Linear Imperfections and Correction

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Organisational matters

- **We will have 3 lectures (2 + 3 + 3 hours), i.e., today and in the next two days**
- **During the lectures, you will have to solve 5 exercises**
  - We will give you ~20 minutes per exercise to work **by yourself or with your neighbors**
  - After each exercise, we will kind ask **one of you** to **show the solution** on the whiteboard (~10 minutes)
- **Please, feel free to interrupt at anytime if something is not clear**

- PS: On the indico page, you can also find a small python notebook used to produce some of the plots presented in this lecture
  - Feel free to use it in your spare time if you think it can be helpful… or use MAD-X!

Outline

Introduction

Closed orbit distortion (steering error)

- ❑ Beam orbit stability
- ❑ Imperfections leading to closed orbit distortion
- ❑ Effect of single and multiple dipole kicks
- ❑ Closed orbit correction methods

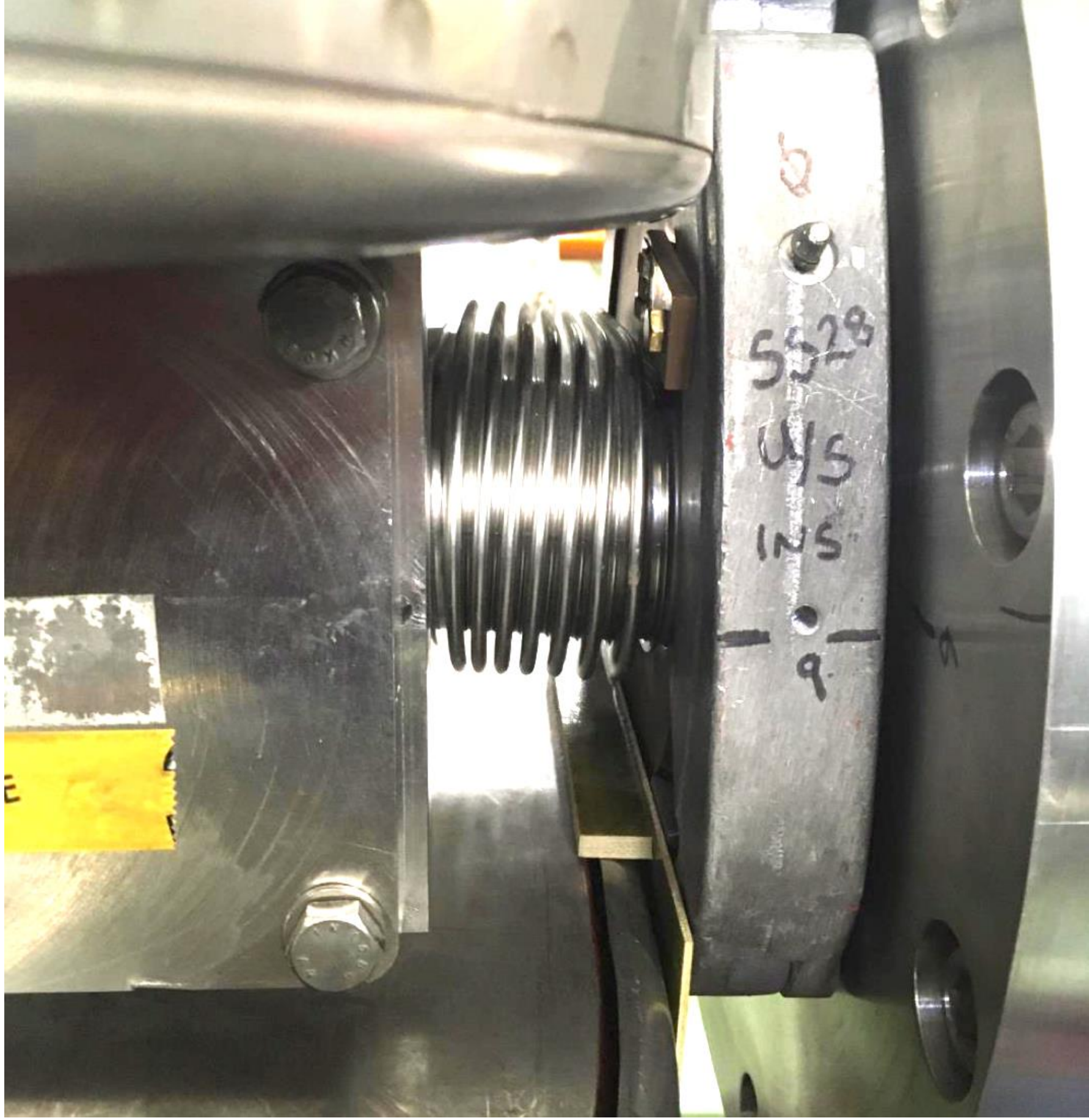
Optics function distortion (gradient error)

- ❑ Imperfections leading to optics distortion
- ❑ Tune-shift and beta distortion due to gradient errors
- ❑ Gradient error correction

Coupling error

- ❑ Coupling errors and their effect
- ❑ Coupling correction

Summary



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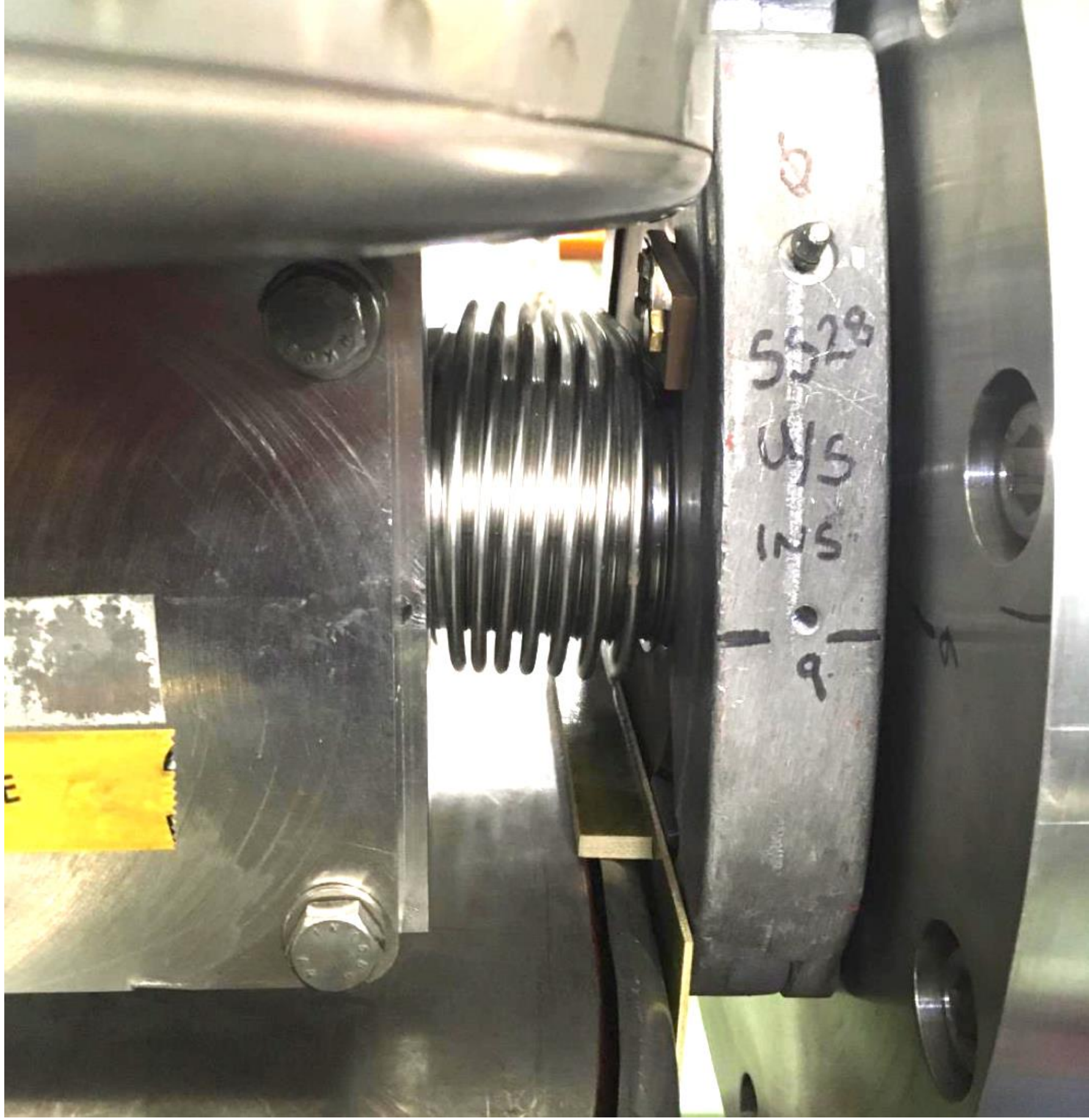


Pre-word

- **We will be discussing *linear imperfections* and their impact on *circular accelerators* ("rings")**
- **These imperfections affect most importantly the *transverse dynamics* of the particle motion**
- **In most parts of the lecture, we will be using normalized magnet strength, i.e. magnetic field B derivatives normalized by the magnetic rigidity *B*ρ**
  - *Remember:* if particle energy, *E*, is given in GeV, and *q* is the particle charge, then the magnetic rigidity in Tesla-meter is :

$$B\rho\,[\mathrm{Tm}] = 3.3356\,\beta_r E\,[\mathrm{GeV}]/q$$

- **Selection of symbols used in this course:**

x, y … denoting the horizontal and vertical plane, respectively

u … denoting either horizontal or vertical plane

ψ … denoting the betatron phase advance in rad

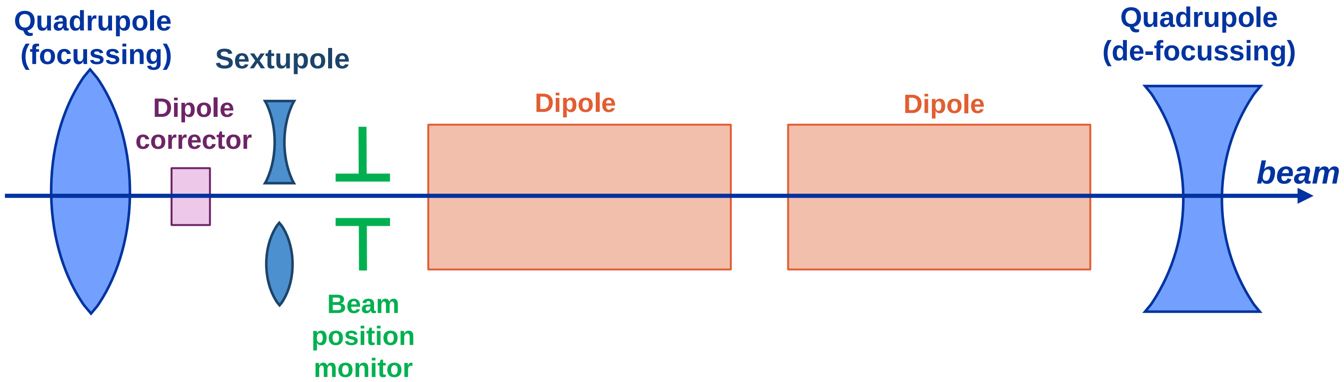
Q … tune in 2π units

C … circumference

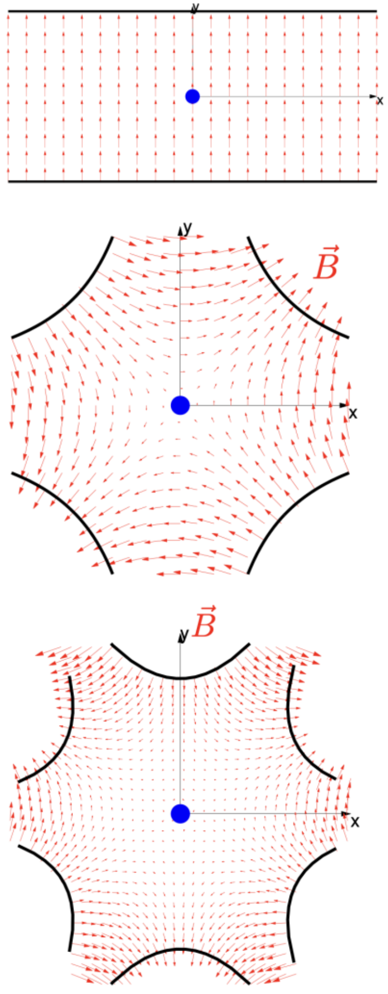
Basics: accelerator lattice

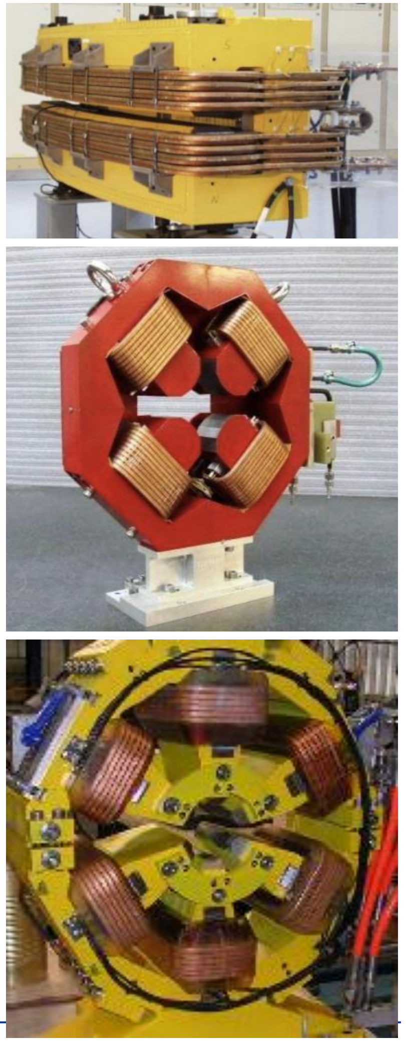
- **An accelerator is usually build using a repetition of basic ‘cells’**

- **A simple <u>FODO cell</u> usually contains:**
  - **Dipole** magnets to bend the beams
  - **Quadrupole** magnets to focus the beams
  - Beam position monitors (**BPM**) to measure the beam position
  - Small dipole **corrector** magnets for beam steering
  - (**Sextupole** magnets to control off-energy focusing)



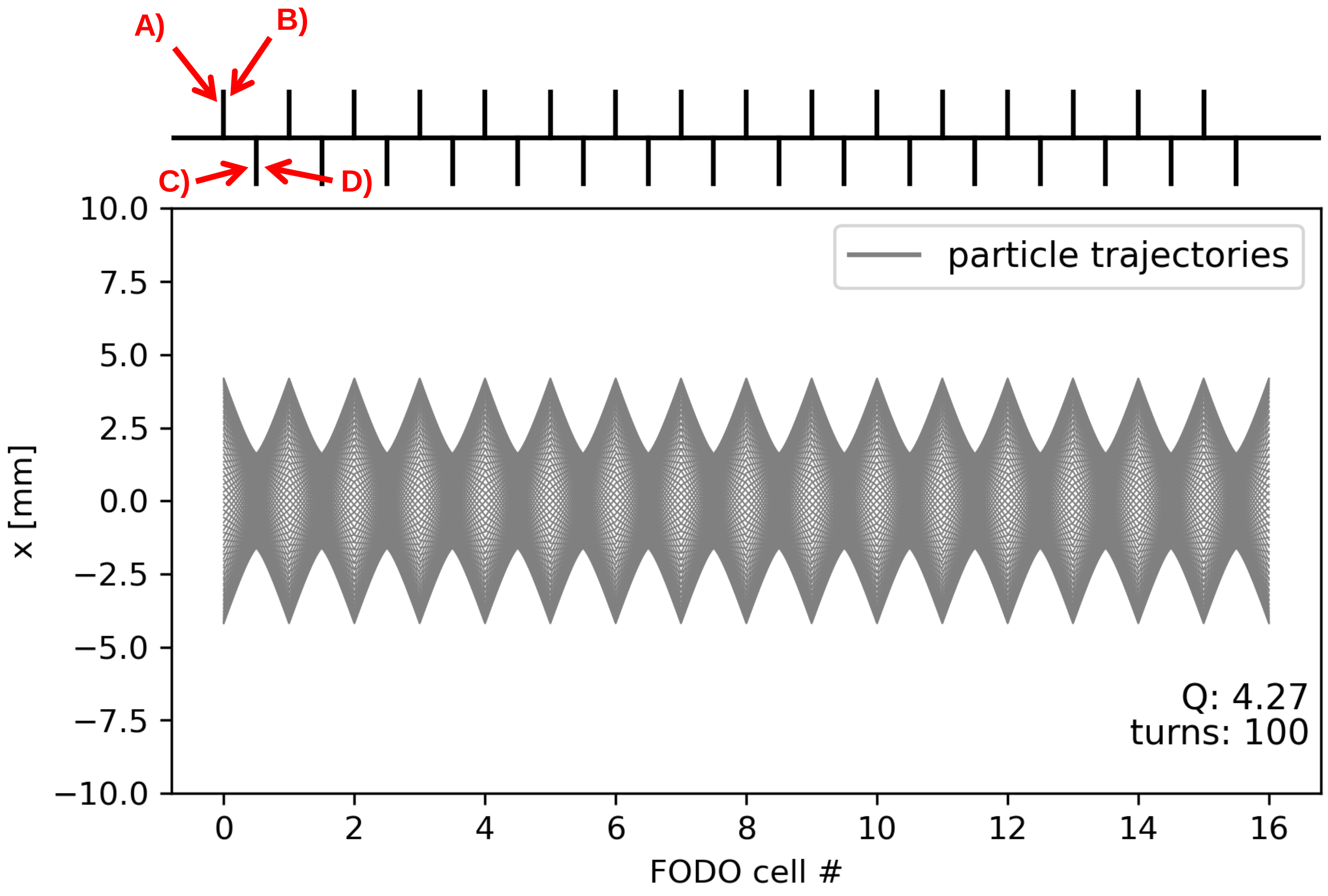
Schematic of a ½ cell (not to scale)

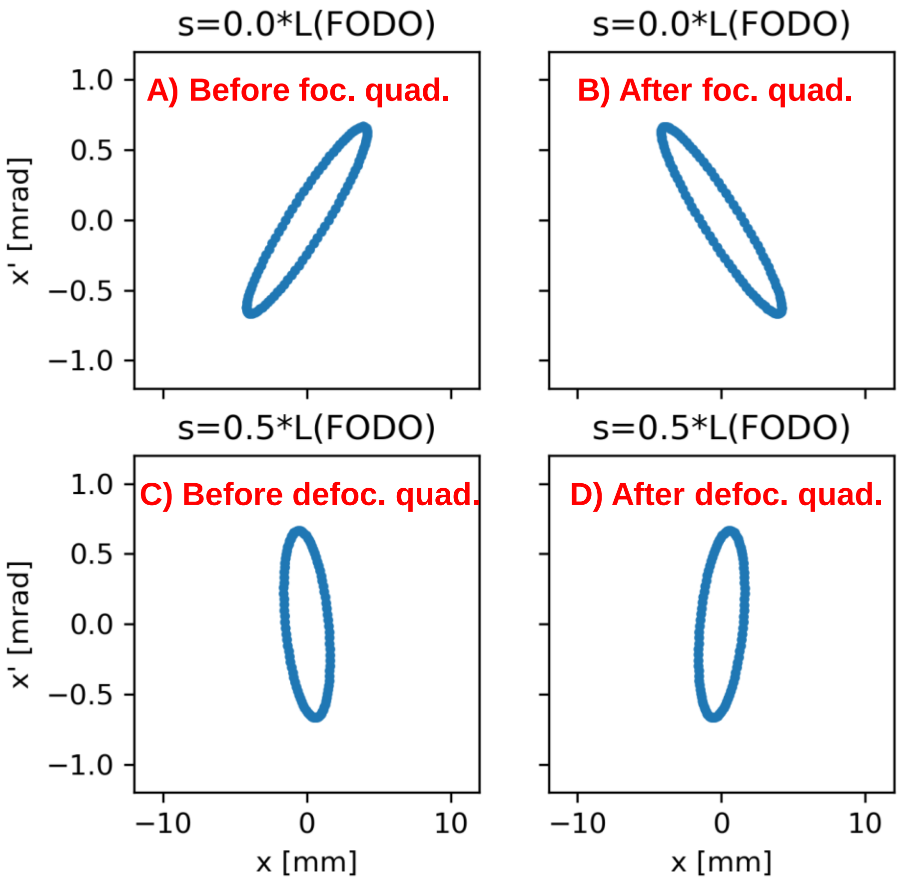




Toy FODO lattice – no errors

- Tracking a single particle, for many turns





From model to reality - fields

- The physical units of the machine model defined by the accelerator physicist must be converted into magnetic fields and eventually into currents for the power converters that feed the magnet circuits.
- **Imperfections** (= errors) in the real accelerator optics can be introduced by uncertainties or errors on:
  - Actual **beam momentum**, **magnet calibration** and **hysteresis, power converter regulation, …**

Magnet strength → **Magnetic field (gradient)** → **Requested current** → **Actual magnet current** ⇢ **Actual field seen by the beam**

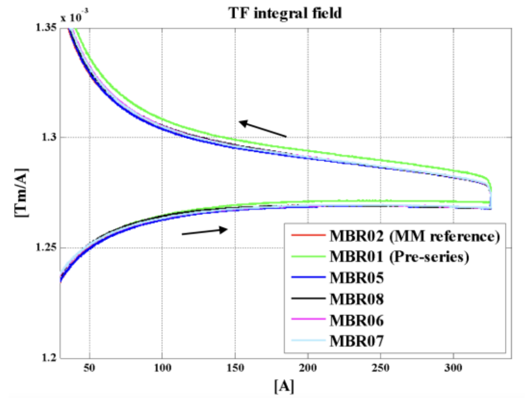
Beam momentum

Magnet Calibration/hysteresis curve **(transfer function)**

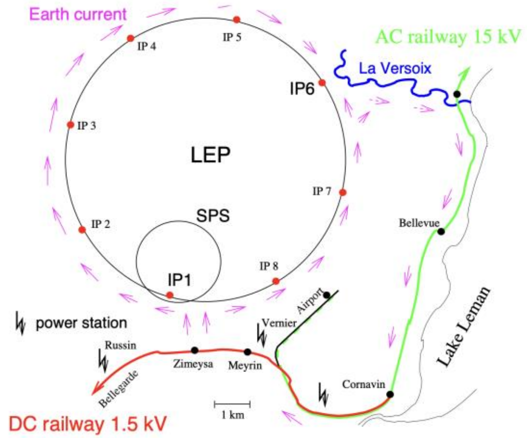
Power converter

Additional effects, e.g. eddy currents

Example of the ELENA main dipoles hysteresis curve

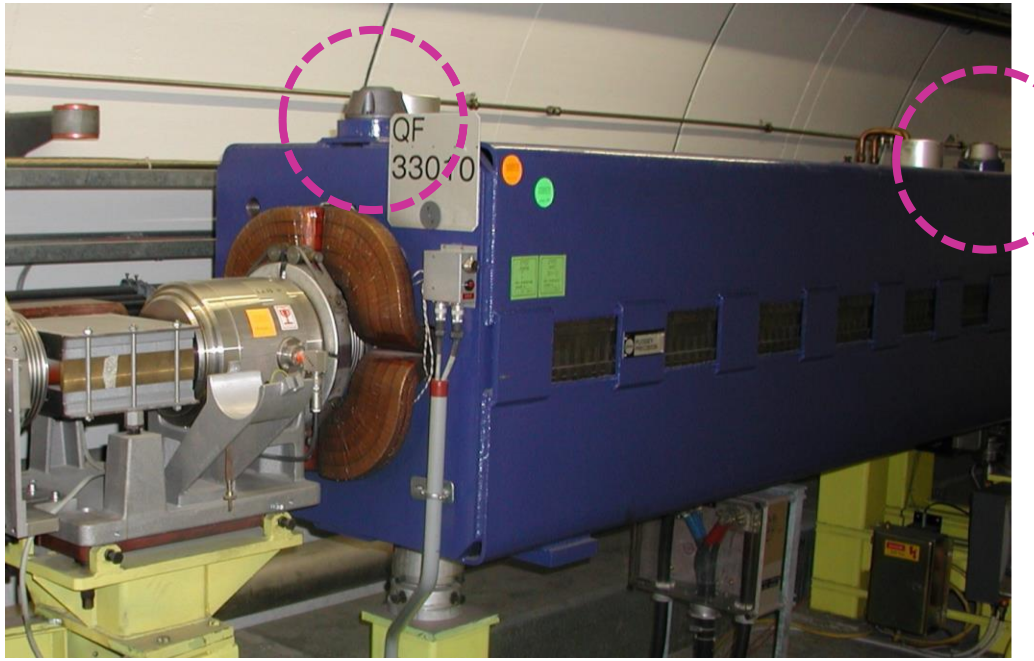


Earth currents flowing over the LEP vacuum chamber that were generated by the DC railway line near Geneva



From model to reality - alignment

- To ensure that the accelerator elements are in the correct position the alignment must be precise – to the level of nanometer for CLIC final focusing !
  - For CERN hadron machines we aim for accuracies of around 0.1 mm.
- The alignment process implies:
  - Precise measurements of the magnetic axis in the laboratory with reference to the element alignment markers used by the survey group.
  - Precise in-situ alignment (position and angle) of the element in the tunnel.
- **Alignment errors are a common source of imperfections**



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Summary

Illustration of closed orbit distortion

1. **Ideal machine toy model (no errors)**
   a) Particle injected on the design (or reference) orbit … remains on the design orbit turn after turn

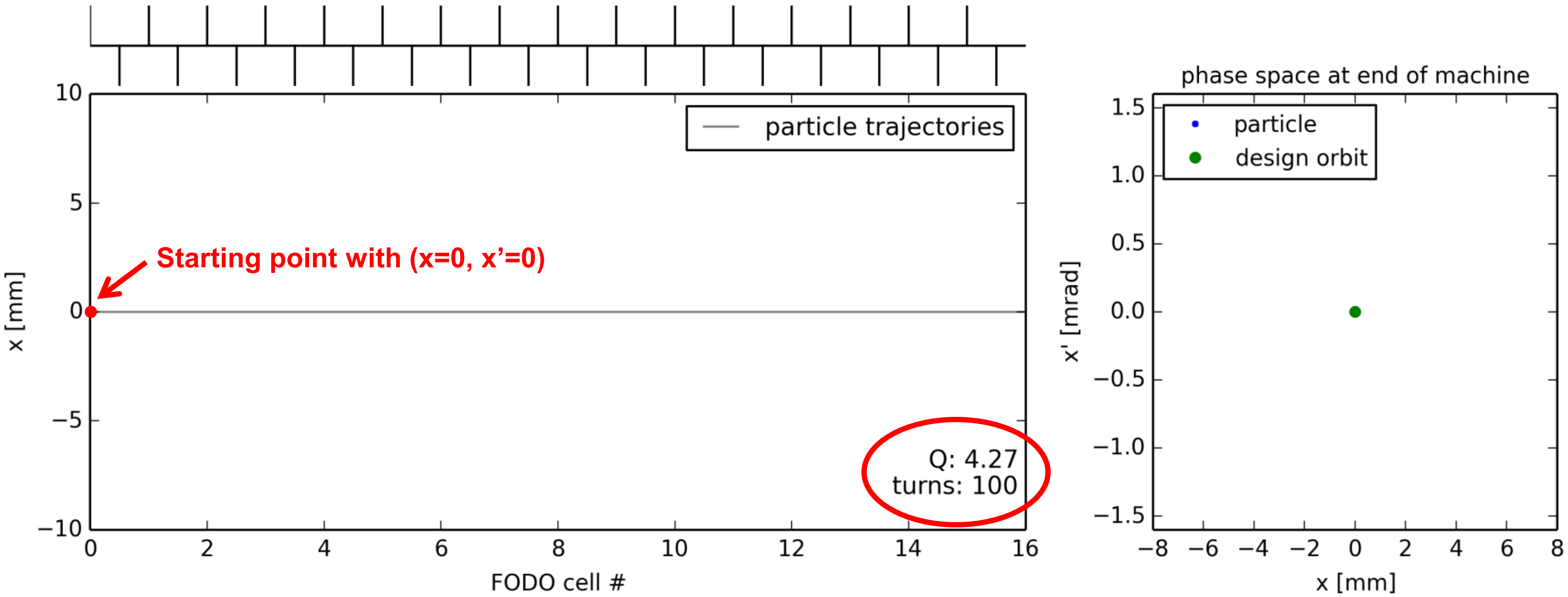
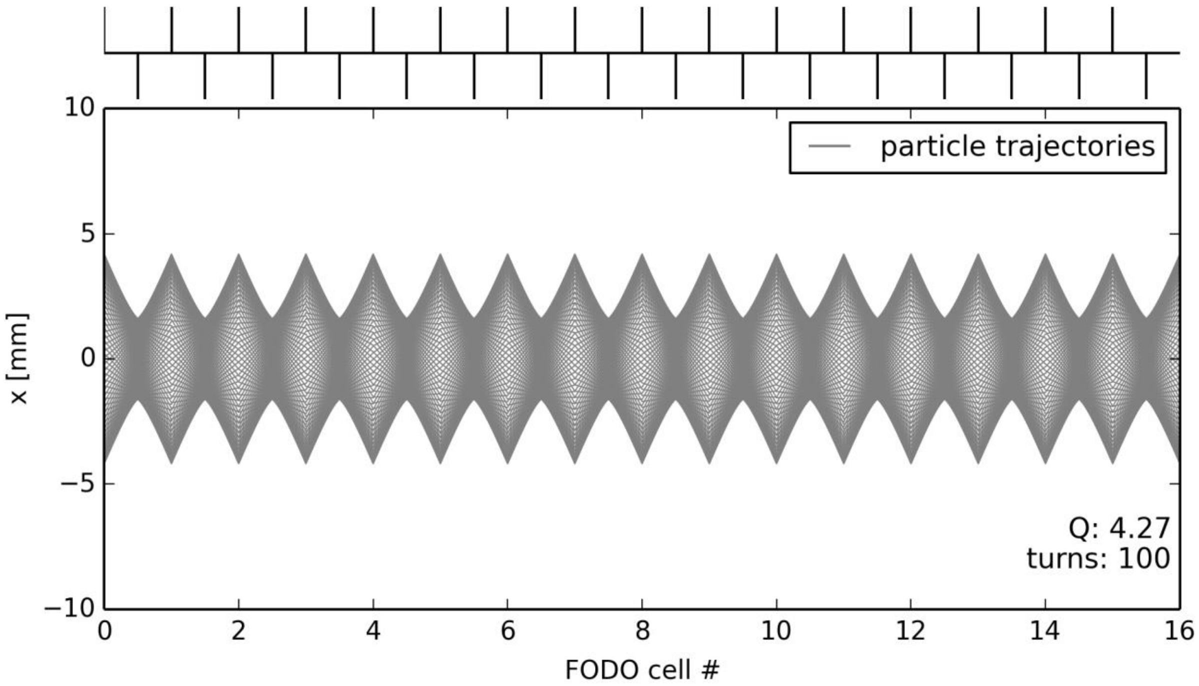


