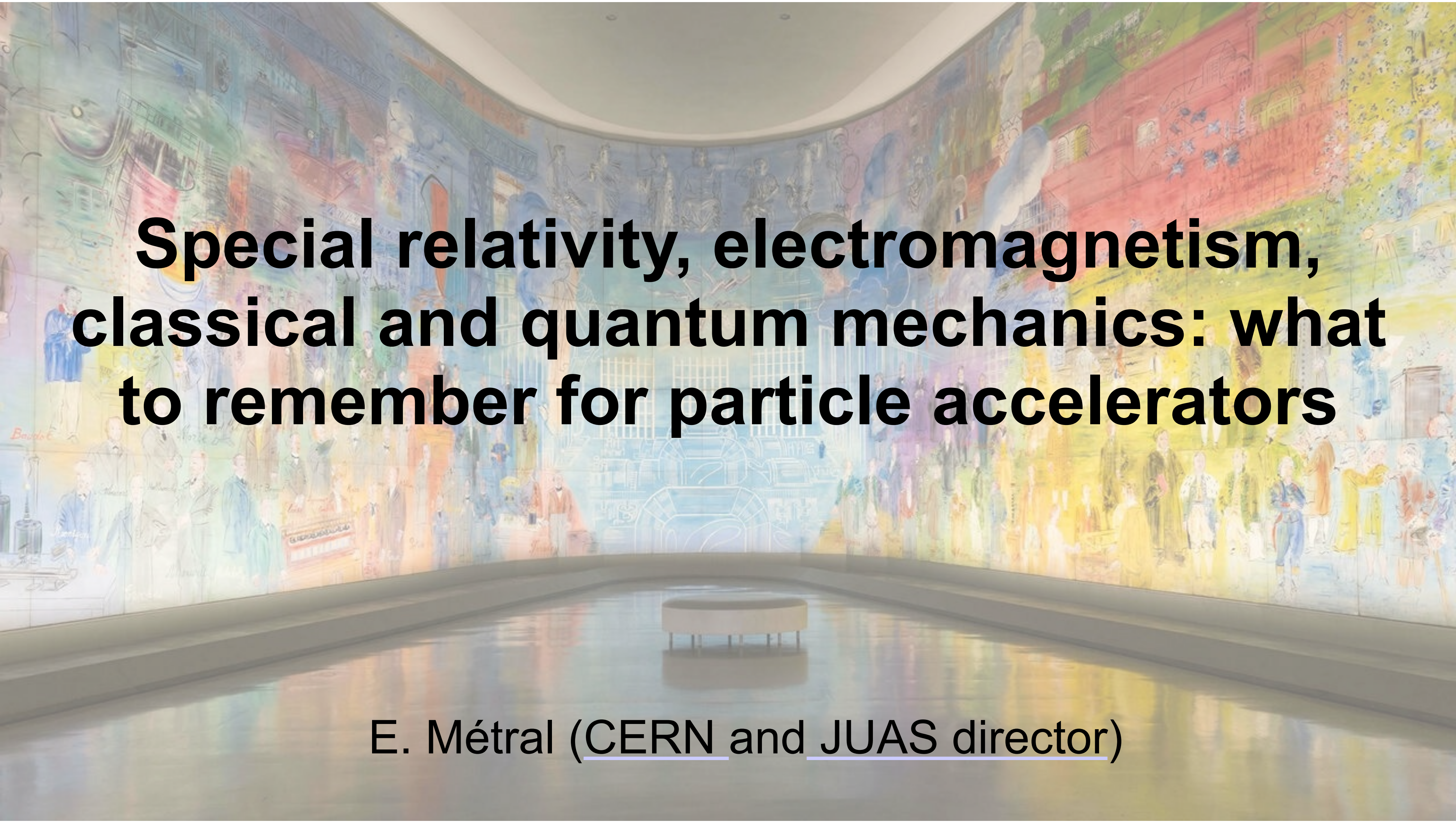
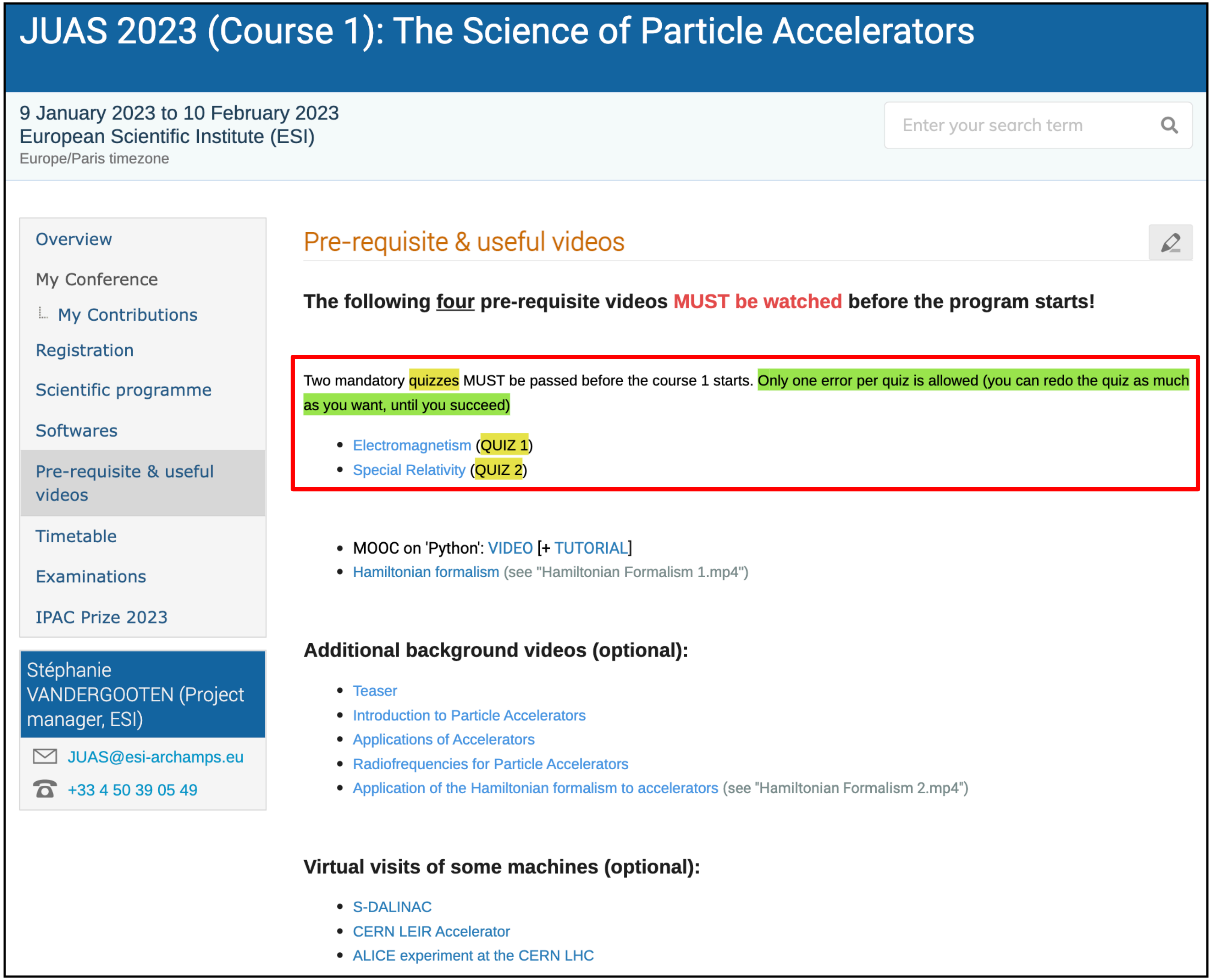


Special relativity, electromagnetism, classical and quantum mechanics: what to remember for particle accelerators

E. Métral (CERN and JUAS director)



JUAS 2023 (Course 1): The Science of Particle Accelerators

9 January 2023 to 10 February 2023
European Scientific Institute (ESI)
Europe/Paris timezone

Stéphanie VANDERGOOTEN (Project manager, ESI)
✉ JUAS@esi-archamps.eu
☎ +33 4 50 39 05 49

Pre-requisite & useful videos

The following four pre-requisite videos MUST be watched before the program starts!

Two mandatory quizzes MUST be passed before the course 1 starts. Only one error per quiz is allowed (you can redo the quiz as much as you want, until you succeed)

- Electromagnetism (QUIZ 1)
- Special Relativity (QUIZ 2)

- MOOC on 'Python': VIDEO [+ TUTORIAL]
- Hamiltonian formalism (see "Hamiltonian Formalism 1.mp4")

Additional background videos (optional):

- Teaser
- Introduction to Particle Accelerators
- Applications of Accelerators
- Radiofrequencies for Particle Accelerators
- Application of the Hamiltonian formalism to accelerators (see "Hamiltonian Formalism 2.mp4")

Virtual visits of some machines (optional):

- S-DALINAC
- CERN LEIR Accelerator
- ALICE experiment at the CERN LHC

What is the link between…?

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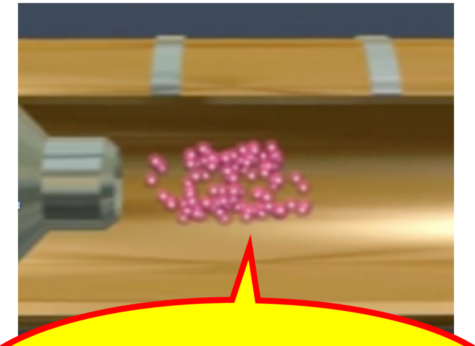

Group ("bunch") of particles (e.g.: p^{+})

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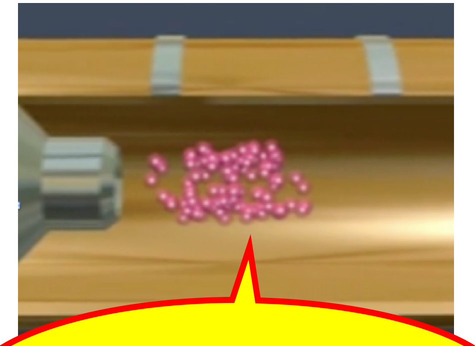

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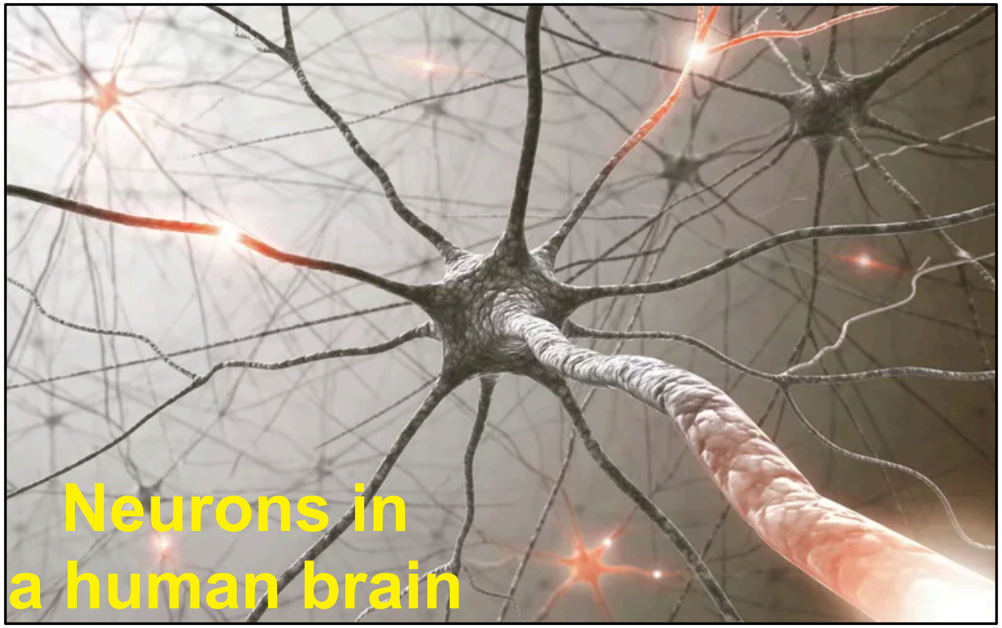

Neurons in a human brain

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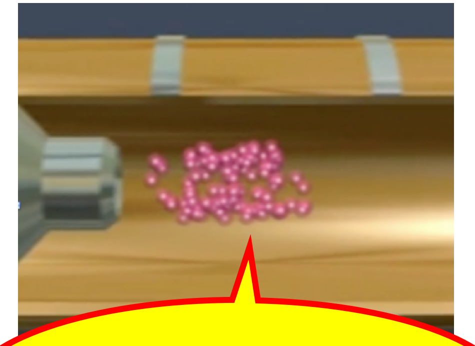

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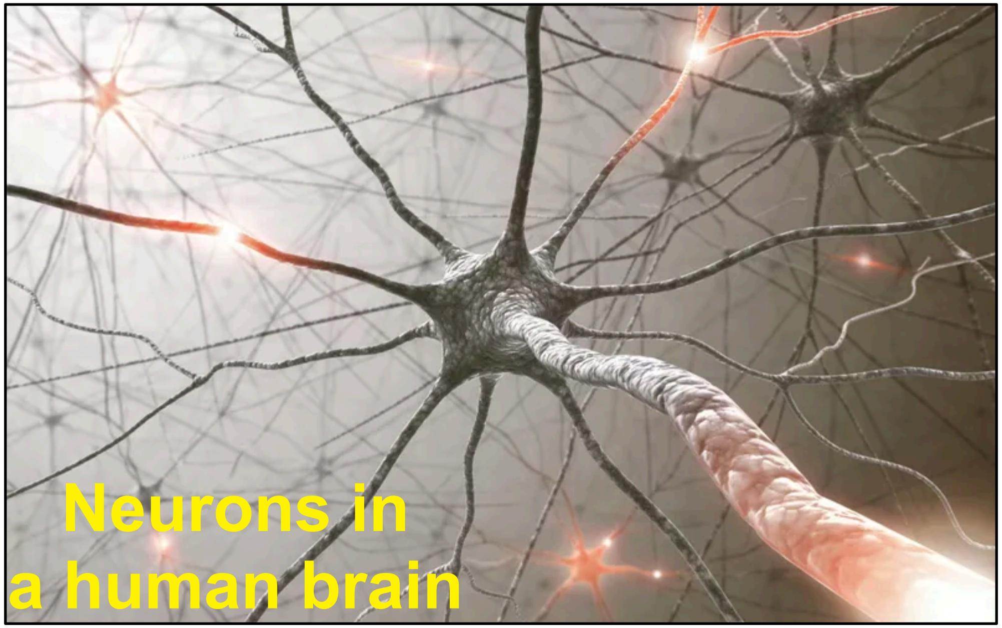

Neurons in a human brain

Stars in our galaxy (Milky Way)

=> The number!

=> The number!: the number of particles (protons) per bunch in the CERN LHC is similar to the number of neurons in a human brain or the number of stars in our galaxy (Milky Way).

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- 10^6
- 10^9
- 10^{11}
- 10^{13}
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- 1 (charged) particle?
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=> How can we keep under control

- ✳1 (charged) particle?
- ✳ ~ 10^{11} particles grouped together in 1 bunch?
- ✳~ 5500 bunches?

We need a force…

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- Do you know the number of the currently known fundamental forces in the universe?

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  - 1
  - 2
  - 4
  - 5
  - 10
  - Infinity

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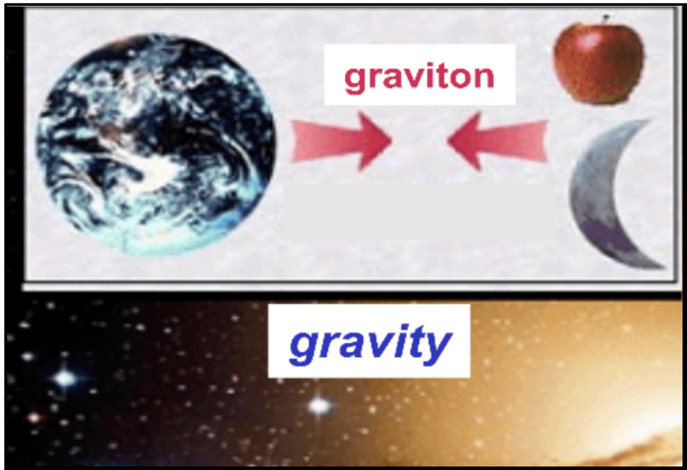

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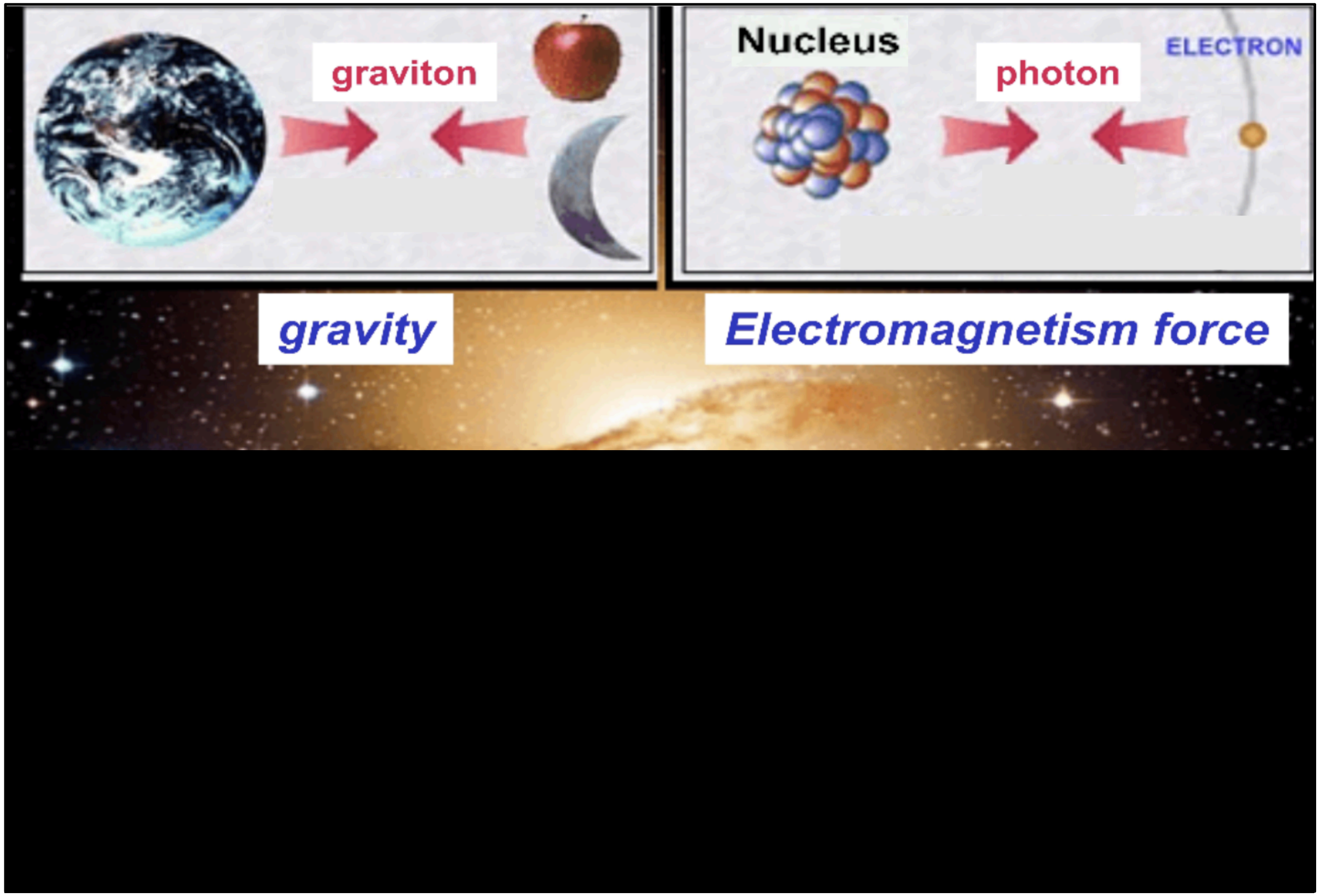

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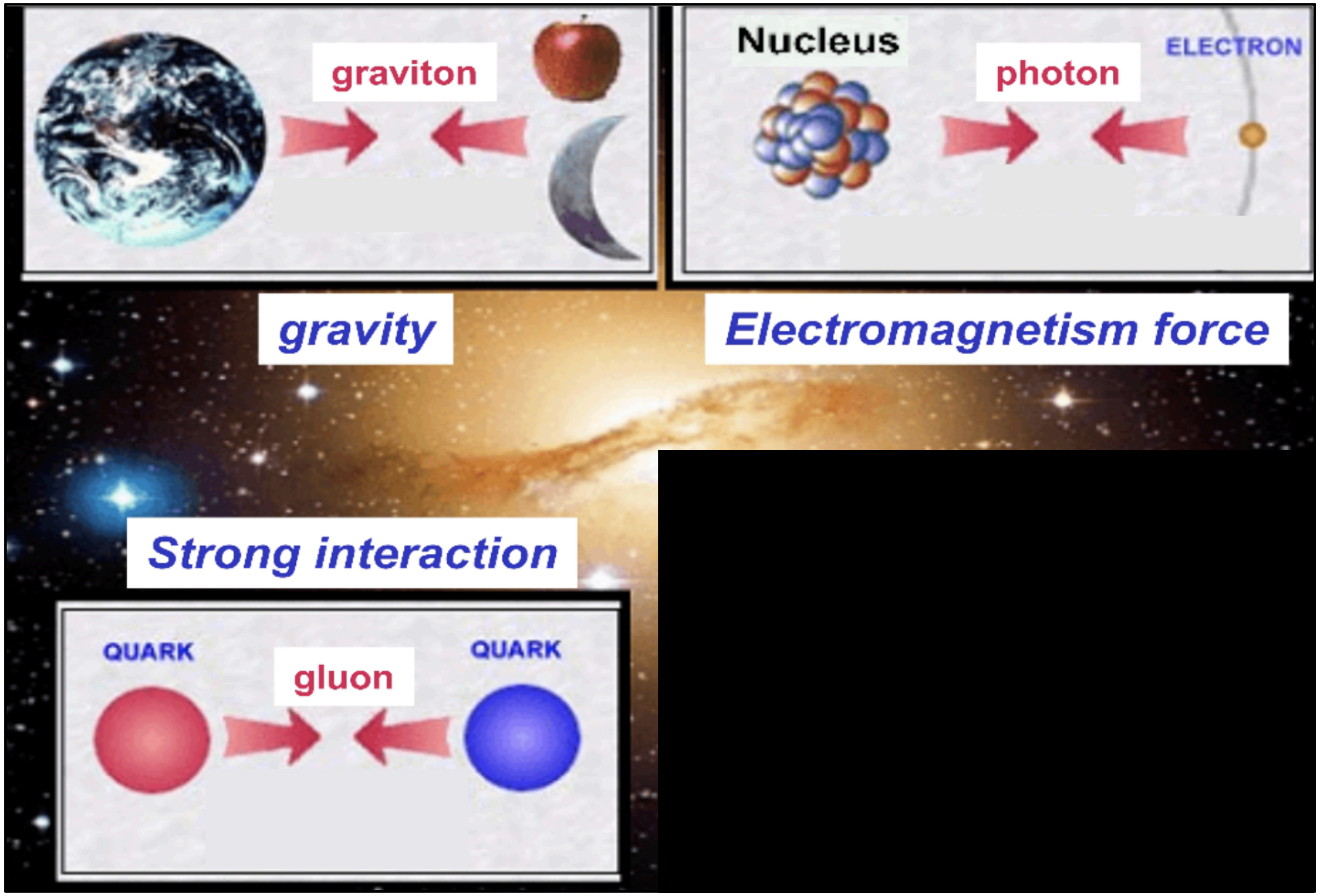

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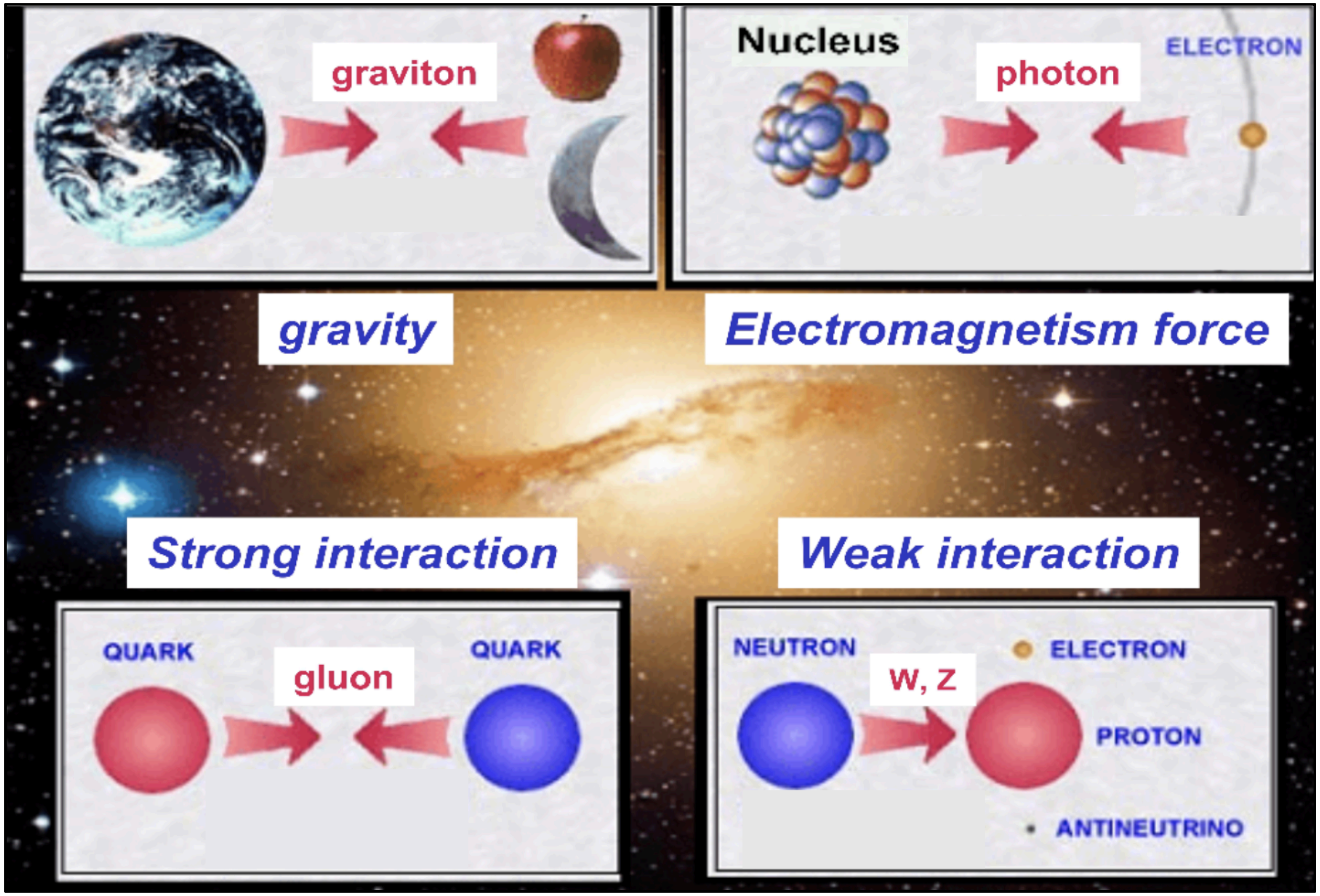

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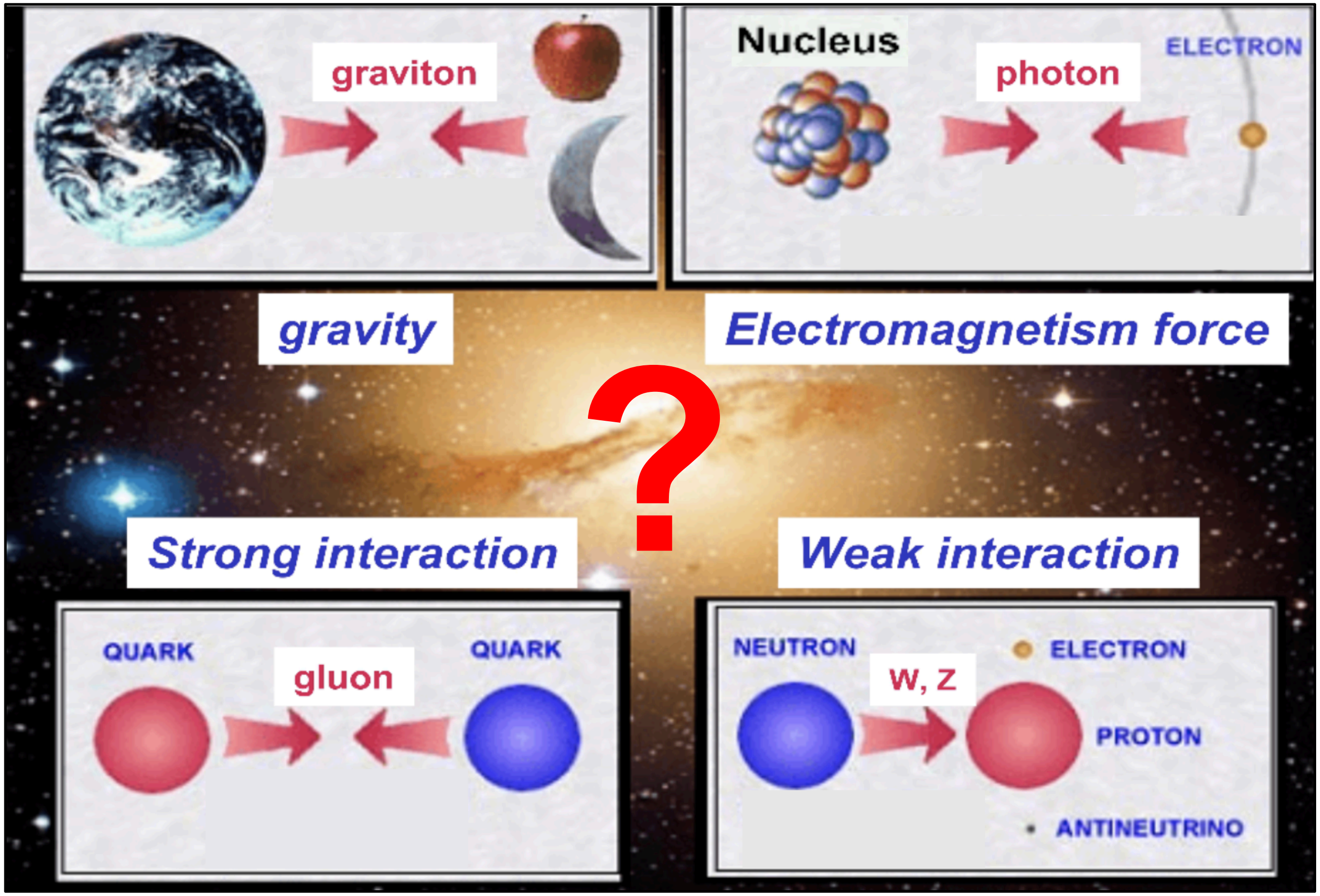

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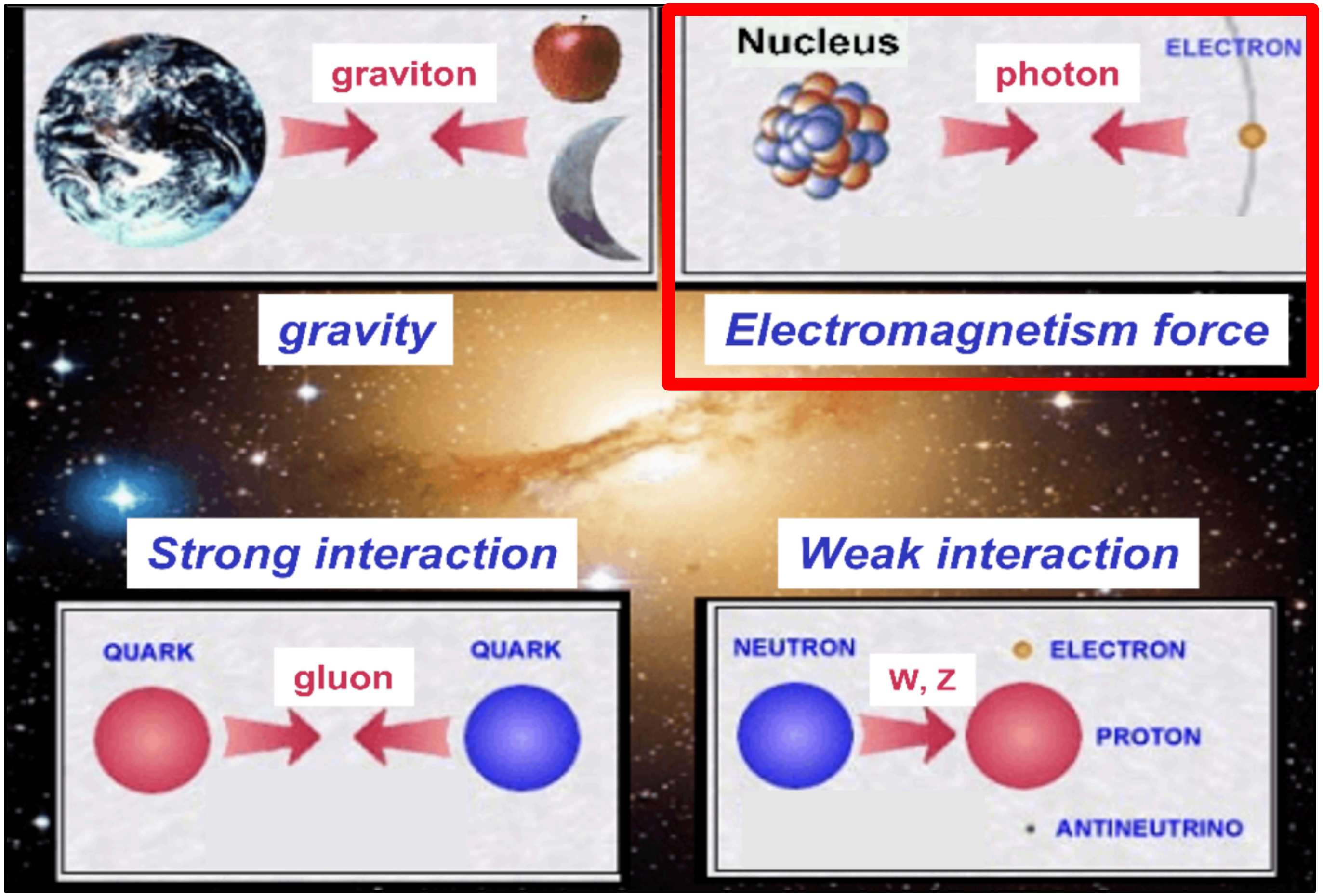

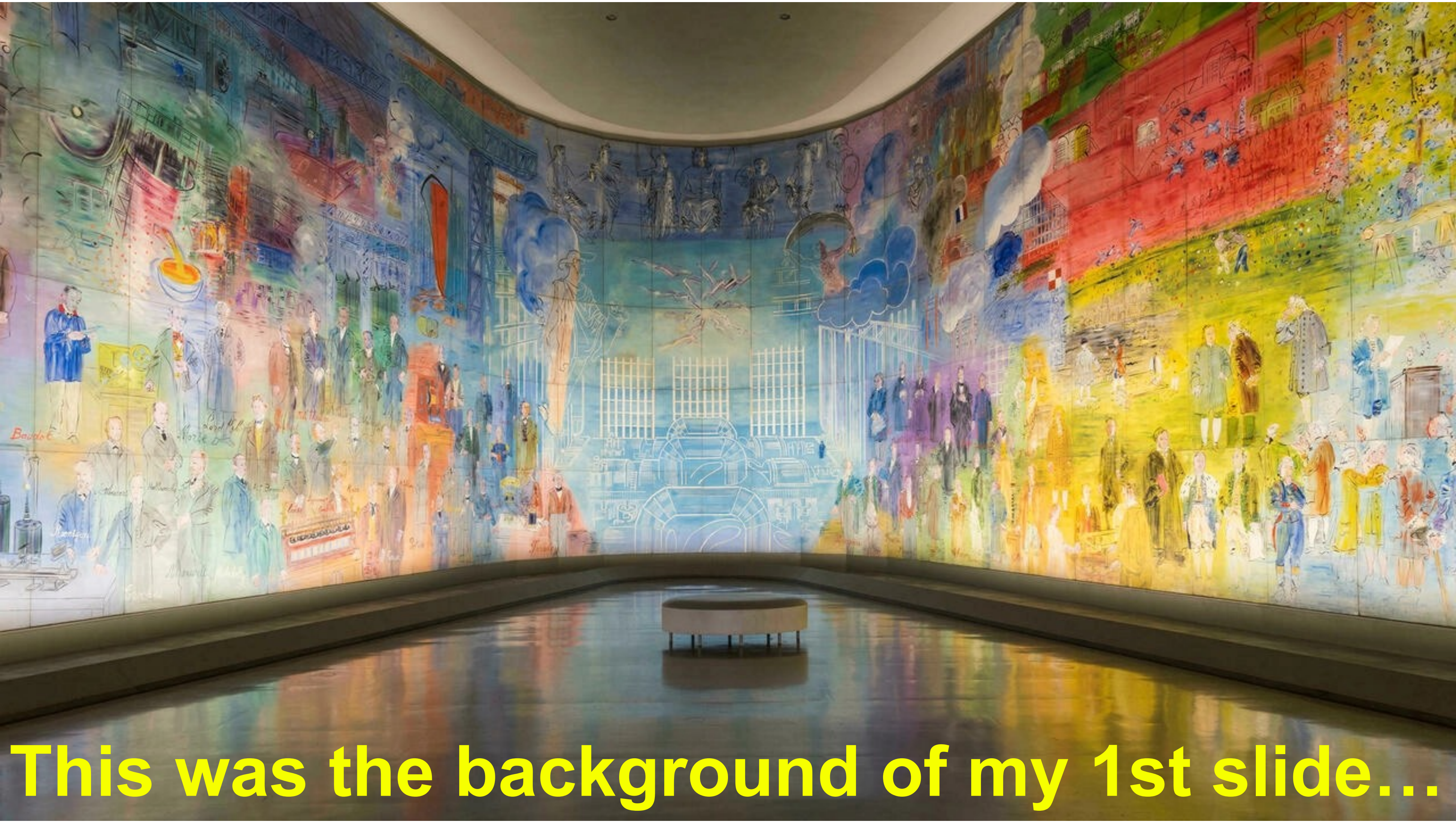

This was the background of my 1st slide…

Do you know what it is?

