Introduction to Transverse Beam Dynamics

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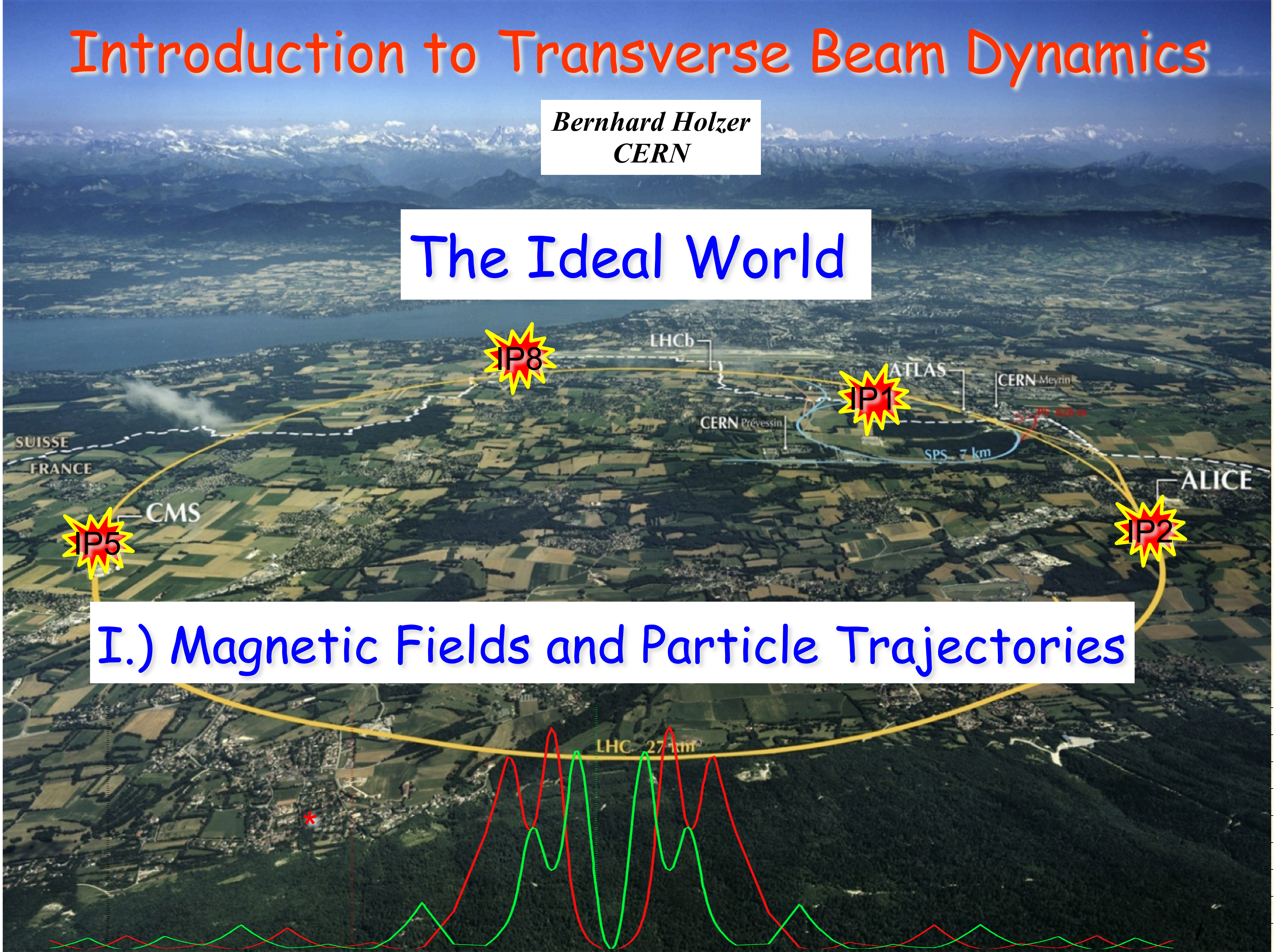
CERN

The Ideal World

I.) Magnetic Fields and Particle Trajectories



0.) Few General Statements

The Main Parts of Beam Dynamics in JUAS

*** Lectures "Transverse Beam Dynamics"**
—> listen and ask intelligent questions

*** Tutorials**
—> think about interesting (!) questions from real life
… and from typical exams ;-)

*** Accelerator Design (Bastian)**
—> learn how to build a real accelerator

*** Mini-Workshop (Adrian)**
—> and actually do it !!

Luminosity Run of a typical storage ring:

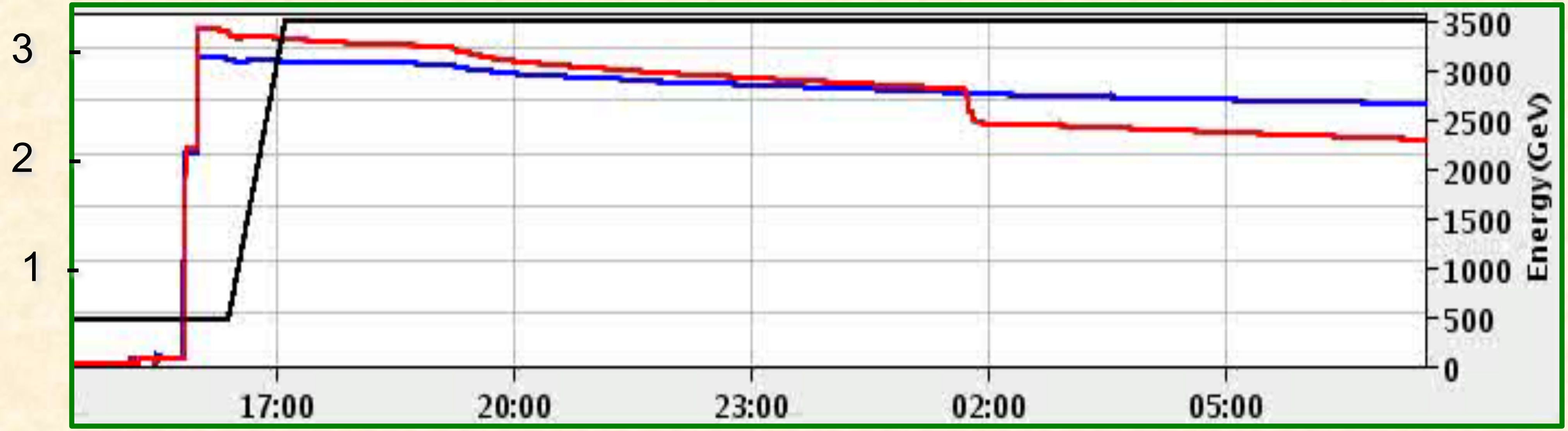
LHC Storage Ring: Protons accelerated and stored for 12 hours

distance of particles travelling at about $v \approx c$

$L = 10^{10}$-10^{11} km

... several times Sun - Pluto and back

intensity (10^{11})



→ *guide the particles on a well defined orbit („design orbit")*

→ *focus the particles to keep each single particle trajectory within the vacuum chamber of the storage ring, i.e. close to the design orbit.*

I.) Introduction and Basic Ideas: The Bending Fields

„ ... in the end and after all it should be a kind of circular machine“

—> need transverse deflecting force

Lorentz force $\vec{F} = q * (\cancel{\vec{E}} + \vec{v} \times \vec{B})$

typical velocity in high energy machines: $v \approx c \approx 3 * 10^8 \ m/s$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q * 3 * 10^8 \frac{m}{s} * 1 \frac{Vs}{m^2}$$

$$F = q * \underbrace{300 \frac{MV}{m}}_{E}$$

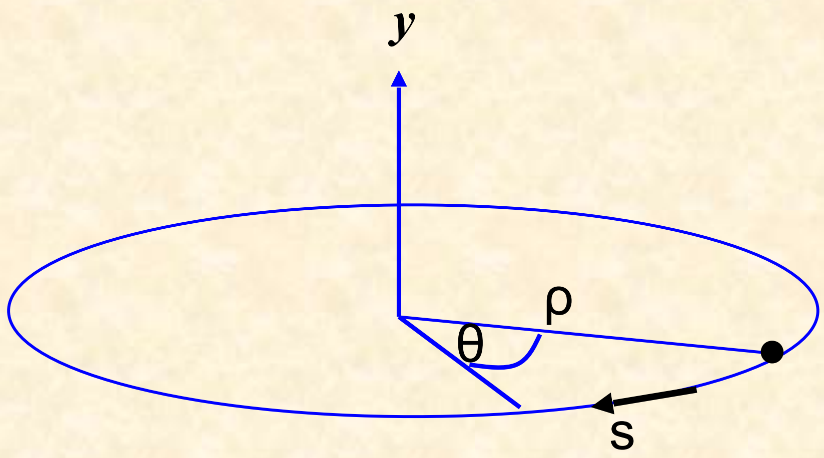
equivalent el. field ... E

technical limit for el. field

$$E \leq 1 \frac{MV}{m}$$

old greek dictum of wisdom:
if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit



circular coordinate system

condition for circular orbit:

Lorentz force $$F_L = e\, v\, B$$

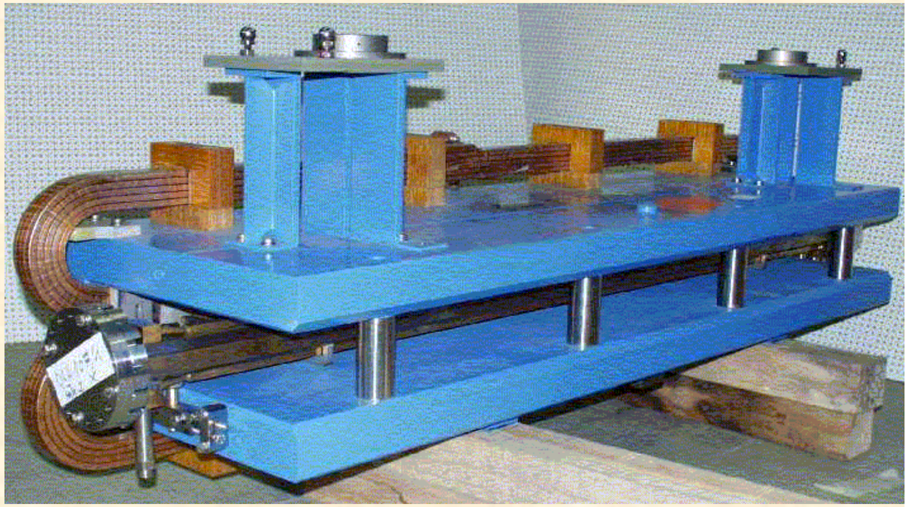
centrifugal force $$F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$$

$$\frac{\gamma\, m_0\, v^{\cancel{2}}}{\rho} = e\, \cancel{v}\, B$$

$$\boxed{\frac{p}{e} = B\,\rho}$$

$B\rho$ = "beam rigidity"

1.) The Magnetic Guide Field



Dipole Magnets:

define the ideal orbit
homogeneous field created
by two flat pole shoes

$$B = \frac{\mu_0 \, n \, I}{h}$$

Normalise magnetic field to momentum:

$$\frac{p}{e} = B\rho \quad \longrightarrow \quad \frac{1}{\rho} = \frac{e B}{p}$$

convenient units:

$$B = [T] = \left[\frac{Vs}{m^2}\right] \qquad p = \left[\frac{GeV}{c}\right]$$

Example LHC:

$$\left. \begin{aligned} B &= 8.33 \; T \\ p &= 7000 \frac{GeV}{c} \end{aligned} \right\}$$

