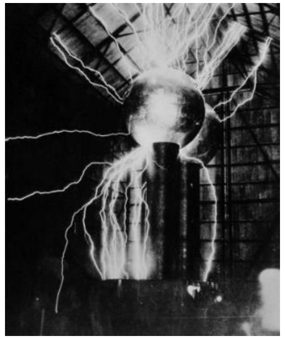
SUMMARY OF THE FIRST LECTURE (2h)

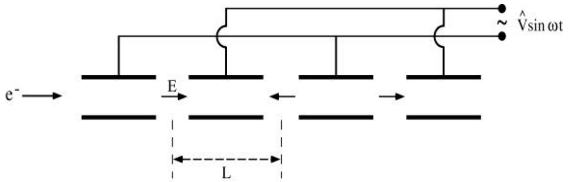
ELECTROSTATIC ACCELERATORS



$$\frac{dE}{dz} = qE_z$$

$$\Delta E = q\Delta V$$

WIDERÖE : RF ACCELERATION



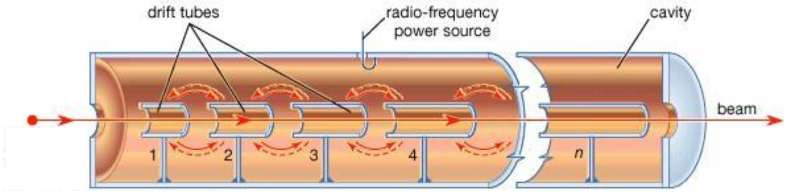
Transit time factor and bunched beam

$$\Delta E = qV_{RF}T\cos(\phi_{inj}) = q\hat{V}_{acc}\cos(\phi_{inj})$$

$$E_z(z,t) = E_{RF}(z)\cos(\omega_{RF}t)$$

ALVAREZ DTL

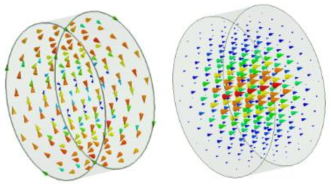
β=0.05-0.5, f_{RF}=50-400 MHz



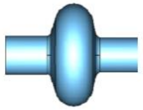
$$L_n = \bar{\beta}_n\lambda_{RF} \qquad \Delta E_n = q\Delta V_n$$

HIGH β RF CYLINDRICAL CAVITIES (PILLBOX-LIKE)

B E



MODE TM_{010}



β>0.5, f_{RF}=0.3-3 GHz (or above)

$$\tau_F = \frac{2Q}{\omega_{RF}} \qquad V_{acc}(t) = \hat{V}_{acc}\left(1 - e^{-\frac{t}{\tau_F}}\right)$$

$$Q = \omega_{RF}\frac{W}{P_{diss}}$$

$$\hat{V}_{acc} = \sqrt{\left(\frac{R}{Q}\right)QP_{diss}} \qquad R = \frac{\hat{V}_{acc}^2}{P_{diss}}\ [\Omega]$$

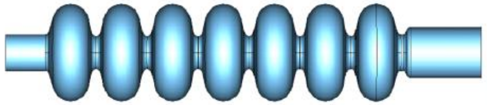
$$R = \frac{\hat{V}_{acc}^2}{P_{diss}}\ [\Omega]$$

$$r = \frac{(\hat{V}_{acc}/L)^2}{P_{diss}/L} = \frac{\hat{E}_{acc}^2}{p_{diss}}\ [\Omega/m]$$

MULTI CELL STRUCTURES

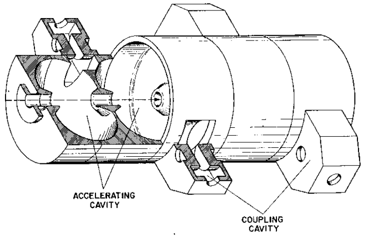
$$d = \frac{\beta\lambda_{RF}}{2} \qquad \pi \text{ MODE}$$

d ⟷



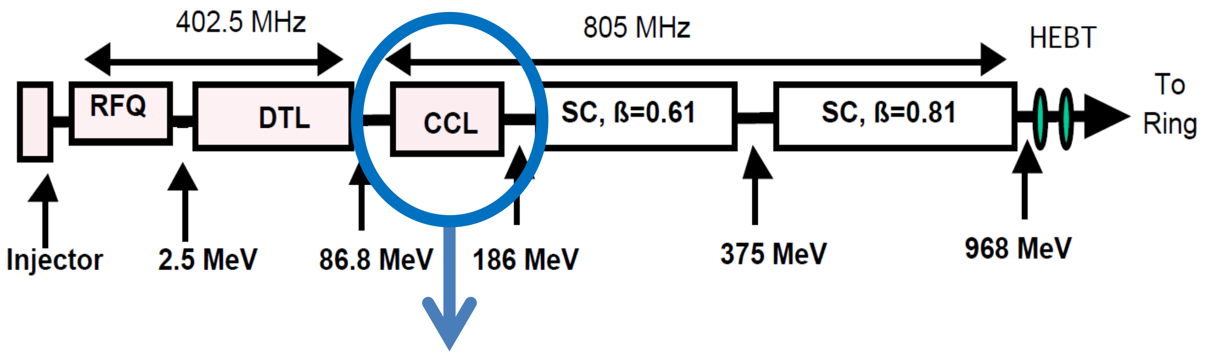
nR

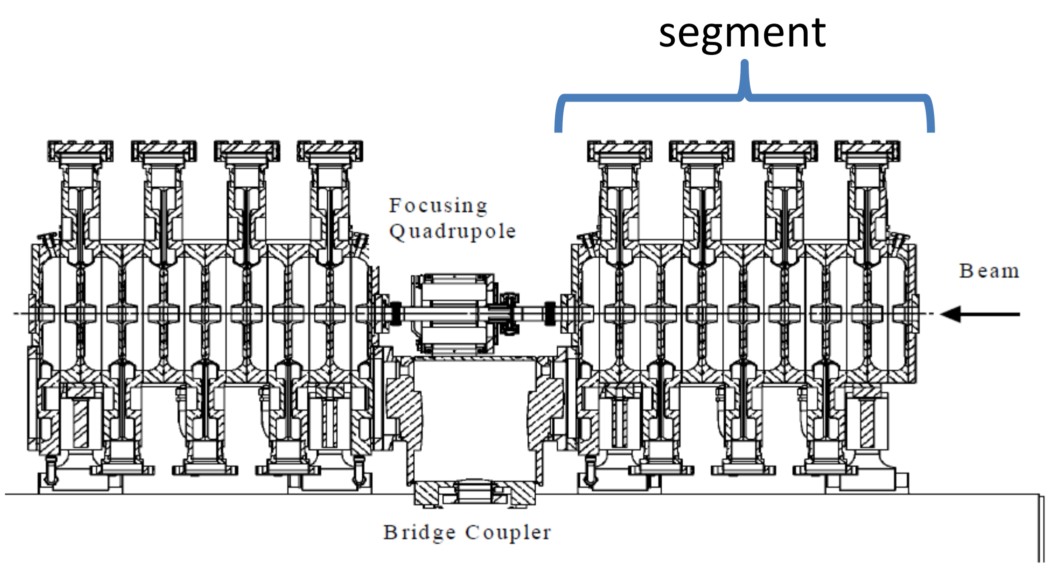
π/2 MODE



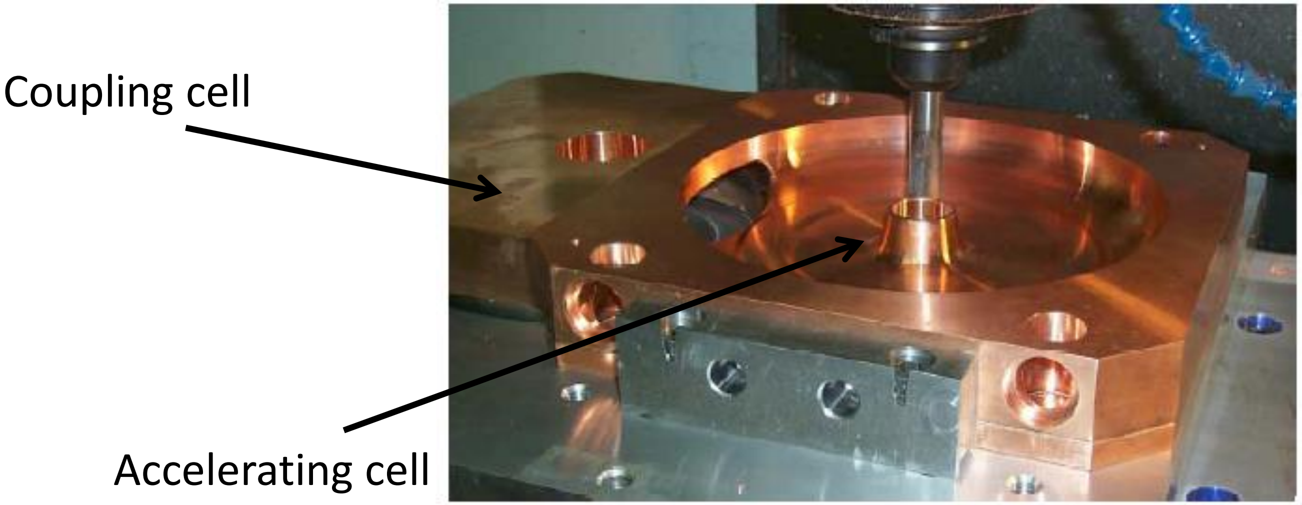
SCC STRUCTURES: EXAMPLES

Spallation Neutron Source Coupled Cavity Linac (protons)



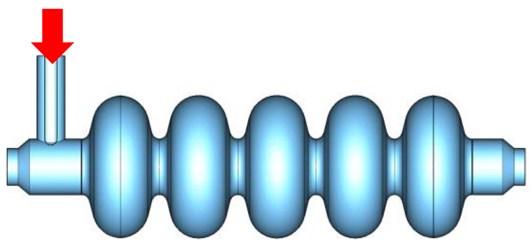


4 modules, each containing 12 accelerator segments CCL and 11 bridge couplers. The CCL section is a RF Linac, operating at **805 MHz** that accelerates the beam **from 87 to 186 MeV** and has a physical installed length of slightly over **55 meters.**



TRAVELLING WAVE (TW) STRUCTURES

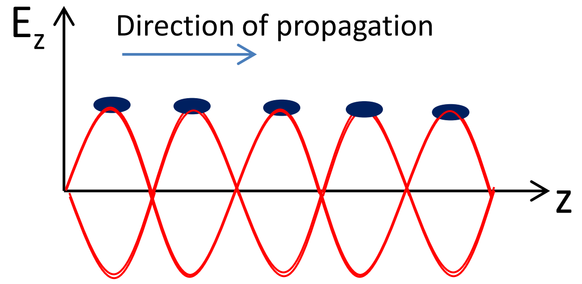
(electrons)



⇒To accelerate charged particles, the electromagnetic field must have an **electric field along the direction of propagation of the particle**.

⇒The field has to be synchronous with the particle velocity.

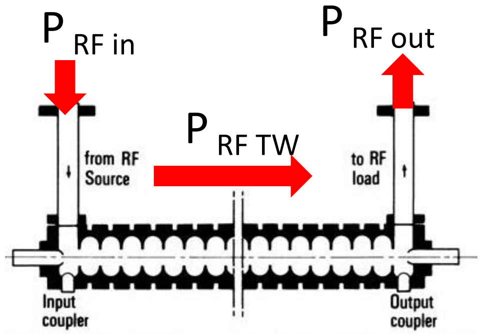
⇒Up to now we have analyzed the standing **standing wave (SW)** structures in which the field has basically a given profile and oscillate in time (as example in DTL or **resonant cavities operating on the** TM_{010}-like).



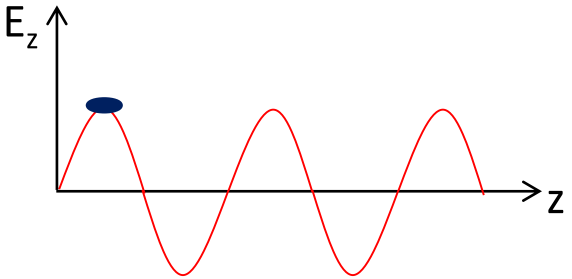
Time oscillation

$$E_z(z,t) = \underbrace{E_{RF}(z)}_{\text{field profile}} \overbrace{\cos(\omega_{RF} t)}^{\text{Time oscillation}}$$

field profile



⇒There is another possibility to accelerate particles: using a **travelling wave (TW)** structure in which the RF wave is **co-propagating** with the beam with a **phase velocity equal to the beam velocity**.



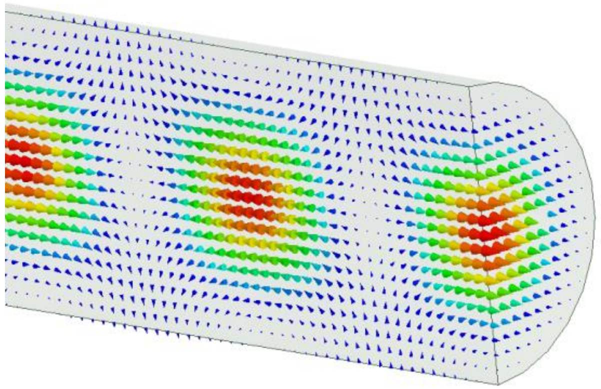
⇒Typically these structures **are used for electrons** because in this case the **phase velocity can be constant** all over the structure and equal to c. On the other hand it is difficult to modulate the phase velocity itself very quickly for a low β particle that changes its velocity during acceleration.

TW CAVITIES: CIRCULAR WAVEGUIDE AND DISPERSION CURVE

(electrons)

In **TW structures** an e.m. wave with **$E_z \neq 0$** travel together with the beam in a special guide in which the **phase velocity of the wave matches the particle velocity (v)**. In this case the beam absorbs energy from the wave and it is **continuously accelerated**.

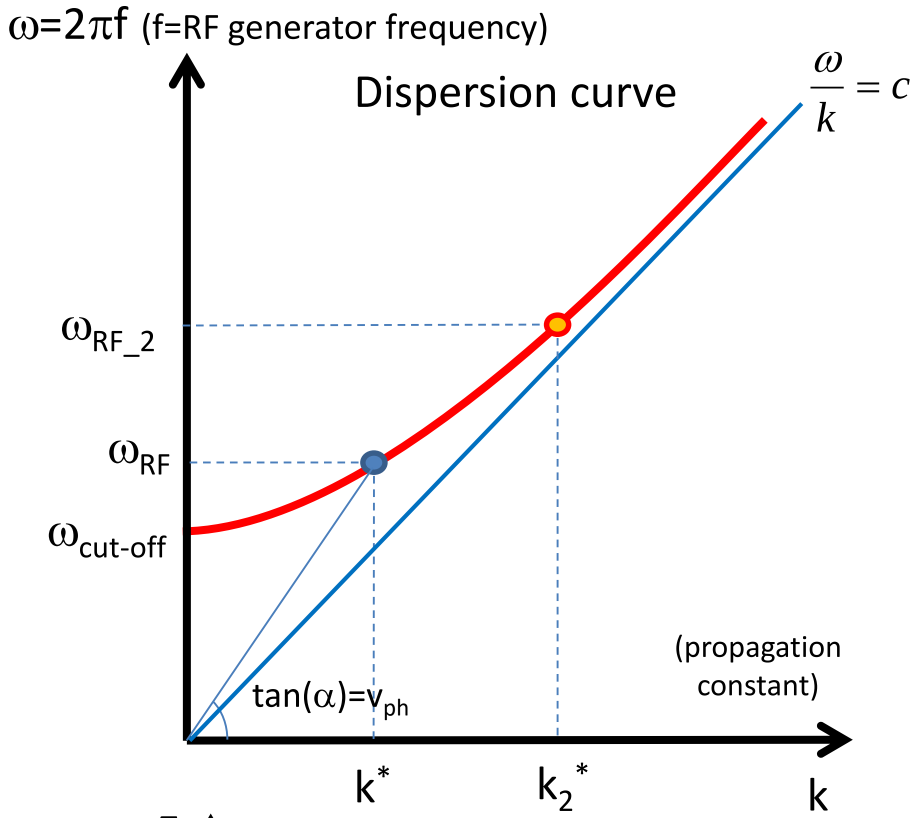
CIRCULAR WAVEGUIDE

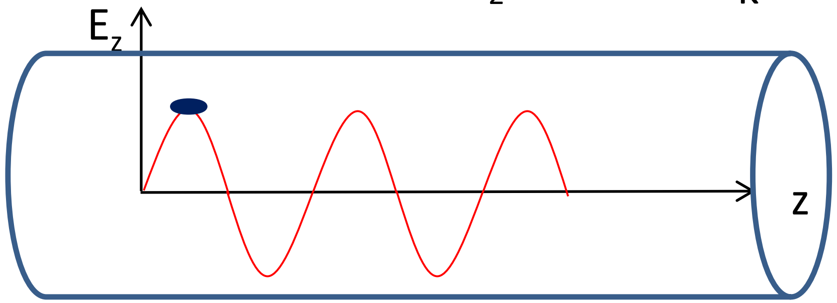


As example if we consider a simple circular waveguide the first propagating mode with **$E_z \neq 0$** is the TM_{01} mode.

Nevertheless by solving the wave equation it turns out that an e.m. wave propagating in this **constant cross section waveguide** will **never be synchronous with a particle beam** since the **phase velocity is always larger than the speed of light c**.

$$E_z\big|_{TM_{01}} = \underbrace{E_0(r)}_{J_0\left(\frac{p_{01}}{a}r\right)}\cos\left(\omega_{RF}t - k^* z\right) \Rightarrow v_{ph} = \frac{\omega_{RF}}{k^*} > c$$



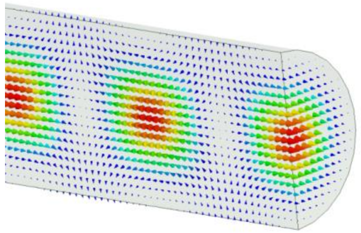


TW CAVITIES: IRIS LOADED STRUCTURES

(electrons)

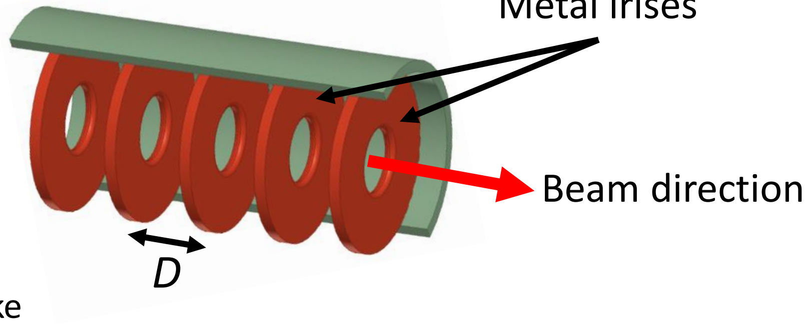
In order to **slow-down the wave phase velocity, iris-loaded periodic structure have to be used**.

CIRCULAR WAVEGUIDE



MODE TM_{01}

IRIS LOADED STRUCTURE



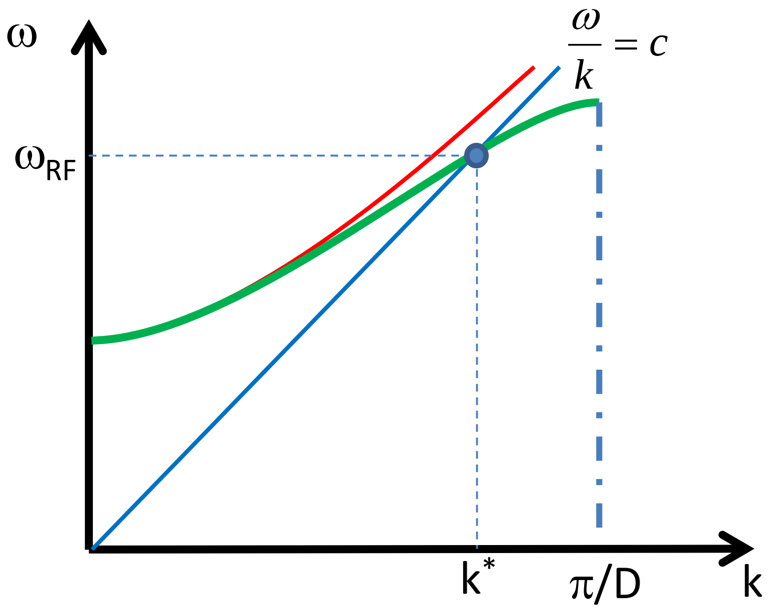
MODE TM_{01}-like

$$E_z\big|_{TM_{01}} = E_0(r)\cos\left(\omega_{RF}t - k^* z\right)$$

Periodic (in z) wih period D
(from Floquet theorem)

$$E_z\big|_{TM_{01}-like} = \hat{E}_{acc}(r,z)\cos\left(\omega_{RF}t - k^* z\right)$$

⇒The field in this type of structures is that of a special wave travelling within a spatial periodic profile.



⇒The structure can be designed to have the **phase velocity equal to the speed of the particles**.

⇒This allows **acceleration over large distances** (few meters, hundred of cells) with just an input coupler and a relatively **simple geometry**.

⇒They are used **especially for electrons** (constant particle velocity→constant phase velocity, same distance between irises, easy realization)

PHASOR NOTATION: RECAP.

