Questions for the Tutorial on Transverse Beam Dynamics

Questions to relax

1.) Can you explain in your own words the meaning of …

phase advance
beam emittance
β-function

…what does it mean, if you are told that the phase advance per cell is 90 degrees in the horizontal and 60 degrees in the vetical plane ?

Concerning the two parameters β-function and beam emittance: they both determine the beam envelope. Can you explain the difference ?

How will the phase space ellipse look like in general ?
And how does it look like at an Interaction point … and why ?

2.) About a real storage ring:
LHC: particle momentum, geometry of a storage ring and thin lenses

The LHC storage ring at CERN will collide proton beams with a maximum momentum of $p = 7\ TeV/c$ per beam. The main parameters of this machine are:

Circumference	$C_0 = 26658.9m$	
particle momentum	$p = 7\ TeV/c$	
main dipoles	$B = 8.392\ T$	$l_B = 14.2m$
main quadrupoles	$G = 235\ T/m$	$l_q = 5.5m$

Calculate the magnetic rigidity of the design beam, the bending radius of the main dipole magnets in the arc and determine the number of dipoles that is needed in the machine. Calculate the k-strength of the quadrupole magnets and compare its focal length to the length of the magnet. Can this magnet be treated as a thin lens?

How does the matrix for such a (foc.) magnet look like?
How would you establish a description of this magnet in thin lens approximation?
Compare the matrix elements.
Nota bene: in our notation a foc. magnet has a negative k-value.

3.) Beam rigidity & particle momentum *(court. Ted Wilson)*

… or the stupid question: after all we have to deal with a relativistic beam!

A synchrotron of 25m radius accelerates protons from a kinetic energy of 50 MeV to 1000 MeV. What is the maximum energy of a deuteron beam (Z=1, A=2) that could be accelerated in the machine ?

4.) Apertures and Beam Envelopes:

The LHC magnet structure in the arcs consists of a symmetric FoDo with 90° phase advance per cell and an aperture radius of r_0 =20mm.

a.) Given the value of β_{max} = 500m in a QF quadrupole lens, what beam emittance would just touch the vacuum chamber ? (We call this value the "acceptance" of the machine).

b.) If now the typical emittance of a real stored beam at 450 GeV injection energy is $\varepsilon \approx 7*10^{-9}$ rad m, how many σ of beam envelope fit into the vacuum chamber for a β_{max} = 500m ?

c) what will happen if – keeping the beam optics constant – you accelerate the beam to an energy of E = 7000 GeV ?

During luminosity operation at this energy we require at least 14 sigma aperture due to background and quench safety reasons. What is the maximum beta function that can be accepted if the aperture of our mini beta quadrupoles is 20mm?

5.) Questions just for the fun of it:

When I did that calculation, I myself was very much surprised !!

(…solution will be given having a good beer in the evening).

Let´s build a real cheap storage ring. Just put it to the north pole and use the magnetic field of the earth whose field lines are perpendicular to the surface at that nice place.

Forget about focusing … that's for nitpickers.

What will be the size of the ring for a 10 keV electron beam if the earth magnetic field is about 0.5 Gauß ?

And now for the beer:

10 keV is similar to the energy in our conventional TV screens. So - if all these considerations are true - given a length of 30cm (for the distance between the TV gun and the screen) the displacement at the screen due to the earth magnetic field is a few millimetres.

So … turning the TV screen around the colours should change !

Do they ?? Or is that all nonsense ???

And in the end, in which direction do you have to circulate the electrons to get stored beam?

6.) Dispersion and Chromaticity

Can you explain in your own words the meaning of …
dispersion
chromaticity

7.) Consider a linear collider: *The general structure of such a machine does not differ too much from the arc of a storage ring. Clearly – it is not a circular machine but concerning the optics it is quite similar …*

Does such a Linac have a chromaticity?
And if so how would you correct it ?

8.) … if not explained already in the lecture.

The beta function in the quadrupoles of a symmetric FoDo cell (i.e. $f_{qd} = -f_{qf}$) depends only on the cell length L and the phase advance μ of the cell:

$$\beta_{QF} = L * \frac{1 + \sin\frac{\mu}{2}}{\sin\mu}$$

$$\beta_{QD} = L * \frac{1 - \sin\frac{\mu}{2}}{\sin\mu}$$

a) Establish the matrix of a FoDo in thin lens approximation and proof these relations.
Hint: remember the trigonometric magic tricks $1 - \cos\mu = 2\sin^2\frac{\mu}{2}$

b) Consider a proton beam where in general the emittances are equal in both planes: $\varepsilon_x = \varepsilon_y$.

Find the optimum phase advance that will give the smallest value for the radius of a particle beam in both planes

$$r^2 = \sigma_x^2 + \sigma_y^2$$

.

9.) it is a wild world …

During the construction phase of a heavy ion storage ring one quadrupole magnet turned out to be too short by 1 mm: The one meter long yoke was stapled by steel plates, 1mm in thickness each, and one of them just was forgotten (this is no joke !).

Calculate the tune change in both planes if this error is not compensated and the beta functions at the location of the quadrupole are $\beta_x = 80m, \quad \beta_y = 20m$ *in the hor. and vert. plane respectively. The quadrupole strength is* $k = 2*10^{-2}$.

Lets assume that the beam will survive this error. (Clearly we corrected the error nevertheless).

Now lets have a look at a typical mini beta insertion:
Given the following parameters:

$$k = 3.4*10^{-2}\frac{1}{m^2}, \quad lq = 1.9\,m, \quad \beta = 1{,}6\,km$$

Assume that the tune shift calculated above is a limit that can be tolerated for beam operation. What is the tolerance for the relative error of the magnet strength, (… or power supply stability, or magnet length …) at such a mini beta section ?

10.) Tuning Quadrupoles:

The main dipole and quadrupole magnets in a storage ring are often powered in series by one power supply. While such a set up facilitates the tracking of the main dipole and quadrupole magnets during acceleration it requires special "tuning" quadrupole circuits for tune adjustments. Assume your machine has one tuning quadrupole per plane, placed at a location where $\beta\, x = 180m$, $\beta\, y = 40m$.
If the overall tune is Q=64.28, what is the maximum tuning range of this system if the maximum acceptable beta-beat in the machine due to the tuning is limited to10% ?
How can the system described above be improved, to obtain a larger tuning range with a beta-beat that is still smaller than 10% ?