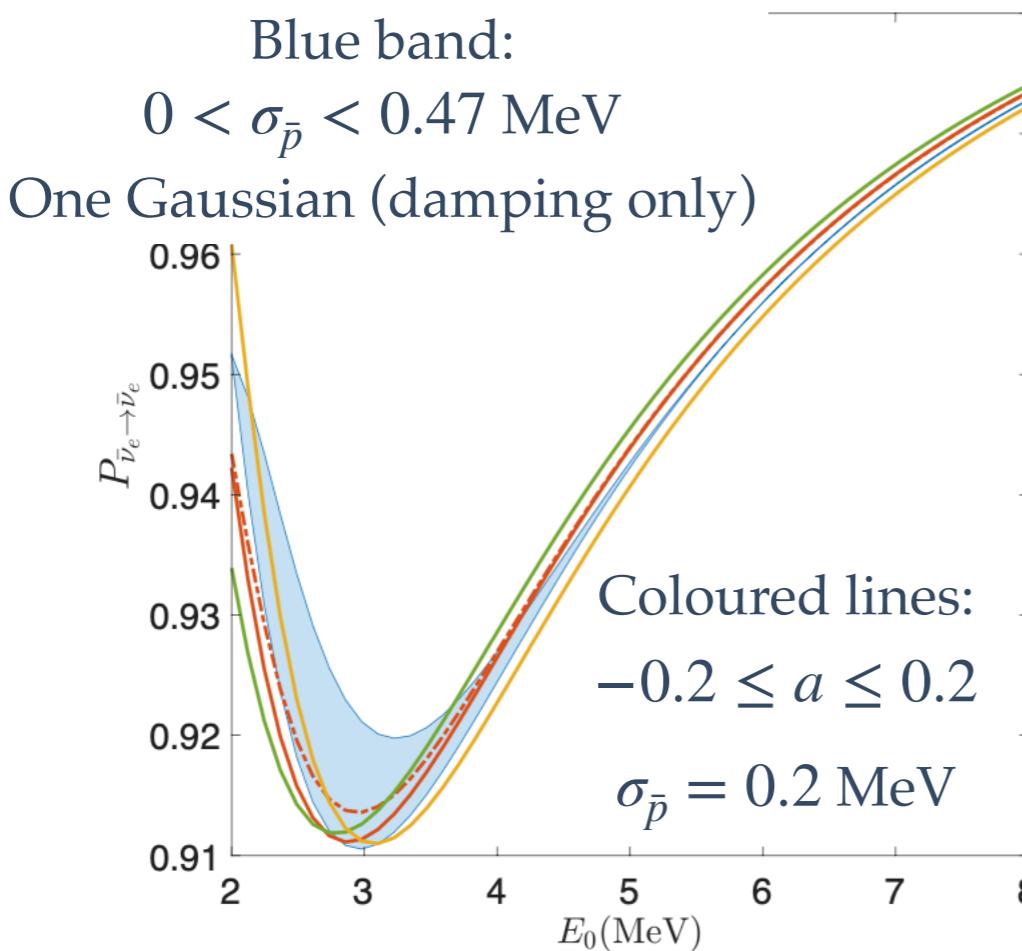
Look for an oscillation min. (e.g. by moving the detector)

$$\psi_{jk}(L_{\text{osc}}^{jk}, E_0) = 2n\pi \rightarrow \psi_{jk}(L_{\min}^{jk}, E_0) + \beta_{jk}(L_{\min}^{jk}, E_0; \vec{\sigma}_n) = 2n\pi,$$

Traditional spectral analysis is not  
(as) sensitive to the phase shift

monochromatic DAR: appearance  $\nu_\mu \rightarrow \nu_e$

Find  $L_{\min}$  with (a moving) 50 m detector(s)



Contour: relative (to JSNS) statistic  
enhancement required for the sensitivity

# Summary

- ❖ Neutrino oscillation experiments are going into a **precision era** — able to see decoherence by damping and / or phase shift signatures
- ❖ Formulate **a structure** to parameterise neutrino decoherence by its quantum or classical origins → **theory input**
- ❖ Find that decoherence can be describe by a **phase averaging effects** (PWO effects) of the oscillation phase or the phase in the Wigner phase space → **allows simple numerical calculation**
- ❖ Observe decoherence by measuring the **oscillation phase, spectrum and traveling distance** → **experimental & analysis design**