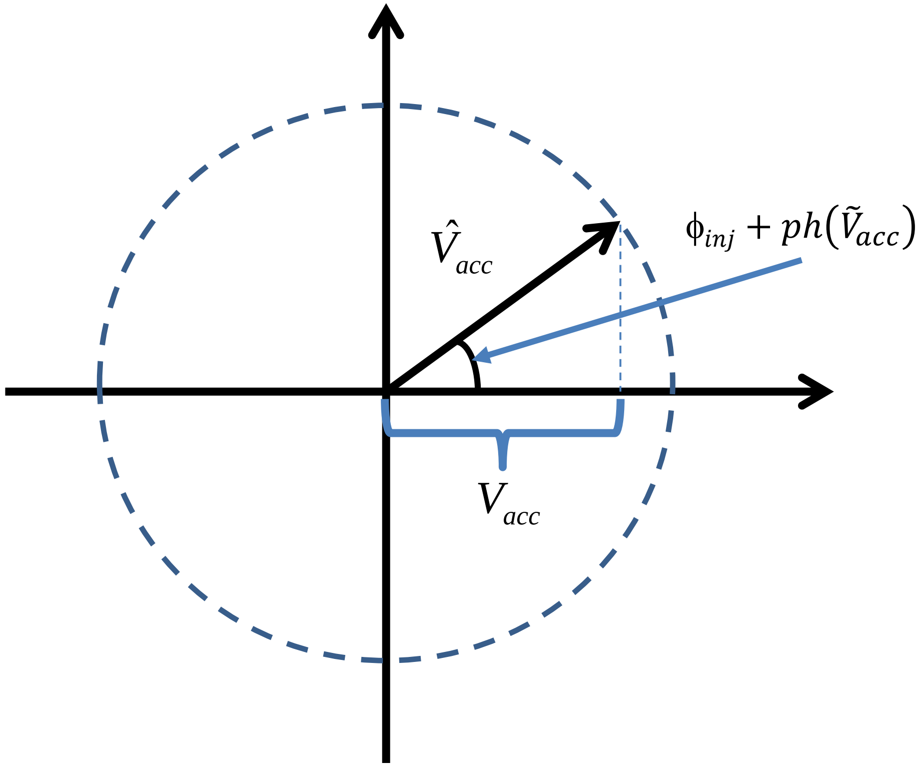
With a **more general notation** we can consider the phasors of the accelerating field.

$$E_z(z,t) = \mathrm{Re}\left[\tilde{E}_z(z)e^{j(\omega_{RF}t+\phi_{inj})}\right]$$

Phasor of the longitudinal electric field (complex quantity)

$$\Delta E = q\int_{gap} E_z(z,t)\Big|_{\substack{seen\\by\\particle}} dz = q\int_{-L/2}^{+L/2}\mathrm{Re}\left[\tilde{E}_z(z)e^{j\left(\omega_{RF}\frac{z}{v}+\phi_{inj}\right)}\right]dz =$$

$$= q\,\mathrm{Re}\left[e^{j\phi_{inj}}\underbrace{\int_{-L/2}^{+L/2}\tilde{E}_z(z)e^{j\omega_{RF}\frac{z}{v}}dz}_{\tilde{V}_{acc}}\right] = \underbrace{q\hat{V}_{acc}\cos\left(\phi_{inj}+ph\left(\tilde{V}_{acc}\right)\right)}_{V_{acc}}$$



Peak accelerating voltage
(i.e.: the maximum accelerating voltage that can be reached for a particular injection phase ($\phi_{inj} + ph(\tilde{V}_{acc})$=0))

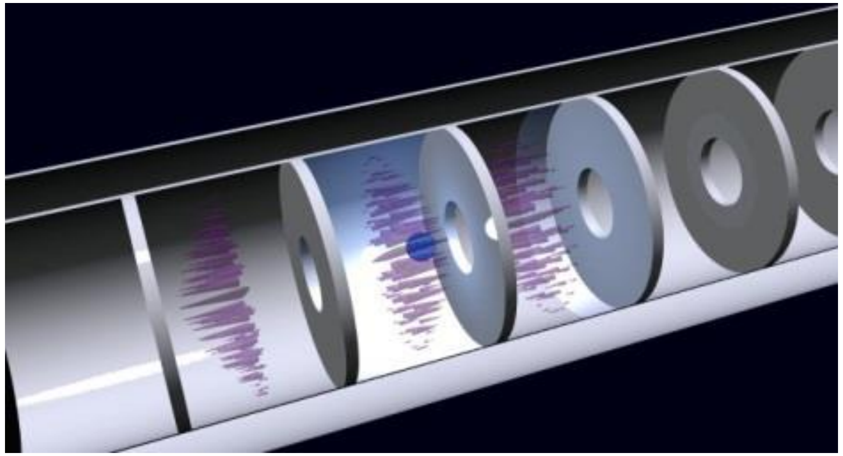
$$\tilde{V}_{acc} = \left|\tilde{V}_{acc}\right|e^{jph\left(\tilde{V}_{acc}\right)}$$

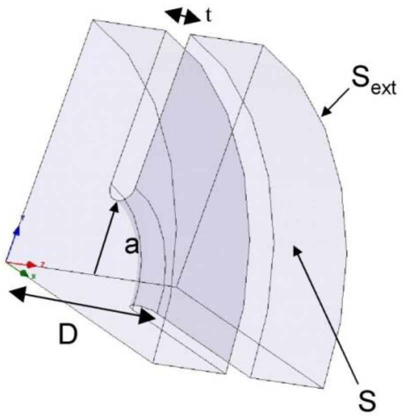
$$\left|\tilde{V}_{acc}\right| = \hat{V}_{acc} = \left|\int_{-L/2}^{+L/2}\tilde{E}_z(z)e^{j\omega_{RF}\frac{z}{v}}dz\right|$$

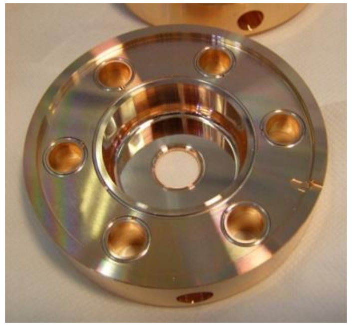
$$ph\left(\tilde{V}_{acc}\right) = ph\left(\int_{-L/2}^{+L/2}\tilde{E}_z(z)e^{j\omega_{RF}\frac{z}{v}}dz\right)$$

TW CAVITIES PARAMETERS: r, α, v_g

Similarly to the SW cavities it is possible to define some figure of merit for the TW structures







$$\hat{V}_{acc} = \left| \int_0^D E_{acc} \cdot e^{j\omega_{RF}\frac{z}{c}} dz \right|$$ single cell accelerating voltage

$$\hat{E}_{acc} = \frac{\hat{V}_{acc}}{D}$$ average accelerating field in the cell

$$P_F = \int_{Section} \frac{1}{2} \mathrm{Re}\left(\vec{E} \times \vec{H}^* \right) \cdot \hat{z} dS$$ flux power

$$P_{diss} = \frac{1}{2} R_s \int_{cavity\ wall} \left| H_{tan} \right|^2 dS$$ average dissipated power in the cell

$$p_{diss} = \frac{P_{diss}}{D}$$ average dissipated power per unit length

$$W = \int_{cavity\ volume} \overbrace{\left(\frac{1}{4} \varepsilon \left| \vec{E} \right|^2 + \frac{1}{4} \mu \left| \vec{H} \right|^2 \right)}^{\text{energy density}} dV$$ stored energy in the cell

$$w = \frac{W}{D}$$ average stored energy per unit length

$$r = \frac{\hat{E}_{acc}^2}{p_{diss}}$$

Shunt impedance per unit length [Ω/m]. Similarly to SW structures the higher is r, the higher the available accelerating field for a given RF power.

$$\alpha = \frac{p_{diss}}{2P_F}$$

Field attenuation constant [1/m]: because of the wall dissipation, the RF power flux and the accelerating field decrease along the structure.

$$v_g = \frac{P_F}{w}$$

Group velocity [m/s]: the velocity of the energy flow in the structure (~1-2% of c).

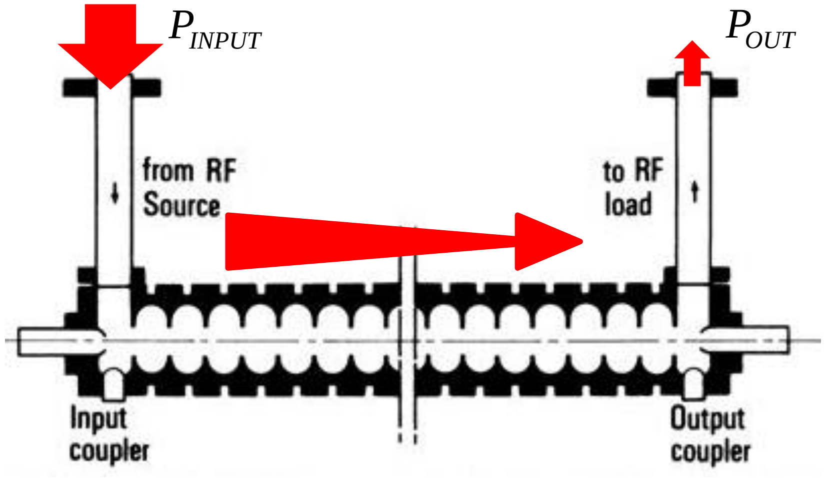
$$Q = \omega_{RF} \frac{w}{p_{diss}}$$

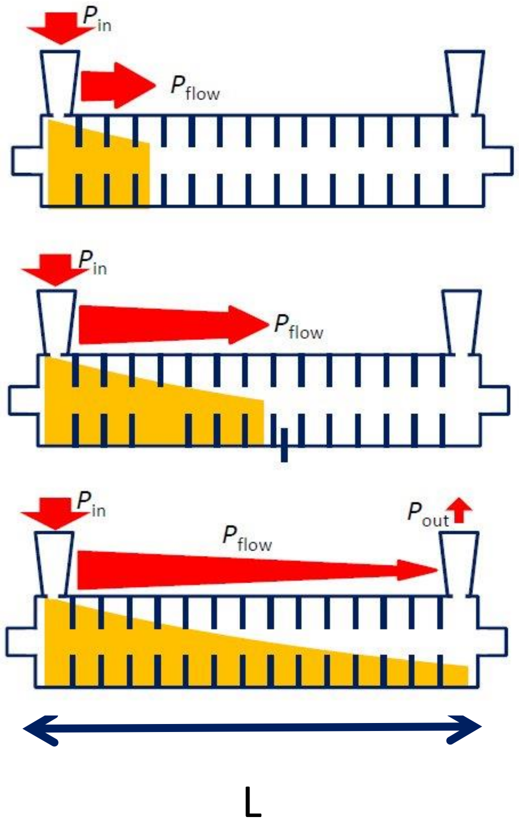
$$\Delta\phi = kD$$

Working mode [rad]: defined as the phase advance over a period *D*. For several reasons the most common mode is the **2π/3**

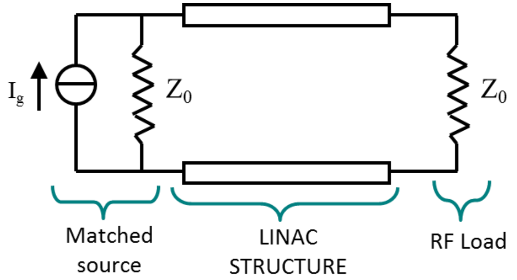
TW CAVITIES: EQUIVALENT CIRCUIT AND FILLING TIME

In a TW structure, the **RF power enters** into the cavity through an **input coupler**, flows (travels) through the cavity in the same direction as the beam and an **output coupler at the end** of the structure is connected to a **matched power load**.
If there is no beam, the input power, reduced by the cavity losses, goes to the power load where it is dissipated.
In the presence of a large beam current, however, a fraction of the TW power is transferred to the beam.





In a purely periodic structure, made by a sequence of **identical cells** (also called "**constant impedance structure**"), α does not depend on z and both the RF power flux and the intensity of the accelerating field decay exponentially along the structure :



$$E_{acc}(z,t) = \underbrace{E_P(r,z)}_{\text{periodic function with period D}} \cos\left(\omega_{RF}t - k_z^* z\right) e^{-\alpha z} \approx E_{IN} \cos\left(\omega_{RF}t - k_z^* z\right) e^{-\alpha z}$$

$$P_F(z) = P_{IN} e^{-2\alpha z} \qquad P_{OUT} = P_{IN} e^{-2\alpha L} \qquad E_{IN} = \sqrt{2\alpha r P_{IN}} \qquad V_{acc} = E_{IN}\frac{1-e^{-\alpha L}}{\alpha}$$

The **filling time** is the time necessary to propagate the RF wave-front from the input to the end of the section of length L is:

$$\tau_F = \frac{L}{v_g}$$

Differently from SW cavities after **one filling time the cavity is completely full of energy**

EXERCISE 6: TW STRUCTURES

A) Demonstrate that if we define the attenuation constant as: $\alpha = \frac{p_{diss}}{2P_F}$

the power flow along the structure scales as: $P_F(z) = P_{IN} e^{-2\alpha z}$

B) Demonstrate that if we define the shunt impedance per unit length as: $r = \frac{\hat{E}_{acc}^2}{p_{diss}}$

the average accelerating field "seen" by an ultrarelativistic particle (z=ct) along the structure can be expressed as:

$$E_{acc}(z) = \sqrt{2\alpha r P_{IN}}\, e^{-\alpha z} = E_{IN} e^{-\alpha_0 z}$$

C) Demonstrate that the total accelerating voltage is given by:

$$V_{acc} = E_{IN} \frac{1 - e^{-\alpha L}}{\alpha}$$

EXERCISE 7: TW STRUCTURES

A SLAC-type TW structure accelerate ultra-relativistic electrons. The structure length is L=3m and it can be simplified as a structure with a group velocity is v_g=1.1% the velocity of light. Calculate:

1) the filling time;
2) if we suppose that the structure has a field attenuation constant α=0.2 m^{-1}, calculate the total accelerating voltage if the accelerating field at the beginning of the structure is E_{INPUT}=20 MV/m;
3) Calculate the average accelerating field
4) if the average dissipated power per unit length in the structure, corresponding to the previous value of the accelerating field, is p_{diss}= 4 MW/m calculate the shunt impedance per unit length.

EXERCISE 8: TW STRUCTURES

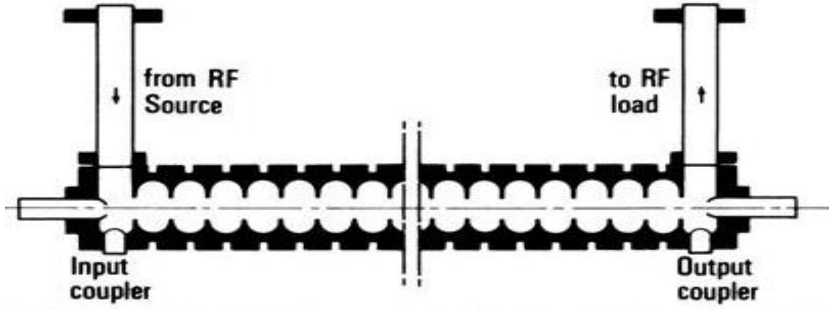
A constant impedance TW structure, accelerates ultra-relativistic electrons (β=1). The cavity has the following parameters: α=0.25 m^{-1} ; shunt impedance r=65 MOhm/m and a total length of 2 m. Calculate:

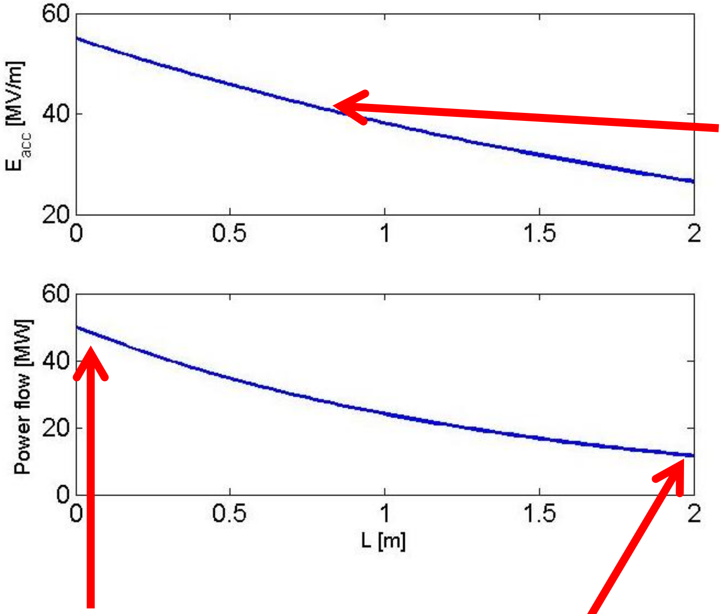
1) the input power to have an energy gain of the particles of 60 MeV
2) if the group velocity v_g is 1% the speed of light, which is the filling time of the structure?

TW CAVITIES: PERFORMANCES (1/2)

Just as an example we can consider a C-band (5.712 GHz) accelerating cavity of L=2 m long made in **copper**.

r=82 [MΩ/m]
α=0.36 [1/m]
v_g/c=1.7%
τ_F=400 ns (**very short if compared to SW**!)



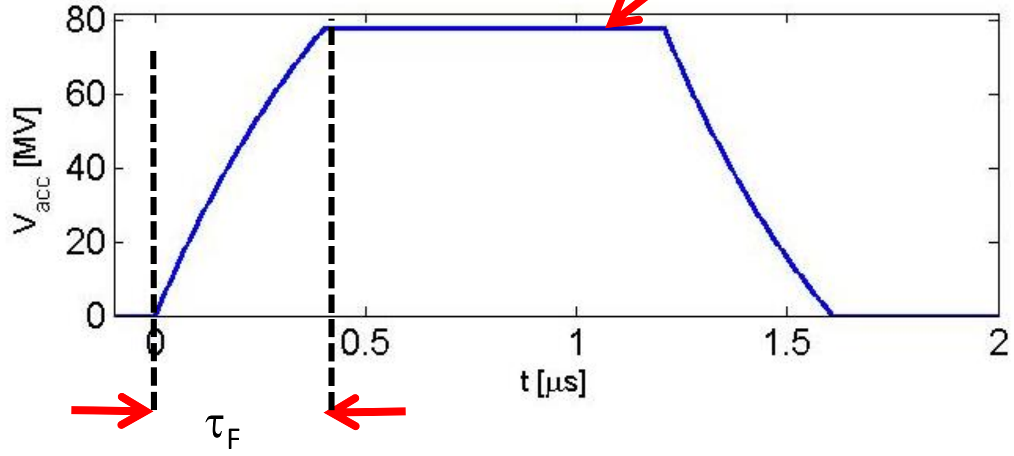


Field attenuation due to the copper dissipations

Input power

Output power (dissipated into the RF load): it is not convenient to have very long RF structures because their efficiency decreases over a certain length (2-3 m depending on the operating frequency).

$$E_{acc}(z)=E_{INPUT}e^{-\alpha z}\Rightarrow V_{acc}=\int_0^L E_{INPUT}e^{-\alpha z}dz=E_{INPUT}\frac{1-e^{-\alpha L}}{\alpha}$$



$$P_F(z)=P_{INPUT}e^{-2\alpha z}\Rightarrow P_{OUT}=P_{INPUT}e^{-2\alpha L}$$

TW CAVITIES: CONSTANT GRADIENT STRUCTURES INTRODUCTION

In order to keep the **accelerating field constant along the LINAC structure,** the group velocity has to be reduced along the structure itself. This can be achieved by a reduction of the iris diameters.

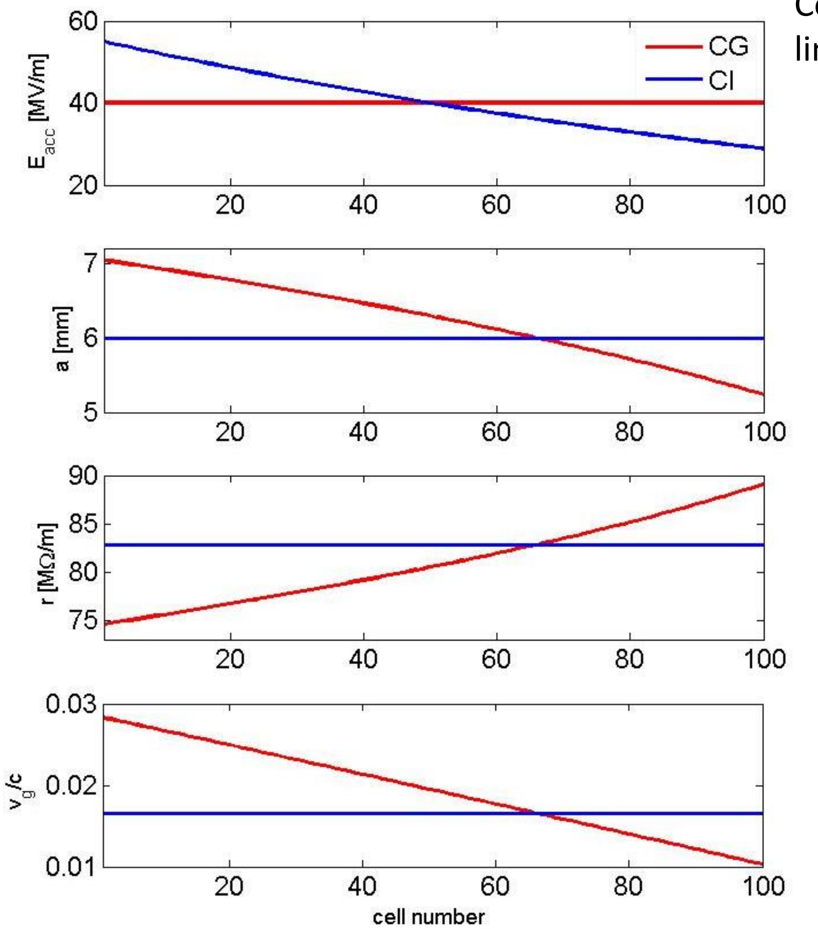
$$v_g = \frac{P_F}{w} \Rightarrow \frac{P_F}{v_g} = w \propto E_{acc}^2$$

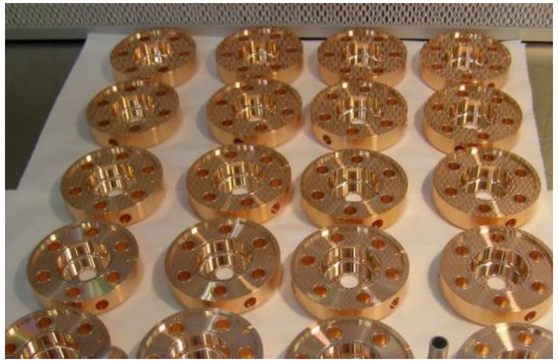
Reduction due to the attenuation (losses) → P_F

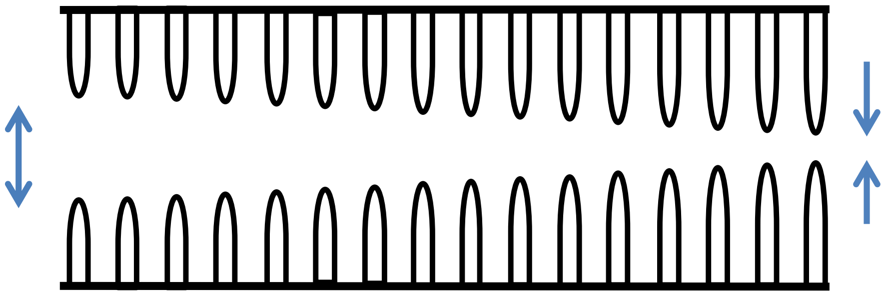
Reduction due to irises modulation → v_g

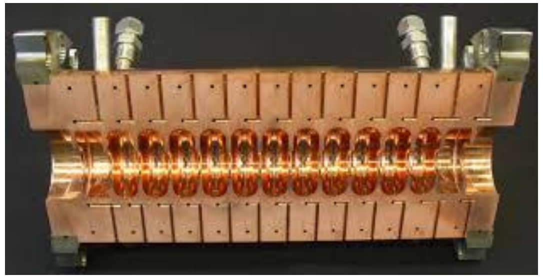
$$\frac{P_F}{v_g} = w \propto E_{acc}^2$$

Constant along the linac → E_{acc}









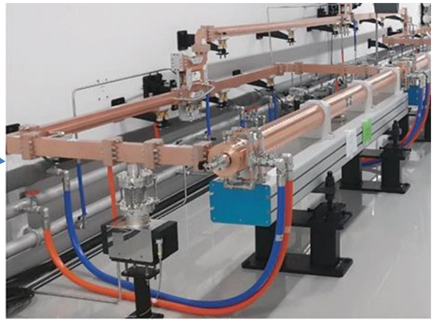
In general constant gradient structures are **more efficient** than constant impedance ones, because of the more uniform distribution of the RF power along them.