Illustration of closed orbit distortion

1. **Ideal machine toy model (no errors)**
   a) Particle injected on the design (or reference) orbit … remains on the design orbit turn after turn
   b) Particle injected with offset …



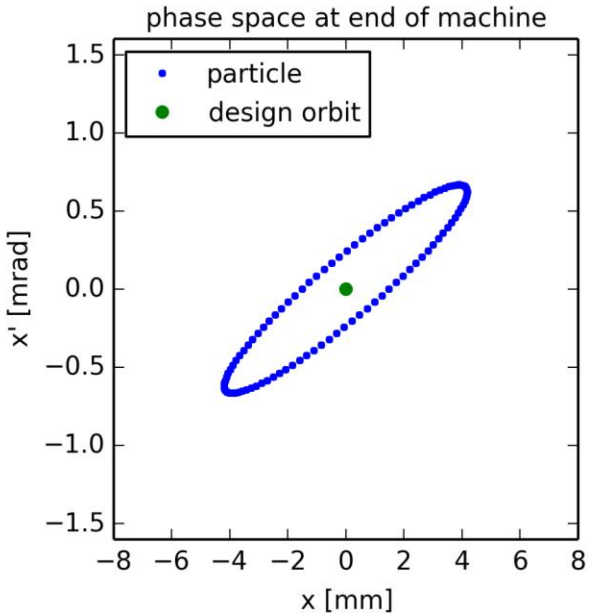


Illustration of closed orbit distortion

1. **Ideal machine toy model (no errors)**
   a) Particle injected on the design (or reference) orbit … remains on the design orbit turn after turn
   b) Particle injected with offset … performs betatron oscillations around the **closed orbit** which is the same as the design orbit as long as there are no imperfections

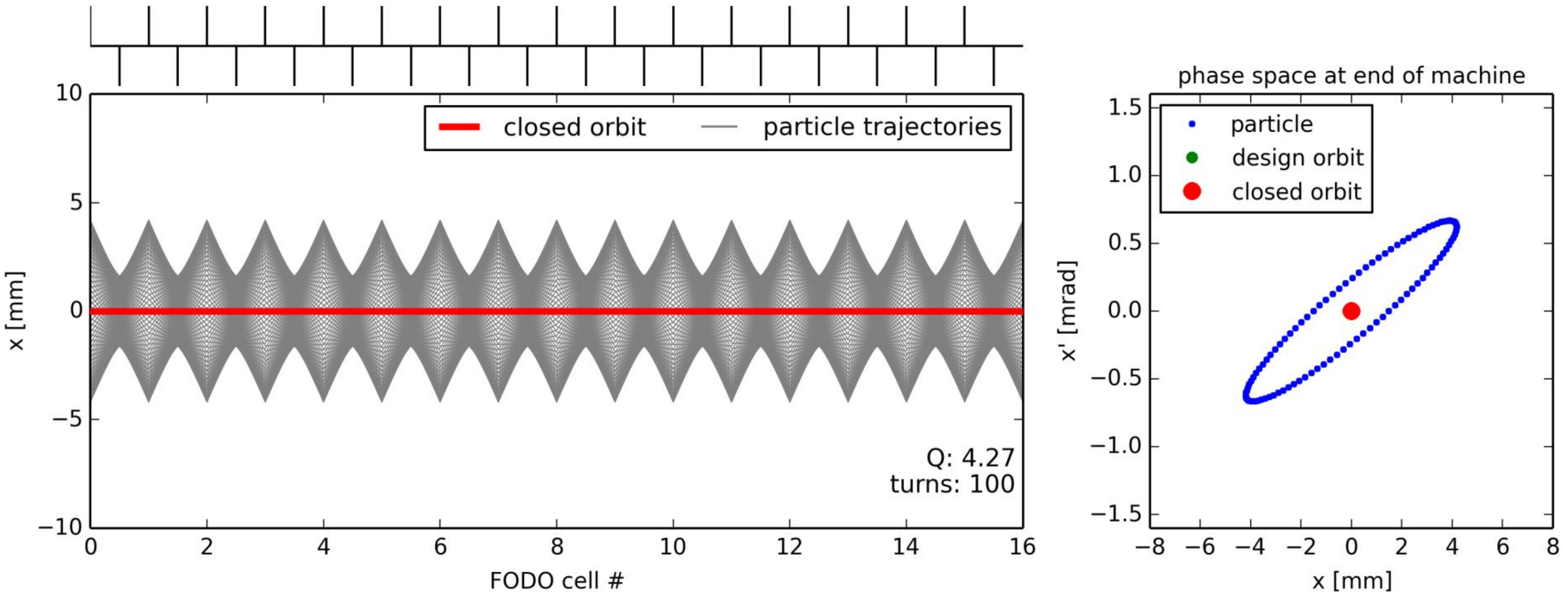


Illustration of closed orbit distortion

2. **Ideal machine toy model with dipole error (unintended deflection) somewhere**
    a) Particle injected on the design orbit … receives dipole kick every turn

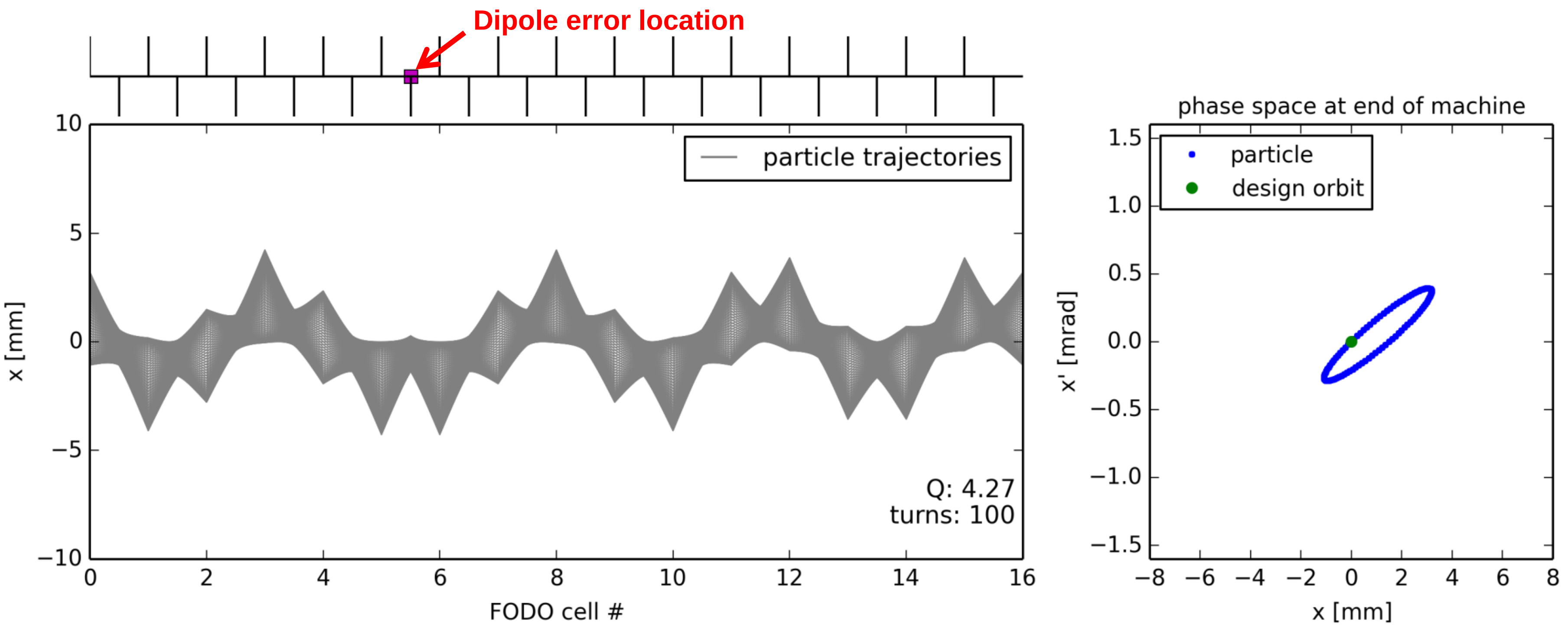


Illustration of closed orbit distortion

2. **Ideal machine toy model with dipole error (unintended deflection) somewhere**

   a) Particle injected on the design orbit … receives dipole kick every turn … and consequently performs betatron oscillation around a **distorted closed orbit**

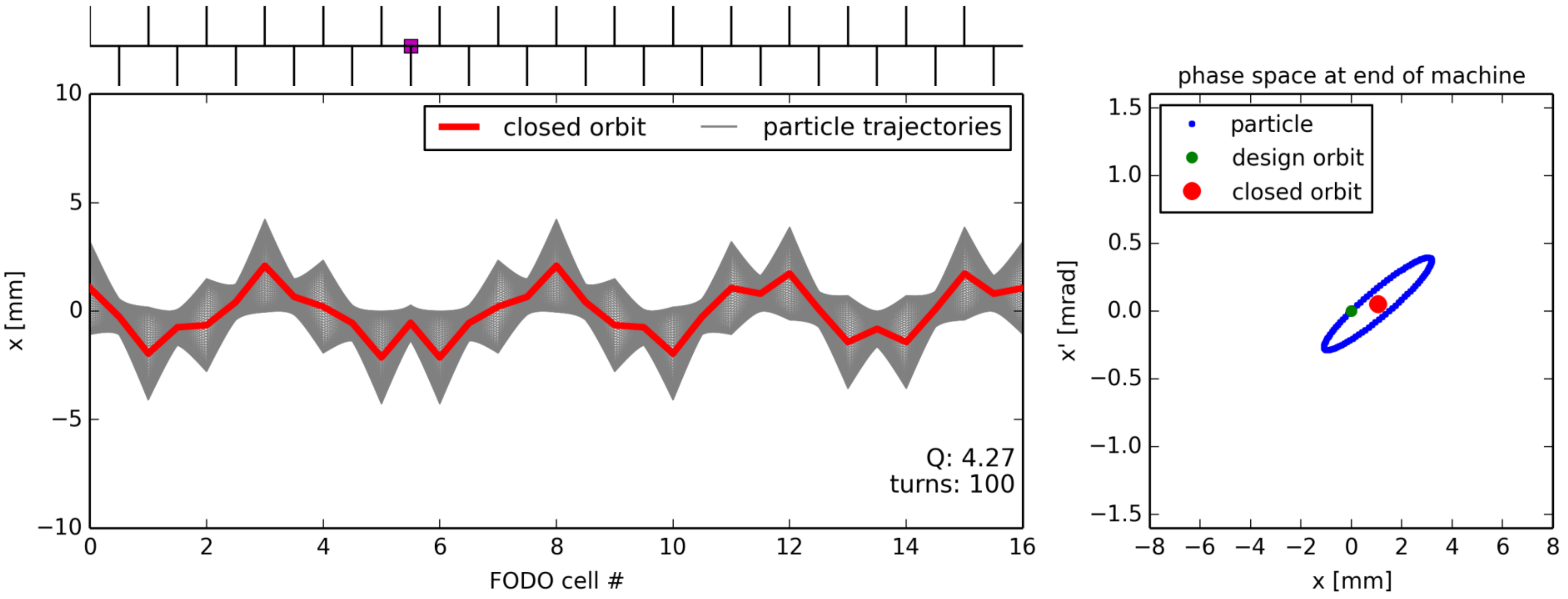
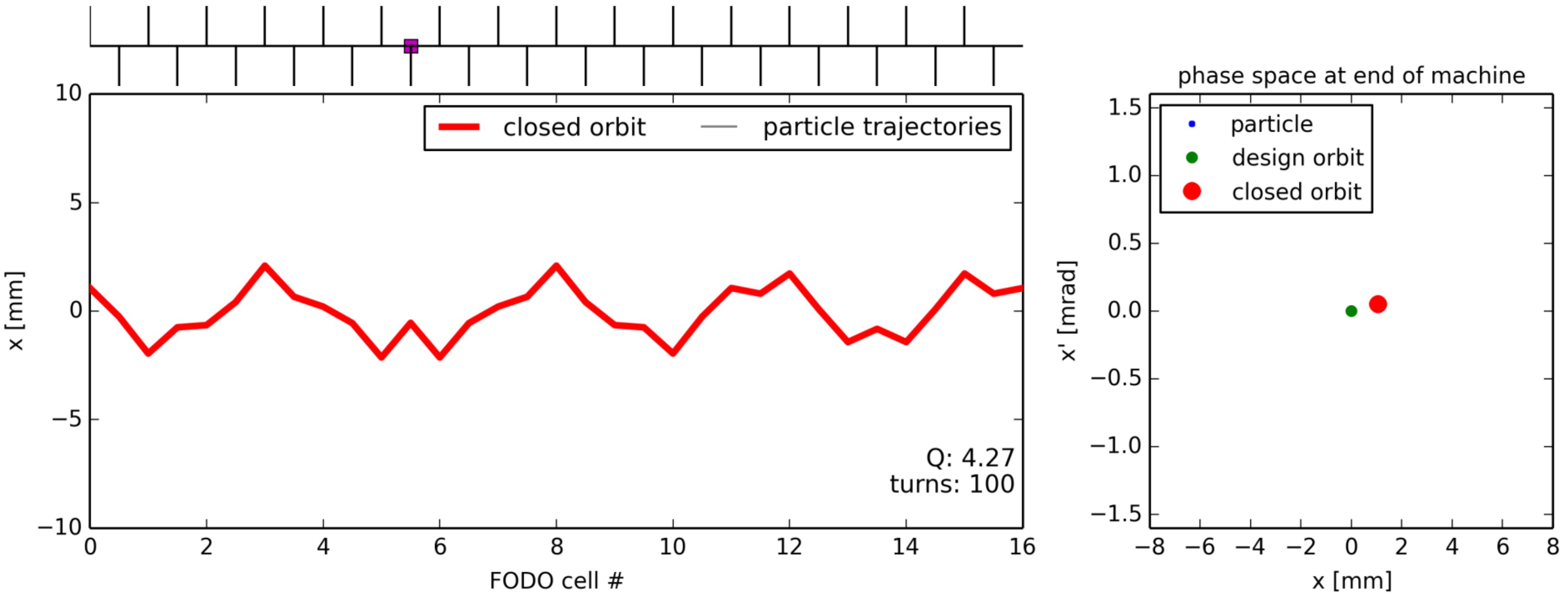


Illustration of closed orbit distortion

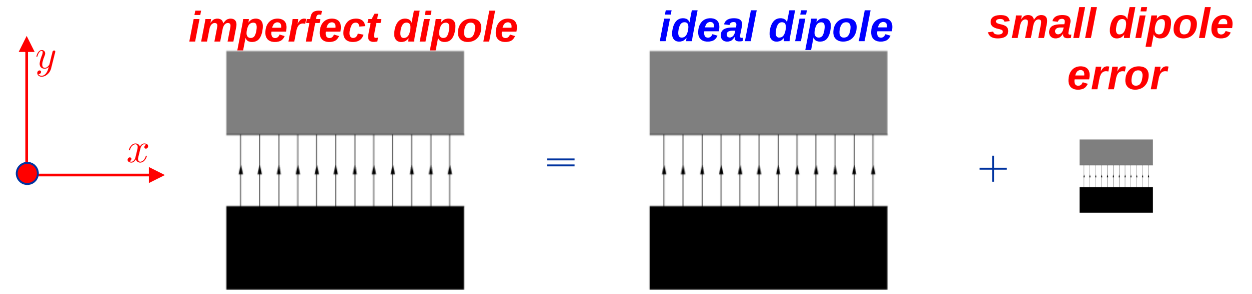
2. **Ideal machine toy model with dipole error (unintended deflection) somewhere**
   a) Particle injected on the design orbit … receives dipole kick every turn … and consequently performs betatron oscillation around a **distorted closed orbit**
   b) Particle injected onto distorted closed orbit remains on closed orbit



Sources of unintended deflections

Field error (deflection error) of a <u>dipole magnet</u>

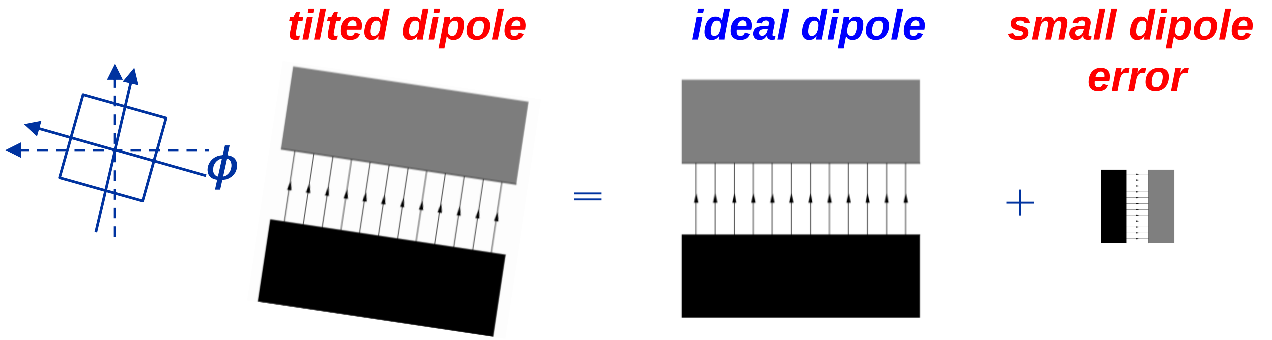
- This can be due to an **error** in the **magnet current** or in the **calibration table** (measurement accuracy etc.)
- The imperfect dipole can be expressed as the ideal one + a small error



→ horizontal kick

$$\theta = \frac{\delta(Bl)}{B\rho}$$

A small **rotation (misalignment)** of a <u>dipole magnet</u> has the same effect, but (mostly) in the "other" plane



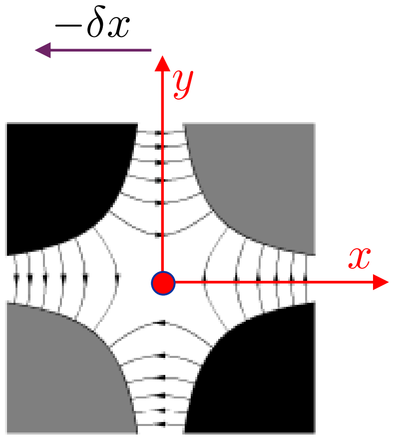
→ vertical kick

$$\theta = \frac{Bl\sin\phi}{B\rho}$$

Misalignments causing feed-down

Horizontal misalignment of a quadrupole magnet

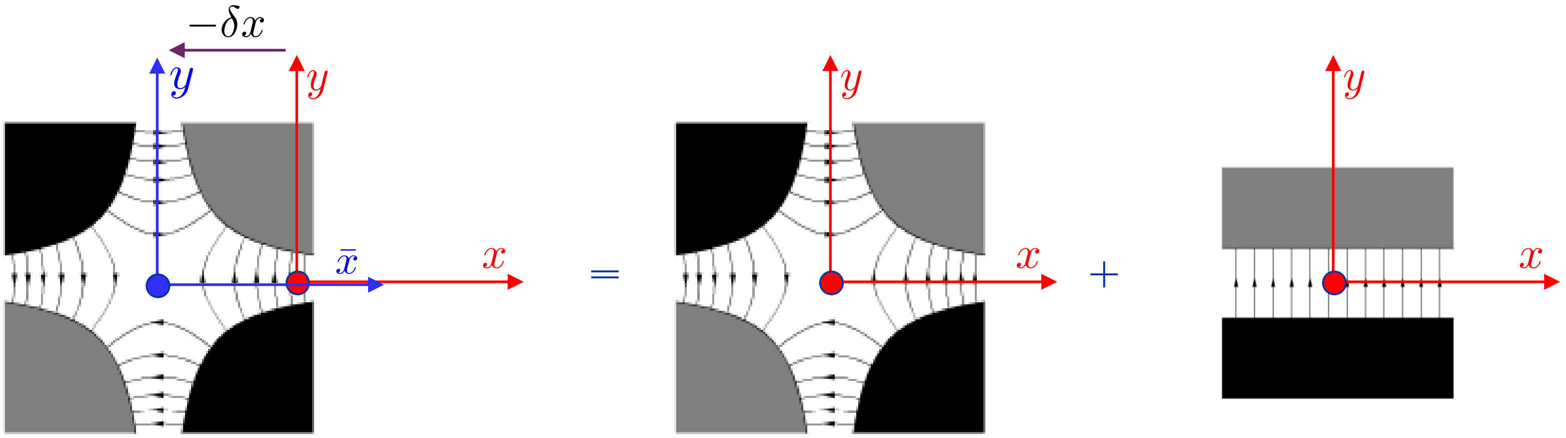
- Equivalent to perfectly aligned quadrupole plus small dipole



Misalignments causing feed-down

Horizontal misalignment of a quadrupole magnet

- Equivalent to perfectly aligned quadrupole plus small dipole



$$B_x(\bar{x}, y) = B_x(x + \delta x, y) = G(y) = \underbrace{Gy}_{\text{quadrupole}}$$

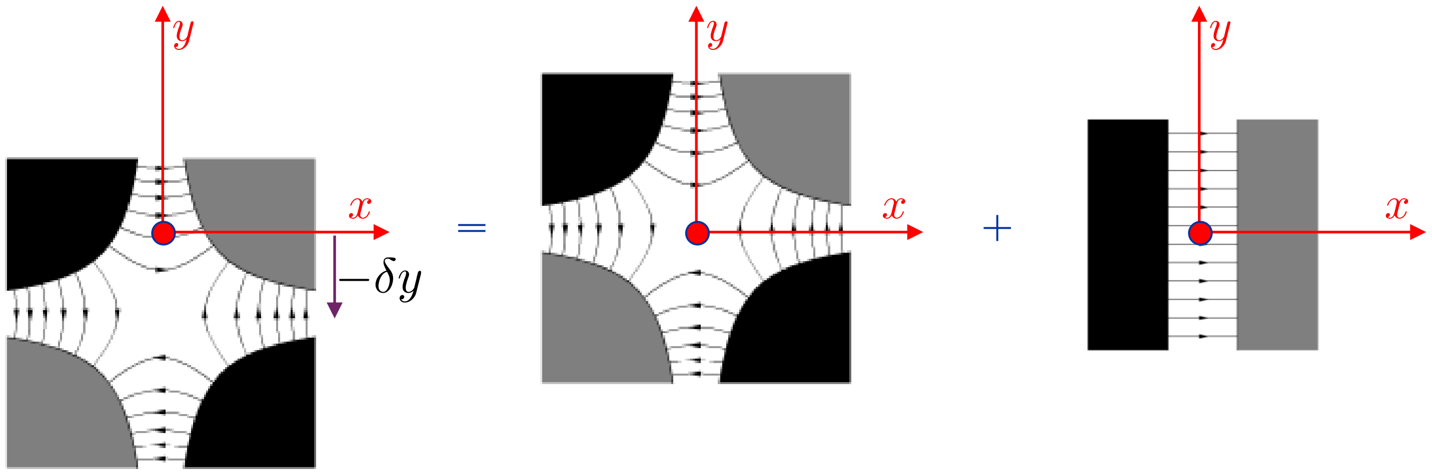
$$B_y(\bar{x}, y) = B_y(x + \delta x, y) = G(x + \delta x) = \overbrace{Gx}^{\text{quadrupole}} + \overbrace{G\delta x}^{\text{dipole}}$$

horizontal offset creates horizontal (normal) dipole

Misalignments causing feed-down

Vertical misalignment of a **quadrupole** magnet

- Equivalent to perfectly aligned quadrupole plus small dipole



$$B_x(x,y+\delta y) = G(y+\delta y) = \underbrace{Gy}_{\text{quadrupole}} + \underbrace{G\delta y}_{\text{dipole}}$$

$$B_y(x,y+\delta y) = G(x) \qquad = \overbrace{Gx}$$

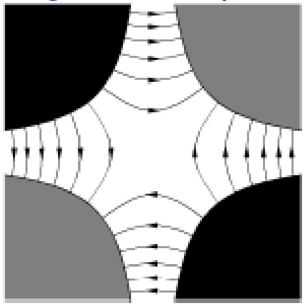
vertical offset creates vertical (skew) dipole

Interlude: multipole expansion

Multipole expansion of transverse magnetic field

- Start from the general expression for the transverse magnetic flux in terms of multipole coefficients

e.g. normal quad



$$\mathbf{B} = B_y + iB_x = \sum_{n=0}^{\infty} (B_n + iA_n) \cdot (x+iy)^n$$

Normal components

("upright" magnets)

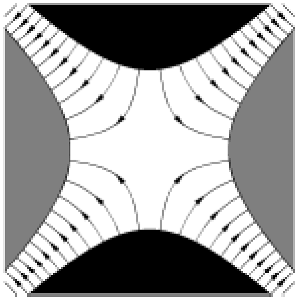
$$B_n = \frac{1}{n!} \frac{\partial^n B_y}{\partial x^n}\Big|_{(0,0)}$$

Skew components

(magnets rotated by $\frac{\pi}{2(n+1)}$)

$$A_n = \frac{1}{n!} \frac{\partial^n B_x}{\partial y^n}\Big|_{(0,0)}$$

e.g. skew quad



- In some cases, it is more convenient to use "**normalized**" components:

Normalized normal components

Normalized skew components

so that:

$$k_n = \frac{1}{B_0\rho_0} \frac{\partial^n B_y}{\partial x^n}\Big|_{(0,0)} = \frac{n!}{B_0\rho_0} B_n\Big|_{(0,0)}$$

$$j_n = \frac{1}{B_0\rho_0} \frac{\partial^n B_x}{\partial y^n}\Big|_{(0,0)} = \frac{n!}{B_0\rho_0} A_n\Big|_{(0,0)}$$

Beam rigidity

$$B_y + iB_x = B_0\rho_0 \sum_{n=0}^{M} (k_n + ij_n) \frac{(x+iy)^n}{n!}$$

Feed-down from multipoles

- Let's **explicitly** write, for example, the **vertical field** as the **sum of all multipole components**

normal skew

$$B_y = \underbrace{B_0}_{\text{dipole}} + \underbrace{B_1 x - A_1 y}_{\text{quadrupole}} + \underbrace{B_2(x^2 - y^2) - 2A_2 xy}_{\text{sextupole}} + \underbrace{B_3(x^3 - 3xy^2) - A_3(-y^3 + 3x^2 y)}_{\text{octupole}} + \dots$$

$$B_x = \overbrace{A_0}^{\text{dipole}} + \overbrace{A_1 x + B_1 y}^{\text{quadrupole}} + \overbrace{A_2(x^2 - y^2) + 2B_2 xy}^{\text{sextupole}} + \overbrace{A_3(x^3 - 3xy^2) + B_3(-y^3 + 3x^2 y)}^{\text{octupole}} + \dots$$

- A **horizontal offset** (-δx) in a normal(skew) magnet of order *n* creates normal(skew) **feed-down components** at y=0 **of all lower orders!**:

(normal)

$$\begin{cases} B_x(y=0) = 0 \\ B_y(y=0) = \underbrace{B_n \bar{x}^n}_{2(n+1)\text{-pole}} = B_n(x+\delta x)^n = B_n\Big(\underbrace{x^n}_{2(n+1)\text{-pole}} + \underbrace{n\delta x x^{n-1}}_{2n\text{-pole}} + \underbrace{\frac{n(n-1)}{2}\delta x^2 x^{n-2}}_{2(n-1)\text{-pole}} + \dots + \underbrace{(\delta x)^n}_{\text{dipole}}\Big) \end{cases}$$

(skew)

$$\begin{cases} B_x(y=0) = \overbrace{A_n \bar{x}^n}^{2(n+1)\text{-pole}} = A_n(x+\delta x)^n = A_n\Big(\overbrace{x^n}^{2(n+1)\text{-pole}} + \overbrace{n\delta x x^{n-1}}^{2n\text{-pole}} + \overbrace{\frac{n(n-1)}{2}\delta x^2 x^{n-2}}^{2(n-1)\text{-pole}} + \dots + \overbrace{(\delta x)^n}^{\text{dipole}}\Big) \\ B_y(y=0) = 0 \end{cases}$$

Feed-down from multipoles

- Let's **explicitly** write, for example, the **vertical field** as the **sum of all multipole components**

normal skew

$$B_y = B_0 + B_1 x - A_1 y + B_2(x^2 - y^2) - 2A_2 xy + B_3(x^3 - 3xy^2) - A_3(-y^3 + 3x^2 y) + \dots$$

dipole quadrupole sextupole octupole

$$B_x = A_0 + A_1 x + B_1 y + A_2(x^2 - y^2) + 2B_2 xy + A_3(x^3 - 3xy^2) + B_3(-y^3 + 3x^2 y) + \dots$$

- A **vertical offset** (-δy) in normal(skew) magnets of order *n* results in **alternating** skew(normal) and normal(skew) **feed-down components** for x=0 **of all lower orders!**, as can be worked out looking at n-order terms are defined (for x=0):

for n = even $\begin{cases} B_y(x{=}0) = i^n B_n \bar{y}^n \\ B_x(x{=}0) = i^n A_n \bar{y}^n \end{cases}$ for n = odd $\begin{cases} B_y(x{=}0) = i^{n+1} A_n \bar{y}^n \\ B_x(x{=}0) = i^{n-1} B_n \bar{y}^n \end{cases}$

Odd exponent, i.e. it must be a skew component

- E.g. **n even**, **normal** magnet: normal

$$B_y(x{=}0) = i^n B_n (y + \delta y)^n = i^n B_n \left(y^n + n\delta y y^{n-1} + \frac{n(n-1)}{2}\delta y^2 y^{n-2} + \dots + (\delta y)^n\right)$$

Even exponent, i.e. a normal component again …

Problem 1

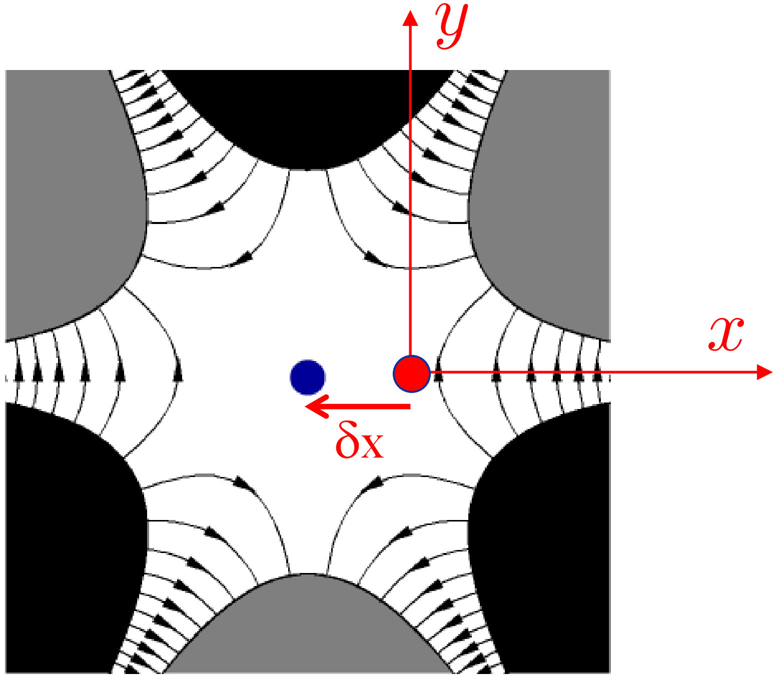
1. Derive an expression for the resulting magnetic field components (B_x and B_y) when the closed orbit in a normal **sextupole** is horizontally displaced by **-δx** from its reference position.
2. Do the same for an **octupole**.

