Linacs

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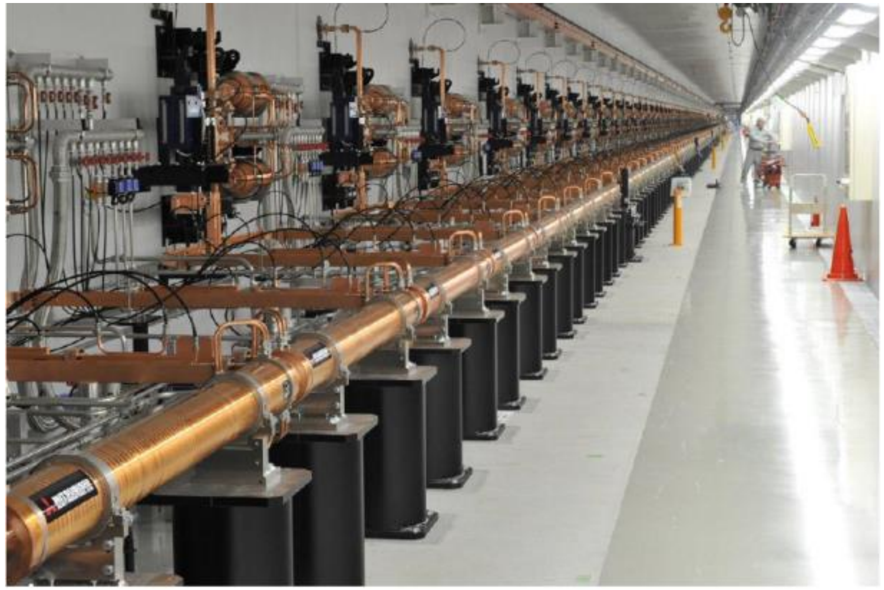
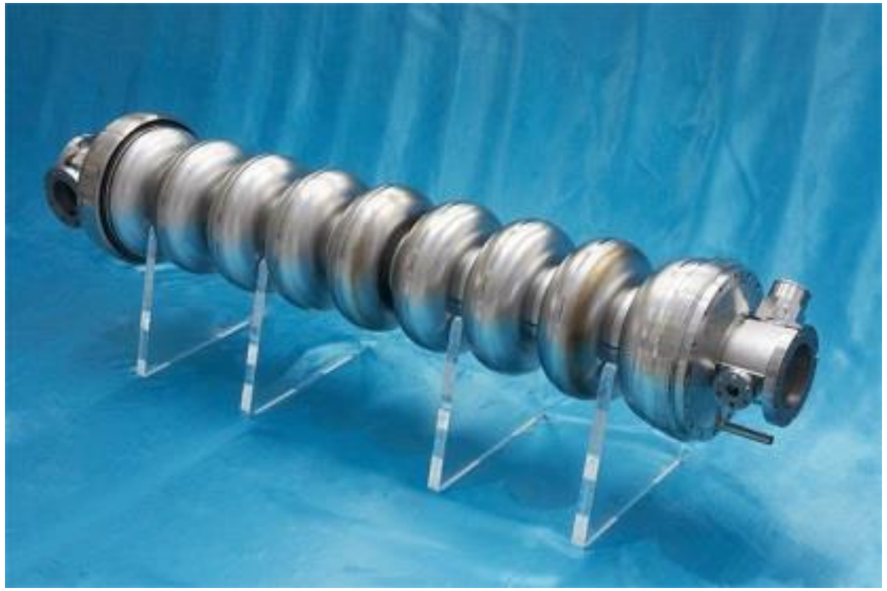




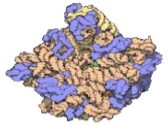
Joint Universities Accelerator School

juas

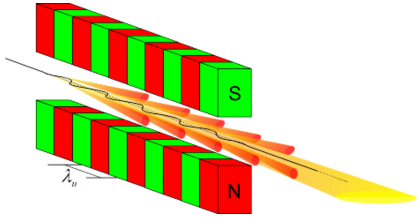
LINAC APPLICATIONS

~10^4 LINACs operating around the world

Injectors for synchrotrons

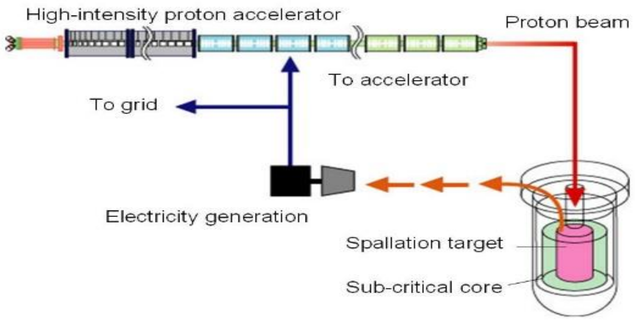


Free Electron Lasers

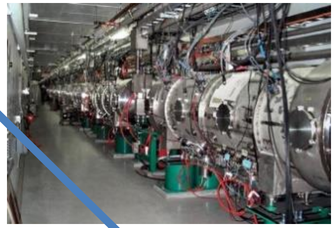


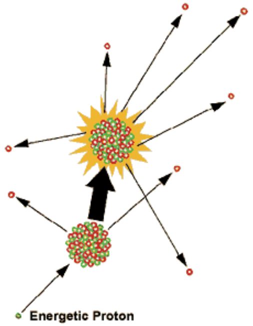
Medical applications: radiotherapy

Nuclear waste treatment and controlled fission for energy production (ADS)



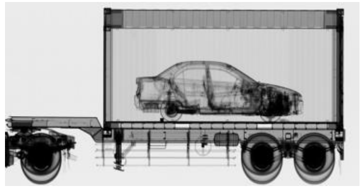
Spallation sources for neutron production





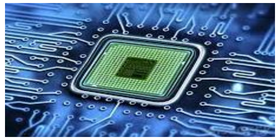
Industrial applications

National security



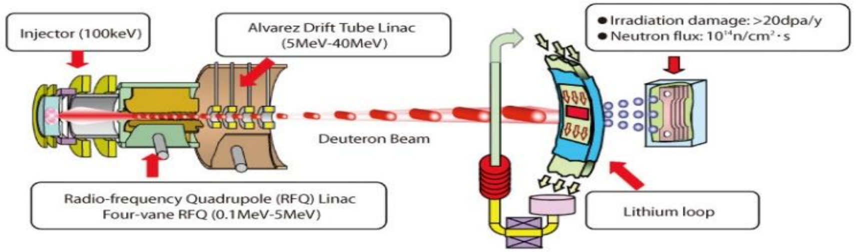
Material treatment

Ion implantation



Material/food sterilization

Material testing for fusion nuclear reactors



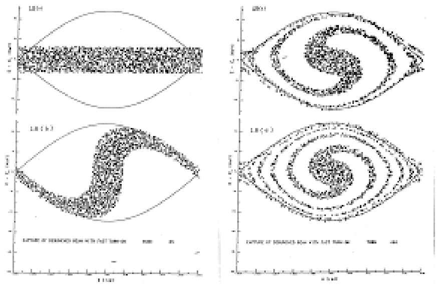
LINAC: BASIC DEFINITION AND MAIN COMPONENTS

LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory** by electromagnetic fields.

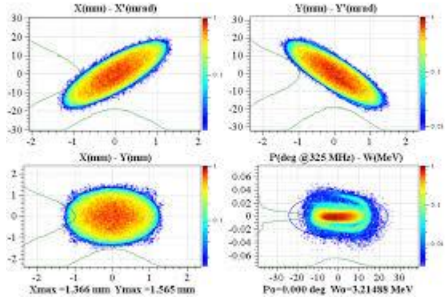
LINAC BEAM DYNAMICS

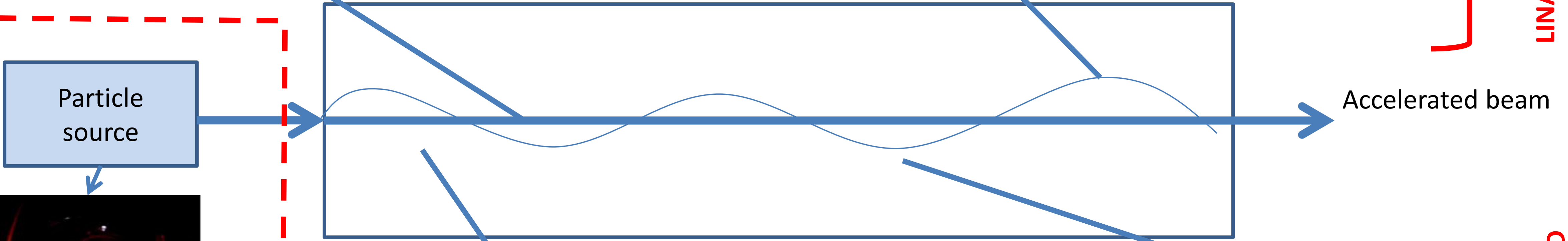
2nd PART OF THE LECTURE

Longitudinal dynamics of accelerated particles



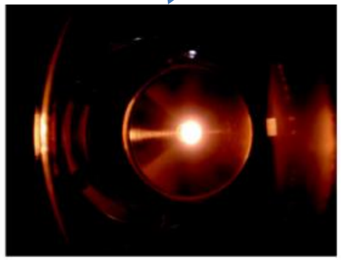
Transverse dynamics of accelerated particles





LINAC COMPONENTS AND TECHNOLOGY

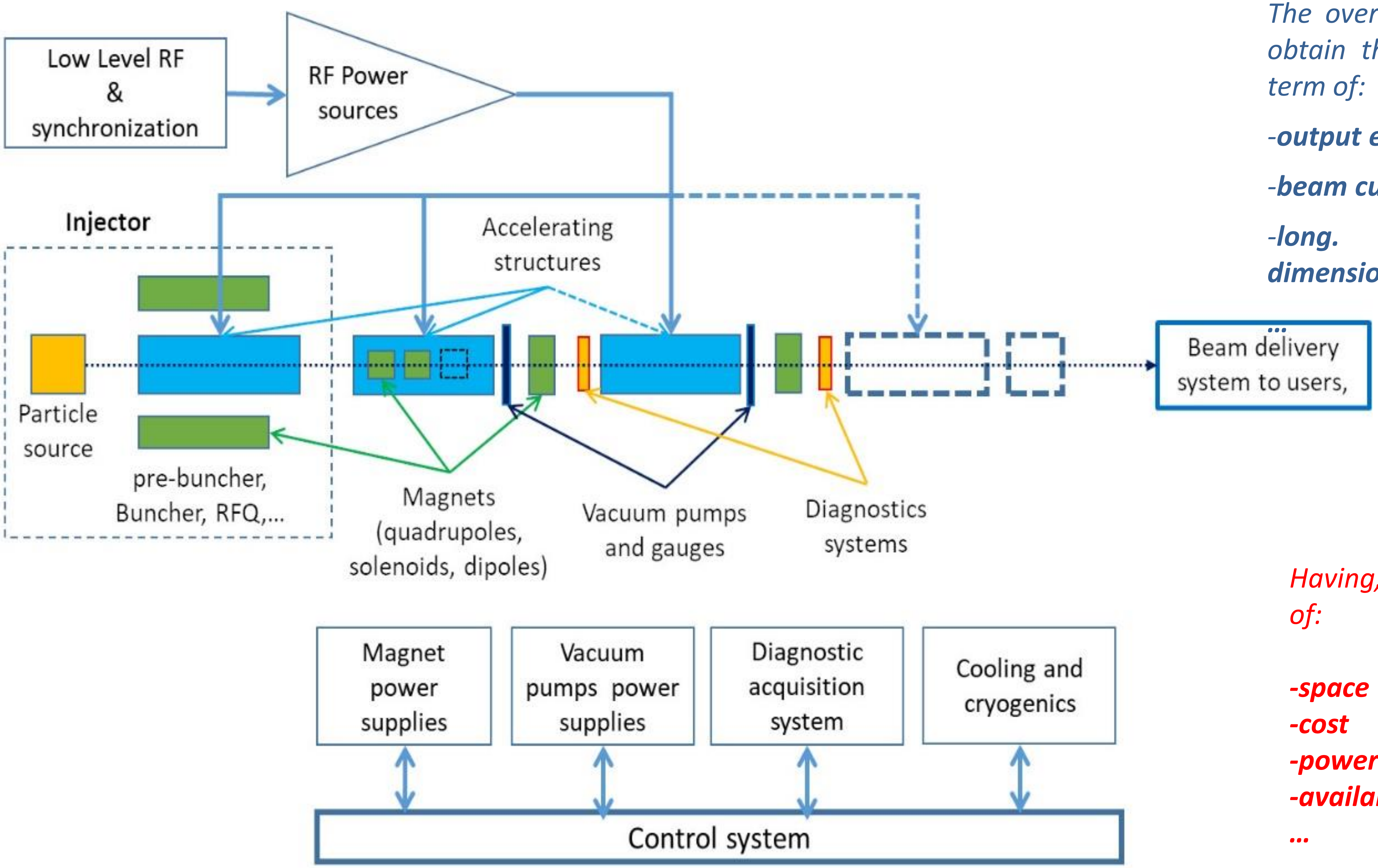
1st PART OF THE LECTURE



Accelerating structures

Focusing elements: quadrupoles and solenoids

LINAC TECHNOLOGY



The overall LINAC has to be designed to obtain the ***desired beam parameters*** *in term of:*

-***output energy/energy spread***

-***beam current (charge)***

-***long. and transverse beam dimensions/divergence (emittance)***

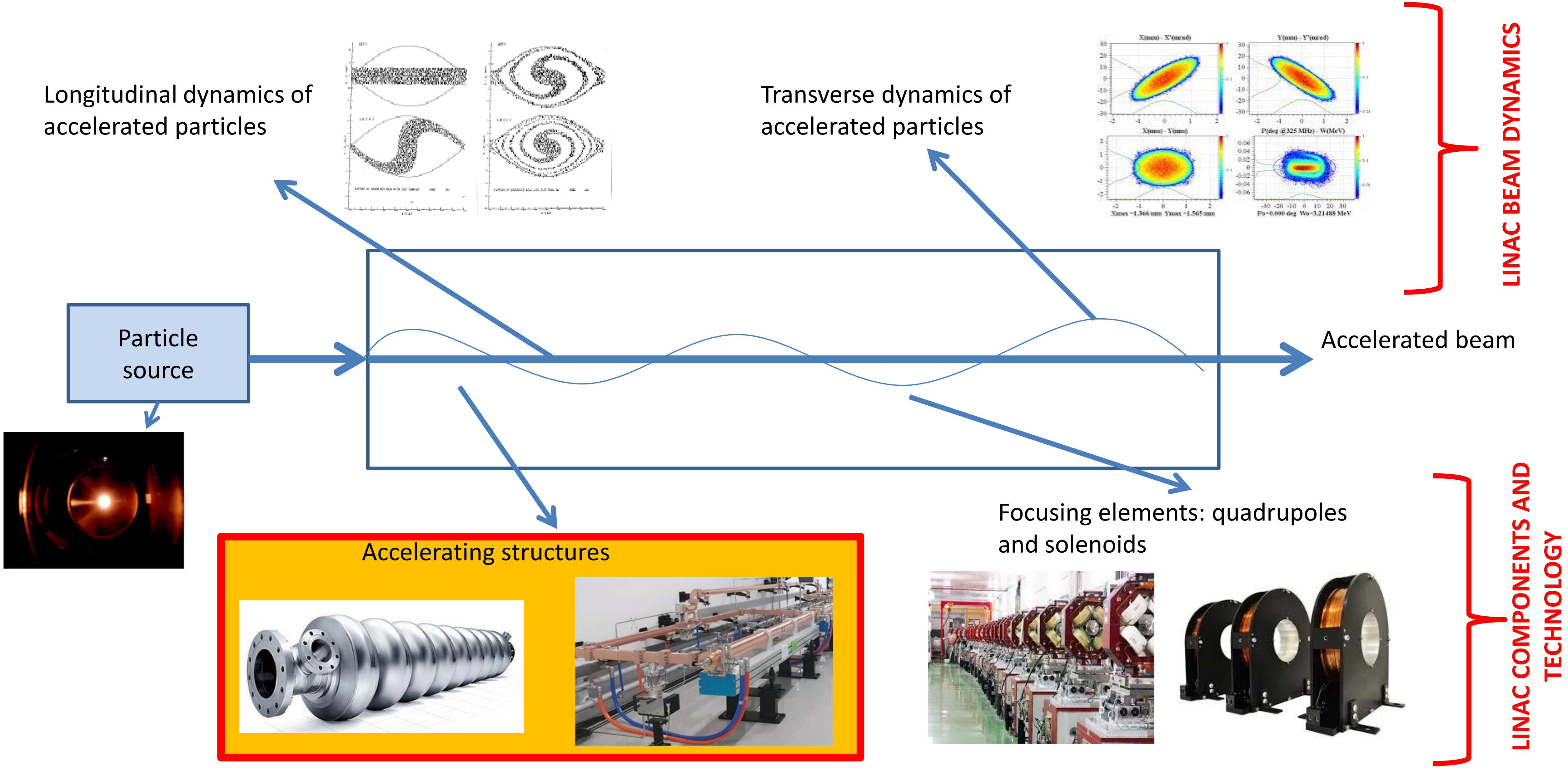
…

Having, in general, ***constraints*** *in term of:*

-space
-cost
-power consumption
-available power sources
…

LINAC: BASIC DEFINITION AND MAIN COMPONENTS

LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory** by electromagnetic fields.



LORENTZ FORCE: ACCELERATION AND FOCUSING

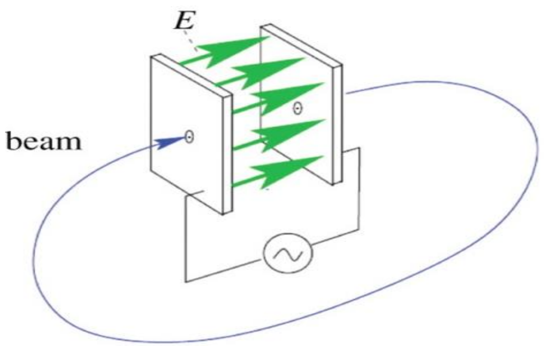
The basic equation that describes the acceleration/bending/focusing processes is the **Lorentz Force**.
Particles are **accelerated through electric** fields and are **bended and focused through magnetic** fields.

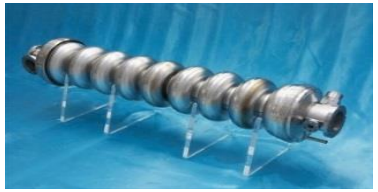
$$\vec{p} = momentum$$
$$m = mass$$
$$\vec{v} = velocity$$
$$q = charge$$

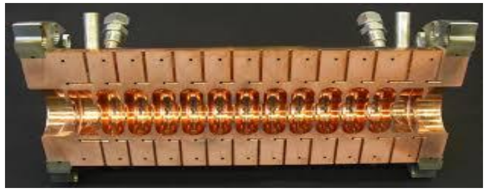
$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

ACCELERATION

To accelerate, we need a force in the direction of motion



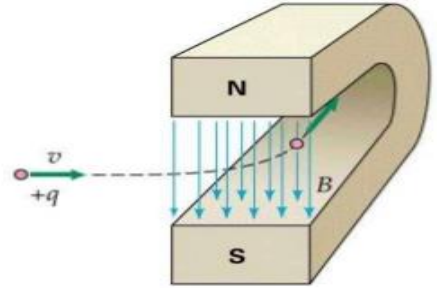


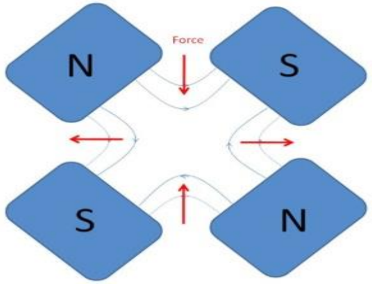


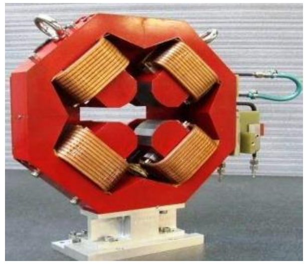
Longitudinal Dynamics

BENDING AND FOCUSING

2nd term always perpendicular to motion => no energy gain



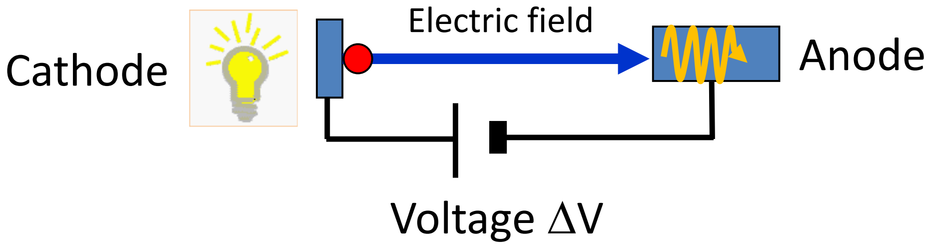




Transverse Dynamics

ACCELERATION: SIMPLE CASE

The **first historical linear particle accelerator** was built by the Nobel prize Wilhelm Conrad Röntgen (1900). It consisted in a vacuum tube containing a cathode connected to the negative pole of a DC voltage generator. **Electrons emitted by the heated cathode** were accelerated while flowing to another electrode connected to the positive generator pole (anode). Collisions between the energetic electrons and the anode produced **X-rays**.