LINAC TECHNOLOGY: MATERIALS

ACCELERATING CAVITY TECHNOLOGY

⇒The cavities (and the related LINAC technology) can be of different material:

- copper for **normal conducting (NC, both SW than TW)** cavities;
- Niobium for **superconducting cavities (SC, SW)**;



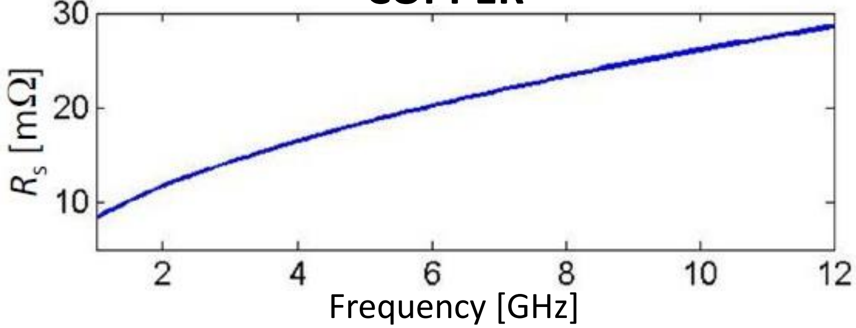
⇒We can choose between NC or the SC technology depending on the required performances in term of:

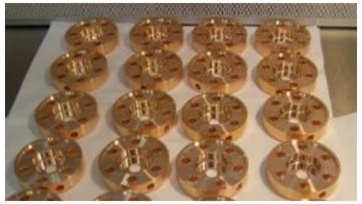
- **accelerating gradient** (MV/m);
- **RF pulse length** (how many bunches we can contemporary accelerate);
- **Duty cycle (see next slide)**: pulsed operation (i.e. 10-100 Hz) or continuous wave (CW) operation;
- **Average beam current**.
- …

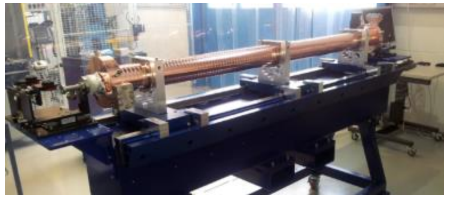
Dissipated power into the cavity walls is related to the surface currents

$$P_{diss} = \int_{\text{cavity wall}} \overbrace{\frac{1}{2} R_s H_{tan}^2}^{\text{power density}} dS$$

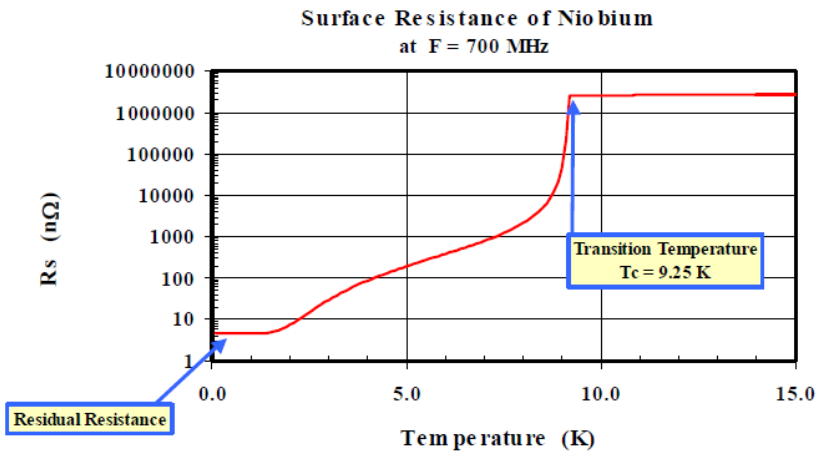
COPPER







NIOBIUM

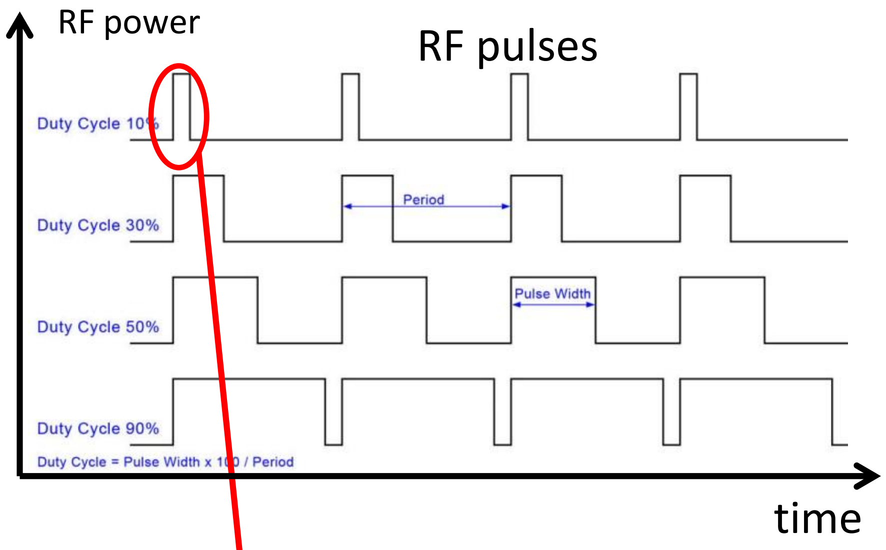


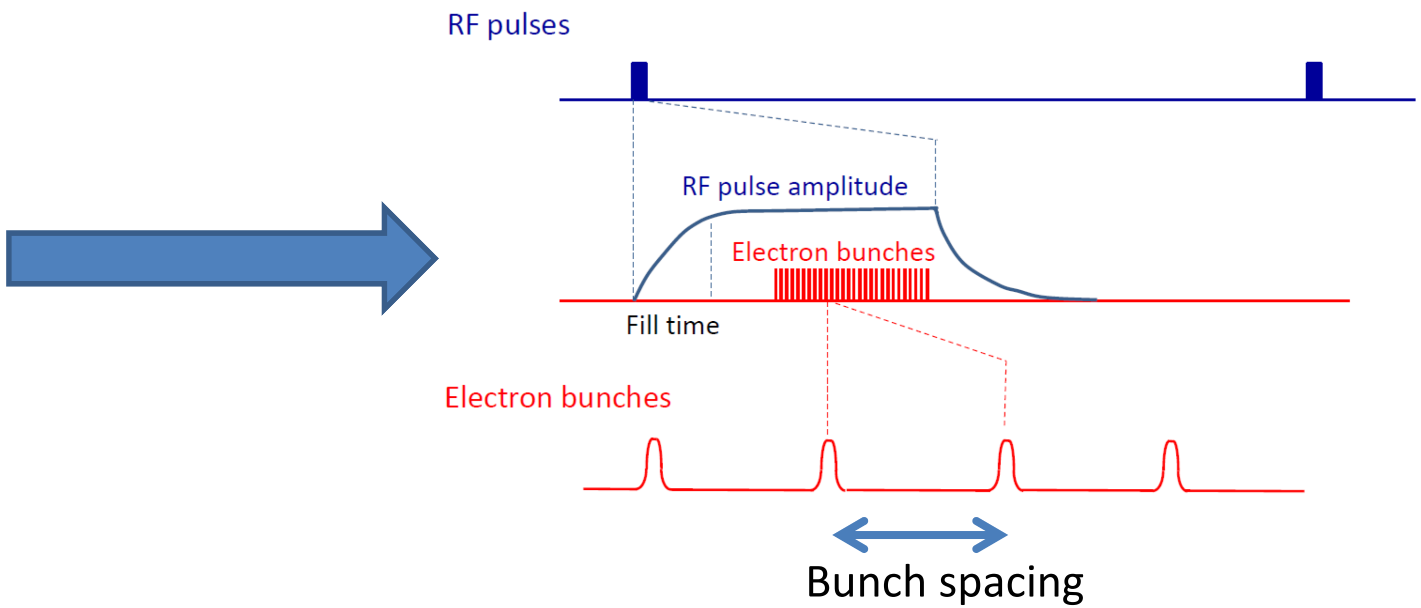
Between copper and Niobium there is a factor 10^5-10^6

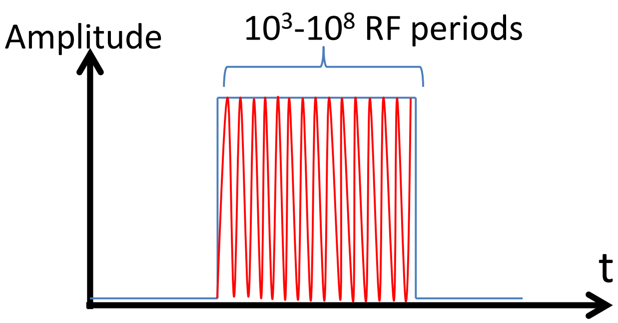
RF STRUCTURE AND BEAM STRUCTURE: NC vs SC

The "**beam structure**" in a LINAC is directly related to the "**RF structure**". There are two possible type of operations:

- **CW** (Continuous Wave) operation ⇒ allow, in principle, to operate with a continuous (bunched) beam
- **PULSED** operation ⇒ there are RF pulses at a certain repetition rate (**Duty Cycle (DC)=pulsed width/period**)







⇒**SC structures allow operation at very high Duty Cycle (>1%) up to a CW operation (DC=100%)** (because of the extremely low dissipated power) **with relatively high gradient (>20 MV/m)**. This means that a continuous (bunched) beam can be accelerated.

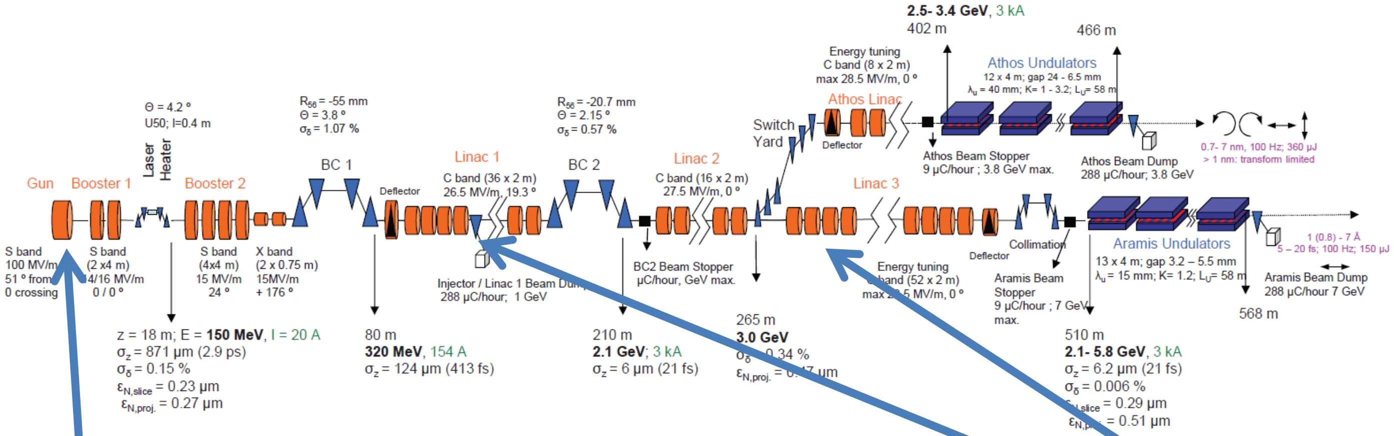
⇒**NC structures can operate in pulsed mode** at **very low DC** (10^{-2}-10^{-1} %) (because of the higher dissipated power) with, in principle, **larger peak accelerating gradient**(>30 MV/m). This means that one or few tens of bunches can be, in general, accelerated. NB: NC structures can also operate in CW but at very low gradient because of the dissipated power.

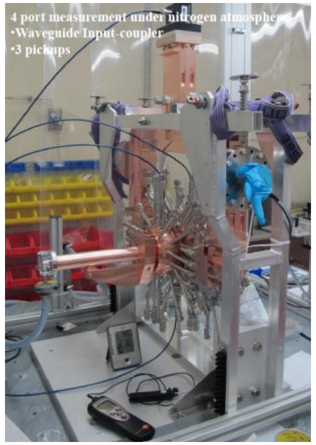
EXERCISE 9: π MODE STRUCTURES AND DUTY CYCLE

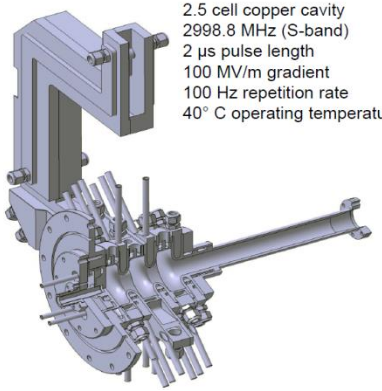
A multi cell SW cavity, operating on the π-mode at 1 GHz, accelerates protons at β=0.5. The cavity is a 9 cell structure. Assuming a negligible variation of the particle velocity through the cavity itself calculate:

1) the distance between the centers of the accelerating cells;
2) assuming a shunt impedance of the single cell (R) of 1 MΩ, calculate the dissipated power to have an effective accelerating voltage on the overall structure of V_{acc}=10 MV;
3) Calculate the average accelerating field;
4) If the cavity is fed by 4 μs rf pulses with a repetition rate of 100 Hz, calculate the Duty Cycle.

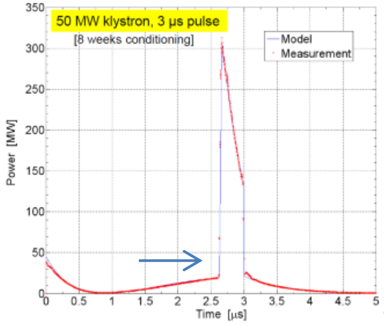
EXAMPLE: SWISSFEL LINAC (PSI)

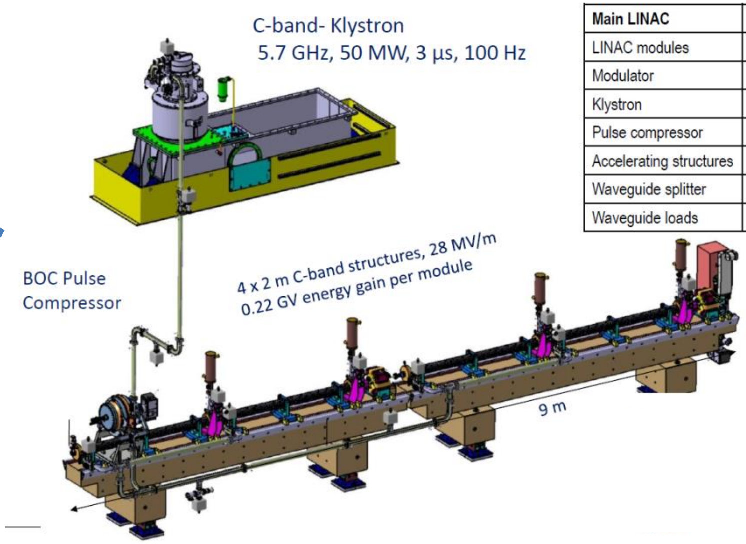




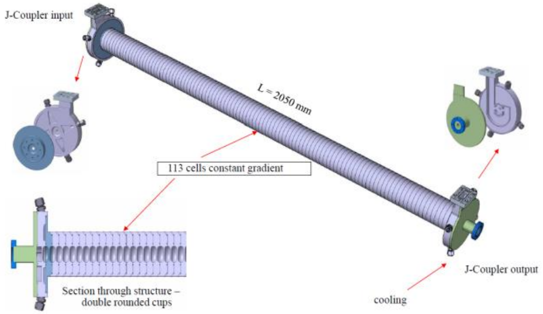


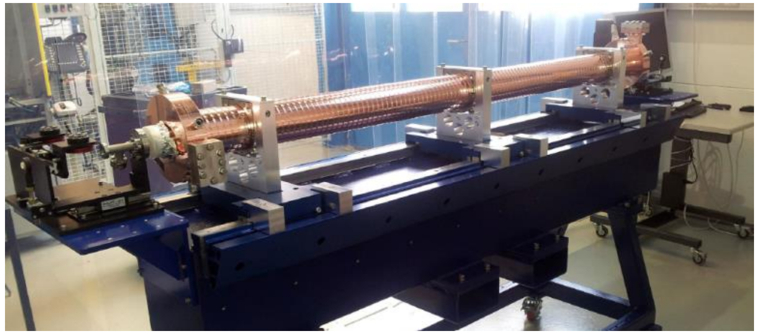
2.5 cell copper cavity
2998.8 MHz (S-band)
2 μs pulse length
100 MV/m gradient
100 Hz repetition rate
40° C operating temperature



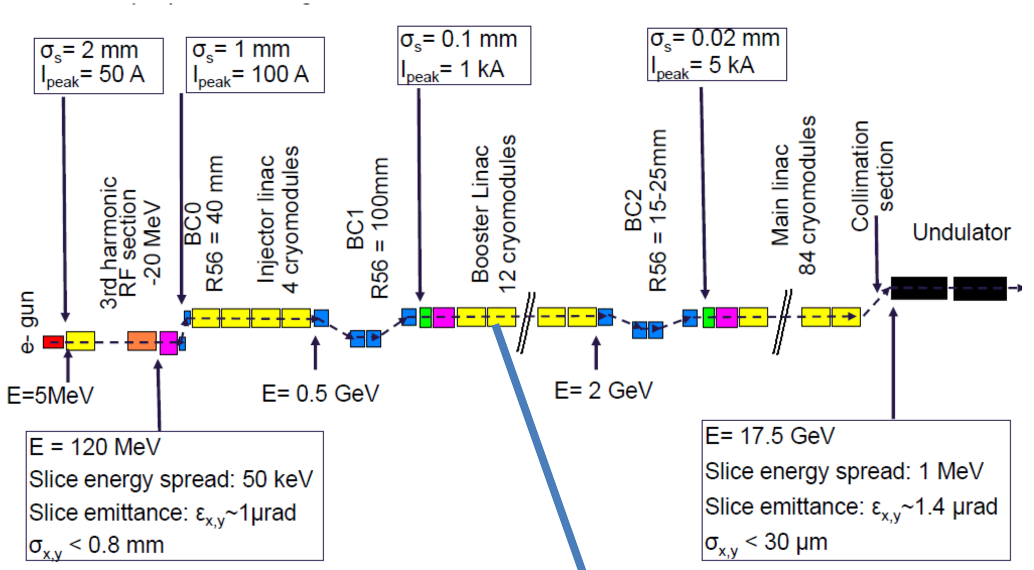


Main LINAC	#
LINAC modules	26
Modulator	26
Klystron	26
Pulse compressor	26
Accelerating structures	104
Waveguide splitter	78
Waveguide loads	104

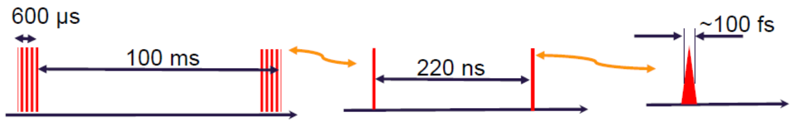




EXAMPLES: EUROPEAN XFEL

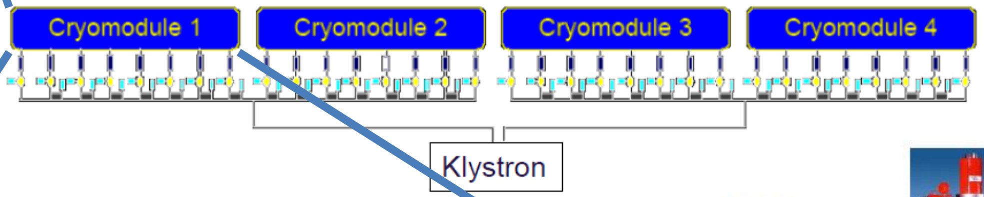


Nominal Energy	GeV	17.5
Beam pulse length	ms	0.60
Repetition rate	Hz	10
Max. # of bunches per pulse		2700
Min. bunch spacing	ns	220
Bunch charge	nC	1
Bunch length, σ_z	μm	< 20
Emittance (slice) at undulator	μrad	< 1.4
Energy spread (slice) at undulator	MeV	1

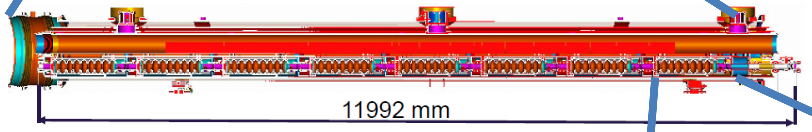


101 cryomodules in total

RF- system: 25 RF units. The unit = 4 cryomodules + RF-power source (klystron)



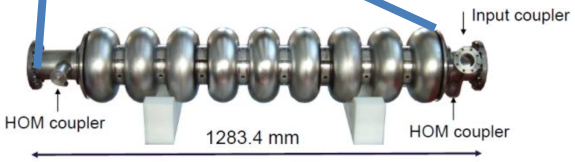
Cryomodule housing: 8 cavities, quadrupole and BPM



25 RF stations
5.2 MW each

800 accelerating cavities

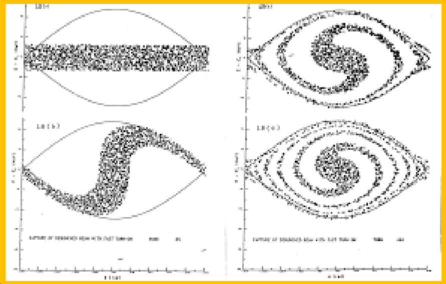
1.3 GHz / 23.6 MV/m



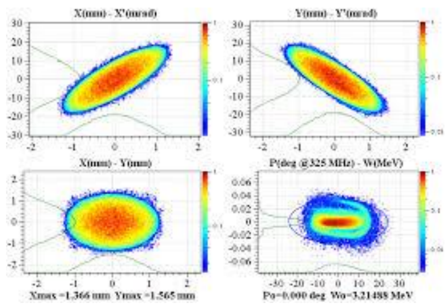
LINAC: BASIC DEFINITION AND MAIN COMPONENTS

LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory** by electromagnetic fields.