The field generated by a sextupole is

$$\begin{cases} B_y(x,y) = B_2(x^2 - y^2) \\ B_x(x,y) = B_2(2xy) \end{cases}$$

The field generated by an octupole is

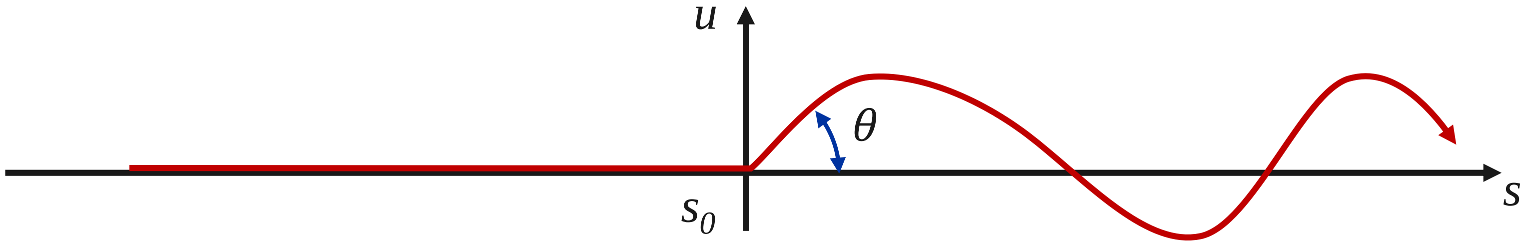
$$\begin{cases} B_y(x,y) = B_3(x^3 - 3xy^2) \\ B_x(x,y) = B_3(-y^3 + 3x^2y) \end{cases}$$



e.g. sextupole

Effect of single dipole kick

- Consider a **single dipole kick** $\theta = \delta u_0' = \delta u'(s_0) = \dfrac{\delta(Bl)}{B\rho}$ at s=s_0



- The **coordinates** of a single particle at a **downstream location *s*** can be computed using the lattice Twiss parameters (***see transverse dynamics course***):

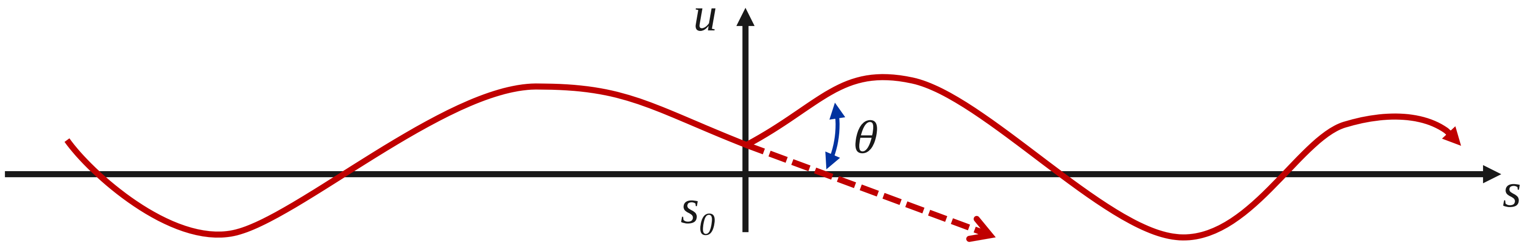
$$\begin{pmatrix} u_s \\ u_s' \end{pmatrix} = \mathbf{M_{s_0 s}} \begin{pmatrix} 0 \\ \theta \end{pmatrix} \quad \textit{where} \quad \mathbf{M_{s_0 s}} = \begin{bmatrix} \sqrt{\frac{\beta_s}{\beta_{s_0}}}(\cos\psi_{s_0 s} + \alpha_{s_0}\sin\psi_{s_0 s}) & \sqrt{\beta_s\beta_{s_0}}\sin\psi_{s_0 s} \\ \frac{\alpha_{s_0}-\alpha_s}{\sqrt{\beta_s\beta_{s_0}}}\cos\psi_{s_0 s} - \frac{1+\alpha_s\alpha_{s_0}}{\sqrt{\beta_s\beta_{s_0}}}\sin\psi_{s_0 s} & \sqrt{\frac{\beta_{s_0}}{\beta_s}}(\cos\psi_{s_0 s} - \alpha_s\sin\psi_{s_0 s}) \end{bmatrix}$$

- **If we want that the orbit closes on itself after one turn, then we must solve:**

$$\begin{bmatrix} \cos(2\pi Q) + \alpha_0\sin(2\pi Q) & \beta_0\sin(2\pi Q) \\ -\frac{1+\alpha_0^2}{\beta_0}\sin(2\pi Q) & \cos(2\pi Q) - \alpha_0\sin(2\pi Q) \end{bmatrix} \begin{pmatrix} u_0 \\ u_0' \end{pmatrix} + \begin{pmatrix} 0 \\ \theta \end{pmatrix} = \begin{pmatrix} u_0 \\ u_0' \end{pmatrix}$$

  - Which only depends on the **Twiss functions** at the s_0 location and the **machine tune *Q***

Closed orbit from single dipole kick



- **The initial conditions of the closed orbit at the location of the kick are therefore obtained as**

$$u_0 = \theta \frac{\beta_0}{2\tan \pi Q} \qquad \textbf{and} \qquad u_0' = \frac{\theta}{2}\left(1 - \frac{\alpha_0}{\tan \pi Q}\right)$$

- **For any location *s* around the ring, the closed orbit distortion *Δu* generated by a kick *θ* in s_0 is**

$$\Delta u_s = \theta_{s_0} \frac{\sqrt{\beta_s \beta_{s_0}}}{2\sin(\pi Q)} \cos(\pi Q - |\psi_s - \psi_{s_0}|)$$

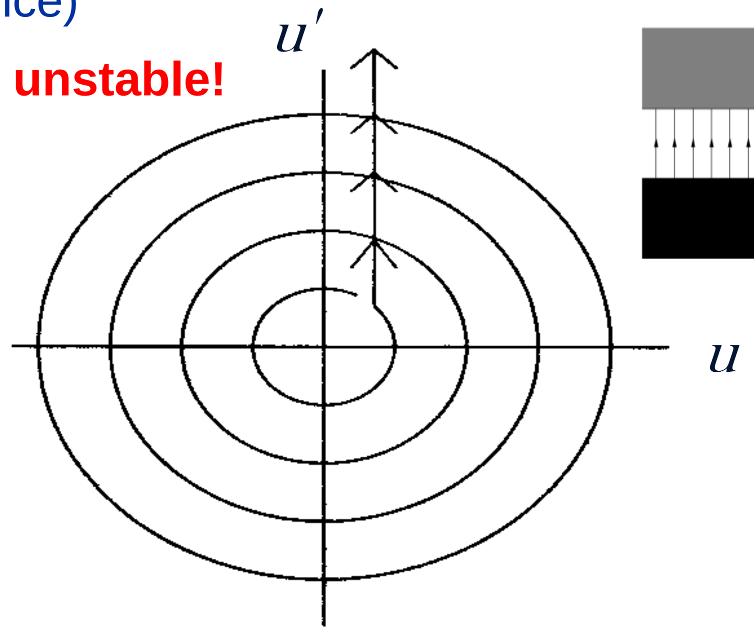
maximum orbit distortion amplitude

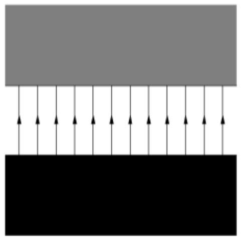
Integer and half integer resonance

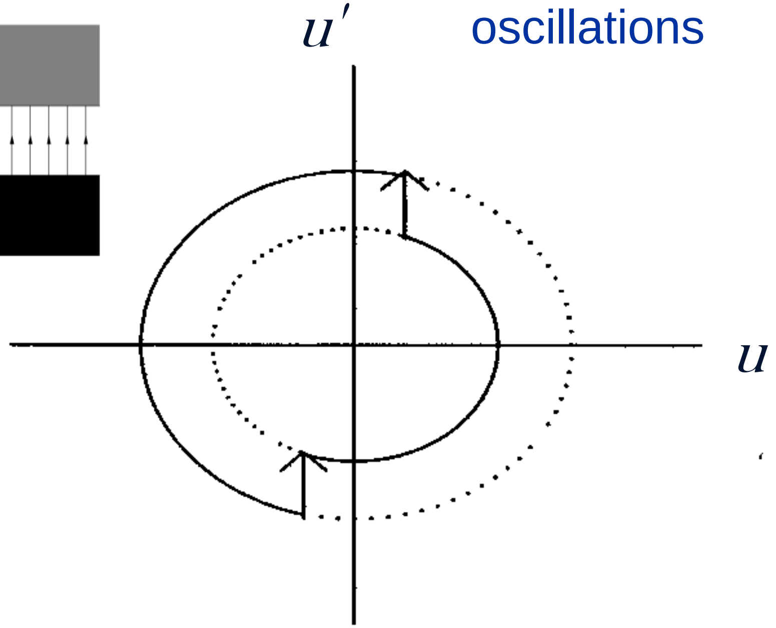
$$\Delta u_s = \theta_{s_0} \frac{\sqrt{\beta_s \beta_{s_0}}}{2\sin(\pi Q)} \cos(\pi Q - |\psi_s - \psi_{s_0}|)$$

- Dipole kicks add-up in consecutive turns for **Q = n**
- Integer tune excites orbit oscillations (resonance)
  - **orbit becomes unstable!**

- Dipole kicks get cancelled in consecutive turns for **Q = n+1/2**
- Half-integer tune cancels orbit oscillations

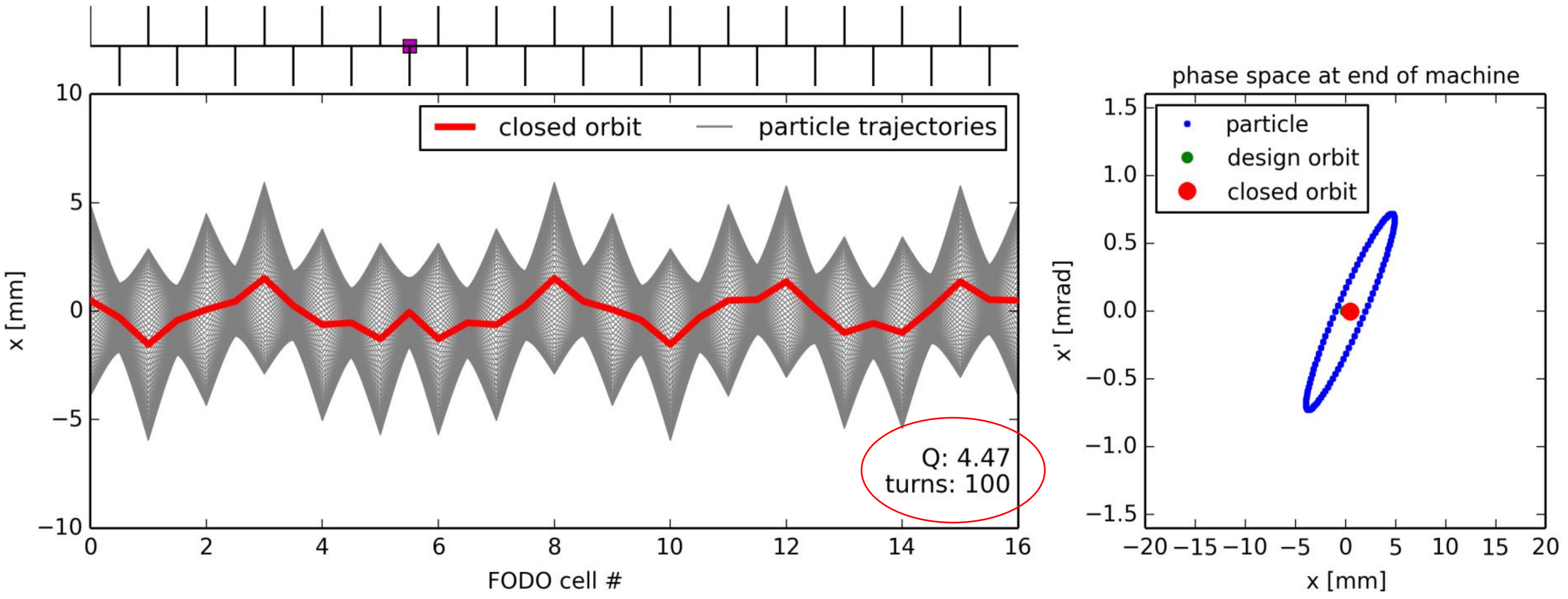






Single dipole kick vs. tune

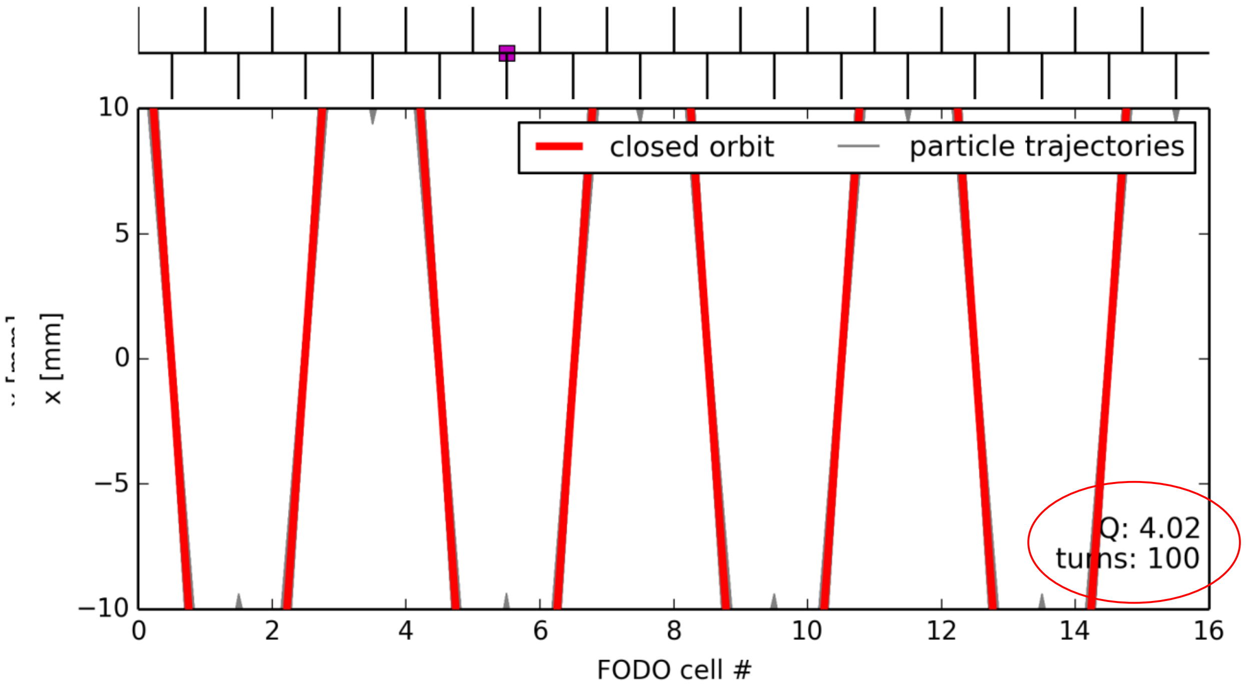
$$\Delta u_s = \theta_{s_0} \frac{\sqrt{\beta_s \beta_{s_0}}}{2\sin(\pi Q)} \cos(\pi Q - |\psi_s - \psi_{s_0}|)$$

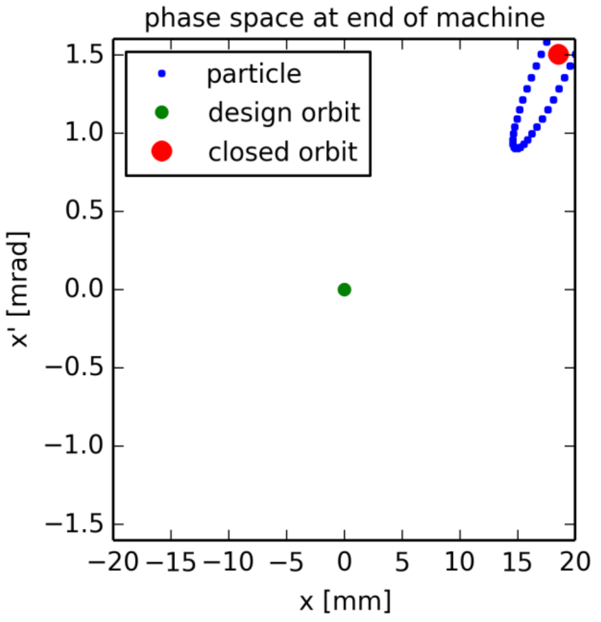


Single dipole kick vs. tune

- Closed orbit distortion is most critical for **tunes close to integer → closed orbit becomes unstable (but beam size not affected)**
- *Note:* the closed orbit distortion propagates with the betatron phase advance (e.g. single kick induces 4 oscillations for a tune of Q=4.x)

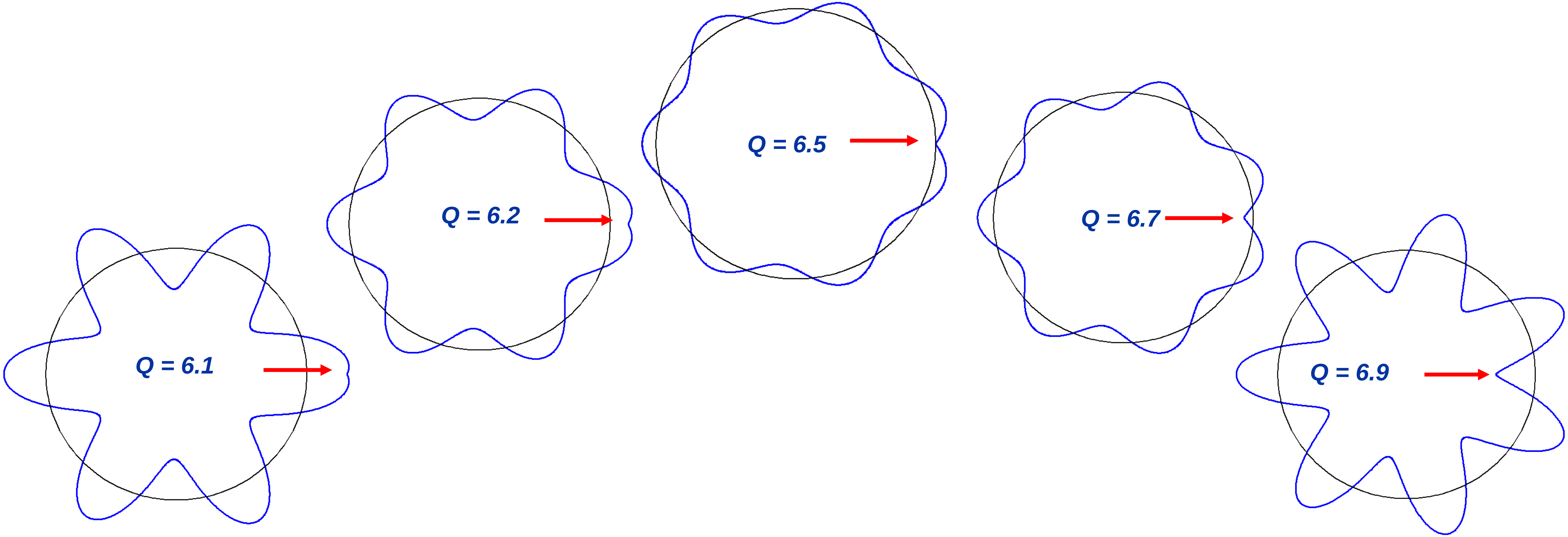
$$\Delta u_s = \theta_{s_0} \frac{\sqrt{\beta_s \beta_{s_0}}}{2\sin(\pi Q)} \cos(\pi Q - |\psi_s - \psi_{s_0}|)$$





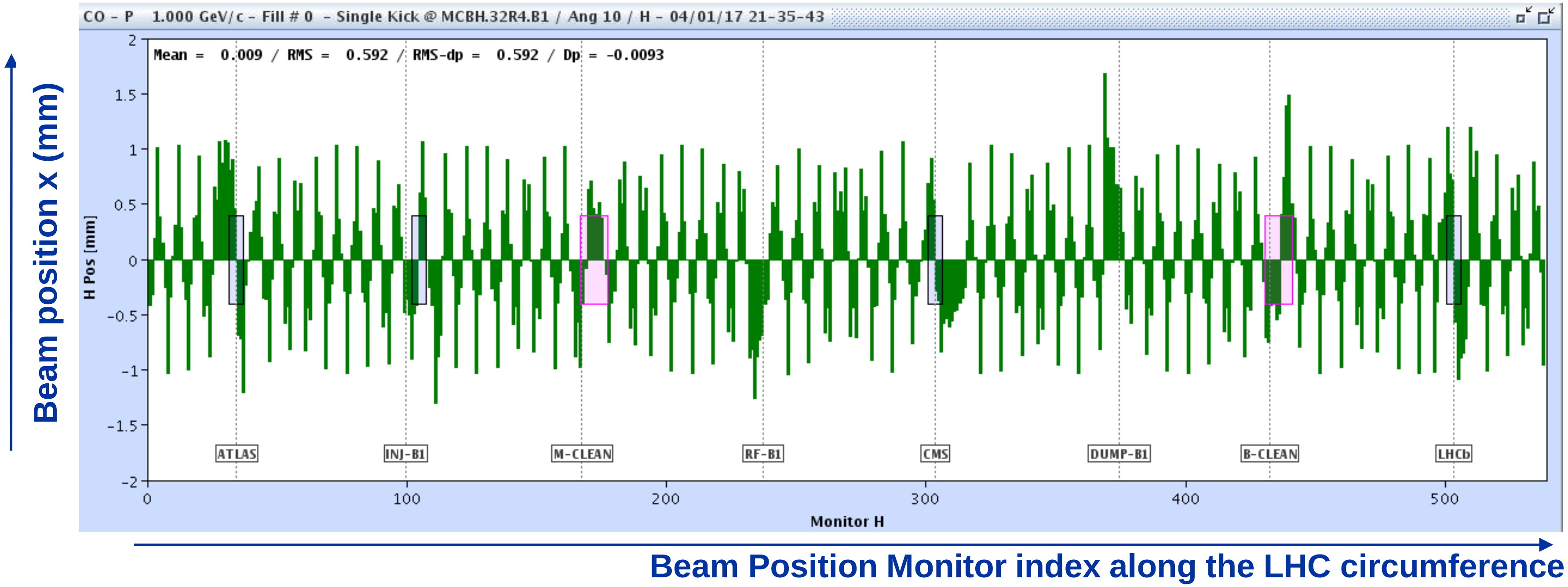
Closed orbit examples

- Example of horizontal closed orbit for a machine with tune Q = 6.x
- The **kink at the location of the deflection** (→) can be used to localize the deflection (if it is not known) → can be used for orbit correction.



A deflection at the LHC

- In the example below for the 26.7km long LHC, there is one undesired deflection, leading to a perturbed closed orbit.

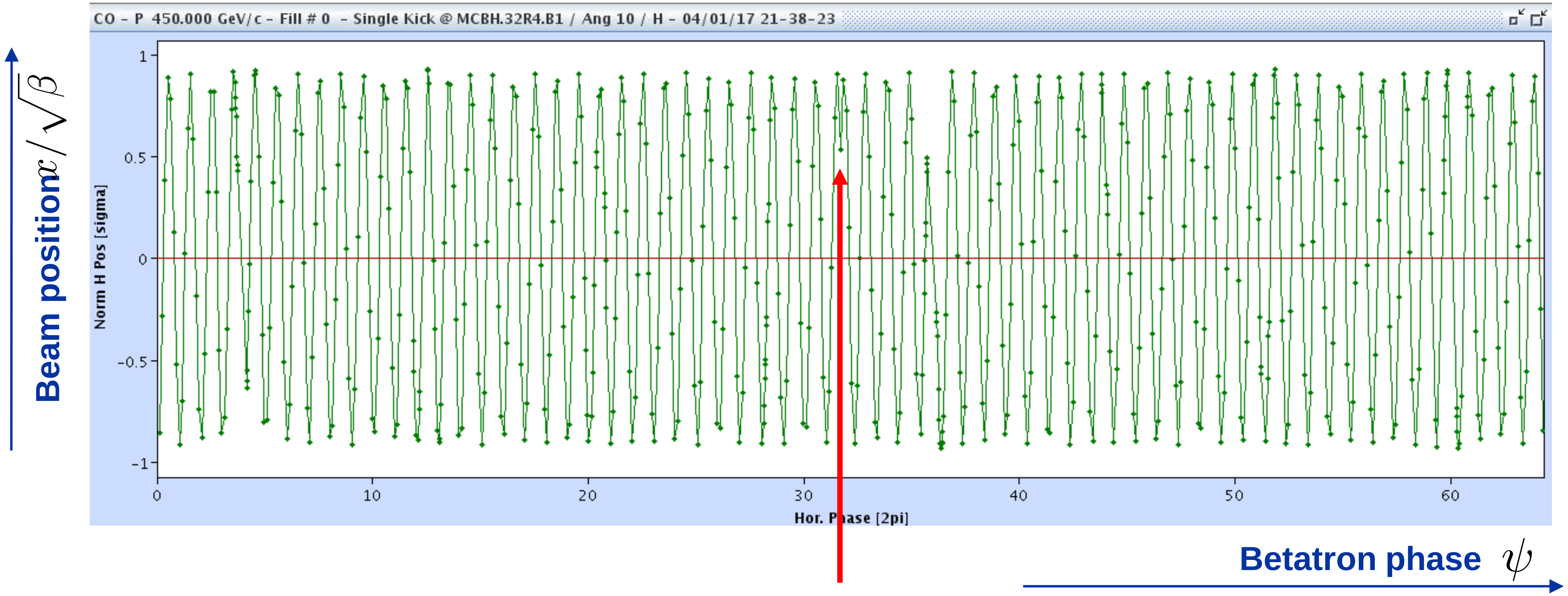


Where is the location of the deflection?