Bertha Röntgen's Hand
8 Nov, 1895

The **energy gained** by the electrons travelling from the cathode to the anode is equal to their charge multiplied the DC voltage between the two electrodes.

$$\frac{d\vec{p}}{dt} = q\vec{E} \quad \Rightarrow \quad \Delta E = q\Delta V$$

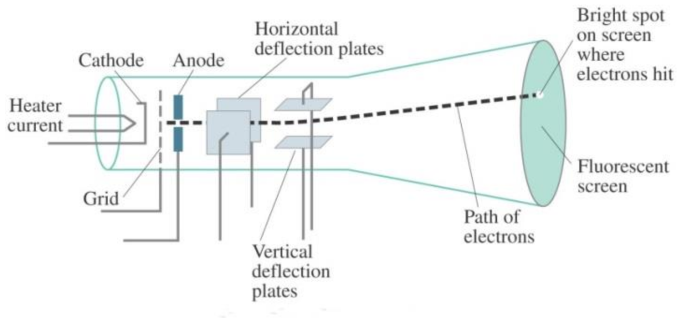
$$\vec{p} = momentum$$

$$q = charge$$

$$E = energy$$

Particle energies are typically expressed in electron-volt [eV], equal to the energy gained by 1 electron accelerated through an electrostatic potential of 1 volt:

1 eV=1.6x10^{-19} J



PARTICLE VELOCITY VS ENERGY: LIGHT AND HEAVY PARTICLES

Single particle
- rest mass m_0
- rest energy E_0 $(=m_0c^2)$
- total energy E
- relativistic mass m
- velocity v
- momentum p $(=mv)$
- Kinetic energy $W=E-E_0$

Relativistic factor
$\beta=v/c$ (<1)

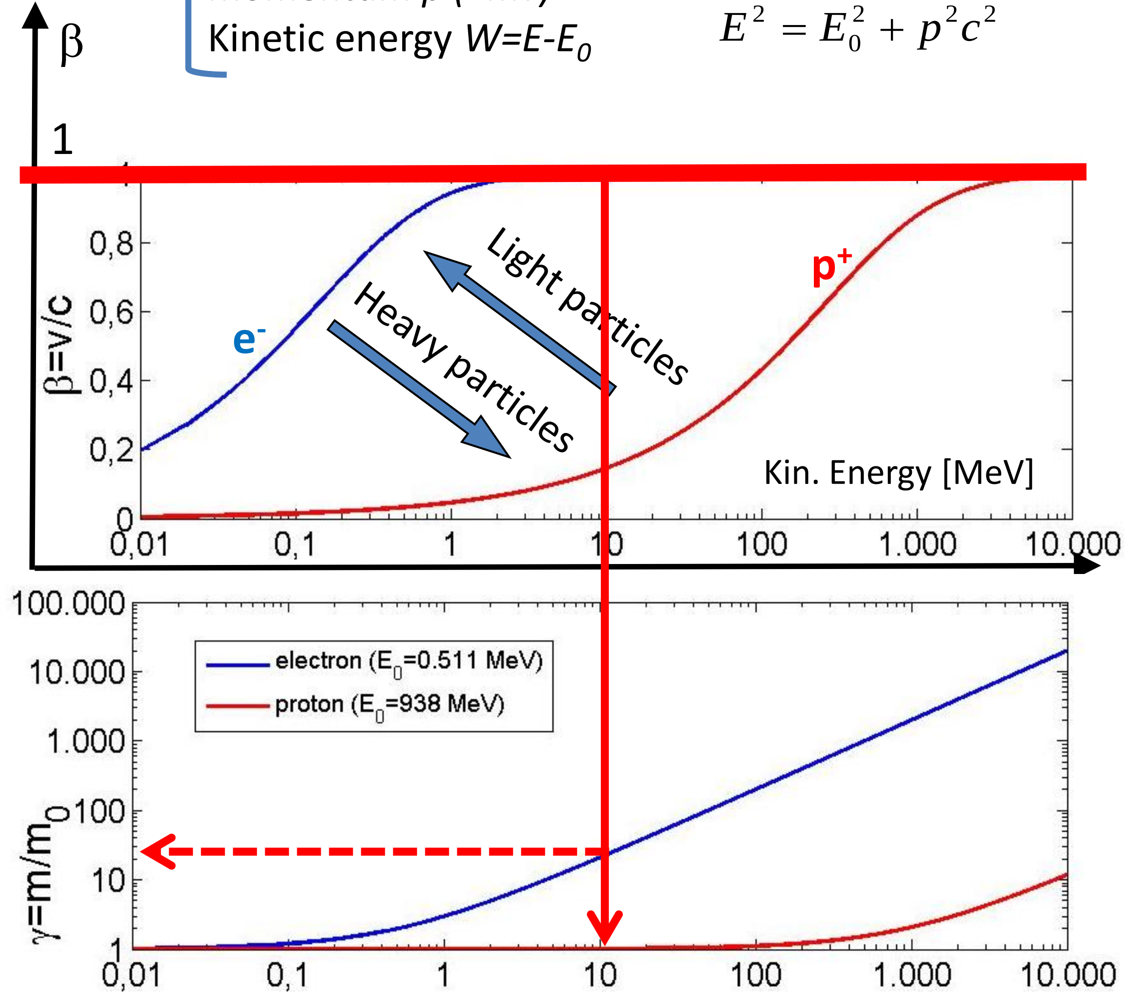
Relativistic factor
$\gamma=E/E_0$ (≥1)

+

$$E^2 = E_0^2 + p^2c^2$$

$$\begin{cases} \beta = \sqrt{1-1/\gamma^2} \\ \gamma = 1/\sqrt{1-\beta^2} \qquad (m = \gamma m_0) \\ W = E - E_0 = (\gamma-1)m_0c^2 \underbrace{\approx}_{\text{if } \beta<<1} \frac{1}{2}m_0v^2 \end{cases}$$

$$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\left(\frac{E_0}{E}\right)^2} = \sqrt{1-\left(\frac{E_0}{E_0+W}\right)^2}$$



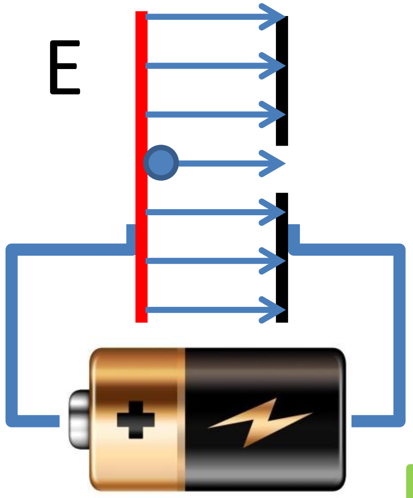
⇒**Light particles** (as **electrons**) are practically fully relativistic (β≅1, γ>>1) at relatively low energy and **reach a constant velocity** (~c). The acceleration process occurs at constant particle velocity

⇒**Heavy particles** (**protons and ions**) are typically weakly relativistic and **reach a constant velocity only at very high energy.** The velocity changes a lot during the acceleration process.

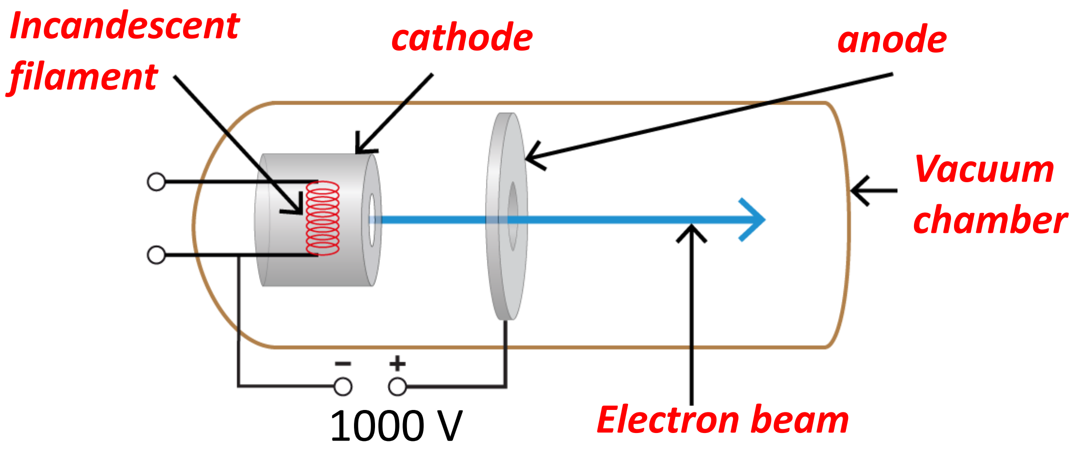
⇒This implies **important differences** in the technical characteristics of the **accelerating structures**. In particular for **protons and ions** we need different types of accelerating structures, **optimized for different velocities** and/or the accelerating structure has to vary its geometry to take into account the velocity variation.

PARTICLE ACCELERATION: ELECTRIC FIELD

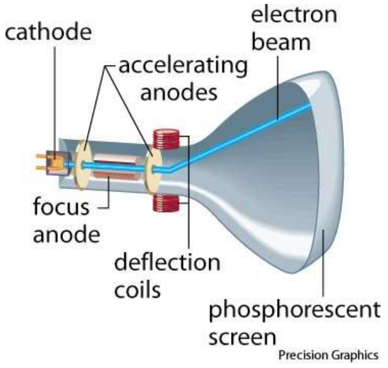
Particles are accelerated through **electric fields**



1 V



Energy gain $\propto$ V



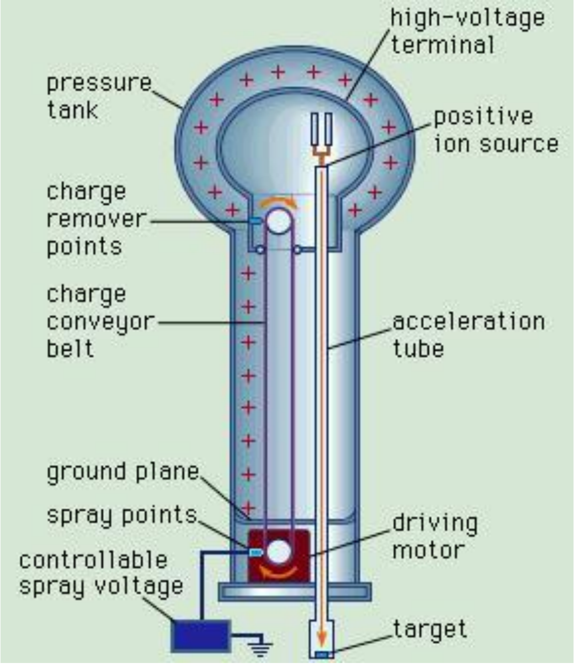
10^9-10^{10} V

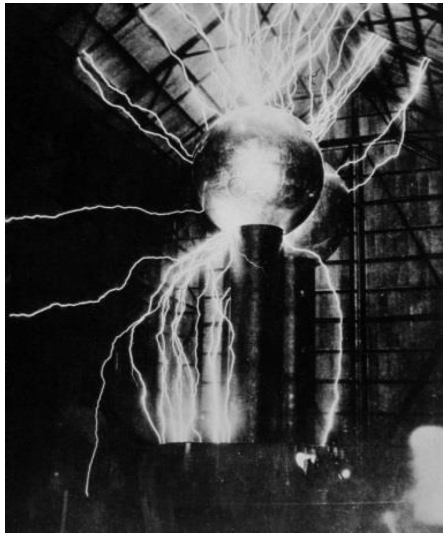
100-200 V

10^5 V

ELECTROSTATIC ACCELERATORS

To increase the achievable maximum energy, Van de Graaff invented an electrostatic generator based on a **dielectric belt** transporting positive charges to an isolated electrode hosting an **ion source**. The positive ions generated in a large positive potential were accelerated toward ground by the static electric field.





LIMITS OF ELECTROSTATIC ACCELERATORS

DC voltage as large as ~10 MV can be obtained (E~10 MeV). The main limit in the achievable voltage is the **breakdown** due to **insulation** problems.

APPLICATIONS OF DC ACCELERATORS

DC particle accelerators are in operation worldwide, typically at V<15MV (E_{max}=15 MeV), I<100mA.

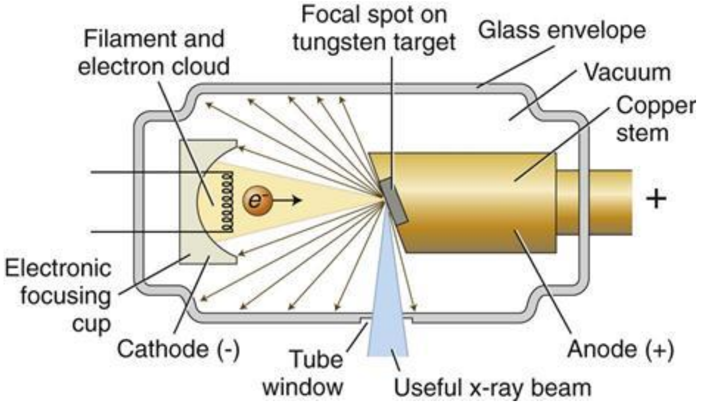
They are used for:

$\Rightarrow$ *material analysis*

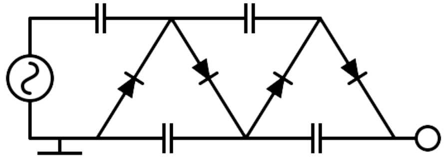
$\Rightarrow$ *X-ray production,*

$\Rightarrow$ *ion implantation for semiconductors*

$\Rightarrow$ *first stage of acceleration (particle sources)*



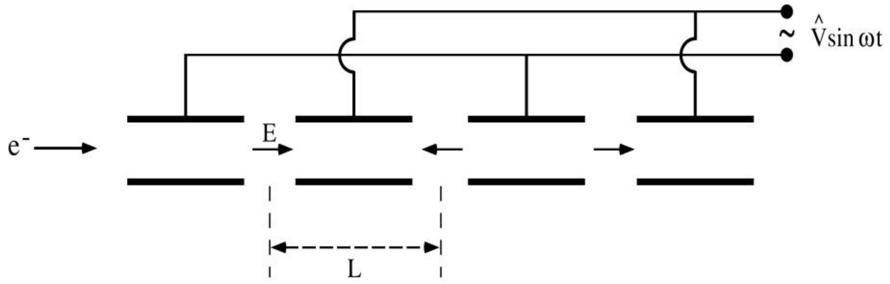
750 kV Cockcroft-Walton
Linac2 injector at CERN from 1978 to 1992



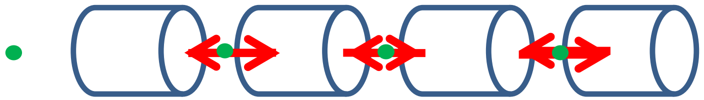
RF ACCELERATORS : WIDERÖE “DRIFT TUBE LINAC” (DTL)

(protons and ions)

Basic idea: the particles are accelerated by the electric field in the gap between electrodes connected alternatively to the poles of an AC generator. This original idea of **Ising** (1924) was implemented by **Wideroe** (1927) who applied a sine-wave voltage to a sequence of **drift tubes**. The particles **do not experience any force while travelling inside the tubes** (equipotential regions) and are **accelerated across the gaps**. This kind of structure is called **Drift Tube LINAC (DTL).**



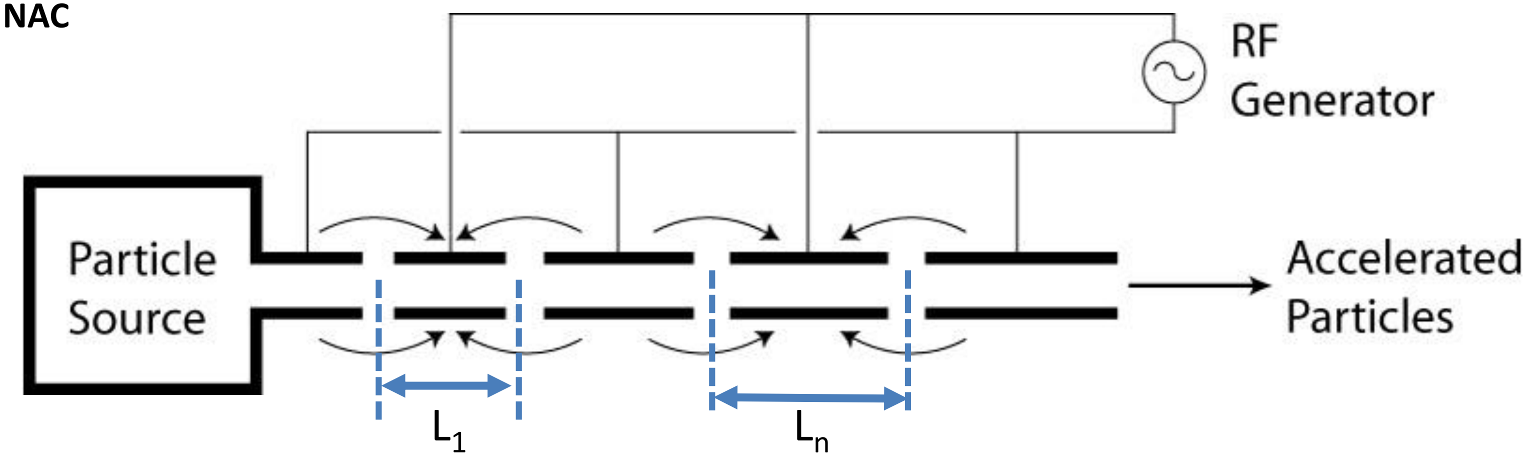

Rolf Wideroe



⇒If the **length of the tubes (or, equivalently, the distances between the centers of the accelerating gaps)** increases with the particle velocity during the acceleration such that the **time of flight between gaps is kept constant and equal to half of the RF period,** the particles are subject to a **synchronous accelerating voltage** and experience an energy gain of $\Delta E = q\Delta V$ at each gap crossing.

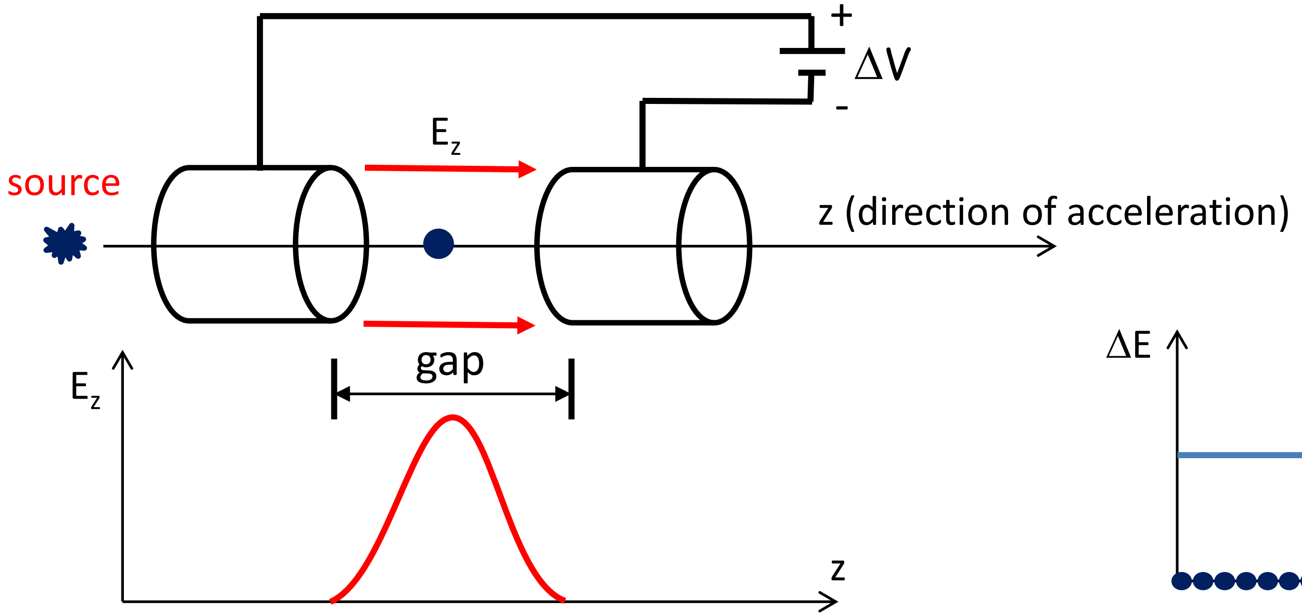
⇒In principle a single **RF generator** can be used to indefinitely accelerate a beam, **avoiding the breakdown limitation** affecting the electrostatic accelerators.

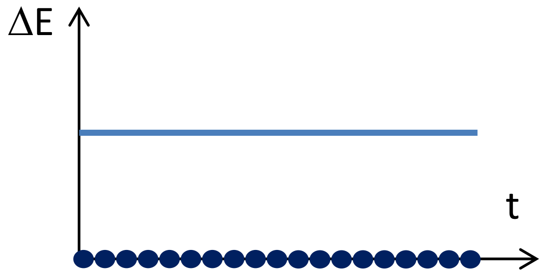
⇒The Wideroe LINAC is the **first RF LINAC**



DC ACCELERATION: ENERGY GAIN

We consider the acceleration between two electrodes in DC.