Longitudinal dynamics of accelerated particles

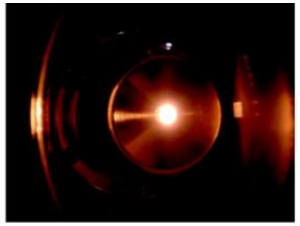


Transverse dynamics of accelerated particles



LINAC BEAM DYNAMICS

Particle source



Accelerated beam

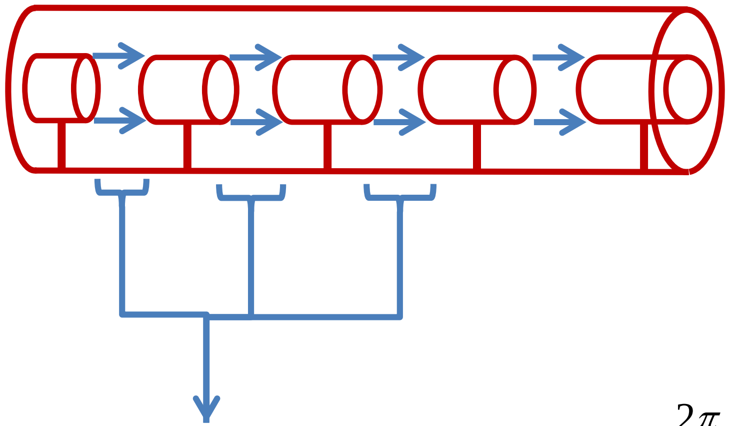
Accelerating structures

Focusing elements: quadrupoles and solenoids

LINAC COMPONENTS AND TECHNOLOGY

SYNCHRONOUS PARTICLE/PHASE

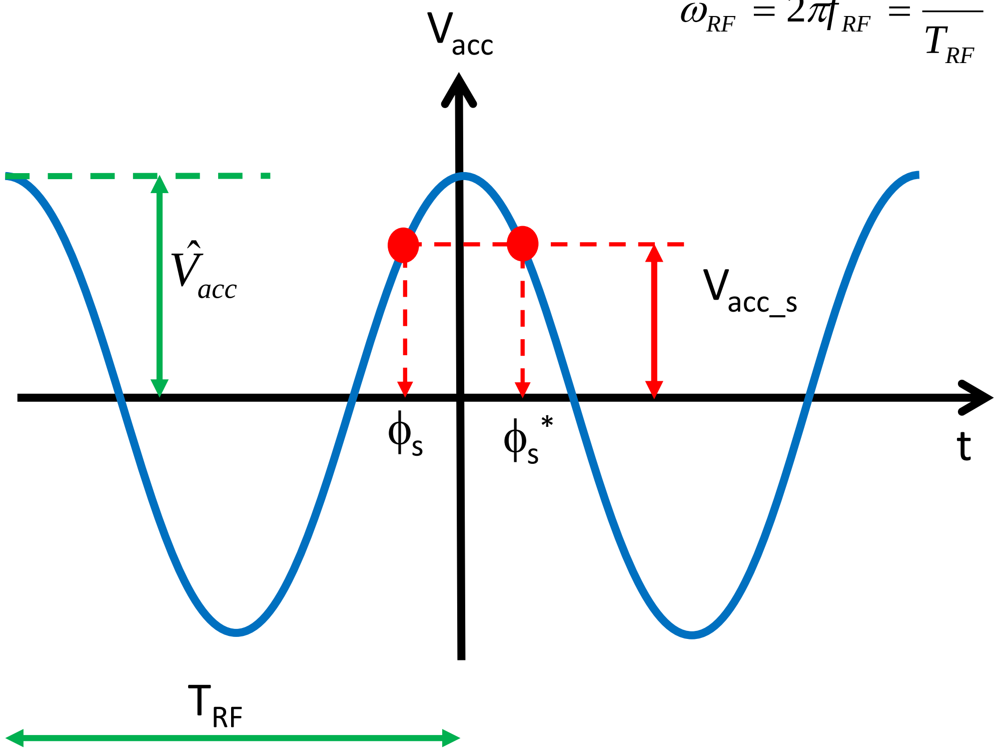
⇒Let us consider a **SW linac structure** made by accelerating **gaps** (like in DTL) or **cavities**.



⇒In **each gap we have an accelerating field** oscillating in time and an integrated accelerating voltage (V_{acc}) still oscillating in time than can be expressed as:

$$V_{acc} = \hat{V}_{acc}\cos(\omega_{RF} t)$$

$$\omega_{RF} = 2\pi f_{RF} = \frac{2\pi}{T_{RF}}$$



⇒Let's assume that **the "perfect" synchronism condition is fulfilled for a phase ϕ_s** (called ***synchronous phase***). This means that a particle (called ***synchronous particle***) entering in a gap with a phase ϕ_s ($\phi_s=\omega_{RF}t_s$) with respect to the RF voltage receive an **energy gain** (and a consequent change in velocity) that allow entering in the subsequent gap with the **same phase ϕ_s** and so on.

⇒for this particle the energy gain in each gap is:

$$\Delta E = q\underbrace{\hat{V}_{acc}\cos(\phi_s)}_{V_{acc\_s}} = qV_{acc\_s}$$

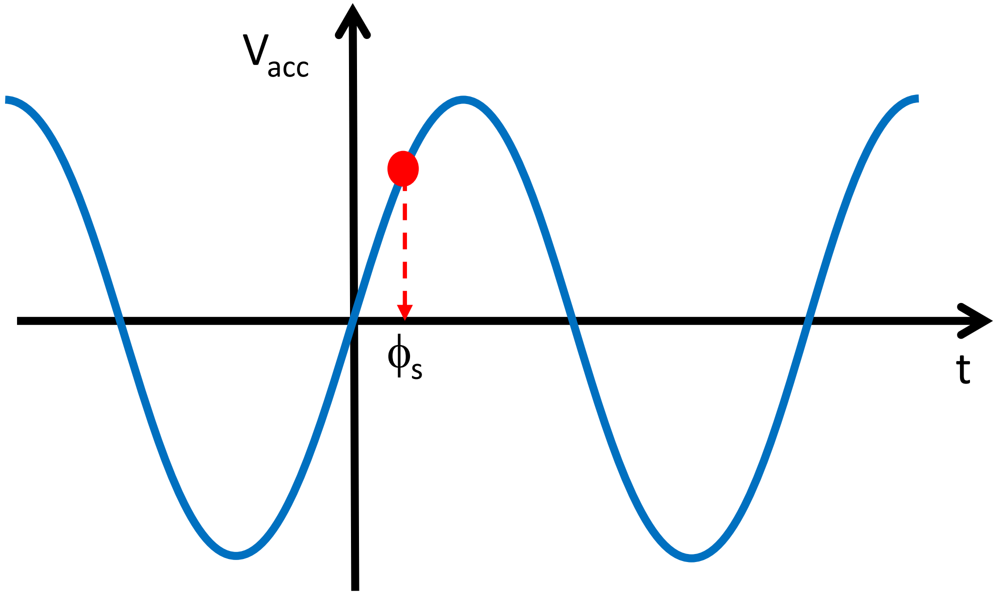
⇒obviously both ϕ_s and ϕ_s^* are synchronous phases.

LINAC-SYNCHROTRON PHASE CONVENTIONS FOR BEAM DYNAMICS CALCULATIONS

⇒ For **circular accelerators**, the origin of time is taken at the zero crossing of the RF voltage with positive slope

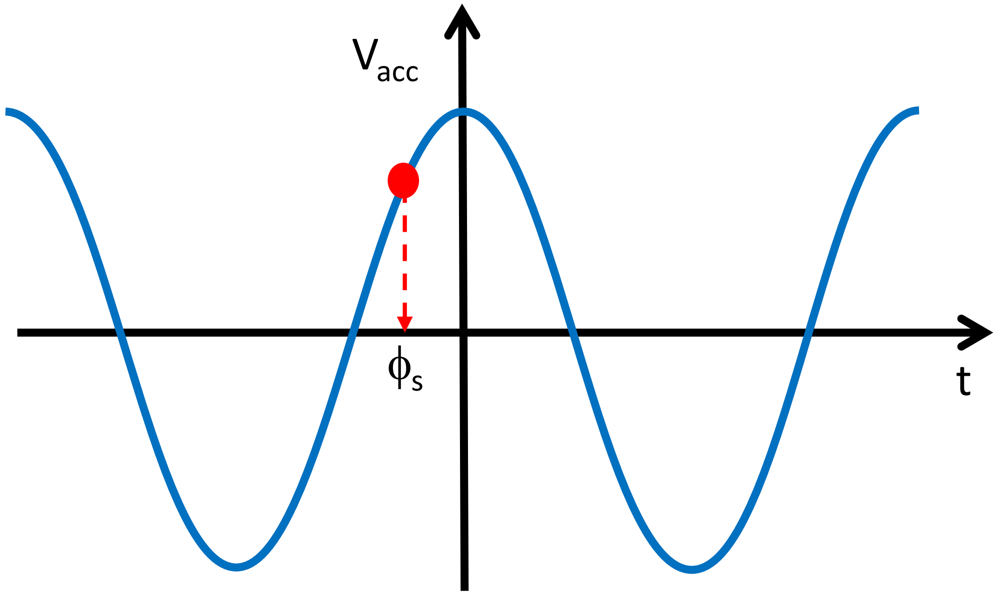
⇒ For **linear accelerators**, the origin of time is taken at the maximum of the positive crest of the RF voltage

circular accelerators



$$V_{acc} = \hat{V}_{acc}\sin(\omega_{RF}t)$$

linear accelerators

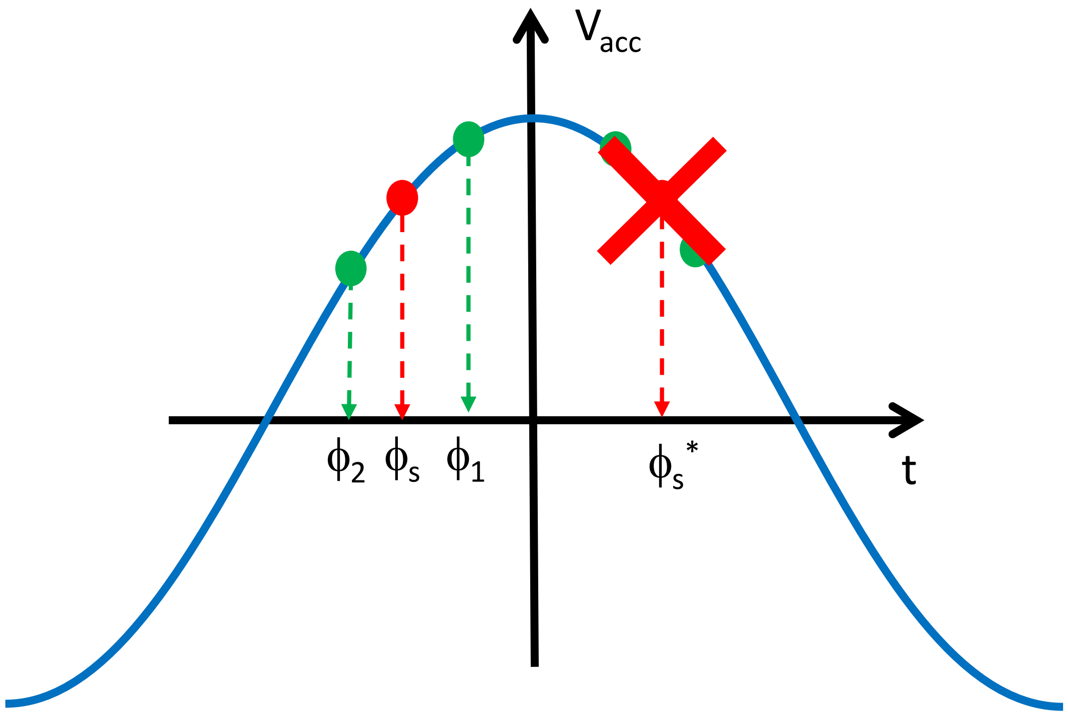


$$V_{acc} = \hat{V}_{acc}\cos(\omega_{RF}t)$$

PRINCIPLE OF PHASE STABILITY

(protons and ions or electrons at extremely low energy)

⇒Let us consider now the first synchronous phase ϕ_s (on the positive slope of the RF voltage). If we consider **another particle** "near" to the synchronous one **that arrives later in the gap** ($t_1>t_s$, $\phi_1>\phi_s$), it will see an higher voltage, it will gain an higher energy and an higher velocity with respect to the synchronous one. As a consequence its time of flight to next gap will be shorter, partially **compensating its initial delay**.



⇒**Similarly** if we consider another particle "near" to the synchronous one that arrives before in the gap ($t_1<t_s$, $\phi_1<\phi_s$), it will see a smaller voltage, it will gain a smaller energy and a smaller velocity with respect to the synchronous one. As a consequence its time of flight to next gap will be longer, compensating the initial advantage.

⇒**On the contrary** if we consider now the synchronous particle at phase ϕ_s^* and another particle "near" to the synchronous one that arrives later or before in the gap, it will receive an energy gain that will increase further its distance form the synchronous one

⇒The choice of the synchronous phase in the positive slope of the RF voltage provides longitudinal focusing of the beam: **phase stability principle**.

⇒The synchronous phase on the negative slope of the RF voltage is, on the contrary, **unstable**

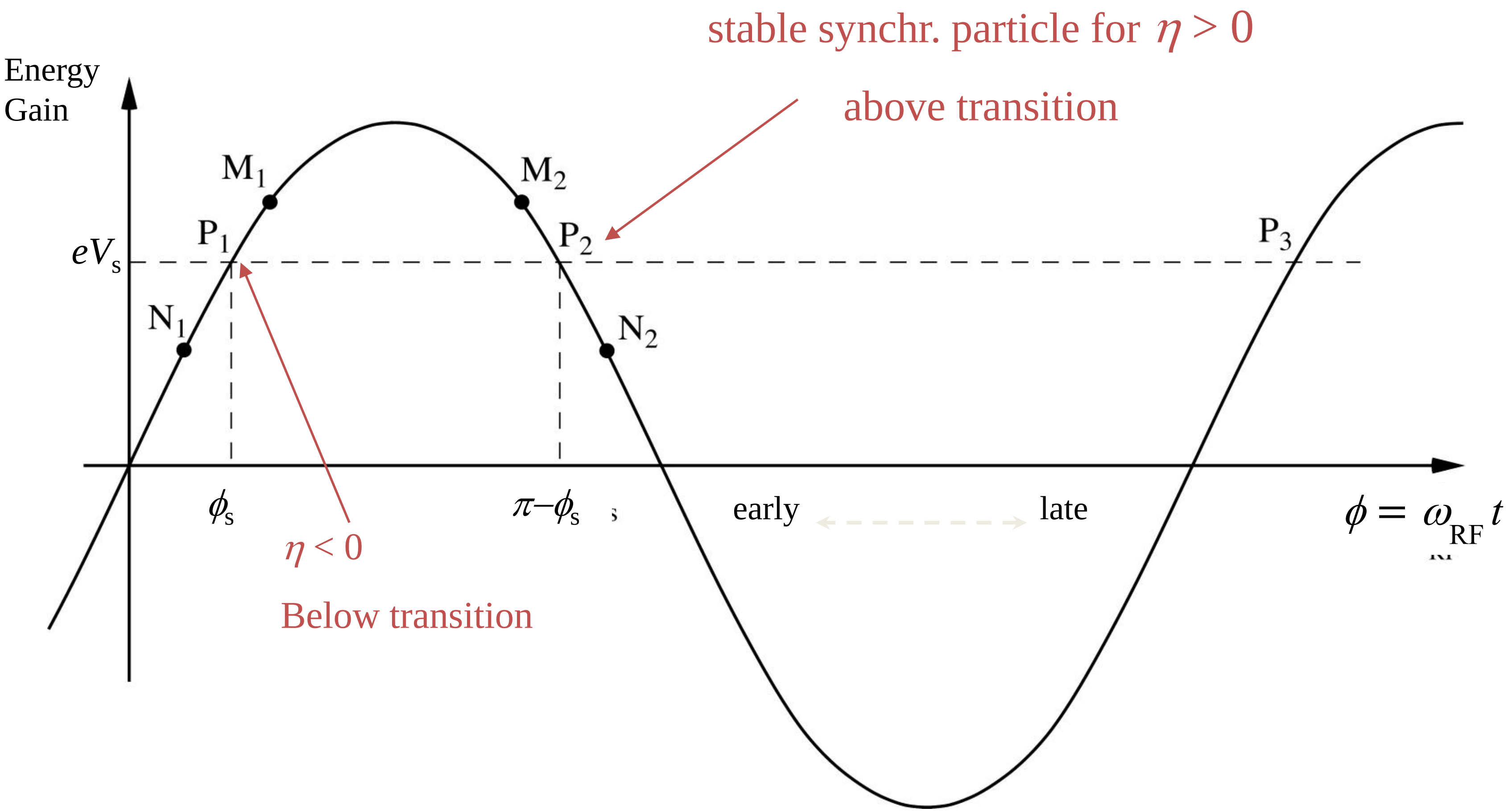
⇒Relying on particle velocity variations, **longitudinal focusing does not work for fully relativistic beams** (electrons). In this case acceleration "on crest" is more convenient.

PHASE STABILITY IN A SYNCHROTRON

From the definition of the **slip factor η** it is clear that an increase in momentum gives, **below transition** (η<0), a higher revolution frequency (increase in velocity dominates) while **above transition** (η>0) a lower revolution frequency (v~c and longer path) where the momentum compaction (generally > 0) dominates. **LINAC phase stability** is similar to the synchrotron phase stability below transition.

$$\frac{dT_{rev}}{T_{rev}} = \eta \frac{dp}{p}$$

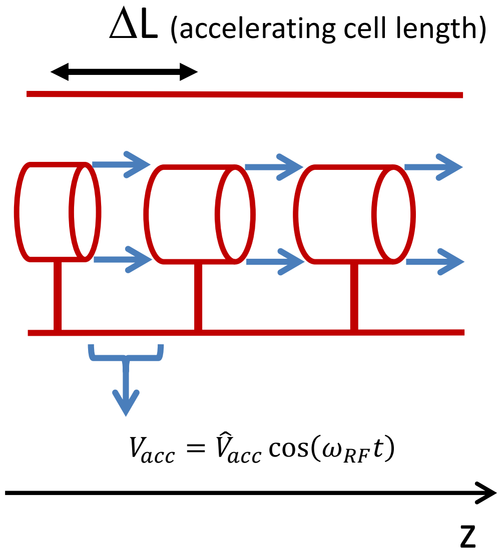
$$\eta = \alpha_c - \frac{1}{\gamma^2}$$



ENERGY-PHASE EQUATIONS (1/2)

(protons and ions or electrons at extremely low energy)

In order to study the **longitudinal dynamics in a LINAC**, the following variables are used, which describe the generic particle **phase** (time of arrival) and **energy with respect to the synchronous particle**:



Arrival time (phase) of a **generic particle** at a certain gap (or cavity)

Arrival time (phase) of the **synchronous particle** at a certain gap (or cavity)

$$\begin{cases} \varphi = \phi - \phi_s = \omega_{RF}(t - t_s) \\ w = E - E_s \end{cases}$$

Energy of the **synchronous particle** at a certain position along the linac

Energy of a **generic particle** at a certain position along the linac

The **energy gain per cell (one gap + tube in case of a DTL)** of a generic particle and of a synchronous particle are:

$$\begin{cases} \Delta E_s = q\hat{V}_{acc}\cos\phi_s \\ \Delta E = q\hat{V}_{acc}\cos\phi = q\hat{V}_{acc}\cos(\phi_s + \varphi) \end{cases}$$

subtracting

$$\Delta w = \Delta E - \Delta E_s = q\hat{V}_{acc}[\cos(\phi_s + \varphi) - \cos\phi_s]$$

Dividing by the accelerating cell length ΔL and assuming that:

$$\frac{\hat{V}_{acc}}{\Delta L} = \hat{E}_{acc}$$

Average accelerating field over the cell (i.e. **average accelerating field**)

$$\frac{\Delta w}{\Delta L} = q\hat{E}_{acc}[\cos(\phi_s + \varphi) - \cos\phi_s]$$

Approximating

$$\frac{\Delta w}{\Delta L} \approx \frac{dw}{dz}$$

$$\frac{dw}{dz} = q\hat{E}_{acc}[\cos(\phi_s + \varphi) - \cos\phi_s]$$

ENERGY-PHASE EQUATIONS (2/2)

(protons and ions or electrons at extremely low energy)

On the other hand we have that the **phase variation per cell** of a generic particle and of a synchronous particle are:

$$\begin{cases}\Delta\phi_s = \omega_{RF}\Delta t_s \\ \Delta\phi = \omega_{RF}\Delta t\end{cases}$$

Δt is basically the **time of flight** between two accelerating cells

subtracting

$$\Delta\varphi = \omega_{RF}(\Delta t - \Delta t_s)$$

Dividing by the accelerating cell length ΔL

v, v_s are the average particles velocities

$$\frac{\Delta\varphi}{\Delta L} = \omega_{RF}\left(\frac{\Delta t}{\Delta L} - \frac{\Delta t_s}{\Delta L}\right) = \omega_{RF}\left(\frac{1}{v} - \frac{1}{v_s}\right) \rightleftarrows \underbrace{\cong}_{MAT} - \frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w$$

Approximating

$$\frac{\Delta\varphi}{\Delta L} \cong \frac{d\varphi}{dz}$$

This system of coupled (non linear) differential equations **describe the motion of a non synchronous particles** in the longitudinal plane with respect to the synchronous one.

$$\frac{dw}{dz} = q\hat{E}_{acc}[\cos(\phi_s + \varphi) - \cos\phi_s]$$

$$\frac{d\varphi}{dz} = -\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w$$

MAT

$$\omega_{RF}\left(\frac{1}{v} - \frac{1}{v_s}\right) = \omega_{RF}\left(\frac{v_s - v}{v v_s}\right) \rightleftarrows \underbrace{\cong}_{\substack{v v_s \cong v_s^2 \\ v - v_s \cong \Delta v}} - \frac{\omega_{RF}}{v_s^2}\Delta v = -\frac{\omega_{RF}}{c}\frac{\Delta\beta}{\beta_s^2}$$ remembering that $\beta = \sqrt{1 - 1/\gamma^2} \Rightarrow \beta d\beta = d\gamma/\gamma^3 \Rightarrow -\frac{\omega_{RF}}{c}\frac{\Delta\beta}{\beta_s^2} \cong -\frac{\omega_{RF}}{c}\frac{\Delta\gamma}{\beta_s^3\gamma_s^3} = -\frac{\omega_{RF}}{c}\frac{\overbrace{\Delta E}^{w}}{E_0\beta_s^3\gamma_s^3}$

SMALL AMPLITUDE ENERGY-PHASE OSCILLATIONS

(protons and ions or electrons at extremely low energy)

$$\begin{cases} \dfrac{dw}{dz} = q\hat{E}_{acc}[\cos(\varphi_s+\phi)-\cos\phi_s] \\ \dfrac{d\varphi}{dz} = -\dfrac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w \rightarrow cE_0\beta_s^3\gamma_s^3\dfrac{d\varphi}{dz} = -\omega_{RF}w \end{cases}$$

Deriving both terms with respect to z

$$cE_0\beta_s^3\gamma_s^3\frac{d^2\varphi}{dz^2} + cE_0\frac{d\beta_s^3\gamma_s^3}{dz}\frac{d\varphi}{dz} = -\omega_{RF}\frac{dw}{dz}$$

General non linear differential equation that gives the phase evolution

$$cE_0\beta_s^3\gamma_s^3\frac{d^2\varphi}{dz^2} + cE_0\left(\frac{d\beta_s^3\gamma_s^3}{dz}\right)\frac{d\varphi}{dz} = -\omega_{RF}q\hat{E}_{acc}[\cos(\varphi_s+\phi)-\cos\phi_s]$$