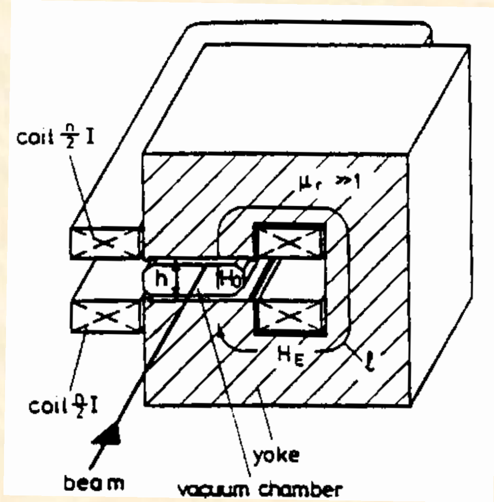
$$1/\rho = e \frac{8.33 \; Vs/m^2}{7000 \cdot 10^9 \; eV/c} = \frac{8.33 \cdot 3 \cdot 10^8 \; m}{7000 \cdot 10^9 m^2}$$

$$1/\rho = 0.000353 \; 1/m$$

$$\rho = 2.83 \; km$$

Dipole Magnets:

homogeneous field created by two flat pole shoes



Field Calculation:

3rd Maxwell equation for a static field: $\vec{\nabla} \times \vec{H} = \vec{j}$

according to Stokes theorem:

$$\int_S (\vec{\nabla} \times \vec{H})\vec{n}\, da = \oint \vec{H}\, d\vec{l} = \int_S \vec{j} \cdot \vec{n}\, da = N \cdot I$$

$$\oint \vec{H}\, d\vec{l} = H_0 * h + H_{Fe} * l_{Fe}$$

in matter we get with $\mu_r \approx 1000$

$$\oint \vec{H}\, d\vec{l} = H_0 * h + \cancel{\frac{H_0}{\mu_r} * l_{Fe}} \approx H_0 * h$$

Magnetic field of a dipole magnet: $H_0 = B_0 / \mu_0$ $\longrightarrow$ $B_0 = \dfrac{\mu_0 N I}{h}$

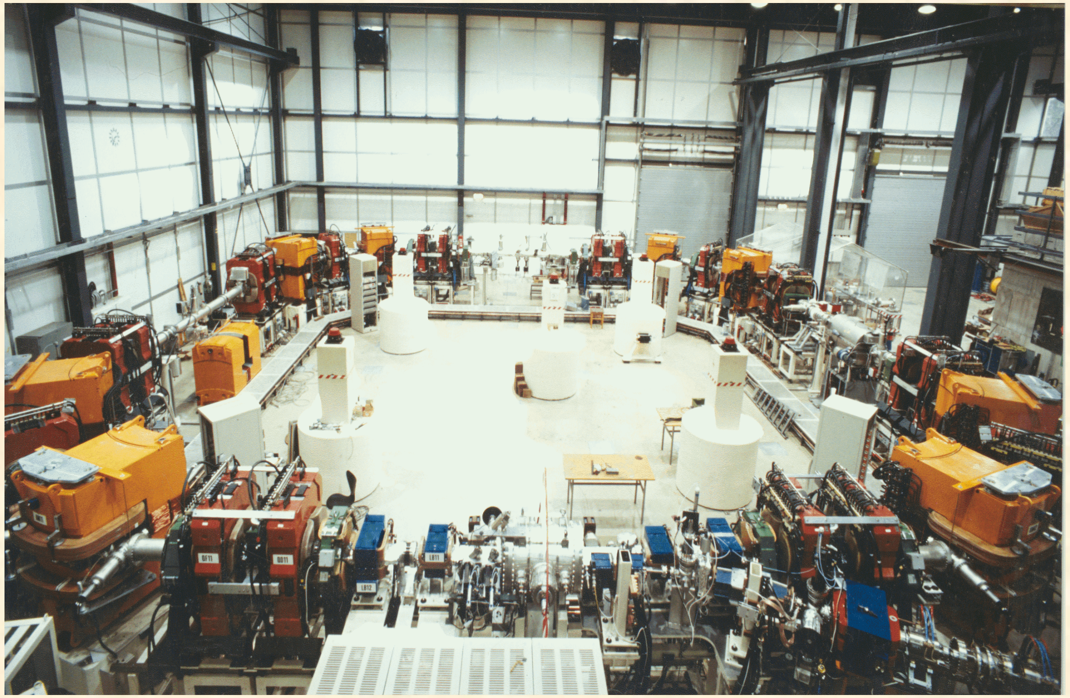
h = gap height

The dipole strength depends on the gap height h , aka aperture "r_0" of the magnet.

$$B_0 = \frac{\mu_0 N I}{h}$$

—> keep the beam dimensions small !!

Example:

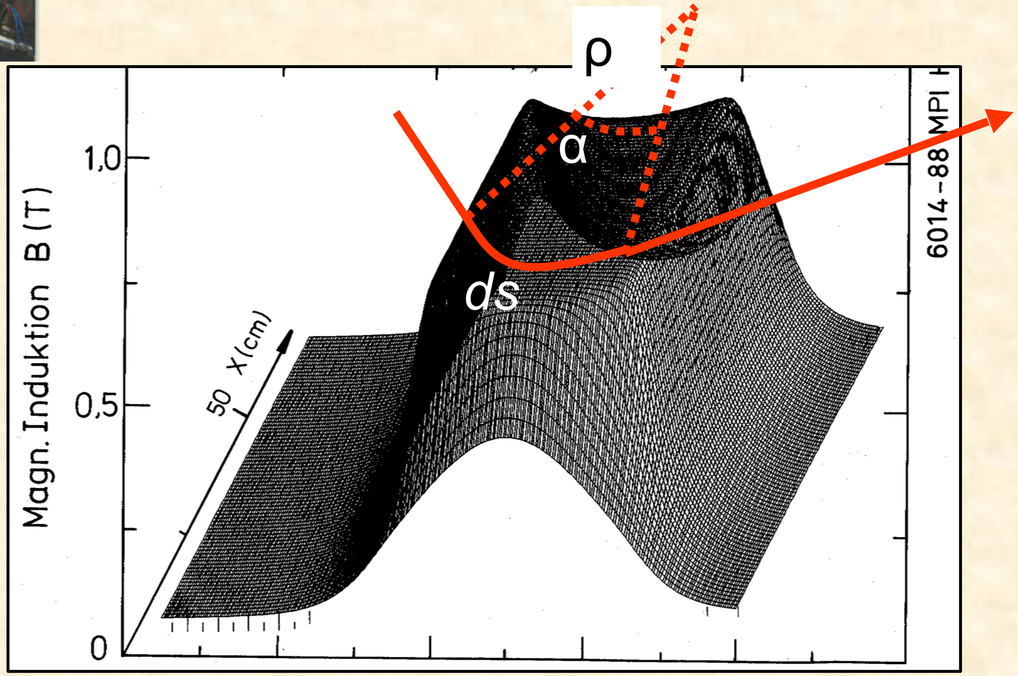


Heavy ion storage ring TSR
8 dipole magnets
of equal bending strength

$$\alpha = \frac{ds}{\rho} \approx \frac{dl}{\rho} \qquad \alpha = \frac{B * dl}{B * \rho}$$

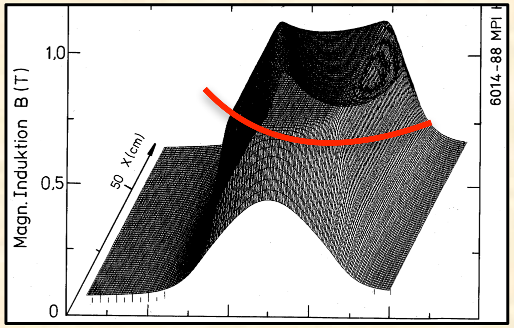
The B fields integrated over the path-length of the beam through all dipole magnets has to add up to give an overall angle of

$$\alpha = 2\pi$$



field map of a storage ring dipole magnet

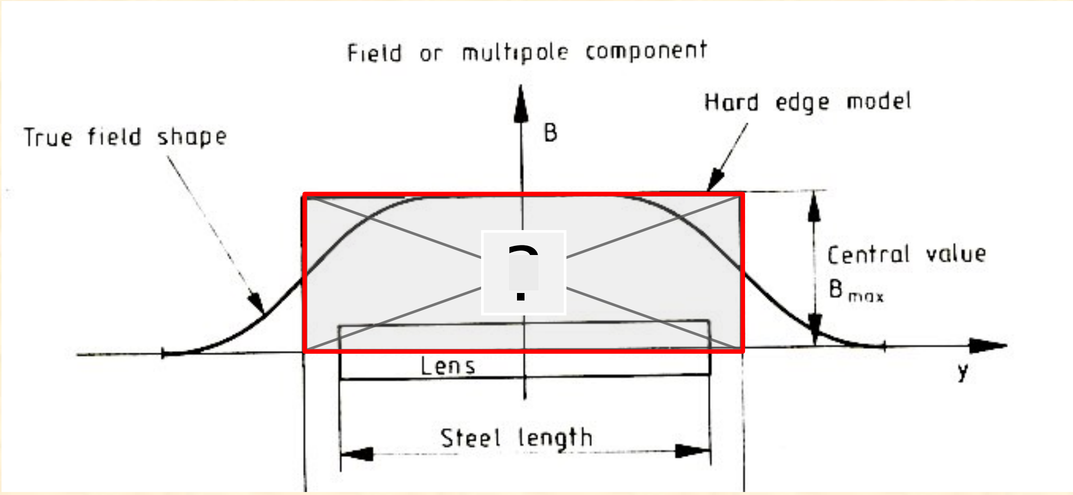
Bending Angle



„integrated field strength"

$$\alpha = \frac{B * dl}{B * \rho}$$

$$B\, l_{eff} = \int_0^{l_{mag}} B\, ds$$



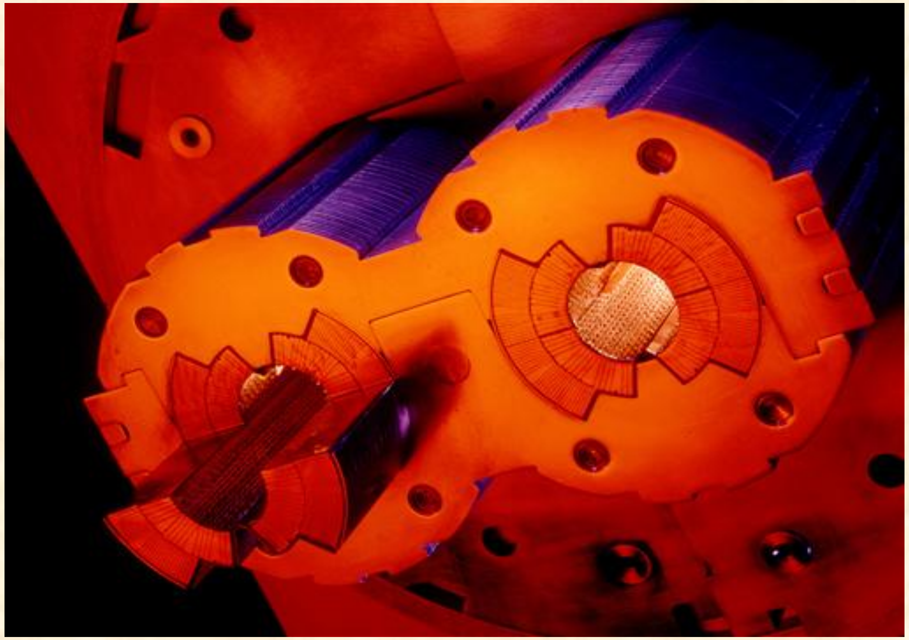
The angle swept out in one revolution must be 2π, so