=> **It’s one of the world’s largest painting (600 m^2)…**

=> It's one of the world's largest painting (600 m^2)… from Raoul Dufy in Paris's Museum of Modern Art…

“The Electricity Fairy”

La Fée Electricité

“The Electricity Fairy”

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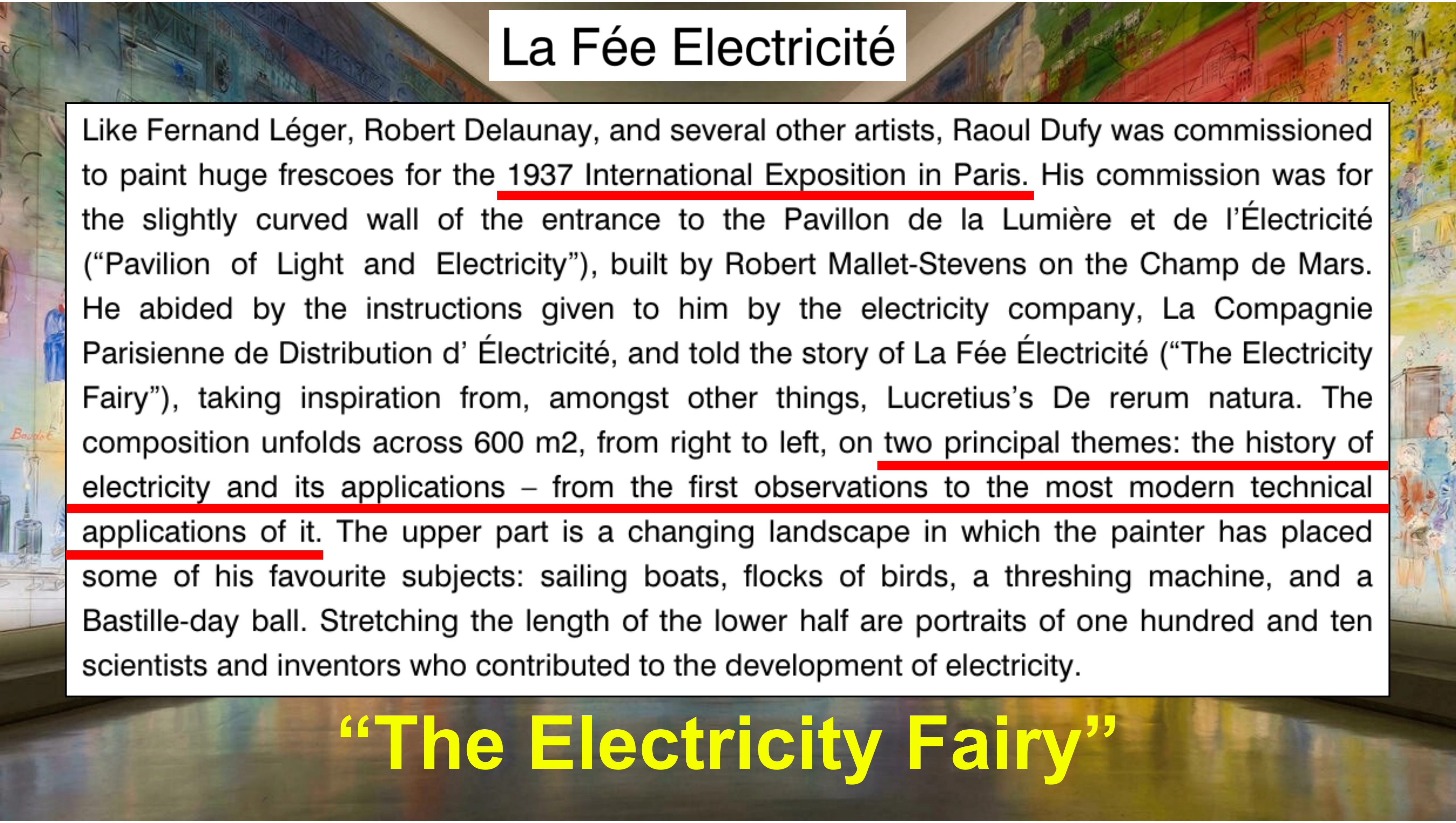
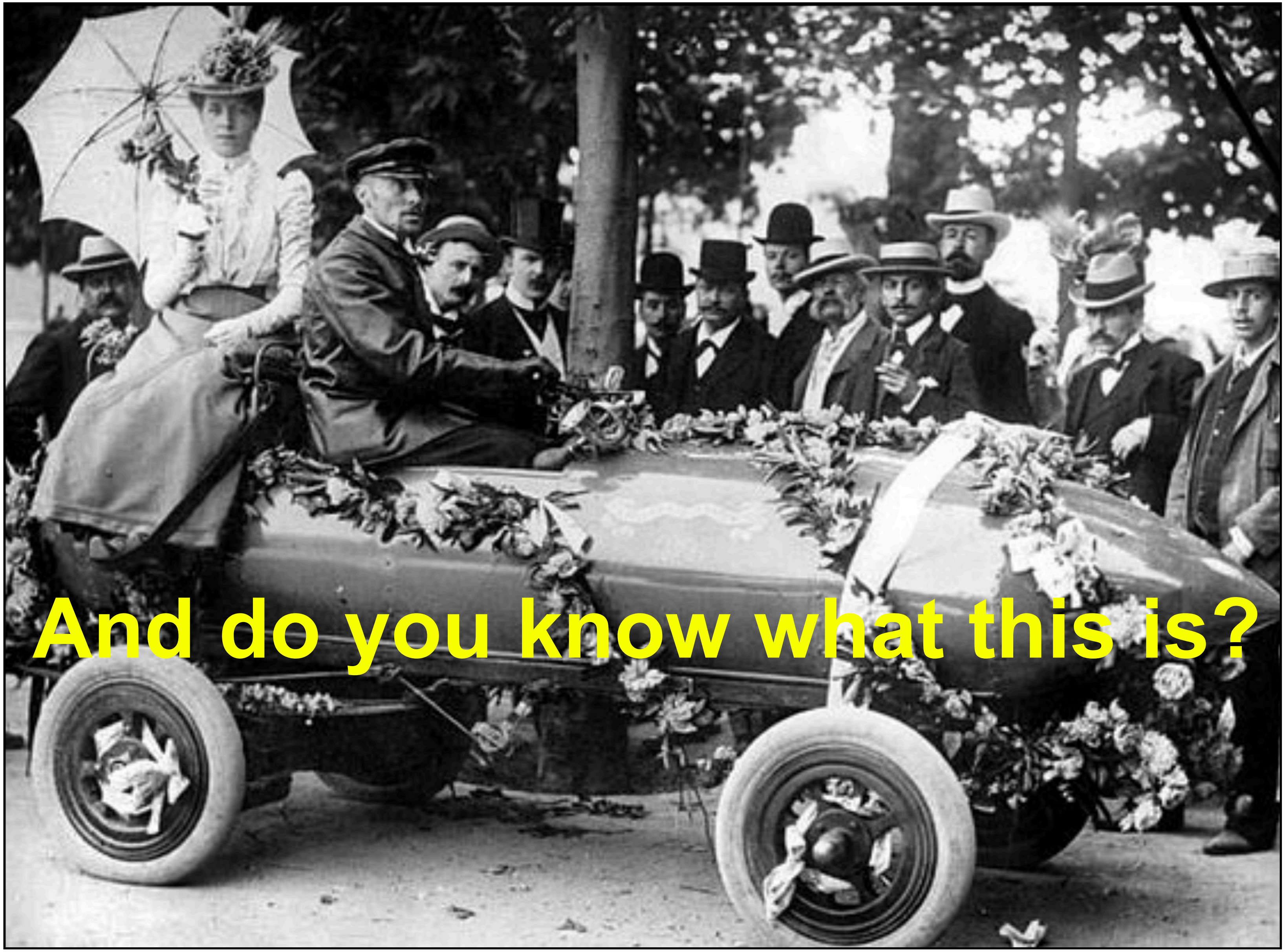


Like Fernand Léger, Robert Delaunay, and several other artists, Raoul Dufy was commissioned to paint huge frescoes for the 1937 International Exposition in Paris. His commission was for the slightly curved wall of the entrance to the Pavillon de la Lumière et de l'Électricité ("Pavilion of Light and Electricity"), built by Robert Mallet-Stevens on the Champ de Mars. He abided by the instructions given to him by the electricity company, La Compagnie Parisienne de Distribution d' Électricité, and told the story of La Fée Électricité ("The Electricity Fairy"), taking inspiration from, amongst other things, Lucretius's De rerum natura. The composition unfolds across 600 m2, from right to left, on two principal themes: the history of electricity and its applications – from the first observations to the most modern technical applications of it. The upper part is a changing landscape in which the painter has placed some of his favourite subjects: sailing boats, flocks of birds, a threshing machine, and a Bastille-day ball. Stretching the length of the lower half are portraits of one hundred and ten scientists and inventors who contributed to the development of electricity.

"The Electricity Fairy"

=> Electricity (and Magnetism), i.e. ElectroMagnetism (EM), is the (only) force which is used for particle accelerators!



And do you know what this is?

"La Jamais Contente" (The Never Contented) was the 1st road vehicle to go over 100 km/h (62 mph) on April 29, 1899. It was a Belgian electric vehicle. Soon after, the internal combustion engine supplanted the electric technology for the next century. Ecological considerations did not appear until much later…and we are now back to electric cars!

And do you know what this is?

Lorentz force

The motion of a charged particle (proton) in a beam transport channel or a circular accelerator is governed by the LORENTZ FORCE

$$\vec{F} = e\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

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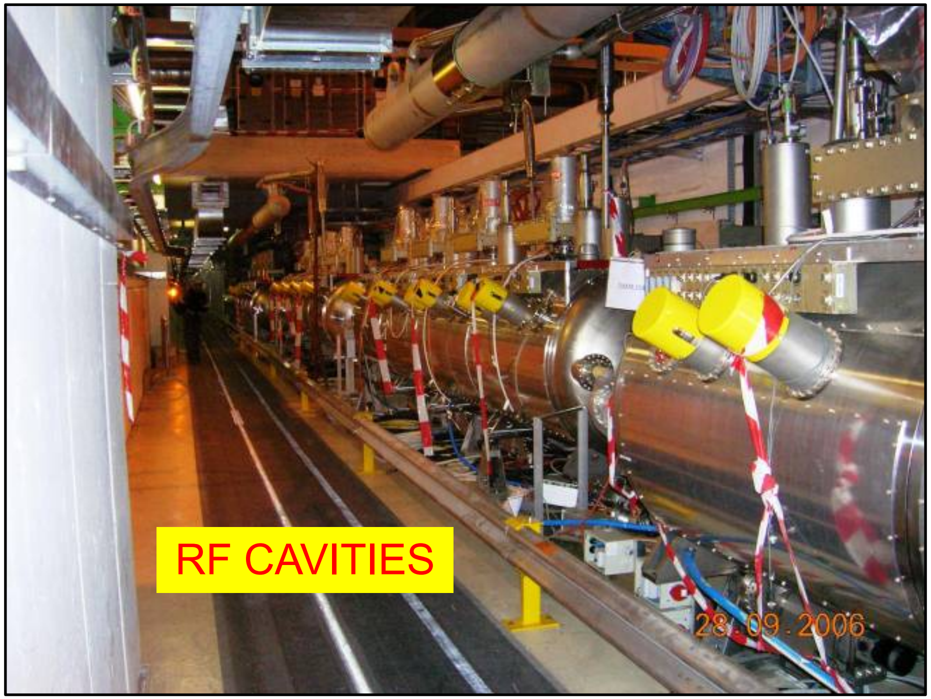

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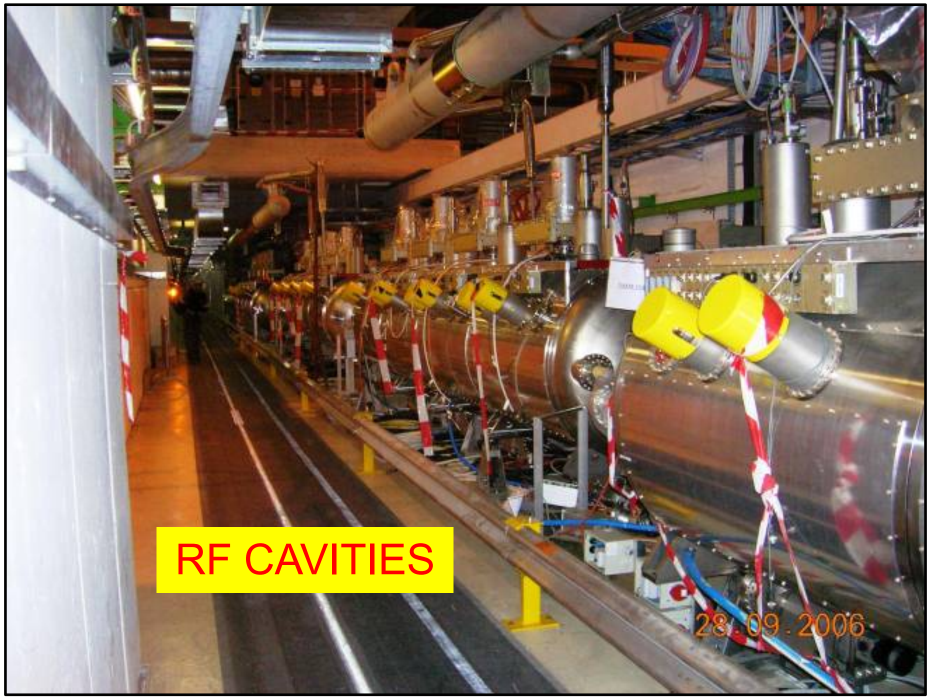

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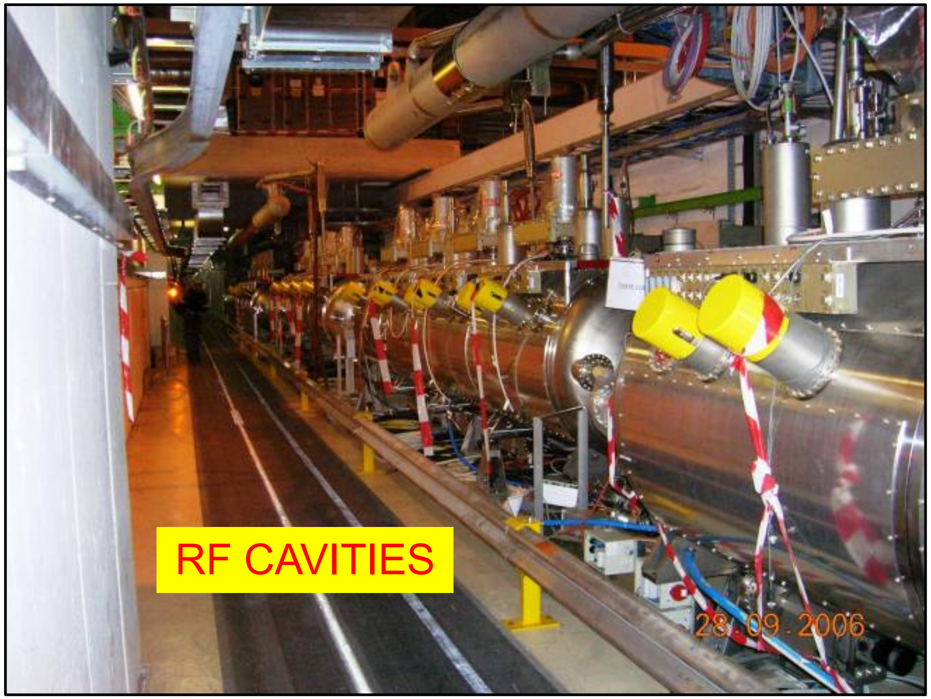

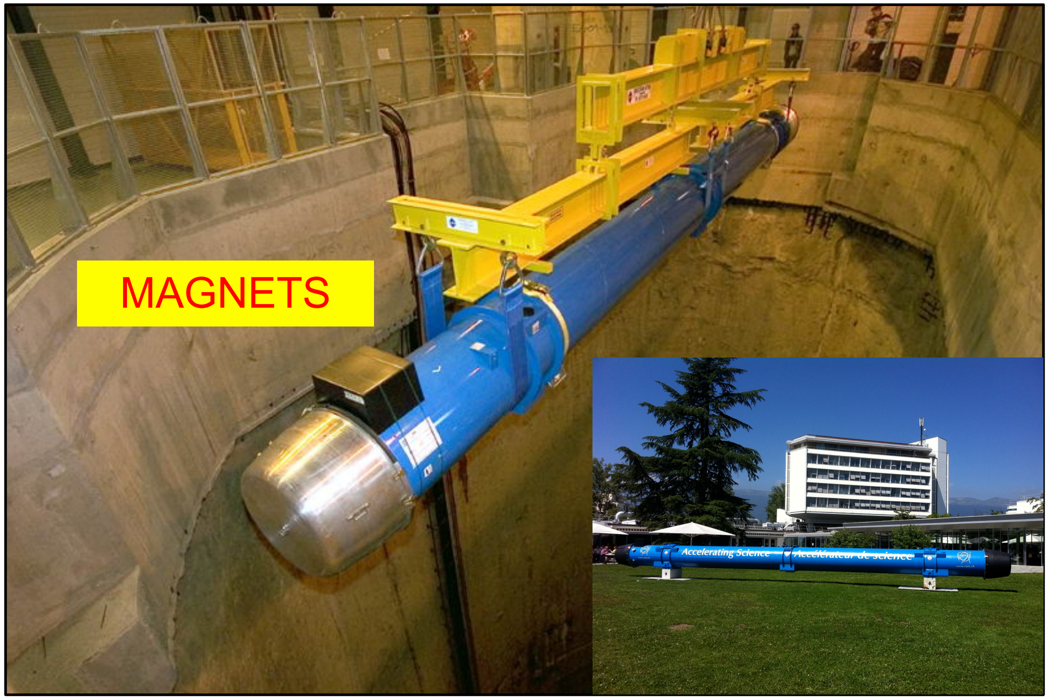

Lorentz force

$$\vec{F} = e \left(\vec{E} + \vec{\upsilon} \times \vec{B} \right)$$

- **Cartesian (x,y,s)**

$$F_x = e \left(E_x - \upsilon B_y \right)$$

$$F_y = e \left(E_y + \upsilon B_x \right)$$

$$F_s = e E_s$$

- **Cylindrical (r,θ,s)**

$$F_r = e \left(E_r - \upsilon B_\vartheta \right)$$

$$F_\vartheta = e \left(E_\vartheta + \upsilon B_r \right)$$

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Transverse magnetic field in MAGNETS to guide and confine the particles

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Transverse magnetic field in MAGNETS to guide and confine the particles

$$F_s = e \, E_s$$

Longitudinal electric field in RF CAVITIES to accelerate (or decelerate) the particles

- **Cylindrical (r,θ,s)**

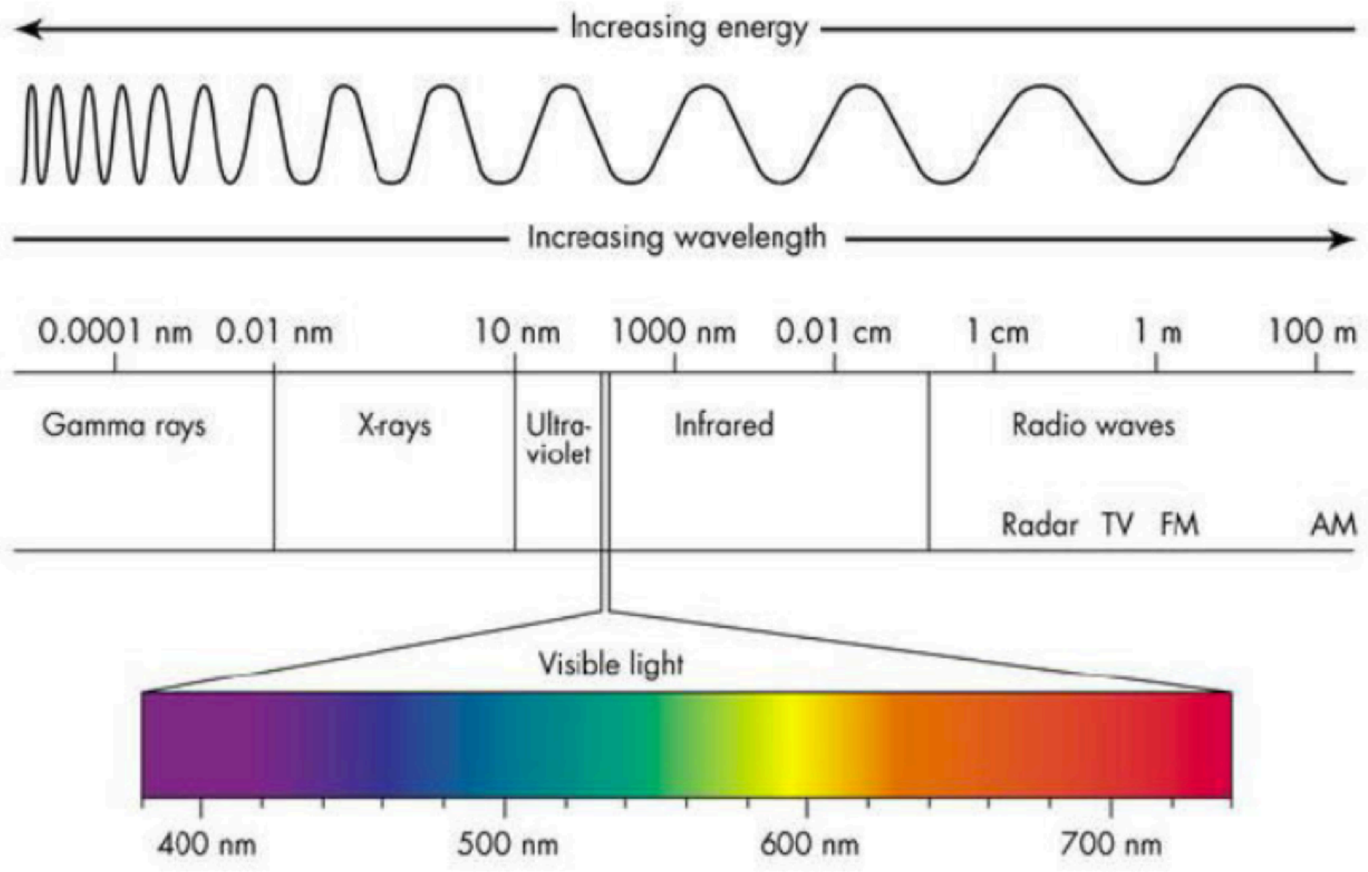
$$F_r = e \left(E_r - \upsilon \, B_\vartheta \right)$$

$$F_\vartheta = e \left(E_\vartheta + \upsilon \, B_r \right)$$

$$F_s = e \, E_s$$

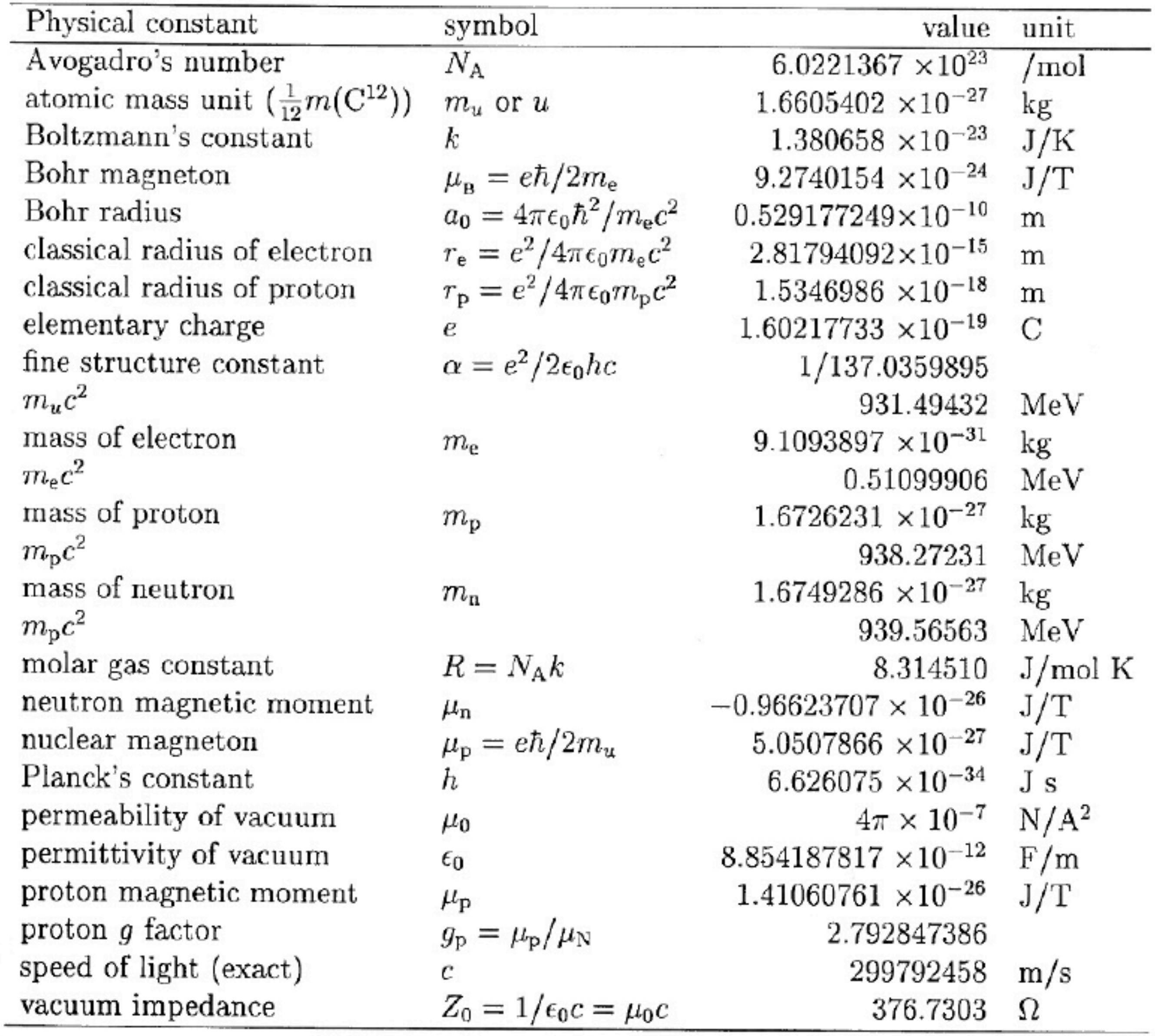
Electromagnetic spectrum

$$E = \frac{h\,c}{\lambda_{wl}}$$



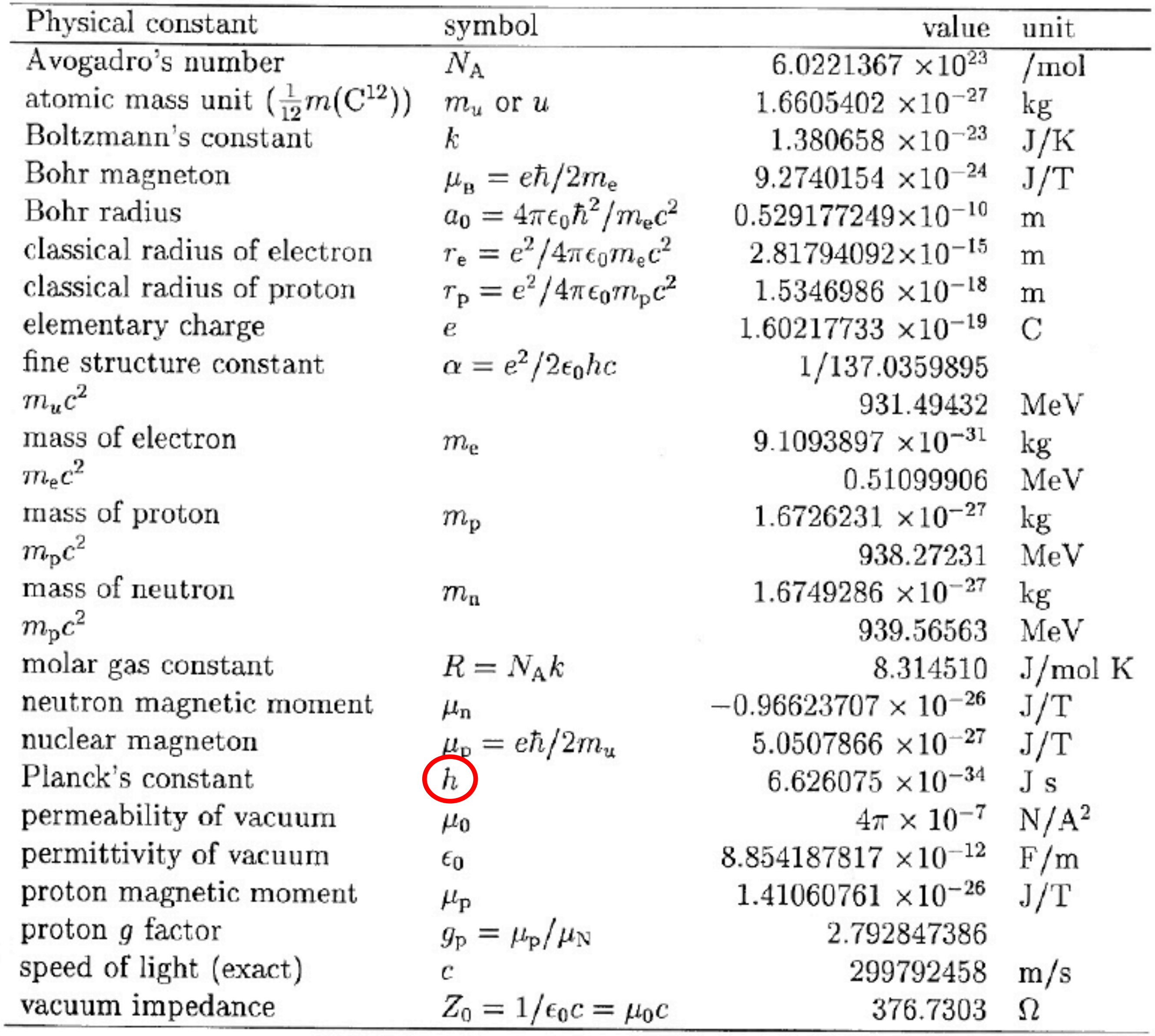
$$\lambda_{wl} = \frac{c}{f}$$

Reminder: Fundamental physical constants



Physical constant	symbol	value	unit
Avogadro's number	N_A	6.0221367×10^{23}	/mol
atomic mass unit ($\frac{1}{12}m(C^{12})$)	m_u or u	$1.6605402 \times 10^{-27}$	kg
Boltzmann's constant	k	1.380658×10^{-23}	J/K
Bohr magneton	$\mu_B = e\hbar/2m_e$	$9.2740154 \times 10^{-24}$	J/T
Bohr radius	$a_0 = 4\pi\epsilon_0\hbar^2/m_ec^2$	$0.529177249 \times 10^{-10}$	m
classical radius of electron	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	$2.81794092 \times 10^{-15}$	m
classical radius of proton	$r_p = e^2/4\pi\epsilon_0 m_p c^2$	$1.5346986 \times 10^{-18}$	m
elementary charge	e	$1.60217733 \times 10^{-19}$	C
fine structure constant	$\alpha = e^2/2\epsilon_0 hc$	1/137.0359895	
m_uc^2		931.49432	MeV
mass of electron	m_e	$9.1093897 \times 10^{-31}$	kg
m_ec^2		0.51099906	MeV
mass of proton	m_p	$1.6726231 \times 10^{-27}$	kg
m_pc^2		938.27231	MeV
mass of neutron	m_n	$1.6749286 \times 10^{-27}$	kg
m_pc^2		939.56563	MeV
molar gas constant	$R = N_A k$	8.314510	J/mol K
neutron magnetic moment	μ_n	$-0.96623707 \times 10^{-26}$	J/T
nuclear magneton	$\mu_p = e\hbar/2m_u$	$5.0507866 \times 10^{-27}$	J/T
Planck's constant	h	6.626075×10^{-34}	J s
permeability of vacuum	μ_0	$4\pi \times 10^{-7}$	N/A^2
permittivity of vacuum	ϵ_0	$8.854187817 \times 10^{-12}$	F/m
proton magnetic moment	μ_p	$1.41060761 \times 10^{-26}$	J/T
proton g factor	$g_p = \mu_p/\mu_N$	2.792847386	
speed of light (exact)	c	299792458	m/s
vacuum impedance	$Z_0 = 1/\epsilon_0 c = \mu_0 c$	376.7303	Ω

Reminder: Fundamental physical constants



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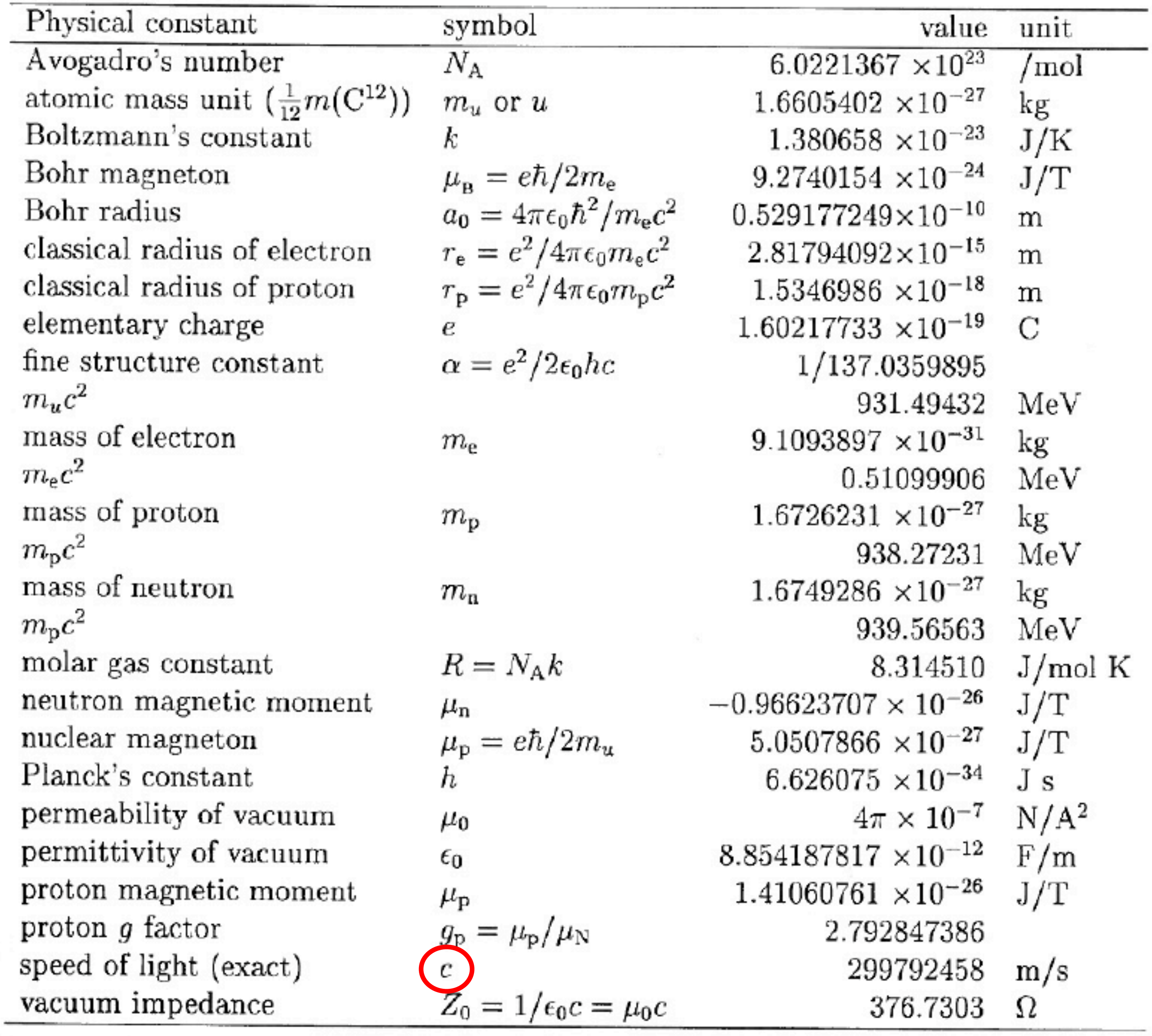