A deflection at the LHC

- To make our life easier we divide the position by $\sqrt{\beta_s}$ and replace the BPM index by its phase ψ_s
  - → transform into **pure sinusoidal oscillation**… **with a kink!**

$$\frac{\Delta u_s}{\sqrt{\beta_s}} = \theta_{s0} \frac{\sqrt{\beta_{s0}}}{2\sin(\pi Q)} \cos(\pi Q - |\psi_s - \psi_{s0}|)$$



Can you localize the deflection now?

Global orbit distortion

Orbit distortion due to many errors:

$$u(s) = \frac{\sqrt{\beta(s)}}{2\sin(\pi Q)} \int_s^{s+C} \theta(\tau)\sqrt{\beta(\tau)}\cos(\pi Q - |\psi(s) - \psi(\tau)|)d\tau$$

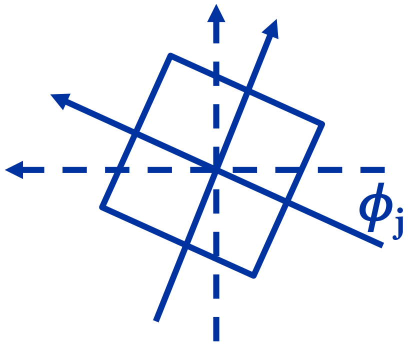
Courant and Snyder, 1957

By approximating the errors as delta functions in n locations, the distortion at ith observation points (Beam Position Monitors) is

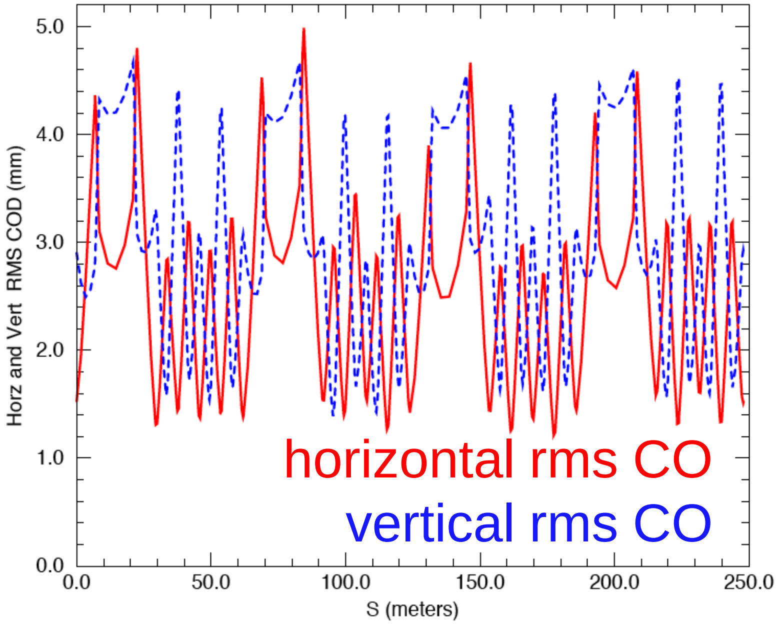
$$u_i = \frac{\sqrt{\beta_i}}{2\sin(\pi Q)} \sum_{j=i+1}^{i+n} \theta_j \sqrt{\beta_j}\cos(\pi Q - |\psi_i - \psi_j|)$$

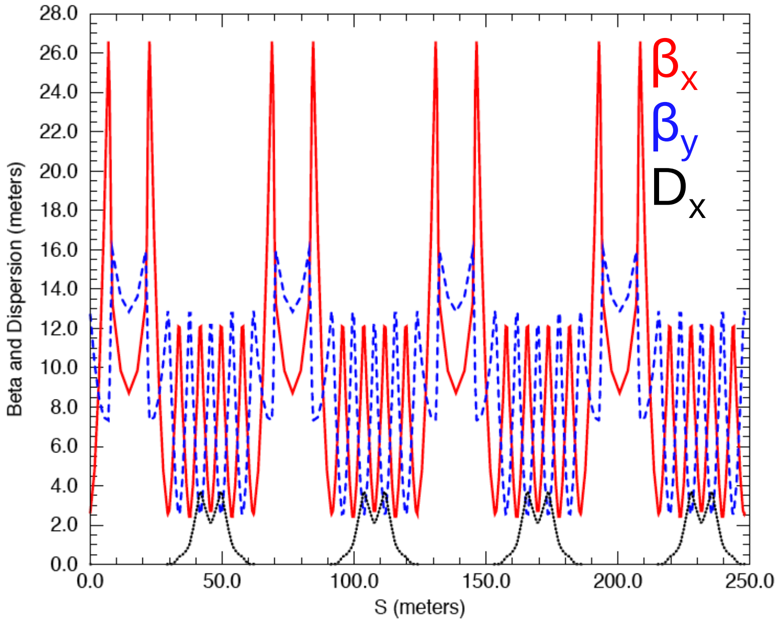
with the kick produced by the jth error:

- **Integrated dipole field error** $\theta_j = \frac{\delta(B_j l_j)}{B\rho}$
- **Dipole roll** $\theta_j = \frac{B_j l_j \sin\phi_j}{B\rho}$
- **Quadrupole displacement** $\theta_j = \frac{G_j l_j \delta u_j}{B\rho}$



Example: Orbit distortion in SNS





- In the SNS accumulator ring, the beta function is about **6 m** in the dipoles and about 30 m in the quadrupoles, the tune is **6.2**
- Consider a single dipole error of **1 mrad**
- The maximum orbit distortion in dipoles is $$u_0 = \frac{\sqrt{6 \cdot 6}}{2\sin(6.2\pi)} \cdot 10^{-3} \approx 5\text{mm}$$
- For quadrupole displacement giving the same **1 mrad** kick (and betas of 30 m) the maximum orbit distortion is 25 mm, to be compared to magnet radius of 105 mm

Statistical estimation of orbit errors

Consider random distribution of errors in N magnets

- By squaring the orbit distortion expression and averaging over the angles (considering uncorrelated errors), the expectation (rms) value is given by

$$u_{\mathrm{rms}}(s) = \frac{\sqrt{\beta(s)}}{2\sqrt{2}|\sin(\pi Q)|}\left(\sum_i \sqrt{\beta_i}\theta_i\right)_{\mathrm{rms}} = \frac{\sqrt{N\beta(s)\beta_{\mathrm{rms}}}}{2\sqrt{2}|\sin(\pi Q)|}\theta_{\mathrm{rms}}$$

Example:

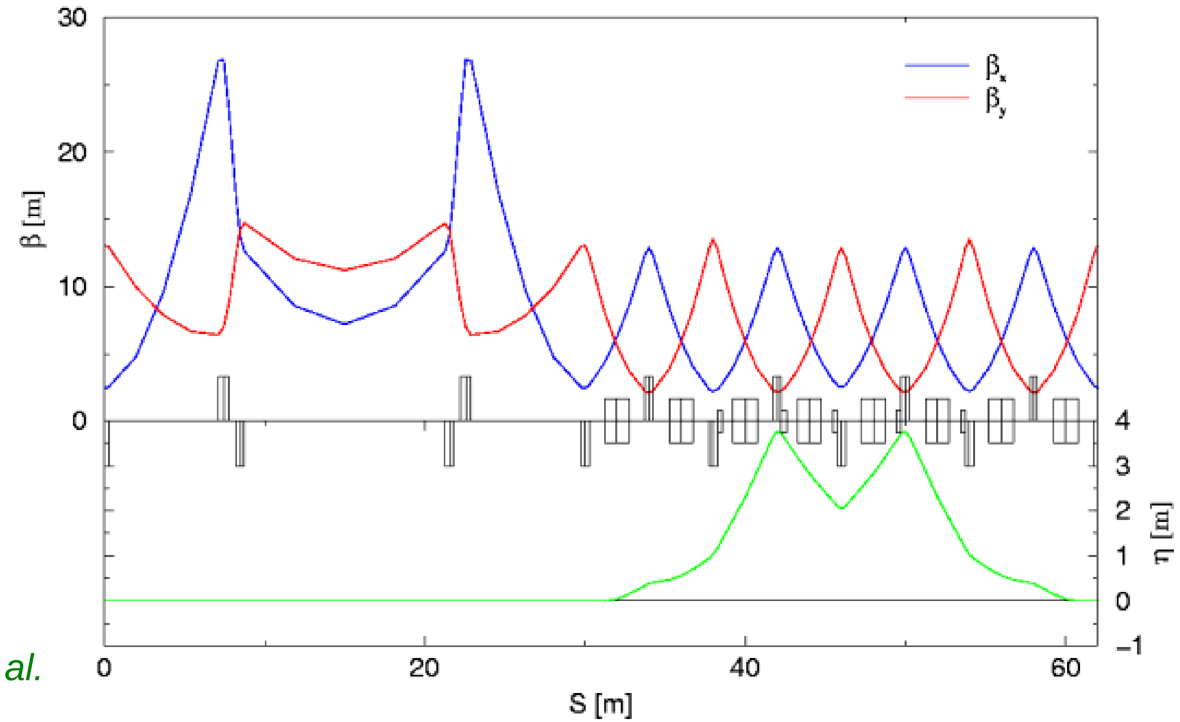
- In the SNS ring, there are **32** dipoles and **54** quadrupoles
  - The rms value of the orbit distortion in the dipoles $u_{\mathrm{rms}}^{\mathrm{dip}} = \frac{\sqrt{6\cdot 6\sqrt{32}}}{2\sqrt{2}\sin(6.2\pi)}\cdot 10^{-3} \approx 2\mathrm{cm}$
  - In the quadrupoles, for equivalent kick $u_{\mathrm{rms}}^{\mathrm{quad}} = \frac{\sqrt{30\cdot 30\sqrt{54}}}{2\sqrt{2}\sin(6.2\pi)}\cdot 10^{-3} \approx 13\mathrm{cm}$

Problem 2

SNS: A **proton** ring with kinetic energy of 1 GeV and a **circumference of 248 m** has **18, 1 m-long** focusing quads with **gradient of 5 T/m**. In one of the quads, the horizontal and vertical **beta function** are **12 m** and **2 m** respectively. The **rms beta** function in both planes on the focusing quads is **8 m**.

1. With a horizontal tune of **6.23** and a vertical of **6.2**, compute the expected horizontal and vertical orbit distortions on a single focusing quad given by **horizontal** and **vertical misalignments of 1 mm in all the quads**.

2. What happens to the horizontal and vertical orbit distortions if the horizontal tune drops to **6.1** and **6.01**?



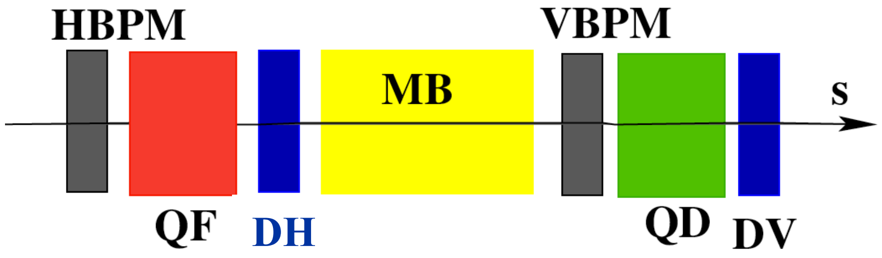
S. Henderson et al.

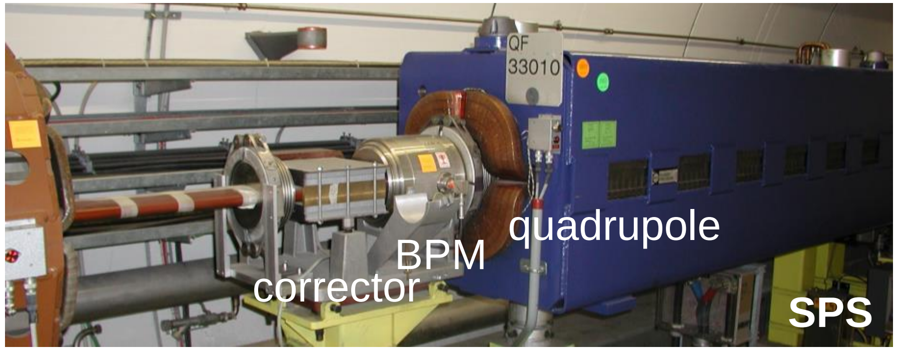
Correcting closed orbit distortion

- **Horizontal/Vertical dipole correctors and BPMs close to focusing/defocusing quads**
  - Highest sensitivity / effect on closed orbit due to beta-function maxima

BPM: Beam Position Monitor

DH, DV: correctors





- **Measure orbit in BPMs and minimize orbit distortion**
  - Locally
    - Closed orbit bumps
  - Globally
    - Singular Value Decomposition (SVD)
    - Harmonic: minimizing components of orbit frequency response from Fourier analysis
    - MICADO: finding the most efficient corrector for minimizing the rms orbit
    - Least square minimization using orbit response matrix of correctors

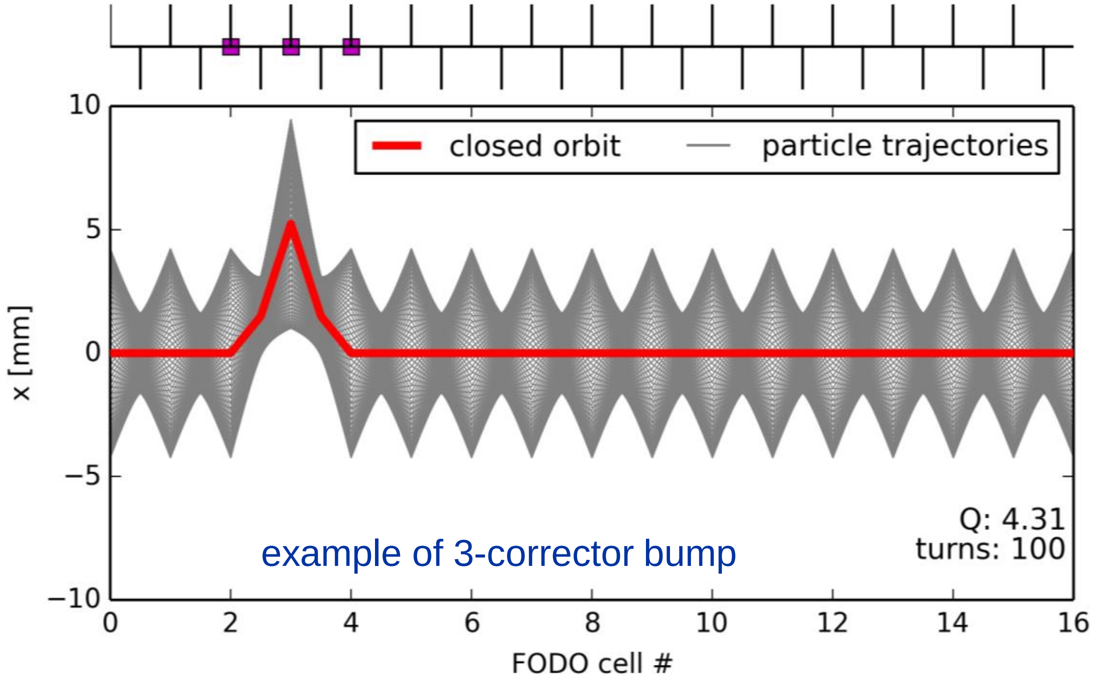
Closed orbit bumps

Often it is needed to steer the closed-orbit away from the nominal trajectory in a localized part of a synchrotron

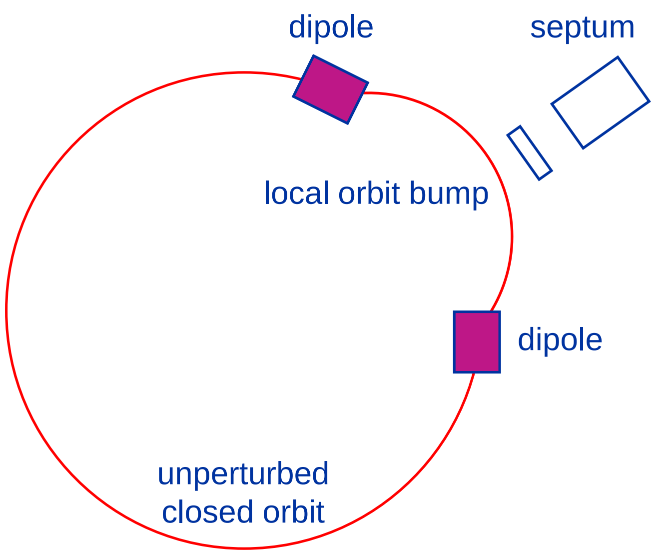
- Injection / extraction
- Local orbit correction (or steering around local aperture restrictions)

Standard bump configurations exist

- π-bump (with 2 correctors)
- 3 and 4-corrector bumps



example of 2-corrector bump



Closed orbit bumps

Often it is needed to steer the closed-orbit away from the nominal trajectory in a localized part of a synchrotron

- Injection / extraction
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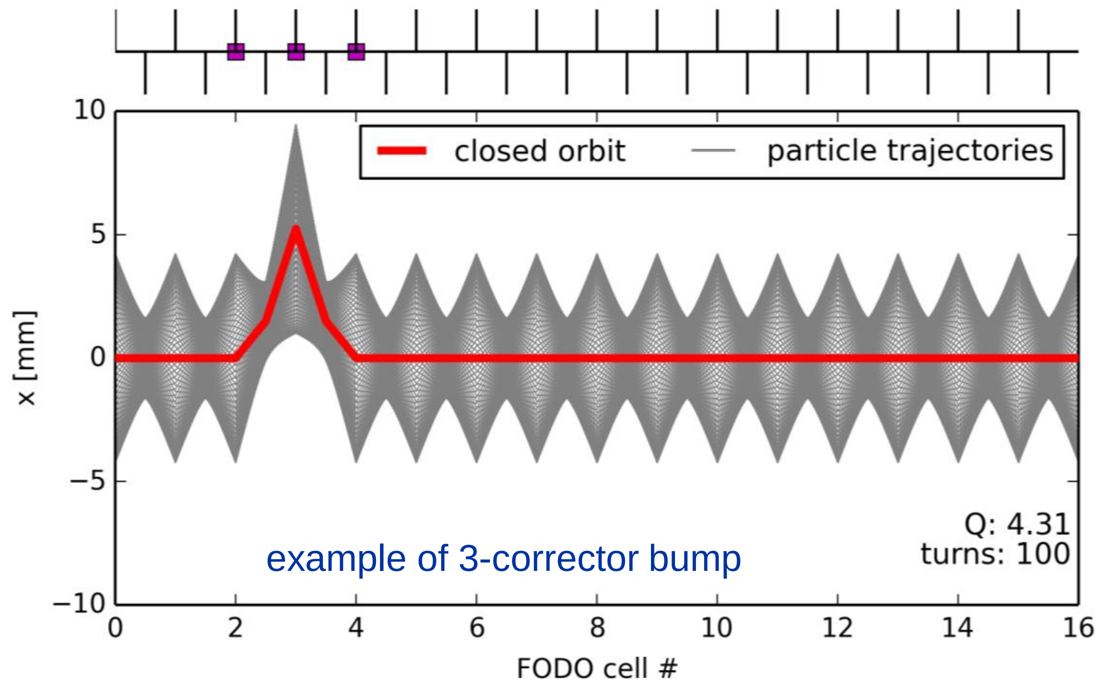
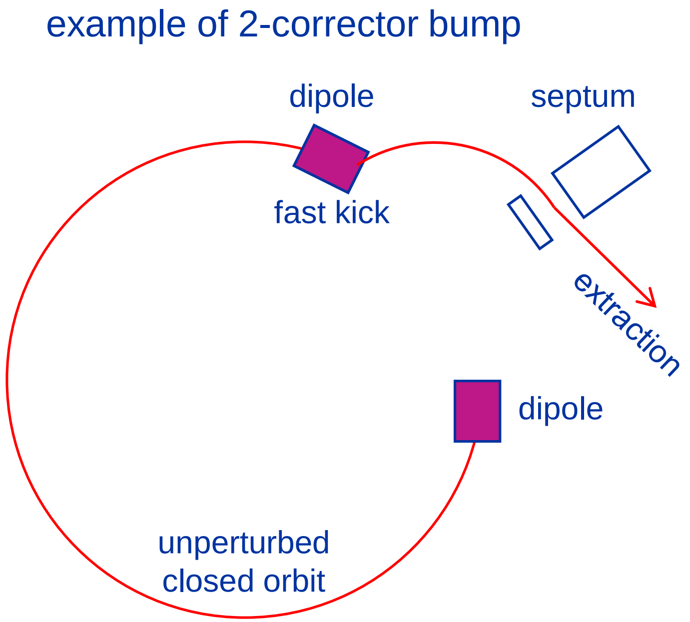




Transport of closed orbit distortion

- Consider a transport matrix between positions 1 and 2

$$\mathcal{M}_{1\to 2} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = \begin{bmatrix} \sqrt{\frac{\beta_2}{\beta_1}}(\cos\psi_{12} + \alpha_1 \sin\psi_{12}) & \sqrt{\beta_2\beta_1}\sin\psi_{12} \\ \frac{\alpha_1-\alpha_2}{\sqrt{\beta_2\beta_1}}\cos\psi_{12} - \frac{1+\alpha_2\alpha_1}{\sqrt{\beta_2\beta_1}}\sin\psi_{12} & \sqrt{\frac{\beta_1}{\beta_2}}(\cos\psi_{12} - \alpha_2\sin\psi_{12}) \end{bmatrix}$$

- Consider a single dipole kick at position 1: $\theta_1 = \frac{\delta(Bl)}{B\rho}$

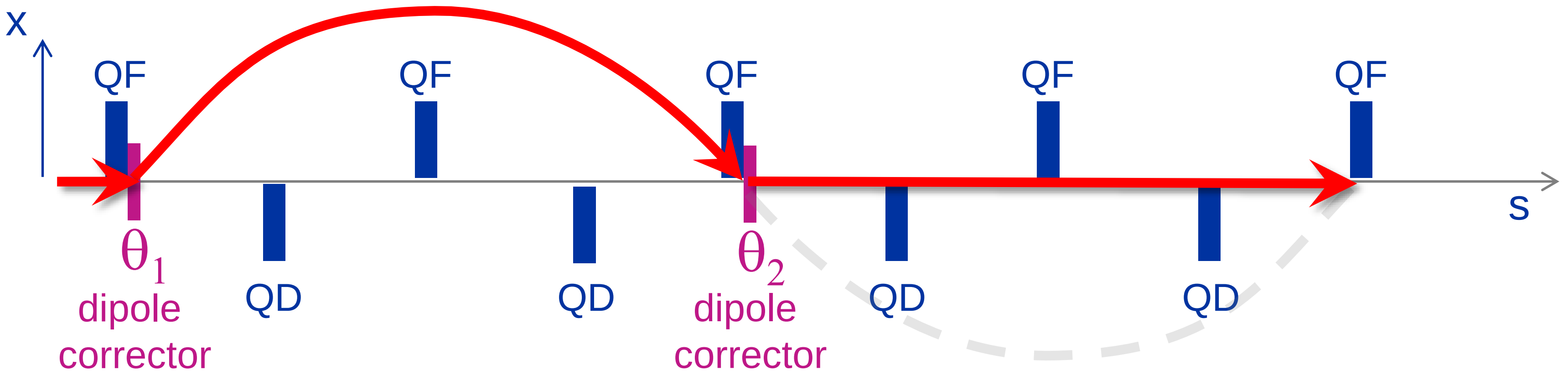
- The variation of position (δu_2) and angle ($\delta u'_2$) at $\begin{pmatrix} \delta u_2 \\ \delta u'_2 \end{pmatrix} = \mathcal{M}_{1\to 2}\begin{pmatrix} 0 \\ \theta_1 \end{pmatrix}$

- Replacing the coefficient from the general betatron matrix, one obtains

$$\delta u_2 = \sqrt{\beta_1\beta_2}\sin(\psi_{12})\theta_1$$

$$\delta u'_2 = \sqrt{\frac{\beta_1}{\beta_2}}[\cos(\psi_{12}) - \alpha_2\sin(\psi_{12})]\theta_1$$

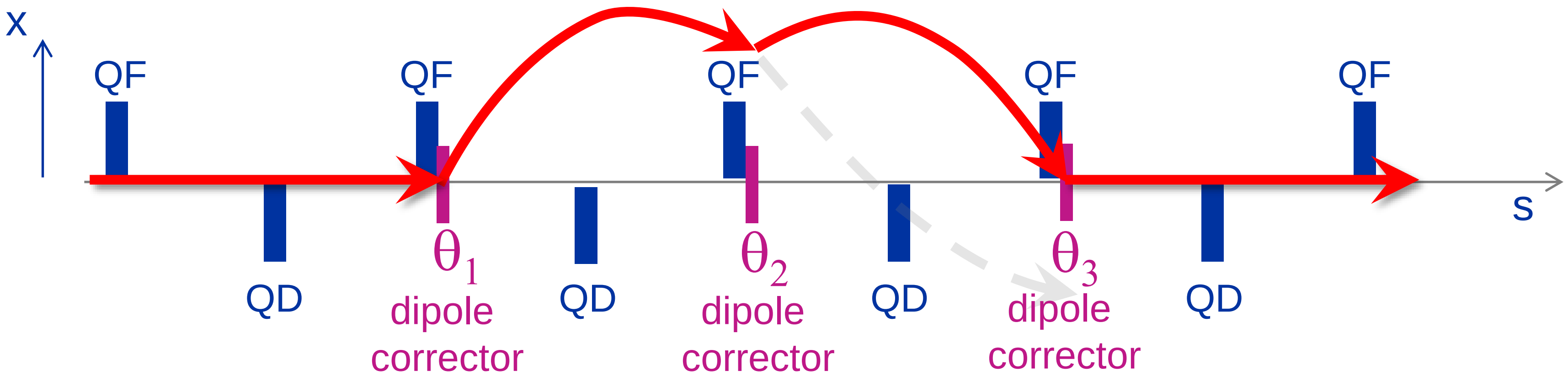
Orbit bumps: 2-corrector bump



- Consider a cell in which correctors are placed close to the focusing quads
- The orbit shift at the 2nd corrector is $\delta u_2 = \sqrt{\beta_1 \beta_2} \sin\left(\psi_{12}\right) \theta_1$
- This orbit bump can be closed by choosing a **phase advance equal to π** between correctors (this is called a "π-bump")
- The angle should satisfy the following equation

$$\theta_2 = \delta u_2' = -\sqrt{\frac{\beta_1}{\beta_2}} \left[\cos(\psi_{12})\theta_1 - \alpha_2 \sin(\psi_{12})\right] = \sqrt{\frac{\beta_1}{\beta_2}} \theta_1$$

Orbit bumps: 3-corrector bump

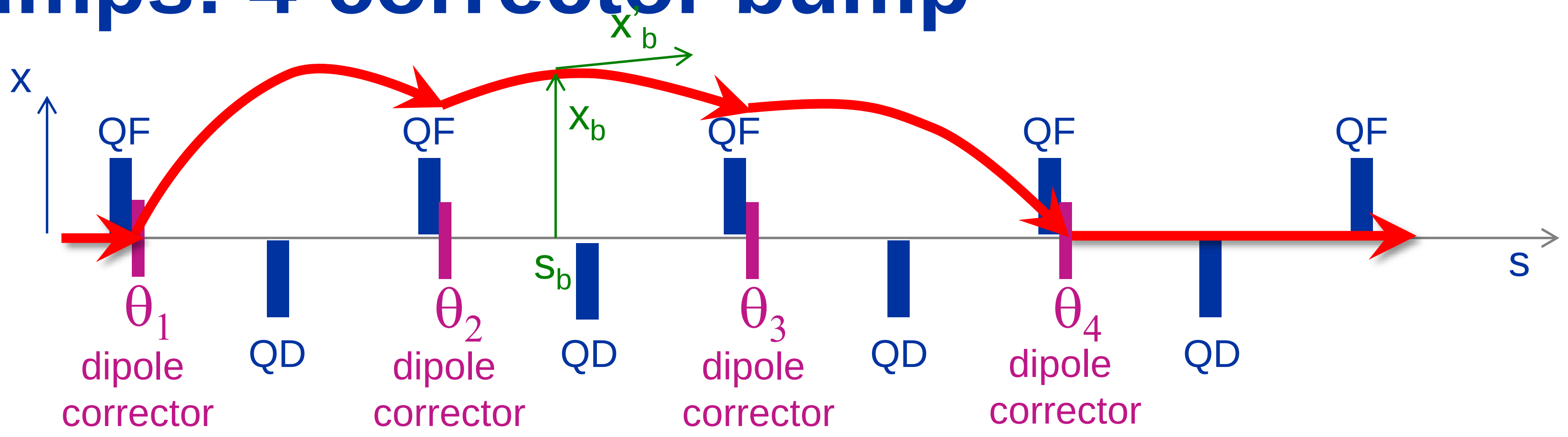


- Works for any phase advance if the three correctors satisfy

$$\frac{\sqrt{\beta_1}}{\sin\psi_{23}}\theta_1 = \frac{\sqrt{\beta_2}}{\sin\psi_{31}}\theta_2 = \frac{\sqrt{\beta_3}}{\sin\psi_{12}}\theta_3$$

- ***Note:*** The **angle** of the closed orbit **in the center** of the bump is defined by the above condition, i.e. it cannot be adjusted independently of bump amplitude

