JUAS 2022
Longitudinal Beam Dynamics Examination
CORRECTION

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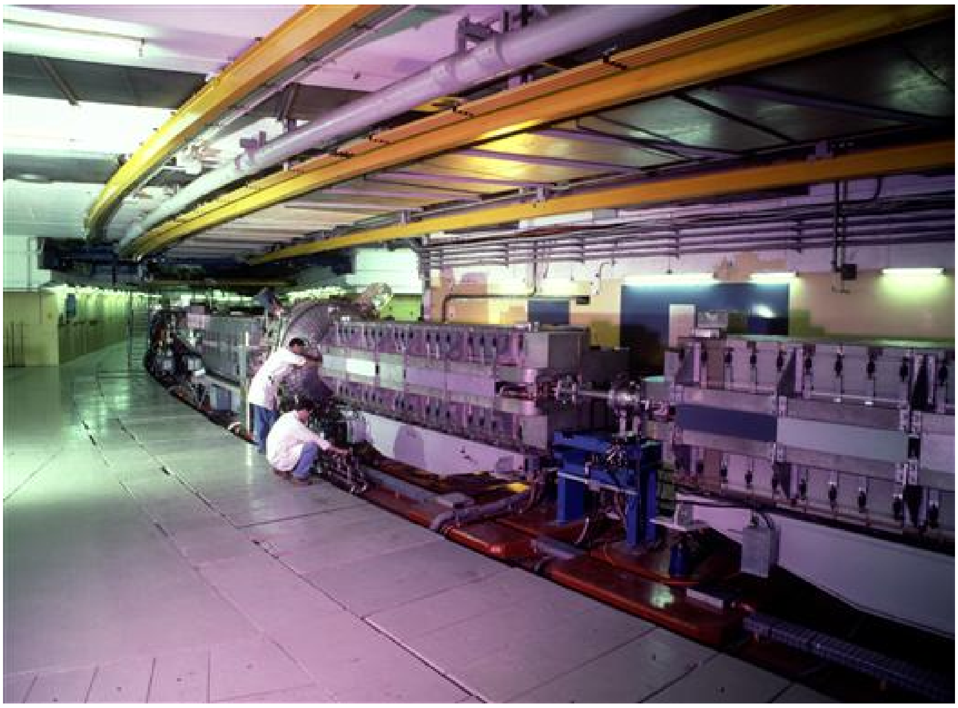
24/01/2022
(11:00 - 12:30)



The theme of the examination is the CERN Proton Synchrotron (PS). The PS is part of the LHC injection chain and accelerates the beam from a kinetic energy $E_{\mathrm{kin}} = 2$ GeV to $E_{\mathrm{kin}} = 25$ GeV. The PS is characterized by its numerous RF systems allowing to perform sequences of RF manipulations. The PS also defines the bunch spacing of 25 ns for the beam going to the LHC.

juas
Joint Universities Accelerator School

Guidelines

The examination consists of three independent exercises. The questions are also made as independent as possible. The total number of points is 100pts. The grades will be normalized to a result /20.

- Exercise 1: Acceleration in the PS (33pts)
- Exercise 2: Bunch splittings (18pts)
- Exercise 3: Bunch-to-bucket transfer to the SPS (49pts)

For numerical computations, please provide at least 4 digits after the period.

Important parameters

Physical constants

- Proton mass: $m_p = 1.672\ (62192369) \times 10^{-27}$ kg
- Proton charge: $+e = 1.602\ (176634) \times 10^{-19}$ C
- Speed of light: $c = 2.997\ (92458) \times 10^{8}$ m/s

PS parameters

- Circumference: $C_0 = 2\pi 100$ m
- Bending radius: $\rho_0 = 70.079$ m
- Harmonic number of the main RF system: $h = 21$
- Momentum compaction factor: $\alpha_p = 2.6874 \times 10^{-2}$
- Ramp time: $T_{\text{acc}} = 0.7$ s
- Protons per bunch at $h = 21$: $N_b = 1.04 \times 10^{12}$

Exercise 1: Computation of the PS parameters (30 min -10min as second column is given - 33pts)

