Cyclotrons, JUAS

Chapter 2 : the different cyclotrons

- AVF cyclotrons
- Edge focusing and AVF cyclotrons
- Spiraled sector cyclotron
- Separated sector cyclotron
- Synchro-cyclotron
- FFAG

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Isochronous Cyclotrons $B_z = f(R, \theta)$

Isochronism condition

(longitudinal)

$B_z = B(R)$

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m}$$

Bz should increase with radius ($B \sim R^{-n}$ with $n<0$)

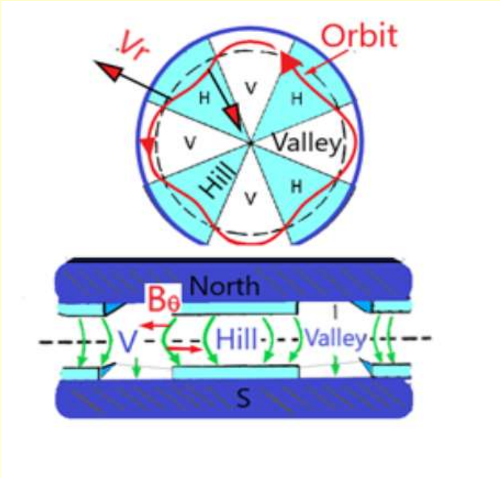
Unstable Vertical oscillations (*Br* defocus in z plane)

Additive Vertical focusing is needed

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\dot{\theta} \\ B_r & B_z & B_\theta \end{vmatrix}$$

B_θ component needed ($F_z = -q\, v_r B_\theta$) : ≪ AVF ≫ Cyclo

AVF cyclotron ~ edge focusing in dipole magnet



AVF cyclotron:

Vertical focusing $\langle F_z \rangle \propto v_r \cdot B_\theta = (dr/dt) \cdot B_\theta$

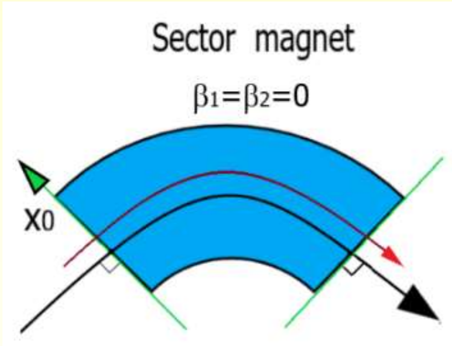
Trajectory is not a circle :

Orbit not perpendicular to hill-valley edge

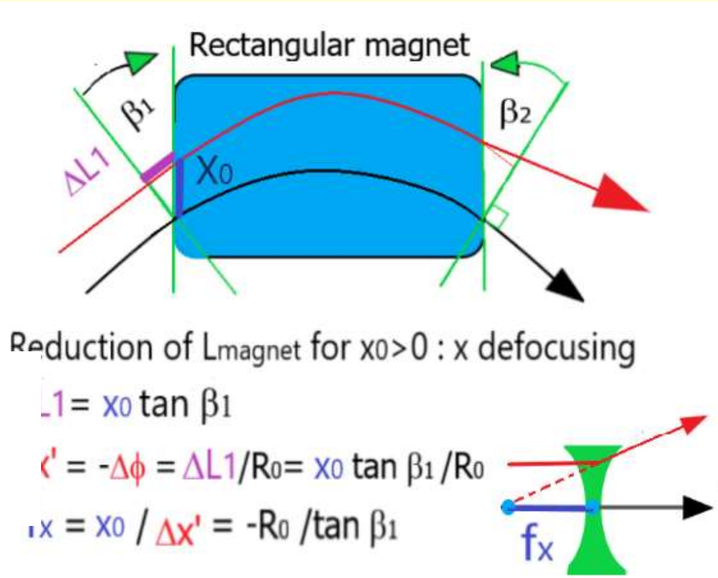
Sector Magnet
(no focusing)
vs

Rectangulat Magnet
edges not perpendicular
to trajectories :

Vertical focusing+
horizontal defocusing
effect



-Slightly focusing
in horizontal
-no focusing in vertical

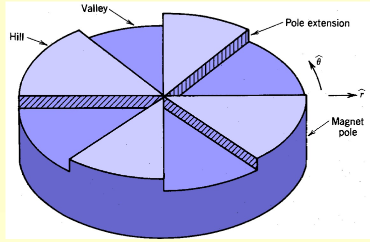


-defocusing in horizontal
- focusing in vertical

Azimuthally varying Field (AVF)

Flutter Fl = evaluation of the focusing effect

- Succession of high field & low field regions : $B_z = f(R, \theta)$



Valley : large gap, weak field

Hill : small gap, strong field

- Flutter function F_l (definition)

$$F_l = \frac{< (B - < B >)^2 >}{\langle B \rangle^2} \longleftrightarrow F_l = \frac{\sigma_B^{\;2}}{\langle B \rangle^2}$$

Example of a field with N sectors

$$B_z(r,\theta) = B_0.[1 + f\cos(N\theta)]$$

$$\langle B_z(R,\theta) \rangle = \frac{1}{2\pi}\int_0^{2\pi} B_z(R,\theta)\, d\theta = B_0$$

$$Fl = f^2/2$$

Effect of AVF $Q_z^2 = < n > + Fl\,\frac{N^2}{N^2 - 1}$

Azimuthally Varying field :
Is it sufficient to guarantee the vertical stability ?