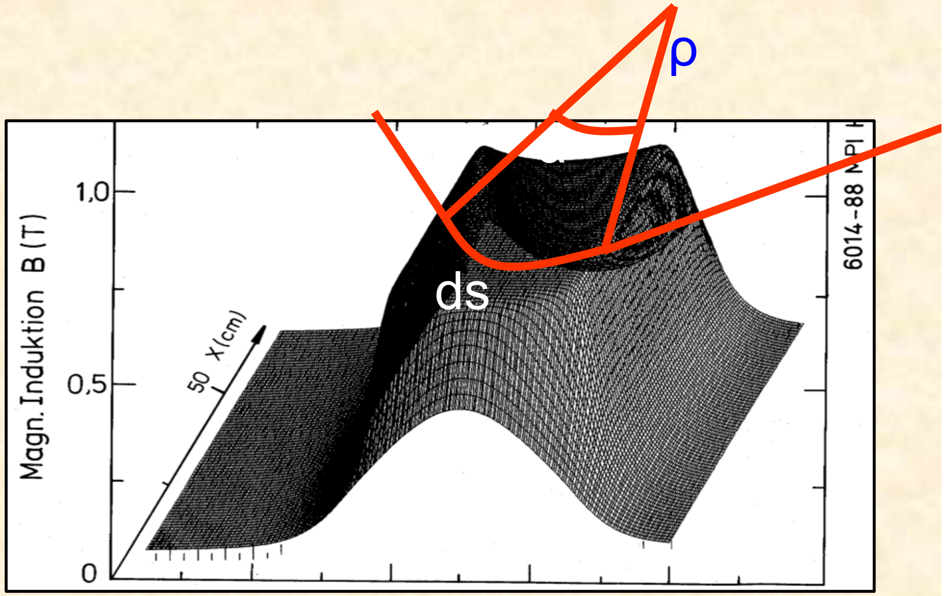
$$\alpha = \frac{\int B dl}{B * \rho} = 2\pi \qquad \rightarrow \qquad \int B dl = 2\pi * \frac{p}{q}$$

… for a full circle

Nota bene: $\frac{\Delta B}{B} \approx 10^{-4}$ *is usually required !!*

The Magnetic Guide Field





field map of a storage ring dipole magnet

$$B \approx 1 \ldots 8 \ \boldsymbol{T}$$

$$\rho = 2.83\ km \longrightarrow 2\pi\rho = 17.6 km$$

$$\approx 66\,\%$$

„normalised bending strength“

$$\frac{1}{\rho} = \frac{e\,B}{p}$$

rule of thumb:

$$\frac{1}{\rho} \approx 0.3\,\frac{B\left[T\right]}{p\left[GeV/c\right]}$$

The integrated dipole strength (along "s") defines the momentum of the particle beam.

$$\alpha = \frac{\int B dl}{B * \rho} = 2\pi \qquad \rightarrow \qquad \int B dl = 2\pi * \frac{p}{q}$$

A Tandem "Van de Graaf" Accelerator

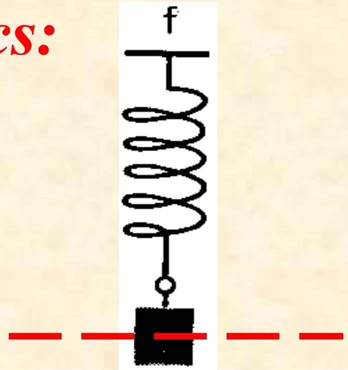
12 MV Voltage DC over 25 m

linear accelerating structure, no dipoles, no focusing,
just straight onto the target.

2.)Particles in Quadrupole Fields:

Focusing Properties of a magnet lattice

Classical Mechanics:
pendulum



there is a restoring force, proportional to the elongation x:

$$F = m \cdot \frac{d^2x}{dt^2} = -k \cdot x$$

Ansatz $x(t) = A \cdot cos(\omega t + \varphi)$

$$\dot{x}(t) = -A\ \omega \cdot sin(\omega t + \varphi)$$

general solution: free harmonic oscillation

$$\ddot{x}(t) = -A\ \omega^2 \cdot cos(\omega t + \varphi)$$

Solution $\omega = \sqrt{k/m}$ $\qquad x(t) = x_0 \cdot cos(\sqrt{\frac{k}{m}}\ t + \varphi)$

Apply this concept to magnetic forces:* we need a *Lorentz force* that rises as a function of the *distance to .... the design orbit

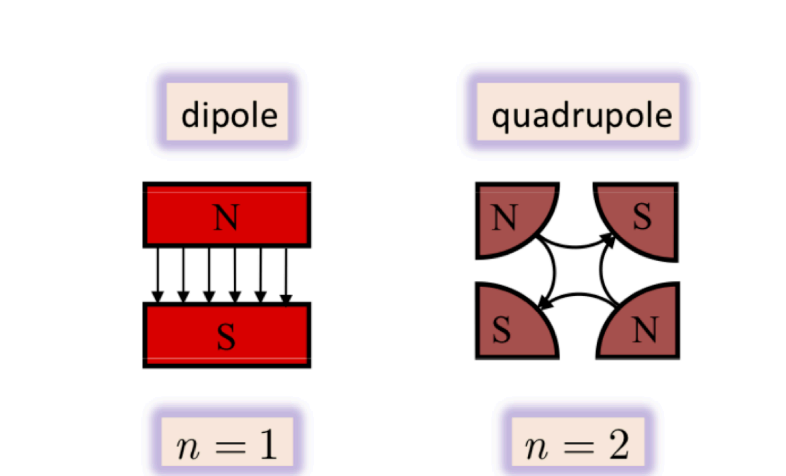
$$F(x) = q * v * B(x)$$

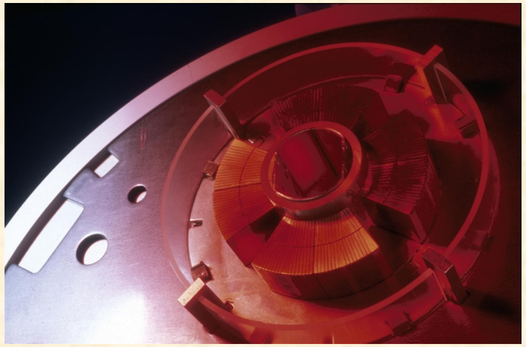
2.)Focusing Forces: Quadrupole Fields

Apply this concept to magnetic forces: we need a Lorentz force that rises as a function of the distance to ...

… the design orbit

$$F(x) = q * v * B(x)$$





Dipoles: Create a constant field $\quad B_y = const$

Quadrupoles: Create a linear increasing magnetic field: $\quad B_y(x) = g \cdot x, \quad B_x(y) = g \cdot y$

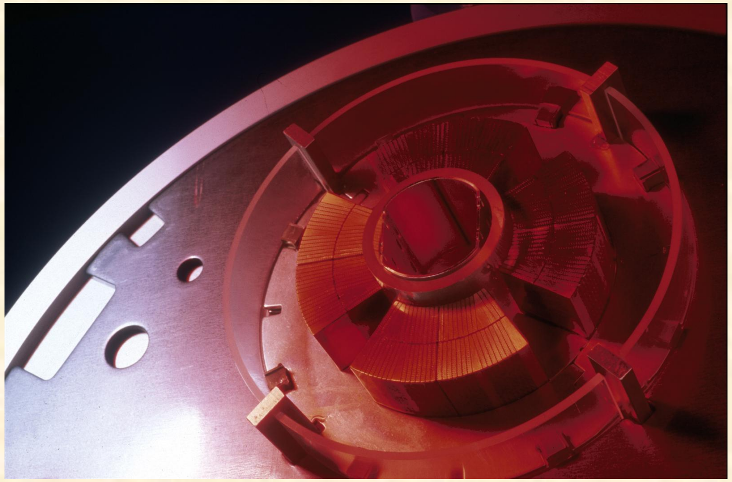
Quadrupole Magnets:

required: *focusing forces to keep trajectories in vicinity of the ideal orbit*

linear increasing Lorentz force

linear increasing magnetic field

$$B_y = g\,x \qquad B_x = g\,y$$



normalised quadrupole field:

gradient of a quadrupole magnet: $$g = \frac{dB_y}{dx}$$

normalised gradient $$k = \frac{g}{p/e}$$

LHC main quadrupole magnet $$g \approx 25 \dots 220 \;\; T/m$$

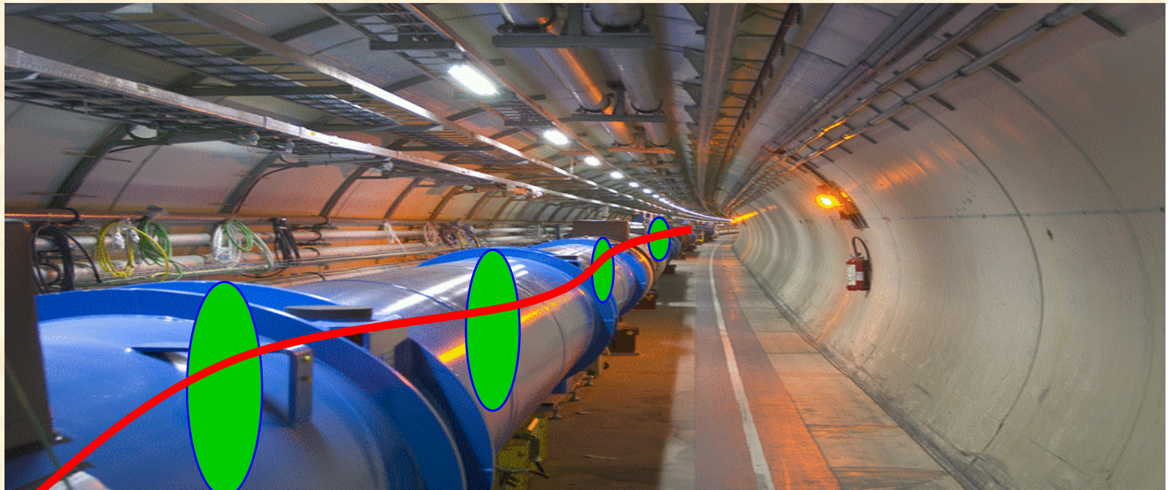
$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

$$k := \frac{g}{p/q}$$

what about the vertical plane:
... Maxwell

$$\vec{\nabla} \times \vec{B} = \cancel{\vec{j}} + \cancel{\frac{\partial \vec{E}}{\partial t}} = 0$$

$$\Rightarrow \qquad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$$



3.) Quadrupole Magnets:

required: ***focusing forces to keep trajectories in vicinity of the ideal orbit***

linear increasing Lorentz force

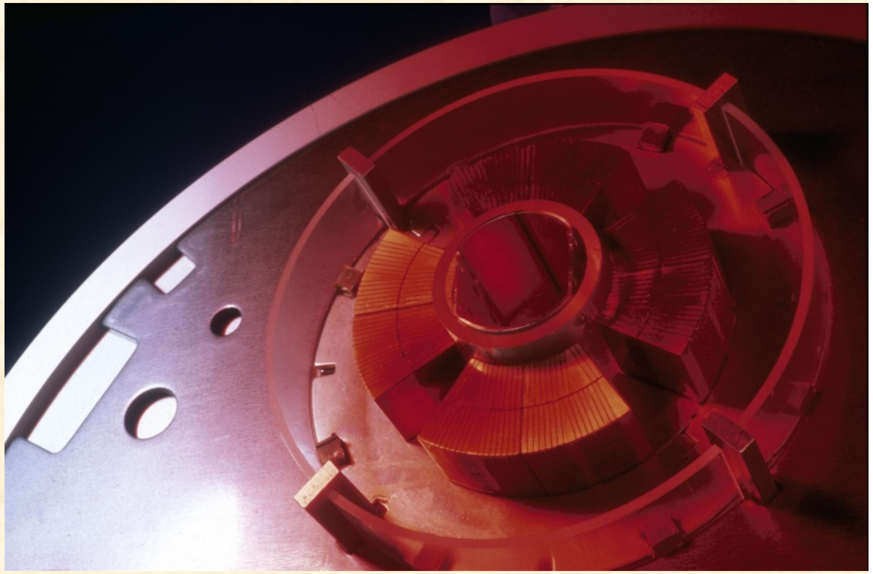
linear increasing magnetic field $B_y = g\,x \qquad B_x = g\,y$

normalised quadrupole field:

gradient of a quadrupole magnet: $g = \dfrac{2\mu_0 n I}{r^2}$

$\longrightarrow \qquad k = \dfrac{g}{p/e}$

simple rule: $\boxed{k = 0.3\,\dfrac{g(T/m)}{p(GeV/c)}}$



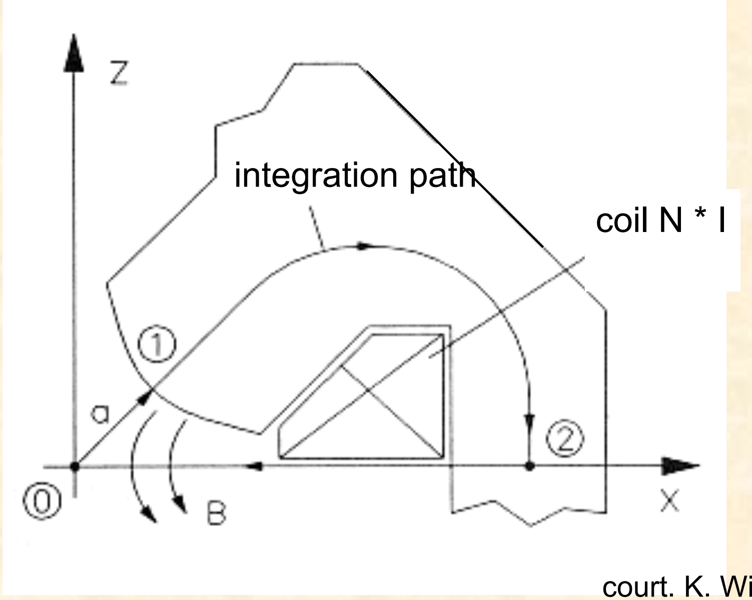
LHC main quadrupole magnet

$g \approx 25 \ldots 220 \;\; T/m$

what about the vertical plane: ... Maxwell

$$\vec{\nabla} \times \vec{B} = \cancel{\vec{j}} + \cancel{\frac{\partial \vec{E}}{\partial t}} = 0 \quad \Rightarrow \quad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$$

Quadrupole Magnets:



court. K. Wille

Calculation of the Quadrupole Field:

$$\oint H ds = N * I$$

$$\oint H ds = \int_0^1 H_0 ds + \underbrace{\cancel{\int_1^2 H_{Fe} ds}}_{H_{Fe}=H_0/\mu_{Fe},\ \mu_{Fe} \approx 1000} + \underbrace{\cancel{\int_2^0 H ds}}_{H\perp ds} = N * I$$

now we know that $H = B/\mu_0$

and we require $B(r) = -g * r$

$$\rightarrow \int_0^1 H_0 ds = \int_0^a \frac{B_0}{\mu_0} dr = \int_0^a \frac{g * r}{\mu_0} dr = N * I$$

gradient of a quadrupole field:

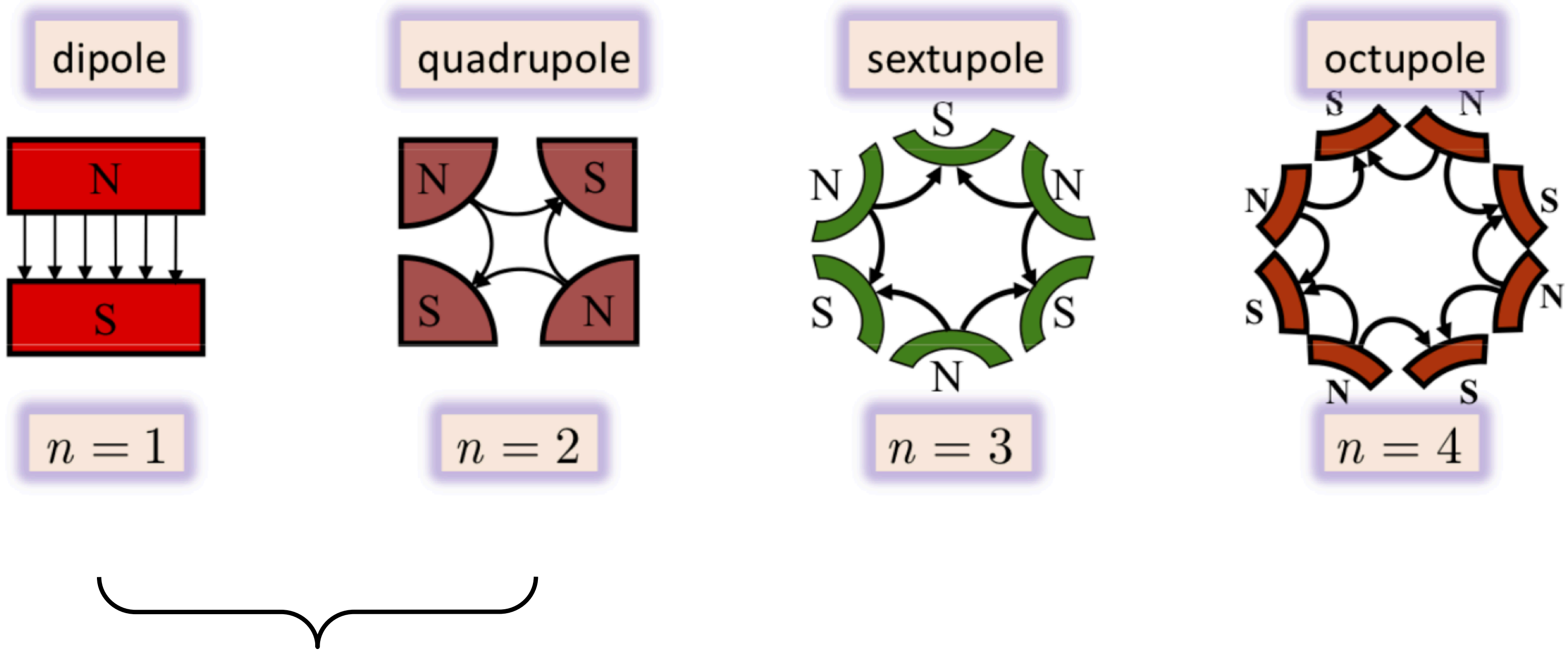
$$g = \frac{2\mu_0 * N * I}{r^2}$$

Linear Lattice:

Dipoles & Quadrupoles …
… and Drifts in between

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \cancel{\frac{1}{2!} m\,x^2} + \cancel{\frac{1}{3!} n\,x^3} + \ldots$$

Magnetic fields used in an accelerator:



Linear Lattice

Chromatic Correction

Landau Damping

The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \cancel{\frac{1}{2!}m\,x^2} + \cancel{\frac{1}{3!}n\,x^3} + \dots$$

only terms linear in x, y taken into account ***dipole fields***
quadrupole fields

Separate Function Machines:

Split the magnets and optimise them according to their job:

bending, focusing etc

Example:
heavy ion storage ring TSR

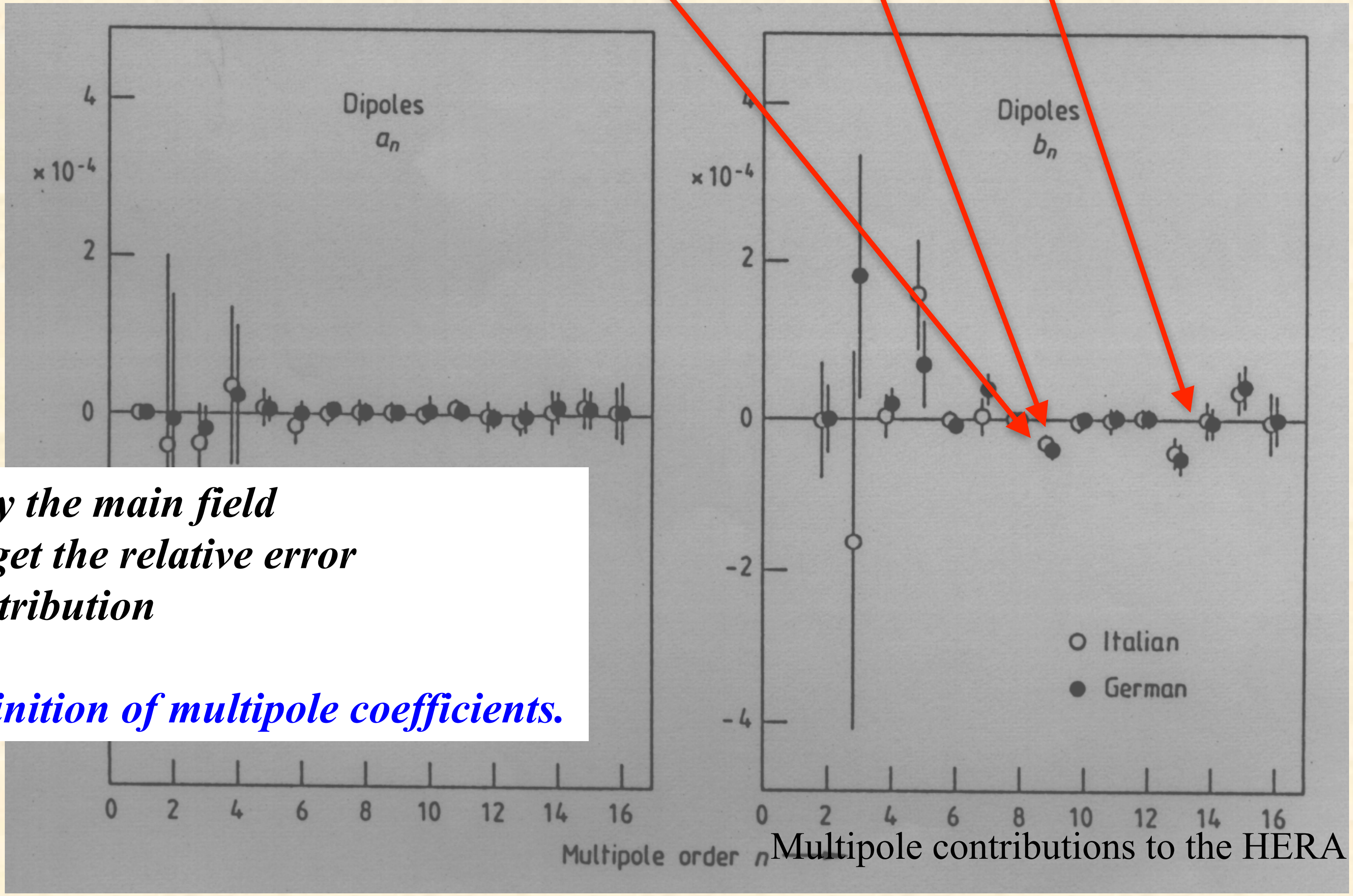
*

man sieht nur dipole und quads —> linear

*** ***Linear Lattice = Dipoles & quads.***
However linbear also means no prominent multipole contributions !!!

Taylor Expansion of the B field:

$$B_y(x) = B_{y0} + \frac{dB_y}{dx}x + \frac{1}{2!}\frac{d^2B_y}{dx^2}x^2 + \frac{1}{3!}\frac{d^3B_y}{dx^3}x^3 + \dots$$

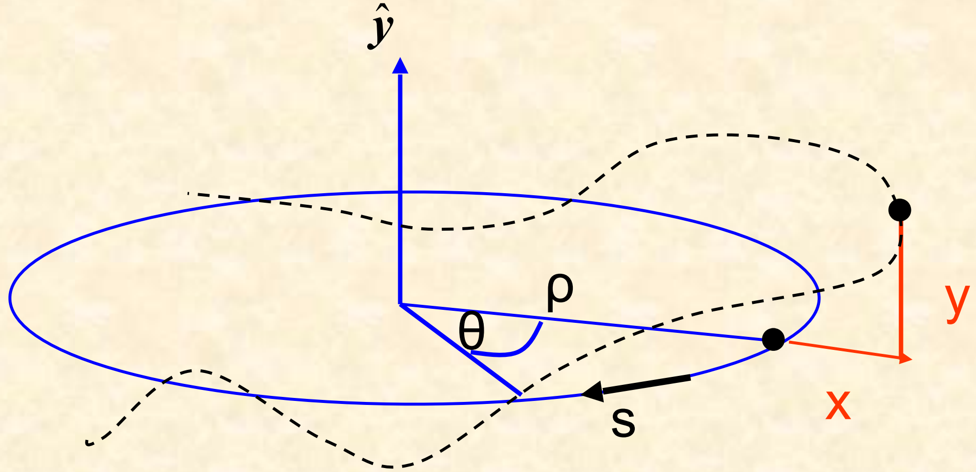


divide by the main field
to get the relative error
contribution

—> definition of multipole coefficients.