Parameter	Injection	Extraction
Kinetic energy E_{kin} [GeV]	2	25
Momentum p [GeV/c]	2.784	25.9213
Total energy E [GeV]	2.938	25.9383
Relativistic velocity β	0.9476	0.9993
Lorentz factor γ	3.132	27.6439
Bending field $\mathcal{B}_y$ [T]	0.1325	1.2338
Revolution period T_0 [μs]	2.211	2.0973
Revolution frequency f_0 [kHz]	452.284	476.7967
Main RF frequency f_r [MHz]	9.498	10.019
Phase slip factor η_0	-0.0751	0.0256

Reminder: Please provide 4 digits after the comma for numerical computations.

1. Explain why the bending radius ρ_0 is not equal to the mean radius R_0 in a synchrotron. What is R_0 for the PS? (1pt)

   A synchrotron is composed of bending and straight sections. The mean radius is obtained from the total circumference taking into account the straight sections.

2. Complete the parameter table above and specify the formulas used (hint: compute first the rest mass energy of the proton in [GeV]). (12pts)

   Mass energy E_0 = 1.67262192369e-27 * 2.99792458**2/1.602176565e-19 = 938.2721 MeV

3. Regarding transition crossing

   (a) Compute the transition γ_t (2pts)

   γ_t = sqrt(1 / 0.02687) = 6.1

   (b) Is transition crossed in the PS? Justify. (2pts)

   Yes as η_0 changes sign

   (c) Compute the total energy in [GeV] at the moment of transition crossing. (2pts)

   E_t = 6.1 * 0.938 = 5.722 GeV

4. At a total energy E = 15 GeV in the acceleration ramp, the RF voltage is $V_{\text{rf}} = 170$ kV.

   (a) Justify from the values you obtained in the parameter table that the magnetic ramp rate is $\dot{\mathcal{B}}_y \approx 1.5733$ [T/s] (linear ramp). (1pt)

   (1.2338 - 0.1325)/0.7 = 1.573 T/s

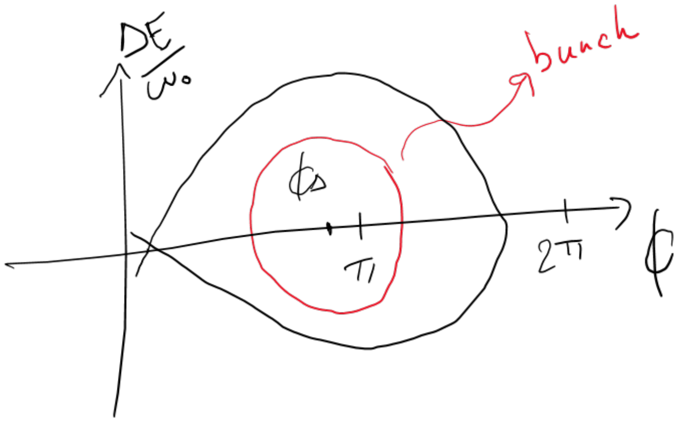
   (b) Compute the energy gain per turn in [keV] and the synchronous phase in [deg]. (3pts)

   Energy gain per turn 2*3.14*70.079*100*1.573 = 69 keV

   Synchronous phase arcsin(2*3.14*70.079*100*1.573/170e3)*180/3.14 = 24 deg

(c) Draw qualitatively the RF bucket and bunch in longitudinal phase space. (3pts)

Accelerating bucket above transition: ϕ_s, shape, orientation, coordinates



5. Evaluate the average beam power delivered to the beam during the acceleration ramp in [kW]. We will assume that all RF buckets are filled with bunches. (3pts)