Not always since : $\langle Bz \rangle = B_0\, \gamma(R)$

$$\gamma(R) = \frac{1}{\sqrt{1 - v^2 / c^2}} = \frac{1}{\sqrt{1 - (R\omega_{rev})^2 / c^2}}$$

in isochronous cyclotron the field index n match the lorentz factor γ :

$$\langle Bz(r) \rangle = \gamma(r)\, B_0 = R^{-n}\, B_0$$

We can demonstrate $n(R) = 1 - \gamma^2$

$$z(t) = z_0 \cos(Q_z\, \omega_{rev}\, t)$$

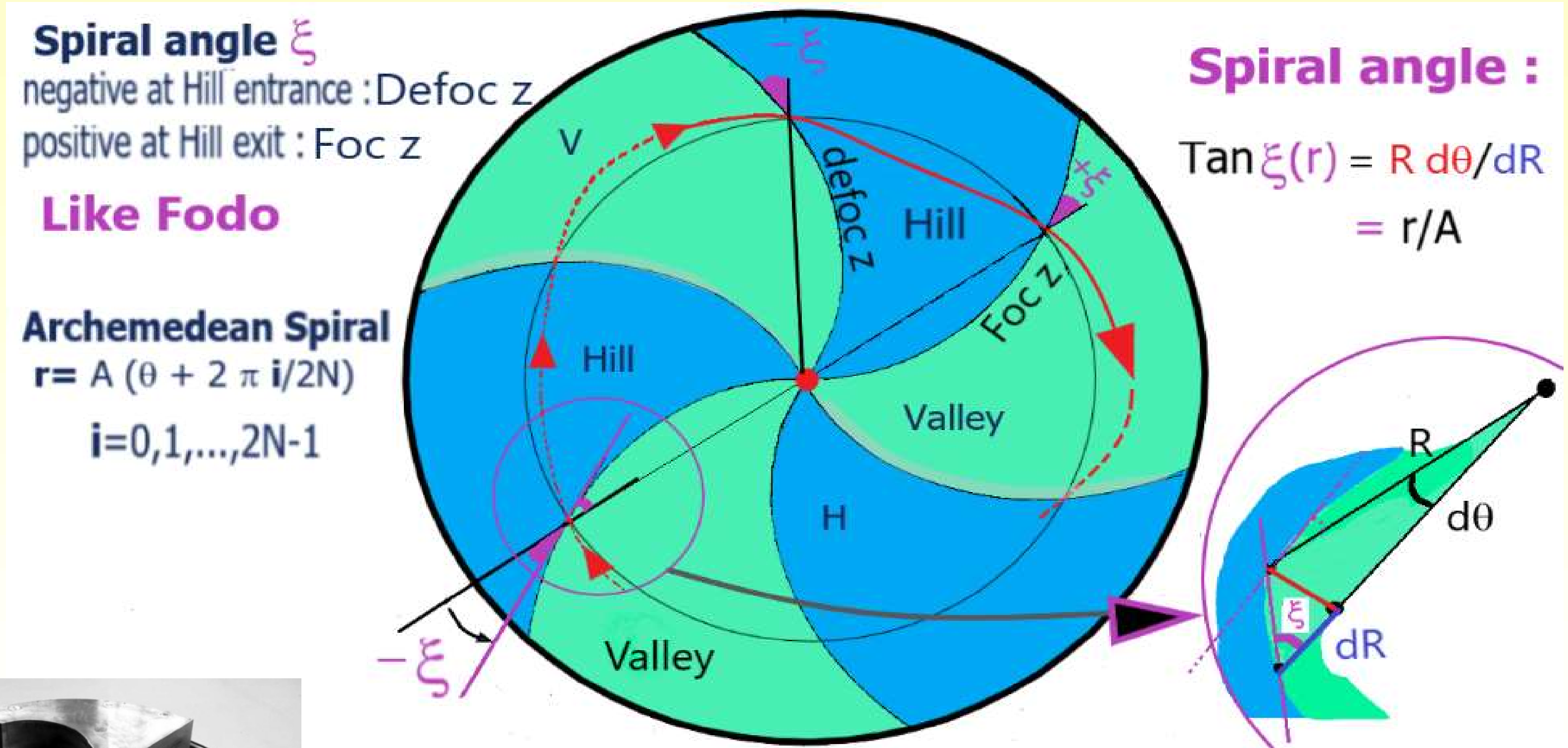
$Q_z^2 = n$ *(negative)* + *AVF terms (positive)*

In high energy cyclotron $\gamma >> 1$, therefore $n << 0$

How to increase AVF terms ?