Assuming **small oscillations** around the synchronous particle

$$\cos(\phi_s+\varphi)-\cos\phi_s \cong \varphi\sin\phi_s$$

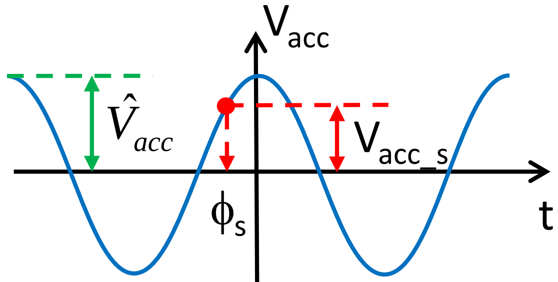
Deriving both terms with respect to z and assuming an **adiabatic acceleration**

$$\frac{d\beta_s^3\gamma_s^3}{dz} << 1$$

$$\frac{d^2\varphi}{dz^2} + \underbrace{q\frac{\omega_{RF}\hat{E}_{acc}\sin(-\phi_s)}{cE_0\beta_s^3\gamma_s^3}}_{\Omega_s^2}\varphi = 0$$

⇒The condition to have stable longitudinal oscillations and acceleration at the same time is:

$$\left.\begin{array}{l}\Omega_s^2 > 0 \Rightarrow \sin(-\phi_s) > 0 \\ V_{acc} > 0 \Rightarrow \cos\phi_s > 0\end{array}\right\} \Rightarrow -\frac{\pi}{2} < \phi_s < 0$$



if we accelerate on the rising part of the positive RF wave we have a **longitudinal force keeping the beam bunched** around the synchronous phase.

$$\begin{cases}\varphi = \hat{\varphi}\cos(\Omega_s z) \\ w = \hat{w}\sin(\Omega_s z)\end{cases}$$

⇒ The angular frequency is simply: $\Omega_T = \Omega_s\beta_s c$;

⇒ **The angular frequency scale with $1/\gamma^{3/2}$** that means that for ultra relativistic electrons shrinks to 0 (the beam is frozen)

COMPARISON LINAC-SYNCHROTRON FREQUENCY

$$\Omega_T^2 = -q\frac{c\omega_{RF}\hat{E}_{acc}\sin(\phi_s)}{E_0\beta_s\gamma_s^3}$$

$$\Omega_{synch}^2 = -\omega_{rev}^2\frac{q\hat{V}_{RF}\eta h}{2\pi\beta^2\gamma_s E_0}\cos\phi_s$$

Below transition

$$\Omega_{synch}^2 = q\frac{c\omega_{RF}\hat{E}_{RF}}{E_0\beta_s\gamma_s^3}\cos\phi_s$$

$$\hat{E}_{RF} = \frac{\hat{V}_{RF}}{L_{sync}}$$

Synchrotron length

LONGITUDINAL BEAM DYNAMICS CONSIDERATIONS

TUPP064 Proceedings of LINAC2014, Geneva, Switzerland

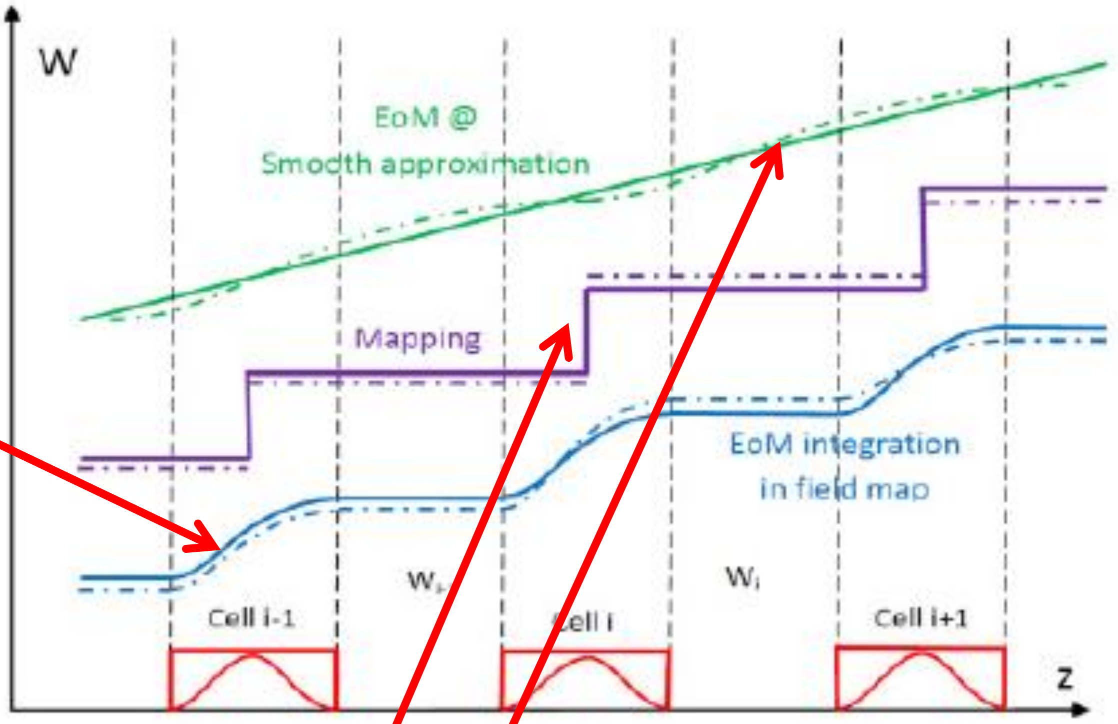
ZERO-CURRENT LONGITUDINAL BEAM DYNAMICS

J-M. Lagniel, GANIL, Caen, France

- The approach we have used so far for the longitudinal beam dynamics calculation is a **simplified approach**
- The most accurate but computer-time consuming, consists in **integrating the equation of motion (EoM)** using field maps giving the amplitude of the rf accelerating field

$$dW = q\,E_z(r(z),z)\,\cos\left(\frac{2\pi z}{\beta(z)\lambda} + \Phi_0\right)dz$$

$$d\phi = \omega_{rf}\,dt = \frac{2\pi}{\beta(z)\lambda}\,dz$$



- Another possible approach is to assume **concentrated energy kicks in the cavities** (Panofsky approach) and integrate the equation of motion.
- The approach we have used assumes basically an average effect of the acceleration (**smooth approximation**)
- For **large amplitude oscillations there are effects that only the correct approach can predict**

LARGE OSCILLATIONS AND SEPARATRIX (SMOOTH APPROX)

To study the longitudinal dynamics **at large oscillations**, we have to consider the **non linear system of differential equations** without small oscillation approximations (but with adiabatic acceleration approximation). It is possible to easily obtain the following relation between w and φ (that is the **Hamiltonian of the system** related to the total particle energy):

$$\frac{1}{2}\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w^2 + q\hat{E}_{acc}[\sin\phi - \phi\cos\phi_s] = \mathrm{H}$$

⇒**For each H we have different trajectories** in the longitudinal phase space

⇒the oscillations are **stable** within a region bounded by a special curve called **separatrix**: its equation is:

$$\frac{1}{2}\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w^2 + q\hat{E}_{acc}[\sin\phi + \sin\phi_s - (\phi+\phi_s)\cos\phi_s] = 0$$

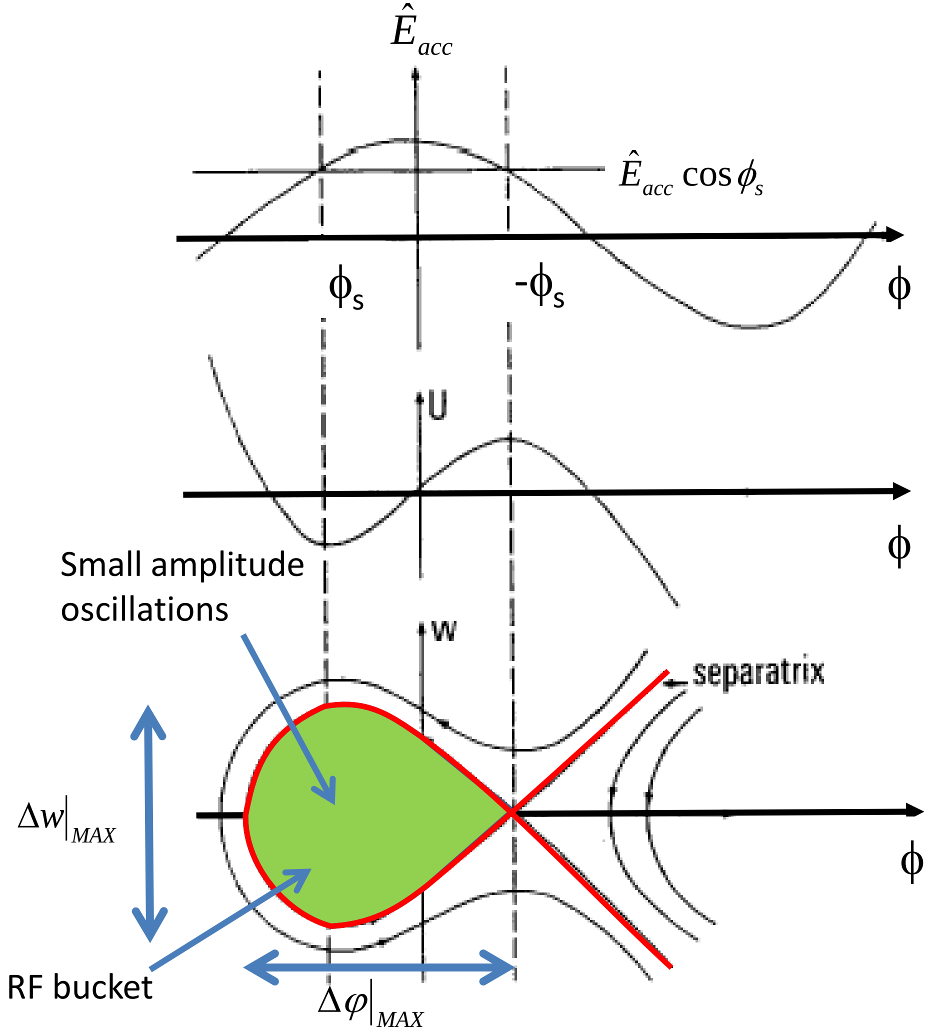
⇒the region inside the separatrix is called **RF bucket**. The dimensions of the bucket shrinks to zero if ϕ_s=0.

⇒trajectories outside the RF buckets are **unstable**.

⇒we can define the **RF acceptance** as the maximum extension in phase and energy that we can accept in an accelerator:

$$\Delta\varphi\big|_{MAX} \cong 3\phi_s$$

$$\Delta w\big|_{MAX} = \pm 2\left[\frac{qcE_o\beta_s^3\gamma_s^3\hat{E}_{acc}(\phi_s\cos\phi_s - \sin\phi_s)}{\omega_{RF}}\right]^{\frac{1}{2}}$$



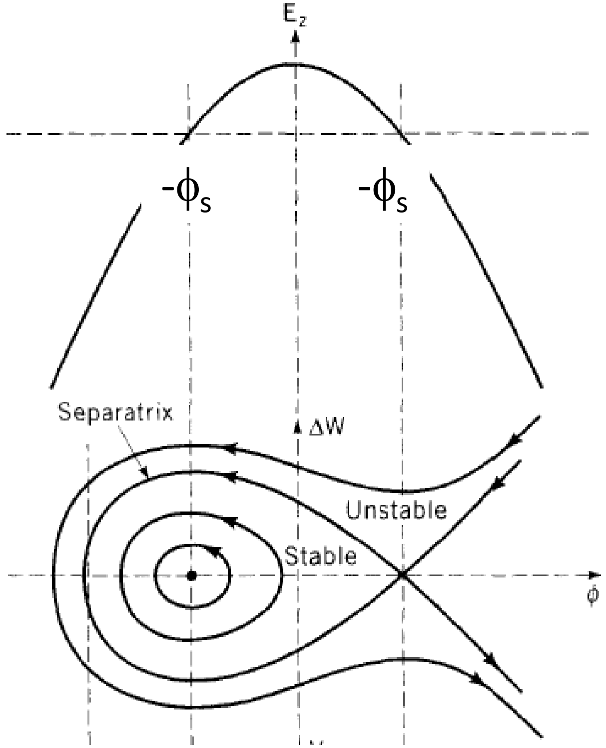
SEPARATRIX EQUATION

To study the longitudinal dynamics at large oscillations, we consider the non linear system of differential equations without the small oscillation approximation. The system also works for the fase ϕ= $(\phi_s + \varphi)$

$$\frac{d^2\phi}{dz^2} = -\frac{\omega_{RF} q\hat{E}_{acc}}{cE_0\beta_s^3\gamma_s^3}[\cos\phi - \cos\phi_s] = F(\phi)$$

The function ***F*** act as a non linear restoring force We can then write.

$$\frac{d}{dz}\left[\left(\frac{d\phi}{dz}\right)^2\right] = 2\frac{d\phi}{dz}\frac{d^2\phi}{dz^2} = 2\frac{d\phi}{dz}\cdot F = 2\frac{d\phi}{dz}\cdot\frac{d}{d\phi}\int_0^\phi F d\phi = 2\frac{d}{dz}\int_0^\phi F d\phi \Rightarrow \frac{d}{dz}\left[\left(\frac{d\phi}{dz}\right)^2 - 2\int_0^\phi F d\phi\right] = 0 \Rightarrow \frac{1}{2}\left(\frac{d\phi}{dz}\right)^2 - \int_0^\phi F d\phi = \text{const}$$



$$\frac{d\phi}{dz} = -\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w$$

$$\frac{1}{2}\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w^2 + q\hat{E}_{acc}[\sin\phi - \phi\cos\phi_s] = \text{const} = \text{H}$$

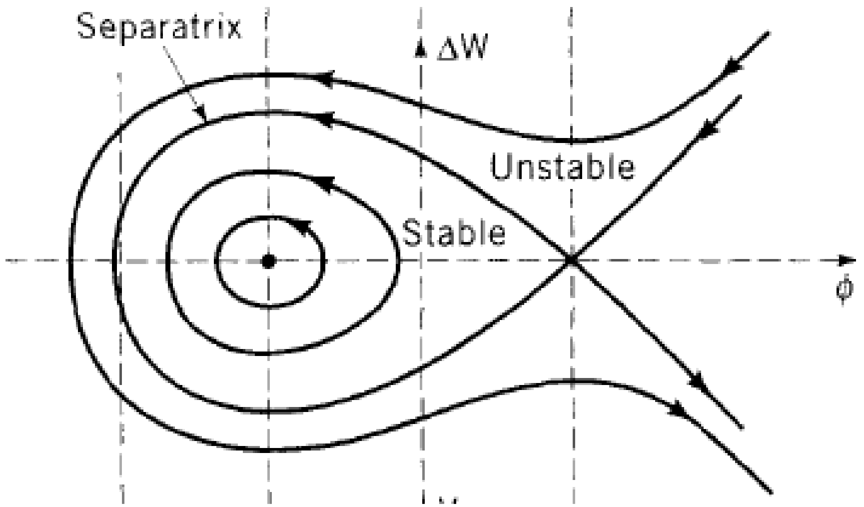
For the separatrix equation we have that when ϕ_s=-ϕ_s w=0

$$q\hat{E}_{acc}[-\sin\phi_s + \phi_s\cos\phi_s] = H_{sep}$$

$$\frac{1}{2}\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w^2 + q\hat{E}_{acc}[\sin\phi + \sin\phi_s - (\phi + \phi_s)\cos\phi_s] = 0$$

ADIABATIC DAMPING

$$\frac{1}{2}\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w^2 + q\hat{E}_{acc}[\sin(\phi_s+\varphi) - (\phi_s+\varphi)\cos\phi_s] = \mathrm{H}$$



For small amplitude oscillations around the synchronous phase we have

$$\frac{1}{2}\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w^2 + q\hat{E}_{acc}\left[\cos(\phi_s)\varphi - \frac{1}{2}sin(\phi_s)\varphi^2 - (\phi_s+\varphi)\cos\phi_s\right] = \mathrm{H}$$

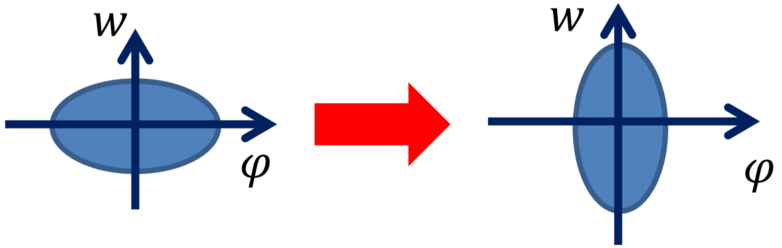
$$\frac{1}{2}\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w^2 - \frac{1}{2}q\hat{E}_{acc}\,sin(\phi_s)\varphi^2 = \mathrm{H}$$

A particle with some initial conditions will perform an ellipse in the phase space. Its maximum energy w_{max} is obtained when $\phi = \phi_s$ (i.e. $\varphi = 0$) and correspondingly its maximum phase excursion is obained when w=0. Then:

$$\varphi = 0 \rightarrow \frac{1}{2}\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w_{max}^2 = \mathrm{H}$$

$$w = 0 \rightarrow -\frac{1}{2}q\hat{E}_{acc}\,sin(\phi_s)\varphi_{max}^2 = \mathrm{H}$$

$$\frac{\varphi_{max}}{w_{max}} = \sqrt{\frac{\omega_{RF}}{-q\hat{E}_{acc}\sin(\phi_s)\,cE_0\beta_s^3\gamma_s^3}}$$



This ratio decrease during acceleration while the area of the ellipse should remain unchanged because there are only conservative forces (Liouville). **This means that the bunch reduce its length and increase its energy spread**

EXERCISE 10: ENERGY ACCEPTANCE

A RF accelerating structure operating at f_{RF}=400 MHz, is used to accelerate protons at an input nominal kinetic energy W_{in}=10 MeV. Assuming that the nominal synchronous phase $\phi_s=-\pi/6$ rad and that the average accelerating field is E_{acc}=2 MV/m, calculate the maximum kinetic energy of the protons that is possible to capture in the RF bucket.

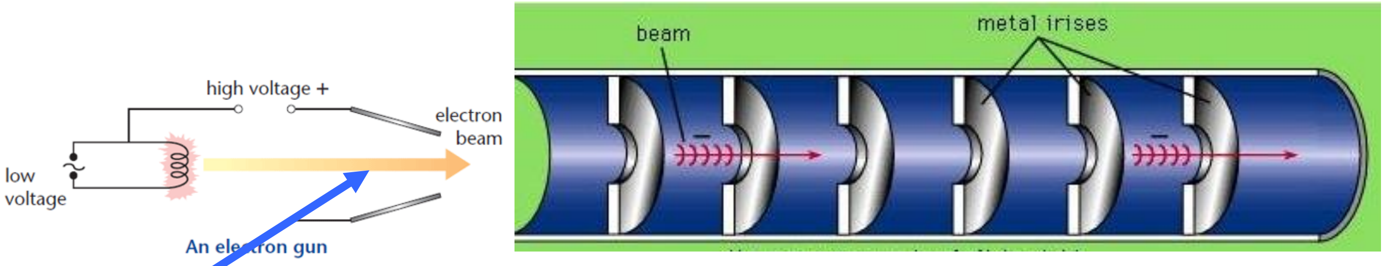
LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS

From previous formulae it is clear that there is **no motion in the longitudinal phase plane for ultrarelativistic particles** (γ>>1).

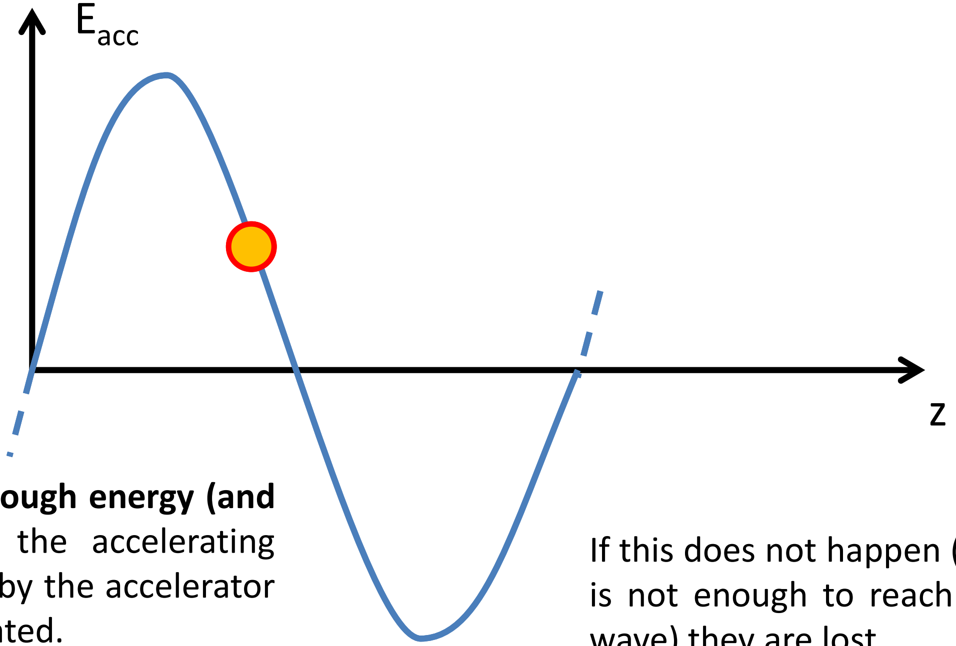
⇒This is the case of **electrons** whose **velocity is always close to speed of light c** even at low energies.

⇒Accelerating structures are designed to provide an accelerating field synchronous with particles moving at v=c. like **TW structures with phase velocity equal to c**.

It is interesting to analyze what happen if we **inject an electron beam produced by a cathode (at low energy) directly in a TW structure** (with v_{ph}=c) and the conditions that allow to **capture** the beam (this is equivalent to consider instead of a TW structure a SW designed to accelerate ultrarelativistic particles at v=c).



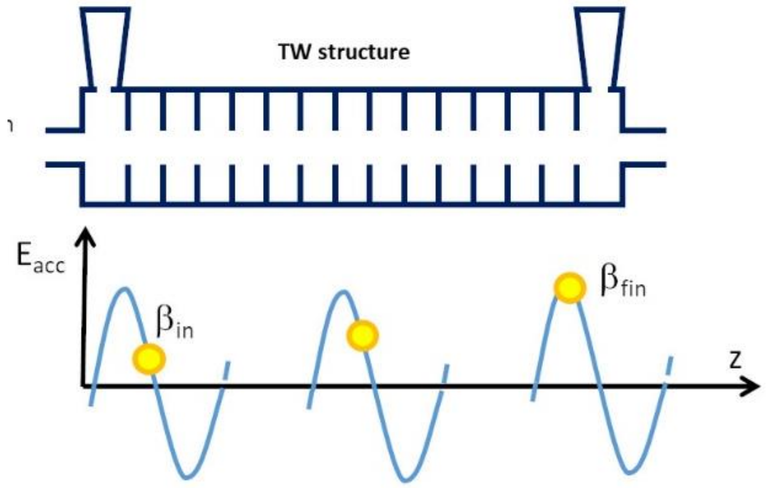
Particles enter the structure with velocity **v<c** and, initially, they are **not synchronous with the accelerating field** and there is a so called slippage.