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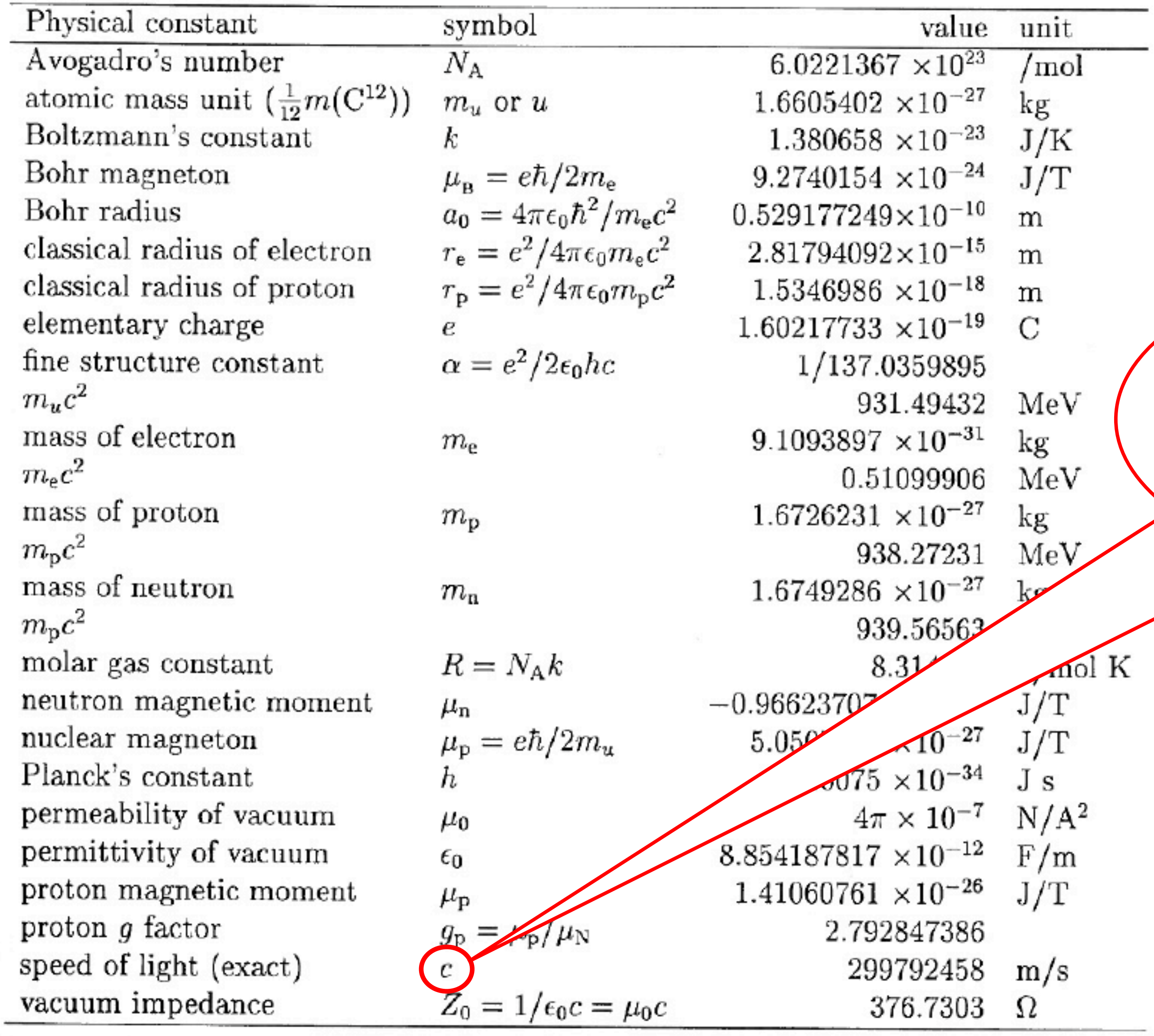

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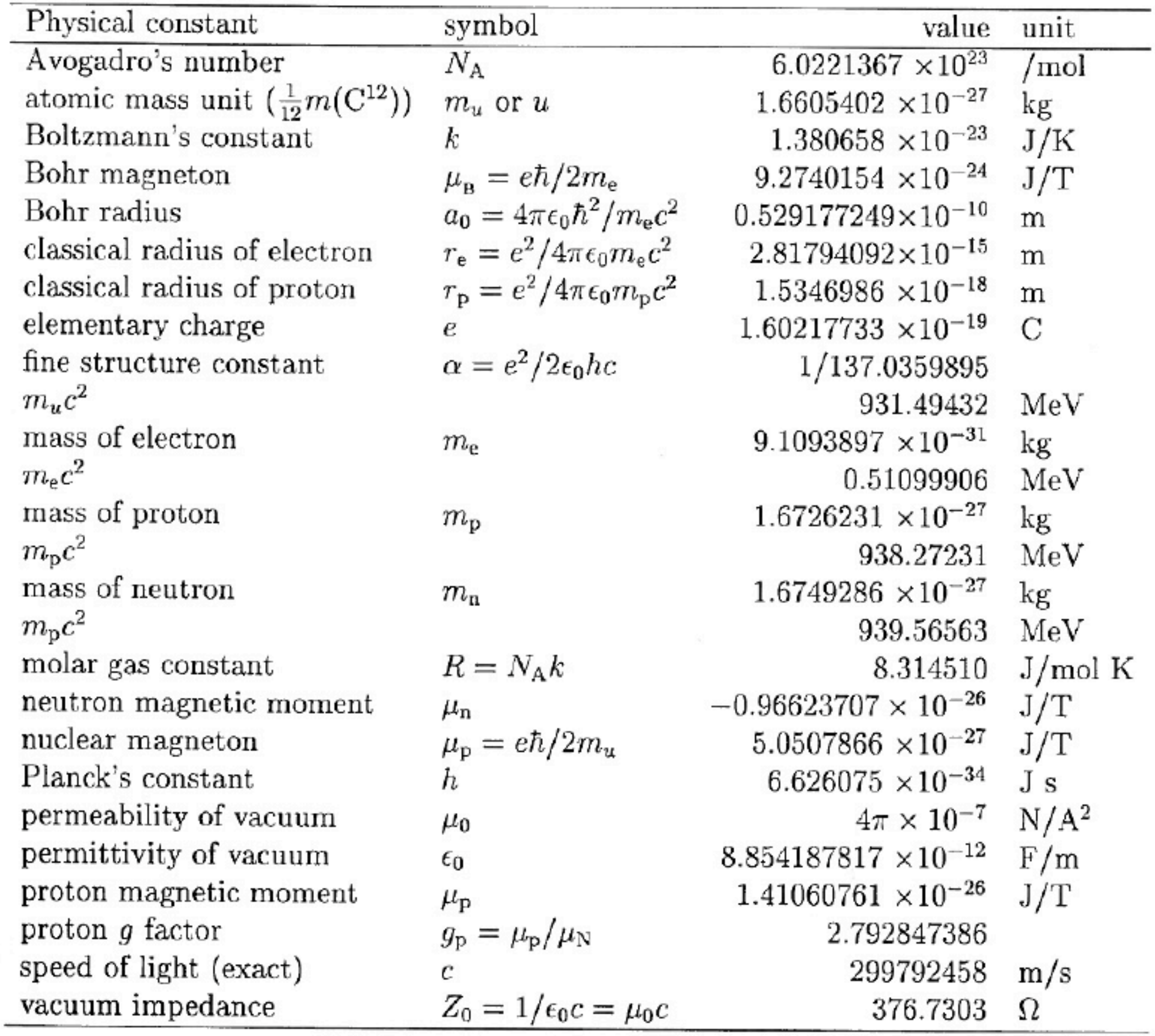
m_pc^2		939.56563	
molar gas constant	$R = N_Ak$	8.31	mol K
neutron magnetic moment	μ_n	−0.96623707	J/T
nuclear magneton	$\mu_p = e\hbar/2m_u$	5.05 $\times10^{-27}$	J/T
Planck's constant	h	075 $\times10^{-34}$	J s
permeability of vacuum	μ_0	$4\pi \times 10^{-7}$	N/A^2
permittivity of vacuum	ϵ_0	$8.854187817 \times10^{-12}$	F/m
proton magnetic moment	μ_p	$1.41060761 \times10^{-26}$	J/T
proton g factor	$g_p = \mu_p/\mu_N$	2.792847386	
speed of light (exact)	c	299792458	m/s
vacuum impedance	$Z_0 = 1/\epsilon_0 c = \mu_0 c$	376.7303	Ω

How can we express the speed of light c as a function of some parameters of this table?

Reminder: Fundamental physical constants



Physical constant	symbol	value	unit
Avogadro's number	N_A	6.0221367×10^{23}	/mol
atomic mass unit $(\frac{1}{12}m(C^{12}))$	m_u or u	1.6605402×10^{-27}	kg
Boltzmann's constant	k	1.380658×10^{-23}	J/K
Bohr magneton	$\mu_B = e\hbar/2m_e$	9.2740154×10^{-24}	J/T
Bohr radius	$a_0 = 4\pi\epsilon_0\hbar^2/m_ec^2$	$0.529177249\times10^{-10}$	m
classical radius of electron	$r_e = e^2/4\pi\epsilon_0m_ec^2$	2.81794092×10^{-15}	m
classical radius of proton	$r_p = e^2/4\pi\epsilon_0m_pc^2$	1.5346986×10^{-18}	m
elementary charge	e	1.60217733×10^{-19}	C
fine structure constant	$\alpha = e^2/2\epsilon_0hc$	1/137.0359895	
m_uc^2		931.49432	MeV
mass of electron	m_e	9.1093897×10^{-31}	kg
m_ec^2		0.51099906	MeV
mass of proton	m_p	1.6726231×10^{-27}	kg
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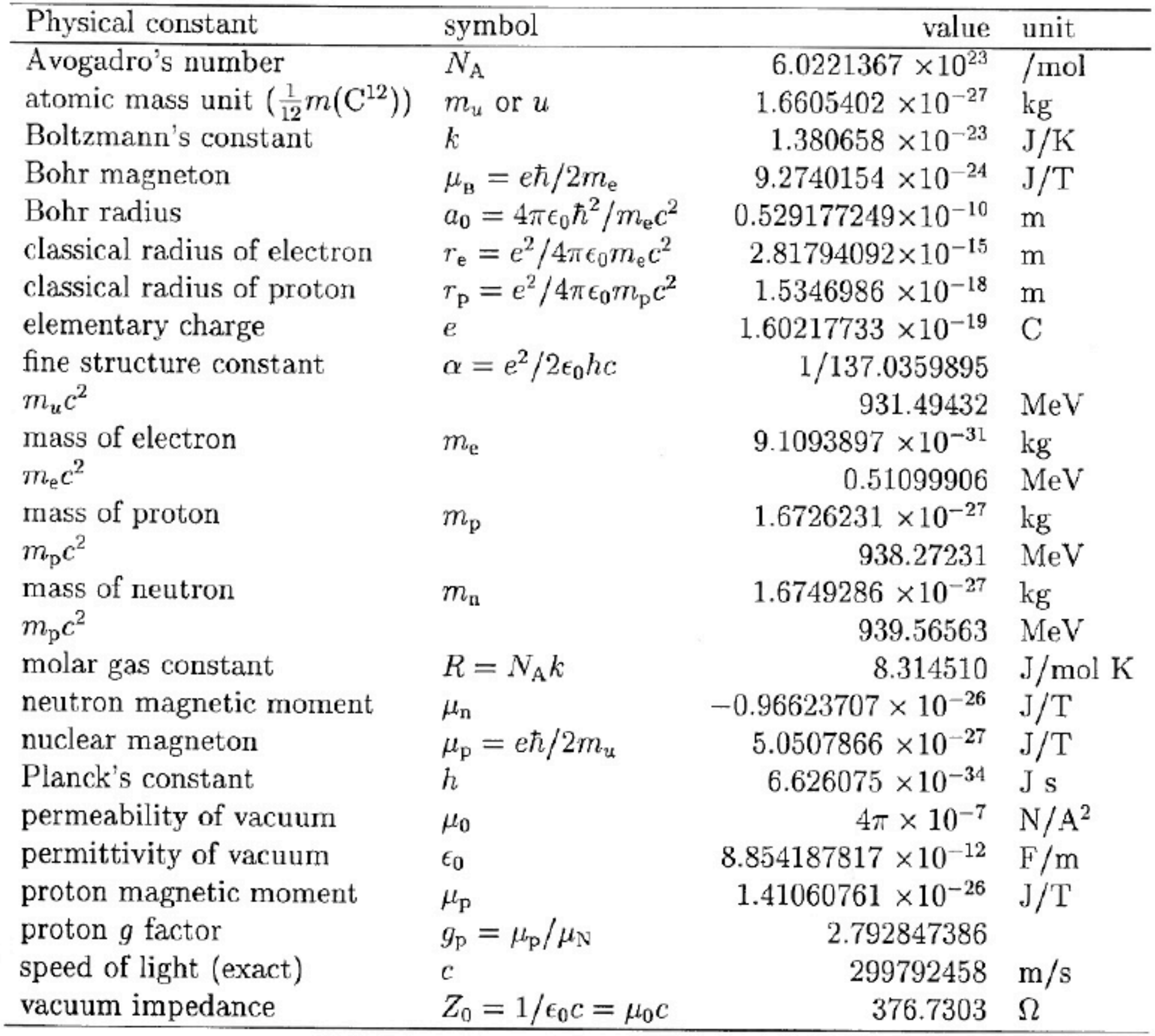

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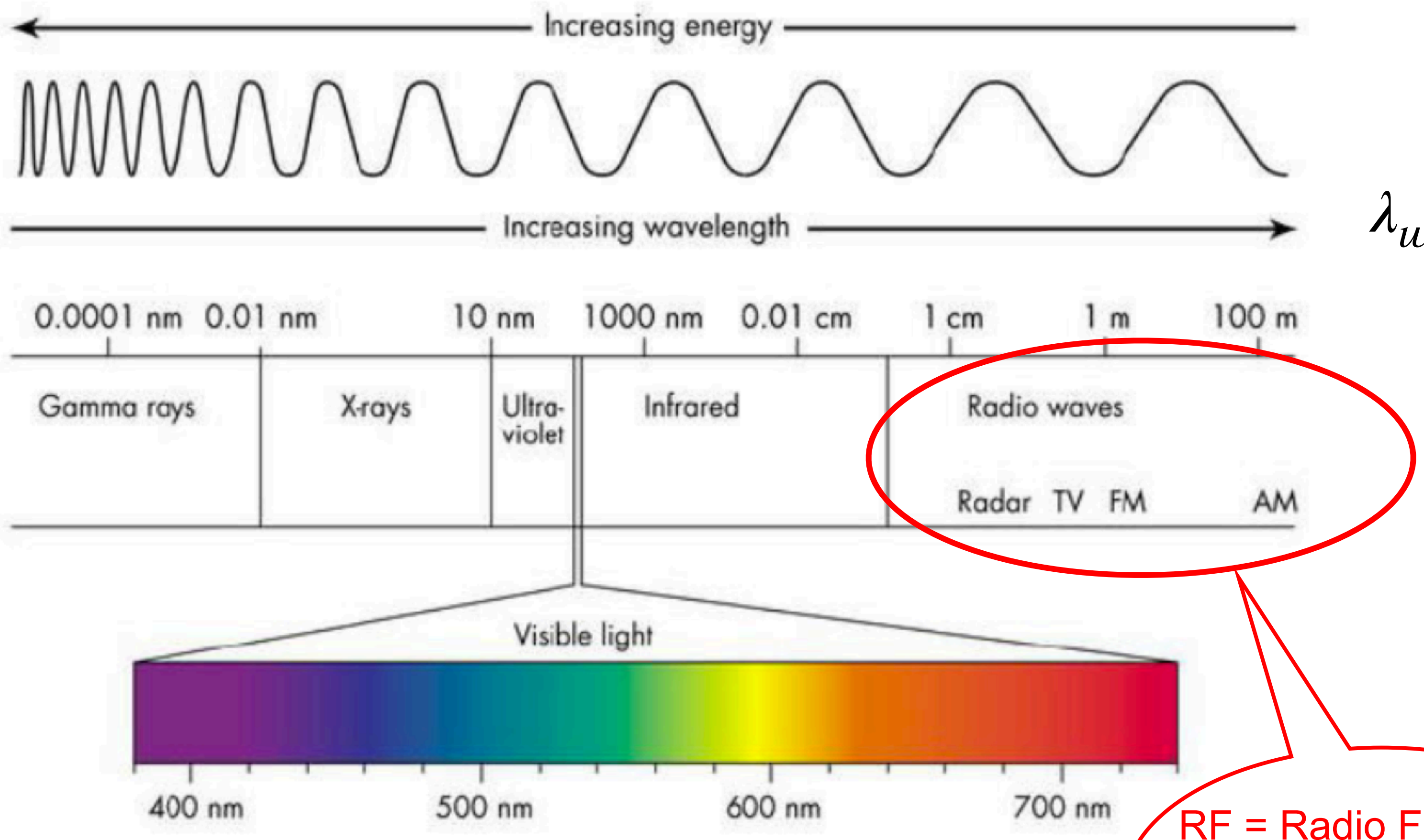
The identification of light with an EM wave (with phase velocity related to the electric permittivity and magnetic permeability) was one of the great achievements of 19th century physics

$$\boxed{c = \frac{1}{\sqrt{\epsilon_0\,\mu_0}} \approx 300\,000 \text{ km/s}}$$

Electromagnetic spectrum

$$E = \frac{h\,c}{\lambda_{wl}}$$



$$\lambda_{wl} = \frac{c}{f}$$

RF = Radio Frequency (from few kHz to hundreds of GHz) used for particle accelerators

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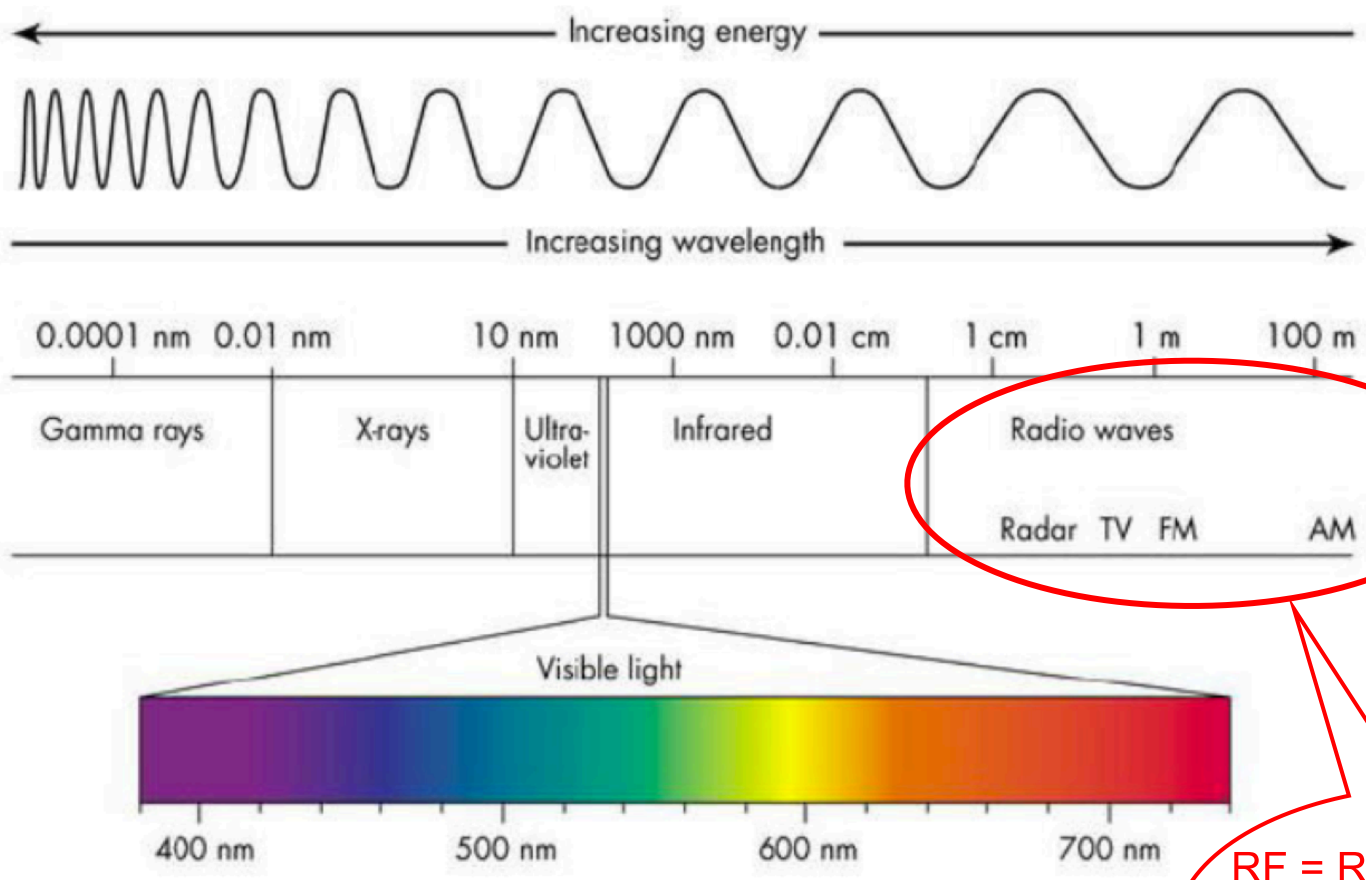

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What is the frequency of your preferred radio?

Electromagnetic spectrum

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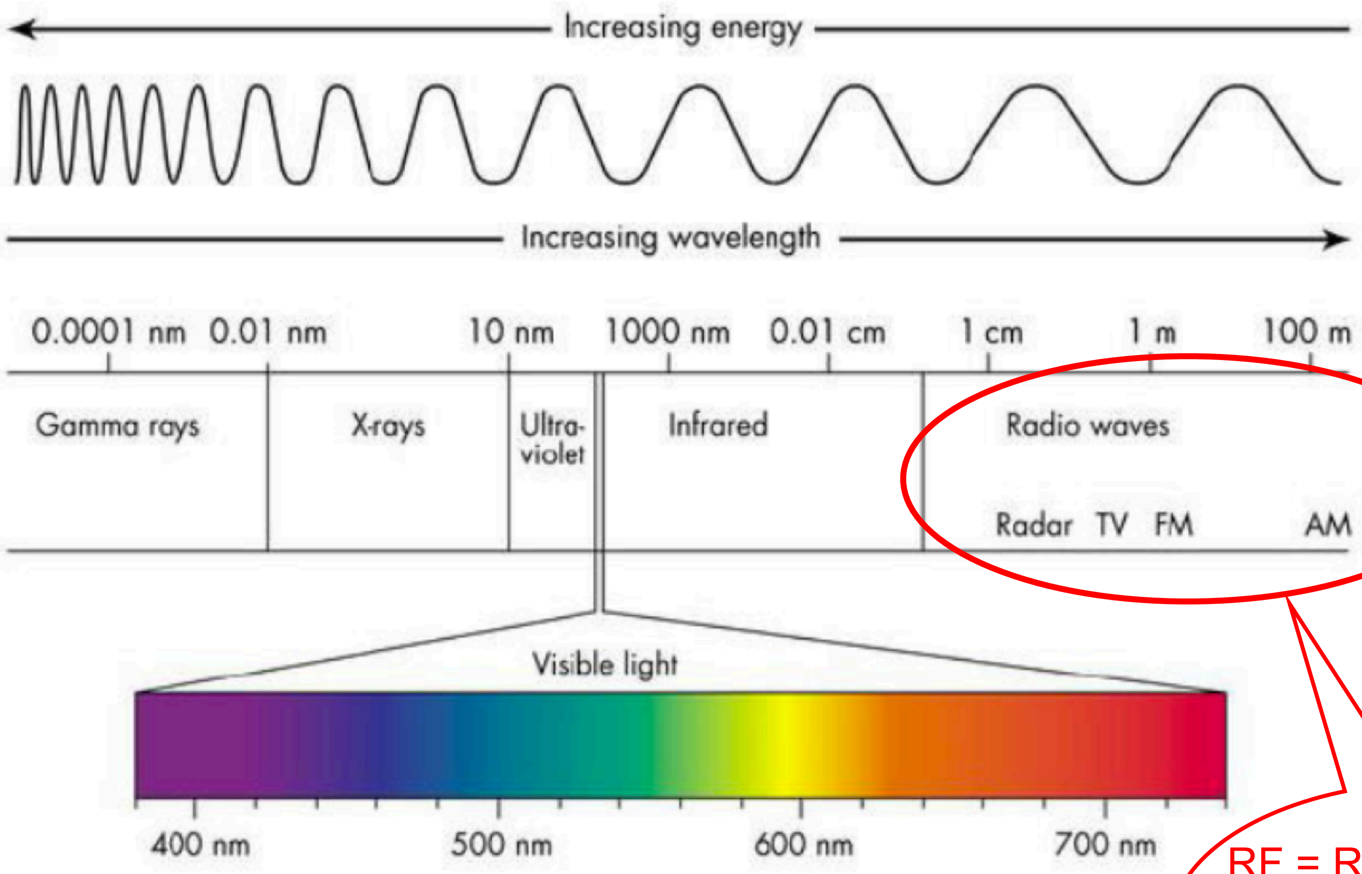

$$\lambda_{wl} = \frac{c}{f}$$

=> For me it is 93.5 MHz (France Inter)

RF = Radio Frequency (from few kHz to hundreds of GHz) used for particle accelerators

Relationship between the force on an object and the motion of this object?

Classical mechanics

Classical mechanics

- Do the Newtonian, Lagrangian and Hamiltonian mechanics describe the same physical mechanisms?
  - Yes
  - No

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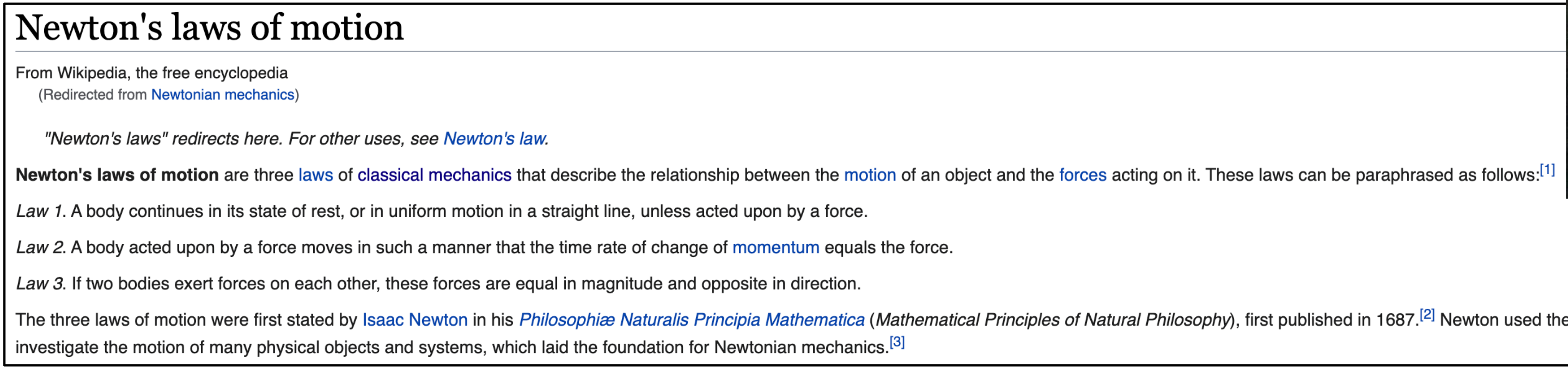

Newton's laws of motion

From Wikipedia, the free encyclopedia
(Redirected from Newtonian mechanics)

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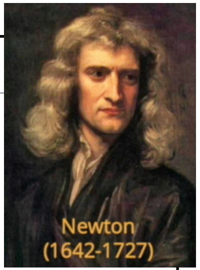
Law 3. If two bodies exert forces on each other, these forces are equal in magnitude and opposite in direction.

The three laws of motion were first stated by Isaac Newton in his *Philosophiæ Naturalis Principia Mathematica* (*Mathematical Principles of Natural Philosophy*), first published in 1687.[2] Newton used them to explain and investigate the motion of many physical objects and systems, which laid the foundation for Newtonian mechanics.[3]


Newton
(1642-1727)

Classical mechanics

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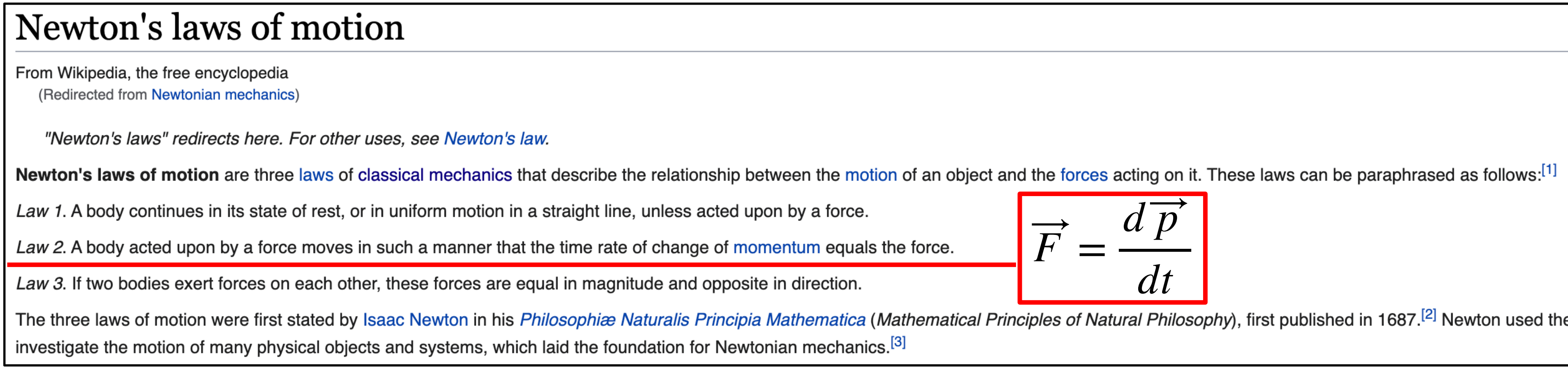

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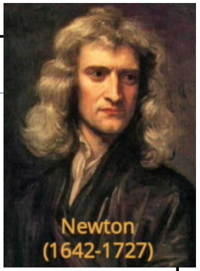
$$\boxed{\vec{F} = \frac{d\,\vec{p}}{dt}}$$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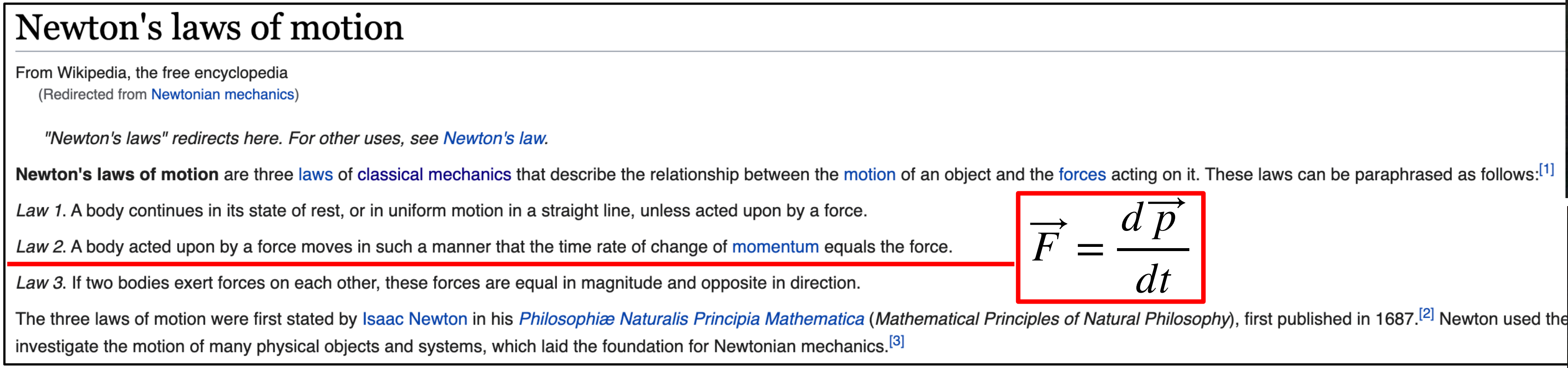

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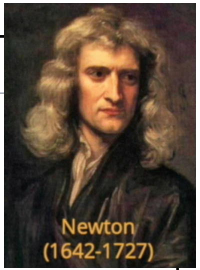

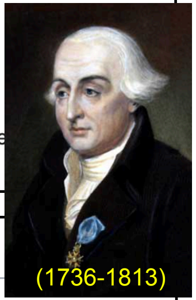

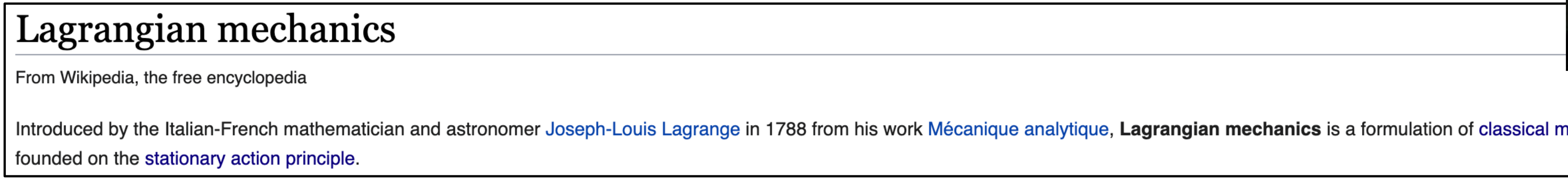

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From Wikipedia, the free encyclopedia

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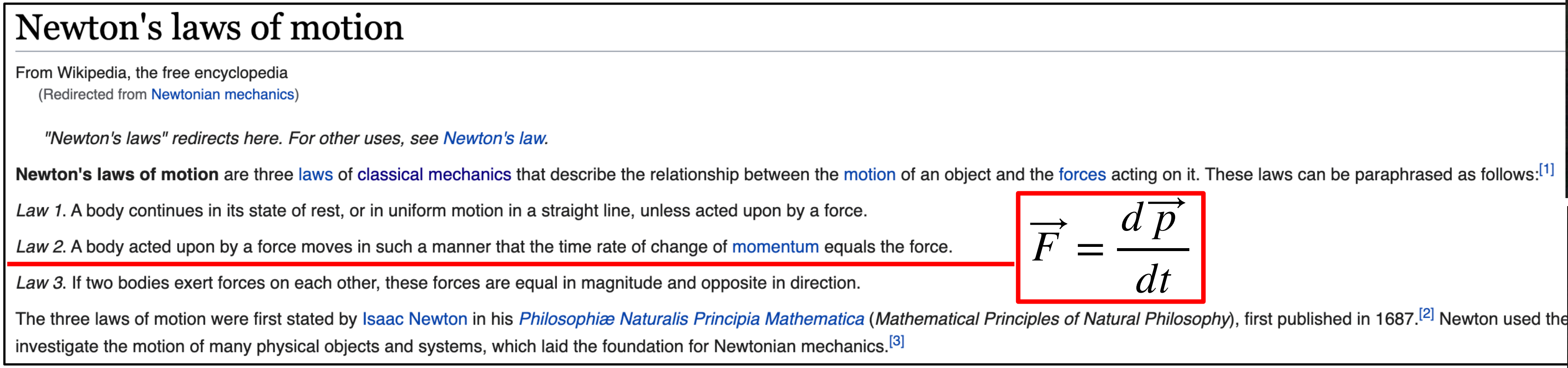

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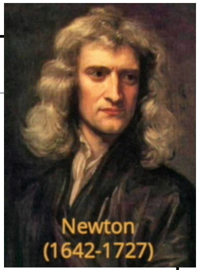

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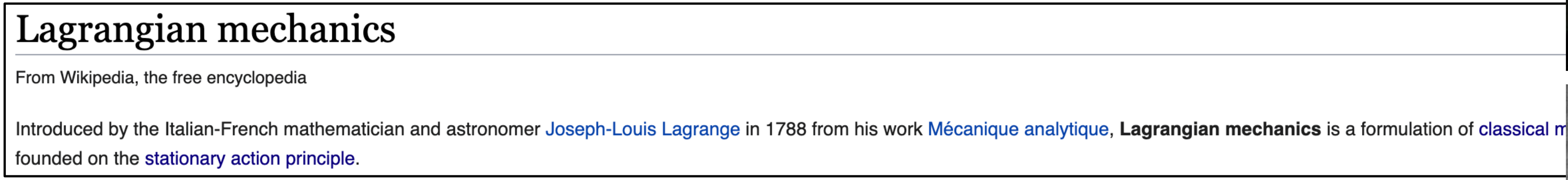

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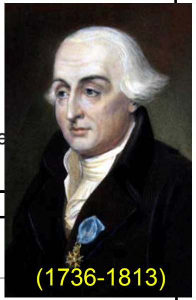
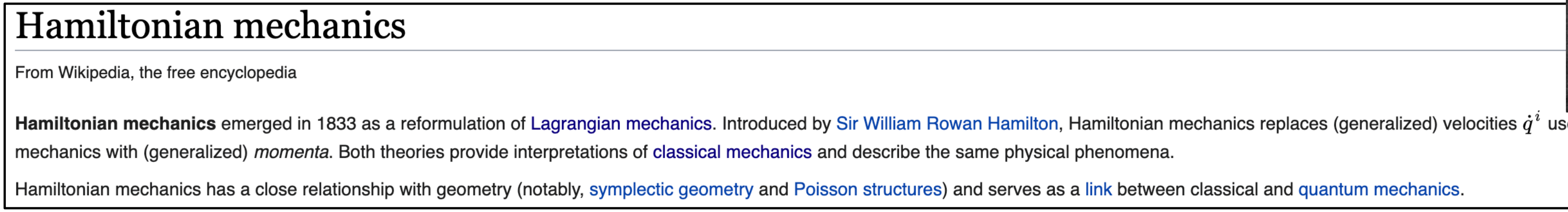


(1736-1813)



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Hamiltonian mechanics has a close relationship with geometry (notably, symplectic geometry and Poisson structures) and serves as a link between classical and quantum mechanics.