Multipole contributions to the HERA s.c. dipole field

4.) The equation of motion:



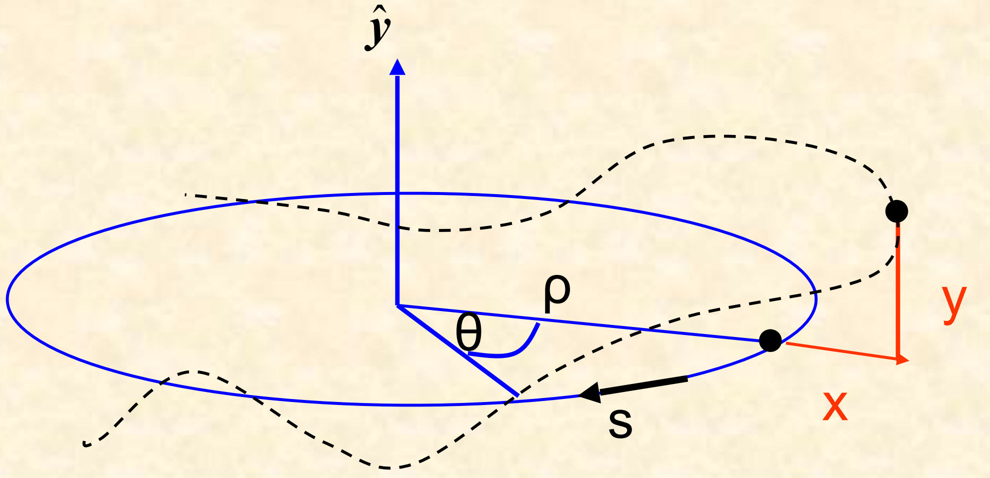
Linear approximation:

* ideal particle —> design orbit

* any other particle —> coordinates x, y small quantities
$x,y << \rho$

* magnetic guide field: only linear terms in x & y of B
have to be taken into account

Equation of Motion:



Consider local segment of a particle trajectory
... and remember the old days:

(Goldstein page 27)

radial acceleration:

$$a_r = \frac{d^2\rho}{dt^2} - \rho\left(\frac{d\theta}{dt}\right)^2$$

Ideal orbit: $\rho = const, \quad \frac{d\rho}{dt} = 0$

centrifugal Force:

$$F = m\rho\left(\frac{d\theta}{dt}\right)^2 = m\rho\omega^2 \quad = mv^2/\rho$$

general trajectory: $\rho \longrightarrow \rho + x$

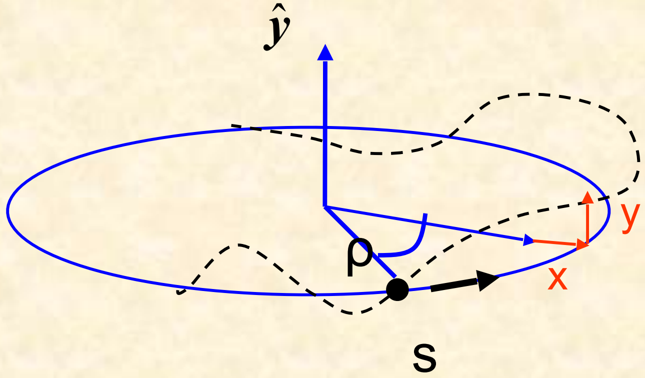
condition for circular orbit:

$$F_{centrifugal} + F_{Lorentz} = 0$$

$$\boxed{F = m\frac{d^2}{dt^2}(x+\rho) - \frac{mv^2}{x+\rho} = -\ eB_y v}$$

$$F = m\frac{d^2}{dt^2}(x+\rho) - \frac{mv^2}{x+\rho} = -\ eB_y v$$

1 2



1 $$\frac{d^2}{dt^2}(x+\rho) = \frac{d^2}{dt^2}x$$ ***… as ρ = const***

2 ***remember: x ≈ mm , ρ ≈ m … —> develop for small x***

$$\frac{1}{x+\rho} \approx \frac{1}{\rho}\left(1-\frac{x}{\rho}\right)$$

Taylor Expansion

$$f(x) = f(x_0) + \frac{(x-x_0)}{1!}f'(x_0) + \frac{(x-x_0)^2}{2!}f''(x_0) + \ldots$$

$$m\frac{d^2x}{dt^2} - \frac{mv^2}{\rho}\left(1-\frac{x}{\rho}\right) = -\ eB_y v$$

guide field in linear approx.

$$m\frac{d^2x}{dt^2} - \frac{mv^2}{\rho}\left(1-\frac{x}{\rho}\right) = -ev\left\{B_0 + x\frac{\partial B_y}{\partial x}\right\}$$

$$\frac{d^2x}{dt^2} - \frac{v^2}{\rho}\left(1-\frac{x}{\rho}\right) = -\frac{evB_0}{m} - \frac{evxg}{m}$$

$: m$

independent variable: $t \rightarrow s$

$$\frac{dx}{dt} = \frac{dx}{ds}\frac{ds}{dt}$$

$$x' = \frac{dx}{ds}$$ = *angle of the particle trajectory*

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{ds}\frac{ds}{dt}\right) = \frac{d}{ds}\left(\underbrace{\frac{dx}{ds}}_{x'}\underbrace{\frac{ds}{dt}}_{v}\right)\frac{ds}{dt}$$

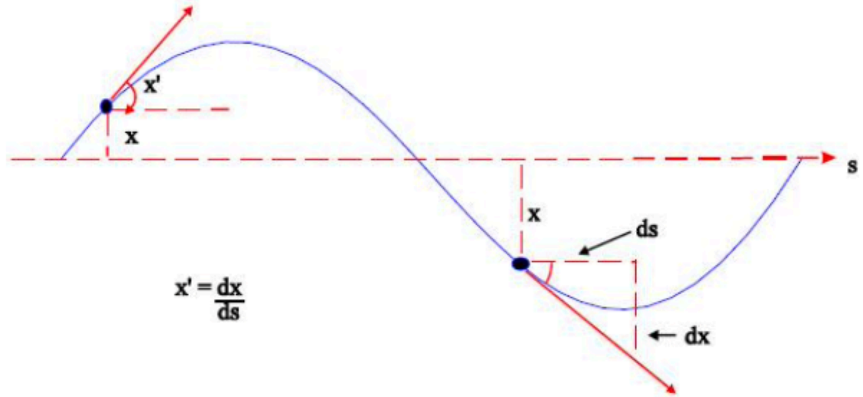
$$\frac{d^2x}{dt^2} = x''v^2 + \cancel{\frac{dx}{ds}\frac{dv}{ds}v}$$

$$x''v^2 - \frac{v^2}{\rho}\left(1-\frac{x}{\rho}\right) = -\frac{evB_0}{m} - \frac{evxg}{m}$$

$: v^2$

Nota bene:

Our coordinates are Amplitude and Angle



hor. Amplitude

x [m]

$$x' = \frac{dx}{ds} = \frac{dx}{dt}\frac{dt}{ds} = \frac{V_x}{V_s} = \frac{P_x}{P_s} \approx \frac{P_x}{P_0}$$ [rad]

vert. Amplitude

y [m]

$$y' = \frac{dy}{ds} = \frac{dy}{dt}\frac{dt}{ds} = \frac{V_y}{V_s} = \frac{P_y}{P_s} \approx \frac{P_y}{P_0}$$ [rad]

$$x'' - \frac{1}{\rho}(1 - \frac{x}{\rho}) = -\frac{eB_0}{mv} - \frac{exg}{mv}$$

$m\,v = p$

$$x'' - \frac{1}{\rho} + \frac{x}{\rho^2} = -\frac{B_0}{p/e} - \frac{xg}{p/e}$$

normalize to momentum of particle

$$\frac{B_0}{p/e} = -\frac{1}{\rho}$$

$$\frac{g}{p/e} = k$$

$$x'' \cancel{-\frac{1}{\rho}} + \frac{x}{\rho^2} = \cancel{-\frac{1}{\rho}} - xk$$

$$x'' + x(\frac{1}{\rho^2} + k) = 0$$

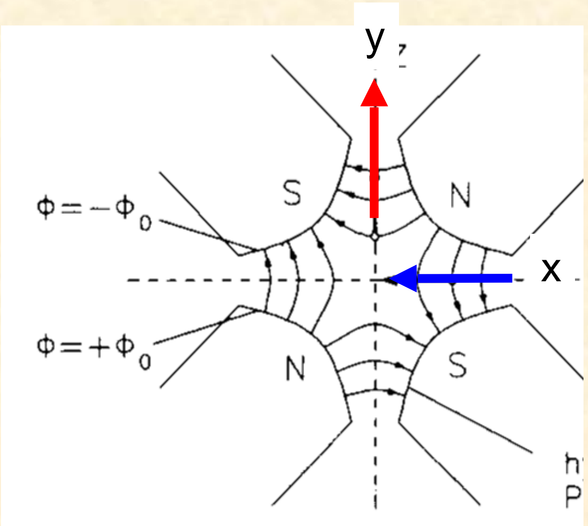
(k > 0 —> foc quad, as defined in MADX)

* ***Equation for the vertical motion:***

$\frac{1}{\rho^2} = 0$ ***no dipoles … in general …***

$\boldsymbol{k \leftrightarrow -k}$ ***quadrupole field changes sign***

$$y'' - k \cdot y = 0$$



!

Remarks:

* $$x'' + (\frac{1}{\rho^2} - k) \cdot x = 0$$

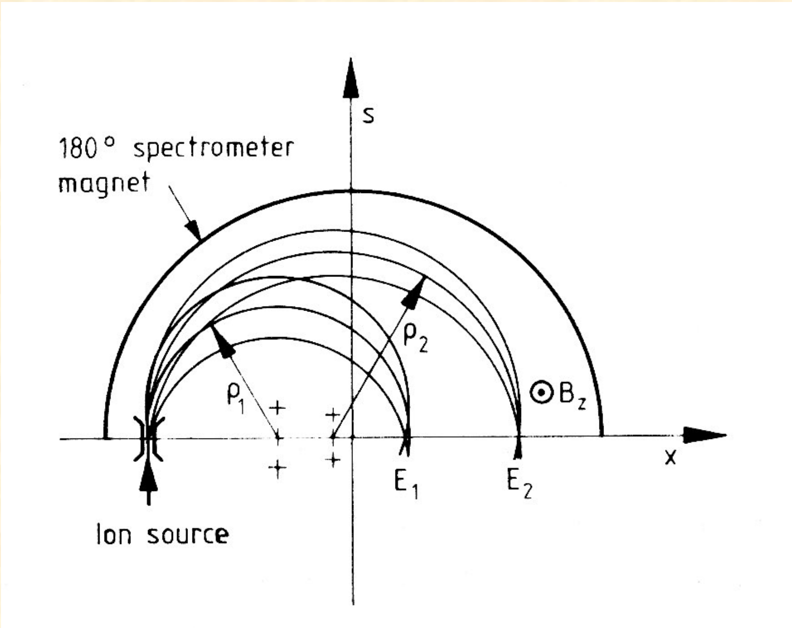
… there seems to be a focusing even without a quadrupole gradient

„weak focusing of dipole magnets‘

$$\boldsymbol{k} = 0 \qquad \Rightarrow \qquad \boldsymbol{x''} = -\frac{1}{\rho^2}\,\boldsymbol{x}$$

even without quadrupoles there is a retriving force (i.e. focusing) in the bending plane of the dipole ma

… in large machines it is weak. (!)



