Exercises on Linear Imperfections

Consider a proton synchrotron with **160 MeV** kinetic energy. The proton rest energy is **938 MeV**. The horizontal tune is **Q_x = 4.25** and the vertical tune is **Q_y = 4.45**. The machine is built of 16 identical cells. The machine has 16 sextupoles (one in each cell), each with a magnetic length of **l=0.3m**. The horizontal dispersion function at the location of the sextupoles is **D_x=1.4m,** the horizontal beta function **β_x** = **6 m** and the vertical beta function **β_y** = **12 m**. The 16 sextupoles are connected in series such that all of them have the same **normalized sextupole strength of K2 = $2B_2/B\rho$ = 2 m^{-3}**.

1. What is the **corresponding sextupole field gradient B_2** (in T/m^2)?
2. **One of the sextupoles is horizontally displaced** such that the **normalized quadrupole field due to feed-down is δK = 0.008 m^{-2}**. Calculate the integrated normalized quadrupole gradient. Does this value depend on the energy of the beam? Calculate the horizontal displacement of the sextupole. What is the integrated dipole kick **Θ** in mrad resulting from the same offset?
3. Compute the **horizontal tune-shift δQ_x** resulting from the quadrupole field induced by the offset in the sextupole. Estimate the resulting **maximum beta-beating in the locations of the sextupoles** for both horizontal and vertical planes.
   <u>BONUS</u>: Estimate the maximum closed orbit distortion at sextupole locations due to the dipole error induced by the displaced sextupole.
4. Will there be an effect on the vertical tune **Q_y**?
5. What is the advantage of having the horizontal tune at 4.25? What will happen if the tune is moved towards 4.5? What will happen if the tune is moved towards 4.0? (please give conceptual answers, do not calculate anything).
6. To minimize the quadrupole error induced by the displaced sextupole one can try to make a closed orbit bump such that the beam orbit is centered in the displaced sextupole. Assume that there is a horizontal dipole corrector in each of the sextupole magnets. The dipole correctors in three consecutive sextupole locations are used to create a horizontal closed orbit bump in the location of the displaced sextupole. **In case the first corrector is set to produce an integrated dipole kick of Θ_1 = 0.66 mrad, calculate the required kicks in the two following sextupole locations in order to achieve a closed 3-bump**. hint: the phase advance between the sextupole locations can be calculated from the tune knowing that the machine has a 16-fold symmetry. (2 points) **<u>BONUS</u>:** What is the resulting amplitude of the closed orbit bump at the location of the central sextupole
7. What would be the leading order multipole effect if a sextupole is shifted vertically? Which resonances should be avoided in that case?

Conceptual questions:

1. Describe in words the concept of closed orbit: what does it represent in relation to a single particle trajectory over multiple passages at a given location of a storage ring?
2. How many orbit correctors are needed to introduce controlled closed orbit bump in a machine?

3. Describe possible sources for closed orbit errors and possible correction schemes. In which locations of a machine would you install closed orbit correctors?
4. Describe how quadrupole (gradient) errors affect a machine.