(1805-1865)

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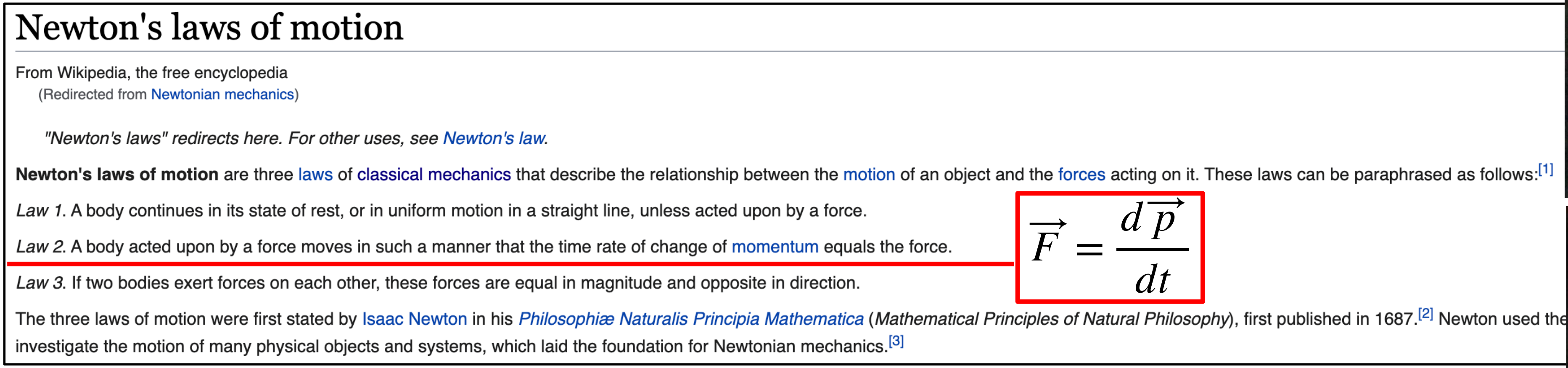

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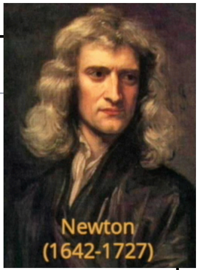

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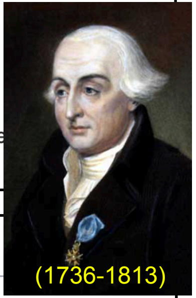

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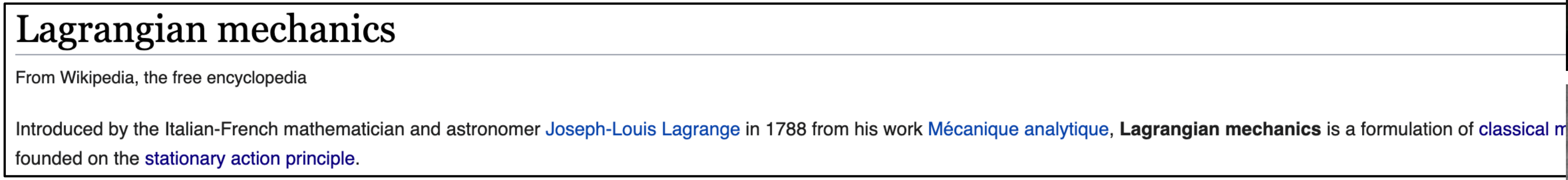

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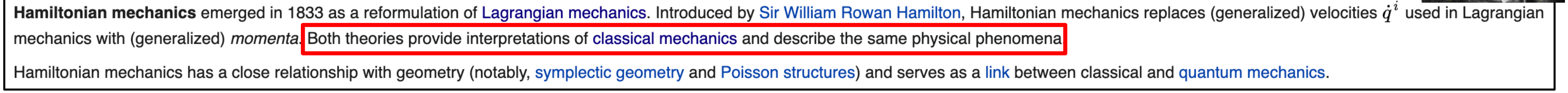

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- For particle accelerators, which one(s) of the following major sub-field of mechanics need to be included?
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  - Special relativity mainly and sometimes quantum mechanics
  - Quantum mechanics, special relativity and general relativity

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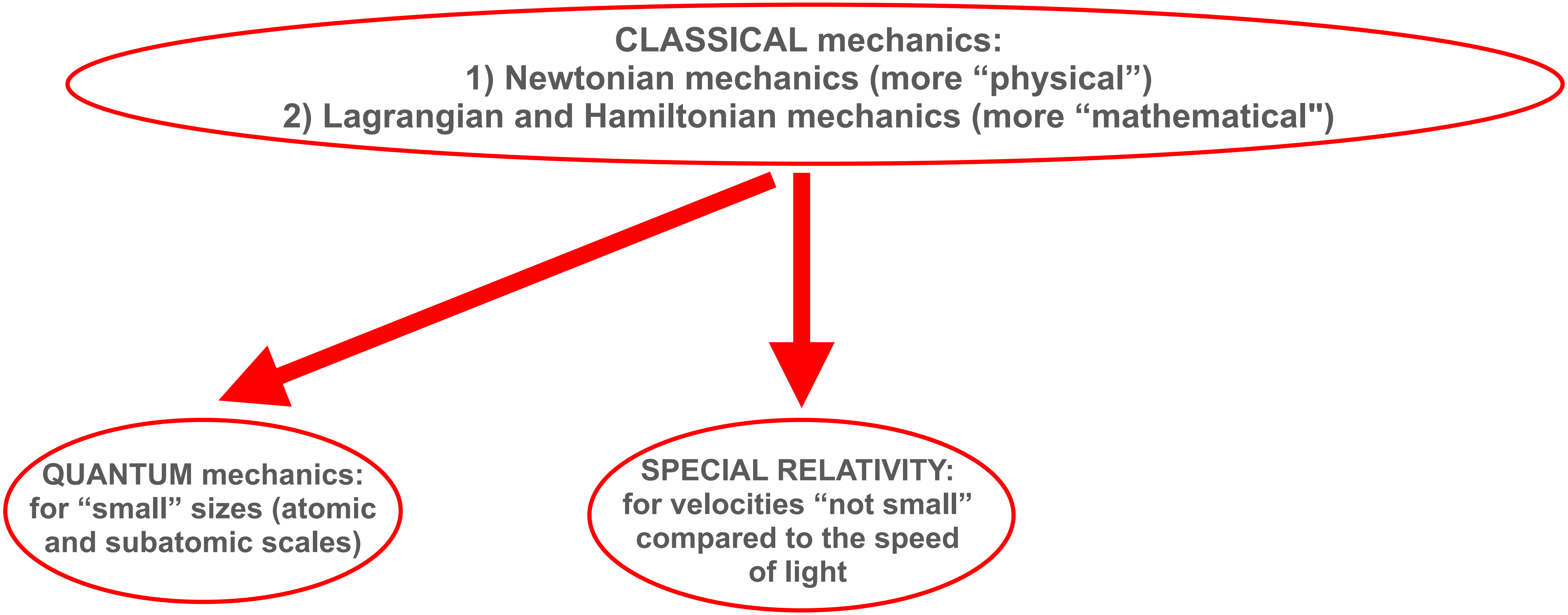
Classical mechanics



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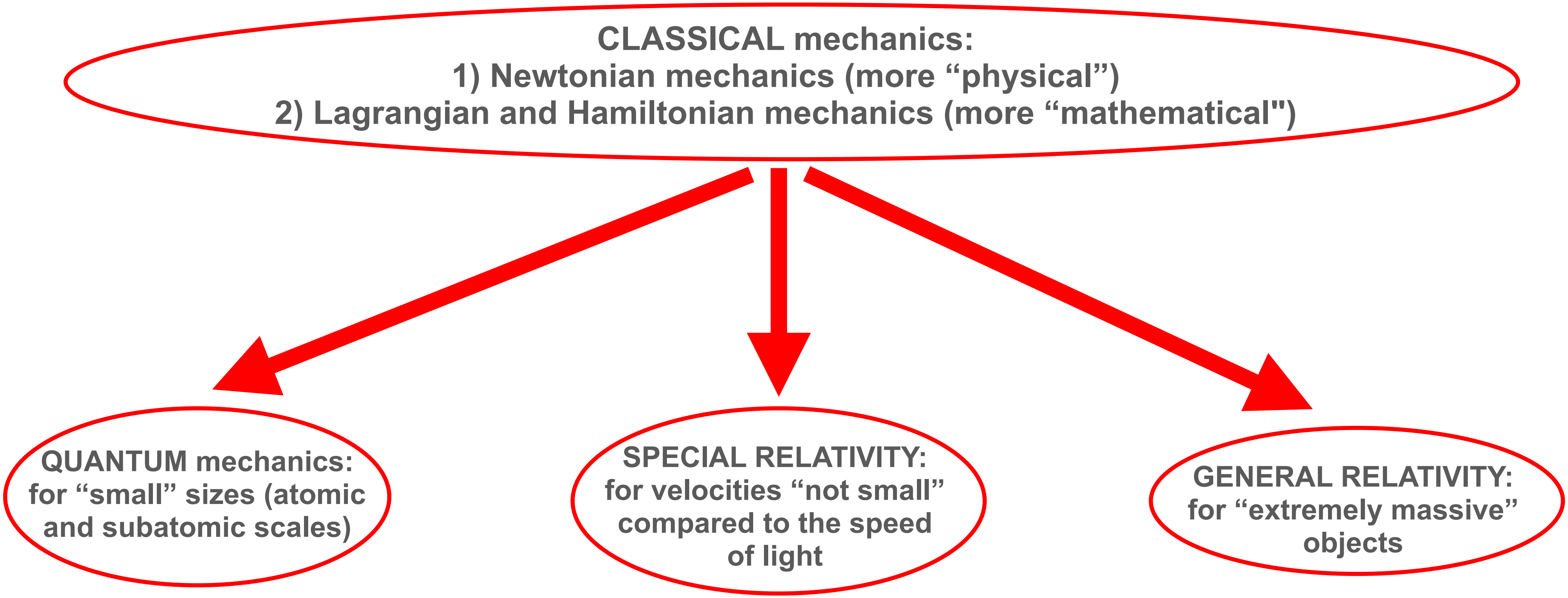
Classical mechanics



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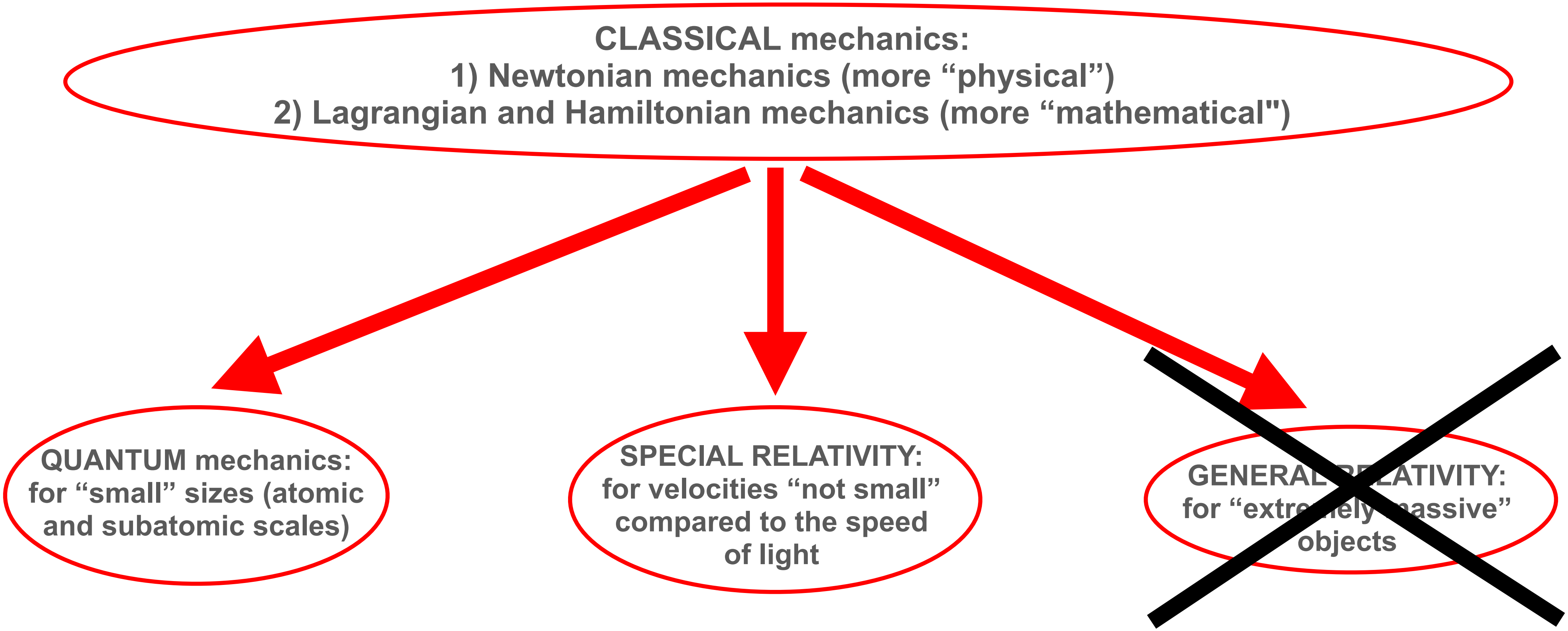
GENERAL RELATIVITY:
for “extremely massive” objects

Classical mechanics



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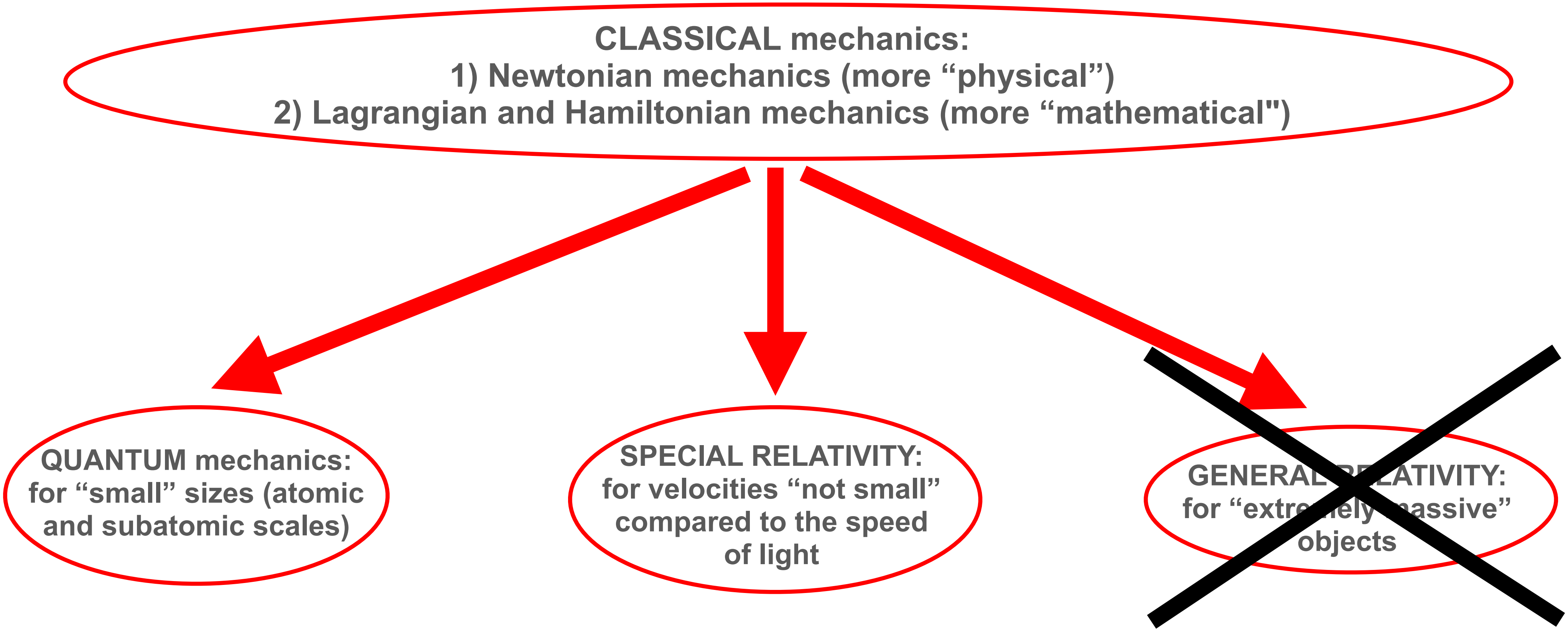

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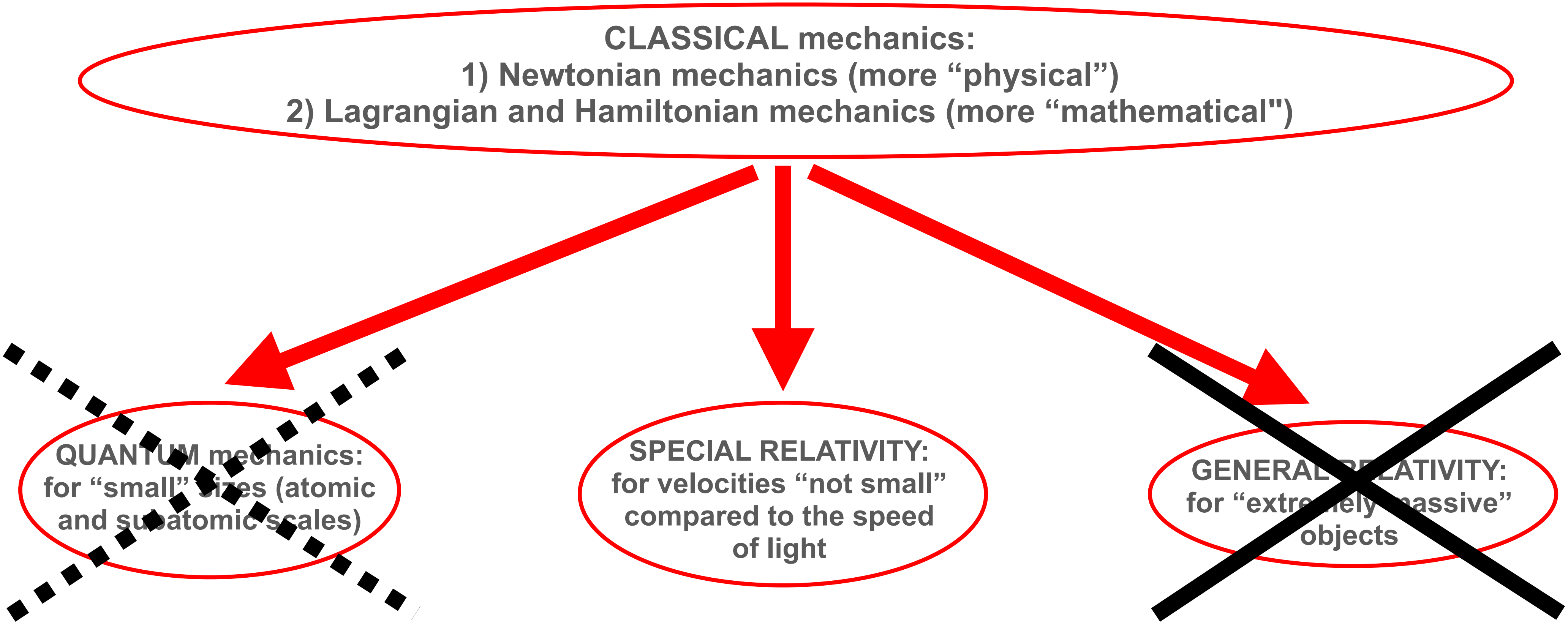
Note that since some time some people want to test general relativity on particle accelerators

Classical mechanics



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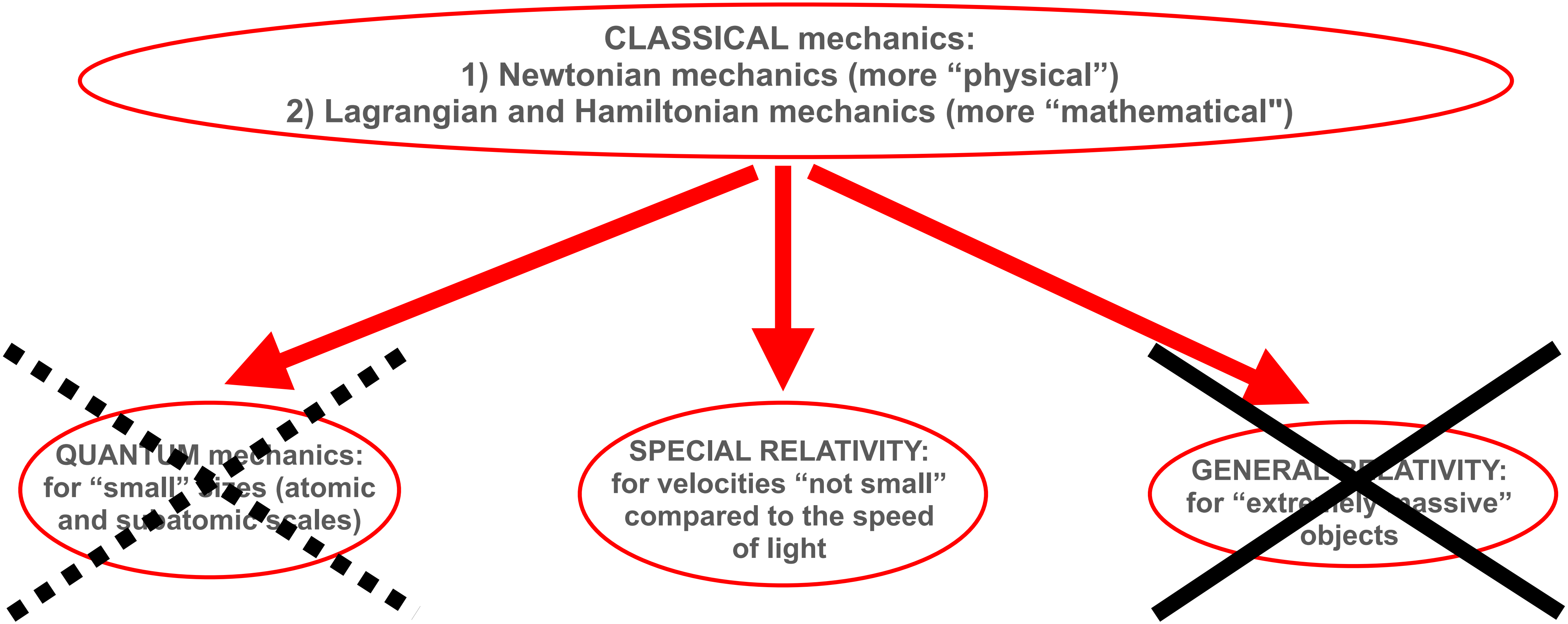

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- For most purposes, the particles can be seen as “hard points” and their motion treated with classical point mechanics (due to the fact that the de Broglie wavelengths of accelerated particles are very small compared to the size of accelerator structures)

Classical mechanics

CLASSICAL mechanics:
1) Newtonian mechanics (more "physical")
2) Lagrangian and Hamiltonian mechanics (more "mathematical")

~~**QUANTUM mechanics: for "small" sizes (atomic and subatomic scales)**~~

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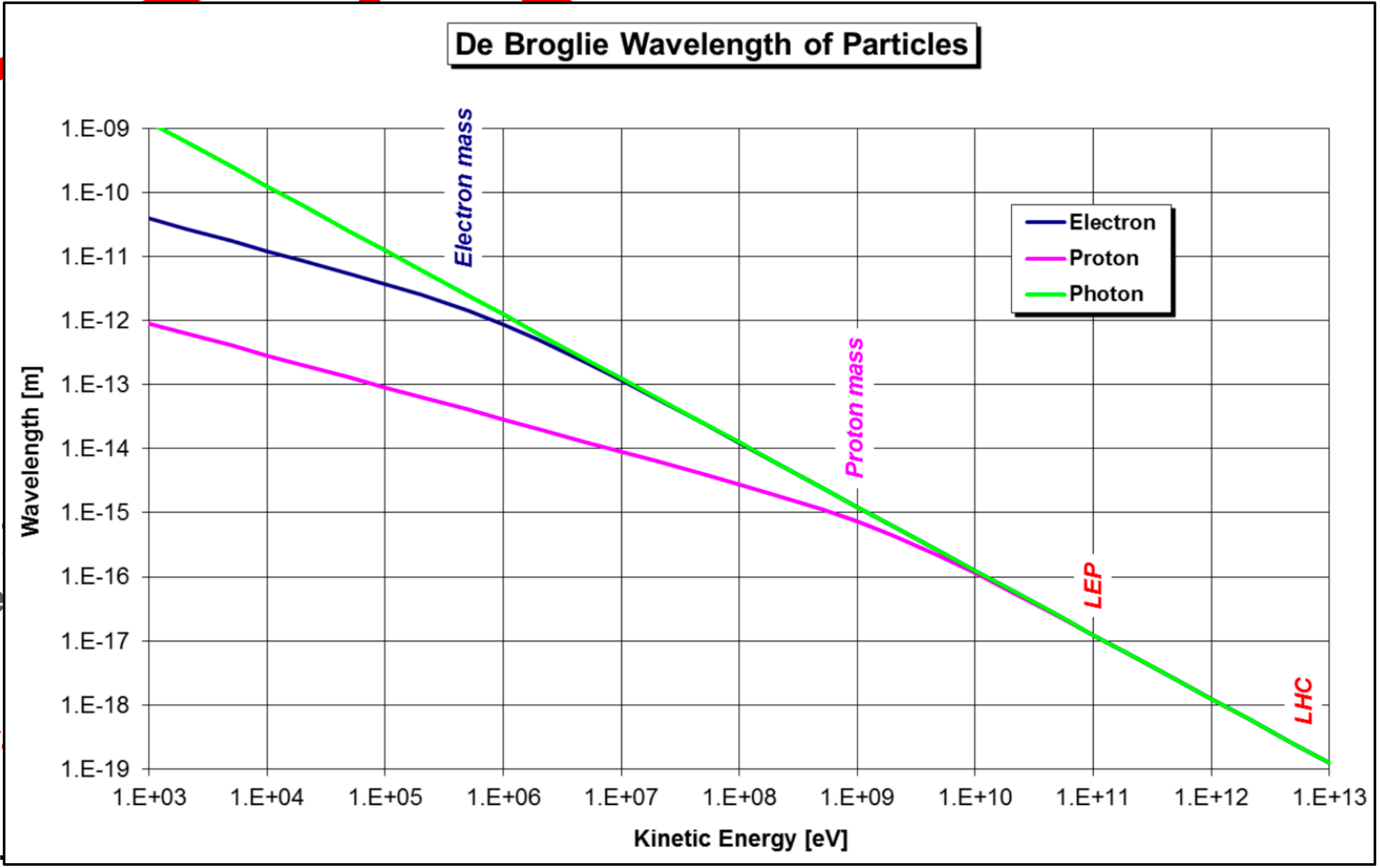

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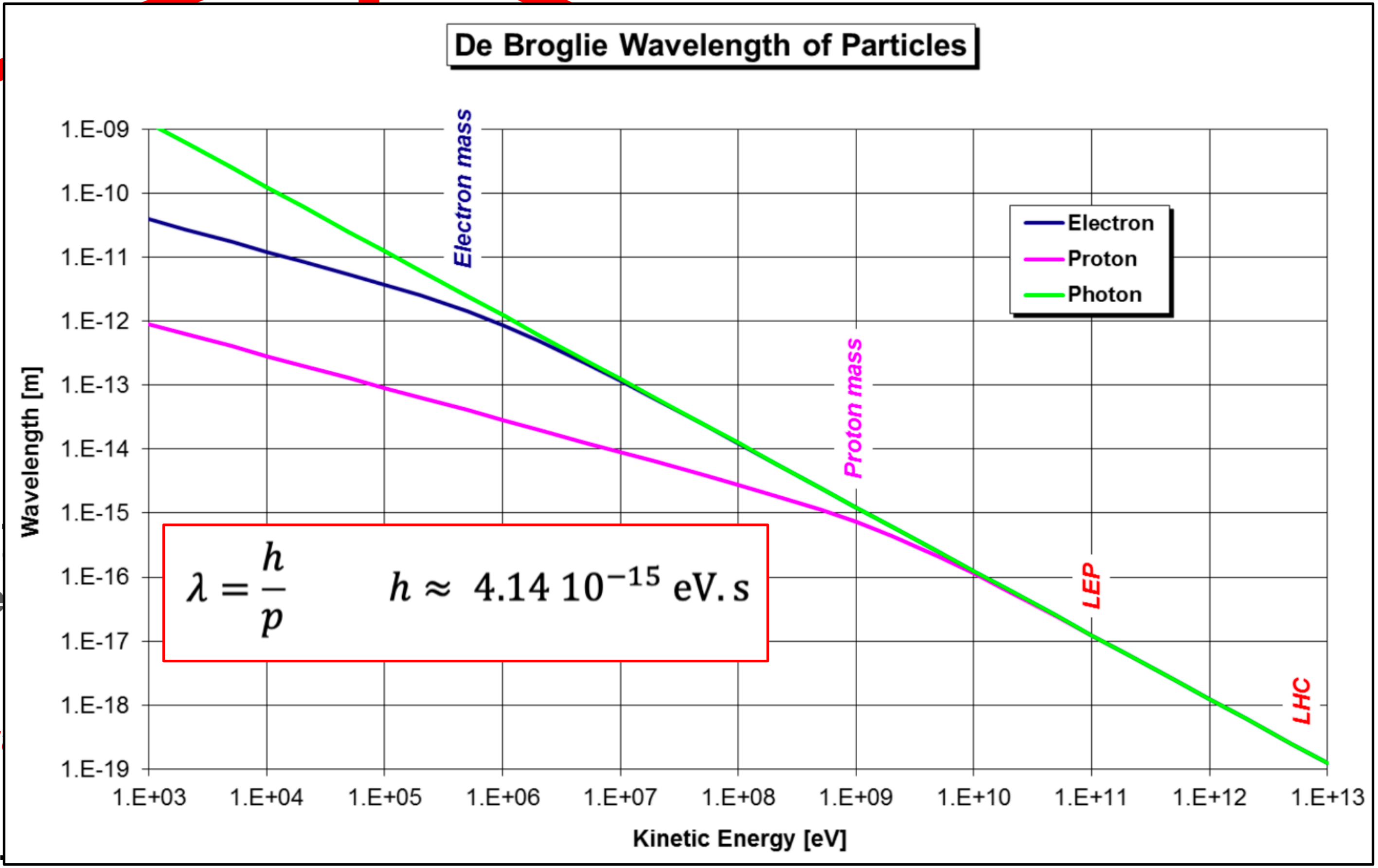

$$\lambda = \frac{h}{p} \qquad h \approx 4.14\,10^{-15}\ \mathrm{eV.s}$$

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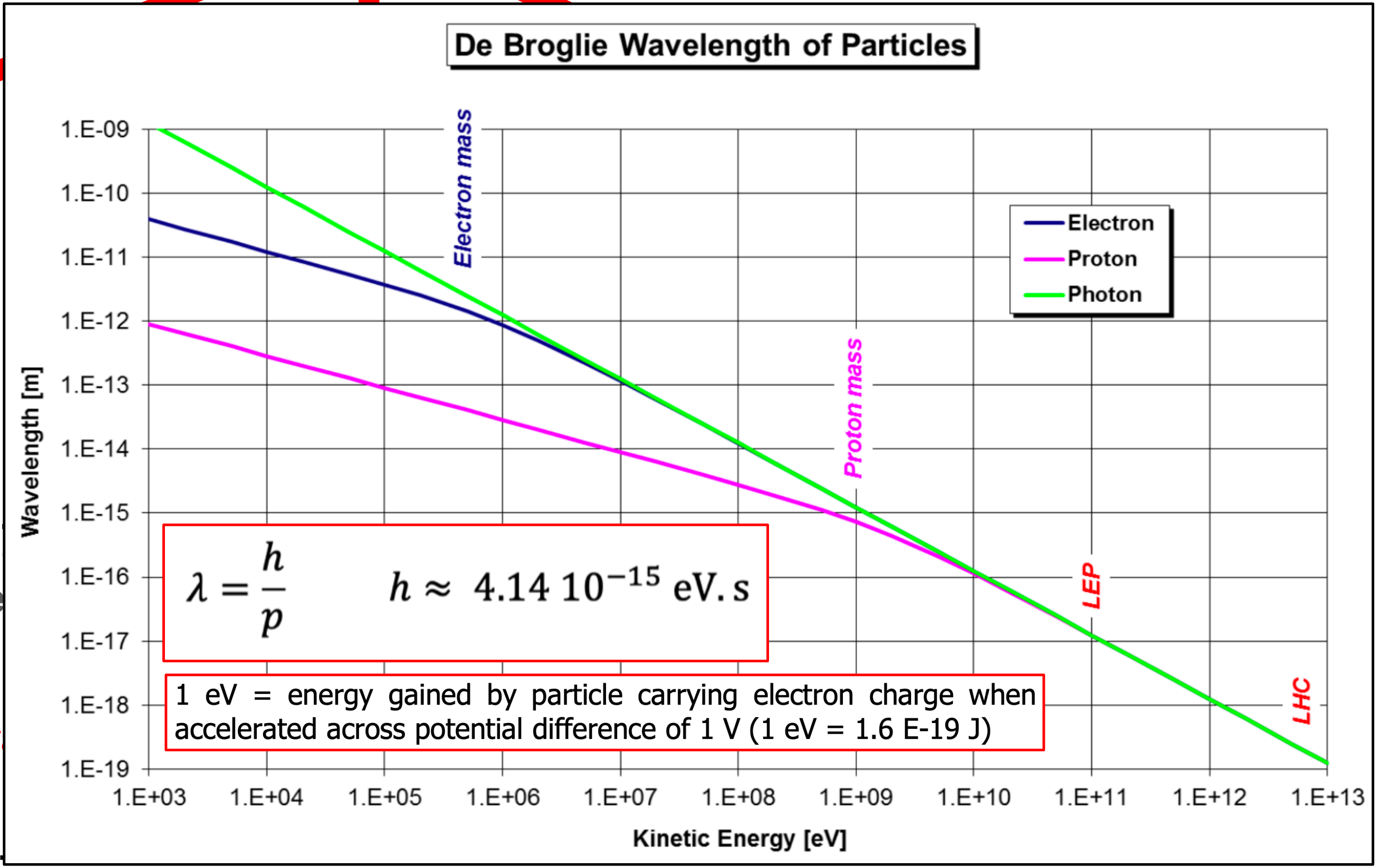
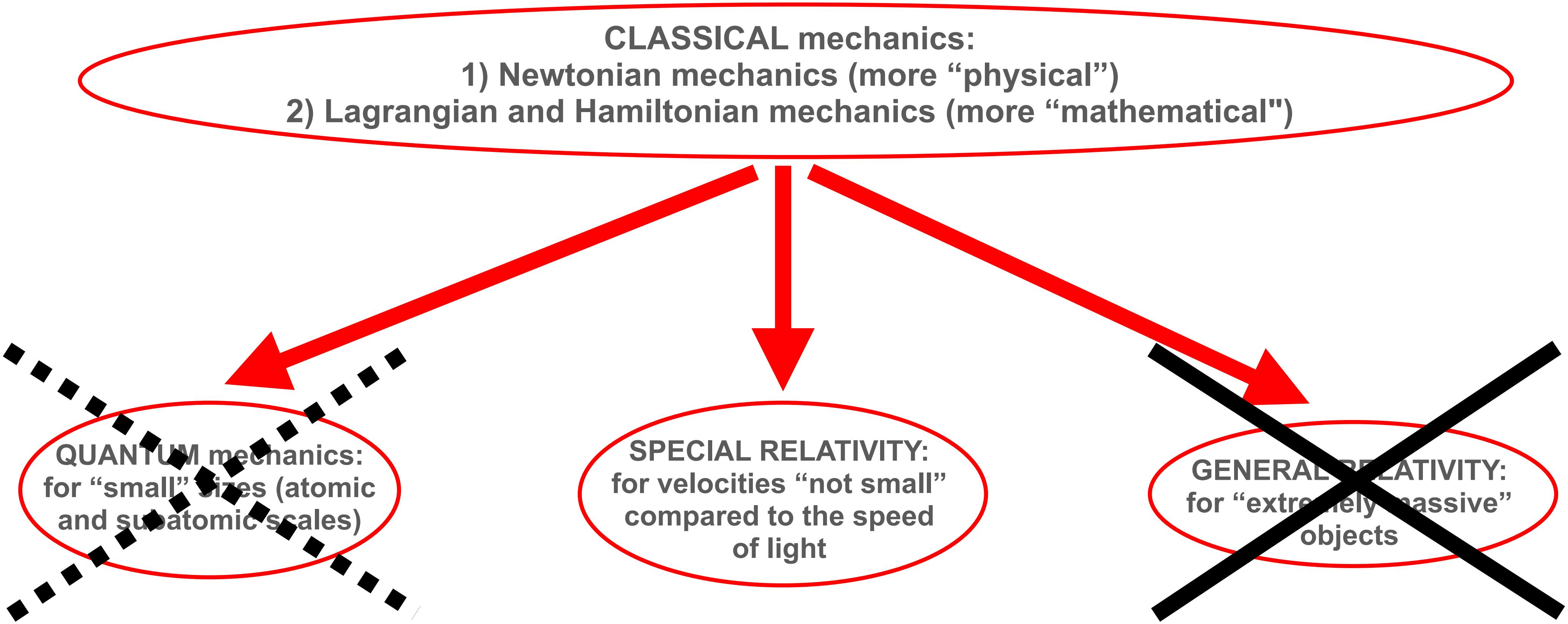


$$\lambda = \frac{h}{p} \qquad h \approx 4.14\, 10^{-15}\ \mathrm{eV.s}$$

1 eV = energy gained by particle carrying electron charge when accelerated across potential difference of 1 V (1 eV = 1.6 E-19 J)