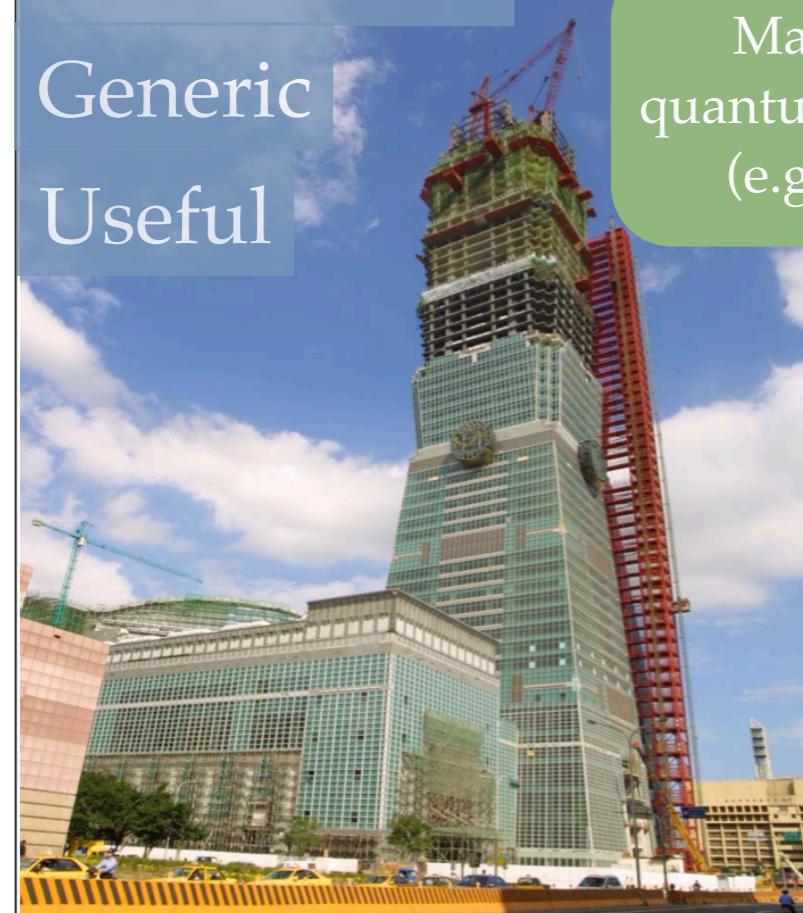
# Outlook

# Neutrino oscillation going to the precision era

## The dream



Nov. 2002



Dec. 2004

# Backup Sildes

# The Microscopic Layer

- ❖ Representation of a layer: how particle states are described

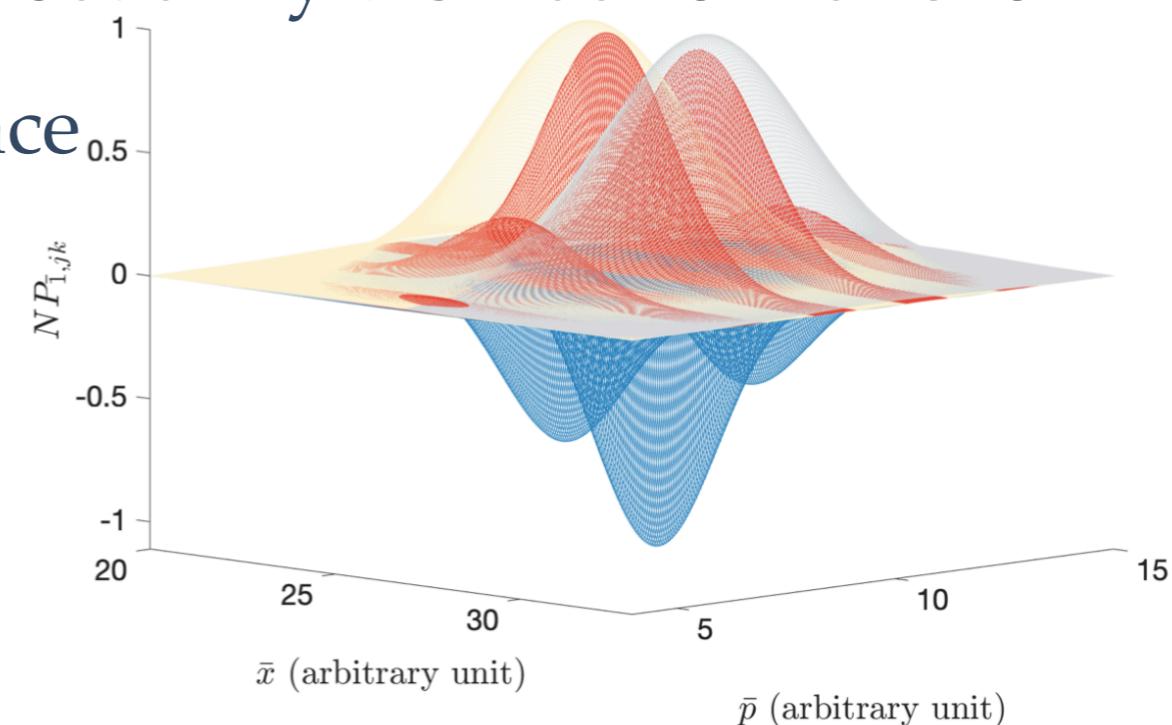
Layer 1: QFT Transition Amplitude,  $A_1(x, p)$

- ❖ Fock phase space — Fock states (second quantisation)