Spiral sectors
to increase the AVF term (vertical focusing)

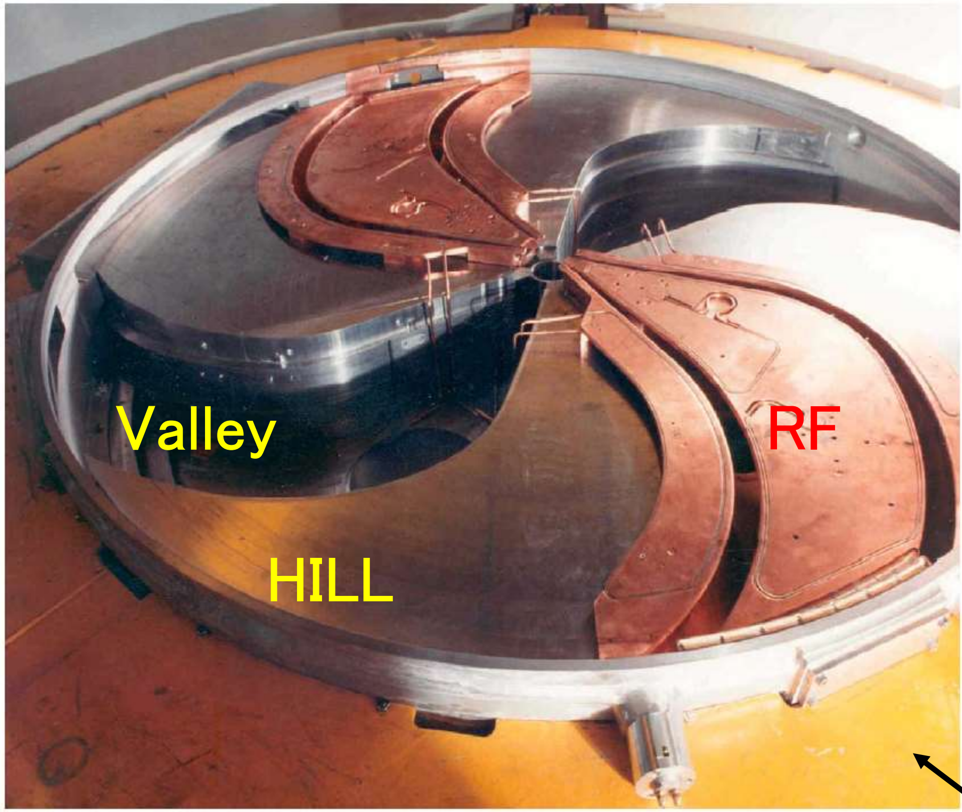
$$B_z(R,\theta) = B_0.[\,1 + f\ \sin(\,N(\theta - g(R)\,)\,)\,]$$



$$Q_z^2 = n + \frac{N^2}{N^2-1}F(1 + 2\tan^2(\xi)\,)+\ldots$$

Example : 235 MeV compact proton cyclotron
4 spiraled sectors (for cancer therapy)