Energy-momentum relation

$$E^2 = E_0^2 + p^2c^2 \Rightarrow 2EdE = 2pdpc^2 \Rightarrow dE = v\frac{mc^2}{E}dp \Rightarrow dE = vdp$$

Lorentz force $\quad \dfrac{dp}{dt} = qE_z \underbrace{\Rightarrow}_{z=vt} v\dfrac{dp}{dz} = qE_z \Rightarrow \boxed{\dfrac{dE}{dz} = qE_z} \quad \left(\text{and also } \dfrac{dW}{dz} = qE_z\right) \quad W=E-E_0$

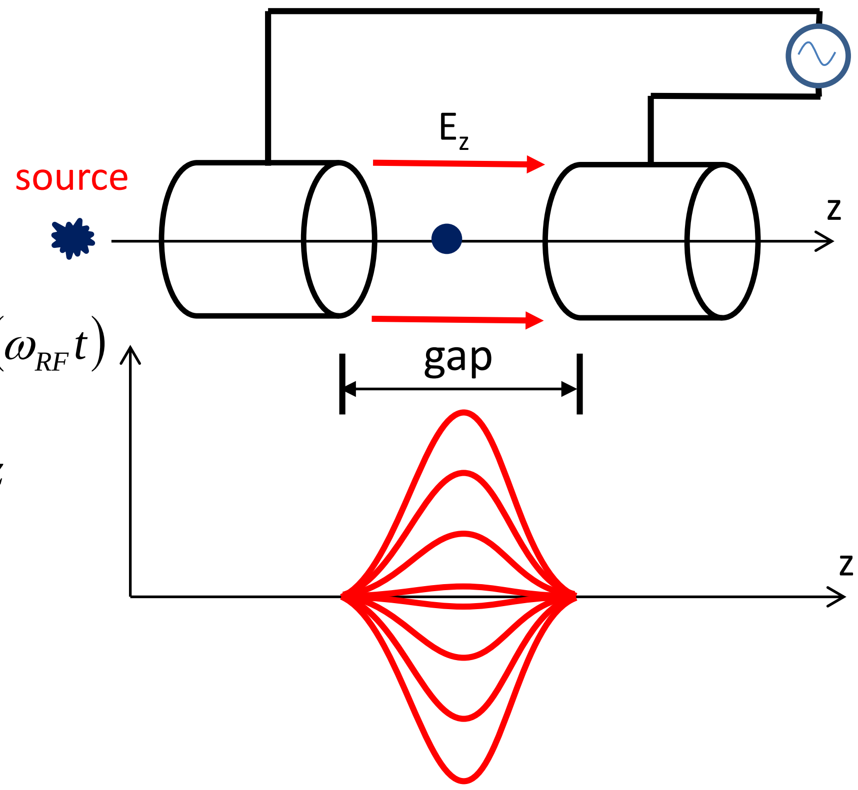
rate of energy gain per unit length

$$\Rightarrow \Delta E = \int_{gap} \frac{dE}{dz}dz = \int_{gap} qE_z dz \Rightarrow \boxed{\Delta E = q\Delta V}$$

energy gain per electrode pair

RF ACCELERATION: BUNCHED BEAM

We consider now the acceleration between two electrodes fed by an RF generator



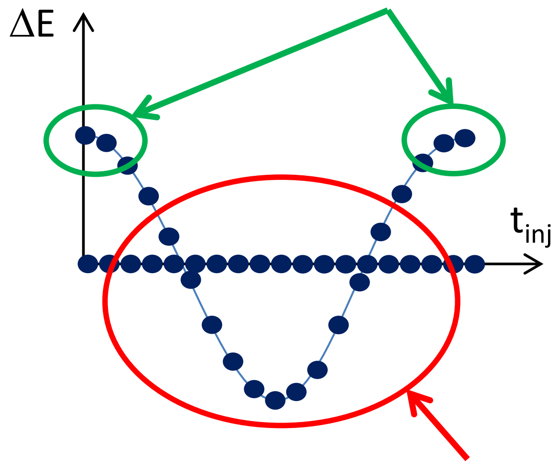
$$\Delta V = V_{RF}\cos(\omega_{RF} t) \qquad \omega_{RF} = 2\pi f_{RF} = \frac{2\pi}{T_{RF}}$$

$$\Rightarrow \Delta E = q\hat{V}_{acc}\cos(\omega_{RF} t_{inj})$$

$$E_z(z,t) = E_{RF}(z)\cos(\omega_{RF} t)$$

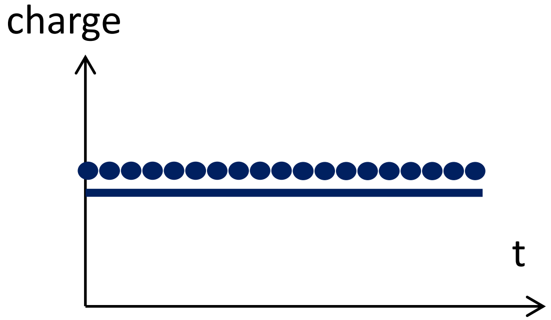
$$V_{RF} = \int_{gap} E_{RF}(z)dz$$

Only these particles are accelerated



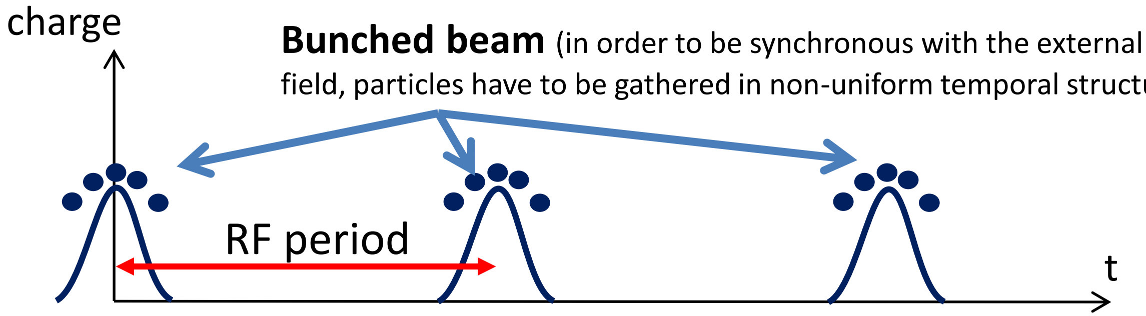
These particles are not accelerated and basically are lost during the acceleration process

DC acceleration



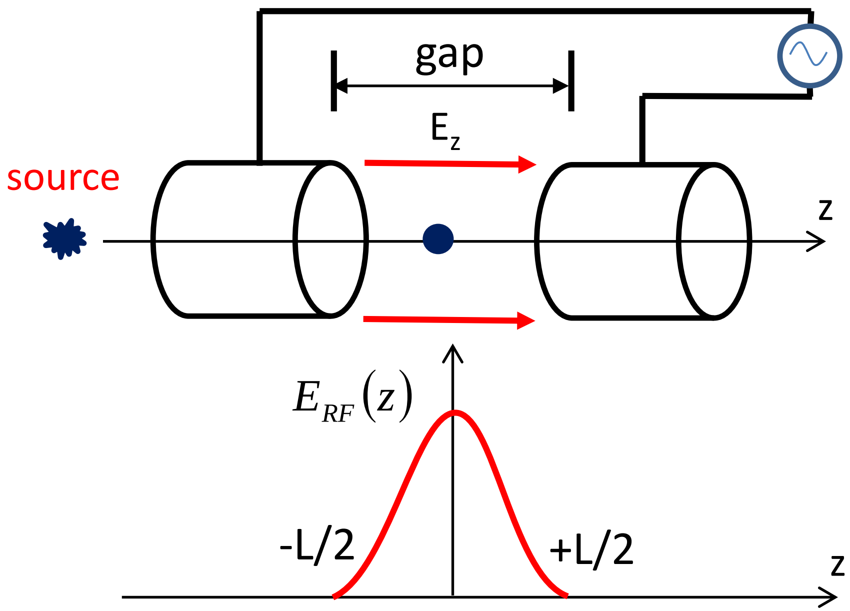
RF acceleration

Bunched beam (in order to be synchronous with the external AC field, particles have to be gathered in non-uniform temporal structure)



RF ACCELERATION: ENERGY GAIN

We consider now the acceleration between two electrodes fed by an RF generator



$$\Delta V = V_{RF}\cos(\omega_{RF} t) \qquad \omega_{RF} = 2\pi f_{RF} = \frac{2\pi}{T_{RF}}$$

$$V_{RF} = \int_{gap} E_{RF}(z)dz \qquad E_z(z,t) = E_{RF}(z)\cos(\omega_{RF} t)$$

$$E_z(z,t)\big|_{\substack{seen\\by\\particle}} = E_{RF}(z)\cos[\omega_{RF}(t+t_{inj})] = E_{RF}(z)\cos(\omega_{RF} t + \phi_{inj})$$

$$\phi_{inj} = \omega_{RF} t_{inj}$$

t=z/v

Hyp. of symmetric accelerating field

$$\Delta E = q\int_{gap} E_z(z,t)\big|_{\substack{seen\\by\\particle}} dz = q\int_{-L/2}^{+L/2} E_{RF}(z)\cos\left(\omega_{RF}\frac{z}{v}+\phi_{inj}\right)dz = q\int_{gap} E_{RF}(z)dz \; \frac{\int_{gap} E_{RF}(z)\cos\left(\omega_{RF}\frac{z}{v}\right)dz}{\int_{gap} E_{RF}(z)dz}\;\cos(\phi_{inj})$$

V_{RF} Peak gap voltage — T<1 Transit time factor — Injection phase

$$\Delta E = q\underbrace{V_{RF} T}_{\hat{V}_{acc}}\cos(\phi_{inj}) = q\underbrace{\hat{V}_{acc}\cos(\phi_{inj})}_{V_{acc}}$$

$\hat{E}_{acc} = \hat{V}_{acc}/L$ Average accelerating field in the gap

$E_{acc} = V_{acc}/L$ Average accelerating field seen by the particle

EXERCISE 1: TRANSIT TIME FACTOR

- Derive the general expression of the transit time factor of an accelerating gap of length L, with constant accelerating field, in which the field is oscillating at f_{RF} and that accelerate particles with relativistic factor β.

- Remembering that the light wavelength in free space is given by $\lambda_{RF}=c/f_{RF}$, for which value of the accelerating gap length L, T is equal to zero?

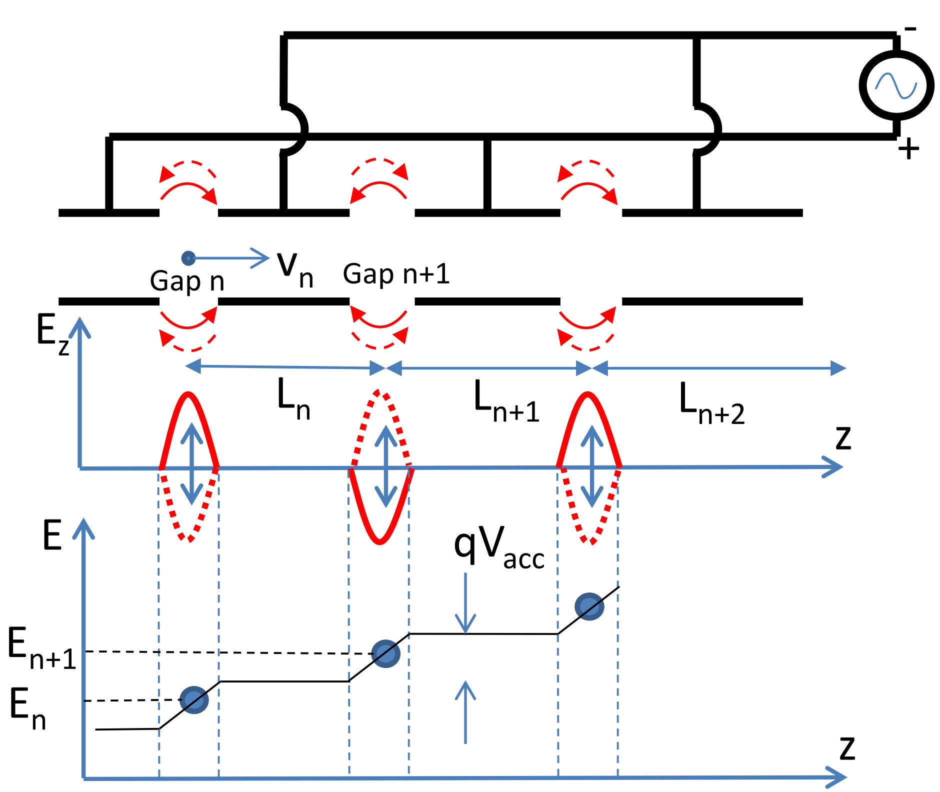
- Calculate the numerical value of T for L=10 cm, f_{RF}=1 GHz and ultra-relativistic electrons (β=1).

- Calculate the accelerating voltage as a function of the gap length L assuming an injection phase on crest ($\phi_{inj}=0$)

DRIFT TUBE LENGTH AND FIELD SYNCHRONIZATION

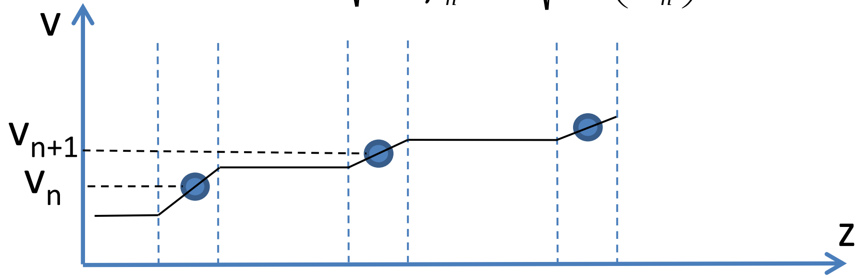
(protons and ions or electrons at extremely low energy)

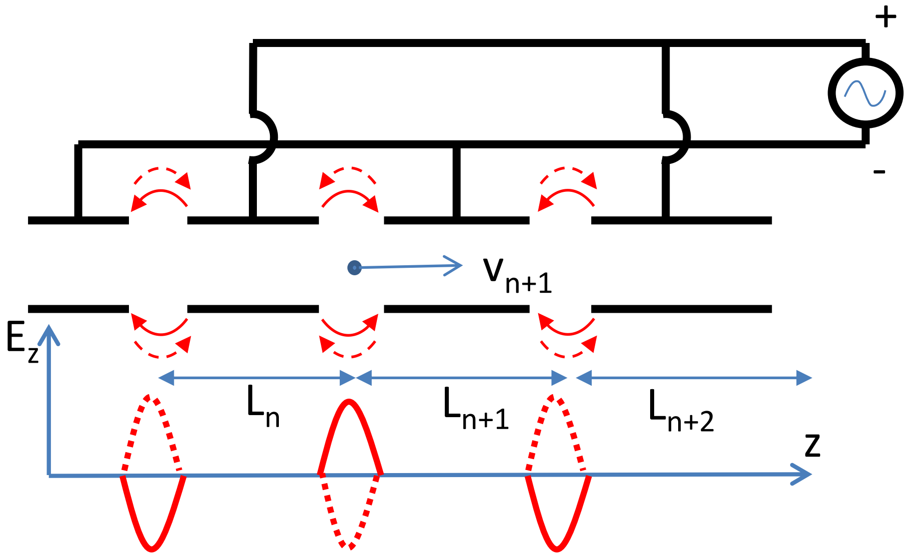
If now we consider a DTL structure, we have that at each gap the maximum energy gain is $\Delta E_n = qV_{acc}$ and the particle increase its velocity accordingly to the previous relativistic formulae. It is convenient to refer to the center of each gap as follows:



$$\Delta V = V_{RF}\cos(\omega_{RF} t)$$

$$v_n = c\beta_n = c\sqrt{1-\frac{1}{\gamma_n^2}} = c\sqrt{1-\left(\frac{E_o}{E_n}\right)^2}$$





⇒ In order to **be synchronous with the accelerating field at the center of each gap**, the distance between the centers of the gaps (L_n) has to be increased as:

$$t_n = \frac{L_n}{\bar{v}_n} = \frac{T_{RF}}{2} \Rightarrow L_n = \frac{1}{2}\bar{v}_n T_{RF} = \frac{1}{2}\bar{\beta}_n \underbrace{cT_{RF}}_{\lambda_{RF}} \quad \Rightarrow \quad \boxed{L_n = \frac{1}{2}\bar{\beta}_n\lambda_{RF}}$$

$\bar{v}_n$=average particle velocity between the gap n and n+1

⇒The **energy gain per unit length** (i.e. the **average accelerating gradient times q**) is given by:

$$\Rightarrow \boxed{\frac{\Delta E}{\Delta L} = \frac{qV_{acc}}{L_n} = \frac{2qV_{acc}}{\lambda_{RF}\bar{\beta}_n}} \quad [\mathrm{eV/m}]$$

ACCELERATION WITH HIGH RF FREQUENCIES: RF CAVITIES

There are two important **consequences** of the previous obtained formulae:

$$L_n = \frac{1}{2}\bar{\beta}_n \lambda_{RF}$$

The condition $L_n \ll \lambda_{RF}$ (necessary to model the tube as an **equipotential region**) requires $\beta \ll 1$. ⇒The Wideröe technique can **not be applied to relativistic particles**.

$$\frac{\Delta E}{\Delta L} = \frac{qV_{acc}}{L_n} = qE_{acc} = \frac{2qV_{acc}}{\lambda_{RF}\bar{\beta}_n}$$

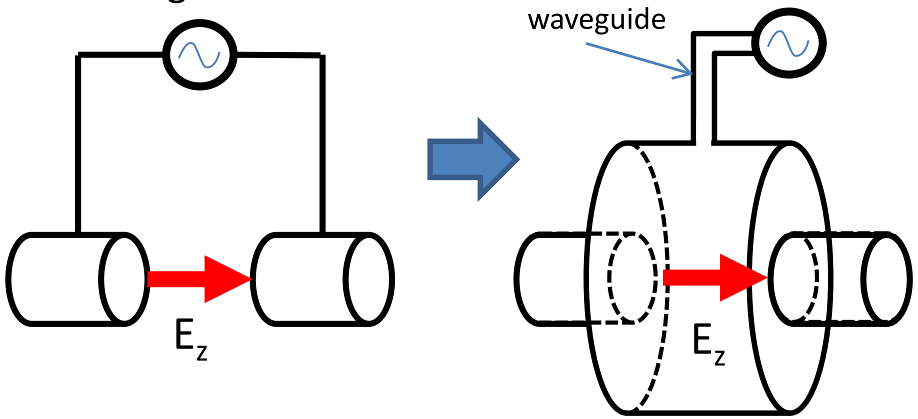
Moreover when particles get high velocities the drift spaces get longer and one looses on the efficiency. The **average accelerating gradient (E_{acc} [V/m]) increase pushes towards small λ_{RF}** (high frequencies).

High frequency, high power **sources** became available after the **2nd world war** pushed by **military** technology needs (such as **radar**). Moreover, the concept of equipotential DT can not be applied at small λ_{RF} and the **power lost by radiation is proportional to the RF frequency.**

As a consequence we must consider **accelerating structures different from drift tubes**.

⇒The solution consists of **enclosing the system in a cavity** which resonant frequency matches the RF generator frequency.

⇒Each cavity can be independently powered from the RF generator



RF CAVITIES

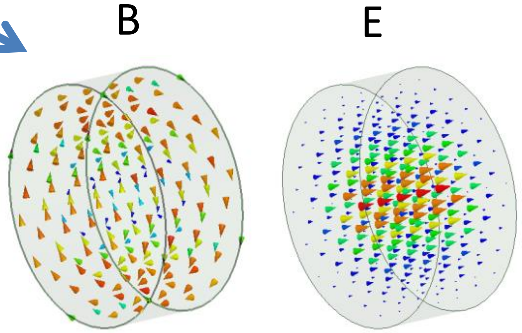
⇒High frequency RF accelerating fields are confined in **cavities**.

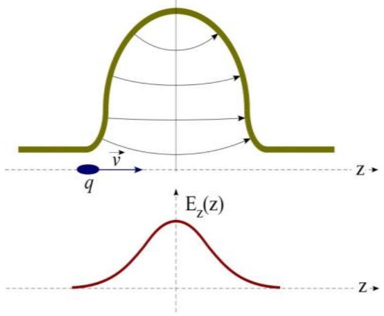
⇒The cavities are **metallic closed volumes** were the e.m fields has a particular spatial configuration (**resonant modes**) whose components, including the accelerating field $\mathbf{E_z}$, oscillate at some specific frequencies $\mathbf{f_{RF}}$ (resonant frequency) characteristic of the mode.

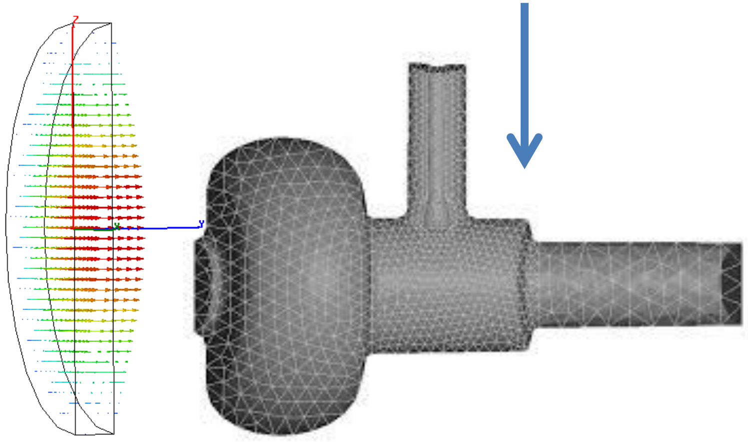
⇒The modes are excited by **RF generators** that are **coupled to the cavities** through waveguides, coaxial cables, etc…

⇒The resonant modes are called **Standing Wave (SW) modes** (spatial fixed configuration, oscillating in time).

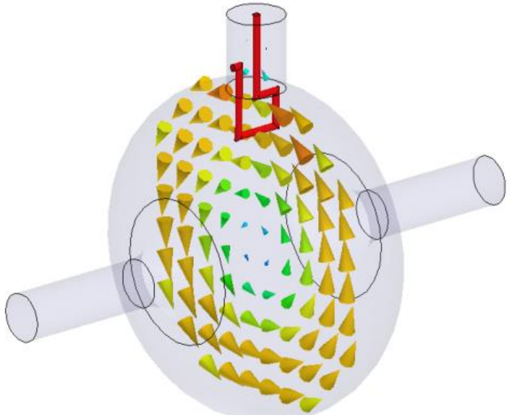
⇒The spatial and temporal field profiles in a cavity have to be computed (analytically or numerically) **by solving the Maxwell equations** with the proper boundary conditions.