Layer  $\bar{1}$ : Wigner Transition Probability,  $P_{\bar{1}}(\bar{x}, \bar{p})$

- ❖ Wigner phase space — Wigner quasi probability distribution function
- ❖ A bridge of QM to statistical phase space

$$\begin{aligned} P_2(T, L, P) &= \mathcal{LMO}^{\bar{1}} P_{\bar{1}}(\bar{x}, \bar{p}) \quad \sigma_{\bar{p}} \quad \sigma_{\bar{x}} \\ &= \mathcal{LMO}^1 A_1^*(x', p') \mathcal{LMO}^1 A_1(x, p). \\ &\quad \sigma_p \quad \sigma_x \quad \sigma_{\bar{p}} \quad \sigma_{\bar{x}} \end{aligned}$$



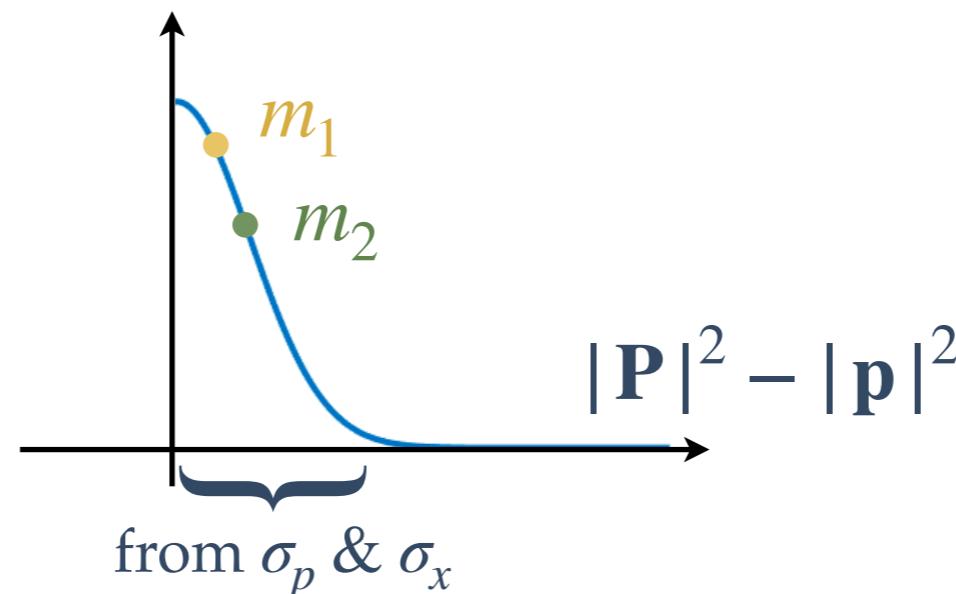
# The Physical Layer

- Relativistic representation — phase space for massless neutrinos
- Different neutrino masses are “allowed” by the uncertainties

$$|\mathbf{P}_j| \equiv E - \delta E_j$$

$$|\mathbf{P}| = E$$

Energy budget



$$\mathbf{P}_j = \mathbf{P} - \vec{\xi}_j \frac{m_j^2}{2E}$$

$$\mathbf{L}_j = \mathbf{L} - \vec{v}_j T$$

Relativistic rep.

- Additional classical uncertainty — In energy ( $\sigma_E$ ) and distance ( $\sigma_L$ )

FTP on its  $i$ th layer:  $P_{i,\nu_\alpha \rightarrow \nu_\beta} = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* P_{i,jk}$

$$X_3 = \{L_0, E_0\},$$

$$P_{3,jk}(X_3) = \int dX_2 P_{2,jk}(X_2) H(X_2 - X_3) \quad \Rightarrow \text{Convolution} \quad X_2 = \{L, E\}$$

$\sigma_p \& \sigma_x \quad \sigma_E \& \sigma_L$

# New Method: direct phase measurement

Look for an oscillation min. (e.g. by moving the detector)

$$\psi_{jk}(L_{\text{osc}}^{jk}, E_0) = 2n\pi \rightarrow \psi_{jk}(L_{\min}^{jk}, E_0) + \beta_{jk}(L_{\min}^{jk}, E_0; \vec{\sigma}_n) = 2n\pi,$$

Binned data: increase both stat. and signal

$$N_i(E_0; \vec{\sigma}_n) = N(E_0) \int_{L_i - \Delta L_{\text{bin}}/2}^{L_i + \Delta L_{\text{bin}}/2} dL_0 \frac{1}{4\pi L_0^2} P_{\nu_\alpha \rightarrow \nu_\beta}(L_0, E_0; \vec{\sigma}_n),$$

Discrete derivative (min @  $F = 0$ )

$$F_i(E_0; \vec{\sigma}_n) = \frac{1}{\bar{N}_i} \frac{N_{i+1}(E_0; \vec{\sigma}_n) - N_i(E_0; \vec{\sigma}_n)}{L_{i+1} - L_i},$$