C235 poles and valleys



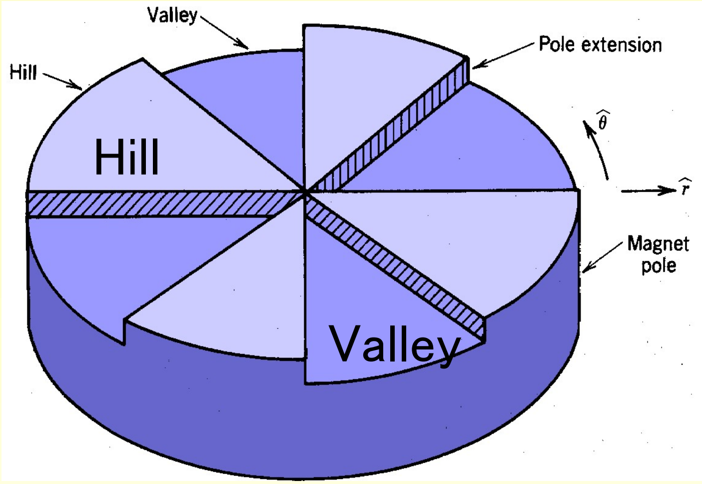
IBA C235 ®

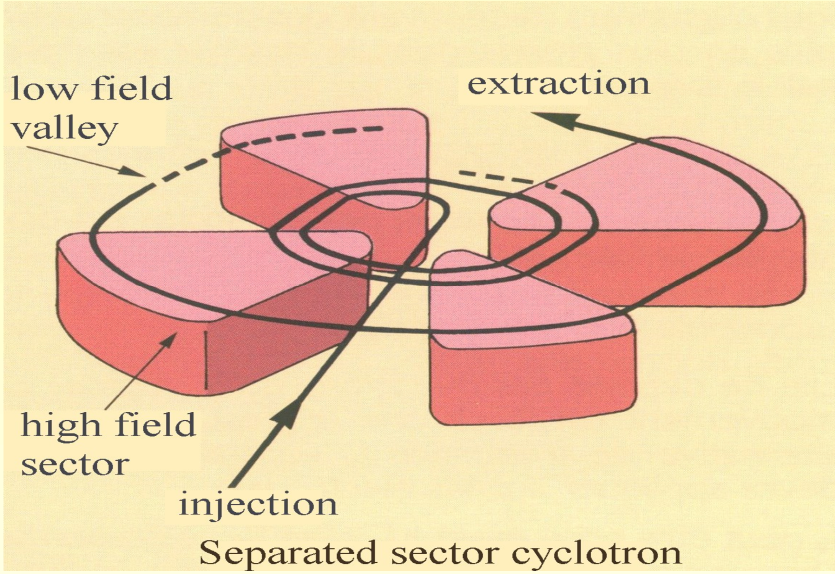
–2 RF cavities (Dees)
Inserted in the valleys

4 Spiraled sectors:

Higher energy =
Higher axial focusing required

Separated sectors
to increase AVF term (vertical focusing)





Compact cyclo: pole modulation
small amplitude field oscillation

Bz= <B0> [1+ f. cos (N θ)]

f = 0.5 (Bhill− Bvalley)/<B0>

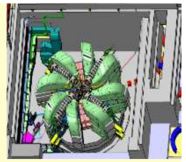
the amplitude **f << 1**
Insufficient vertical focusing

Separated magnets generate large field oscillations in θ

Bz= <B0 > [1+ f cos (N θ)]

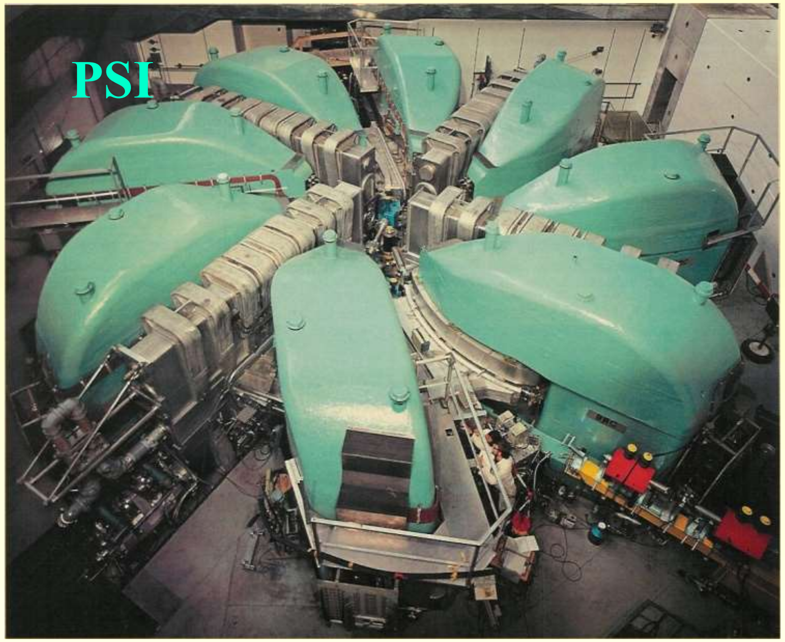
Separated sector cyclotron

With f~1 , The Flutter *Fl* is larger =>
Larger vertical focusing



Example PSI: ring cyclotron

Separated Sectors Cyclotron with spiral edges



$$E_K = (\gamma - 1).mc^2$$

γ = 1+ mc^2 /EK = 1+ 590 MeV/ 930MeV

<n> = 1- γ^2 = -1,65 < 0

$$Q_z^2 = < n > + \frac{N^2}{N^2-1} Fl.(1 + 2\tan^2(\xi))+ \ldots$$

PSI= 590 MeV proton γ=1.63

Separated sectors cyclotron + Spiral edge
Is needed at "High energies" (n(R) =1-γ^2 << 0)
for Q_z^2 >0 (vertical stability)

Tutorial 1 :