Mass spectrometer: particles are separated according to their energy and focused due to the 1/ρ effect of the dipole

5.) Solution of Trajectory Equations

Define … hor. plane: $K = 1/\rho^2 - k$

… vert. Plane: $K = k$

$$\boldsymbol{x'' + K\ x = 0}$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: $x(s) = a_1 \cdot \cos(\omega s) + a_2 \cdot \sin(\omega s)$

general solution: linear combination of two independent solutions

$$x'(s) = -a_1\omega \sin(\omega s) + a_2\omega \cos(\omega s)$$

$$x''(s) = -a_1\omega^2 \cos(\omega s) - a_2\omega^2 \sin(\omega s) = -\omega^2 x(s) \longrightarrow \omega = \sqrt{K}$$

general solution:

$$x(s) = a_1 \cos(\sqrt{K}s) + a_2 \sin(\sqrt{K}s)$$

determine a_1 , a_2 by boundary conditions:

$$s = 0 \longrightarrow \begin{cases} x(0) = x_0 \quad , \quad a_1 = x_0 \\ x'(0) = x'_0 \quad , \quad a_2 = \dfrac{x'_0}{\sqrt{K}} \end{cases}$$

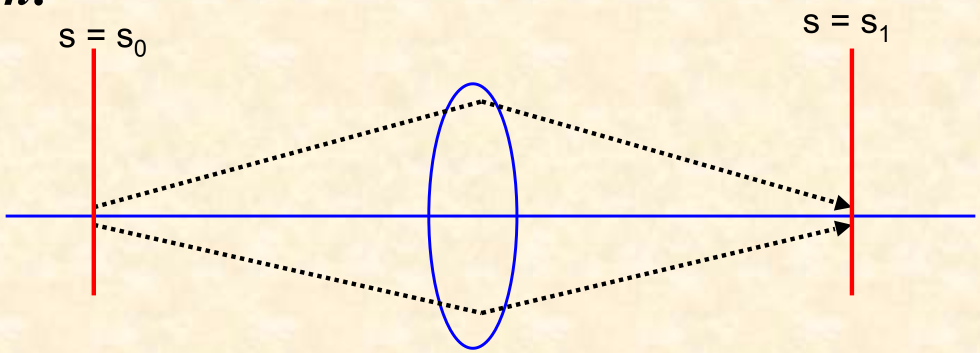
Hor. Focusing Quadrupole $K > 0$:

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x'_0 \cdot \cos(\sqrt{|K|}s)$$

For convenience expressed in matrix formalism:

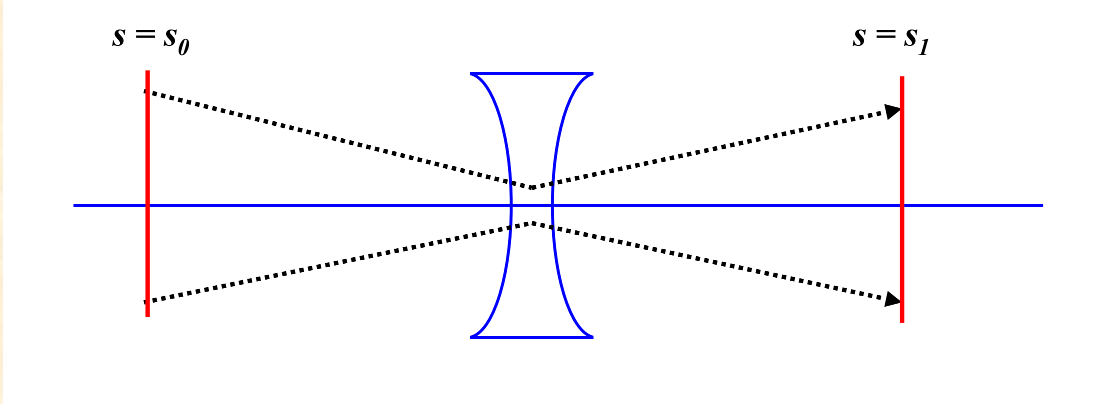
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



$$\boldsymbol{M}_{foc} = \begin{pmatrix} \cos\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) & \dfrac{1}{\sqrt{|\boldsymbol{K}|}} \sin\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) \\ -\sqrt{|\boldsymbol{K}|} \sin\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) & \cos\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) \end{pmatrix}$$

hor. defocusing quadrupole:

$$x'' - K\,x = 0$$



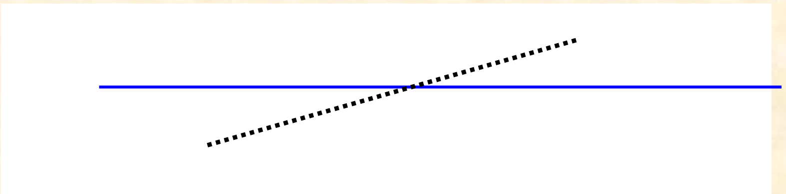
Remember from school:

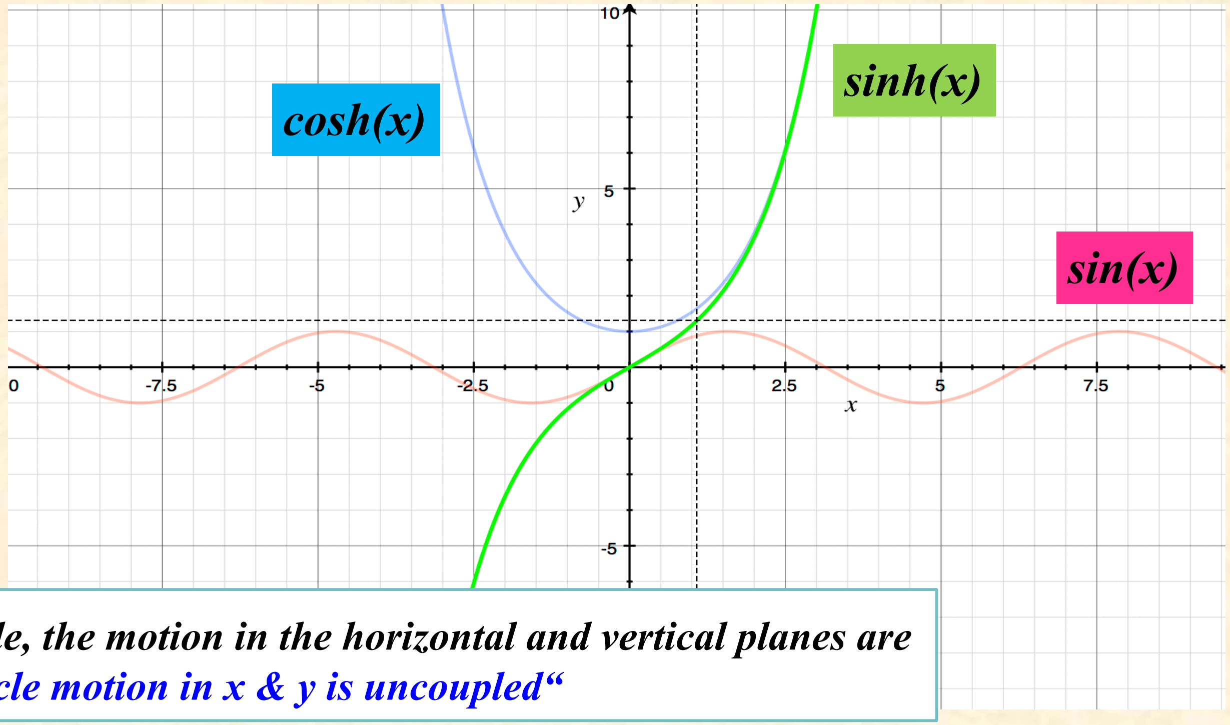
$$f(s) = \cosh(s) \quad , \quad f'(s) = \sinh(s)$$

Ansatz: $$x(s) = a_1 \cdot \cosh(\omega\, s) + a_2 \cdot \sinh(\omega\, s)$$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}\,l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}\,l \\ \sqrt{|K|}\sinh\sqrt{|K|}\,l & \cosh\sqrt{|K|}\,l \end{pmatrix}$$

drift space: ***K = 0***





$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

! ***with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“***

One word for the Math Lovers

We talk about a differential equation of second order.
... which has two independent solutions.

$$\boldsymbol{x'' + K\, x = 0}$$

e.g. hor. Focusing Quadrupole K > 0:

$$x(s) = \underbrace{x_0 \cdot \cos(\sqrt{|K|}s)}_{C} + \underbrace{x_0' \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)}_{S}$$

$$x'(s) = \underbrace{-x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s)}_{C'} + \underbrace{x_0' \cdot \cos(\sqrt{|K|}s)}_{S'}$$

Wronski tells us:
The two solutions are independent of each other if the Wronski determinant ≠ 0.

Each of the two solutions fulfils

$$C'' + K(s)C = 0 \qquad S'' + K(s)S = 0$$

$$W = \left| \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \right| \quad \longrightarrow \quad \frac{d}{ds}W = CS'' - SC'' = -K(CS - SC) = 0$$

So, W= const.
We can choose the initial values at s=0

$$\left.\begin{matrix} C_0 = 1 & S_0 = 0 \\ C_0' = 0 & S_0' = 1 \end{matrix}\right\} \longrightarrow$$

W=1 for all linear accelerator element matrices

$$W = det \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} = 1$$

Thin Lens Approximation:

matrix of a quadrupole lens

$$M = \begin{pmatrix} \cos\sqrt{|k|}l & \frac{1}{\sqrt{|k|}}\sin\sqrt{|k|}l \\ -\sqrt{|k|}\sin\sqrt{|k|}l & \cos\sqrt{|k|}l \end{pmatrix}$$

in many practical cases we have the situation:

$f = \frac{1}{kl_q} >> l_q$ ***... focal length of the lens is much bigger than the length of the magnet***

limes: $l_q \rightarrow 0$ ***while keeping*** $k\, l_q = const$

$$M_x = \begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix} \qquad M_y = \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix}$$

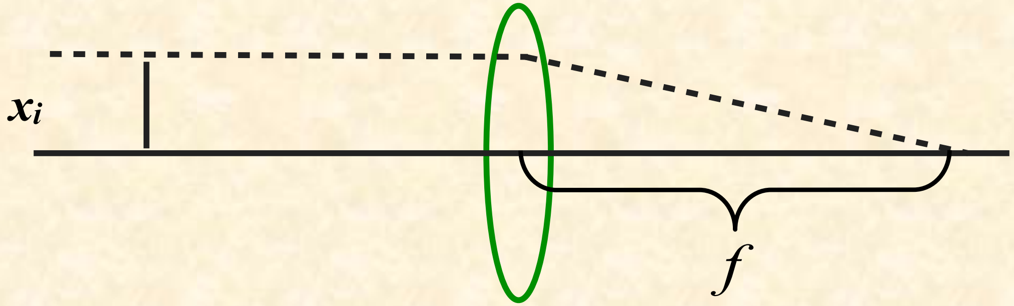
... useful for fast (and in large machines still quite accurate) „back on the envelope calculations" ... and for the guided studies !