Orbit bumps: 4-corrector bump



$$\theta_1 = +\frac{1}{\sqrt{\beta_1\beta_b}}\frac{\cos\psi_{2b} - \alpha_b\sin\psi_{2b}}{\sin\psi_{12}}x_b - \sqrt{\frac{\beta_b}{\beta_1}}\frac{\sin\psi_{2b}}{\sin\psi_{12}}x'_b$$

$$\theta_2 = -\frac{1}{\sqrt{\beta_2\beta_b}}\frac{\cos\psi_{1b} - \alpha_b\sin\psi_{1b}}{\sin\psi_{12}}x_b + \sqrt{\frac{\beta_b}{\beta_2}}\frac{\sin\psi_{1b}}{\sin\psi_{12}}x'_b$$

$$\theta_3 = -\frac{1}{\sqrt{\beta_3\beta_b}}\frac{\cos\psi_{b4} + \alpha_b\sin\psi_{b4}}{\sin\psi_{34}}x_b - \sqrt{\frac{\beta_b}{\beta_4}}\frac{\sin\psi_{b4}}{\sin\psi_{34}}x'_b$$

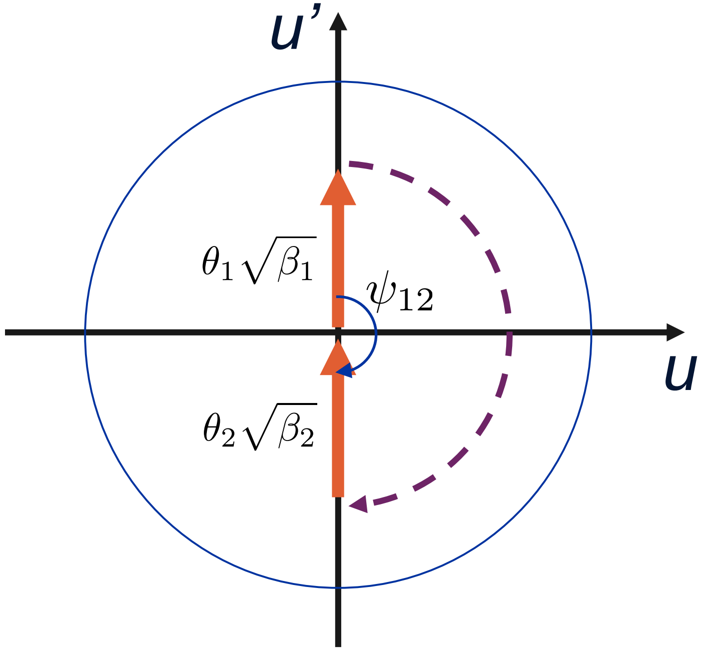
$$\theta_4 = +\frac{1}{\sqrt{\beta_4\beta_b}}\frac{\cos\psi_{b3} + \alpha_b\sin\psi_{b3}}{\sin\psi_{34}}x_b + \sqrt{\frac{\beta_b}{\beta_4}}\frac{\sin\psi_{b3}}{\sin\psi_{34}}x'_b$$

- Works for any phase advance
- Position x_b and angle x'_b of the bump at location s_b can be adjusted independently
- Can be used for aperture scanning, extraction bumps, …

Visualisation of simplest orbit bumps

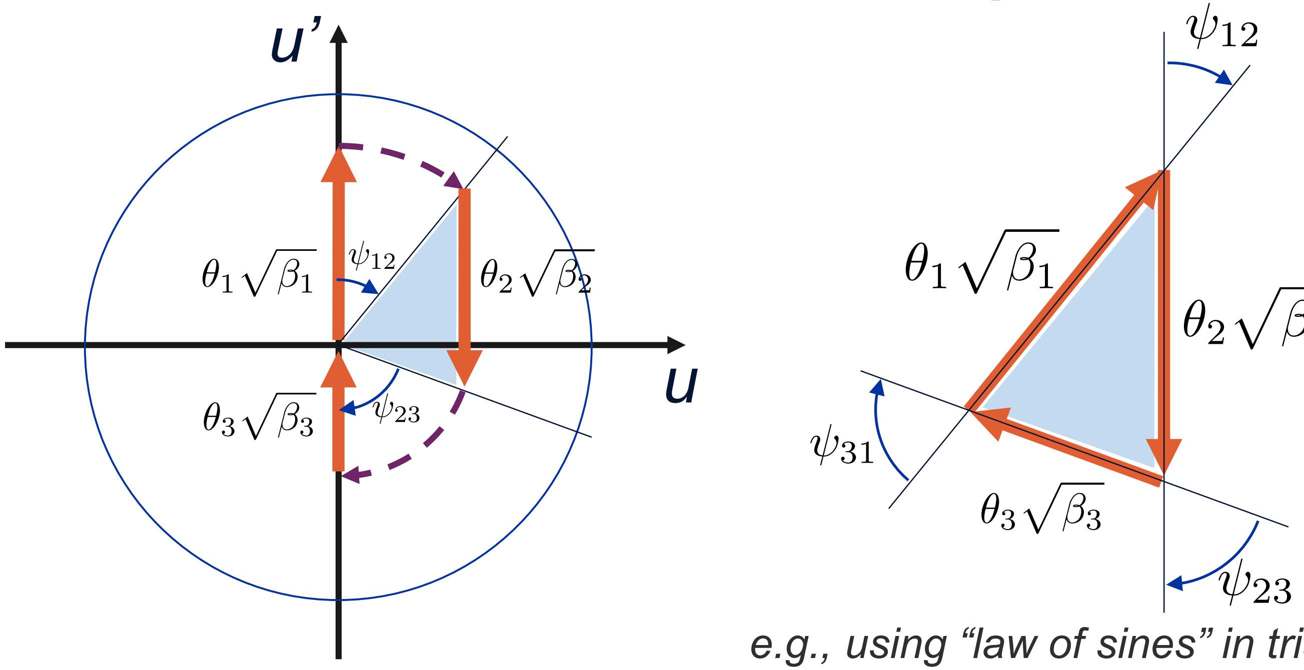
- **Let's consider normalised phase-space (see linear dynamics lectures) where phase spaces are simple circles**

pi-bump



$$\theta_1\sqrt{\beta_1} = \theta_2\sqrt{\beta_2}$$

3 corrector-bump

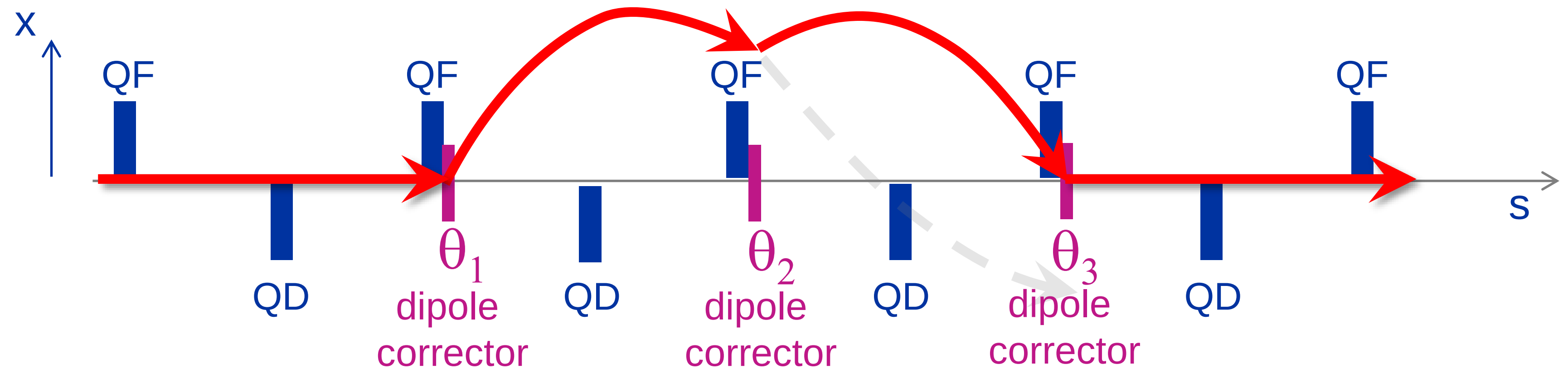


e.g., using "law of sines" in triangles

$$\frac{\sqrt{\beta_1}}{\sin\psi_{23}}\theta_1 = \frac{\sqrt{\beta_2}}{\sin\psi_{31}}\theta_2 = \frac{\sqrt{\beta_3}}{\sin\psi_{12}}\theta_3$$

Problem 3

Three correctors are placed at locations with phase advance of $\pi/4$ between them and beta functions of **12, 2** and **12 m**. How are the corrector kicks related to each other in order to achieve a closed **3-corrector bump** (i.e. what is the relative strength between the three kicks)?



Closed orbit correction: MICADO

- The problem of correcting the orbit deterministically came up a long time ago in the first CERN machines.
- B. Autin and Y. Marti published a note in 1973 describing an algorithm that is still in use today (but in JAVA/C/C++ instead of FORTRAN) at ALL CERN machines: **MICADO***

* MInimisation des CArrés des Distortions d'Orbite.

(Minimization of the quadratic orbit distortions)

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN ISR-MA/73-17

CLOSED ORBIT CORRECTION OF A.G. MACHINES
USING A SMALL NUMBER OF MAGNETS

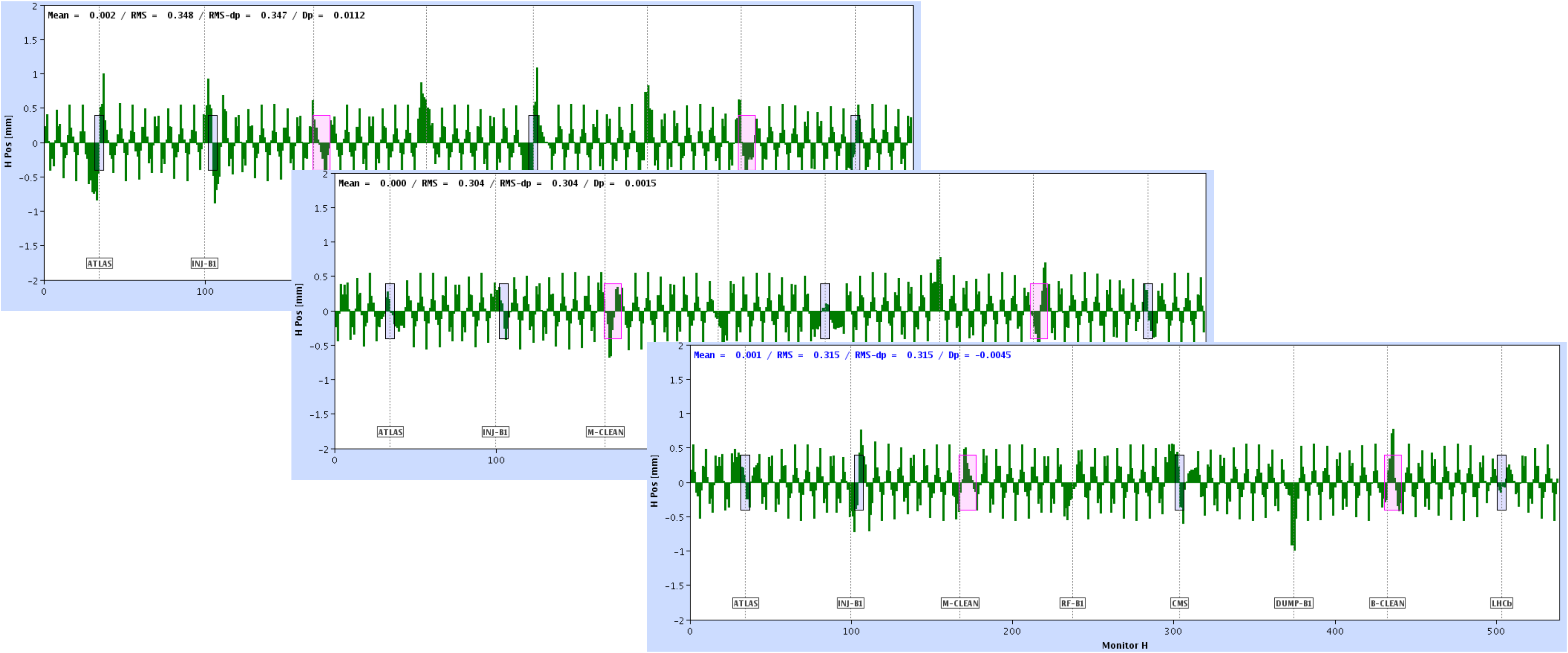
by

B. Autin & Y. Marti

MICADO – how does it work?

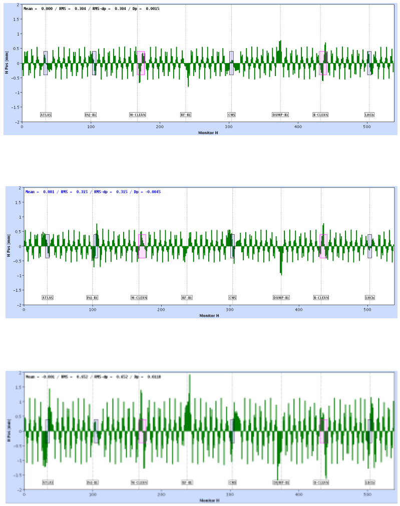
- The intuitive principle of MICADO is rather simple:

 1. It requires a **model of the machine**
 2. It computes for **each orbit corrector** what the effect (**response**) is expected to be on the orbit



MICADO – how does it work?

3. MICADO compares the response of every corrector with the raw orbit in the machine



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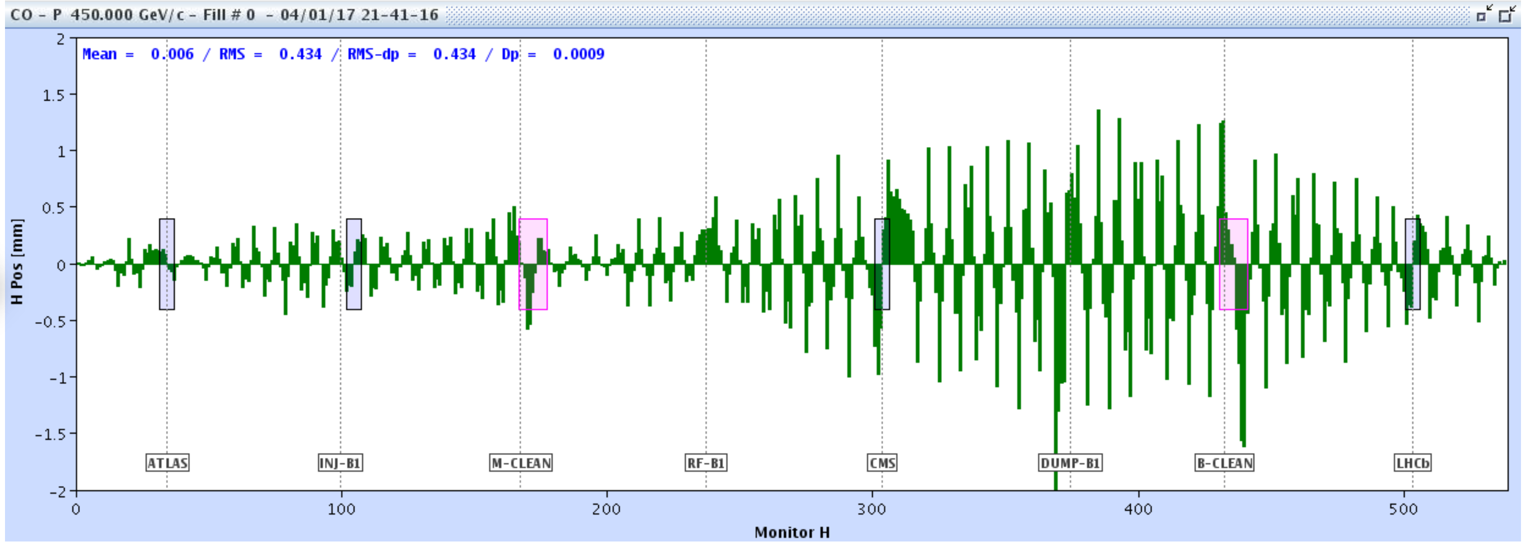
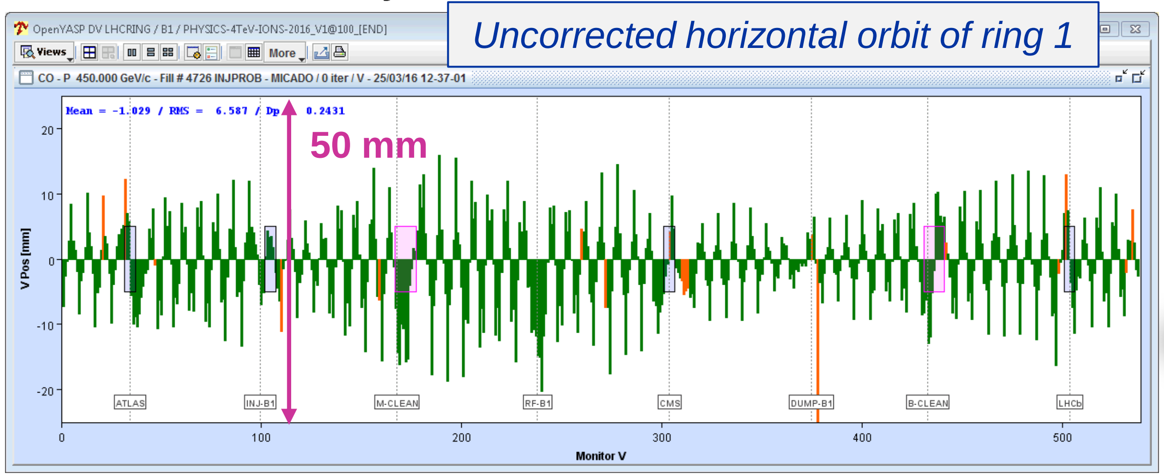
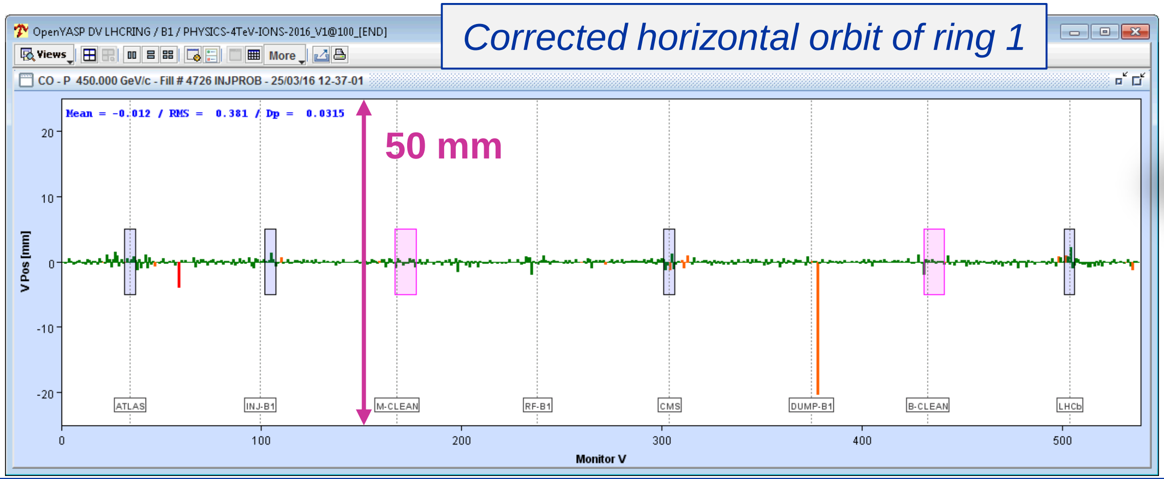


4. MICADO **picks the corrector that has the best match with the raw orbit**, i.e. that will give the largest improvement to the orbit deviation rms
5. The procedure can be **iterated** to the second-best corrector and so on, until the orbit is good enough (or as good as it can be)

MICADO – LHC Orbit example

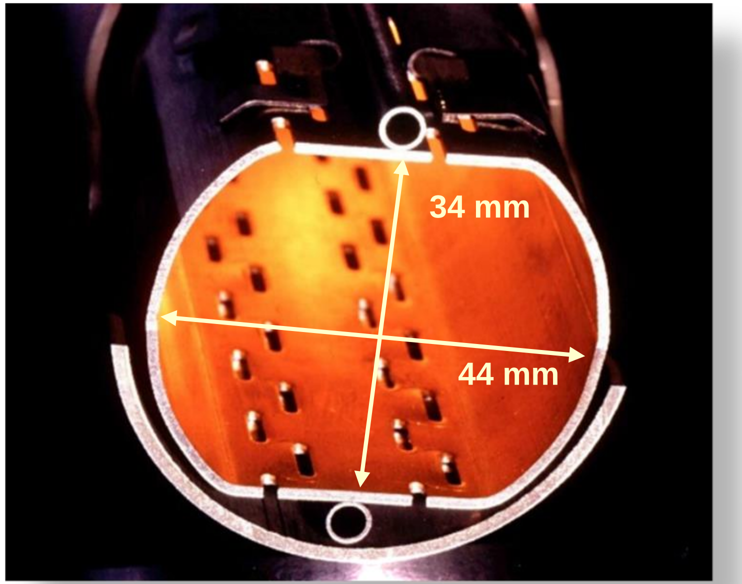
- The raw orbit at the LHC can have huge errors, but the correction (based partly on MICADO) brings the deviations down by a factor 20





MICADO & Co

LHC vacuum chamber



At the LHC a good orbit correction is vital !

Response matrix approach

This approach works for orbit correction when using the measured orbit distortion (but also for beta-beating when using $\Delta\beta/\beta$, etc.)

- Consider an available set of correctors: $\vec{c}$
- Consider the available observables (here the orbit at the Beam Position Monitors): $\vec{m}$
- Assume (or verify) that the linear approximation is good enough (small corrections): $\mathbf{A}\vec{c} = \vec{m}$
- Use optics model to compute the response matrix $\mathbf{A}$ (i.e. the orbit change in the i^{th} monitor due to a unit kick from the j^{th} corrector):

$$A_{i,j} = \frac{\sqrt{\beta_i \beta_j}\cos(\pi Q - |\psi_i - \psi_j|)}{2\sin(\pi Q)}$$

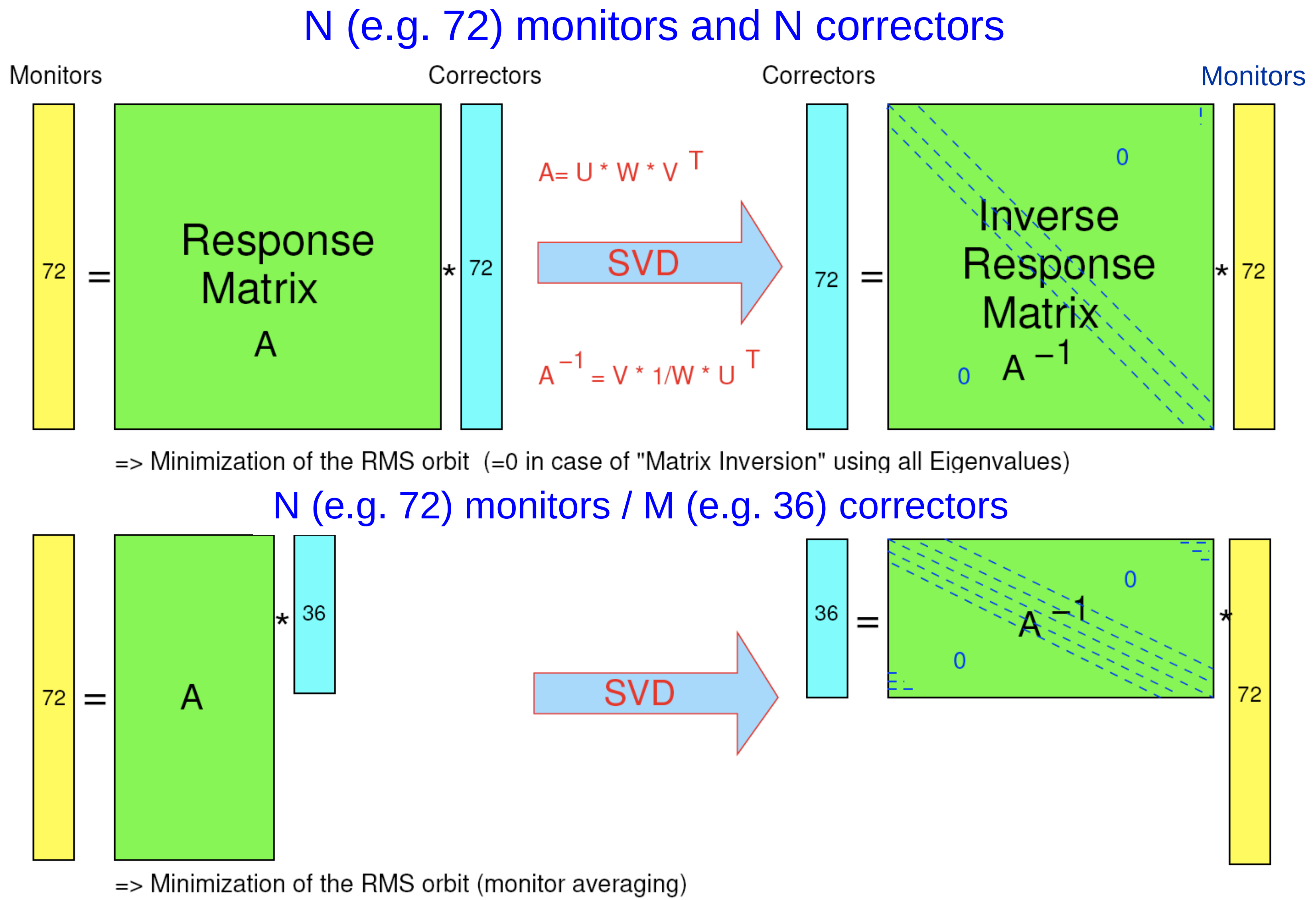
… or use, e.g., MAD-X, or measure it directly in the machine…

- Invert the matrix $\mathbf{A}$ to compute a **global correction** to obtain the desired orbit variation $\Delta\vec{m}$:

$$\Delta\vec{c} = \mathbf{A}^{-1}\Delta\vec{m}$$

- **In case the number of correctors is not the same as the number of Beam Position Monitors** one has to perform a pseudo matrix inversion, for example using the “**Singular Value Decomposition (SVD)**” algorithm

Singular Value Decomposition

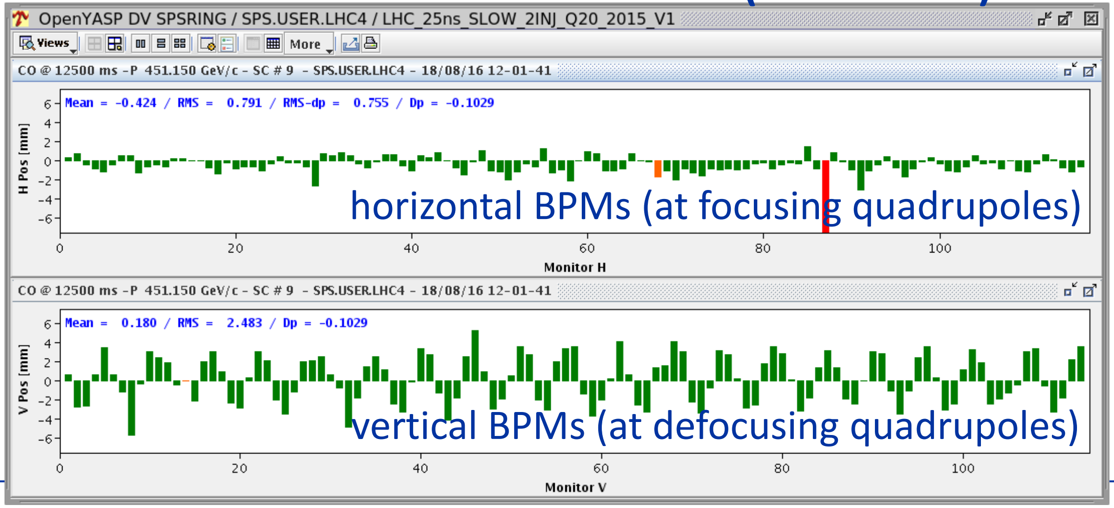


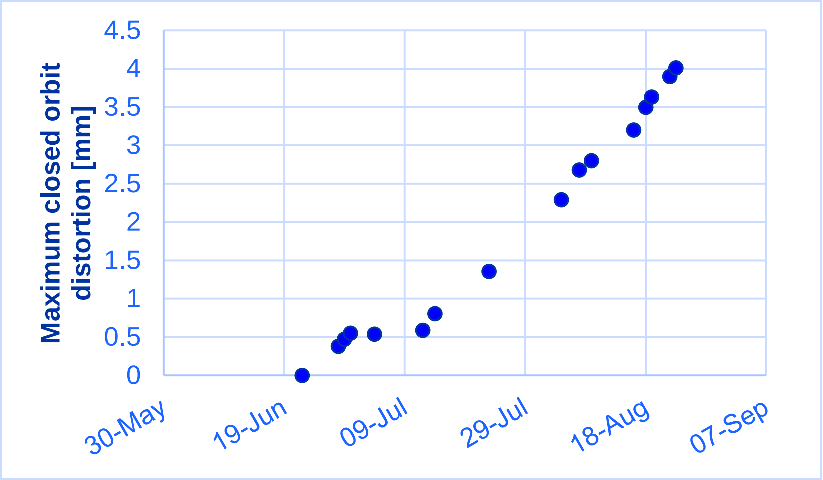
Problem 4

The SPS is a **400 GeV proton synchrotron** with a FODO lattice consisting of **108** focusing and **108** defocusing quadrupoles of length **3.22 m** and a gradient of **15 T/m**, with a **horizontal and vertical beta of 108 m and 30 m** in the focusing quads (**30 m and 108 m** for the defocusing ones). The tunes are **Q_x=20.13** and **Q_y=20.18**. Due to a mechanical problem, a focusing quadrupole was sinking down in 2016, resulting in an increasing closed orbit distortion compared to a reference taken earlier in the year.