After a certain distance they can **reach enough energy (and velocity) to become synchronous** with the accelerating wave. This means that they are captured by the accelerator and from this point they are stably accelerated.

If this does not happen (the energy increase is not enough to reach the velocity of the wave) they are lost

LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS: PHASE SPLIPPAGE



The **accelerating field** of a TW structure can be expressed by

$$E_{acc} = \hat{E}_{acc}\cos\underbrace{(\omega_{RF}t - kz)}_{\phi(z,t)}$$

The **equation of motion of a particle with a position z at time t accelerated by the TW** is then

$$\frac{d}{dt}(mv) = q\hat{E}_{acc}\cos\phi(z,t) \;\Rightarrow\; m_0 c\frac{d}{dt}(\gamma\beta) = m_0 c\gamma^3\frac{d\beta}{dt} = q\hat{E}_{acc}\cos\phi$$

It is useful to find which is the relation between β and ϕ from an initial condition (in) to a final one (fin)

$$\sin\phi_{fin} = \sin\phi_{in} + \frac{2\pi E_0}{\lambda_{RF} q\hat{E}_{acc}}\left(\sqrt{\frac{1-\beta_{in}}{1+\beta_{in}}} - \sqrt{\frac{1-\beta_{fin}}{1+\beta_{fin}}}\right)$$

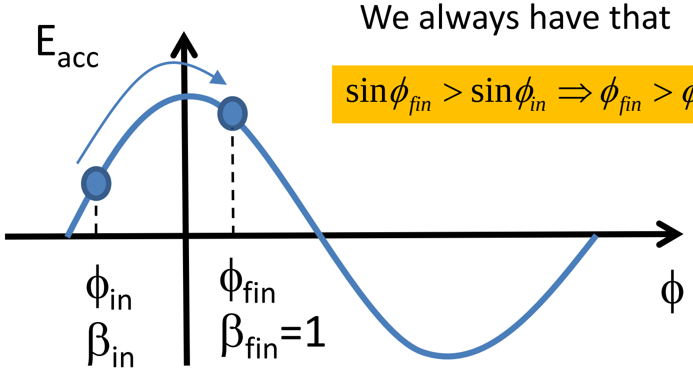
Suppose that the particle reach asymptotically the value β_{fin}=1 we have:

Should be in the interval [-1,1] to have a solution for ϕ_{fin}

$$\sin\phi_{fin} = \sin\phi_{in} + \frac{2\pi E_0}{\lambda_{RF} q\hat{E}_{acc}}\sqrt{\frac{1-\beta_{in}}{1+\beta_{in}}}$$

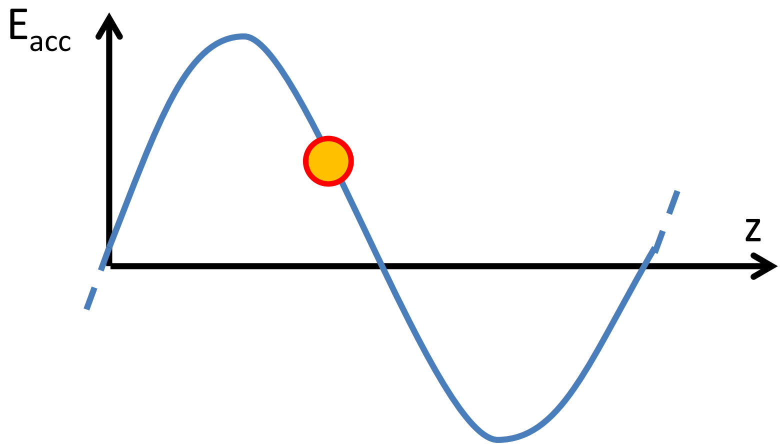
This limits the possible injection phases (i.e. the phase of the electrons that is possible to capture)

This quantity is >0



We always have that

$$\sin\phi_{fin} > \sin\phi_{in} \Rightarrow \phi_{fin} > \phi_{in}$$



LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS: CAPTURE ACCELERATING FIELD CALCULATION

$\Rightarrow$For a given injection energy (β_{in}) and phase (ϕ_{in}) we can find which is the accelerating field (E_{acc}) that is necessary, to have the completely relativistic beam at phase fin (that is necessary to **capture the beam at phase ϕ_{in}**)

$$\hat{E}_{acc} = \frac{2\pi E_0}{\lambda_{RF} q\left(\sin\phi_{fin} - \sin\phi_{in}\right)}\sqrt{\frac{1-\beta_{in}}{1+\beta_{in}}}$$

Example:

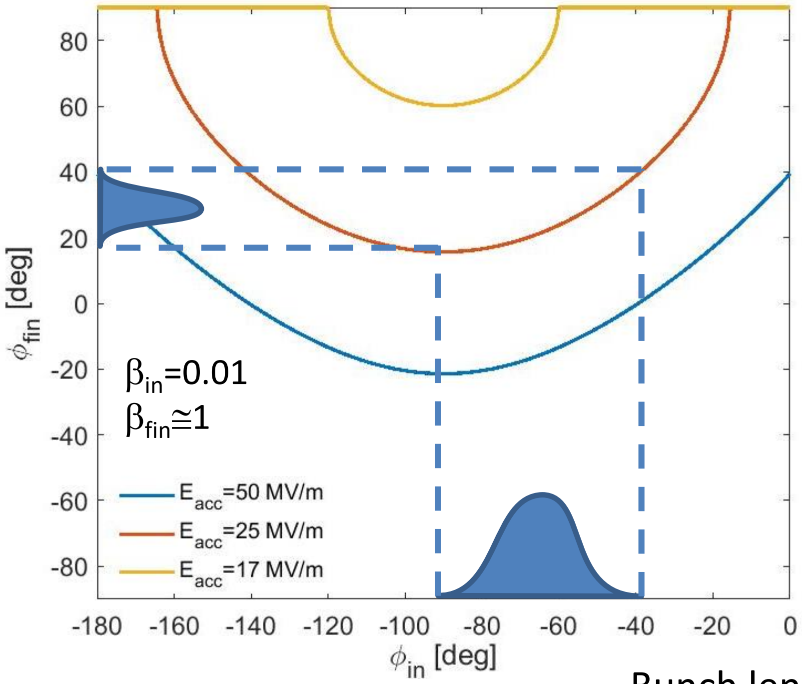
E_{in} = *50 keV*, (kinetic energy), $\phi_{in}=-\pi/2$,
$\phi_{fin}=0 \Rightarrow \gamma_{in} \approx 1.1$; $\beta_{in} \approx 0.41$
f_{RF}= *2856 MHz* $\Rightarrow \lambda_{RF} \approx$ *10.5 cm*

We obtain $E_{acc} \cong 20$MV/m;

LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS: CAPTURE EFFICIENCY AND BUNCH COMPRESSION

During the capture process, as the injected beam moves up to the crest, the beam is also bunched, which is caused by **velocity modulation** (velocity bunching). This mechanism can be used to compress the electron bunches (FEL applications).

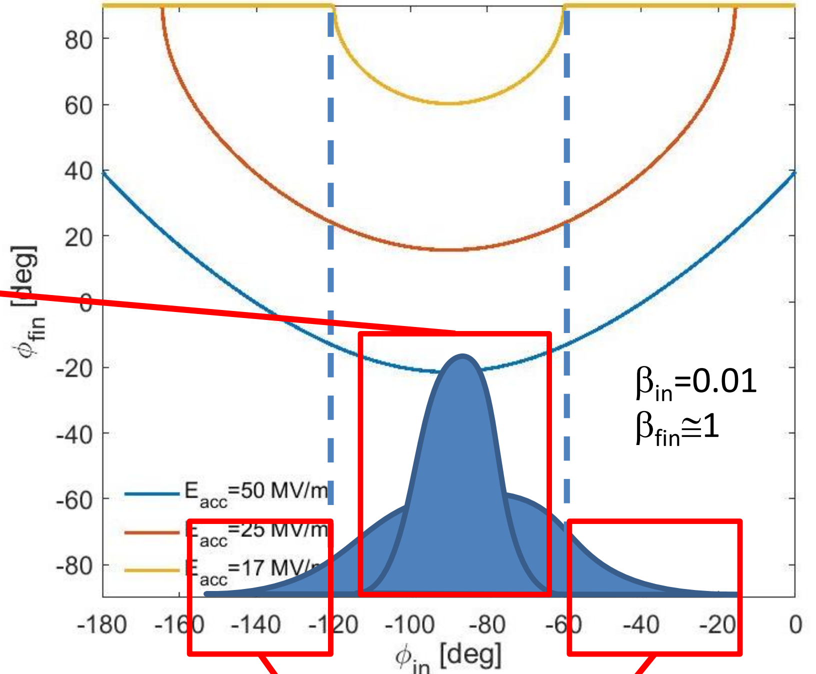
BUNCH COMPRESSION



$$\sin\phi_{fin} = \sin\phi_{in} + \frac{2\pi E_0}{\lambda_{RF} q\hat{E}_{acc}}\sqrt{\frac{1-\beta_{in}}{1+\beta_{in}}}$$

All particles are captured

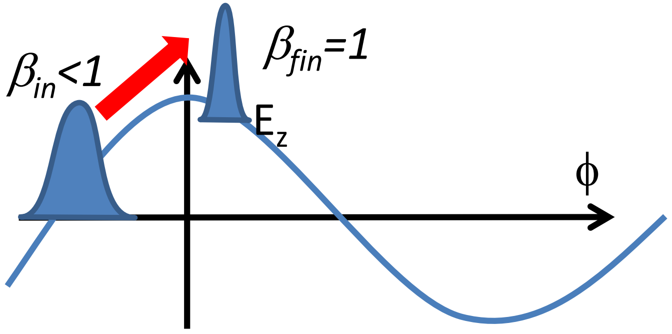
CAPTURE EFFICIENCY



These particle are lost during the capture process

Bunch lenght variation

$$\sin\phi_{fin} = \sin\phi_{in} + \frac{2\pi E_0}{\lambda_{RF} q\hat{E}_{acc}}\sqrt{\frac{1-\beta_{in}}{1+\beta_{in}}} \Rightarrow \Delta\phi_{fin} = \Delta\phi_{in}\frac{\cos\phi_{in}}{\cos\phi_{fin}}$$

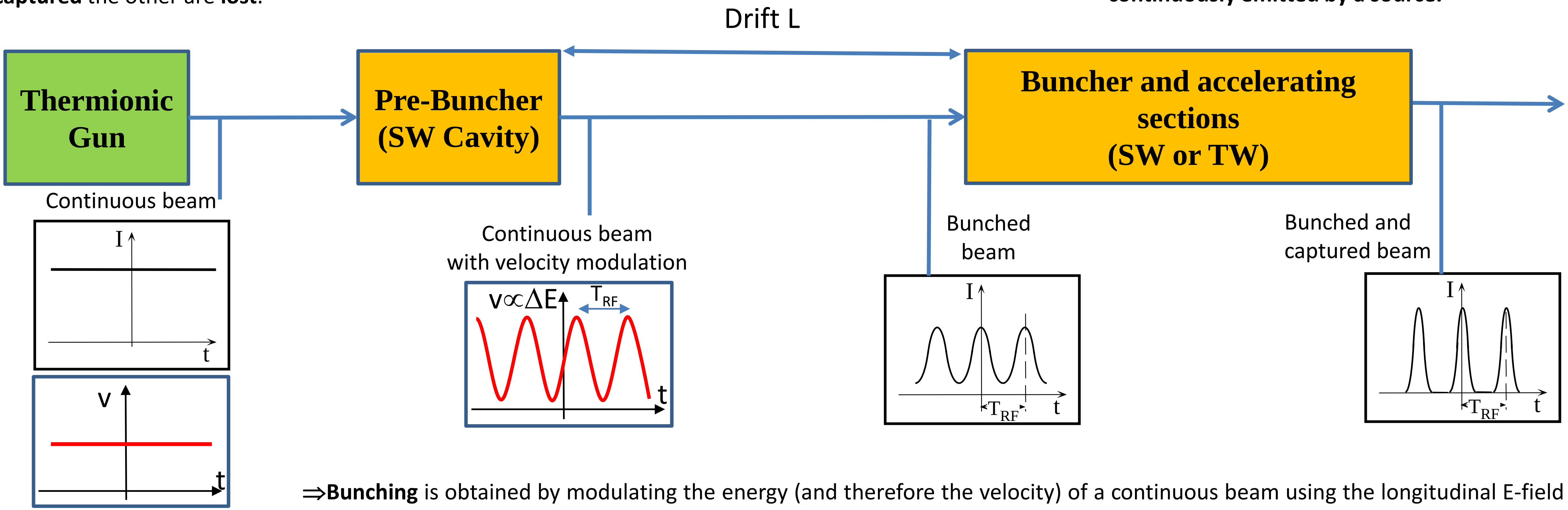


Depending on the injection phase we can have bunch compression or expansion

BUNCHER AND CAPTURE SECTIONS
(electrons)

Once the capture condition $E_{RF}>E_{RF\_MIN}$ is fulfilled the fundamental equation of previous slide sets the **ranges of the injection phases ϕ_{in} actually accepted**. Particles whose injection phases are within this range can be **captured** the other are **lost**.

In order **to increase the capture efficiency** of a traveling wave section, **pre-bunchers** are often used. They are SW cavities aimed at **pre-forming particle bunches gathering particles continuously emitted by a source.**



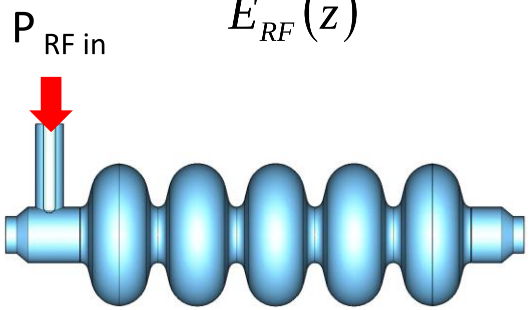
⇒**Bunching** is obtained by modulating the energy (and therefore the velocity) of a continuous beam using the longitudinal E-field of a SW cavity. After a certain **drift space** the **velocity modulation is converted in a density charge modulation**. The density modulation depletes the regions corresponding to injection phase values incompatible with the capture process

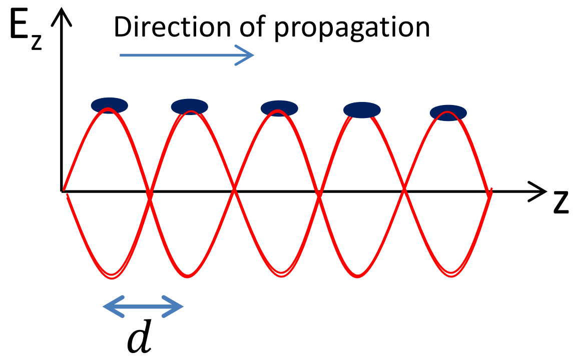
⇒A TW accelerating structure (**capture section**) is placed at an **optimal distance from the pre-buncher**, to capture a large fraction of the charge and accelerate it till relativistic energies. The **amount of charge lost is drastically reduced**, while the capture section provide also further beam bunching.

SW AS A SUM OF TWO TW: RF NON-SYNCRONOUS HARMONICS

Let us consider the case of a multi-cell SW cavity working on the π-mode. The Accelerating field can be expressed as:

$$E_z = \underbrace{\hat{E}_{RF}\cos(kz)}_{E_{RF}(z)}\cos(\omega_{RF} t)$$





In order to have synchronism between the accelerating field and and ultrarelativistic particle we have to satisfy the following relation (supposing electrons β=1):

$$d = \frac{c}{2f_{RF}} = \frac{\lambda_{RF}}{2}$$

$$k = \frac{2\pi}{2d} = \frac{2\pi}{\lambda_{RF}} = \frac{\omega_{RF}}{c}, \ \lambda_{RF} = cT_{RF}$$

The accelerating field seen by the particle is given by (t=z/c):

$$E_z\Big|_{\substack{seen\\by\\particle\\z=ct}} = \hat{E}_{RF}\cos(kz)\cos\left(\omega_{RF}\frac{z}{c}\right) = \hat{E}_{RF}\cos^2(kz) = \frac{\hat{E}_{RF}}{2} + \underbrace{\frac{\hat{E}_{RF}}{2}\cos(2kz)}$$

Oscillating term that has an average value equal to 0

On the other hand we can the SW can be written as the sum of two TWs in the form:

$$E_z = \hat{E}_{RF}\cos(kz)\cos(\omega_{RF} t) = \underbrace{\frac{\hat{E}_{RF}}{2}\cos(\omega_{RF} t - kz)} + \underbrace{\frac{\hat{E}_{RF}}{2}\cos(\omega_{RF} t + kz)}$$

Synchronous wave co-propagating with beam

NON-Synchronous wave (called RF non-synchronous harmonic) counter-propagating with beam (opposite direction)

The accelerating field seen by the particle is given by $k = \omega_{RF}/c$:

$$E_z\Big|_{\substack{seen\\by\\particle\\z=ct}} = \frac{\hat{E}_{RF}}{2}\cos\left(\omega_{RF}\frac{z}{c} - kz\right) + \frac{\hat{E}_{RF}}{2}\cos\left(\omega_{RF}\frac{z}{c} + kz\right) = \frac{\hat{E}_{RF}}{2} + \frac{\hat{E}_{RF}}{2}\cos(2kz)$$

Oscillating field that does not contribute to acceleration but that gives RF focusing

$$E_z\Big|_{\substack{seen\\by\\particle\\z=ct}} = \frac{\hat{E}_{RF}}{2} + \frac{\hat{E}_{RF}}{2}\cos(2kz) = \frac{\hat{E}_{RF}}{2} + \frac{\hat{E}_{RF}}{2}\cos(2\omega_{RF} t)$$

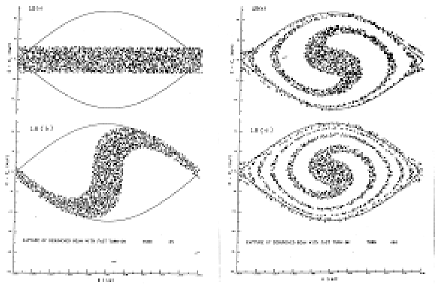
t=z/c

Synchronous wave: acceleration

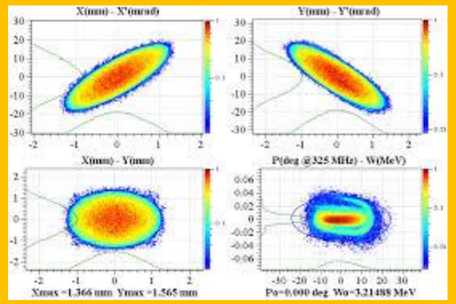
LINAC: BASIC DEFINITION AND MAIN COMPONENTS

LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory** by electromagnetic fields.

Longitudinal dynamics of accelerated particles

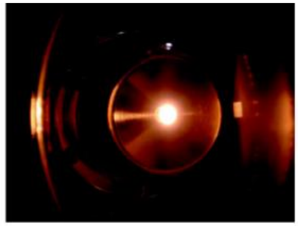


Transverse dynamics of accelerated particles



LINAC BEAM DYNAMICS

Particle source



Accelerated beam

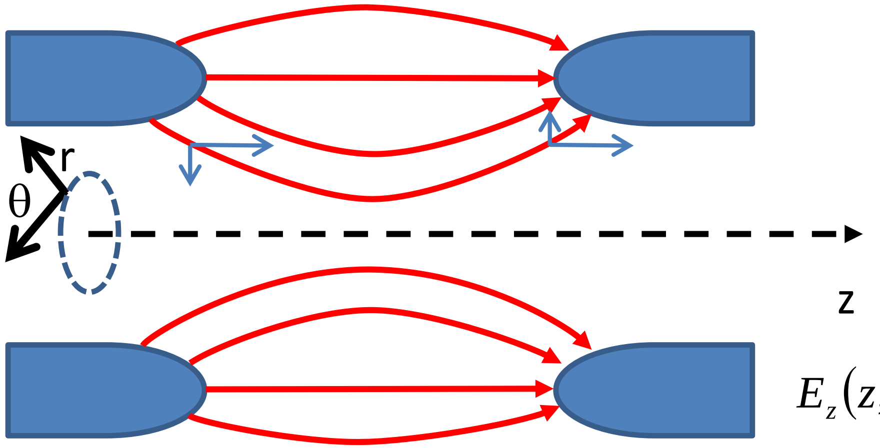
Accelerating structures

Focusing elements: quadrupoles and solenoids

LINAC COMPONENTS AND TECHNOLOGY

RF TRANSVERSE FORCES

The **RF fields act on the transverse beam dynamics** because of the transverse components of the E and B field



⇒According to Maxwell equations the **divergence of the field is zero** and this implies that in traversing one accelerating gap there is a focusing/defocusing term