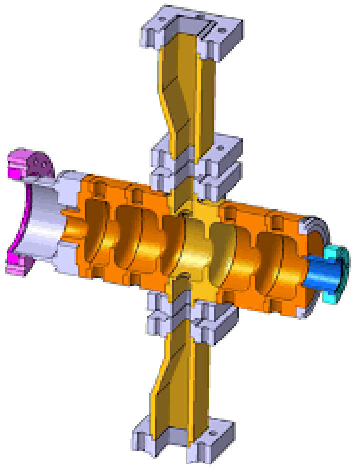






Courtesy E. Jensen

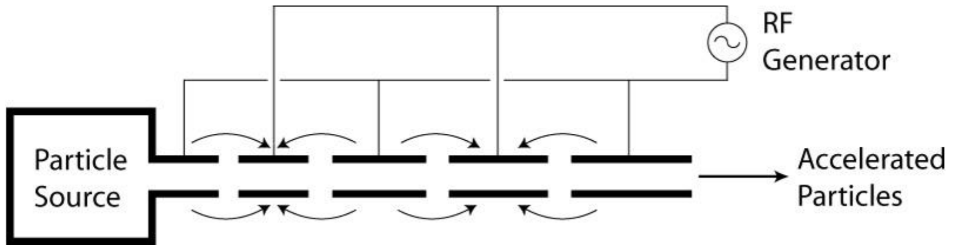


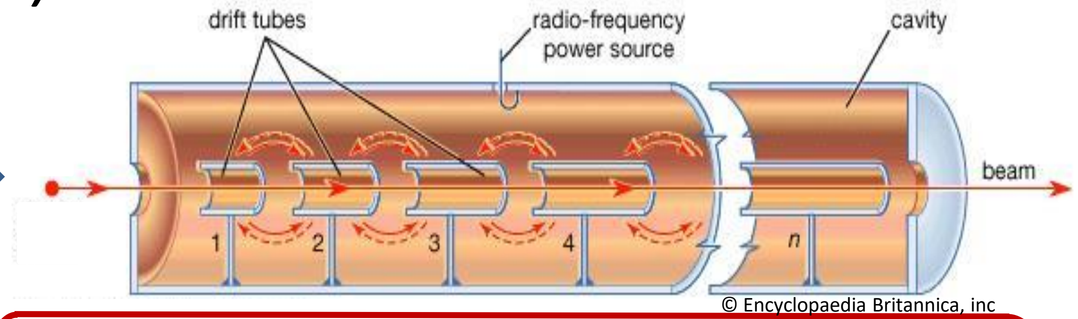


ALVAREZ STRUCTURES

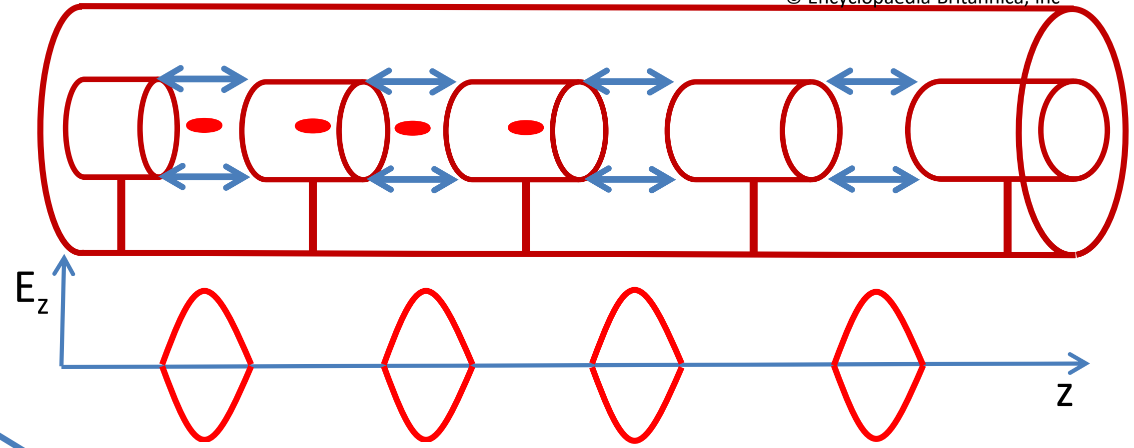
(protons and ions)

Alvarez's structure can be described as a special DTL in which the electrodes are part of a **resonant macrostructure.**



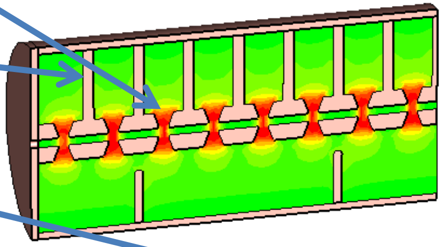


⇒The DTL operates in **0 mode** for **protons and ions** in the range β=0.05-0.5 (f_{RF}=50-400 MHz, λ_{RF}=6-0.7 m) 1-100 MeV;



⇒The beam is inside the "**drift tubes**" when the electric field is decelerating. The **electric field** is concentrated between gaps;

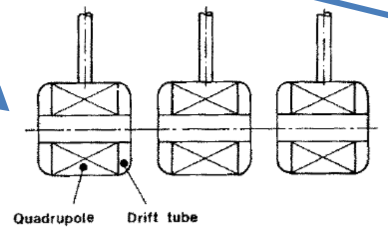
⇒The drift tubes are suspended by **stems**;

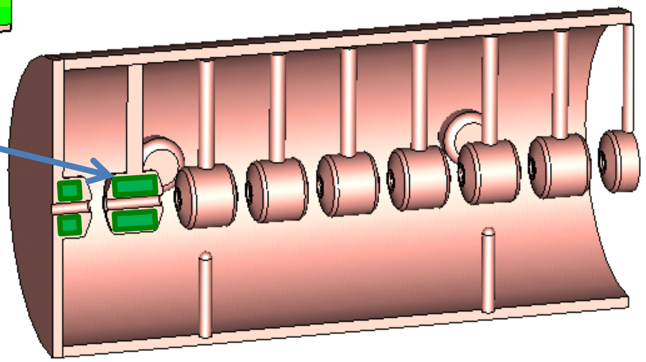


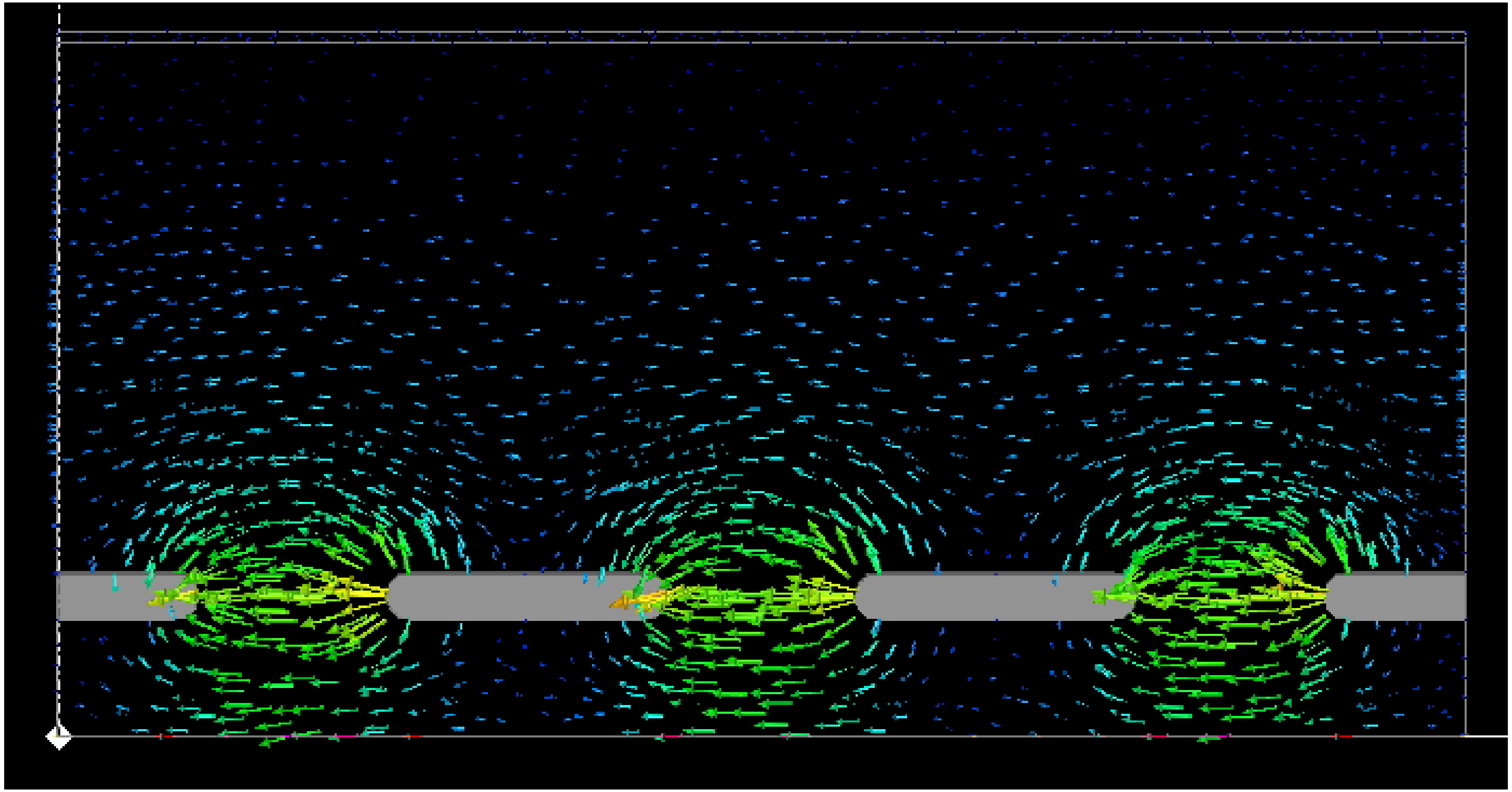
⇒**Quadrupole** (for transverse focusing) can fit inside the drift tubes.

⇒In order to be synchronous with the accelerating field at each gap the **length of the n-th drift tube** L_n has to be:

$$L_n = \overline{\beta}_n \lambda_{RF}$$

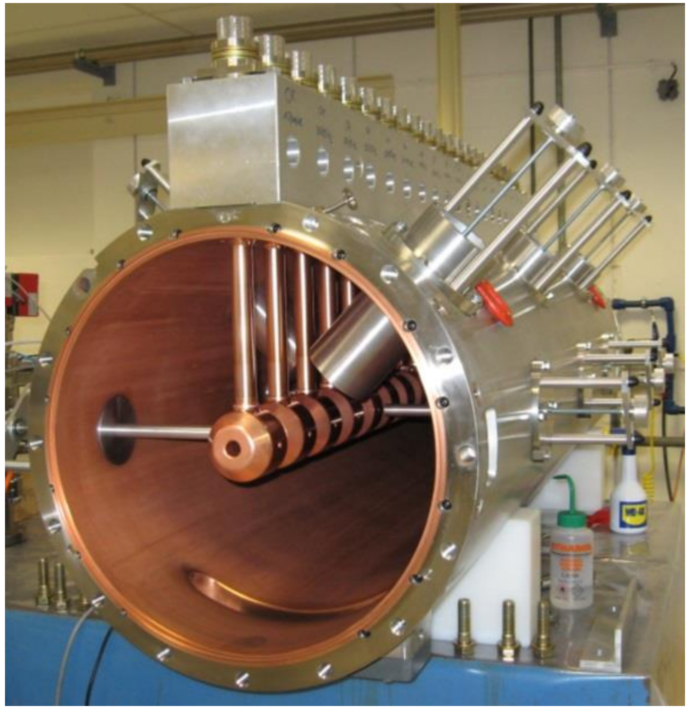




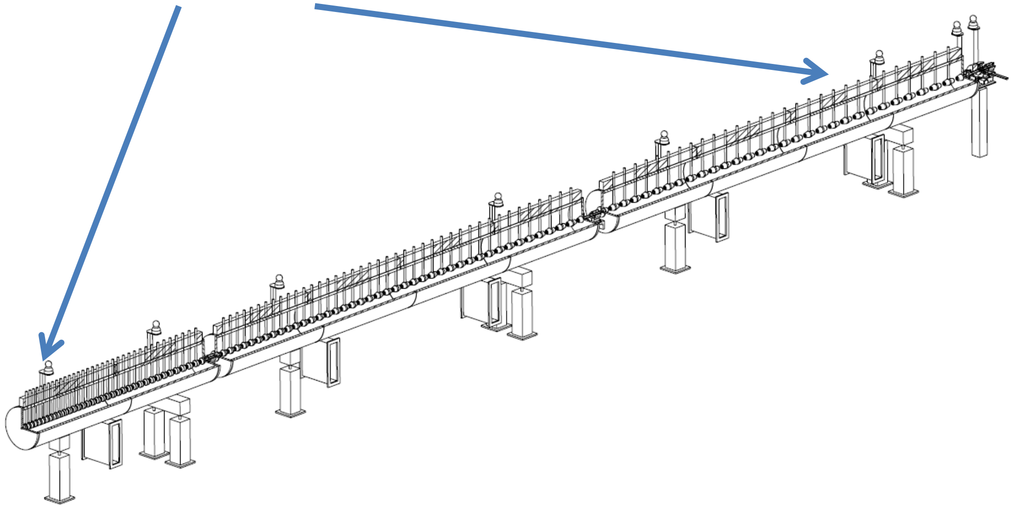


ALVAREZ STRUCTURES: EXAMPLES

CERN LINAC 2 tank 1:
200 MHz 7 m x 3 tanks, 1 m diameter, final energy 50 MeV.



CERN LINAC 4: 352 MHz frequency, Tank diameter 500 mm, 3 resonators (tanks), Length 19 m, 120 Drift Tubes, Energy: 3 MeV to 50 MeV, β=0.08 to 0.31 $\rightarrow$ cell length from 68mm to 264mm.



EXERCISE 2: ALVAREZ STRUCTURES

A proton beam is injected into a DTL (Alvarez structure) working at f_{RF}=300 MHz, with a kinetic energy W_{in}=4 MeV. Calculate:

1) the distance between the first two centers of the accelerating gaps (L_{gaps}) assuming a constant velocity of the proton beam between the first two gaps and a negligible increase of the velocity due to the accelerating field;

2) if the structure is composed by 40 accelerating gaps (N_{gaps}) and the average accelerating voltage per gap is V_{acc}=0.5 MV, calculate final proton beam kinetic energy.

REMEMBER

proton rest energy $m_{0\_p}c^2=E_{0\_p}=938$ *MeV*
velocity of light c=2.998e8

EXERCISE 3: ALVAREZ STRUCTURES AND TRANSIT TIME FACTOR

Particles at β=0.5 are accelerated through an ideal DTL operating at f_{RF}=400 MHz. Assuming a uniform accelerating RF field (E_{RF}) along the gap, calculate the accelerating gap length (L) that maximize the energy gain of the accelerated particles.

HIGH β CAVITIES: CYLINDRICAL STRUCTURES

(electrons or protons and ions at high energy)

⇒When the β of the particles increases (>0.5) one has to use **higher RF frequencies** (>400-500 MHz) to increase the accelerating gradient per unit length

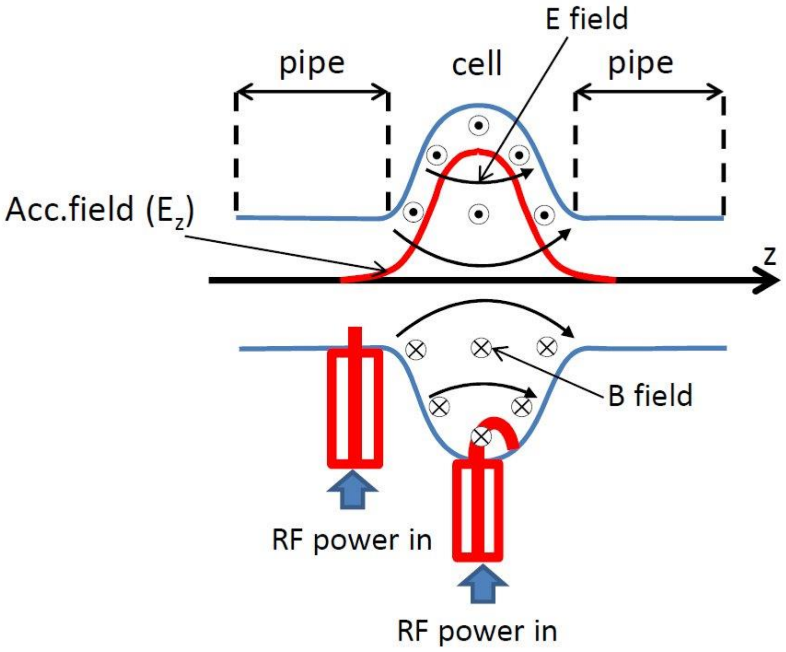
⇒the **DTL structures became less efficient** (effective accelerating voltage per unit length for a given RF power);

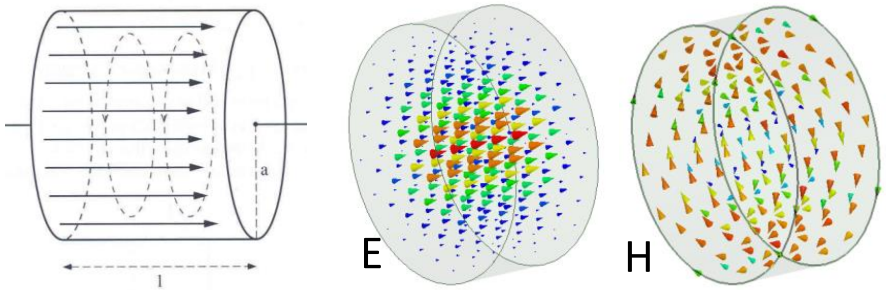
Cylindrical single (or multiple cavities) working on the **TM_{010}-like** mode are used

For a **pure cylindrical structure** (also called **pillbox cavity**) the first accelerating mode (i.e. with non zero longitudinal electric field on axis) is the **TM_{010} mode**. It has a well known analytical solution from Maxwell equation.

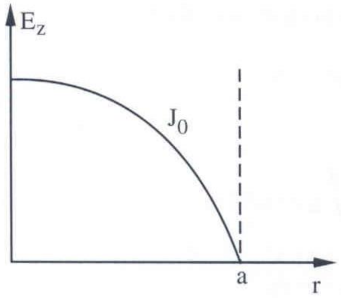
Real cylindrical cavity

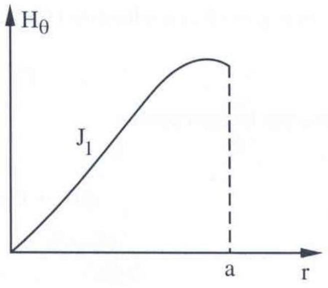
(TM_{010}-like mode because of the shape and presence of beam tubes and couplers)





$$f_{res} = \frac{2.405c}{2\pi a}$$





$$E_z = AJ_0\left(2.405\frac{r}{a}\right)\cos(\omega_{RF}t) \qquad H_\theta = A\frac{1}{Z_0}J_1\left(2.405\frac{r}{a}\right)\sin(\omega_{RF}t)$$

SW CAVITIES PARAMETERS: V_{acc}, P_{diss}, W

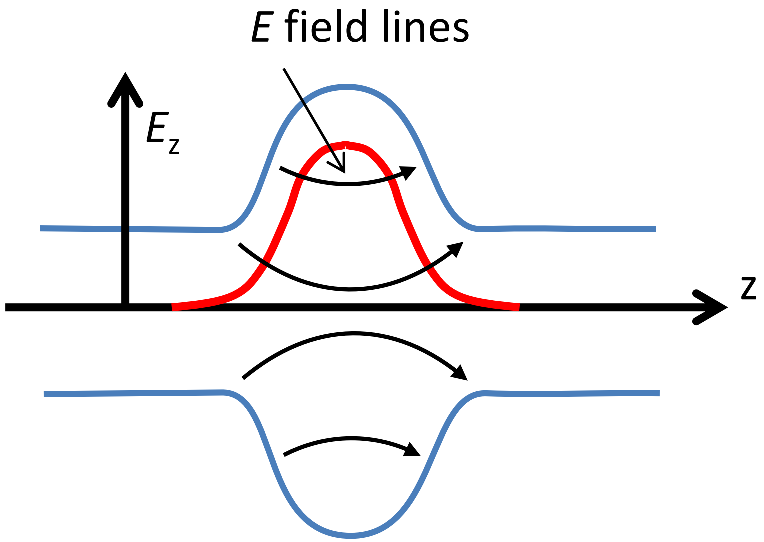
To compare and qualify different cavities is necessary to define some parameter that characterize each accelerating structure.

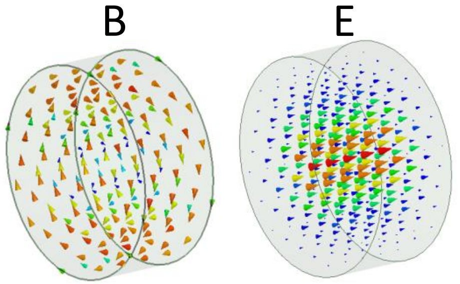
ACCELERATING VOLTAGE

We suppose that the cavities are powered at a **constant frequency f_{RF}**. The **maximum energy gain** of a particle crossing the cavity at a velocity v (~c for electrons) is obtained integrating the time-varying accelerating field sampled by the particle along the trajectory:

$$\hat{V}_{acc} = \int_{gap} E_{RF}(z)\cos\left(\omega_{RF}\frac{z}{v}\right)dz$$

More general form $\hat{V}_{acc} = \left| \int_{cavity} \tilde{E}_z(z) e^{j\omega_{RF}\frac{z}{v}} dz \right|$





MODE TM_{010}

DISSIPATED POWER

Real cavities have **losses**.

Surface currents (related to the surface magnetic field $\vec{J} = \vec{n} \times \vec{H}$) "sees" a **surface resistance R_s** and dissipate energy, so that a certain amount of RF power must be provided from the outside to keep the accelerating field at the desired level. The total dissipated power is:

$$P_{diss} = \int_{\substack{cavity\\wall}} \overbrace{\frac{1}{2}R_s|H_{tan}|^2}^{\text{power density}} dS$$

NC cavity (Cu $R_s \approx 10$ mΩ at 1 GHz)

SC cavity (Nb at 2 K $R_s \approx 10$ nΩ at 1 GHz)

STORED ENERGY

The total e.m. energy stored in the cavity:

$$W = \int_{\substack{cavity\\volume}} \overbrace{\left(\frac{1}{4}\varepsilon|\vec{E}|^2 + \frac{1}{4}\mu|\vec{H}|^2\right)}^{\text{energy density}} dV$$

SW CAVITIES PARAMETERS: R, Q, R/Q

ACCELERATING VOLTAGE (V_{acc}) ***DISSIPATED POWER (P_{diss})*** ***STORED ENERGY (W)***

SHUNT IMPEDANCE

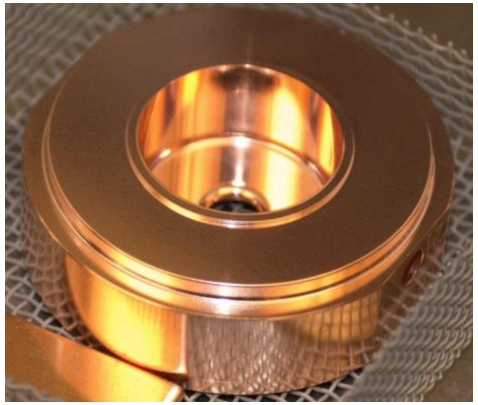
The shunt impedance is the parameter that qualifies the **efficiency of an accelerating mode**. The highest is its value, the larger is the obtainable accelerating voltage for a given power. Traditionally, it is the quantity to optimize in order to **maximize the accelerating field for a given dissipated power**:

$$R = \frac{\hat{V}_{acc}^2}{P_{diss}} \; [\Omega]$$

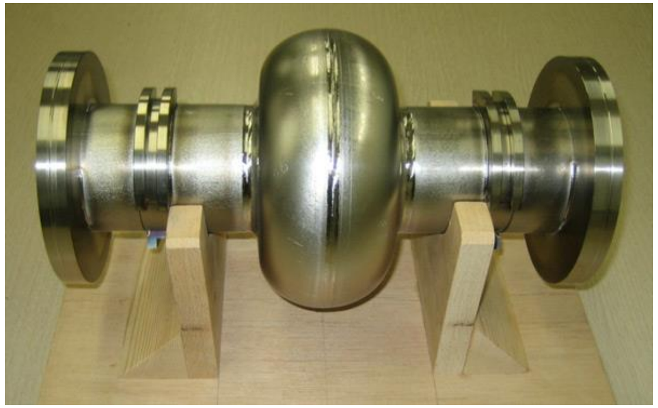
SHUNT IMPEDANCE PER UNIT LENGTH

$$r = \frac{\left(\hat{V}_{acc}/L\right)^2}{P_{diss}/L} = \frac{\hat{E}_{acc}^2}{p_{diss}} \; [\Omega/m]$$

NC cavity R~1MΩ



SC cavity R~1TΩ



QUALITY FACTOR

$$Q = \omega_{RF} \frac{W}{P_{diss}}$$

NC cavity $Q \sim 10^4$

SC cavity $Q \sim 10^{10}$

$$\frac{R}{Q} = \frac{\hat{V}_{acc}^2}{\omega_{RF} W}$$

The R/Q is a **pure geometric qualification factor**. It does not depend on the cavity wall conductivity. R/Q of a single cell is of the order of 100.