Classical mechanics



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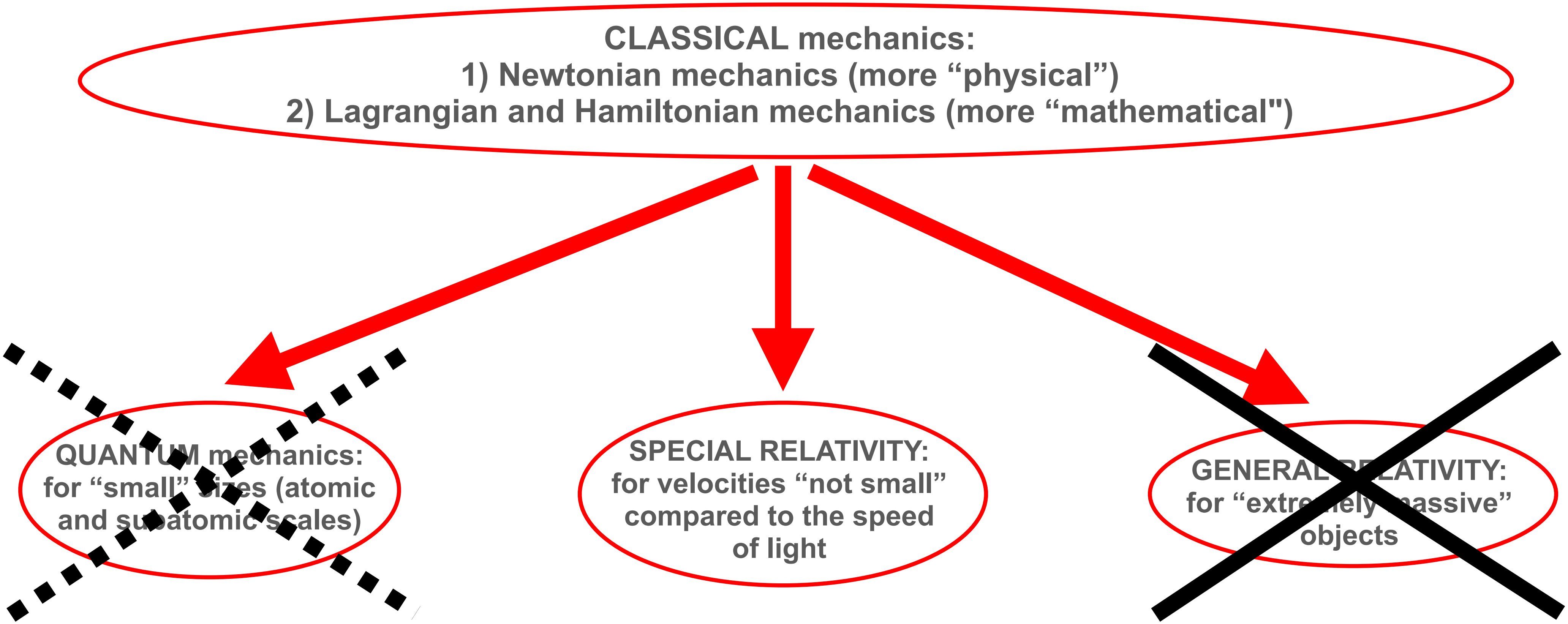

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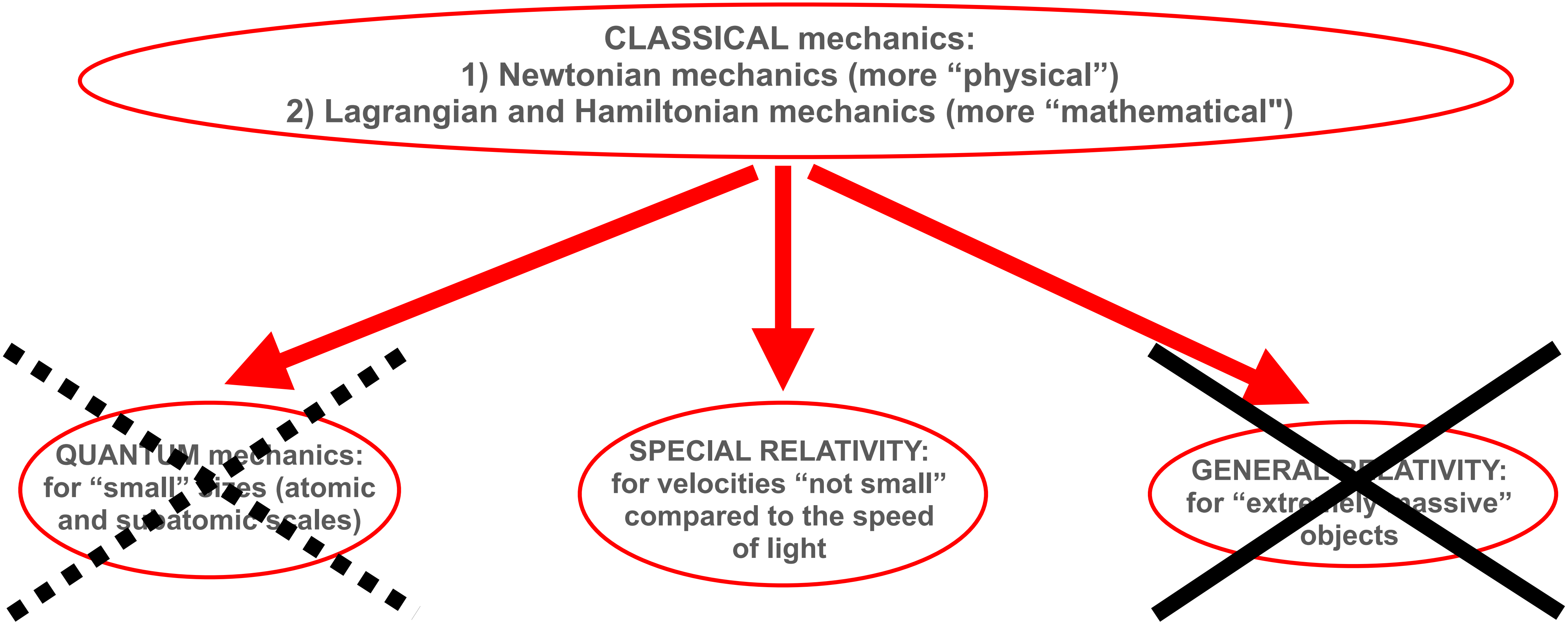
=> See for instance "Quantum aspects of beam physics" from 1999 (https://accelconf.web.cern.ch/p99/PAPERS/TUCR1.PDF)

Classical mechanics



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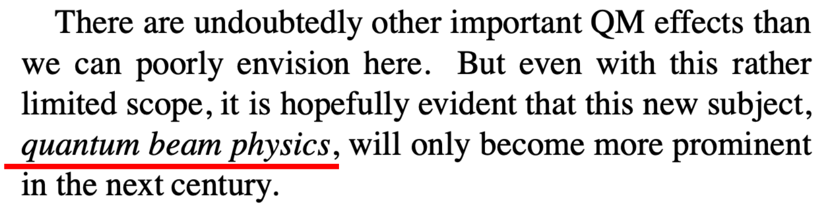
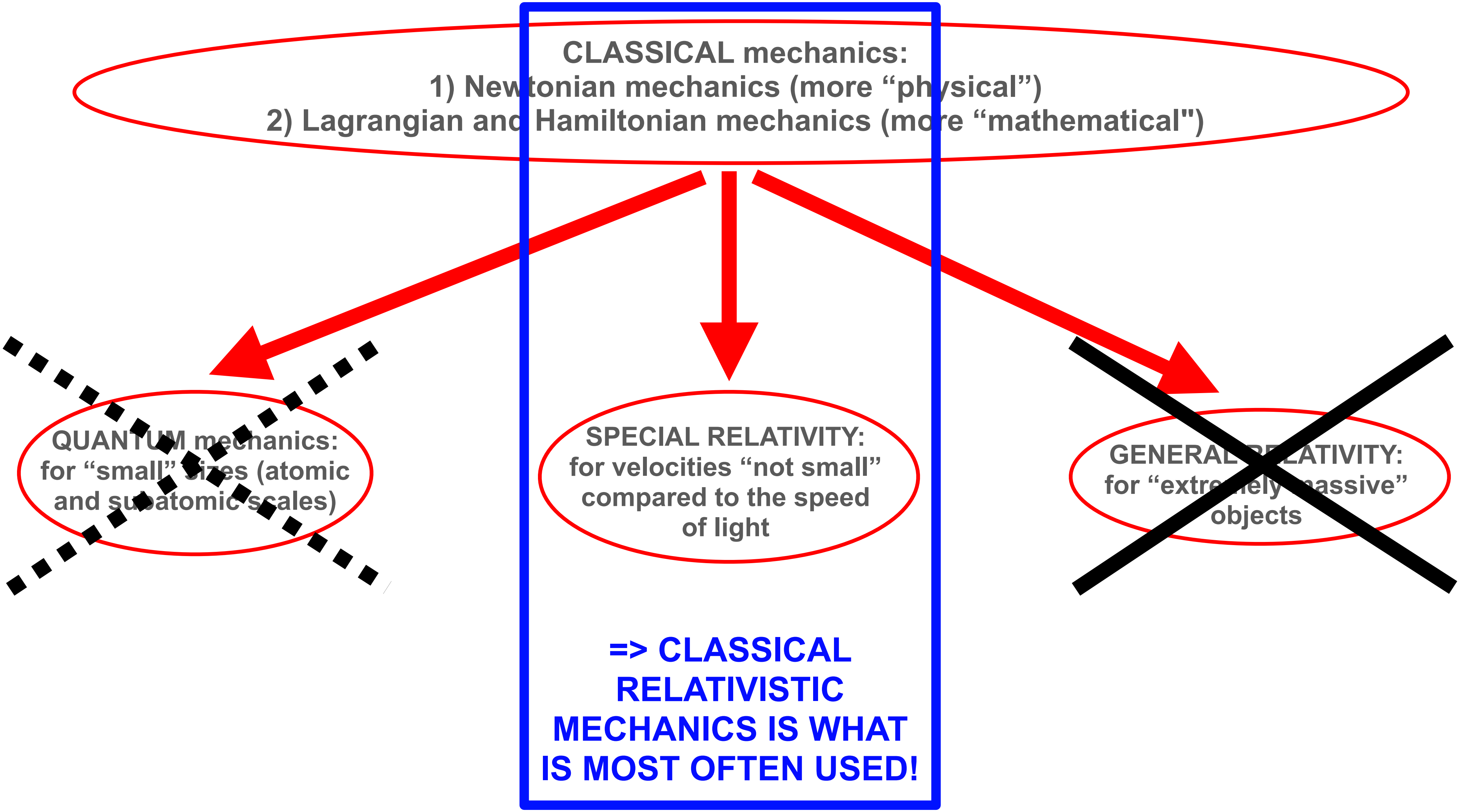


There are undoubtedly other important QM effects than we can poorly envision here. But even with this rather limited scope, it is hopefully evident that this new subject, *quantum beam physics*, will only become more prominent in the next century.

Classical mechanics



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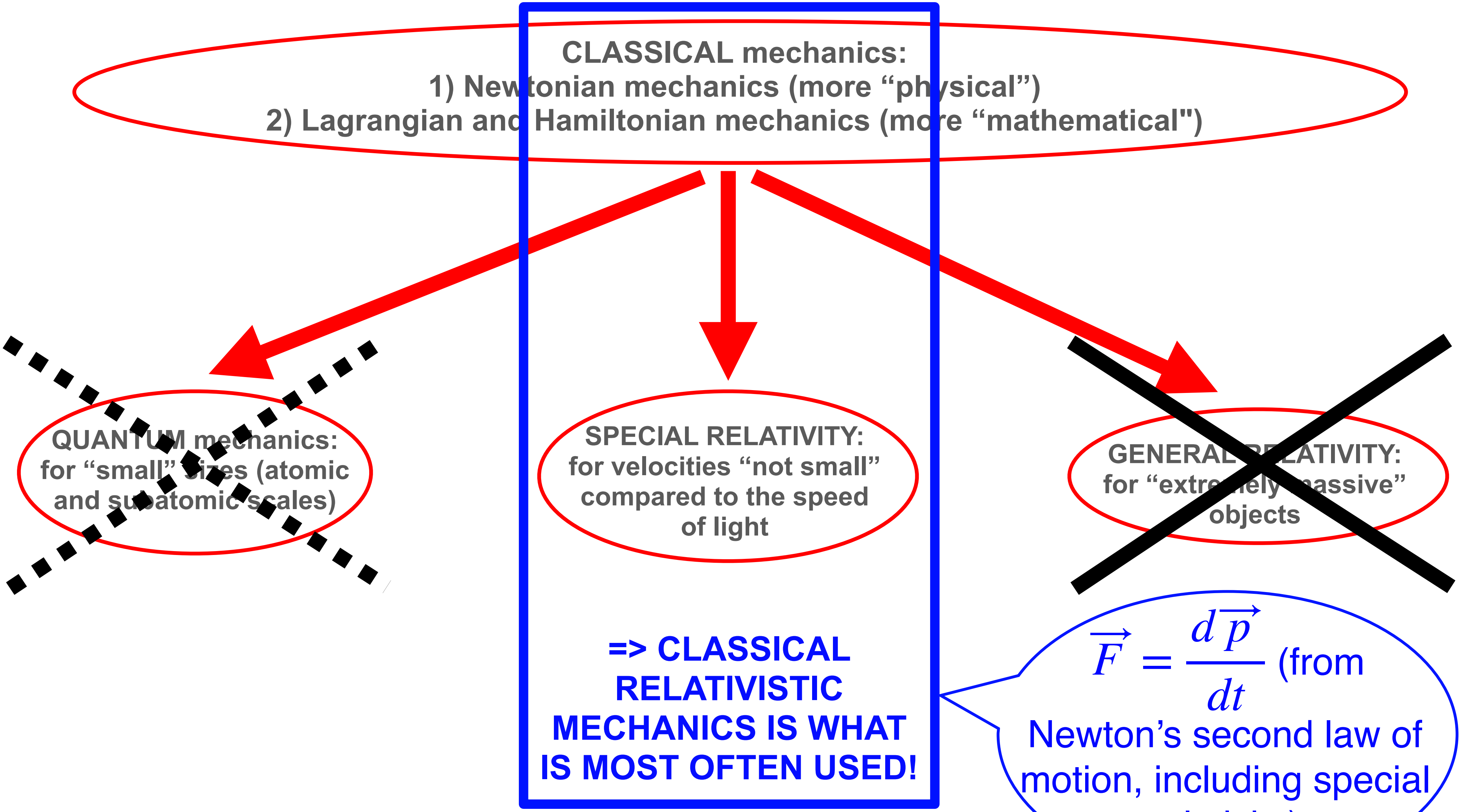
=> CLASSICAL RELATIVISTIC MECHANICS IS WHAT IS MOST OFTEN USED!

Classical mechanics



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=> CLASSICAL RELATIVISTIC MECHANICS IS WHAT IS MOST OFTEN USED!

$$\vec{F} = \frac{d\,\vec{p}}{dt}$$ (from Newton's second law of motion, including special relativity)

Particle accelerators

- Particle accelerators are devices that handle the motion of particles by means of **EM fields**

Particle accelerators

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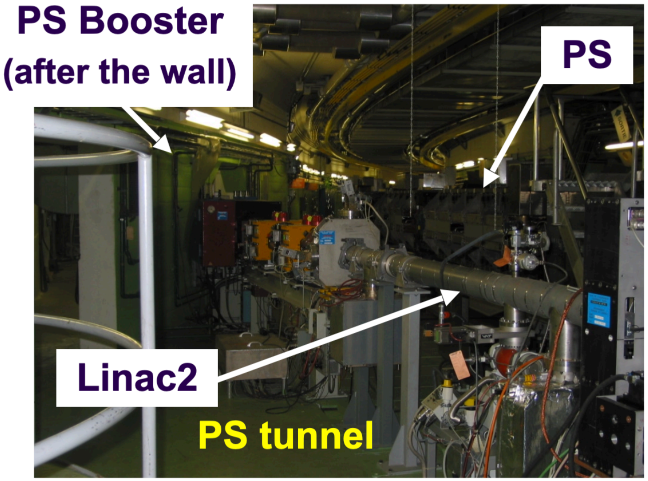
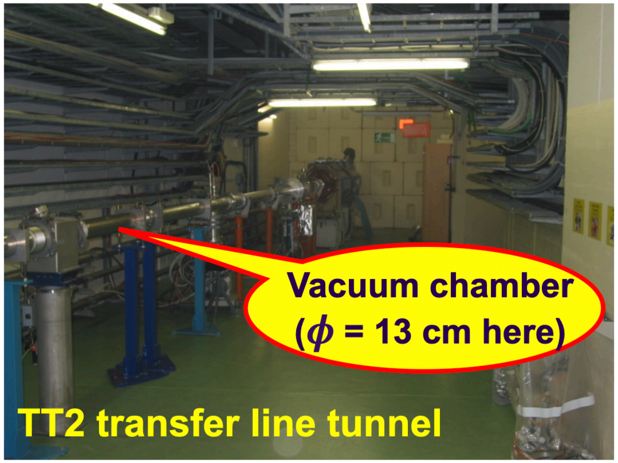
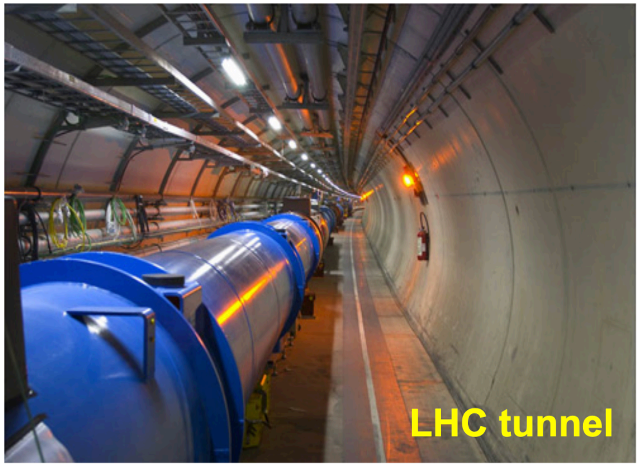
Example of some particle accelerators from CERN







Particle accelerators

- 3 conditions must be satisfied: **which ones?**

Particle accelerators

- 3 conditions must be satisfied
  - **Charged particles** (e.g. **p+**, **e-**, **ions** or **anti-particles**)

Particle accelerators

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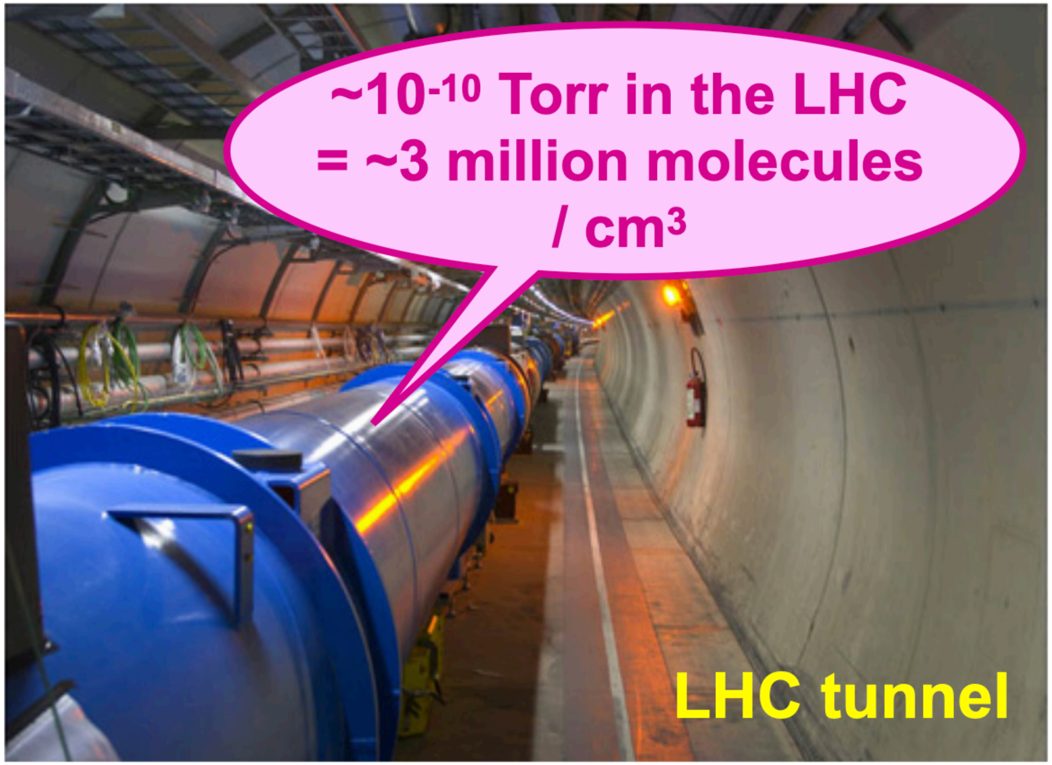

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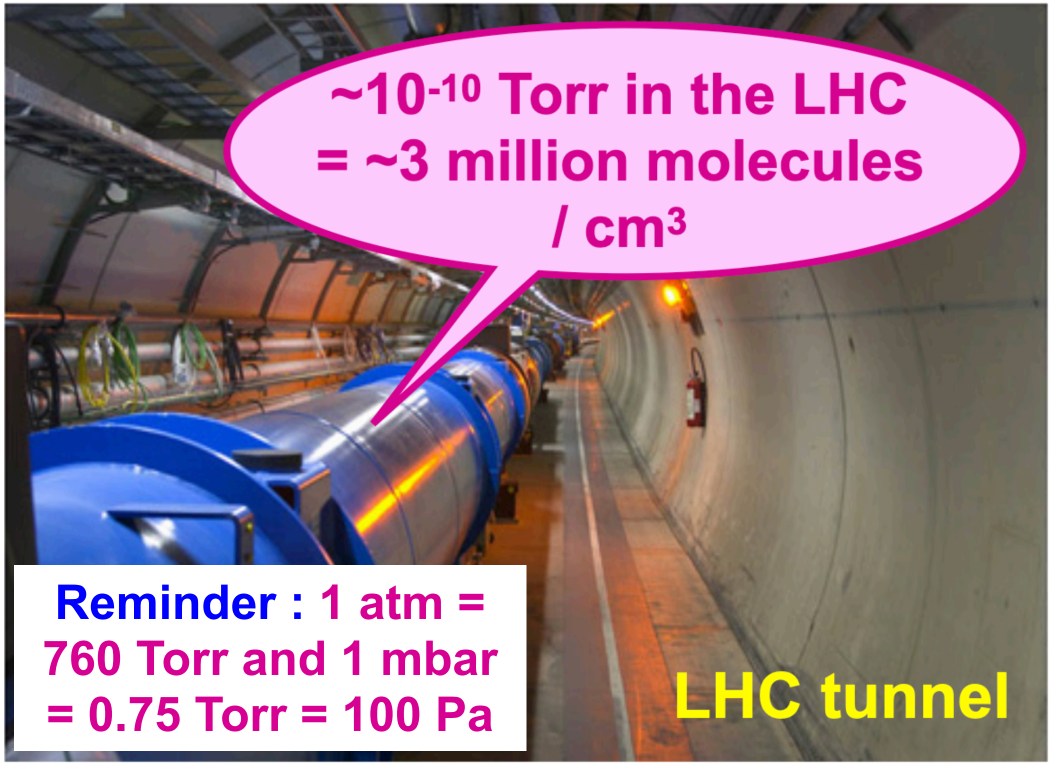

~10^{-10} Torr in the LHC = ~3 million molecules / cm^3

Reminder : 1 atm = 760 Torr and 1 mbar = 0.75 Torr = 100 Pa

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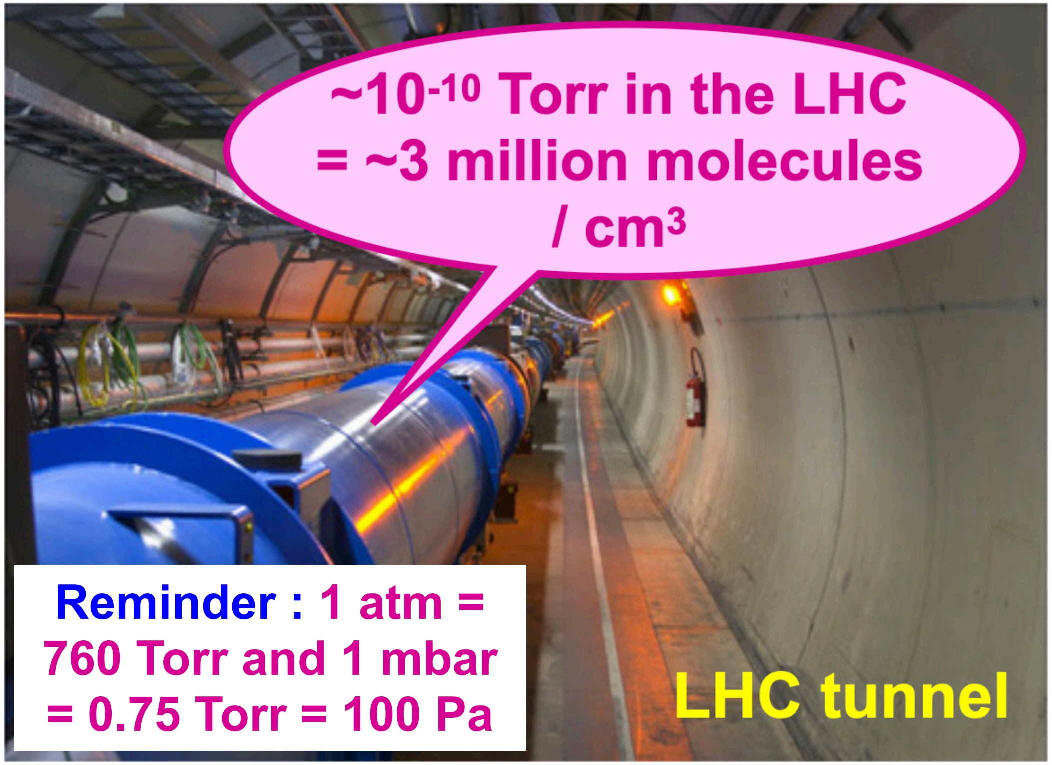

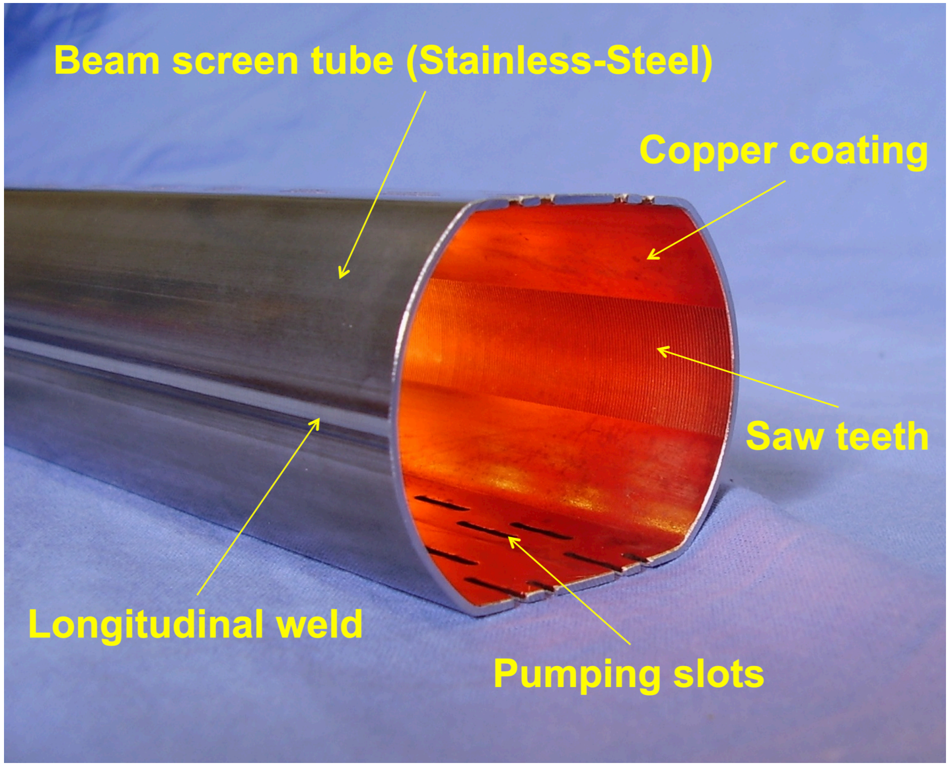

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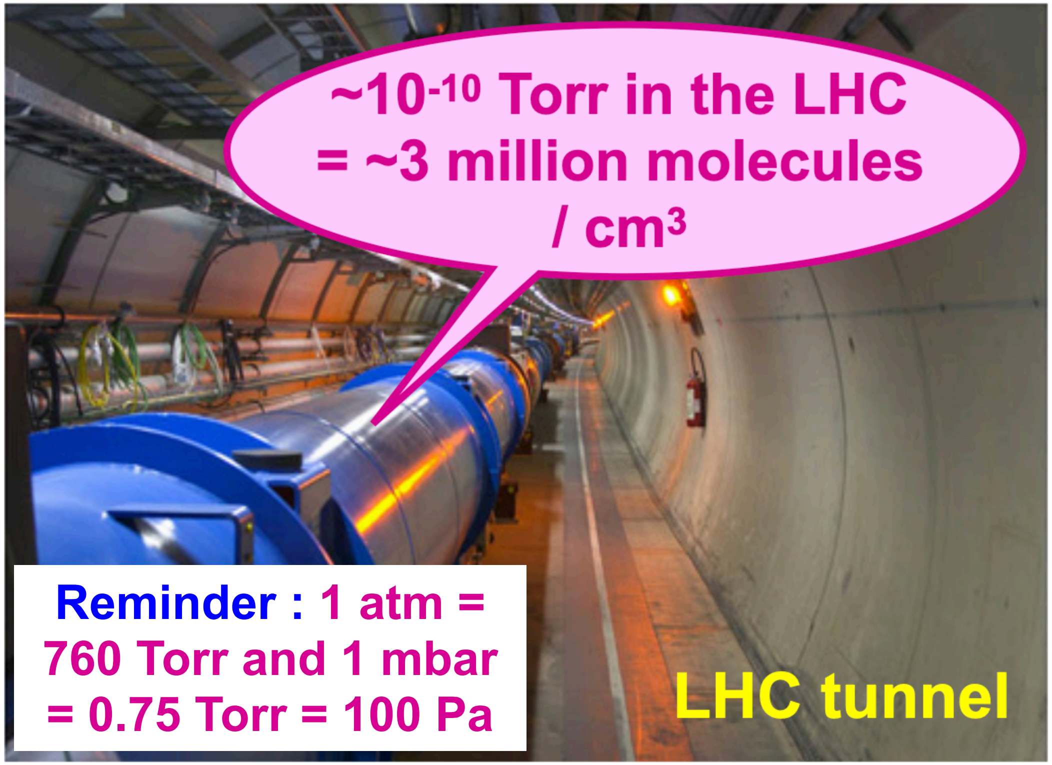

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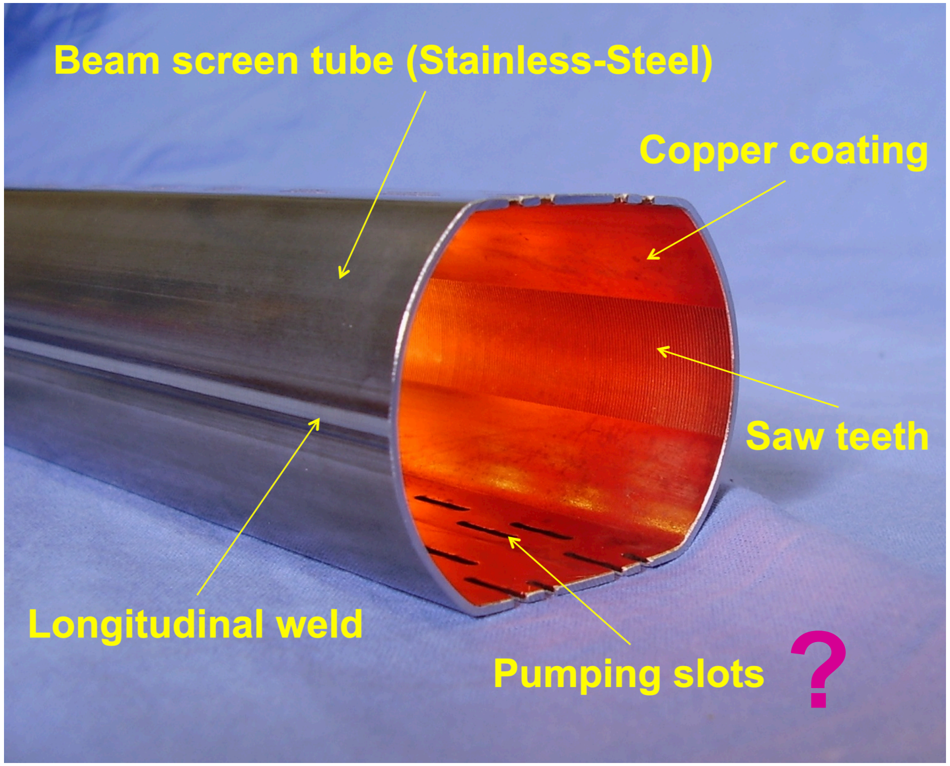