Give the **Lorentz force** in a **cyclotron**
and explain the focusing and defocusing effect in Vertical plane
of the **B**r (radial) and **B**θ (azimuthal) components

$$m\gamma \frac{d^2\mathbf{r}}{dt^2} = m\gamma \frac{d^2(r\mathbf{e_r} + z\mathbf{e_z})}{dt^2} = q(\mathbf{v} \times B) = ?$$

$$m\gamma \frac{d^2 z}{dt^2} = F_z = ?$$

$$B_z = B_{0z} R^{-n}$$

$$B_r = ?$$

Remenber Curl B =0

Tutorial 1 : Give the Lorentz force in a cyclotron and explain the focusing and defocusing effect of the **Br** (radial) and **Bθ** (azimuthal) components

$$m\gamma \frac{d^2 z}{dt^2} = F_z = q\,(\mathrm{v}\times B)_z = -q(\mathrm{v}_r B_\theta - \mathrm{v}_\theta B_r)$$

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \mathrm{v_r} & \mathrm{v_z} & \mathrm{v}_\theta \\ B_r & B_z & B_\theta \end{vmatrix}$$

$$\frac{d^2 z}{dt^2} + \frac{q}{m\gamma}(\mathrm{v}_r B_\theta + R\omega_{rev} n.\frac{B_z}{R} z) = 0$$

$$\mathrm{v_r} = \frac{dr}{dt} \qquad \mathrm{v}_\theta = R\frac{d\theta}{dt} = R\omega_{rev}$$

Isochronous cyclotron n <0
since Bz (R) increases

$B_z = f(R, \theta)$ and $\nabla \times B = 0$

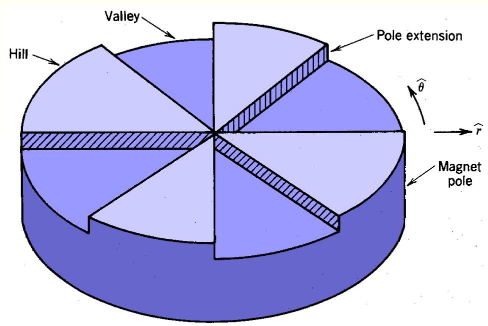
$$\left(\frac{dB_\theta}{dz} - \frac{dB_z}{Rd\theta}\right) = 0$$

$$B_\theta = z.\frac{dB_z}{Rd\theta}.e_\theta$$

$B_r = -z.n\,B_z/R.\,e_r$

n<0 Br : Defocusing in vertical plan

Induced by Sectors (AVF)
Focus in vertical plan
it compensates n < 0



Beam dynamics in the Isochronous cyclotrons

B= Constant ≠ Isochronism condition

A STRONG LIMITIATION in energy γ=1

to get the ions synchrone With RF

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m}$$

$B_z = B_0 . g(R)$

B_z increase with R (field index n < 0)

Unstable Vertical oscillations

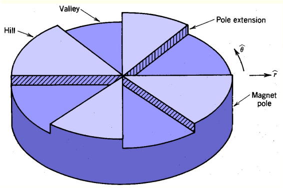
strong limitation in transmission

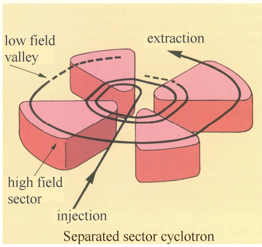
Additive Vertical focusing is needed : –N sectors (Hills//valleys)

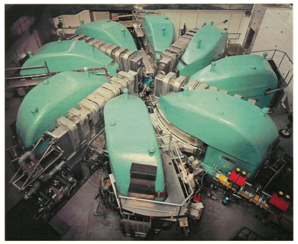
–separated straight sectors

–spiralled sectors

– separated spiralled sectors







$B_z = B_0 . g(R,\theta)$

4 techniques

The problems of isochronous cyclotron

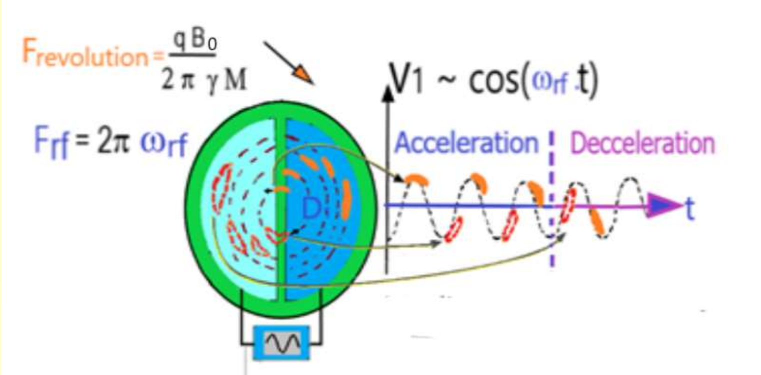
Fixed RF frequency requires increasing field Bz, and final azimuthal modulation (100 % duty cycle)
BUT
1) **Very Weak vertical focu**sing (even Complex AVF magnet)
2) **Complex magnet**
3) **Limitation in energy** (γ <2)
Fixed RF frequency is it really needed ? ? ?
(in Synchrotron the RF is cycle during the acceleration)