   21 * 10.4e12 * (25e9-2e9) * 1.602e-19 / 0.7 = 115 kW

6. If the main RF system was a pillbox cavity.

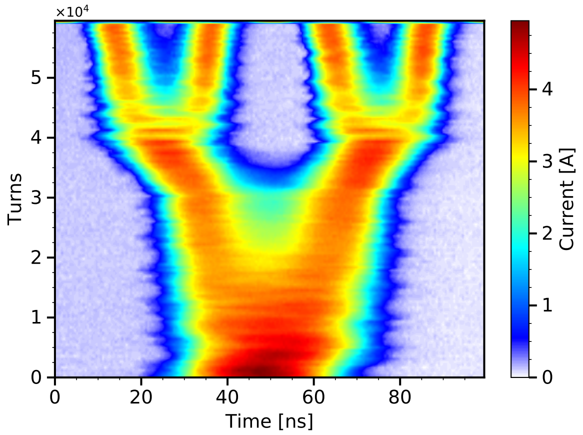
   (a) What would be the radius of the cavity in [m], based on the obtained RF frequency? (2pts)

   2.405 * 2.998e8 / (2*3.14*10e6) = 11.5 m

   (b) Justify with the help of the image on the first page that the actual RF system design is unlikely to be a pillbox cavity. (2pt)

   The cavity would be more than 20 m high and would not fit in the tunnel

Exercise 2: Bunch splittings in the PS (15 min - 18pts)



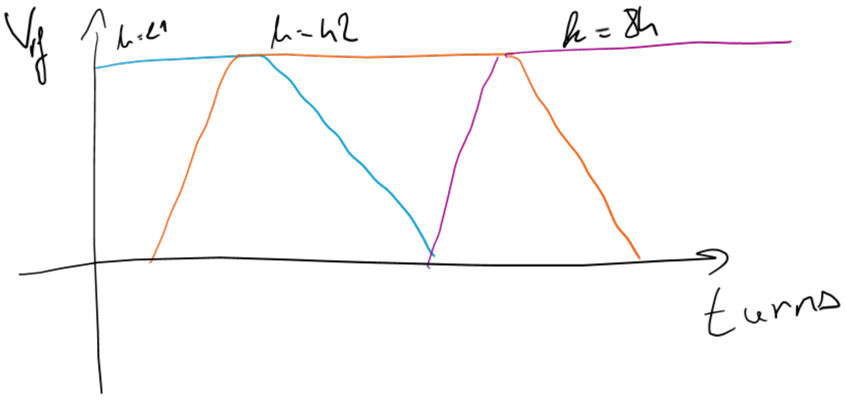
After acceleration to $E_{\mathrm{kin}} = 25$ GeV, the bunches are split in four using RF cavities tuned at $h = 42$ and then $h = 84$. The RF voltage before splitting is $V_{\mathrm{rf,h=21}} = 20$ kV. We start with all RF buckets filled with bunches in $h = 21$. The longitudinal emittance for each bunch after the splitting is $\varepsilon_l = 0.35$ eVs. The two splitting instances are independent.

1. What is
   - (a) The total number of bunches in the PS before and after the splittings? (1pt)

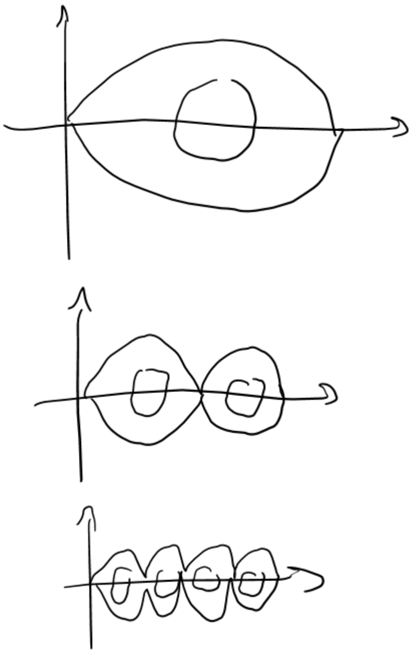
     21 split into 84
   - (b) The longitudinal emittance and number of protons per bunch before and after the first splitting, assuming that the splitting is done adiabatically? (3pts)

     Emittance: 1.4→0.7→0.35 eVs

     Intensity: $1.04 \times 10^{12} \to 5.2 \times 10^{11} \to 2.6 \times 10^{11}$ ppb
2. Draw qualitatively
   - (a) The RF programs vs. number of turns for each RF harmonic during the splittings. (3pts)



(b) The bunch distributions and RF buckets in longitudinal phase space before and after the splittings. (3pts)



3. What happens if there is a relative phase error between the RF cavities at different harmonics during the splittings? (3pts)

   Bunch by bunch variation in intensity and longitudinal emittance.