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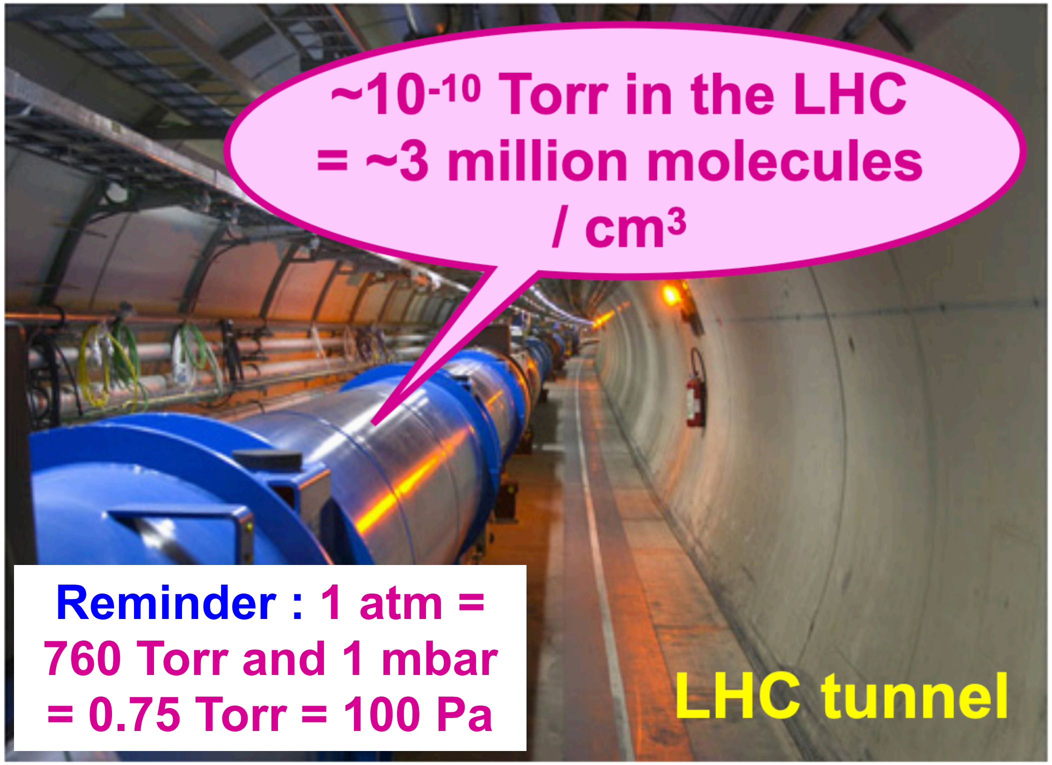

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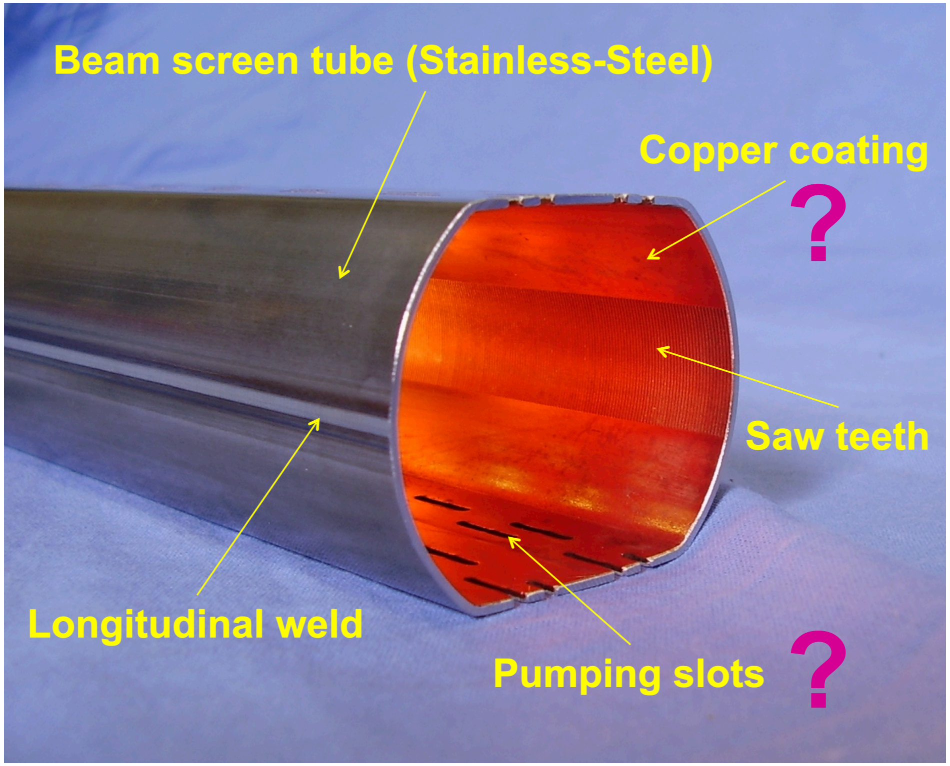

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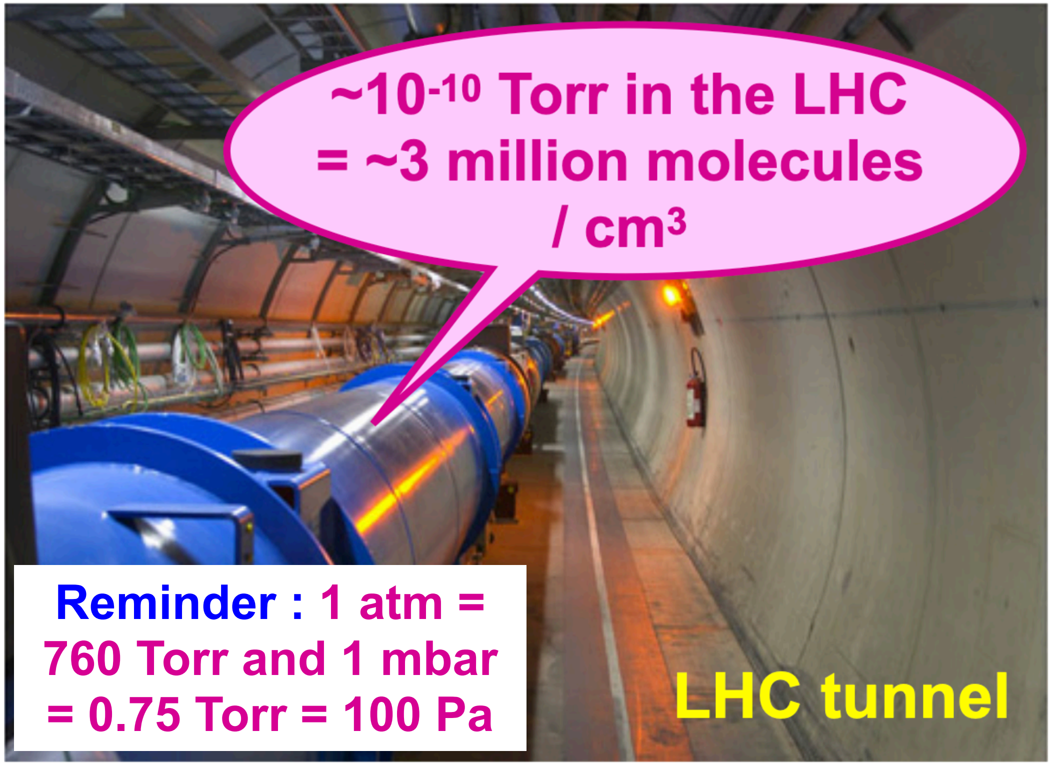
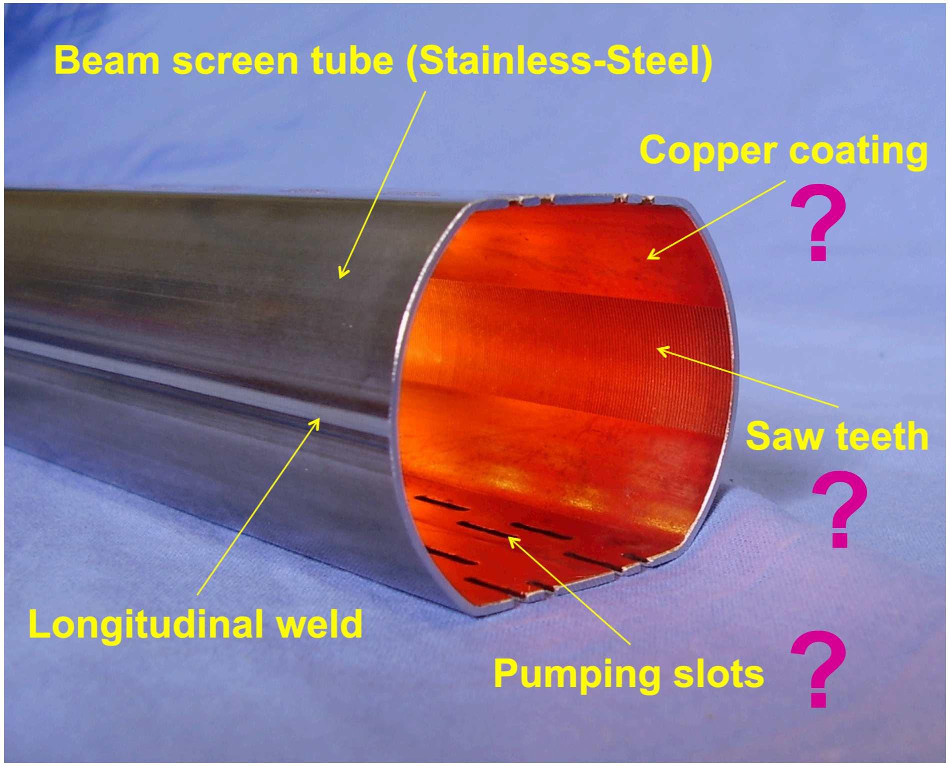


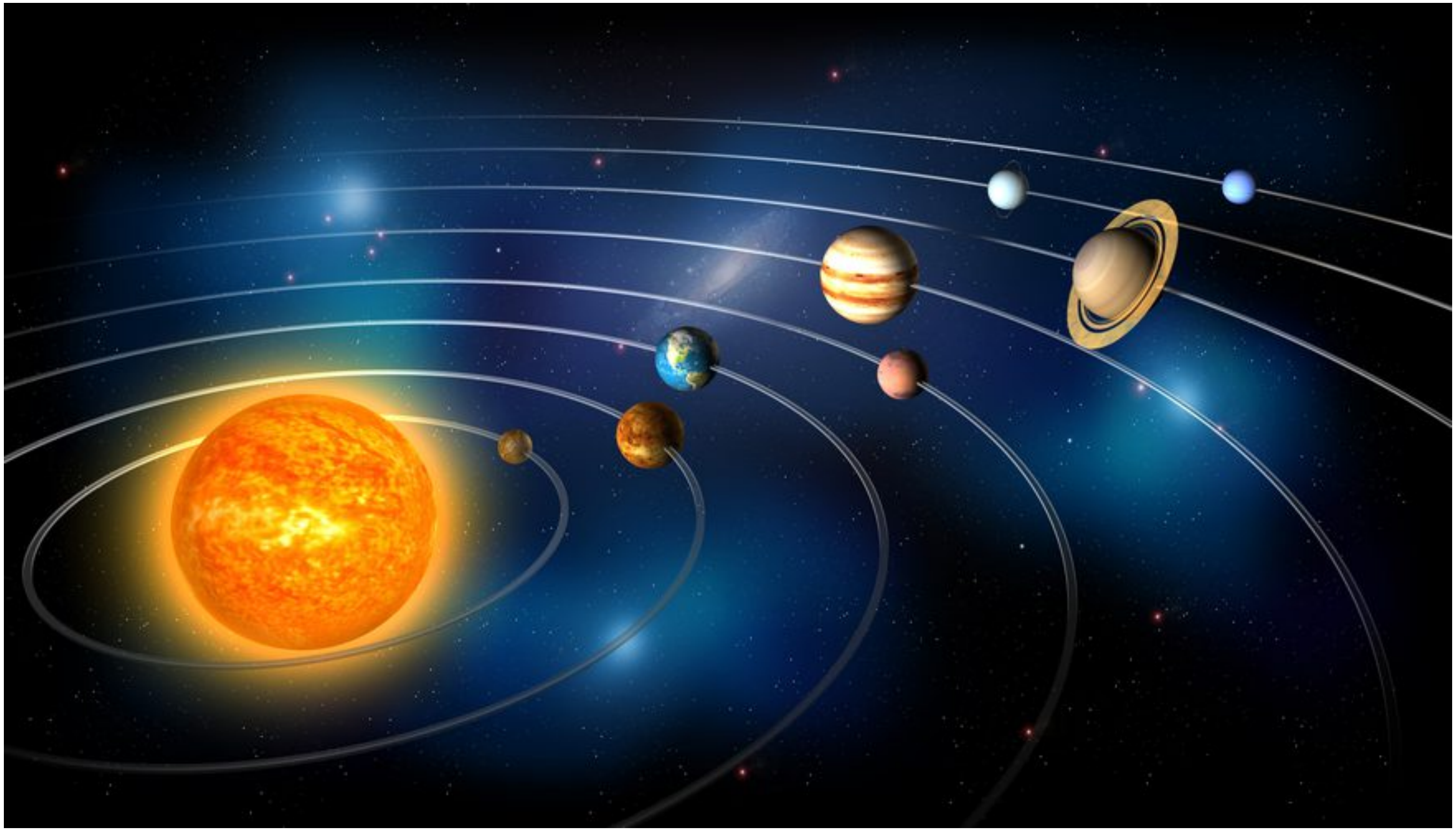


Particle accelerators

- TRICK of particle accelerators: ?

Particle accelerators

- **TRICK of particle accelerators**: the best way to keep something (here particles) under control (i.e. stable) is to **make it oscillate**! And this is what we are doing…in the 3 planes



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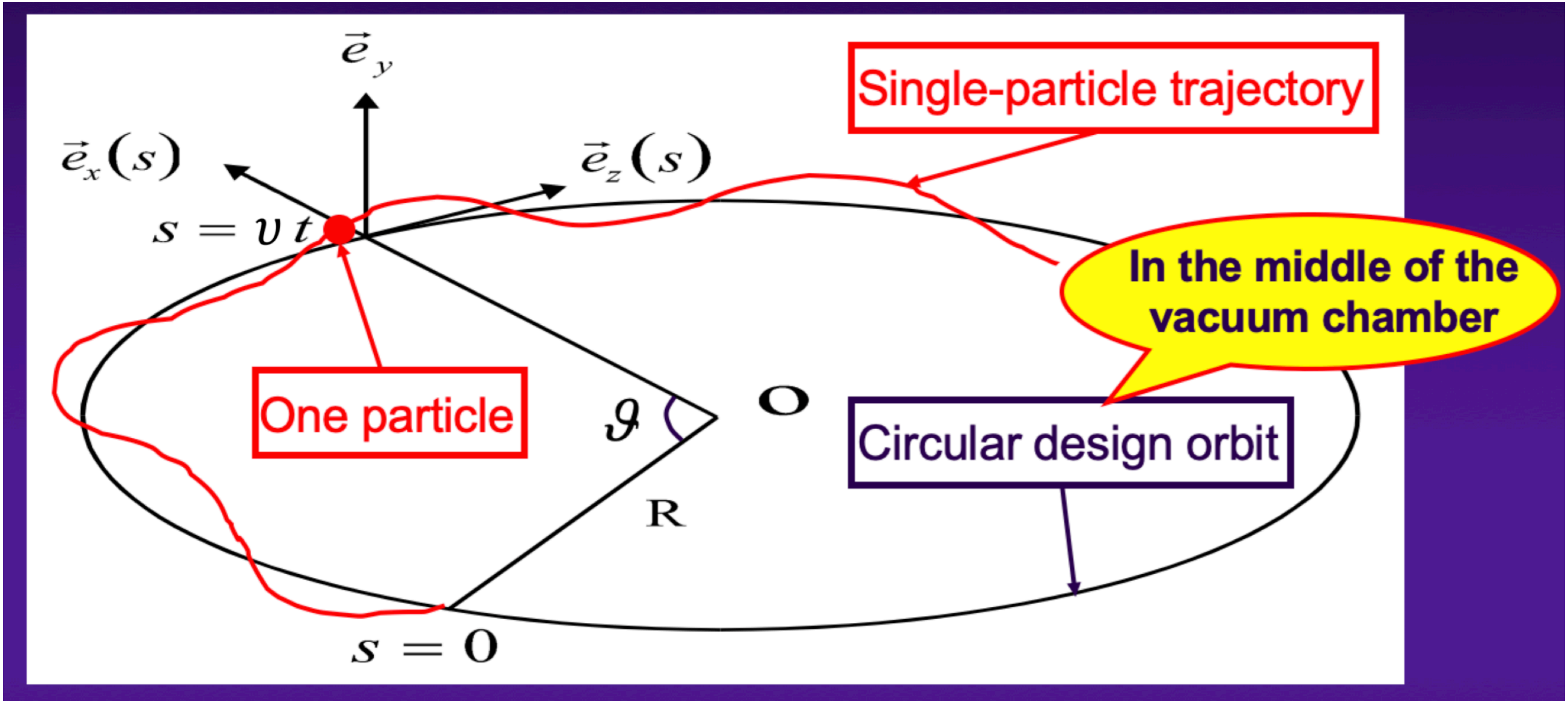


Single-particle trajectory

One particle

In the middle of the vacuum chamber

Circular design orbit

Case here of a “synchrotron”

$$1\ \mathrm{eV} = 1.6\ \times\ 10^{-19}\ \mathrm{J}$$

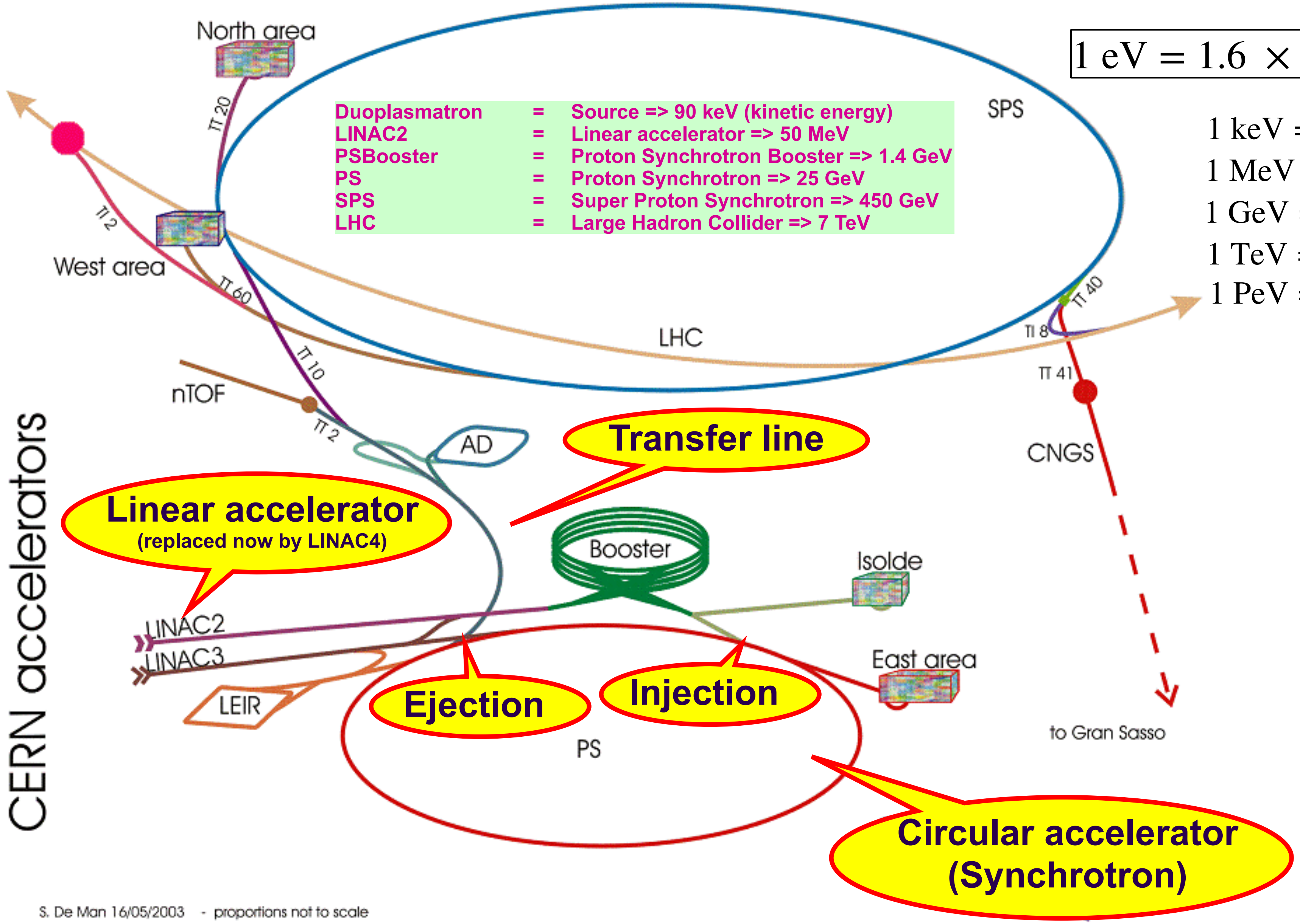
$$1\ \mathrm{keV} = 10^{3}\ \mathrm{eV}$$

$$1\ \mathrm{MeV} = 10^{6}\ \mathrm{eV}$$

$$1\ \mathrm{GeV} = 10^{9}\ \mathrm{eV}$$

$$1\ \mathrm{TeV} = 10^{12}\ \mathrm{eV}$$

$$1\ \mathrm{PeV} = 10^{15}\ \mathrm{eV}$$



Duoplasmatron = Source => 90 keV (kinetic energy)
LINAC2 = Linear accelerator => 50 MeV
PSBooster = Proton Synchrotron Booster => 1.4 GeV
PS = Proton Synchrotron => 25 GeV
SPS = Super Proton Synchrotron => 450 GeV
LHC = Large Hadron Collider => 7 TeV

Transfer line

Linear accelerator
(replaced now by LINAC4)

Ejection

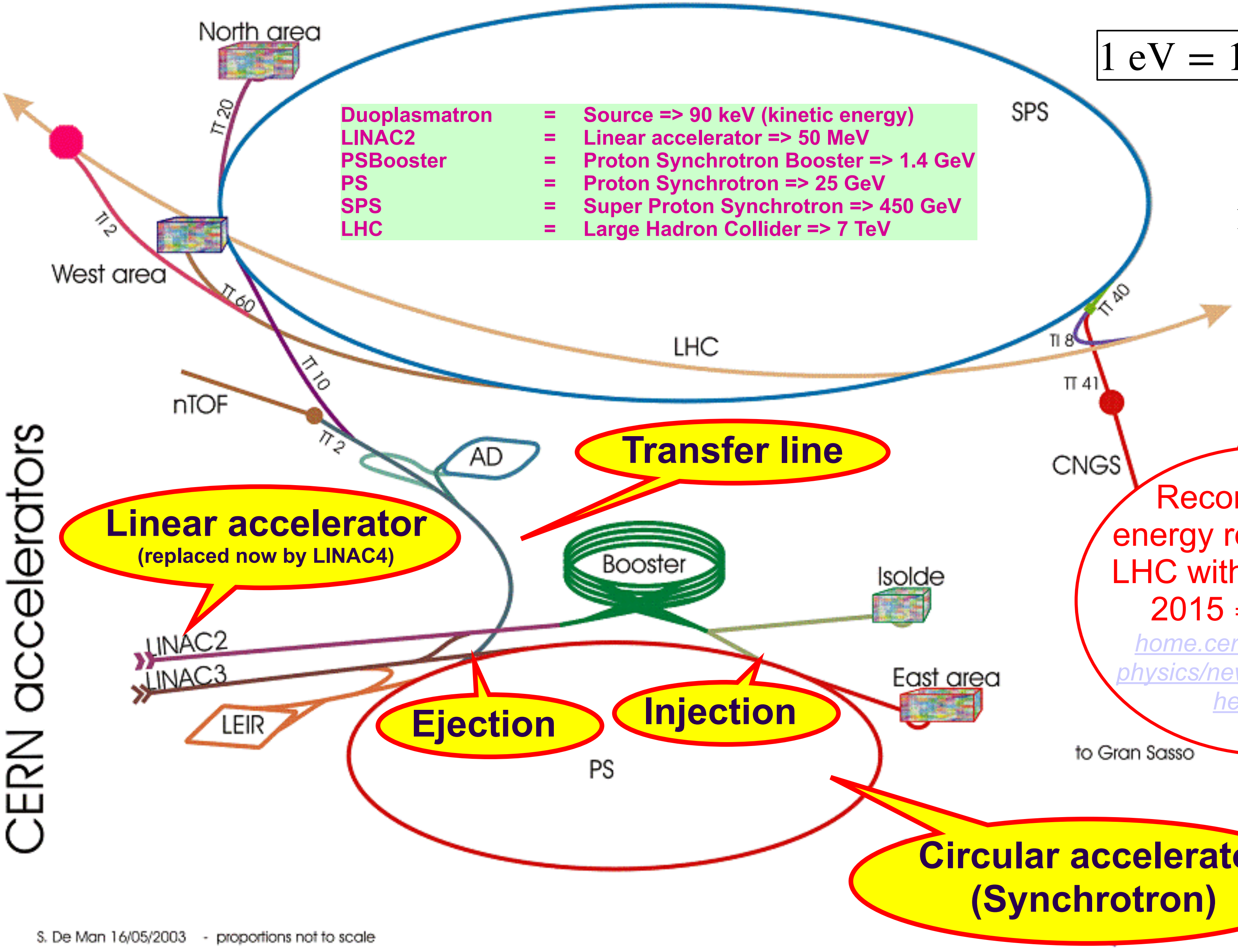
Injection

Circular accelerator (Synchrotron)

S. De Man 16/05/2003 - proportions not to scale

LHC proton beam in the injector chain

LHC proton beam in the injector chain



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Transfer line

Linear accelerator
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Injection

Circular accelerator (Synchrotron)

Record collision energy reached in the LHC with Lead ions in 2015 => *See* https://home.cern/news/opinion/physics/new-energy-frontier-heavy-ions

S. De Man 16/05/2003 - proportions not to scale

$$\boxed{1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}}$$

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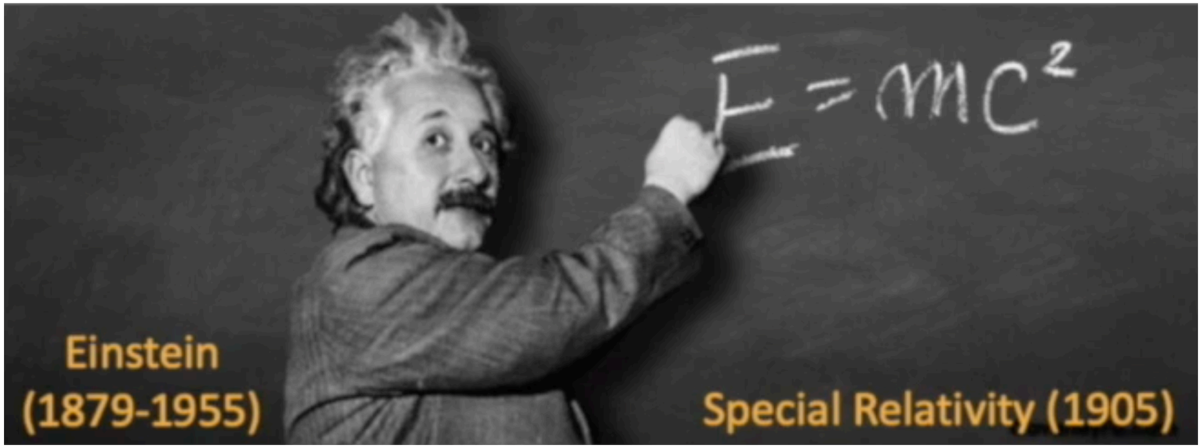
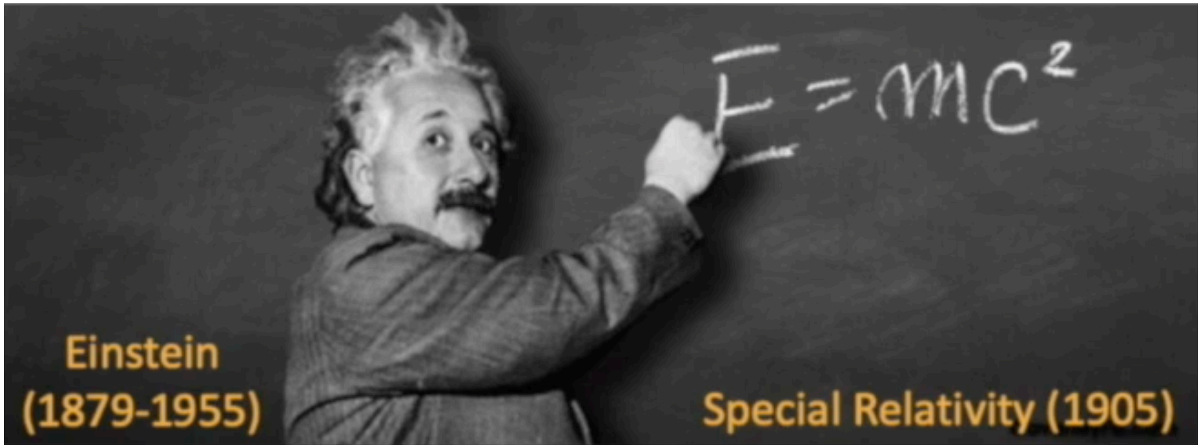
$1 \text{ TeV} = 10^{12} \text{ eV}$
$1 \text{ PeV} = 10^{15} \text{ eV}$

Special Relativity

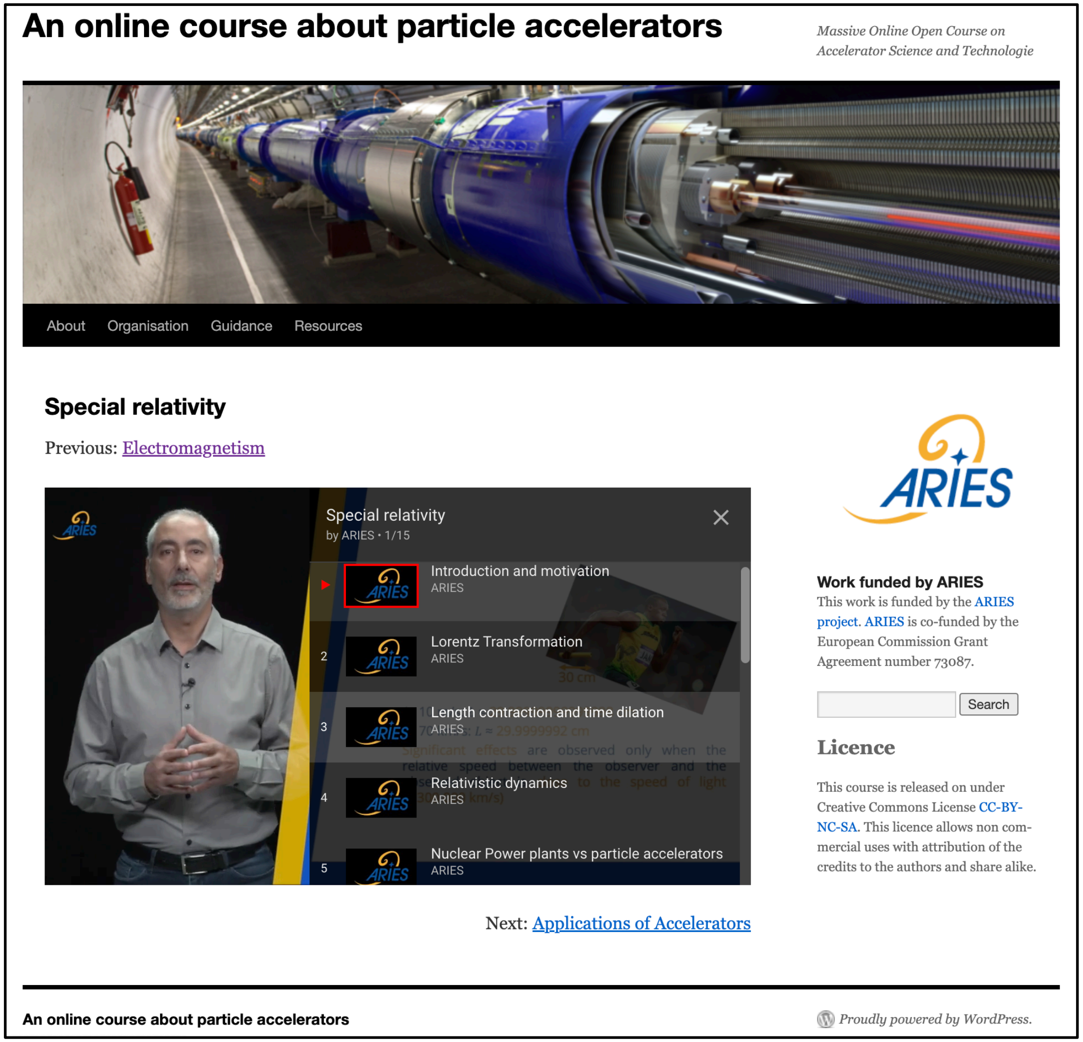


Special Relativity



=> See **MOOC** (Massive Open Online Course) **on Special Relativity (SR)**: http://mooc.particle-accelerators.eu/special-relativity/

Special Relativity



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Special relativity

Previous: Electromagnetism

Special relativity
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1. Introduction and motivation — ARIES
2. Lorentz Transformation — ARIES
3. Length contraction and time dilation — ARIES
4. Relativistic dynamics — ARIES
5. Nuclear Power plants vs particle accelerators — ARIES

Next: Applications of Accelerators

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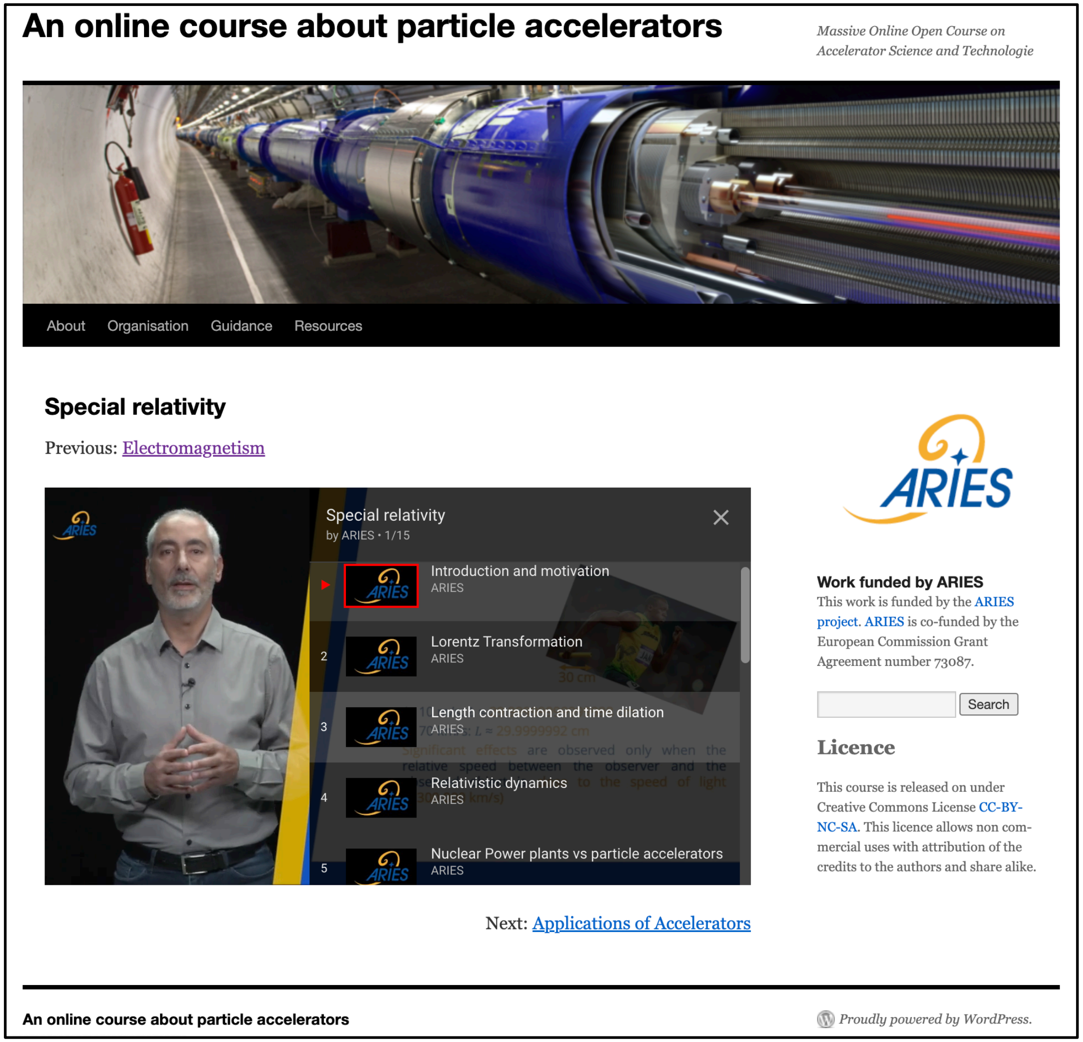
Erratum: in concept 5, at t=1:27′, the term (m01 + m02) should be replaced by “invariant mass”.