Example:

R~1MΩ

P_{diss}=1 MW

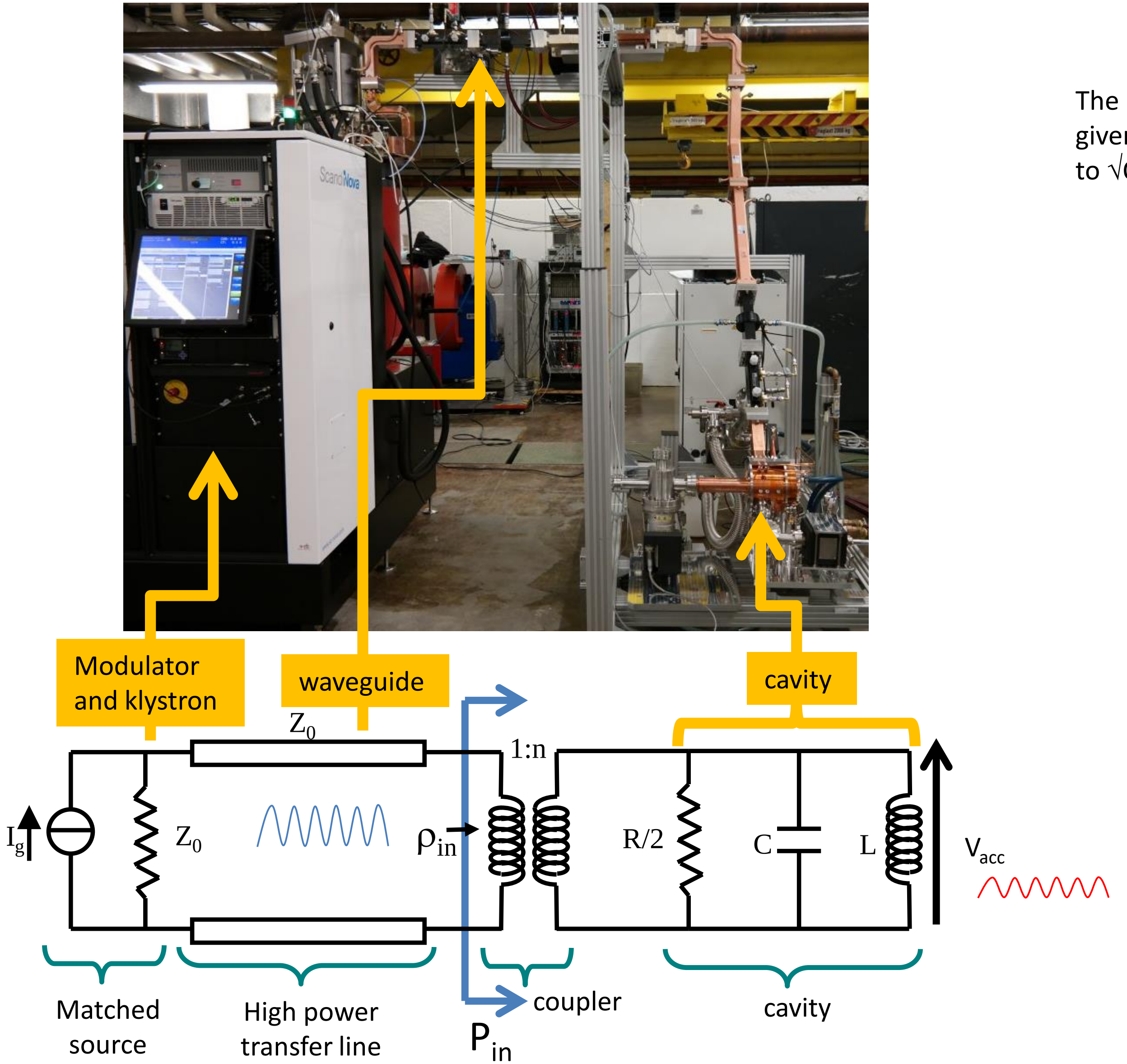
V_{acc}=1MV

For a cavity working at 1 GHz with a structure length of 10 cm we have an average accelerating field of 10 MV/m

SW CAVITIES : EQUIVALENT CIRCUIT AND BANDWIDTH

The previous quantities plays crucial roles in the evaluation of the **cavity performances**. Let us consider the case of a cavity powered by a source (klystron) at a constant frequency in CW and at a fixed power (P_{in}).

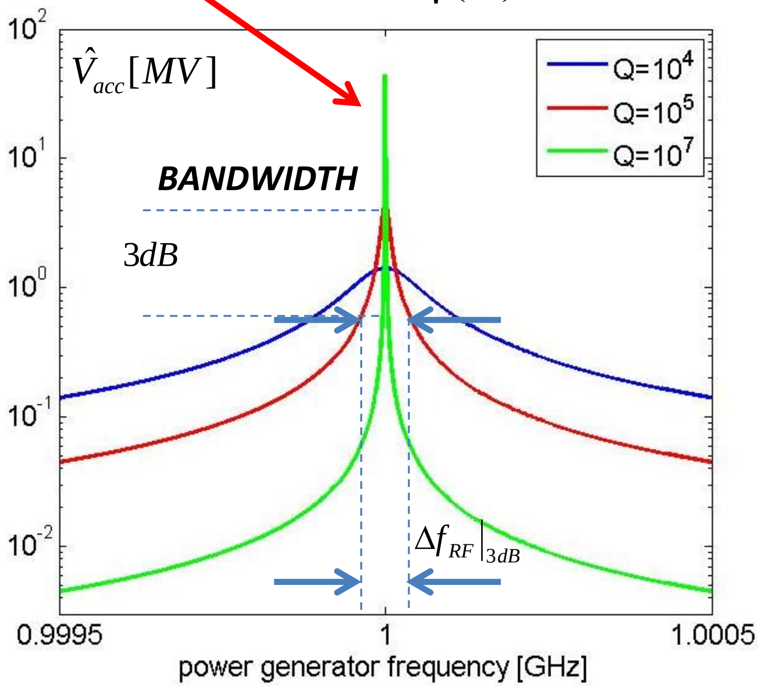
P_{in}=1 MW
R/Q=100
Critical coupling ($P_{diss}=P_{in}$)
f_{res}=1 GHz



The reachable V_{acc} for a given power is proportional to √Q

Frequency domain

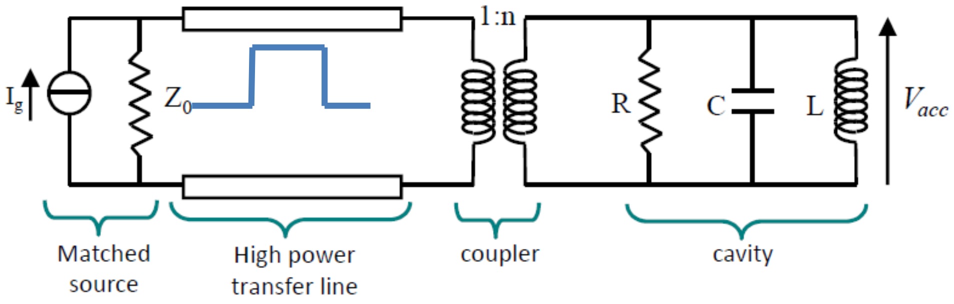
$$\hat{V}_{acc} = \sqrt{RP_{diss}} = \sqrt{\left(\frac{R}{Q}\right)QP_{in}} \propto \sqrt{Q}$$



$$\frac{\Delta f_{RF}|_{3dB}}{f_{RF}} = \frac{1}{Q} \Rightarrow \begin{cases} \Delta f_{RF}|_{3dB}|_{NC} = 100kHz \\ \Delta f_{RF}|_{3dB}|_{SC} < 1Hz \end{cases}$$

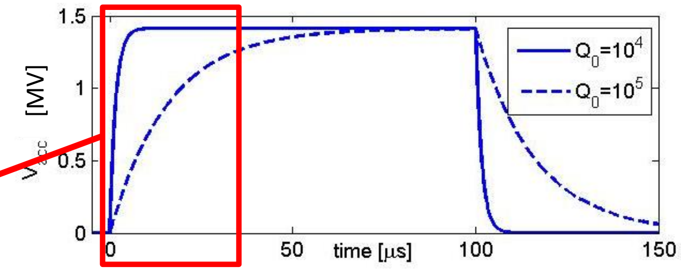
SW CAVITIES : FILLING TIME AND DISSIPATED POWER

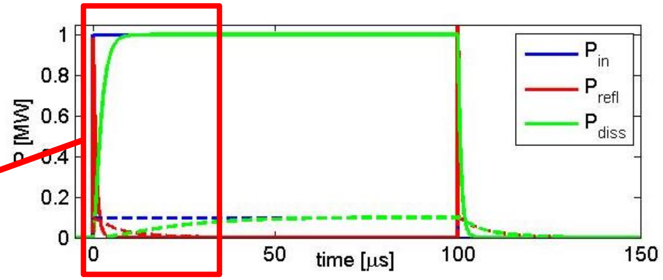
Let us now consider the case of a cavity powered by a source (klystron) in **pulsed mode** at a frequency $f_{RF}=f_{res}$. The accelerating voltage has an exponential behavior and reach the steady state regime asymptotically with a certain filling time (τ_F) that is proportional to the quality factor of the resonator.



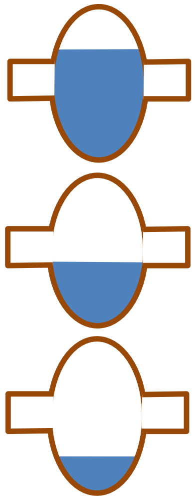
$$V_{acc}(t)=\hat{V}_{acc}\left(1-e^{-\frac{t}{\tau_F}}\right)$$

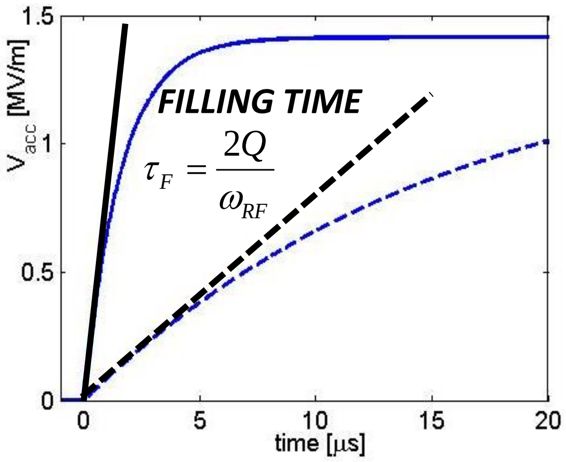
Time domain





One needs several filling times to completely fill the cavity



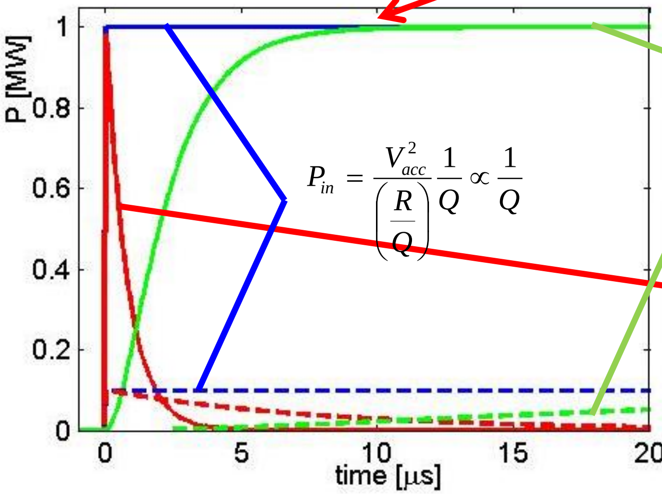


The reachable V_{acc} for a given power is proportional to $\sqrt{Q}$ but, on the other hand, the filling time is $\propto Q$

$$\tau_F\big|_{NC}\approx \mu s$$

$$\tau_F\big|_{SC}>100ms$$

Example:
Q~10000
f_{RF}=1 GHz
τ_F=3.1 μs



$P_{diss}\propto 1/Q$

To reach a given voltage the required power is inversely proportional to the Q-factor

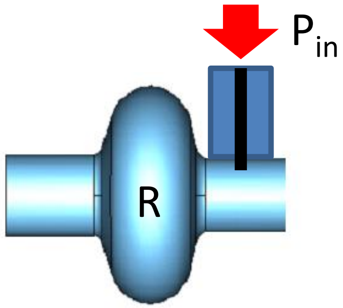
P_{refl} to the generator (it that has to be protected!)

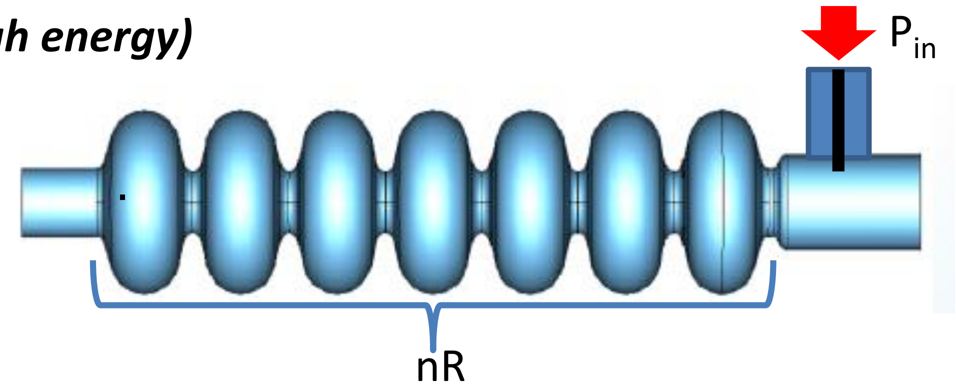
EXERCISE 4: FILLING TIME

A SW cavity is feed by an RF generator with constant power, calculate after how much time the field in the cavity is 90% the field at full regime supposing that the cavity operate at f_{RF}=1.3 GHz and has an equivalent Q factor of 10000.

MULTI-CELL SW CAVITIES

(electrons or protons and ions at high energy)



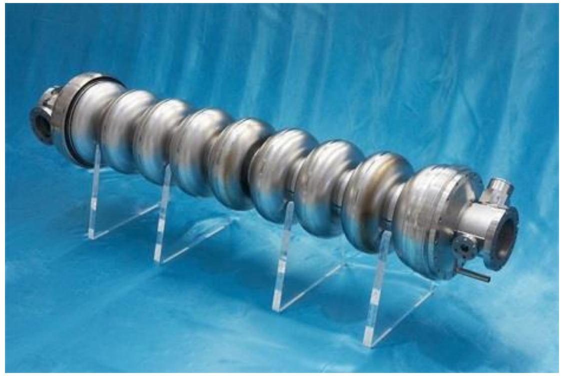


- In a multi-cell structure there is **one RF input coupler**. As a consequence the **total number of RF sources is reduced**, with a **simplification of the layout and reduction of the costs;**

- The **shunt impedance is n time** the impedance of a single cavity

- They are **more complicated** to fabricate than single cell cavities;

- The fields of adjacent cells couple through the cell **irises** and/or through properly designed coupling **slots**.



MULTI-CELL SW CAVITIES: π MODE STRUCTURES

(electrons or protons and ions at high energy)

- The N-cell structure behaves like a system composed by **N coupled oscillators** with **N coupled multi-cell resonant modes**.

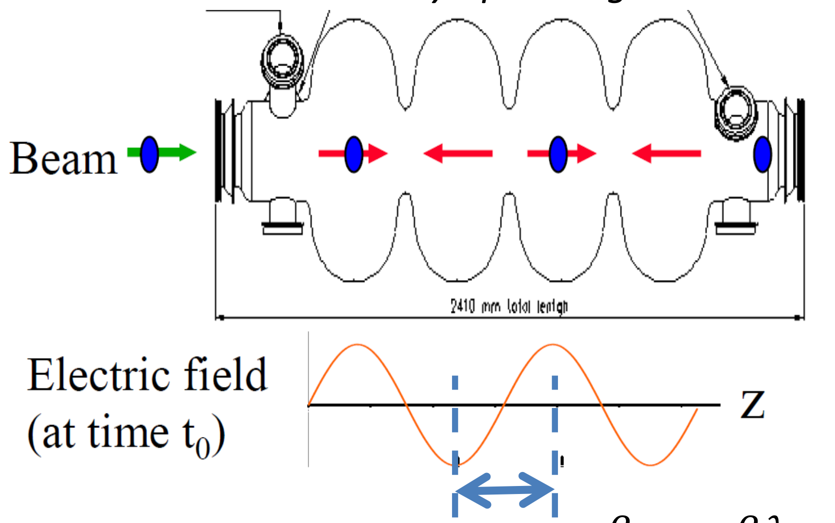
- The modes are characterized by a cell-to-cell phase advance given by:

$$\Delta\phi_n = \frac{n\pi}{N-1} \qquad n = 0,1,\ldots,N-1$$

- The multi cell mode generally used for acceleration is the **π, π/2 and 0 mode** (DTL as example operate in the 0 mode).

- The cell lengths have to be chosen in order to synchronize the accelerating field with the particle traveling into the structure at a certain velocity

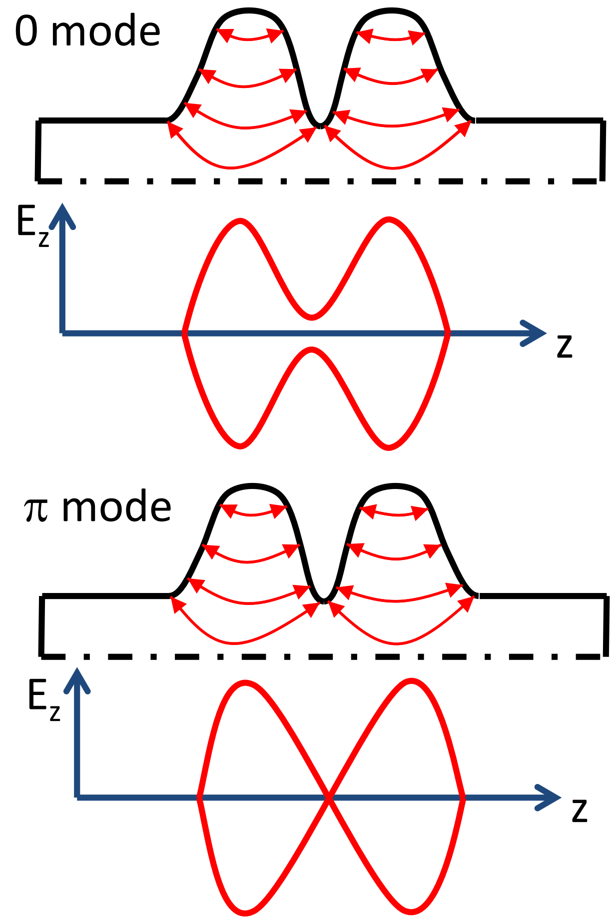
EXAMPLE: 4 cell cavity operating on the π-mode



$$d = \frac{\beta c}{2f_{RF}} = \frac{\beta\lambda_{RF}}{2}$$

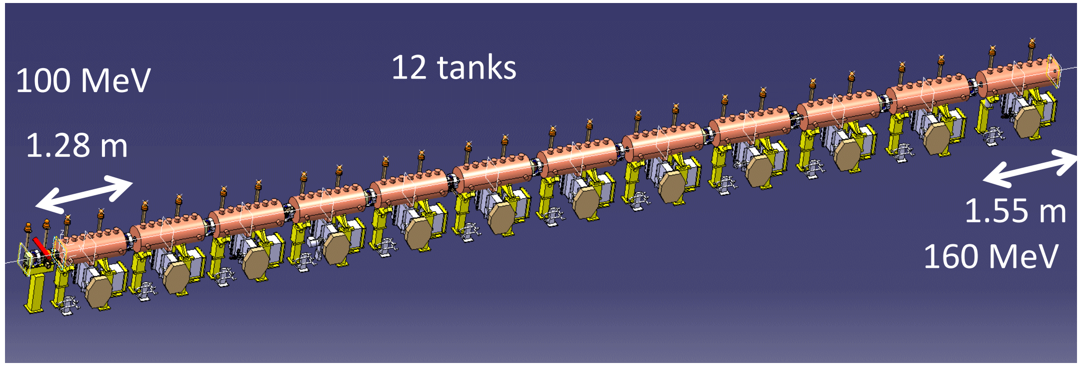
⇒For **ions and protons** the cell lengths have to be increased along the linac that will be a sequence of different accelerating structures matched to the ion/proton velocity.

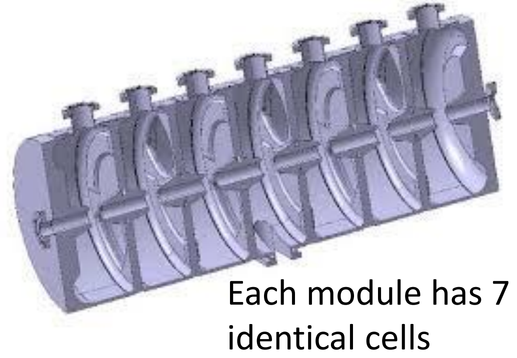
⇒For **electron**, β=1, $d=\lambda_{RF}/2$ and the linac will be made of an injector followed by a series of identical accelerating structures, with cells all the same length.

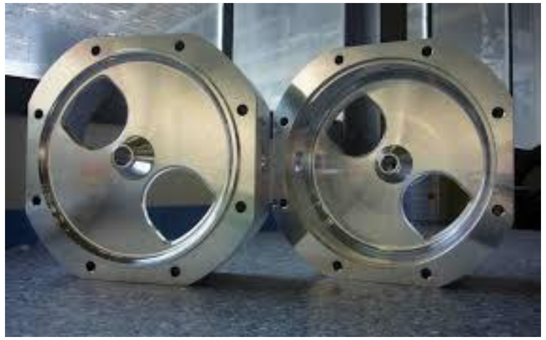


π MODE STRUCTURES: EXAMPLES

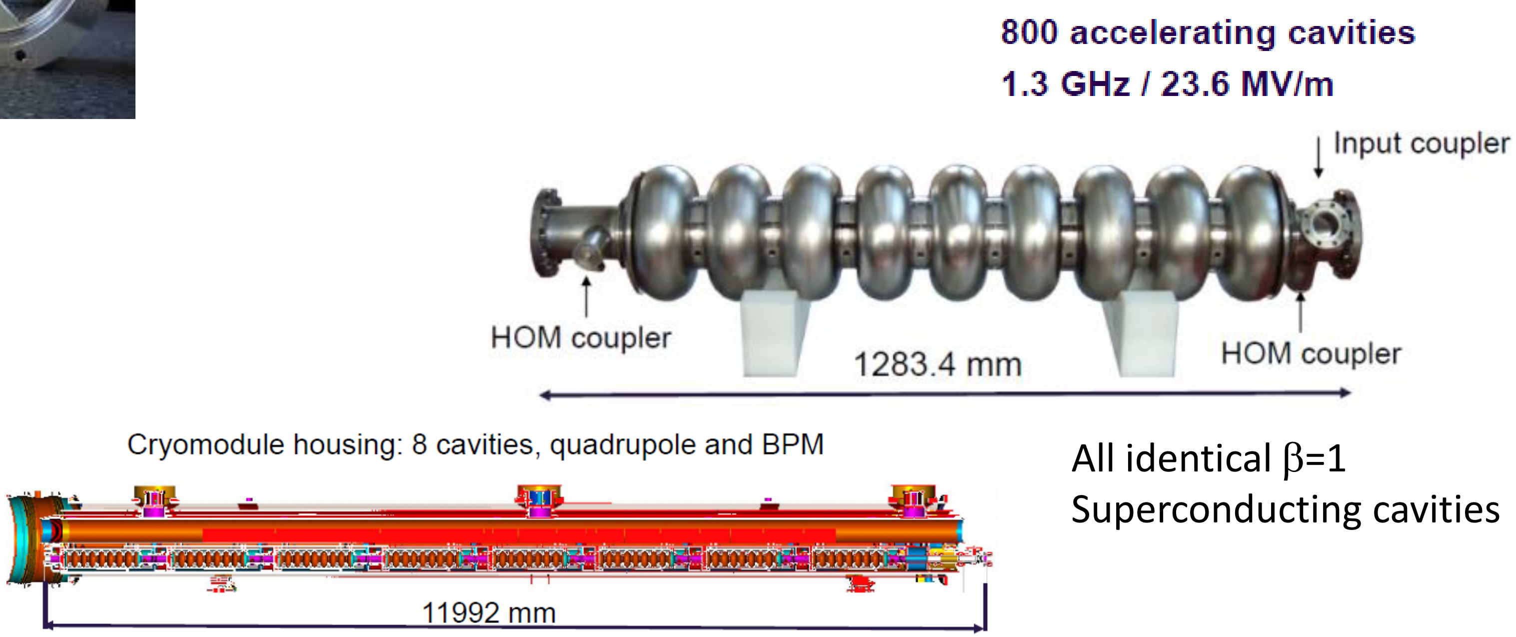
LINAC 4 (CERN) PIMS (PI Mode Structure) for protons: f_{RF}=352 MHz, β>0.4







European XFEL (Desy): electrons



All identical β=1
Superconducting cavities

EXERCISE 5: π MODE STRUCTURES

A π-mode structure, operating at f_{RF}=400 MHz, is made up of N=8 cells and is used to accelerate protons. Each single cell has a shunt impedance R=3 MΩ and a length L_{cell}=15 cm. Calculate:

1) the total shunt impedance of the π-mode structure;
2) the accelerating voltage if the total dissipated power into the cavity is P_{diss}=1 MW;
3) the average accelerating field;
4) the average β of the proton beam accelerated by this structure.