Can we cycle the RF acceleration
Can we use uniform magnet => SYNCHRO CYCLOTRON

Can we cycle the RF acceleration
Can we improve the focusing even at large energy (γ >>1)

=> F.F.A.G.
(Fixed field alternate gradient accelerator)

Synchro-cyclotron RF cycled , with simpler magnet



If you want to construct a cyclotron
With constant field B_0 (magnet is simpler)

BUT the revolution Frequency evolve with radius
ωrev = F(Radius) :

RF has to synchronized

$$\frac{qB_0}{m\,\gamma(R)} = f(time) = \omega_{rf}(t\,)/H$$

You could cycle the RF frequency

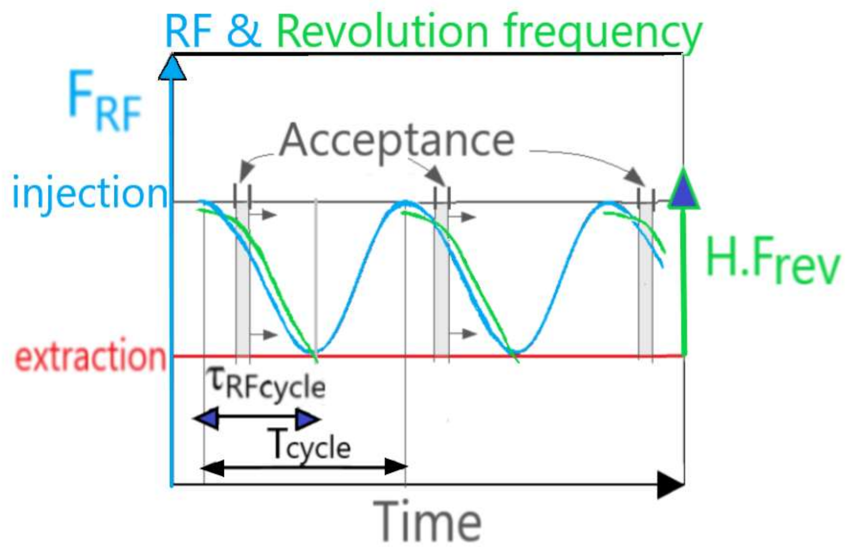
1) Inject few bunches : Frf= Finjection
2) synchronize these bunches with RF :
Accelerate the ions with a decresing frequency up to extraction
Frf= Finjection / γ
3) Go back at higher frequency Frf= Finjecti, Inject few bunches : on

Synchro-cyclotron RF cycled , with simpler magnet

If you want to construct a cyclotron
With constant field B_0 (magnet is simpler)