4. What should be the RF voltage at $h = 42$ and then at $h = 84$ to keep the RF bucket filling factor constant (emittance/acceptance)? Adapt your answer to the question 2.(a) to have a constant filling factor. (5pts)

$$A_{bk} \propto \frac{1}{h}\sqrt{\frac{V}{h}} \qquad \frac{V_2}{V_1} = \left(\frac{A_{bk,2}}{A_{bk_1}}\right)^2 \left(\frac{h_2}{h_1}\right)^3 \tag{1}$$

   Increase h by factor 2 and reduce A_{bk} by factor 2, requires increase in voltage at each step by a factor 2

Exercise 3: PS-SPS transfer (30 min 49pts)

After acceleration and splittings at $E_{\mathrm{kin}} = 25$ GeV, the beam is transferred to the next synchrotron in the LHC injector chain, the SPS. After the splittings in the PS, the main RF harmonic is $h = 84$ (all other RF systems switched off). The PS RF voltage is set to $V_{\mathrm{rf,h=84}} = 100$ kV. The full bunch length at that stage is $\tau_l = 12$ ns and the longitudinal emittance $\varepsilon_l = 0.35$ eVs. The maximum available voltage in the PS is $V_{\mathrm{rf,h=84}} = 600$ kV, and the SPS rf frequency at injection is $f_{r,\mathrm{SPS}} = 200$ MHz.

Reminder: Please provide 4 digits after the comma for numerical computations.

1. Parameters of the RF bucket in the PS

   (a) What is the RF frequency in [MHz] at h=84? (2pts)

   40 MHz

   (b) Give an estimation of the total momentum spread of the bunch in the PS (in [MeV/c], and relative to the beam energy). (3pts)

   Using $\pi \frac{\Delta E_b}{2} \frac{\tau_b}{2} \varepsilon_l \rightarrow$ 4*0.35/3.14/12e-9=37.16 MeV

   In relative=37.16e6/25.938e9 = 1.43e-3

   (c) Compute the RF bucket area in [eVs] and the RF bucket height in [MeV]. (4pts)

   Using $A_{\mathrm{bk}} = \frac{16\beta}{\omega_{\mathrm{rf}}} \sqrt{\frac{qV_{\mathrm{rf}}E}{2\pi h \eta_0}}$ = 16*0.9993/(2*3.14*4*10.019e6) * sqrt(1*100e3*25.938e9 / (2*3.14*84*0.0256)) = 0.88 eVs

   Using $H = A_{\mathrm{bk}}/8 * \omega_{\mathrm{rf}}$ = 0.88 / 8 * (2*3.14*4*10.019e6) = 27.70 MeV (full 55.40 MeV)

   (d) Compute the linear synchrotron frequency in [Hz] and the linear synchrotron tune. (4pts)

   Using $f_{s0} = \frac{1}{2\pi} \sqrt{\frac{\omega_{\mathrm{rf}}^2 \eta_0 q V_{\mathrm{rf}}}{2\pi h \beta^2 E}} = 1/(2 * 3.14) * sqrt((2 * 3.14 * 4 * 10.019e6) * *2 * 0.0256 * 1 * 100e3/(2 * 3.14 * 84 * 0.9993 * *2 * 25.938e9)) = 549$ Hz

   Using Q_{s0}= 549/477e3 = 1.15e-3

   (e) What is the filling factor of the RF bucket (emittance/acceptance)? (2pts)

   0.35/0.88 = 40% in emittance/acceptance

2. Basic transfer to the SPS RF bucket

   (a) What is the RF bucket length in the SPS in [ns]? (2pts)

   1/200e6 = 5 ns

   (b) Would the PS bunch fit in a single SPS bucket if the PS RF voltage was kept at 100 kV? (2pts)

   No as the bunch length is bigger than the bucket length

   (c) What would be the consequences of sending a $\tau_l = 12$ ns bunch to the SPS? (3pts)