MULTI-CELL SW CAVITIES: π/2 MODE STRUCTURES

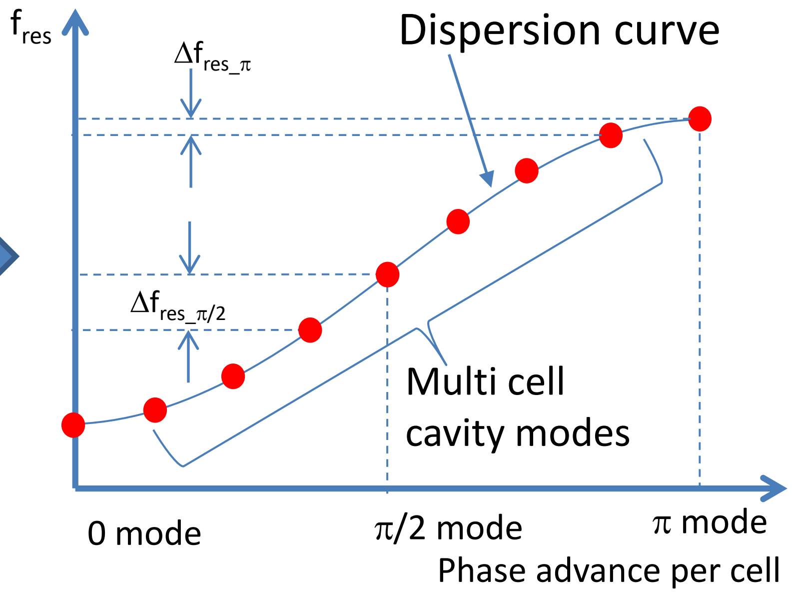
(electrons or protons and ions at high energy)

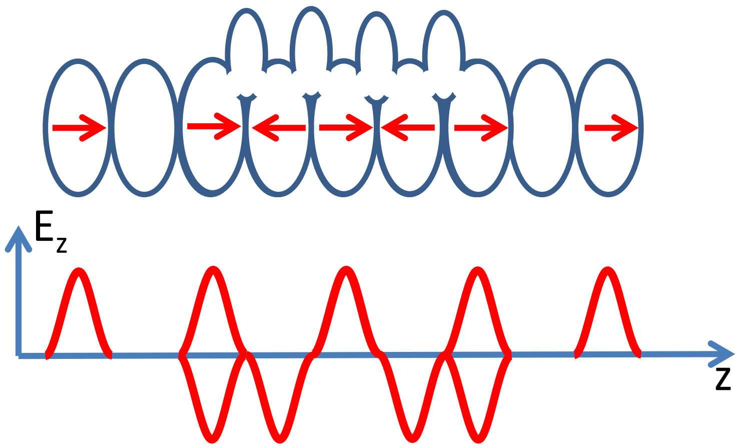
⇒It is possible to demonstrate that **over a certain number of cavities** (>10) working on the π mode, the **overlap between adjacent modes** can be a problem (as example the field uniformity due to machining errors is difficult to tune).

⇒The criticality of a working mode depend on the **frequency separation between the working mode and the adjacent mode**

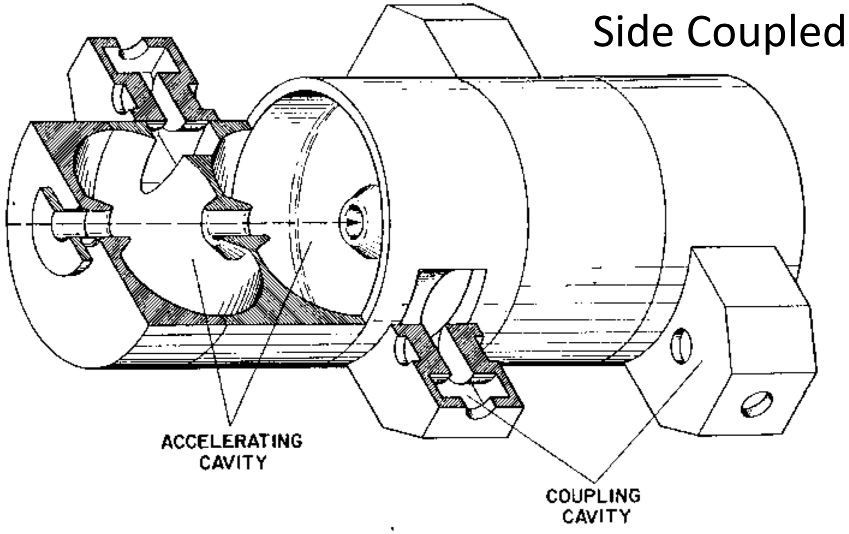
⇒the **π/2 mode from this point of view is the most stable mode**. For this mode it is possible to demonstrate that the accelerating field is zero every two cells. For this reason, the empty cells are put of axis and coupling slots are opened from the accelerating cells to the empty cells.

⇒this allow to increase the number of cells to >20-30 without problems



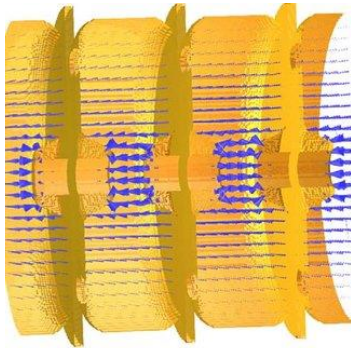


Side Coupled Cavity (SCC)



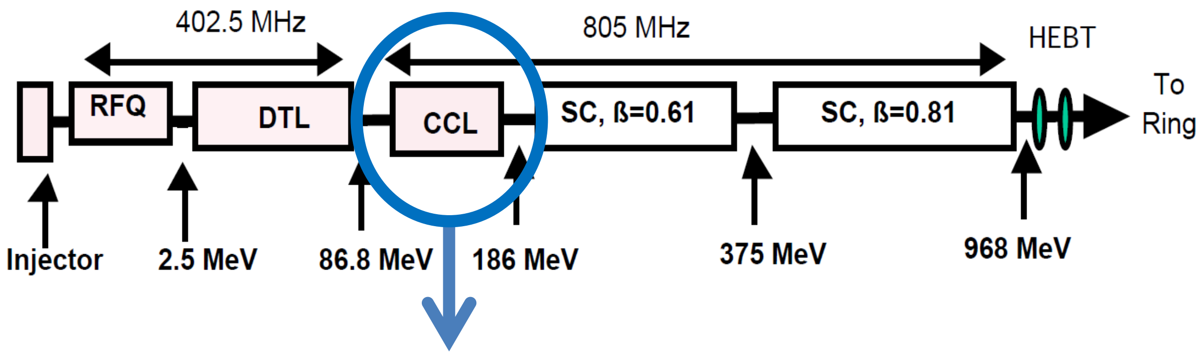
f_{RF}=800 - 3000 MHz for proton (β=0.5-1) and electrons

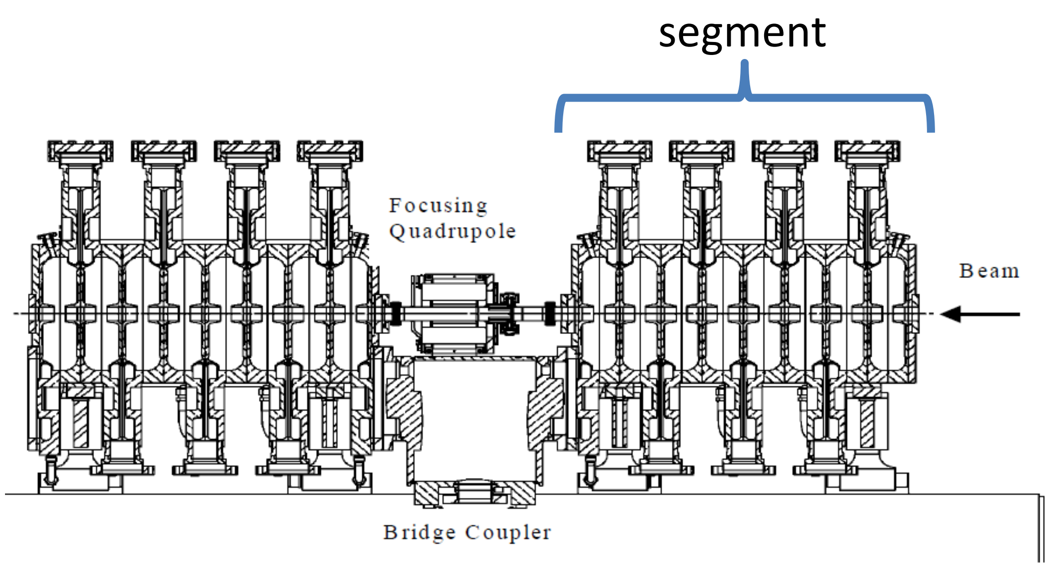
Axial coupled



SCC STRUCTURES: EXAMPLES

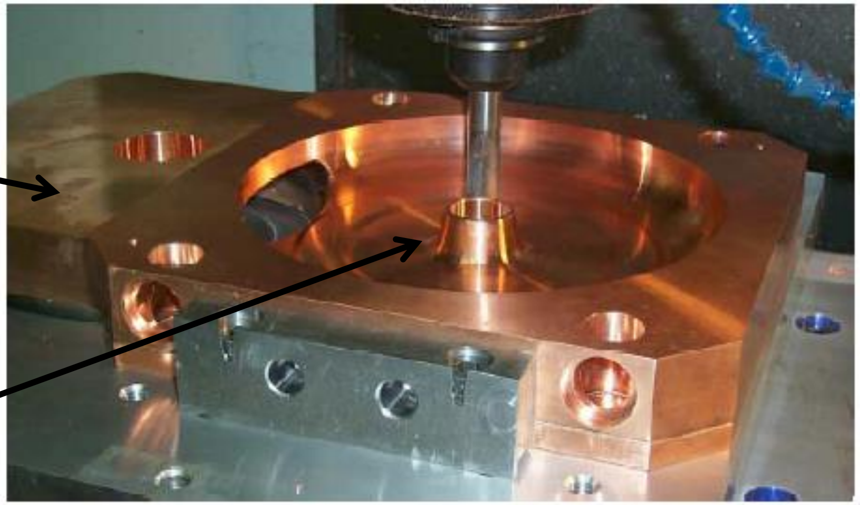
Spallation Neutron Source Coupled Cavity Linac (protons)





4 modules, each containing 12 accelerator segments CCL and 11 bridge couplers. The CCL section is a RF Linac, operating at **805 MHz** that accelerates the beam **from 87 to 186 MeV** and has a physical installed length of slightly over **55 meters.**

Coupling cell



Accelerating cell

TRAVELLING WAVE (TW) STRUCTURES

(electrons)

⇒To accelerate charged particles, the electromagnetic field must have an **electric field along the direction of propagation of the particle**.

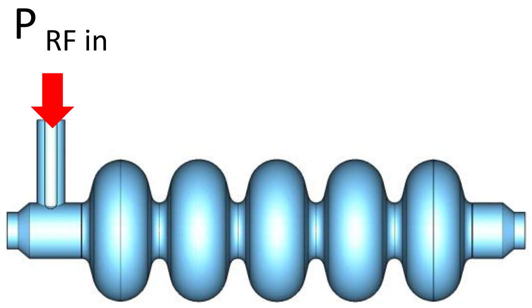
⇒The field has to be synchronous with the particle velocity.

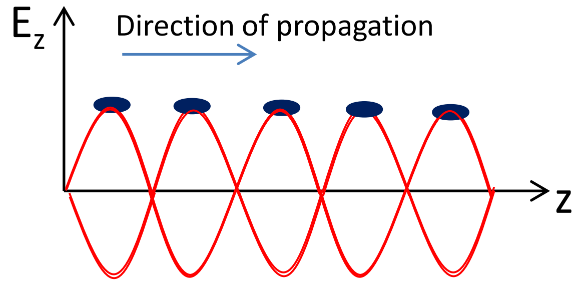
⇒Up to now we have analyzed the standing **standing wave (SW)** structures in which the field has basically a given profile and oscillate in time (as example in DTL or **resonant cavities operating on the** TM_{010}-like).

Time oscillation

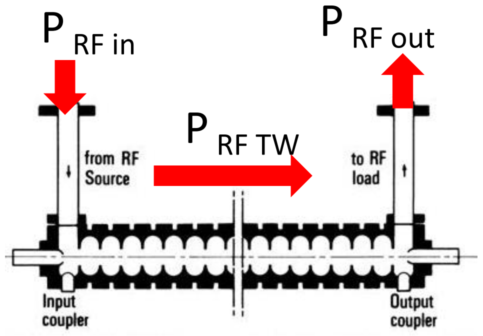
$$E_z(z,t) = \underbrace{E_{RF}(z)}_{\text{field profile}}\overbrace{\cos(\omega_{RF} t)}^{\text{Time oscillation}}$$

field profile

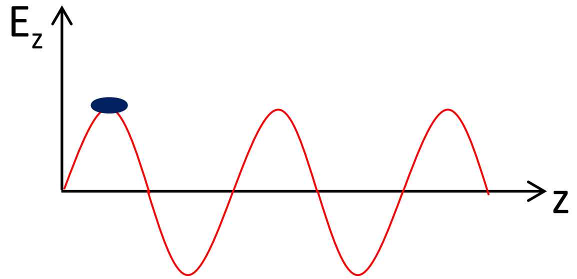




⇒There is another possibility to accelerate particles: using a **travelling wave (TW)** structure in which the RF wave is **co-propagating** with the beam with a **phase velocity equal to the beam velocity**.



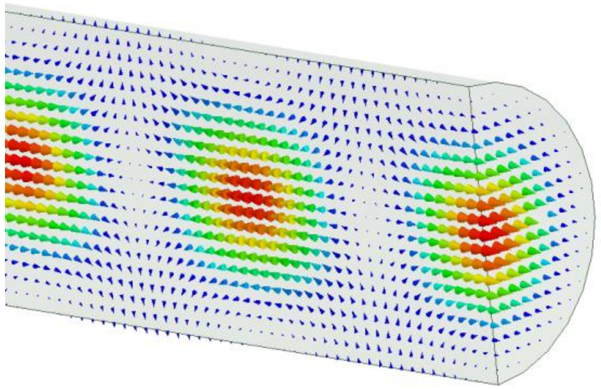
⇒Typically these structures **are used for electrons** because in this case the **phase velocity can be constant** all over the structure and equal to c. On the other hand it is difficult to modulate the phase velocity itself very quickly for a low β particle that changes its velocity during acceleration.

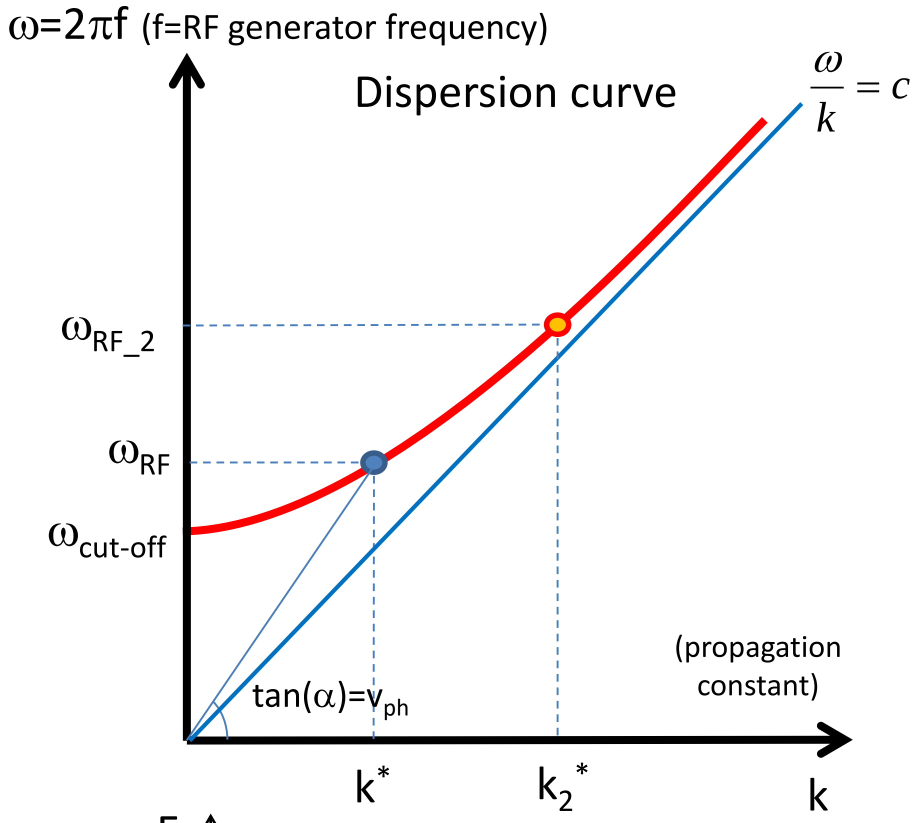


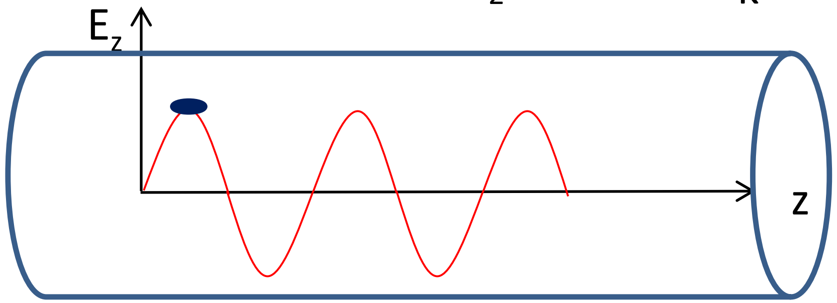
TW CAVITIES: CIRCULAR WAVEGUIDE AND DISPERSION CURVE
(electrons)

In **TW structures** an e.m. wave with **$E_z \neq 0$** travel together with the beam in a special guide in which the **phase velocity of the wave matches the particle velocity (v)**. In this case the beam absorbs energy from the wave and it is **continuously accelerated**.

CIRCULAR WAVEGUIDE







As example if we consider a simple circular waveguide the first propagating mode with **$E_z \neq 0$** is the TM_{01} mode.

Nevertheless by solving the wave equation it turns out that an e.m. wave propagating in this **constant cross section waveguide** will **never be synchronous with a particle beam** since the **phase velocity is always larger than the speed of light c**.

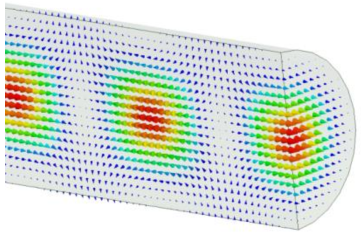
$$E_z\big|_{TM_{01}} = \underbrace{E_0(r)}_{J_0\left(\frac{p_{01}}{a}r\right)}\cos\left(\omega_{RF}t - k^* z\right) \Rightarrow v_{ph} = \frac{\omega_{RF}}{k^*} > c$$

TW CAVITIES: IRIS LOADED STRUCTURES

(electrons)

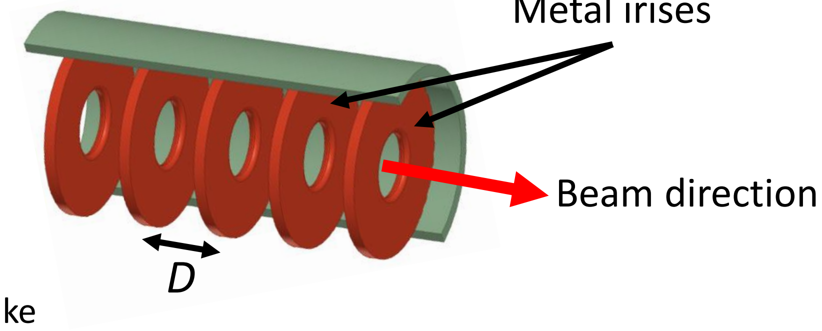
In order to **slow-down the wave phase velocity, iris-loaded periodic structure have to be used**.