RF has to synchronized

$$\frac{qB_0}{m\,\gamma(R)} = f(time) = \omega_{rf}(t)/H$$

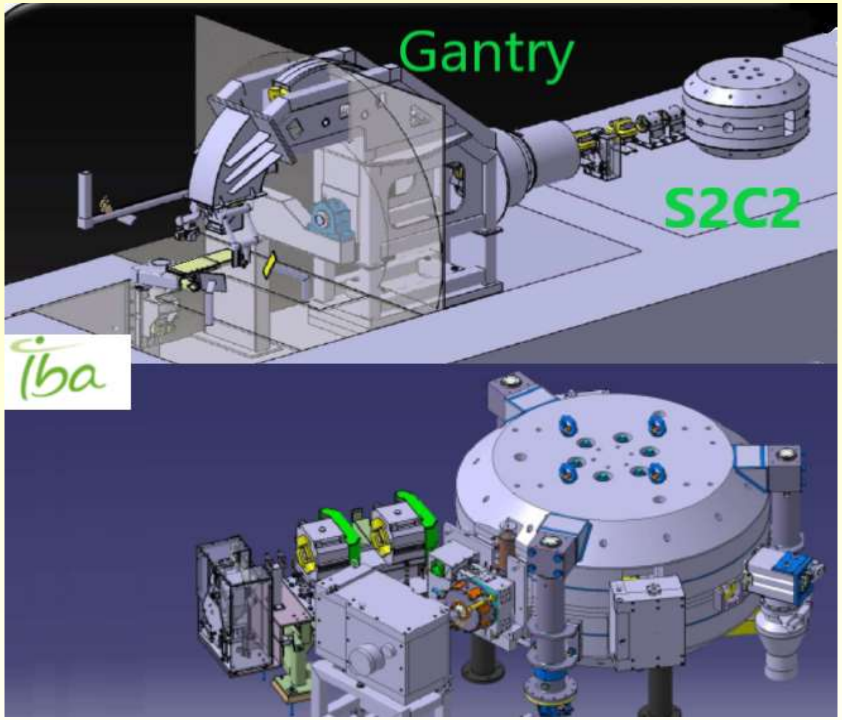


RF Cycle feasible ~1Khz : a pulse of particle every 1 millseconds

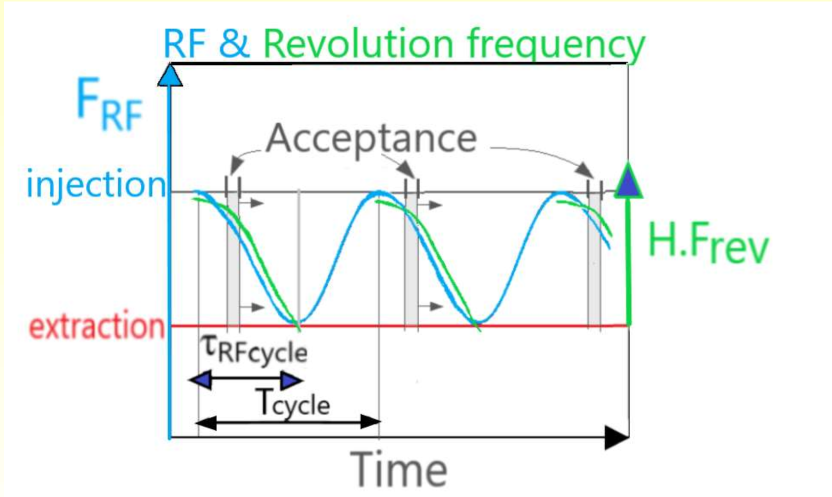
- Injection
- then acceleration Frf follows Frev
- Go back Frf = F_0
- Injection

Particles are accelerated during the decreasing part of the frequency ~ 1ms
Very slow acceleration : RF Voltage very low
Large number of turn in the cyclotron 10000-100000 turn

SYNCHRO CYCLOTRON S2C2 (IBA) the most compact accelerator for Proton cancer therapy at 230 MeV



$$\frac{qB_0}{m\,\gamma(R)} = f(time) = \omega_{rf}(t\,)/H$$



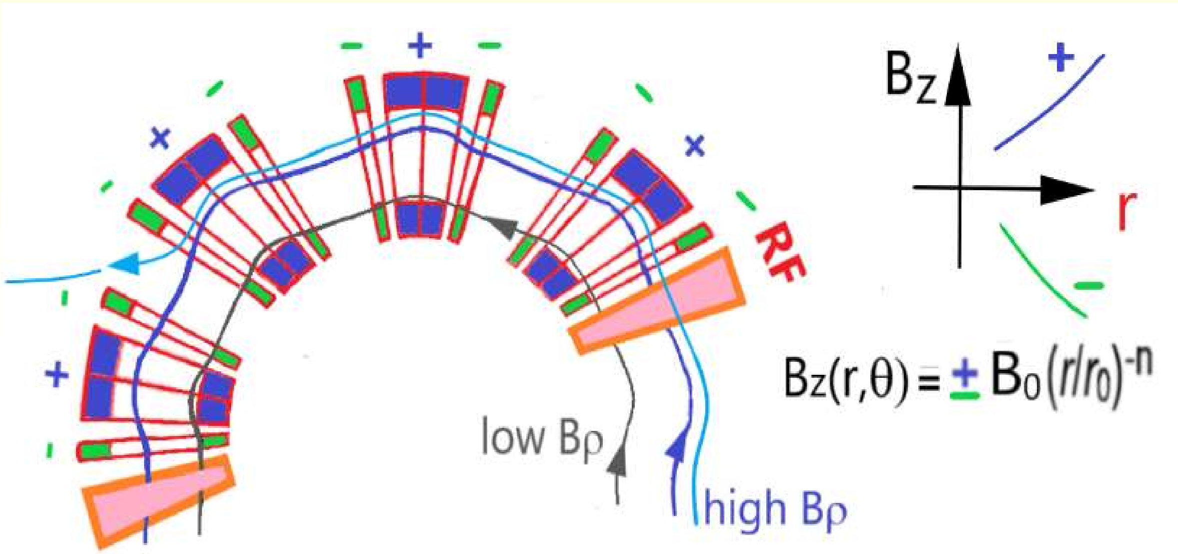
Very compact 5T superconducting magnet (No AVF)
R= 49 cm
40000 turns
duty cycle=0.7%) . Frf =[93 Mhz , 63 Mhz] (cycled) . Vrf= 10kV
Already 30 S2C2s constructed by IBA for cancer therapy centre in 2022
0.7% duty cycle

FFAG RF cycled , with complex magnets

F.F.A.G. (Fixed field alternate gradient accelerator)

- How improve the reputation rate of a Synchrotron (10-50 Hz) ?
keeping fixed magnetic field, RF is faster to cycle than magnet (1KHz feasible)

How improve the focusing of cyclotron even at large energy (γ >>1) ?
Using dipolar magnet with alternate Gradient