1. By how much the quadrupole had shifted down when the maximum vertical closed orbit distortion amplitude in defocusing quadrupoles reached 4 mm?
2. Why was there no change of the horizontal orbit measured?
3. How big would have been the maximum closed orbit distortion amplitude if it would have been a defocusing quadrupole?

Difference orbit wrt reference (18.08.2016)





Final words on beam orbit stability

Beam orbit stability is very critical

- Injection and extraction efficiency of synchrotrons
- Stability of collision point in colliders
- Stability of the synchrotron light spot in the beam lines of light sources

Consequences of orbit distortion

- Miss-steering of beams, modification of dispersion function, resonance excitation, aperture limitations, lifetime reduction, coupling, modulation of lattice functions, poor injection/extraction efficiency

Sources for closed orbit drifts

- **Long term (years - months)**: ground settling, season changes, …
- **Medium term (days - hours)**: sun and moon, day-night variations (thermal), rivers, rain, wind, refills and start-up, sensor motion, drift of electronics, local machinery, filling patterns, …
- **Short term (minutes - seconds)**: ground vibrations, power supplies, experimental magnets, air conditioning, refrigerators/compressors, …

Outline

Introduction

Closed orbit distortion (steering error)

- ❑ Beam orbit stability
- ❑ Imperfections leading to closed orbit distortion
- ❑ Effect of single and multiple dipole kicks
- ❑ Closed orbit correction methods

Optics function distortion (gradient error)

- ❑ Imperfections leading to optics distortion
- ❑ Tune-shift and beta distortion due to gradient errors
- ❑ Gradient error correction

Coupling error

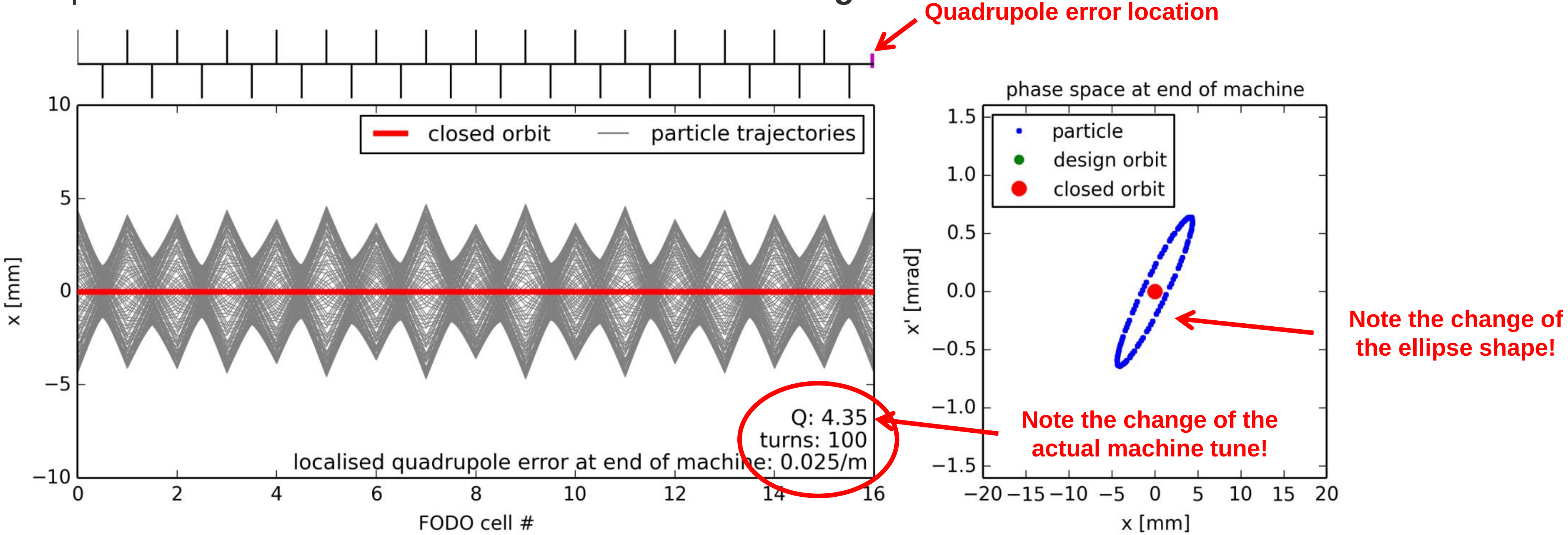
- ❑ Coupling errors and their effect
- ❑ Coupling correction

Summary

Illustration of optics distortion

Ideal machine toy model with regular FODO lattice and quadrupole error at the end of circumference

- Particle injected with offset performs betatron oscillations but gets additional focusing from quadrupole error
  - There is a **tune-shift** (additional de-/focusing)
  - Beam envelope is distorted around the machine … "**beta-beating**"



Gradient error and optics distortion

Optics functions perturbation can induce aperture restrictions

Tune perturbation can lead to reduced beam stability (dynamic aperture)

Broken super-periodicity → excitation of all resonances

- In a ring made of *N* identical cells, only resonances with harmonics being integer multiples of *N* can be excited

Sometimes control of optics is critical for machine performance

- Beta functions at collision points or at collimators (e.g. LHC)

Sources

- Errors in quadrupole strengths (random and systematic)
- Injection elements
- Higher-order multi-pole magnets and errors

Observables

- Tune-shift
- Beta-beating
- Excitation of integer and half integer resonances, beam losses

Gradient error: some math…

- Consider the **1-turn** transfer matrix:

$$\mathcal{M}_0 = \begin{pmatrix} \cos(2\pi Q) + \alpha_0 \sin(2\pi Q) & \beta_0 \sin(2\pi Q) \\ -\gamma_0 \sin(2\pi Q) & \cos(2\pi Q) - \alpha_0 \sin(2\pi Q) \end{pmatrix}$$

- Consider a **gradient error in a quad**. We can take the error into account by adding a thin lens quadrupole to the one turn matrix. The new 1-turn matrix is

$$\mathcal{M} = \begin{pmatrix} 1 & 0 \\ -\delta K ds & 1 \end{pmatrix} \mathcal{M}_0$$

which yields

$$\mathcal{M} = \begin{pmatrix} \cos(2\pi Q) + \alpha_0 \sin(2\pi Q) & \beta_0 \sin(2\pi Q) \\ -\delta K ds(\cos(2\pi Q) + \alpha_0 \sin(2\pi Q)) - \gamma_0 \sin(2\pi Q) & \cos(2\pi Q) - (\delta K ds \beta_0 + \alpha_0) \sin(2\pi Q) \end{pmatrix}$$

Gradient error and tune-shift

- Can also be written as a new matrix with a new tune $\chi = 2\pi(Q + \delta Q)$

$$\mathcal{M}^\star = \begin{pmatrix} \cos(\chi) + \alpha_0 \sin(\chi) & \beta_0 \sin(\chi) \\ -\gamma_0 \sin(\chi) & \cos(\chi) - \alpha_0 \sin(\chi) \end{pmatrix}$$

- The traces of the two matrices describing the 1-turn should be equal, i.e. $\mathrm{trace}\,(\mathcal{M}^*) = \mathrm{trace}\,(\mathcal{M})$ which gives:

$$2\cos(2\pi Q) - \delta K ds \beta_0 \sin(2\pi Q) = 2\cos(2\pi(Q + \delta Q))$$

- Developing the right-hand side: $$\cos(2\pi(Q + \delta Q)) = \cos(2\pi Q)\underbrace{\cos(2\pi\delta Q)}_{\approx 1} - \sin(2\pi Q)\underbrace{\sin(2\pi\delta Q)}_{\approx 2\pi\delta Q}$$

- Which finally gives: $4\pi\delta Q = \delta K ds \beta_0$

  - I.e., for a quadrupole of length *l* the tune shift is: $$\delta Q = \frac{1}{4\pi}\int_{s_0}^{s_0+l} \delta K \beta_0 ds$$

  - For distributed quadrupole errors, the tune shift is: $$\boxed{\delta Q = \frac{1}{4\pi}\oint \delta K(s)\beta(s)ds}$$

Gradient error and beta distortion

- Consider the unperturbed transfer matrix for one turn

$$M_0 = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = B \cdot A \quad \text{with} \quad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

- Introduce a gradient perturbation between the two matrices

$$\mathcal{M}_0^\star = \begin{pmatrix} m_{11}^\star & m_{12}^\star \\ m_{21}^\star & m_{22}^\star \end{pmatrix} = B \begin{pmatrix} 1 & 0 \\ -\delta K ds & 1 \end{pmatrix} A$$

- Recall that $m_{12} = \beta_0 \sin(2\pi Q)$ and write the perturbed term as

a) $$m_{12}^\star = (\beta_0 + \delta\beta)\sin(2\pi(Q+\delta Q)) = m_{12} + \delta\beta \sin(2\pi Q) + 2\pi\delta Q \beta_0 \cos(2\pi Q)$$

- where we used $\sin(2\pi\delta Q) \approx 2\pi\delta Q$ and $\cos(2\pi\delta Q) \approx 1$ and

$$\sin(2\pi(Q+\delta Q)) = \sin(2\pi Q)\cos(2\pi\delta Q) + \cos(2\pi Q)\sin(2\pi\delta Q)$$

Gradient error and beta distortion

- On the other hand $a_{12} = \sqrt{\beta_0 \beta(s_1)} \sin\psi, \; b_{12} = \sqrt{\beta_0 \beta(s_1)} \sin(2\pi Q - \psi)$

b) $$m^{\star}_{12} = \underbrace{b_{11}a_{12} + b_{12}a_{22}}_{m_{12}} - a_{12}b_{12}\delta K ds = m_{12} - a_{12}b_{12}\delta K ds$$

- Equating the two terms **a) = b)**

$$\delta\beta \sin(2\pi Q) + 2\pi\delta Q \beta_0 \cos(2\pi Q) = -\beta_0\beta(s_1)\sin\psi \sin(2\pi Q - \psi)\delta K ds$$
$$\delta\beta \sin(2\pi Q) + \frac{1}{2}\delta K ds \beta_0 \beta(s_1)\cos(2\pi Q) = -\beta_0\beta(s_1)\sin\psi \sin(2\pi Q - \psi)\delta K ds$$

- using $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$ and integrating yields

$$\frac{\delta\beta}{\beta_0} = -\frac{1}{2\sin(2\pi Q)}\int_{s_1}^{s_1+C}\beta(s)\delta K(s)\cos(2\psi - 2\pi Q)ds$$

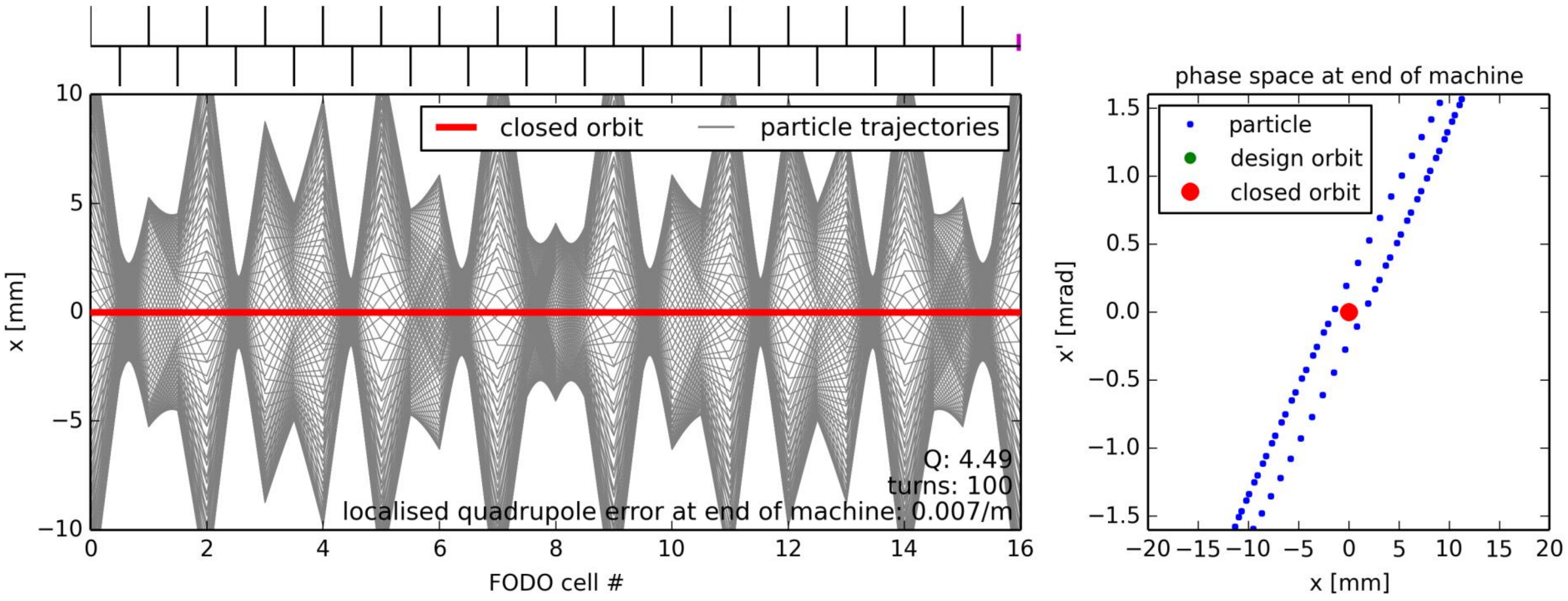
- for distributed errors around the machine

$$\boxed{\frac{\delta\beta(s)}{\beta(s)} = -\frac{1}{2\sin(2\pi Q)}\int_{s}^{s+C}\beta(s_1)\delta K(s_1)\cos(|2\psi(s_1)) - 2\psi(s)| - 2\pi Q)ds_1}$$

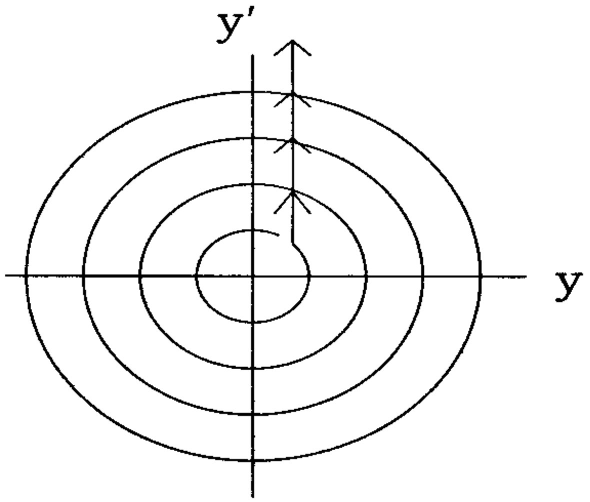
Optics distortion vs. tune

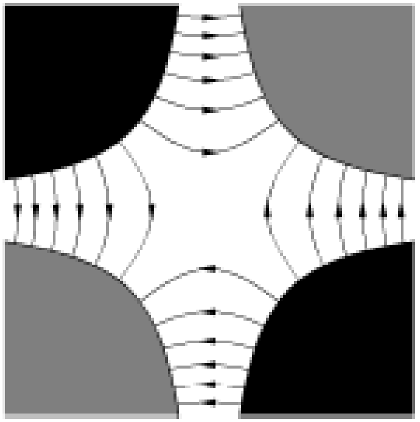
- Quadrupole errors have biggest impact **close to integer and half integer tunes → envelope (or beam size) becomes unstable**

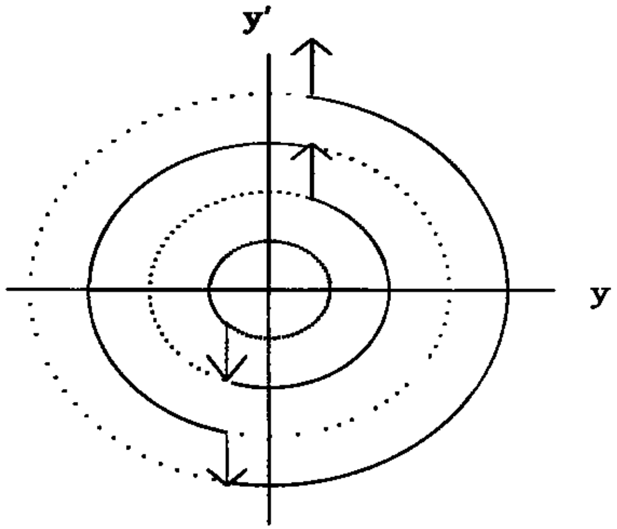
$$\frac{\delta\beta}{\beta_0} = -\frac{1}{2\sin(2\pi Q)}\int_{s_1}^{s_1+C}\beta(s)\delta K(s)\cos(2\psi - 2\pi Q)ds$$



Quadrupole error in phase space







Q = n (integer)
→ kicks from quadrupoles add up
(same as for kicks from dipoles)

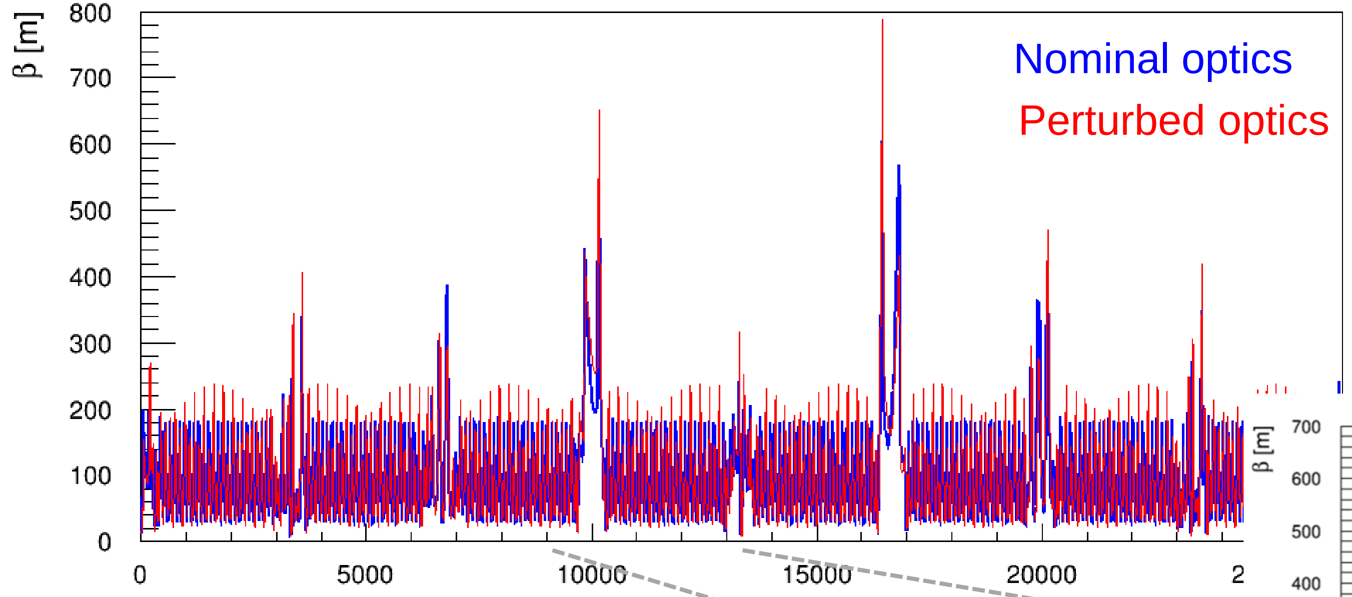
Q = n/2 (half integer)
→ kicks from quadrupoles also add up
(while kicks from dipoles cancel)

- Therefore, **integer tunes and half integer tunes need to be avoided for machine operation** to avoid beam envelope becoming unstable due to quadrupole errors
- *Recall:* for integer tunes dipole errors drive the closed orbit unstable, but for half integer tunes they have minimum effect

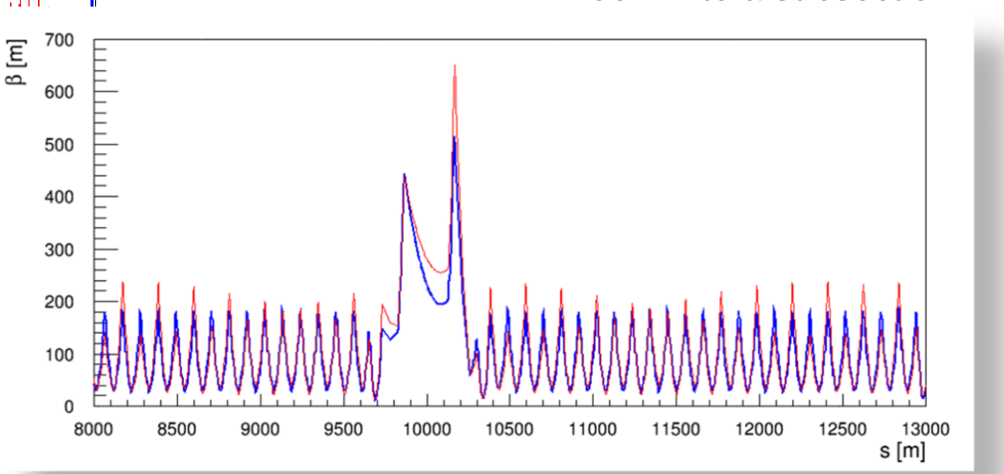
Optics distortion characteristics

- Let's take a look at the LHC …

Example: one quadrupole gradient is incorrect

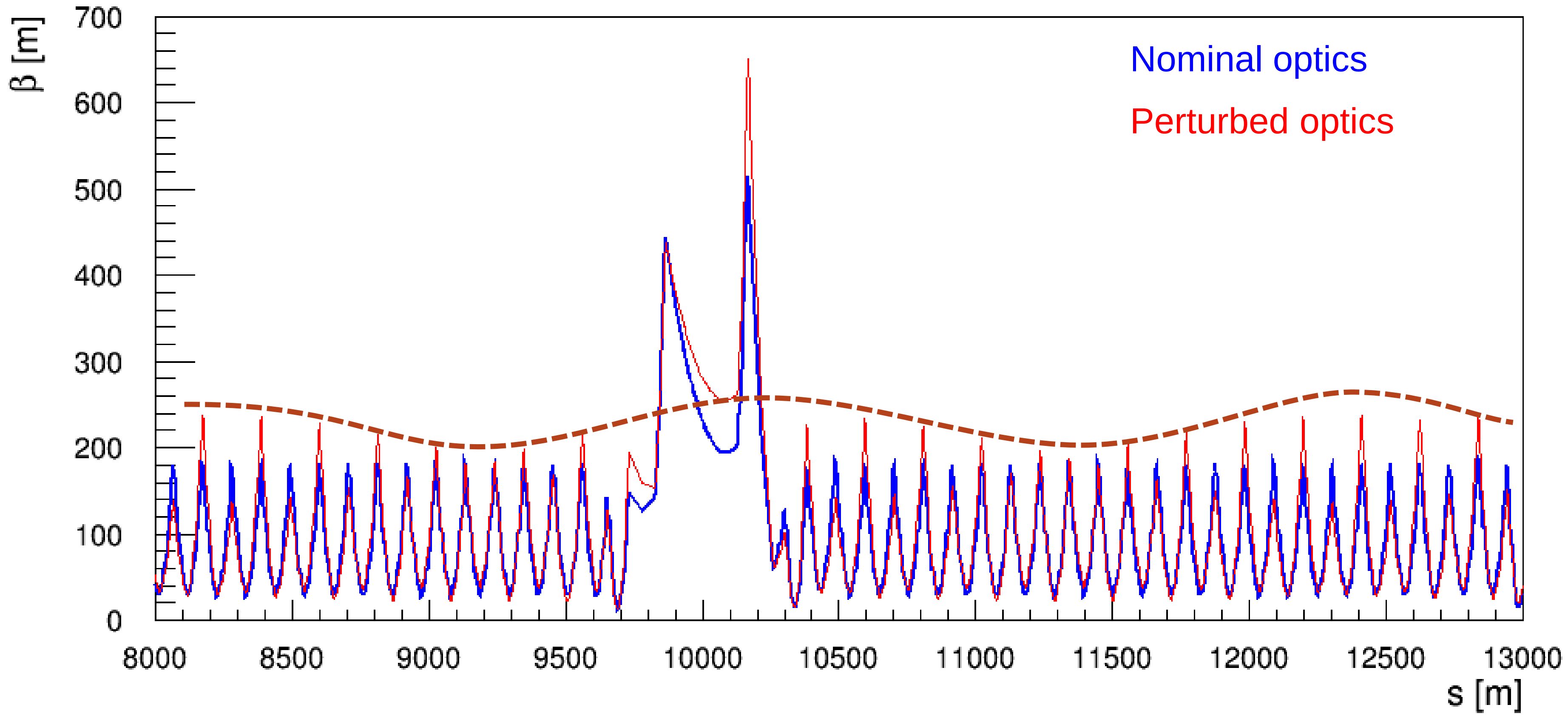


Zoom into a subsection



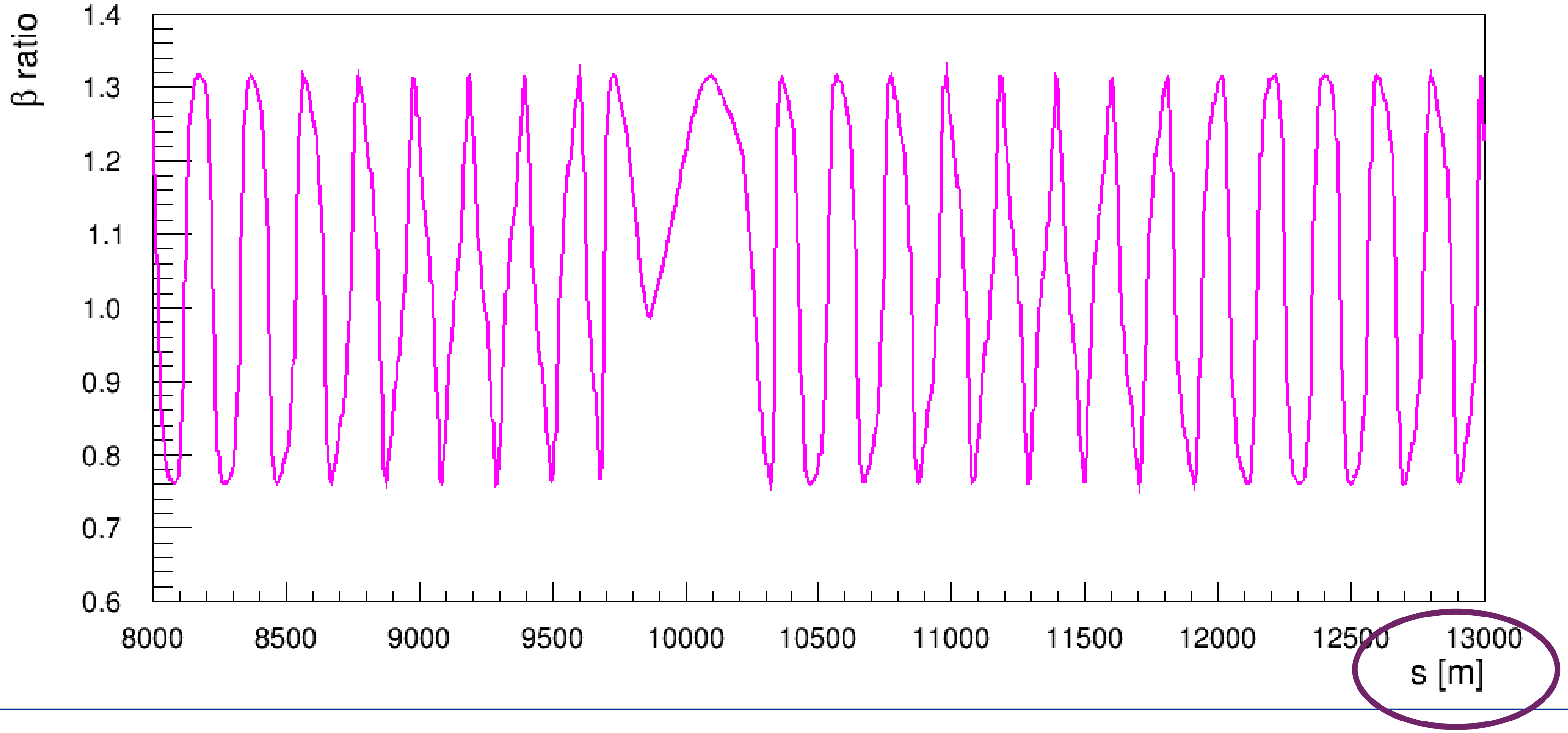
Optics distortion characteristics

- The local beam optics perturbation
- … note the oscillating pattern



Optics distortion characteristics

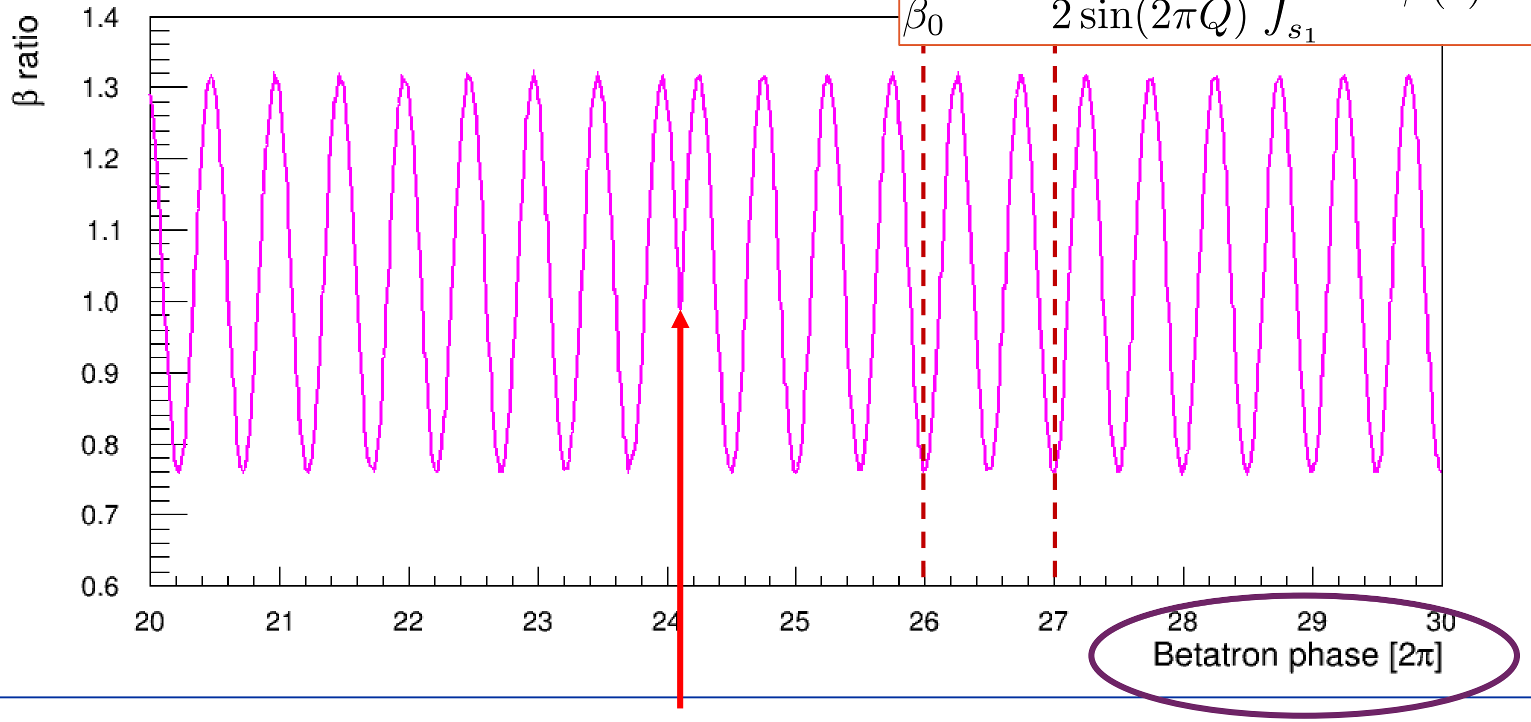
- The error is easier to analyse and diagnose if one considers the ratio of the betatron function perturbed/nominal.
- The ratio reveals an oscillating pattern called the **betatron function beating** (‘beta-beating’).
  - **Note: the amplitude of the perturbation is the same all over the ring!**