   Losses and capture to the neighboring buckets

3. Adiabatic bunch shortening in the PS

(a) Justify that if the RF voltage is increased very slowly (adiabatically), the bunch length would scale as $\tau_{\text{after}} = \tau_{\text{before}} \left(V_{\text{before}}/V_{\text{after}}\right)^{1/4}$ (3pts)

Conservation of the longitudinal emittance, $\tau_l \propto \frac{1}{V^{1/4}}$

(b) Estimate the bunch length obtained by increasing the PS RF voltage slowly to the maximum possible. (2pts)

12*(100/600)**(1/4) = 7.67 ns

(c) Would the PS bunch fit in the SPS RF bucket after a slow increase of the PS RF voltage? (2pts)

The bunch would still be too long.

4. If the RF voltage is increased instantaneously (non-adiabatically), the bunch rotates in phase space. When the bunch is the shortest, the bunch length would scale as $\tau_{\text{after}} = \tau_{\text{before}} \left(V_{\text{before}}/V_{\text{after}}\right)^{1/2}$

   (a) Estimate the bunch length obtained by increasing the PS RF voltage rapidly to the maximum possible. (2pts)

   12*(100/600)**(1/2) = 4.89 ns

   (b) Would the PS bunch fit in the SPS RF bucket after a rapid increase of the PS RF voltage? (2pts)

   The bunch fits in the SPS RF bucket, at the very limit.

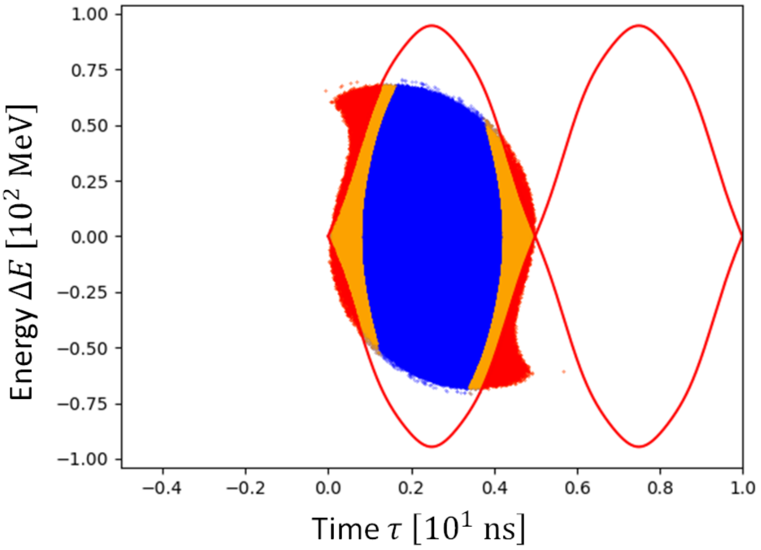
5. Capture of rotated bunches in the SPS

   (a) The bunch in phase space after the increase of the RF voltage as a step in the PS in represented in the figure below, together with the SPS separatrix. Can you explain the "Z-shape" of the bunch? (5pts)

   The shape is due to the non-linearities of the RF bucket.

   (b) What would be the consequence of an increased longitudinal emittance in the PS, regarding SPS injection? (3pts)

   Increased losses and capture in neighboring buckets.



6. The SPS operators noticed that the bunch relative momentum is too large by $\Delta p/p_0 = +10^{-3}$ at SPS injection with respect to the expected value.

   (a) How should the PS adjust the bending field $\mathcal{B}_y$ at extraction in [mT], while keeping the RF frequency constant, to fine tune the beam momentum? (4pts)

   Using synchrotron differential equation (3). We want to substract $\Delta p/p_0 = -10^{-3}$

   $\Delta B = B\frac{\gamma^2-\gamma_t^2}{\gamma^2}\Delta p/p_0 =$ 1.2338 * (27.652**2 - 6.1**2) / 27.652**2 * (-1e-3) = -1.174 mT

   (b) What is the resulting orbit change in [mm]? (4pts)

   Using synchrotron differential equation (2) or (4).

   $\Delta R = R\frac{1}{\gamma^2}\Delta p/p_0 =$ 100 / 27.652**2 * (-1e-3) = -0.131 mm