FFAG RF cycled , with complex magnets

Two Families of F.F.A.G.
(Fixed field alternate gradient accelerator)

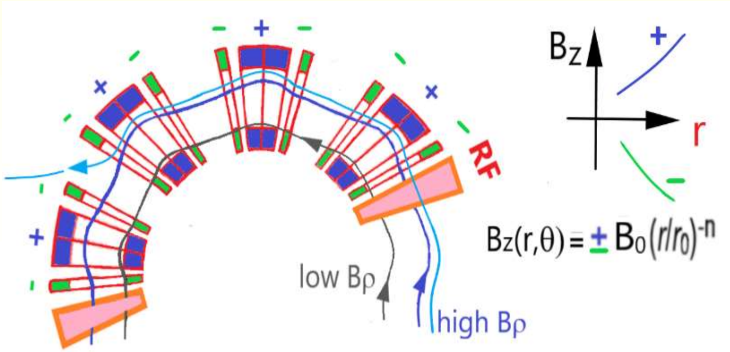
Non-Scaling F.F.A.G.

Tunes Qz , Qr dependent of Radius
But good longitudinal porperties
Strongest focusing
Small compaction factor

EMMA
(2010)
20 MeV
electron

Scaling F.F.A.G.

Tunes Qz , Qr independent of Radius
No resonance crossing, No beam losses
n =constant in the two dipole kind

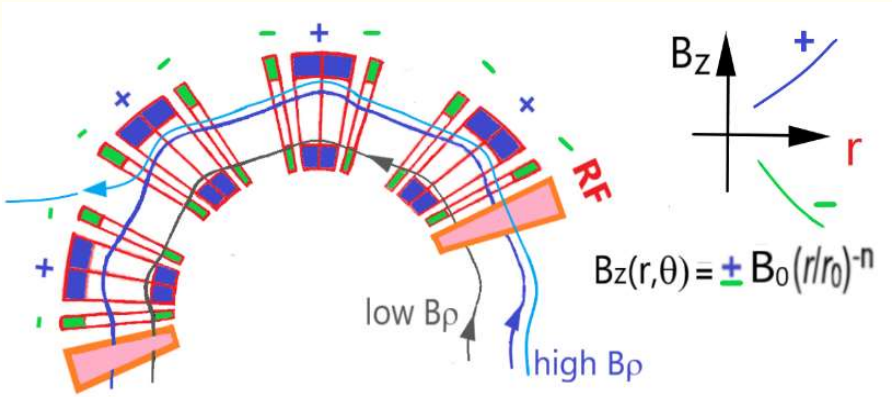


FFAG RF cycled , with complex magnets

FFAG : advantages
No limit in energy
Faster cycling than a synchrotron
Transverse focusing with alternate gradient magnets

BUT
Huge size compare to a cyclotron of equivalent energy
Very complex magnet compare to synchrotron

Possible future Application :
Muon acceleration project (20 GeV),
Accelerator Driven nuclear Reactor



Few other slides for questions

Tutorial 3 : What is the field index n(R) ?

n(R) : it gives the radial evolution of *Bz*

$$n = -\frac{R}{B_0}\frac{\partial B_z}{\partial R}$$

⟺

Equivalent definition

$$Bz \sim B_0\,(r/R_0)^{-n}$$

The field index is not constant in a cyclotron n = n (radius)

Isochronous cyclotron n(r) <0 : $\langle Bz(r,\theta)\rangle_{turn}$ increases with R

$$Bz(R) = \gamma(r)\;B_0$$
$$= k\,r^{-n}$$

$$< B_z(R) >= \frac{B_0}{\sqrt{1-[R\;\omega/c]^2}} = B_0.R^{-n(R)}$$

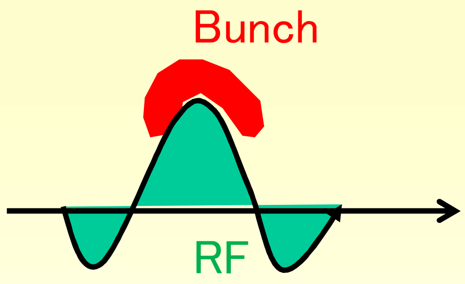
$$\gamma = \frac{1.}{\sqrt{1-[v/c]^2}} = \frac{1.}{\sqrt{1-[R\omega/c]^2}}$$

Dynamics in cyclotron

summary

$Qe_0 \hat{V} \cos\phi . N_{gap}$ ***Energy gain per turn***

$\phi_0 \approx 0°$ ***Central RF phase ,***
Ion bunches are centered at 0°



$\omega_{RF} = h\omega_{rev} = const$

RF synchronism = Isochronism
H - harmonic number)

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m} = const$$

$R = R(t) = R(N°turn \quad)$ ***Orbit evolving***

Average Magnetic field

$B\rho(t) = P/q \Rightarrow \langle B \rangle = B\rho / R$