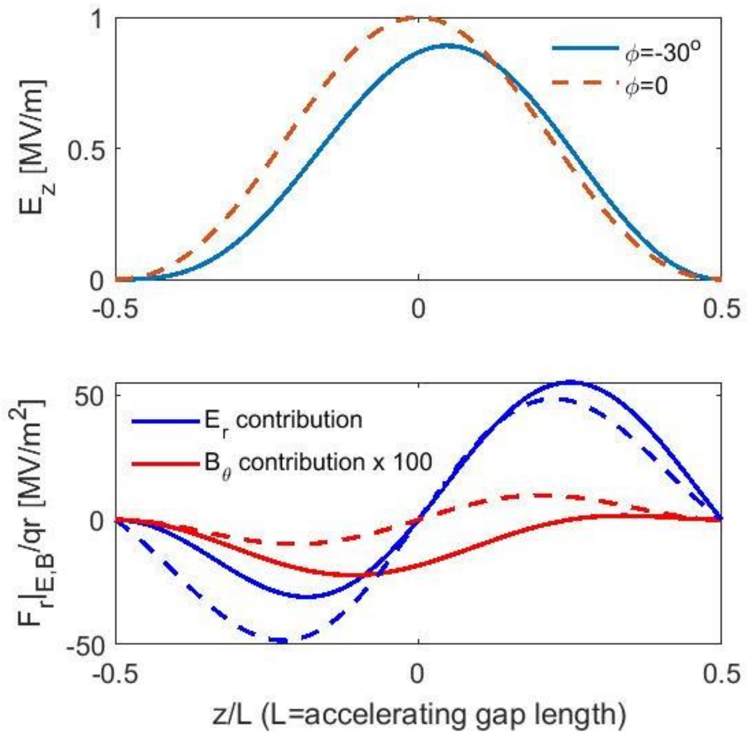
$$E_z(z,t) = E_{RF}(z)\cos(\omega_{RF} t)$$

$$\nabla \vec{E} = 0 \qquad \nabla \times \vec{B} = \frac{1}{c^2}\vec{E} \Rightarrow \begin{cases} E_r = -\dfrac{r}{2}\dfrac{\partial E_z}{\partial z} \\ B_\theta = \dfrac{r}{2c^2}\dfrac{\partial E_z}{\partial t} \end{cases}$$

$$F_r = q(E_r - vB_\theta) = -q\frac{r}{2}\left(\frac{\partial E_z}{\partial z} - \frac{\beta}{c}\frac{\partial E_z}{\partial t}\right)$$

$$F_r\big|_E = -q\frac{r}{2}\frac{\partial E_{RF}(z)}{\partial z}\cos\left(\omega_{RF}\frac{z}{\beta c} + \phi_{inj}\right)$$

$$F_r\big|_B = q\frac{r}{2}\omega_{RF}\frac{\beta}{c}E_{RF}(z)\sin\left(\omega_{RF}\frac{z}{\beta c} + \phi_{inj}\right)$$



f_{RF}=350 MHz
β=0.1
L=3cm

RF TRANSVERSE FORCES IN MULTI-CELL STRUCTURES: CONTRIBUTION OF THE FORWARD AND BACKWARD WAVES

For a multi-cell structure let us suppose that the field can be written as:

$$E_z = \underbrace{\hat{E}_{RF}\cos(kz)}_{E_{RF}(z)}\cos(\omega_{RF}t)$$

The Lorentz force is then given by the two contributions

To have synchronism

$$d = \frac{\beta c}{2f_{RF}} = \frac{\beta\lambda_{RF}}{2}$$

$$k = \frac{2\pi}{2d} = \frac{2\pi}{\beta\lambda_{RF}} = \frac{\omega_{RF}}{\beta c},\ \lambda_{RF} = cT_{RF}$$

$$\gamma = 1\Big/\sqrt{1-\beta^2}$$

$$F_r|_E = -q\frac{r}{2}\frac{\partial E_{RF}(z)}{\partial z}\cos\left(\omega_{RF}\frac{z}{\beta c}+\varphi_s\right) = q\frac{r}{2}\hat{E}_{RF}\sin(kz)\cos\left(\omega_{RF}\frac{z}{\beta c}+\varphi_s\right) =$$
$$q\frac{r}{2}\frac{\omega_{RF}}{\beta c}\hat{E}_{RF}\sin\left(\omega_{RF}\frac{z}{\beta c}z\right)\cos\left(\omega_{RF}\frac{z}{\beta c}+\varphi_s\right) = q\frac{r}{4}\frac{\omega_{RF}}{\beta c}\hat{E}_{RF}\left[\sin(-\varphi_s)+\cos\left(2\omega_{RF}\frac{z}{\beta c}+\varphi_s\right)\right]$$

$$F_r|_B = q\frac{r}{2}\omega_{RF}\frac{\beta}{c}\hat{E}_{RF}\cos(kz)\sin\left(\omega_{RF}\frac{z}{\beta c}+\varphi_s\right) = q\frac{r}{4}\omega_{RF}\frac{\beta}{c}\hat{E}_{RF}\left[\sin(\varphi_s)+\cos\left(2\omega_{RF}\frac{z}{\beta c}+\varphi_s\right)\right]$$

$$F_r\Big|_E + F_r\Big|_B = \underbrace{q\frac{r}{2}\frac{\pi\hat{E}_{RF}}{\lambda_{RF}\beta\gamma^2}\sin(-\varphi_s)}_{} + q\frac{r}{2}\frac{\pi\hat{E}_{RF}}{\lambda_{RF}}\left(\frac{\beta^2+1}{\beta}\right)\underbrace{\cos\left(2\omega_{RF}\frac{z}{\beta c}+\varphi_s\right)}_{}$$

Positive term (i.e. defocusing force) because φ_s<0. This term is given by the synchronous harmonic (forward wave)

The average integrated effect is zero. This term is given by the non synchronous harmonic (backward wave). In electron linacs this gives a focusing force

RF DEFOCUSING

From previous formulae it is possible to calculate the **transverse momentum increase** due to the RF transverse forces. Assuming that the velocity and position changes over the gap are small we obtain to the first order:

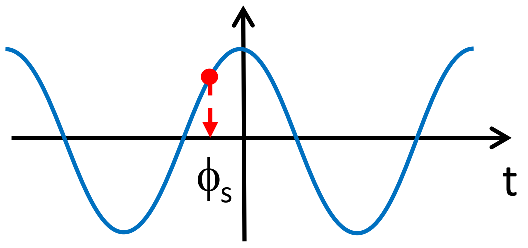
Transverse momentum increase

Defocusing force since sinϕ<0

Gap length

Defocusing effect

$$\Delta p_r = \int_{-L/2}^{+L/2} F_r \frac{dz}{\beta c} = -\frac{\pi q \hat{E}_{acc} L \sin\phi}{c\gamma^2\beta^2\lambda_{RF}} r$$



$\hat{E}_{acc} = \hat{E}_{RF}/2$

⇒ transverse **defocusing scales as ~1/γ²** and **disappears at relativistic regime (electrons). In this case we have a compensation between the electric deflection and the magnetic one.**

⇒ At relativistic regime (**electrons**), moreover, we have, in general, **ϕ=0 for maximum acceleration** and this completely cancel the defocusing effect

⇒ Also in the **non relativistic regime** for a correct evaluation of the defocusing effect we have to:

⇒ take into account the **velocity change across the accelerating gap**

⇒ the **transverse beam dimensions changes across the gap** (with a general reduction of the transverse beam dimensions due to the focusing in the first part)

Both effects give a **reduction of the defocusing force**

COLLECTIVE EFFECTS: SPACE CHARGE AND WAKEFIELDS

Collective effects are **all effects related to the number of particles** and they can play a crucial role in the longitudinal and transverse beam dynamics of intense beam LINACs

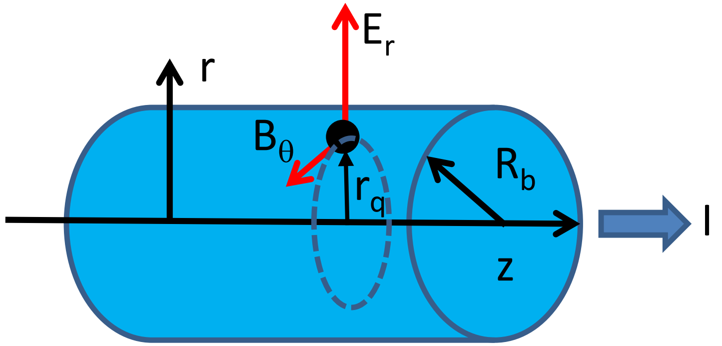
SPACE CHARGE

⇒ **Effect of Coulomb repulsion between particles (space charge).**

⇒ These effects cannot be neglected especially at **low energy and at high current** because the space charge forces **scales as 1/γ^2 and with the current I.**

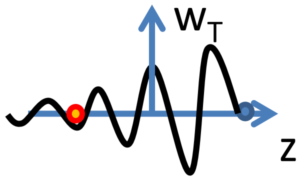
EXAMPLE: Uniform and infinite cylinder of charge moving along z

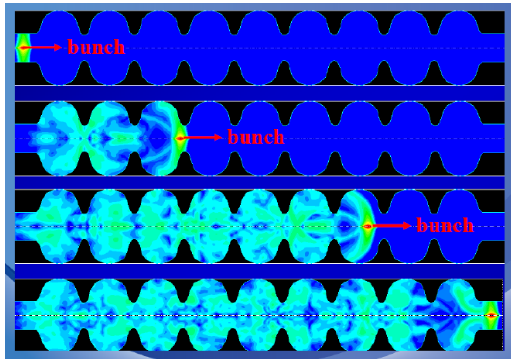
$$\vec{F}_{SC} = q\frac{I}{2\pi\varepsilon_0 R_b^2 \beta c \gamma^2} r_q \hat{r}$$



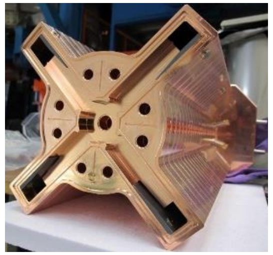
WAKEFIELDS

The other effects are due to the **wakefield**. The passage of bunches through accelerating structures excites electromagnetic **field**. This field can have longitudinal and transverse components and, interacting with subsequent bunches (long range wakefield), **can affect the longitudinal and the transverse beam dynamics**. In particular the **transverse wakefields**, can drive an instability along the train called **multibunch beam break up** (BBU).





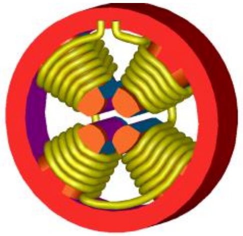
Several approaches are used to absorb these field from the structures like **loops** couplers, **waveguides**, Beam pipe **absorbers**



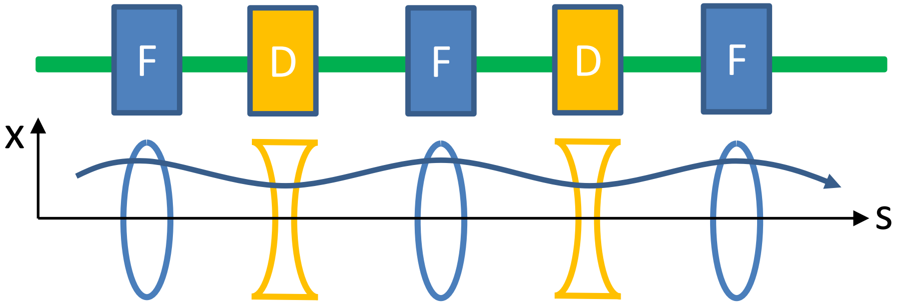
MAGNETIC FOCUSING AND CONTROL OF THE TRANSVERSE DYNAMICS

⇒**Defocusing RF forces, space charge** or the natural divergence (emittance) of the beam need to be **compensated** and controlled by **focusing forces**.

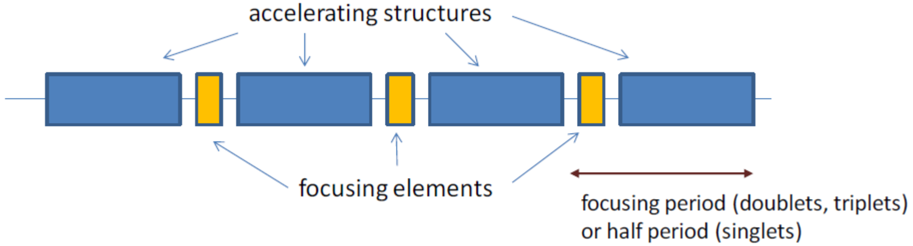
This is provided by **quadrupoles** along the beam line.
At low energies also **solenoids** can be used



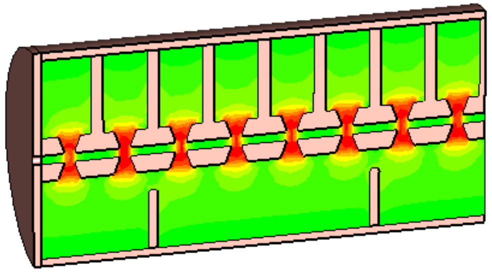
⇒Quadrupoles are focusing in one plane and defocusing on the other. A global focalization is provides by **alternating quadrupoles** with opposite signs



⇒In a linac one **alternates accelerating structures with focusing sections**.

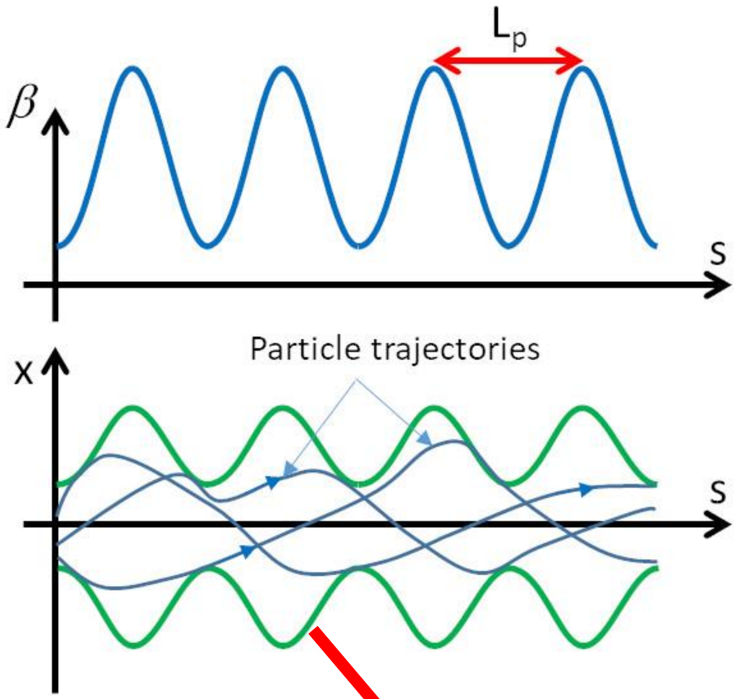


⇒The type of magnetic configuration and magnets type/distance depend on the type of particles/energies/beam parameters we want to achieve.



TRANSVERSE OSCILLATIONS AND BEAM ENVELOPE

Due to the **alternating quadrupole focusing system** each particle perform transverse oscillations along the LINAC.



Focusing period (L_p)= length after which the structure is repeated (usually as Nβλ).

The final transverse beam dimensions ($\sigma_{x,y}(s)$) vary along the linac and are contained within an **envelope**

The **equation of motion in the transverse plane** is of the type:

Term depending on the magnetic configuration

RF defocusing/focusing term

$$\frac{d^2x}{ds^2} + \underbrace{\left[\kappa^2(s) - k_{RF}^2(s)\right]}_{K^2(s)} x - F_{SC} = 0$$

Space charge term

The **single particle trajectory is a pseudo-sinusoid** described by the equation:

$$x(s) = \sqrt{\varepsilon_0 \beta(s)} \cos\left[\int_{s_0}^{s} \frac{ds}{\beta(s)} + \phi_0\right]$$

Characteristic function (Twiss β-function [m]) that depend on the magnetic and RF configuration

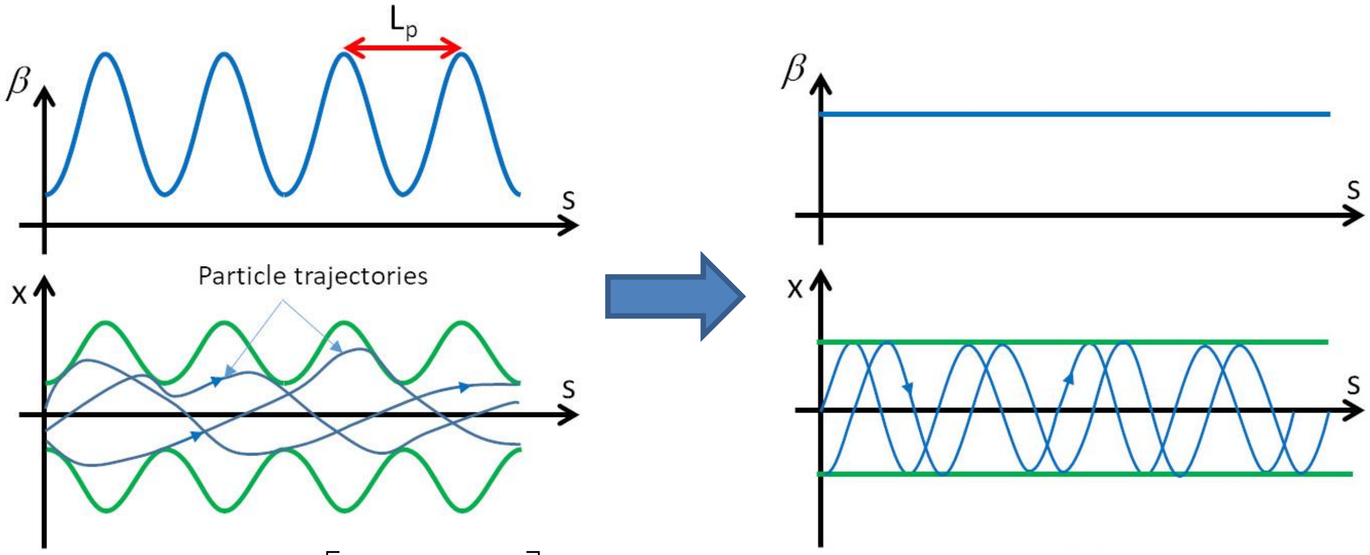
Depend on the initial conditions of the particle

$$\sigma = \int_{L_p} \frac{ds}{\beta(s)} \approx \frac{L_p}{\langle \beta \rangle}$$

Transverse phase advance per period L_p. For stability should be $0<\sigma<\pi$

SMOOTH APPROXIMATION OF TRANSVERSE OSCILLATIONS

⇒In case of "**smooth approximation**" of the LINAC (we consider an average effect of the quadrupoles and RF) we obtain a simple harmonic motion along s of the type (β is constant):

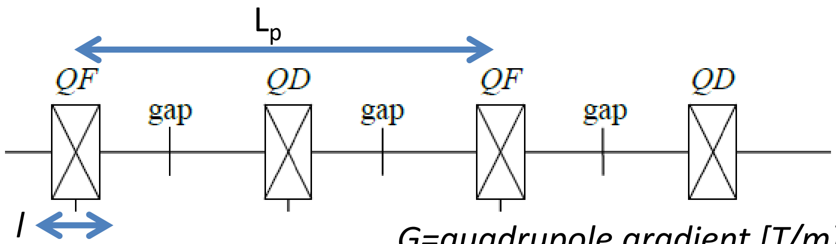


$$x(s)=\sqrt{\varepsilon_o\beta(s)}\cos\left[\int_{s_0}^{s}\frac{ds}{\beta(s)}+\phi_0\right]$$

$$\sigma=\int_{L_p}\frac{ds}{\beta(s)}\approx\frac{L_p}{\langle\beta\rangle}$$

$$x(s)=\sqrt{\varepsilon_o}\sqrt{1/K_0}\cos\left(K_0 s+\phi_0\right)$$

$$\sigma=\int_{L_p}\frac{ds}{\beta(s)}=K_0L_p$$



G=quadrupole gradient [T/m]
l=quadrupole length

Phase advance per unit length (σ/L_p)

$$K_0=\sqrt{\left(\frac{qGl}{2m_0c\gamma_s\beta_s}\right)^2-\frac{\pi q\hat{E}_{acc}\sin(-\phi_s)}{m_0c^2\lambda_{RF}(\gamma_s\beta_s)^3}}$$

Magnetic focusing elements (for a FODO)

RF defocusing term

NB: the RF defocusing term ∝f sets a higher limit to the working frequency

If we consider also the **Space Charge contribution** in the simple case of an **ellipsoidal beam** (linear space charges) we obtain:

$$K_0=\sqrt{\left(\frac{qGl}{2m_0c\gamma_s\beta_s}\right)^2-\frac{\pi q\hat{E}_{acc}\sin(-\phi_s)}{m_0c^2\lambda_{RF}(\gamma_s\beta_s)^3}-\frac{3Z_0qI\lambda_{RF}(1-f)}{8\pi m_0c^2\beta_s^2\gamma_s^3r_xr_yr_z}}$$

Space charge term

I= average beam current (Q/T_{RF})
$r_{x,y,z}$=ellipsoid semi-axis
f= form factor (0<f<1)
Z_0=free space impedance (377 Ω)

For ultrarelativistic **electrons RF defocusing and space charge disappear** and the external focusing is required to control the emittance and to stabilize the beam against instabilities.

GENERAL CONSIDERATIONS ON LINAC OPTICS DESIGN (1/2)

PROTONS AND IONS

⇒ Beam dynamics dominated by **space charge and RF defocusing forces**

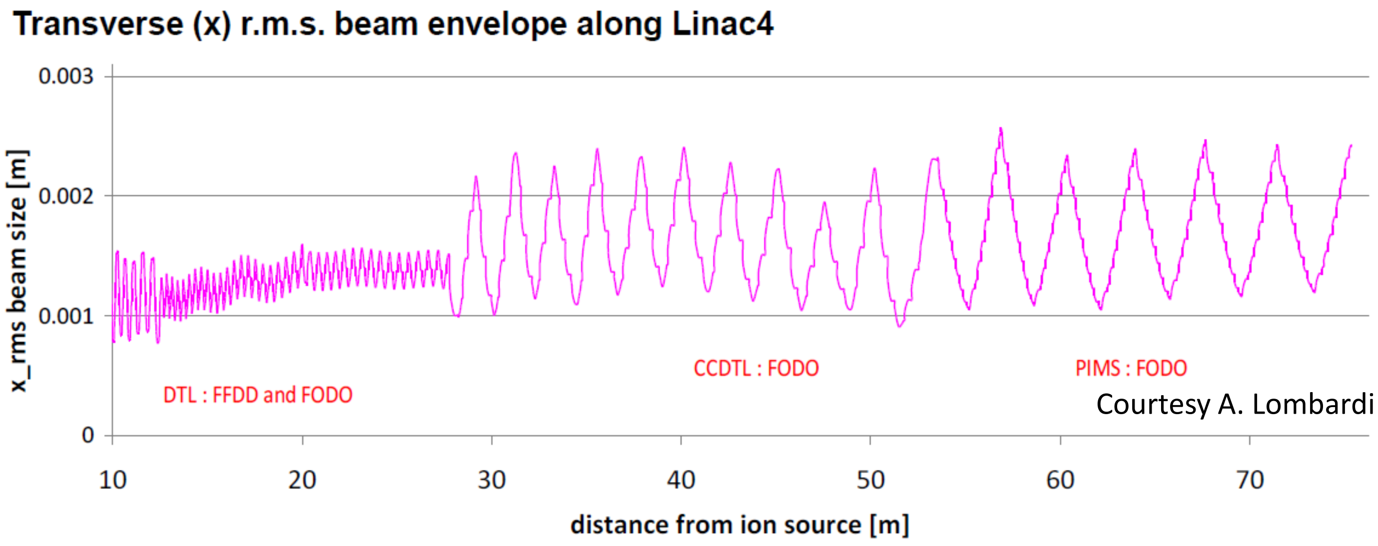
⇒ Focusing is usually provided by **quadrupoles**

⇒ Phase advance per period (σ) should be, in general, in the range 30-80 deg, this means that, at low energy, we need a strong focusing term (**short quadrupole distance and high quadrupole gradient**) to compensate for the rf defocusing, but the limited space ($\beta\lambda$) limits the achievable G and beam current

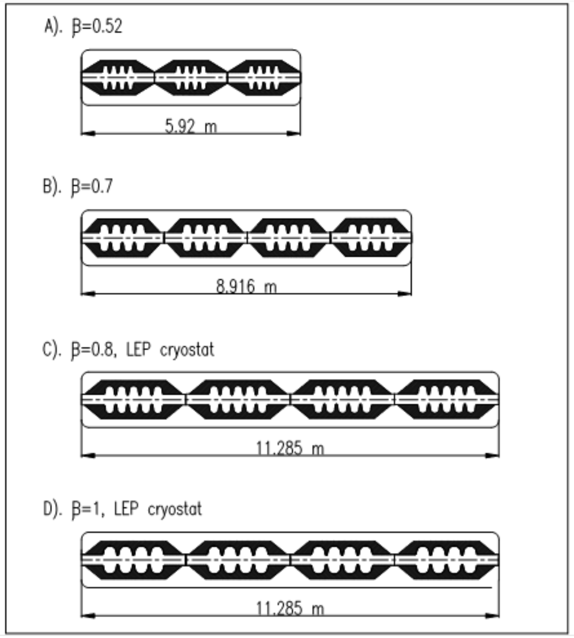
⇒ **As β increases, the distance between focusing elements can increase** ($\beta\lambda$ in the DTL goes from ~70mm (3 MeV, 352 MHz) to ~250mm (40 MeV), and can be increased to 4-10$\beta\lambda$ at higher energy (>40 MeV).