CIRCULAR WAVEGUIDE



MODE TM_{01}

IRIS LOADED STRUCTURE



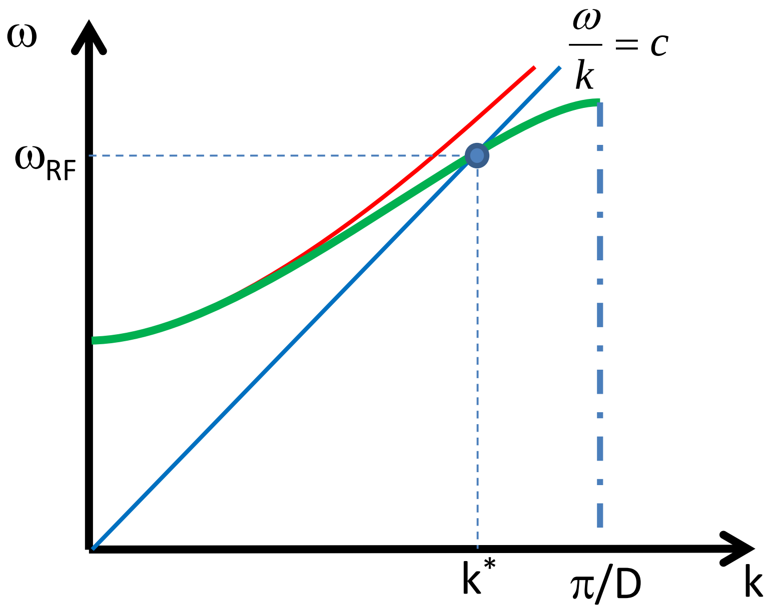
MODE TM_{01}-like

$$E_z\big|_{TM_{01}} = E_0(r)\cos\left(\omega_{RF}t - k^* z\right)$$

Periodic (in z) wih period D
(from Floquet theorem)

$$E_z\big|_{TM_{01}-like} = \hat{E}_{acc}(r,z)\cos\left(\omega_{RF}t - k^* z\right)$$

⇒The field in this type of structures is that of a special wave travelling within a spatial periodic profile.



⇒The structure can be designed to have the **phase velocity equal to the speed of the particles**.

⇒This allows **acceleration over large distances** (few meters, hundred of cells) with just an input coupler and a relatively **simple geometry**.

⇒They are used **especially for electrons** (constant particle velocity→constant phase velocity, same distance between irises, easy realization)

PHASOR NOTATION: RECAP.

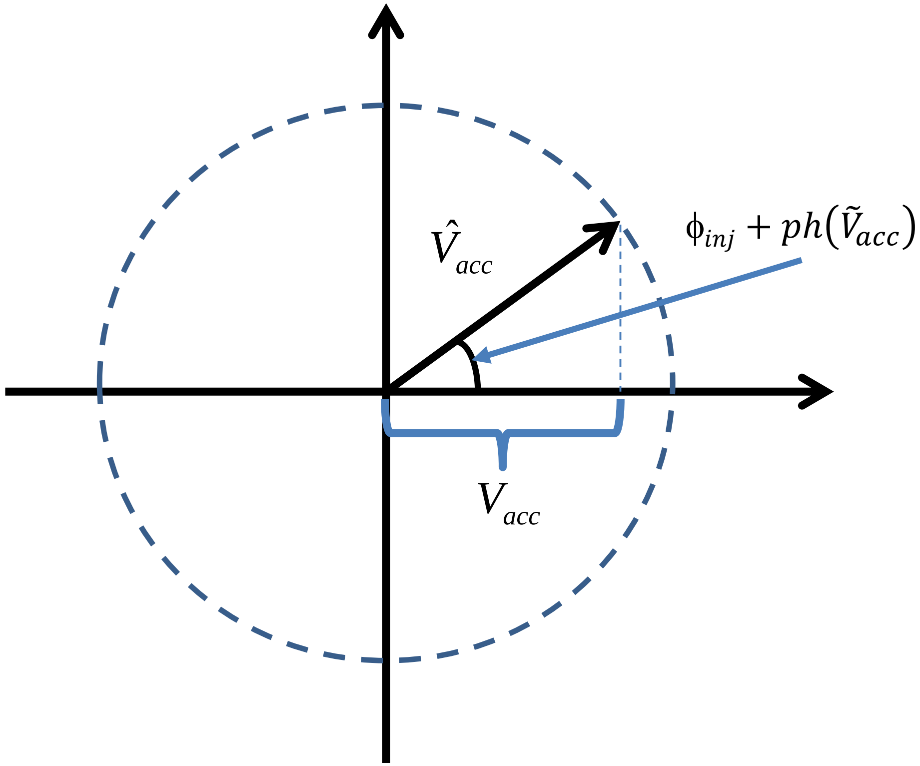
With a **more general notation** we can consider the phasors of the accelerating field.

$$E_z(z,t) = \mathrm{Re}\left[\tilde{E}_z(z)e^{j(\omega_{RF}t+\phi_{inj})}\right]$$

Phasor of the longitudinal electric field (complex quantity)

$$\Delta E = q\int_{gap} E_z(z,t)\Big|_{\substack{seen\\by\\particle}} dz = q\int_{-L/2}^{+L/2}\mathrm{Re}\left[\tilde{E}_z(z)e^{j\left(\omega_{RF}\frac{z}{v}+\phi_{inj}\right)}\right]dz =$$

$$= q\,\mathrm{Re}\left[e^{j\phi_{inj}}\underbrace{\int_{-L/2}^{+L/2}\tilde{E}_z(z)e^{j\omega_{RF}\frac{z}{v}}dz}_{\tilde{V}_{acc}}\right] = \underbrace{q\hat{V}_{acc}\cos\left(\phi_{inj}+ph\left(\tilde{V}_{acc}\right)\right)}_{V_{acc}}$$



Peak accelerating voltage
(i.e.: the maximum accelerating voltage that can be reached for a particular injection phase ($\phi_{inj} + ph(\tilde{V}_{acc})$=0))

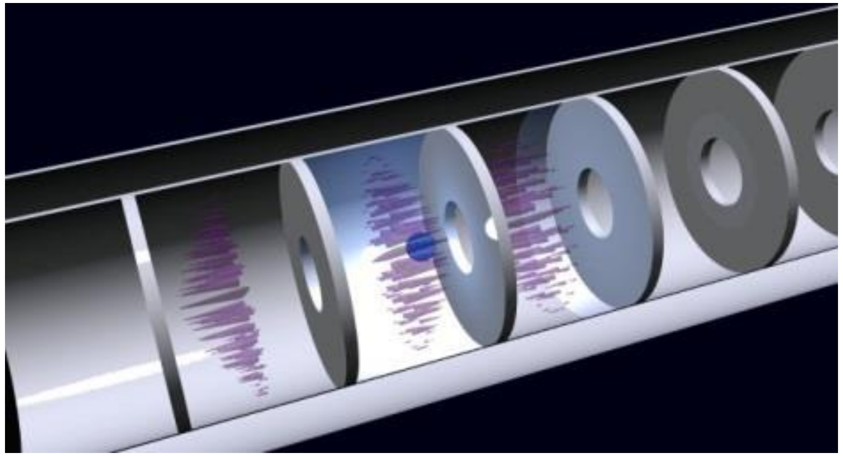
$$\tilde{V}_{acc} = \left|\tilde{V}_{acc}\right|e^{jph\left(\tilde{V}_{acc}\right)}$$

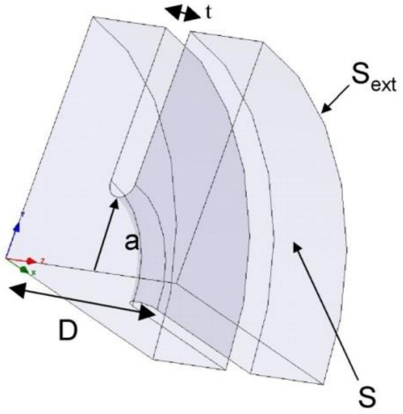
$$\left|\tilde{V}_{acc}\right| = \hat{V}_{acc} = \left|\int_{-L/2}^{+L/2}\tilde{E}_z(z)e^{j\omega_{RF}\frac{z}{v}}dz\right|$$

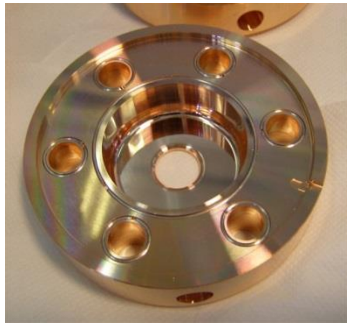
$$ph\left(\tilde{V}_{acc}\right) = ph\left(\int_{-L/2}^{+L/2}\tilde{E}_z(z)e^{j\omega_{RF}\frac{z}{v}}dz\right)$$

TW CAVITIES PARAMETERS: r, α, v_g

Similarly to the SW cavities it is possible to define some figure of merit for the TW structures







$$\hat{V}_{acc} = \left| \int_0^D E_{acc} \cdot e^{j\omega_{RF}\frac{z}{c}} dz \right|$$ single cell accelerating voltage

$$\hat{E}_{acc} = \frac{\hat{V}_{acc}}{D}$$ average accelerating field in the cell

$$P_F = \int_{Section} \frac{1}{2} \mathrm{Re}\left(\vec{E} \times \vec{H}^*\right) \cdot \hat{z} dS$$ flux power

$$P_{diss} = \frac{1}{2} R_s \int_{\substack{cavity\\wall}} \left| H_{\tan} \right|^2 dS$$ average dissipated power in the cell

$$p_{diss} = \frac{P_{diss}}{D}$$ average dissipated power per unit length

$$W = \int_{\substack{cavity\\volume}} \overbrace{\left(\frac{1}{4} \varepsilon \left| \vec{E} \right|^2 + \frac{1}{4} \mu \left| \vec{H} \right|^2 \right)}^{\text{energy density}} dV$$ stored energy in the cell

$$w = \frac{W}{D}$$ average stored energy per unit length

$$r = \frac{\hat{E}_{acc}^2}{p_{diss}}$$

Shunt impedance per unit length [Ω/m]. Similarly to SW structures the higher is r, the higher the available accelerating field for a given RF power.

$$\alpha = \frac{p_{diss}}{2P_F}$$

Field attenuation constant [1/m]: because of the wall dissipation, the RF power flux and the accelerating field decrease along the structure.

$$v_g = \frac{P_F}{w}$$

Group velocity [m/s]: the velocity of the energy flow in the structure (~1-2% of c).

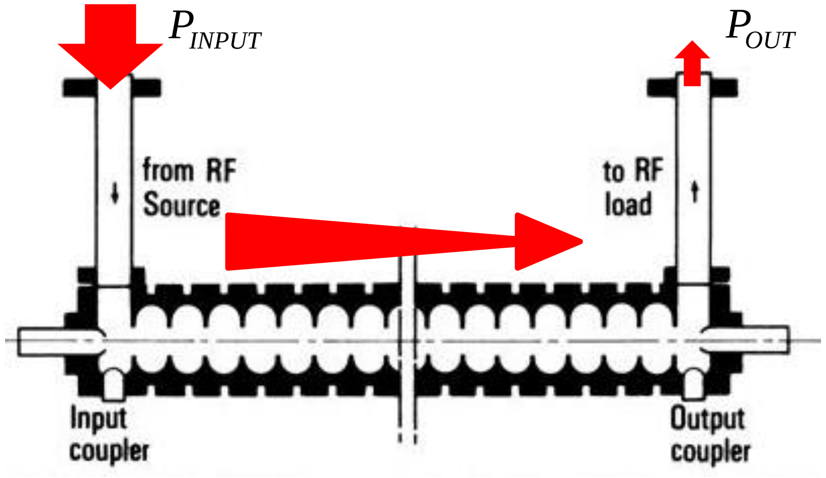
$$Q = \omega_{RF} \frac{w}{p_{diss}}$$

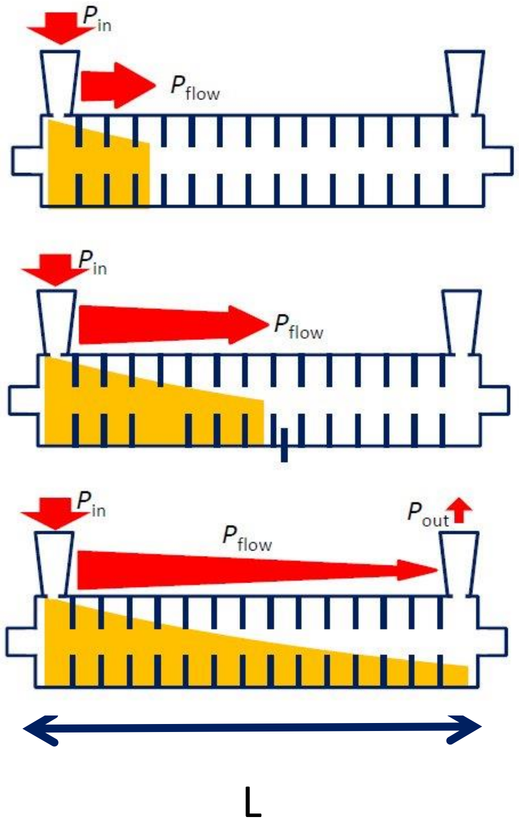
$$\Delta\phi = kD$$

Working mode [rad]: defined as the phase advance over a period *D*. For several reasons the most common mode is the **2π/3**

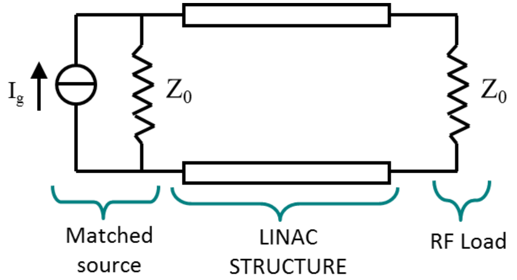
TW CAVITIES: EQUIVALENT CIRCUIT AND FILLING TIME

In a TW structure, the **RF power enters** into the cavity through an **input coupler**, flows (travels) through the cavity in the same direction as the beam and an **output coupler at the end** of the structure is connected to a **matched power load**.
If there is no beam, the input power, reduced by the cavity losses, goes to the power load where it is dissipated.
In the presence of a large beam current, however, a fraction of the TW power is transferred to the beam.





In a purely periodic structure, made by a sequence of **identical cells** (also called "**constant impedance structure**"), α does not depend on z and both the RF power flux and the intensity of the accelerating field decay exponentially along the structure :



$$E_{acc}(z,t) = \underbrace{E_P(r,z)}_{\text{periodic function with period D}} \cos(\omega_{RF}t - k_z^* z)e^{-\alpha z} \approx E_{IN}\cos(\omega_{RF}t - k_z^* z)e^{-\alpha z}$$

$$P_F(z) = P_{IN}e^{-2\alpha z} \qquad P_{OUT} = P_{IN}e^{-2\alpha L} \qquad E_{IN} = \sqrt{2\alpha r P_{IN}} \qquad V_{acc} = E_{IN}\frac{1-e^{-\alpha L}}{\alpha}$$

The **filling time** is the time necessary to propagate the RF wave-front from the input to the end of the section of length L is:

$$\tau_F = \frac{L}{v_g}$$

Differently from SW cavities after **one filling time the cavity is completely full of energy**

EXERCISE 6: TW STRUCTURES

A) Demonstrate that if we define the attenuation constant as: $\alpha = \frac{p_{diss}}{2P_F}$

the power flow along the structure scales as: $P_F(z) = P_{IN} e^{-2\alpha z}$

B) Demonstrate that if we define the shunt impedance per unit length as: $r = \frac{\hat{E}_{acc}^2}{p_{diss}}$

the average accelerating field "seen" by an ultrarelativistic particle (z=ct) along the structure can be expressed as:

$$E_{acc}(z) = \sqrt{2\alpha r P_{IN}} e^{-\alpha z} = E_{IN} e^{-\alpha_0 z}$$

C) Demonstrate that the total accelerating voltage is given by:

$$V_{acc} = E_{IN} \frac{1 - e^{-\alpha L}}{\alpha}$$

EXERCISE 7: TW STRUCTURES

A SLAC-type TW structure accelerate ultra-relativistic electrons. The structure length is L=3m and it can be simplified as a structure with a group velocity is v_g=1.1% the velocity of light. Calculate:

1) the filling time;
2) if we suppose that the structure has a field attenuation constant α=0.2 m^{-1}, calculate the total accelerating voltage if the accelerating field at the beginning of the structure is E_{INPUT}=20 MV/m;
3) Calculate the average accelerating field
4) if the average dissipated power per unit length in the structure, corresponding to the previous value of the accelerating field, is p_{diss}= 20 MW/m calculate the shunt impedance per unit length.

EXERCISE 8: TW STRUCTURES

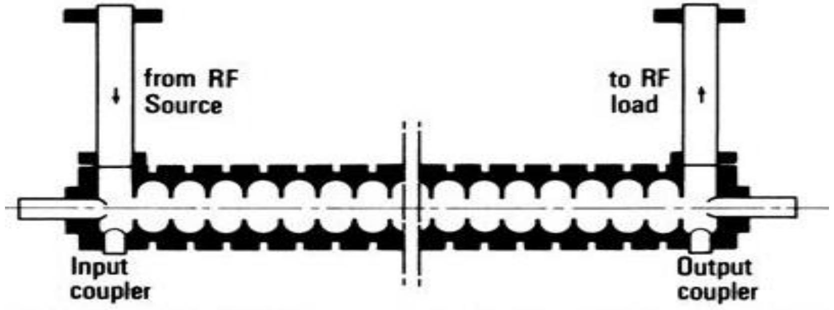
A constant impedance TW structure, accelerates ultra-relativistic electrons (β=1). The cavity has the following parameters: α=0.25 m^{-1} ; shunt impedance r=65 MOhm/m and a total length of 2 m. Calculate:

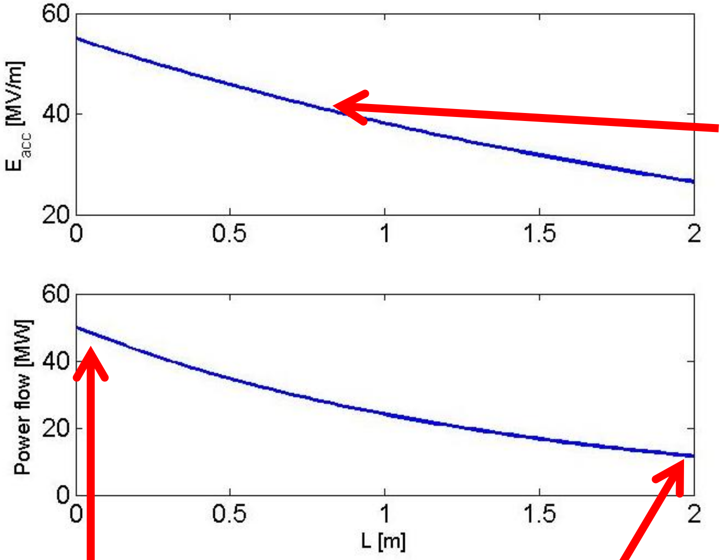
1) the input power to have an energy gain of the particles of 60 MeV
2) if the group velocity v_g is 1% the speed of light, which is the filling time of the structure?

TW CAVITIES: PERFORMANCES (1/2)

Just as an example we can consider a C-band (5.712 GHz) accelerating cavity of L=2 m long made in **copper**.

r=82 [MΩ/m]
α=0.36 [1/m]
v_g/c=1.7%
τ_F=400 ns (**very short if compared to SW**!)



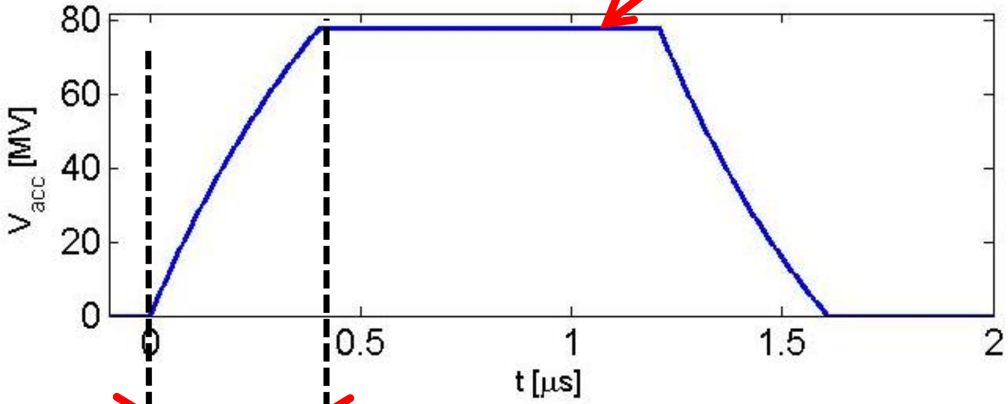


Field attenuation due to the copper dissipations

Input power

Output power (dissipated into the RF load): it is not convenient to have very long RF structures because their efficiency decreases over a certain length (2-3 m depending on the operating frequency).

$$E_{acc}(z)=E_{INPUT}e^{-\alpha z}\Rightarrow V_{acc}=\int_0^L E_{INPUT}e^{-\alpha z}dz=E_{INPUT}\frac{1-e^{-\alpha L}}{\alpha}$$



τ_F

$$P_F(z)=P_{INPUT}e^{-2\alpha z}\Rightarrow P_{OUT}=P_{INPUT}e^{-2\alpha L}$$

TW CAVITIES: CONSTANT GRADIENT STRUCTURES INTRODUCTION

In order to keep the **accelerating field constant along the LINAC structure,** the group velocity has to be reduced along the structure itself. This can be achieved by a reduction of the iris diameters.