Focal Length of a Quadrupole:

matrix of a (thin) quadrupole lens

$$\boldsymbol{M_x} = \begin{pmatrix} 1 & 0 \\ \frac{-1}{\boldsymbol{f}} & 1 \end{pmatrix}$$

$$f = \frac{1}{kl_q}$$



Definition of focal length:

a trajectory with amplitude x_0 parallel to "s" will be focussed to $x_i = 0$ within the length f

$$x_i - x'_f \cdot f = 0 \qquad x'_f = \frac{l}{\rho} = \frac{Bl}{B\rho}$$

$$x'_f = \frac{x_i\ g\ l_q}{B\ \rho} = x_i k l_q$$

$$x_i - x_i \cdot kl_q \cdot f = 0 \qquad 1 - kl_q \cdot f = 0 \longrightarrow \boxed{f = \frac{1}{kl_q}}$$

„veni vidi vici ...“ .... or in english .... „we got it !“

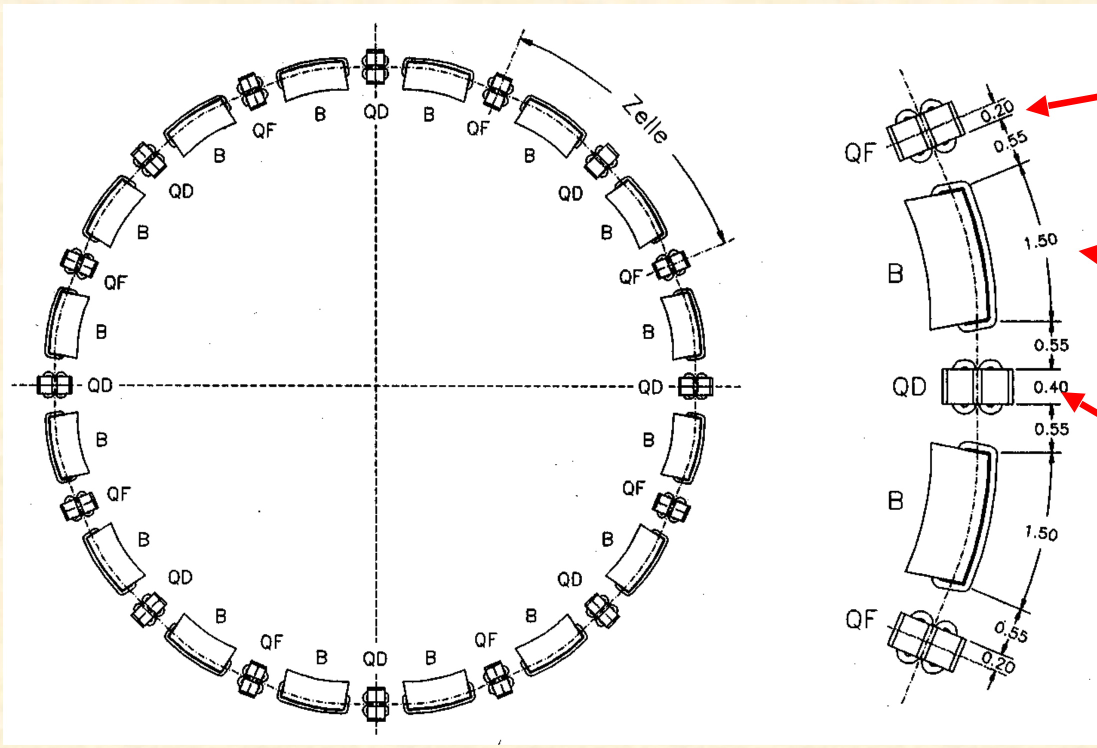
** we can calculate the trajectory of a single particle, inside a storage ring magnet (lattice element)*

** for arbitrary initial conditions $x_{0,}\ x'_0$*

** we can combine these trajectory parts (also mathematically) and so get the complete transverse trajectory around the storage ring*

$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D*} .....$$

Example:
Toy storage ring
for the kids
(court. K.Wille)



horizontal focussing quadrupole lens

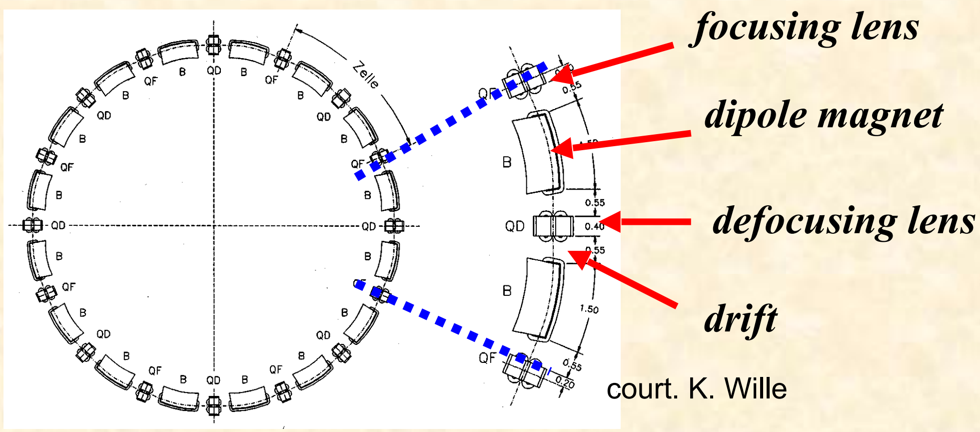
dipole magnet

horizontal defocussing quadrupole lens

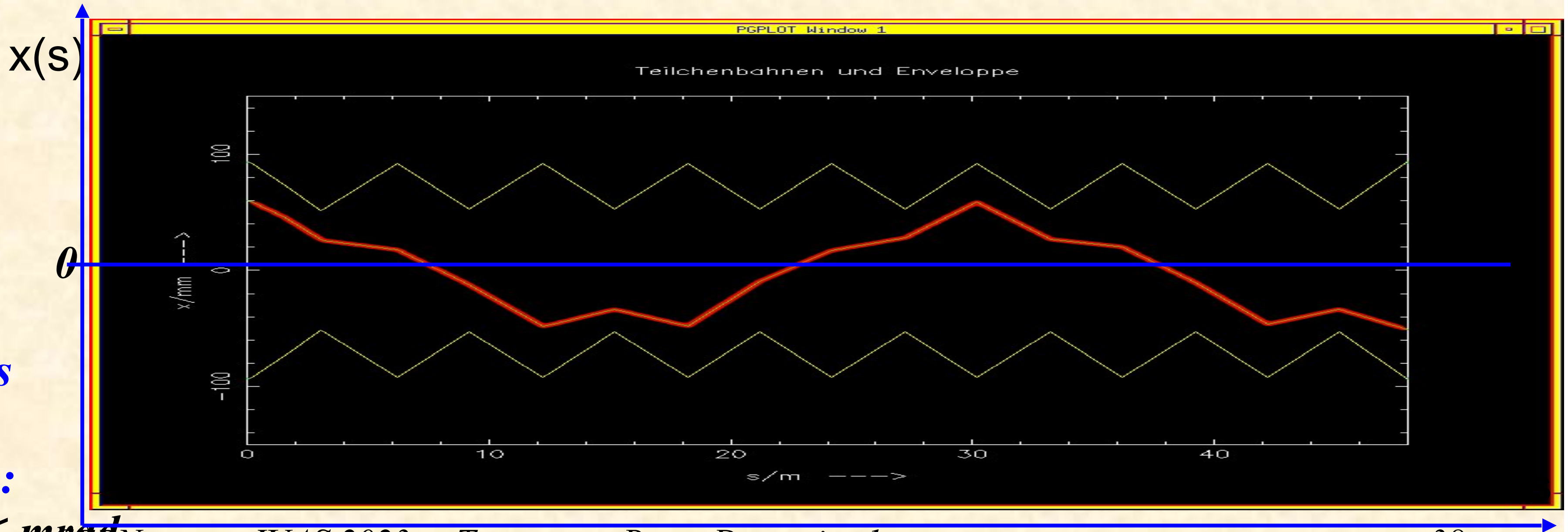
Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D*.....}$$



in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „



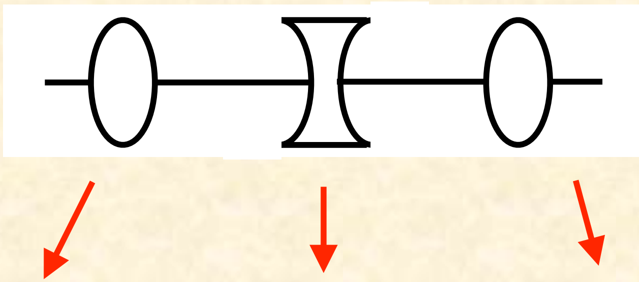
typical values in a strong foc. machine:
$x \approx$ mm, $x' \leq$ mrad

Can we understand what the optics code is doing ?

matrices $$M_{QF} = \begin{pmatrix} \cos(\sqrt{K}*l_q) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}*l_q) \\ -\sqrt{K}\sin(\sqrt{K}*l_q) & \cos(\sqrt{K}*l_q) \end{pmatrix}, \qquad M_{Drift} = \begin{pmatrix} 1 & l \\ 0 & 1_d \end{pmatrix}$$

strength and length of the FoDo elements K = +/- 0.54102 m^{-2}
lq = 0.5 m
ld = 2.5 m

The matrix for the complete cell is obtained by multiplication of the element matrices

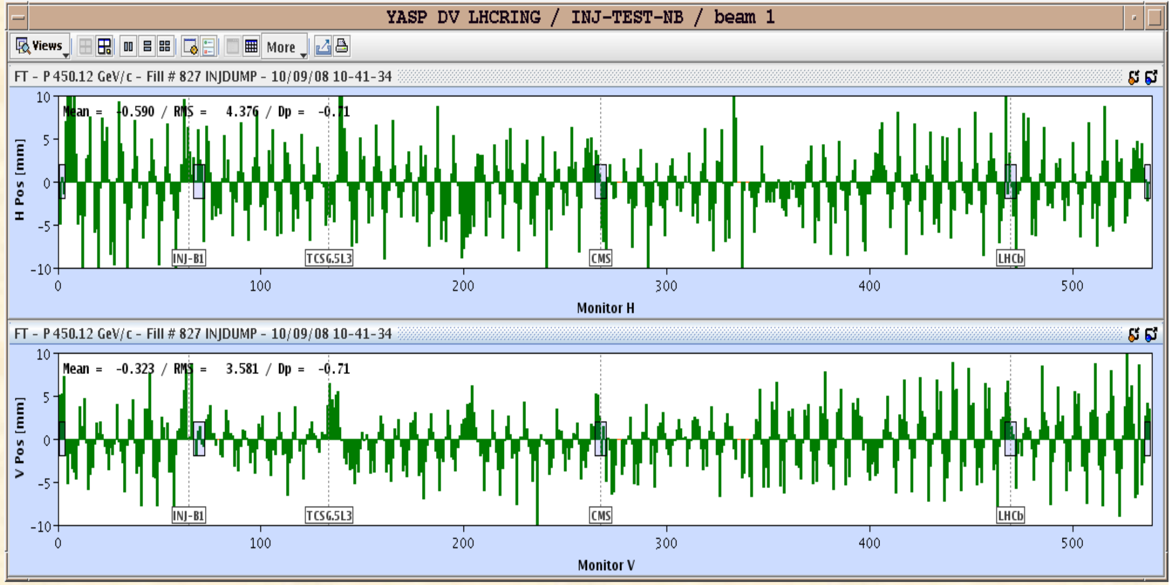


$$M_{FoDo} = M_{qfh} * M_{ld} * M_{qd} * M_{ld} * M_{qfh}$$

Putting the numbers in and multiplying out ...

$$M_{FoDo} = \begin{pmatrix} 0.707 & 8.206 \\ -0.061 & 0.707 \end{pmatrix}$$

6.) Orbit & Tune:



Tune: number of oscillations per turn

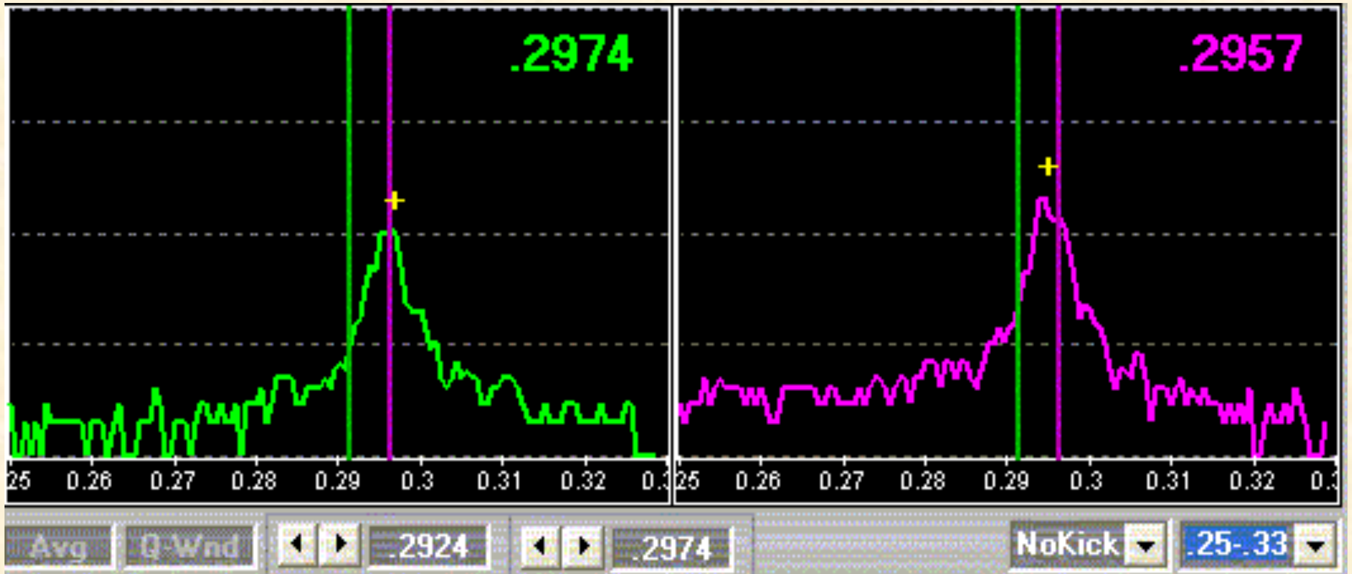
64.31
59.32

Relevant for beam stability:

non integer part

LHC revolution frequency: 11.3 kHz

$0.31*11.3 = 3.5kHz$



Ok … ok ... it's a bit complicated and cosh and sinh and all that is a pain.
BUT ... compare ...

Weak Focusing / Strong Focusing

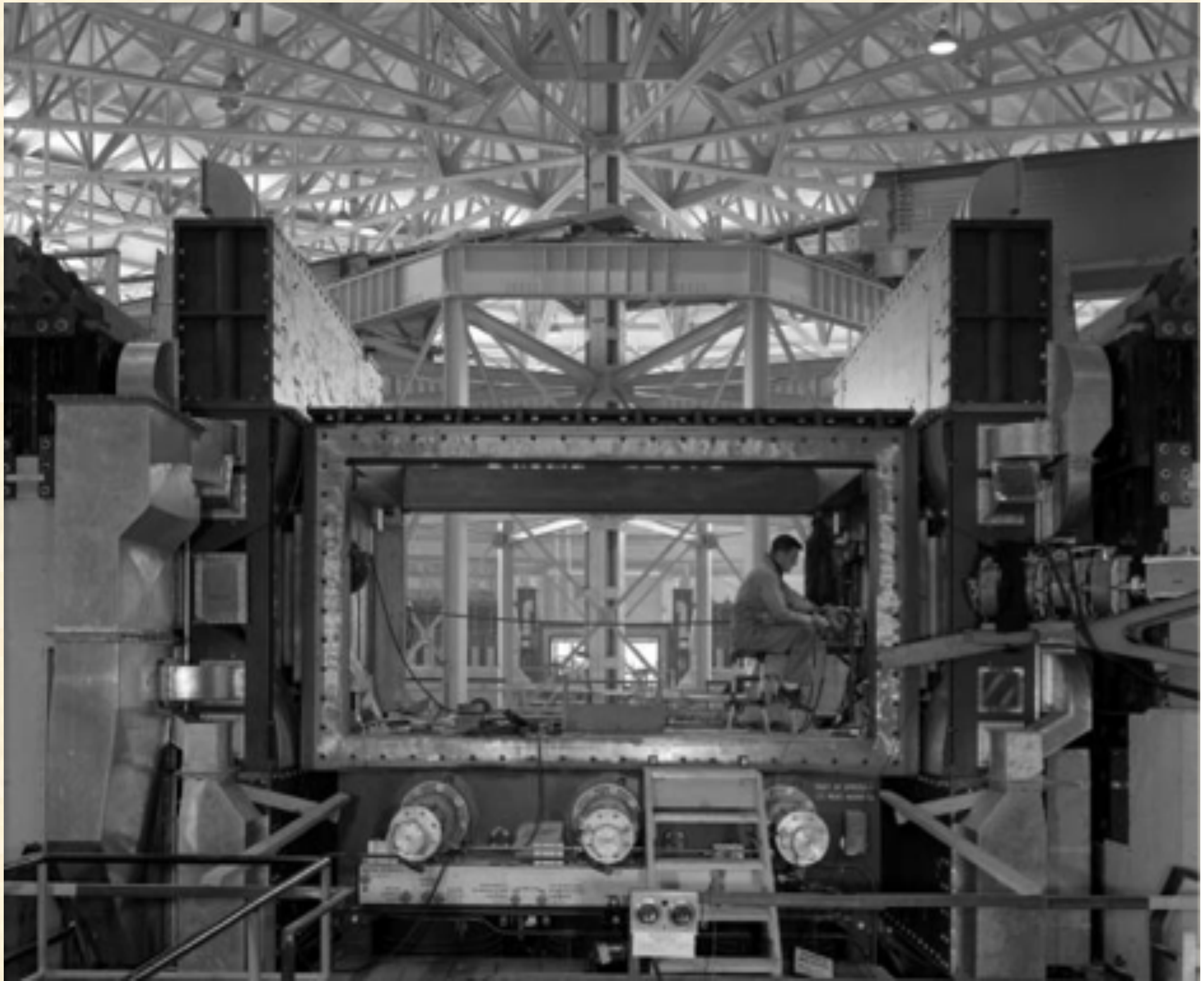
weak focusing term = 1/ρ²

$$x'' + x\left(\frac{1}{\rho^2} + \cancel{k}\right) = 0$$

Problem: the higher the energy, the larger the machine

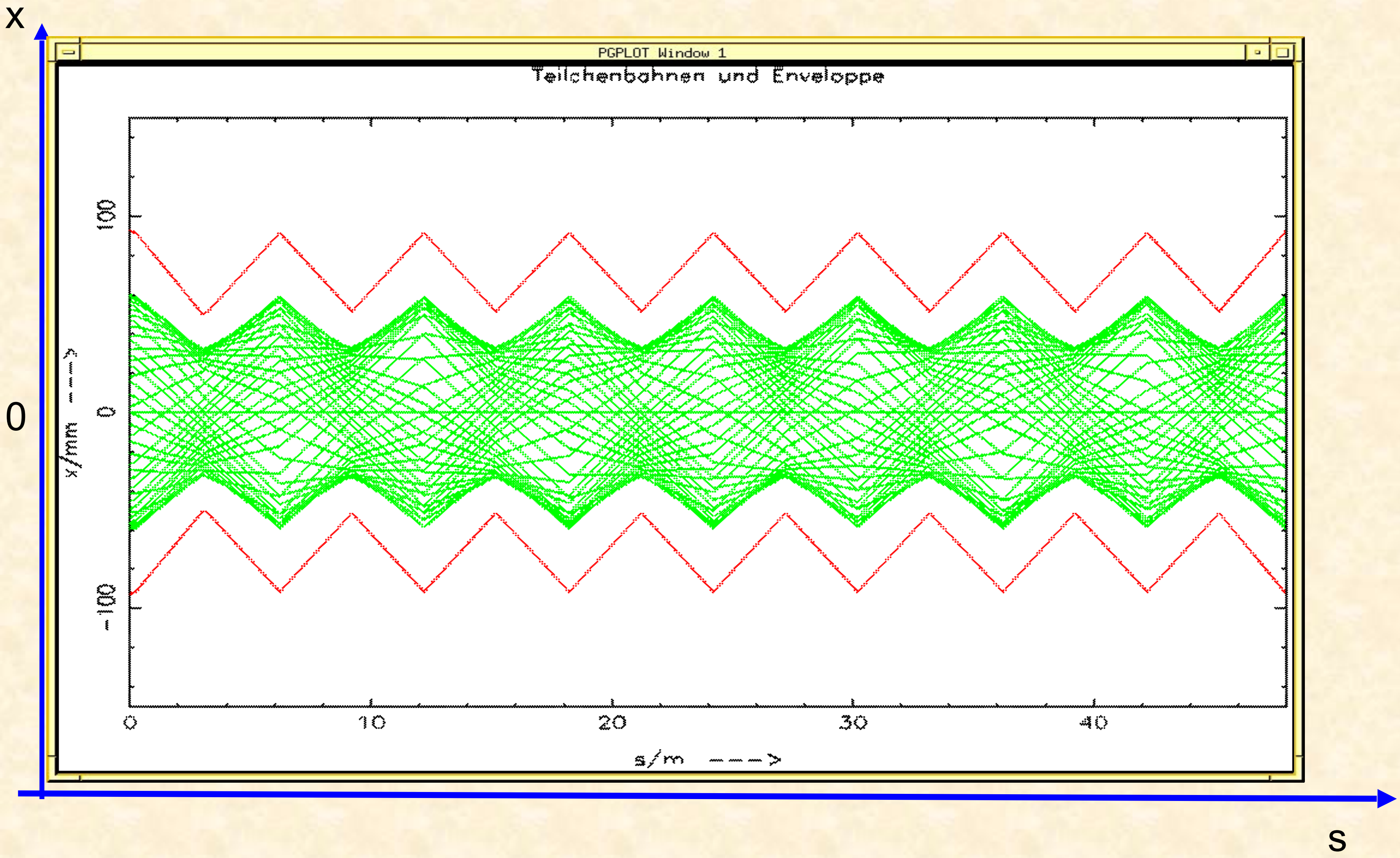
The last weak focusing high energy machine … BEVATRON

→ *large apertures needed*
→ *very expensive magnets*



Question: what will happen, if the particle performs a second turn ?

... or a third one or ... 10^{10} turns



Once more … the crucial question is …

Gretchen Frage (J.W. Goethe, Faust)

Do they actually drop ?

???

Résumé:

beam rigidity: $B \cdot \rho = p/q$

bending strength of a dipole: $\frac{1}{\rho}\left[m^{-1}\right] = \frac{0.2998 \cdot B_0(T)}{p(GeV/c)}$

focusing strength of a quadrupole: $k\left[m^{-2}\right] = \frac{0.2998 \cdot g}{p(GeV/c)}$

focal length of a quadrupole: $f = \frac{1}{k \cdot l_q}$

equation of motion: $x'' + Kx = \frac{1}{\rho}\frac{\Delta p}{p}$

matrix of a foc. quadrupole: $x_{s2} = M \cdot x_{s1}$

$$M = \begin{pmatrix} \cos\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sin\sqrt{|K|}l \\ -\sqrt{|K|}\sin\sqrt{|K|}l & \cos\sqrt{|K|}l \end{pmatrix}, \qquad M = \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix}$$

6.) Bibliography:

1.) Edmund Wilson: Introd. to Particle Accelerators
Oxford Press, 2001

2.) Klaus Wille: Physics of Particle Accelerators and Synchrotron
Radiation Facilicties, Teubner, Stuttgart 1992

3.) Peter Schmüser: Basic Course on Accelerator Optics, CERN Acc.
School: 5th general acc. phys. course CERN 94-01

4.) Bernhard Holzer: Lattice Design, CERN Acc. School: Interm.Acc.phys course,
http://cas.web.cern.ch/cas/ZEUTHEN/lectures-zeuthen.htm

5.) M.S. Livingston, J.P. Blewett: Particle Accelerators, Mc Graw-Hill,
New York,1962

6.) The CERN Accelerator School (CAS) Proceedings

7.) Frank Hinterberger: Physik der Teilchenbeschleuniger, Springer Verlag 1997

8.) Mathew Sands: The Physics of e+ e- Storage Rings, SLAC report 121, 1970

9.) D. Edwards, M. Syphers : An Introduction to the Physics of Particle
Accelerators, SSC Lab 1990