Special Relativity



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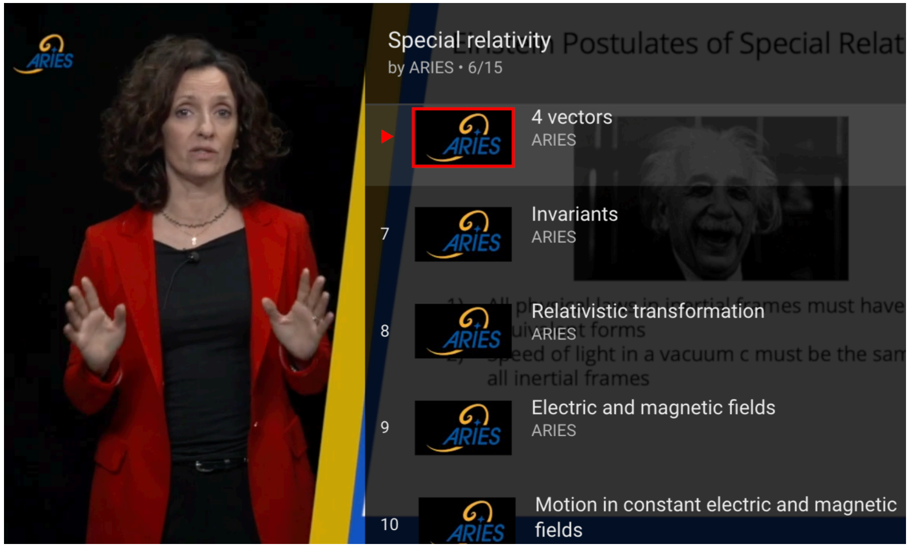

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4 vectors
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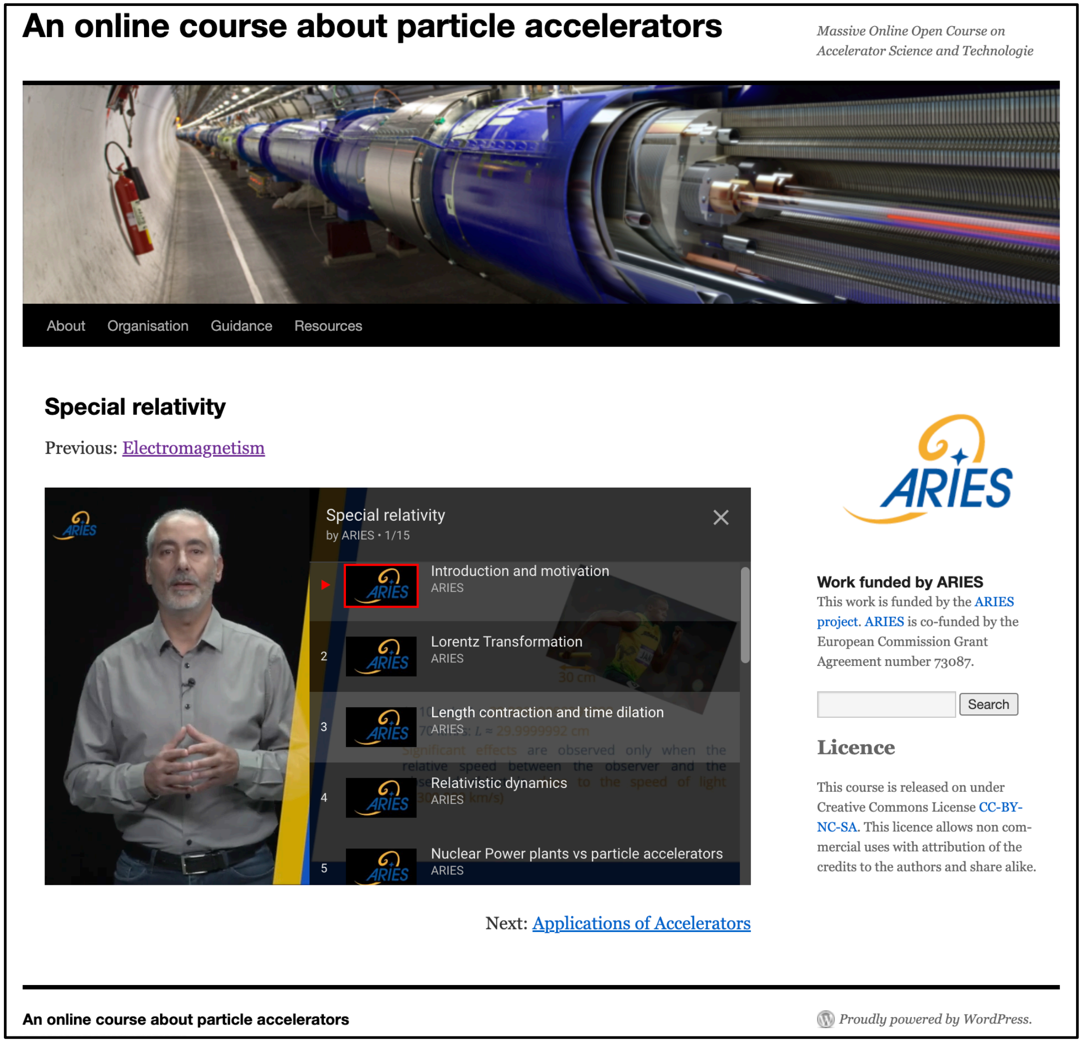
7 Invariants
ARIES

8 Relativistic transformation
ARIES

9 Electric and magnetic fields
ARIES

10 Motion in constant electric and magnetic fields

Special Relativity



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Special relativity

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- Introduction and motivation ARIES
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Next: Applications of Accelerators

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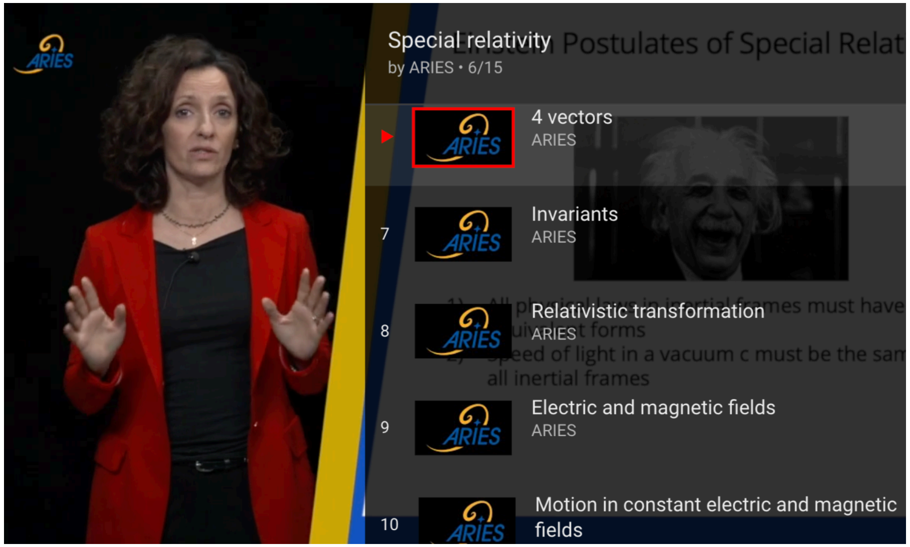

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- 4 vectors ARIES
- 7 Invariants ARIES
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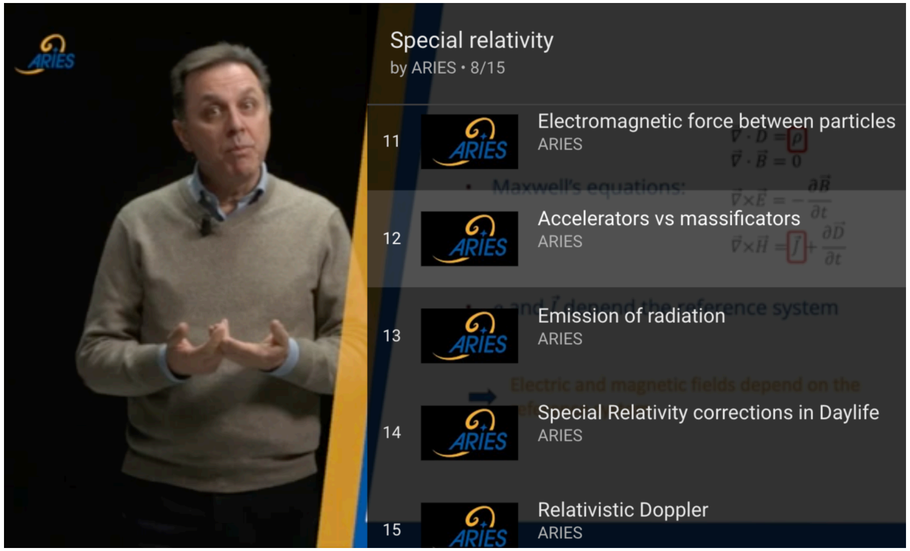
- 10 Motion in constant electric and magnetic fields



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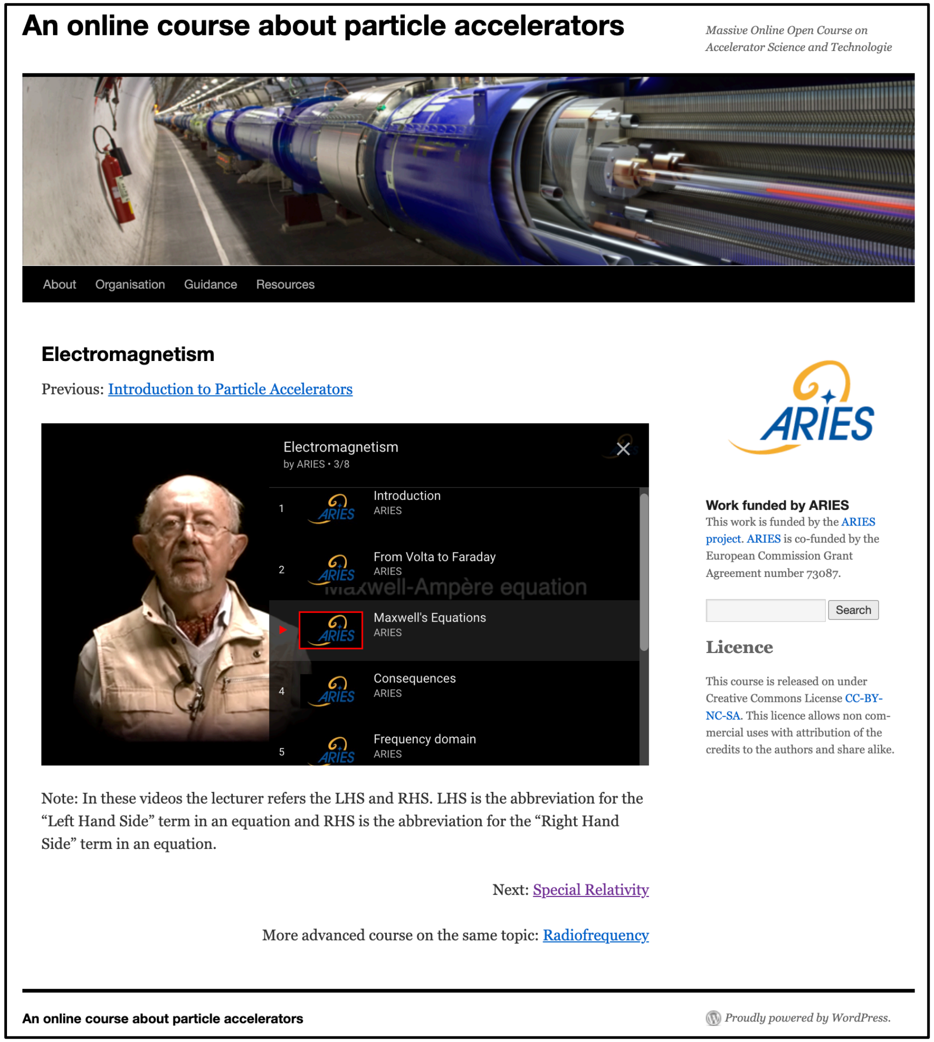
- 11 Electromagnetic force between particles ARIES
- 12 Accelerators vs massificators ARIES
- 13 Emission of radiation ARIES
- 14 Special Relativity corrections in Daylife ARIES
- 15 Relativistic Doppler ARIES

Erratum: in concept 5, at t=1:27′, the term (mo1 + mo2) should be replaced by “invariant mass”.

ElectroMagnetism

=> See **MOOC on ElectroMagnetism**: http://mooc.particle-accelerators.eu/electromagnetism/

ElectroMagnetism



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Electromagnetism

Previous: Introduction to Particle Accelerators

Electromagnetism
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1 Introduction
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2 From Volta to Faraday
ARIES

Maxwell-Ampère equation

Maxwell's Equations
ARIES

4 Consequences
ARIES

5 Frequency domain
ARIES

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Note: In these videos the lecturer refers the LHS and RHS. LHS is the abbreviation for the “Left Hand Side” term in an equation and RHS is the abbreviation for the “Right Hand Side” term in an equation.

Next: Special Relativity

More advanced course on the same topic: Radiofrequency

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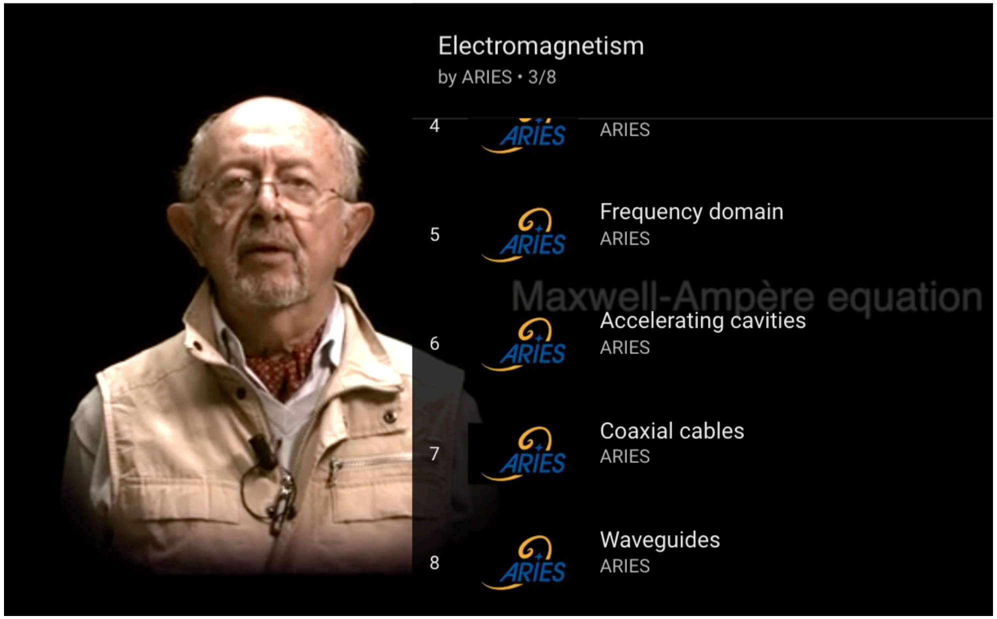
Prepared by Vittorio Vaccaro and Andrea Passarelli.



Electromagnetism
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4 ARIES

5 Frequency domain
ARIES

Maxwell-Ampère equation

6 Accelerating cavities
ARIES

7 Coaxial cables
ARIES

8 Waveguides
ARIES

Results of the quiz on SR (as of this morning)

Course 1

29/30 passed the quiz: congratulations! (only the person coming “à la carte” did not take the quiz)

=> Any comment?

Results of the quiz on EM (as of this morning)

Course 1

29/30 passed the quiz: congratulations! (only the person coming "à la carte" did not take the quiz)

=> Any comment?

Special Relativity

$$\boxed{E = m\,c^2}$$

normalized velocity

$$\beta = \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$$

energy

$$E = E_{kin} + E_0$$

total

kinetic

rest

$$\frac{\text{total energy}}{\text{rest energy}}$$

$$\gamma = \frac{E}{E_0} = \frac{m}{m_0} = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}}$$

momentum

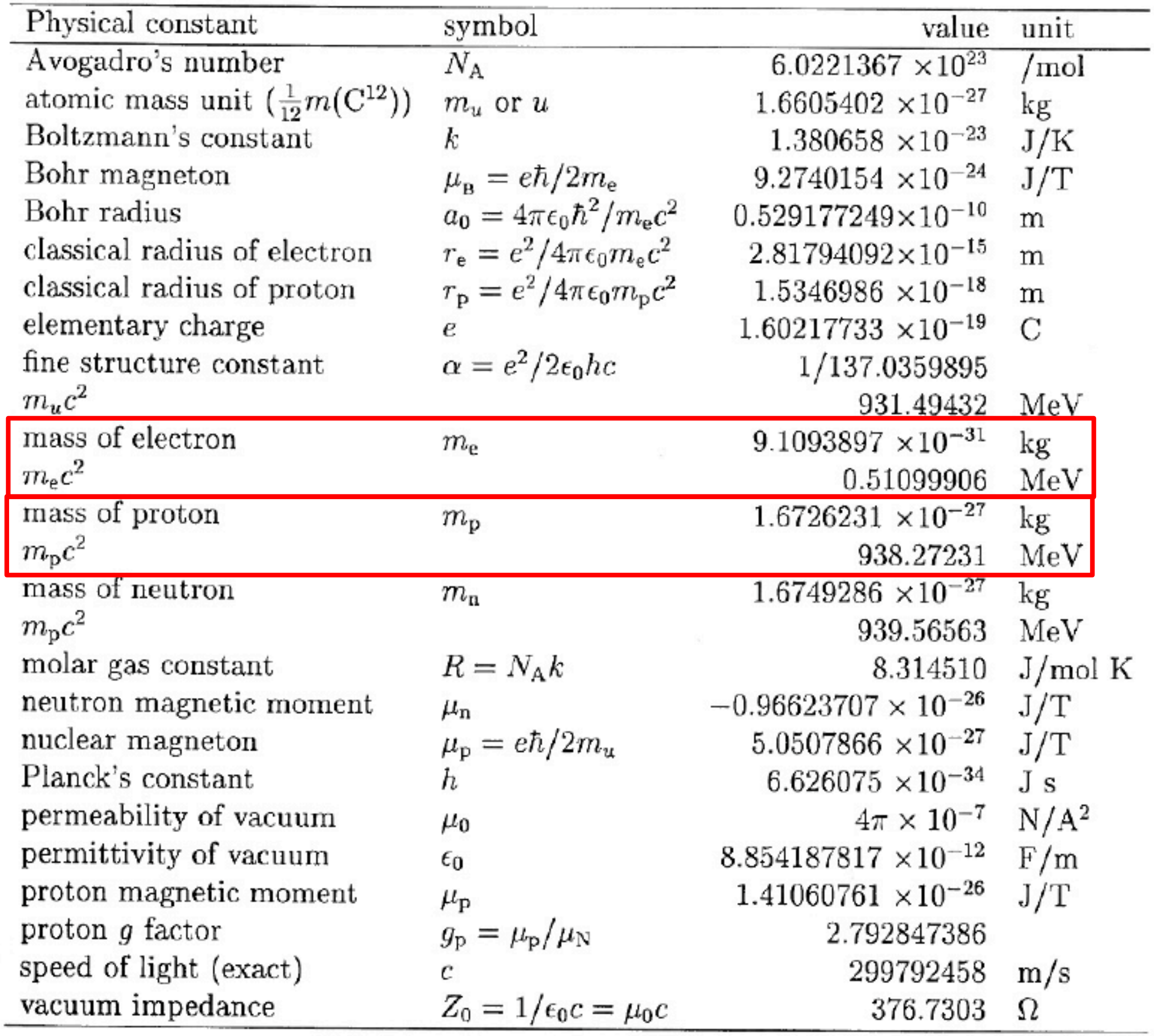
$$p = mv = \beta\frac{E}{c} = \beta\,\gamma\,m_0\,c$$

energy	momentum	mass
eV	eV/c	eV/c^2

$$p^2c^2 = E^2 - E_0{}^2$$

$$\gamma = 1 + \frac{E_{kin}}{E_0}$$

Special Relativity



Physical constant	symbol	value	unit
Avogadro's number	N_A	6.0221367×10^{23}	/mol
atomic mass unit ($\frac{1}{12}m(C^{12})$)	m_u or u	$1.6605402 \times 10^{-27}$	kg
Boltzmann's constant	k	1.380658×10^{-23}	J/K
Bohr magneton	$\mu_B = e\hbar/2m_e$	$9.2740154 \times 10^{-24}$	J/T
Bohr radius	$a_0 = 4\pi\epsilon_0\hbar^2/m_e c^2$	$0.529177249 \times 10^{-10}$	m
classical radius of electron	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	$2.81794092 \times 10^{-15}$	m
classical radius of proton	$r_p = e^2/4\pi\epsilon_0 m_p c^2$	$1.5346986 \times 10^{-18}$	m
elementary charge	e	$1.60217733 \times 10^{-19}$	C
fine structure constant	$\alpha = e^2/2\epsilon_0 hc$	$1/137.0359895$	
$m_u c^2$		931.49432	MeV
mass of electron	m_e	$9.1093897 \times 10^{-31}$	kg
$m_e c^2$		0.51099906	MeV
mass of proton	m_p	$1.6726231 \times 10^{-27}$	kg
$m_p c^2$		938.27231	MeV
mass of neutron	m_n	$1.6749286 \times 10^{-27}$	kg
$m_p c^2$		939.56563	MeV
molar gas constant	$R = N_A k$	8.314510	J/mol K
neutron magnetic moment	μ_n	$-0.96623707 \times 10^{-26}$	J/T
nuclear magneton	$\mu_p = e\hbar/2m_u$	$5.0507866 \times 10^{-27}$	J/T
Planck's constant	h	6.626075×10^{-34}	J s
permeability of vacuum	μ_0	$4\pi \times 10^{-7}$	N/A^2
permittivity of vacuum	ϵ_0	$8.854187817 \times 10^{-12}$	F/m
proton magnetic moment	μ_p	$1.41060761 \times 10^{-26}$	J/T
proton g factor	$g_p = \mu_p/\mu_N$	2.792847386	
speed of light (exact)	c	299792458	m/s
vacuum impedance	$Z_0 = 1/\epsilon_0 c = \mu_0 c$	376.7303	Ω

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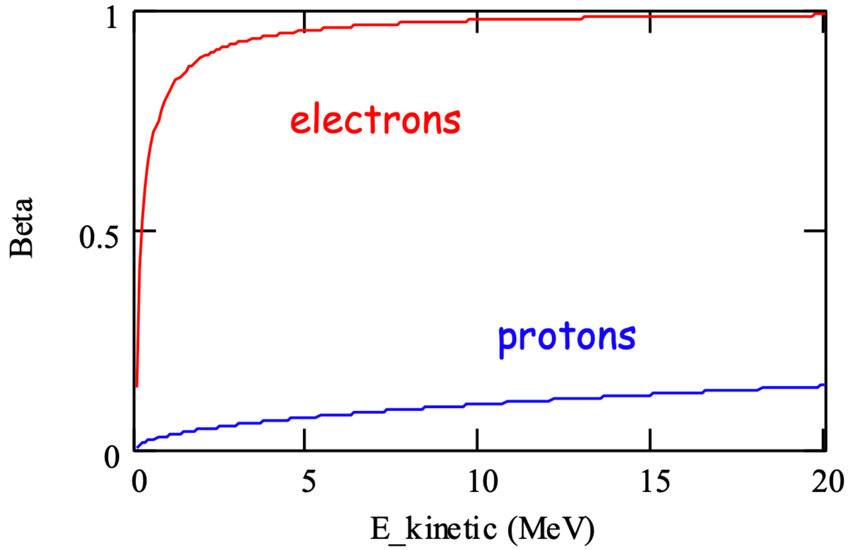
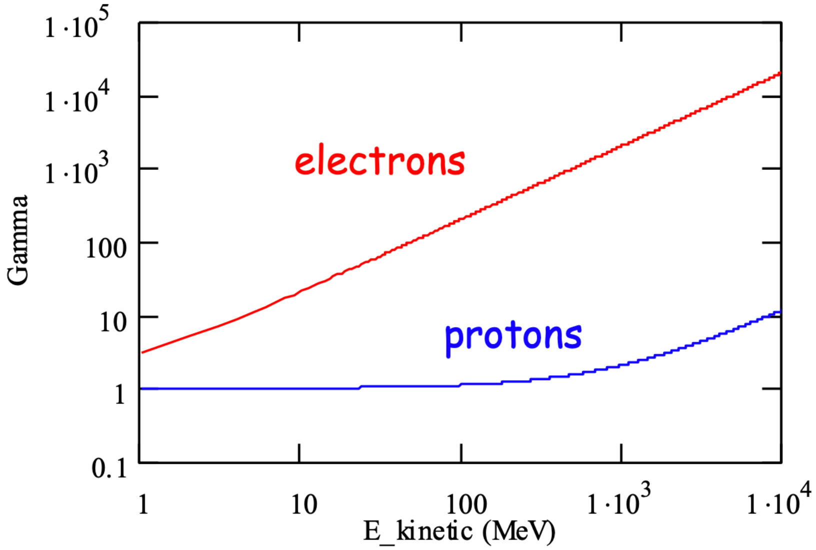


$\frac{\text{total energy}}{\text{rest energy}}$

$$\gamma = \frac{E}{E_0} = \frac{m}{m_0} = \frac{1}{\sqrt{1 - {v^2}/{c^2}}} = \frac{1}{\sqrt{1-\beta^2}}$$



EM: the 4 Maxwell equations

- 4 “coupled” equations, which combine the work of **Gauss**, **Faraday**, **Lenz** and **Ampere**

Maxwell

(1831-1879)

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- Apply to all electric and magnetic phenomena and describe the behavior of the electric and magnetic fields, and electric charges and currents (the magnetic charge does not exist) **=> Framework for all calculations involving EM fields**

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- Predicted **EM waves**
- Led Einstein to discover special relativity (together with the “failed” Michelson-Morley experiment)

EM: the 4 Maxwell equations

- **Differential forms**

$$(1) \quad div\ \vec{E} = \frac{\rho}{\varepsilon}$$

Gauss's law for electric charge

$$(2) \quad div\ \vec{H} = 0$$

Gauss's law for magnetic charge

$$(3) \quad \overrightarrow{rot}\ \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

Faraday's and Lenz law

$$(4) \quad \overrightarrow{rot}\ \vec{H} = \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}$$

Ampere's law

- **Integral forms**

$$\iiint div\ \vec{E}\ dV = \iint \vec{E} \,.\, d\vec{S} = \frac{1}{\varepsilon} \iiint \rho\ dV$$

$$\iiint div\ \vec{H}\ dV = \iint \vec{H} \,.\, d\vec{S} = 0$$

$$\iint \overrightarrow{rot}\, \vec{E} \,.\, d\vec{S} = \oint \vec{E} \,.\, d\vec{s} = -\mu \iint \frac{\partial \vec{H}}{\partial t} \,.\, d\vec{S}$$

$$\iint \overrightarrow{rot}\, \vec{H} \,.\, d\vec{S} = \oint \vec{H} \,.\, d\vec{s} = \iint \vec{J} \,.\, d\vec{S} + \varepsilon \iint \frac{\partial \vec{E}}{\partial t} \,.\, d\vec{S}$$

with

$$\vec{B} = \mu\ \vec{H}$$

$$\vec{D} = \varepsilon\ \vec{E}$$

$$\vec{J} = \rho \vec{\upsilon} + \sigma \vec{E}$$

Maxwell equations valid in homogeneous, isotropic, continuous media

EM: the 4 Maxwell equations

permeability

conductivity

$$\vec{B} = \mu \, \vec{H}$$

$$\mu = \mu_0 \, \mu_1 = \mu_0 \, \mu_r \left(1 - j \tan \vartheta_{\mathrm{M}}\right)$$

$$\vec{D} = \varepsilon \, \vec{E}$$

$$\varepsilon = \varepsilon_0 \, \varepsilon_1 = \varepsilon_0 \left(\varepsilon'_r - j \, \varepsilon''_r\right) = \varepsilon_0 \, \varepsilon_b + \frac{\sigma}{j 2 \pi f}$$

permittivity

Imaginary parts describe the losses

EM: the 4 Maxwell equations

- q: electric charge [C] => $q = e$ for a proton
- ρ: electric charge density [C/m^3]
- I, $\overrightarrow{J}$: electric current [A], electric current density [A/m^2]
- $\overrightarrow{E}$: electric field [V/m]
- $\overrightarrow{H}$: magnetic field [A/m]
- $\overrightarrow{D}$: electric displacement [C/m^2]
- $\overrightarrow{B}$: magnetic induction or magnetic flux density [T] => But, beware: it is often called “magnetic field”

EM: the 4 Maxwell equations

- **Cartesian (x,y,s)**

$$\vec{\nabla} \equiv \left| \begin{matrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial s} \end{matrix} \right.$$

- **Cylindrical (r,θ,s)**

$$\vec{\nabla} \equiv \left| \begin{matrix} \dfrac{\partial}{\partial r} \\ \dfrac{1}{r}\left(\dfrac{\partial}{\partial \vartheta}\right) \\ \dfrac{\partial}{\partial s} \end{matrix} \right.$$

Also noted
$\overrightarrow{curl}\,\vec{\mathrm{E}}$ or $\vec{\nabla} \wedge \vec{\mathrm{E}}$

$$\overrightarrow{grad}\ \rho \equiv \vec{\nabla}\ \rho = \left| \begin{matrix} \dfrac{\partial \rho}{\partial x} \\ \dfrac{\partial \rho}{\partial y} \\ \dfrac{\partial \rho}{\partial s} \end{matrix} \right.$$

$$\overrightarrow{rot}\ \vec{E} \equiv \vec{\nabla} \times \vec{E} = \left| \begin{matrix} \dfrac{\partial E_s}{\partial y} - \dfrac{\partial E_y}{\partial s} \\ \dfrac{\partial E_x}{\partial s} - \dfrac{\partial E_s}{\partial x} \\ \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} \end{matrix} \right.$$

$$\overrightarrow{grad}\ \rho = \left| \begin{matrix} \dfrac{\partial \rho}{\partial r} \\ \dfrac{1}{r}\left(\dfrac{\partial \rho}{\partial \vartheta}\right) \\ \dfrac{\partial \rho}{\partial s} \end{matrix} \right.$$

$$\overrightarrow{rot}\ \vec{E} = \left| \begin{matrix} \dfrac{1}{r}\left(\dfrac{\partial E_s}{\partial \vartheta}\right) - \dfrac{\partial E_\theta}{\partial s} \\ \dfrac{\partial E_r}{\partial s} - \dfrac{\partial E_s}{\partial r} \\ \dfrac{1}{r}\left[\dfrac{\partial \left(r\, E_\theta\right)}{\partial r} - \dfrac{\partial E_r}{\partial \theta}\right] \end{matrix} \right.$$

$$div\ \vec{E} \equiv \vec{\nabla} . \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_s}{\partial s}$$

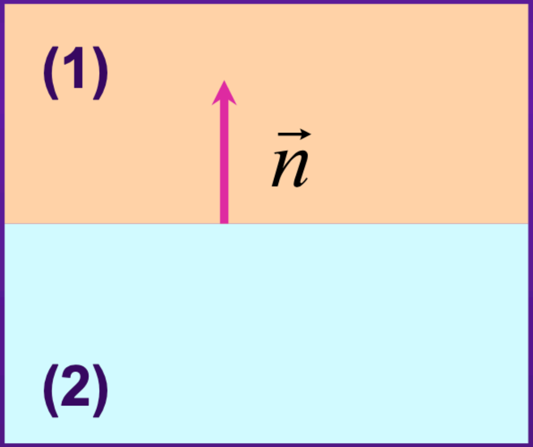
$$div\ \vec{E} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\, E_r\right) + \frac{1}{r}\frac{\partial E_\theta}{\partial \theta} + \frac{\partial E_s}{\partial s}$$

$$\Delta \rho \equiv \nabla^2 \rho = \text{Laplacian operator}$$
$$= \frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} + \frac{\partial^2 \rho}{\partial s^2}$$

$$\Delta \rho = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \rho}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \rho}{\partial \theta^2} + \frac{\partial^2 \rho}{\partial s^2}$$

Field matching

Consider a surface separating two media “1” and “2”. The following boundary conditions can be derived from Maxwell equations for the normal (⊥) and parallel (//) components of the fields at the surface



$$\vec{E}^{1}_{//} = \vec{E}^{2}_{//}$$

$$\vec{H}^{1}_{//} - \vec{H}^{2}_{//} = \vec{K}$$

$$D^{1}_{\perp} - D^{2}_{\perp} = \Sigma$$

$$B^{1}_{\perp} = B^{2}_{\perp}$$

where Σ is the surface charge density and $\vec{K}$ is the surface current density

Energy of EM waves

- **Poynting vector**: $\vec{S} = \vec{E} \times \vec{H}$

=> It points in the direction of propagation and describes the "energy flux", i.e. the energy crossing a unit area per second

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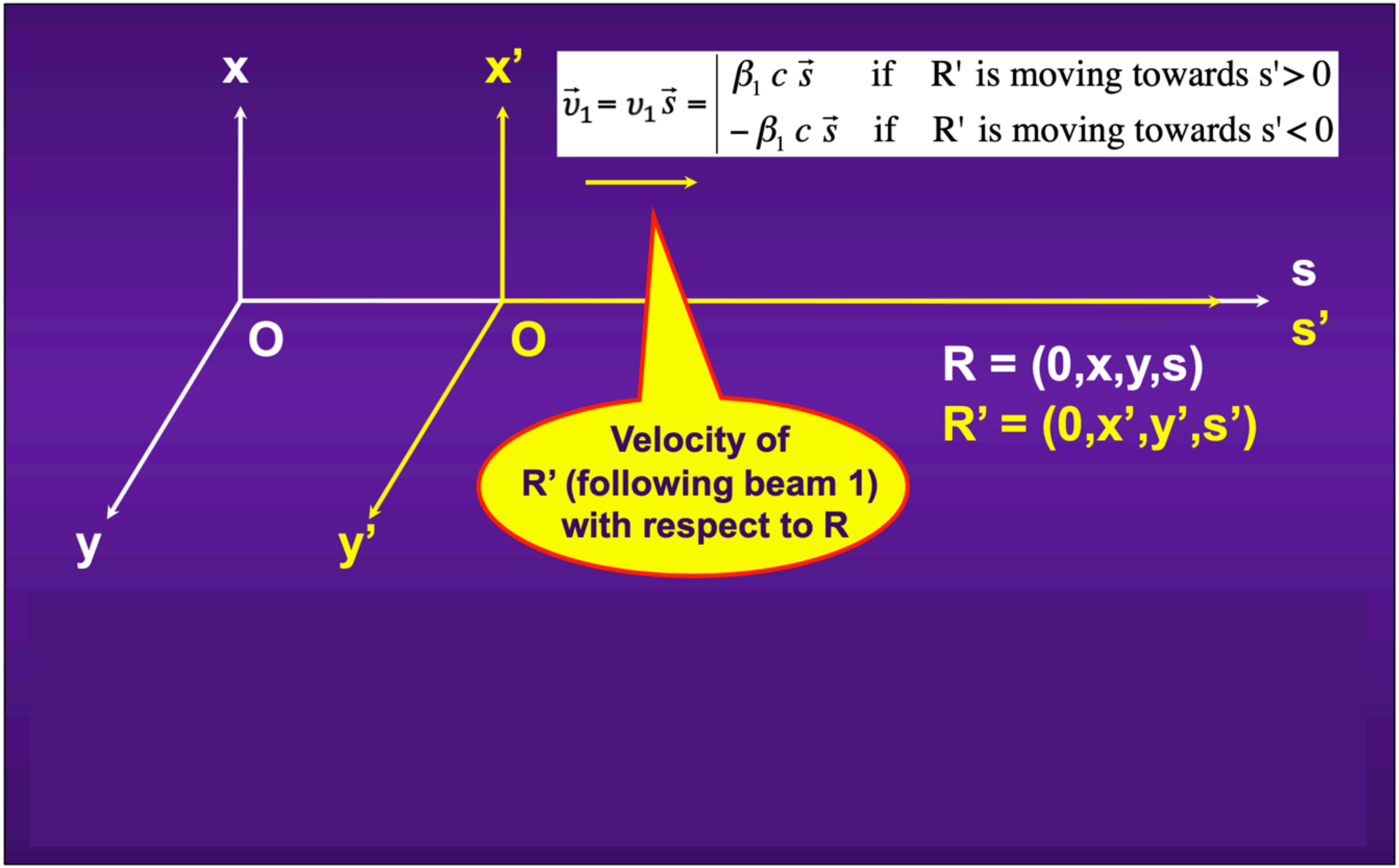
- **Remark on complex notations for vectors**
  - As long as we deal with linear equations, we can carry out all the algebraic manipulations using complex field vectors, where **it is implicit that the physical quantities are obtained by taking the real parts of the complex vectors**
  - However, when using the complex notation, particular care is needed when taking the product of two complex vectors: to be safe, one should always take the real part before multiplying two complex quantities, the real parts of which represent physical quantities