⇒ A linac is made of a **sequence of structures, matched to the beam velocity**, and where the length of the focusing period increases with energy. As β increases, longitudinal phase error between cells of identical length becomes small and we can have **short sequences of identical cells** (lower construction costs).

⇒ Keep sufficient safety **margin between beam radius and aperture**



Courtesy A. Lombardi



GENERAL CONSIDERATIONS ON LINAC OPTICS DESIGN (2/2)

ELECTRONS

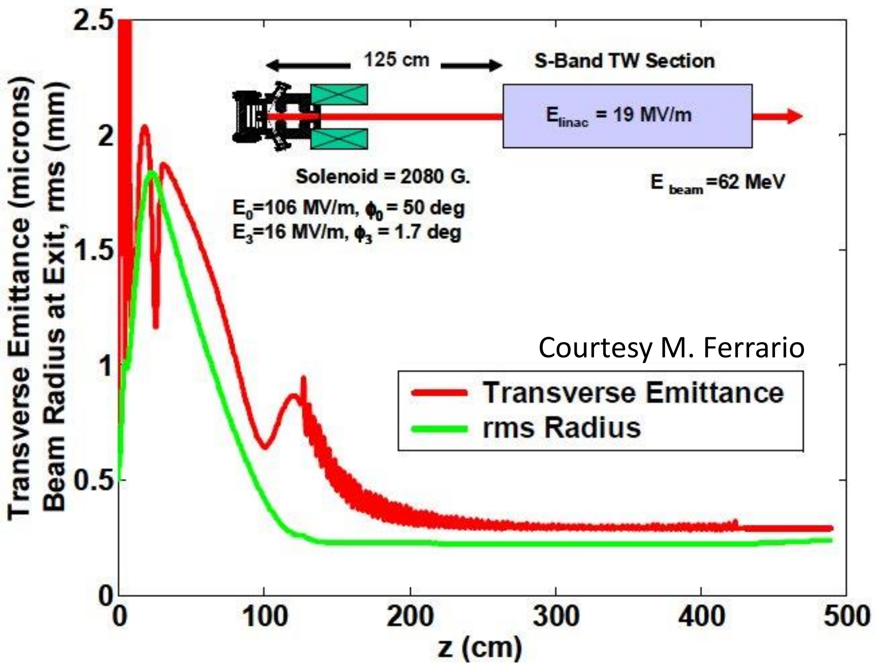
⇒ **Space charge only at low energy and/or high peak current**: below 10-20 MeV (injector) the beam dynamics optimization has to include emittance compensation schemes with, typically solenoids;

⇒ At higher **energies no space charge and no RF defocusing effects** occur but we have **RF focusing due to the ponderomotive force: focusing periods up to several meters**

⇒ Optics design has to take into account **longitudinal and transverse wakefields** (due to the **higher frequencies used for acceleration**) that can cause energy spread increase, head-tail oscillations, multi-bunch instabilities,…

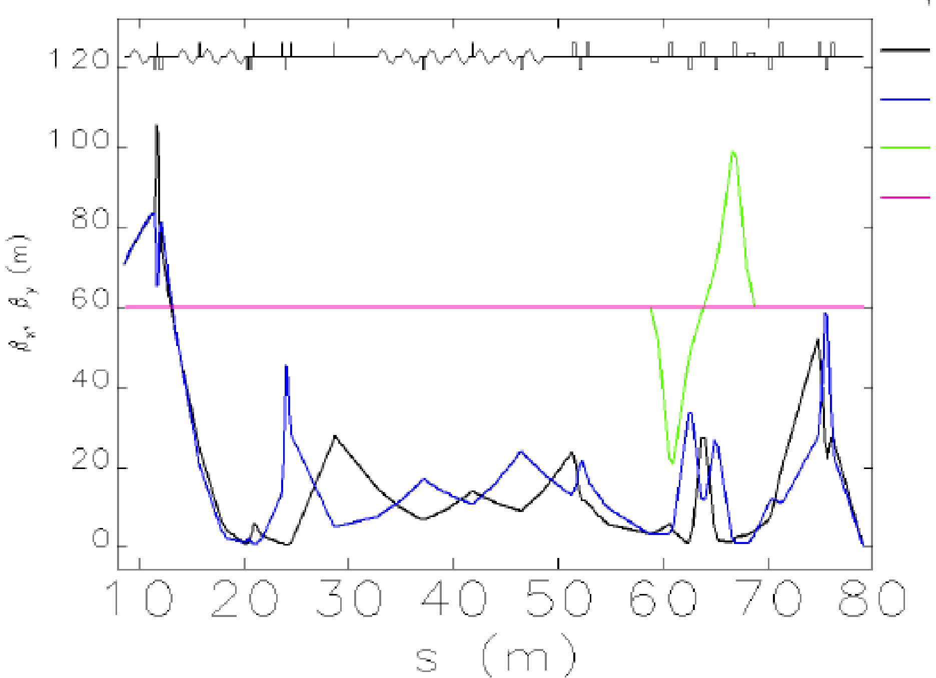
⇒ Longitudinal bunch compressors schemes based on magnets and chicanes have to take into account, for short bunches, the interaction between the beam and the emitted synchrotron radiation (**Coherent Synchrotron Radiation effects**)

⇒ All these effects are important especially in LINACs for **FEL that requires extremely good beam qualities**



Courtesy M. Ferrario

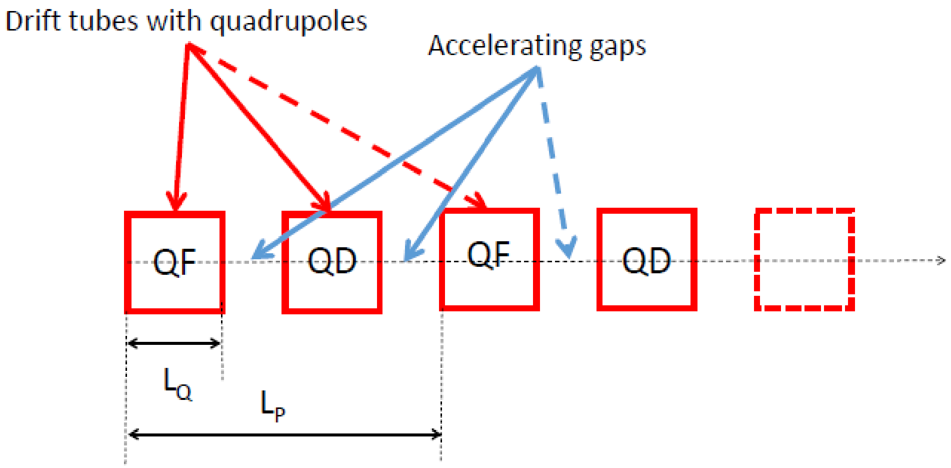
Courtesy C. Vaccarezza



EXERCISE 11: TRANSVERSE BEAM DYNAMICS

A DTL (Alvarez structure) working at f_{RF}=300 MHz accelerate protons with an injection energy W_{in}=4 MeV, permanent magnet quadrupoles are inside the drift tubes and the focusing system is equivalent to a FODO lattice, as sketched below. The quadrupoles inside the drift tubes have a length L_Q=5 cm.

If the average accelerating field per cell is E_{acc}=2 MV/m and the nominal synchronous phase $\phi_S=-\pi/6$, calculate, using the "smooth approximation" approach, the quadrupole gradient (G) that is necessary to have, in the first cells, of the structure in order to achieve a transverse phase advance per period (σ) equal to $\pi/3$, supposing that the period of the FODO (L_P) is exactly twice the distance between two accelerating gaps.

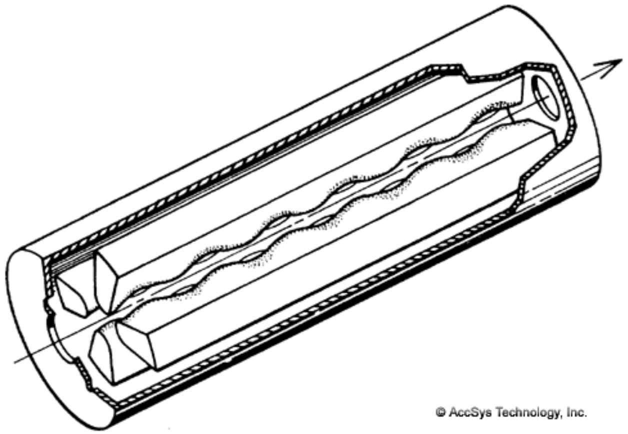


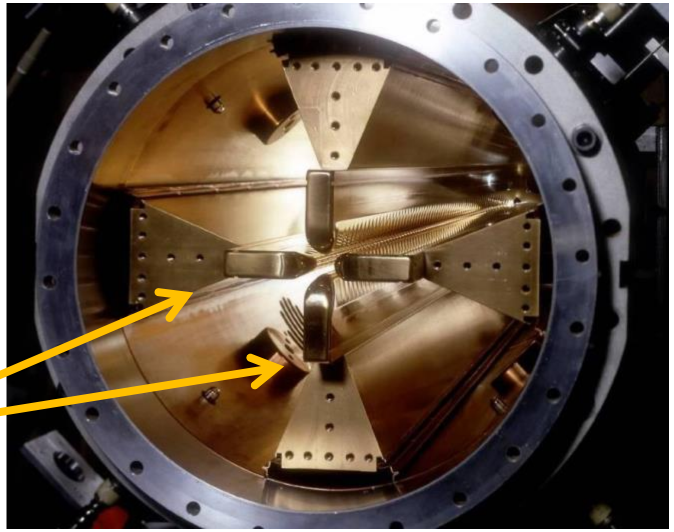
proton rest energy $m_0c^2=E_0$=938 MeV

velocity of light c=2.998e8

RADIO FREQUENCY QUADRUPOLES (RFQ)

At low proton (or ion) energies (β~0.01), **space charge defocusing is high and quadrupole focusing is not very effective**. Moreover cell length becomes small and conventional accelerating structures **(DTL) are very inefficient**. At this energies it is used a (relatively) new structure, the **Radio Frequency Quadrupole** (1970).



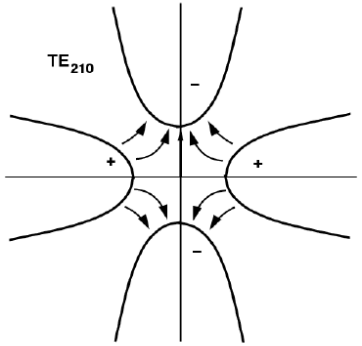


Electrodes

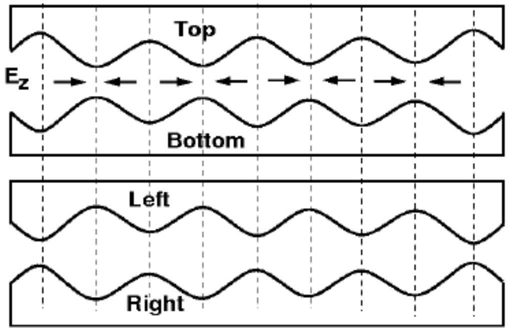
Courtesy M. Vretenar

These structures allow to simultaneously provide:

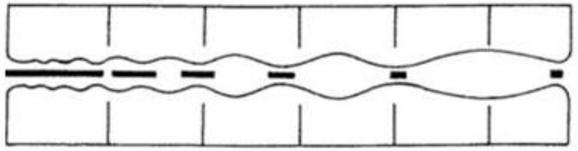
Transverse Focusing



Acceleration



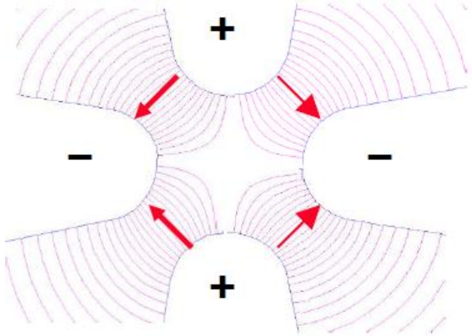
Bunching of the beam

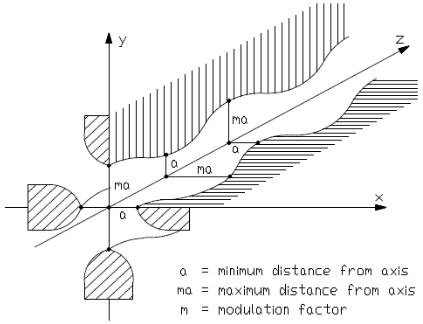


RFQ: PROPERTIES

1-Focusing

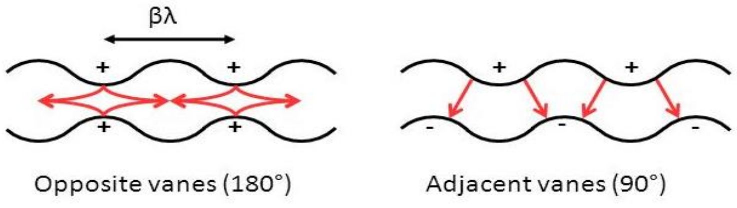
The resonating mode of the cavity (between the four electrodes) is a **focusing mode**: **Quadrupole mode (TE_{210})**. The alternating voltage on the electrodes produces an **alternating focusing channel** with the period of the RF (**electric focusing** does not depend on the velocity and is ideal at low β)





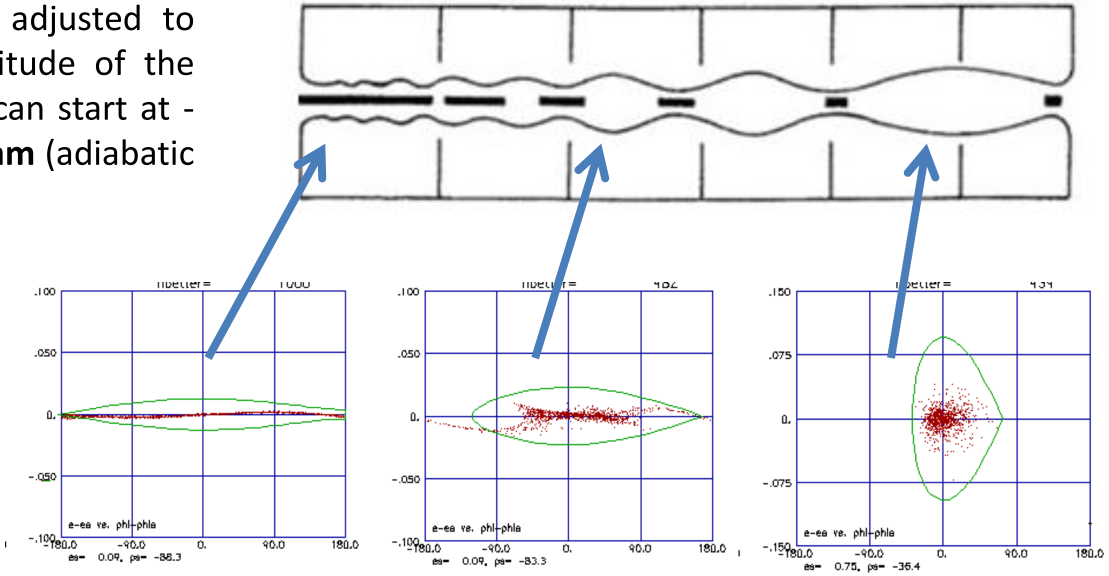
2-Acceleration

The vanes have a **longitudinal modulation** with period = $\beta\lambda_{RF}$ this creates a **longitudinal component of the electric field** that accelerate the beam (the modulation corresponds exactly to a series of RF gaps).



3-Bunching

The modulation period (distance between maxima) can be slightly adjusted to change the phase of the beam inside the RFQ cells, and the amplitude of the modulation can be changed to change the accelerating gradient. One can start at -90° phase (linac) with some **bunching cells**, progressively **bunch the beam** (adiabatic bunching channel), and only in the last cells switch on the **acceleration**.

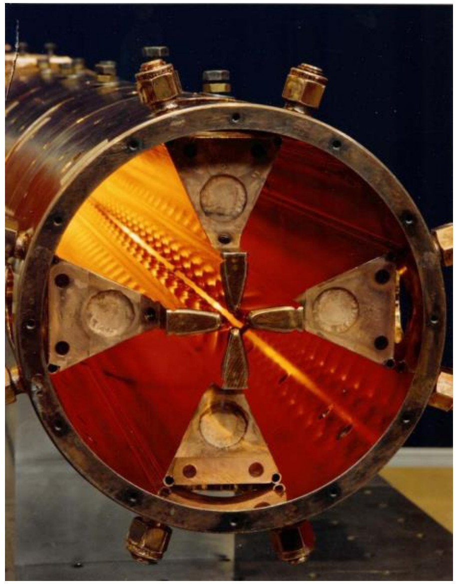


The RFQ is the only linear accelerator that can accept a low energy continuous beam.

Courtesy M. Vretenar and A. Lombardi

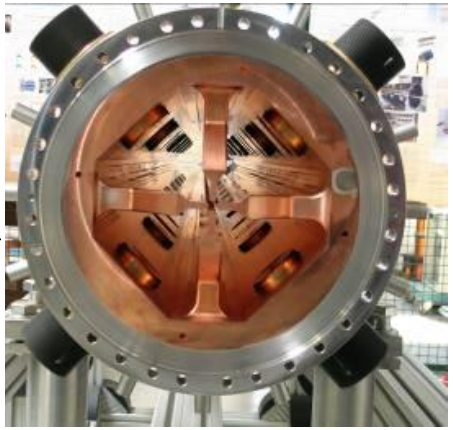
RFQ: EXAMPLES

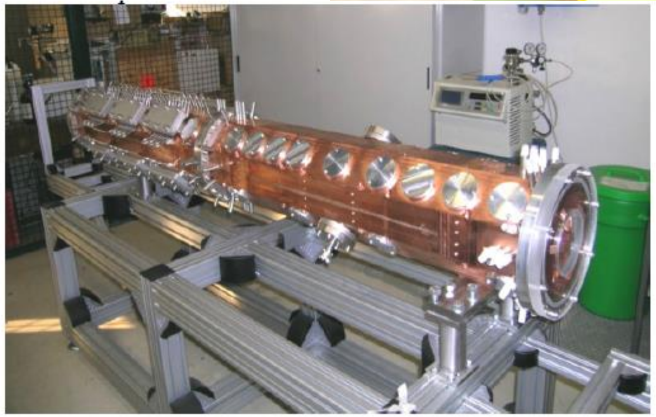
The 1st 4-vane RFQ, Los Alamos
1980: 100 KeV - 650 KeV, 30 mA , 425 MHz



The CERN Linac4 RFQ
45 keV – 3 MeV, 3 m
80 mA H-, max. 10% duty cycle

TRASCO @ INFN Legnaro
Energy In: 80 keV
Energy Out: 5 MeV
Frequency 352.2 MHz
Proton Current (CW) 30 mA





THE CHOICE OF THE ACCELERATING STRUCTURE

In general **the choice of the accelerating structure depends on**:

⇒ **Particle type**: mass, charge, energy
⇒ **Beam current**
⇒ **Duty cycle** (pulsed, CW)
⇒ **Frequency**
⇒ **Cost** of fabrication and of operation

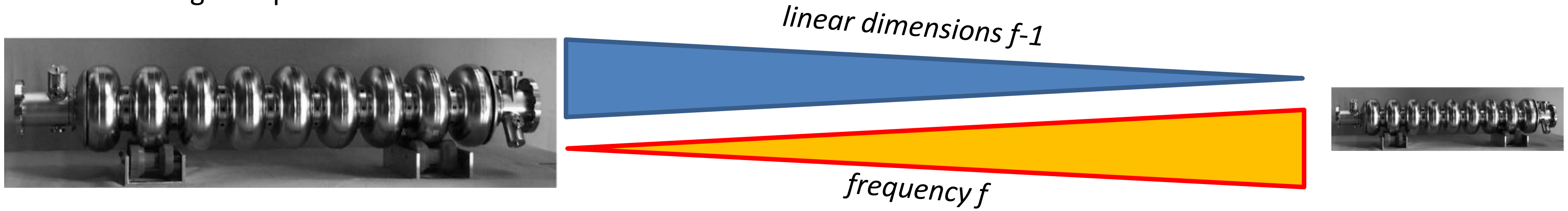
Moreover **a given accelerating structure has also a curve of efficiency** (shunt impedance) with respect to the particle energies and the choice of one structure with respect to another one depends also on this.

As example a very general scheme is given in the Table (absolutely not exhaustive).

Cavity Type	β Range	Frequency	Particles
RFQ	0.01– 0.1	40-500 MHz	Protons, Ions
DTL	0.05 – 0.5	100-400 MHz	Protons, Ions
SCL	0.5 – 1	600 MHz-3 GHz	Protons, Electrons
SC Elliptical	> 0.5-0.7	350 MHz-3 GHz	Protons, Electrons
TW	1	3-12 GHz	Electrons

STRUCTURE PARAMETERS SCALING WITH FREQUENCY

We can analyze how all parameters (r, Q) scale with frequency and what are the advantages or disadvantages in accelerate with low or high frequencies cavities.



parameter	NC	SC
R_s	$\propto f^{1/2}$	$\propto f^2$
Q	$\propto f^{-1/2}$	$\propto f^{-2}$
r	$\propto f^{1/2}$	$\propto f^{-1}$
r/Q	$\propto f$	
$w_{//}$	$\propto f^2$	
$w_{\perp}$	$\propto f^3$	

Wakefield intensity: related to BD issues ($w_{//}$, $w_{\perp}$)

⇒**r/Q increases at high frequency**

⇒for **NC structures** also r increases and this push to adopt **higher frequencies**

⇒for SC structures the power losses increases with f^2 and, as a consequence, r scales with 1/f this push to adopt **lower frequencies**

⇒On the other hand at very high frequencies (>10 GHz) **power sources** are less available

⇒Beam interaction (**wakefield**) became more critical at high frequency

⇒Cavity fabrication at very high frequency requires **higher precision** but, on the other hand, at low frequencies one needs more material and **larger machines**

⇒**short bunches** are easier with higher f

Compromise between several requirements

SW SC: 500 MHz-1500 MHz
TW NC: 3 GHz-6 GHz
SW NC: 0.5 GHz-3 GHz