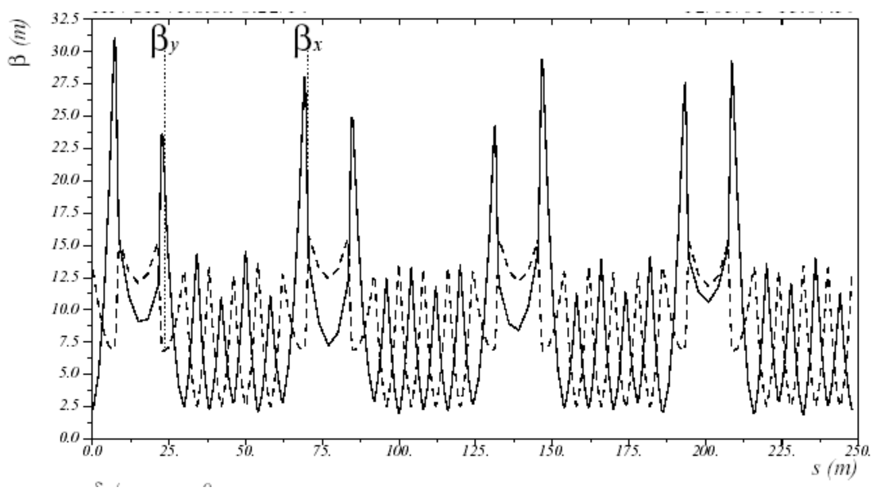
Optics distortion characteristics

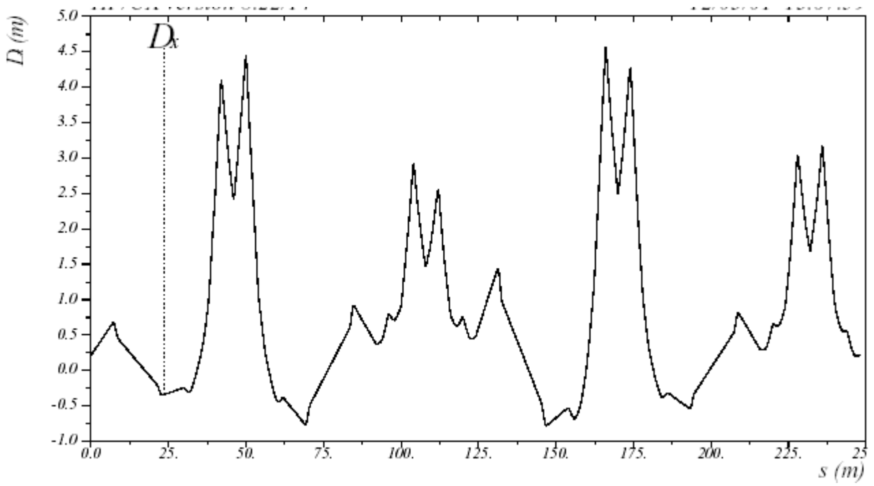
- The beta-beating pattern comes out even more clearly if we replace the longitudinal coordinate with the betatron phase advance
  - The result is very similar to the case of the closed orbit kick, **the error reveals itself by a kink**!
  - If you watch closely, you will observe that there **are two oscillation periods per 2π (360 deg) phase**. The beta-beating frequency is twice the frequency of the orbit!

$$\frac{\delta\beta}{\beta_0} = -\frac{1}{2\sin(2\pi Q)}\int_{s_1}^{s_1+C}\beta(s)\delta K(s)\cos(2\psi - 2\pi Q)ds$$



Example: Gradient error in SNS





- Consider **18 focusing quads** in the SNS ring with **0.01 T/m** systematic gradient error (wrt to **nominal 5 T/m**). In this location ***β* = 12 m**. The length of the quads is **1 m,** the magnetic rigidity is **5.6567 Tm,** and the tune is **Q = 6.2**
  - The tune-shift is: $\delta Q = \frac{1}{4\pi}\sum_i \delta k_i(s)\beta_i(s) = \frac{1}{4\pi}18\cdot 12\frac{0.01}{5.6567}1 = 0.03$
- For a random distribution of errors of 1% of the nominal gradient, the beta beating is:

$$\left(\frac{\delta\beta}{\beta_0}\right)_{\rm rms} = \frac{1}{2\sqrt{2}|\sin(2\pi Q)|}\left(\sum_i \delta k_i^2\beta_i^2\right)^{1/2} = \frac{1}{2\sqrt{2}|\sin(2\pi 6.2)|}\sqrt{18}\cdot 12\frac{0.05}{5.6567}1 = 0.17$$

  - Optics functions beating **~20%** by 1% random errors (1% of gradient)!

Gradient error correction

Quadrupole correctors

- Individual correction magnets
- Windings on the core of the quadrupoles (trim windings)
- Pairs of correctors at well-chosen locations for minimizing resonance

Methods & approaches

- Compute tune-shift and optics function beta distortion
- Move working point close to integer and half integer resonance to increase sensitivity
- Minimize beta wave or quadrupole resonance width with trim windings
- Individual powering of trim windings can provide flexibility and beam-based alignment of BPM

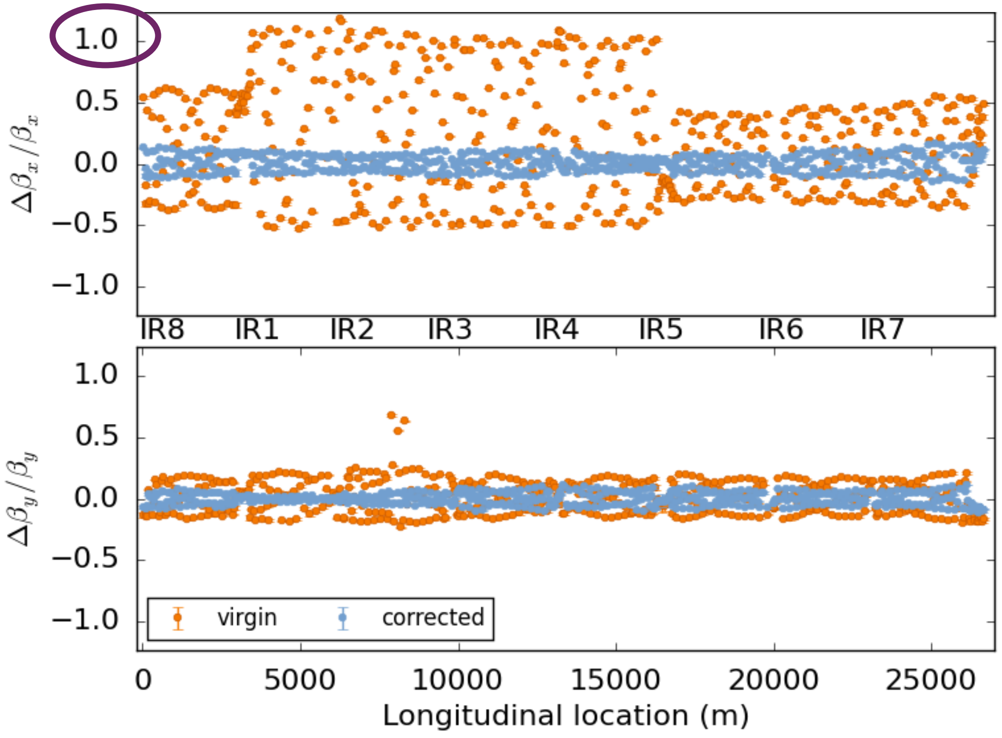
Modern methods of response matrix analysis (LOCO) can fit optics model to real machine and correct optics distortion

Example: LHC optics corrections

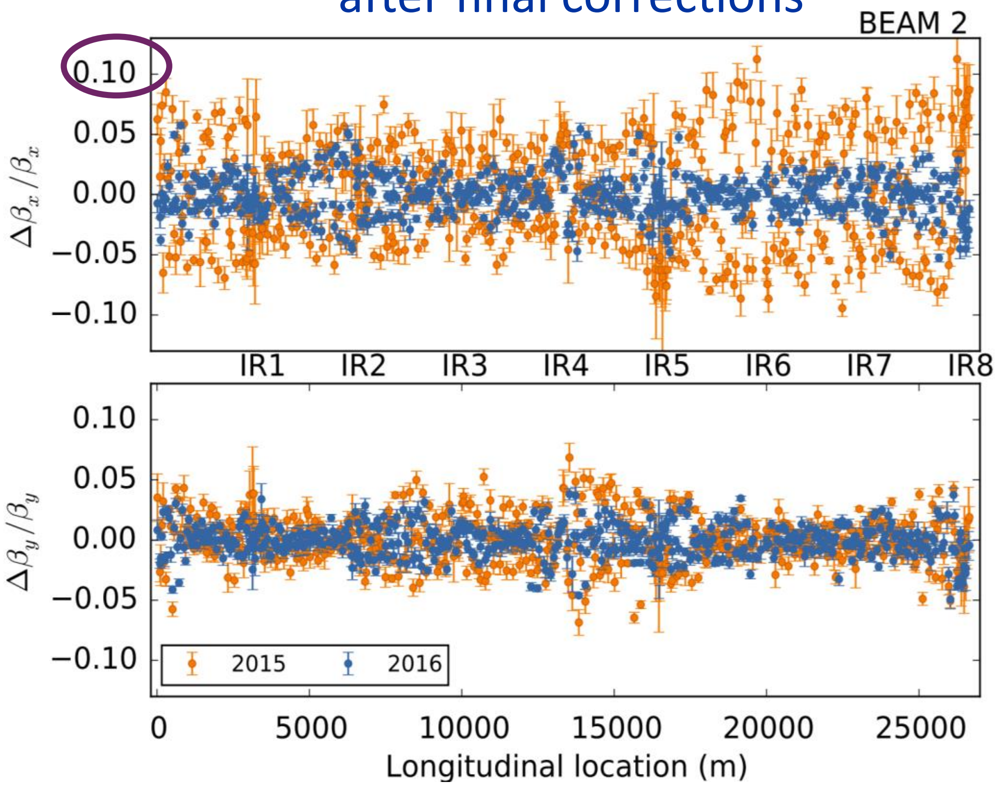
- At β*=40cm, the bare machine has a beta-beat of more than 100%
- After global and local corrections, β-beating was reduced to few %

R. Tomas et al.

before and after local correction



after final corrections



Example: PSB half integer resonance correction

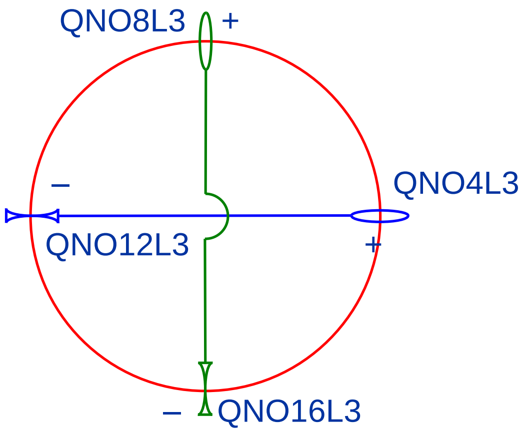
Compensation of quadrupole errors at half integer Q_y=4.5

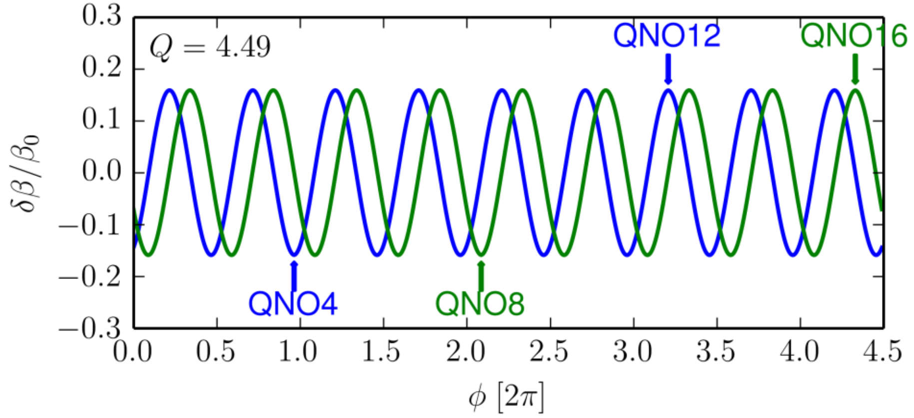
- PSB has 16-fold symmetry
- 2 families of normal quadrupole correctors
  - **+QNO4 and –QNO12** with $\Delta\mu_y = 2.25 * 2\pi$
  - **+QNO8 and –QNO16** with $\Delta\mu_y = 2.25 * 2\pi$

$$\frac{\delta\beta(s)}{\beta(s)} = -\frac{1}{2\sin(2\pi Q)}\int_s^{s+C}\beta(s_1)\delta K(s_1)\cos(|2\psi(s_1)) - 2\psi(s)| - 2\pi Q)ds_1$$

$$\delta Q = \frac{1}{4\pi}\oint \delta K(s)\beta(s)ds$$

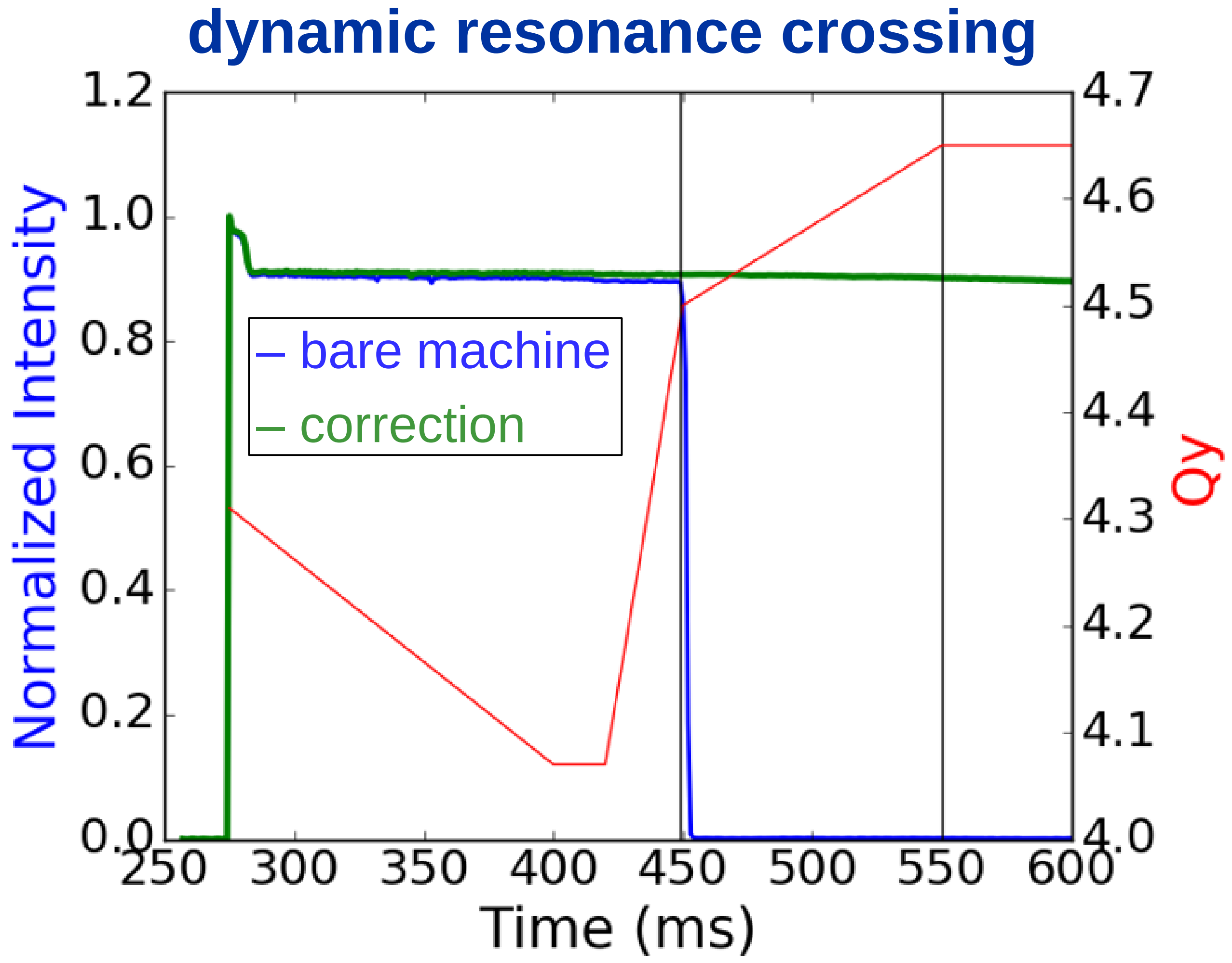
- **Due to opposite polarity within each family, their contribution on beta-beating adds up** (beta-beat frequency is twice the tune!) **while there is no change of tune** (same change of focusing & defocusing)
- The two families are orthogonal with respect to the half integer resonance driving term (every phase achievable)





PSB half integer resonance correction

Experimental data!

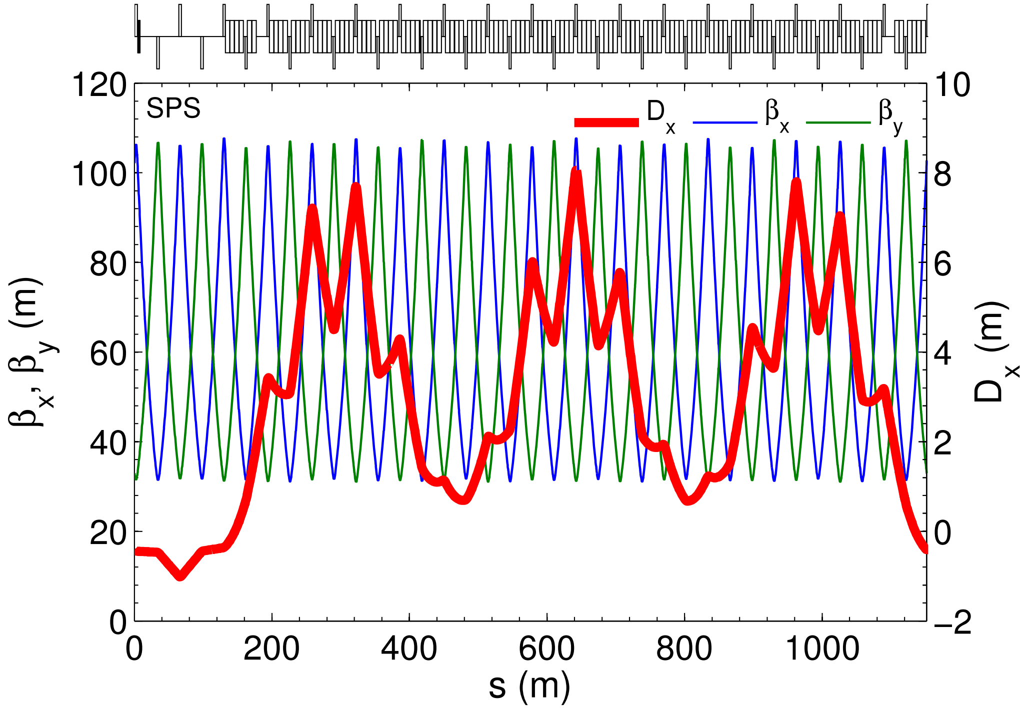


F. Asvesta

Problem 5

The SPS is a **400 GeV proton synchrotron** with a FODO lattice consisting of **108** focusing and **108** defocusing quadrupoles of length **3.22 m** and a gradient of **15 T/m**, with a **horizontal and vertical beta of 108 m and 30 m** in the focusing quads (**30 m and 108 m** for the defocusing ones). The tunes are **Q_x=20.13** and **Q_y=20.18**.

1. Find the tune shift for **systematic** gradient errors of 1% in the focusing and 0.5% in the defocusing quads
2. Find the $\boldsymbol{\beta}_x$ and $\boldsymbol{\beta}_y$ rms beating for **rms** gradient errors of 1% in both focusing and defocusing quads



Outline

Introduction

Closed orbit distortion (steering error)

- ❑ Beam orbit stability
- ❑ Imperfections leading to closed orbit distortion
- ❑ Effect of single and multiple dipole kicks
- ❑ Closed orbit correction methods

Optics function distortion (gradient error)

- ❑ Imperfections leading to optics distortion
- ❑ Tune-shift and beta distortion due to gradient errors
- ❑ Gradient error correction

Coupling error

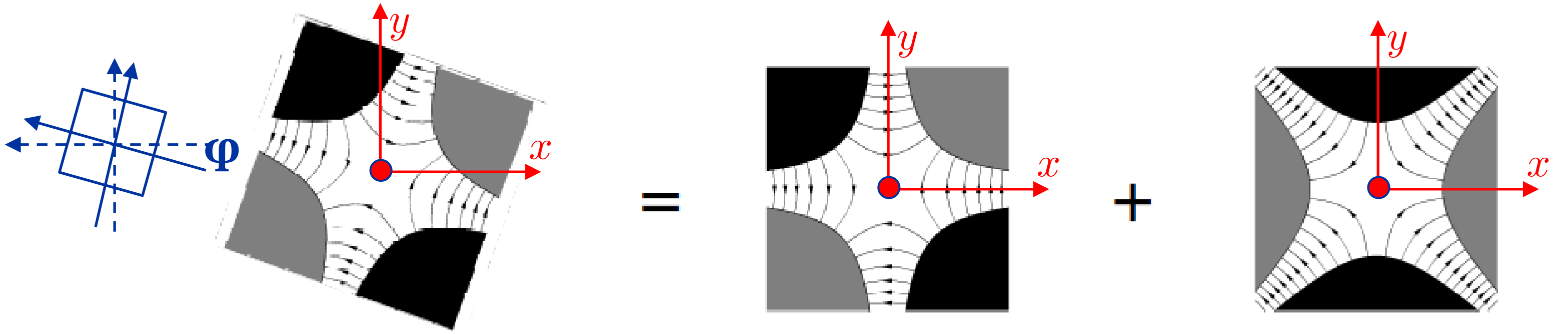
- ❑ Coupling errors and their effect
- ❑ Coupling correction

Summary

Coupling errors

- Coupling may result from **rotation of a quadrupole**, so that the field contains a skew quadrupole component

tilted quadrupole *normal quadrupole* *skew quadrupole*

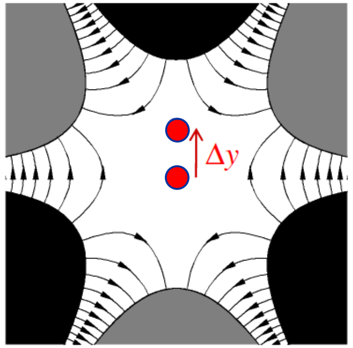


- A **systematic vertical offset in a sextupole** has the same effect as a skew quadrupole.
  - For a displacement of δy the field becomes

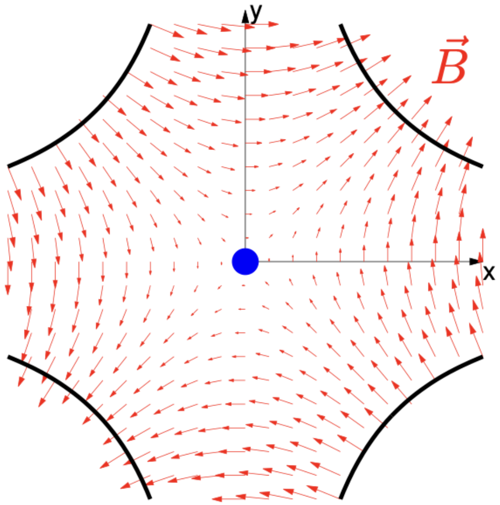
$$B_x = 2B_2 x\bar{y} = 2B_2 xy + \underbrace{2B_2 x\delta y}$$

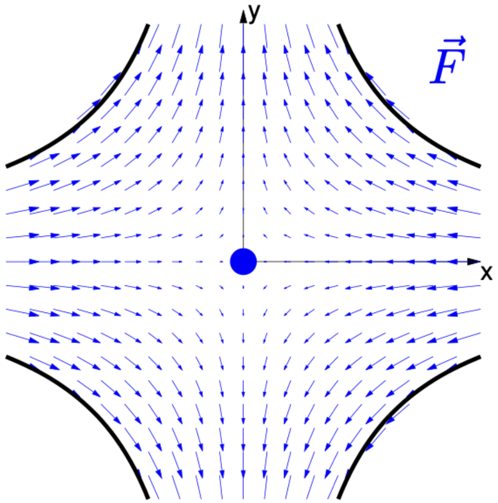
skew quadrupole

$$B_y = B_2(x^2 - \bar{y}^2) = \overbrace{-2B_2 y\delta y} + B_2(x^2 - y^2) - B_2(\delta y)^2$$



Normal vs. skew quadrupole



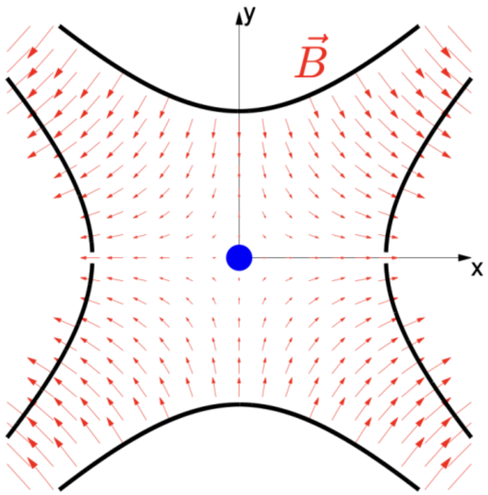


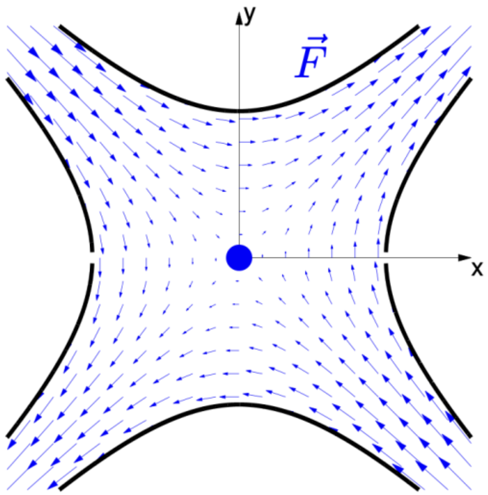
normal quadrupole

$$F_x = -kx$$

$$F_y = +ky$$

horizontal force depends on horizontal position (and likewise for vertical)





skew quadrupole

$$F_x = k_s y$$

$$F_y = k_s x$$

horizontal force depends on vertical position (and vice versa)