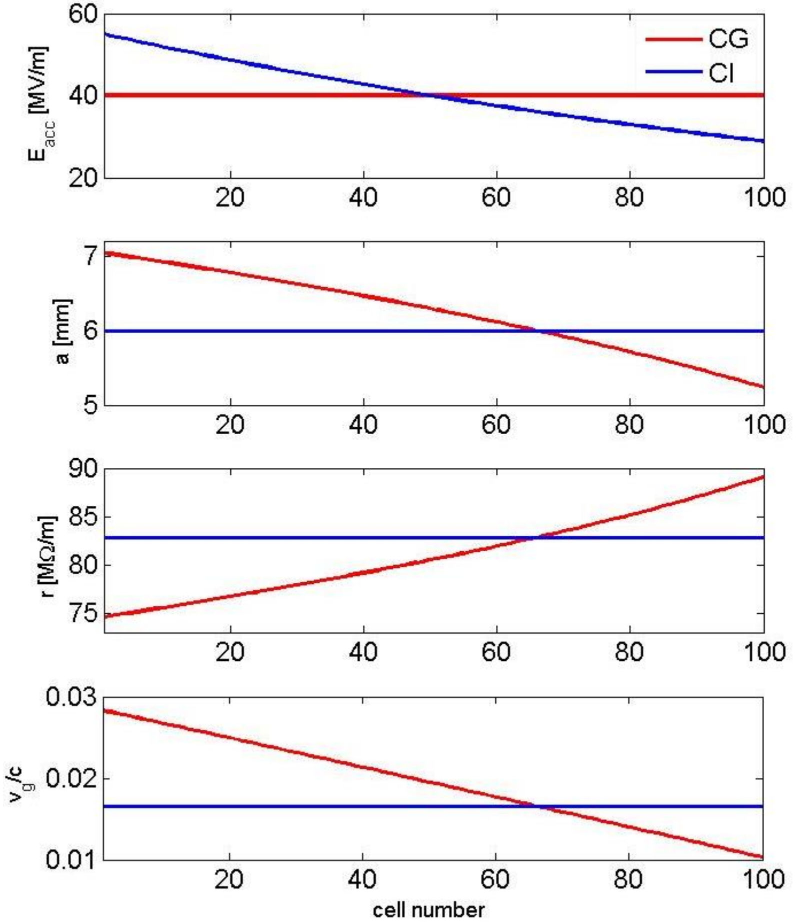
$$v_g = \frac{P_F}{w} \Rightarrow \frac{P_F}{v_g} = w \propto E_{acc}^2$$

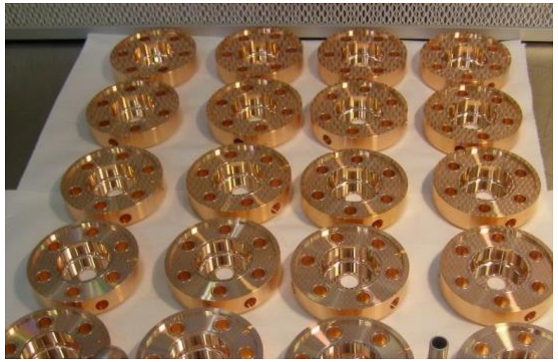
Reduction due to the attenuation (losses) → P_F

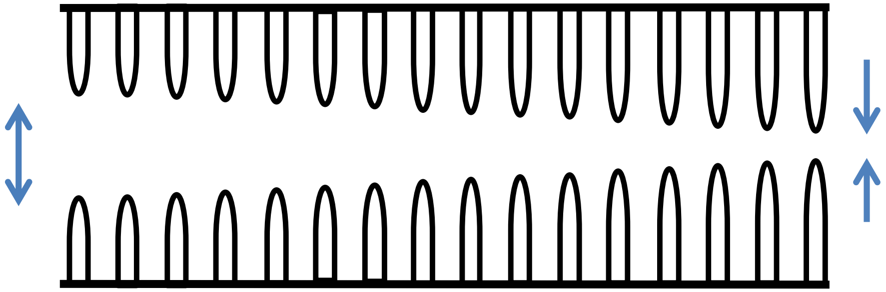
Reduction due to irises modulation → v_g

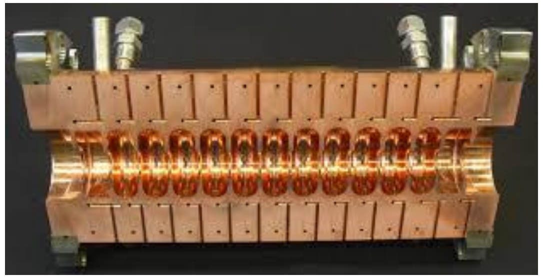
$$\frac{P_F}{v_g} = w \propto E_{acc}^2$$

Constant along the linac → E_{acc}^2









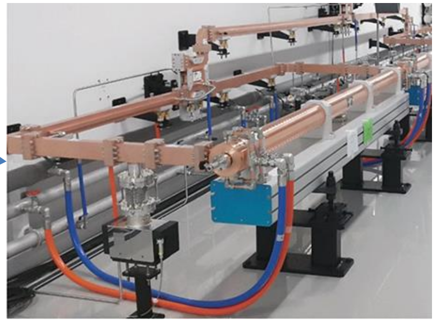
In general constant gradient structures are **more efficient** than constant impedance ones, because of the more uniform distribution of the RF power along them.

LINAC TECHNOLOGY: MATERIALS

ACCELERATING CAVITY TECHNOLOGY

⇒The cavities (and the related LINAC technology) can be of different material:

- copper for **normal conducting (NC, both SW than TW)** cavities;
- Niobium for **superconducting cavities (SC, SW)**;



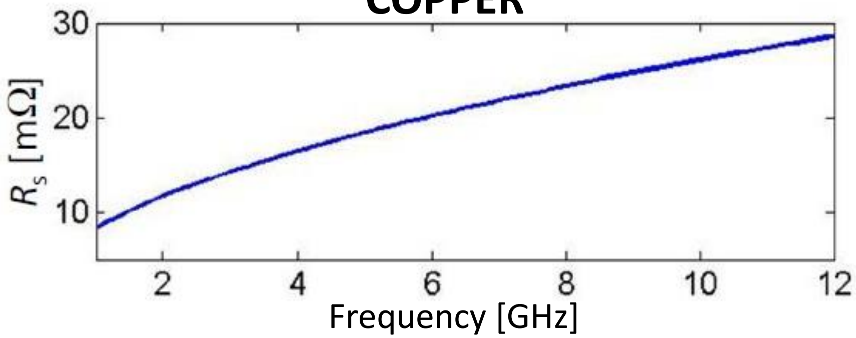
⇒We can choose between NC or the SC technology depending on the required performances in term of:

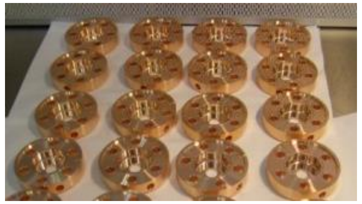
- **accelerating gradient** (MV/m);
- **RF pulse length** (how many bunches we can contemporary accelerate);
- **Duty cycle (see next slide)**: pulsed operation (i.e. 10-100 Hz) or continuous wave (CW) operation;
- **Average beam current**.
- …

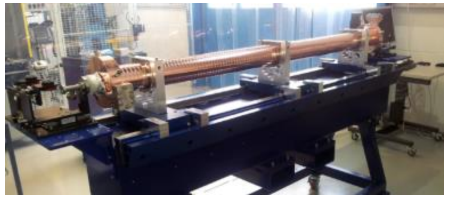
Dissipated power into the cavity walls is related to the surface currents

$$P_{diss} = \int_{\text{cavity wall}} \overbrace{\frac{1}{2} R_s H_{tan}^2}^{\text{power density}} dS$$

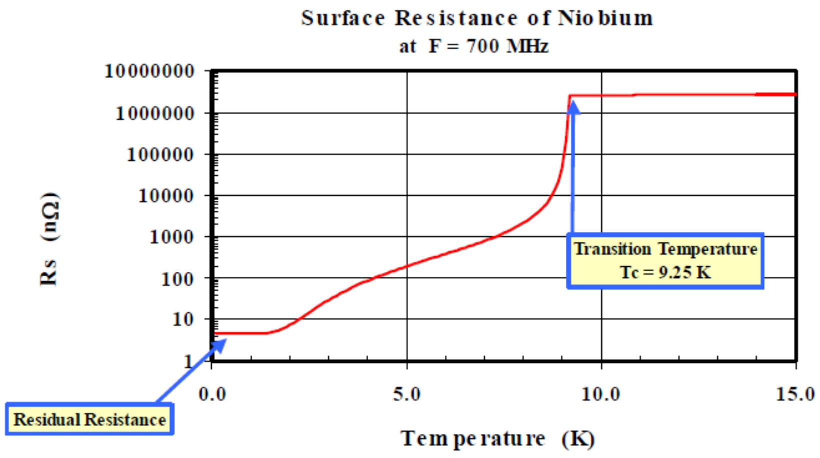
COPPER







NIOBIUM

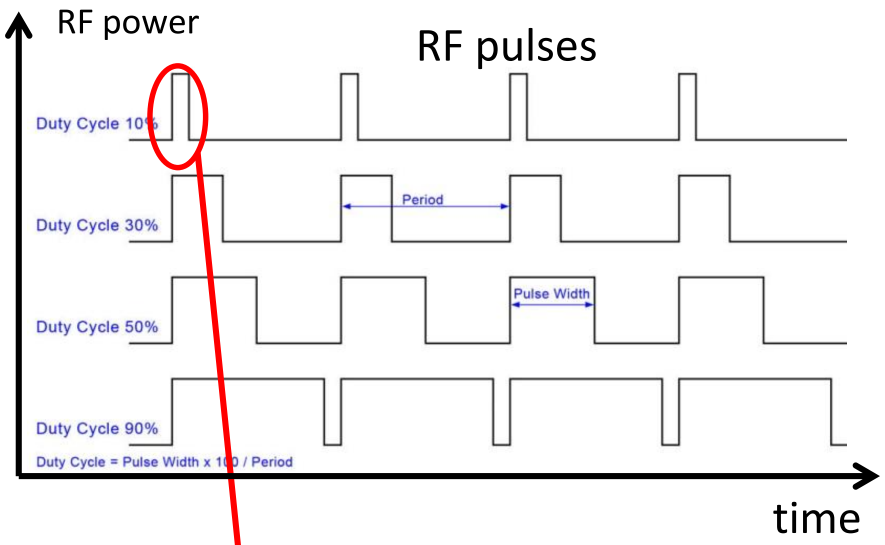


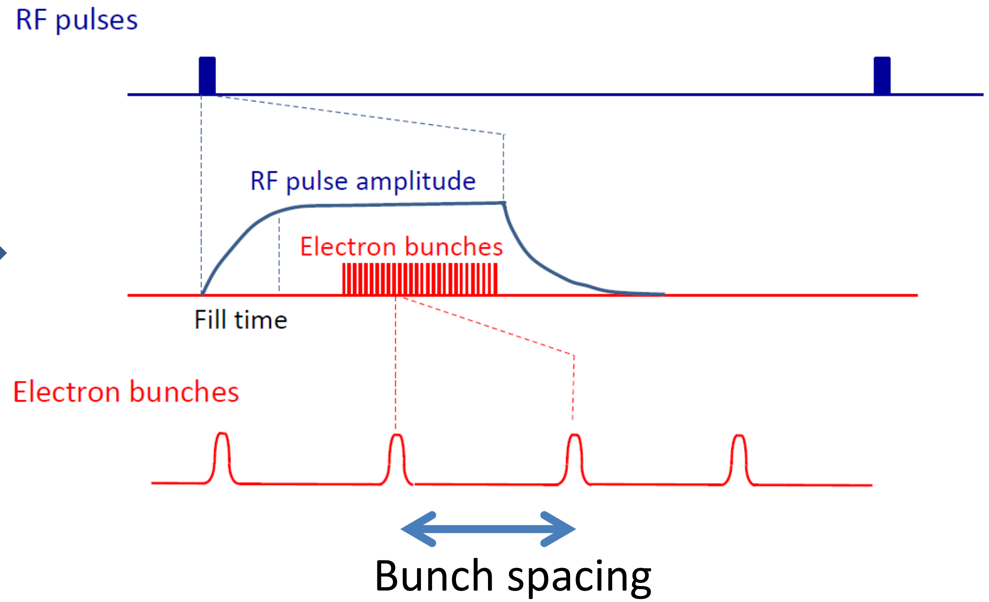
Between copper and Niobium there is a factor 10^5-10^6

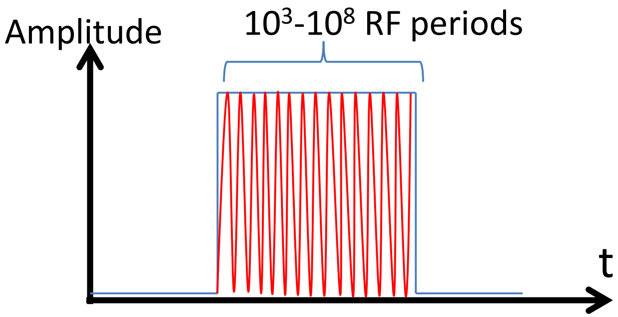
RF STRUCTURE AND BEAM STRUCTURE: NC vs SC

The “**beam structure**” in a LINAC is directly related to the “**RF structure**”. There are two possible type of operations:

- **CW** (Continuous Wave) operation ⇒ allow, in principle, to operate with a continuous (bunched) beam
- **PULSED** operation ⇒ there are RF pulses at a certain repetition rate (**Duty Cycle (DC)=pulsed width/period**)







⇒**SC structures allow operation at very high Duty Cycle (>1%) up to a CW operation (DC=100%)** (because of the extremely low dissipated power) **with relatively high gradient (>20 MV/m)**. This means that a continuous (bunched) beam can be accelerated.

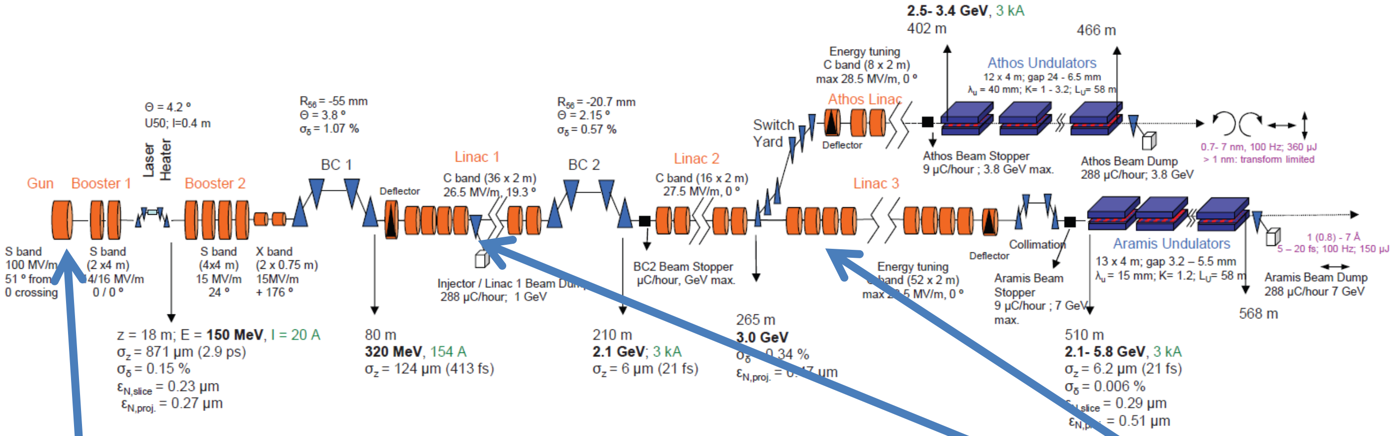
⇒**NC structures can operate in pulsed mode** at **very low DC** (10^{-2}-10^{-1} %) (because of the higher dissipated power) with, in principle, **larger peak accelerating gradient**(>30 MV/m). This means that one or few tens of bunches can be, in general, accelerated. NB: NC structures can also operate in CW but at very low gradient because of the dissipated power.

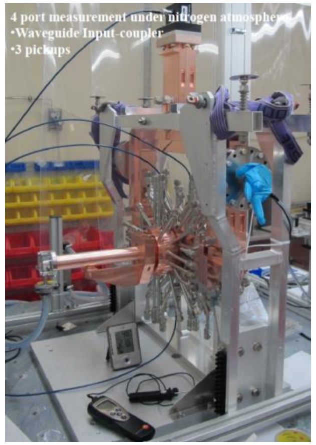
EXERCISE 9: π MODE STRUCTURES AND DUTY CYCLE

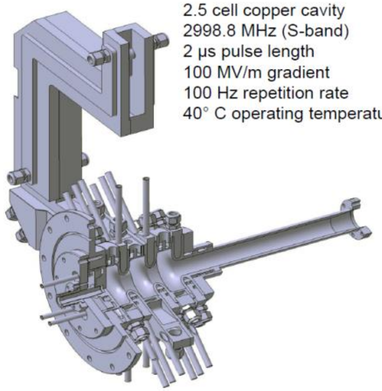
A multi cell SW cavity, operating on the π-mode at 1 GHz, accelerates protons at $\beta=0.5$. The cavity is a 9 cell structure. Assuming a negligible variation of the particle velocity through the cavity itself calculate:

1) the distance between the centers of the accelerating cells;
2) assuming a shunt impedance of the single cell (R) of 1 MΩ, calculate the dissipated power to have an effective accelerating voltage on the overall structure of V_{acc}=10 MV;
3) Calculate the average accelerating field;
4) If the cavity is fed by 4 μs rf pulses with a repetition rate of 100 Hz, calculate the Duty Cycle.

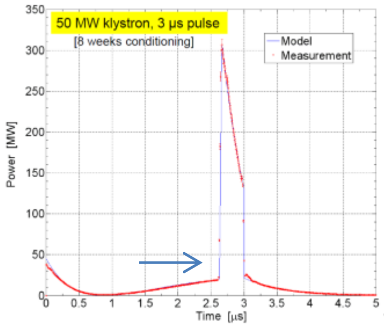
EXAMPLE: SWISSFEL LINAC (PSI)

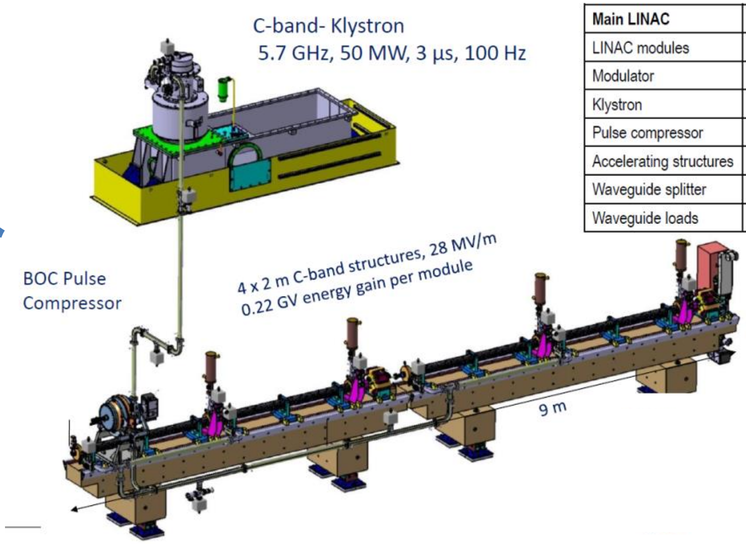




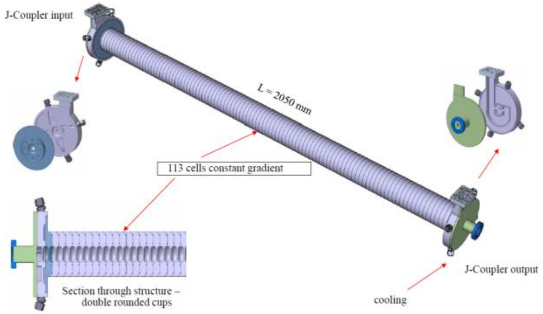


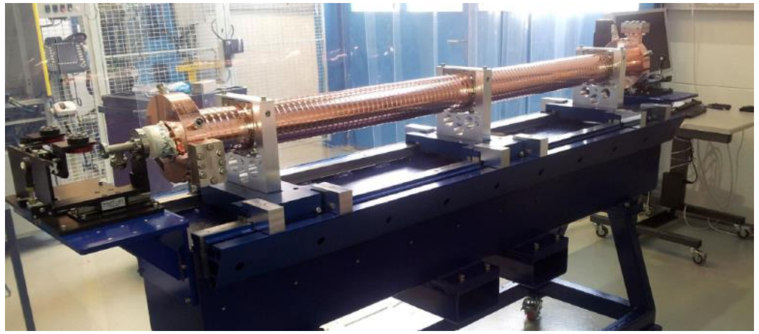
2.5 cell copper cavity
2998.8 MHz (S-band)
2 μs pulse length
100 MV/m gradient
100 Hz repetition rate
40° C operating temperature



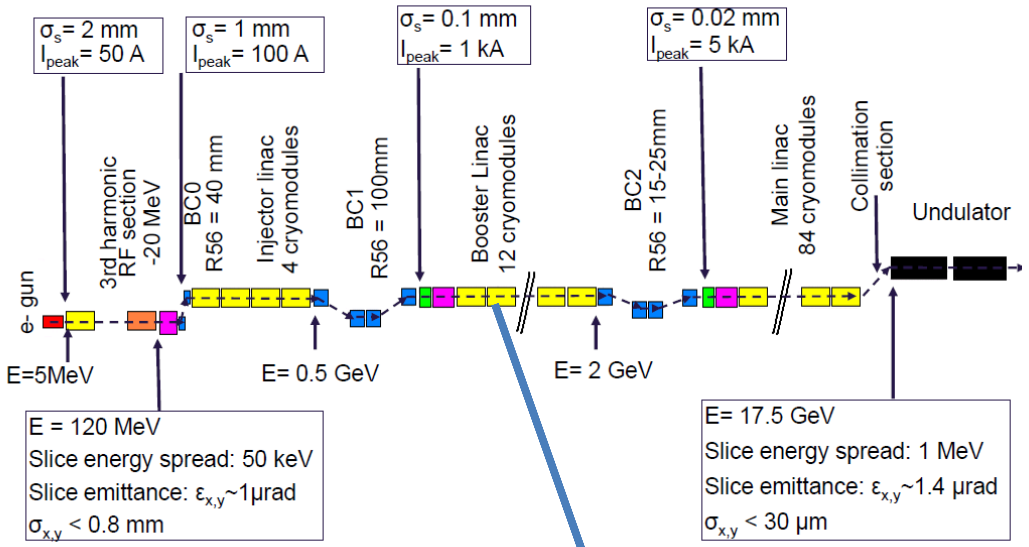


Main LINAC	#
LINAC modules	26
Modulator	26
Klystron	26
Pulse compressor	26
Accelerating structures	104
Waveguide splitter	78
Waveguide loads	104

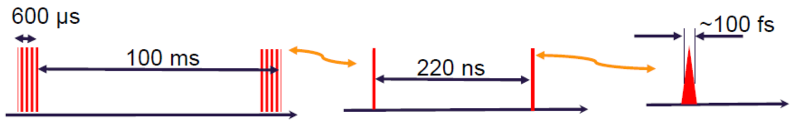




EXAMPLES: EUROPEAN XFEL

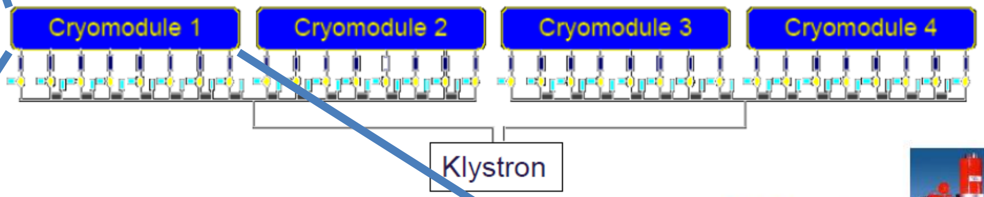


Nominal Energy	GeV	17.5
Beam pulse length	ms	0.60
Repetition rate	Hz	10
Max. # of bunches per pulse		2700
Min. bunch spacing	ns	220
Bunch charge	nC	1
Bunch length, σ_z	µm	< 20
Emittance (slice) at undulator	µrad	< 1.4
Energy spread (slice) at undulator	MeV	1

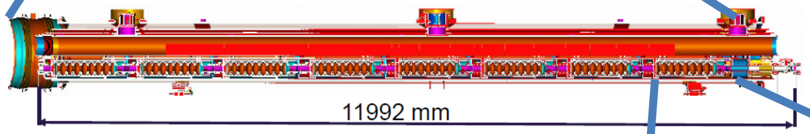


101 cryomodules in total

RF- system: 25 RF units. The unit = 4 cryomodules + RF-power source (klystron)



Cryomodule housing: 8 cavities, quadrupole and BPM



25 RF stations
5.2 MW each

800 accelerating cavities

1.3 GHz / 23.6 MV/m