Relativistic transformation of EM fields



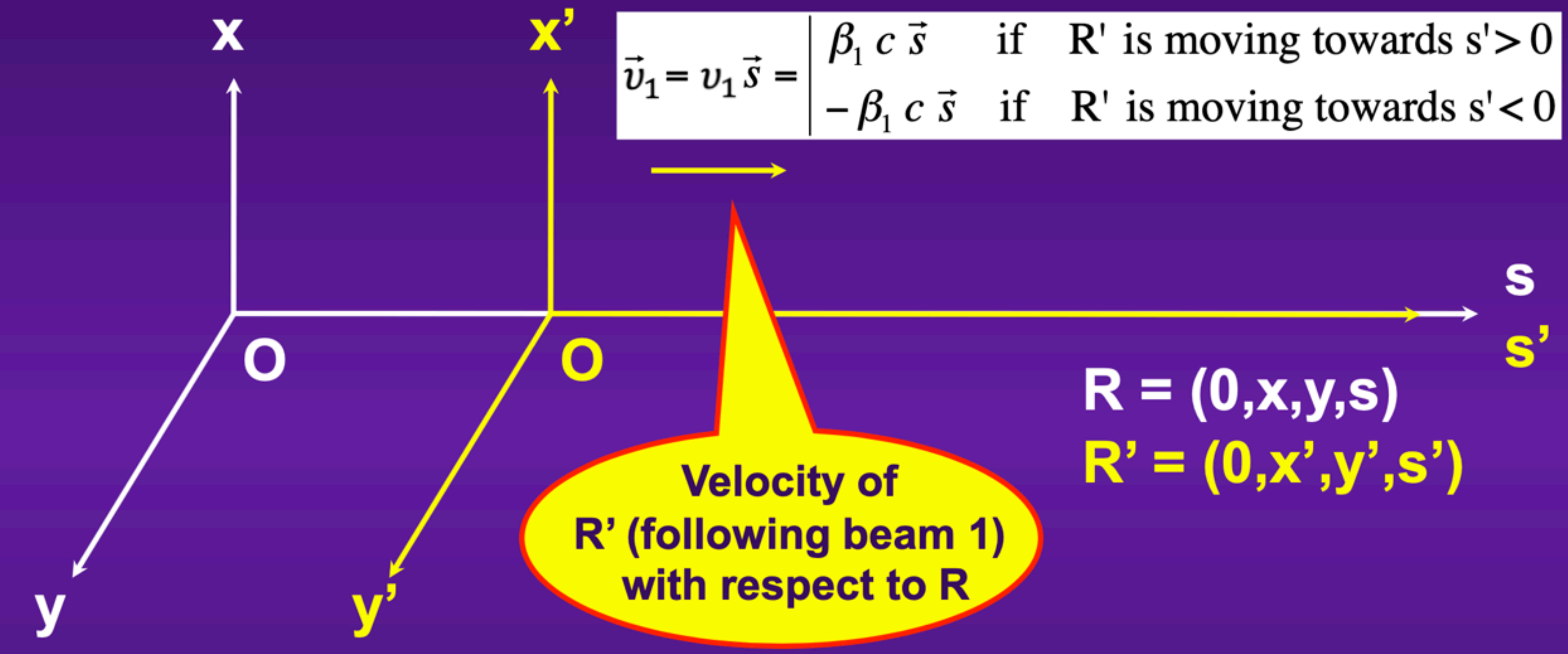
$$\vec{v}_1 = v_1\,\vec{s} = \begin{vmatrix} \beta_1\, c\, \vec{s} & \text{if} \quad \text{R' is moving towards s'} > 0 \\ -\beta_1\, c\, \vec{s} & \text{if} \quad \text{R' is moving towards s'} < 0 \end{vmatrix}$$

Velocity of R' (following beam 1) with respect to R

R = (0,x,y,s)

R' = (0,x',y',s')

Relativistic transformation of EM fields



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Velocity of R' (following beam 1) with respect to R

R = (0,x,y,s)

R' = (0,x',y',s')

$$E'_x = \gamma_1 \left(E_x - v_1 B_y \right)$$

$$E'_y = \gamma_1 \left(E_y + v_1 B_x \right)$$

$$E'_s = E_s \qquad B'_s = B_s$$

$$B'_x = \gamma_1 \left(B_x + \frac{v_1}{c^2} E_y \right)$$

$$B'_y = \gamma_1 \left(B_y - \frac{v_1}{c^2} E_x \right)$$

Relativistic transformation of EM fields

- **Lorentz force on the particle 2 moving with velocity** $\vec{v}_2 = v_2 \, \vec{s}$

$$\vec{F} = e\left(\vec{E} + \vec{v}_2 \times \vec{B}\right)$$

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=> $B_x = - \dfrac{v_1}{c^2} E_y$ $\quad B_y = \dfrac{v_1}{c^2} E_x$ $\quad B_s = 0$

=> $$F_{x,y} = e \, E_{x,y} \left| \begin{array}{ll} \left(1 - \beta_1 \, \beta_2 \right) & \text{if 2 moves in same direction as 1} \\ \left(1 + \beta_1 \, \beta_2 \right) & \text{if 2 moves in oppo. direction as 1} \end{array} \right.$$

Space charge

Beam beam

Relativistic transformation of EM fields

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=> $$\boxed{F_{x,y} = e\,E_{x,y}\left(1 - \beta^2\right) = e\,\frac{E_{x,y}}{\gamma^2}}$$

Electric part

Magnetic part

Relativistic transformation of EM fields

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=> $$F_{x,y} = e\, E_{x,y} \left(1 - \beta^2\right) = e\, \frac{E_{x,y}}{\gamma^2}$$

Electric part **Magnetic part**

and $$E'_{x,y} = \frac{E_{x,y}}{\gamma}$$

$$B_x = -\frac{\beta}{c} E_y$$

$$E'_s = E_s$$

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(RF) cavities & waveguides

- At the surface of **an ideal (or perfect) conductor** (i.e. with **no energy dissipation**), the normal component of $\vec{B}$ and the tangential component of $\vec{E}$ must both vanish => **Standing waves that can persist within the cavity are determined by the shape of the cavity**

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- Usually, **the energy stored in an RF cavity is needed to manipulate a charged particle beam in a particular way**
  - **Accelerate** the beam => Most of the time
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- The effect on the beam is determined by the field pattern. Therefore, it is important to design the shape of the cavity, so that the fields in the cavity interact with the beam in the desired way; and that undesirable interactions (which always occur to some extent) are minimized

(RF) cavities & waveguides

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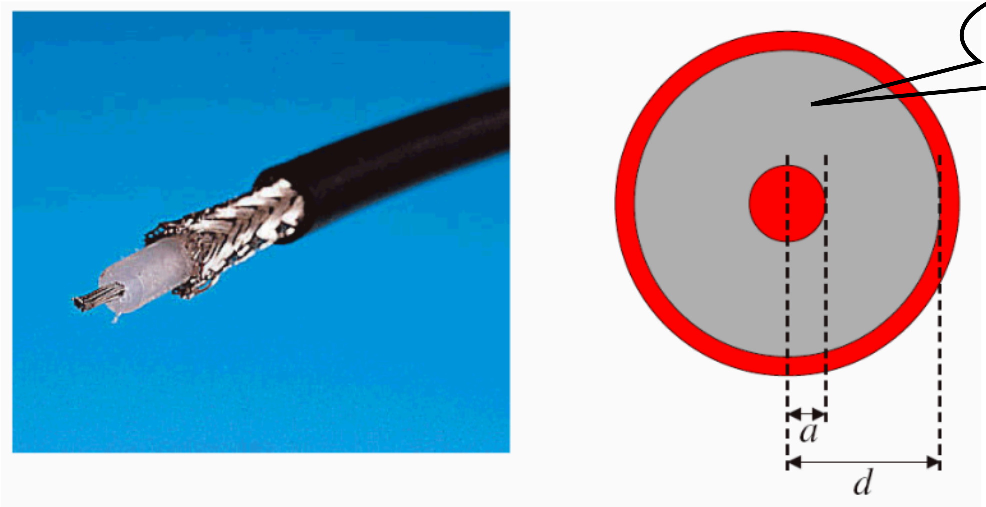

Dieletric (ε, μ)

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- Although the basic physics in waveguides and transmission lines is the same – both involve EM waves propagating through bounded regions – different formalisms are used for their analysis, depending on the geometry of the boundaries
- As was the case for cavities, the patterns of the fields in the resonant modes are determined by the geometry of the boundary

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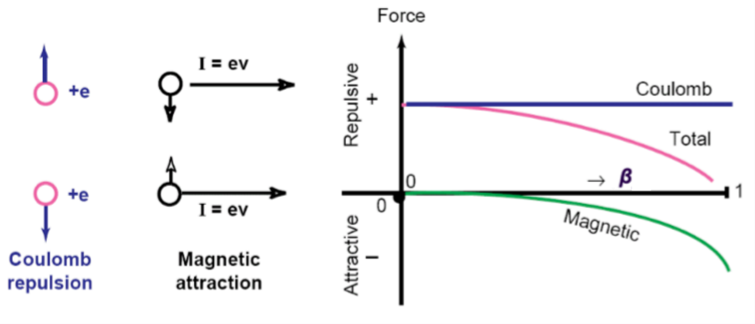
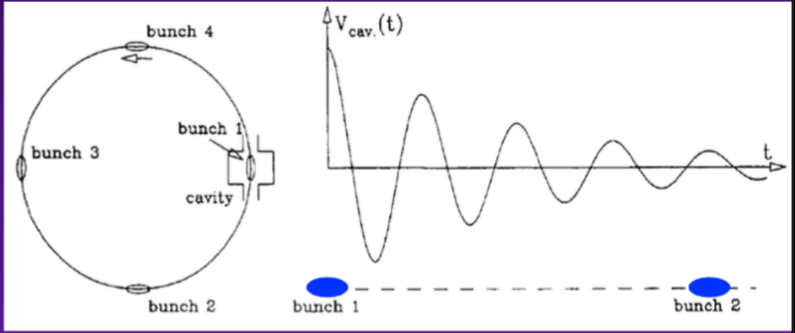
  - Etc. **=> To correctly describe the dynamics of a beam of particles, all the wanted and unwanted EM interactions need to be taken into account!**

Conclusions on EM & SR

SPACE CHARGE

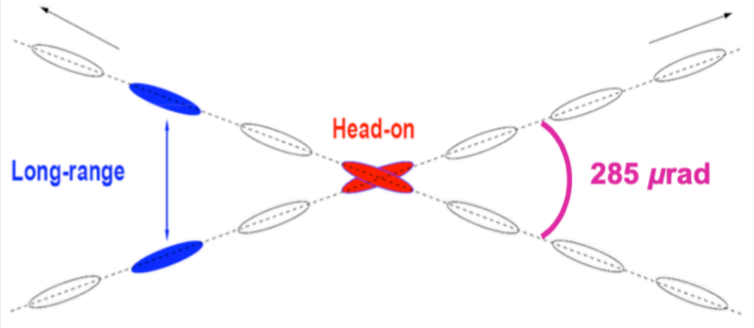


WAKE FIELD **(or IMPEDANCE)**

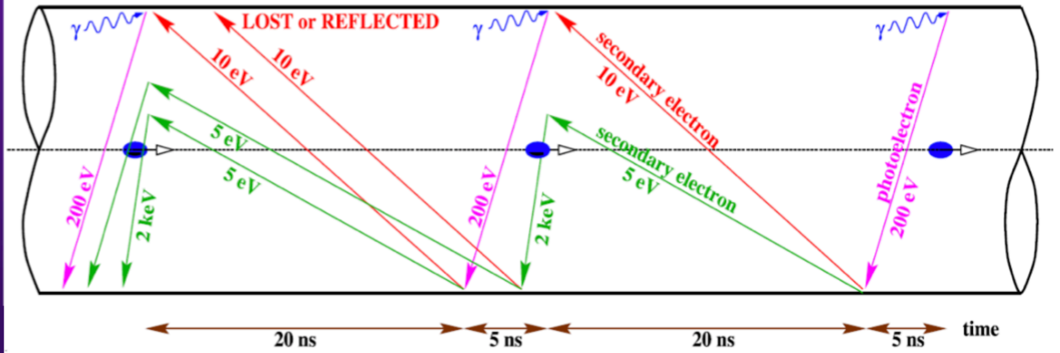


IBS = INTRA-BEAM SCATTERING
(due to collisions between the particles), etc.

BEAM-BEAM



ELECTRON CLOUD



2 modes of particle accelerators: Fixed-target vs. Collider

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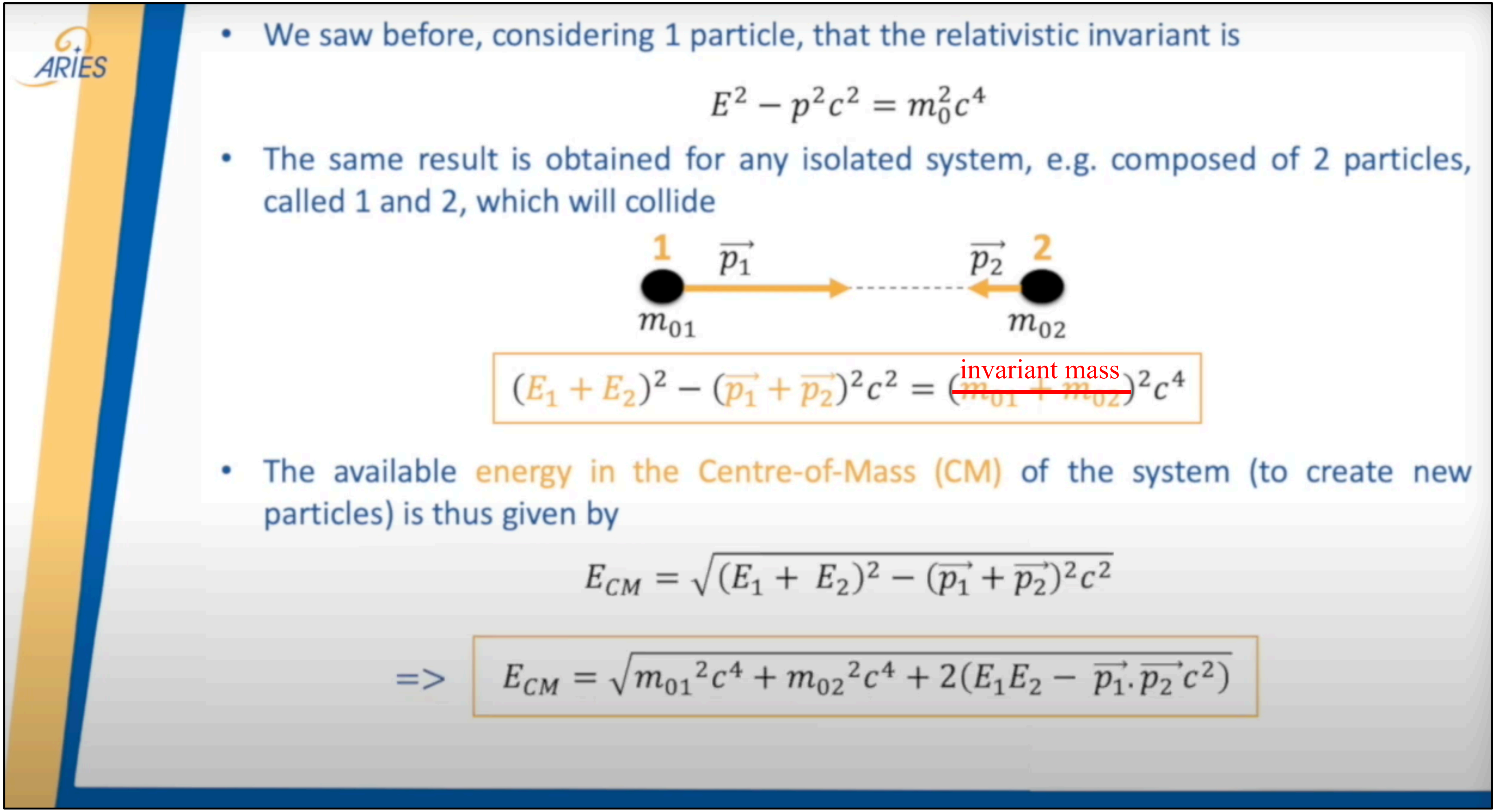


- We saw before, considering 1 particle, that the relativistic invariant is

$$E^2 - p^2c^2 = m_0^2c^4$$

- The same result is obtained for any isolated system, e.g. composed of 2 particles, called 1 and 2, which will collide

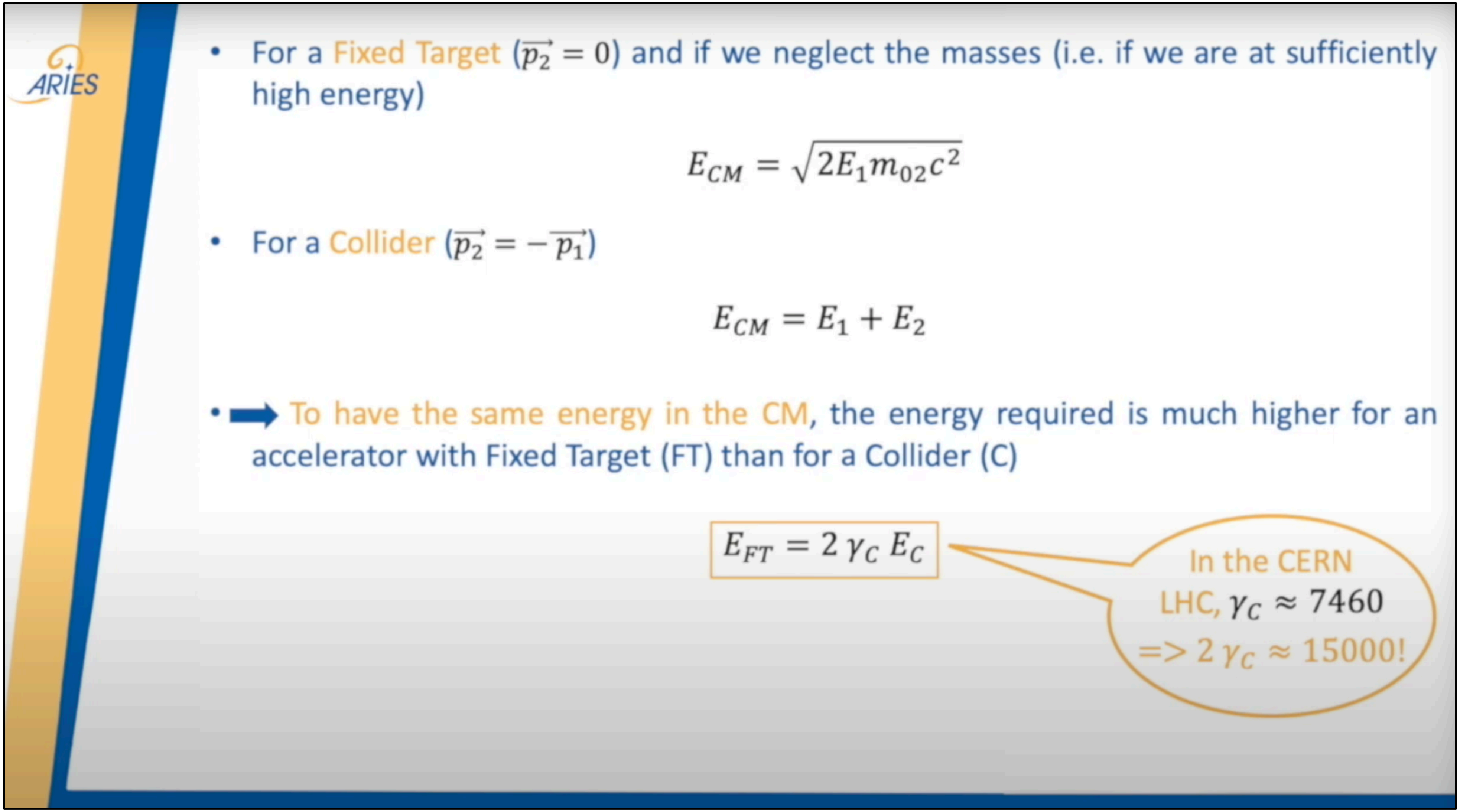
$$\boxed{(E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2c^2 = (\overset{\text{invariant mass}}{\cancel{m_{01} + m_{02}}})^2c^4}$$

- The available energy in the Centre-of-Mass (CM) of the system (to create new particles) is thus given by

$$E_{CM} = \sqrt{(E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2c^2}$$

$$=> \boxed{E_{CM} = \sqrt{m_{01}{}^2c^4 + m_{02}{}^2c^4 + 2(E_1E_2 - \vec{p_1}.\vec{p_2}\,c^2)}}$$

2 modes of particle accelerators: Fixed-target vs. Collider



- For a Fixed Target ($\overrightarrow{p_2} = 0$) and if we neglect the masses (i.e. if we are at sufficiently high energy)

$$E_{CM} = \sqrt{2E_1 m_{02} c^2}$$

- For a Collider ($\overrightarrow{p_2} = -\overrightarrow{p_1}$)

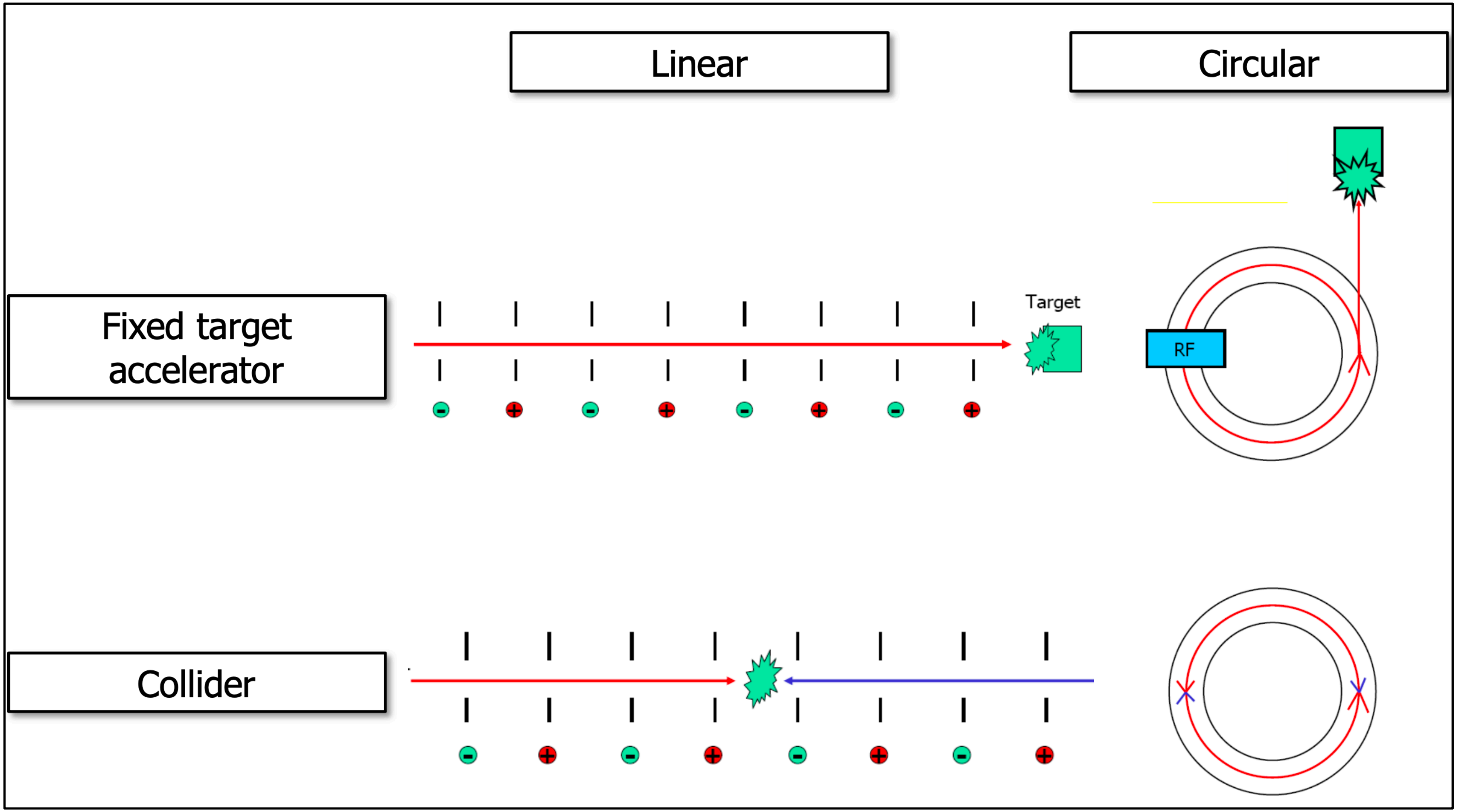
$$E_{CM} = E_1 + E_2$$

- ➡ To have the same energy in the CM, the energy required is much higher for an accelerator with Fixed Target (FT) than for a Collider (C)

$$\boxed{E_{FT} = 2\,\gamma_C\,E_C}$$

In the CERN LHC, $\gamma_C \approx 7460$ => $2\,\gamma_C \approx 15000!$

2 modes of particle accelerators: Fixed-target vs. Collider



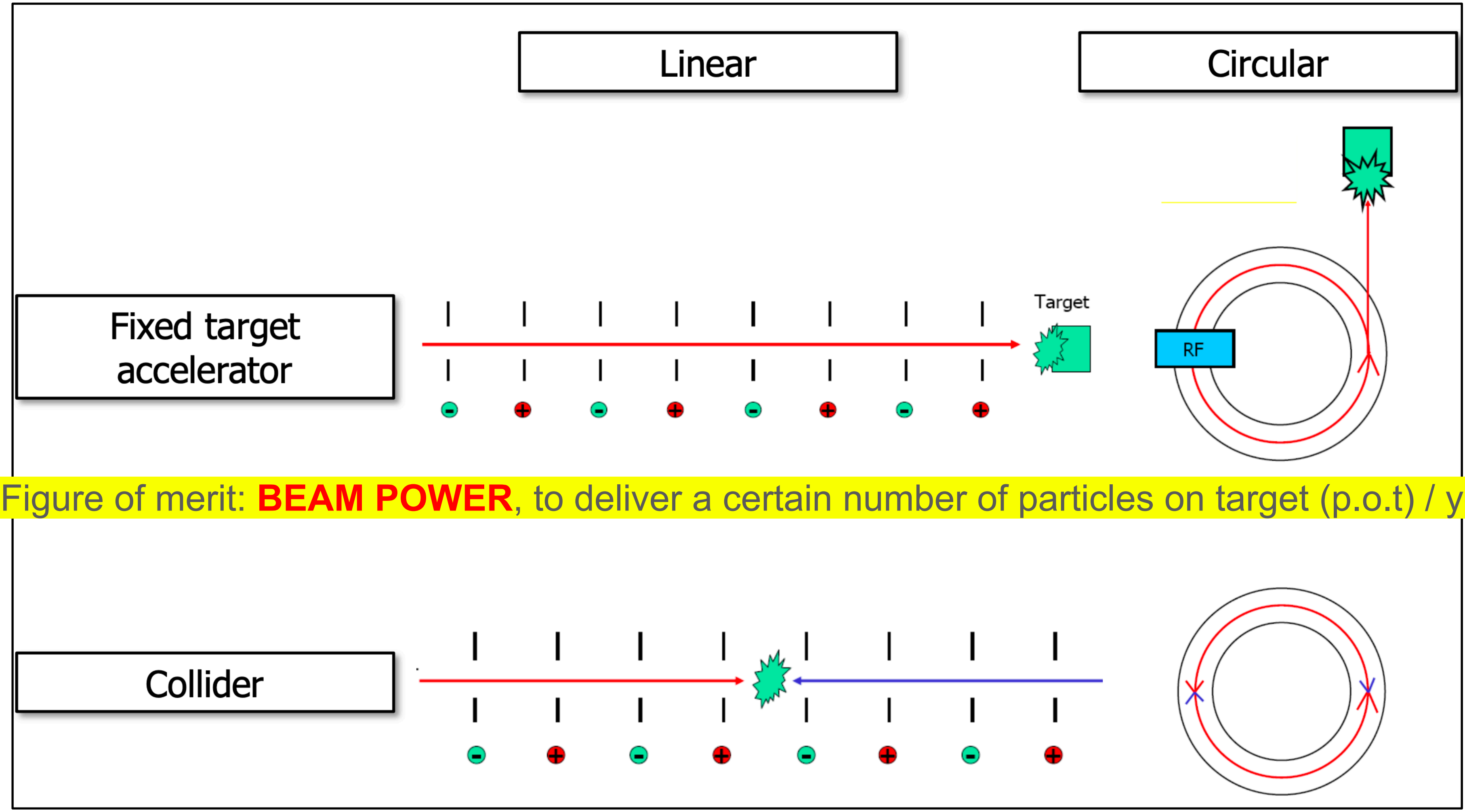
Linear

Circular

Fixed target accelerator

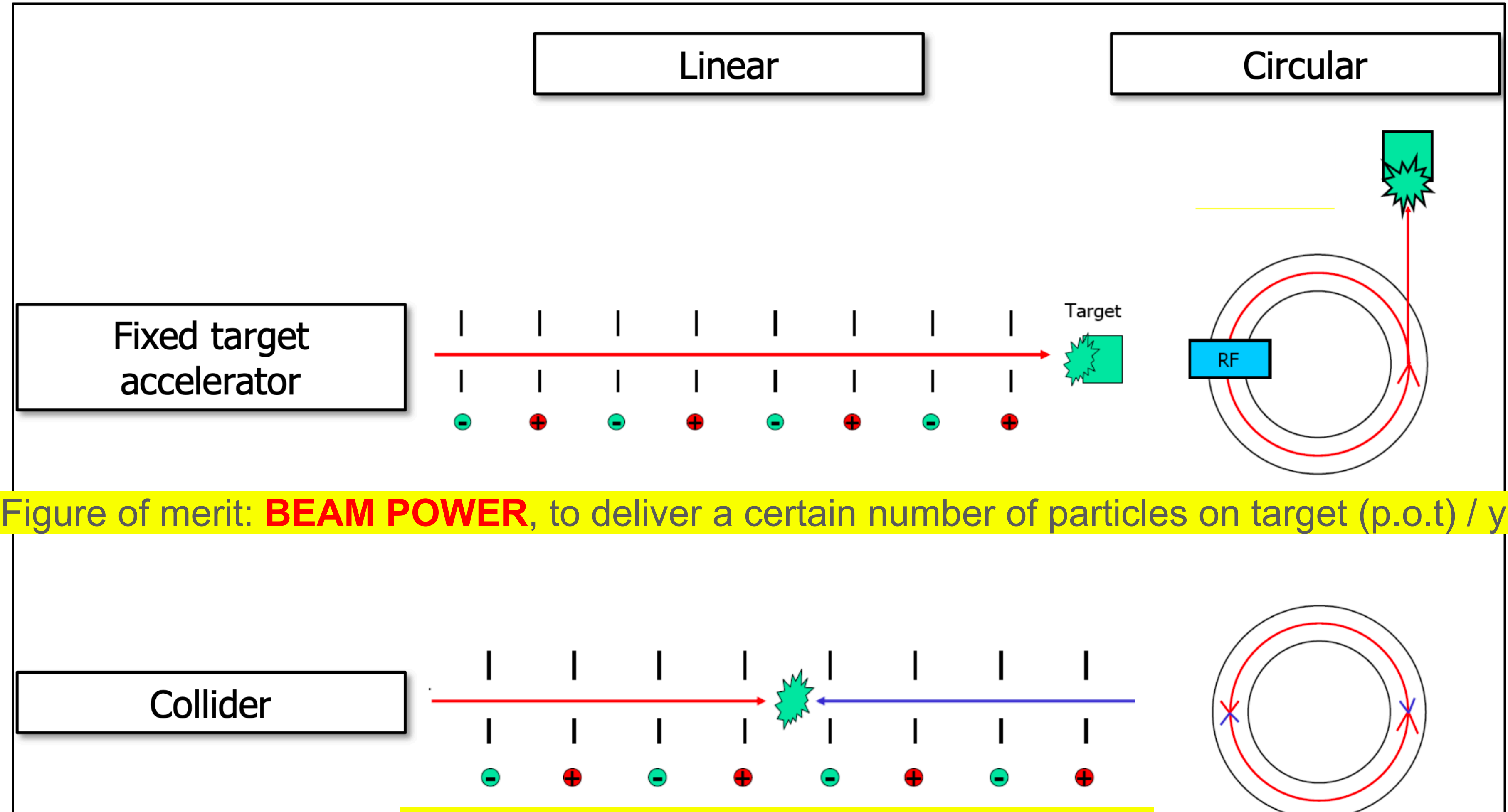
Collider

2 modes of particle accelerators: Fixed-target vs. Collider



=> Figure of merit: **BEAM POWER**, to deliver a certain number of particles on target (p.o.t) / year

2 modes of particle accelerators: Fixed-target vs. Collider



=> Figure of merit: **BEAM POWER**, to deliver a certain number of particles on target (p.o.t) / year

=> Figure of merit: **LUMINOSITY** (see after)

Luminosity: figure of merit of a collider

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The number of events N_{exp} is the product of the cross-section of interest σ_{exp} and the time integral over the instantaneous luminosity $L(t)$

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Detector

Nature

Luminosity:
figure of merit of a collider

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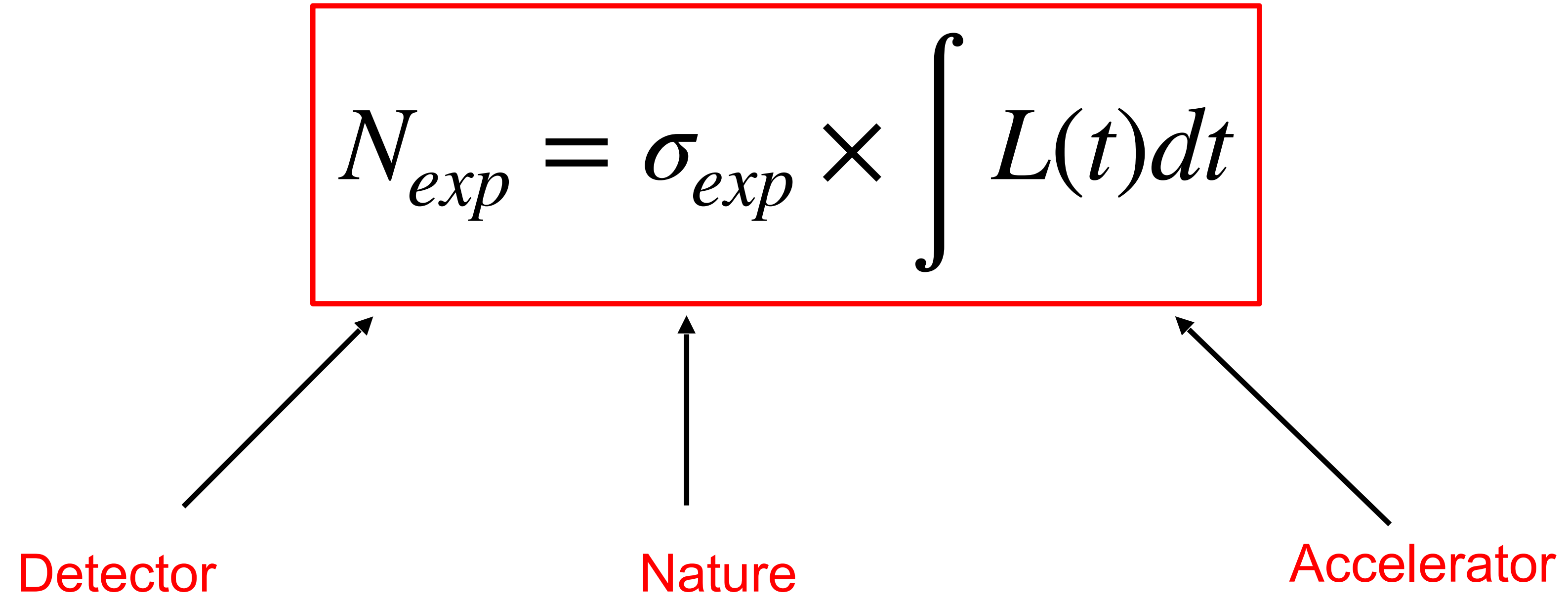

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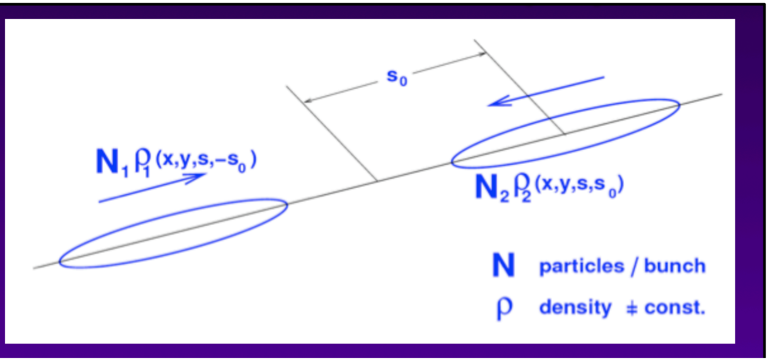
Detector

Nature

Accelerator

Luminosity for the **SIMPLEST case**

- **Collision without crossing angle**



Number of bunches

$$\boxed{L = \frac{M\, N_1\, N_2\, f_{rev}}{4\, \pi\, \sigma_x\, \sigma_y}}$$

Transverse beam sizes

=> More will be said during the “Introduction on colliders” on Monday 23/01/23 and during the colliders’ session on Tuesday 24/01/23 afternoon

Many thanks for your attention and I wish you again a great JUAS-2023!