→ resulting forces couples the motion in the two planes

4x4 Matrices - uncoupled

- Starting from uncoupled motion, we have the transport matrices for each transverse plane, which we can write as

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \begin{pmatrix} C_x(s) & S_x(s) \\ C'_x(s) & S'_x(s) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

$$\begin{pmatrix} y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C_y(s) & S_y(s) \\ C'_y(s) & S'_y(s) \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

- We can write a 4x4 matrix describing the uncoupled motion

$$\begin{pmatrix} x(s) \\ x'(s) \\ y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C_x(s) & S_x(s) & 0 & 0 \\ C'_x(s) & S'_x(s) & 0 & 0 \\ 0 & 0 & C_y(s) & S_y(s) \\ 0 & 0 & C'_y(s) & S'_y(s) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \end{pmatrix}$$

Uncoupled motion – no cross-talk between planes

4x4 Matrices – with skew quadrupole

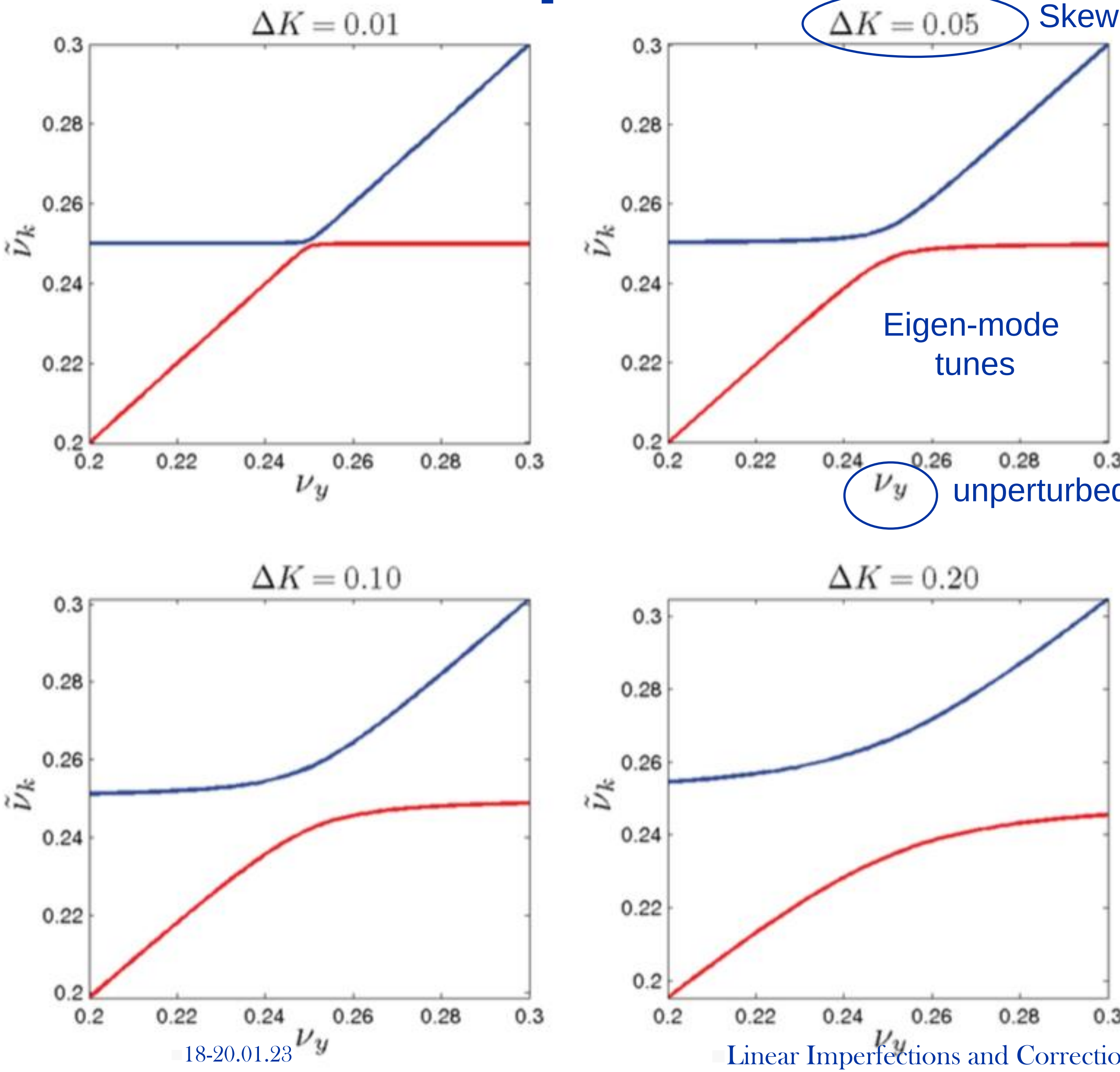
- In presence of a **thin skew quadrupole** the transport matrix becomes

$$\begin{pmatrix} x(s) \\ x'(s) \\ y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C_x(s) & S_x(s) & 0 & 0 \\ C'_x(s) & S'_x(s) & 0 & 0 \\ 0 & 0 & C_y(s) & S_y(s) \\ 0 & 0 & C'_y(s) & S'_y(s) \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\delta K_s L & 0 \\ 0 & 0 & 1 & 0 \\ -\delta K_s L & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \end{pmatrix}$$

These terms from the skew quadrupole
couples the motion in the two planes

- In the presence of coupling, the motion is still linear but has **two new eigen-mode tunes**
- By adjusting quadrupole strengths, we can adjust the tunes. Let us suppose we keep the horizontal tune fixed and vary the vertical tune. Then we plot the eigen-mode tunes (as obtained from the eigenvalues of the new one-turn matrix) as a function of the unperturbed vertical tune for a fixed value $\delta k_s ds$ …

Eigen-mode tunes computation

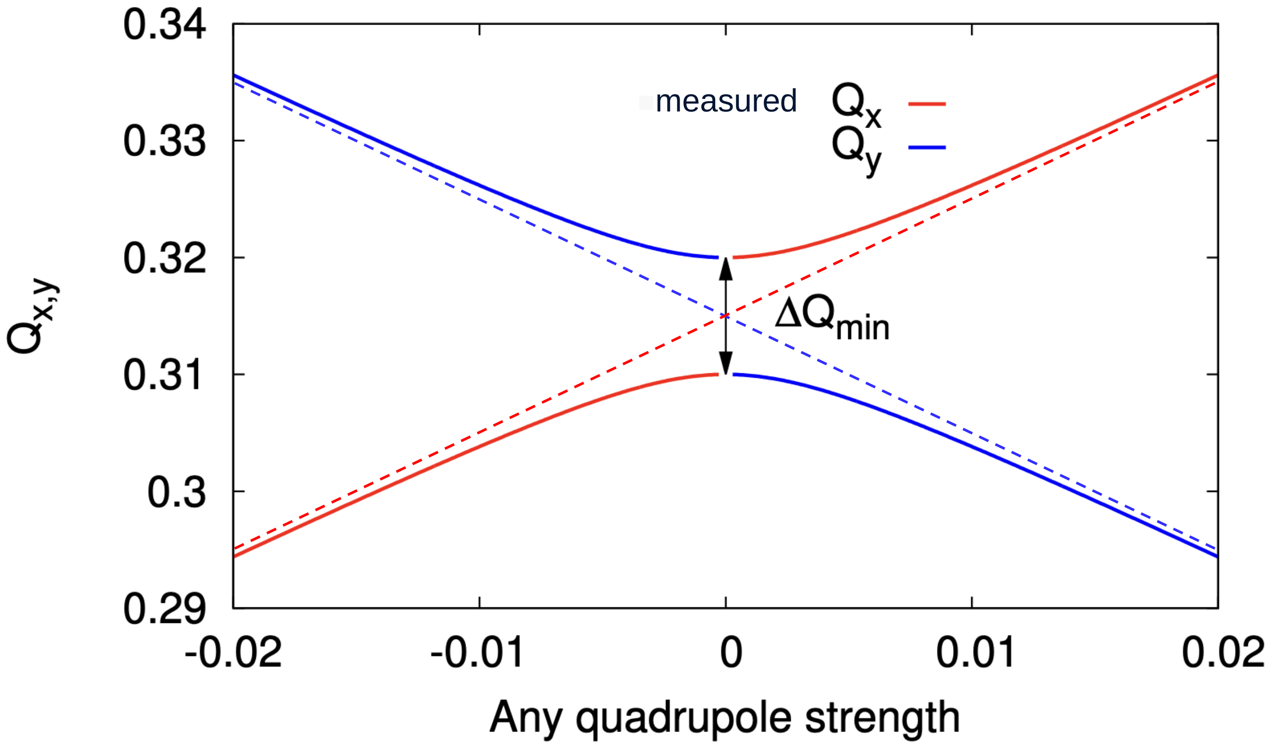


For increasing skew quadrupole strength, the distance between the two eigen-mode tunes increases!

Minimum tune separation

- It can be shown that the distance between the “perturbed” tunes at equal unperturbed tunes (i.e. “on the coupling resonance”) is given by

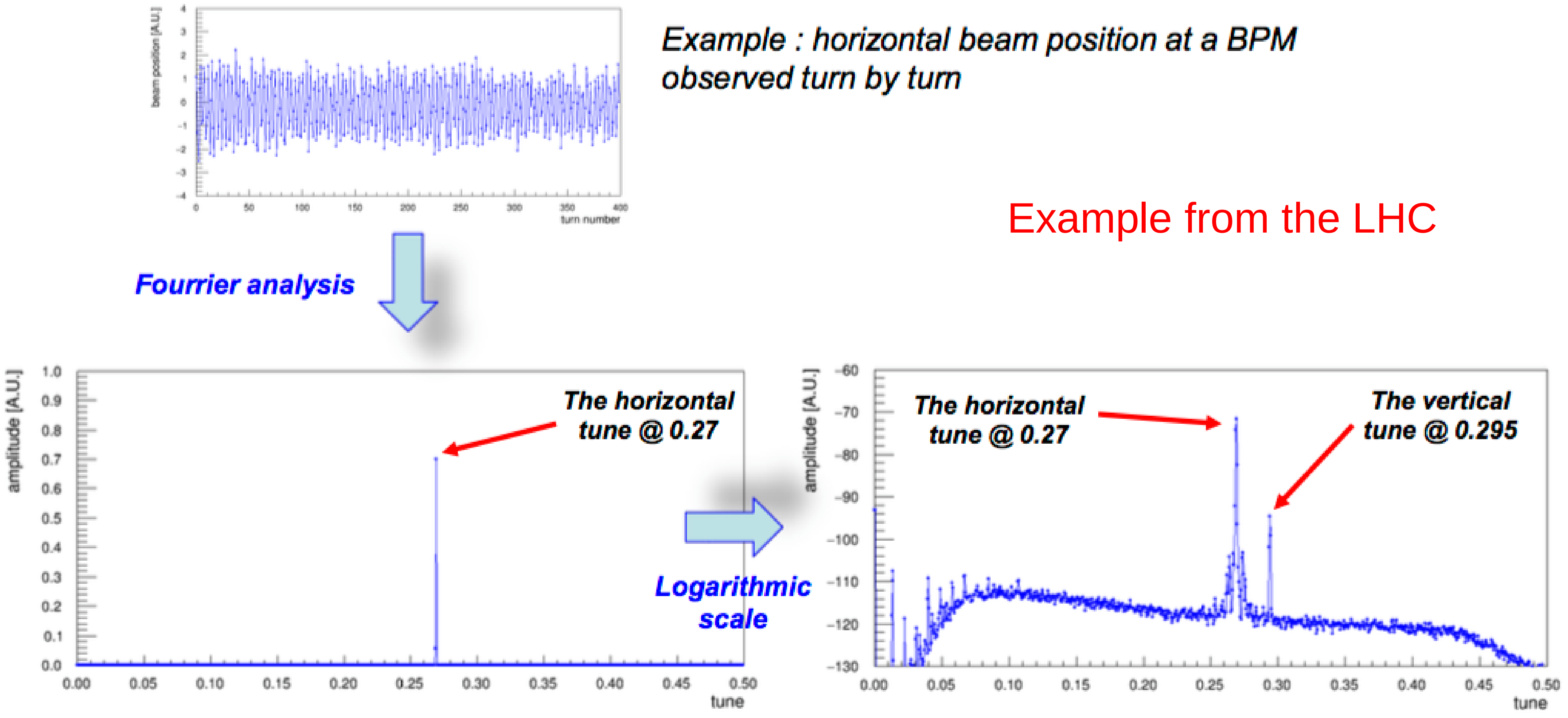
$$\Delta Q_{\min} \approx \frac{\sqrt{\beta_x \beta_y}}{2\pi} |\delta K_s| L$$



- In other words, there is a minimum tune separation between the eigen-mode tunes (which is determined by the so-called *coupling coefficient*)

Coupling and tune observation

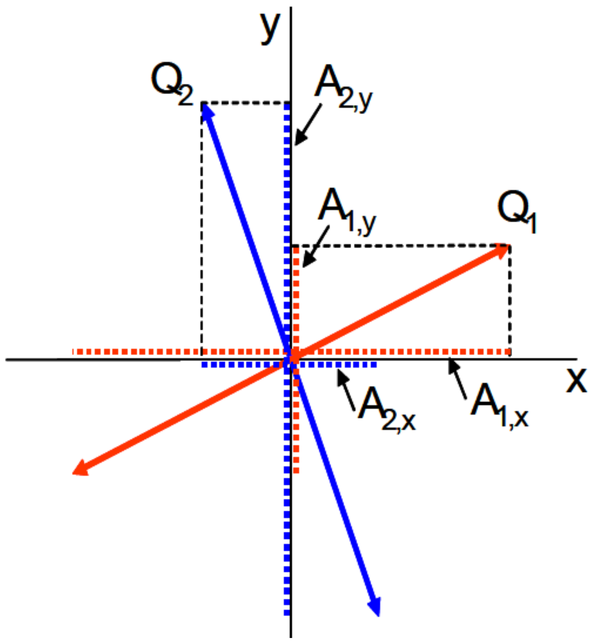
- The simplest method to determine if there is coupling is to excite a beam oscillation in one plane (by kicking the beam), and then observe the oscillations or the frequency content
- If coupling is present, then for a horizontal kick there will be a small vertical oscillation (and vice- versa).



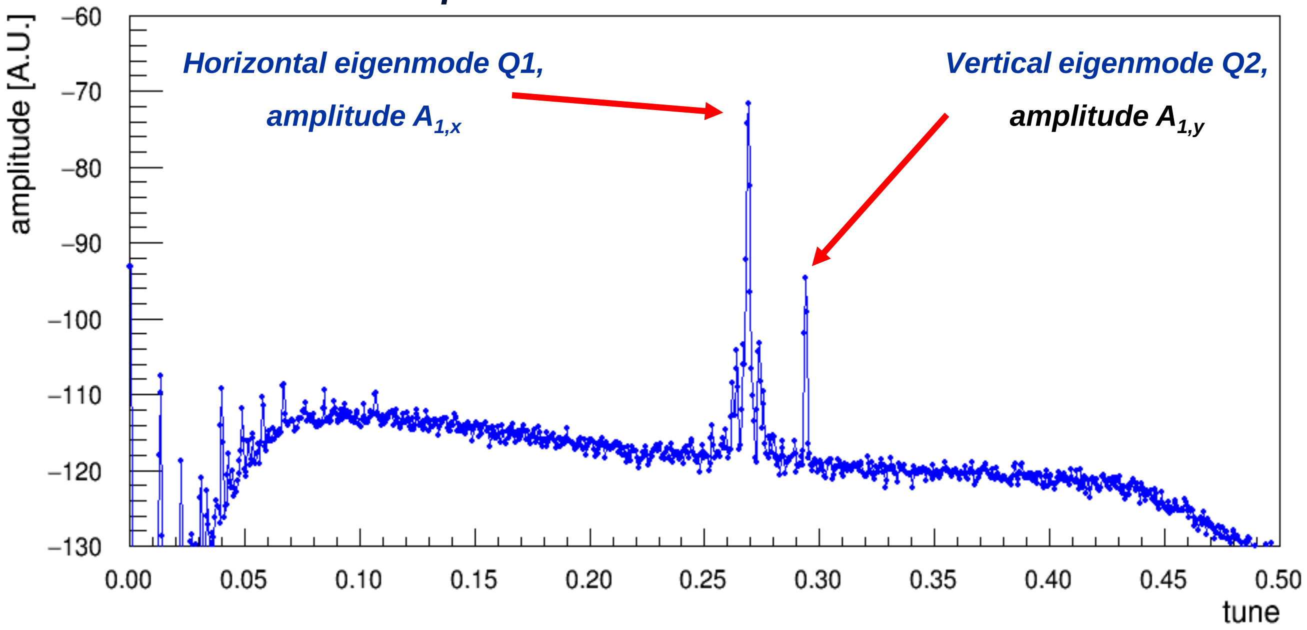
Local coupling measurement

A **first technique** to characterise the coupling coefficient C^- consists in measuring the **crossed tune peak amplitudes**:

- Vertical tune in horizontal spectrum and vice-versa.
- Simple measurement, but no phase information.
- **Only the local coupling** is obtained, which **can differ from the global coupling**.



Horizontal tune spectrum at the LHC



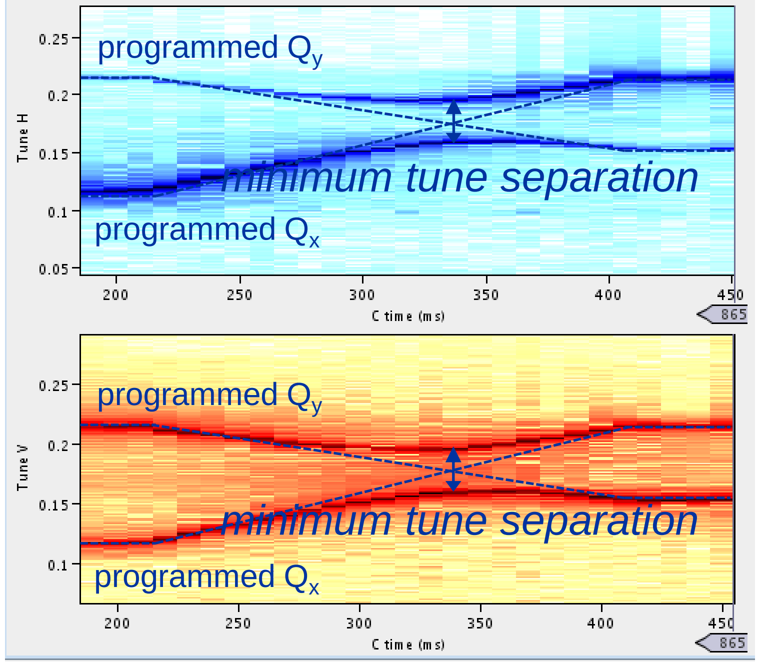
$$\|C^-\| = \frac{2\sqrt{r_1 r_2}\,|Q_1 - Q_2|}{1 + r_1 r_2}$$

$$r_1 = \frac{A_{1,y}}{A_{1,x}} \qquad r_2 = \frac{A_{2,x}}{A_{2,y}}$$

Closest tune approach

Tune measurement in the CERN PS

quadrupole setting changed dynamically during storage time



tune peaks from both planes visible in Fourier spectra of horizontal and vertical motion

- Coupling makes it impossible to approach the tunes below $\Delta Q_{min} = |C^-|$ where C^- is the coupling coefficient
- The coupling coefficient C^- can be measured by trying to approach the tunes and measure the minimum distance

Linear coupling correction

Coupling correctors

- Introduce skew quadrupoles into the lattice
- If skew quadrupoles are not available, one can make vertical closed orbit bumps in sextuple magnets (used in JPARC main ring until installation of skew quadrupole correctors)

Methods & approaches

- Correct globally/locally coupling coefficient (or resonance driving term)
- Correct optics distortion (e.g. vertical dispersion)
- Move working point close to coupling resonances and repeat

Remarks

- Correction especially important for beams with unequal emittances “flat beams” (coupling leads to emittance exchange)
- The (vertical) orbit correction may be critical for reducing coupling (e.g. due to feed-down sextupoles)

Problem 6

The SPS is a **400 GeV proton synchrotron** with a FODO lattice consisting of **108** focusing and **108** defocusing quadrupoles of length **3.22 m** and a gradient of **15 T/m**, with a **horizontal and vertical beta of 108 m and 30 m** in the focusing quads (**30 m and 108 m** for the defocusing ones). The tunes are **Q_x=20.13** and **Q_y=20.18**.
In order to correct its natural chromaticity, several **0.42 m long sextupoles** are installed next to focusing and defocusing quadrupoles at locations with high dispersion.
Assume that **one** of those sextupoles installed next to a **focusing quad** has a gradient of **60.3 T/m^2** and it is **vertically misaligned by δy=10 mm.** Assume that the **beta functions at the sextupole** are equal to the one at the **nearby quadrupole**

1. What is the **normalized sextuple strength**?
2. Compute the impact of the vertical misalignment on: **tune**, **max beta beating**, **minimum tune separation**, **max closed orbit** deviation
(neglect next order effect of such an orbit on transverse optics due to other machine sextupoles…)
3. Repeat for the case in which the **sextupole is displaced horizontally**
4. What would be the **maximum closed orbit deviation** if only one **focusing quadrupole** would be **vertically displaced by -δy=10 mm**? Qualitatively, would you expect some effect on coupling or tune or beta-beating in such a case?

Outline

Introduction

Closed orbit distortion (steering error)

- ❑ Beam orbit stability
- ❑ Imperfections leading to closed orbit distortion
- ❑ Effect of single and multiple dipole kicks
- ❑ Closed orbit correction methods

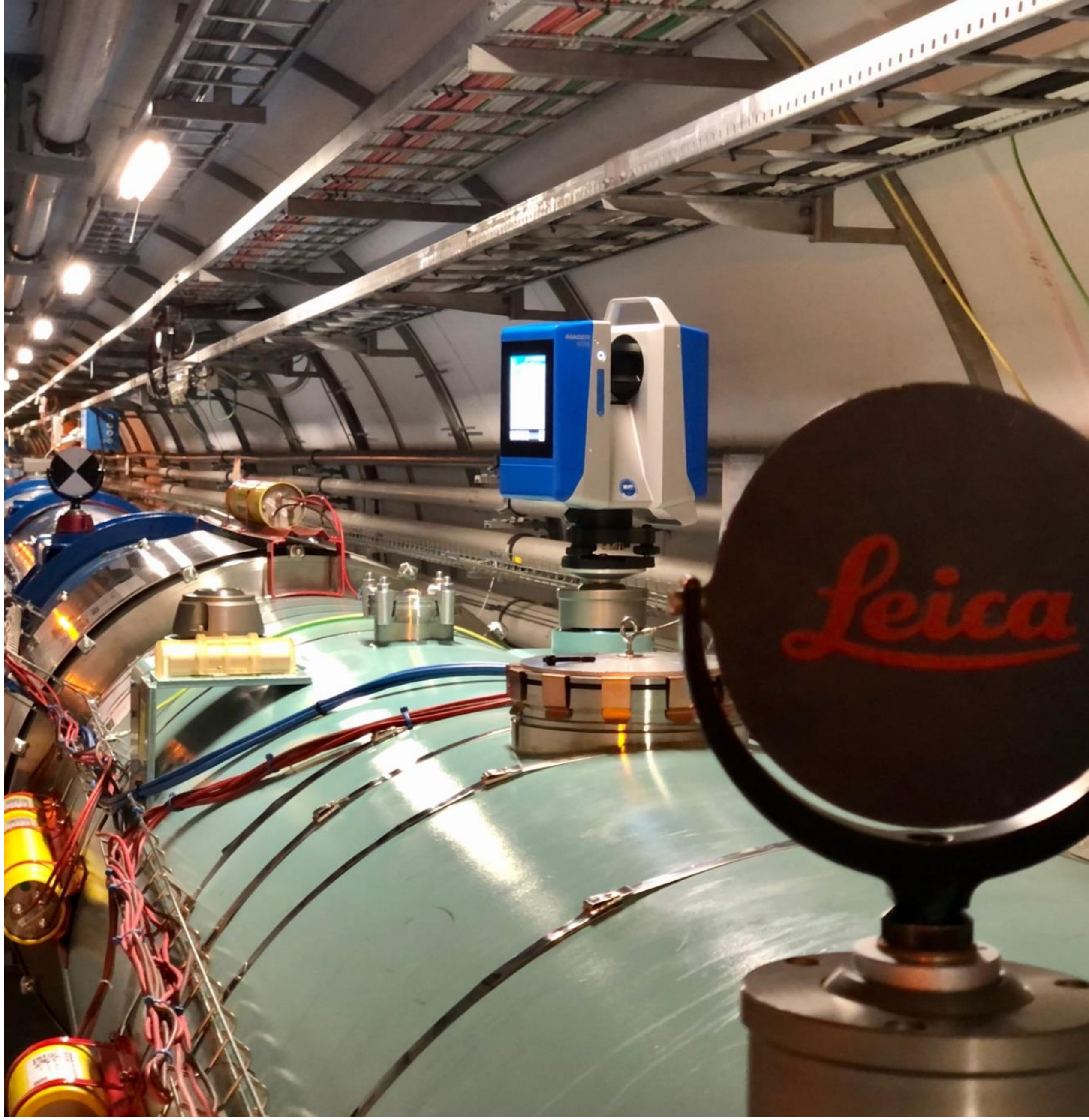
Optics function distortion (gradient error)

- ❑ Imperfections leading to optics distortion
- ❑ Tune-shift and beta distortion due to gradient errors
- ❑ Gradient error correction

Coupling error

- ❑ Coupling errors and their effect
- ❑ Coupling correction

Summary



Summary of linear imperfections

- **Linear imperfections,** such as magnet misalignments and field errors, **are unavoidable** in a real accelerator, but they can be corrected to some extent as summarized in this table:

Error	Effect	Cure
fabrication imperfections	unwanted multipolar components	better fabrication / multipolar correctors coils
transverse misalignments	feed-down effect	better alignment / correctors
dipole kicks	orbit distortion / residual dispersion	corrector dipoles
quadrupole field errors	tune shift, beta-beating	trim special quadrupoles
quadrupole tilts	coupling x − y	better alignment / skew quads
power supplies	closed orbit distortion / tune shift / modulation	improve power supplies and their calibration

A few useful formulas

Beam Rigidity

$$B\rho\,[\mathrm{Tm}] = 3.3356\,\beta_r E\,[\mathrm{GeV}]/q$$
$$= 3.3356\,p\,[\mathrm{GeV/c}]c/q$$

Closed orbit variation due to single kick

$$\Delta u_s = \theta_{s0}\frac{\sqrt{\beta_s\beta_{s0}}}{2\sin(\pi Q)}\cos(\pi Q - |\psi_s - \psi_{s0}|)$$

rms orbit due to rms kicks

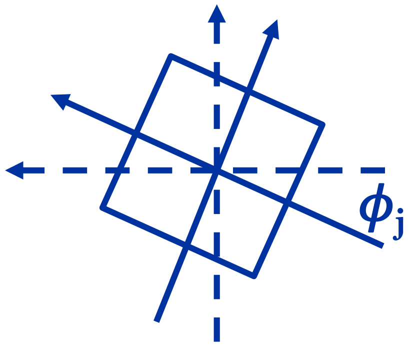
$$u_{\mathrm{rms}}(s) = \frac{\sqrt{N\beta(s)\beta_{\mathrm{rms}}}}{2\sqrt{2}|\sin(\pi Q)|}\theta_{\mathrm{rms}}$$

3-bump kicks

$$\frac{\sqrt{\beta_1}}{\sin\psi_{23}}\theta_1 = \frac{\sqrt{\beta_2}}{\sin\psi_{31}}\theta_2 = \frac{\sqrt{\beta_3}}{\sin\psi_{12}}\theta_3$$

Kicks from field error/misalignments

- **Integrated dipole field error** $$\theta_j = \frac{\delta(B_j l_j)}{B\rho}$$
- **Dipole roll** $$\theta_j = \frac{B_j l_j \sin\phi_j}{B\rho}$$
- **Quadrupole displacement** $$\theta_j = \frac{G_j l_j \delta u_j}{B\rho}$$



Tune change due to quad error

$$\delta Q_u = \frac{1}{4\pi}\sum_i \beta_u K_i\left(\frac{\delta K}{K}\right)_i l_i$$

Minimum tune approach due to skew error

$$\Delta Q_{\min} \approx \frac{\sqrt{\beta_x\beta_y}}{2\pi}|\delta K_s|L$$

Beta beat due to several quad errors

$$\frac{\delta\beta(s)}{\beta(s)} = -\frac{1}{2\sin(2\pi Q)}\int_s^{s+C}\beta(s_1)\delta K(s_1)\cos(|2\psi(s_1)) - 2\psi(s)| - 2\pi Q)ds_1$$

Rms beta beating

$$\left(\frac{\delta\beta}{\beta_0}\right)_{\mathrm{rms}} = \frac{1}{2\sqrt{2}|\sin(2\pi Q)|}\left(\sum_i \delta k_i^2\beta_i^2\right)^{1/2}$$